

$$\Delta_B(f) = \{i \mid p\{x := y a_i\} \leq f_i\}$$

$$Q_B(i) = \{f \mid p\{x := y a_i\} \leq f_i\}$$

不通过点

$G_B \subseteq \mathbb{N}^{\mathbb{N}}$

$$\text{Fact: } \# Q_B(i) \leq 1$$

$$\exists i \text{ s.t. } \# Q_B(i) \geq 2 \text{ 当且仅当 } p\{x := x y\} \leq f_i$$

$$(1) \#A = \#B = \#Q = k \quad A' \cap B' = \emptyset$$

$$\text{W.l.o.g., } A' = \{a_{k+1}, a_{k+2}\}, B' = \{a_1, a_2\}$$

$$\exists \leq j \leq k+2 \quad 1 \leq i \leq 2 \quad \# \Delta_B(f) = 1$$

$$1 \leq i \leq k \quad 1 \leq i \leq 2 \quad \# Q_B(i) = 1$$

$$G_B: \{1, \dots, k\} \rightarrow \{3, \dots, k+2\}$$

$$Q_B(i) = \{G_B(i)\}$$

$$(2) \#A = k, \#Q = k, \#B = k-1 \quad A' \cap B' = \emptyset$$

$$A' = \{a_{k+1}, a_{k+2}\}, B' = \{a_1, a_2, a_3\}$$

$$Q_B = \{f_i \mid \exists f \text{ s.t. } p\{x := y a_i\} \leq f_i\}$$

$$(2-1) \#Q_B = k$$

$$G_B: \{1, \dots, k\} \rightarrow \{4, \dots, k+2\}$$

$$(2-2) \#Q_B = k-1 \quad \text{W.l.o.g., } f_k \notin Q_B$$

$$G_B: \{1, \dots, k-1\} \rightarrow \{4, \dots, k+2\}$$

$i \quad 1 \quad 2 \quad \dots \quad k \quad \text{w.l.o.g.} \quad \boxed{1}$

$$\sigma_A(i) \quad \sigma_A(1) \quad \sigma_A(2) \quad \dots \quad \sigma_A(k) \quad A' = \{k+1, k+2\}$$

$$\sigma_B(i) \quad \sigma_B(1) \quad \sigma_B(2) \quad \dots \quad \sigma_B(k) \quad B' = \{1, 2\}$$

$$\sigma_A: \{1, 2, \dots, k\} \rightarrow \{1, 2, \dots, k\}$$

$$\sigma_B: \{1, 2, \dots, k\} \rightarrow \{3, \dots, k+1, k+2\}$$

$$A = \{a_j \in \Sigma \mid \exists i \text{ s.t. } p\{x := a_j\} \leq f_i\}$$

$$B = \{a_j \in \Sigma \mid \exists i \text{ s.t. } p\{x := y a_j\} \leq f_i\}$$

$$R = \{a_{k+1}a_1, a_{k+1}a_2, a_{k+2}a_1, a_{k+2}a_2\}$$

$$r \in R \quad k+1 < i < k+2 \quad i(r) \in [1 \leq i(r) \leq k+2]$$

$$p\{x := r\} \leq f_{i(r)} \quad \text{exists } 1 \leq i \leq k+2 \text{ s.t.}$$

$$i(r) \quad 1) \quad 3 \leq \sigma_A(i(r)), \sigma_B(i(r)) \leq k \quad \text{Lem 6}$$

$$\sigma_A(i(r)) \quad 2-1) \quad 1 \leq \sigma_A(i(r)) \leq 2 \quad \& \quad \sigma_B(i(r)) \leq k$$

$$\sigma_B(i(r)) \quad 2-2) \quad 3 \leq \sigma_A(i(r)), k+1 \leq \sigma_B(i(r)) \leq k+2 \quad \text{Lem 7}$$

$$\left\{ \begin{array}{l} p\{x := a_{\sigma_A(i(r))}\} \leq f_{i(r)} \\ p\{x := y a_{\sigma_B(i(r))}\} \leq f_{i(r)} \\ p\{x := r\} \leq f_{i(r)} \end{array} \right.$$

$$3) \quad 1 \leq \sigma_A(i(r)) \leq 2 \quad \& \quad k+1 \leq \sigma_B(i(r)) \leq k+2$$

$$\sigma_A, \sigma_B \neq$$

$$\text{big. } \text{fuzz. } \text{Lem 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 794, 795, 796, 797, 798, 799, 800, 801, 802, 803, 804, 805, 806, 807, 808, 809, 810, 811, 812, 813, 814, 815, 816, 817, 818, 819, 820, 821, 822, 823, 824, 825, 826, 827, 828, 829, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 920, 921, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 948, 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969, 970, 971, 972, 973, 974, 975, 976, 977, 978, 979, 980, 981, 982, 983, 984, 985, 986, 987, 988, 989, 990, 991, 992, 993, 994, 995, 996, 997, 998, 999, 1000$$

$i \quad 1 \quad 2 \quad \dots \quad k \quad \text{w.l.o.g.} \quad [2]$

$$\sigma_A(i) \quad \sigma_A(1) \quad \sigma_A(2) \quad \dots \quad \sigma_A(k) \quad A' = \{k+1, k+2\}$$

$$\sigma_B(i) \quad \sigma_B(1) \quad \sigma_B(2) \quad \dots \quad \sigma_B(k) \quad B' = \{1, 2, 3\}$$

$$\sigma_A: \{1, 2, \dots, k\} \rightarrow \{1, 2, \dots, k\}$$

$[2-1]$ $\sigma_B: \{1, 2, \dots, k\} \rightarrow \{4, \dots, k+1, k+2\}$

$$A = \{a_j \in \Sigma \mid \exists i \text{ s.t. } p\{x := a_j\} \leq f_i\}$$

$$B = \{a_j \in \Sigma \mid \exists i \text{ s.t. } p\{x := y a_j\} \leq f_i\}$$

$$R = \{ \underset{a_{k+1} a_3}{a_{k+1} a_1}, a_{k+1} a_2, \underset{a_{k+2} a_3}{a_{k+2} a_1}, a_{k+2} a_2 \}$$

$$r \in R \quad k \geq 1 \quad i(r) \in [1 \leq i(r) \leq k+1]$$

$$p\{x := r\} \leq f_{i(r)} \quad \text{where } 1 \leq i(r) \leq k+1$$

$$i(r) \quad 1) \quad 4 \leq \sigma_A(i(r)), \sigma_B(i(r)) \leq k \quad \text{Lem 6}$$

$$\sigma_A(i(r)) \quad 2-1) \quad 1 \leq \sigma_A(i(r)) \leq 3 \quad \& \quad \overset{4,2}{\sigma_B(i(r)) \leq k}$$

$$\sigma_B(i(r)) \quad 2-2) \quad 3 \leq \sigma_A(i(r)) \leq k, \quad k+1 \leq \sigma_B(i(r)) \leq k+2 \quad \text{Lem 7}$$

$$\left\{ \begin{aligned} p\{x := a_{\sigma_A(i(r))}\} &\leq f_{i(r)} \\ p\{x := y a_{\sigma_B(i(r))}\} &\leq f_{i(r)} \end{aligned} \right.$$

$$\left\{ \begin{aligned} p\{x := r\} &\leq f_{i(r)} \end{aligned} \right.$$

$$3) \quad 1 \leq \sigma_A(i(r)) \leq 3 \quad \& \quad k+1 \leq \sigma_B(i(r)) \leq k+2$$

\square $\text{and } \text{Lem 6}$
 $\text{Lem 7} \text{ is not needed}$

$$V_1 = A_{k+1} A_1$$

$$V_4 = A_{k+2} A_1$$

$$V_2 = A_{k+1} A_2$$

$$V_5 = A_{k+2} A_2$$

$$V_3 = A_{k+1} A_3$$

$$V_6 = A_{k+2} A_3$$

$$\begin{cases} p \{ x := A_{\delta_A}(\bar{c}(v_1)) \} \leq f_{\bar{c}(v_1)} \\ p \{ x := y A_{\delta_B}(\bar{c}(v_1)) \} \leq f_{\bar{c}(v_1)} \\ p \{ x := v \} \leq f_{\bar{c}(v_1)} \end{cases}$$

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$$(\delta_A(\bar{c}(v_1)), \delta_B(\bar{c}(v_1))) = (1, k+1) \text{ 1x7+1'k} \quad \text{--- (1)}$$

$$p \{ x := xy \} \leq f_{\bar{c}(v_1)} \text{ 673}$$

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$$(\delta_A(\bar{c}(v_4)), \delta_B(\bar{c}(v_4))) = (1, k+2) \text{ 1x7+1'k} \quad \text{--- (2)}$$

$$p \{ x := xy \} \leq f_{\bar{c}(v_4)} \text{ 673}$$

①② 673 2 2 3 6 2 3 6

$$\delta_A(\bar{c}(v_1)) = \delta_A(\bar{c}(v_4)) \text{ 2 6 3 6 3}$$

$$\delta_A \text{ 673, 2 6 3 6 3 } \bar{c}(v_1) = \bar{c}(v_4)$$

$$- \bar{c} \delta_B(\bar{c}(v_1)) \neq \delta_B(\bar{c}(v_4)) \text{ 2 6 3}$$

$$= \text{1x7 } \delta_B \text{ 673 2 6 3 6 3 2 6 3}$$

2-3

$$\sigma_A: \{1, 2, \dots, k\} \rightarrow \{1, 2, \dots, k\}$$

$$\sigma_B: \{1, 2, \dots, k-2\} \rightarrow \{5, \dots, k+1, k+2\}$$

σ_B^{-1}

$$A' = \{k+1, k+2\}$$

$$B' = \{1, 2, 3, 4\}$$

$$(k \geq 4)$$

$$r_1 = a_{k+1} a_1$$

$$r_2 = a_{k+1} a_2$$

$$r_3 = a_{k+1} a_3$$

$$r_4 = a_{k+1} a_4$$

$$r_5 = a_{k+2} a_1$$

$$r_6 = a_{k+2} a_2$$

$$r_7 = a_{k+2} a_3$$

$$r_8 = a_{k+2} a_4$$

$$1 \leq \tilde{c}(v) \leq k-2 \text{ and } \exists$$

$$1) \quad 5 \leq \sigma_A(\tilde{c}(v)) \leq k, \quad \sigma_B(\tilde{c}(v)) \leq k \quad \text{Lem 6}$$

$$2-1) \quad 1 \leq \sigma_A(\tilde{c}(v)) \leq 4 \text{ and } \sigma_B(\tilde{c}(v)) \leq k$$

$$2-2) \quad 5 \leq \sigma_A(\tilde{c}(v)) \leq k, \quad k+1 \leq \sigma_B(\tilde{c}(v)) \leq k+2$$

$$3) \quad 1 \leq \sigma_A(\tilde{c}(v)) \leq 4 \text{ and}$$

$$k+1 \leq \sigma_B(\tilde{c}(v)) \leq k+2$$

$$k-1 \leq \tilde{c}(v) \leq k \text{ and } \exists$$

σ_B^{-1} function is not injective.

a b c d Lem b

① a b c a Lem b

② a a c d Lem 7

③ a b b d Lem b

④ a b c c Lem 7

⑤ a b b a Lem b

a b a d
└────────┘

┌────────┐
a b c b

1) a a c b Lem 7

a c a b
└────────┘

a c c b Lem b

2) ② a a b c Lem 7

a c b a Lem b

a c b c
└────────┘

3) a c b b Lem 7

4) a b a c

① a b c a Lem b

④ a b c c Lem 7

5) a b c b

③ a b b c Lem b

a a a a
└────────┘

1) a a a b
└────────┘

2) a a b a
└────────┘

3) a a b b Lem 7 x

4) a b a a
└────────┘

5) a b a b
└────────┘

⑤ 6) a b b a Lem b

7) a b b b
└────────┘

B A' B' A

a b c d Lem b

① a b c a Lem b

② a a c d Lem 7

③ a b b d Lem b

④ a b c c Lem 7

⑤ a b b a Lem b

Lem 8 zür 12. 7-2

a b a d

Lem 8 zür 12. 7-2

a b c b

B A' B' A
a a b b

Cor 2

a a a b

a b a

a b b

② a b c

a b a a

b a b

b a c

⑤ a b b a

b b b

③ b b c

① a b c a

b c b

b c c

$$\mathbb{Q} \quad 1 \quad 2 \quad 3 \quad \dots \quad k$$

$$\sigma_A(1) \quad \sigma_A(2) \quad \sigma_A(3) \quad \dots \quad \sigma_A(k)$$

$$\sigma_B(1) \quad \sigma_B(2) \quad \sigma_B(3) \quad \dots \quad \sigma_B(k)$$

$$\sigma_A: \overset{I_A}{\{1, 2, \dots, k\}} \longrightarrow \overset{I_\Sigma \cup \{0\}}{\{0, 1, 2, \dots, k, k+1, k+2\}}$$

$$\sigma_B: \{1, 2, \dots, k\} \longrightarrow \{0, 1, 2, \dots, k, k+1, k+2\}$$

$$\sigma_A, \sigma_B: I_{\mathbb{Q}} \rightarrow I_\Sigma \cup \{0\}$$

$$A = \sigma_A(I_{\mathbb{Q}}) \setminus \{0\} \quad A' = I_\Sigma \setminus A$$

$$B = \sigma_B(I_{\mathbb{Q}}) \setminus \{0\} \quad B' = I_\Sigma \setminus B$$

$$3) \quad \#A \leq k-2, \quad \#B \leq k-2 \quad (A' \cap B' = \emptyset)$$

$$l_A = k - \# \sigma_A^{-1}(0) - \#A \quad \text{重複した記号数}$$

$$l_B = k - \# \sigma_B^{-1}(0) - \#B$$

$$k - \# \sigma_A^{-1}(0) \geq \#A$$

$$\exists i_1, i_2, i_3 \in I_{\mathbb{Q}}; \sigma_A(i_1) = \sigma_A(i_2) = q(i_3)$$

$$\# \sigma_A^{-1}(0) \leq k - \#A$$

$$\exists i_1, i_2, i_3, i_4 \in I_{\mathbb{Q}}; \sigma_A(i_1) = \sigma_A(i_2) \text{ \& \& } \sigma_A(i_3) = \sigma_A(i_4)$$

$$\#A' \geq 4 \quad \#B' \geq 4$$

$$\#A = k - \# \sigma_A^{-1}(0) - l_A \leq k-2$$

$$\# \sigma_A^{-1}(0) + l_A \geq 2 \quad (l_A \geq 0) \quad \dots \text{--- 結局 } l_A = 0 \text{ なる}$$

$$\text{同様に } \# \sigma_B^{-1}(0) + l_B \geq 2$$

$$\sigma_A^{-1}(0) = \{i_0, i_1, i_2, i_3\}$$

$$I_{\mathbb{Q}}^{(2,0)} = \{i \in I_{\mathbb{Q}} \mid i \in \sigma_A^{-1}(0) \cap \sigma_B^{-1}(0)\}$$

$$I_{\mathbb{Q}}^{(2,1)} = \{i \in I_{\mathbb{Q}} \mid i \in \sigma_A^{-1}(0) \setminus \overset{\sigma_B^{-1}(0) \cap \sigma_A^{-1}(0)}{\sigma_B^{-1}(0)}\}$$

$$I_a^{(0,0)} = \{i \in I_a \mid i \in \sigma_A^{-1}(0) \cap \sigma_B^{-1}(0)\}$$

$$I_a^{(0,1)} = \{i \in I_a \mid i \in \sigma_A^{-1}(0) \setminus \sigma_B^{-1}(0)\}$$

$$I_a^{(1,0)} = \{i \in I_a \mid i \in \sigma_B^{-1}(0) \setminus \sigma_A^{-1}(0)\}$$



$$\#A' \geq 4, \#B' \geq 4$$

$$R = \{a_\alpha a_\beta \mid \alpha \in A' \text{ and } \beta \in B'\}$$

$$\forall a_\alpha a_\beta \in R \quad i \neq j \neq k$$

$$\sigma_A(i) \in A \cap B$$

$$\text{or } A \setminus B$$

$$\sigma_B(i) \in A \cap B$$

$$\text{or } B \setminus A$$

$$i(\alpha, \beta) = \{i \in I_a \mid p\{x = a_\alpha a_\beta\} \leq g_i\}$$

if $\exists \alpha \in A', \beta \in B'$ and

$$\exists i \in i(\alpha, \beta) \text{ s.t. } \sigma_A(i) \in A \text{ and } \sigma_B(i) \in B$$

$$\text{i.e. } p\{x = a_{\sigma_A(i)} y\} \leq g_i$$

$$\text{and } p\{x = y a_{\sigma_B(i)}\} \leq g_i$$

$$= a \neq i$$

$$p\{x = xy\} \leq g_i \text{ and } \sigma_B(i) \in A$$

$$\forall i \in i(\alpha, \beta) \Rightarrow \sigma_A(i) = 0 \text{ or } \sigma_B(i) = 0$$

$$\# \sigma_A^{-1}(0) = k - \#A - Q_A$$

$$\# \sigma_B^{-1}(0) = k - \#B - Q_B$$

$$\#A' = k + 2 - \#A \text{ or } \#A = k + 2 - \#A'$$

$$\# \sigma_A^{-1}(0) = k - (k + 2 - \#A') - Q_A$$

$$= \#A' - Q_A - 2 \leq \#A' - 2$$

$$\# \sigma_B^{-1}(0) \leq \#B' - 2$$

$$I_a^{(0,0)} = \{z \in I_a \mid z \in \sigma_A^{-1}(0) \cap \sigma_B^{-1}(0)\}$$

$$I_a^{(0,1)} = \{z \in I_a \mid z \in \sigma_A^{-1}(0) \setminus \sigma_B^{-1}(0)\}$$

$$I_a^{(1,0)} = \{z \in I_a \mid z \in \sigma_B^{-1}(0) \setminus \sigma_A^{-1}(0)\}$$

$$\begin{aligned} \# \sigma_A^{-1}(0) &= k - (k+2 - \#A') - l_A \\ &= \#A' - l_A - 2 \leq \#A' - 2 \end{aligned}$$

$$l_A = l_B = 0$$

note

$$\# \sigma_B^{-1}(0) \leq \#B' - 2$$

$$4\#I_a^{(0,0)} + 2\#I_a^{(0,1)} + 2\#I_a^{(1,0)}$$

$$= 2(\#I_a^{(0,0)} + \#I_a^{(0,1)}) + 2(\#I_a^{(0,0)} + \#I_a^{(1,0)})$$

$$= 2\#\sigma_A^{-1}(0) + 2\#\sigma_B^{-1}(0)$$

$$\leq 2(\#A' + \#B') - 8 < \#A' \times \#B'$$

$$\therefore \#A' \times \#B' - 2(\#A' + \#B') + 8$$

$$= (\#A' - 2)(\#B' - 2) + 4 > 0$$

$$I_a^{(1,1)} = \{z \in I_a \mid \sigma_A(z) \in A \text{ and } \sigma_B(z) \in B\}$$

$$\# \Delta_A^{-1}(0) = k - \#A - l_A$$

$$\# \Delta_B^{-1}(0) = k - \#B - l_B$$

$$l_A \in l_B \in \text{集合} \mathbb{Z}$$

$$\# \Delta_A^{-1}(0)$$

$$\begin{aligned} & 4 \# I_a^{(0,0)} + 2 \# I_a^{(0,1)} + 2 \# I_a^{(1,0)} \\ &= 2 (\# I_a^{(0,0)} + \# I_a^{(0,1)}) + 2 (\# I_a^{(0,0)} + \# I_a^{(1,0)}) \\ &= 2 \# \Delta_A^{-1}(0) + 2 \# \Delta_B^{-1}(0) \end{aligned}$$

$$\# A' = k + 2 - \#A \quad \#A = k + 2 - \#A'$$

$$\# \Delta_A^{-1}(0) = k - (k + 2 - \#A') - l_A$$

$$= \#A' - l_A - 2$$

$$\# \Delta_B^{-1}(0) = \#B' - l_B - 2$$

$$= 2 (\#A' - l_A - 2) + 2 (\#B' - l_B - 2)$$

$$= 2 (\#A' + \#B') - 8 - 2 (l_A + l_B)$$

$$\#A' \times \#B' - (2 (\#A' + \#B') - 8 - 2 (l_A + l_B))$$

$$= \#A' \times \#B' - 2 (\#A' + \#B') + 8 + 2 (l_A + l_B)$$

$$= (\#A' - 2) (\#B' - 2) + 2 (l_A + l_B + 2)$$

これは $I_a^{(0,0)}$ に 5 つ以上 3 次元 $I_a^{(0,1)}$, $I_a^{(1,0)}$ に 3 つ以上 $Q_a Q_B$ の交わり \mathbb{R}^n .

$$l_A = k - \# \vec{\Delta}_A(0) - \# A$$

$$l_B = k - \# \vec{\Delta}_B(0) - \# B$$

$$\# A = \# (\Delta_A(Ia) / \{0\})$$

$$\# A + \# A' = k + 2$$

$$\# A + l_A + \# \vec{\Delta}_A(0) = k$$

$$\# A' - l_A - \# \vec{\Delta}_A(0) = 2$$

	A	A'
B	A ∩ B	B \ A
B'	A \ B	∅

$$l_A + l_B = 2k - (\# A + \# B) - (\# \vec{\Delta}_A(0) + \# \vec{\Delta}_B(0))$$

$$2(\# A' + \# B') - 8 - 2(l_A + l_B)$$

$$\min(\#(A \setminus B) + l_A, \#(B \setminus A) + l_B)$$

$$\leq \min(\# B' + l_A, \# A' + l_B)$$

$$\leq \frac{\# A' + \# B' + l_A + l_B}{2}$$

$$\begin{aligned} \# A' &\geq 3, \# B' \geq 3 \\ n \in \mathbb{Z} \\ \# A &\leq k-1 \\ \# B &\leq k-1 \\ n \in \mathbb{Z} \end{aligned}$$

$$\# A' \times \# B' - 2(\# A' + \# B') + 8 + 2(l_A + l_B)$$

$$= \# A' \times \# B' - \frac{5}{2}(\# A' + \# B') + 8 + \frac{3}{2}(l_A + l_B)$$

$$= \left(\# A' - \frac{5}{2}\right) \left(\# B' - \frac{5}{2}\right) + \frac{9}{4} + \frac{3}{2}(l_A + l_B)$$