1. Background

A priority encoder with input length 2^n, $\operatorname{penc}(2^n)$ is defined as follows.

Input: $y[2^n - 1 : 0] \in \{0, 1\}^{2^n}$

Output: $x[n - 1 : 0] \in \{0, 1\}^n$, *valid* $\in \{0, 1\}$

Functionality:

$$valid = \begin{cases} 1 & ; \quad y \neq 0^{2^n} \\ 0 & ; \quad y = 0^{2^n} \end{cases}$$

Let i denote the largest index i such that $y[i] = 1$. If *valid* $= 1$, then $\vec{x}$ should satisfy $\langle \vec{x} \rangle = i$.

Formally:

$$\vec{y} \neq 0^{2^n} \implies y\, 2^n - 1 : \langle \vec{x} \rangle = 0^{2^n - 1 - \langle x \rangle} \circ 1$$