In [1]:
```
import itertools # product()
import numpy as np
import qutip
import qutip.states
import tomographer
import tomographer.querrorbars
import tomographer.jpyutil
from IPython.display import display, Markdown
```

In [2]:
```
rho_target_Bell = qutip.states.ket2dm(qutip.Qobj(np.array([0,1,1j,0]/np.sqrt(2)))
)
display(Markdown('rho_target_Bell = '))
display(rho_target_Bell)
# The data below were simulated from the following true state:
rho_sim = 0.95*rho_target_Bell + 0.05*qutip.qeye(4)/4;
display(Markdown('rho_sim = '))
display(rho_sim)
```

rho_target_Bell =

Quantum object: dims = [[4], [4]], shape = [4, 4], type = oper, isherm = True

$$\begin{pmatrix} 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.500 & -0.500j & 0.0 \\ 0.0 & 0.500j & 0.500 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{pmatrix}$$

rho_sim =

Quantum object: dims = [[4], [4]], shape = [4, 4], type = oper, isherm = True

$$\begin{pmatrix} 0.013 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.487 & -0.475j & 0.0 \\ 0.0 & 0.475j & 0.487 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.013 \end{pmatrix}$$

In [3]:
```
# All POVM effects when measuring Pauli X, Y, or Z on a single qubit
MeasEffects1Qubit = [ [
        np.array([[.5, .5],[.5, .5]]),     # X, +1 outcome
        np.array([[.5, -.5],[-.5, .5]]),   # X, -1 outcome
    ], [
        np.array([[.5, -.5j],[.5j, .5]]),  # Y, +1 outcome
        np.array([[.5, .5j],[-.5j, .5]]),  # Y, -1 outcome
    ], [
        np.array([[1,0],[0,0]]),           # Z, +1 outcome
        np.array([[0,0],[0,1]]),           # Z, -1 outcome
    ]
]

# Listing of all POVM effects of product Paulis on two qubits (with individual ou
tcomes on each qubit)
Emn = [ None ] * 36 # prepare 36 elements
for i in range(3):
    for j in range(3):
        for s in range(2):
            for t in range(2):
                idx = j*3*2*2 + i*2*2 + t*2 + s
                Emn[idx] = np.kron(MeasEffects1Qubit[i][s], MeasEffects1Qubit[j][
t])

# These are the measurement counts. Nm[k] is the number of times the POVM effect
# Emn[k] was observed. The numbers here were obtained by simulating measurements
# from the state `rho_sim` given above using the described measurement settings.
Nm = np.array([
   122,   105,   135,   138, # counts for XX for outcomes (+1, +1), (+1, -1), (-1
, +1), (-1, -1)
   248,     7,     5,   240, # counts for XY for outcomes (+1, +1), (+1, -1), (-1
, +1), (-1, -1)
   102,   131,   119,   148, # counts for XZ for outcomes (+1, +1), (+1, -1), (-1
, +1), (-1, -1)
     7,   252,   240,     1, # counts for YX for outcomes (+1, +1), (+1, -1), (-1
, +1), (-1, -1)
   125,   135,   127,   113, # counts for YY for outcomes (+1, +1), (+1, -1), (-1
, +1), (-1, -1)
   140,   124,   118,   118, # counts for YZ for outcomes (+1, +1), (+1, -1), (-1
, +1), (-1, -1)
   122,   119,   135,   124, # counts for ZX for outcomes (+1, +1), (+1, -1), (-1
, +1), (-1, -1)
   126,   123,   134,   117, # counts for ZY for outcomes (+1, +1), (+1, -1), (-1
, +1), (-1, -1)
     9,   233,   253,     5, # counts for ZZ for outcomes (+1, +1), (+1, -1), (-1
, +1), (-1, -1)
]);
```

In [4]:
```
# An entanglement witness which is appropriate for our target state, as a qutip.Q
obj
EntglWitness = (- qutip.qeye(4)
    # how do you "collapse systems together" with qutip?? we could do this with n
p.kron() also...
    - qutip.Qobj(qutip.tensor(qutip.sigmax(),qutip.sigmay()).data,dims=[[4],[4]])
    + qutip.Qobj(qutip.tensor(qutip.sigmay(),qutip.sigmax()).data,dims=[[4],[4]])
    - qutip.Qobj(qutip.tensor(qutip.sigmaz(),qutip.sigmaz()).data,dims=[[4],[4]])
 )
display(EntglWitness)
```

Quantum object: dims = [[4], [4]], shape = [4, 4], type = oper, isherm = True

$$\begin{pmatrix} -2.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & -2.0j & 0.0 \\ 0.0 & 2.0j & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & -2.0 \end{pmatrix}$$

In [5]:

```
# Value for rho_target_Bell maximally entangled state: +2
display(qutip.expect(EntglWitness, rho_target_Bell))
```

```
1.9999999999999996
```

In [6]:

```
# but you can show that for any separable state this value is <= 0. For example:
display(qutip.expect(EntglWitness, qutip.qeye(4)/4))
display(qutip.expect(EntglWitness, qutip.Qobj(np.array([1,0,0,0]))))
display(qutip.expect(EntglWitness, 0.5*qutip.ket2dm(qutip.Qobj(np.array([0,1,0,0]
)))
             + 0.5*qutip.ket2dm(qutip.Qobj(np.array([0,0,1,0])))))
```

```
-1.0

-2.0

0.0
```

In [7]:

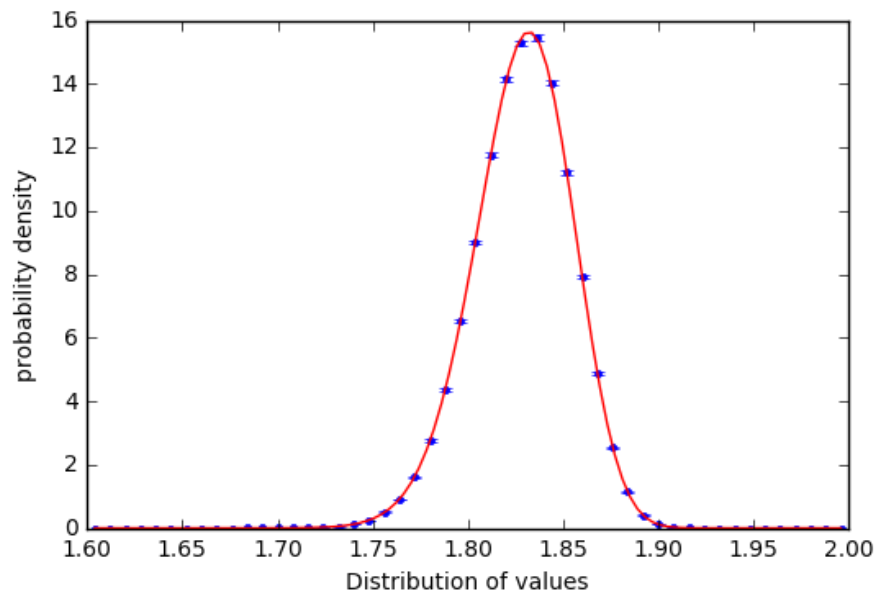
```
# Now, we're ready to run our tomography procedure. We'll be estimating
# the expectation value of the entanglement witness.

r = None # global variable

with tomographer.jpyutil.RandWalkProgressBar() as prg:
    r = tomographer.tomorun.tomorun(
        # the dimension of the quantum system
        dim=4,
        # the tomography data
        Nm=Nm,
        Emn=Emn,
        # Histogram: values in [1.8, 2.0] split into 50 bins
        hist_params=tomographer.UniformBinsHistogramParams(1.6,2,50),
        # Random Walk parameters: step size, sweep size, number of thermalization
 sweeps, number of live sweeps
        mhrw_params=tomographer.MHRWParams(0.009,120,500,32768),
        # figure of merit:
        fig_of_merit="obs-value",
        observable=EntglWitness.data.toarray(),
        #num_repeats=12, # default value = auto-detect number of CPU's
        progress_fn=prg.progress_fn
    )
    prg.displayFinalInfo(r['final_report_runs'])
```

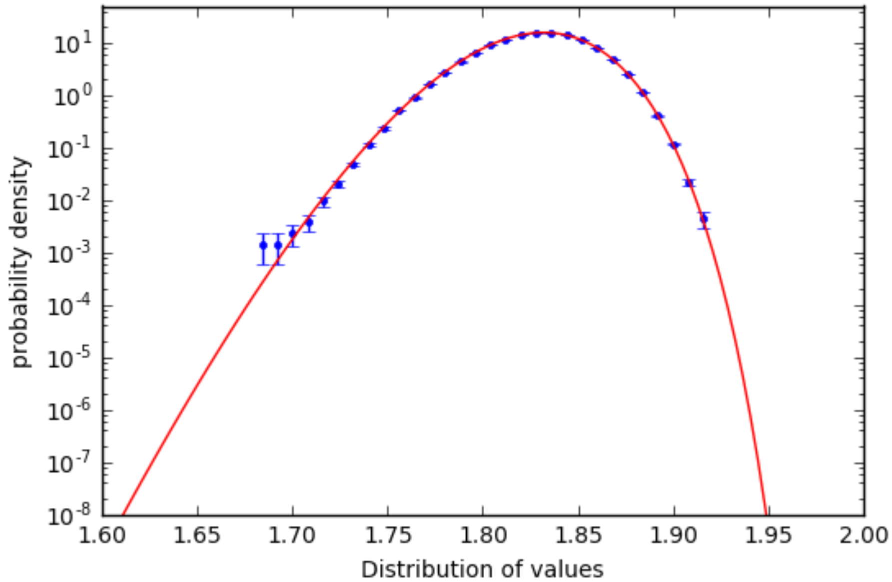
In [8]:

```
# Collect the histogram
final_histogram = r['final_histogram']
```

In [9]:
```
# Do the analysis and get the quantum error bars
analysis = tomographer.querrorbars.HistogramAnalysis(final_histogram, ftox=(2,-1)
)
analysis.printFitParameters()
analysis.printQuantumErrorBars()
# linear scale plot
analysis.plot()
# log scale plot (adjust scale before showing plot)
p = analysis.plot(log_scale=True, show_plot=False)
p.ax.set_ylim([1e-8, 50])
p.show()
```

```
Fit parameters:
          a2 = 22.6201
          a1 = 243.756
           m = 42.2292
           c = 119.668
Quantum Error Bars:
          f0 = 1.832
       Delta = 0.03602
       gamma = 0.002499
```




probability density
$10^{1}$
$10^{0}$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
$10^{-6}$
$10^{-7}$
$10^{-8}$
1.60 1.65 1.70 1.75 1.80 1.85 1.90 1.95 2.00
Distribution of values

```
In [ ]:
```