

1. Hubble law: $v = H_0 \cdot d$ Redshift $z = \frac{\Delta \lambda}{\lambda_0} = \frac{\lambda_{\text{obs}} - \lambda_0}{\lambda_0}$

Lyman- α : $\lambda_{\alpha} = 121,5 \text{ nm} = 1215 \text{ \AA}$

a) from spectrum $\lambda_{\text{obs}} = 1214 \text{ \AA} \Rightarrow \Delta \lambda = -1 \text{ \AA}$

\Rightarrow redshift of andromeda: $z = -\frac{1}{1215} = -0,0008 = -8 \cdot 10^{-4}$

\Rightarrow radial velocity: $v = c \cdot z = 3 \cdot 10^8 \frac{\text{m}}{\text{s}} \cdot (-8 \cdot 10^{-4})$
 $= -2,5 \cdot 10^5 \frac{\text{m}}{\text{s}}$
 $= -247 \frac{\text{km}}{\text{s}}$

b) $d = 780 \text{ kpc} = 0,78 \text{ Mpc}$

$v = 71 \cdot 0,78 \cdot \frac{\text{km}}{\text{s}} \cdot \frac{\text{Mpc}}{1} \cdot \frac{1}{\text{Mpc}} = 55,4 \frac{\text{km}}{\text{s}}$

radial velocity from Hubble law: ~~$v = 71 \cdot 0,78 \cdot \frac{\text{km}}{\text{s}} \cdot \frac{\text{Mpc}}{1} \cdot \frac{1}{\text{Mpc}} = 55,4 \frac{\text{km}}{\text{s}}$~~

Large discrepancy! This is because galaxies also possess a certain peculiar motion. Andromeda actually moves towards the Milky Way (as shown in a).

The Hubble law produces exacter radial velocities the further away a galaxy is. For these galaxies the peculiar motion of a galaxy becomes smaller compared to the Hubble flow velocity and is negligible. For close-by galaxies like Andromeda, however, the peculiar velocity is dominant.

c) $z = 0,05$ radial velocity: $v = c \cdot z = 15 \cdot 10^6 \frac{\text{m}}{\text{s}} = 15 \cdot 10^3 \frac{\text{km}}{\text{s}}$

distance: $d = \frac{v}{H_0}$
 $= \frac{15 \cdot 10^3}{71} \frac{\text{km}}{\text{s}} \cdot \frac{\text{s} \cdot \text{Mpc}}{\text{km}}$
 $= 211,3 \text{ Mpc}$

2. $E \approx k_B T$ $\left(\frac{T}{\text{K}}\right) \approx 1,5 \cdot 10^{10} \left(\frac{t}{\text{s}}\right)^{-1/2}$

a) electron - positron pair production: $\gamma + \gamma \rightarrow e^- + e^+$

Electrons / Positrons have a restmass of $0,511 \text{ MeV}$. Therefore, pair production requires a minimum photon energy of $1,02 \text{ MeV}$.

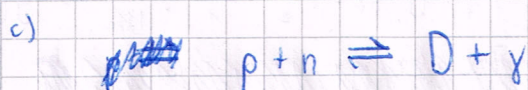
$$\Rightarrow \text{minimum temperature: } T = \frac{E}{k_B} = 1,02 \cdot 10^6 \text{ eV} \cdot \frac{1}{8,62 \cdot 10^{-5} \frac{\text{K}}{\text{eV}}} \\ = 12 \cdot 10^9 \text{ K}$$

Below this temperature the photons will not carry sufficient energy to enable pair production

b) At this stage the photon - baryon ratio was 10^9 to 1.

$$T = 1,5 \cdot 10^{10} \left(\frac{t}{s}\right)^{-\frac{1}{2}} \text{ K} \quad \rightarrow \text{ where we have inserted } t = 1s \\ = 1,5 \cdot 10^{10} \text{ K}$$

$$E = k_B T = 1,3 \cdot 10^6 \text{ eV} = 1,3 \text{ MeV}$$



The deuteron has a binding energy of $2,2 \text{ MeV}$. A photon must ^{carry} ~~have~~ this energy to be able to break apart a deuteron. Thus the photon frequency is

$$\nu = \frac{E}{h} = \frac{2,2 \cdot 10^6 \text{ eV} \cdot \frac{1,6}{10^{19}}}{4,14 \cdot 10^{-15} \text{ eV} \cdot s} \\ = 5,31 \cdot 10^{20} \frac{1}{s}$$

As the deuteron's binding energy is relatively small, the photons in the early universe were well able to break them apart, i.e. frequent disintegration occurred. At some point ($t \approx 3s$) the deuterons start surviving but are then immediately used for further nucleosynthesis. Thus, deuterons never got the chance to accumulate until much later ($t \approx 200s$)