Aerospace Structures

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EXTERNAL LOADS

Aircraft loads

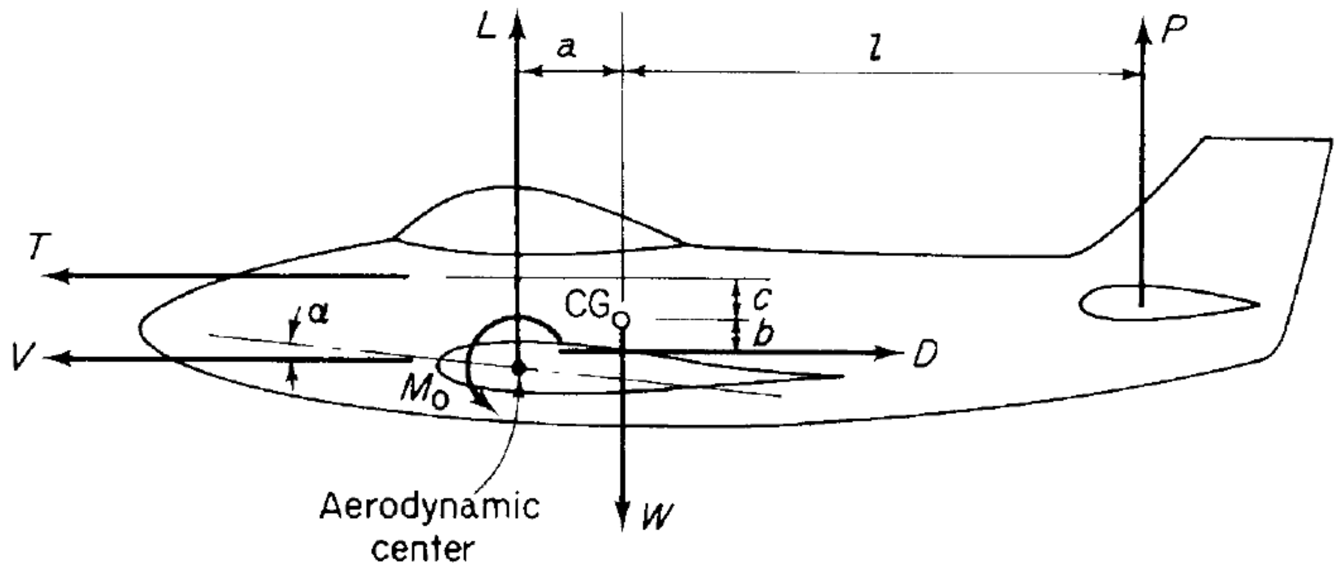
Maximum components loads of an aircraft’s structure generally occur when the aircraft is undergoing **some form of acceleration or deceleration**

Aircraft loads

Before a structural component can be designed, **the inertia loads corresponding to acceleration and decelerations must be calculated**

Aircraft loads

- Most of the resultant loads come from interactions between the airflow and aircraft’s aerodynamic surfaces
- These forces allow the pilot to fly and control the aircraft
- The study of these loads belongs to aerodynamicists



Aerodynamic forces

Lift: $$L = C_L \, \rho \, \frac{V^2}{2} \, A$$

Drag: $$D = C_D \, \rho \, \frac{V^2}{2} \, A$$

Pitching moment coefficient: $$C_m = \frac{M}{q \, S \, c}$$

$$q = \frac{1}{2} \rho V^2$$

Load Factor

An increase or reduction in loads is going to be measured in terms of the load factor n

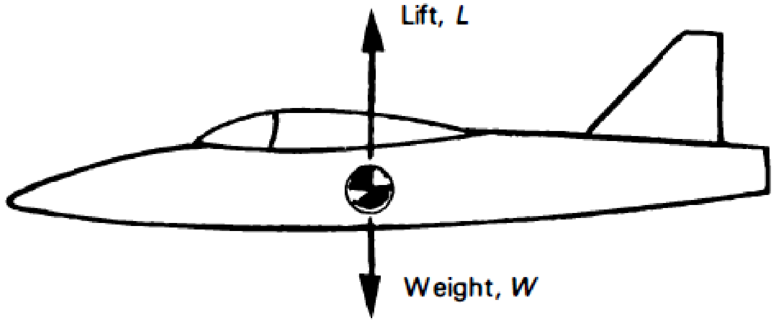
- Any aircraft's load will produce a structural stress
- In a steady flight, the lift produced by wings will support the weight of the plane
- During a flight, maneuvers and gusty air are presented. The severity of these actions or situations will produce a change in loads

Load Factor

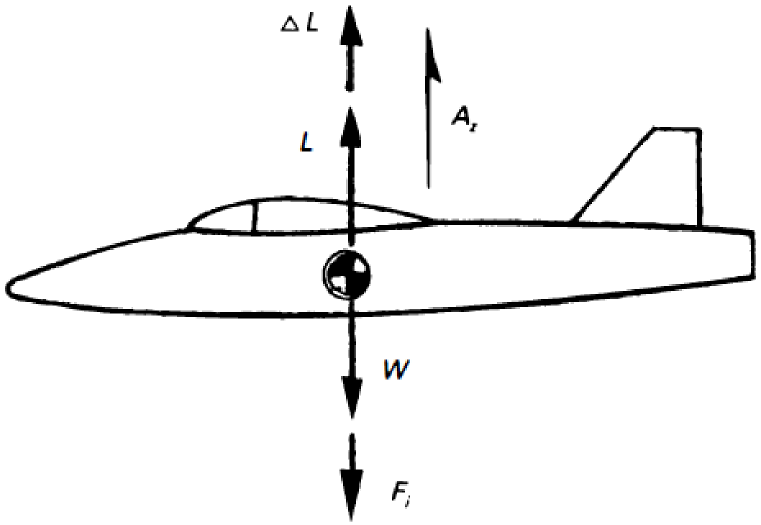
Load factor is a multiplying factor which defines a load in terms of weight

The force ΔL is an increment of lift which is caused either by encountering a gust or because of an intentional maneuver. This additional force causes an acceleration and equilibrium is provided by inertial forces F_i.

Load factor is measured in terms of G's



(a) $L = W$ case



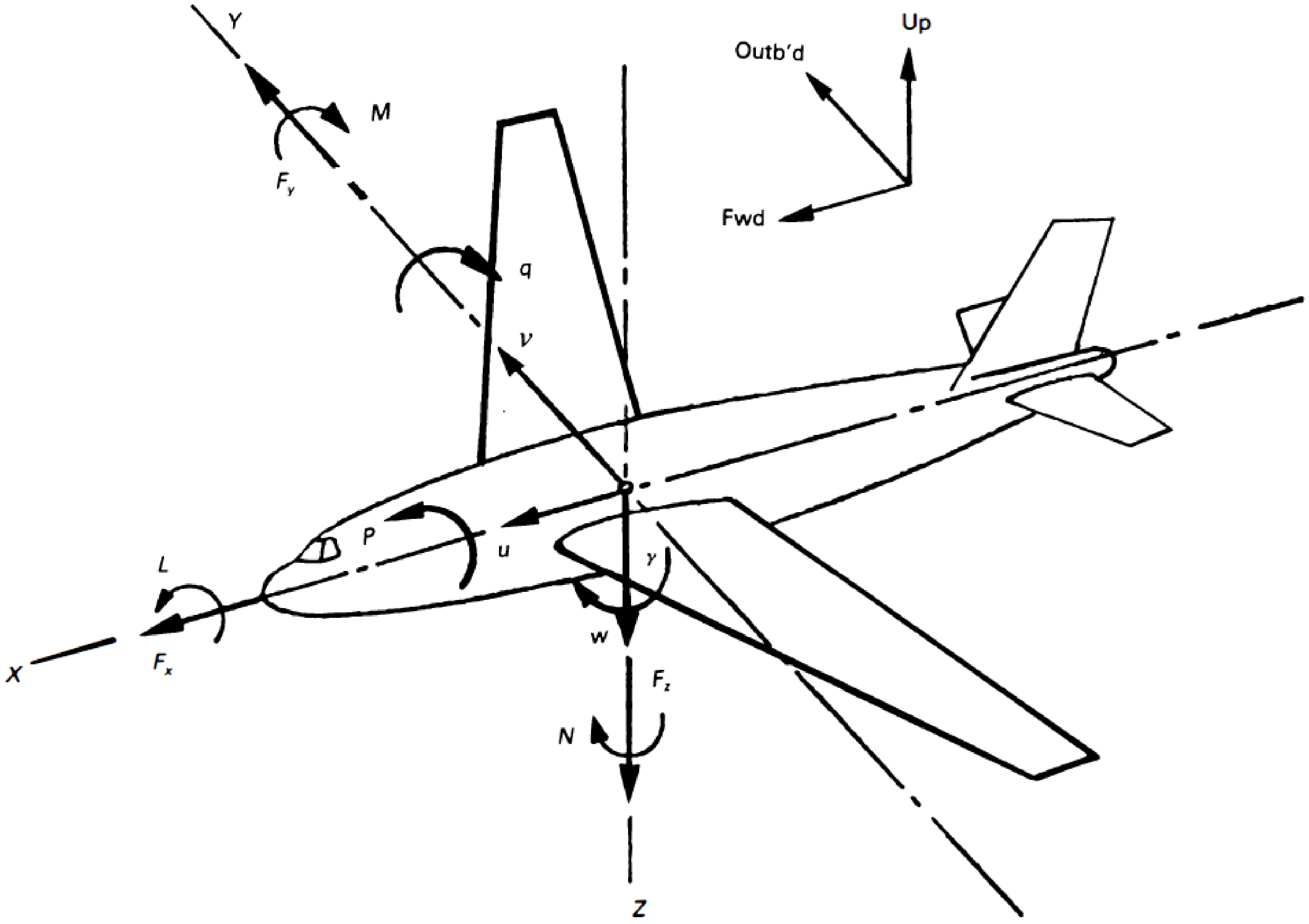
(b) $L + \Delta L = W + F_i$ case where $L = W$ and $\Delta L = F_i$

Load Factor

Load factor:

$$n = 1 + \frac{A_Z}{g}$$

$$n = \frac{L}{W}$$



Load Factor

Load factor is important due to the following

- If the load factor is unknown, the pilot could overload the structure
- A variation in the load factor may affect the stall speed

Load Factor

The maneuvering load factor to which an airplane is designed depends on its intended usage

Fighters, are designed to withstand loads corresponding to the accelerations that a pilot can physically resist

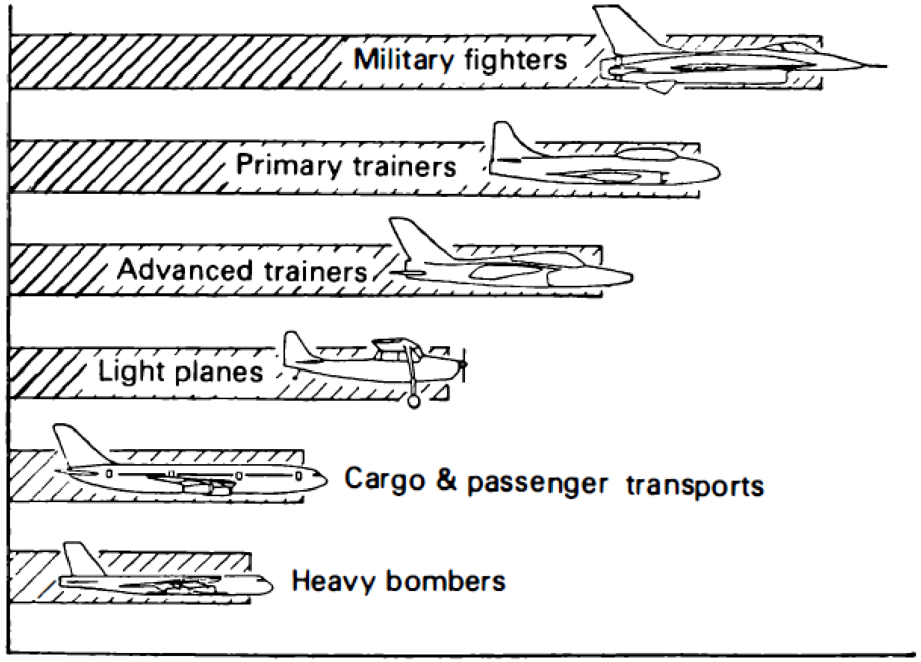


Fig. 3.1.2 Design load factor.

Load Factor

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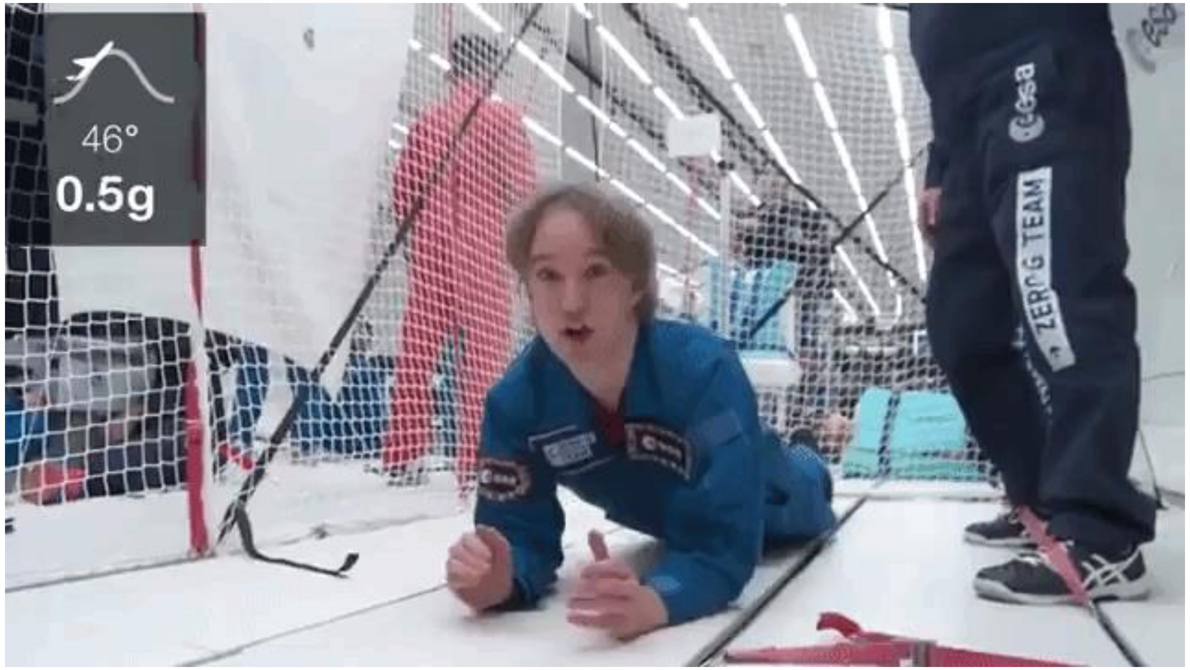
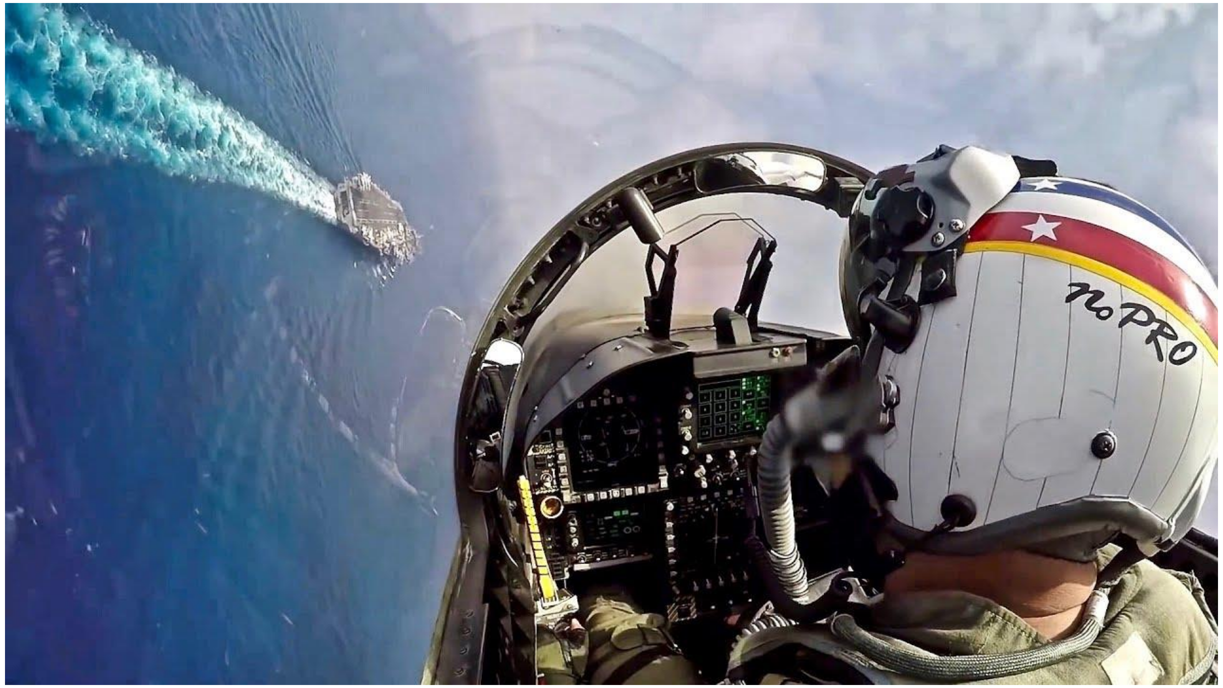


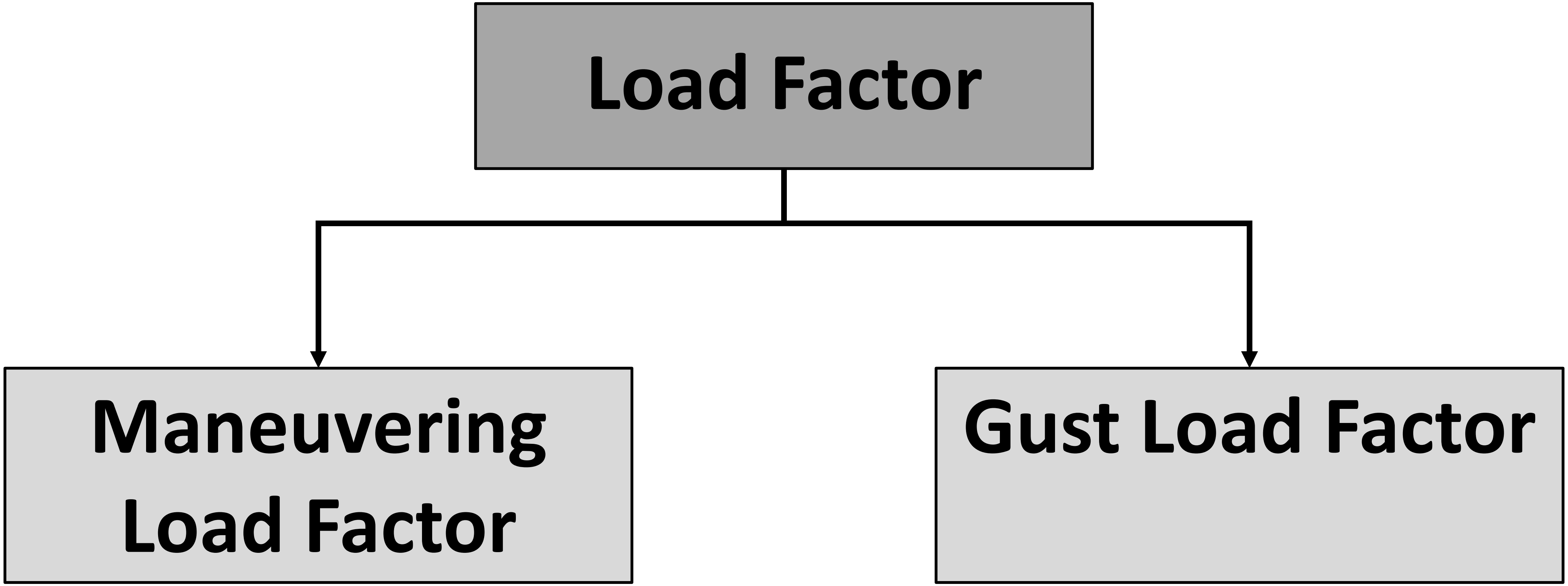
https://laughingsquid.com/wp-content/uploads/2017/12/an-inside-look-at-how-zero-gravity-airplanes-work.gif?w=750

Load Factor

A load factor of three means the total load on an aircraft's structure is three times its weight (3G).

The pilot Will also be subjected to 3 G's, he or she would be pressed down into the seat with a force equal to three times his or her weight.




Load Factor
Maneuvering Load Factor
Gust Load Factor

Design Loads

AERONÁUTICA CIVIL
Unidad Administrativa Especial

FEDERAL AVIATION ADMINISTRATION

Design loads for FAR23 (?)

1955

2005

1955

2005

1956

2006

Design loads for FAR23 (?)

Means of compliance (MOC)

§ 23.2010 Accepted means of compliance.

(a) An applicant must comply with this part using a means of compliance, which may include consensus standards, accepted by the Administrator.

(b) An applicant requesting acceptance of a means of compliance must provide the means of compliance to the FAA in a form and manner acceptable to the Administrator.

ASTM INTERNATIONAL

Design loads for ASTM F3116 (FAR23)

The positive limit maneuvering load factor $\boldsymbol{n}$ may not be less than*:

$\boldsymbol{n = 2.1 + \frac{24000}{W+10000}}$, for normal, utility, commuter. Where W = design maximum take-off weight (lb), except that n need not be more than 3.8.

$\boldsymbol{n = 6.0}$ for airplanes approved for aerobatics.

*Certification in the normal category applies to airplanes with a passenger-seating configuration of 19 or less and a maximum certificated takeoff weight of 19,000 pounds or less.

Design loads for ASTM F3116 (FAR23)

CATEGORY	POSITIVE LOAD LIMIT FACTOR	NEGATIVE LIMIT LOAD FACTOR
Normal, commuter and utility	3.8	-1.52 (0.4 times PLF)
Aerobatics	6.0	-3.0 (0.5 times PLF)

Maneuvering load factors lower than those specified in this section may be used if the airplane has design features that make it impossible to exceed these values in flight.

Design loads for FAR25

$\boldsymbol{n = 2.1 + \frac{24000}{W+10000}}$, $\boldsymbol{n}$ may not be less than 2.5 and not be greater than 3.8.

Negative load factor may not be less than 1.0

Must vary linearly with speed from the value at V_C Cruising speed to zero at V_D dive speed

Stall

- In a stall, the wing does not totally stop producing lift. Rather, it cannot generate adequate lift to sustain level flight
- A stall can occur at any pitch, attitude or airspeed
- The wing root reaches its critical AOA first making the stall progress outward toward the wingtip. By having the wing root stall first, aileron effectiveness is maintained at the wingtips, maintaining controllability of the aircraft

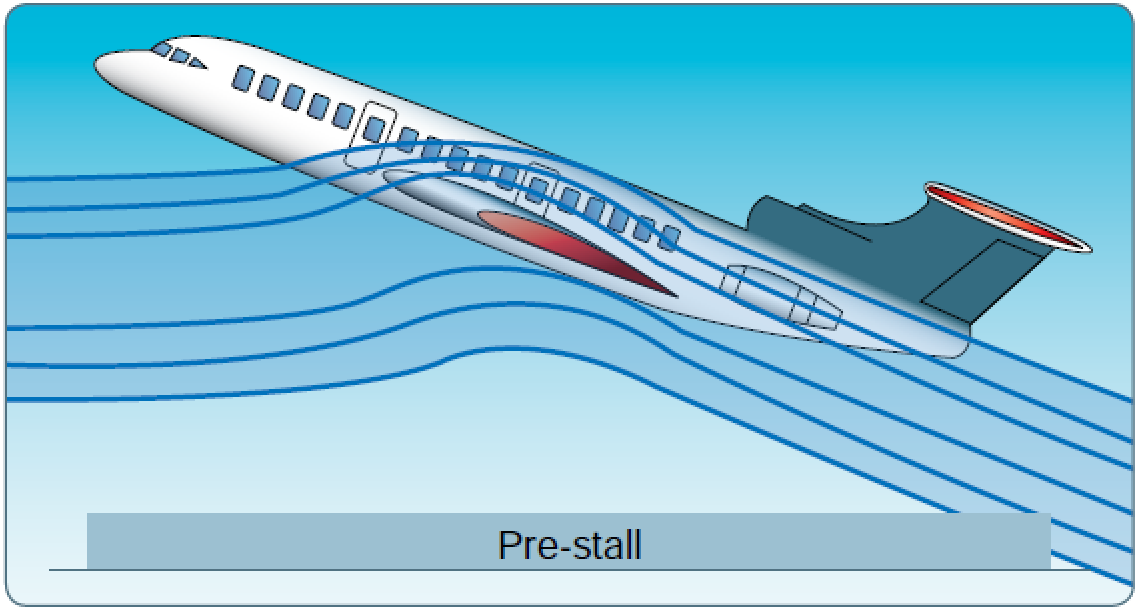


Figure 5-69. *Wingtip pre-stall.*

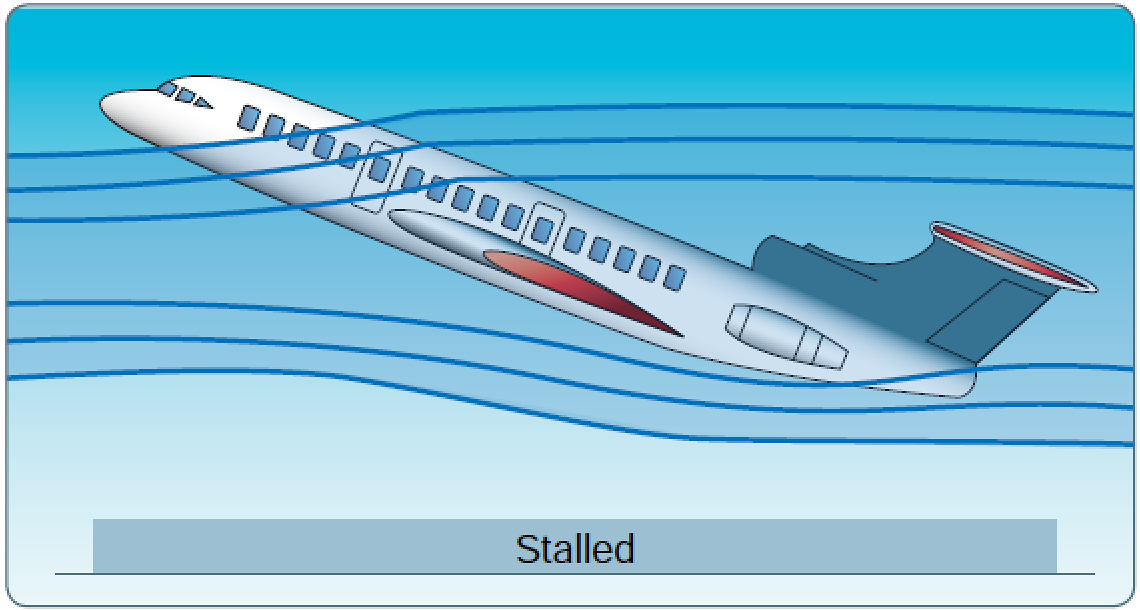


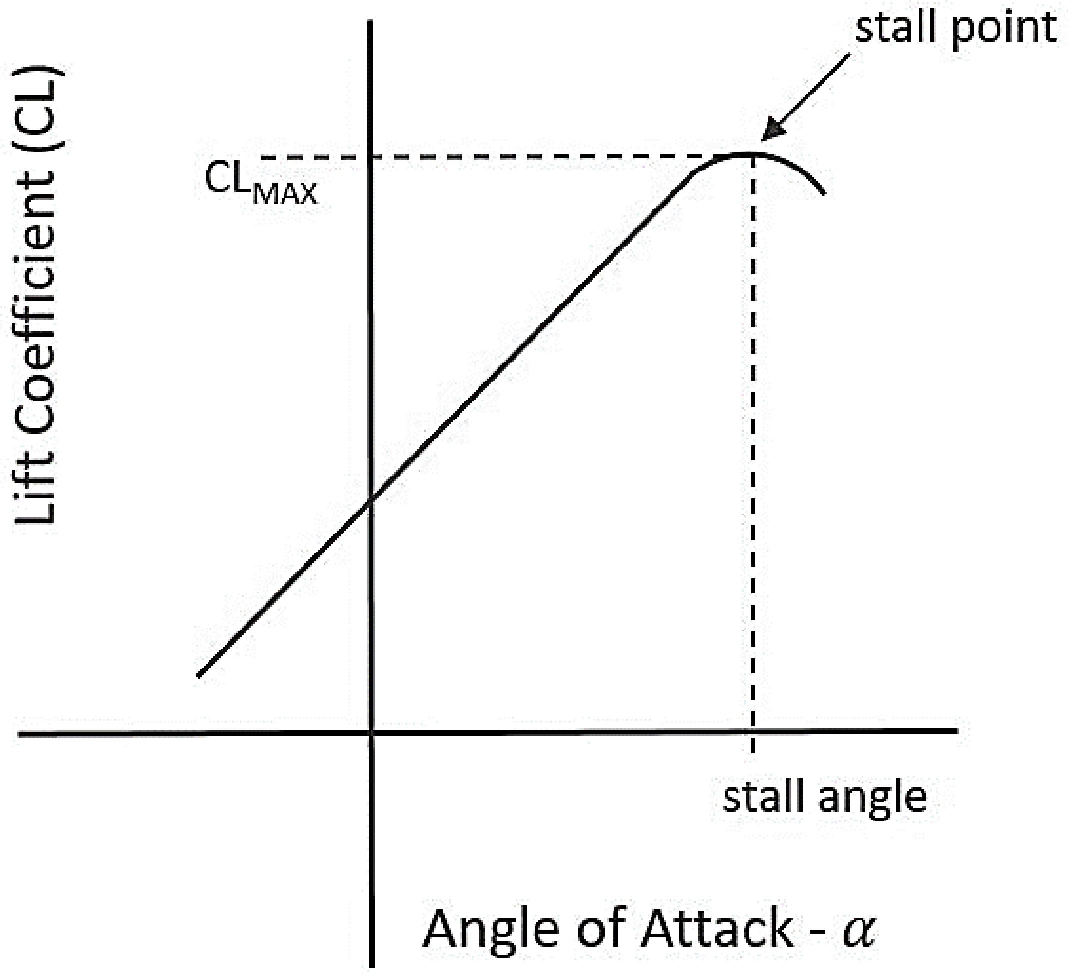
Figure 5-70. *T-tail stall.*

Stall speed

The lift coefficient C_L increases, with an increment in the AOA

After the aircraft reaches the maximum AOA, $(\alpha = \alpha_S)$ and $C_L = C_{Lmax}$ the lift force decrease rapidly

$$V_S = \sqrt{\frac{2W}{\rho S C_{Lmax}}}$$



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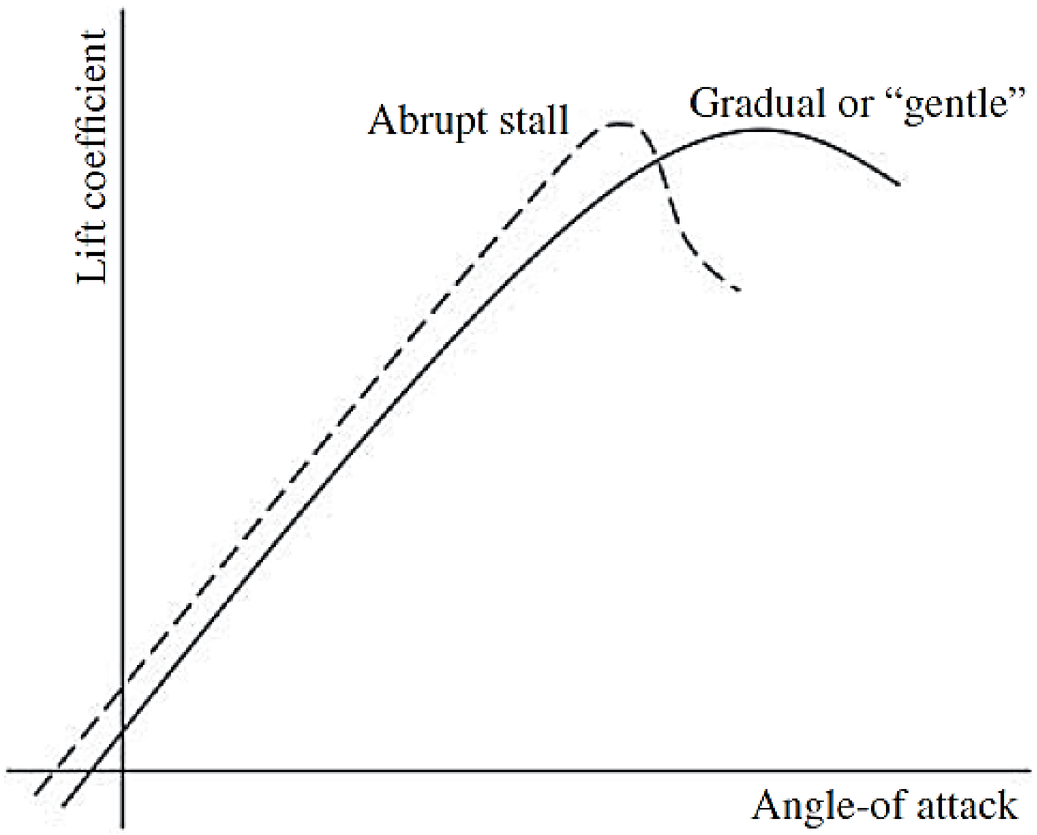


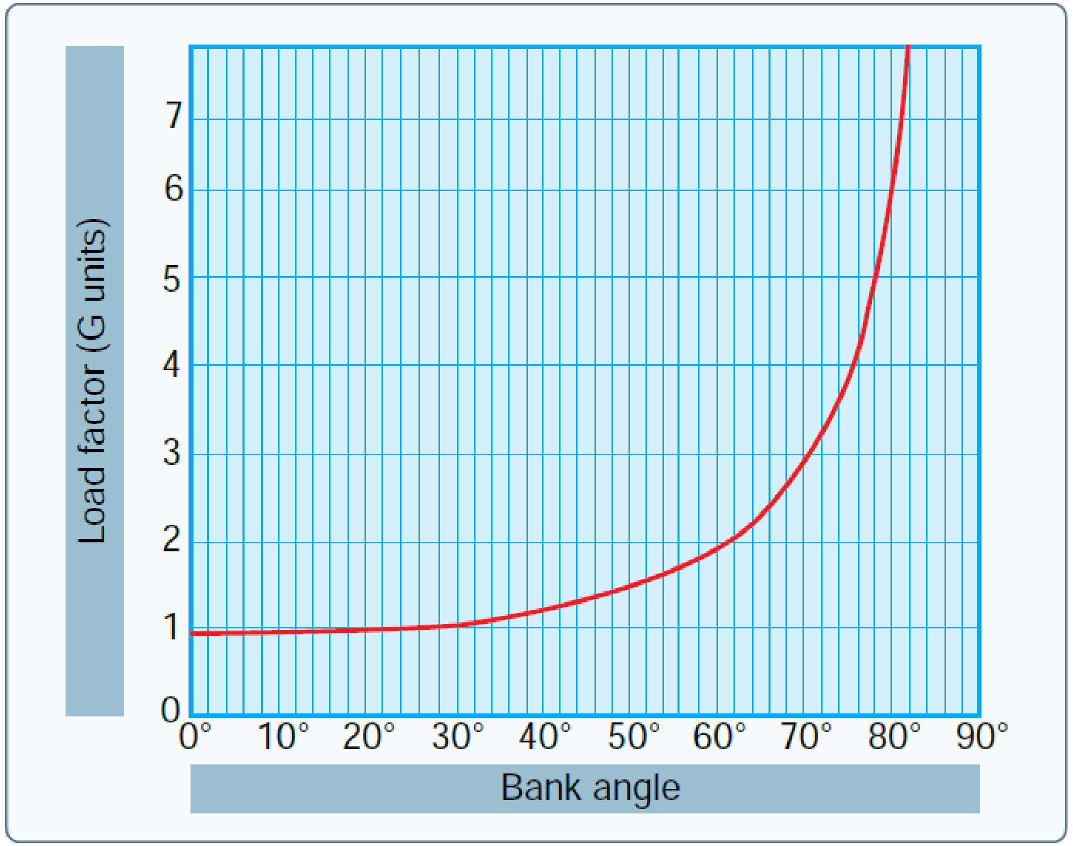
Stall speed and load factor

$$\boldsymbol{V_{Sm} = V_{SL}\sqrt{n}}$$

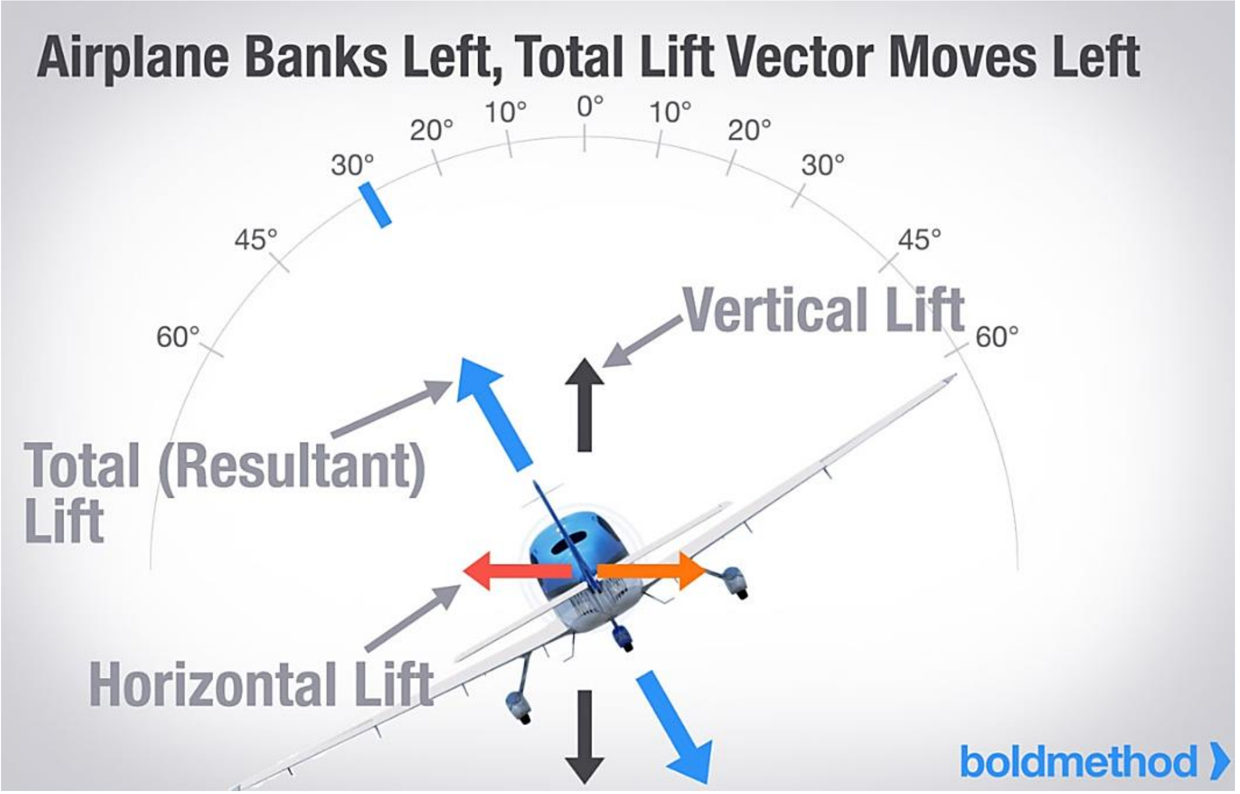
$V_{Sm} = stall\ speed\ in\ maneuvering$

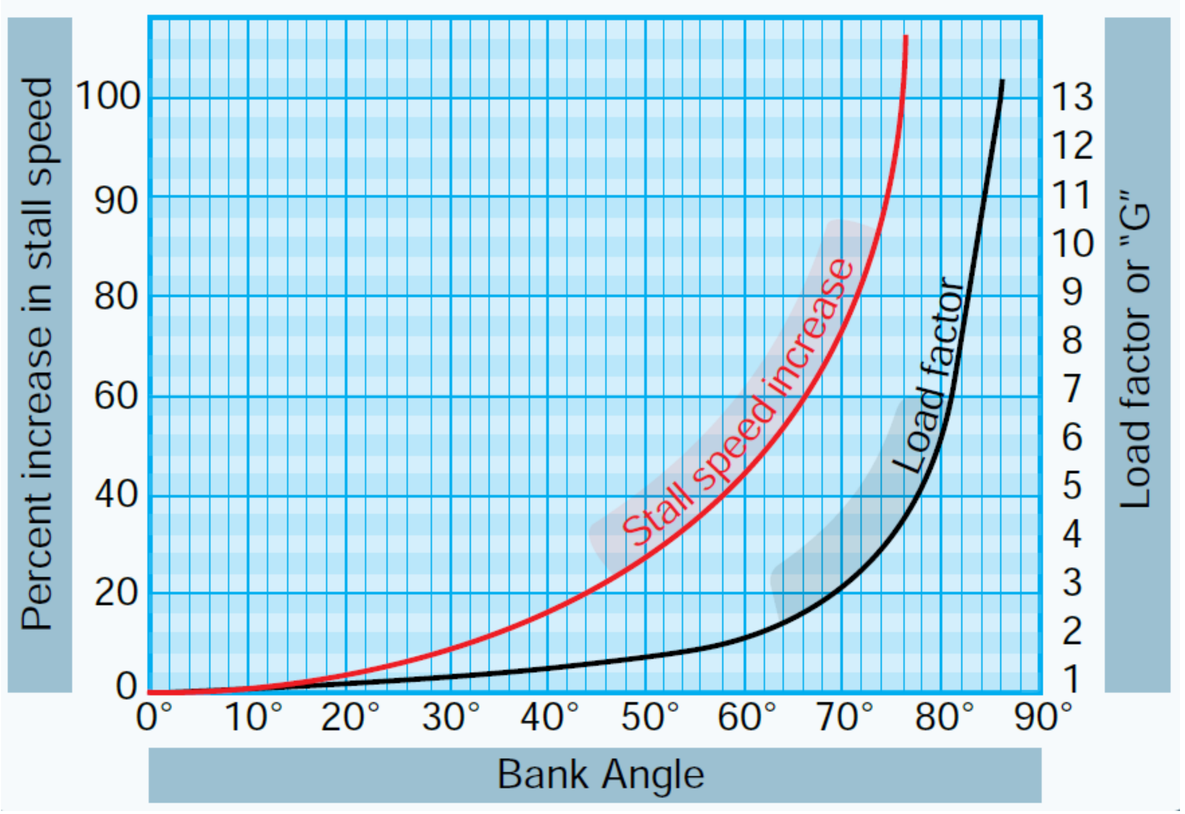
$V_{SL} = stall\ speed\ in\ level\ flight$

$n = load\ factor$



Stall speed and load factor





Stall speed and load factor

Table 3.16 Wing loadings and stall speeds of selected aircraft.

Aircraft	Primary function	Wing loading, W/S	Stall speed, V_s
Wright *Flyer I*	1st airplane	1.47 lb/ft^2 (7.18 kg$_f$/m^2)	22 mph (35 km/h)
Schweitzer 2–33	Glider trainer	4.74 lb/ft^2 (23.1 kg$_f$/m^2)	36 mph (58 km/h)
Beechcraft A36 *Bonanza*	General aviation	20.2 lb/ft^2 (98.6 kg$_f$/m^2)	59 mph (95 km/h)
North American P-51 *Mustang*	WW II fighter	41.2 lb/ft^2 (201 kg$_f$/m^2)	95.4 mph (154 km/h)
Northrop T-38 *Talon*	Military jet trainer	69.5 lb/ft^2 (339 kg$_f$/m^2)	146 mph (235 km/h)
Boeing 777–300	Commercial airliner	143 lb/ft^2 (698 kg$_f$/m^2)	165 mph (265 km/h)

Stall speed and load factor

Example: an aircraft has a stall speed of 40 knots, and you put a load factor of 4G. ¿What is the new stall speed?

$$V_{Sm} = V_{SL}\sqrt{G}$$

$$V_{Sm} = 40\,\sqrt{4}$$

$$V_{Sm} = 80\;knots$$

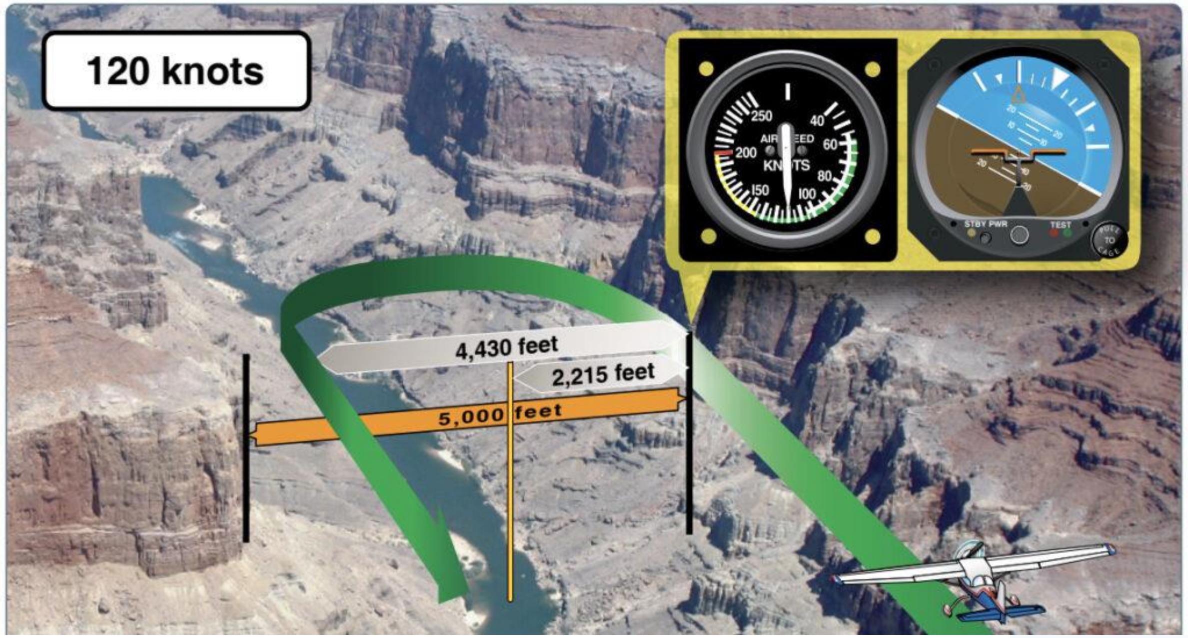
Unsymmetrical loads

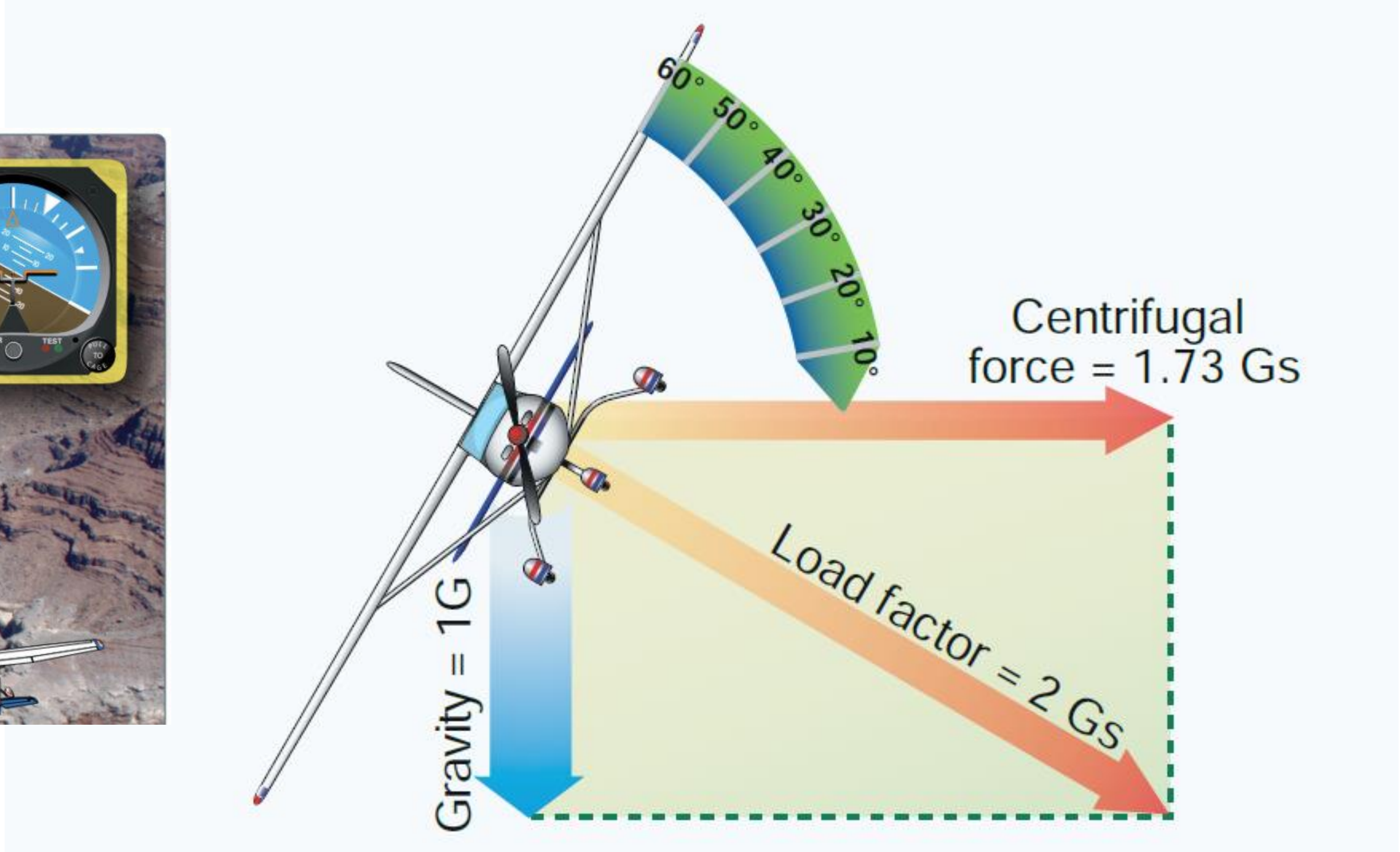
Let's consider three cases involving **radial acceleration** which generates curved trajectories.

Three cases are considered:

- Level turn
- Pull up
- Pull down

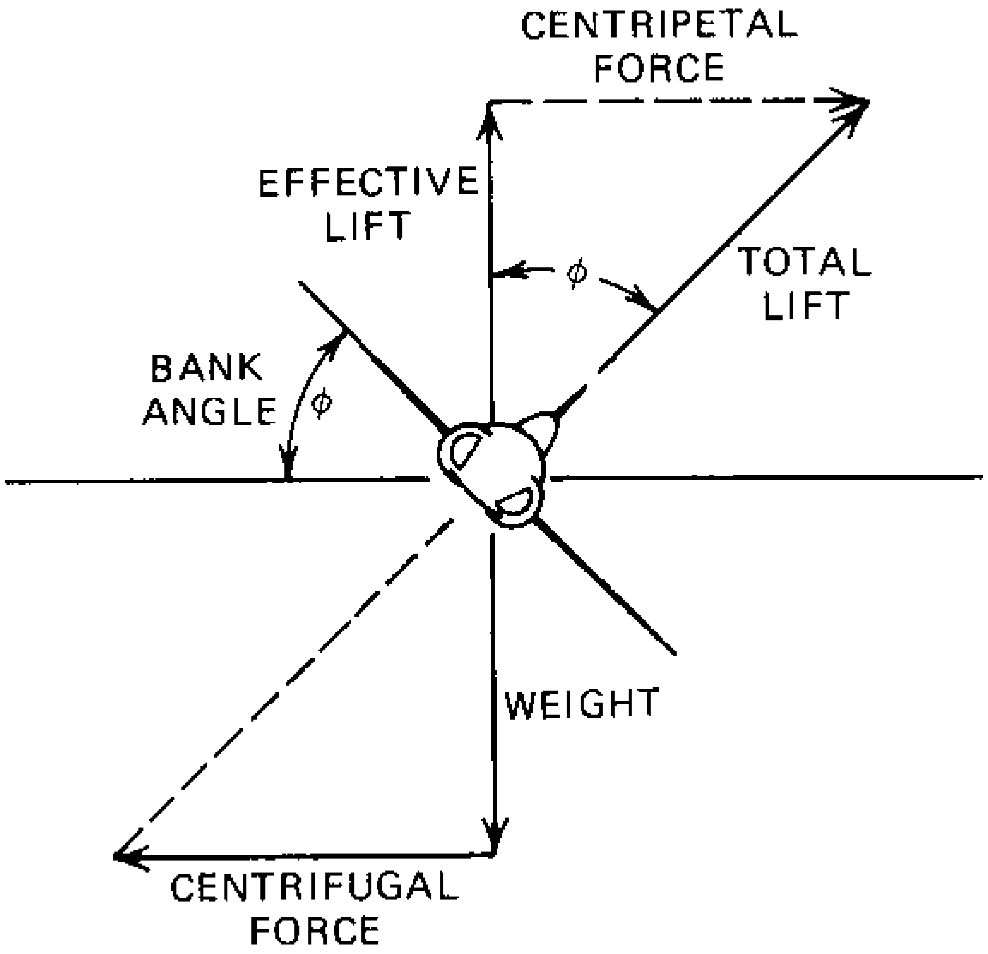
Level turn


120 knots
4,430 feet
2,215 feet
5,000 feet


60°
50°
40°
30°
20°
10°
Centrifugal force = 1.73 Gs
Load factor = 2 Gs
Gravity = 1G

Level turn

- The wings of the airplane are banked through the angle ϕ bank angle)
- Therefore, the lift vector is also tilted at the angle ϕ to the vertical
- The lift vertical component equals the weight



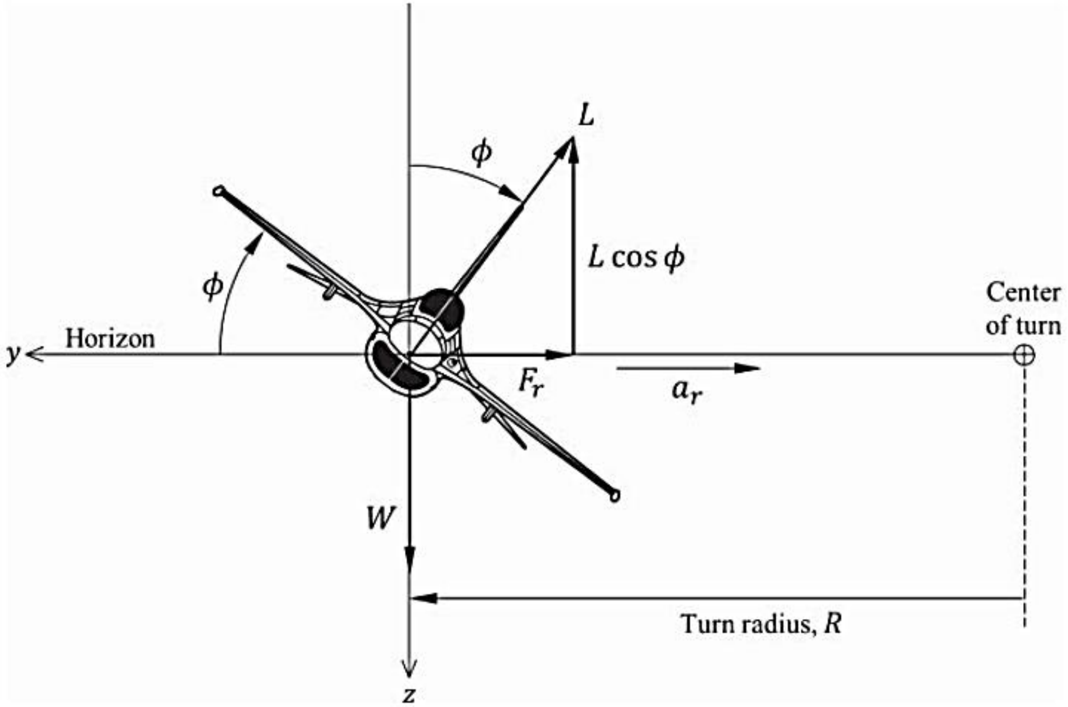
$$L\cos\phi = W \qquad n = \frac{1}{\cos\phi} = \sec\phi$$

Level turn

The resultant of $\boldsymbol{L}$ and $\boldsymbol{W}$,leads to a resultant force $\boldsymbol{F_r}$, which acts in the horizontal plane.

This resultant force is perpendicular to the flight path, causing the airplane to turn in a circular path with a curvature radius $\boldsymbol{R}$ and a turn rate $\boldsymbol{d\theta/dt}$

The resultant force is:



$$\boldsymbol{F_r = \sqrt{L^2 - W^2}}$$

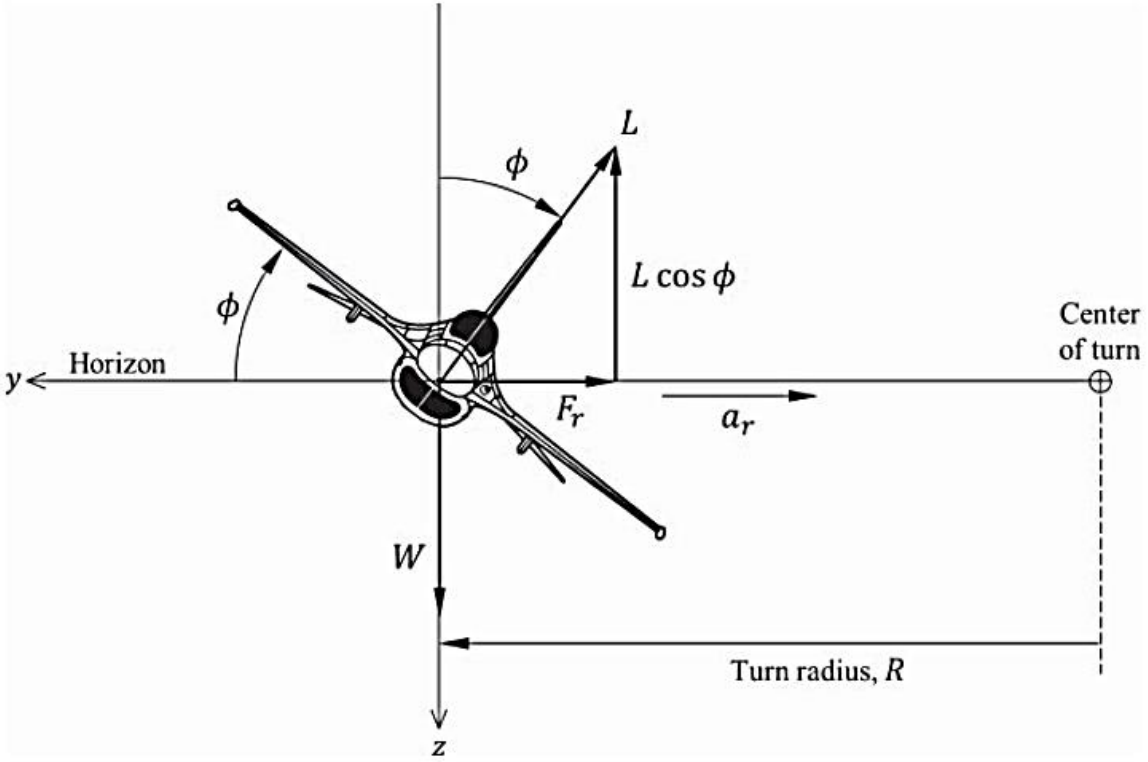
Level turn

Using trigonometric functions:

$$\boldsymbol{tan\phi = \sqrt{n^2 - 1}}$$

Therefore, the **turn rate** will also be:

$$\boldsymbol{\omega = \frac{g\,tan\phi}{V_\infty} = \frac{g\,\sqrt{n^2 - 1}}{V_\infty}}$$



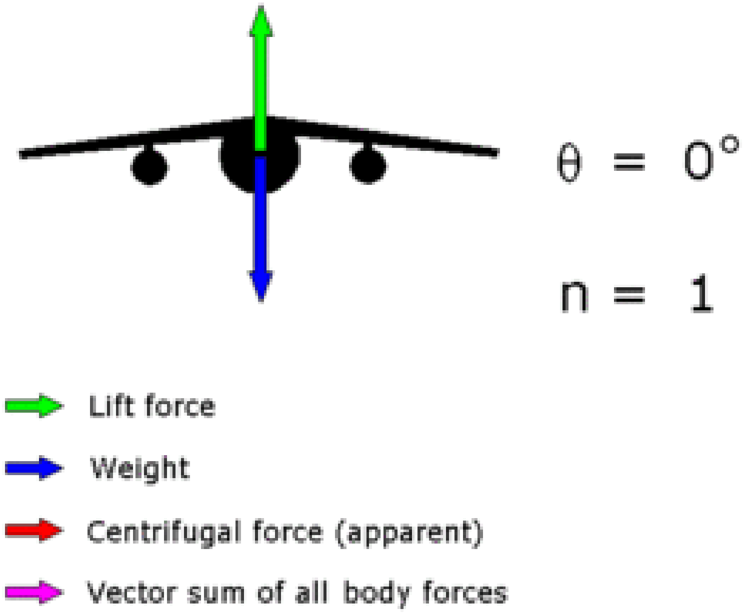
Level turn

Regardless the maneuver performance of an airplane, both, military and civil, it is advantage to have:

- **The smallest possible radius $\boldsymbol{R}$**
- **The largest possible turn rate $\boldsymbol{\omega}$**

The previous information is obtained with:

- **The highest possible load factor**
- **The lowest possible velocity**

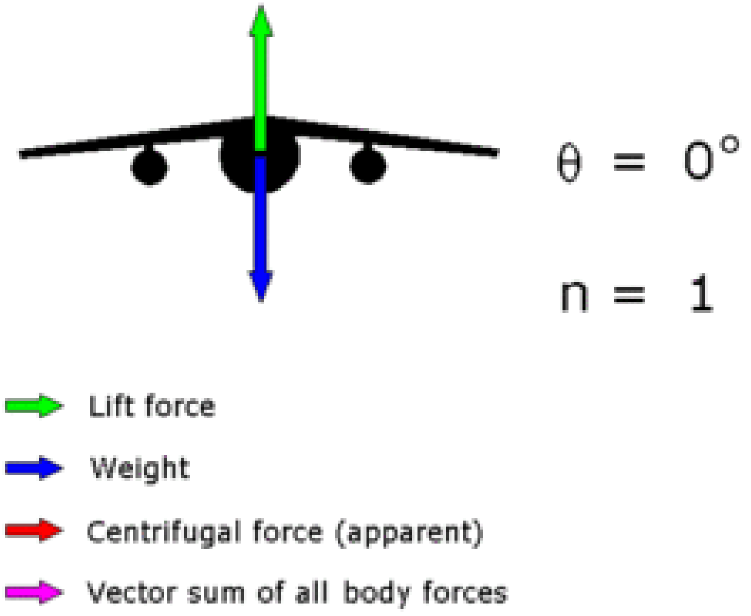


Level turn

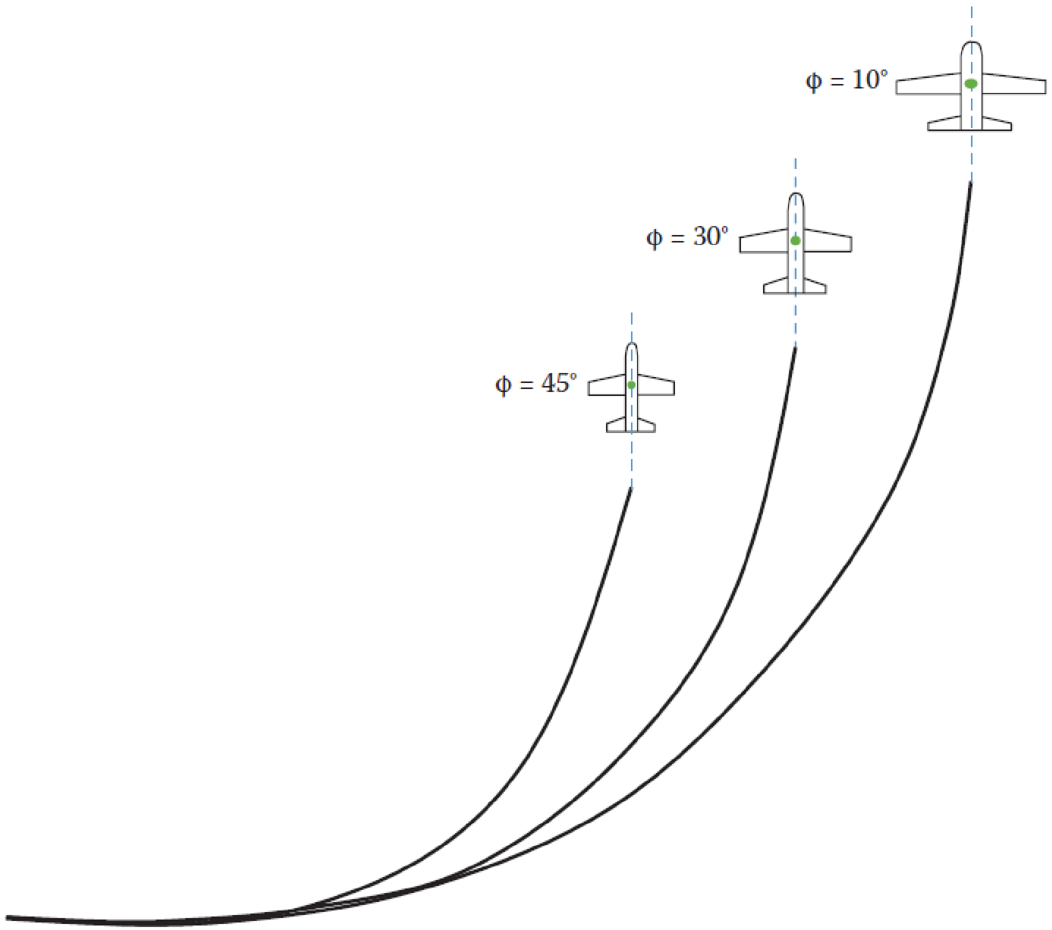
Turn radius

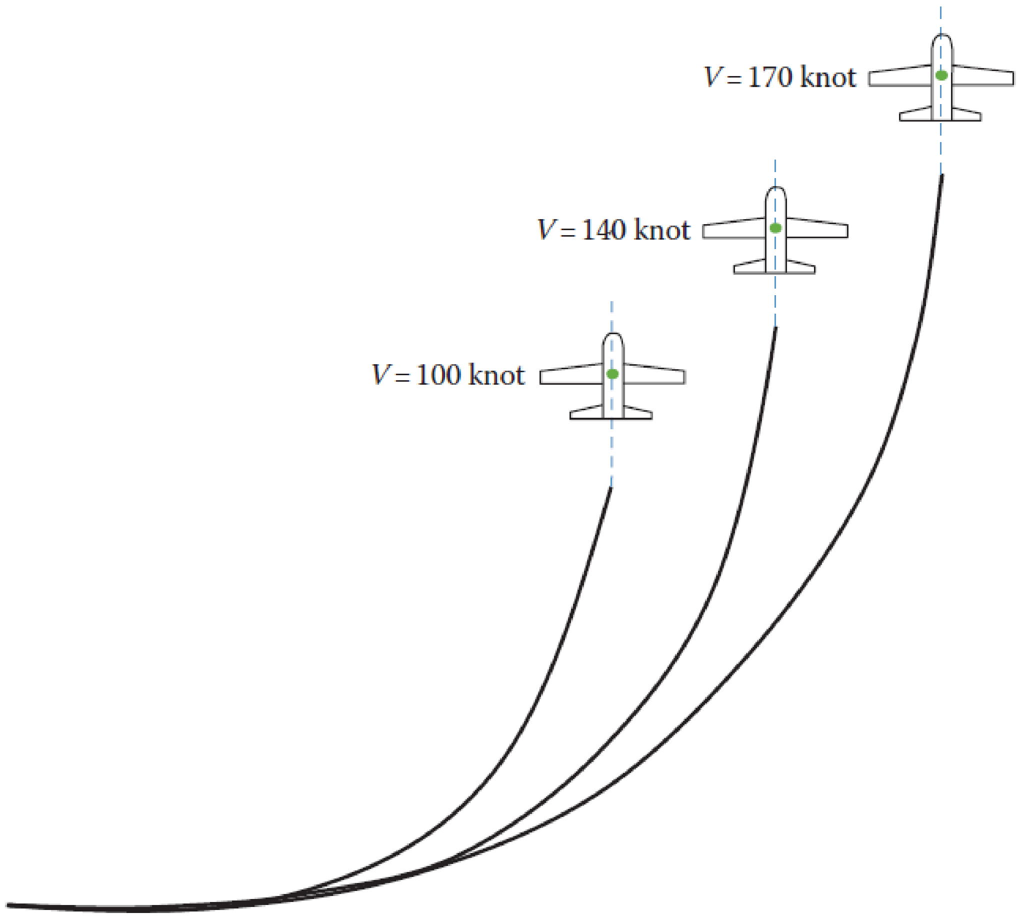
$$R = \frac{V^2}{g\sqrt{n^2 - 1}}$$

$$R = \frac{V^2}{g\ tan\phi}$$



Level turn



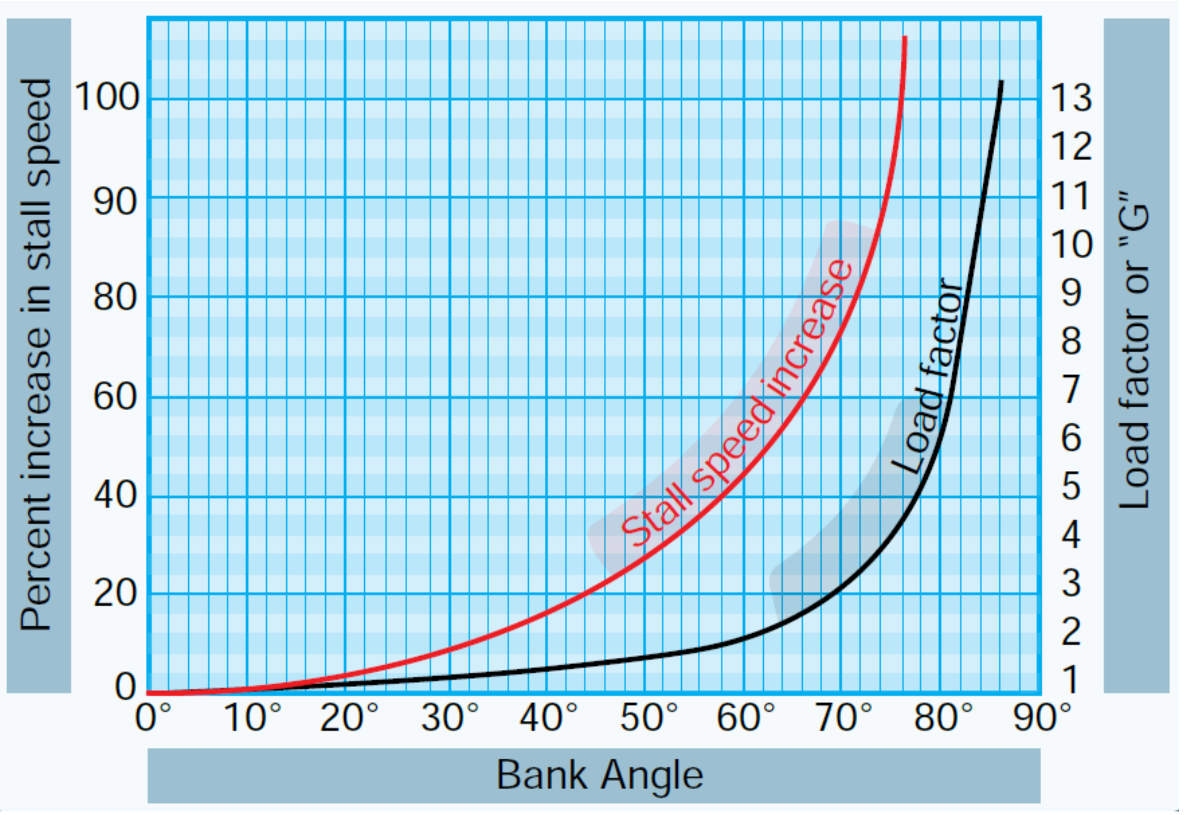


Level turn

The stalling speed increases in proportion to the square root of the load factor

Level turn stall speed:

$$V_{stall\_turn} = \sqrt{\frac{2\,W}{\rho\, S\, C_{Lmax}\, cos\phi}}$$



Example 1

Determine the stall speed of a utility aircraft with a mass of 4500 kg, a wing area of 19.5 m^2, and the maximum lift coefficient of 2.5. Perform this analysis at the sea level for the following two flight conditions:

a. Cruising flight
b. Turning flight with a 30° bank angle

Example 2

Consider a very light aircraft (VLA) with a mass of 750 kg and an airspeed of 100 knots.

a. If the maximum permissible load factor is 3.8, what is the equivalent maximum bank angle?
b. Determine the corresponding turn radius for a coordinated turn with such a bank angle.
c. If the aircraft is turning coordinately with a radius of 300 m and a bank angle of 30°, calculate the airspeed and load factor.

a. $\mathbf{74.74° \approx 75°}$ **b.** $\mathbf{73.6}\boldsymbol{m}$ **c.** $\mathbf{41.2}\ \boldsymbol{m/s}$

Example 3

An aircraft enters a level turn at a constant airspeed of 315 mph. It completes a full 360∘ circle in 27 s, maintaining the entry speed constant throughout the turn. Calculate the turn rate, turn radius, load factor, and bank angle of the level turn.

$$\boldsymbol{\omega = 0.2327\ rad/s}$$

$$\boldsymbol{R = 605.123\ \mathrm{m}}$$

$$\boldsymbol{n = 3.487}$$

$$\boldsymbol{\phi = 73.33^\circ}$$

Level turn

The standard turn rate is approximately 3°/s or 0.052 rad/s, therefore 360° turn is completed in around 2 minutes.

ICAO suggest $\boldsymbol{\phi = 25°}$ *

* Aircraft Operations, Volume I Flight Procedures

Segment or fix of turn location	*Speed (IAS)*[1]	*Altitude/height*	*Wind*	*Bank angle*[2]
Departure	Final missed approach IAS + 10%, see Table I-4-1-1 or Table I-4-1-2 [3]	Turn at altitude/height: Specified altitude/height Turn at turn point: A/D elevation + height based on 10% climb from DER	95% omnidirectional wind or 56 km/h (30 kt) for wind spirals	15° until 305 m (1°000 ft) 20° between 305 m (1 000 ft) and 915 m (3 000 ft) 25° above 915 m (3 000 ft)
En route	585 km/h (315 kt)	Specified altitude	95% probability wind or ICAO standard wind[4]	15°
Holding	Tables I-6-1-1 and I-6-1-2[1]	Specified altitude	ICAO standard wind [4]	23°
Initial approach – reversal and racetrack procedures	Table I-4-1-1 or Table I-4-1-2	Specified altitude	ICAO standard wind[4] or statistical wind	25°
Initial approach – DR track procedures	CAT A, B: 165 to 335 km/h (90 to 180 kt) CAT C, D, E: 335 to 465 km/h (180 to 250 kt)	CAT A, B: 1 500 m (5 000 ft) CAT C, D, E: 3 000 m (10 000 ft)	ICAO standard wind[4] DR leg: 56 km/h (30 kt)	25°
IAF, IF, FAF	See Tables I-4-1-1 and I-4-1-2 Use Initial approach speed for turn at IAF or IF Use maximum final approach speed for turn at FAF	Specified altitude	95% omnidirectional wind or 56 km/h (30 kt)	25°

Segment or fix of turn location	*Speed (IAS)*[1]	*Altitude/height*	*Wind*	*Bank angle*[2]
Missed approach	Table I-4-1-1 or Table I-4-1-2[3]	A/D elevation + 300 m (1 000 ft)	56 km/h (30 kt)	15°
Visual manoeuvring using prescribed track	See Tables I-4-1-1 and I-4-1-2	A/D elevation + 300 m (1 000 ft)	46 km/h (25 kt)	25°
Circling	See Tables I-4-1-1 and I-4-1-2	A/D elevation + 300 m (1 000 ft)	46 km/h (25 kt)	20°

GENERAL NOTES:

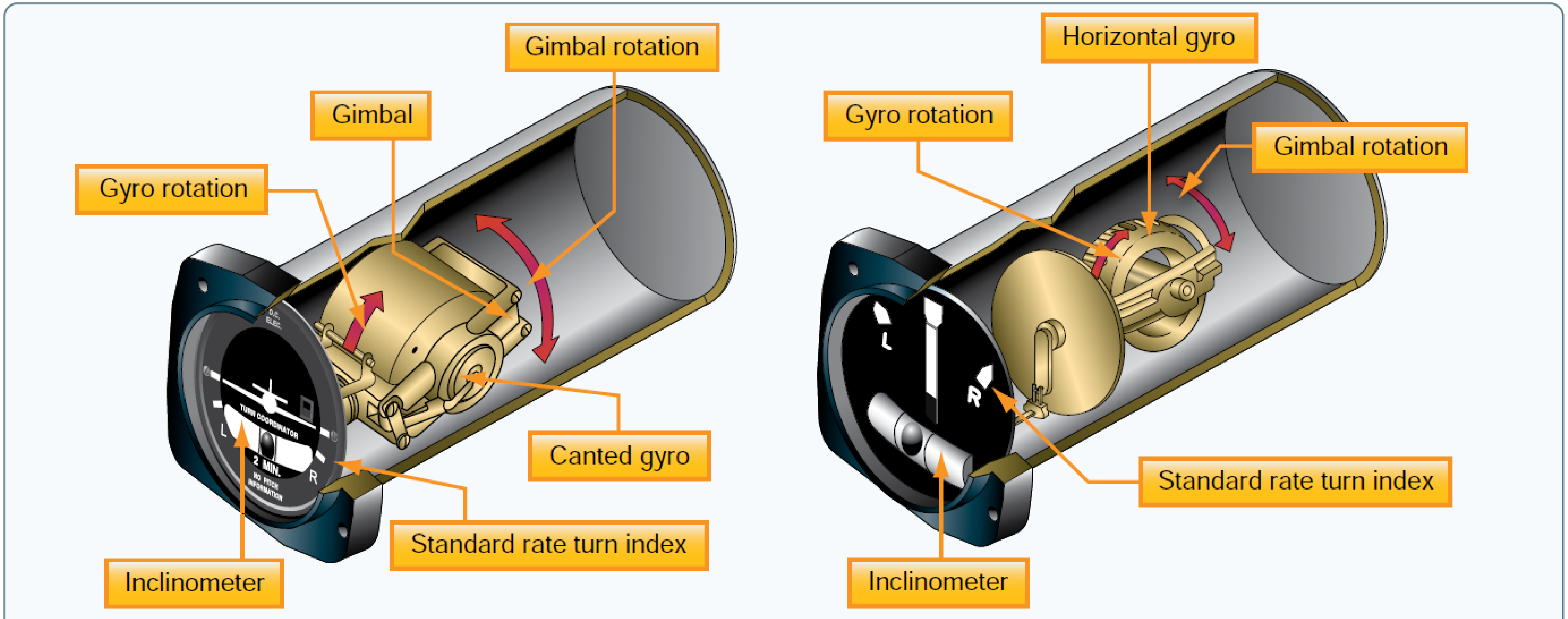
1. *For the specific application of the parameters in the table, see the applicable chapters in this document.*
2. *The rate of turn associated with the stated bank angle values in this table shall not be greater than 3°/s.*

Turn indicator

Turn coordinator

Turn and slip indicator

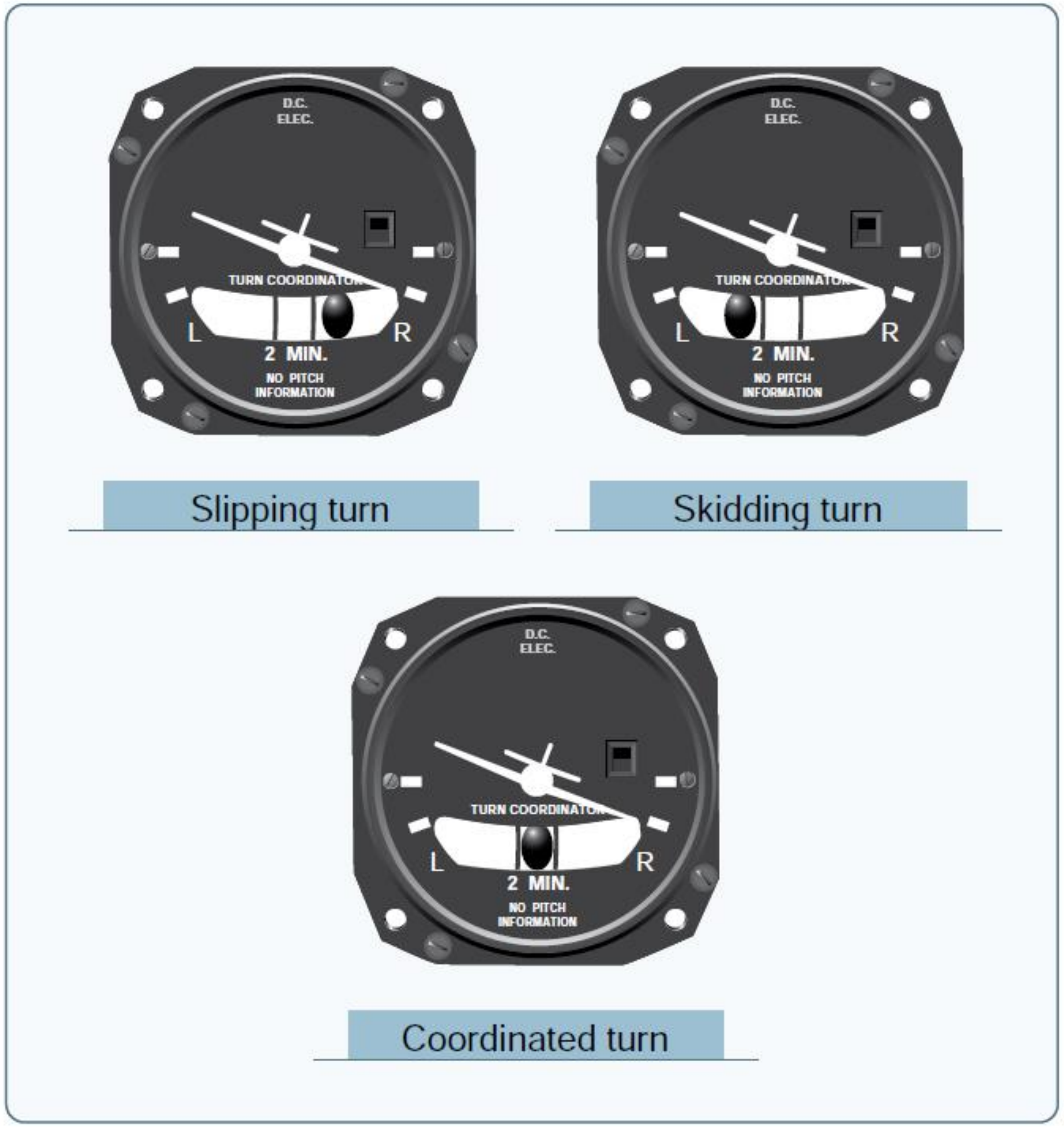
Turn indicator



Turn coordinator

Turn-and-slip indicator

Turn indicator

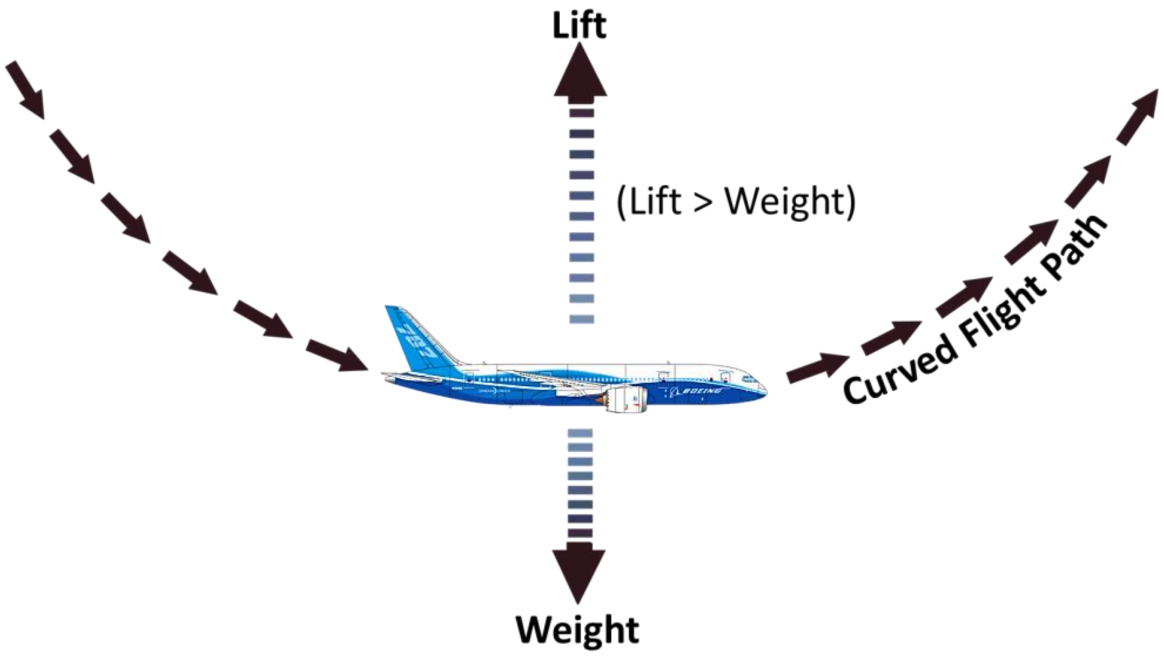


Pull-up

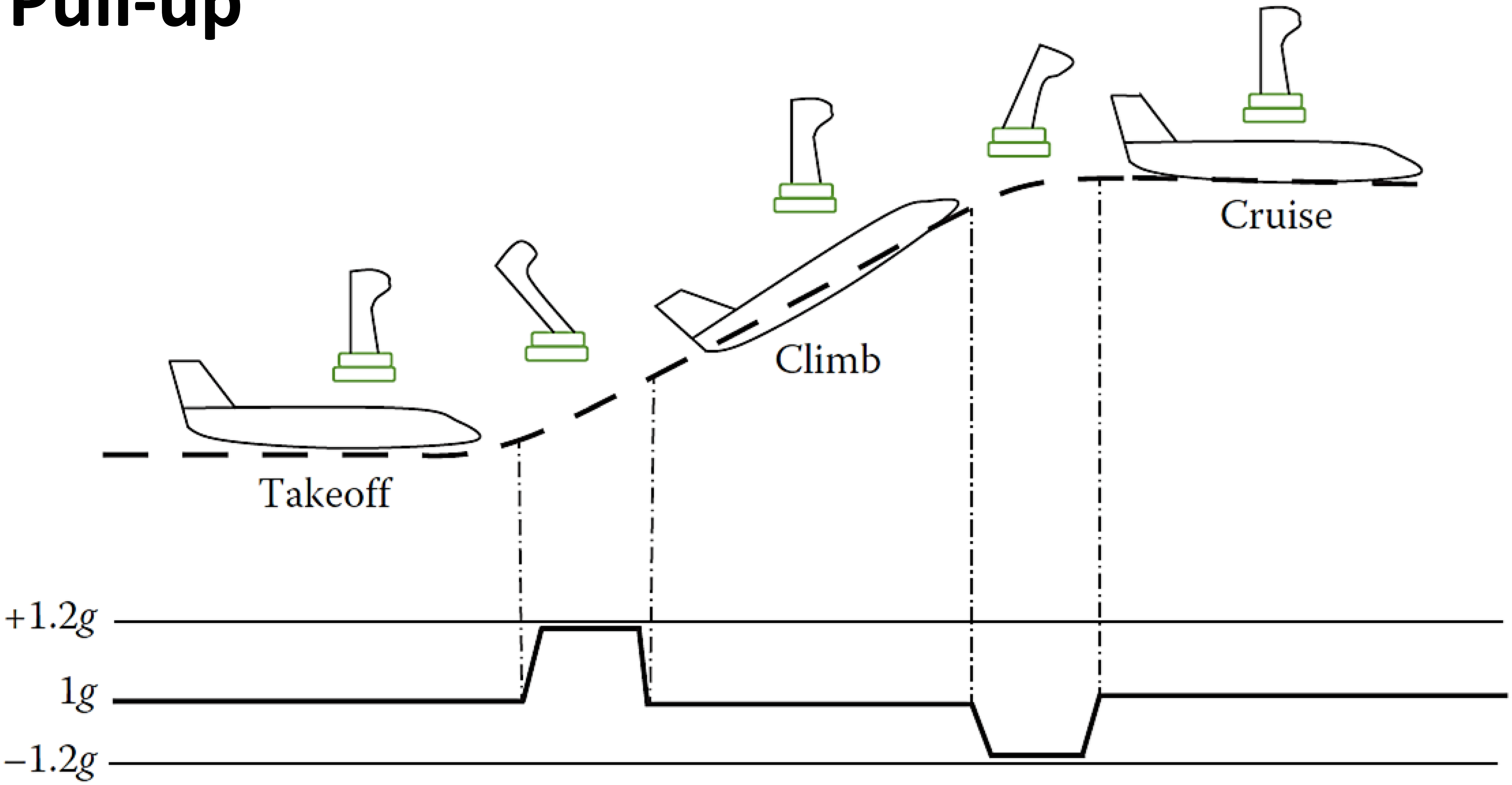
Consider another case of turning flight, where an airplane initially in straight level $(L = W)$ suddenly experiences an **increase in lift $L > W$**

The airplane will begin to turn upward

The flight path becomes curved in the vertical plane, with a turn rate $\omega = d\theta/dt$



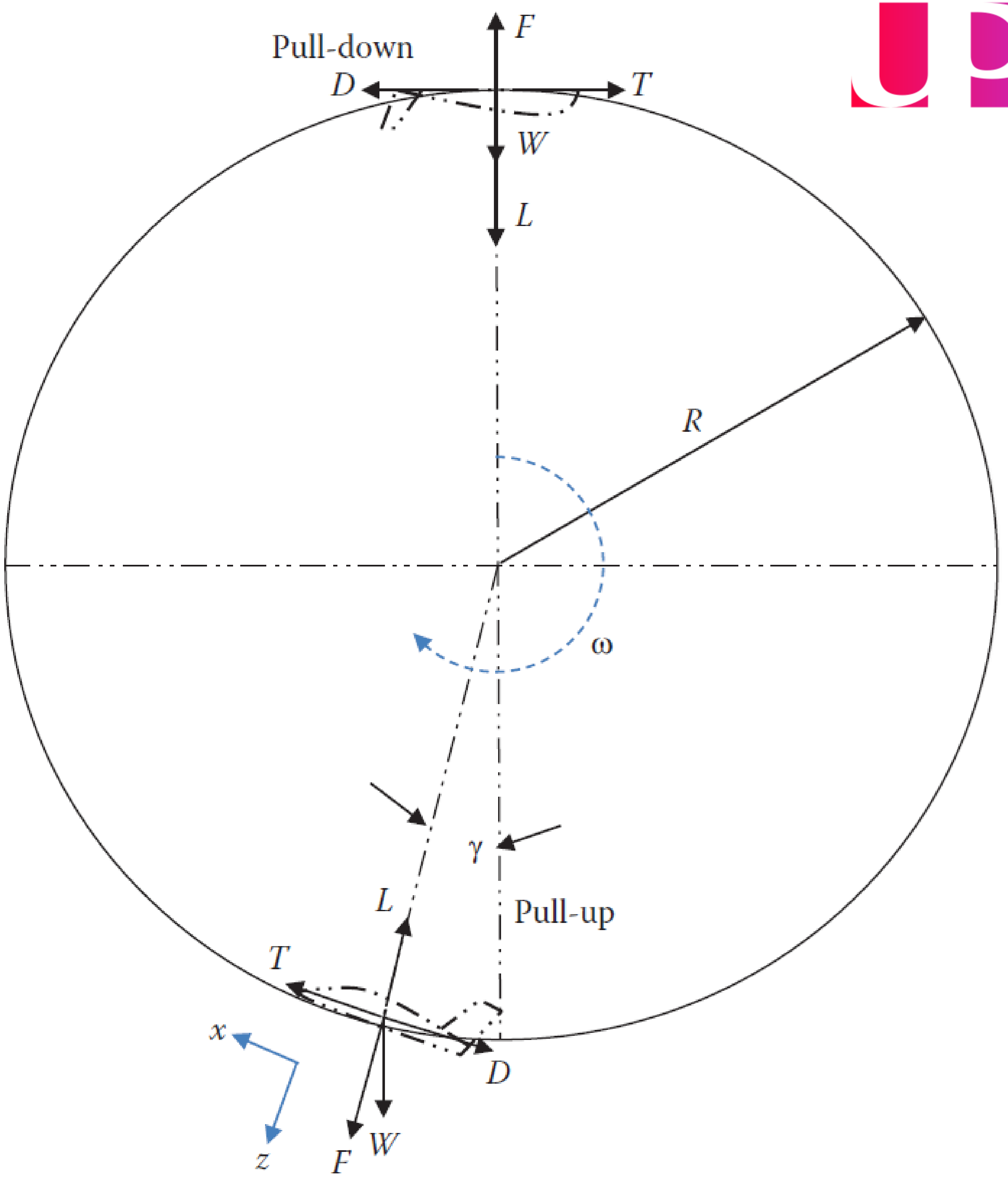
Pull-up



Pull-up

There is an important distinction between vertical maneuvers and a coordinated level turn.

In a **level coordinated turn**, **no force** (other than human's weight) **is felt** by human onboard, while in a **vertical maneuver**, a **centrifugal force** (via a centripetal acceleration) **is felt**



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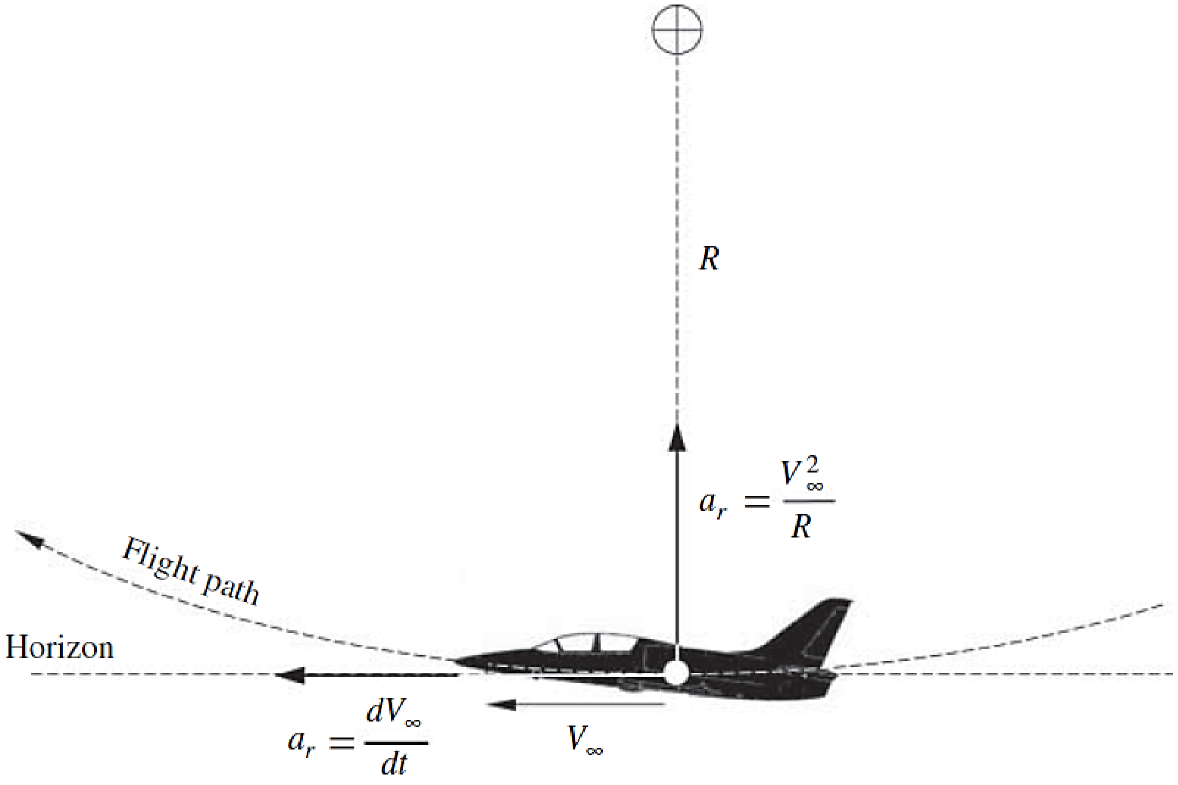
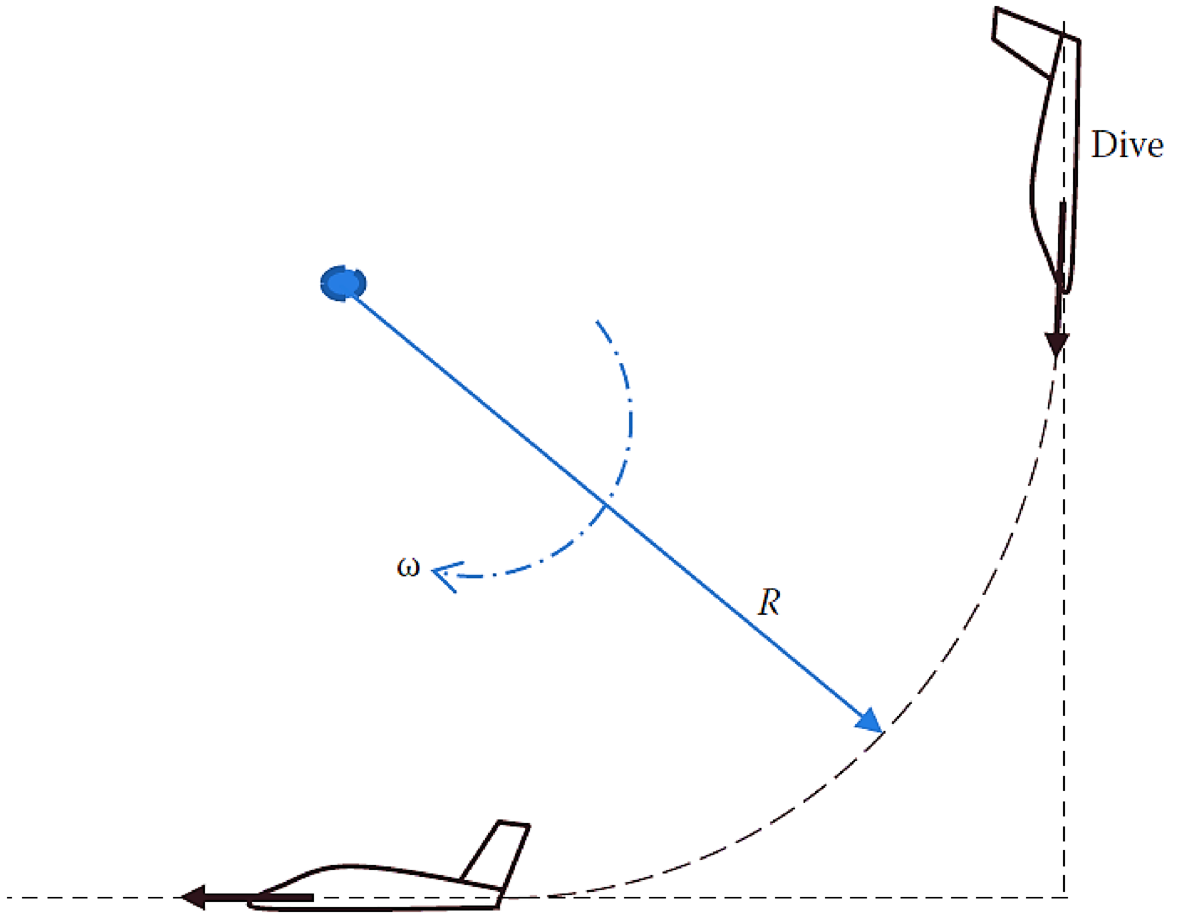


Pull out

The flight path of a pull out from a dive resembles the lower right quarter of a circle.

In the case of a pull out of a dive, the radius is limited by:

- The dive speed ($<V_{NE}$)
- The maximum allowable load factor



Pull out

The dive speed should not pass the never exceed V_{NE} speed. Otherwise, the structure will break.

With a given dive speed, the minimum radius in a pull out is determined by:

$$\boldsymbol{R_{min} = \frac{V_{dive}^2}{g\,(n_{max} - 1)}}$$

And the load factor will be:

$$\boldsymbol{n = \frac{V^2}{R\,g} + 1}$$

Example

Consider an acrobatic aircraft with a mass of 1,500 kg diving with a velocity of 100 knots (1 knot = 0.514 m/s)

a. If the aircraft pulls out of the dive with a radius of 150 m, determine the load factor.

b. If the maximum allowable load factor is 6, determine the minimum radius or a pull-out of this dive.

a. $\mathbf{n = 2.8}$ **b.** $\boldsymbol{R_{min} = 54\ \mathrm{m}}$

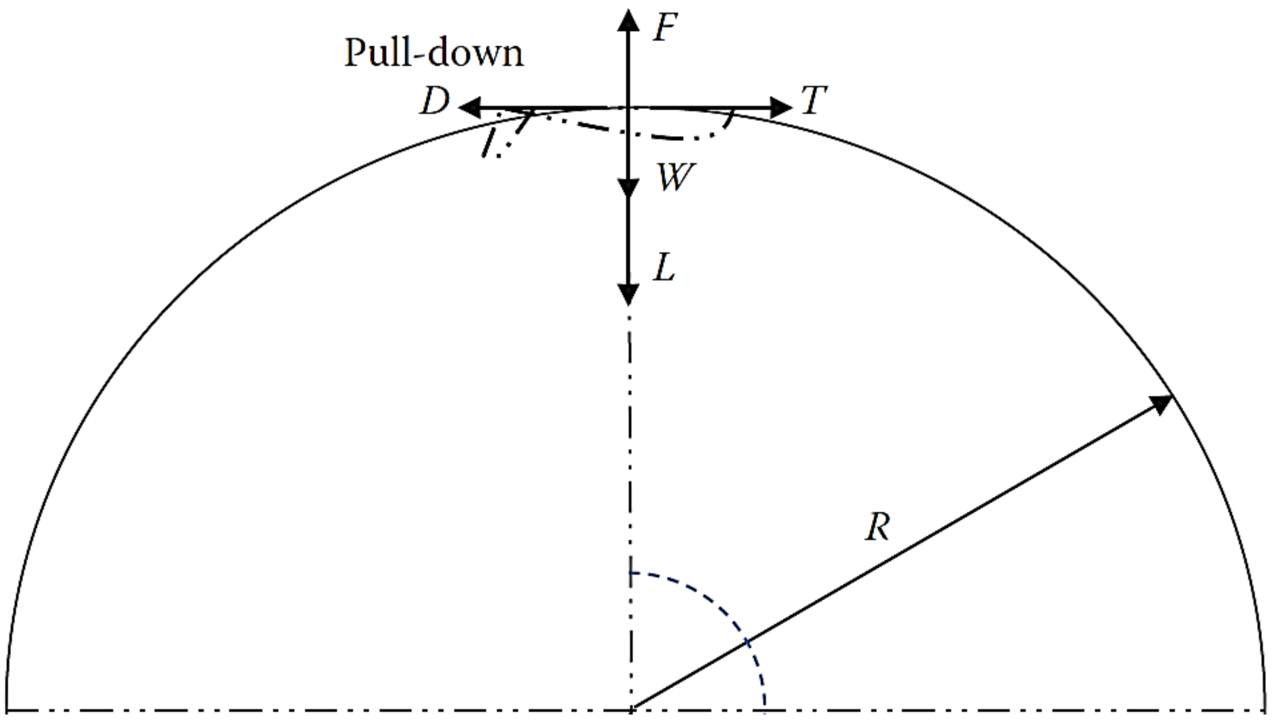
Pull down (inverted position)

Similar analysis for Pull up maneuver:

$$R = \frac{V^2}{g\,(n+1)}$$

Turn rate is defined by:

$$\omega = \frac{g\,(n+1)}{V}$$



Another flight maneuvers

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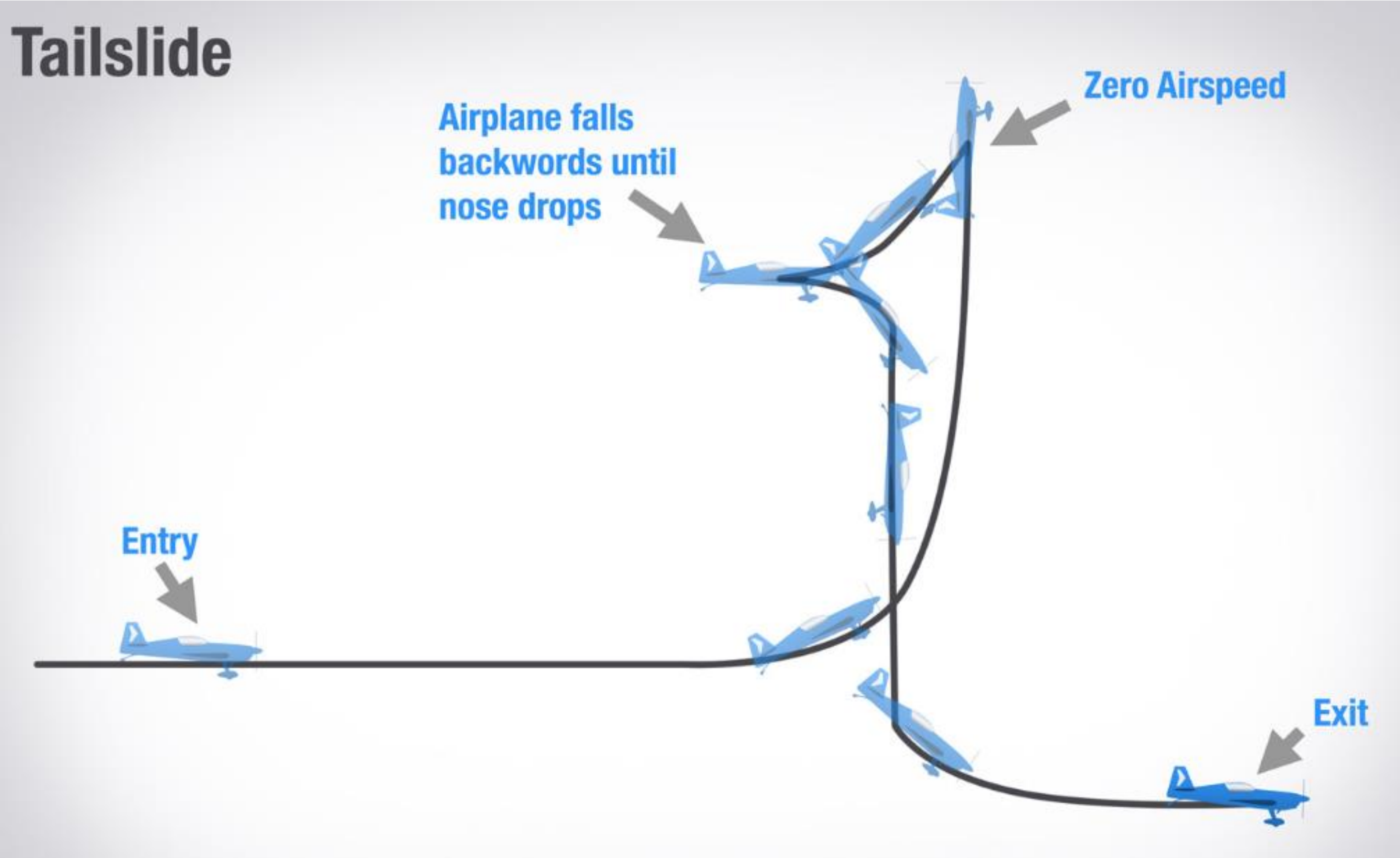


V speeds

V_1	The speed beyond which takeoff should no longer be aborted
V_A	Design maneuvering speed. This is the speed above which it is unwise to make full application of any single flight control (or "pull to the stops") as it may generate a force greater than the aircraft's structural limitations
V_C	Design cruise, also known as the optimum cruise speed
V_D	Design diving speed, the highest speed planned to be achieved in testing
V_{MO}	Maximum operating limit speed. Exceeding V_{MO} may trigger an overspeed alarm
V_{NE}	Never exceed speed
V_R	Rotation speed. The speed at which the pilot begins to apply control inputs to cause the aircraft nose to pitch up, after which it will leave the ground
V_S	Stall speed or minimum steady flight speed for which the aircraft is still controllable

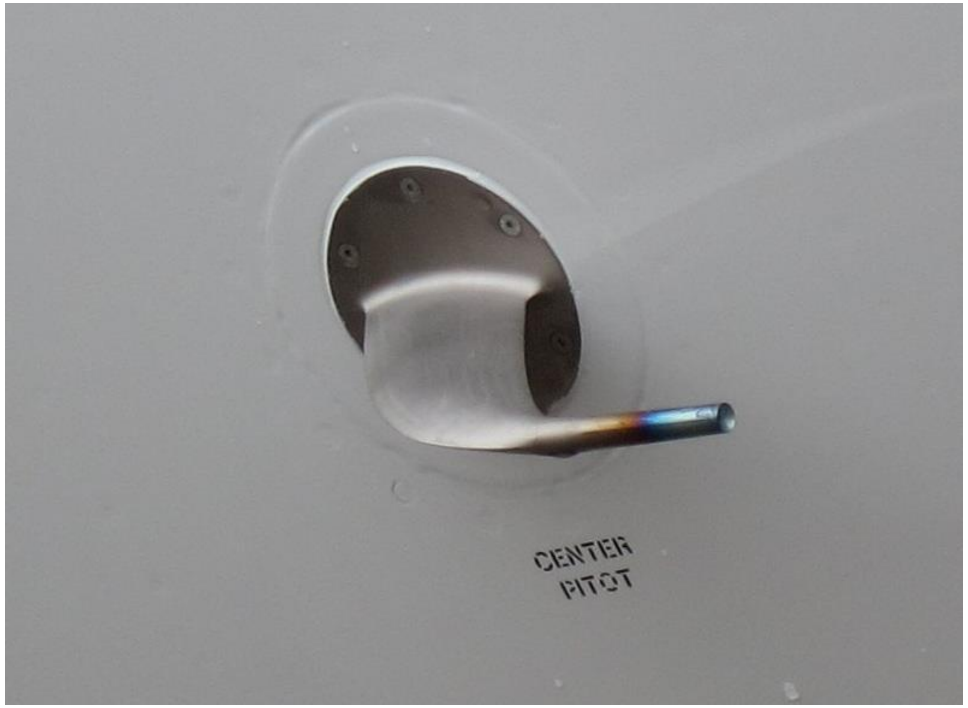
https://www.ecfr.gov/current/title-14/chapter-I/subchapter-A/part-1/section-1.2

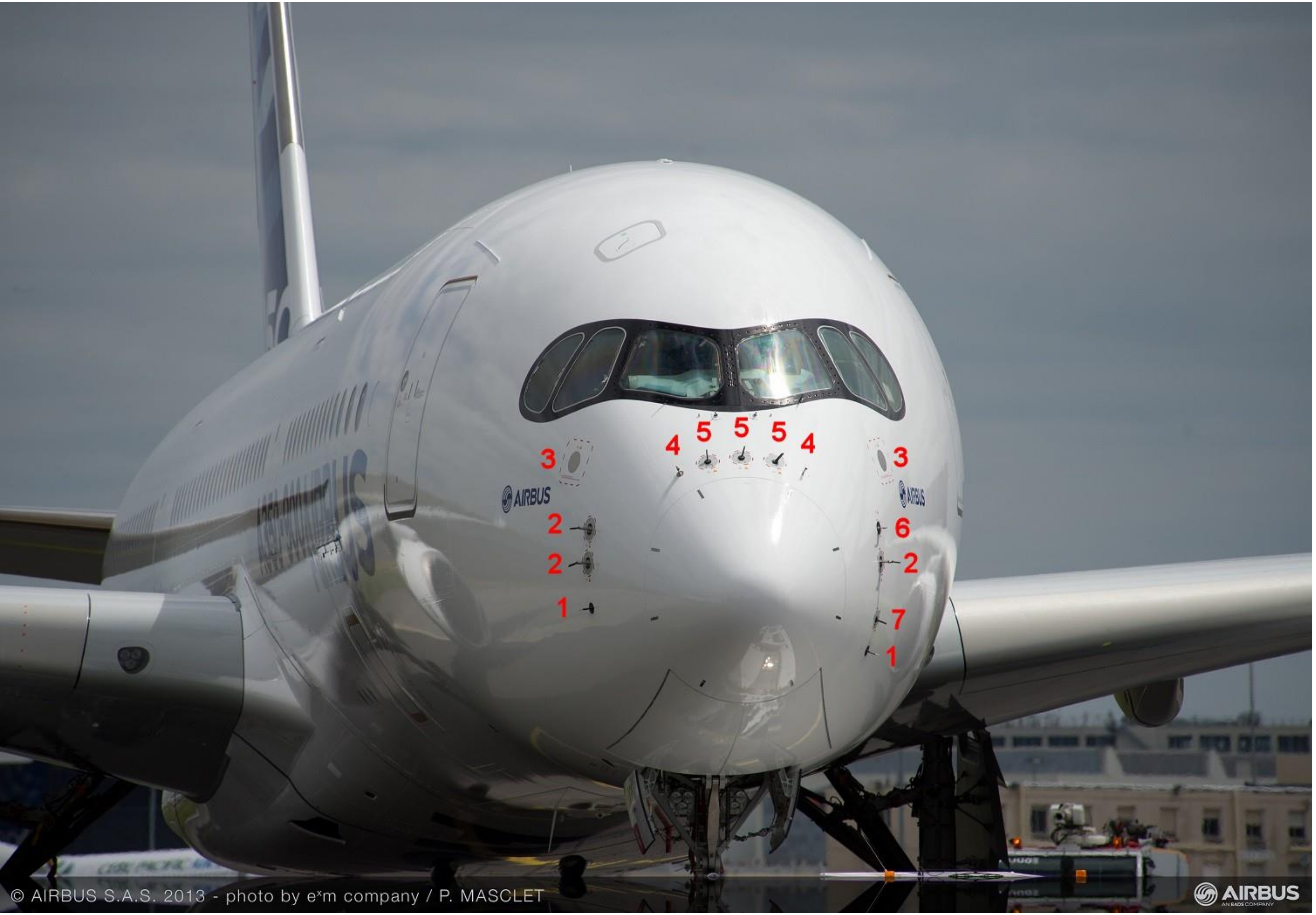
Atmospheric properties and Airspeed

As altitude increases the ρ of the air decreases as does temperature

In order that all aerodynamic calculations should be made on the same basis, a standard variation with altitude of the various parameters has been established

At sea level $\rho = 1.225\ kg/m^3$





1) Ice detector
2) Multi-function probe
Pitot, AOA, TAT
3) Static port
4) TAT probe
5) Side-slip vane
6) Pitot probe
7) Angle-of-attack vane

Atmospheric properties and Airspeed

The difference in air density by changing altitude must be considered:

$$q = \frac{1}{2}\rho_{alt} V_t^2$$

Where ρ_{alt} is air density at the altitude being considered and V_t **is true airspeed (KTAS true air speed in KNOTS)**

Atmospheric properties and Airspeed

TAS (True Airspeed)

TAS is the actual speed of an aircraft relative to the air mass in which it is flying. It is the speed that is crucial for navigation and flight planning since it indicates how fast the aircraft is moving through the air. TAS increases with altitude if the indicated airspeed (IAS) remains constant, due to the decreasing air density at higher altitudes.

Atmospheric properties and Airspeed

EAS (Equivalent Airspeed)

EAS is the airspeed at sea level that would produce the same dynamic pressure as the aircraft is experiencing at its current altitude and speed. EAS is used to correct for the compressibility effects of air at high speeds, particularly in high-performance aircraft.

As airspeed increases and altitude increases, compressibility becomes significant, and EAS provides a more accurate measurement for aerodynamic calculations.

Atmospheric properties and Airspeed

CAS (Calibrated Airspeed)

CAS is the indicated airspeed (IAS) corrected for instrument and positional errors. These errors are caused by the inaccuracy of the airspeed indicator and the position of the pitot-static system, which measures the air pressure difference used to calculate speed.

CAS is crucial when more accurate airspeed readings are needed, particularly at lower speeds and in certain configurations such as during takeoff and landing.

Atmospheric properties and Airspeed

IAS (Indicated Airspeed)

IAS is the speed shown on the aircraft's airspeed indicator, uncorrected for instrument, position, and compressibility errors. IAS is directly measured using the pitot-static system and is the primary speed reference for pilots during flight.

It is essential for determining the aircraft's performance, including stall speeds, approach speeds, and flap deployment speeds, all of which are indicated by the aircraft's IAS.

Atmospheric properties and Airspeed

GS (Indicated Airspeed)

GS is the speed at which an aircraft is moving relative to the ground. Unlike airspeeds, which measure the aircraft's speed relative to the surrounding air, ground speed takes into account the effects of wind.

Ground speed is crucial for navigation and determining the actual time it will take to reach a destination.

Atmospheric properties and Airspeed

- **IAS** is the raw data shown on the cockpit's airspeed indicator.

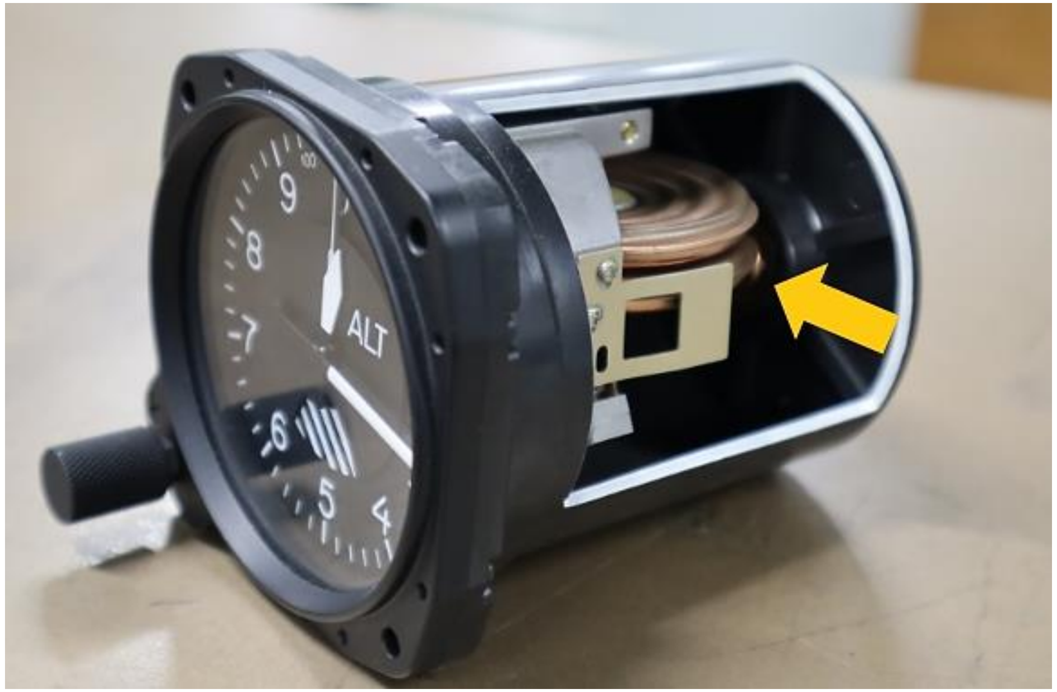
- **CAS** corrects **IAS** for any errors in measurement and is typically close to **IAS** at lower speeds and altitudes.

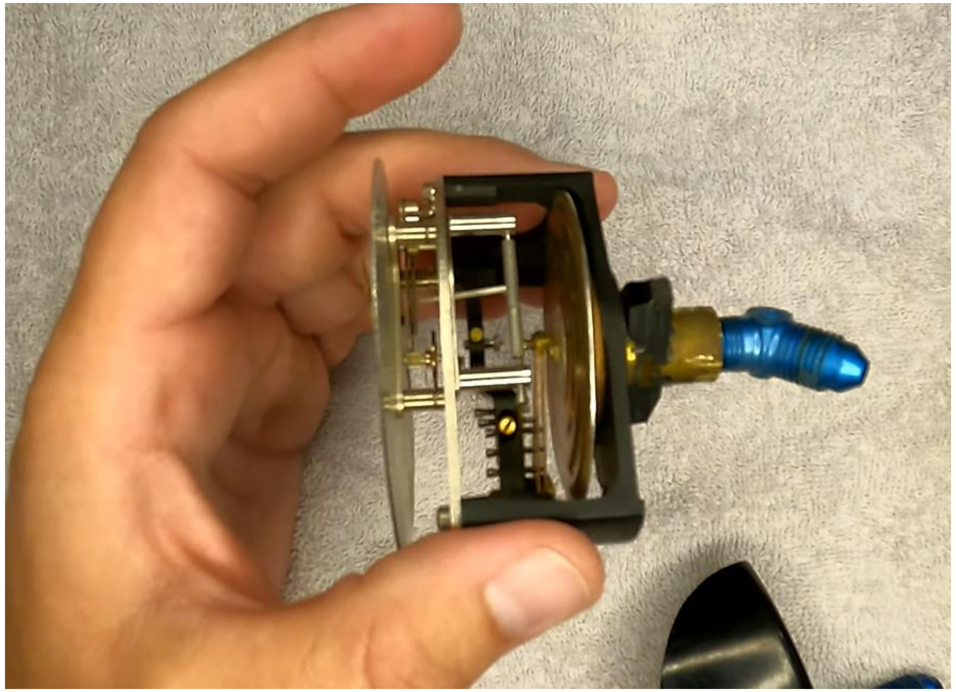
- **EAS** adjusts **CAS** for the effects of air compressibility, making it relevant at high speeds and altitudes.

- **TAS** then adjusts **EAS** for the actual air density at the aircraft's altitude, providing the true speed relative to the air mass.

Atmospheric properties and Airspeed

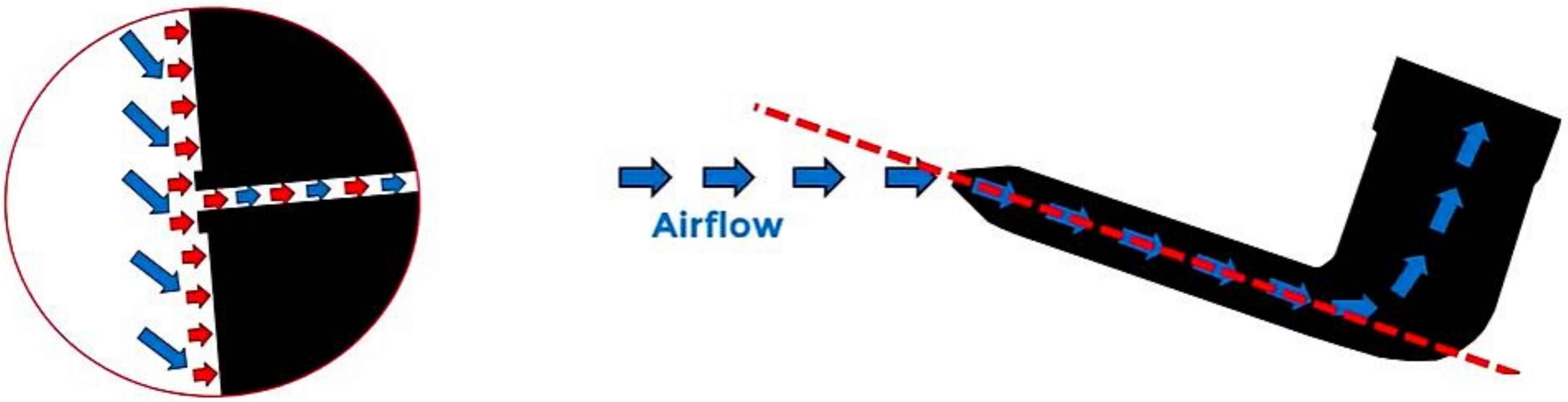
Instrument error: manufacturing imperfections and gear wear of the instruments.





Atmospheric properties and Airspeed

Position error: pitot-static system error for pressure measurements due to maneuver or changes in aircraft configuration.



Atmospheric properties and Airspeed

Equivalent airspeed V_e(KEAS equivalent air speed in KNOTS):

$\boldsymbol{V_e}$ is the true airspeed corrected by the differences between the sea level density and the density at a determined altitude.

$$\boldsymbol{TAS = EAS\sqrt{\frac{\rho_0}{\rho}}}$$

$\boldsymbol{\rho_0}$ is the standard air density at sea level

$\boldsymbol{\rho}$ is the air density at aircraft flight altitude

Atmospheric properties and Airspeed

At sea level:

$$V_t = V_e$$

¡NORMALLY KEAS IS USED!

Atmospheric properties and Airspeed

Ratios of pressure, temperature and density:

$$\delta = \frac{P}{P_0} \qquad \theta = \frac{T}{T_0} \qquad \sigma = \frac{\rho}{\rho_0}$$

Mach number:

$$M = \frac{u}{c}$$

u is local flow velocity TAS (moving aircraft) and c is the speed of sound at given altitude

Example

For 325 KEAS at 10000 ft altitude, calculate:

KTAS (true airspeed)	$\mathbf{378.2\ knots}$
Mach number	$\mathbf{M = 0.592}$
Dynamic pressure	$\mathbf{q = 17092.12\ Pa}$

Primary sources of basic loads

- Airplane geometry
- Aerodynamic data
- Weight data
- Design speeds
- Stiffness data
- Miscellaneous systems data
- Operational data
- V-n diagram

Airplane weight data

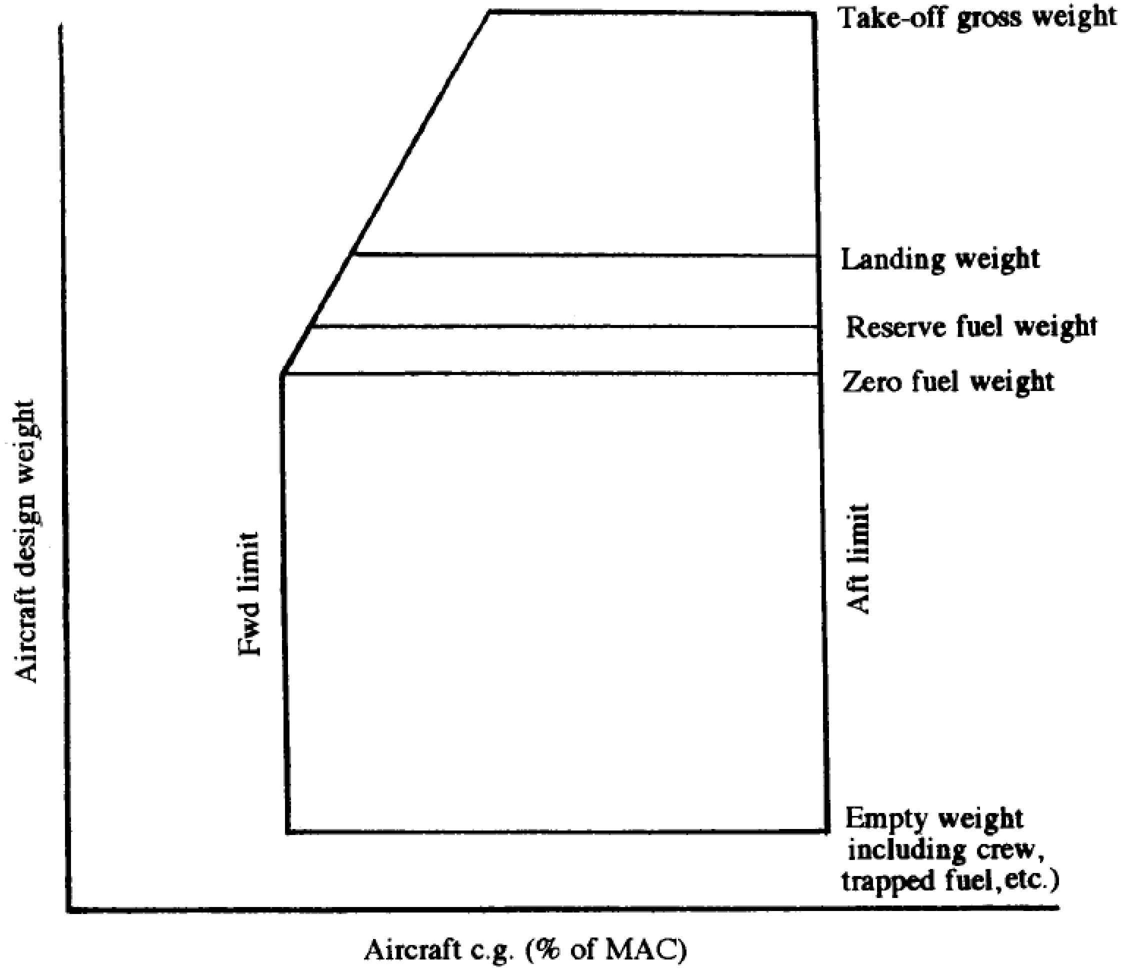
Design loads affect the **weight of the airframe structure**, this interferences the magnitude of design loads.

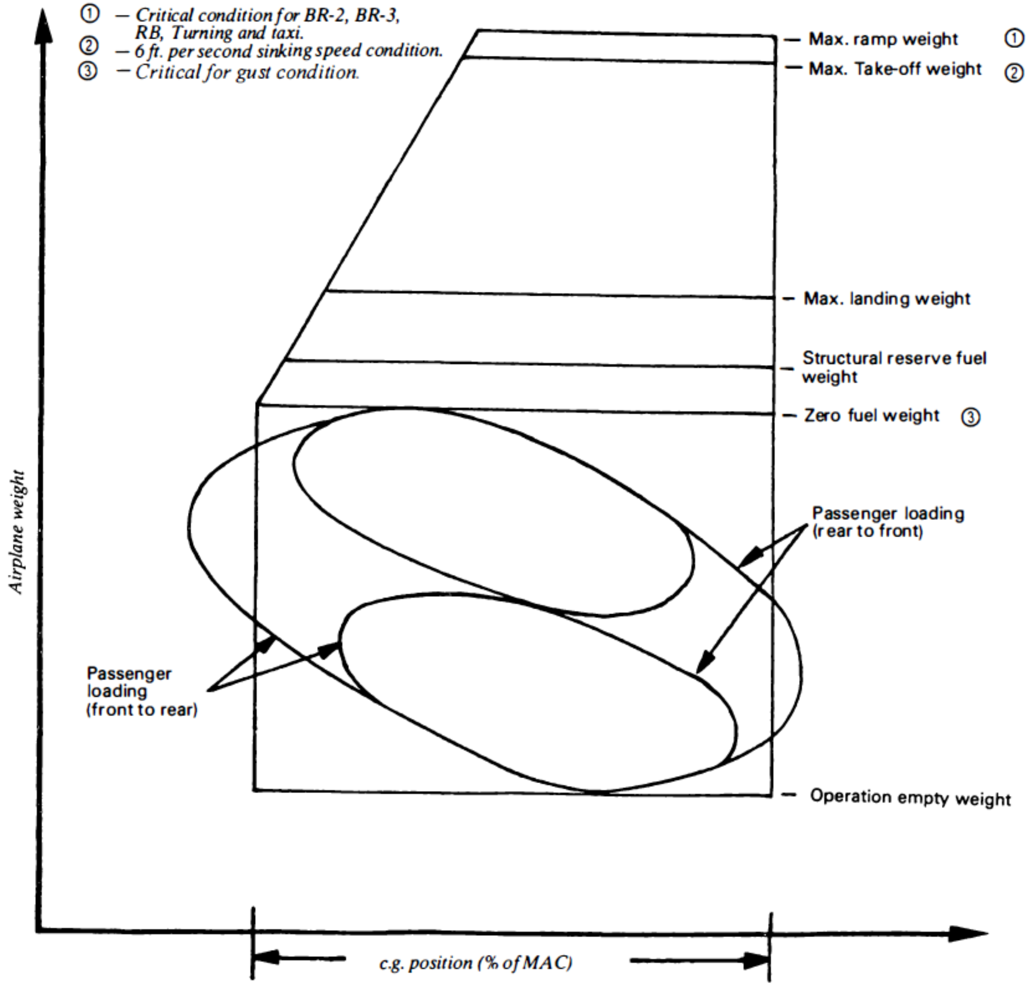
Center of gravity envelope: Gross weight vs center of gravity location

Airplane weight data

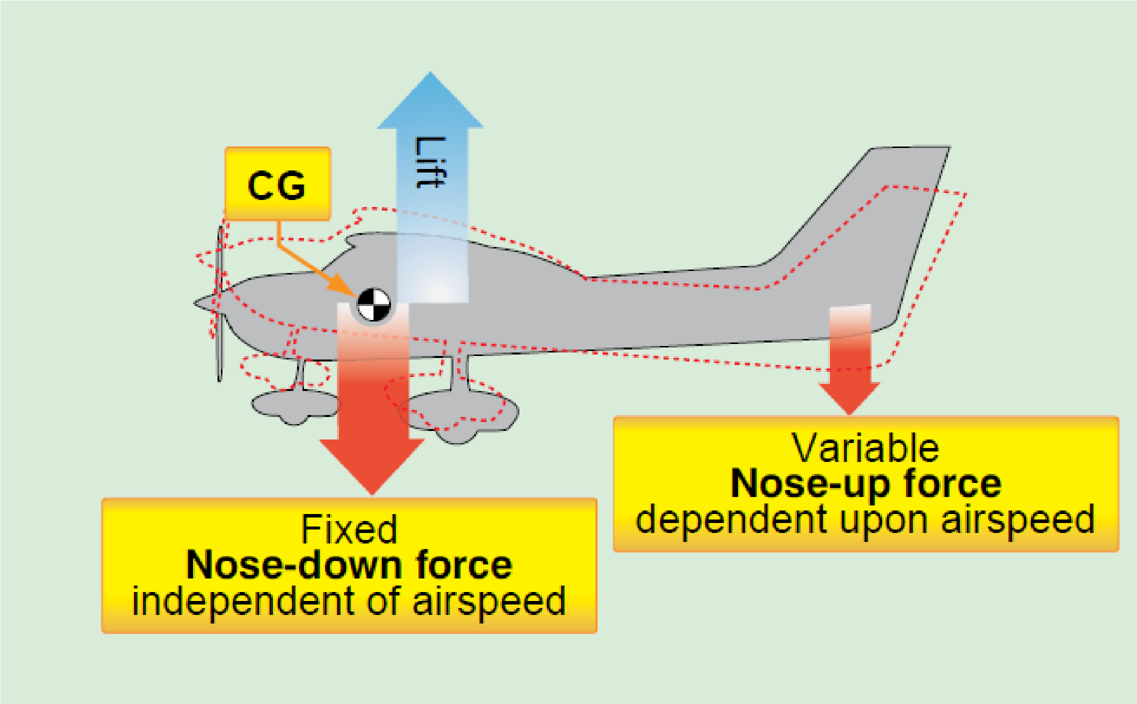
- **MEW manufacturer empty weight**: includes weight of the structure, power plant, furnishing, installations, systems and other equipment
- **ZFW Zero fuel weight**: weight of the aircraft minus weight of the usable fuel.
- **Payload weight**: is the carrying capacity of the aircraft.
- **OEW operating empty weight**: is the basic weight of the aircraft including crew, fluids (excluding usable fuel and payload)
- **MTOW Maximum weight**: at which the pilot is allowed to attempt a take off
- **MLW Maximum landing weight**: at which an aircraft is permitted to land
- **Gross Weight**: total weight at any moment

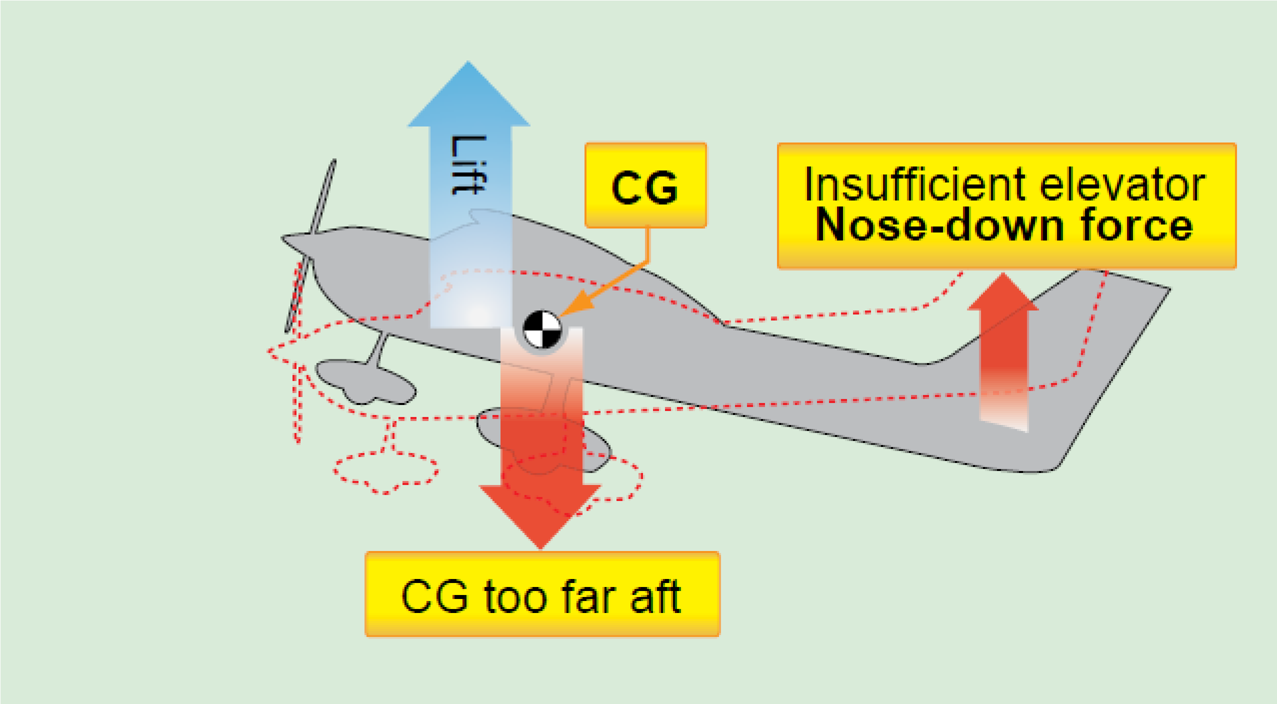
Airplane weight data



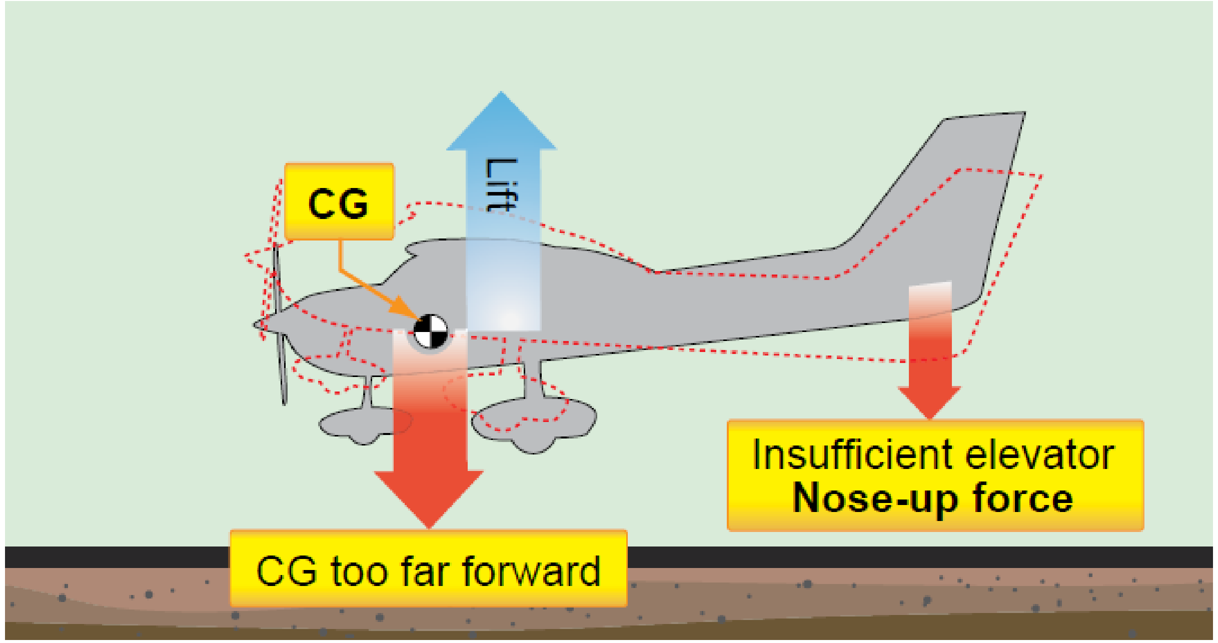


Airplane weight data

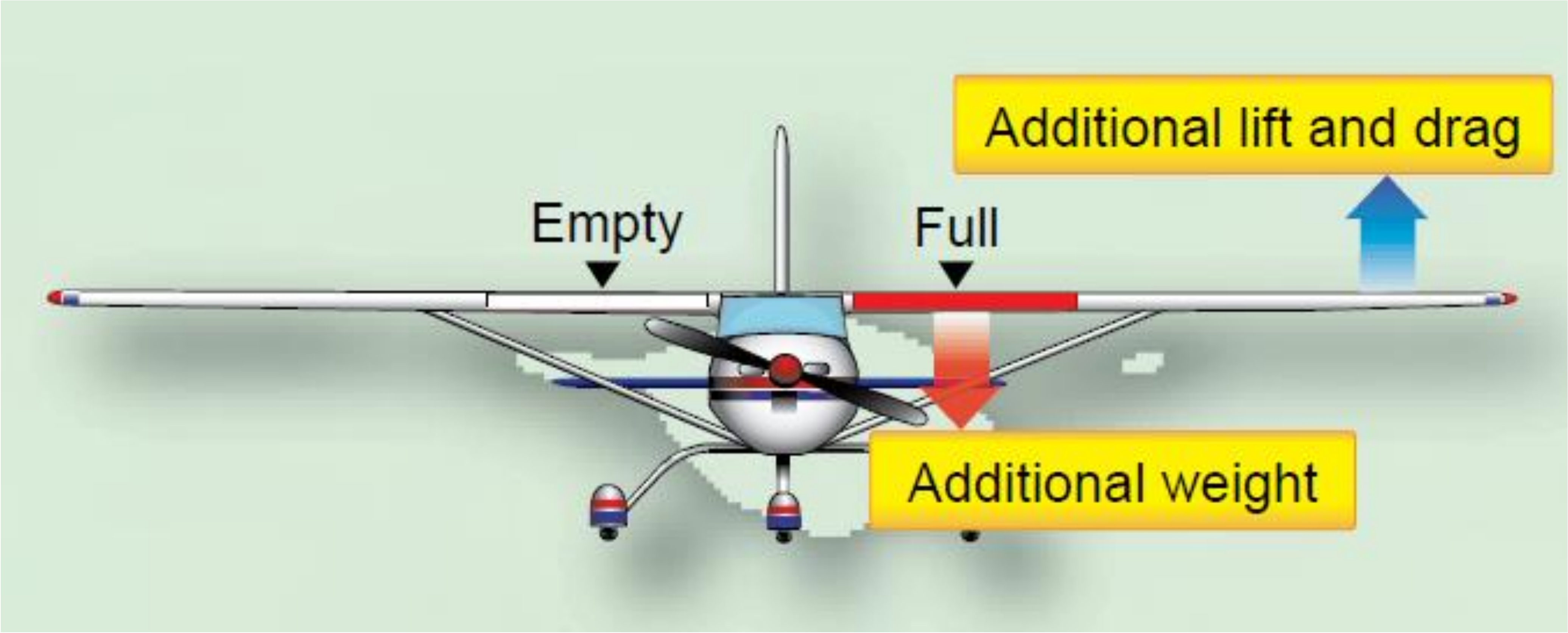




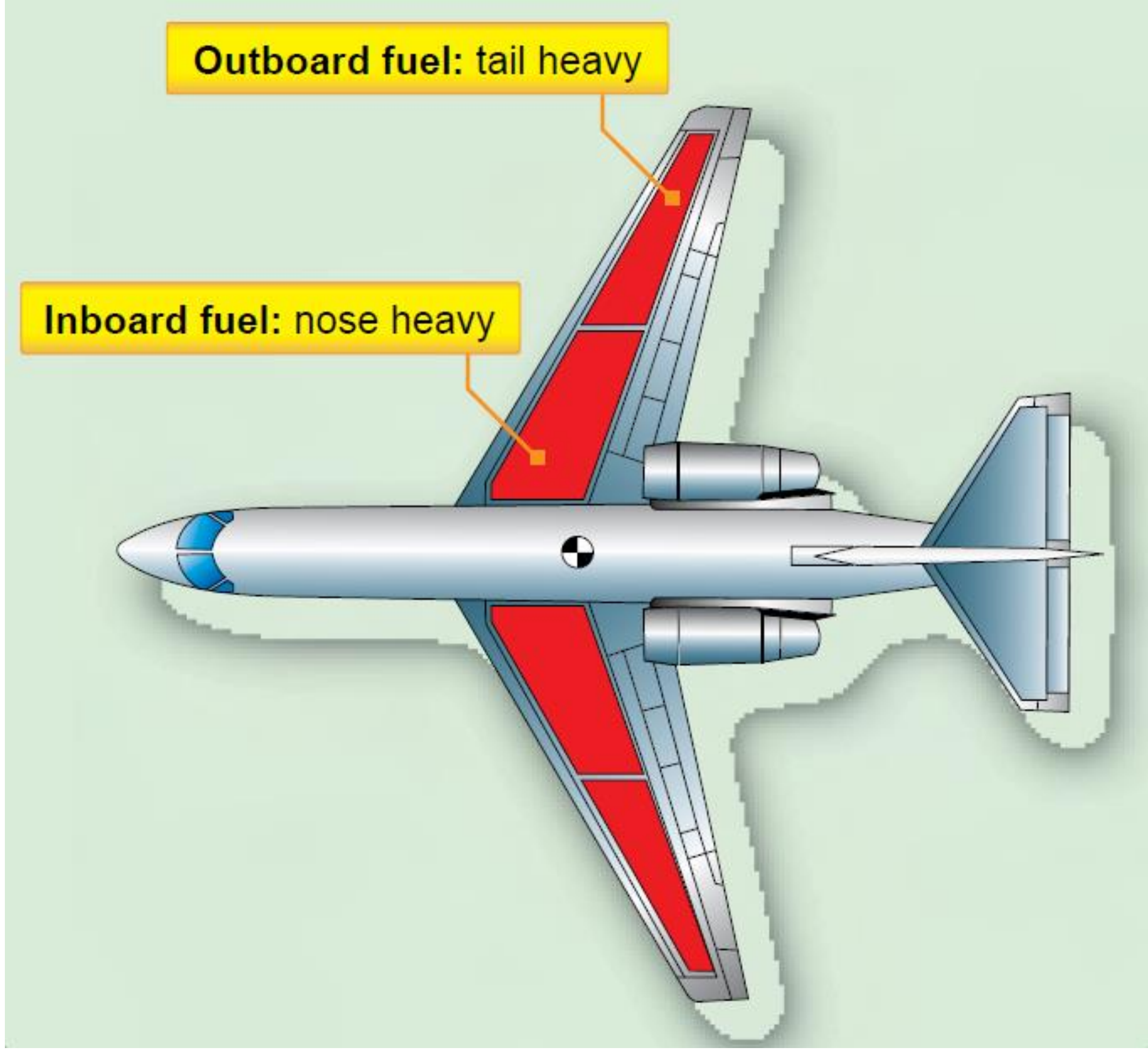
Airplane weight data



Airplane weight data



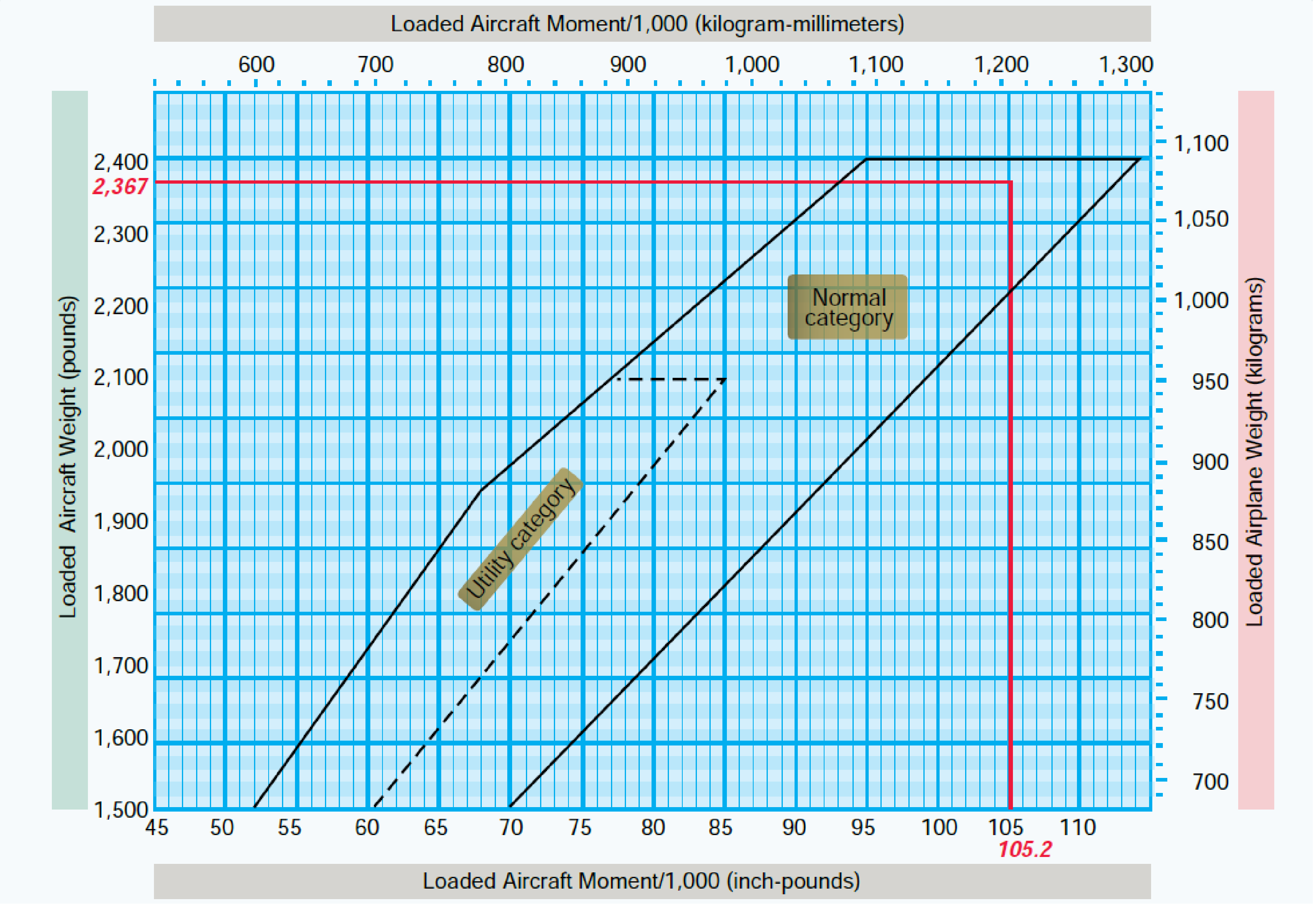
Airplane weight data



CG Envelope

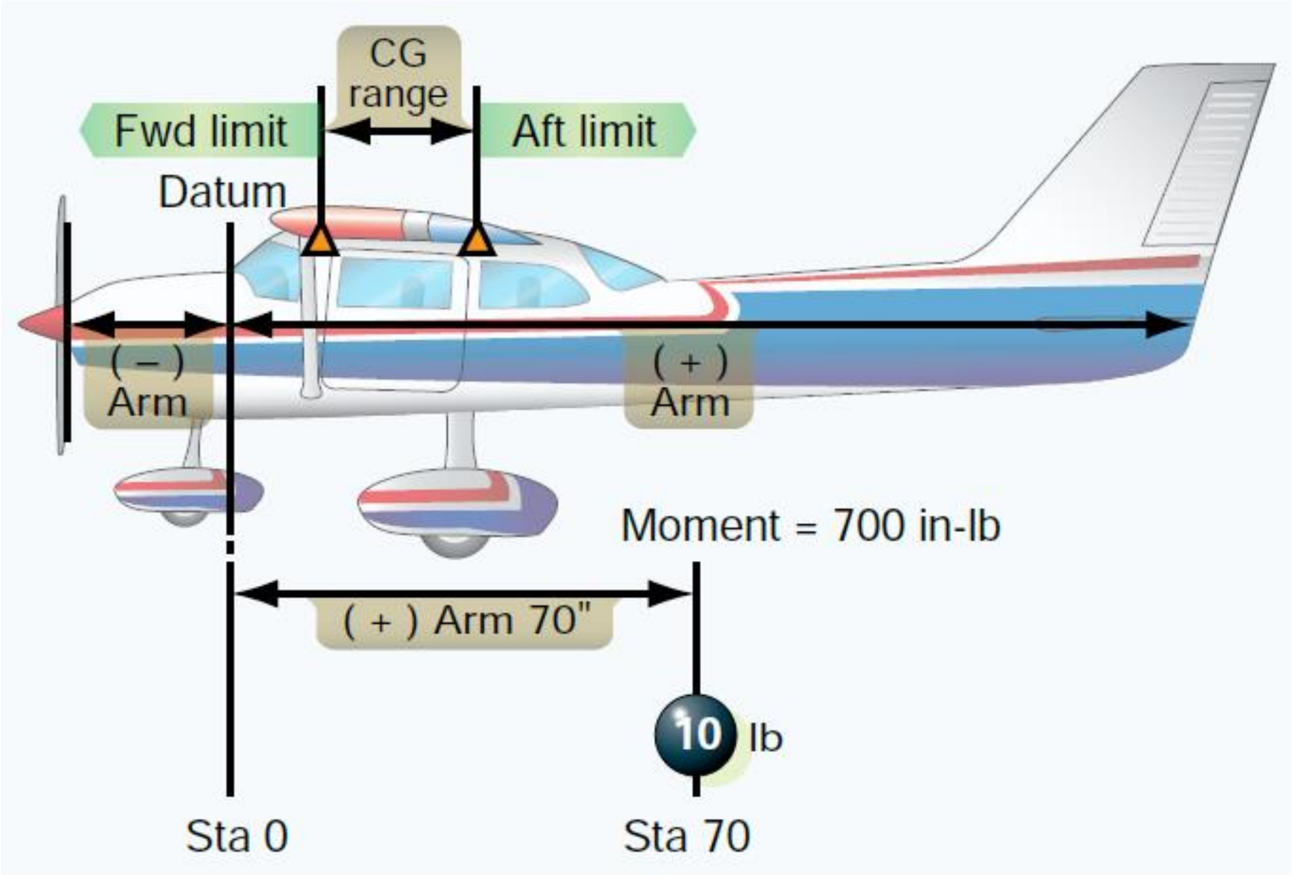
It is a graphical method to determine the loaded weight and CG.

The graphs are provided by the manufacturers.

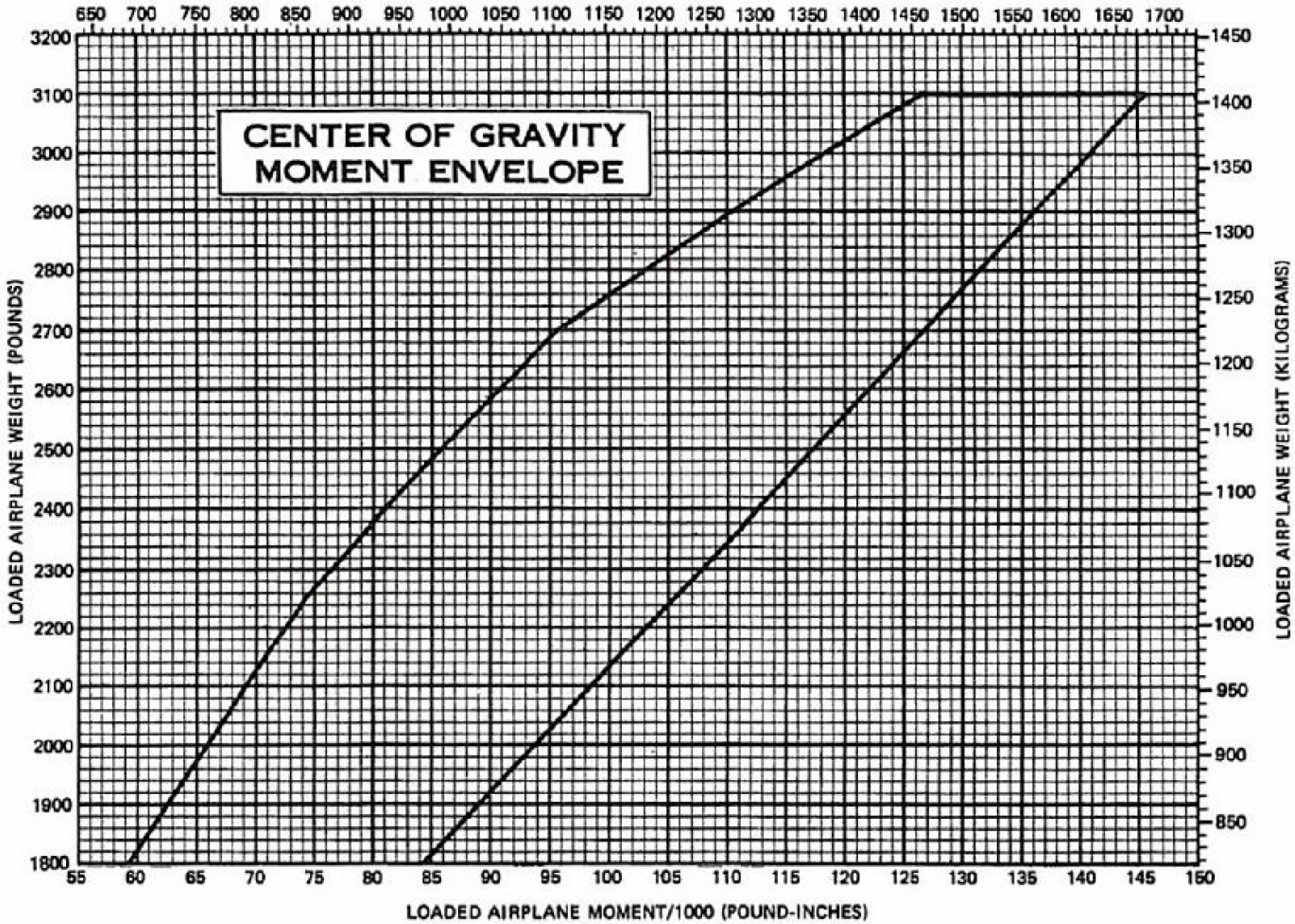


CG Envelope

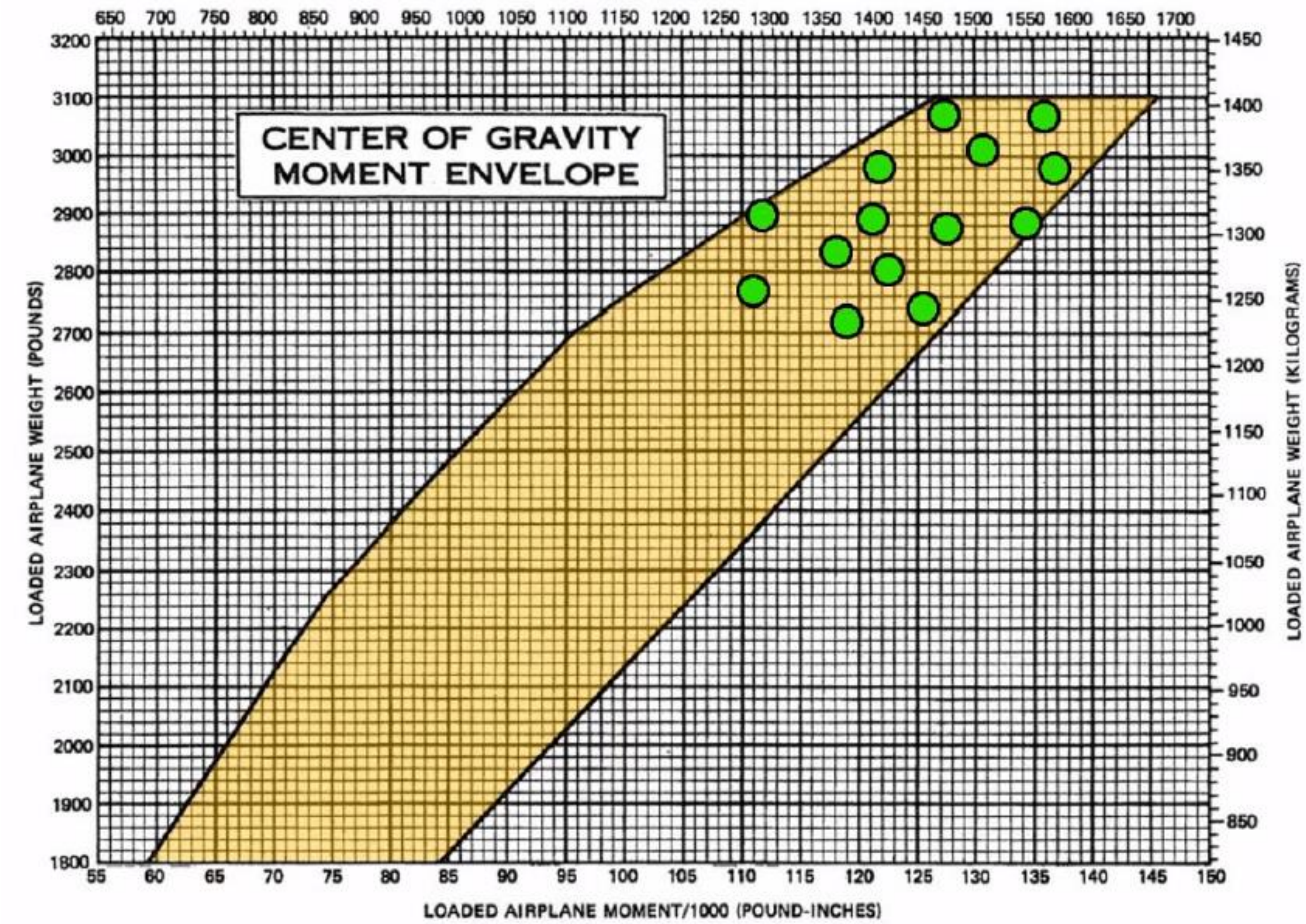
Sample Loading Problem	Weight (lb)	Moment (in-lb/1,000)
1. Basic empty weight (Use data pertaining to aircraft as it is presently equipped) includes unusable fuel and full oil	1,467	57.3
2. Usable fuel (At 6 lb/gal)		
▪ Standard tanks (40 gal maximum)	240	11.5
▪ Long range tanks (50 gal maximum)		
▪ Integral tanks (62 gal maximum)		
▪ Integral reduced fuel (42 gal)		
3. Pilot and front passenger (Station 34 to 46)	340	12.7
4. Rear passengers	300	21.8
5. Baggage area 1 or passenger on child's seat (Station 82 to 108, 120 lb maximum)	20	1.9
6. Baggage area 2 (Station 108 to 142, 50 lb maximum)		
7. Weight and moment	2,367	105.2



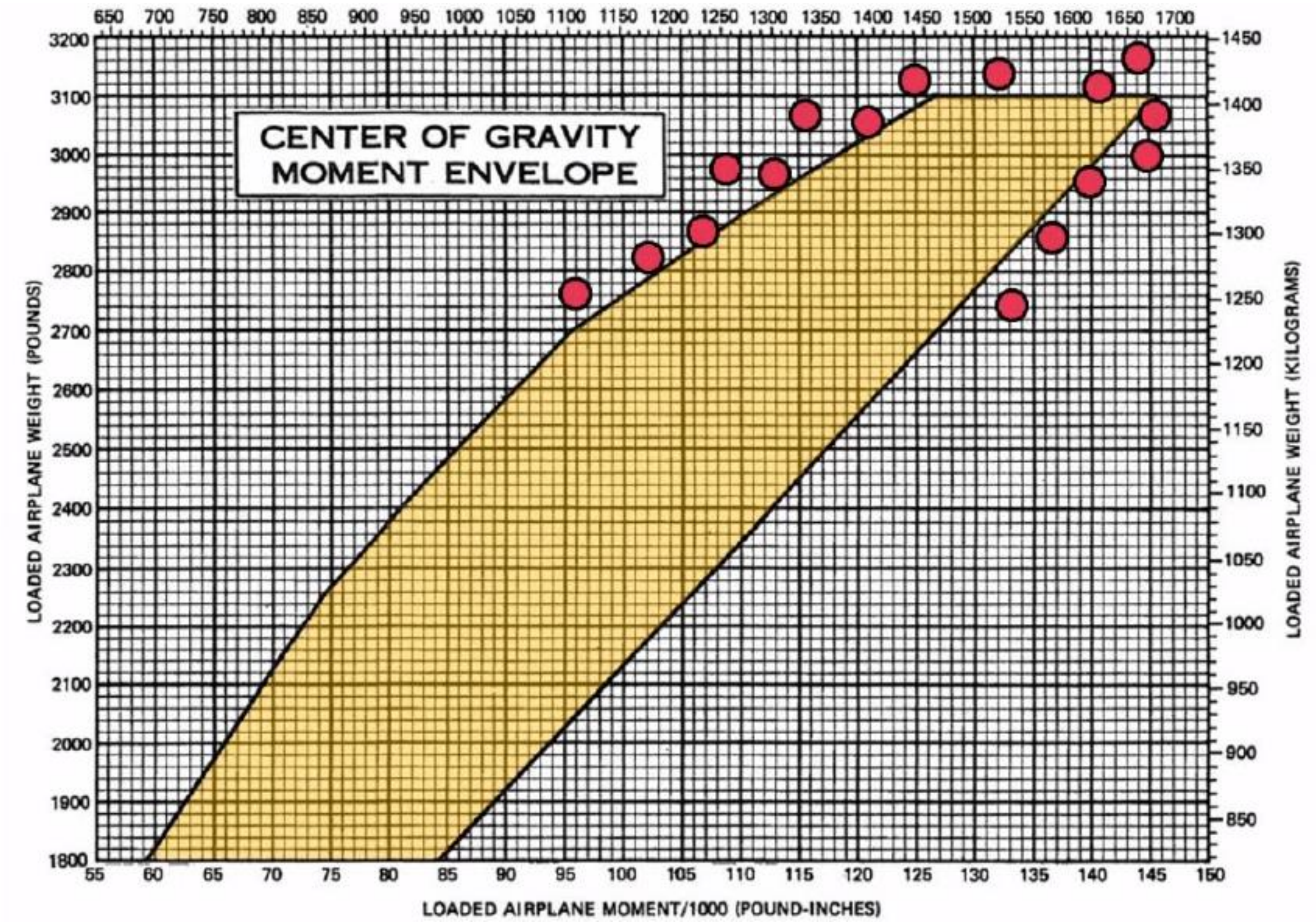
CG Envelope


CENTER OF GRAVITY MOMENT ENVELOPE
650 700 750 800 850 900 950 1000 1050 1100 1150 1200 1250 1300 1350 1400 1450 1500 1550 1600 1650 1700
3200 3100 3000 2900 2800 2700 2600 2500 2400 2300 2200 2100 2000 1900 1800
LOADED AIRPLANE WEIGHT (POUNDS)
1450 1400 1350 1300 1250 1200 1150 1100 1050 1000 950 900 850
LOADED AIRPLANE WEIGHT (KILOGRAMS)
55 60 65 70 75 80 85 90 95 100 105 110 115 120 125 130 135 140 145 150
LOADED AIRPLANE MOMENT/1000 (POUND-INCHES)

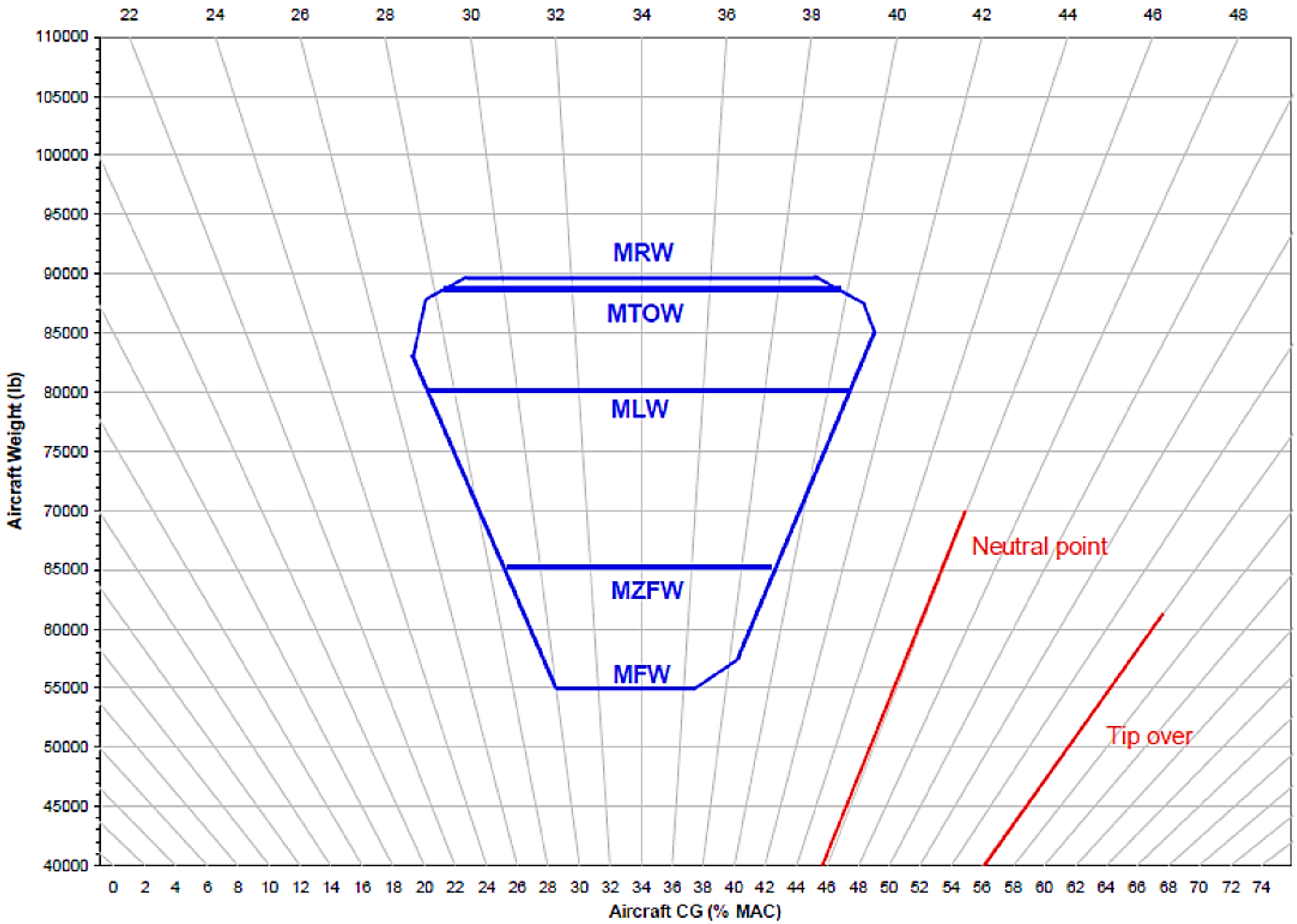
CG Envelope (Ok)



CG Envelope (Not ok)

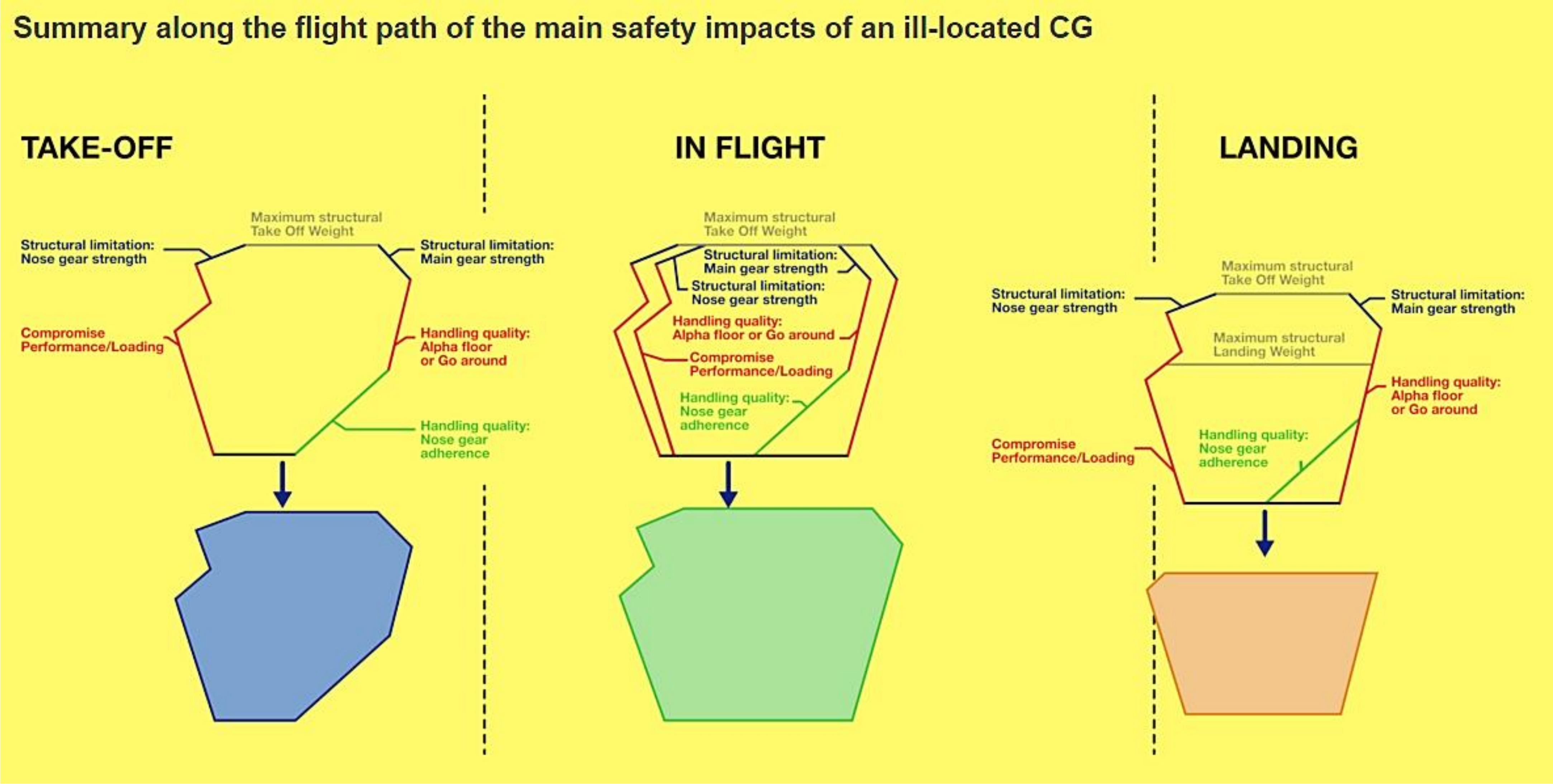


CG Envelope



MRW
MTOW
MLW
MZFW
MFW
Neutral point
Tip over
Aircraft Weight (lb)
Aircraft CG (% MAC)
110000
105000
100000
95000
90000
85000
80000
75000
70000
65000
60000
55000
50000
45000
40000
22 24 26 28 30 32 34 36 38 40 42 44 46 48
0 2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32 34 36 38 40 42 44 46 48 50 52 54 56 58 60 62 64 66 68 70 72 74

CG Envelope


Summary along the flight path of the main safety impacts of an ill-located CG
TAKE-OFF
Maximum structural Take Off Weight
Structural limitation: Nose gear strength
Structural limitation: Main gear strength
Compromise Performance/Loading
Handling quality: Alpha floor or Go around
Handling quality: Nose gear adherence
IN FLIGHT
Maximum structural Take Off Weight
Structural limitation: Main gear strength
Structural limitation: Nose gear strength
Handling quality: Alpha floor or Go around
Compromise Performance/Loading
Handling quality: Nose gear adherence
LANDING
Maximum structural Take Off Weight
Structural limitation: Nose gear strength
Structural limitation: Main gear strength
Maximum structural Landing Weight
Handling quality: Alpha floor or Go around
Handling quality: Nose gear adherence
Compromise Performance/Loading

Design speeds

Structural design speeds must be chosen such that they are compatible with the airplane performance and operational condition requirements

- Design speeds must meet the design requirements of the certifying agency
- Usually the design speeds are initially defined in EAS

Design speeds

Design cruising speed V_C: a minimum value of the maximum speed to be considered for straight and level flight, not greater than 0.8 V_D.

Design dive speed V_D: the maximum speed for structural design. Design dive speed is sufficiently greater than V_C to provide safe recovery from inadvertent upsets. Aeroelasticity problems

Stall speed V_S

Stalling speed with flaps retracted V_{S1}

Design speeds

Never exceed speed V_{NE}: not greater than $0.9\,V_D$

Design flap speed V_F: $1.15\,V_S$

Design maneuver speed V_A: less than V_C

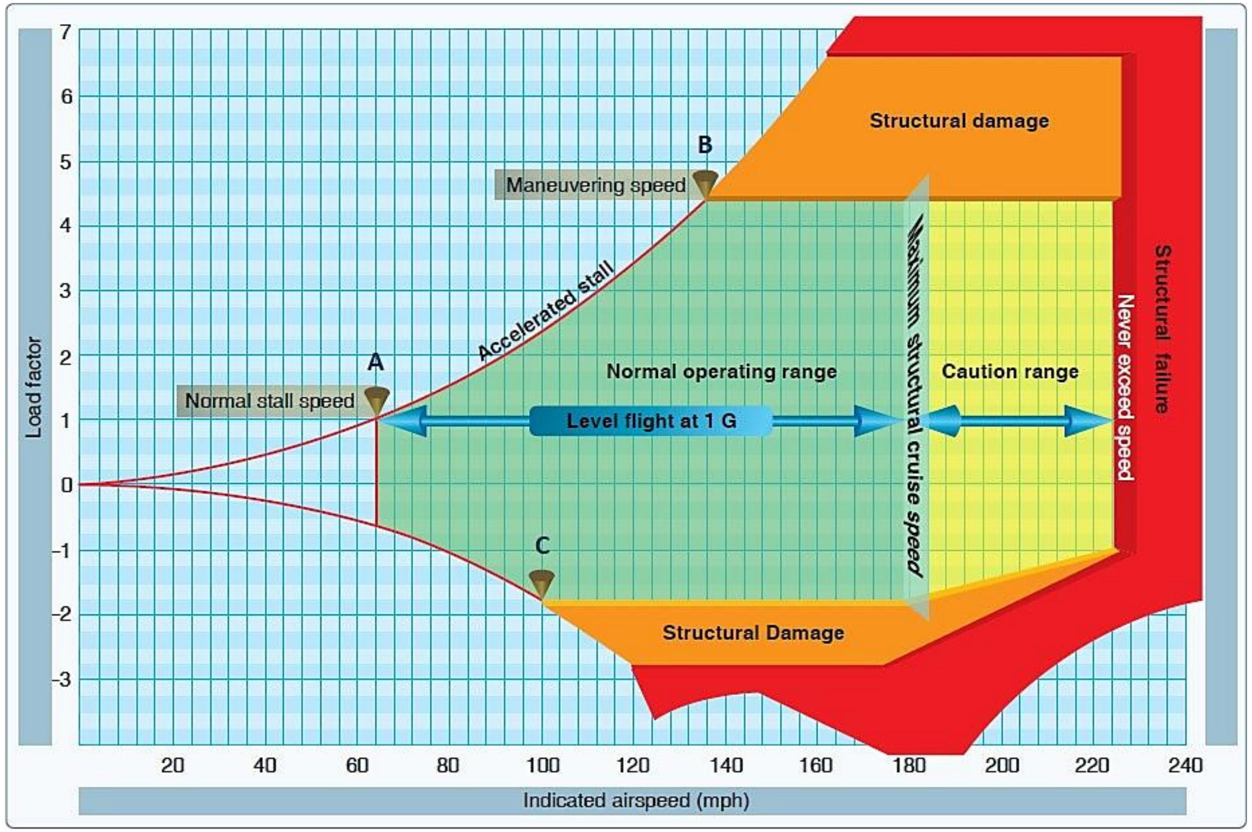
Design speed for maximum gust intensity V_B

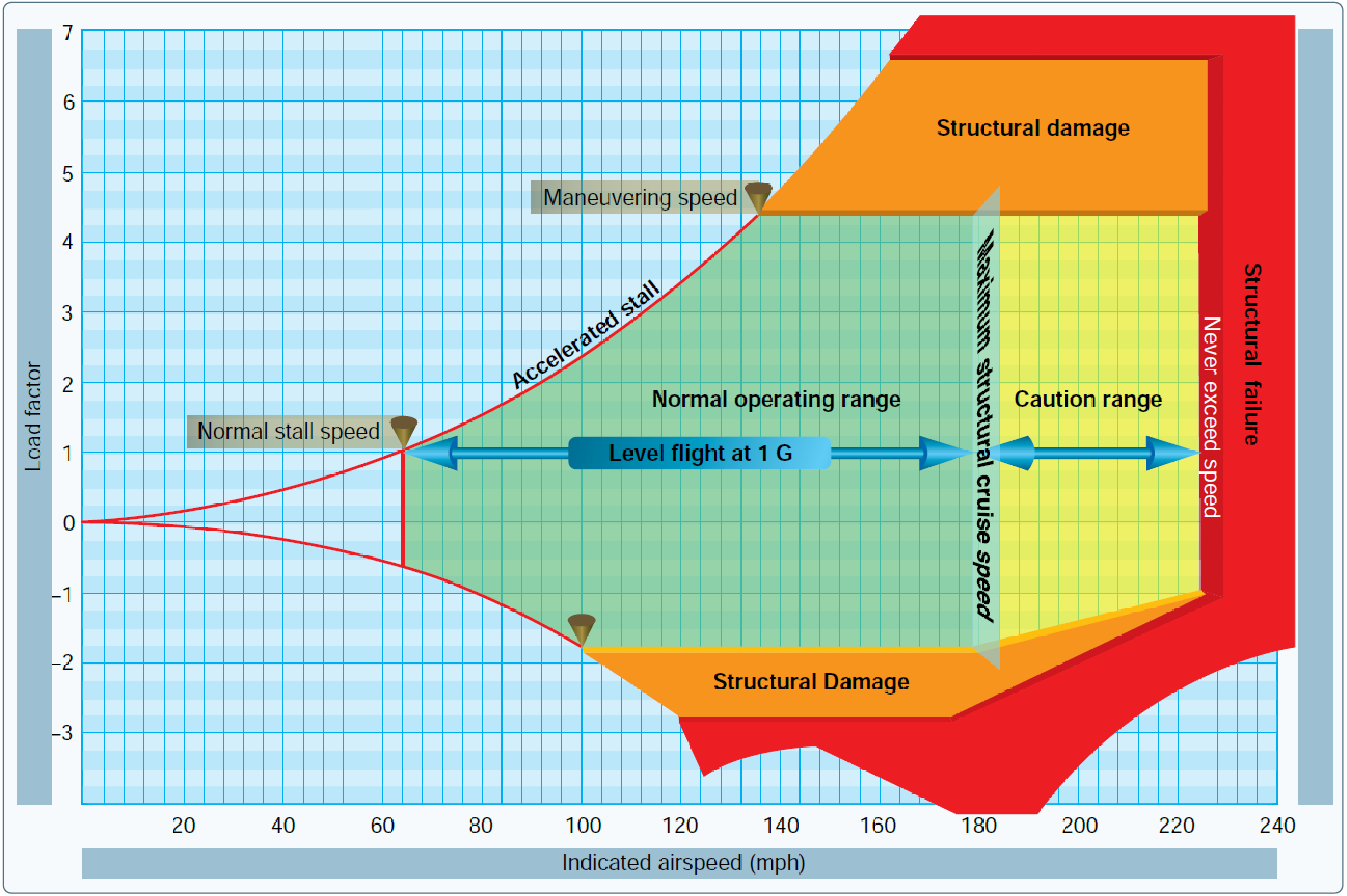
Flight envelope

The **flight regime** of an aircraft **must allow all combinations** of speed, altitude, weights, CG's and other configurations.

This regime is preliminarily established by **aerodynamics, propulsion, structures and flight dynamics**.

The safety of the aircraft is guaranteed as long as it flies within the limits established in the flight envelope.





Structural damage
Maneuvering speed
Accelerated stall
Normal stall speed
Normal operating range
Level flight at 1 G
Maximum structural cruise speed
Caution range
Never exceed speed
Structural failure
Structural Damage
Load factor
7
6
5
4
3
2
1
0
–1
–2
–3
20
40
60
80
100
120
140
160
180
200
220
240
Indicated airspeed (mph)

Flight envelope

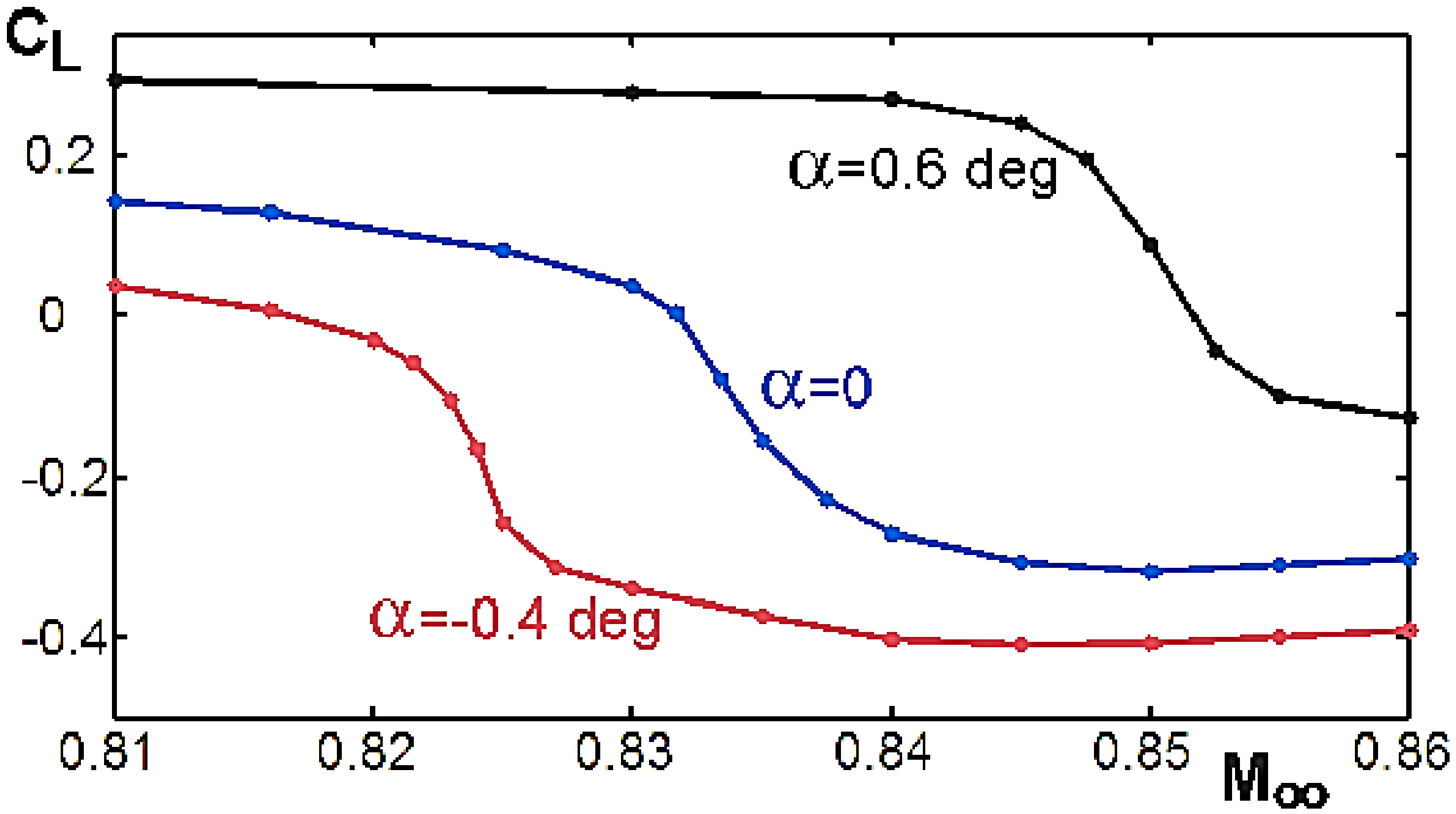
- The limits of the flight regime of the aircraft are set at the **flight envelope or maneuvering envelope**.

- If the aircraft exceeds the limits it will be **uncontrollable**, or it may exceed the structural limits sufficient to make it airworthy.

- **Very useful for pilots**

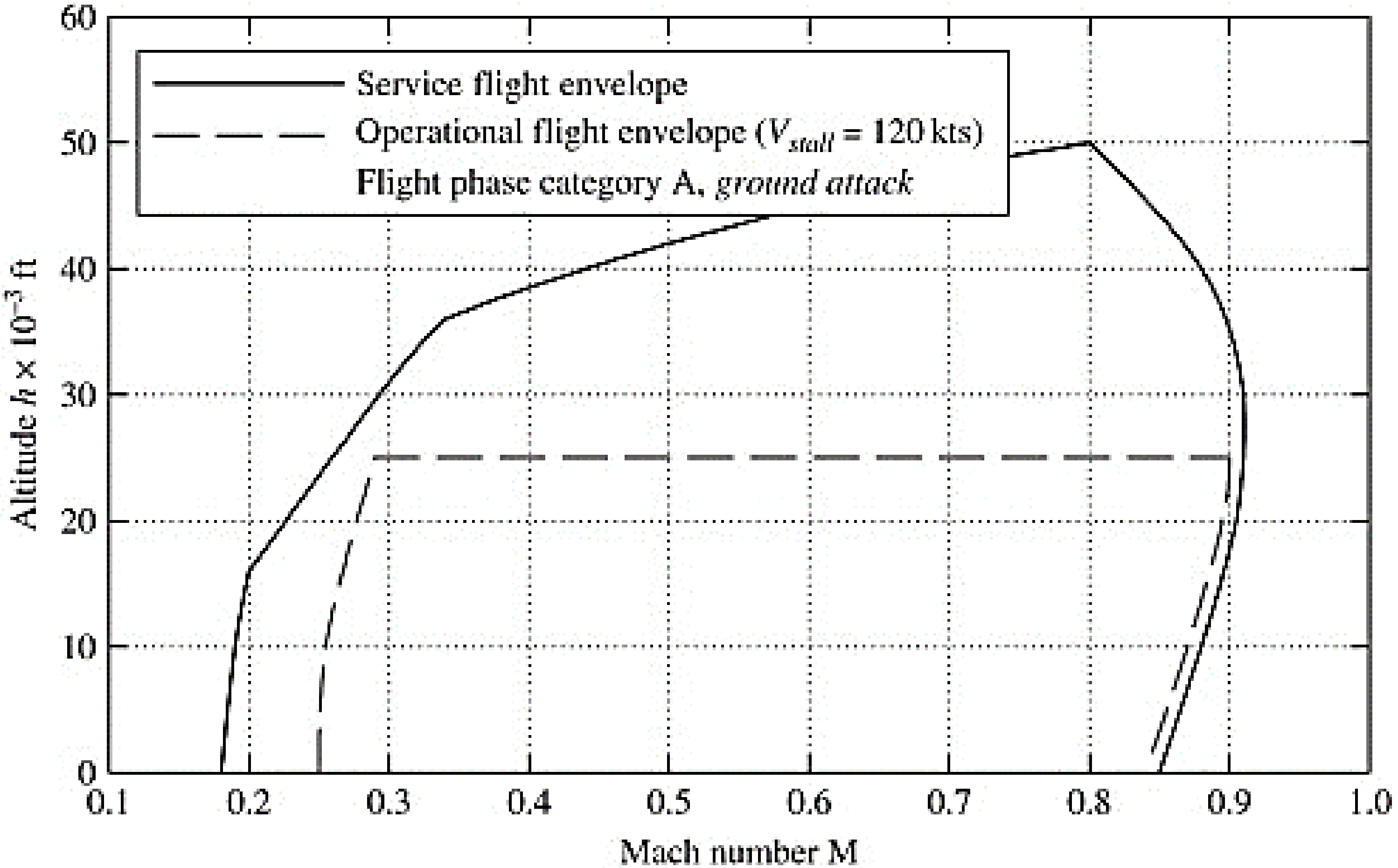
Flight envelope (Diagrams)

Diagram of variations of aircraft lift coefficient vs Mach number ($C_L - M$)



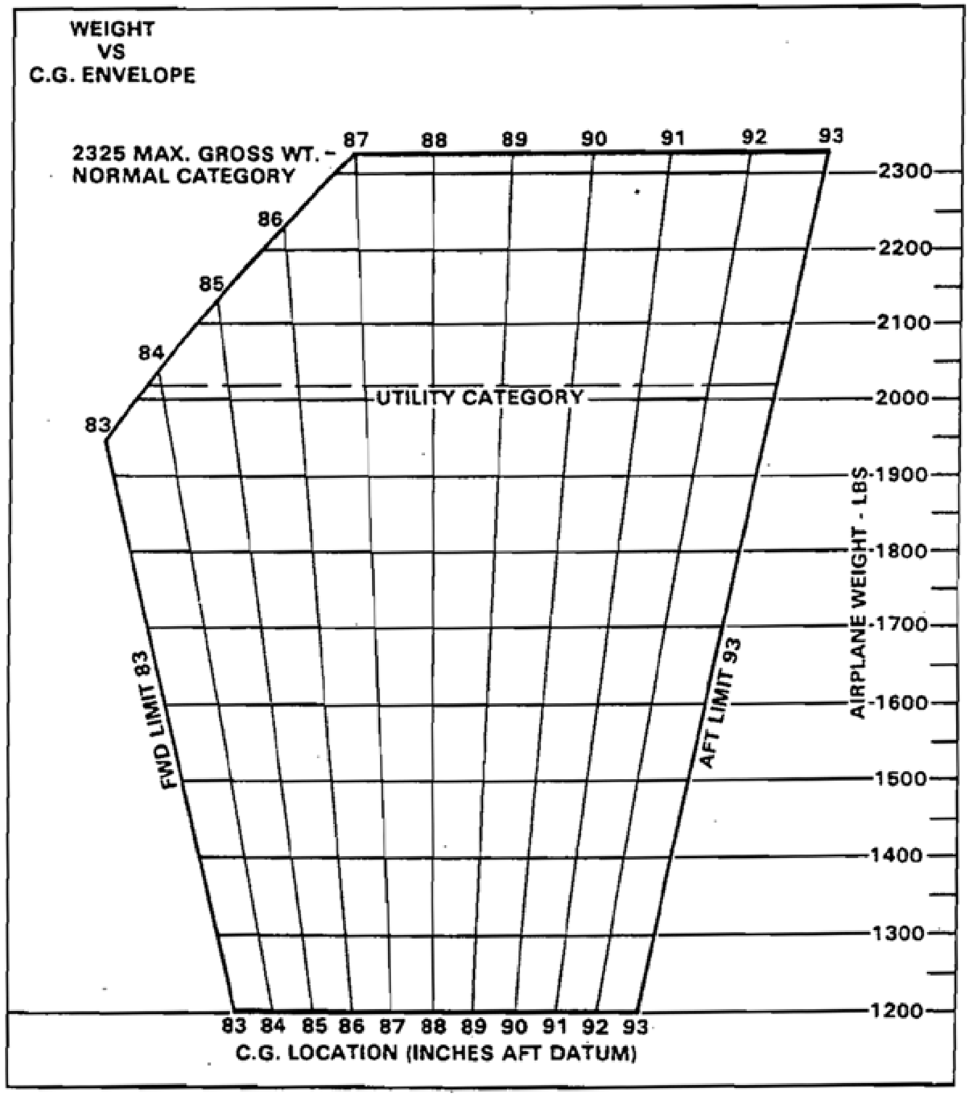
Flight envelope (Diagrams)

Diagram of variations of airspeed vs altitude $(V - h)$



Flight envelope (Diagrams)

Diagram of variations of CG location versus weight $(X_{CG} - M)$



Flight manual

An FAA approved document that contains information (**operating limitations, operating procedures, performance information**, etc.) necessary to operate the airplane at the level of safety established by the airplane's certification basis.

Safe Operation: for the purposes of this AC, safe operation means operation of the airplane in a manner that is mandatory, or is recommended, for compliance with the airworthiness requirements.

Limitation: for the purposes of this AC, an AFM limitation establishes the approved bounds of operation of the airplane or its systems

Source: https://www.faa.gov/regulations_policies/advisory_circulars/index.cfm/go/document.information/documentID/22692

Flight manual

BOEING

747-441
Operations Manual
International Lease Finance Corp.

AIRBUS INDUSTRIE

A320

MODEL : 320 - 214

FLIGHT MANUAL

ALL FLIGHTS MUST BE DONE IN ACCORDANCE
WITH THE LIMITATIONS INCLUDED IN THIS MANUAL

APPROVED BY D.G.A.C.

J. ANDRE

10 MARS 1995

Airbus Industrie
FLIGHT DIVISION
31707 Blagnac Cedex
FRANCE

T.O. GR1F-16CJ-1

FLIGHT MANUAL

HAF SERIES AIRCRAFT

F-16C/D

BLOCKS 50 AND 52+

LOCKHEED MARTIN CORPORATION

F33657-90-C-2002
F42620-01-D-0058

Commanders are responsible for bringing this publication to the attention of all Air Force personnel cleared for operation of subject aircraft.

15 OCTOBER 2002

CHANGE 1 15 JUNE 2003

V-n Diagram

In summary, the load factor can be classified into four different scenarios:

- **Maximum producible load factor** by the lift using the maximum engine thrust/power
- **Maximum producible load factor** by the centrifugal force in the extreme case of the pull out of a dive
- **Maximum producible load factor** in an undesired motion produced by the lift/inertial force due to a gust/turbulence
- **Maximum allowable load factor** (tolerable by the aircraft structure and the human onboard)

V-n Diagram

The absolute maximum producible load factor (**Maximum of the maximum load factor**) will be as follows:

$$\boldsymbol{n_{max_{max}} = \frac{T_{max}}{2W\sqrt{KC_{Do}}}}$$

Where,

T_{max} is the maximum engine thrust

K is the induced drag factor

C_{Do} Is the Zero-lift drag coefficient

W is the aircraft weight

Note: Aircraft structure and human onboard dictate the maximum allowable load factor

V-n Diagram

The airspeed that corresponds to the maximum of the maximum load factor is only a function of four parameters:

$$V_{n_{max}} = \sqrt{\frac{T_{max}}{\rho S C_{Do}}}$$

1 Altitude

2 Wing area

3 Zero-lift drag coefficient

4 Maximum engine thrust

Example

$$\mathrm{K} = \frac{1}{\pi\, e\, AR} \qquad \mathrm{AR} = \frac{b^2}{S}$$

The single-engine Lockheed Martin F-16 Fighting Falcon fighter jet aircraft has the following characteristics:

m=12000 kg S=27.87 m^2 b=9.96 m T_{max}=127 kN

Assume: e = 0.85, C_{Do} = 0.017 (low subsonic), C_{Do} = 0.032 (transonic), C_{Do} =0.04 (supersonic), CL_{max} = 2.

Note, the given value for the mass is for the aircraft loaded mass, and the given value for engine thrust is for the thrust with an afterburner.

For sea level determine:

a. Corresponding velocity to the maximum producible load factor $\mathbf{341\ m/s}$

b. Maximum producible load factor $\mathbf{9.91}$

Safety factor

Aircraft are designed to operate adequately **under certain speeds and accelerations** where the pilot must remain operational.

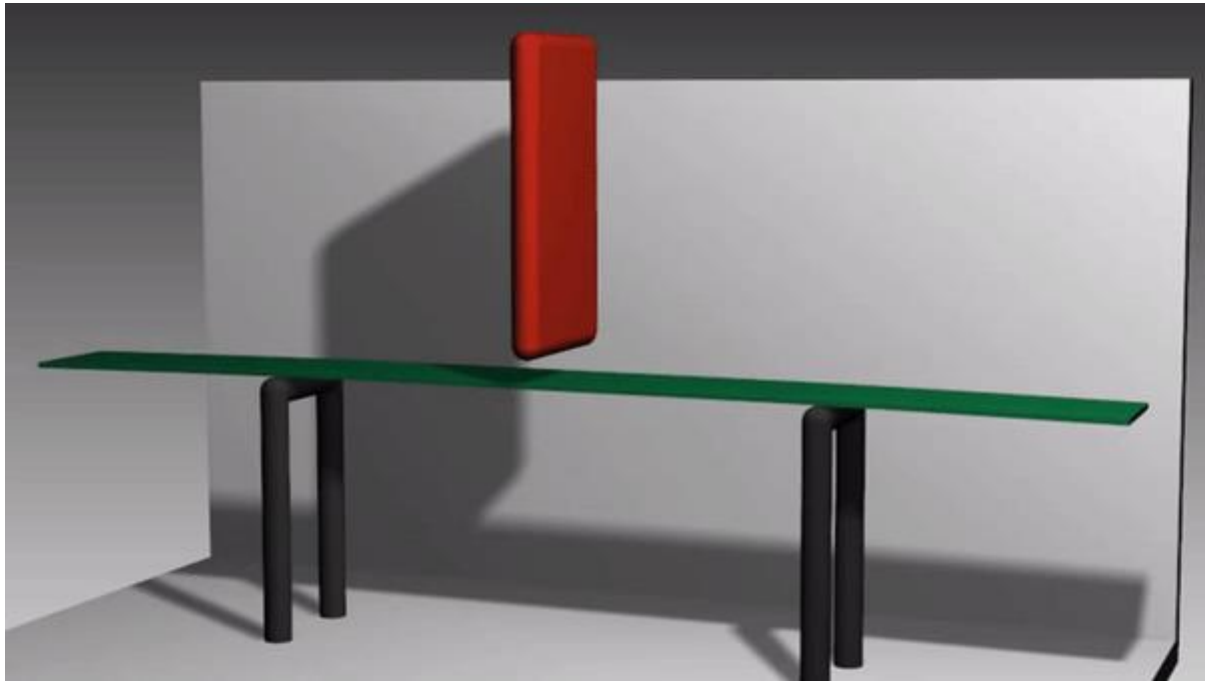
However, these accelerations may be exceeded due to some emergency flight conditions (engine failure, stalling, gusts).

Limit load

Limit load refers to the loads that the aircraft is expected to experience **during its service**.

The aircraft must withstand the limit load **without suffering damage due to permanent deformation**.

The structure must support a load greater than the limit load **without affecting safe operation**.



https://yasincapar.com/wp-content/uploads/2020/12/ezgif.com-video-to-gif-1.gif

Ultimate load

The ultimate load is also called the design load and is calculated as follows:

$$ultimate\ load = limit\ load * SF$$

A SF value of 1.5 is commonly used.

It is not that common that ultimate loads are exerted on aircraft.

This load is defined to guarantee uncertainties in calculations and maneuvers

https://www.youtube.com/watch?v=WlqSha2Ve7I&ab_channel=DGFlugzeugbauGmbH

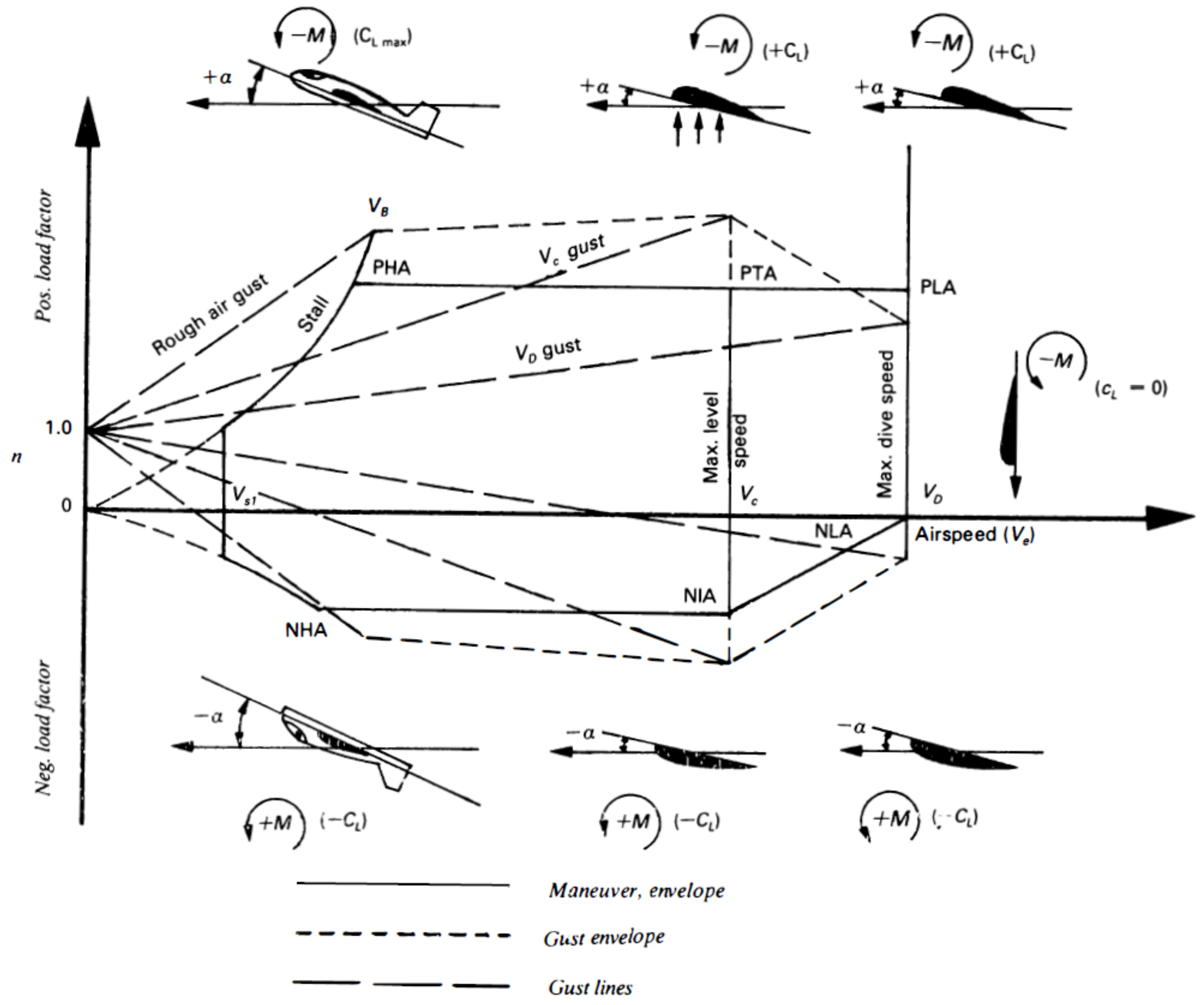
https://youtu.be/Q9ins-nbsGY

https://youtu.be/T_XukF0sOpY

https://youtu.be/GzVGWsRrEvo?t=29

https://youtu.be/HUeyZ2wuZBM

V-n Diagram

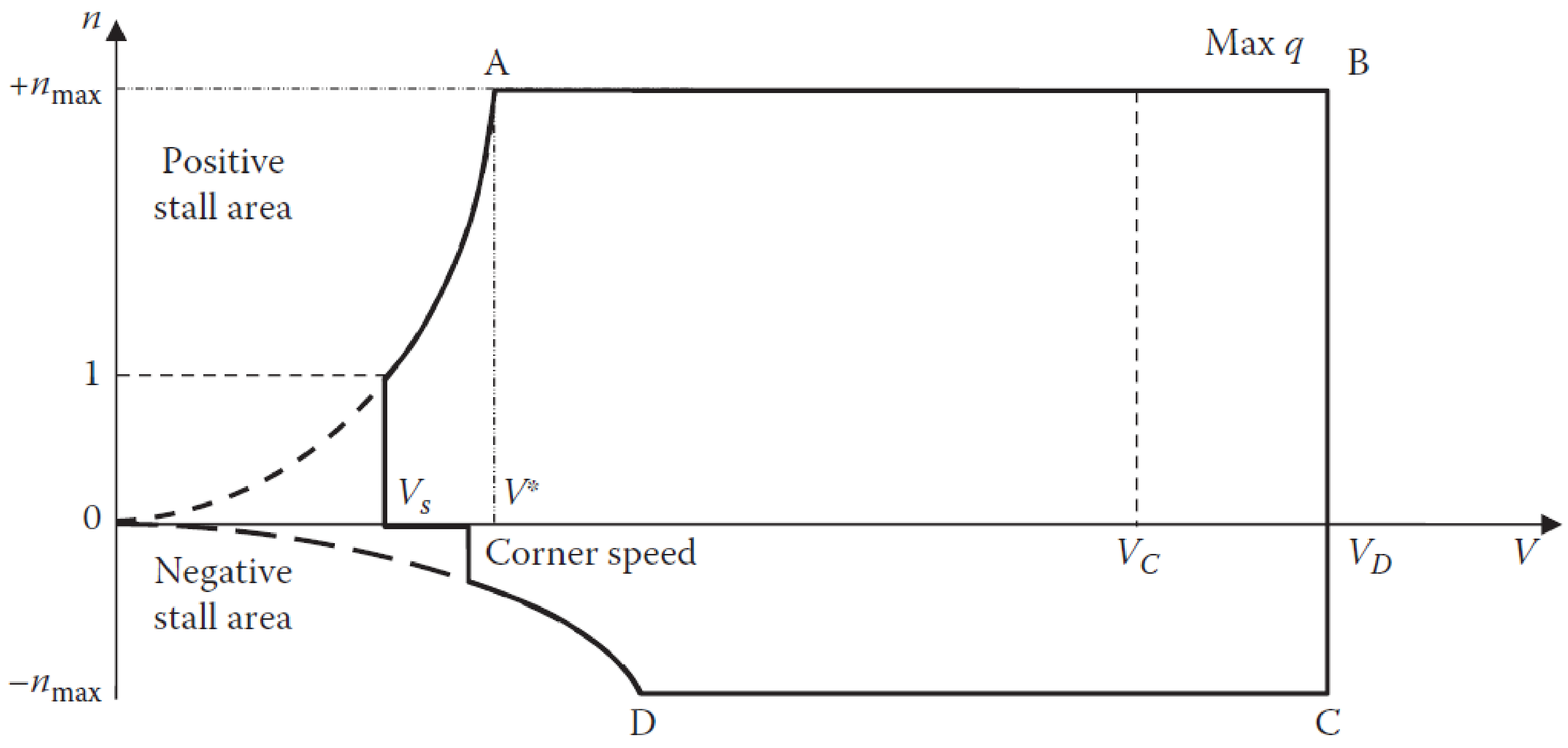


−M
$(C_{L\,max})$
+α
−M
$(+C_L)$
+α
−M
$(+C_L)$
+α
Pos. load factor
V_B
PHA
V_c gust
PTA
PLA
Rough air gust
Stall
V_D gust
−M
$(c_L = 0)$
1.0
n
0
V_{s1}
Max. level speed
V_c
Max. dive speed
V_D
NLA
Airspeed (V_e)
NIA
NHA
Neg. load factor
−α
−α
−α
+M
$(-C_L)$
+M
$(-C_L)$
+M
$(-C_L)$
Maneuver, envelope
Gust envelope
Gust lines

V-n Diagram

In this V-n diagram, gust are neglected.

X-axis correspond to design speeds and Y-axis are the load factors.



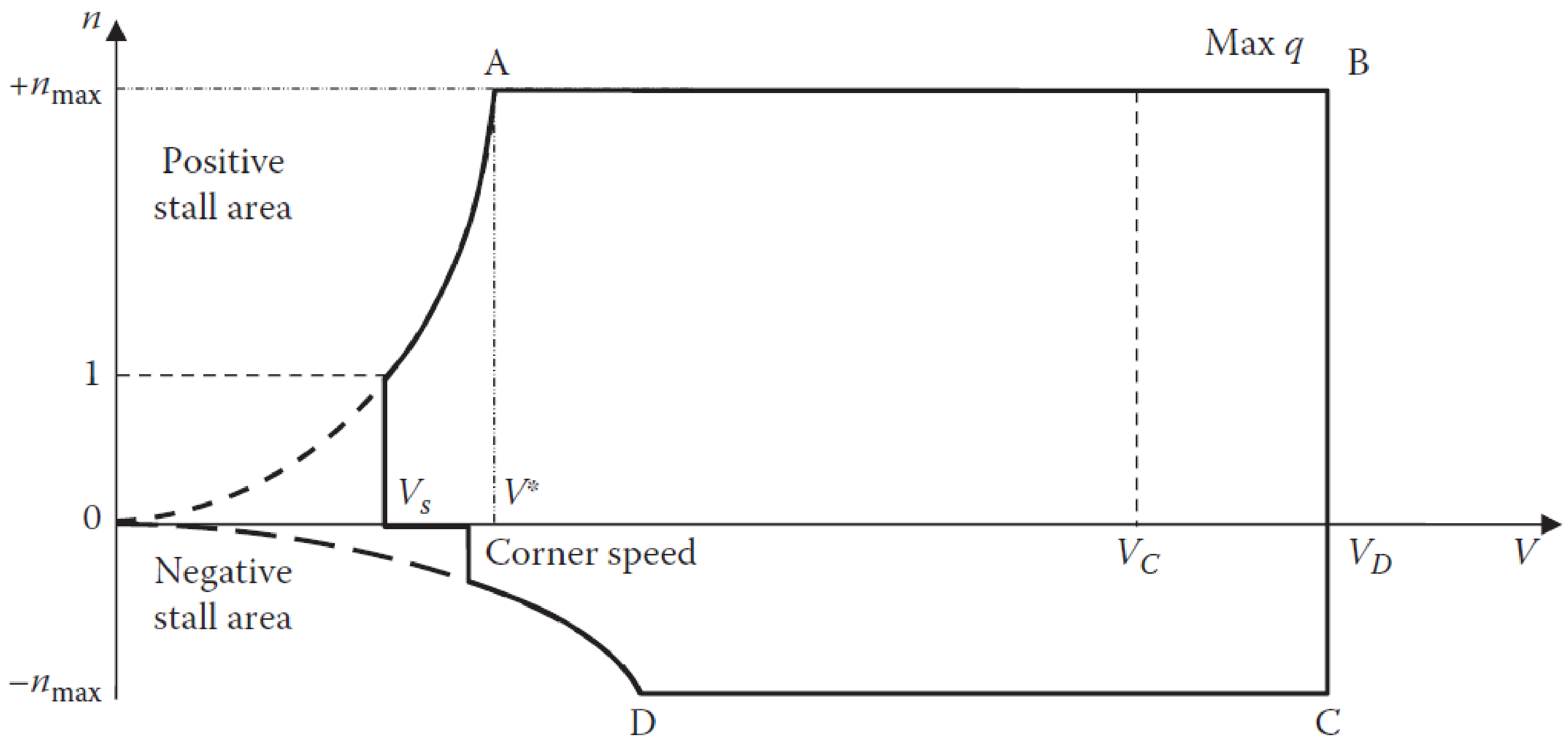
V-n Diagram

First two curves represent aerodynamical limitations +n and –n due to $C_{L\ stall}$

$$V_S = \sqrt{\frac{2nmg}{\rho S C_{Lmax}}}$$

Region above and below this curves represent aircraft stall.

V_s is set at $n = 1$



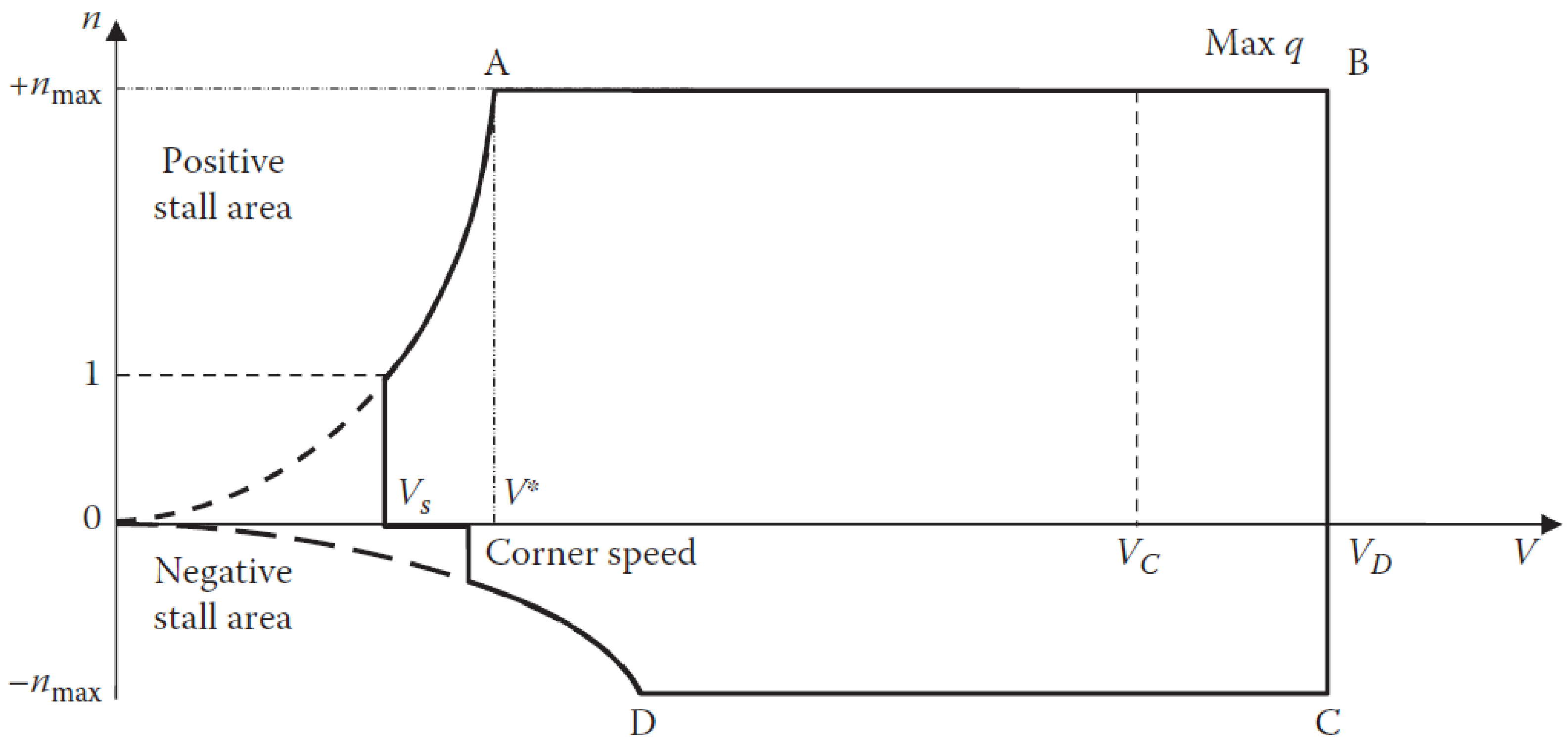
V-n Diagram

The V_S equation can be solved for the maximum load factor for a given velocity:

$$n = \frac{\rho V^2 S C_{L\,max}}{2mg}$$

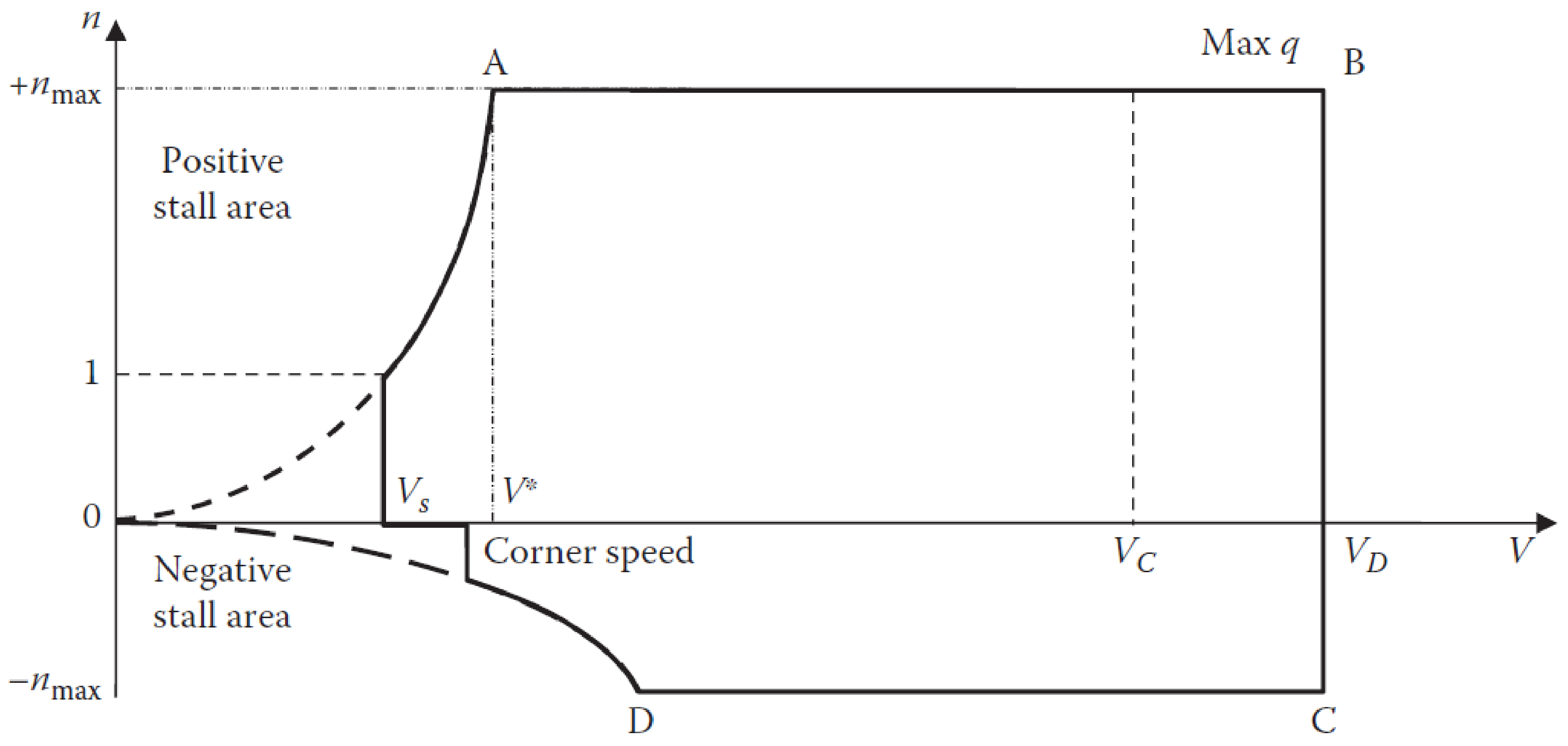
$$n = \frac{q C_{L\,max}}{W/S}$$

Where, W/S is the wing loading (carga alar)



V-n Diagram

The intersection line between positive stall curve and load factor 0 it is known as “cornering speed” or “maneuvering speed” V_A

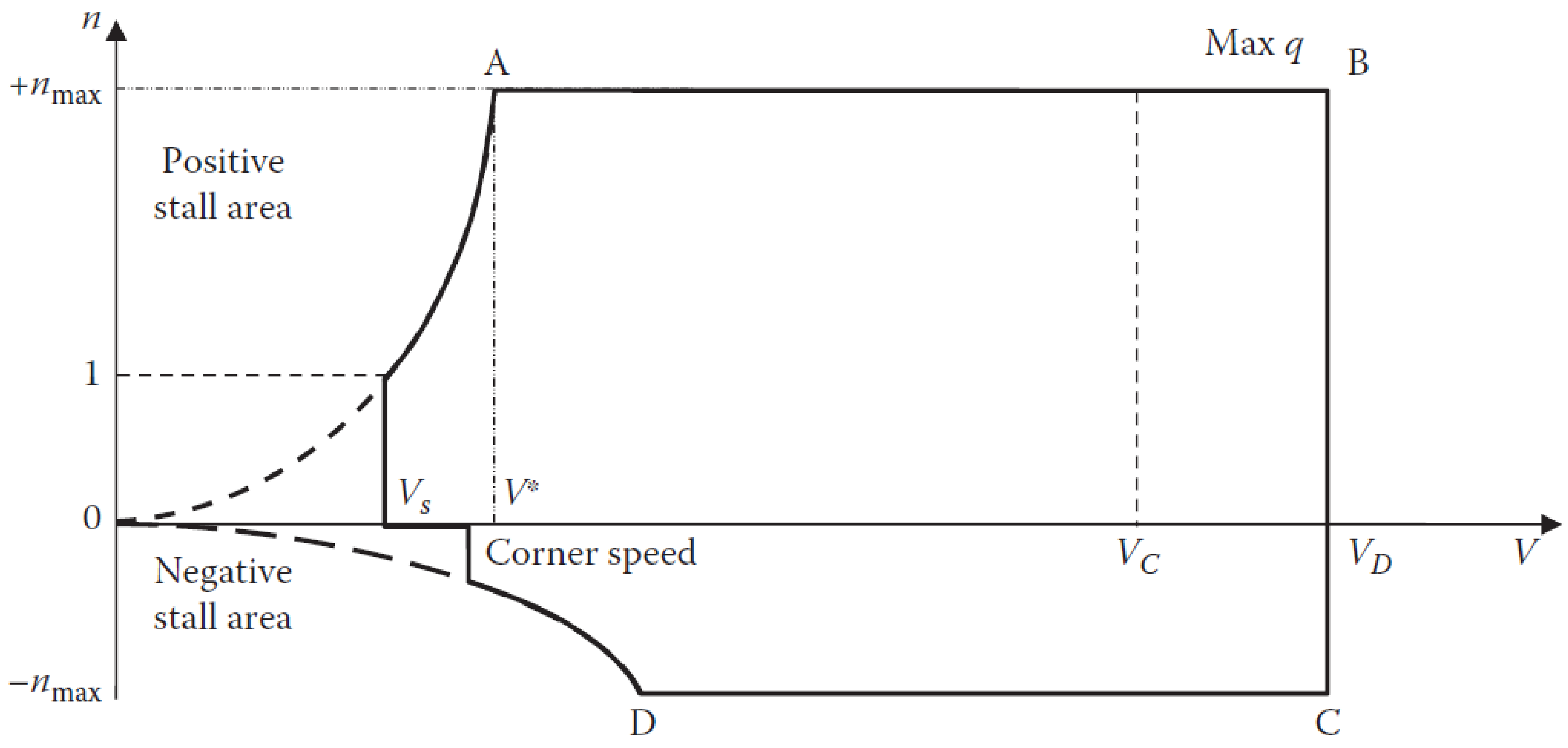


$$V^* = V_A = \sqrt{\frac{2n_{max}mg}{\rho S C_{Lmax}}}$$

$$V_A = V_S\sqrt{n_{max}}$$

V-n Diagram

At lower speeds than $V^* = V_A$, pilots can perform maneuvers.

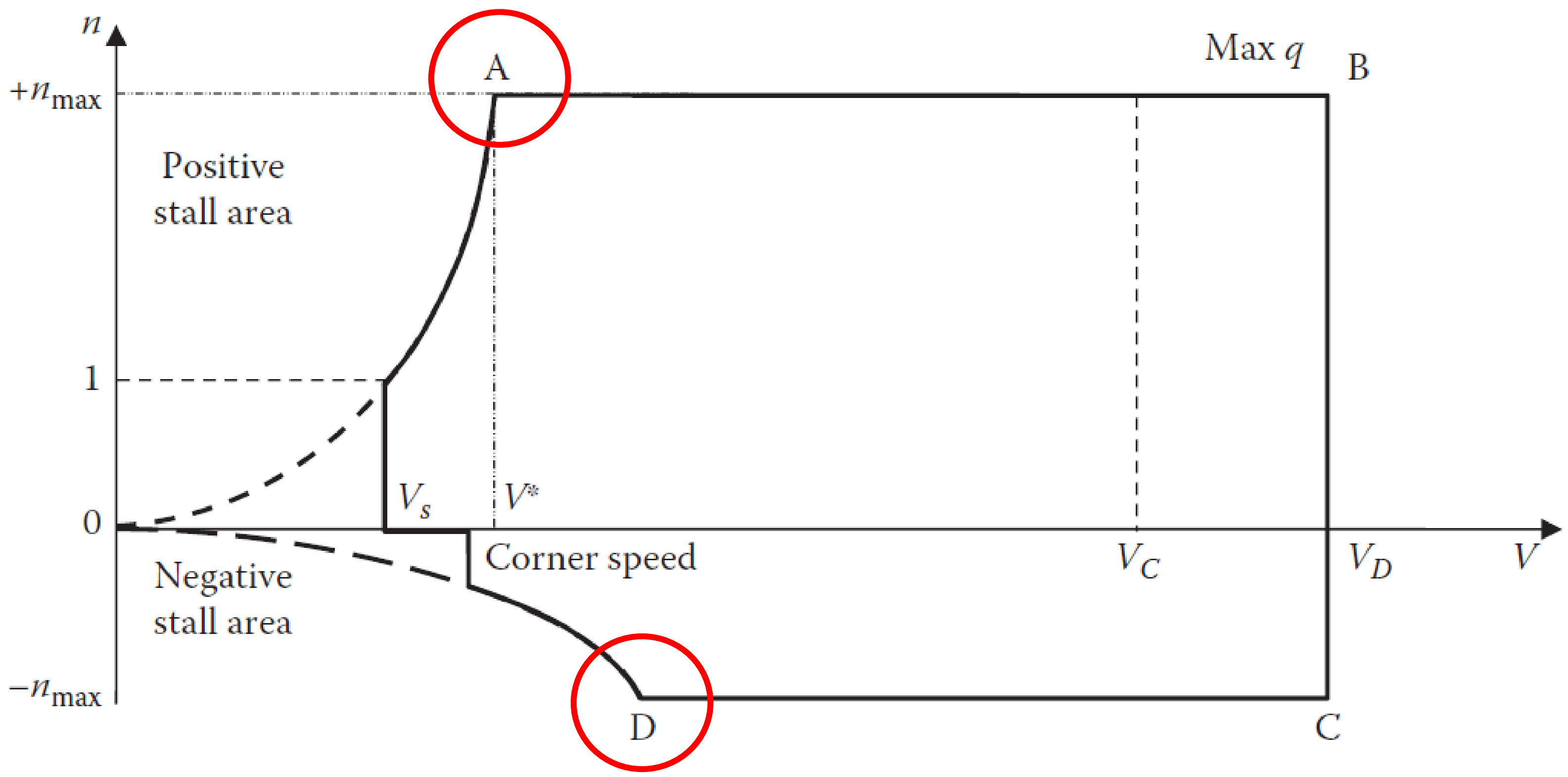


V-n Diagram

At lower speeds than $V^* = V_A$, pilots can perform maneuvers.

As the speed increases above V^* the maneuvers become more limited.

These points correspond simultaneously to the tightest possible turn and the fastest possible turn.

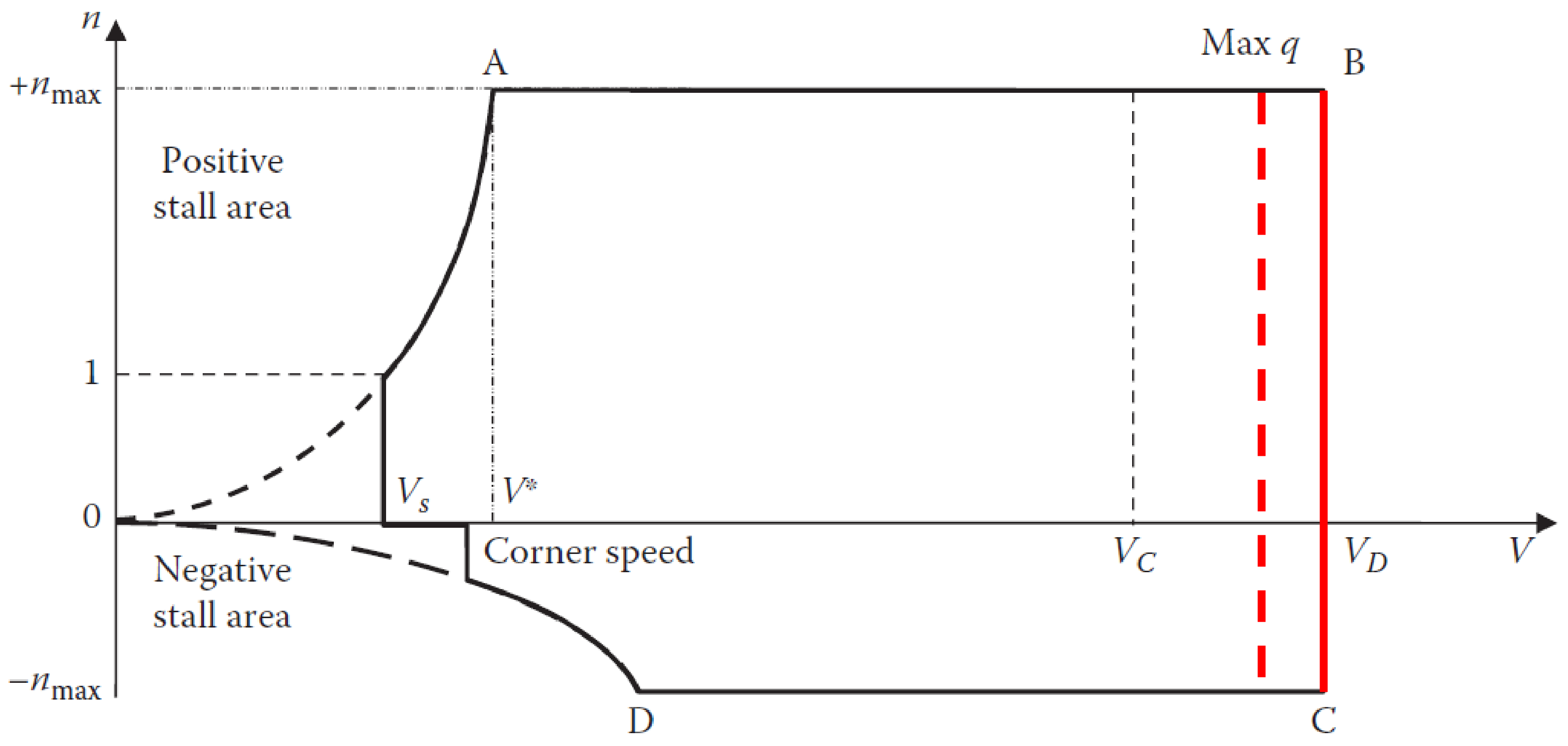


V-n Diagram

B-C line represents the highest achievable speed

This speed is also known as dive speed V_D

Dynamic pressure q is the greatest value in which aircraft is design to withstand or resist (Flutter, aileron reversal)

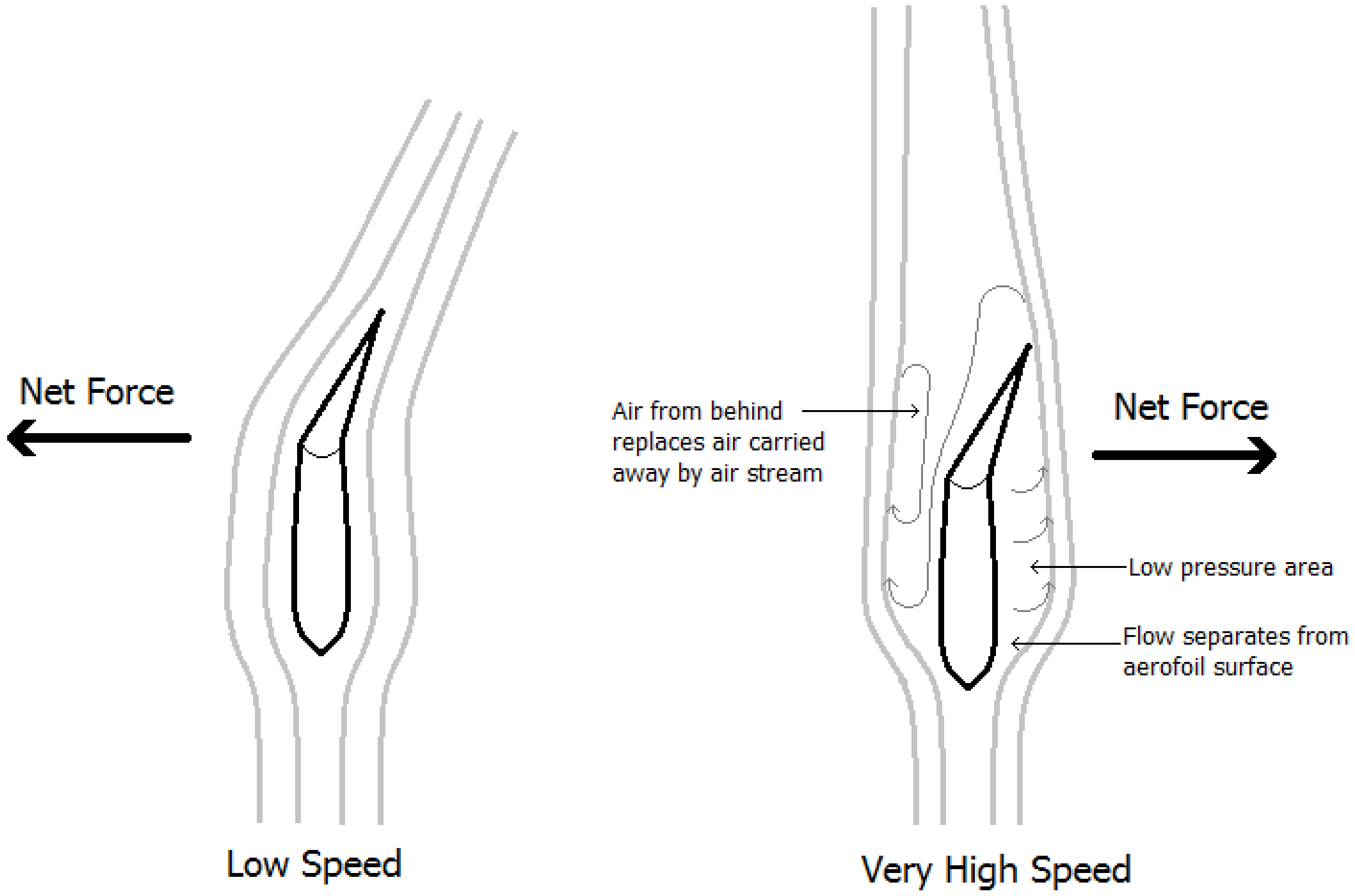


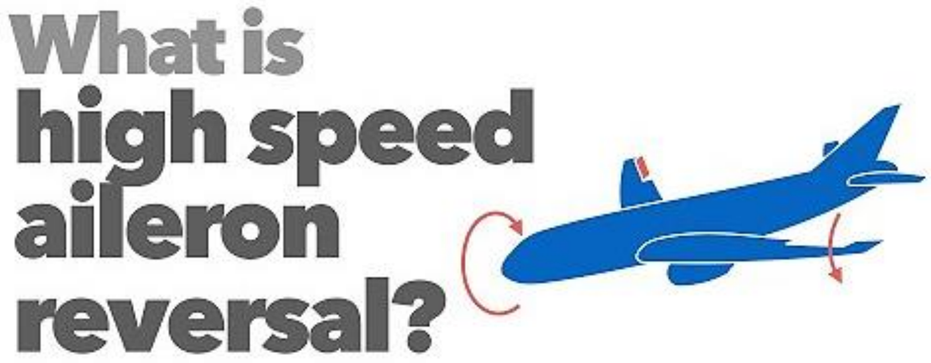
Flutter

https://i.makeagif.com/media/11-10-2015/_tvFel.gif

https://64.media.tumblr.com/594e477e18bec748593764311cdb3c22/tumblr_oc8t4zlUa21us21qco2_r1_400.gif

Aileron reversal





https://www.youtube.com/watch?v=X8OpuijN4sA&ab_channel=flight-club

V-n Diagram

FAR 23

$V_D \geq 1.4V_C$ (Normal, utility)

$V_D \geq 1.55V_C$ (Aerobatics)

FAR 25

$V_C \leq 0.8V_D$ (Transport)

$V_D \geq 1.25V_C$

FAR 23 (OLD)

$V_D \geq 1.4V_C$ (Normal)

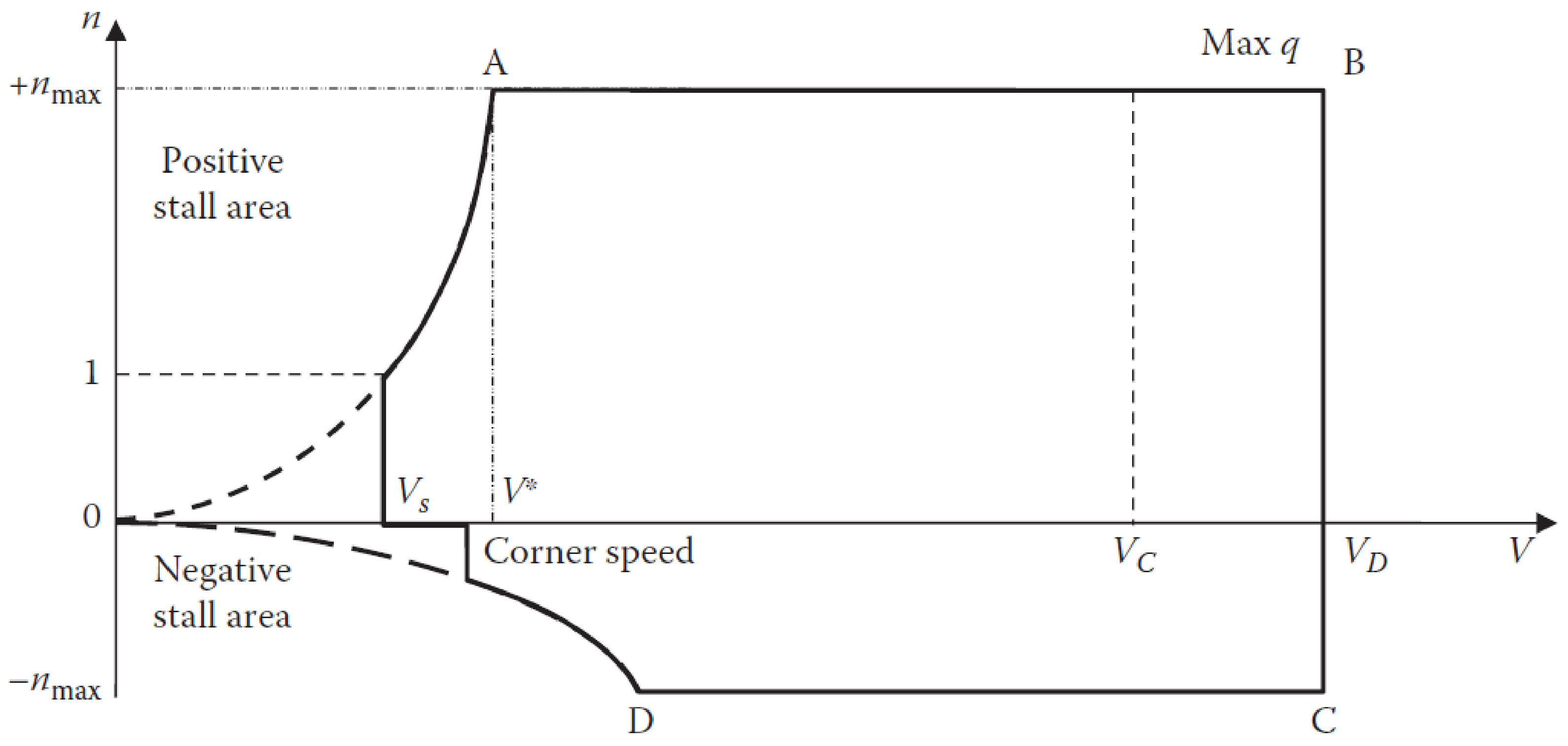
$V_D \geq 1.5V_C$ (Utility)

$V_D \geq 1.55V_C$ (Aerobatics)

V-n Diagram

The lower line C-D corresponds to the maximum negative load factor that can be reached in situations such as **inverted flight**.

The lower left curve corresponds to the negative stall speed.

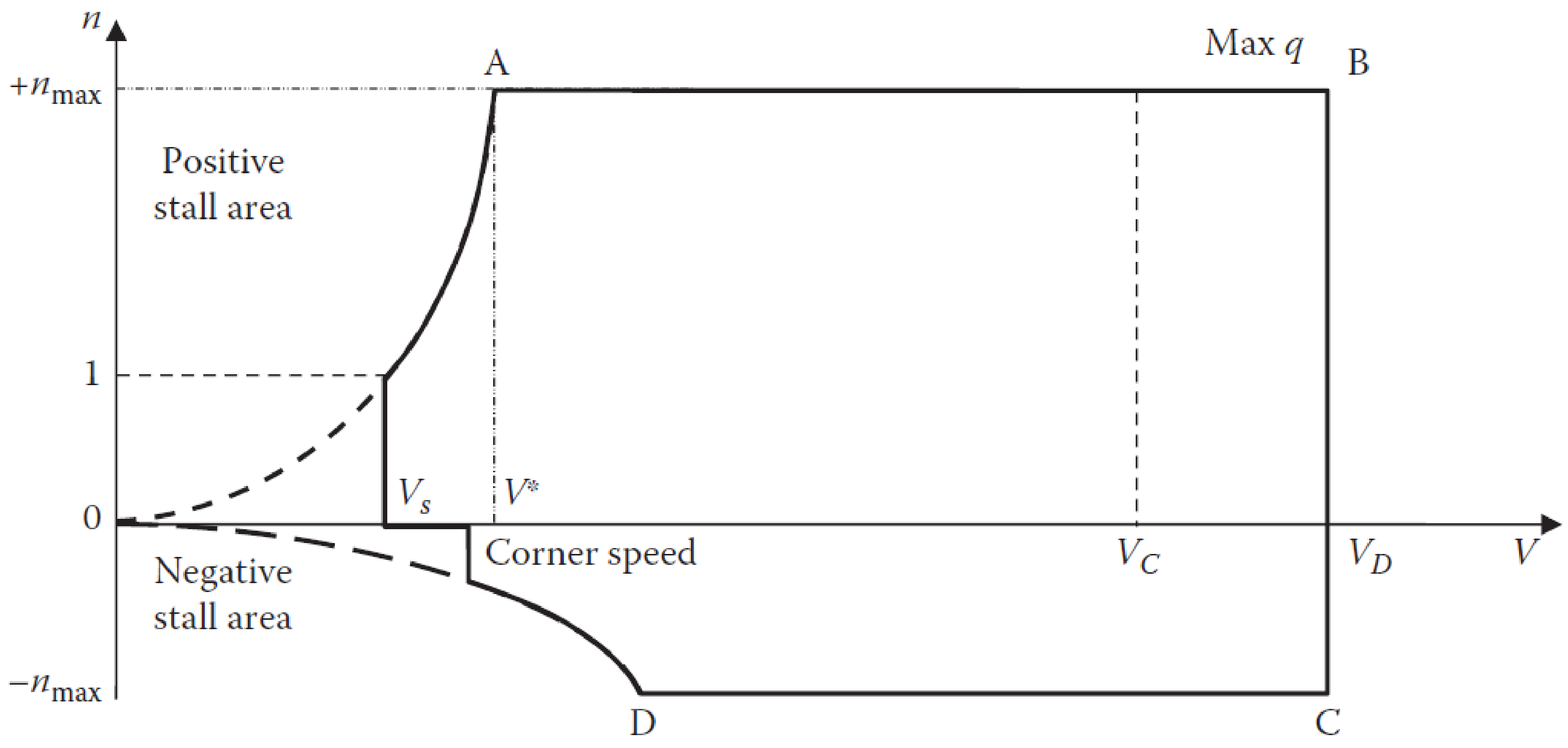


V-n Diagram

When V-n diagram is calculated, it is common not to find certain values from airplanes. In this case assume this values for similar aircrafts or assume with FAR 23 o FAR 25.

$-C_{Lmax}$ can be approximated by:

$$-C_{Lmax} = 0.5C_{Lmax}$$



Load levels

$n_1 = n_{max}$ (positive limit load)

$n_2 = 0.75\, n_{max}$

$n_3 = n_{min}$ (negative limit load)

Proof load and Ultimate load

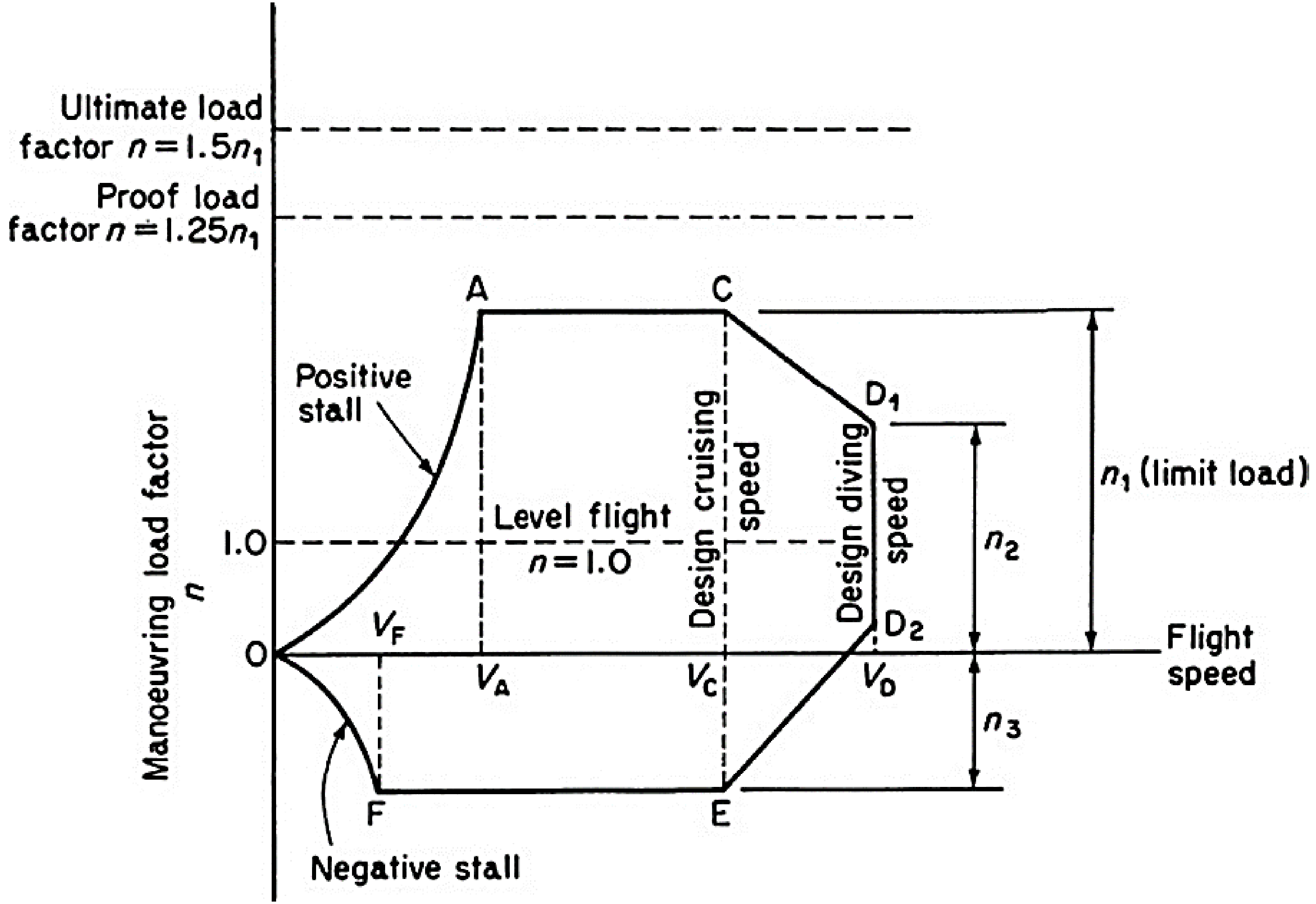
We have to use a factor safety in the actual strength of any designed component. This factor is considered in two steps. In the first step, we **multiply n_{max} by a factor of 1.25 to obtain the proof load factor**.

In the second step, we **multiply n_{max} by a factor of 1.5 to get the ultimate load**.

The factors 1.25 and 1.5 are the recommended values from the cumulative experience of aircraft design over the years. A component is expected to survive up to the ultimate load.

The structure must be able to support ultimate loads without failure for at least 3 seconds.

Proof load and Ultimate load



Proof load and Ultimate load

https://www.youtube.com/watch?v=Kx5YeqTBcDI&ab_channel=NASALangleyResearchCenter

Gust V-n diagram

- The atmosphere is a dynamic system that can bring with it unwanted phenomena.

- These phenomena include shear winds, turbulent flows of other aircraft, thermal, among others.

- In this section we focus on turbulent phenomena because they are not easy to predict but it is very common to encounter them during a normal flight.

- These types of gusts can be added to the V-n diagram.

Gust V-n diagram

- Load factor due to gusts is caused by **atmospheric disturbances** and this is beyond the pilot's control.

- **There is no** exact way to determine the **appropriate resistance** for this type of loads but hundreds of flight hours ensure safe operation.

- Statistical information provided by agencies is used

Gust V-n diagram

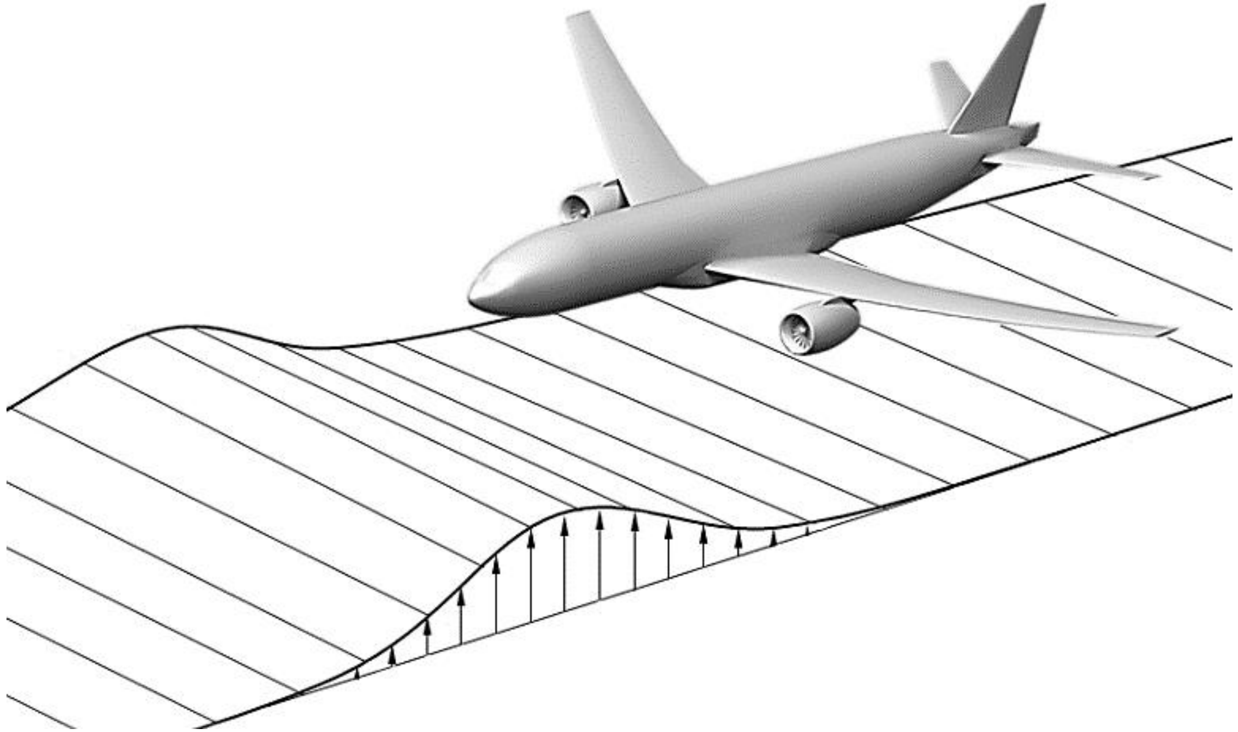
- We then need to know the speed of the gusts to determine the loads due to these.

- Typically, gust speed information is determined from statistical studies based on experimental flights.

https://www.nasa.gov/centers/armstrong/features/flying_high_at_60.html

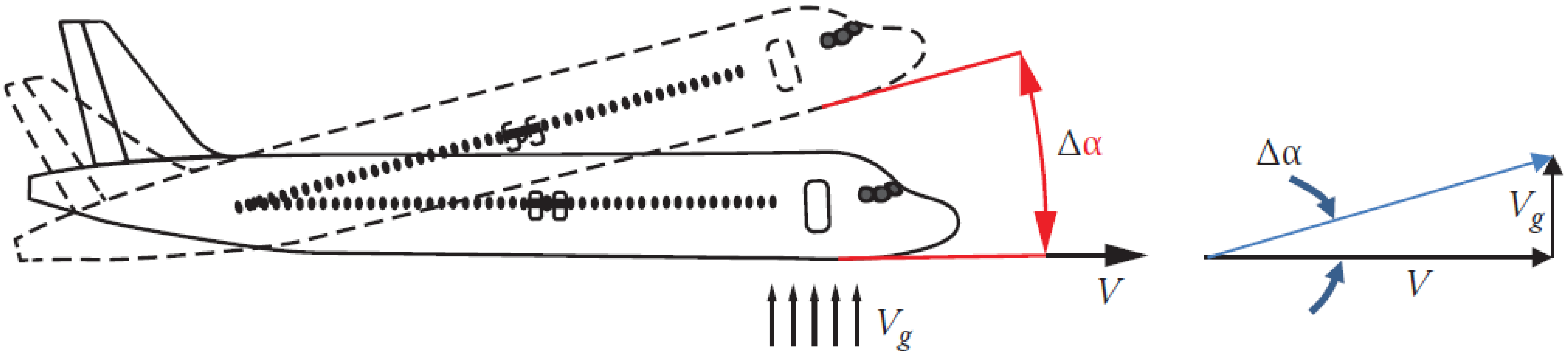
Gust V-n diagram

- Gusts can impact the aircraft in any direction.
- These can be upward, downward or lateral gusts which lead to uncontrolled movements.
- The immediate effect is an increase or decrease in the aircraft's angle of attack.



Gust V-n diagram

- Assuming a gust speed V_g that impacts the aircraft flying at a speed V.
- This caused incidence can be modeled as a change in the angle of attack.

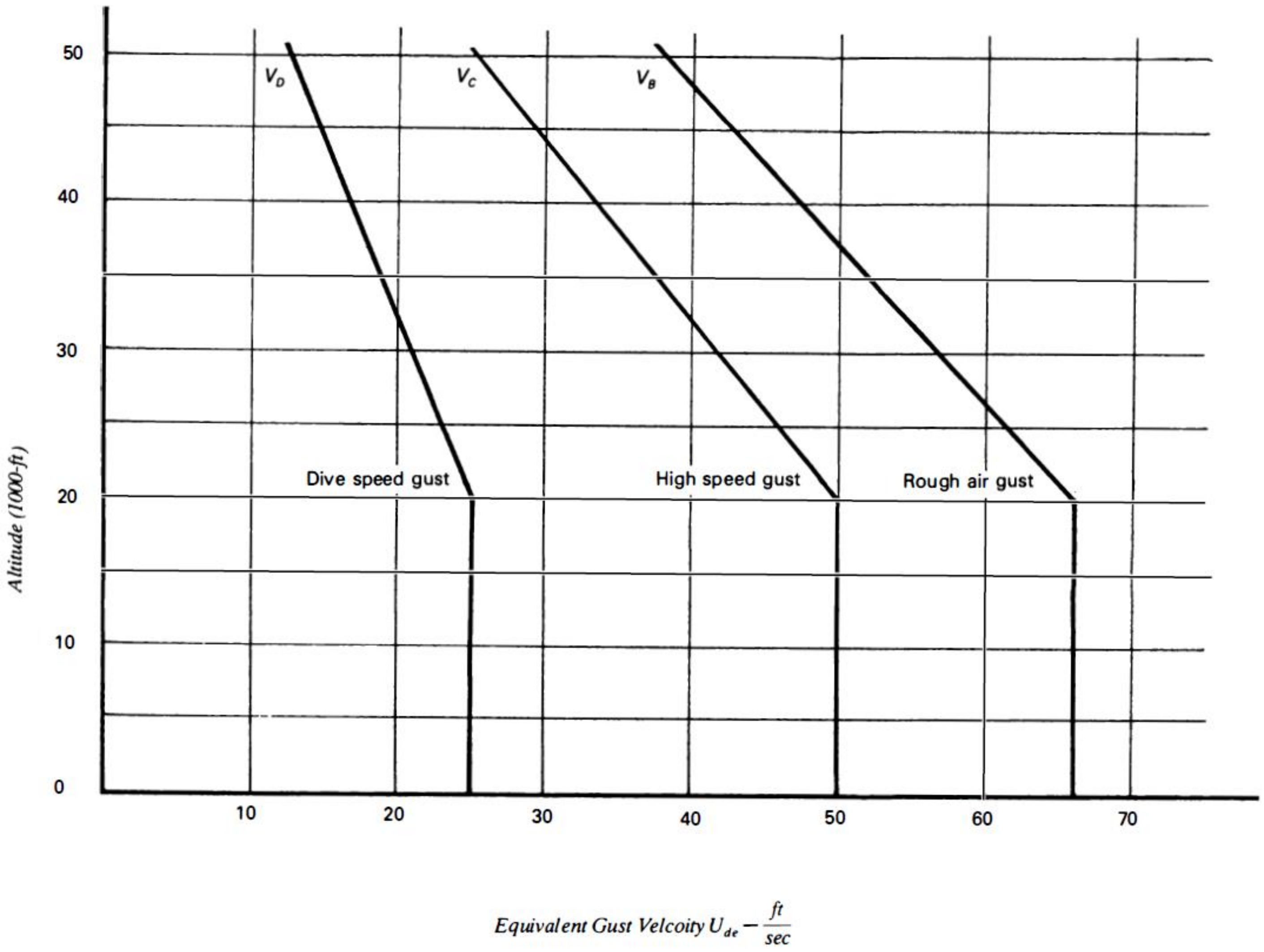


Gust V-n diagram

- Any change (increase) in the angle of attack produces a change in the lift coefficient, therefore a change in lift as such.

- Consequently there will also be a change in the load factor.

Gust V-n diagram



Gust V-n diagram

Gust Speed for Constructing Gust-Induced Load				
No.	**Aircraft Type**	**Airspeed**	**Sea Level up to 20,000 ft**	**20,000–50,000 ft**
1.	GA normal, utility, acrobatic	Cruise speed	50 ft/s	50 ft/s decreases linearly to 25 ft/s
		Dive speed	25 ft/s	25 ft/s decreases linearly to 12.5 ft/s
2.	Commuter	Design speed for maximum gust intensity	66 ft/s	66 ft/s decreases linearly to 38 ft/s
No.	**Aircraft Type**	**Airspeed**	**Sea Level up to 30,000 ft**	**30,000–80,000 ft**
3.	Transport	Cruise speed	85 ft/s	85 ft/s decreases linearly to 30 ft/s
		Design speed for maximum gust intensity	112 ft/s	112 ft/s decreases linearly to 40 ft/s

Example

Airplane data
Geometry (See Fig. 3.11.1).
Wing:
Area, $S = 1200$ sq ft
Span, $b = 98$ ft
Aspect ratio, AR (or A) $= 8.0$
Taper ratio, $\lambda = \dfrac{C_T}{C_R} = 0.4$
Root chord, $C_R = 17.5$ ft
Tip chord, $C_T = 7.0$ ft

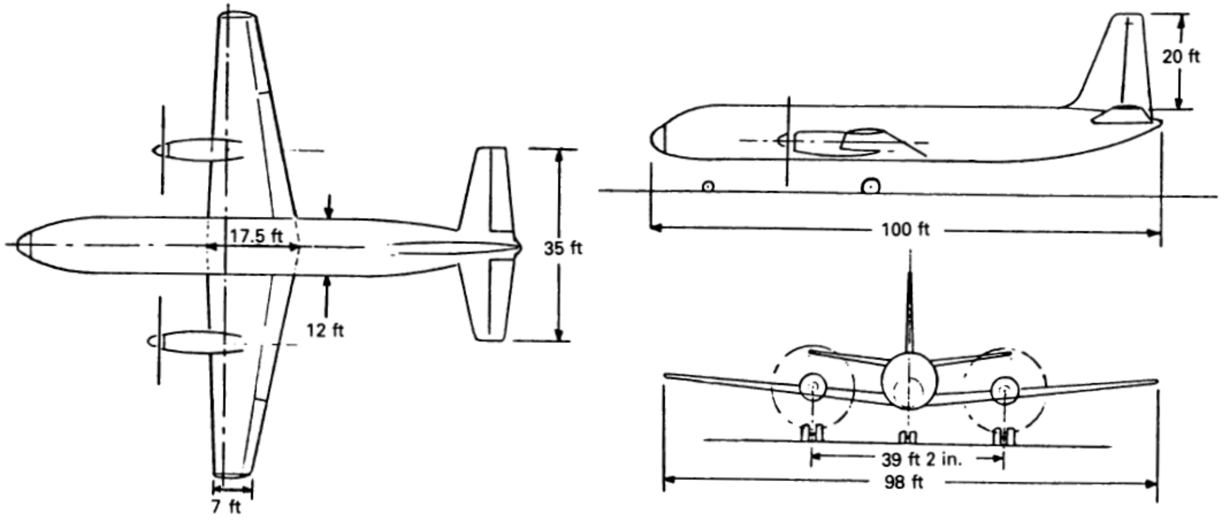
Mean aerodynamic chord, $\bar{c} = 13.0$ ft
Vertical Tail:
Area, $S_{vt} = 220$ sq ft
Tail length, $\ell_{vt} = 45$ ft
Rudder area, $S_{\gamma} = 60$ sq ft
Rudder mean chord, $\bar{c}_{\gamma} = 3.5$ ft
Horizontal Tail:
Area, $S_{ht} = 200$ sq ft
Tail length, $\ell_{ht} = 48.0$ ft
Weight Data
Maximum take-off gross weight = 108 000 lb
Landing design gross weight = 88000 lb
Maximum zero fuel weight = 84 000 lb
Aerodynamic Data
$C_{L\ \max}$ (complete airplane) = 1.3
$C_{L_{\alpha_A}} = 1.10\ C_{L_{\alpha_W}}$
$C_{h_{\delta_\gamma}} = 0.01$ per degree
Rudder hinge moment output = 7520 ft-lb
Landing Gear Data
Strut stroke, $X_S = 18$ in
Tire defection, $X_t = 4.5$ in
Tire spring constant = 160000 lb/ft
Tire efficiency, $\eta_t = 0.8$

Example

Calculate and sketch the V-n diagram for maneuver and gust for a gross weight of 88 000 lbs at 10 000 ft altitude assuming that $V_c = 325$ KEAS and $V_D = 400$ KEAS. Use commercial airplane requirements. Obtain wing lift slope ($C_{L\alpha}$) from Ref. 3.5. Assume $C_{L_{max}}$ negative is equal to $C_{L_{max}}$ positive.

Refer to tabular form in Fig. 3.11.6:

Step ②: $M = \frac{V_e}{C_{se}}$; where $C_{se} = 548.6$ KEAS at $h = 10\,000$ ft from Fig. 3.1.5



Gust V-n diagram

Design speed for maximum gust intensity may not be less than:

$$V_B = V_{S1}\left[1 + \frac{K_g V_{gE} V_C \rho C_{l\alpha A} S}{2mg}\right]^{1/2}$$

$$V_B = V_{S1}\left[1 + \frac{K_g V_{gE} V_C C_{l\alpha A}}{498\frac{W}{S}}\right]^{1/2}$$

Where:

V_{S1} stall speed with retracted flaps

K_g gust alleviation factor

V_{gE} rough air gust speed

V_C cruise speed

ρ air density

$C_{l\alpha A}$ aircraft lift slope

S wing area

W aircraft weight

Gust V-n diagram

Load factor induced by gusts is calculated as follows:

$$\mathbf{n} = \mathbf{1} \pm \frac{\boldsymbol{K_g V_{gE} V_C \rho C_{l\alpha A} S}}{\boldsymbol{2mg}}$$

If working in imperial system, with sea level density:

$$\mathrm{n} = 1 \pm \frac{K_g V_{gE} V_E C_{l\alpha A}}{498 \frac{W}{S}}$$

$$W \ in \ lbf, S \ in \ ft^2, V_E \ KEAS$$

Gust V-n diagram

The gust alleviation factor K_g is determined as follows:

$$\boldsymbol{K_g = \frac{0.88\,\mu_g}{5.3 + \mu_g}}$$

$\boldsymbol{\mu_g}$ is the **“aircraft mass ratio”**

$$\boldsymbol{\mu_g = \frac{2\frac{W}{S}}{g\rho C_{MAC} C_{l\alpha A}} = \frac{2m}{\rho\, C_{MAC}\, C_{l\alpha A}\, S}}$$

Where:

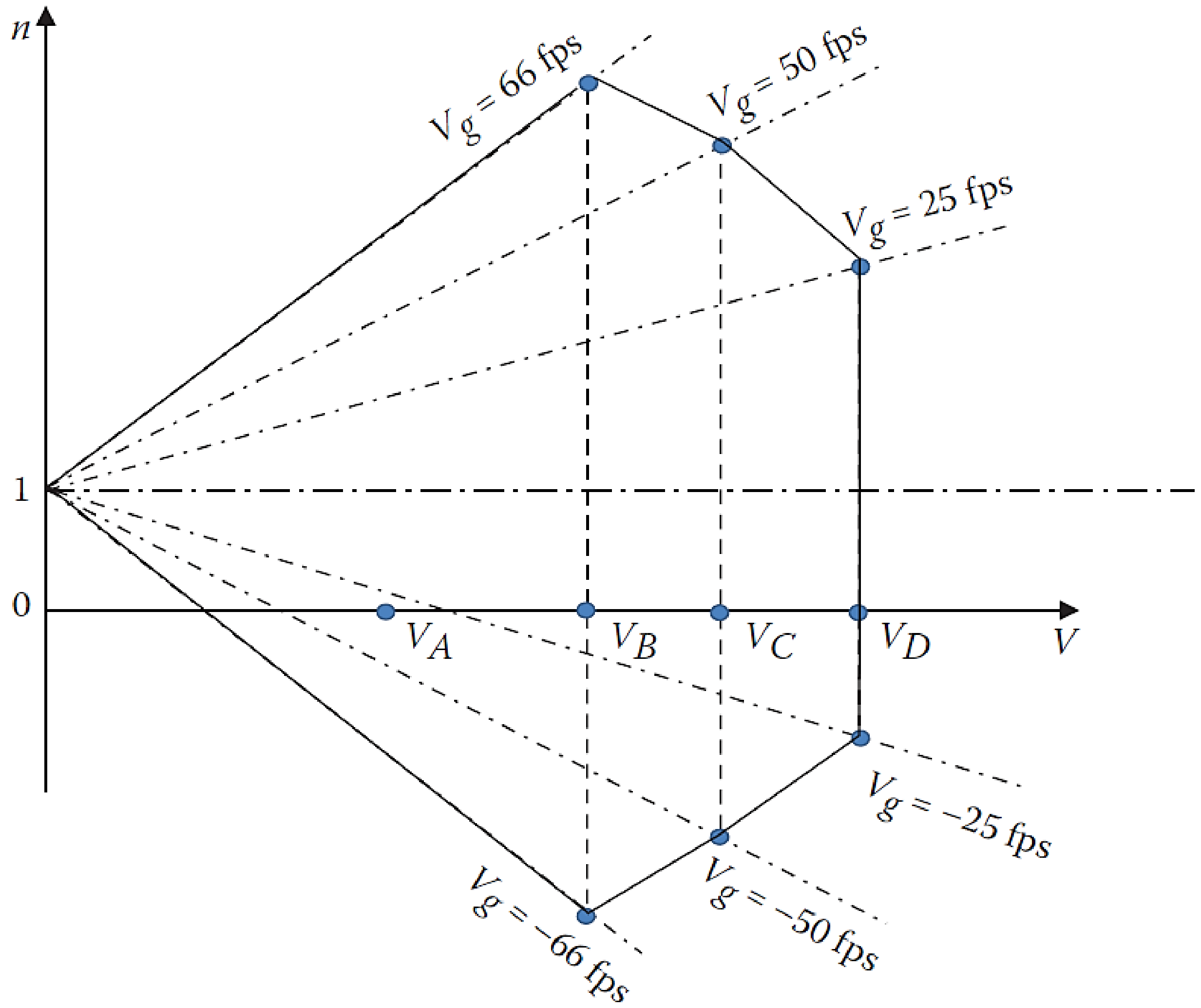
g gravity

ρ air density at indicated altitude

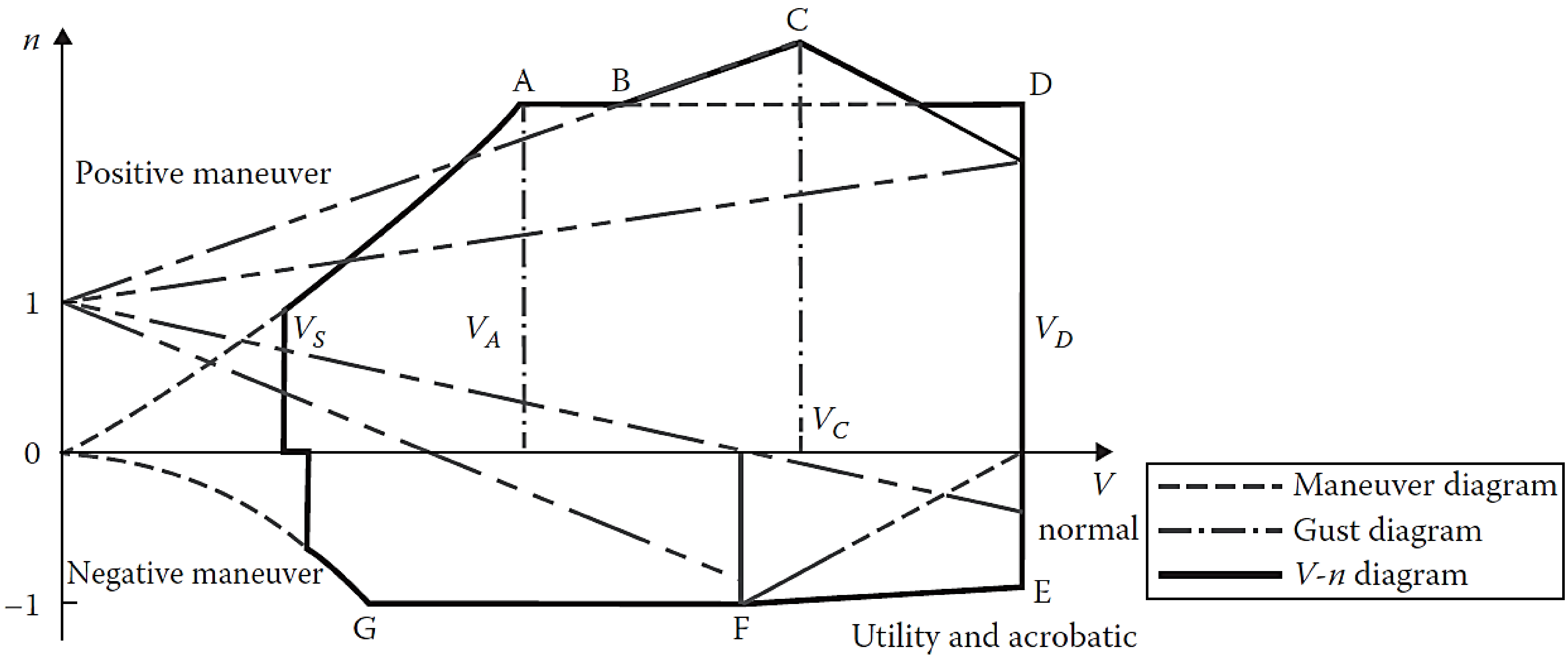
C_{MAC} wing mean aerodynamic chord

W aircraft weight

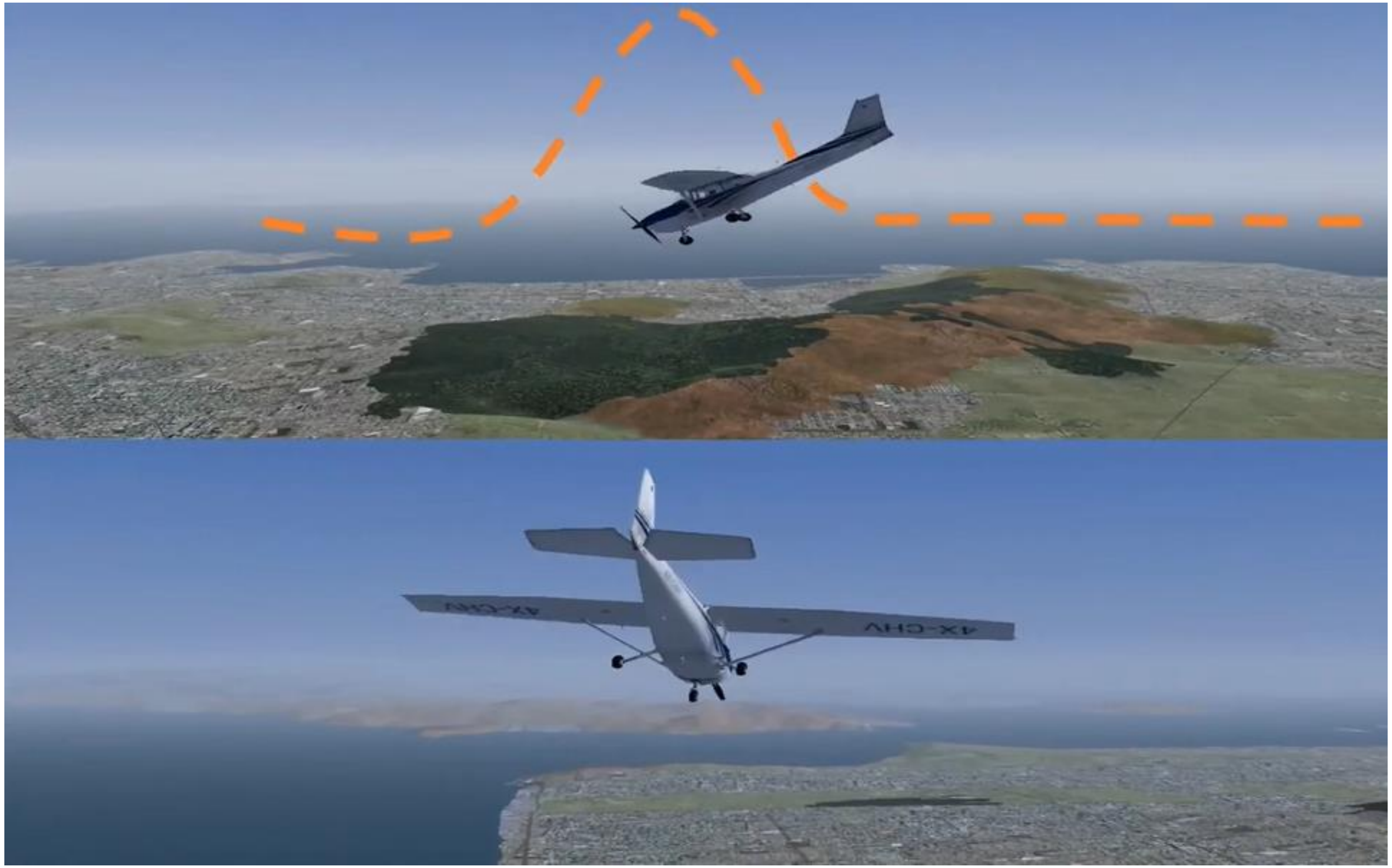
Gust V-n diagram



Gust V-n diagram



Gust V-n diagram



https://www.youtube.com/watch?v=u8dlFdRAQM4&ab_channel=ControlOVGU

https://www.youtube.com/watch?v=4LyiULX51MM&ab_channel=THEDonMaxwell

https://www.youtube.com/watch?v=fQCo47AZGD8&ab_channel=yani992

Compressible Parameters

Compressibility sweep parameter:

$$\Lambda_\beta = \tan^{-1}\frac{\tan\Lambda}{\beta}$$

Compressibility parameter:

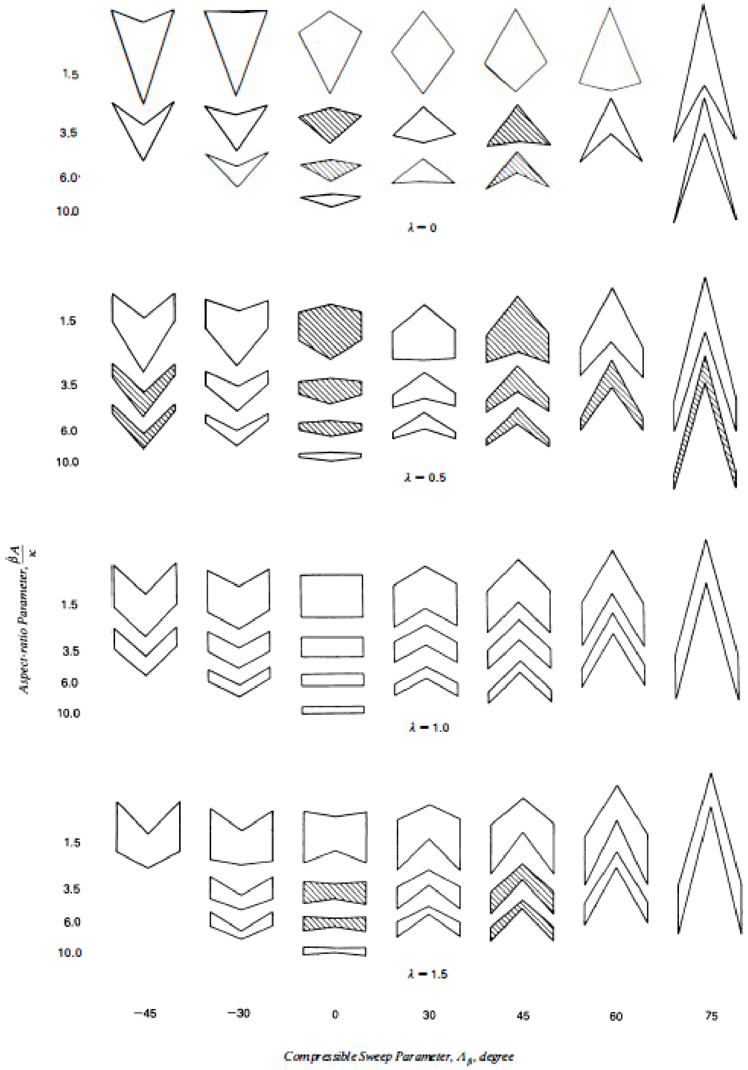
$$\beta = \sqrt{1 - M^2}$$

Lift-curve-slope parameter:

$$\frac{\beta C_{L\alpha}}{k}$$

Where k is the ratio of $C_{L\alpha}$ to $(C_{L\alpha})$ theory

Assume $k = 1$



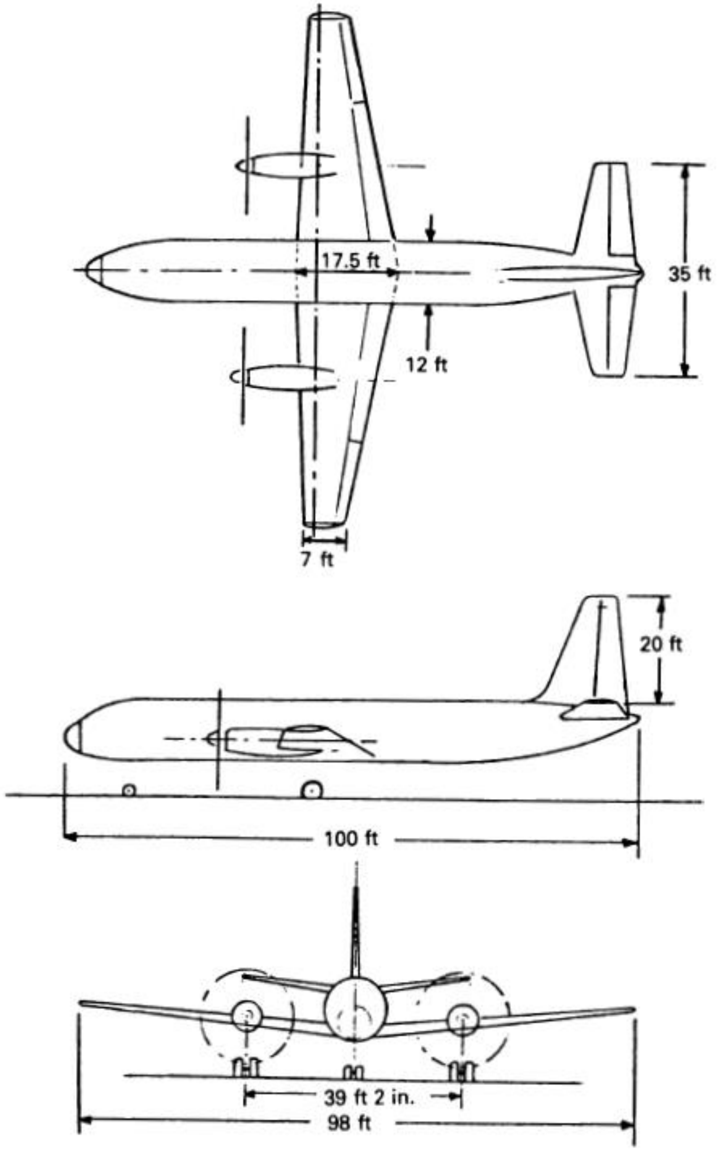
Example 2

$AR = 8$

$\lambda = 0.4$ (tapper ratio)

$\Lambda = 0$ (sweep angle of wing quarter-chord line, degrees)

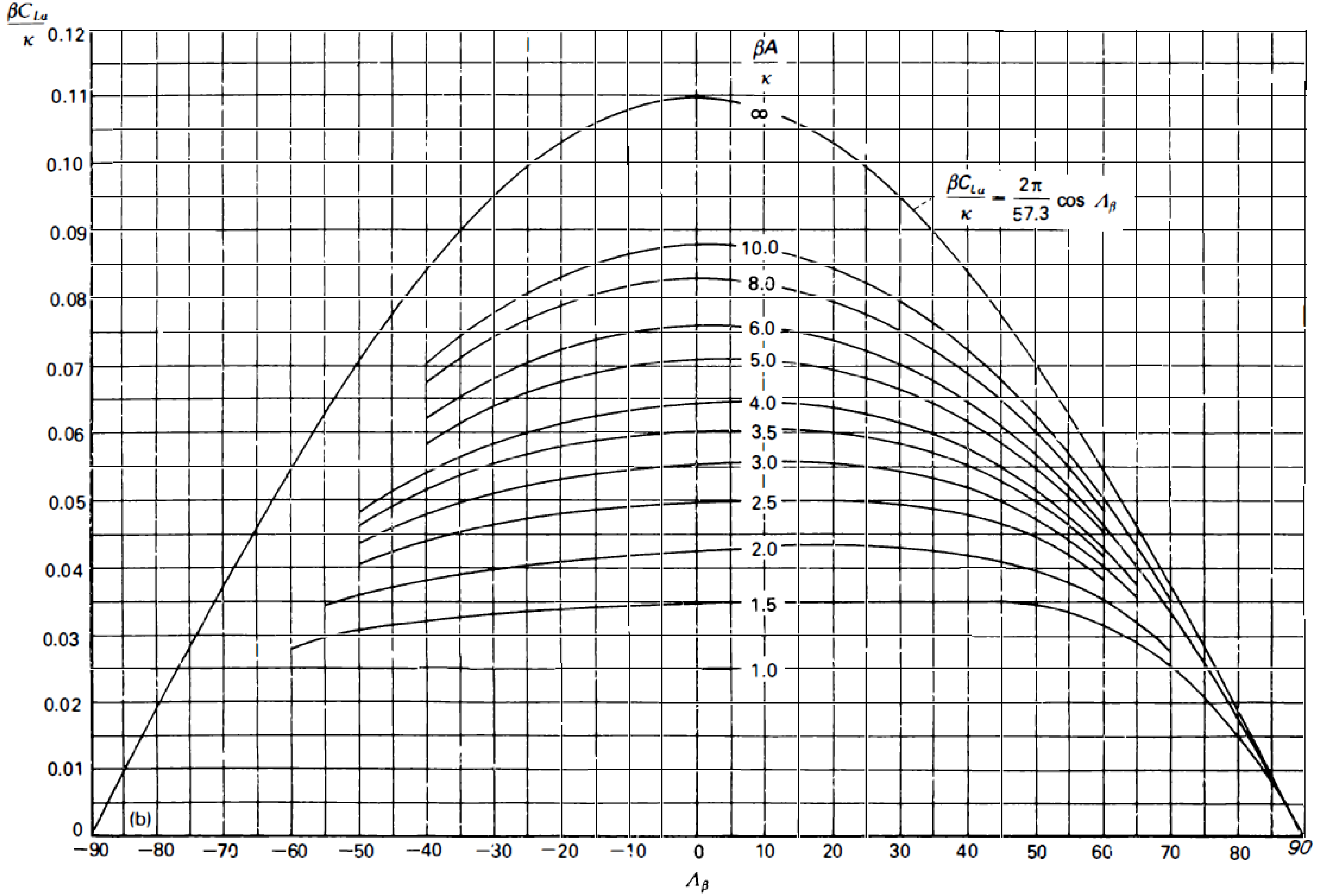
$C_{L\alpha A} = 1.1\, C_{L\alpha w}$



Calculate

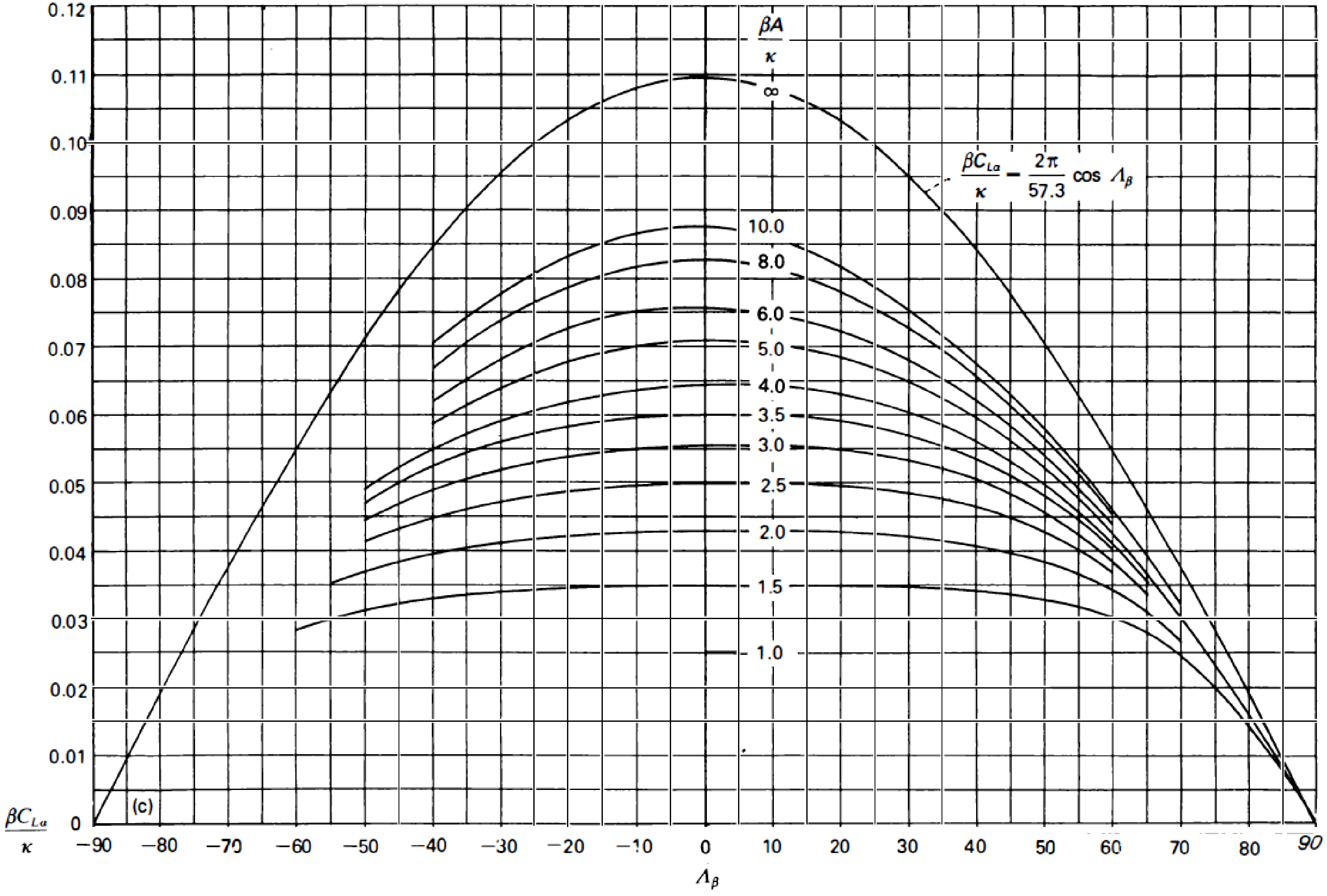
$$C_{L\alpha A}$$

Example 2



(b) $\lambda = 0.25$

Example 2



(c) $\lambda = 0.50$

Example 2

$\Lambda_\beta = 0$

At $M = 0$

$\beta = 1$

$k = 1$

$$\frac{\beta A}{k} = 8$$

Interpolation

$$y = y_1 + (x - x_1)\frac{(y_2 - y_1)}{(x_2 - x_1)}$$

$$C_{L\alpha w} = \frac{0.0827}{degree} = \frac{4.74}{rad}$$

$$C_{L\alpha A} = 1.1 * 4.74 = 5.214/rad$$

Example 3

Plot the flight envelope (combined V-n diagram) for the following GA acrobatic aircraft. Then, determine the maximum load factor.

$m = 2300\,kg$

$\mathrm{S} = 19.33\,m^2$

$C_{Lmax} = 2$

$-C_{Lmax} = -1.2$

$\mathrm{AR} = 7$

$C_{L\alpha} = 6.31/rad$

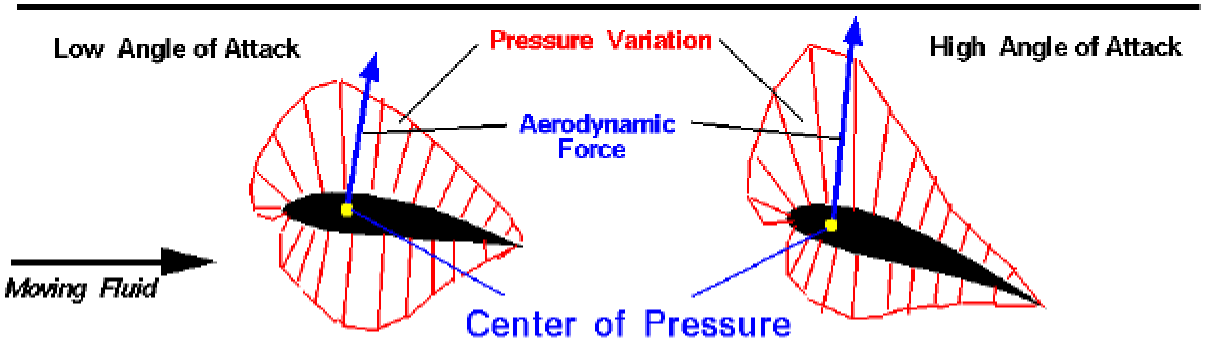
Vc KEAS = 310

Wing design loads

- The loads on the wing are the result of inertial and aerodynamic forces and can be summarized in bending, shear, and torsional moments.
- They are expected to be within the maneuvering range of the aircraft defined by the V-n diagram.
- Other loading conditions depend on control surfaces

Wing design loads

Wing spanwise loading can be divided into two elements: **additional loading and basic loading**



Additional loading: It is caused by the change in the lift distribution that varies depending on $\Delta\alpha$

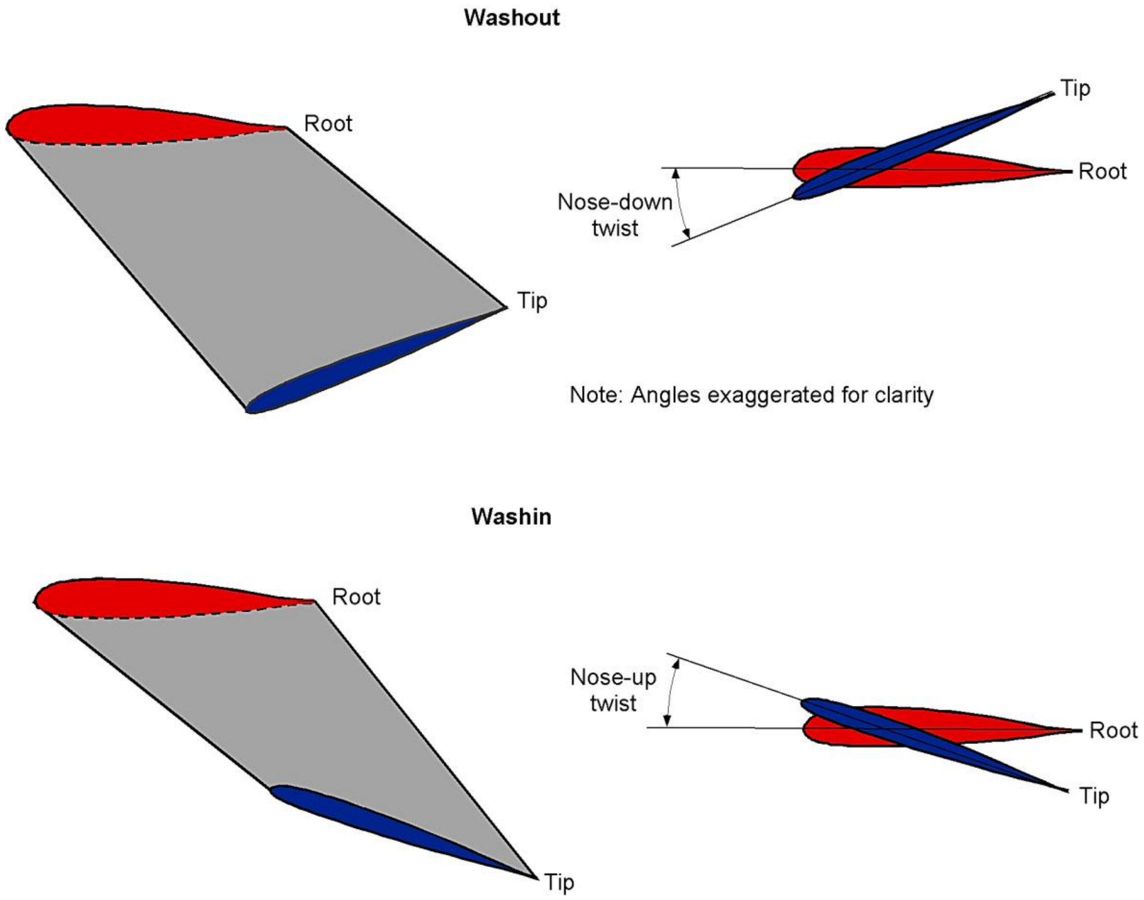
Basic loading: It is caused by the twist of the wing

Wing design loads

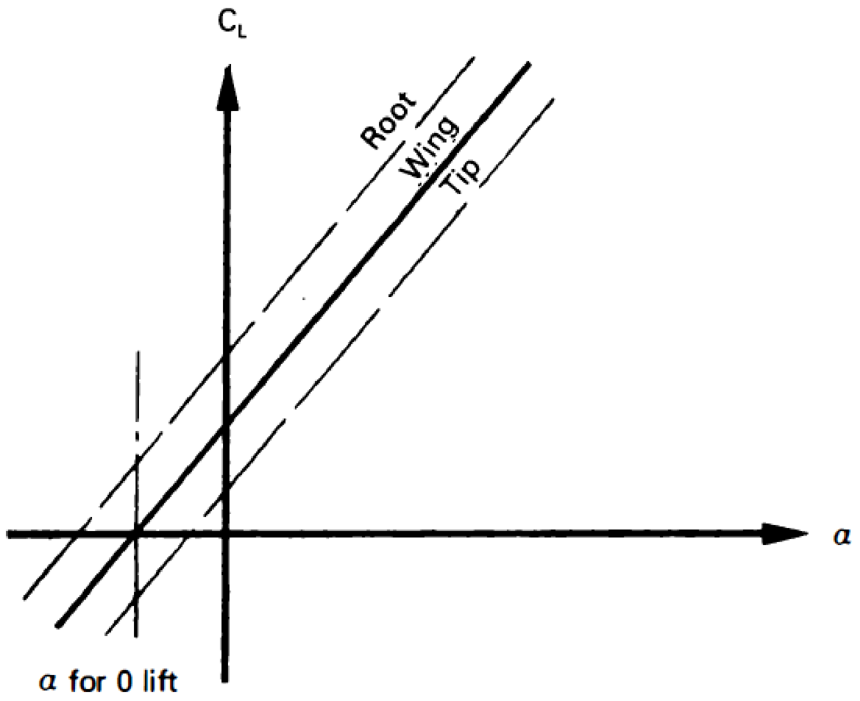
Wing spanwise loading can be divided into two elements: **additional loading and basic loading**

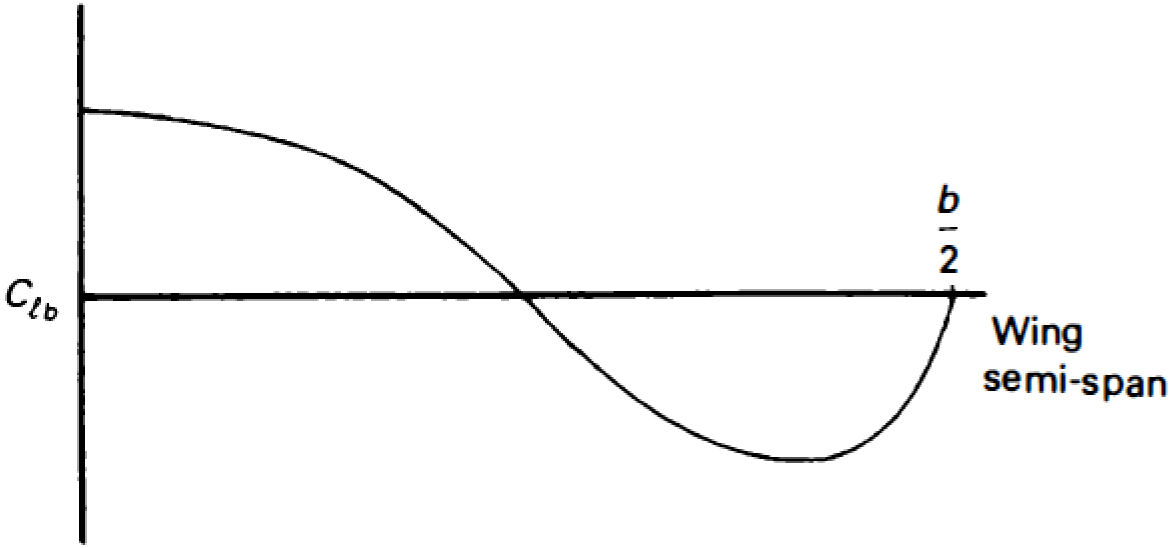
Additional loading: It is caused by the change in the lift distribution that varies depending on $\Delta\alpha$

Basic loading: It is caused by the twist of the wing

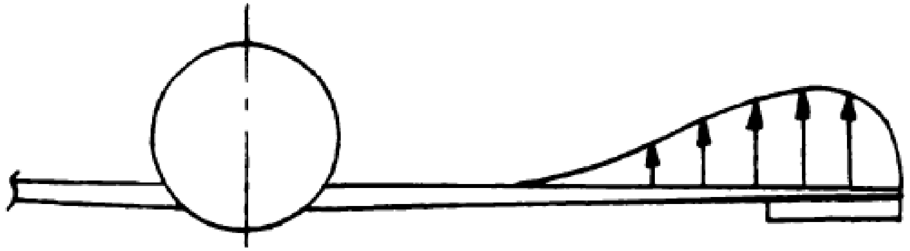


Wing design loads



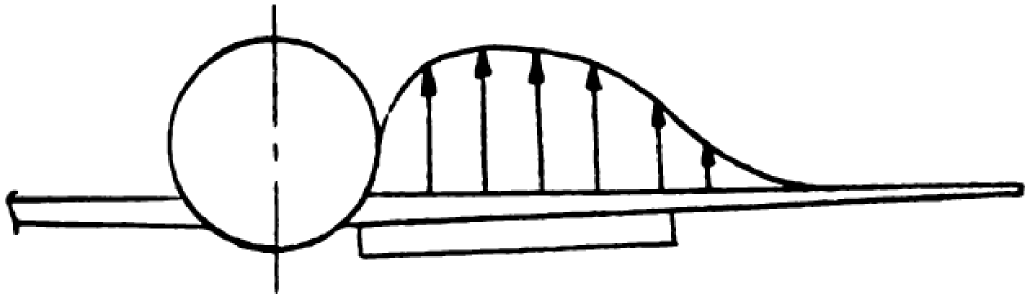


Wing design loads



Load distribution from aileron

Fig. 3.5.10 Wing load distribution due to deflected ailerons.



(a) Load distribution from flap

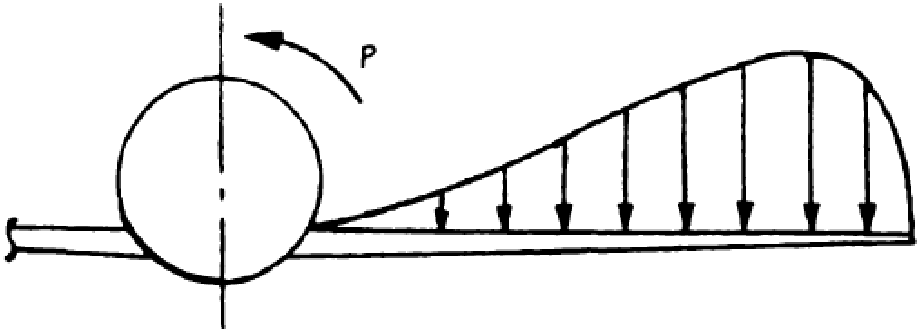
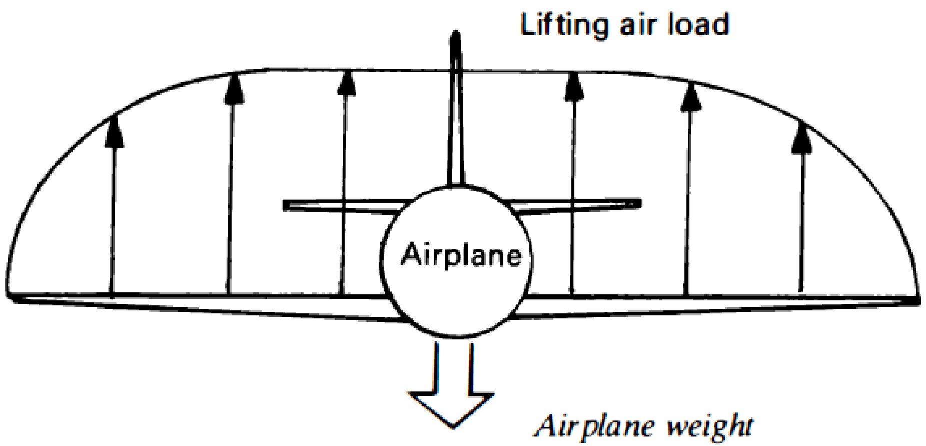


Fig. 3.5.11 Damping load distribution on wing span due to rolling.



Wing design loads

Fuselage, Nacelle and Wing Stores Effect on Wing Loads

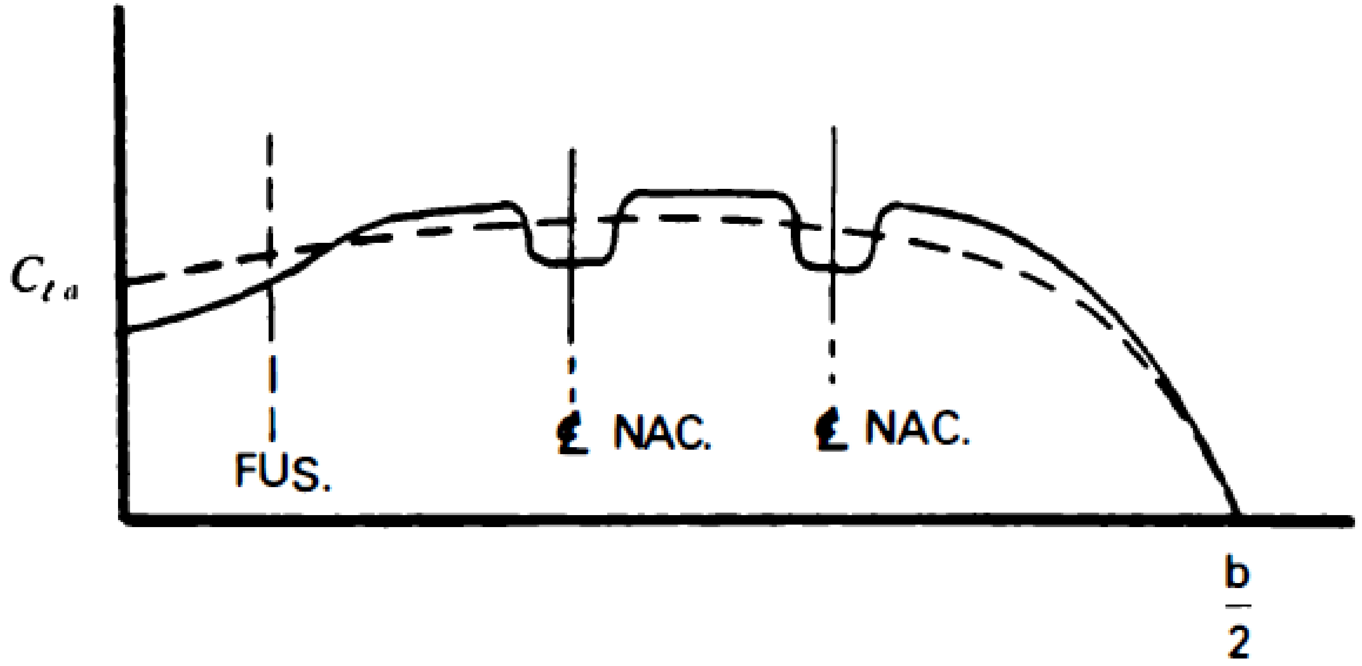
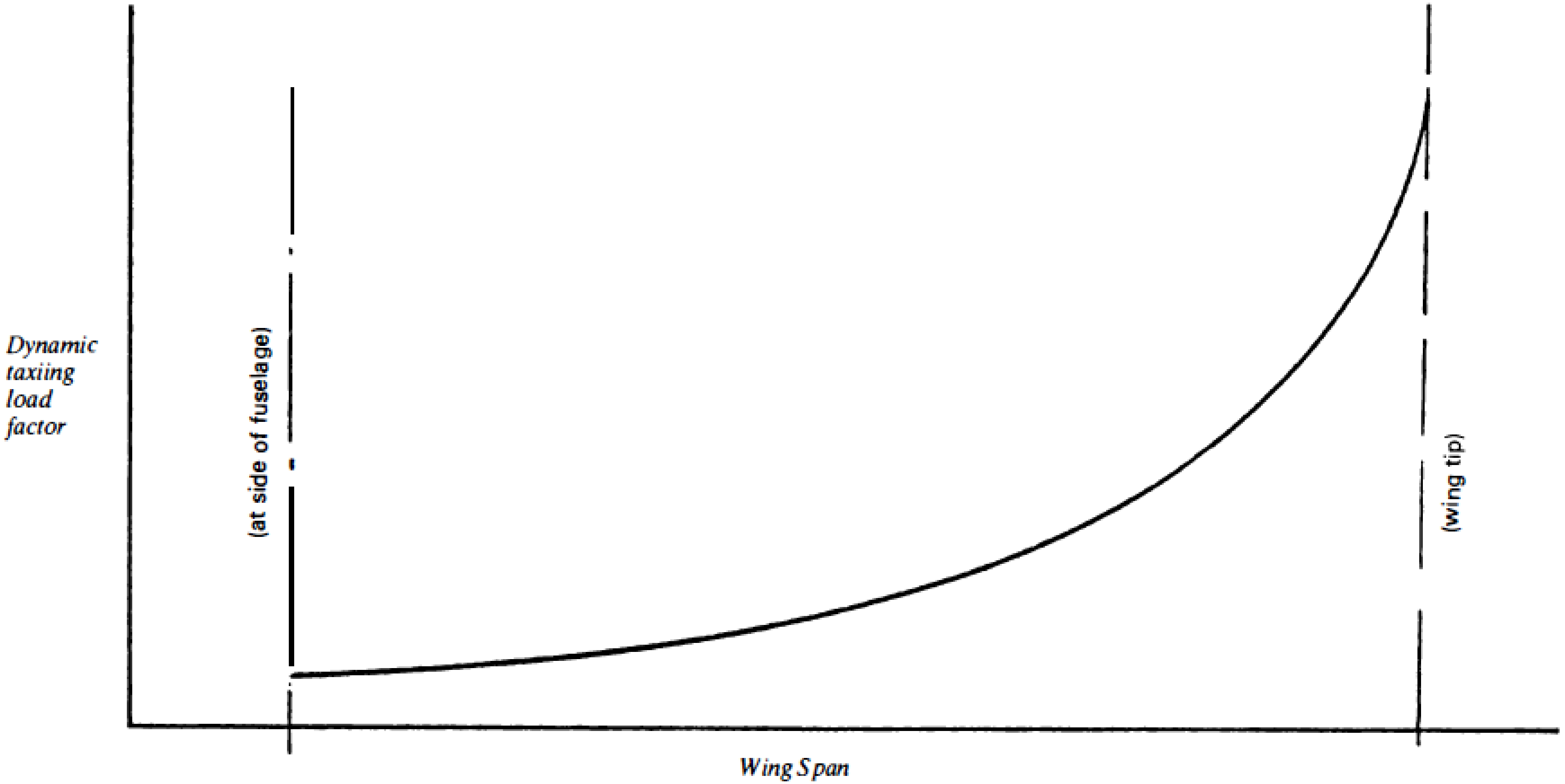


Fig. 3.5.5 Additional lift distribution due to nacelle or stores.

Wing taxiing loads

Loads can be up to 1.7 to 2G



Empennage loads

Horizontal Tail

- **Balance the moments caused by aerodynamic and inertia forces** of other parts of the airplane
- **Provides control** to the pitching axis

Vertical Tail

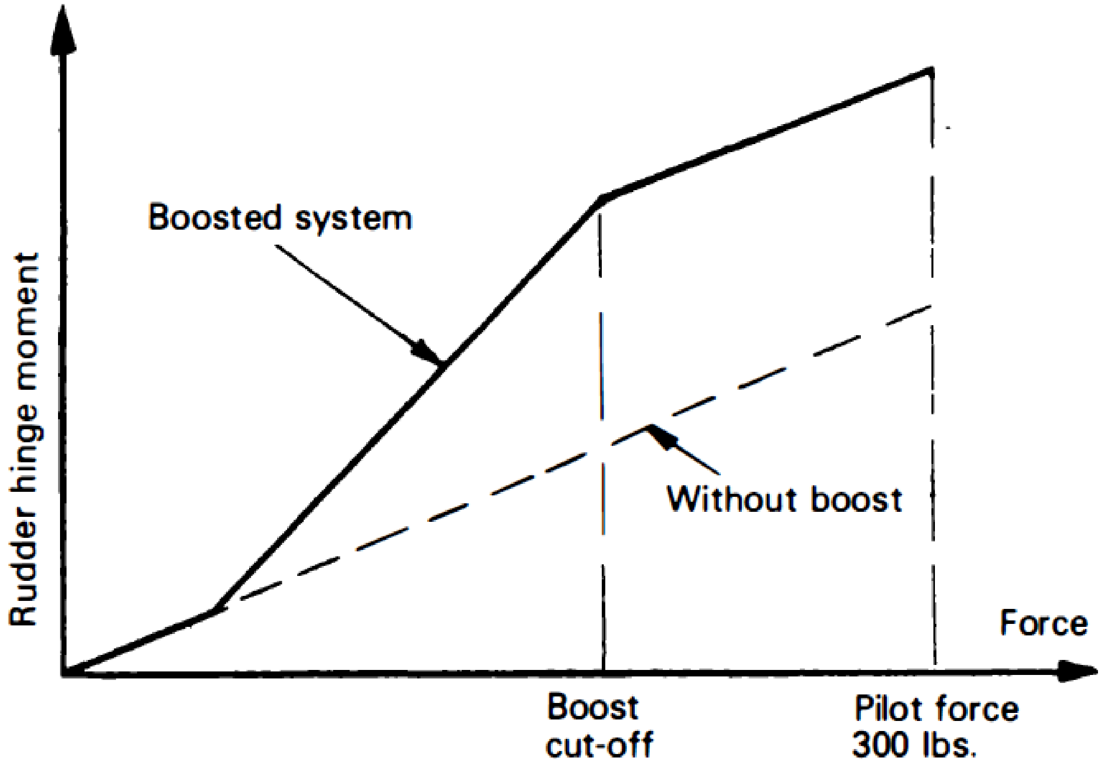
- Rudder Aileron and deflection
- Lateral gusts
- Asymmetric engine thrust

Vertical tail loads

- Rudder deflection is a function of rudder power
- Rudder effectiveness must be included in considering rudder power

Vertical tail loads

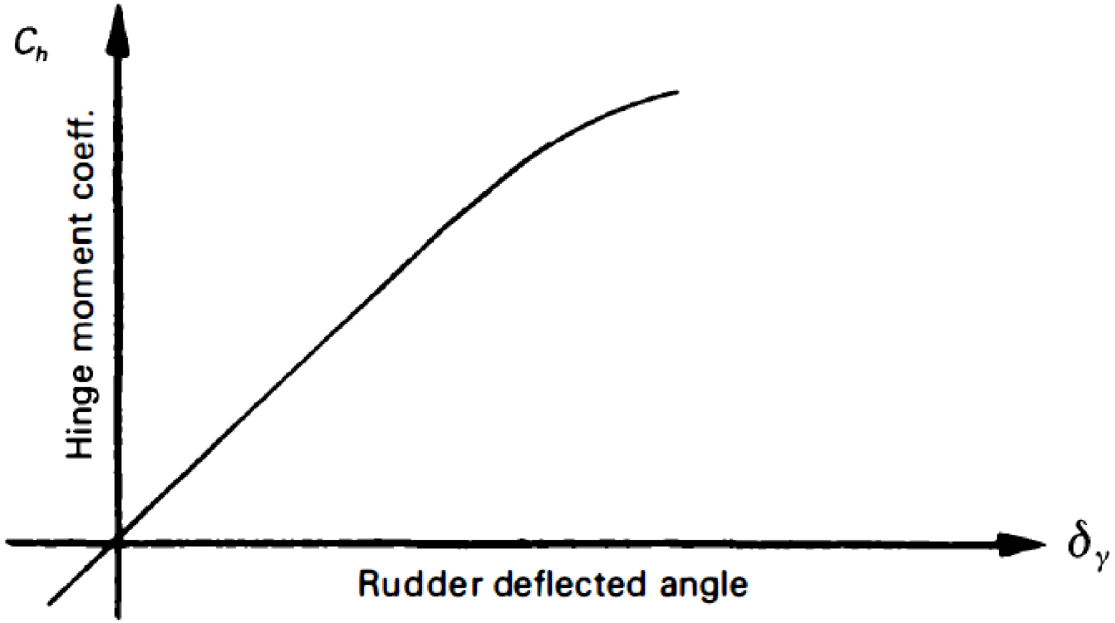
- The 300 lb pedal force is the maximum pedal force considered for structural design
- The boost cutoff is about 160 to 180 lb



Rudder Hinge Moment Coefficient

Rudder angle at any speed:

$$\delta_\gamma = \frac{HM}{C_{h\delta_\gamma}\, q\, S_\gamma\, \bar{C}_\gamma}$$



HM is the hinge moment

δ_γ is the rudder angle

$C_{h\delta\gamma}$ is the slope of C_h vs δ_γ curve

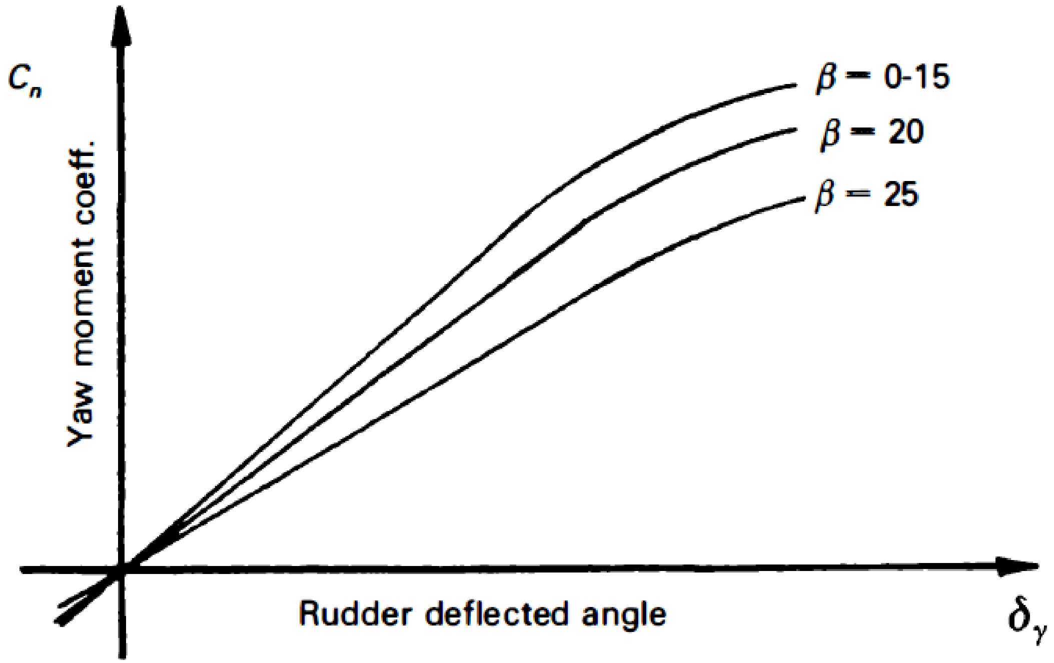
S_γ is the rudder area

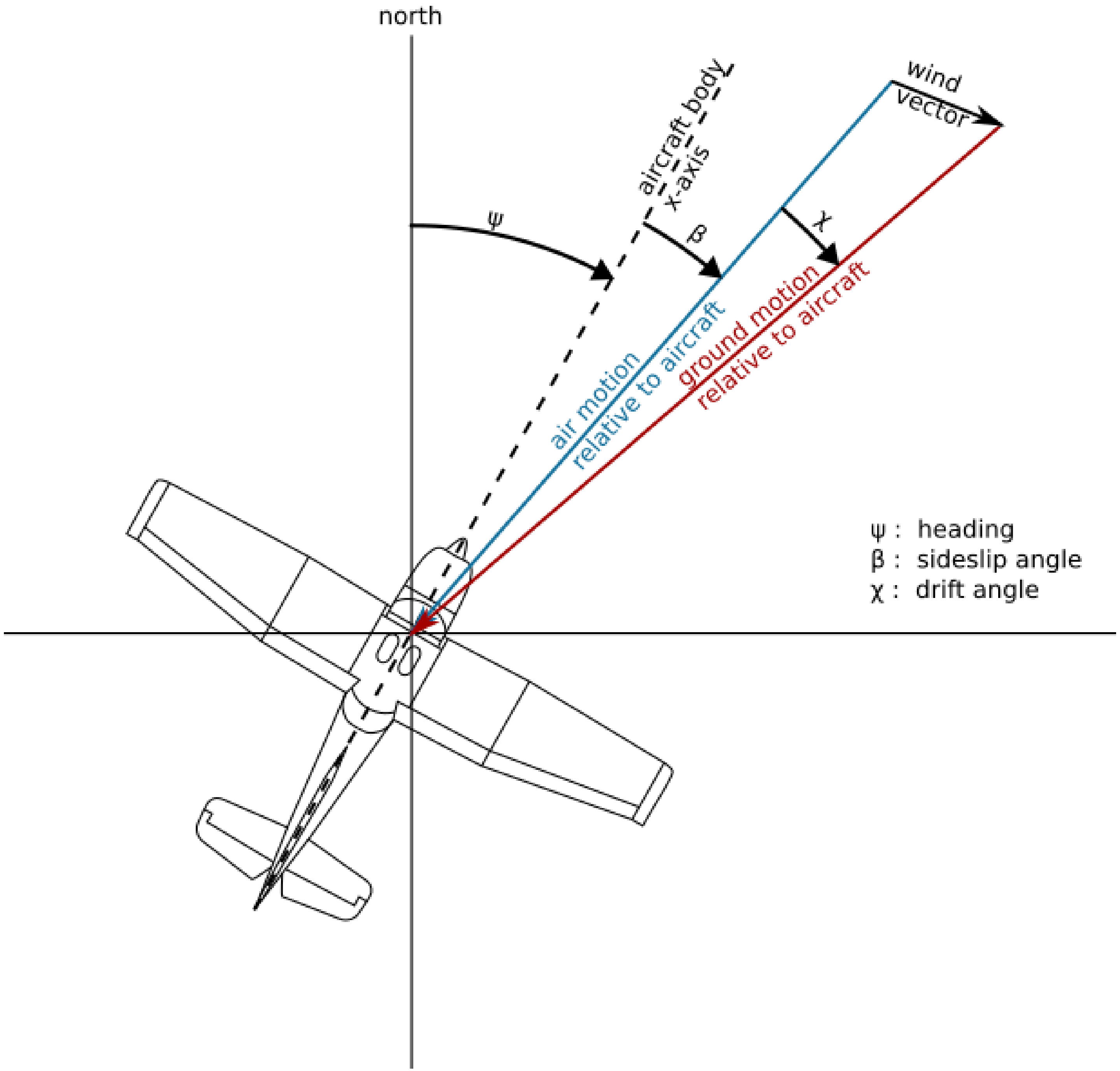
$\bar{C}_\gamma$ is the rudder MAC

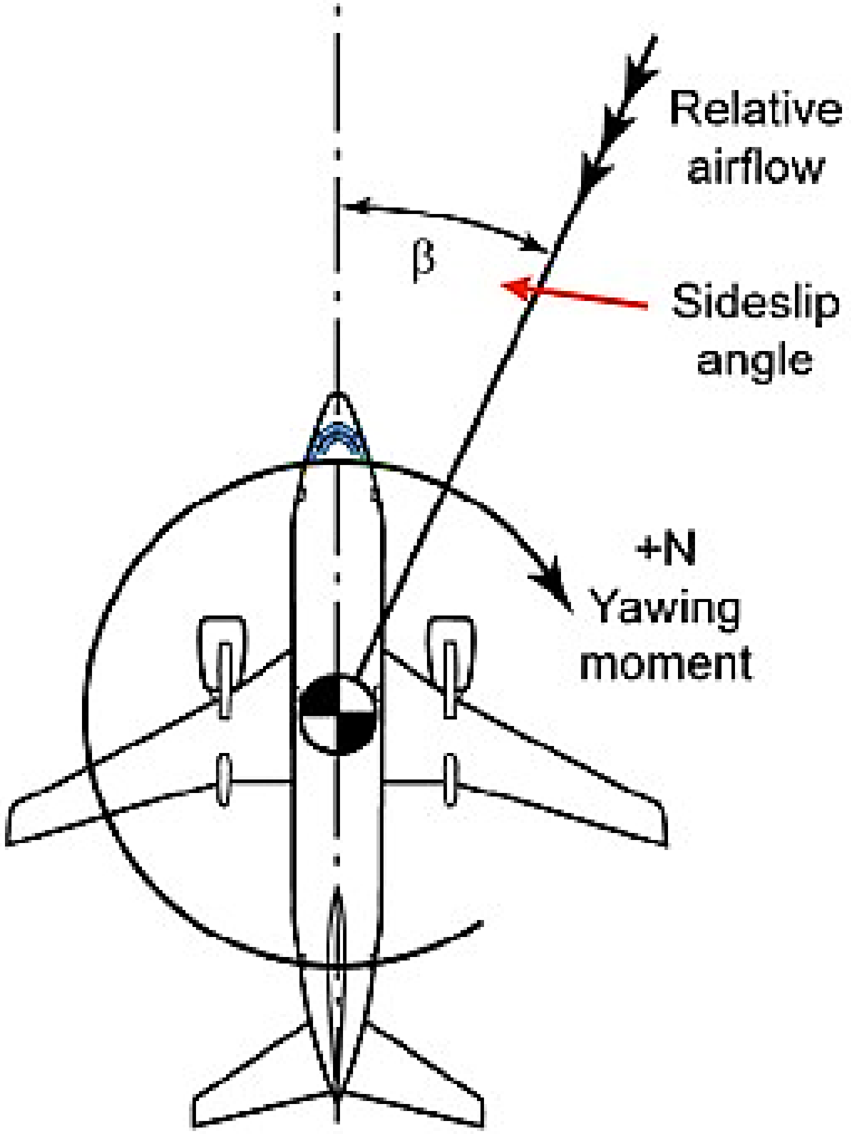
Rudder effectiveness

The primary loading on the vertical tail is caused by the resulting sideslip angle β when rudder is deflected.

This curves indicate the efficiency of the rudder in producing aircraft's sideslip angles



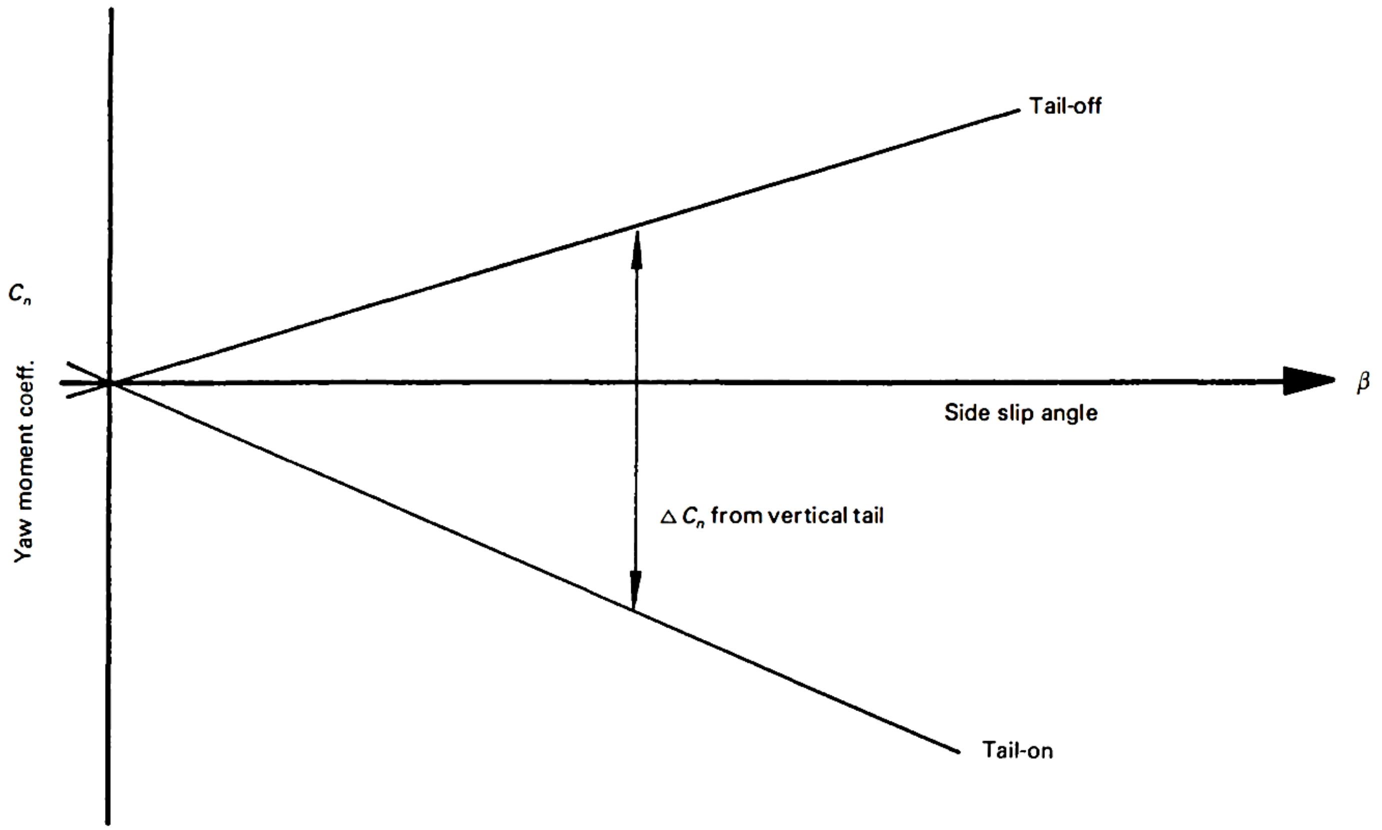

north
aircraft body x-axis
wind vector
ψ
β
χ
air motion relative to aircraft
ground motion relative to aircraft
ψ : heading
β : sideslip angle
χ : drift angle


Relative airflow
β
Sideslip angle
+N Yawing moment

Yawing Moment Coefficient

The equilibrium yaw angle β_0 is found equating the yaw moment caused by the rudder and the yaw moment of the plane:

$$\beta_0 = \frac{C_{n\delta\gamma}\,\delta_\gamma}{C_{n\beta A}}$$



Initiation of maneuver load (onset)

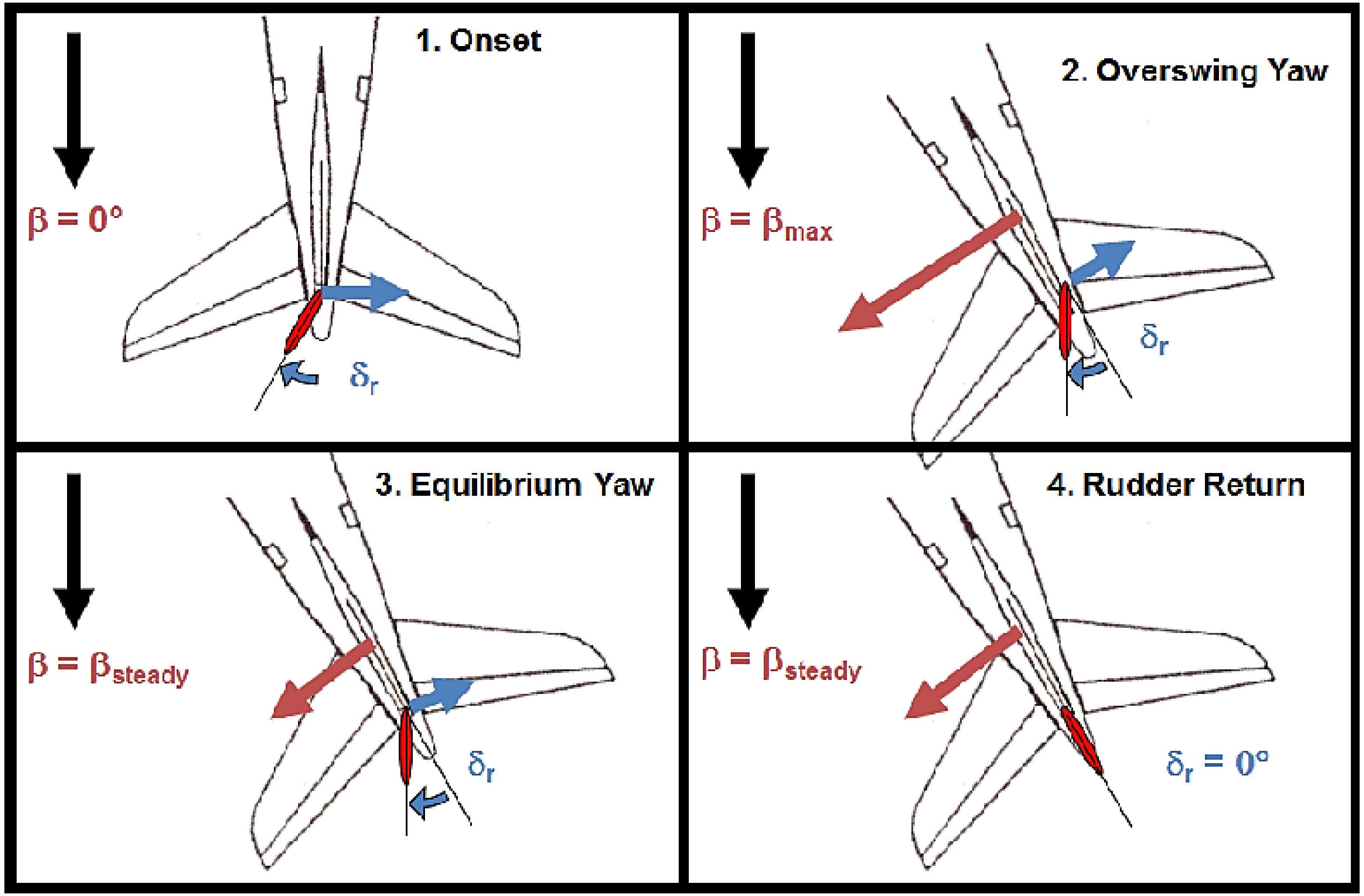
$$P_{yvt} = C_{n\delta\gamma}\,\delta_\gamma\, q\,\frac{S_{vt}}{\bar{V}_{vt}}$$

$C_{n\delta\gamma}$ is the slope of C_n (yaw moment coeff) vs δ_γ curve
δ_γ is the rudder angle
S_γ is the rudder area
S_{vt} is the vertical tail area
$\bar{V}_{vt}$ is the vertical tail volume

Onset: Starting from level flight, the rudder is deflected by a sudden pilot pedal command

Initiation of maneuver load (onset)

$$P_{yvt} = C_{n\delta\gamma}\, \delta_\gamma\, q\, \frac{S_{vt}}{\bar{\bar{V}}_{vt}}$$



Dynamic overswing load

$$P_{yvt} = \left[(\Delta C_{n\beta})_{vt}\, K_\beta \beta_0 - C_{n\delta\gamma}\delta_\gamma\right] q\, \frac{S_{vt}}{\bar{\bar{V}}_{vt}}$$

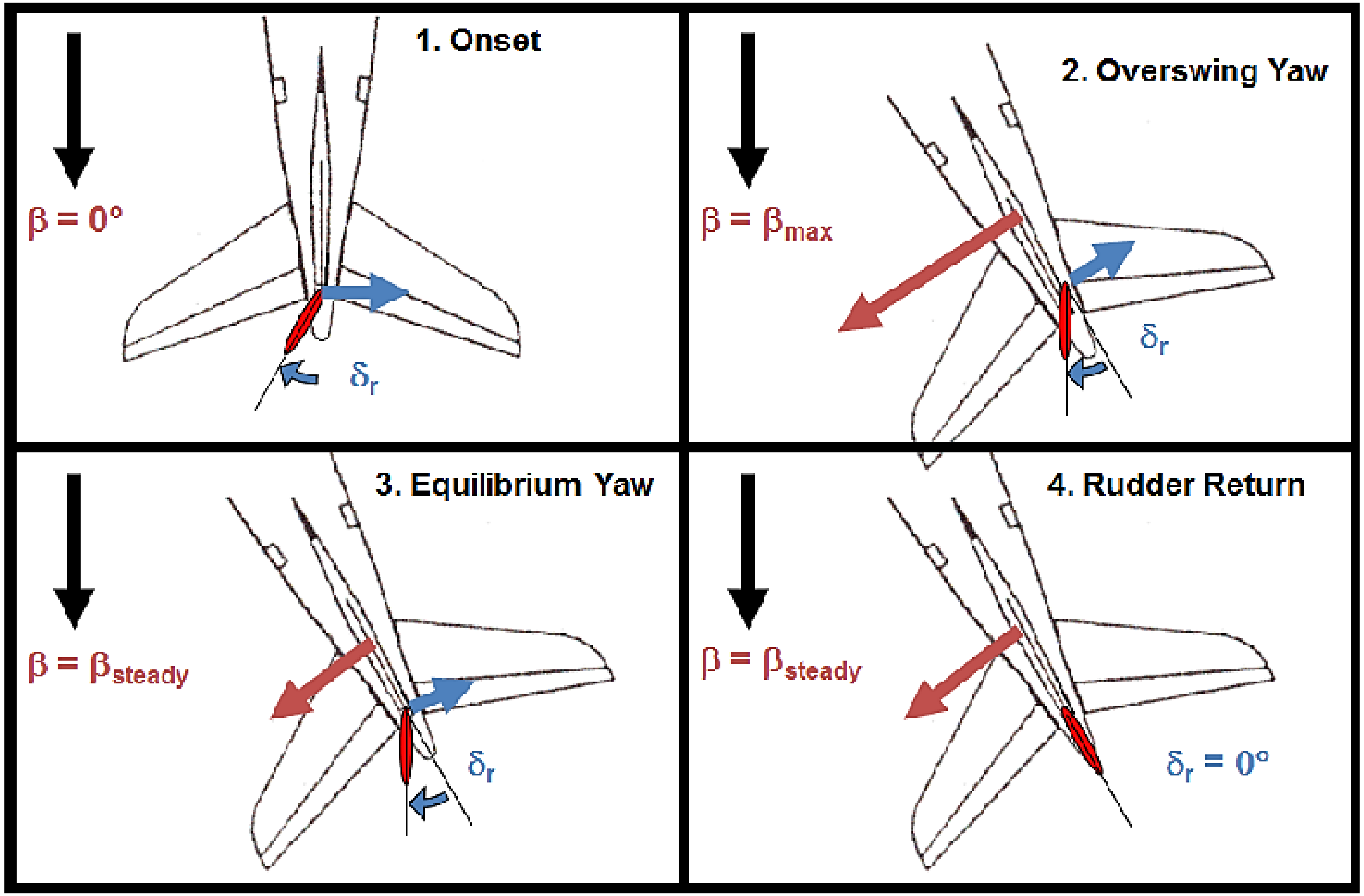
$(\Delta C_{n\beta})_{vt}$ is the slope of the tail lift curve

K_β magnification of sideslip from 1.4 to 1.6

“Overswing”: As a result of the rudder command the aircraft starts to yaw and a dynamic overswing resulting in a maximum sideslip angle occurs

Dynamic overswing load

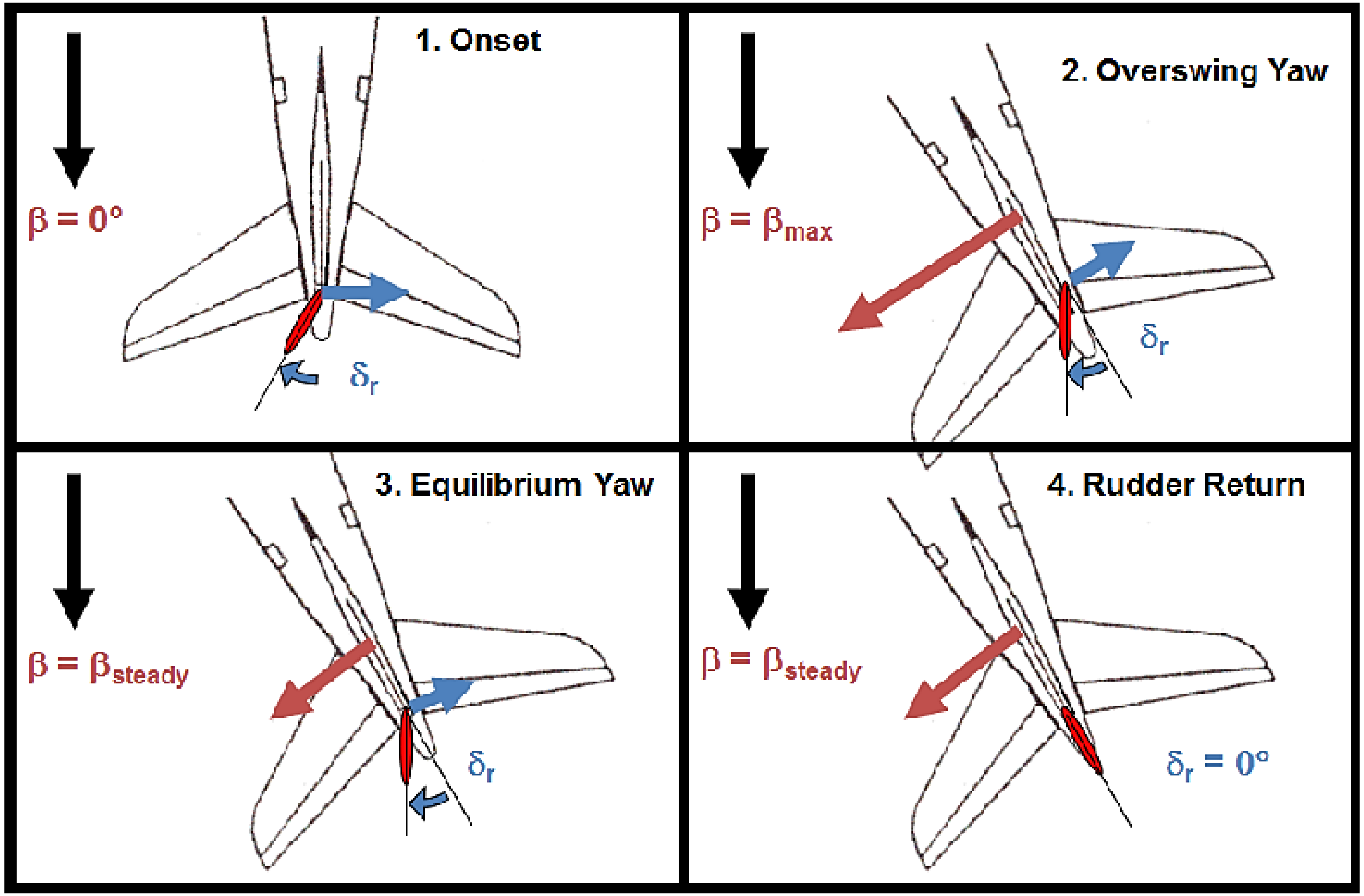
$$P_{yvt} = \left[(\Delta C_{n\beta})_{vt}\, K_{\beta}\beta_0 - C_{n\delta\gamma}\delta_{\gamma}\right] q\, \frac{S_{vt}}{\bar{\bar{V}}_{vt}}$$



Return to neutral load

$$P_{yvt} = (\Delta C_{n\beta})_{vt}\, \beta_0\, q\, \frac{S_{vt}}{\bar{V}_{vt}}$$

Rudder Return Form the steady sideslip condition, the rudder command is returned to zero.



Example 4

Estimate the loads on the vertical tail for a speed of 325 KEAS for a rudder kick maneuver. Solve for three points in time:

1. Initiation of maneuver assuming instantaneous rudder deflection
2. Dynamic over swing loading holding constant rudder from initiation of maneuver (use dynamic over swing factor K_{β})
3. Instantaneous return of rudder to neutral while at equilibrium yaw angle. Neglect the effect of yaw velocity, make use of the curves C_{nA} and $C_{nA} - t$versus β, assuming that the difference between the airplane and airplane less tail is entirely due to load on the vertical tail and the air load due to rudder deflection is entirely on the vertical tail

Given airplane data: $C_{h\delta\gamma} = 0.01/degree$, $S_{\gamma} = 60\ ft^2$, $\bar{C}_{\gamma} = 3.5\ ft$, $\mathrm{HM} = 7520\ ft - lb$

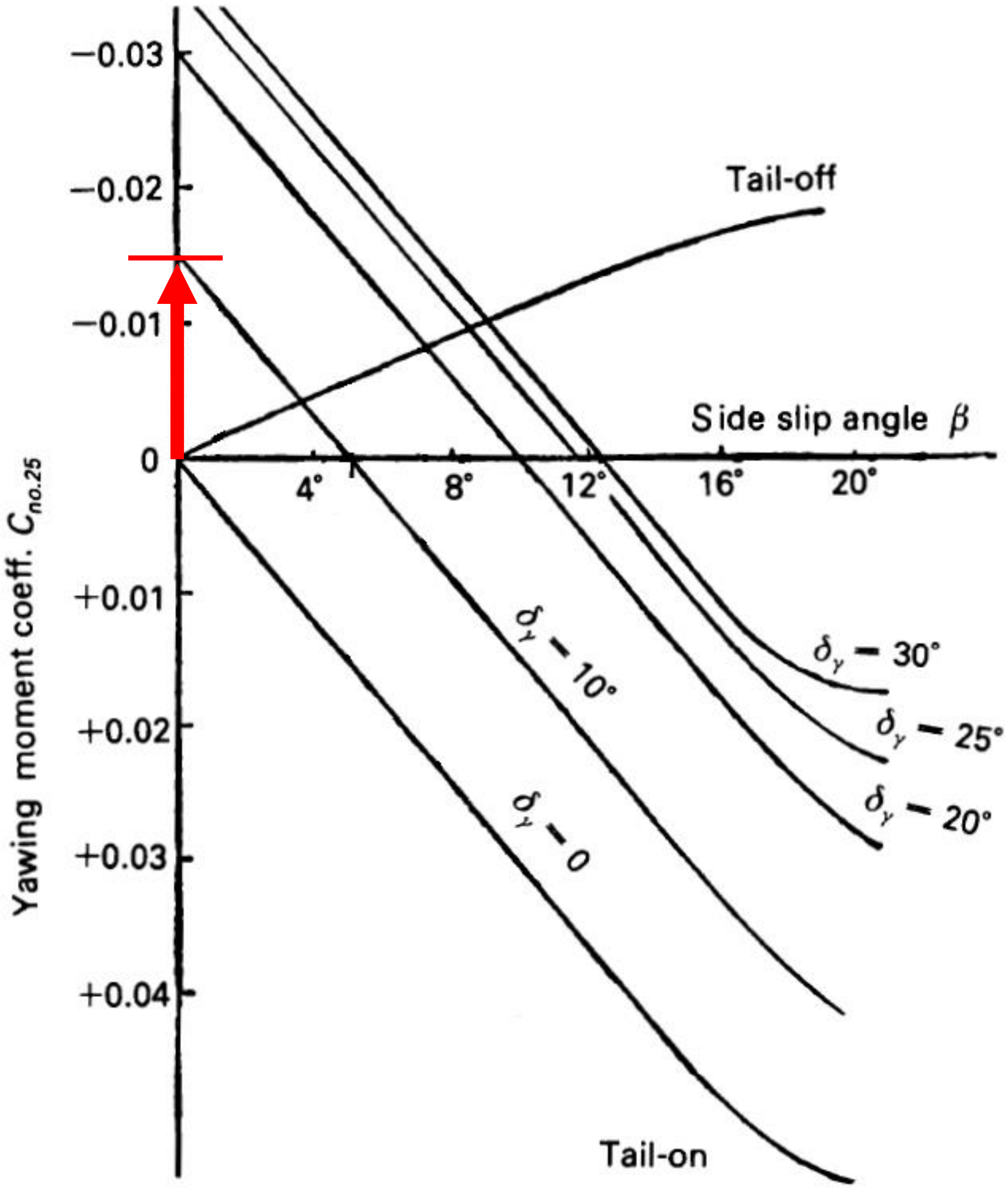
Example 4

$$\delta_\gamma = \frac{HM}{C_{h\delta_\gamma}\, q\, S_\gamma\, \bar{C}_\gamma}$$

$\delta_\gamma = 10°$

$\beta = 0$

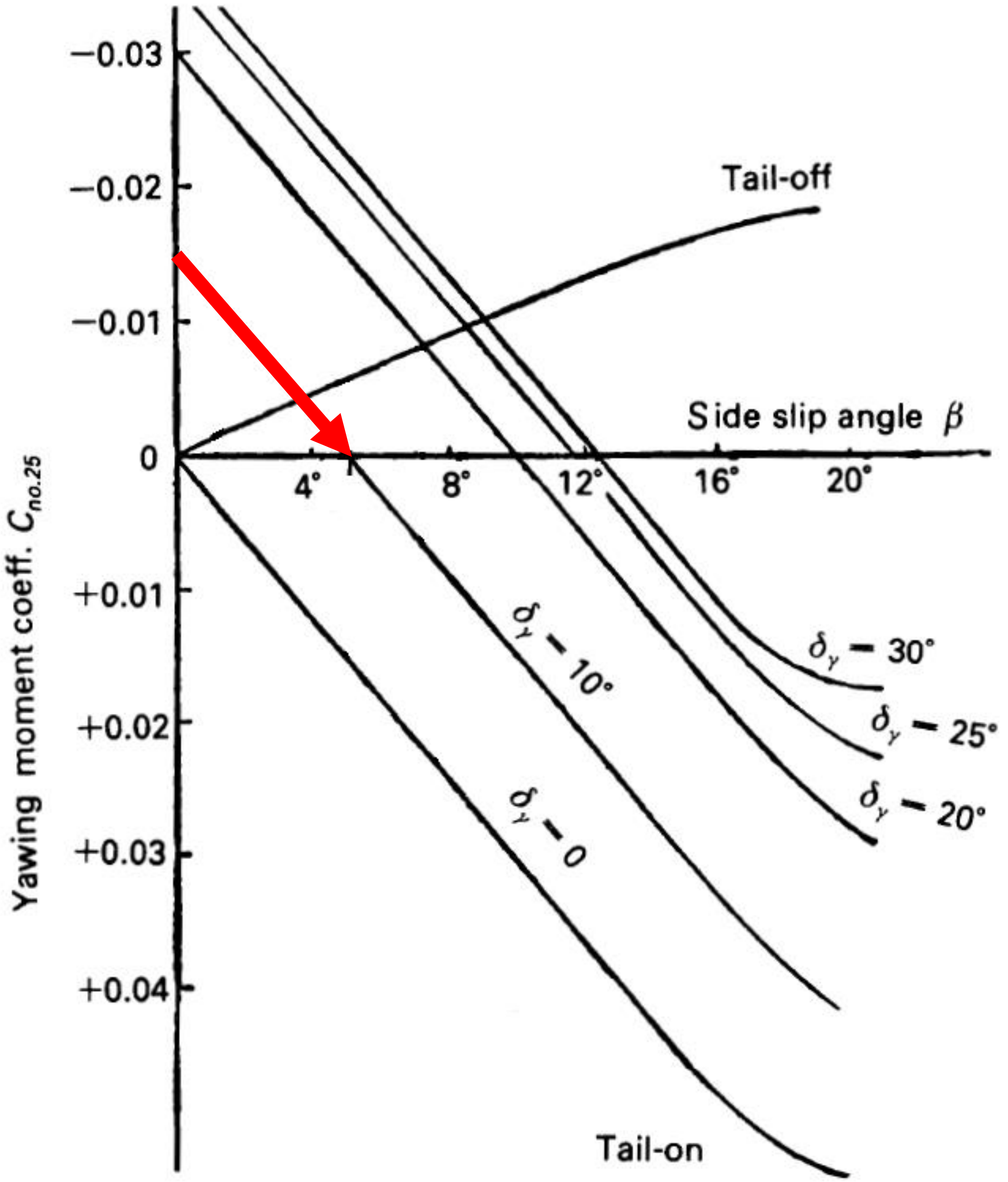
$\Delta C_n = 0 - 0.015 = -0.015$



Example 4

$\beta_0 = 5.1°$ due to $\delta_\gamma = 0°$
$K_\beta * \beta_0 = 7.14°$

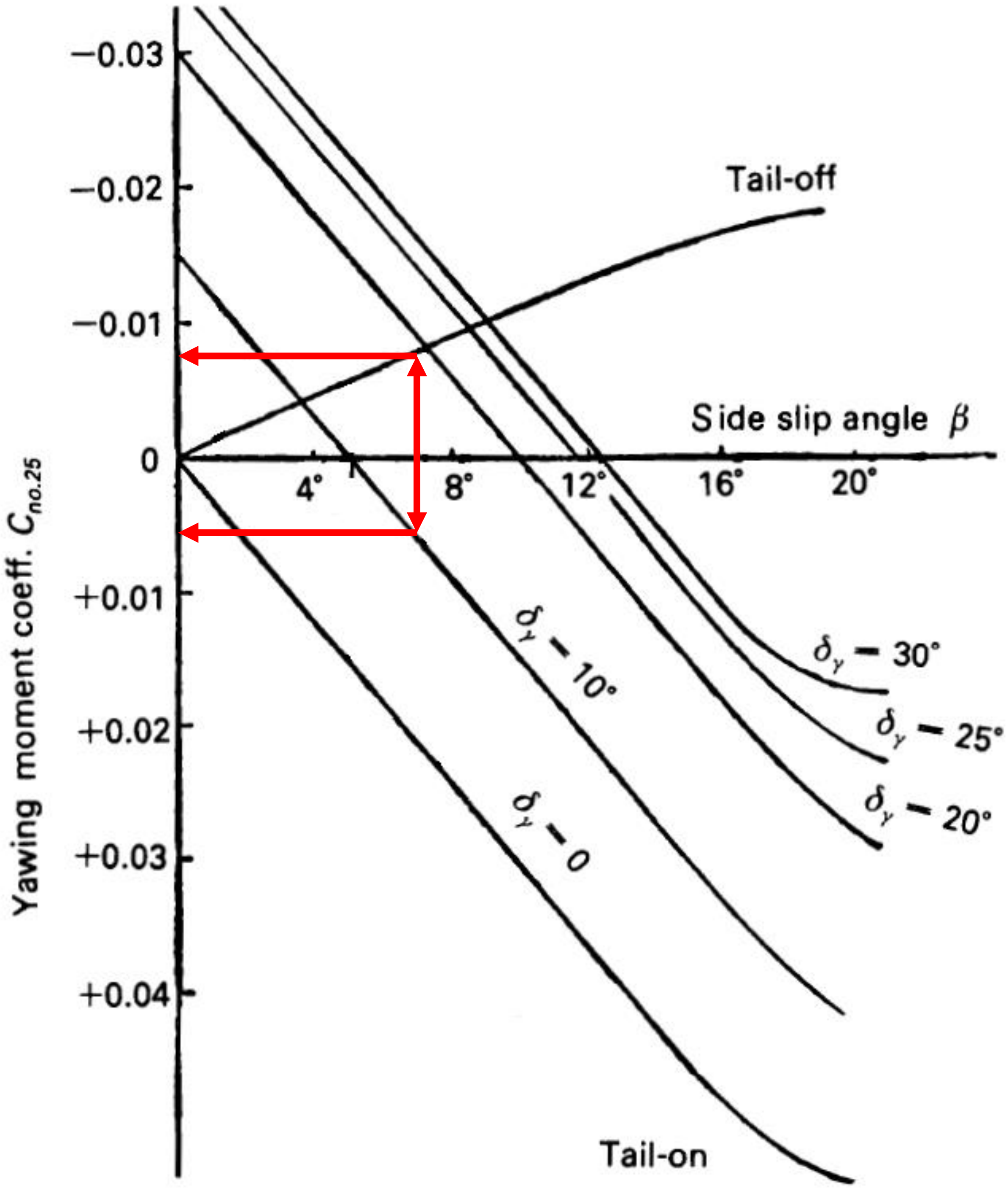
Maximum overswing angle 7.14°



Example 4

Maximum overswing angle 7.14°

$$\Delta C_n = 0.008 + 0.006 = 0.014$$

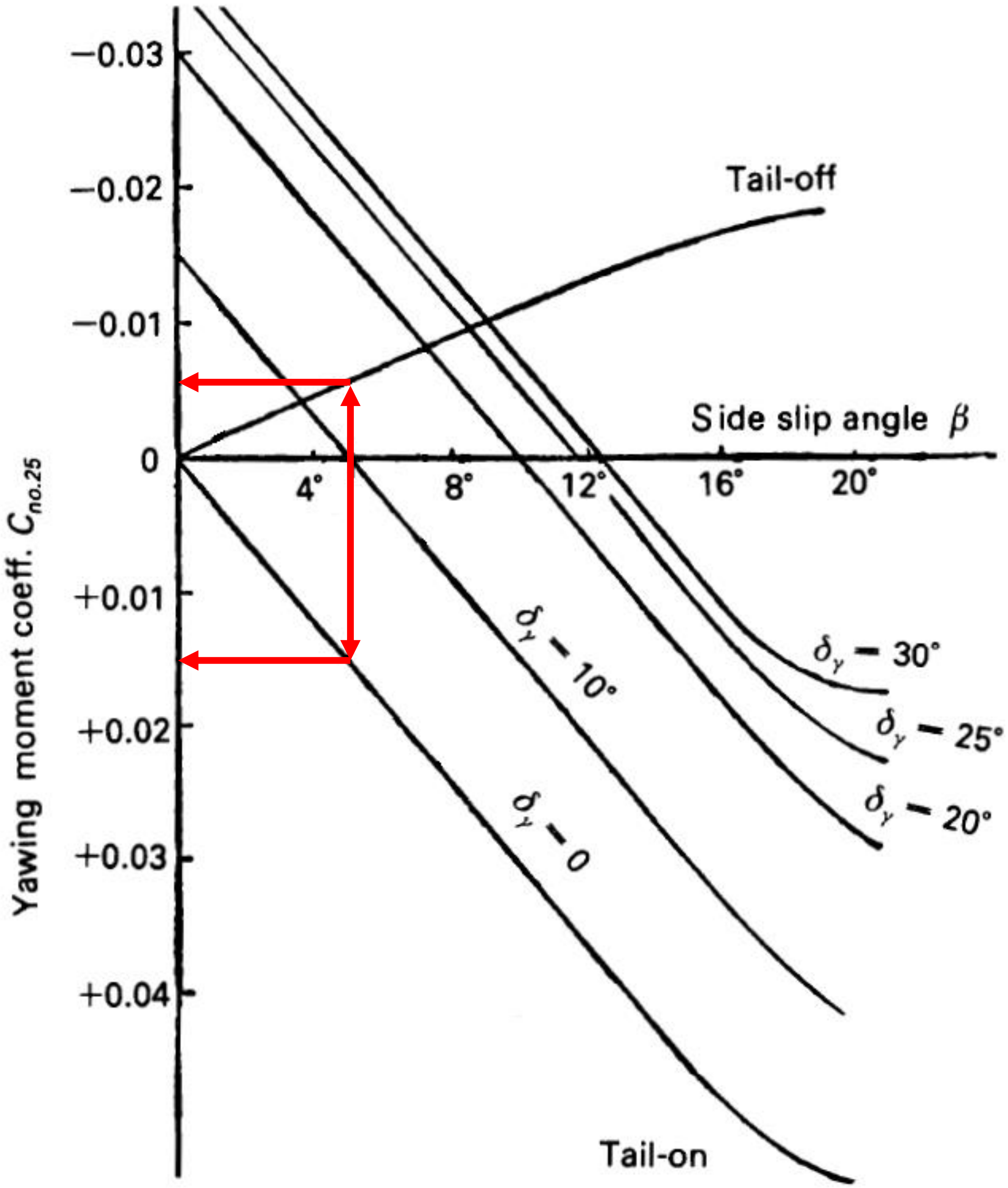


Example 4

Return to neutral

$\beta_0 = 5.1°$

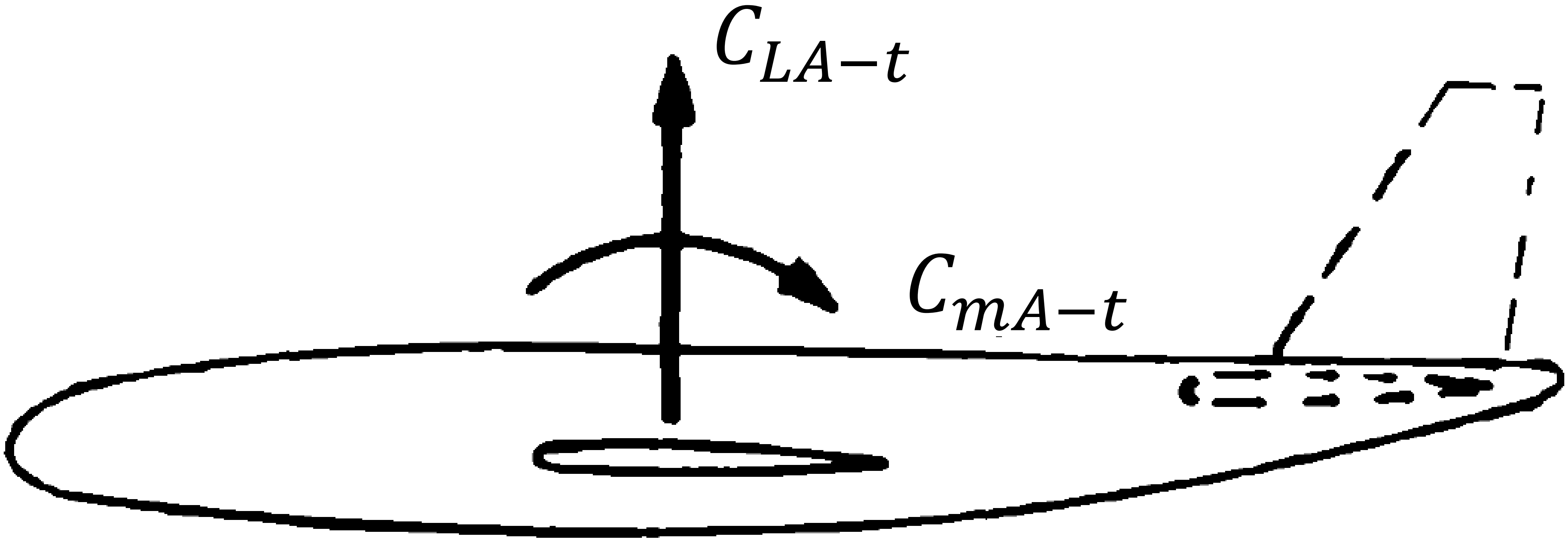
$\Delta C_n = 0.006 + 0.015 = 0.0021$



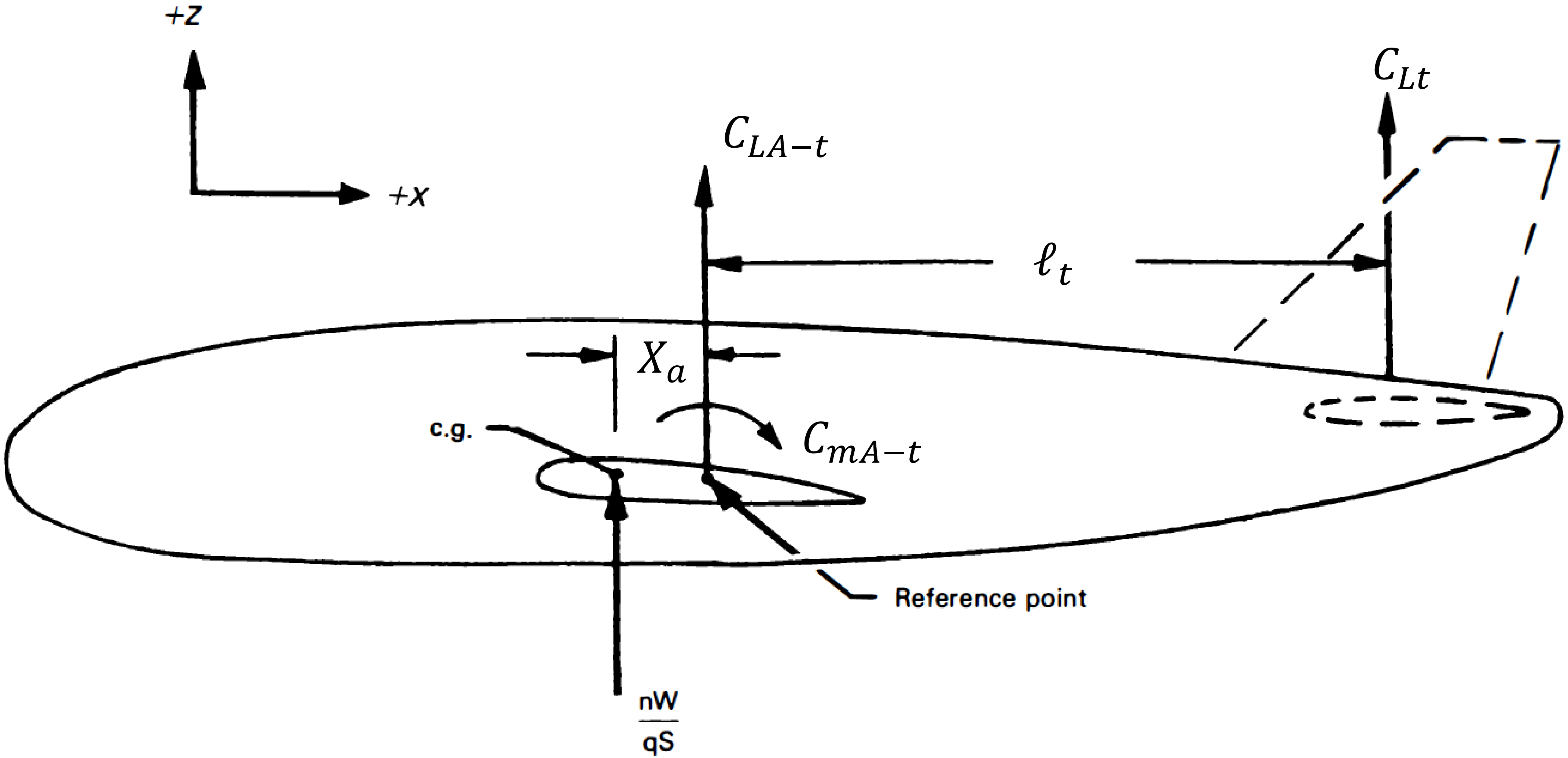
Horizontal tail loads

- Aerodynamic Forces
- Aerodynamic Moments
- Wind Tunnel Testing

Horizontal tail loads



Horizontal tail loads



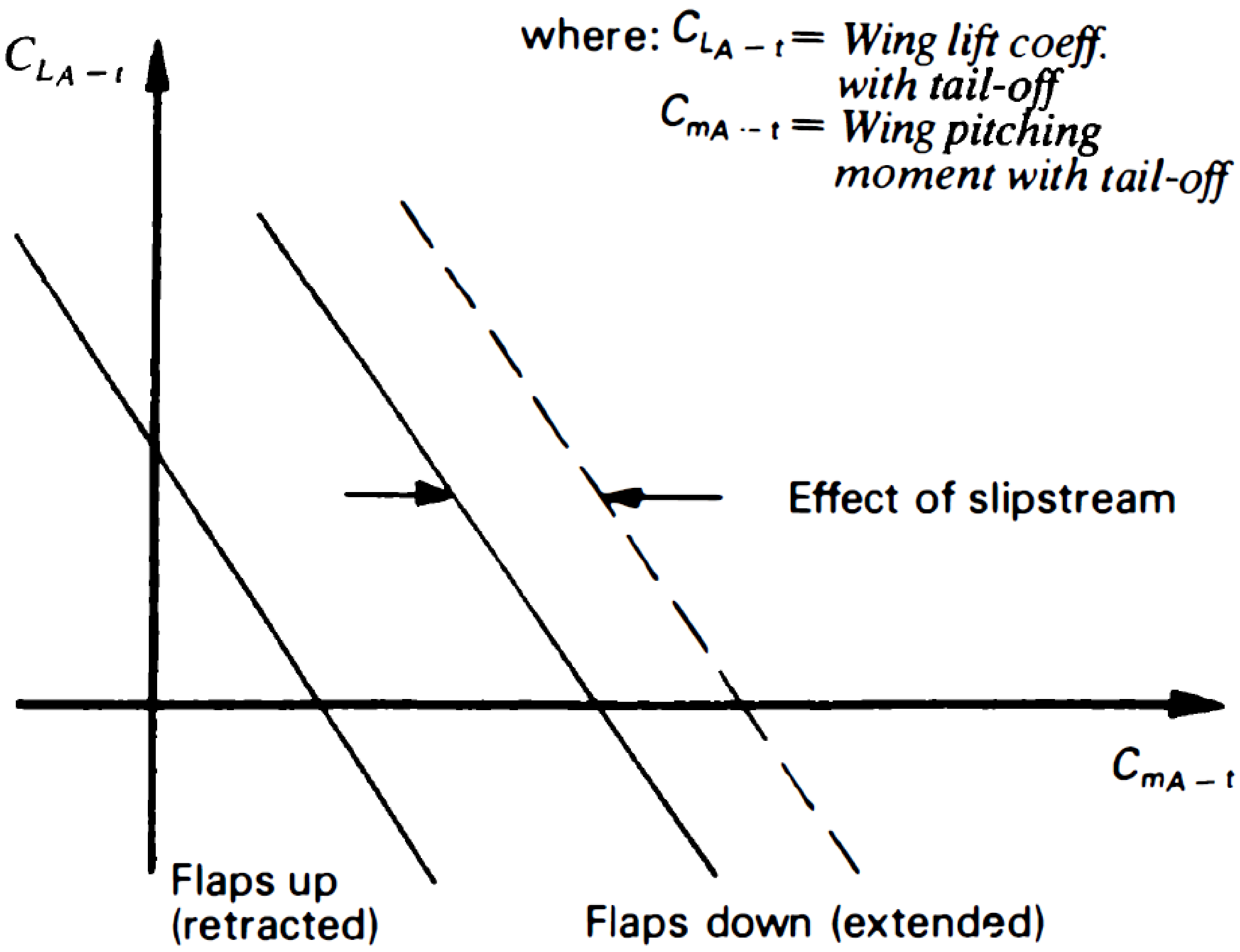
Horizontal tail loads

$$\sum F_Z = 0 \rightarrow C_{LA} = C_{LA-t} + C_{Lt} = \frac{n\dfrac{W}{S}}{q}$$

$$\sum M_{cg} = 0 = -C_{mA-t} + C_{LA-t}\left(\frac{X_a}{\bar{c}}\right) + C_{Lt}\left(\frac{\ell_t}{\bar{c}}\right)$$

$$C_{Lt} = \frac{C_{mA-t} - C_{LA-t}\left(\dfrac{X_a}{\bar{c}}\right)}{\dfrac{\ell_t}{\bar{c}}}$$

Horizontal tail loads



Horizontal tail loads