Lecture – 7
CS 372 (Computer Graphics)

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2D TRANSFORMATIONS & MATRICES

- Transformation in 2D is basically matrix transformation.
- With transformation we can move a point, change shape, etc.
- By application of basic matrix manipulation techniques.

$$[B] = [T][A]$$

[A] = co-ordinate of points on which we apply transformation
[B] = co-ordinate of transformed points.
[T] = geometric transformation matrix / operator

So, if [A] and [T] are known, transformed points are obtained by calculation of B.

GENERAL TRANSFORMATION OF 2D POINTS

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} ; \quad \begin{aligned} x' &= ax + cy \\ y' &= bx + dy \end{aligned}$$

[T] is the transformation matrix with 4 scalar parameters. [x, y] are the points that are to be transformed and (x', y') are the transformed co-ordinates of (x, y).

So, we are pre-multiplying operator [T] with [A].
We can also do post-multiplication i.e. [B] = [A][T] but we have to keep the solution intact i.e., $x' = ax + cy$; $y' = bx + dy$.

$$\begin{bmatrix} x' \\ y' \end{bmatrix}^T = \begin{bmatrix} x \\ y \end{bmatrix}^T \begin{bmatrix} a & b \\ c & d \end{bmatrix} ; \quad \begin{aligned} x' &= ax + cy \\ y' &= bx + dy \end{aligned}$$

SOLID BODY TRANSFORMATION

Transformation equation is valid for all set of points and lines of the object being transformed.

A solid transformation preserves distances between every pair of points.

SPECIAL CASES OF 2D MATRIX.

1] When $a = d = 1$, $b = c = 0$, so. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

$\therefore$ T = Identity matrix and $x' = x$; $y' = y$.

So, when $[T]$ = Identity, transformation do not change the structure of the solid body.

SCALING

$a = d \neq 0$ & $b = c = 0$, so. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow x' = ax \,;\ y' = dy$

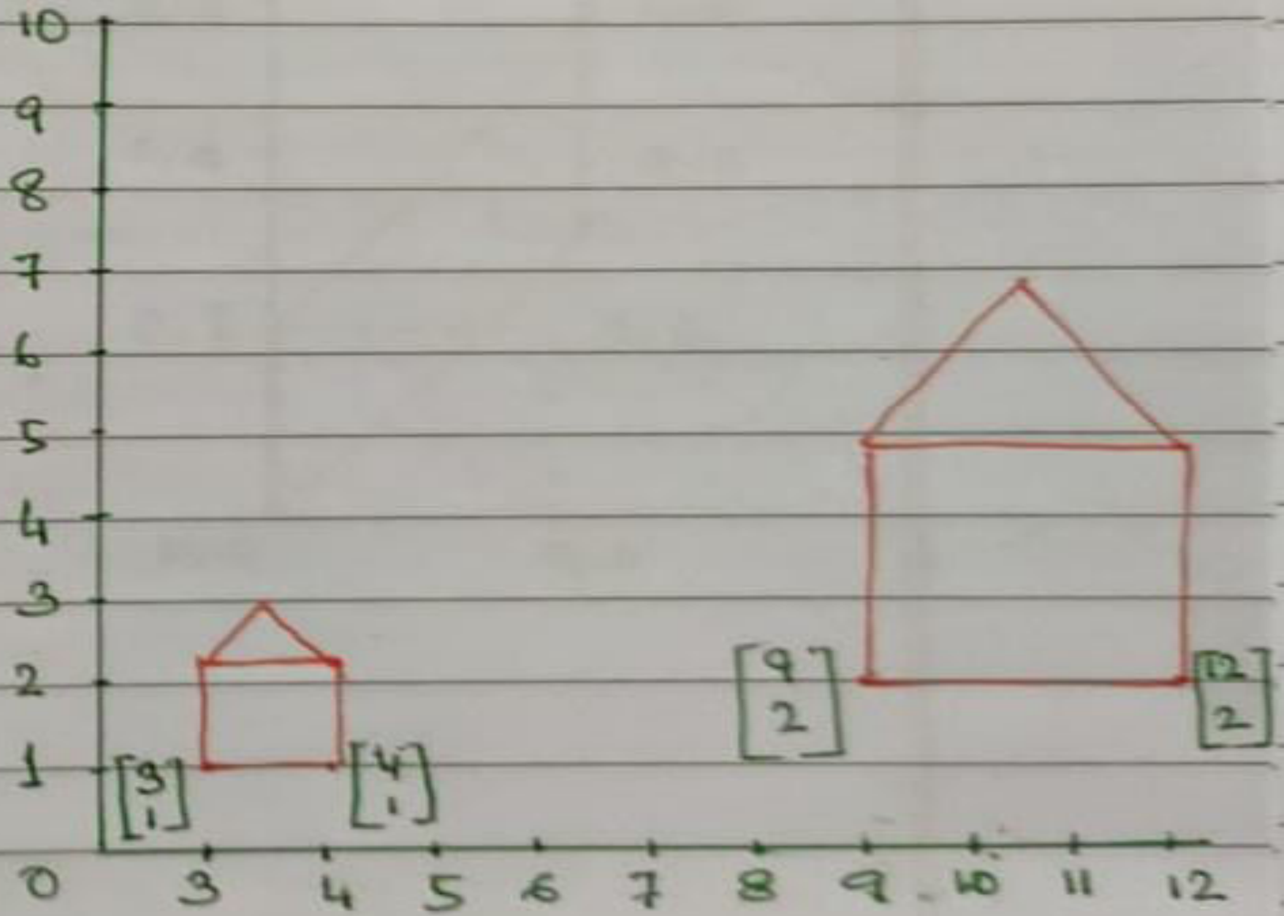
So, x is now scaled by a factor 'a' and y by a factor 'd'.

$a = d > 1$, ENLARGEMENT

$0 < a = d < 1$, COMPRESSION

If a & d are same, we will have UNIFORM SCALING.

If $a \neq d$, NON-UNIFORM SCALING.



EXAMPLE: $a = 3, d = 2$

Non uniform scaling $a \neq d$

Expansion $a, d > 0$

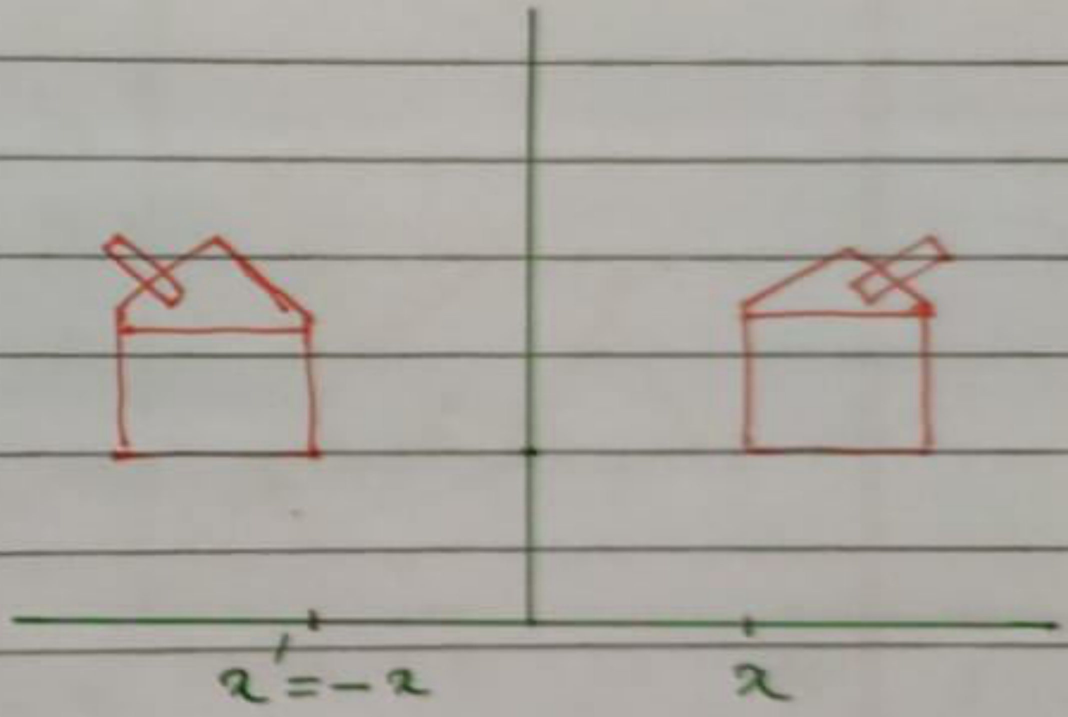
Applicable on all points and lines of the object.

REFLECTION

When a and/or d < 0, reflection along an axis or plane (3D).

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}; \quad x' = -x$$

Reflection around X-axis



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}; \quad y' = -y$$

Reflection around Y-axis.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}; \quad \text{Special Case}$$

Reflection around a plane (3D case).

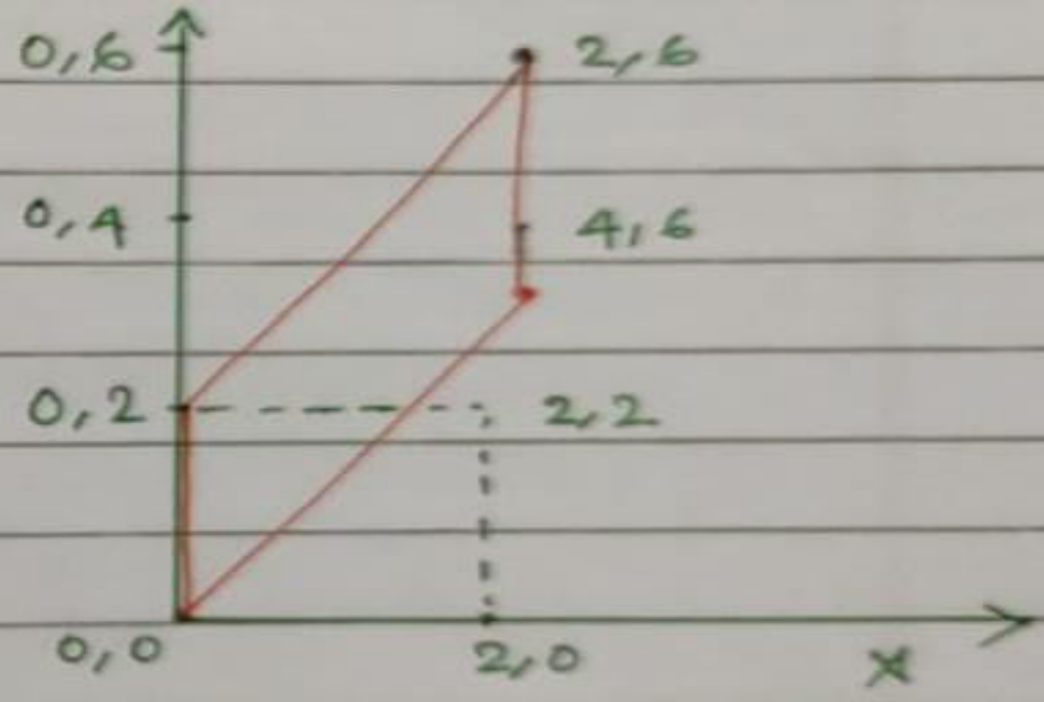
SHEAR

$a = d = 1$. Let $c = 0$, $b = 2$

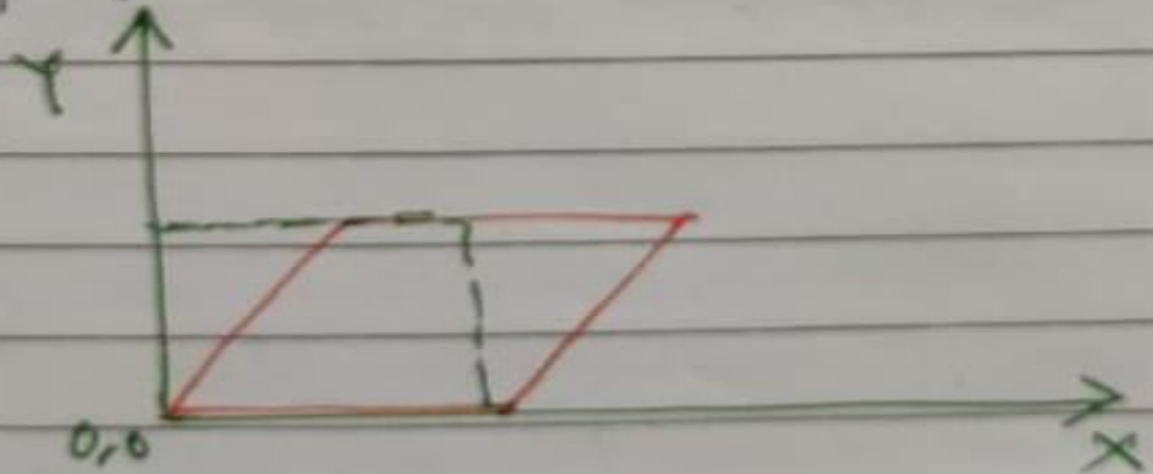
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$x' = x$; $y' = 2x + y$

So, y' depends linearly on x. This effect is called Shear.



If $c = 2$, $b = 0$, shear will be proportional to x-axis



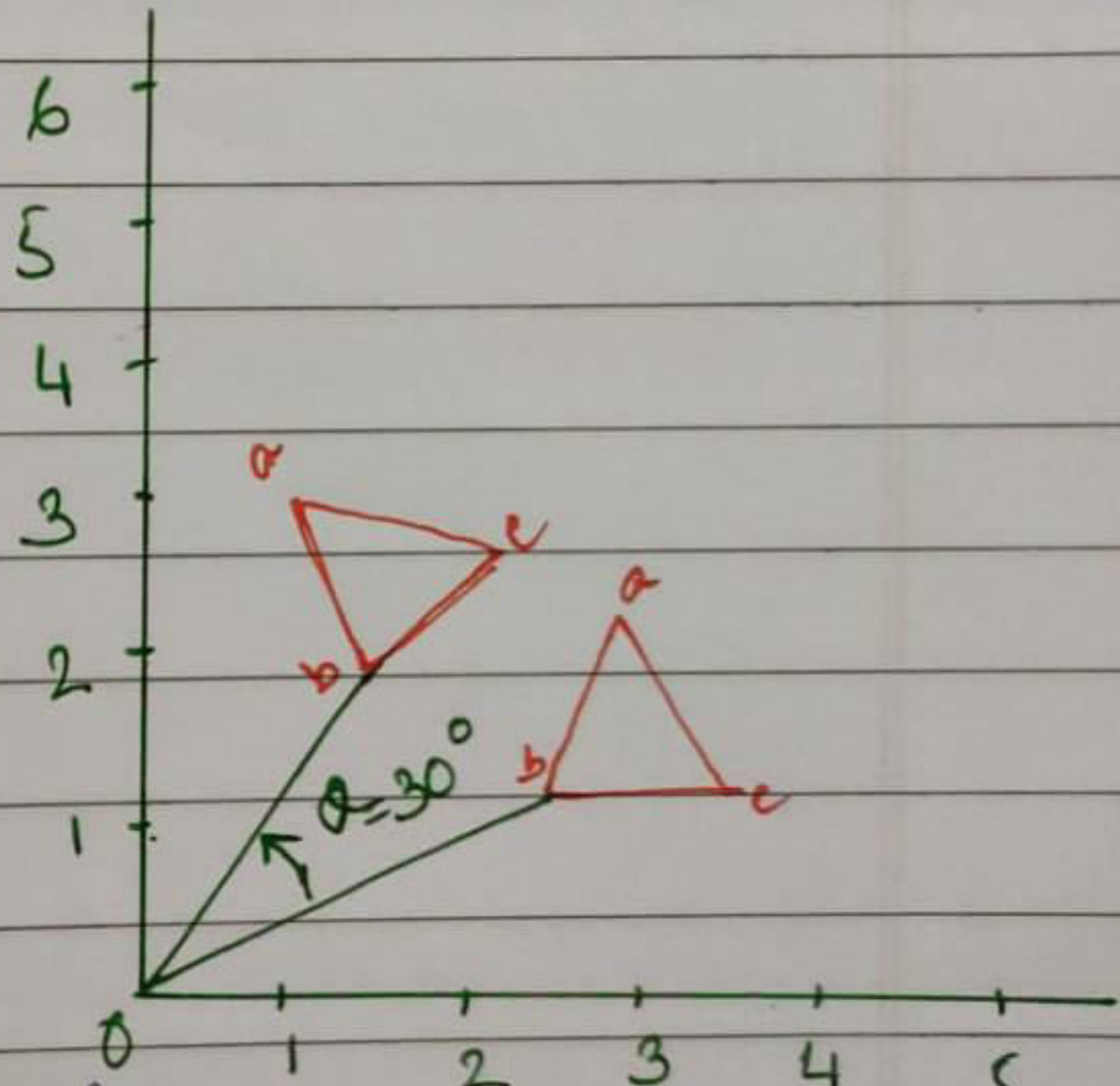
ROTATION

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

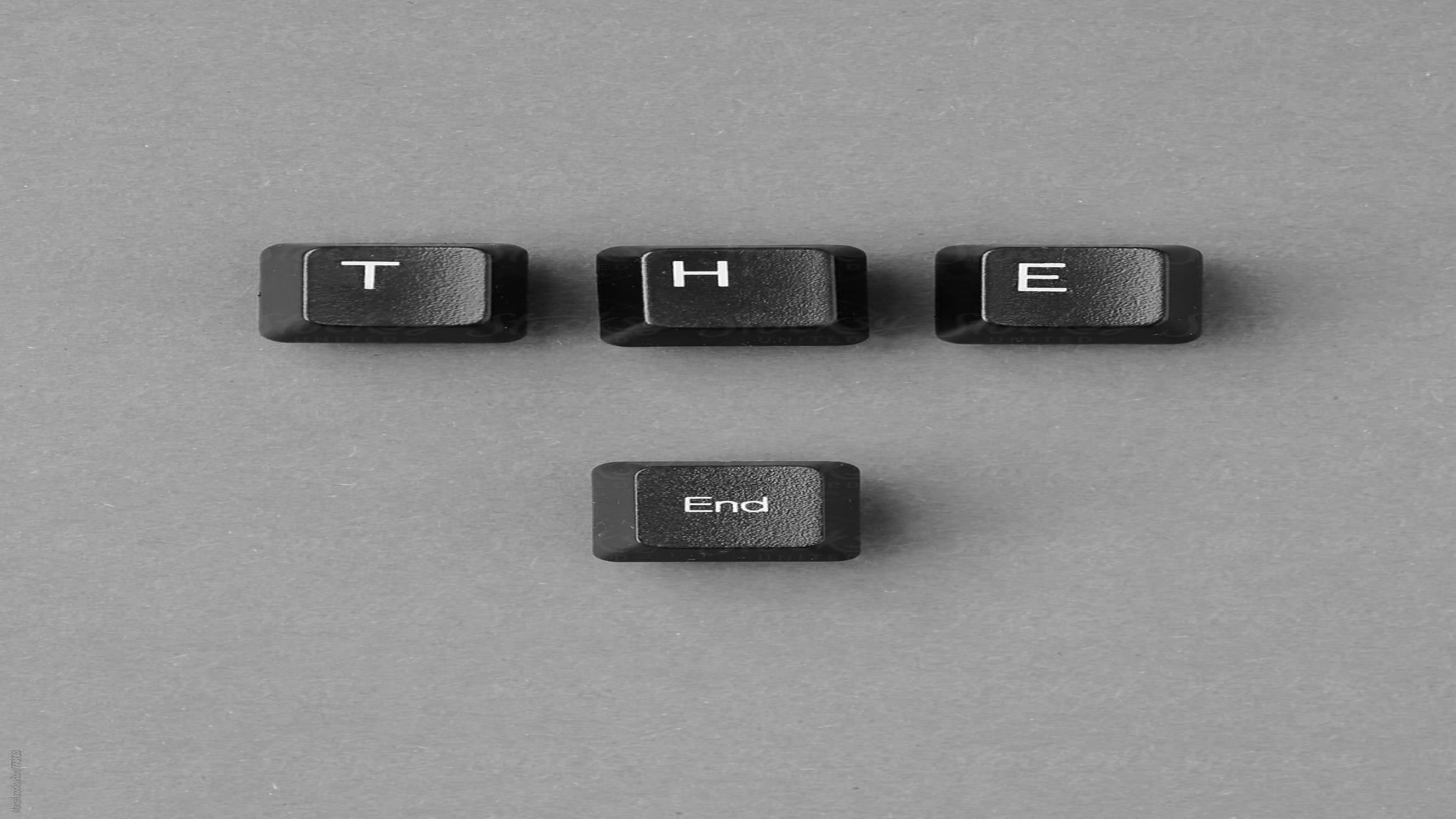
Rotation always around origin.

Counter-clockwise direction is positive

$|T| = 1$ and $[T]^T = [T]^{-1}$



Rotation matrices are orthogonal.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