Conjecture 2
The Central Pricing Problem

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Problem Description

We extend the Central Price Problem from a single asset to a system of interacting assets, with explicit microstructural and mean-field effects.

Setup

Fix an integer $N > 3$ denoting the number of traded assets. For each $i \in \{1, \ldots, N\}$, let $\{P_t^{(i)}\}_{t \geq 0}$ denote the (mid-)price process of asset i, observed on a discrete time grid

$$0 = t_0 < t_1 < \cdots < t_T < t_{T+1} < \ldots,$$

with prices known for all $t_k \leq T$ and unknown for $t_k > T$. We write

$$\mathbf{P}_t := \left(P_t^{(1)}, \ldots, P_t^{(N)}\right)^\top$$

for the vector of prices at time t. For concreteness, one may think of:

- an underlying asset A, its option A^* and swap $\tilde{A}$,

- another underlying B with derivatives B^* and $\tilde{B}$,

- and, more generally, underlyings $A, B, C, D, \ldots$ with associated derivatives $A^*, \tilde{A}, B^*, \tilde{B}, \ldots$.

The assets may be related through payoff structure (underlying–derivative), sector or factor exposure, common fundamentals, or purely through flow and correlation.

Assumption 1 (Information set and observables). *At time T the modeller observes an information set $\mathcal{I}_T$ that includes at least:*

1. *The joint price and volume history*

$$\left\{(\mathbf{P}_{t_k}, \mathbf{V}_{t_k}) : k = 0, \ldots, T\right\},$$

where $\mathbf{V}_{t_k}$ collects traded volumes or order-flow statistics for all N assets at time t_k.

2. *Cross-sectional and cross-asset features, e.g. empirical correlation matrices of returns, realised volatilities, liquidity measures, and regime indicators extracted from the joint history.*

3. *Fundamental data, such as financial statements, sector/industry classifications and macroeconomic indicators, at least for the issuers associated with a subset of the assets.*

4. *Aggregate information about other market participants, encoded as a (possibly approximate) distribution over their states,*

$$m_T \in \mathcal{P}(\mathcal{X}),$$

where $\mathcal{P}(\mathcal{X})$ denotes the set of probability measures on a suitable state space $\mathcal{X}$ (e.g. inventory, risk limits, trading style, latency).

Assumption 2 (Immediate vs historical dependence)**.** *For each asset i and time T, the one-step-ahead price admits a decomposition of the form*

$$P_{T+1}^{(i)} = \alpha_T^{(i)} I_T^{(i)} + \left(1 - \alpha_T^{(i)}\right) H_T^{(i)} + \varepsilon_{T+1}^{(i)}, \tag{1}$$

where:

- $I_T^{(i)}$ *is an* immediate impact *term, measurable with respect to the short-horizon order-book state and flow around T;*

- $H_T^{(i)}$ *is a* historical dynamics *term, measurable with respect to the longer-horizon price/volume/fundamental history up to time T;*

- $\alpha_T^{(i)} \in [0,1]$ *is a (possibly random) weight capturing the relative importance of immediate versus historical effects, allowed to vary in time and across assets;*

- $\varepsilon_{T+1}^{(i)}$ *is a noise term with mean zero (under an appropriate physical or risk-neutral measure), capturing residual unpredictable variation.*

We assume that $\varepsilon_{T+1}^{(i)}$ is small in a sense to be made precise (e.g. sub-Gaussian, or with bounded conditional variance).

Assumption 3 (Structure of historical component)**.** *For each i, the historical component $H_T^{(i)}$ is a measurable functional of $\mathcal{I}_T$, and may depend on:*

- *past prices $\{P_{t_k}^{(j)} : 0 \le k \le T,\ 1 \le j \le N\}$,*

- *cross-asset and cross-derivative correlations,*

- *volatility and liquidity regimes (e.g. estimated from realised volatility, bid–ask spreads, depth, etc.),*

- *macro/fundamental signals extracted from financial statements and economic data,*

- *medium- to long-horizon features such as drift, mean reversion, or factor loadings.*

In particular, $H_T^{(i)}$ may encode stylised facts such as:

- *partial mean reversion after extreme returns or all-time highs/lows;*

- *reversion towards fundamentals (e.g. valuation ratios);*

- *persistent drift components over longer horizons.*

Assumption 4 (Structure of immediate impact)**.** *The immediate impact term $I_T^{(i)}$ is a functional of the short-horizon order-book state and flow around T, including:*

- *incoming market and limit orders (sizes, sides, time priority),*

- *the current limit-order-book shape (depth at each price level, imbalance),*

- *observed behaviour of other agents (e.g. clustering of aggressive trades, herding).*

When the modeller submits an order of price $p^{(i)}$ and size (or normalised volume) $q^{(i)}$ at time T, the mid-price of asset i is shifted according to an impact rule

$$mid_{T+}^{(i)} = mid_T^{(i)} + \left(p^{(i)} - mid_T^{(i)}\right) \phi^{(i)}\left(q^{(i)}, \mathcal{I}_T\right), \tag{2}$$

for some impact function $\phi^{(i)}$, which in a toy specification may take the form

$$\phi^{(i)}\left(q^{(i)}, \mathcal{I}_T\right) = \sqrt{q^{(i)}}.$$

More refined impact functions (e.g. concave in volume, state-dependent) are allowed and encouraged.

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Assumption 5 (Mean-field interaction)**.** *Other participants are modelled collectively via the distribution m_T over their states. Their joint behaviour feeds back into prices through both $H_T^{(i)}$ and $I_T^{(i)}$. In particular, for a given policy of the modeller, the evolution of $\mathbf{P}_t$ is coupled with the evolution of m_t, leading to a mean-field type interaction in the sense of mean-field games or mean-field control.*

Central Price and Confidence Regions

Given the information set $\mathcal{I}_T$ and the mean-field state m_T, define the *central price map*

$$F_T : (\mathcal{I}_T, m_T) \longmapsto \widehat{\mathbf{P}}_{T+1} := \big(\widehat{P}_{T+1}^{(1)}, \ldots, \widehat{P}_{T+1}^{(N)}\big)^\top,$$

where $\widehat{P}_{T+1}^{(i)}$ is a model-based estimate of $P_{T+1}^{(i)}$ (e.g. the conditional expectation or a robust location functional).

For a fixed horizon $H \geq 5$, we also define a *confidence-region map*

$$\mathcal{C}_T : (\mathcal{I}_T, m_T) \longmapsto \mathcal{R}_T \subset \mathbb{R}^{N \times H},$$

where $\mathcal{R}_T$ is interpreted as a confidence set for the joint future price paths

$$\big\{\mathbf{P}_{T+1}, \mathbf{P}_{T+2}, \ldots, \mathbf{P}_{T+H}\big\}.$$

Conjecture 1 (Central Price Problem, Multi-Asset Extension)**.** *Under Assumptions 1–5 and under realistic market conditions, there exist:*

1. *A non-trivial, empirically calibratable family of central price maps*

$$F_T : (\mathcal{I}_T, m_T) \mapsto \widehat{\mathbf{P}}_{T+1},$$

   such that for each asset i,

$$P_{T+1}^{(i)} - \widehat{P}_{T+1}^{(i)}$$

   is statistically small (e.g. sub-Gaussian with variance bounded uniformly in T) and the resulting central price estimates are practically useful for trading, market-making or risk management.

2. *A family of confidence-region maps*

$$\mathcal{C}_T : (\mathcal{I}_T, m_T) \mapsto \mathcal{R}_T \subset \mathbb{R}^{N \times H}, \qquad H \geq 5,$$

   such that the true joint future prices satisfy

$$\big(\mathbf{P}_{T+1}, \ldots, \mathbf{P}_{T+H}\big) \in \mathcal{R}_T$$

   with probability at least $1 - \delta$ for some prescribed level $0 < \delta < 1$, and such that this coverage property holds with empirical *accuracy when tested on out-of-sample data from realistic markets.*

Equivalently, there should exist an empirically testable mapping from:

- *historical dynamics (prices, volumes, correlations, volatility and liquidity regimes, fundamentals), and*

- *immediate impact (order-book state, flows, participant positions, the modeller's own trades),*

to:

- *short-term central prices $\widehat{\mathbf{P}}_{T+1}$, and*

- *medium-horizon confidence regions $\mathcal{R}_T$ for a system of $N \geq 4$ interacting assets,*

with non-trivial predictive power beyond naive baselines (e.g. random walk or constant-volatility models).

Remark 1 (Unified pricing framework). Special interest is reserved for solutions to Conjecture 1 that embed the above construction into a unified theoretical framework covering simultaneously:

- the market-maker's very short-horizon pricing and inventory-control problem,

- the high-frequency trader's short- to medium-horizon alpha and execution problem, and

- the medium- to long-term asset-pricing and portfolio-allocation problem,

under a common set of assumptions linking:

- microstructure (order-book mechanics, impact, immediacy),

- participant behaviour and mean-field interactions, and

- longer-term price formation, drift, and risk premia.

Such a framework should make explicit how the decomposition (1) and the impact rule (2) (or their generalisations) propagate across time scales, from tick-by-tick dynamics to daily, weekly, or longer horizons.