GBW-BK-S Saturation Model

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March 30, 2022

Contents

1 overall notes, general stuff etc.

for flavour f and polarization p [1][2],

$$\sigma_{f,\,p} = \int_0^1 dz \int d^2\mathbf{r}|\Psi_{f,\,p}|^2\sigma_{DP}. \tag{1.1}$$

The dipole cross section σ_{DP} is written [1]

$$\sigma_{DP} = \int d\mathbf{b}(1 - S(\mathbf{r}_1, \mathbf{r}_2)) \tag{1.2}$$

$$= \sigma_0(1 - \mathcal{S}(\mathbf{r})), \tag{1.3}$$

where $\mathbf{b} = \frac{\mathbf{r_1}+\mathbf{r_2}}{2}$, and $S(\mathbf{r}_1, \mathbf{r}_2) = \left\langle \mathrm{tr}\left(U^n(\mathbf{r}_1)U^{n\dagger}(\mathbf{r}_2)\right)\right\rangle / N_c$[1], and $\mathbf{r} = (\mathbf{r}_1 - \mathbf{r}_2)/2$ as defined in [1].
Photon wave functions $\Psi_{f,\,p}$ are avaiable in [2] [3].

Now, B. W. Xiao et al [4] define

$$xG^{(2)}(x, k_\perp) = \frac{q_\perp^2 N_c}{2\pi^2\alpha_s} S_\perp \int \frac{d^2\mathbf{r}_\perp}{(2\pi)^2} e^{-i\mathbf{k}_\perp\cdot\mathbf{r}_\perp} \frac{1}{N_c}\left\langle \mathrm{tr}\, U(0)U^\dagger(\mathbf{r}_\perp)\right\rangle_x \tag{1.4}$$

Nikolaev et al write[3],

$$\sigma(x, r) = \frac{\pi\alpha_s(r)r^2}{3} \int \frac{d^2\mathbf{k}}{k^2} \frac{4\left(1 - e^{i\mathbf{k}\cdot\mathbf{r}}\right)}{k^2r^2} \frac{\partial G(x_g, k^2)}{\partial \log(k^2)} \tag{1.5}$$

$$= \frac{4\pi\alpha_s(r)}{3} \int \frac{d^2\mathbf{k}}{k^4} \left(1 - e^{i\mathbf{k}\cdot\mathbf{r}}\right) \frac{\partial G(x_g, k^2)}{\partial \log(k^2)} \tag{1.6}$$

where gluon structure function $G(x, k^2) = xg(x, k^2)$.

where, at leading order [3],

$$\frac{\partial G(x_g, k^2)}{\partial \log(k^2)} = \frac{4}{\pi}\alpha_s(k^2)\left(1 - \langle N| e^{i\mathbf{k}\cdot(\mathbf{r_1}-\mathbf{r_2})} |N\rangle\right). \tag{1.7}$$

Which is also presented in [5],

$$\sigma(x, \mathbf{r}) = \frac{2\pi}{3} \int \frac{d^2\mathbf{l}}{l^4} \alpha_s f(x, l^2)(1 - e^{i\mathbf{l}\cdot\mathbf{r}})(1 - e^{-i\mathbf{l}\cdot\mathbf{r}}) \tag{1.8}$$

which is[2] ,

$$= \frac{4\pi}{3} \int \frac{d^2\mathbf{l}}{l^4} \alpha_s f(x, l^2)(1 - e^{i\mathbf{l}\cdot\mathbf{r}}) \tag{1.9}$$

note ‘gluon amplitude’ f is related to the gluon distribution g somehow... at leading order that’d be $f(x, k^2) \sim \frac{\partial G(x_g,k^2)}{\partial \log(k^2)}$

Eq. 1.9 is inverted to yield [5],

$$\frac{\alpha_s f(x, l^2)}{l^4} = \frac{3}{4\pi} \int \frac{d^2\mathbf{r}}{(2\pi)^2} e^{i\mathbf{l}\cdot\mathbf{r}} \left(\sigma(x, \infty) - \sigma(x, r)\right). \tag{1.10}$$

[1] $U^n(\mathbf{r}) = P\exp\left(ig\int_{-\infty}^{+\infty} dun_\mu A_\mu^c(u\mathbf{n} + \mathbf{r})t^c\right)$.

[2] cf [6] eq. 4

2 something else

bessels function of first kind $J_n(z)$ for $n = 0$.

$$J_0(z) = \frac{1}{\pi}\int_0^\pi e^{iz\cos(\theta)}d\theta \tag{2.1}$$

hence,

$$\int d^2\mathbf{r} f(r)e^{i\mathbf{r}\cdot\mathbf{k}} = 2\pi\int dr\; rJ_0(kr)f(r) \tag{2.2}$$

3 Xiao et al [4]

With sudakov

$$\begin{aligned} &xG^{(2)}(x, k_\perp, \zeta_c = \mu_F = Q) \\ &\qquad = -\frac{2}{\alpha_s}\int \frac{d^2x_\perp d^2y_\perp}{(2\pi)^4} e^{i\mathbf{k}_\perp\cdot\mathbf{r}_\perp}\mathcal{H}^{DP}(\alpha_s)e^{-\mathcal{S}_{sud}(Q^2,r_\perp^2)}\nabla^2_{r_\perp}\mathcal{F}^{DP}_{Y=1/x}(x_\perp, y_\perp). \end{aligned} \tag{3.1}$$

Again $\mathcal{F}^{DP}$ is related to σ_{DP} by

$$\sigma_{DP} = \int d\mathbf{b}(1 - \mathcal{F}(x_\perp, y_\perp)) \tag{3.2}$$

4 notations, relevant identities relations etc

4.1 Cambridge core Quantum chromodynamics [1] chapter 9

$f^g(x, Q^2)$

gluon distribution.

$= G(x, Q^2)$.

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$\mathcal{F}(x, \mathbf{k}^2)$

unintegrated gluon density[3].

$$xG(x, Q^2) = \int \mathcal{F}(x, \mathbf{k}^2) d\mathbf{k}^2 \theta(Q^2 - \mathbf{k}^2)/\mathbf{k}^2. \tag{4.1}$$

See also; gluon distribution G.

$S(\mathbf{r}_1, \mathbf{r}_2; Y)$

dipole elastic scattering S-matrix.

$$\sigma_{dp}(r, Y) = \int d^2b \left(1 - S(\mathbf{r}_1, \mathbf{r}_2; Y)\right). \tag{4.2}$$

$$S(\mathbf{r}_1, \mathbf{r}_2; Y) = \left\langle \mathrm{tr}\left(U^n(\mathbf{r}_1) U^{n\dagger}(\mathbf{r}_2)\right)\right\rangle / N_c \tag{4.3}$$

see also; dipole cross-section σ_{dp}

σ_{dp}

dipole coross section

$$\sigma_{dp}(r, Y) = \int d^2b \left(1 - S(\mathbf{r}_1, \mathbf{r}_2; Y)\right). \tag{4.4}$$

[3]note sometimes in the chapter $\mathcal{F}(x, \mathbf{k})$.

$$\sigma = \int d^2r \int_0^1 dz |\Psi_{\gamma^*}(r,z,Q^2)|^2 \sigma_{dp}(r,Y) \tag{4.5}$$

$$\sigma_{dp}(r,Y) = \int \frac{d^2q}{2\pi \mathbf{q}^2} |1 - e^{-i\mathbf{q}\mathbf{r}}|^2 \mathcal{F}(\mathbf{q}) \tag{4.6}$$

but note $\mathcal{F}(\mathbf{q})$ is not the same as unintegrated gluon distrubution $\mathcal{F}(x, \mathbf{k}^2)$. [1]-eq.(9.197)

References

[1] B. L. Ioffe, V. S. Fadin, and L. N. Lipatov, "Bfkl approach," in *Quantum Chromodynamics: Perturbative and Nonperturbative Aspects* (Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology), Cambridge Monographs on Particle Physics, Nuclear Physics and Cosmology. Cambridge University Press, 2010, 448–532. DOI: 10.1017/CBO9780511711817.010.

[2] K. Golec-Biernat and M. Wüsthoff, "Saturation effects in deep inelastic scattering at low Q^2 and its implications on diffraction," *Phys. Rev. D*, vol. 59, p. 014 017, 1 1998. DOI: 10.1103/PhysRevD.59.014017. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevD.59.014017.

[3] N. Nikolaev and B. Zakharov, "On determination of the large-1x gluon distribution at hera," *Physics Letters B*, vol. 332, no. 1, pp. 184–190, 1994, ISSN: 0370-2693. DOI: https://doi.org/10.1016/0370-2693(94)90877-X. [Online]. Available: https://www.sciencedirect.com/science/article/pii/037026939490877X.

[4] B.-W. Xiao, F. Yuan, and J. Zhou, "Transverse momentum dependent parton distributions at small-x," *Nuclear Physics B*, vol. 921, pp. 104–126, 2017, ISSN: 0550-3213. DOI: https://doi.org/10.1016/j.nuclphysb.2017.05.012. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0550321317301761.

[5] J. Bartels, K. Golec-Biernat, and H. Kowalski, "Modification of the saturation model: Dokshitzer-gribov-lipatov-altarelli-parisi evolution," *Phys. Rev. D*, vol. 66, p. 014 001, 1 2002. DOI: 10.1103/PhysRevD.66.014001. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevD.66.014001.

[6] I. P. Ivanov, N. N. Nikolaev, W. Schafer, B. G. Zakharov, and V. R. Zoller, "Lectures on diffraction and saturation of nuclear partons in DIS off heavy nuclei," in *36th Annual Winter School on Nuclear and Particle Physics (PINP 2002) and 8th St. Petersburg School on Theoretical Physics*, Dec. 2002. arXiv: hep-ph/0212161.