Conjecture 1
Dynamic reallocation between strategies under model updates

Tomorrow Capital Research

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Problem Description

Consider a single risky asset with price process $(S_t)_{t\in[0,T]}$, observed at discrete times

$$0 = t_0 < t_1 < \ldots < t_N = T.$$

We are given:

- A family of models $\{M_k\}_{k\geq 1}$ for the asset dynamics, each of which can be used to generate Monte Carlo scenario paths.
- A finite collection of trading strategies $\{\pi^1, \ldots, \pi^K\}$ (e.g. mean-reversion, trend-following, etc.). For each strategy π^k and model M_k, we obtain a position process $(\theta_t^{(k)})_{t\in[0,T]}$ (e.g. number of units held) and associated P&L along a given price path.

Initial phase (regime 1: $[0, T_1]$). At time $t_0 = 0$, choose an initial model M_1 (e.g. mean-reverting) and simulate a family of paths

$$\{X_t^{(1,i)}\}_{i=1}^M, \quad t \in [0, T_1].$$

Using these paths and a given objective (expected utility, risk-adjusted return, drawdown, etc.), select a "best" strategy $\pi^{(1)}$ (e.g. a mean-reversion strategy) and trade according to its position process

$$\left(\theta_t^{(1)}\right)_{t\in[0,T_1]}.$$

Ex post, once realised prices on $[0, T_1]$ are known, we can assess the fit of both the model M_1 and the associated strategy $\pi^{(1)}$ to the realised path via some loss function L_1.

Model update and regime change (event at T_2). Suppose that within the next interval, at time $T_2 \in (T_1, T)$, a significant event or new information arrives (macro shock, regime shift, structural break, etc.). As a result:

- The original model M_1 and strategy $\pi^{(1)}$ are no longer believed to be "optimal" beyond T_2.
- We adopt a new model M_2 (e.g. trend-following regime) and consider a new strategy $\pi^{(2)}$ with position process

  $$\left(\theta_t^{(2)}\right)_{t\in[T_2,T_3]},$$

  where $T_3 > T_2$ is the time by which the transition to the new regime is intended to be complete.

Starting from the realised price S_{T_2}, we simulate a new set of paths

$$\{X_t^{(2,j)}\}_{j=1}^N, \quad t \in [T_2, T_3],$$

under M_2, and obtain corresponding candidate positions for $\pi^{(2)}$.

The transition problem (core question). On the time interval $[T_2, T_3]$, we now face a transition problem:

- We currently hold a position $\theta^{(1)}_{T_2}$ implied by the original strategy $\pi^{(1)}$.
- We wish to gradually move to the new position trajectory $\big(\theta^{(2)}_t\big)_{t\in[T_2,T_3]}$ implied by $\pi^{(2)}$.
- Trading is subject to frictions (transaction costs, liquidity constraints, risk limits).

The informal goal is to construct an algorithm or mathematical framework that prescribes, at each intermediate time

$$t_k \in [T_2, T_3], \quad k = 0, 1, \dots,$$

the allocation between the old and new strategies, or equivalently the position θ_{t_k}, such that:

1. The resulting trajectory $(\theta_{t_k})_k$ is in some sense the "shortest" or most coherent adjustment from the old regime to the new one (e.g. minimal cumulative trading cost, minimal deviation from the new optimal position, or minimal path-wise distortion).
2. Risk and capital constraints are respected throughout the transition.
3. The procedure is dynamically consistent: if we revisit the problem at an intermediate time t_k with updated information, the algorithm's prescription from t_k to T_3 coincides with the restriction of the original plan.

More concrete formulation. One possible concrete formulation is as follows. Let $\theta^{(1)}_t$ be the "legacy" position (e.g. mean-reversion) and $\theta^{(2)}_t$ the "target" position (e.g. trend-following). Define a control process $w_t \in [0, 1]$ on $[T_2, T_3]$ representing the fraction of capital allocated to the new regime, so that the actual position is

$$\theta_t = (1 - w_t)\,\theta^{(1)}_t + w_t\,\theta^{(2)}_t, \quad t \in [T_2, T_3].$$

Given:

- a loss functional $\mathcal{L}\big(\{\theta_t\}, \{\theta^{(2)}_t\}\big)$, measuring deviation from the new "optimal" strategy, and
- a cost functional $\mathcal{C}\big(\{\theta_t\}\big)$, capturing transaction costs, turnover, and risk,

The problem is to find an admissible control w_t (or discrete actions at $t_1, \dots, t_n$) that minimises

$$\mathbb{E}\big[\,\mathcal{L} + \mathcal{C}\,\big]$$

over a set of simulated paths $\{X^{(2,j)}\}$ and/or under the model M_2, subject to trading and risk constraints.