

Discretized Berry phase

$$\mathcal{P}_n = \oint A_n(k) dk \approx \sum_{j=0}^{N-1} A_n(k_j) \Delta k$$

$$= - \sum_{j=0}^{N-1} \text{Im} \langle n(k_j) | \partial_k n(k_j) \rangle \Delta k$$

$x \approx \ln(1+x)$

$$\approx - \sum_{j=0}^{N-1} \text{Im} \ln \left[1 + \underbrace{\langle n(k_j) | \partial_k n(k_j) \rangle}_{\langle n(k_j) | n(k_j) \rangle} \Delta k \right]$$

$$= - \sum_{j=0}^{N-1} \text{Im} \ln \left[\underbrace{\langle n(k_j) | \left(|n(k_j)\rangle + \Delta k |\partial_k n(k_j)\rangle \right)}_{\approx |n(k_j + \Delta k)\rangle = |n(k_{j+1})\rangle} \right]$$

$$= - \sum_{j=0}^{N-1} \text{Im} \ln \langle n(k_j) | n(k_{j+1}) \rangle$$

phase of $\langle n(k_j) | n(k_{j+1}) \rangle$

$$= - \text{Im} \ln \prod_{j=0}^{N-1} \langle n(k_j) | n(k_{j+1}) \rangle \mod 2\pi$$