Lecture – 11
CS 372 (Computer Graphics)

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PROJECTION

Projection of a 3D object can be defined by straight projection rays (projectors) emanating from the object, passing / intersecting through the projection plane and meeting / converging to the center of projection (COP).

Broadly classified into 2 types

1. Parallel projection — a. Orthographic
   b. Oblique

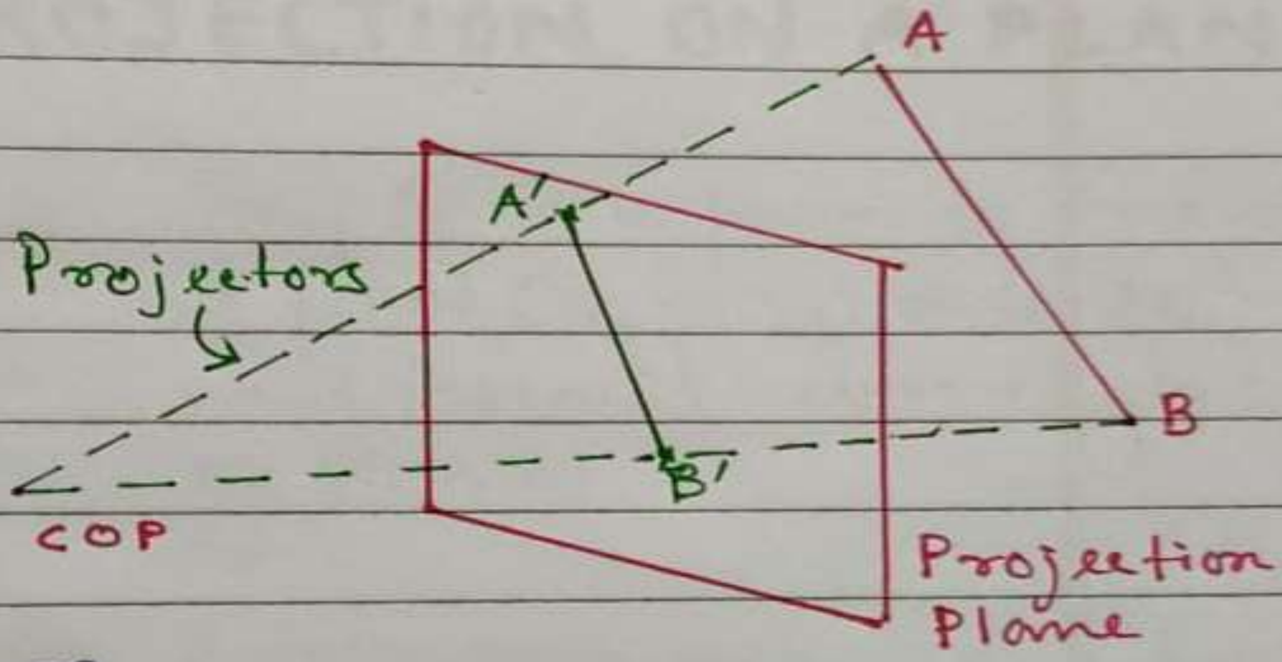
2. Perspective projection.

COP is a finite point in 3D space.

PERSPECTIVE PROJECTION

1. Distance between the COP and projection plane is finite.

2. Projectors are not parallel.

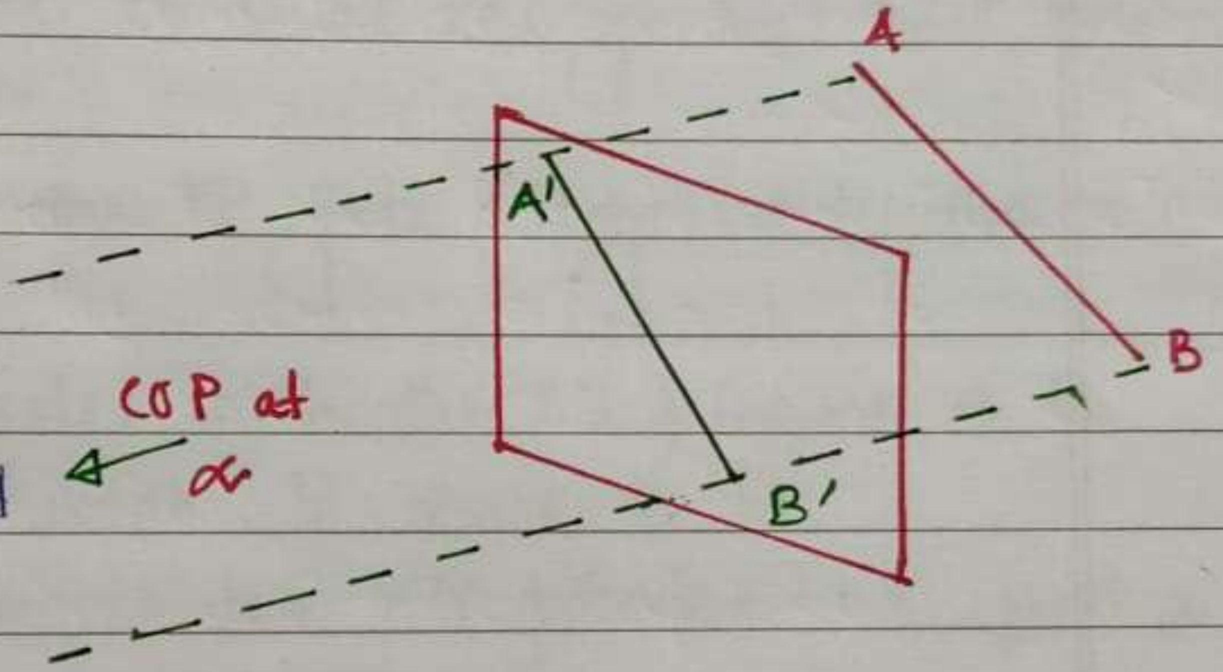


3. Perspective foreshortening
- Closer is the object to the COP, larger is its projection. Further the object from the COP, smaller is its projection

4. Vanishing points - Two parallel railway tracks appear to meet at a point on the horizon. This point is called the VP.

PARALLEL PROJECTION

Distance between COP and projection plane (PP) is infinite.

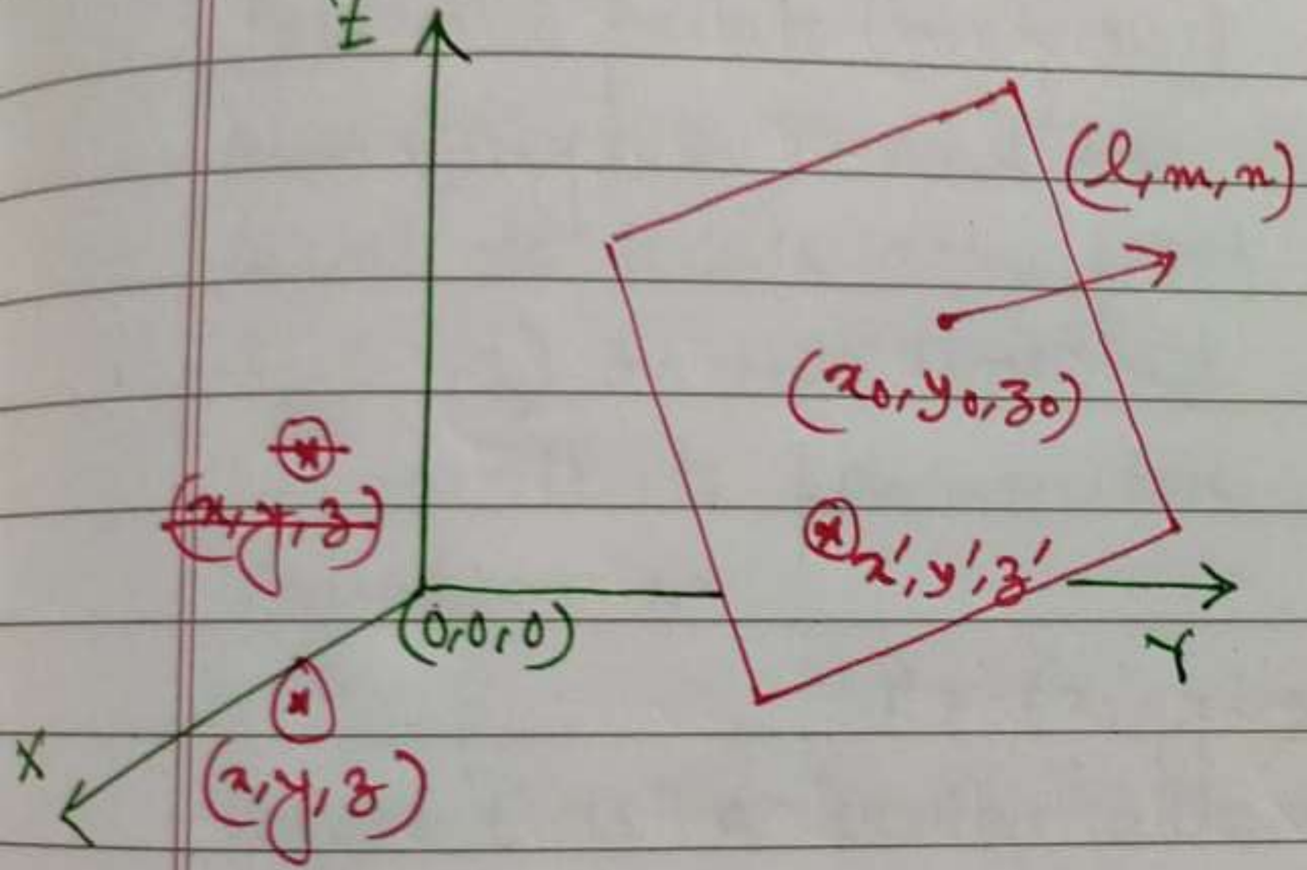


Co-ordinate position of the object are transferred to the PP along parallel lines. So, u convey true size of the object

Oblique projections : Projectors are at an angle to the PP.

Orthographic projections : Projectors are perpendicular to PP.

ORTHOGRAPHIC PROJECTION ON A PLANE



The plane is passing through a point (x_0, y_0, z_0) and its unit normal vector is given by (l, m, n).

I take any arbitrary point (x, y, z) and I project it onto the plane orthographi-cally.
What will be the steps involved?

SOLUTION

~~Let the~~ STEP1: Translate so that (x_0, y_0, z_0) becomes the new origin $\equiv T_1(x_0, y_0, z_0)$.

STEP2: Rotate such that the PP gets aligned to the XY plane.

Rotate s.t. l, m, n coincides with the XZ plane. $\equiv T_2$

Rotate s.t. it coincides with Z-axis. $\equiv T_3$

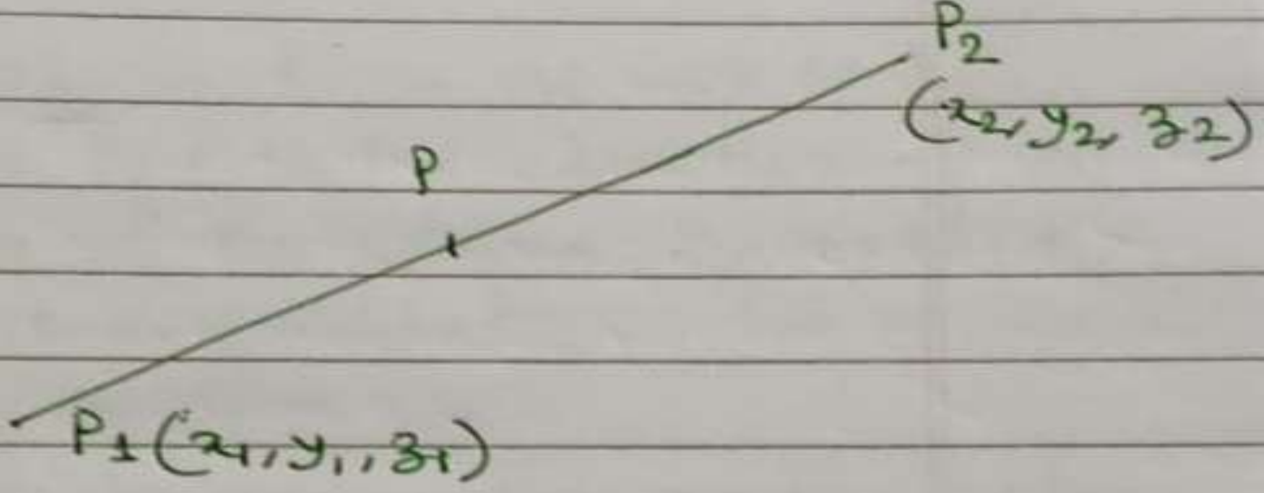
STEP3: Project (x, y, z) on to the XY plane i.e, put $z = 0$

$$T_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

STEP4: Reverse STEP2 & STEP1.

PARAMETRIC EQUATION OF A LINE

Given 2 points (x_1, y_1, z_1) and (x_2, y_2, z_2) and we want to locate any point $P(x, y, z)$ on this line, we can write its parametric equation as.



$$P = (x_1, y_1, z_1) + (P_2 - P_1)t = P_1 + (P_2 - P_1)t.$$

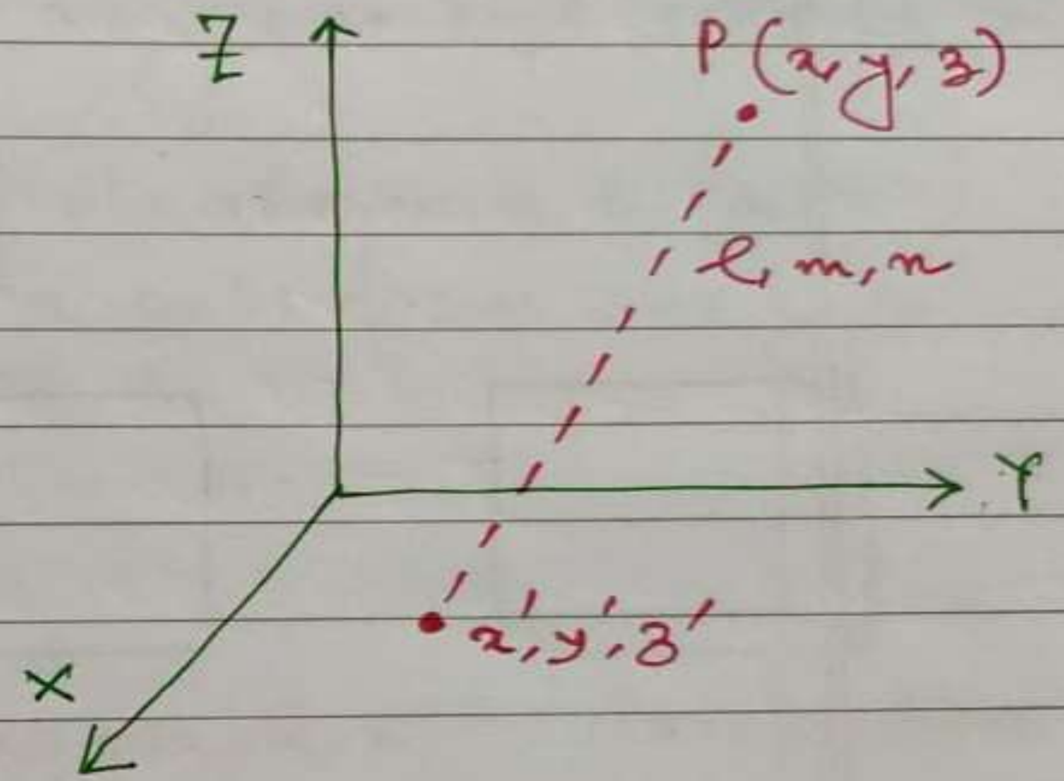
where 't' is a scalar quantity.

Note: Since P_1P_2 and P_1P are parallel, P_1P is simply a scalar multiple of P_1P_2.

Direction cosines are the cosines of the angle between the vector and each of the axes.

GENERAL PARALLEL PROJECTION

Given any arbitrary point $P(x, y, z)$ we want to project it on the XY plane.



My projecting direction is given by (l, m, n). So if I project in the direction of (l, m, n) I will get some point in the XY plane, say (x', y', z'). What will be the co-ordinates of x', y', z'?

We are talking about general parallel projection, not necessarily orthographic.

SOLUTION

We know the initial position of the line as well as the direction cosines. So, we can write the equation of the line & we know the equation of the plane. Intersection of the two will give us the co-ordinates.

Any point 'P' on this line will be given by $x+tl, y+tm,$ $z+tn$.

For intersection with XY plane $z+tn=0 \Rightarrow t=-z/n$

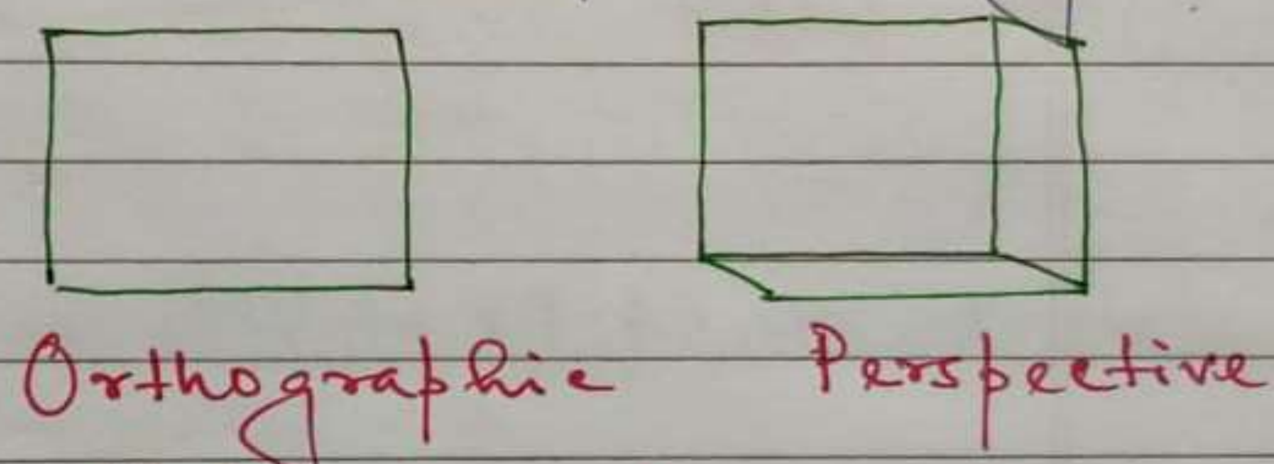
$x'=x-lz/n, \; y'=y-mz/n, \; z'=0.$

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & -l/n & 0 \\ 0 & 1 & -m/n & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

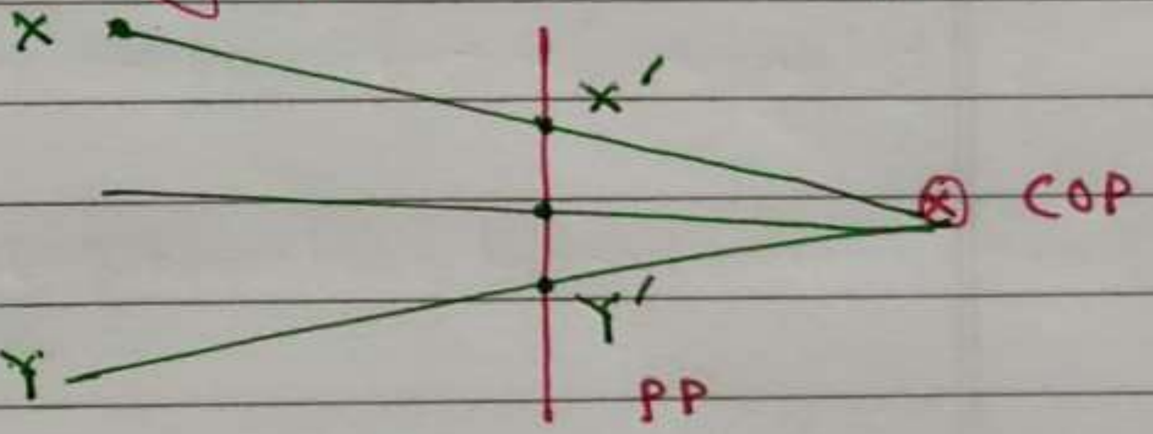
PERSPECTIVE PROJECTION

We try to draw an object the way our eye sees it.

For ex. the orthographic view of a cube will be a rectangle. But when we view a cube from our eyes we see the sides also.



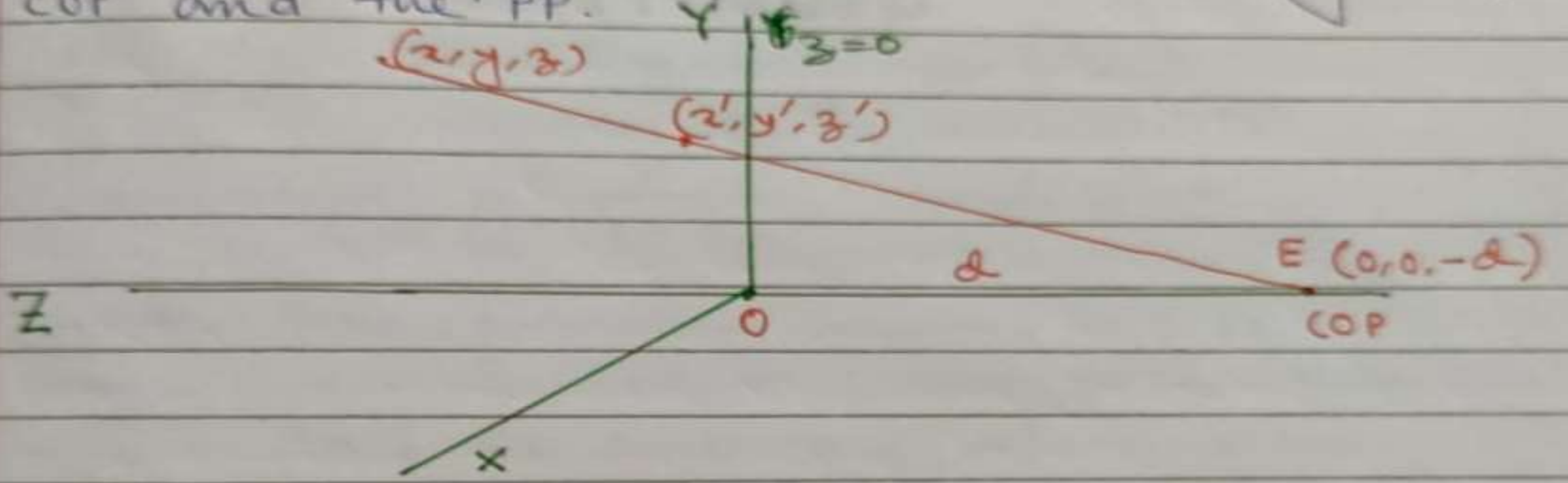
Because projectors are not parallel, they are going to be focussed on the eye (COP).



Hence a point X will get transformed to X' on the PP.

CONTINUED

Hence in perspective projection, we always define the COP and the PP.



XY plane is the projecting plane, so here $z=0$.
COP is located at $(0,0,-d)$ where 'd' is the distance between origin and COP.
I want to project any arbitrary point (x,y,z) on the XY plane.
What will be the co-ordinates of the transformed points?

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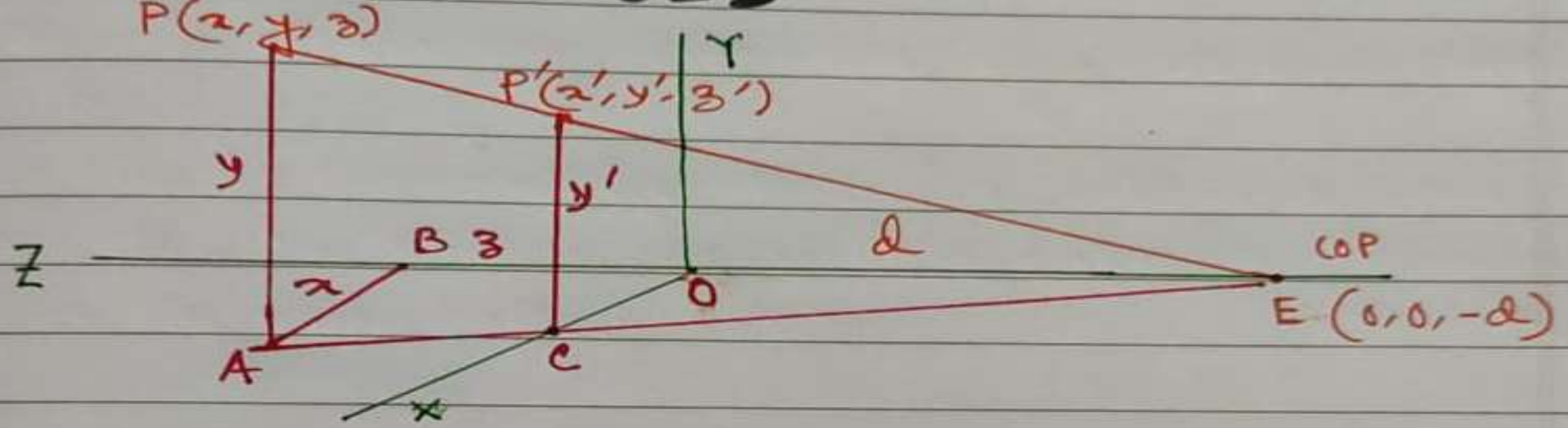


From 'P' I draw a perpendicular to XZ-plane, distance 'y'.
AB is parallel to X-axis, distance = x.
BO = z.

SIMILAR TRIANGLES.

ECP' & EAP : $\frac{EC}{EA} = \frac{CP'}{AP} = \frac{y'}{y}$

ECO & EAB : $\frac{EC}{EA} = \frac{CO}{AB} = \frac{EO}{EB}$ $\quad \frac{CO}{AB} = \frac{x'}{x}$; $\frac{EO}{EB} = \frac{d}{EO + OB} = \frac{d}{d+z}$

CONTINUED

$$\frac{x'}{x} = \frac{d}{d+z} \Rightarrow x' = x \cdot \frac{d}{z+d} \Rightarrow x' = \frac{x}{1+z/d}$$

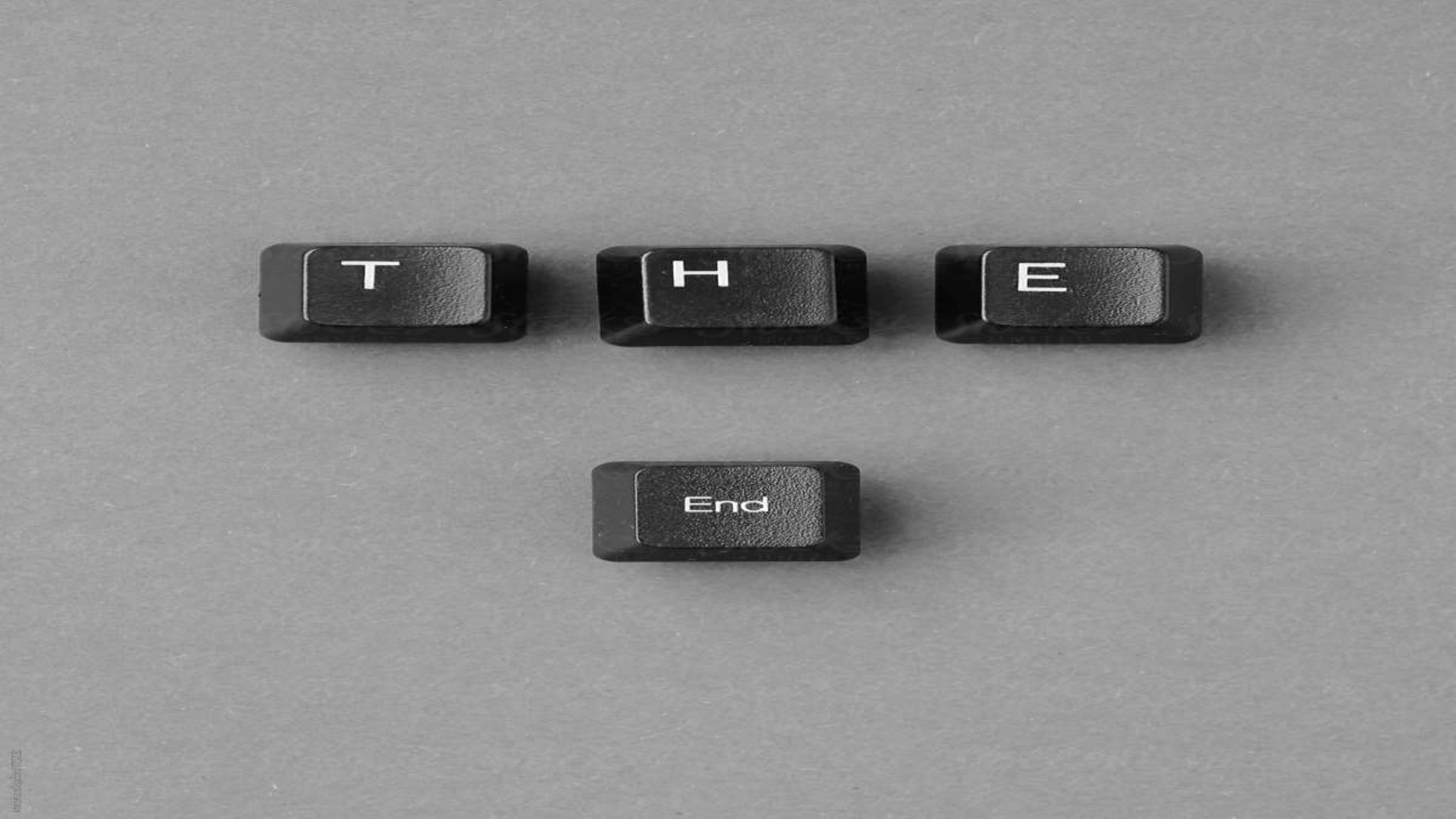
Similarly $y' = \dfrac{y}{1+z/d}$ and $z' = 0$.

So, the transformation matrix will be

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/d & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \qquad \begin{aligned} x' &= x \\ y' &= y \\ z' &= 0 \\ w &= 1+z/d \end{aligned}$$

$$\begin{array}{cc} \text{COP} & \text{COP} \\ -X & -Y \end{array}$$

We set the 'w' cordinate $= 1+z/d$, to convert to cartesian you will get the desired values.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