Lecture – 12
CS 372 (Computer Graphics)

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PERSPECTIVE PROJECTION ABOUT A PLANE

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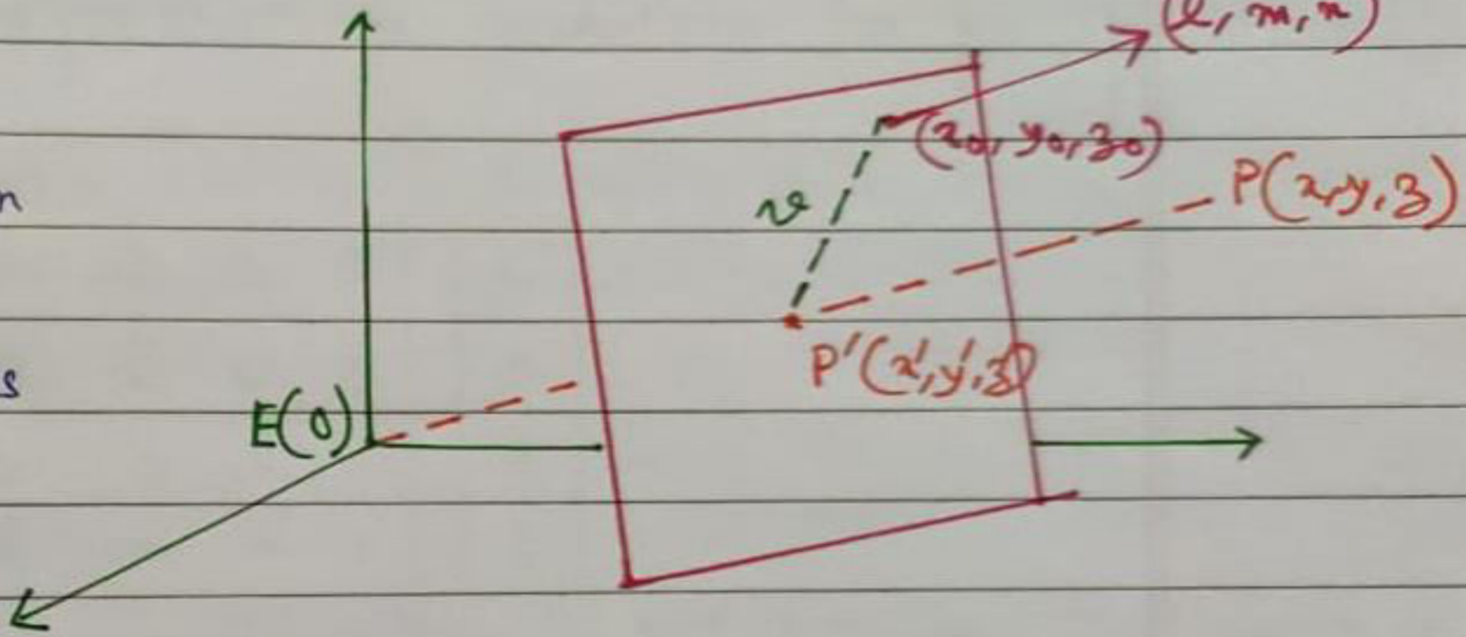
- Eye (COP) is at origin.
- Plane passes through (x_0, y_0, z_0) in the direction of (l, m, n) vectors.
- What is the co-ordinates (x', y', z') for the point $P(x, y, z)$?



Let v be a vector joining P' and (x_0, y_0, z_0). Then the dot product of v and (l, m, n) is zero, as they are $\perp^r$ to each other.

$$v \cdot (l, m, n) = 0$$

$$(x' - x_0, y' - y_0, z' - z_0) \cdot (l, m, n) = 0$$

$$l(x' - x_0) + m(y' - y_0) + n(z' - z_0) = 0 \quad \text{—— ①}$$

CONTINUED

14 Again, the line joining P to the COP is a straight line. Any intermediate point (x', y', z') can be written as

$$x' = t.x \; ; \; y' = t.y \text{ and } z' = t.z$$

Substituting the values in (1), gives us

$$l(tx - x_0) + m(ty - y_0) + n(tz - z_0) = 0$$

$$\Rightarrow t(lx + my + nz) - lx_0 - my_0 - nz_0 = 0$$

$$\Rightarrow t = \frac{lx_0 + my_0 + nz_0}{lx + my + nz} = \frac{d}{lx + my + nz}$$

So, $x' = \dfrac{dx}{lx + my + nz}$; $y' = \dfrac{dy}{lx + my + nz}$ & $z' = \dfrac{dz}{lx + my + nz}$

CONTINUED

15 So, the general transformation matrix for perspective projection will be

$$\begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix} = \begin{bmatrix} d & 0 & 0 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & d & 0 \\ l & m & n & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Recall the general transformation matrix for MT.

NOTE: When COP is at any arbitrary point (a, b, c).

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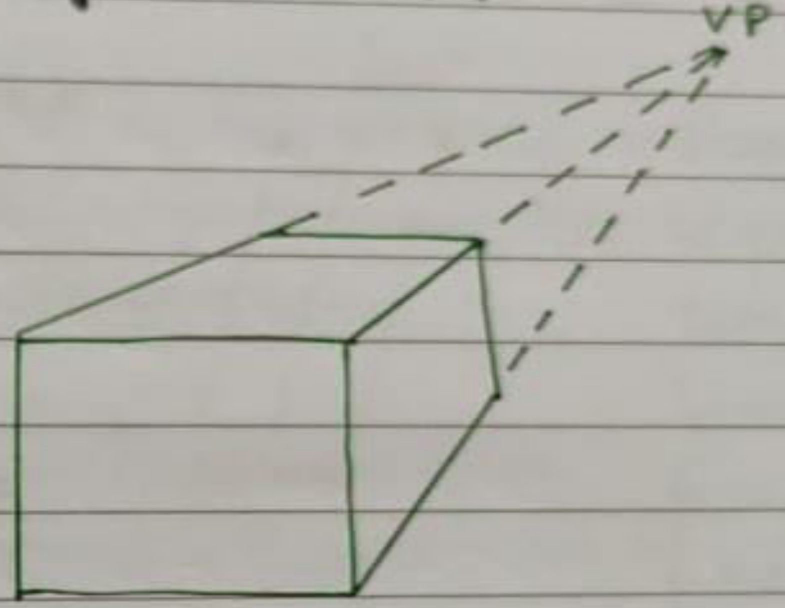
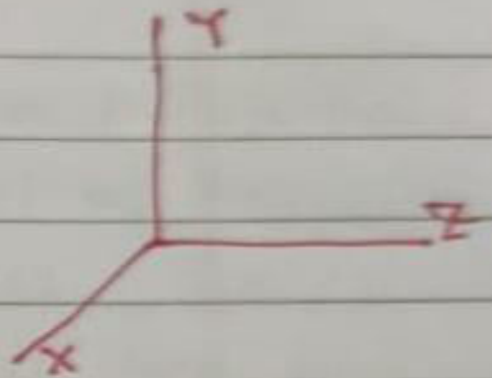
VANISHING POINTS

Projection Plane parallel to YZ.

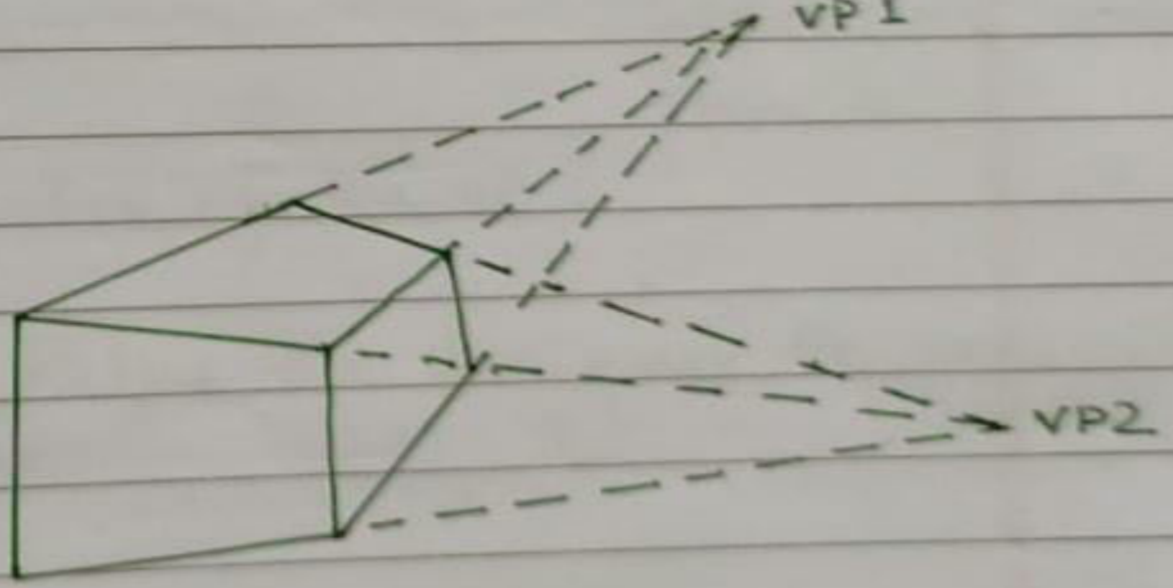
- All lines in Y & Z will remain parallel.
- But 2 parallel lines in X will appear to come from a point.





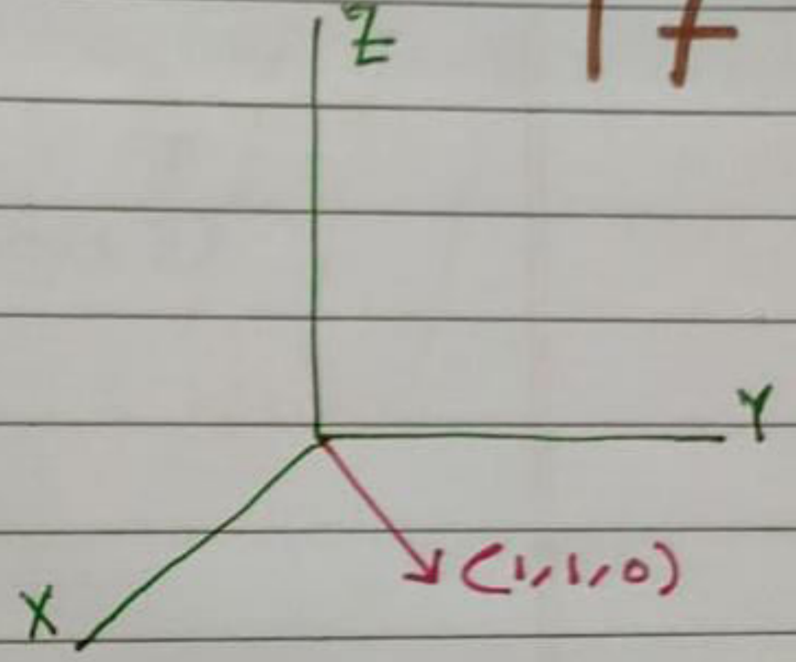
Slightly at an angle may lead to 2 VP.

- Lines parallel to X and Z appear to come from a finite point.



VP CONTINUED

Vanishing points corresponding (parallel) to the principal axis X, Y, Z are called the principal VP or PVP.



We can also have VP let's say in the direction of the vector (1, 1, 0). This will be a VP but not a PVP

A VP is a point from where a set of parallel lines seem to emerge in a perspective transformation.

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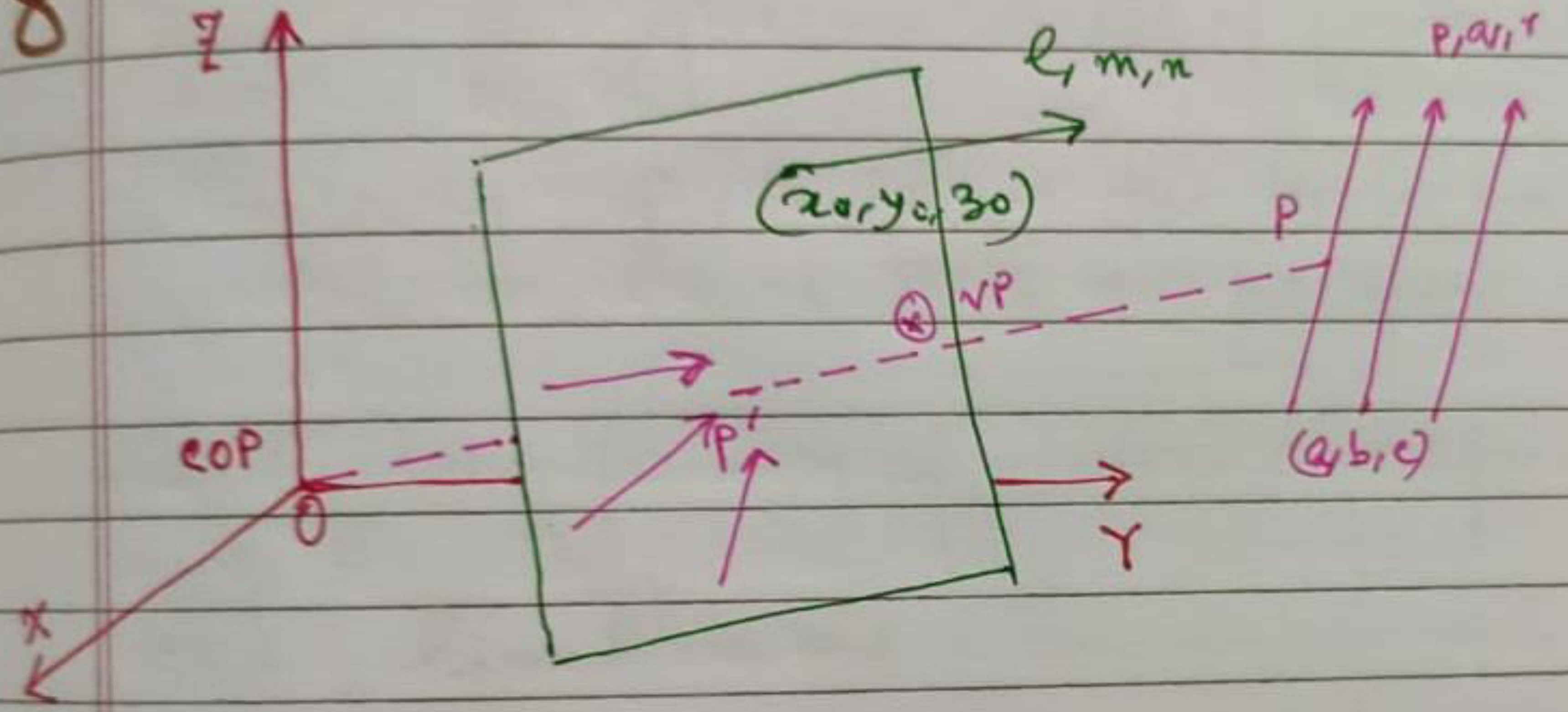
VP FOR GENERAL PERSPECTIVE PROJECTION



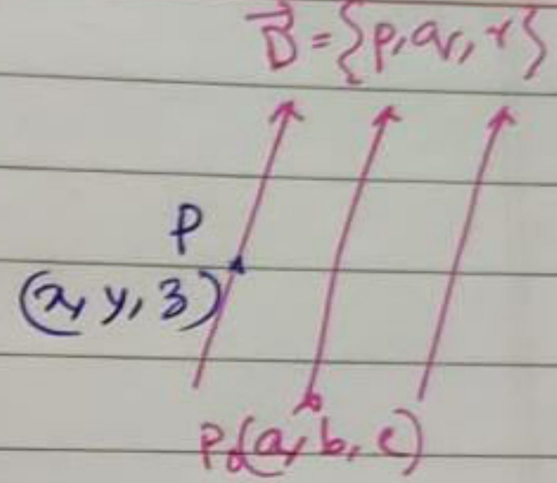
From P, we have a set of parallel lines. These lines will get projected on the plane and will appear to come from a VP.

What will be the co-ordinates of the VP?

Information of the VP helps the designer to recreate the view in a 2D plane.

SOLUTION

19 The equation of a line passing through (a, b, c) in the direction of p, q, r will be:



$$\left.\begin{aligned} x &= pt + a \\ y &= qt + b \\ z &= rt + c. \end{aligned}\right\} \quad \vec{P} = \vec{D}t + \vec{P_0}$$

This set of equation, for different a, b, c will give me a family of straight lines parallel to the direction of p, q, r.

And (x, y, z) is any point lying on these lines.

For a point to be at infinity on these lines, $\lim\limits_{t \to \infty} \vec{P}$

SOLUTION CONTINUED

20 $\lim_{t \to \alpha} \vec{P}$ will give me a set of points at infinity on these lines

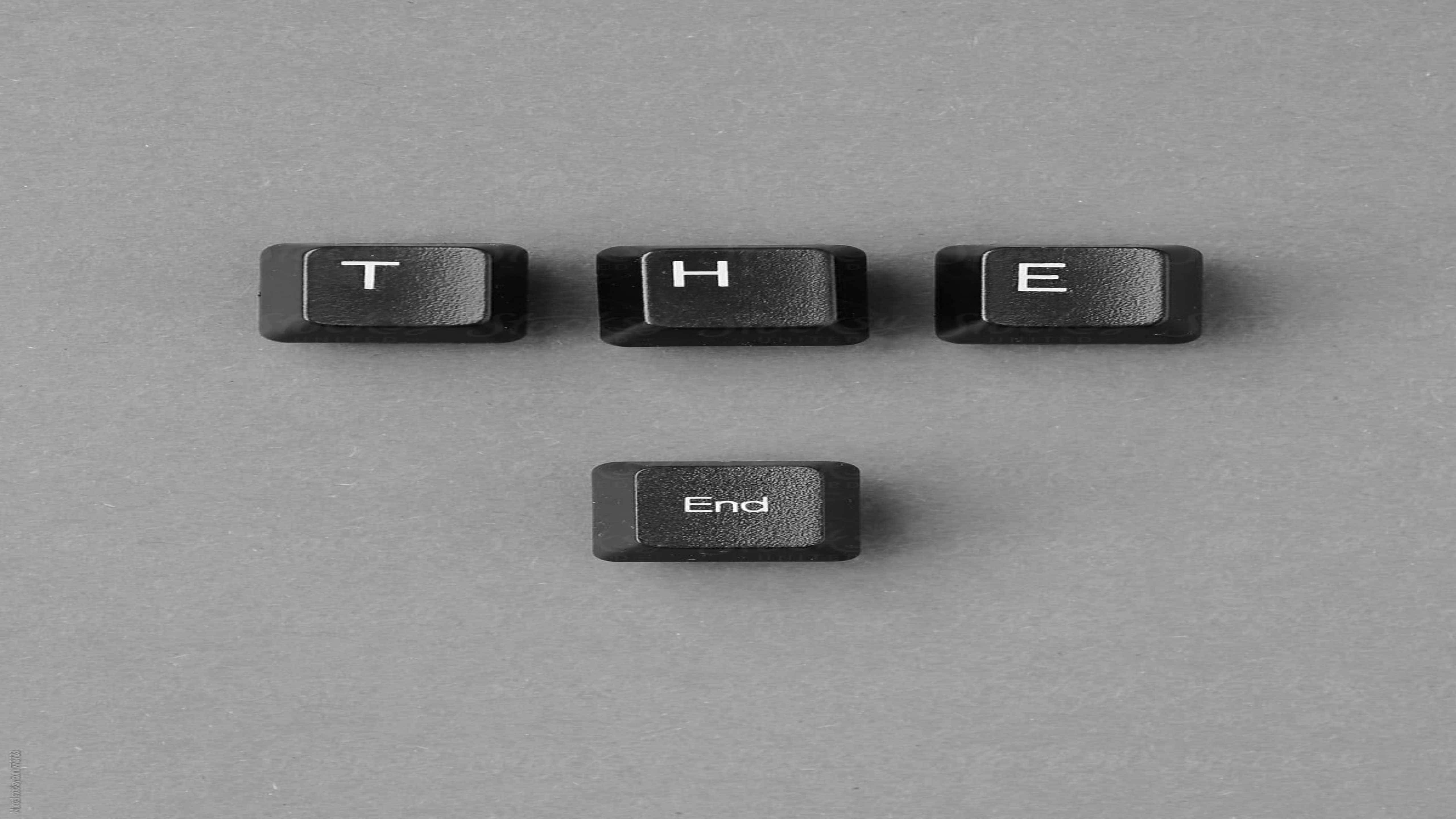
Similarly. $\vec{P}' = T\vec{P}$ is the transformed point for $\vec{P}$ on the plane, where T is the perspective transform matrix.

So, $\vec{P}'$ is the family of vectors corresponding to the parallel lines $\vec{P}$.

And. $\lim_{t \to \alpha} \vec{P}' = $ V.P

$$\lim_{t \to \alpha} \vec{P}' = \lim_{t \to \alpha} \vec{P} T = \lim_{t \to \alpha} (\vec{D}t + \vec{P}_0) T$$

For PVP, $\vec{D}$ in X will be $(1, 0, 0)$, in Y will be $(0, 1, 0)$ and in Z will be $(0, 0, 1)$.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