Fall 2016, CSE/CS 383, Homework 3
Due 9:30am, 9/22. Also submit electronic PDF (scans are ok)

Reading: Textbook lectures 7,8,9

1. Let $A \in \mathbb{R}^{m\times n}$ be full rank and let $\{q_j\}_{j=1}^{n}$ be the orthonormal vectors from classical the Gram-Schmidt orthogonalization of the columns of A. Let $P_j = q_j q_j^*$ The modified Gram-Schmidt algorithm is

$$w = \left[\prod_{j=1}^{i-1}(I - P_j)\right] A(:,i),\ \ q_i = w/\|w\|_2, \quad i = 1,\dots,m.$$

   Show that, in exact arithmetic, this algorithm is mathematically equivalent to the classical Gram-Schmidt algorithm (i.e., it produces the same q_j).

2. Write a single MATLAB function `[Q,R] = gramschmidt(A,flag)` that computes a **reduced** QR factorization $A = QR$ of an $m \times n$ full rank matrix A with $m \geq n$ using the classical Gram Schmidt orthogonalization when `flag == true` and the modified Gram Schmidt orthogonalization when `flag == false` or `flag` is left unspecified. Q should be an $m \times n$ matrix and R an $n \times n$ matrix.

3. Suggest a test (or tests) to (quantitatively) check the accuracy of your QR factorization. Apply the test(s) to QR factorizations obtained by the two Gram Schmidt variants and MATLAB's `[Q,R]=qr(A)` to the following matrices:
   `A = gallery('randsvd',100,kappa);` for `kappa`=1,1E3,1E6, 1E9. Discuss your results.