The University of Texas at Austin
Department of Electrical and Computer Engineering

**EE381K: Large Scale Optimization — Fall 2015**

Problem Set Seven

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**Reading Assignments**

1. Reading: Boyd & Vandenberghe: Chapters 7 & 8.

**Matlab and Computational Assignments**. Please provide a printout of the Matlab code you wrote to generate the solutions to the problems below.

1. **Sums of Squares and SDP**. Construct a 4th (or higher) degree non-convex univariate polynomial that is bounded from below (just make sure the leading term is even degree, with positive coefficient), and show that there is a starting point from which gradient descent will get trapped in a local minimum. Then formulate the problem as a sum of squares, and solve using SDP via CVX.

2. (?) Try solving the above SDP via an interior point method. You can try a generic approach as discussed in class, or something more carefully tailored to SDPs.

3. **MaxCut**. The MAXCUT problem is as follows: Given a graph $G = (V, E)$ with nonnegative edge weights $W = \{w_{ij}\}$, find a partition of the vertices $V = V_1 \cup V_2$ (where $V_1 \cap V_2 = \emptyset$) so that the sum of the edge weights from $V_1$ to $V_2$ is maximized.

   This can be formulated as a binary integer programming problem as follows:

$$\begin{aligned} \max : \quad & \sum_{i,j} \frac{w_{ij}}{2}(1 - x_i x_j) \\ \text{s.t.} : \quad & x_i \in \{-1, 1\}. \end{aligned}$$

   Written problem 1 below asks you to show that this is equivalent to the following rank-constrained SDP:

$$\begin{aligned} \max : \quad & \sum_{i,j} \frac{w_{ij}}{2}(1 - X_{ij}) \\ \text{s.t.} : \quad & X_{ii} = 1 \\ & X \succeq 0 \\ & \operatorname{rank}(X) = 1. \end{aligned}$$

   This is still a non-convex problem, because of the rank constraint. The SDP relaxation of this comes from dropping the rank one constraint, and hence obtaining:

$$\begin{aligned} \max : \quad & \sum_{i,j} \frac{w_{ij}}{2}(1 - X_{ij}) \\ \text{s.t.} : \quad & X_{ii} = 1 \\ & X \succeq 0 \end{aligned}$$

which is a convex optimization problem. Use CVX to formulate and solve MaxCut on:

(a) Any two planar graphs of your choice (with a reasonable number of nodes).

(b) The Petersen Graph: `http://en.wikipedia.org/wiki/Petersen_graph`.

**Written Problems**

1. Show that the rank-constrained SDP is equivalent to the binary variable formulation of MaxCut.

2. Find the dual of the following convex quadratically constrained quadratic program, and show that the dual objective is concave and hence the problem is convex (the dual is always convex – I am asking you to check this explicitly for this particular case). In the problem below, the matrices $P_0$ and $P_i$ are positive definite.

$$\begin{aligned} \underset{x}{\text{minimize}} \quad & \frac{1}{2}\mathbf{x}^\top P_0 \mathbf{x} + \mathbf{p}_0^\top \mathbf{x} + r_0 \\ \text{subject to} \quad & \frac{1}{2}\mathbf{x}^\top P_i \mathbf{x} + \mathbf{p}_i^\top \mathbf{x} + r_i \leq 0, \ i = 1, \ldots, m. \end{aligned} \tag{1}$$

3. Problem 7.12 form Boyd & Vandenberghe.

4. Problem 7.13 from Boyd & Vandenberghe.

5. Problem 8.8 from Boyd & Vandenberghe.

6. Problem 8.9 from Boyd & Vandenberghe.

7. Problem 8.23 from Boyd & Vandenberghe.

8. Problem 8.24 from Boyd & Vandenberghe.

9. Problem 8.25 from Boyd & Vandenberghe.

10. (?) 8.2 from Boyd & Vandenberghe.

11. (?) 8.13 from Boyd & Vandenberghe.