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In [3]: import itertools # product()
import numpy as np
import qutip
import qutip.states
import tomographer
import tomographer.tools.densedm
import tomographer.querrorbars
import tomographer.jpyutil
from IPython.display import display, Markdown
```

```
In [4]: rho_target_Bell = qutip.states.ket2dm(qutip.Qobj(np.array([0,1,1j,0]/np.sqrt(2))))
)
display(Markdown('rho_target_Bell = '))
display(rho_target_Bell)
# The data below were simulated from the following true state:
rho_sim = 0.95*rho_target_Bell + 0.05*qutip.qeye(4)/4;
display(Markdown('rho_sim = '))
display(rho_sim)
```

rho_target_Bell =

Quantum object: dims = [[4], [4]], shape = [4, 4], type = oper, isherm = True

$$\begin{pmatrix} 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.500 & -0.500j & 0.0 \\ 0.0 & 0.500j & 0.500 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{pmatrix}$$

rho_sim =

Quantum object: dims = [[4], [4]], shape = [4, 4], type = oper, isherm = True

$$\begin{pmatrix} 0.013 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.487 & -0.475j & 0.0 \\ 0.0 & 0.475j & 0.487 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.013 \end{pmatrix}$$

```

In [5]: # All POVM effects when measuring Pauli X, Y, or Z on a single qubit
MeasEffects1Qubit = tomographer.tools.densedm.PauliMeasEffectsQubit

# Listing of all POVM effects of product Paulis on two qubits (with individual outcomes on each qubit)
Emn = [ None ] * 36 # prepare 36 elements
for i in range(3):
    for j in range(3):
        for s in range(2):
            for t in range(2):
                idx = j*3*2*2 + i*2*2 + t*2 + s
                Emn[idx] = np.kron(MeasEffects1Qubit[i][s], MeasEffects1Qubit[j][t])

# These are the measurement counts. Nm[k] is the number of times the POVM effect
# Emn[k] was observed. The numbers here were obtained by simulating measurements
# from the state `rho_sim` given above using the described measurement settings.
Nm = np.array([
    122, 105, 135, 138, # counts for XX for outcomes (+1, +1), (+1, -1), (-1, +1), (-1, -1)
    248, 7, 5, 240, # counts for XY for outcomes (+1, +1), (+1, -1), (-1, +1), (-1, -1)
    102, 131, 119, 148, # counts for XZ for outcomes (+1, +1), (+1, -1), (-1, +1), (-1, -1)
    7, 252, 240, 1, # counts for YX for outcomes (+1, +1), (+1, -1), (-1, +1), (-1, -1)
    125, 135, 127, 113, # counts for YY for outcomes (+1, +1), (+1, -1), (-1, +1), (-1, -1)
    140, 124, 118, 118, # counts for YZ for outcomes (+1, +1), (+1, -1), (-1, +1), (-1, -1)
    122, 119, 135, 124, # counts for ZX for outcomes (+1, +1), (+1, -1), (-1, +1), (-1, -1)
    126, 123, 134, 117, # counts for ZY for outcomes (+1, +1), (+1, -1), (-1, +1), (-1, -1)
    9, 233, 253, 5, # counts for ZZ for outcomes (+1, +1), (+1, -1), (-1, +1), (-1, -1)
]);

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In [6]: # An entanglement witness which is appropriate for our target state, as a qutip.Qobj
EntglWitness = (- qutip.qeye(4)
    # how do you "collapse systems together" with qutip?? we could do this with n
    # p.kron() also...
    - qutip.Qobj(qutip.tensor(qutip.sigmax(),qutip.sigmay()).data,dims=[[4],[4]])
    + qutip.Qobj(qutip.tensor(qutip.sigmay(),qutip.sigmax()).data,dims=[[4],[4]])
    - qutip.Qobj(qutip.tensor(qutip.sigmaz(),qutip.sigmaz()).data,dims=[[4],[4]])
)
display(EntglWitness)

```

Quantum object: dims = [[4], [4]], shape = [4, 4], type = oper, isherm = True

$$\begin{pmatrix} -2.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -2.0j & 0.0 \\ 0.0 & 2.0j & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -2.0 \end{pmatrix}$$

```

In [7]: # Value for rho_target_Bell maximally entangled state: +2
display(qutip.expect(EntglWitness, rho_target_Bell))

```

1.9999999999999996

```
In [8]: # but you can show that for any separable state this value is <= 0. For example:
display(qutip.expect(EntglWitness, qutip.qeye(4)/4))
display(qutip.expect(EntglWitness, qutip.Qobj(np.array([1,0,0,0])))
display(qutip.expect(EntglWitness, 0.5*qutip.ket2dm(qutip.Qobj(np.array([0,1,0,0]
)))
          + 0.5*qutip.ket2dm(qutip.Qobj(np.array([0,0,1,0])))))

-1.0
-2.0
0.0
```

```
In [9]: # Now, we're ready to run our tomography procedure. We'll be estimating
# the expectation value of the entanglement witness.

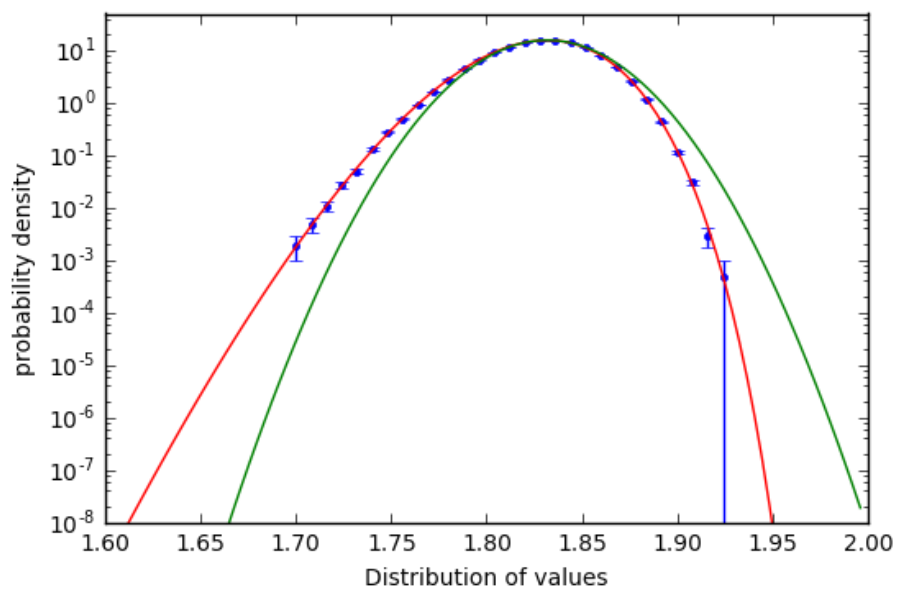
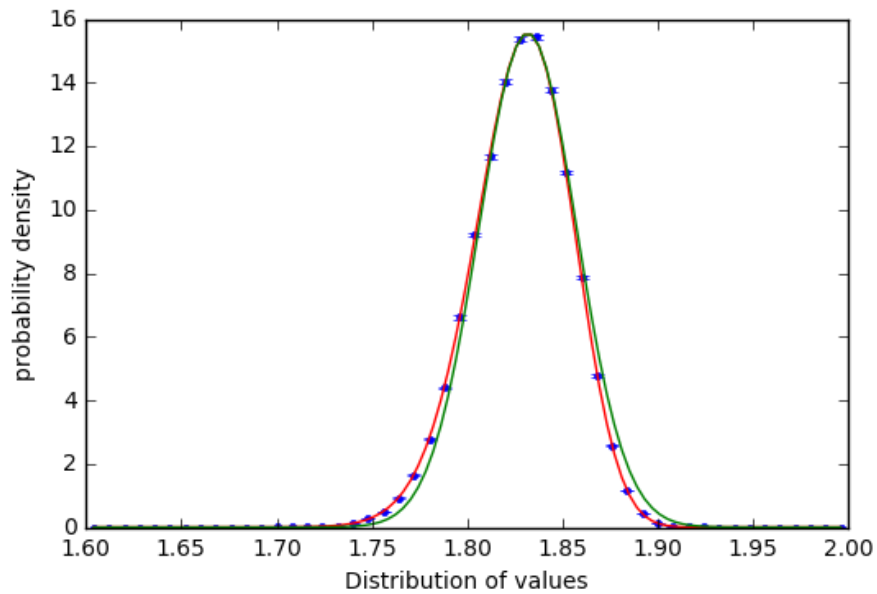
r = None # global variable

with tomographer.jpyutil.RandWalkProgressBar() as prg:
    r = tomographer.tomorun.tomorun(
        # the dimension of the quantum system
        dim=4,
        # the tomography data
        Nm=Nm,
        Emn=Emn,
        # Histogram: values in [1.6, 2.0] split into 50 bins
        hist_params=tomographer.UniformBinsHistogramParams(1.6,2,50),
        # Random Walk parameters: step size, sweep size, number of thermalization
        sweeps, number of live sweeps
        mhrw_params=tomographer.MHRWParams(0.009,120,500,32768),
        # figure of merit:
        fig_of_merit="obs-value",
        observable=EntglWitness.data.toarray(),
        #num_repeats=12, # default value = auto-detect number of CPU's
        progress_fn=prg.progress_fn
    )
    prg.displayFinalInfo(r['final_report_runs'])
```

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In [11]: # Collect the histogram
final_histogram = r['final_histogram']
```

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In [12]: # Do the analysis and get the quantum error bars
analysis = tomographer.querrorbars.HistogramAnalysis(final_histogram, ftox=(2,-1)
)
analysis.printFitParameters()
analysis.printQuantumErrorBars()
# linear scale plot
analysis.plot()
# log scale plot (adjust scale before showing plot)
p = analysis.plot(log_scale=True, show_plot=False)
p.ax.set_ylim([1e-8, 50])
p.show()
```

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Fit parameters:
  a2 = 43.1956
  a1 = 227.108
  m = 40.6498
  c = 114.628
Quantum Error Bars:
  f0 = 1.832
  Delta = 0.03624
  gamma = 0.002455
```



In []: