Lecture – 8
CS 372 (Computer Graphics)

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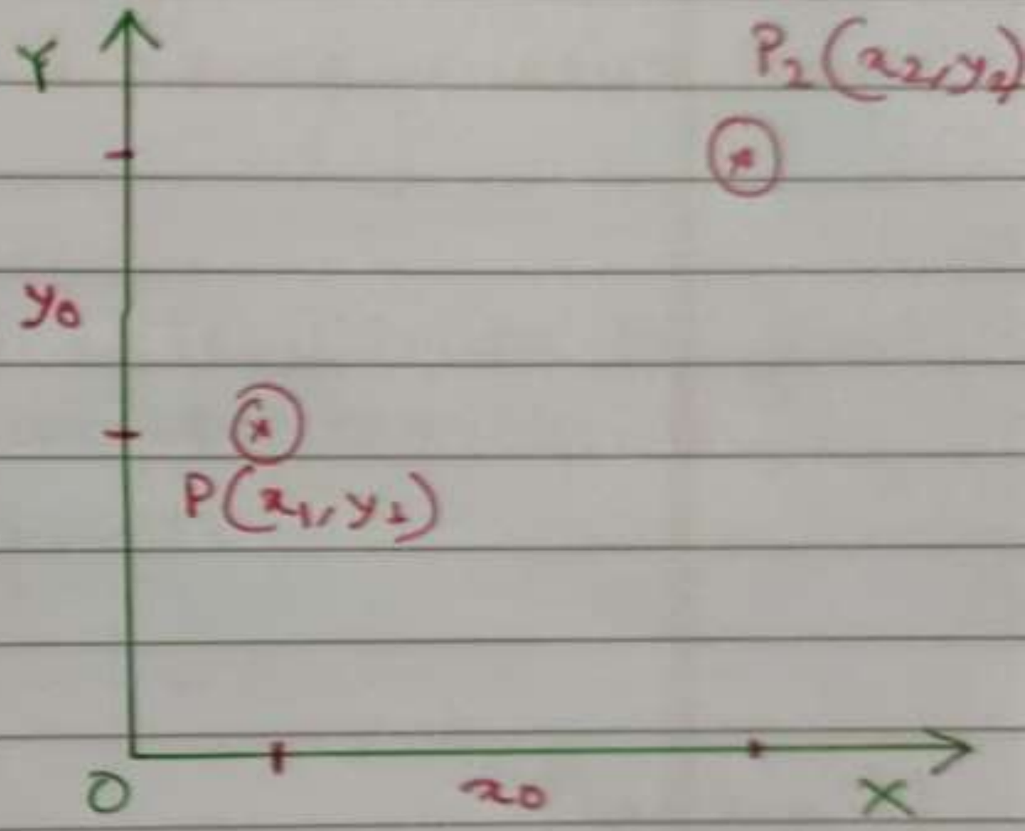
TRANSLATION

Translate a point $P_1(x_1, y_1)$ to another point $P_2(x_2, y_2)$.



Translation by x_0 in X and y_0 in Y i.e, $x_2 = x_0 + x_1$, $y_2 = y_0 + y_1$.

General 2D Matrix eauation

$$[B] = [T][A]$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix}$$

Translations are used in Scaling and Rotation, when objects/lines are not centered around origin.

HOMOGENEOUS COORDINATES

In this co-ordinate system a 2D point (x, y) is represented using a triplet (x, y, w) and the 2D transformation matrix [T] will be a 3×3 matrix.

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} a & c & x_0 \\ b & d & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

So, (x, y, w) are co-ordinates before transformation and after transformation we get (x', y', w').

$$\begin{cases} x' = ax + cy + \underline{x_0 w} \\ y' = bx + dy + \underline{y_0 w} \\ w' = w \end{cases}$$

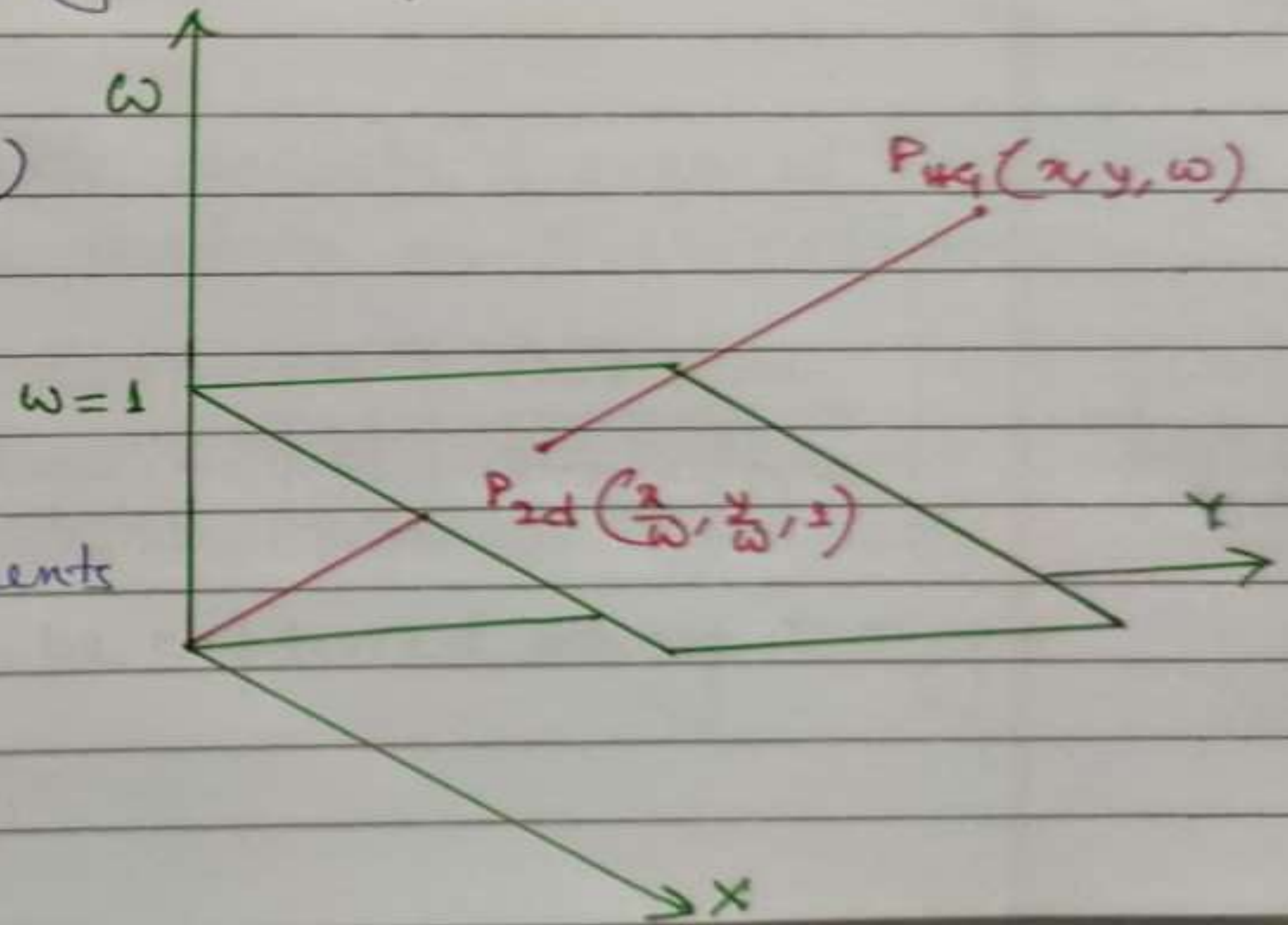
Remember. We are not in 3D space. We are still talking about 2D transformations.

What is the role of w, also called homogeneous term.

HG CONTD...

(x, y, ω) is a point in the HG coordinate. Divide the first 2 elements by ω i.e, $\{x/\omega, y/\omega\}$ gives the cartesian co-ordinates for the homogeneous points.

So (x, y, ω) and (x', y', ω') are the Homogeneous representation. To get the Cartesian co-ordinates back, we divide the first 2 elements by the third.



PROPERTY

1. 2 Homogeneous co-ordinates (x_1, y_1, w_1) and (x_2, y_2, w_2) may represent the same point iff they are multiples of one another. Ex: $(1, 2, 3)$ & $(3, 6, 9)$.

2. Hence, there is no unique homogeneous representation of a point.

3. All triplets of the form $\{tx, ty, tw\}$ form a line in the x, y, w space.

4. Cartesian co-ordinates are just the plane $w=1$ in this space.

5. $w=0$? Points at infinity. $(x, y, 0)$ is the "Ideal Point".

TRANSLATION.

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad \omega = 1.$$

Homogeneous Co-ordinate concept is created to capture translation as matrix multiplication.

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/s \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1/s \end{bmatrix} \xrightarrow[\text{Transform}]{\text{Cartesian}} \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} s.x \\ s.y \\ 1 \end{bmatrix}$$

Uniform scaling can be captured using a single parameter.

2D TRANSFORMATION MATRIX

$$T = \begin{bmatrix} a & c & p \\ b & d & q \\ m & n & s \end{bmatrix}$$

Parameters involved in scaling, rotation, reflection & shear: a, c, b, d.

If $B = TA$, translation parameters: p, q

If $B = AT$, translation parameters: m, n

s: Special case for uniform scaling.

If $B = TA$, what is the role of m, n? Perspective transform

ADVANTAGES OF HG COORDINATE

1. Translation can be captured in the usual matrix multiplication framework.

2. Uniform scaling can be captured by a single parameter

3. Transformations can be combined.

4. Points at infinity can be captured.

5. Perspective transforms can be captured.

COMPOSITE TRANSFORMATIONS

If we want to apply a series of transformations T_1, T_2, T_3 to a point 'p', we can do it in 2 ways.

1. $p' = T_1 * p \longrightarrow p'' = T_2 * p' \longrightarrow p''' = T_3 * p''$

2. Calculate $T = T_3 * T_2 * T_1$ and then $p''' = T * p$.

Method 2 saves large no. of computational time.

Note: $T_3 * T_2 * T_1$. As per convention, since T_1 is applied first it has to be the right most transformation, and then T_2 and so on.

SOME EXAMPLES

Translate by tx_1, ty_1 and then by tx_2, ty_2

$$\begin{bmatrix} 1 & 0 & tx_1 + tx_2 \\ 0 & 1 & ty_1 + ty_2 \\ 0 & 0 & 1 \end{bmatrix}$$

Scale by a_1, b_1 and then by a_2, b_2

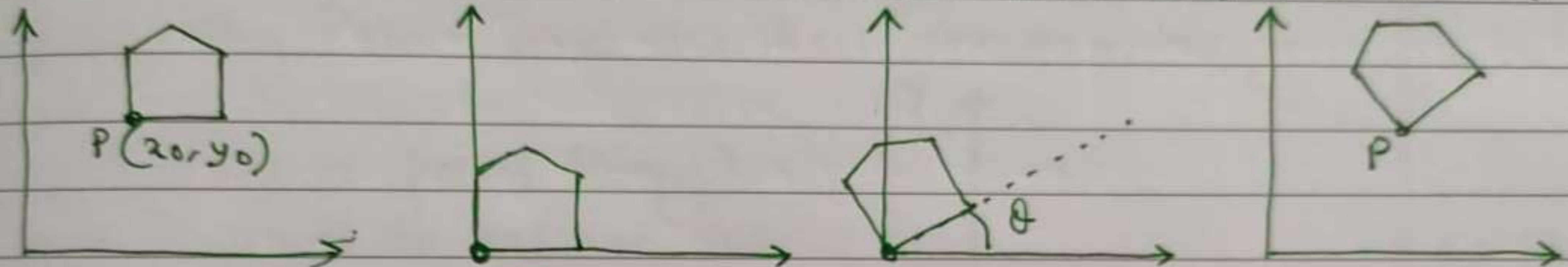
$$\begin{bmatrix} a_1 + a_2 & 0 & 0 \\ 0 & b_1 + b_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotate by θ_1 and then by θ_2.

- Replace θ by $(\theta_1 + \theta_2)$
- Calculate T_1 for θ_1 and T_2 for θ_2 and multiply them.

$$\begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ROTATION ABOUT AN ARBITRARY POINT P



House at $P(x_0, y_0)$ | Translate P to origin T_1 | Rotation by θ T_2 | Translate back to P T_3

What will be the transformation matrix? T

$$T = \underbrace{\begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix}}_{T_3} * \underbrace{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{T_2} * \underbrace{\begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix}}_{T_1}$$

SCALING ABOUT AN ARBITRARY POINT

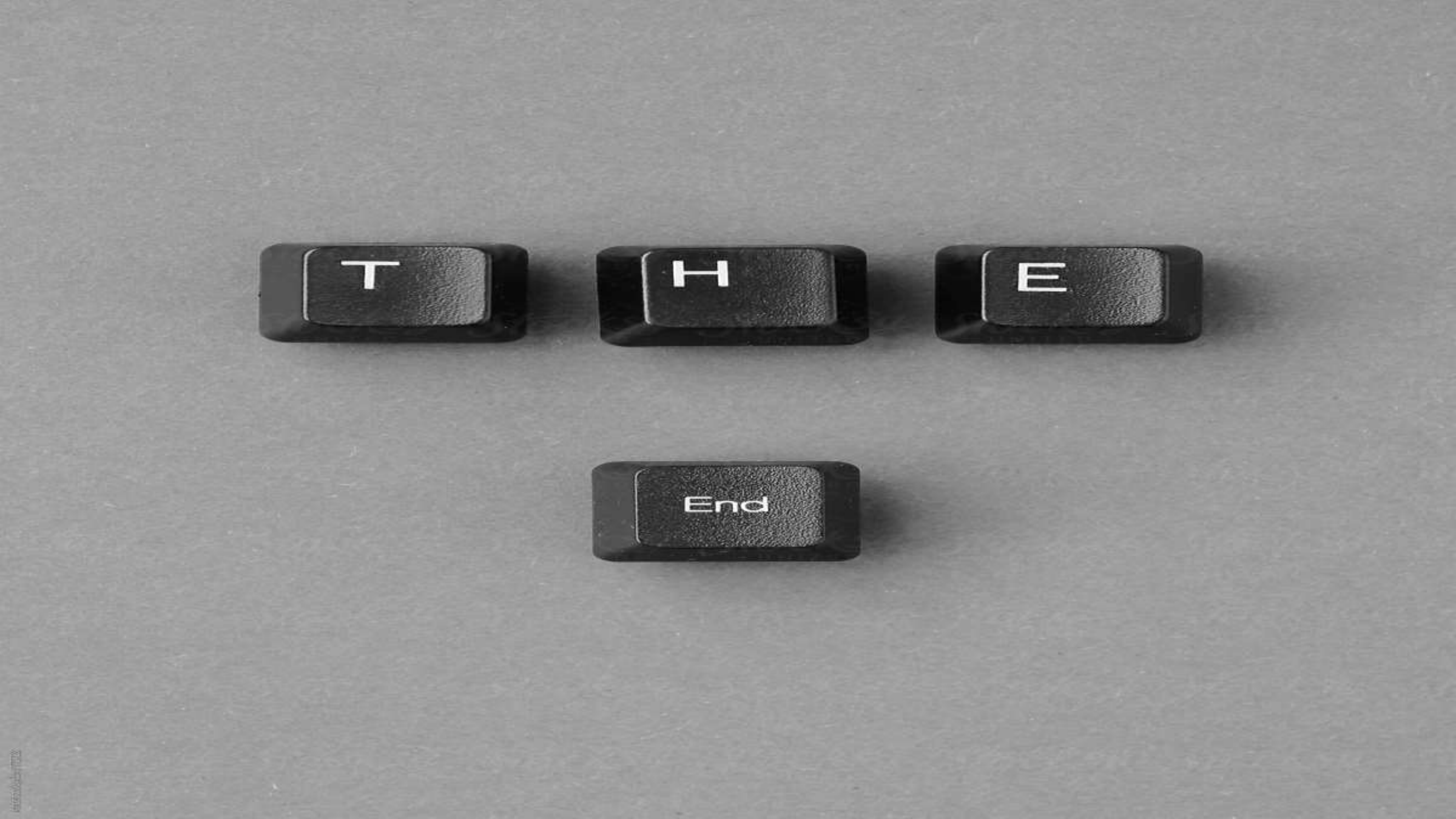
T_1 : Translate P to origin

T_2 : Scale

T_3 : Translate P back.

$$T = \underbrace{\begin{bmatrix} 1 & 0 & x_0 \\ 0 & 1 & y_0 \\ 0 & 0 & 1 \end{bmatrix}}_{T_3} * \underbrace{\begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{T_2} * \underbrace{\begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ 0 & 0 & 1 \end{bmatrix}}_{T_1}$$

$$T = T_3(x_0, y_0) * T_2(S_x, S_y) * T_1(-x_0, -y_0).$$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