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Vigilada Mineducación

$V = \omega . R$

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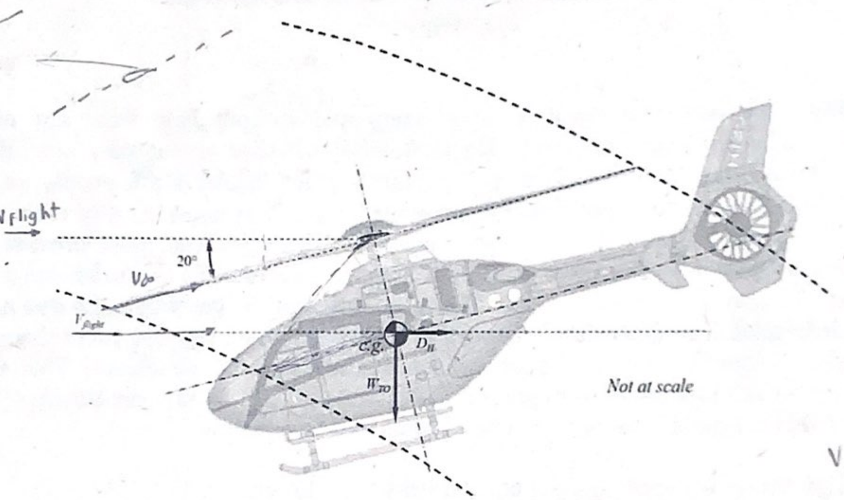
Helicopters need to pitch down their own boy in order to get forward speed, to do that, the main rotor has to be tilted down at a tilt-angle as shown in the following figure:

V_∞



En el lado que baja se suman y en el que baja se restan

$V_\infty = V_{flight} + V_{tangencial}$

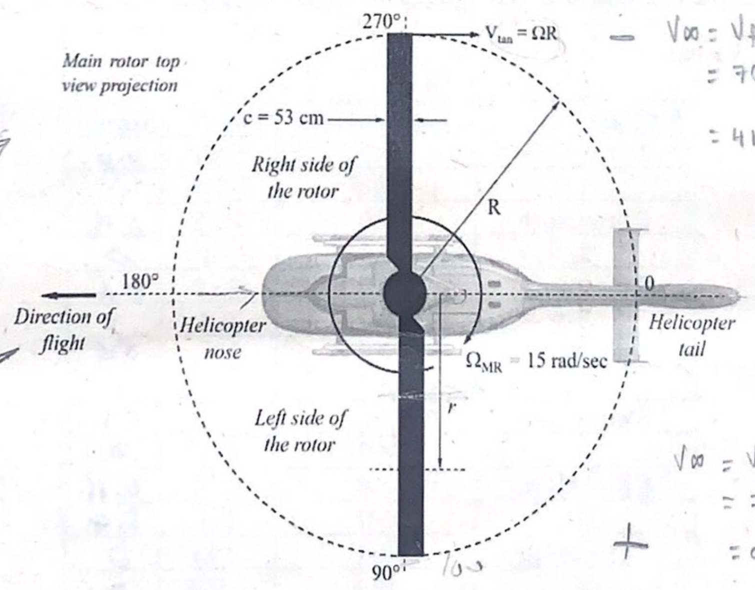
The forward movement of the helicopter creates an undesired effect called dissymmetry of lift, meaning that the left and right sides of the main rotor will experience different amounts of lift force. To counteract this, the rotating blade must have the same amount of lift when the blades are in the 90- and 270-degree positions, as shown in the following figure:

V_∞

V_F

V_∞

V_F 20



$-$ $V_\infty = V_{flight} - V_{tan}$ $V_{tan} =$

$= 70{,}5 - \left(15 \frac{rad}{s} \times 1{,}92\right)$

$= 41{,}7\ m/s$ //

$V_\infty = V_{flight} + V_{tan}$

$+$ $= 70{,}5 + \left(15 \frac{rad}{s} \times 1{,}92\ m\right)$

$= 99{,}13\ m/s$ //

The helicopter shown above is flying at a forward speed (V_{flight}) of 253.8 km/h at standard sea level conditions (R = 287.0 [J/(kg*K)]). The rotor diameter is 9.6 [m].

$R = 4{,}8 \times 0{,}4 = 1{,}92\ m$

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The local velocity (in km/h) distribution over a blade element (section of a two-dimensional blade) in the advancing region (left side of the rotor e.g. 90 [deg]) at a radial position (r/R) of 40% is measured as follows:

- Upper surface: V increases from the leading edge to 392.4 at 3% of the chord distance and to 427.5 at 8%. Then, the velocity decreases to 394.2 and 379.8 at 20% and 60% of the chord distance, respectively. At the trailing edge, the local velocity decreases to 261.9.
- Lower surface: V increases from the leading edge to 353.25 and 334.08 at 11% and 70% of the chord distance, respectively.

It is also known that the test was performed at sea-level conditions. The wing section's zero-lift angle of attack is -2.0 [deg]. The advancing blade pitch angle (angle between the plane of rotation and the airfoil's chord) is 14.36 [deg]. The graph of its drag coefficient versus the angle of attack (Re = 3.6 x 10^6) can be plotted from the following functions:

- $c_d = 3.05 \times 10^{-5}\alpha^2 + 2.48 \times 10^{-5}\alpha + 0.00695$; for: $-10.9\,[deg] \le \alpha \le -3.78[deg]$
- $c_d = 8.5 \times 10^{-7}\alpha^3 + 4.74 \times 10^{-5}\alpha^2 + 8.07 \times 10^{-5}\alpha + 0.00713$; for: $-3.78\,[deg] \le \alpha \le 1.56[deg]$
- $c_d = 6.23 \times 10^{-5}\alpha^2 + 9.54 \times 10^{-5}\alpha + 0.00728$; for: $1.56\,[deg] \le \alpha \le 8.68[deg]$

You are asked to find:

1. The value of the angle of attack of the blade element at which this test is performed $\alpha_{\text{left blade element}}$ = ________ [deg] / X
2. The value of the lift force per unit of span L' = 676,6832 [N] / X
3. The value of the aerodynamic efficiency: L/D = / ________ X
4. Moment coefficient with respect to the leading edge if $c_{m\_c/4}$ is equal to -0.032 and the airfoil is at the same angle of attack: c_{m_LE} = -0,08485 X
5. If the airfoil is inverted, calculate the angle of attack to obtain the same value of L' of point three: α = ________ [deg] / X

For the retreating side (right blade element: 270 deg) at the same radial position (r/R) and assuming that the local blade lift force (per unit of span) is the same as point 2, calculate:

6. The value of the lift coefficient $c_{l\_\text{right blade element}}$ = 1,198757 X
7. The value of the angle of attack of the blade element at which this test is performed $\alpha_{\text{right blade element}}$ = ________ [deg] X

$\frac{V}{c}$

0,03

$C_{mLE} = C_{m\frac{c}{4}} - \frac{Cl}{4}$

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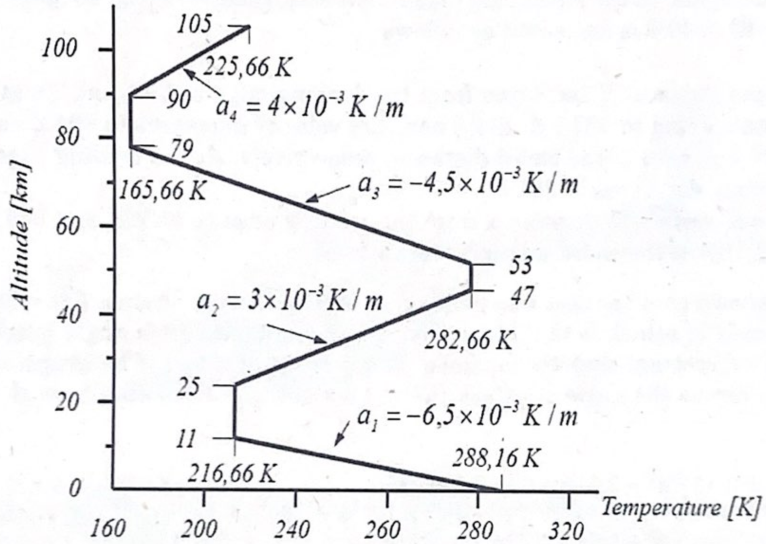
Sheet of useful formulas:



(Hints: $\rho_{@SL} = 1.225$ [kg/m³]; $p_{@SL} = 101325.0$ [Pa]; $R_{air} = 287.0$ [J/(kg*K)]; $\mu_{@SL} = 1.789 \times 10^{-5}$ [kg/(m*seg)]; $g_0 = 9.81$ [m/s²])

Isothermal:

- $\dfrac{p}{p_1} = e^{-\left(\frac{g_0}{R\cdot T}\right)(h-h_1)}$
- $\dfrac{\rho}{\rho_1} = e^{-\left(\frac{g_0}{R\cdot T}\right)(h-h_1)}$

Gradient:

- $T = T_1 + a\,(h - h_1)$
- $\rho = \rho_1\left(\dfrac{T}{T_1}\right)^{-\left(\frac{g_0}{aR}+1\right)}$
- $\dfrac{p}{p_1} = \left(\dfrac{T}{T_1}\right)^{-\frac{g_0}{a\cdot R}}$

Power law relation: $\mu = \mu_0\left(\dfrac{T}{T_0}\right)^{\left(\frac{2}{3}\right)}$

$$q_\infty = \frac{\rho_\infty \cdot V_\infty^2}{2}$$

$$L_{airfoil} = q_\infty S c_l$$

$$D_{airfoil} = q_\infty S c_d$$

$$C_p = \frac{p - p_\infty}{q_\infty}$$

$$p + \frac{1}{2}\rho_\infty V_\infty^2 = p_1 + \frac{1}{2}\rho_\infty V_1^2 = const.$$

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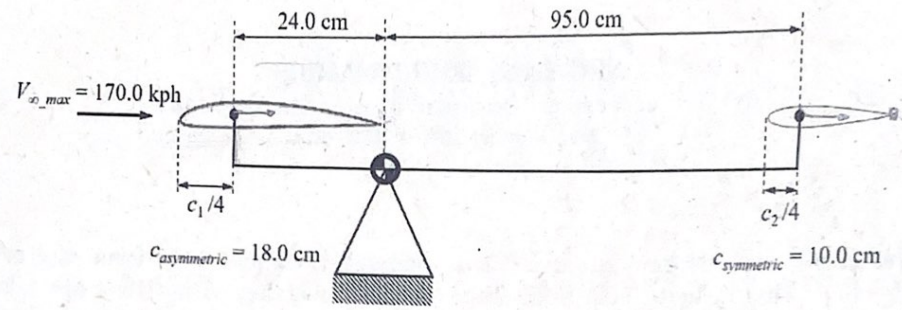

This arrangement must be studied before being placed in a wind tunnel test section. Both wingspans are 52.0 cm, which covers the distance between both lateral walls inside the test section. The main idea of the arrangement is always to achieve equilibrium in the system. The test will be performed at sea level conditions (ρ_∞ = 1.225 [kg/m³]; μ_∞ = 1.81x10⁻⁵ [kg/(m*sec)]) and with the maximum velocity given by the wind tunnel in its test section. The points of attachment of the structure to the airfoil are located at the quarter chord for both the cambered and the uncambered sections. The nomenclature for the cambered and uncambered sections are NACA 3715 and NACA 0009, respectively.

1. If the cambered wing section is set at an angle of attack of 4°, calculate the value of the angle of attack of the other wing section to create a static equilibrium in the whole arrangement.

   $\alpha_{\text{uncambered wing section}}$ = 2,398° [deg] X

2. The drag polar results for the symmetrical airfoil with the conditions set previously are described in the following polynomial function:

   $$c_{d\_total} = -0.0004c_l^6 + 0.0007c_l^5 + 0.0003c_l^4 - 0.0006c_l^3 + 0.0035c_l^2 + 0.0001c_l + 0.0051$$

   If the critical Reynolds number for this test is 3.25×10^5, calculate:

   a. The drag coefficient due to form and attitude – c_{d_p} = $7{,}9699 \times 10^{-4}$ X
   b. The percentage of skin friction drag compared with the total drag value.
   D_f = 0,33 ✓ [N] → 85,359 ✓ %
   c. Conclude about the results obtained in pints a. and b.:

   In the symmetrical airfoil the X_crit is equal to the Chord of the airfoil which means the flow along the airfoil is laminar, there are no point of transition and there are no flow turbulent.

   The drag of friction is more than the pressure, due to along the airfoil the flow is laminar for this the drag pressure is less.

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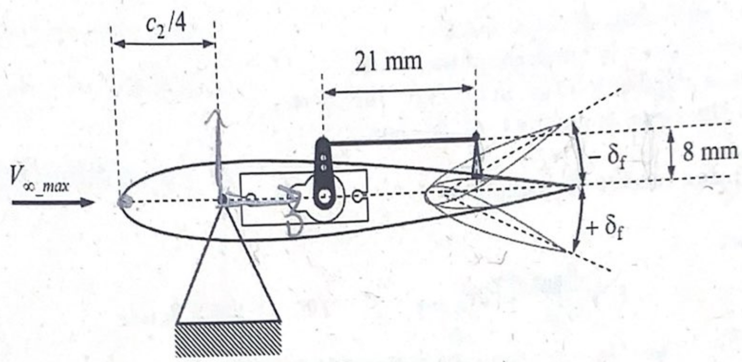
3. Now the symmetrical wing section is tilted to an angle of attack of zero degrees, and a flapped section (plain flap type) is added, as shown in the following figure:



(Note: figure not at scale)

A servo motor will be used for the flap deflection control. If the flap hinge position location is placed at 70.0% of the chord length from the leading edge, calculate the value of the angle of the flap deflection to create a static equilibrium of the whole arrangement δ_f = 0 [deg] X

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Sheet of useful formulas:

Airfoil nomenclature:

NACA 4 digits – NACA z_{max} x_{z_max} tt
z_{max} – max. camber in hundreds of the chord
x_{z_max} – max. camber location in tenths of the chord
tt – max. thickness in hundreds of the chord

Aerofoil mean camber line equation:

Front part:

$$z = \frac{z_{max}}{x_{z\_max}^2} x \left(2x_{z\_max} - \frac{x}{c}\right) \quad for \quad 0 \leq x \leq x_{z\_max}$$

Rear part:

$$z = z_{max} \frac{(c-x)}{(1-x_{z\_max})^2} \left(1 + \frac{x}{c} - 2x_{z\_max}\right) \quad for \quad x_{z\_max} \leq x \leq c$$

Fourier sine coefficients (airfoil):

$$A_0 = \alpha - \frac{1}{\pi} \int_0^{\pi} \frac{dz}{dx} d\theta$$

$$A_n = \frac{2}{\pi} \int_0^{\pi} \frac{dz}{dx} \cos n\,\theta d\theta$$

$$c_{m,le} = -\frac{\pi}{2}\left(A_0 + A_1 - \frac{A_2}{2}\right) = -\left[\frac{c_l}{4} + \frac{\pi}{4}(A_1 - A_2)\right]$$

$$c_{m,c/4} = \frac{\pi}{4}(A_2 - A_1)$$

TAT generalized equations:

$$c_l = \pi(2A_0 + A_1) = a_0(\alpha - \alpha_{L=0}); \ \alpha[rad]$$

$$a_0 = 2\pi\left[{}^{1}/_{rad}\right]$$

$$\alpha_{L=0} = -\frac{1}{\pi} \int_0^{\pi} \frac{dz}{dx} (\cos\theta - 1) d\theta$$

Airfoil transformation x to θ:

$$x = \frac{c}{2}(1 - \cos\theta)$$

Fourier sine coefficients (airfoil):

$$A_0 = \alpha - \frac{1}{\pi} \int_0^{\pi} \frac{dz}{dx} d\theta$$

$$A_n = \frac{2}{\pi} \int_0^{\pi} \frac{dz}{dx} \cos n\,\theta d\theta$$

Fourier sine coefficients (airfoil with flap):

$$A_0 = \alpha - \frac{1}{\pi}\left(\int_0^{\phi} \frac{dz}{dx} d\theta + \int_{\phi}^{\pi} \delta_f d\theta\right)$$

$$A_n = \frac{2}{\pi}\left(\int_0^{\phi} \frac{dz}{dx} \cos n\,\theta d\theta + \int_{\phi}^{\pi} \delta_f \cos n\,\theta d\theta\right)$$

Flapped surface transformation F to ϕ:

$$(1 - F) = \frac{1}{2}(1 - \cos\phi); \ \delta_f = -\frac{h}{F}$$

$\int_{\phi}^{\pi} \frac{dz}{dx}$

$\frac{-1}{\pi} \int_{\phi}^{\pi} \delta_F \cos(\theta - 1)$

$\phi = \cos^{-1}(1 - 2(1-F))$ 1,9823

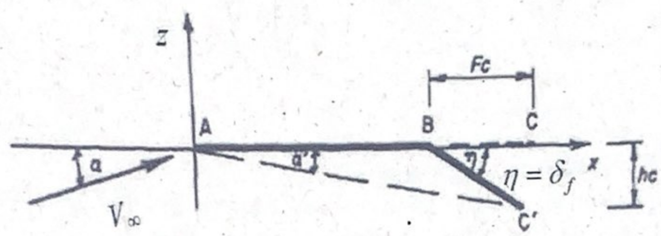
Reynolds number:

$$Re_x = \frac{\rho_\infty \cdot V_\infty \cdot x}{\mu_\infty}$$

For laminar boundary layer: $\delta_{lam} = \frac{5x}{\sqrt{Re_x}}$; $C_{f,lam} = \frac{1.328}{\sqrt{Re_x}}$

For turbulent boundary layer: $\delta_{tur} = \frac{0.37x}{Re_x^{1/5}}$; $C_{f,tur} = \frac{0.074}{Re_x^{1/5}}$

- Nomenclature



Integrals solution:

$$\int \cos\theta \, d\theta = \sin\theta$$

$$\int \cos^2\theta \, d\theta = \frac{1}{2}\sin\theta\cos\theta + \frac{\theta}{2}$$

$$\int \cos^3\theta \, d\theta = \frac{1}{3}\sin\theta\left(\cos^2\theta + 2\right)$$

$$\int \cos^4\theta \, d\theta = \frac{1}{4}\cos^3\theta\sin\theta + \frac{3}{8}(\sin\theta\cos\theta + \theta)$$

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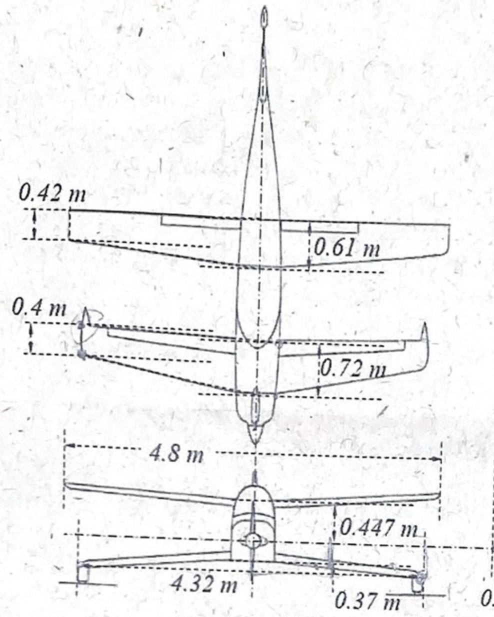
The Ruttan's Quickie airplane is a unique and rare airplane designed as a low-cost personal vehicle solution with enhanced performance characteristics. The following figure shows the 3 main views of the aircraft with some geometric characteristics:



This aircraft is considered a biplane and a lifting tandem wing canard type. Both surfaces act to give the maximum lift force to the entire airplane. The front wing is also considered as a horizontal stabilizer; the rear one is established to be the main wing. The following table gives some characteristics for both wings:

Wing	e	a_0 [1/deg]	$C_{M,ac,\,wf}$
Front (canard)	0.98	0.112	-0.08
Rear (main)	0.95	N/A	-0.18

The total wing lift coefficient for the rear wing is 0.33, and the profile drag is 5.7×10^{-3}. It is known that the following function gives the profile drag coefficient for the front wing:

$$c_{d\_front} = 4.556\times10^{-5}(c_l)^2 - 8.11\times10^{-5}(c_l) + 5.08\times10^{-3}$$

The aircraft's cruise speed is 51.41 m/s, and the maximum weight is 220.0 [kgf], assuming sea level conditions ($\rho_\infty = 1.225$ [kg/m³], $\mu_\infty = 1.79\times10^{-5}$ [kg/(m*seg)]).

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With all this information, determine:

The value of the total wing lift and drag coefficients:

1. C_{L,w_front} = 0,2138818 X
2. C_{D,w_front} = $6{,}9879 \times 10^{-3}$ X

The percentage of lift force exerted by each wing:

3. L_{front} = 38,81 [%] X
4. L_{rear} = 61,19 [%] X

Analyze and conclude about the lift percentage distribution in this airplane:

The rear wing has a taper ratio more than the front wing, because of that the lift slope of the rear wing its close to the rear profile, and, the rear wing has a wing area more than front wing, more area = more lift force

An airplane with an MTOW of 20022.21 [N] has a wing's asymmetrical airfoil whose lift slope is 6.436 [1/deg] and a zero-lift angle of attack of -4.1 [deg], a taper ratio of 1.0 with a wing-span of 12.9 [m] and a c_{root} of 1.3 [m]. The wing has no geometric or aerodynamic twist.

This aircraft's cruise speed is 67.0 [m/s] at sea level. The wing does not have any aerodynamic or geometric twist. The following results for the Fourier coefficient are obtained using LLT with 4 points of control (divisions) on the wing at a geometric angle of attack of 4 [deg]:

Fourier series coefficient	Value
A(1)	2.312×10^{-2}
A(2)	2.79×10^{-3}

Assuming that the wing lift force opposes the total weight of the aircraft, calculate the wing's incidence angle for cruise flight conditions:

5. i_W = 4 [deg] X

$i_W = \alpha_{geo} + \beta$

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The general characteristics of an airplane are a maximum take-off weight of 600 [kgf], a maximum available power of 95.7 [hp], a wing area of 18.17 [m^2], a wingspan of 8.65 [m], and an Oswald efficiency factor of 0.96. The performance characteristics are a cruise speed of 62.5 [m/s] and a stall speed of 20.0 [m/s].

If the total parasite drag coefficient for cruise flight condition is 1.953×10^{-2} (corrected value), calculate:

C_{D0} $C_{D0\_1c?}$ = C_{Di}

6. The power required for cruise flight condition: 12.494 [hp] X
   Note: ($P_{req} = D*V_{cruise}$); hint: 1 [Watt] = 1.341×10^{-3} [hp]

7. The ratio of power spent for this flight condition (cruise) and the available power in percentage: 13.055 [%] X

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Sheet of useful formulas:

$$\bar{c} = \frac{2}{3} c_{root} \frac{(1+\lambda+\lambda^2)}{(1+\lambda)}$$

$$\bar{Y} = \frac{b}{6}\left[\frac{(1+2\lambda)}{(1+\lambda)}\right]$$

Reynolds number:

$$Re_x = \frac{\rho_\infty \cdot V_\infty \cdot x}{\mu_\infty}$$

Aerodynamic characteristics of a wing:

$$\alpha_i = \sin^{-1}\left(\frac{D_{w\_i}}{L_w}\right) = C_{L\_w} \cdot K; \quad K = \frac{1}{\pi \cdot e \cdot AR}$$

$\bar{c}_{faron} =$

$$a = \frac{a_0}{1 + \frac{57.3 \cdot a_0}{\pi \cdot e \cdot AR}}$$; where: a_0 [1/deg]

LLT:

$\tan(\theta) \times h =$

$$C_{L\_w} = \pi \cdot AR \cdot A(1)$$

$$e = (1+\delta)^{-1}; \text{ where: } \delta = \sum_{n=2}^{N} n\left(\frac{A(n)}{A(1)}\right)^2$$

$$C_{Di} = k \cdot C_L^2; \text{ where: } k = \frac{1}{\pi \cdot e \cdot AR}$$

$\tan(\beta) = \frac{h}{\bar{y}}$

$\tan(\theta) = \frac{h}{\bar{c}_{root}}$

$\text{sen}(\theta) = \frac{\beta}{\ }$

$\beta_c = \beta$

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The Airbus A-350 flies at a cruise service ceiling of 13.1 [km], where the air temperature is 203.01 [K], and the air density is 0.2756 [kg/m³]. The wing has the following characteristics: an aspect ratio of 9.49, an area of 442 square meters, and a taper ratio of 0.3. λ

The supercritical airfoil ($(t/c)_{max}$ = 12%) used has a zero-lift angle of attack of -1.5 [deg]. An aerodynamic study was done with a speed value of 42.13 [m/s] at an angle of attack of 4 [deg], giving the following results for the local velocity (in [km/h]) distribution over the two-dimensional section in a wind tunnel:

- Upper surface: V continuously increases from the leading edge to 102.6, 180, and 203.4 at 1.0%, 15.0%, and 48% of the chord distance, respectively. Then, it decreases to 124.2 at 78% of the chord distance. At the trailing edge, the local velocity decreases to 92.7.
- Lower surface: V increases from the leading edge to 94.05 and 127.8 at 2% and 40% of the chord distance, respectively. Then, it decreases to 126 at 70% of the chord distance.

The following relations with the angle of attack describe the drag coefficient value for this airfoil:

- $c_d = 2.56 \times 10^{-4}\alpha^2 - 1.2 \times 10^{-4}\alpha + 4.9 \times 10^{-2}$; for: $-11.22\,[deg] \le \alpha \le -3.44[deg]$
- $c_d = 1.32 \times 10^{-4}\alpha^3 + 2.69 \times 10^{-3}\alpha^2 + 2.1 \times 10^{-3}\alpha + 3.36 \times 10^{-2}$; for: $-3.44\,[deg] \le \alpha \le 2.39[deg]$
- $c_d = 5.23 \times 10^{-4}\alpha^2 + 1.59 \times 10^{-4}\alpha + 5.25 \times 10^{-2}$; for: $2.39\,[deg] \le \alpha \le 10.16[deg]$

Using the most recent approximation for the compressibility correction, determine:

1. The critical Mach number value for this airfoil: M_{crit} = 0,6133

If the aircraft wing (and the airfoil) flies at a geometric angle of attack of 0.2 [deg] and knowing that the incompressible quarter chord moment coefficient is equal to -0.075 times the $c_{l,0}$, calculate the following airfoil's compressible values of:

2. Aerodynamic efficiency: $(L/D)_{comp}$ = 8,394
3. Quarter chord moment coefficient: $c_{m,c/4\_comp}$ = -0,07583

If a value of M_{crit} = 0.85 is wanted for the wing (assume e = 0.92), determine:

4. The leading edge swept angle: Λ_{LE} = 40,815 [deg]
5. The Mach drag divergence: M_{dd} = 0,9569
6. The value of the compressible wing lift coefficient: $C_{L,comp}$ = 0,3965 $cl = a_0(\alpha - \alpha_{l=0})$
7. The value of the wing's total compressible lift force: L_{comp} = 9,369 [Ton]

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The Lockheed Martin F-22 Raptor has a reported weight of 29410 [kg], a wingspan of 13.56 [m], and a wing area of 78.04 [m^2]. The aircraft is designed to reach a maximum Mach number of 2.25. Assuming sea level conditions (ρ_0 = 1.225 [kg/m^3]; T_0 = 288 [K]), calculate the following:

8. The geometric angle of attack: α_{geo} = 0,297 [deg]

At this flight condition, the aircraft encounters a sudden attitude change in pitch, reaching an angle of attack of 3 [deg]; calculate the load factor:

9. n = L/W = 98,926 [g's]

Note: For all exercises, assume R_{air} = 287 [(N)(m) /(kg)(K)], and γ_{air} = 1.4

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Sheet of useful formulas:

Wing geometry:

$$AR = \frac{b^2}{S}$$

$$c_{root} = \frac{2 \cdot S}{b(1+\lambda)}$$

$$\lambda = \frac{c_{tip}}{c_{root}}$$

$$\bar{c} = \frac{2}{3} c_{root} \frac{(1+\lambda+\lambda^2)}{(1+\lambda)}$$

$$\bar{Y} = \frac{b}{6}\left[\frac{(1+2\lambda)}{(1+\lambda)}\right]$$

Incompressible flow:

$A_1V_1 = A_2V_2$ *continuity*

$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$ *Bernoulli's equation*

$$C_p = \frac{p - p_\infty}{q_\infty} = 1 - \left(\frac{V}{V_\infty}\right)^2$$

$$c_n = \frac{1}{c}\int_0^c (C_{p,l} - C_{p,u})dx \approx c_l$$

Compressible flow:

$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$ *continuity*

$\frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^\gamma = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$ *isentropic relations*

$c_pT_1 + \frac{1}{2}V_1^2 = c_pT_2 + \frac{1}{2}V_2^2$ *energy*

$p_1 = \rho_1 R T_1$ *equation of state*

$p_2 = \rho_2 R T_2$ *equation of state*

$$\frac{\rho_0}{\rho_1} = \left(1 + \frac{\gamma-1}{2}M_1^2\right)^{\frac{1}{\gamma-1}}$$

$$\frac{p_0}{p_1} = \left(1 + \frac{\gamma-1}{2}M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{T_0}{T_1} = 1 + \frac{\gamma-1}{2}M_1^2$$

Compressibility corrections (high subsonic):

$$c_l = \frac{c_{l,0}}{\sqrt{1 - M_\infty^2}}$$

$$c_d = \frac{c_{d,0}}{\sqrt{1 - M_\infty^2}}$$

$$c_{m,c/4} = \frac{\left(c_{m\_c/4}\right)_0}{\sqrt{1 - M_\infty^2}}$$

$$M_1^2 = \frac{2}{\gamma - 1}\left[\left(\frac{p_0}{p_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$a_\infty = \sqrt{\gamma R T_\infty}$$

$$M_\infty = \frac{V_\infty}{a_\infty}$$

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2}} \quad \textit{Prandt'l} - \textit{Glauert rule}$$

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2} + \left(\frac{M_\infty^2}{1 + \sqrt{1 - M_\infty^2}}\right)\frac{C_{p,0}}{2}} \quad \textit{Karman} - \textit{Tsien rule}$$

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2} + \left\{\frac{M_\infty^2\left[\left(1 + \left(\frac{\gamma - 1}{2}\right)M_\infty^2\right)\right]}{2\sqrt{1 - M_\infty^2}}\right\}C_{p,0}} \quad \textit{Laiton's rule}$$

$M_{dd} = \frac{\kappa_A}{\cos\Lambda_{LE}} - \frac{(t/c)_{max}}{(\cos\Lambda_{LE})^2} - \frac{c_{l_{comp}}}{10(\cos\Lambda_{LE})^3}$ *Korn's approximation*; where $\kappa_A = 0.87$ for NACA 6-series airfoil and 0.95 for supercritical airfoils

$$\boxed{C_{p,crit} = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{2 + (\gamma - 1)M_\infty^2}{\gamma + 1}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]}$$

Supersonic:

$$\sin\mu = \frac{1}{M_\infty}$$

$$c_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$

$$c_{d,w} = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}}$$

$$\frac{C_{D,w}}{\alpha^2} = \frac{4}{\sqrt{M_\infty^2 - 1}}\left[1 - \frac{1}{2\left(AR\sqrt{M_\infty^2 - 1}\right)}\right]$$