2. Cálculo diferencial — Aulas TP+P: Folha 2

Nota: A numeração indicada é a do formulário de regras de derivação da Unidade Curricular.

Comandos **Geogebra:**

- Para calcular a derivada de $y = f(x)$: `derivada( <expressão de f(x)> )`

Reproduza a regra indicada no cálculo da derivada de cada uma das seguintes funções:

D7: $(f^p)' = p\, f^{p-1}\, f'$ D3: $(k\, f)' = k\, f'$

a) $f(x) = x^9$; b) $f(x) = \frac{1}{2}x^9$; c) $f(x) = \frac{\sqrt[4]{x^5}}{3}$; d) $f(x) = \frac{1}{(3x)^2}$;

D4: $(f+g)' = f' + g'$

a) $f(x) = x^3 + \frac{3x}{5} - \frac{1}{x^2}$;

D5: $(f \times g)' = f' \times g + f \times g'$

a) $f(x) = \left(x^2 - 2\right)\left(3 - \frac{1}{x^2}\right)$;

D6: $\left(\frac{f}{g}\right)' = \frac{f' \times g - f \times g'}{g^2}$

a) $f(x) = \frac{3x - 2x^2}{x - 1}$;

D8: $\left(a^f\right)' = f' a^f\, \ln(a)$

a) $f(x) = \frac{1}{e^{2x}}$; b) $f(x) = e^{\sqrt{x}}$; c) $f(x) = e^{\frac{1}{x}}$;

D9: $\left(\log_a(f)\right)' = \frac{f'}{f\ln(a)}$

a) $f(x) = \ln(2x)$; b) $f(x) = \ln(x^2+2)$; c) $f(x) = \frac{\ln^3(x^2+1)}{5}$;

D10: $\left(\sin(f)\right)' = f'\cos(f)$ D11: $\left(\cos(f)\right)' = -f'\sin(f)$

a) $f(x) = \sin(2x^2)$; b) $f(x) = \cos\left(\ln(x)\right)$; c) $f(x) = \ln\left(\cos(2x)\right)$;

D12: $\left(\operatorname{tg}(f)\right)' = f'\sec^2(f)$ D13: $\left(\operatorname{cotg}(f)\right)' = -f'\operatorname{cosec}^2(f)$

a) $f(x) = \operatorname{tg}(x^3)$; b) $f(x) = 2\operatorname{tg}(e^x)$; c) $f(x) = \operatorname{tg}\left(\ln(x^2+1)\right)$;

D14: $\left(\sec(f)\right)' = f'\sec(f)\operatorname{tg}(f)$ D15: $\left(\operatorname{cosec}(f)\right)' = -f'\operatorname{cosec}(f)\operatorname{cotg}(f)$

a) $f(x) = \operatorname{cosec}(2x^2)$; b) $f(x) = \ln\left(\sec(2x)\right)$;

D16: $\left(\arcsin(f)\right)' = \frac{f'}{\sqrt{1-f^2}}$ D17: $\left(\arccos(f)\right)' = -\frac{f'}{\sqrt{1-f^2}}$

a) $f(x) = \arcsin\left(\operatorname{tg}(x)\right)$; b) $f(x) = \arcsin^3\left(\ln(x)\right)$;

D18: $\left(\operatorname{arctg}(f)\right)' = \frac{f'}{1+f^2}$

a) $f(x) = \operatorname{arctg}\left(\ln(x)\right)$; b) $f(x) = \operatorname{arctg}(e^x)$; c) $f(x) = \operatorname{arctg}^2(x^2)$;

D7 & D3

a) $f'(x) = \underbrace{(x^9)'}_{D7} = 9\,x^8 \underbrace{(x)'}_{D2} = 9x^8$

b) $f'(x) = \underbrace{\left(\frac{1}{2}\,x^9\right)}_{D3} = \frac{1}{2}\underbrace{(x^9)}_{D7} = \frac{1}{2}\,9x^8 \underbrace{(x)'}_{D2} = \frac{9}{2}\,x^8$

c) $f'(x) = \underbrace{\left(\frac{\sqrt[4]{x^5}}{3}\right)'}_{D3} = \frac{1}{3}\left(\sqrt[4]{x^5}\right)' = \frac{1}{3}\underbrace{\left(x^{\frac{5}{4}}\right)'}_{D7+D2} = \frac{1}{3}\,\frac{5}{4}\,x^{\frac{1}{4}} = \frac{5}{12}\,\sqrt[4]{x}$

d) $f'(x) = \left(\frac{1}{(3x)^2}\right)' = \underbrace{\left((3x)^{-2}\right)'}_{D7} = -2\,(3x)^{-3} \underbrace{(3x)'}_{D3+D2} = -\frac{6}{(3x)^3}$

D4

a) $f'(x) = \underbrace{\left(x^3 + \frac{3x}{5} - \frac{1}{x^2}\right)'}_{D4} = \underbrace{(x^3)'}_{D7+D2} + \underbrace{\left(\frac{3x}{5}\right)'}_{D3+D2} - \underbrace{(x^{-2})'}_{D7+D2} = 3x^2 + \frac{3}{5} + 2\,x^{-3}$

D5

a) $f'(x) = \underbrace{\left(\left(x^2-2\right)\left(3-\frac{1}{x^2}\right)\right)'}_{D5} = \underbrace{(x^2-2)'}_{D4}\left(3-\frac{1}{x^2}\right) + \left(x^2-2\right)\underbrace{\left(3-\frac{1}{x^2}\right)'}_{D4}$

$= \left(\underbrace{(x^2)'}_{D7+D2} - \underbrace{(2)'}_{D1}\right)\left(3-\frac{1}{x^2}\right) + \left(x^2-2\right)\left(\underbrace{(3)'}_{D1} - \underbrace{(x^{-2})'}_{D7+D2}\right) = 2x\left(3-\frac{1}{x^2}\right) + \left(x^2-2\right)\left(2\,x^{-3}\right)$

$= 6x - \frac{4}{x^3}$

R6

a) $f'(x) = \left(\frac{3x-2x^2}{x-1}\right)' = \frac{\overbrace{(3x-2x^2)'}^{D4}(x-1) - (3x-2x^2)\overbrace{(x-1)'}^{D4}}{(x-1)^2}$

$= \frac{\left(\overbrace{(3x)}^{D3+D2} - \overbrace{(2x^2)'}^{D3+D7+D2}\right)(x-1) - (3x-2x^2)\left(\overbrace{(x)'}^{D2} - \overbrace{(1)'}^{D1}\right)}{(x-1)^2}$

$= \frac{(3-4x)(x-1) - (3x-2x^2)}{(x-1)^2} = \frac{3x-3-4x^2+4x-3x+2x^2}{(x-1)^2} = \frac{-2x^2+4x-3}{(x-1)^2}$

D8

a) $f'(x) = \left(\frac{1}{e^{2x}}\right)' = \underbrace{(e^{-2x})'}_{D8} = e^{-2x}\underbrace{(-2x)'}_{D3+D2} = e^{-2x}\,(-2) = -2\,e^{-2x}$

b) $f'(x) = \underbrace{\left(e^{\sqrt{x}}\right)'}_{D8} = e^{\sqrt{x}}\underbrace{\left(x^{\frac{1}{2}}\right)'}_{D7+D2} = e^{\sqrt{x}}\,\frac{1}{2}\,x^{-\frac{1}{2}} = \frac{1}{2}\,\frac{e^{\sqrt{x}}}{\sqrt{x}}$

c) $f'(x) = \underbrace{\left(e^{\frac{1}{x}}\right)'}_{D8} = e^{\frac{1}{x}}\underbrace{\left(x^{-1}\right)'}_{D7+D2} = e^{\frac{1}{x}}\left(-x^{-2}\right) = -\frac{e^{\frac{1}{x}}}{x^2}$

D9

a) $f'(x) = \underbrace{\left(\ln(2x)\right)'}_{D9} = \frac{\overbrace{(2x)'}^{D3+D2}}{2x} = \frac{2}{2x} = \frac{1}{x}$

b) $f'(x) = \underbrace{\left(\ln(x^2+2)\right)'}_{D9} = \frac{\overbrace{(x^2+2)'}^{D4}}{x^2+2} = \frac{\overbrace{(x^2)'}^{D7+D2} + \overbrace{(2)'}^{D1}}{x^2+2} = \frac{2x}{x^2+2}$

$$\text{c)}\quad f'(x) = \underbrace{\left(\frac{\ln^3(x^2+1)}{5}\right)'}_{D3} = \frac{1}{5}\underbrace{\left(\ln^3(x^2+1)\right)'}_{D7} = \frac{1}{5}\,3\,\ln^2(x^2+1)\underbrace{\Big(\ln(x^2+1)\Big)'}_{D9}$$

$$= \frac{3}{5}\ln^2(x^2+1)\,\frac{\overbrace{(x^2+1)'}^{D4}}{x^2+1} = \frac{3}{5}\ln^2(x^2+1)\,\frac{\overbrace{(x^2)'}^{D7+D2} + \overbrace{(1)'}^{D2}}{x^2+1} = \frac{3}{5}\ln^2(x^2+1)\,\frac{2x}{x^2+1}$$

D10 & D11

$$\text{a)}\quad f'(x) = \underbrace{\Big(\sin(2x^2)\Big)'}_{D10} = \underbrace{(2x^2)'}_{D3+D7+D2}\cos(2x^2) = 4x\cos(2x^2)$$

$$\text{b)}\quad f'(x) = \underbrace{\Big(\cos\big(\ln(x)\big)\Big)'}_{D11} = -\underbrace{\Big(\ln(x)\Big)'}_{D9+D2}\sin\big(\ln(x)\big) = -\frac{1}{x}\sin\big(\ln(x)\big)$$

$$\text{c)}\quad f'(x) = \underbrace{\Big(\ln\big(\cos(2x)\big)\Big)'}_{D9+D11} = \frac{-\overbrace{(2x)'}^{D3+D2}\sin(2x)}{\cos(2x)} = -\frac{2\sin(2x)}{\cos(2x)} = -2\,\mathrm{tg}\,(2x)$$

D12 & D13

$$\text{a)}\quad f'(x) = \underbrace{\left(\mathrm{tg}\,(x^3)\right)'}_{D12} = \underbrace{(x^3)'}_{D7+D2}\sec^2(x^3) = 3x^2\sec^2(x^3)$$

$$\text{b)}\quad f'(x) = \underbrace{\left(2\,\mathrm{tg}\,(e^x)\right)'}_{D3} = 2\underbrace{\left(\mathrm{tg}\,(e^x)\right)'}_{D12} = 2\underbrace{(e^x)'}_{D8+D2}\sec^2(e^x) = 2\,e^x\sec^2(e^x)$$

$$\text{c)}\quad f'(x) = \underbrace{\Big(\mathrm{tg}\,\big(\ln(x^2+1)\big)\Big)'}_{D12} = \underbrace{\left(\ln(x^2+1)\right)'}_{D10}\sec^2\big(\ln(x^2+1)\big)$$

$$= \frac{\overbrace{(x^2+1)'}^{D4+D7+D2+D1}}{x^2+1}\sec^2\big(\ln(x^2+1)\big) = \frac{2x}{x^2+1}\sec^2\big(\ln(x^2+1)\big)$$

D14 & D15

$$\text{a)}\quad f'(x) = \underbrace{\left(\mathrm{cosec}\,(2x^2)\right)'}_{D15} = -\underbrace{(2x^2)'}_{D3+D7+D2}\,\mathrm{cosec}\,(2x^2)\,\mathrm{cotg}\,(2x^2)$$

$$\text{b)}\quad f'(x) = \underbrace{\Big(\ln\big(\sec(2x)\big)\Big)'}_{D9} = \frac{\overbrace{\big(\sec(2x)\big)'}^{D14}}{\sec(2x)} = \frac{\overbrace{(2x)'}^{D3+D2}\sec(2x)\,\mathrm{tg}\,(2x)}{\sec(2x)} = 2\,\mathrm{tg}\,(2x)$$

D16 & D17

$$\text{a)}\quad f'(x) = \underbrace{\Big(\arcsin\big(\mathrm{tg}\,(x)\big)\Big)'}_{D16} = \frac{\overbrace{\big(\mathrm{tg}\,(x)\big)'}^{D12+D2}}{\sqrt{1-\big(\mathrm{tg}\,(x)\big)^2}} = \frac{\sec^2(x)}{\sqrt{1-\mathrm{tg}^2(x)}}$$

$$\text{b)}\quad f'(x) = \underbrace{\Big(\arcsin^3\big(\ln(x)\big)\Big)'}_{D7} = 3\arcsin^2\big(\ln(x)\big)\underbrace{\Big(\arcsin\big(\ln(x)\big)\Big)'}_{D16} = 3\arcsin^2\big(\ln(x)\big)\frac{\overbrace{\big(\ln(x)\big)'}^{D9+D2}}{\sqrt{1-\ln^2(x)}}$$

$$= 3\arcsin^2\big(\ln(x)\big)\frac{\frac{1}{x}}{\sqrt{1-\ln^2(x)}}$$

D18 a) $$f'(x) = \underbrace{\Big(\operatorname{arctg}\big(\ln(x)\big)\Big)'}_{D18} = \frac{\overbrace{\big(\ln(x)\big)'}^{D9+D2}}{1+\ln^2(x)} = \frac{1}{x\left(1+\ln^2(x)\right)}$$

b) $$f'(x) = \underbrace{\big(\operatorname{arctg}(e^x)\big)'}_{D18} = \frac{\overbrace{(e^x)'}^{D8+D2}}{1+(e^x)^2} = \frac{e^x}{1+e^{2x}}$$

c) $$f'(x) = \underbrace{\big(\operatorname{arctg}^2(x^2)\big)'}_{D7} = 2\operatorname{arctg}(x^2)\,\underbrace{\big(\operatorname{arctg}(x^2)\big)'}_{D18} = 2\operatorname{arctg}(x^2)\,\frac{\overbrace{(x^2)'}^{D7+D2}}{1+(x^2)^2} = 2\operatorname{arctg}(x^2)\,\frac{2x}{1+x^4}$$

$$= \operatorname{arctg}(x^2)\,\frac{4x}{1+x^4}$$