Aerospace Structures

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INTERNAL LOADS

Internal loads

Aircraft structures are typified by **arrangements of thin, load bearing skins, frames and stiffeners**, fabricated from lightweight and high strength materials.

An aircraft is basically an **assembly of stiffened shell structures** raging from a single cell closed section fuselage (thin-walled tube comprising skin, transverse frames and stringers; transverse frames which extend completely across the fuselage are known as bulkheads) to multicell wing and tail surfaces each subjected to bending, shear, torsional and axial loads.

Internal loads

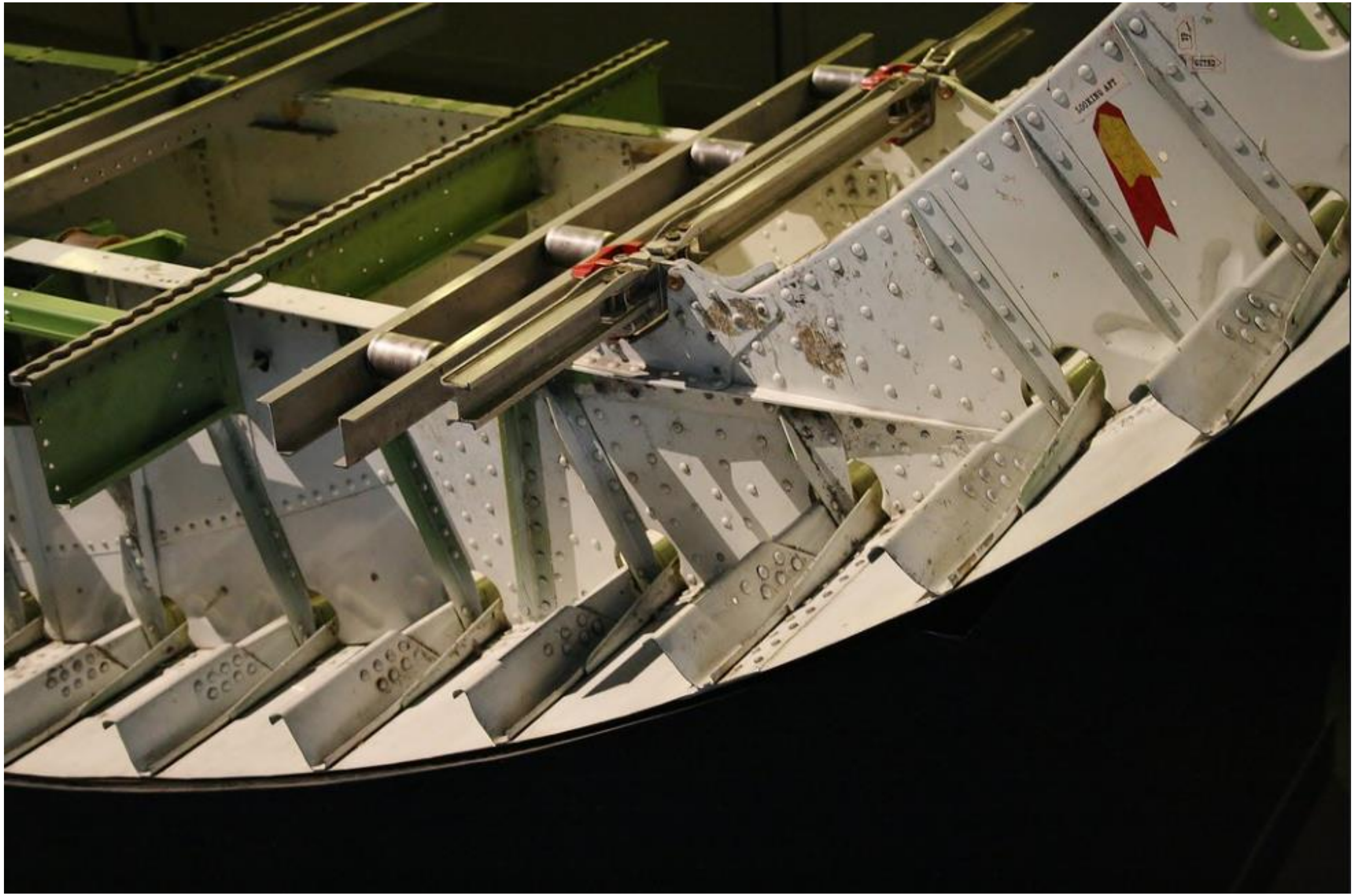
Internal loads

- Smaller portions of the structure consist of **thin-walled channel, T-, Z-, "top-hat" – or I-sections**, which are used to **stiffen the thin skins of the cellular components** and **provide support** for **internal loads** from floors, engine mountings, etc. Structural members such as these are known as **open section beams**.

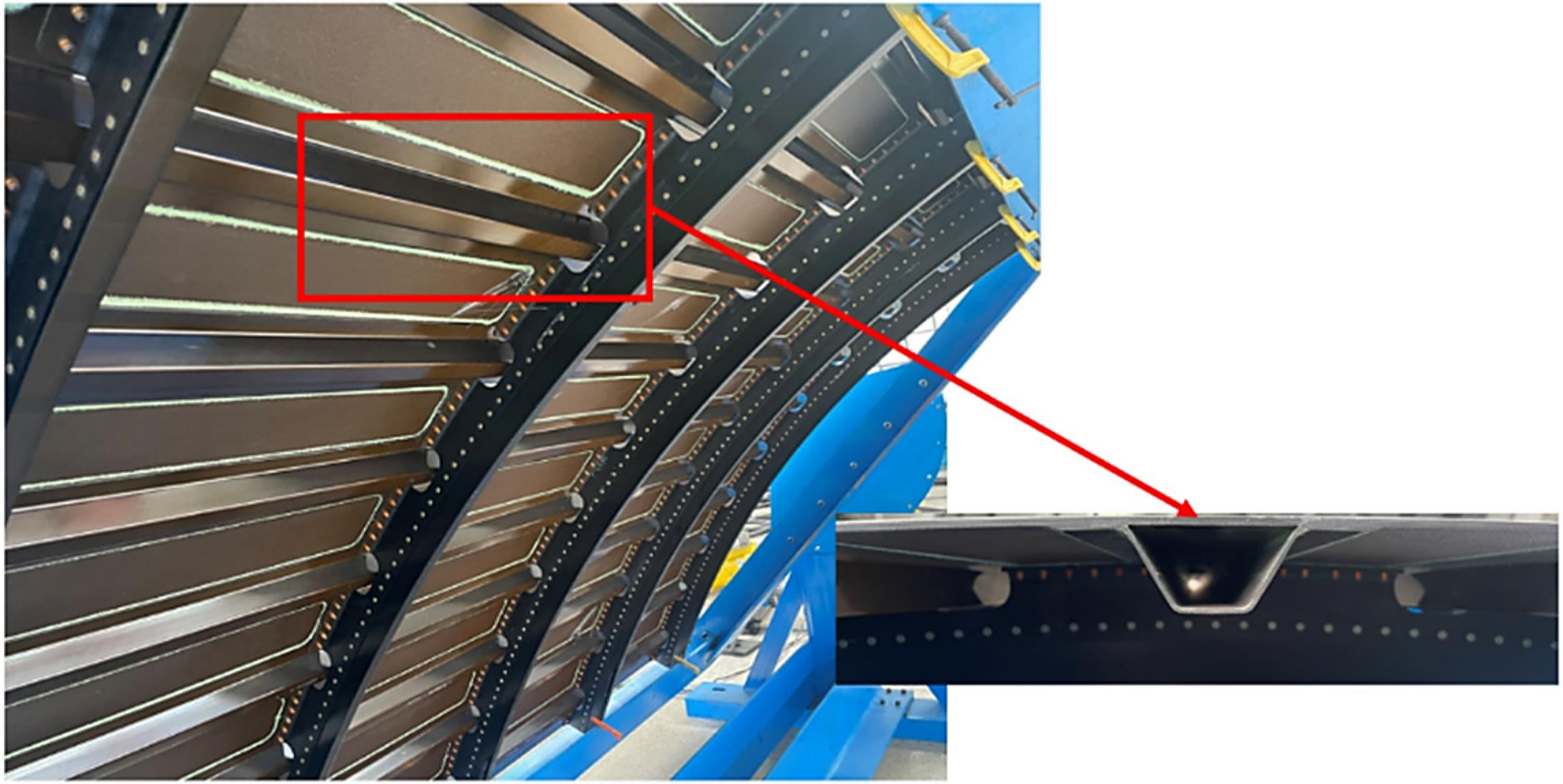
- While the cellular components are called **closed section beams**.

Clearly, both types of beam are subjected to axial, bending, shear and torsional loads.

Internal loads



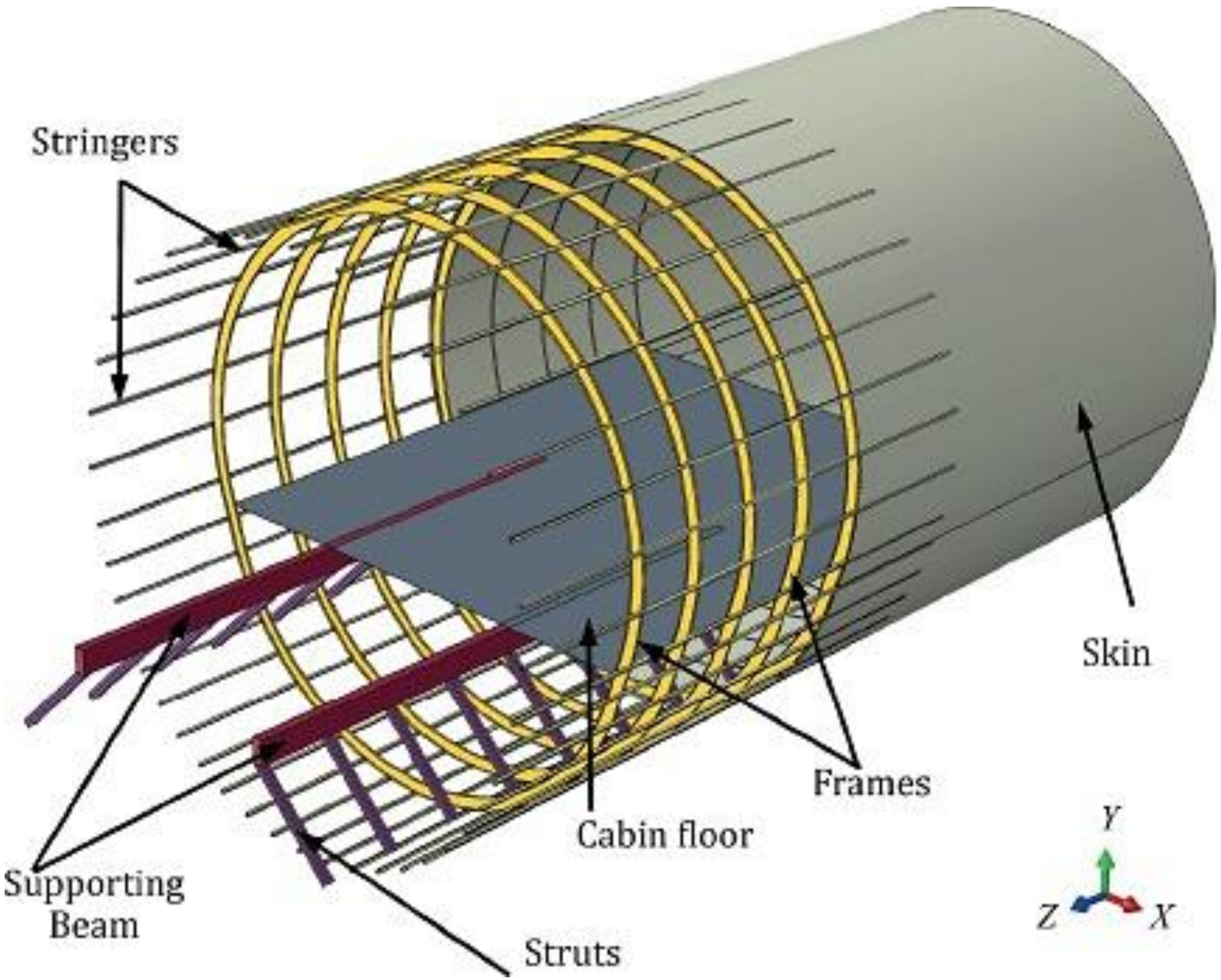
Internal loads



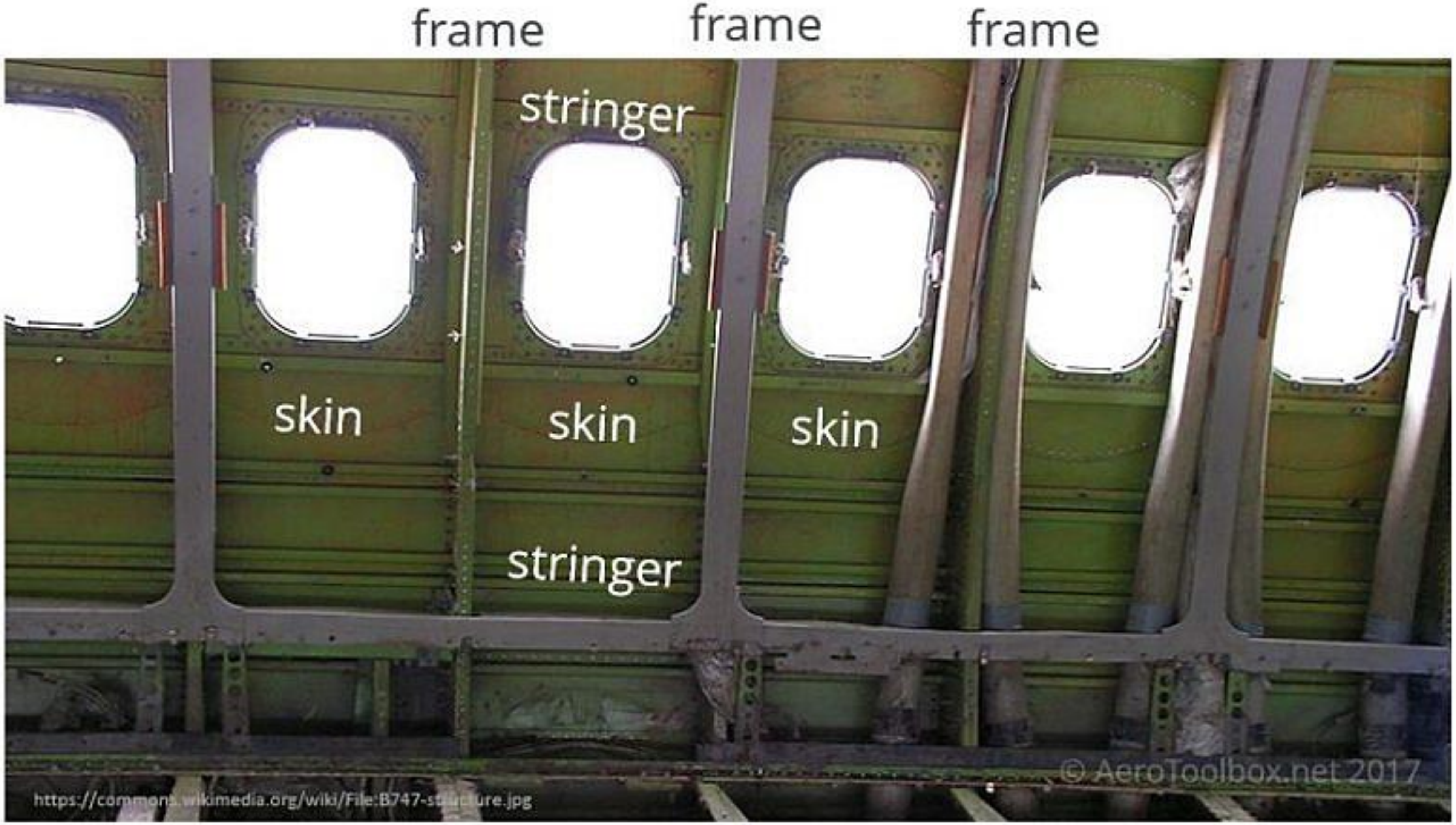
Structural instability

Columns in aircrafts

Stringers


Stringers
Skin
Frames
Cabin floor
Supporting Beam
Struts
Y
Z
X

Stringers


frame
frame
frame
stringer
skin
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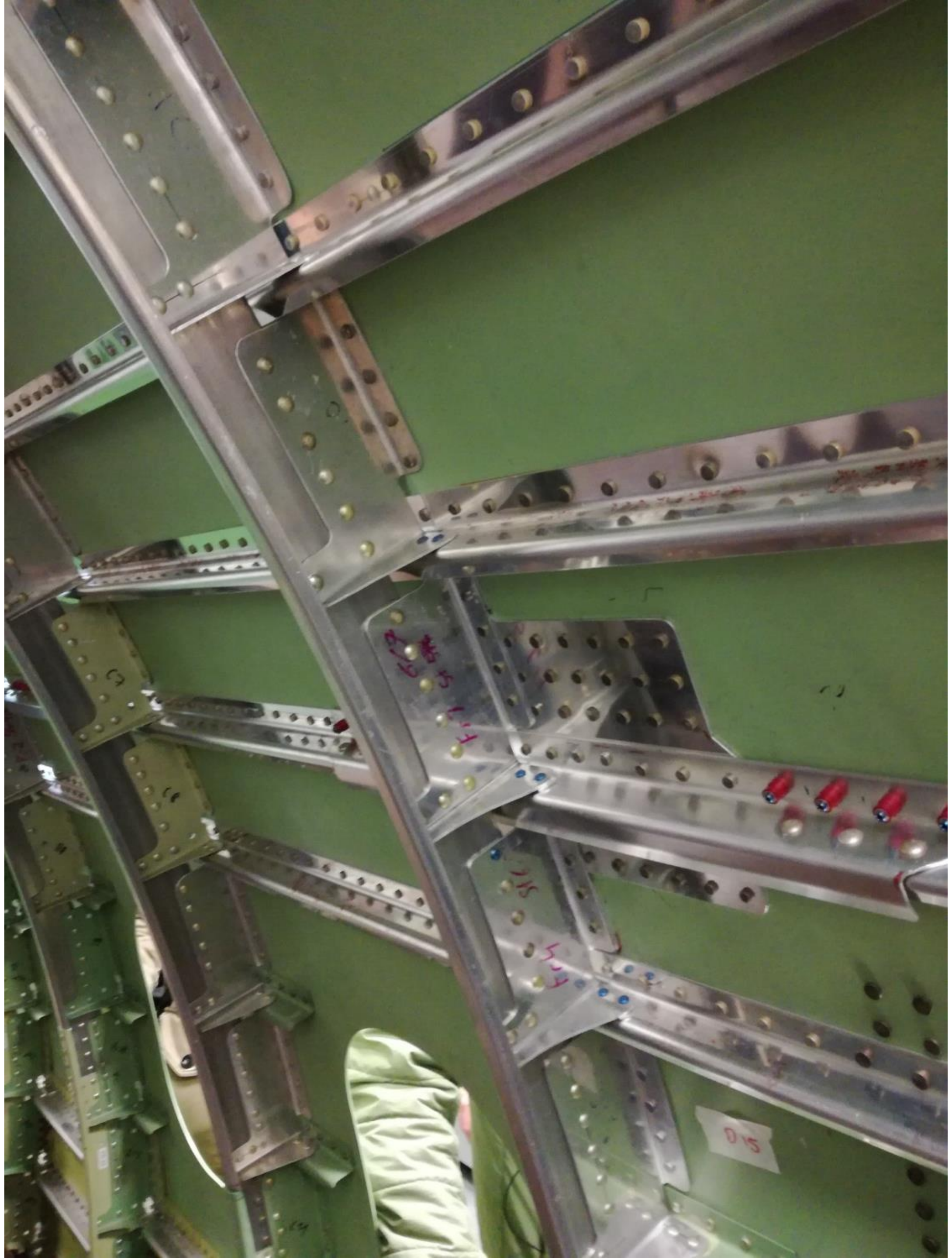
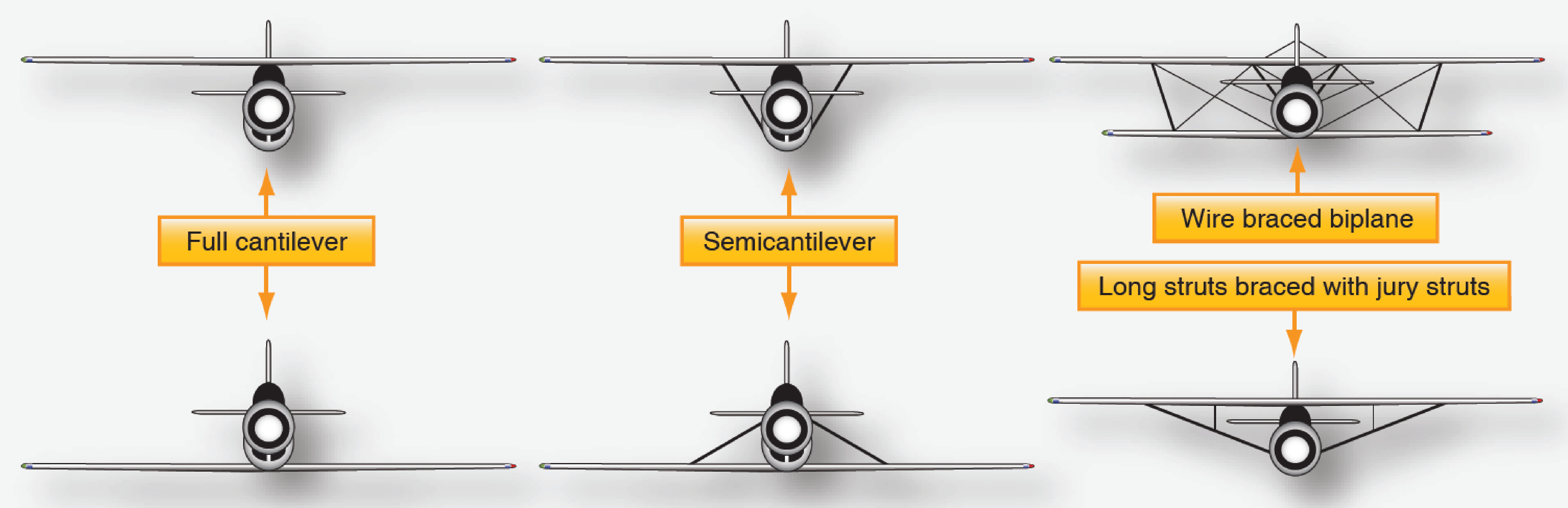
skin
stringer
© AeroToolbox.net 2017
https://commons.wikimedia.org/wiki/File:B747-structure.jpg

Stringers



Struts and wired braced wings



Struts and wired braced wings

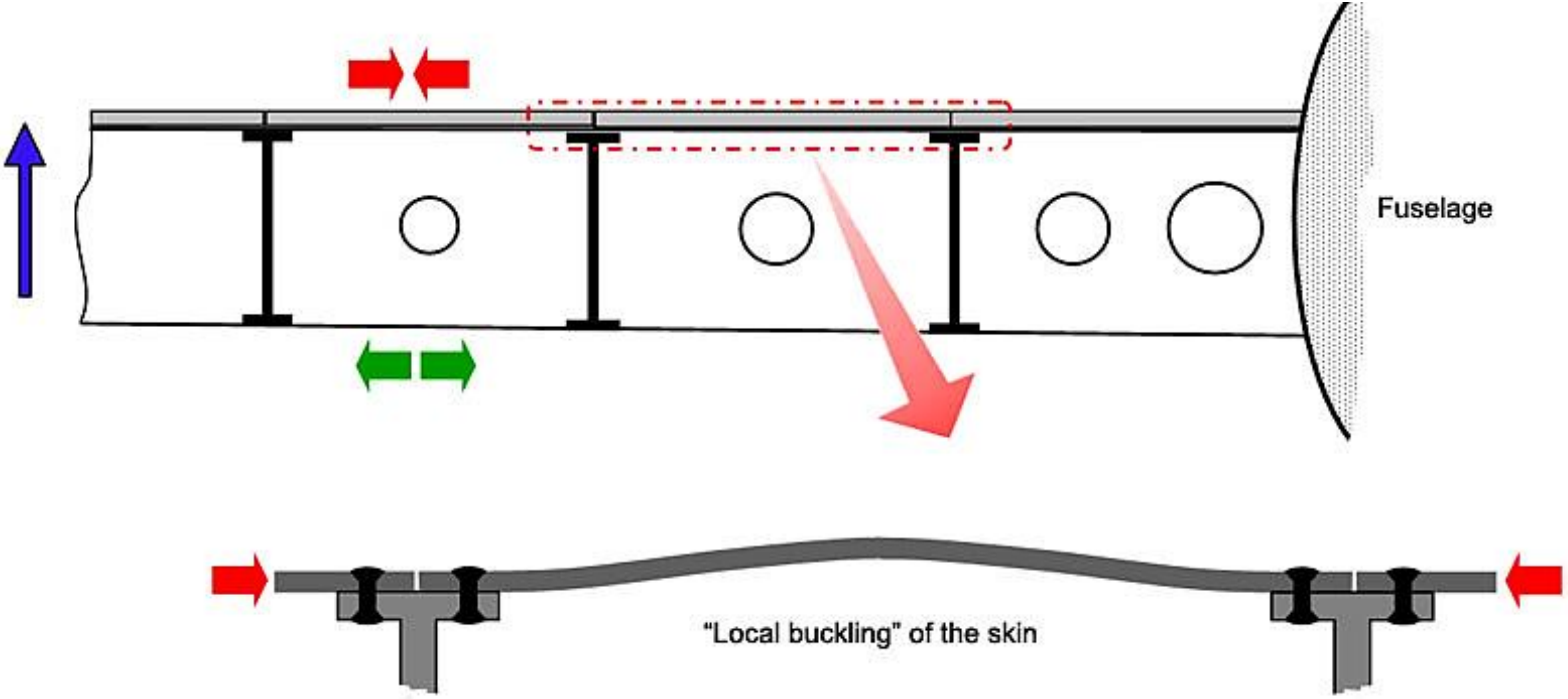
Column stability

- Columns are considered elements subjected to compressive loads.

- These elements are studied under the premise of stability.

- Stability: the ability of the structure to support a specified load without undergoing unacceptable deformations.

Buckling



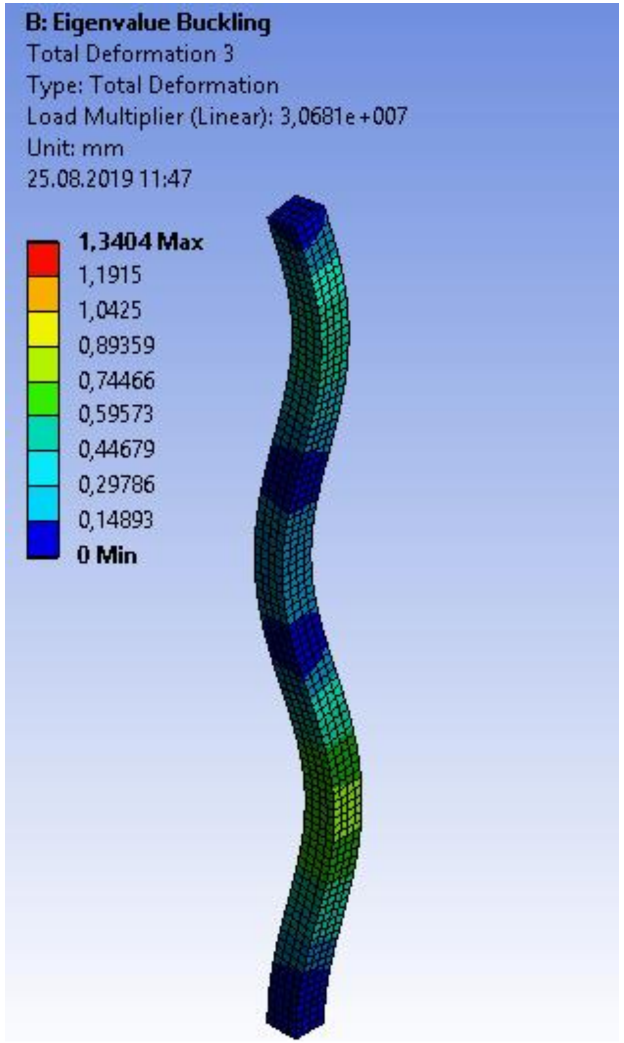
Buckling (Pandeo)

A significant contribution for the theory of the buckling in columns was made by Euler.

If an increasing axial compressive load is applied to a slender column, there is a value of the load at which the column will suddenly bow or buckle in some undefined direction.

P_{cr} is the critical load

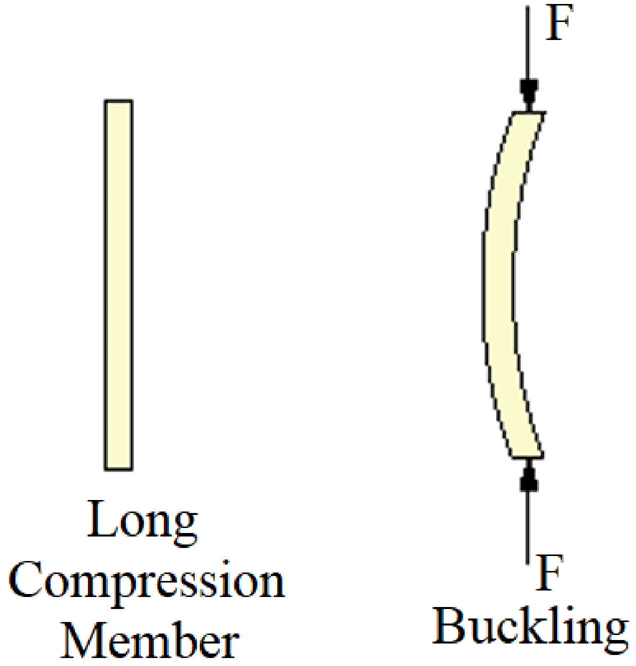
(L/r) Slenderness ratio



Buckling

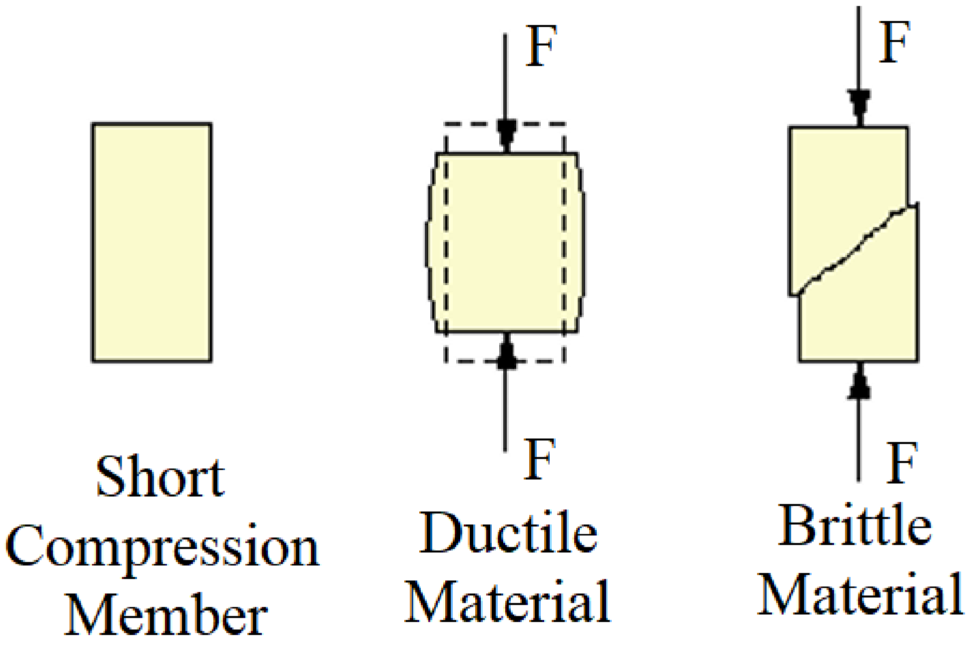
Columns

Fail by buckling



$$R = \frac{effective\ length\ (l_e)}{least\ lateral\ dimension} > 12$$

Fail by crushing load



$$R = \frac{effective\ length\ (l_e)}{least\ lateral\ dimension} < 12$$

Slenderness ratio

The slenderness ratio is defined by:

$$S = \frac{L}{r}$$

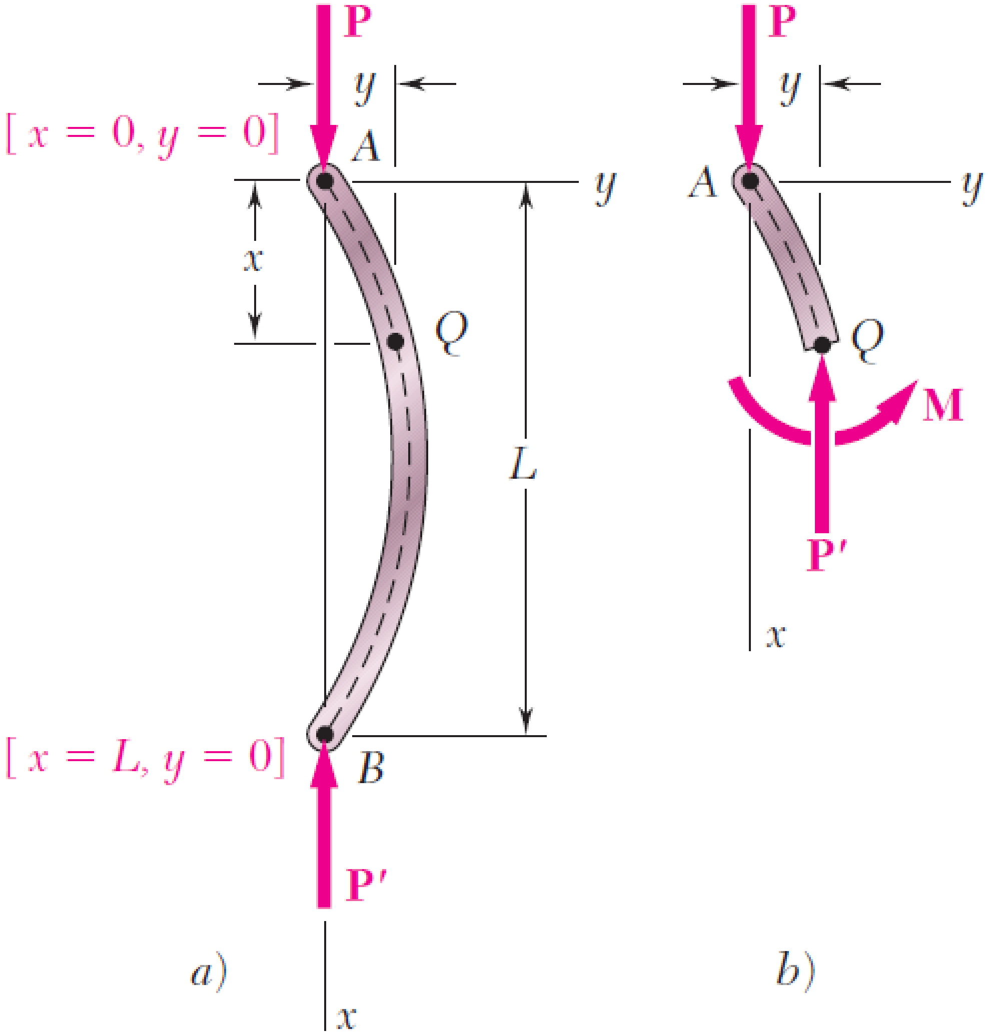
L is the length of column, and r the cross-sectional radius of gyration:

$$r = \sqrt{\frac{I}{A}}$$

Where I is the moment of inertia and A is the cross-sectional area.
Larger the radius of gyration, weaker the buckling resistance

Euler formulation for columns

Consider a distance x from A to a point Q belonging to the elastic curve, and y the column's deflection.



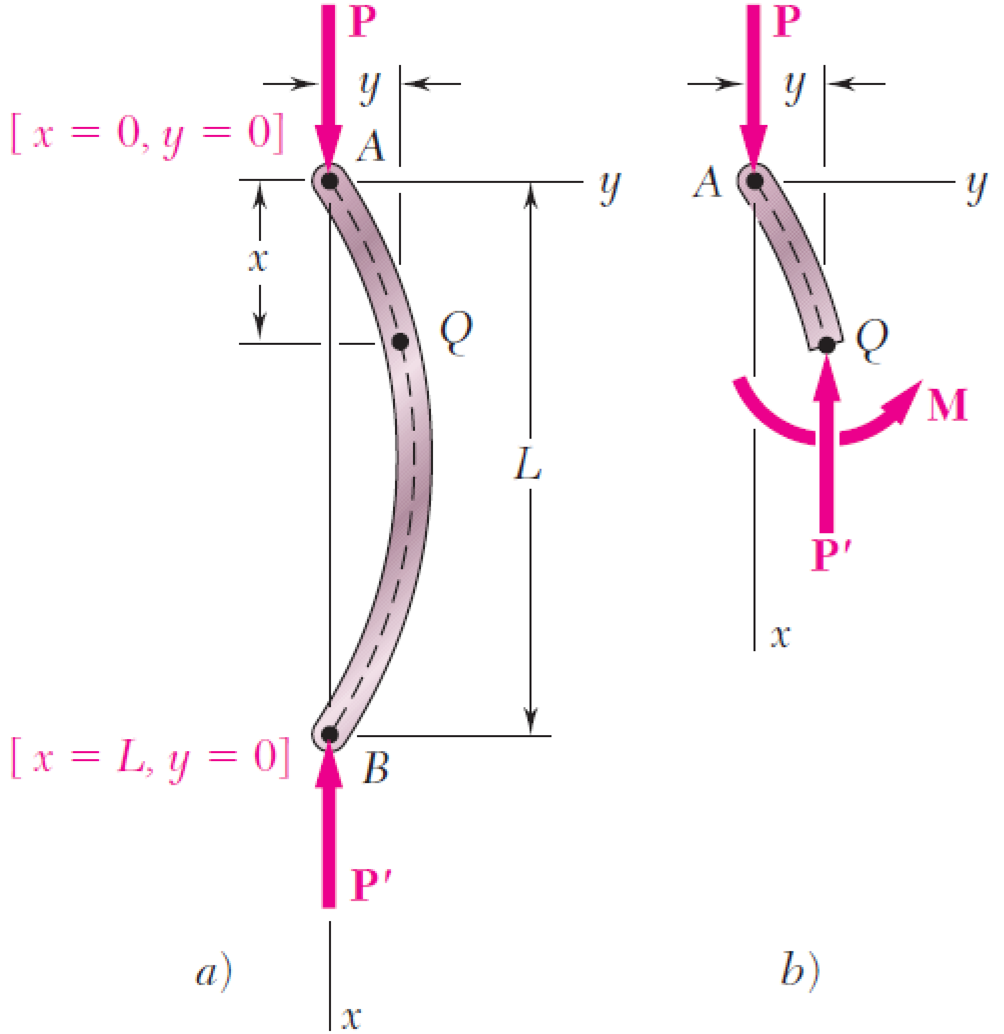
Euler formulation for columns

Critical Load

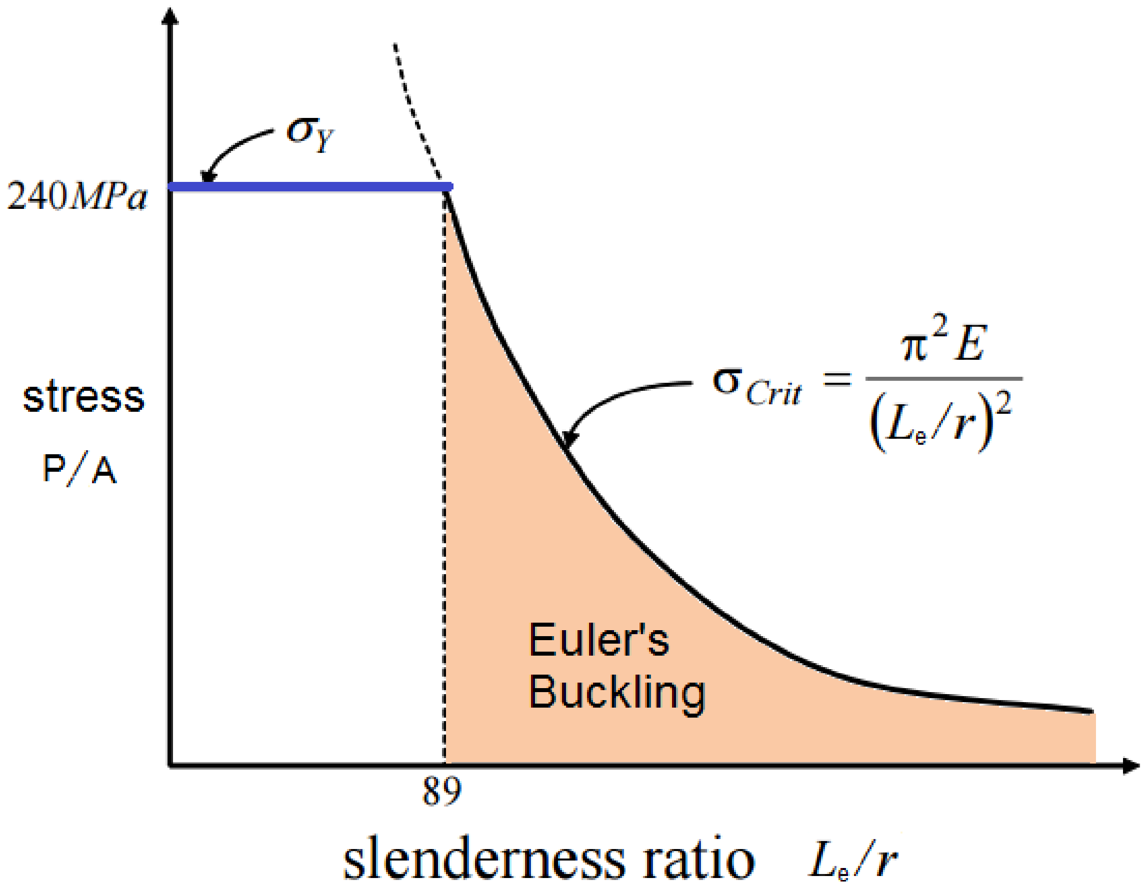
$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

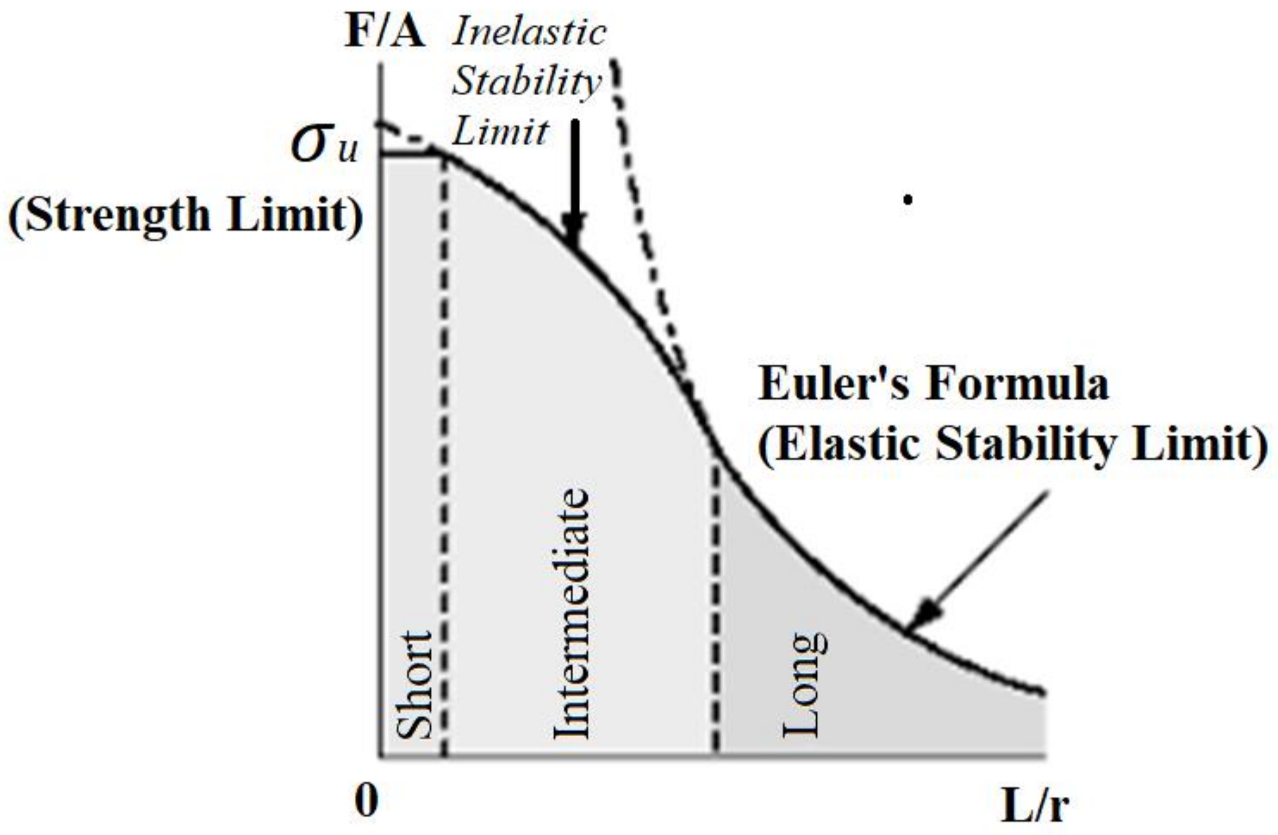
Critical stress

$$\sigma_{cr} = \frac{\pi^2 E}{(L_e/r)^2}$$

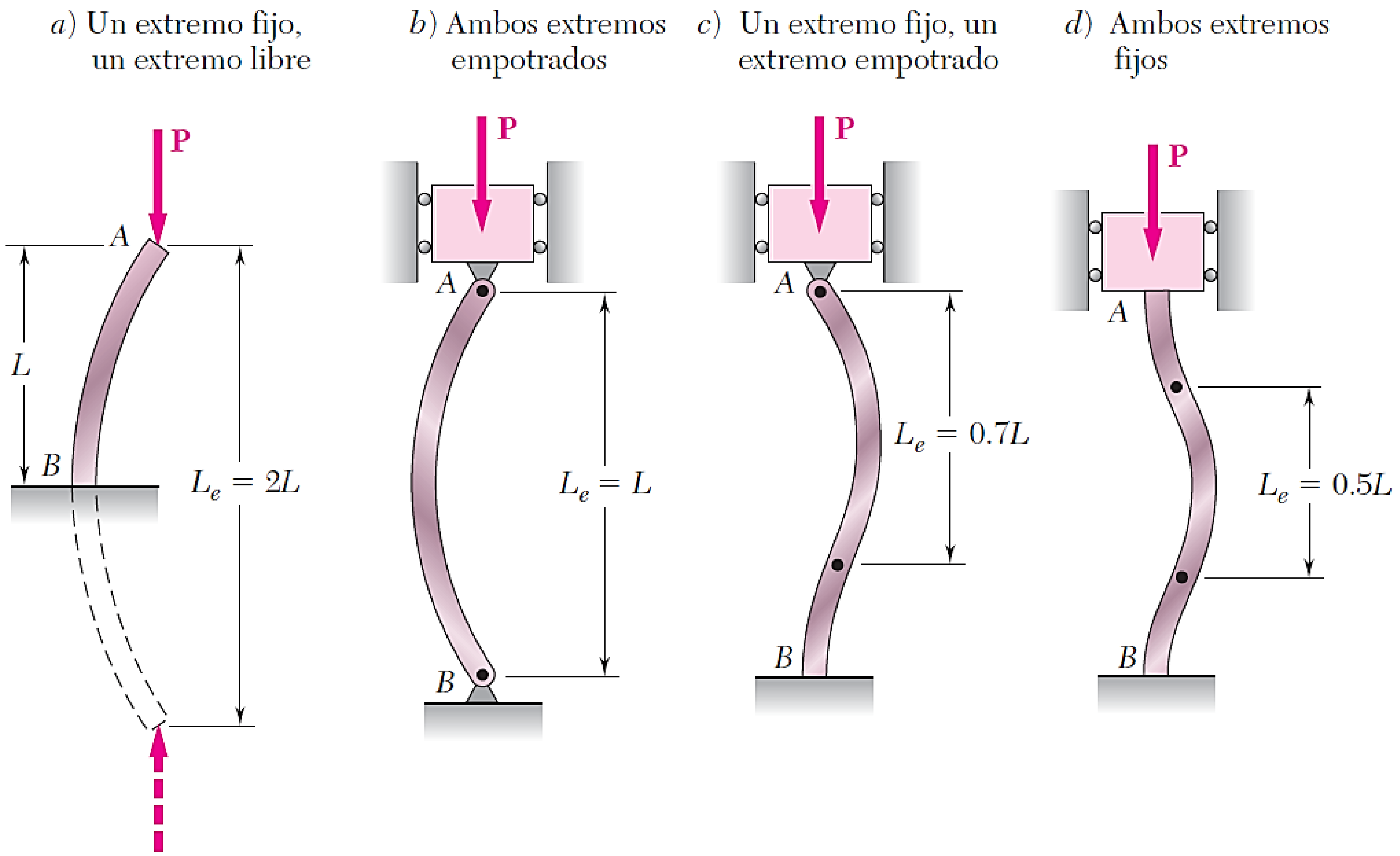


Euler formulation for columns



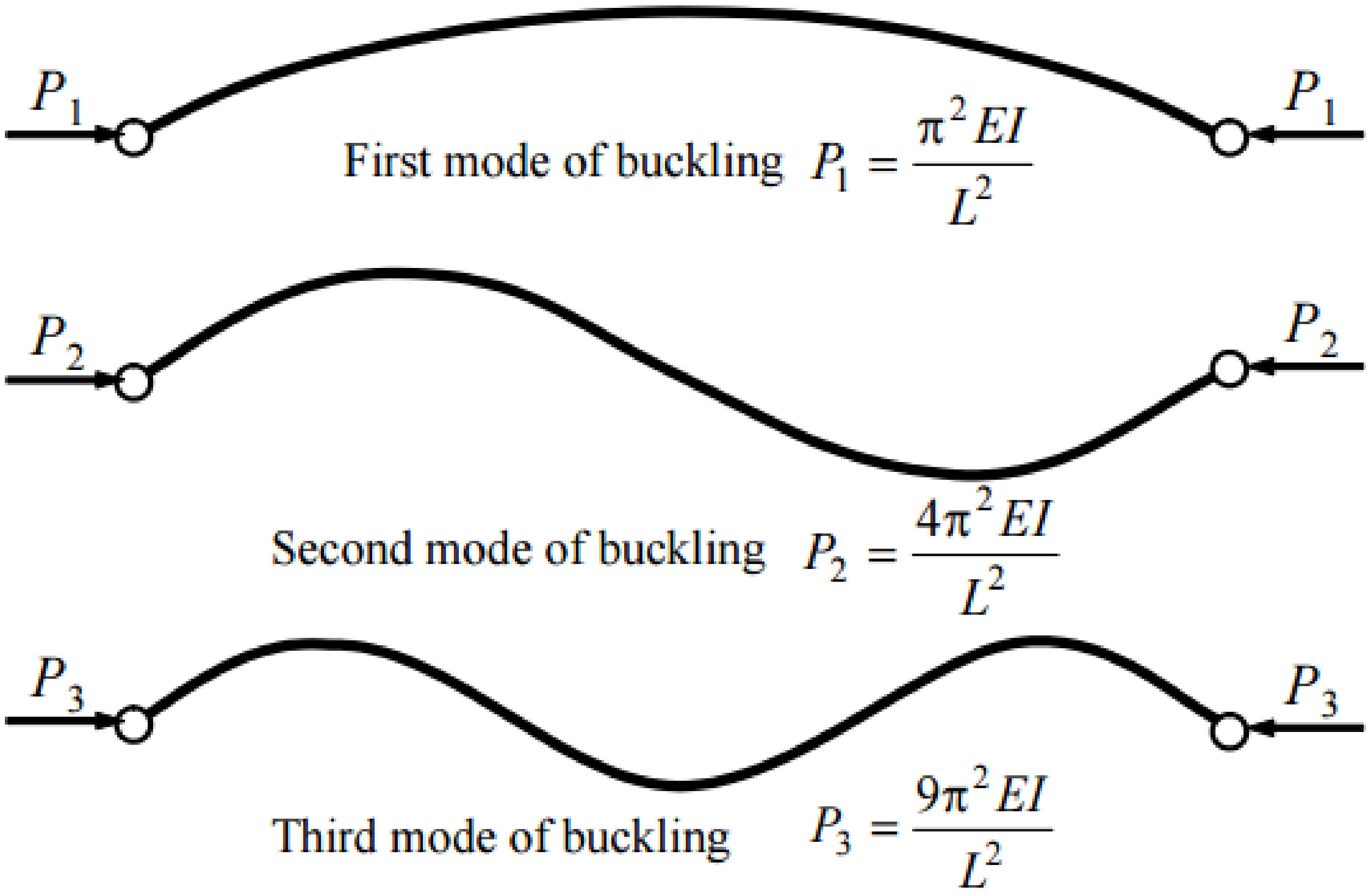


Euler formulation for columns



Euler formulation for columns

$$P_{cr} = \frac{n^2\pi^2 EI}{L_e^2}$$



Example

Assume a 2 m long pin ended column of square cross section with $E = 13\ GPa$ and $\sigma allowable = 12\ MPa$ and using a factor of safety of 2.5 in computing Euler's critical load for buckling, **determine the size of the cross section** of the column to safely support:

(a) $100\ kN$ load

(b) $200\ kN$ load

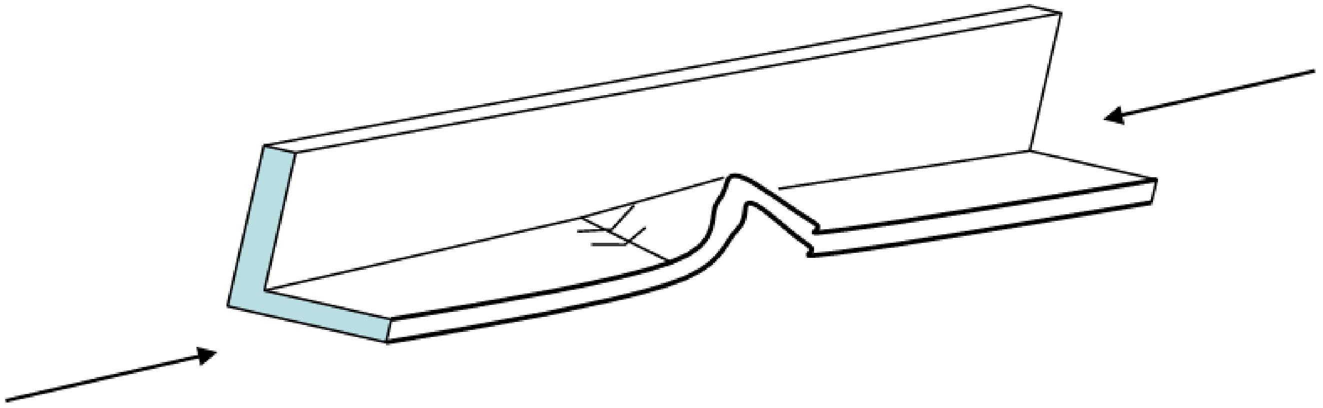
Crippling stress

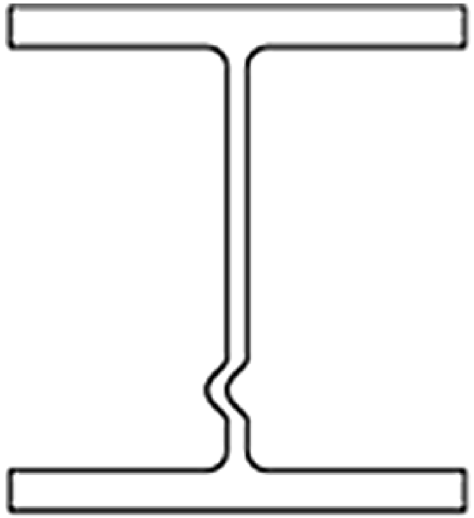
- Crippling stress is a form of instability involving inelastic axial strain of the more stable portions of a structural element resulting in permanent deformations of the section.

- Crippling stress is assumed to be independent of the component length.

- In reality, parts of the section buckles at a load below the critical crippling load with the result that stable areas such as intersections and corners reach a higher stress than the buckled elements.

Crippling stress



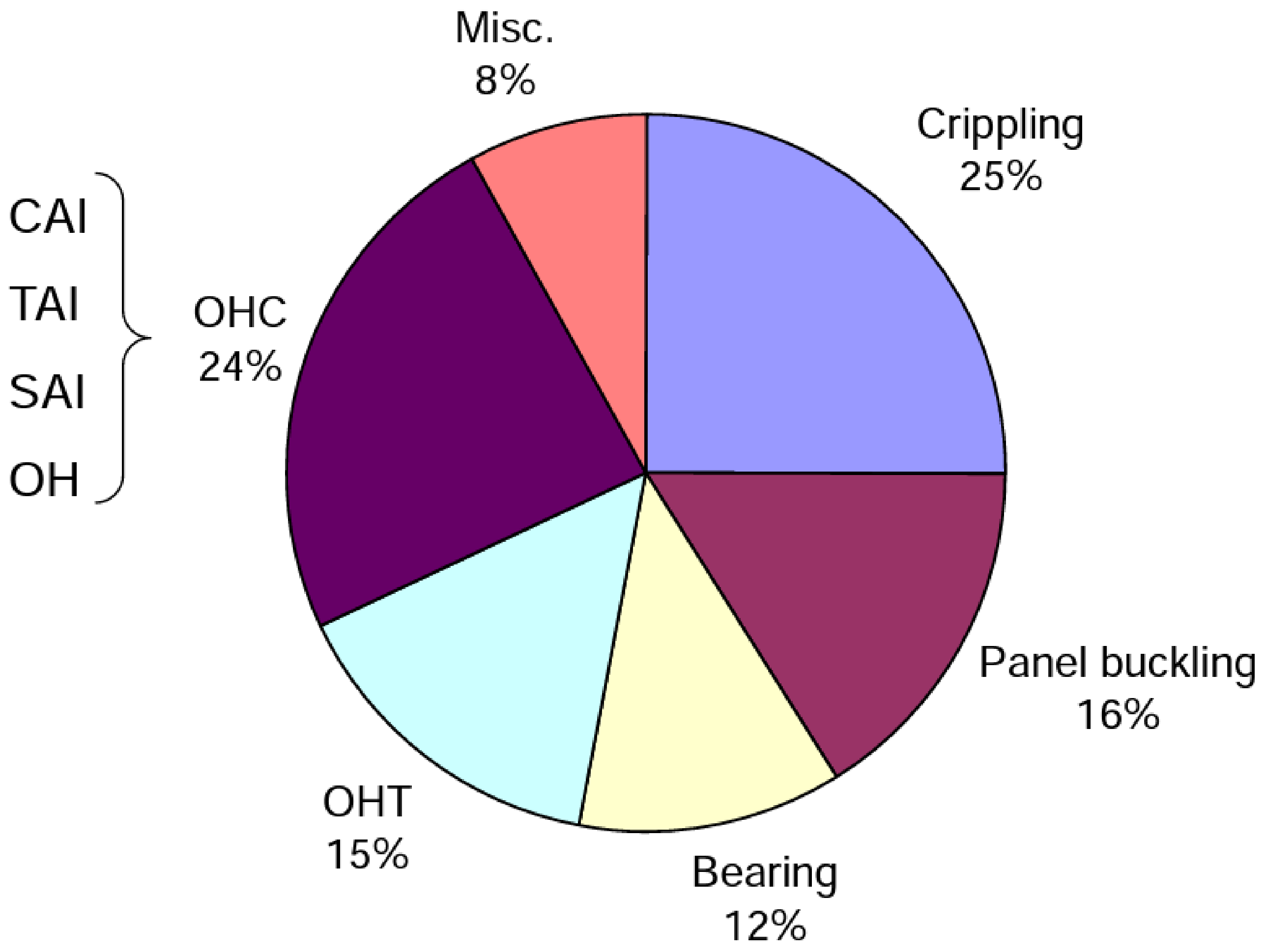


web crippling

web Buckling

Crippling stress

Fuselage failure modes



Stiffener crippling

- In a good design, **stiffeners do not fail by column buckling** but by flange crippling.
- If column buckling is an issue, the effective beam length is decreased and/or the bending stiffness of the stiffener is increased until flange crippling becomes the primary failure mode.

Reason: in column buckling the entire stiffener is “gone”. In stiffener crippling, local flange failure occurs which may be confined in one flange, without immediate failure of entire stiffener, to absorb enough of a crash load to protect passengers.

Crippling stress

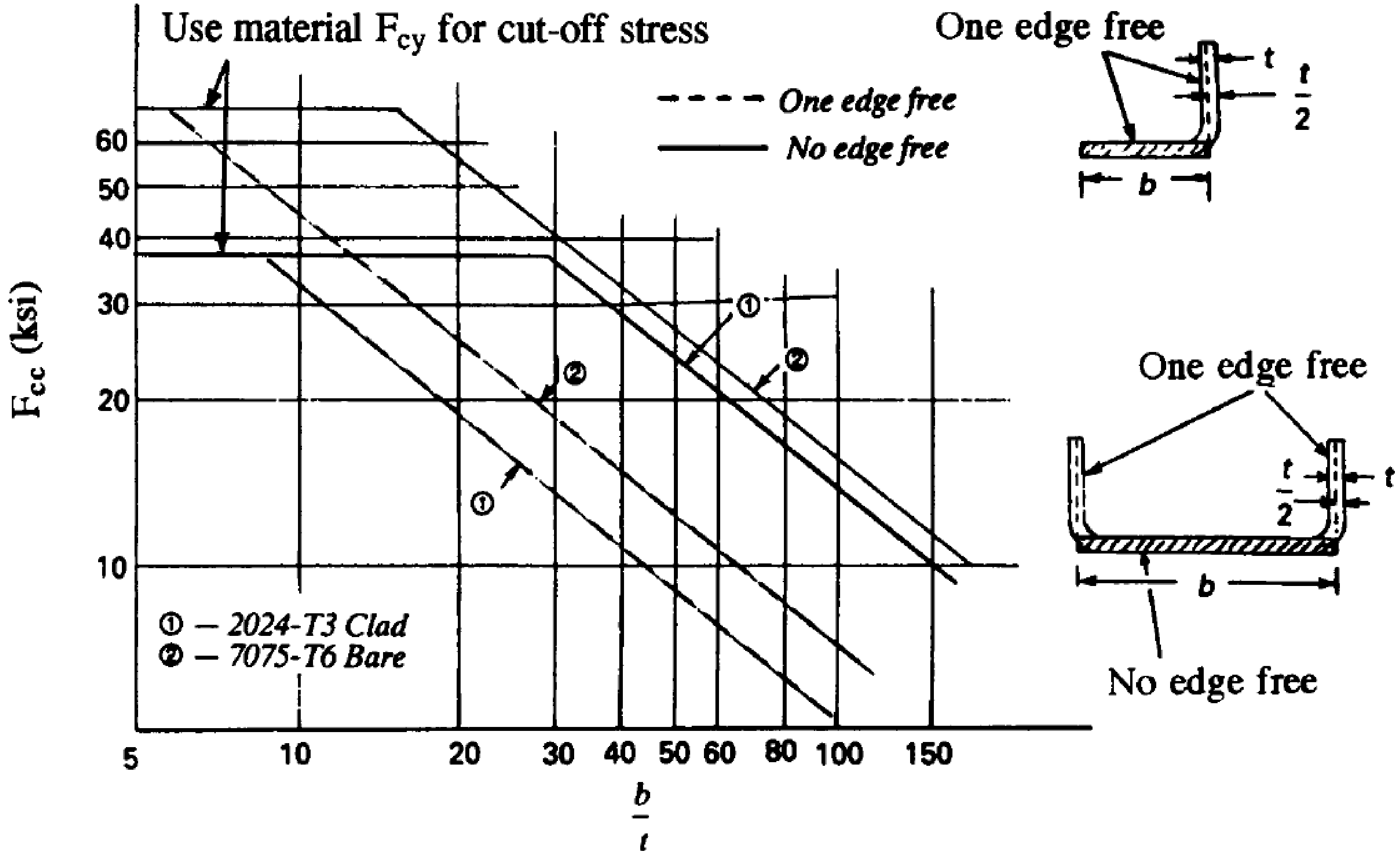
The following is a method to analyze the crippling stress allowable for extruded, machined and formed sections:

- The section is broken into individual segments. Each segment has a width b, a thickness t, and will have either no edge free or one edge free.
- The crippling stress is calculated as follow:

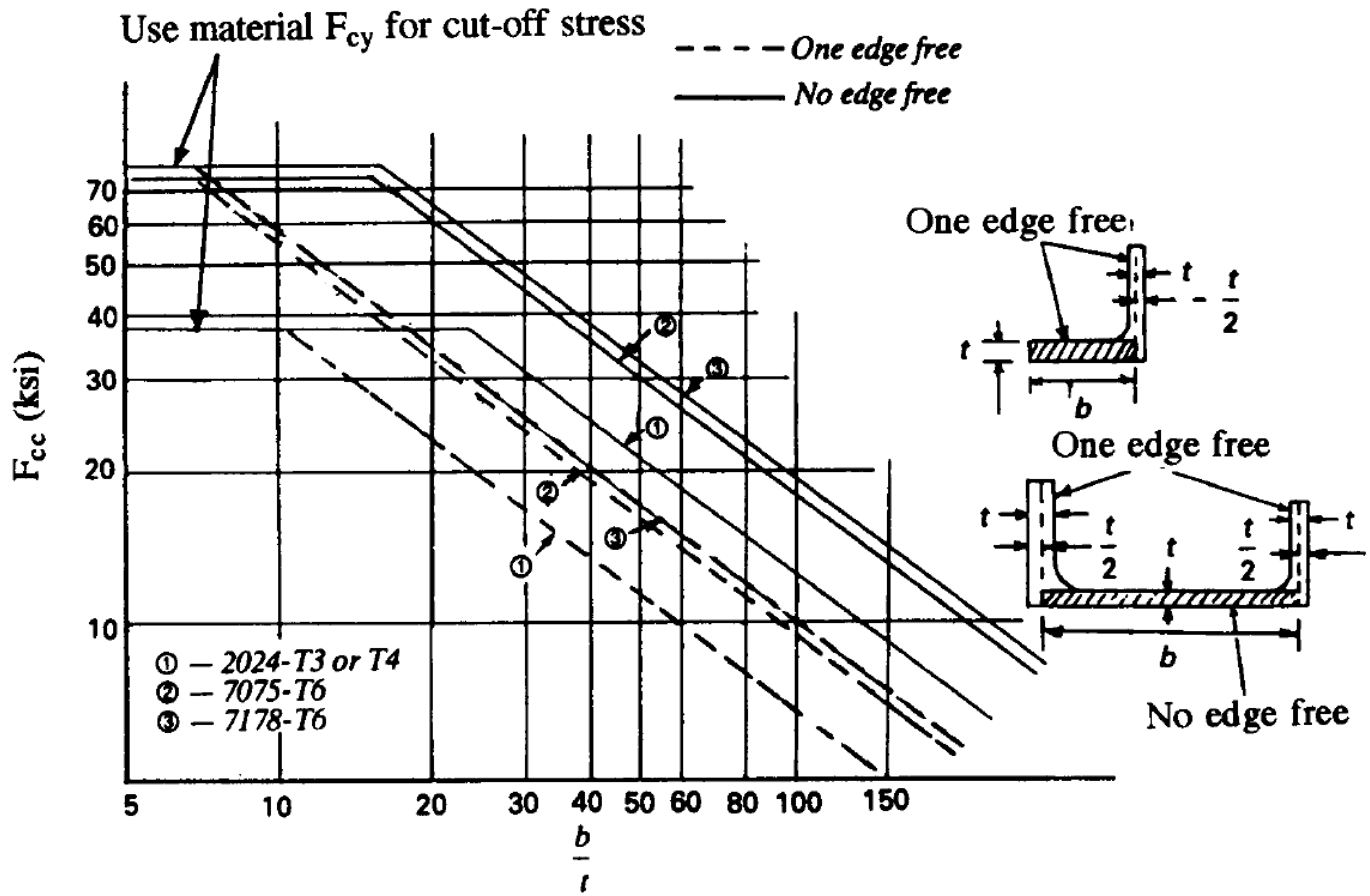
$$\sigma_{cc} = \frac{\sum b_n t_n \sigma_{ccn}}{\sum b_n t_n}$$

σ_{ccn} is the allowable stress value corresponding to computed b/t values of individual segments.

Crippling stress (sheets)



Crippling stress (extrusions)



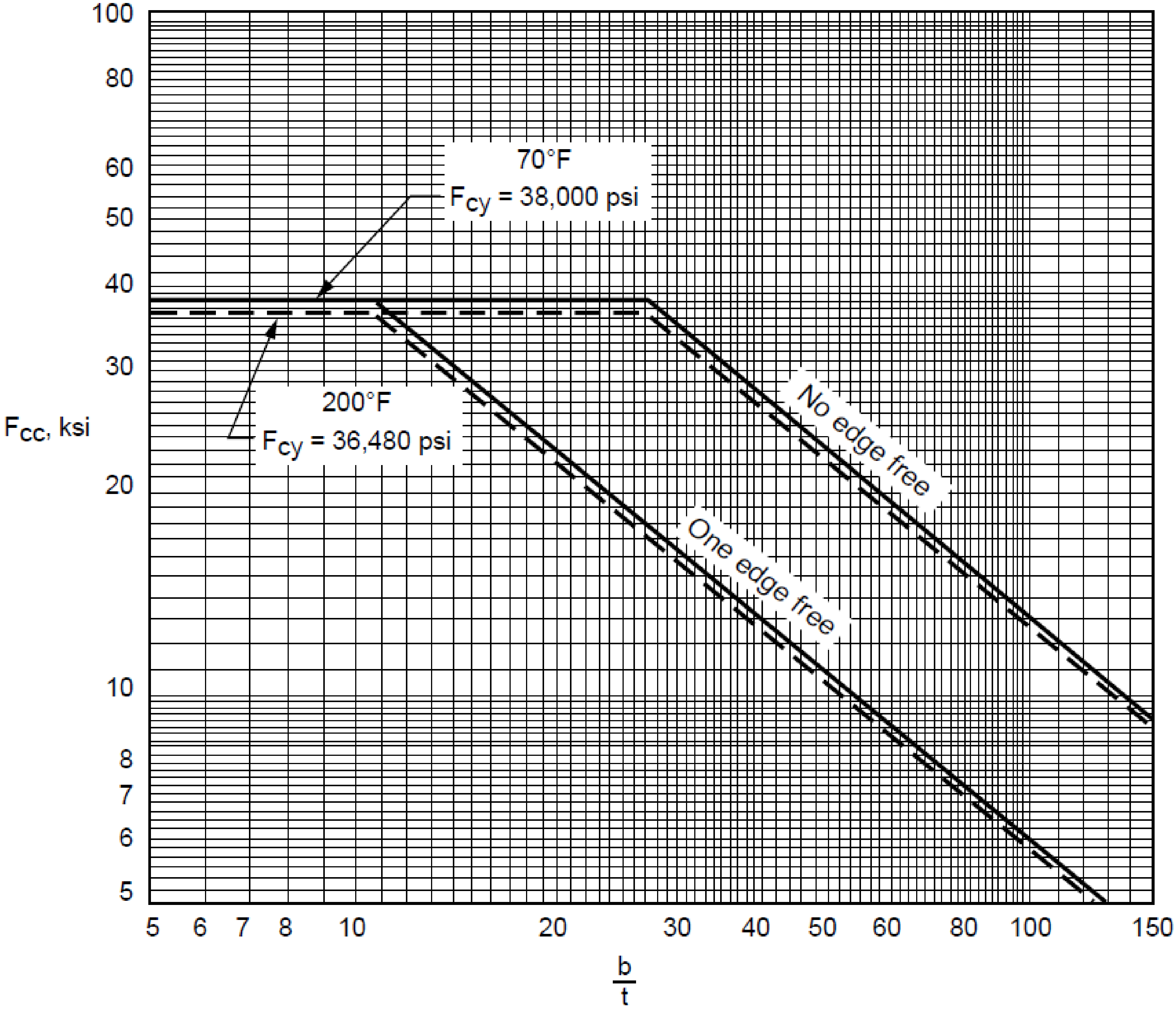


Figure 11. Compressive Crippling of Formed Sections, 775-T6, -T651, Bare Aluminum

Lip to flange stability

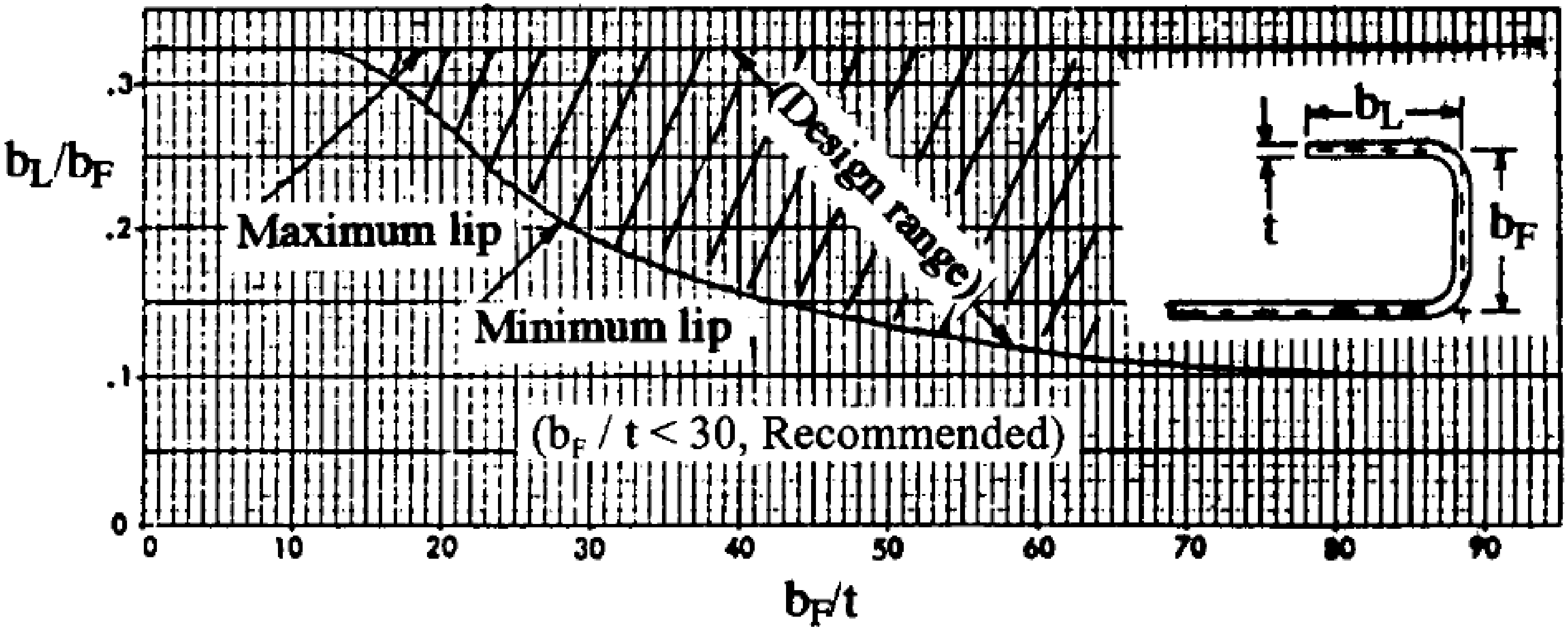
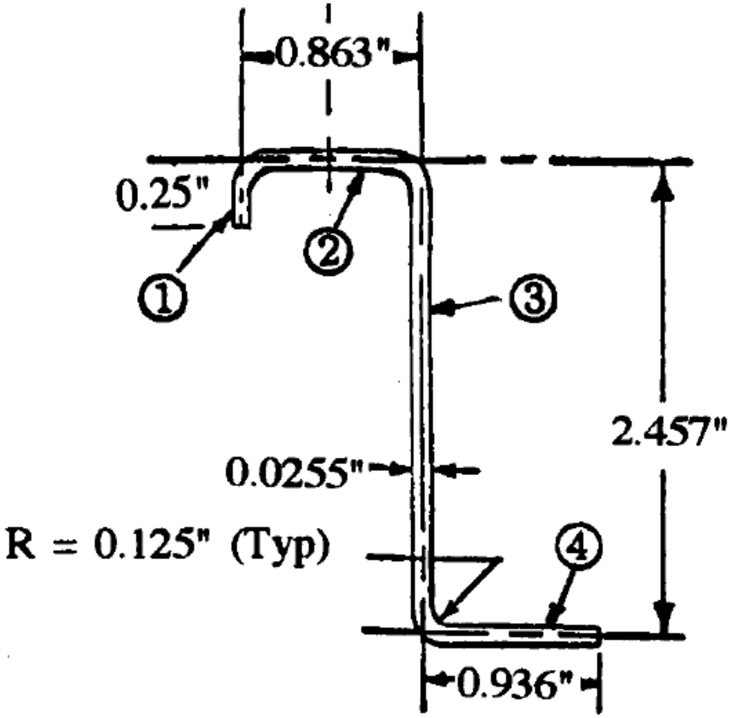


Fig. 10.7.5 Lip Criteria for Formed Sections

Example

Find crippling stress of the following formed section and material 2024-T3 clad ($\sigma_y = 38\ ksi$)



Example

Find crippling stress of the following formed section and material 2024-T3 clad ($\sigma_y = 38\ ksi$)

Segment	Free edges	b_n	t_n	$\frac{b_n}{t_n}$	$b_n t_n$	σ_{ccn} (ksi)	$b_n t_n \sigma_{ccn}$
1							
2							
3							
4							

Thin plates

- When we examine the structural components of aircrafts, they consist mainly of **thin plates stiffened** by arrangements of ribs and/or stringers.

- **Thin plates** under relatively small compressive loads are **prone to buckle** and so must be stiffened to prevent this.

- The determination of buckling loads for thin plates in isolation is **relatively straightforward but when stiffened by ribs and stringers, the problem becomes complex** and frequently relies on an empirical solution.

Thin wall theory or plate theory

- A thin plate is a sheet of material whose thickness is small compared with its other dimensions, but which is capable of resisting forces like bending.

- This kind of structure is common in aircrafts (stressed skin).

- We shall investigate the effect of a variety of loadings and support conditions on the small deflection of regular plates.

Plate theory

Two theories are accepted in continuum mechanics:

- Kirchhoff-love (classical plate theory). *it’s an extension of Euler-Bernoulli beam theory.

- Mindlin-Reissner (first order shear plate theory).

Thin wall theory

Two approaches can be presented:

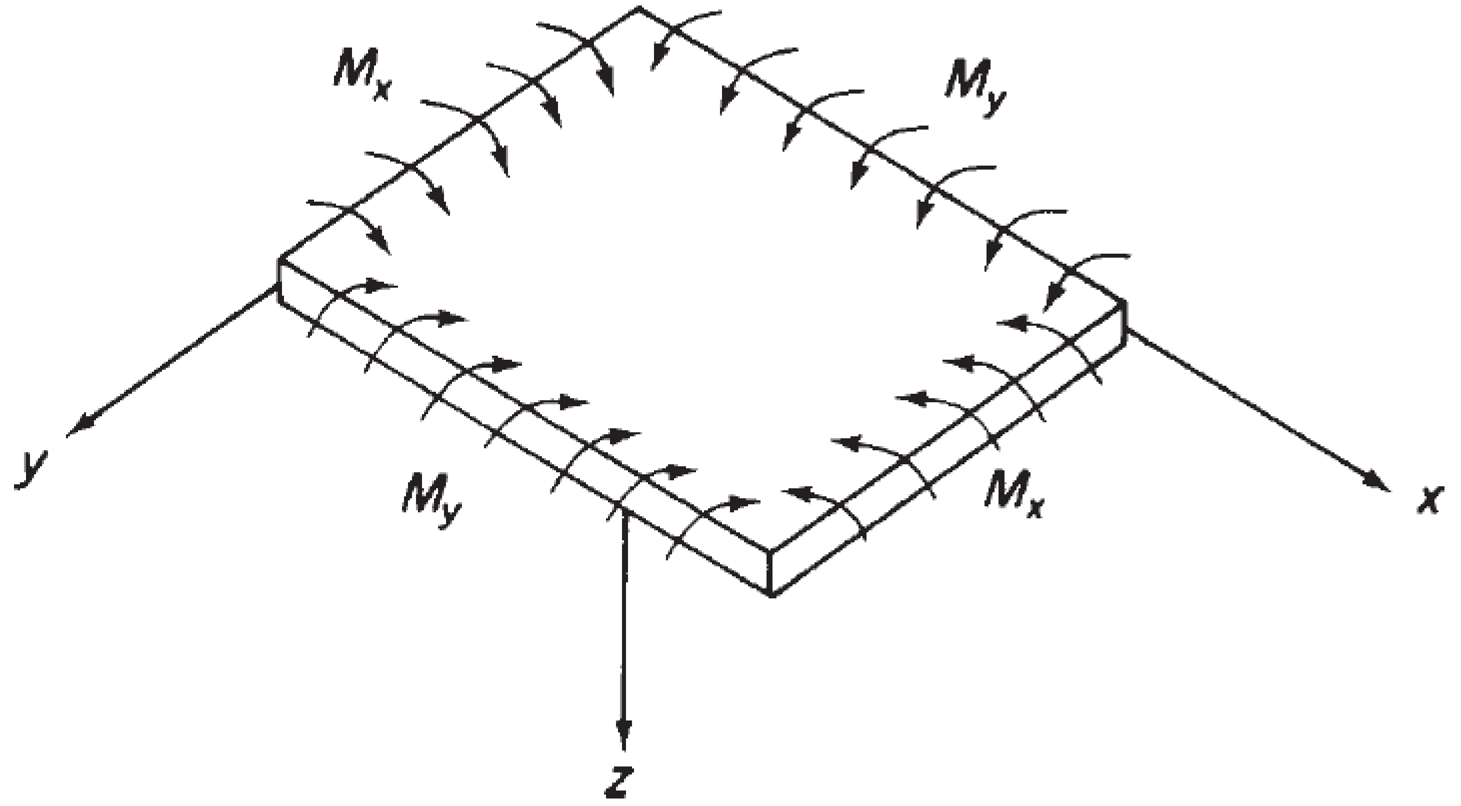
An exact theory based on the solution of a differential equation.

An energy method relying on the principle of the stationary value of the total potential energy and the plate’s applied loading.


CAUTION CAUTION

Pure bending of thin plates

The thin plate is subjected to bending moments M_x and M_y



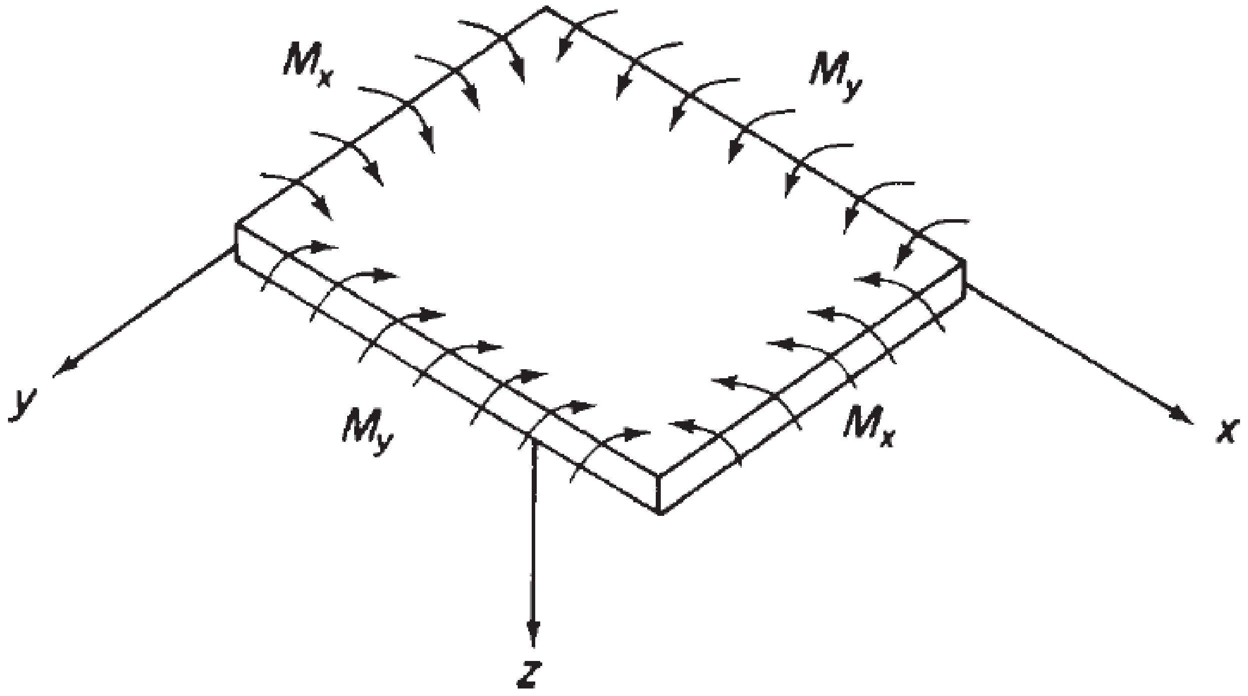
Pure bending of thin plates

Assumptions:

- The material is homogeneous and isotropic
- The deflection is small
- The normal to middle surface of the plate before deformation remains normal to the plane after deformation. This shows that strain in z-direction is zero.
- The in-plane forces are neglected in bending of thin plates
- The stresses $\sigma_z = \tau_{xz} = \tau_{zx} = \tau_{xy} = \tau_{yz} = 0$

Pure bending of thin plates

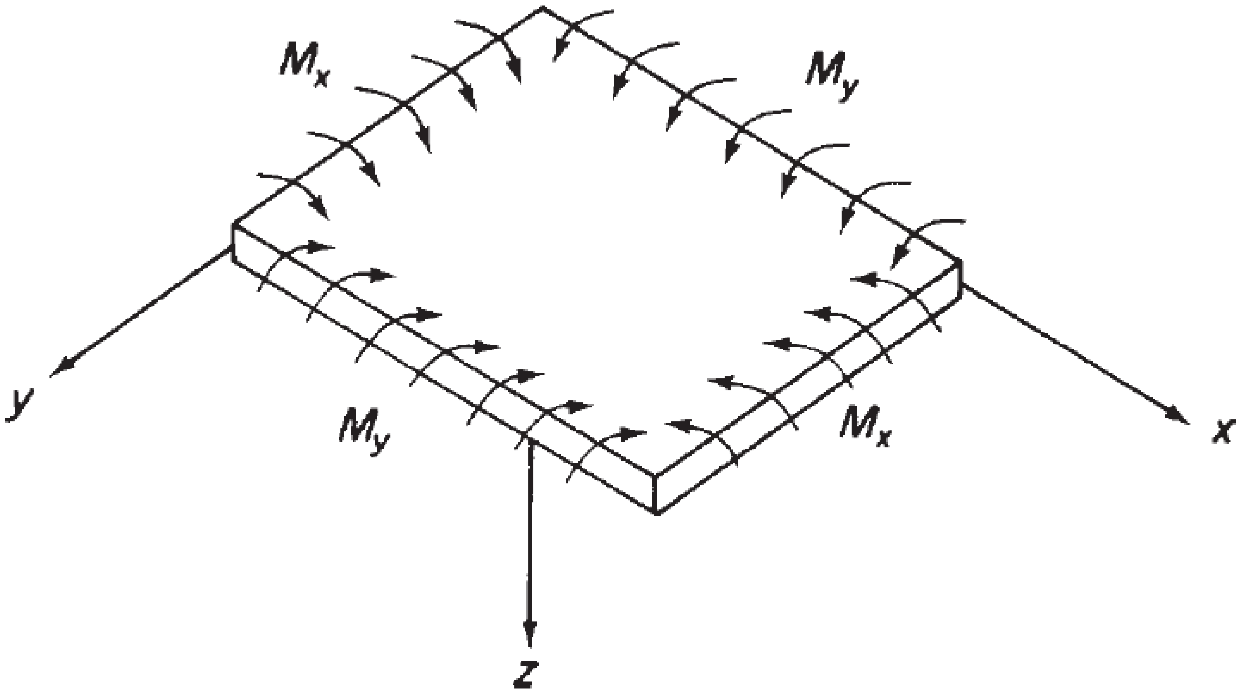
We shall assume that these bending moments are positive when they produce compression at the upper surface of the plate and tension at the lower one.



Pure bending of thin plates

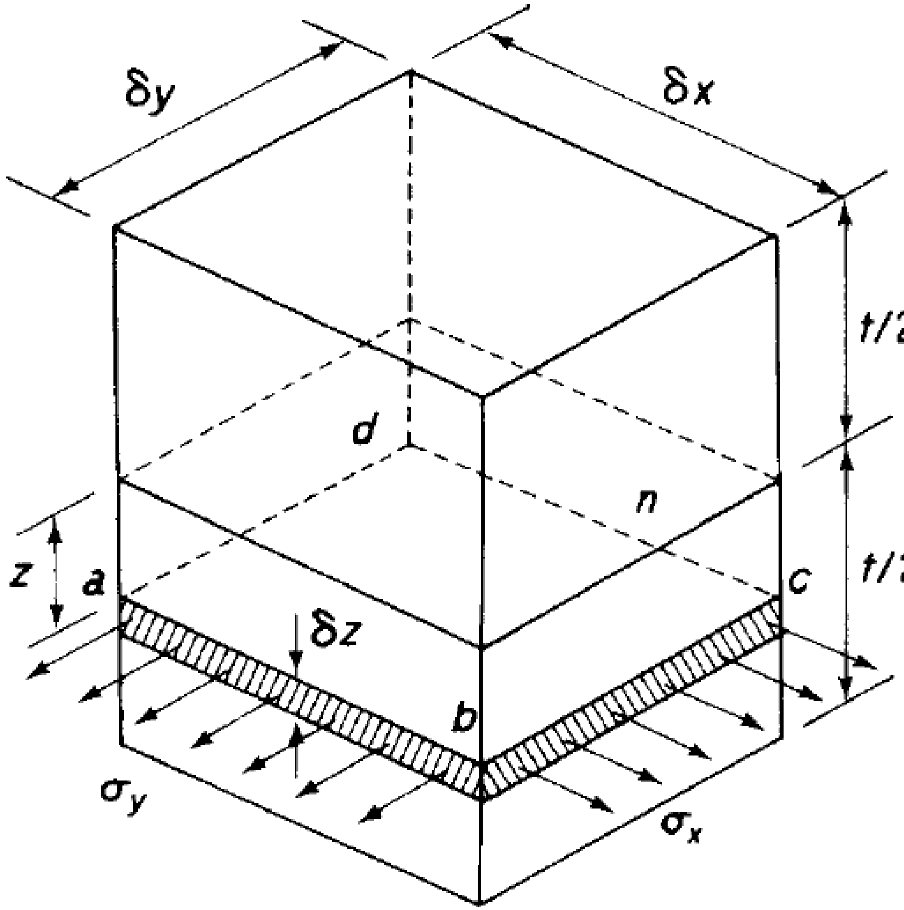
We assume that the displacement of the plate in a direction parallel to the z axis is small compared with its thickness t.

The middle plane of the plate does not deform during the bending and is therefore a neutral plane (REFERENCE PLANE).



Pure bending of thin plates

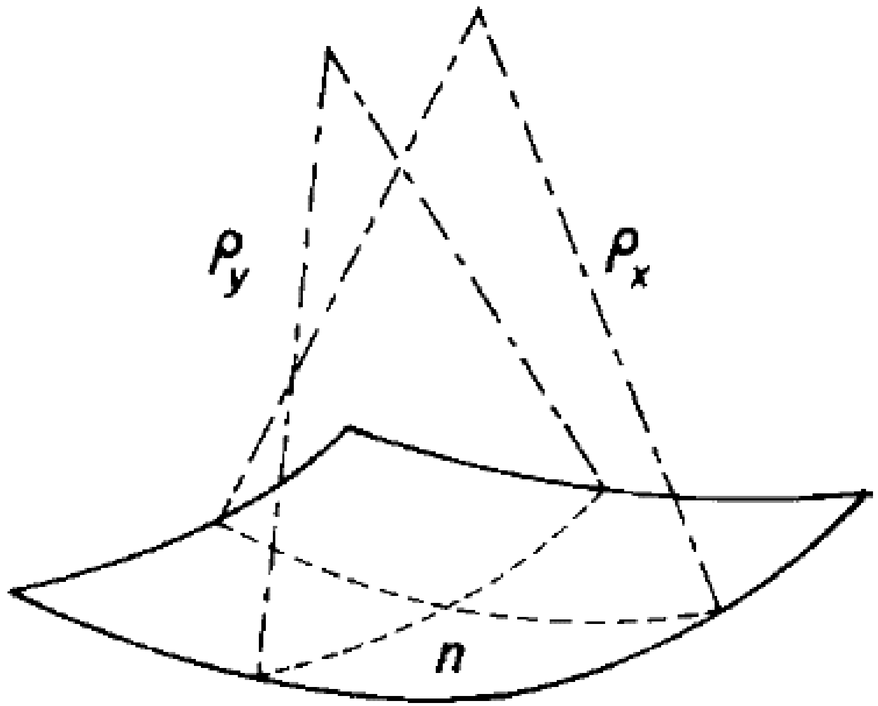
Let us consider an element of the plate of side $\delta x$and $\delta y$and having a depth equal to the thickness t of the plate.



Pure bending of thin plates

Suppose that the radii of curvature of the neutral plane n are:

- ρ_x in the xz plane
- ρ_y in the yz plane



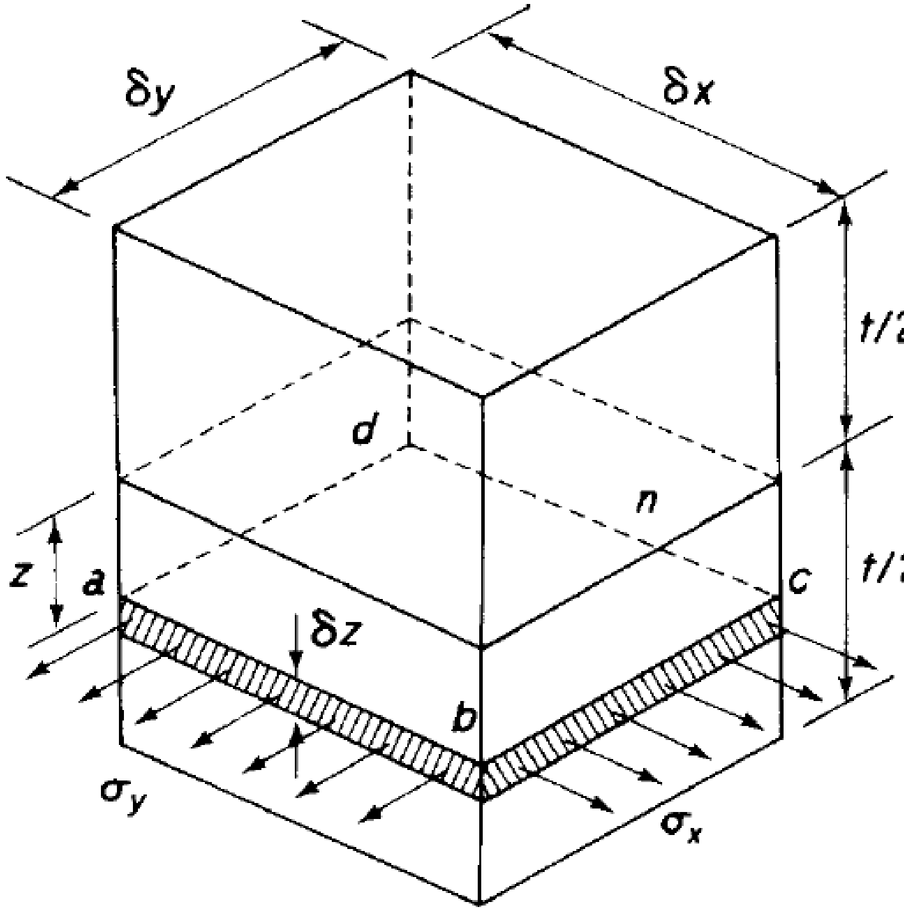
Positive curvature of the plate corresponds to the positive bending moments that produce displacements in the positive direction of the z axis.

Pure bending of thin plates

The direct strains ε_x and ε_y of an elemental lamina of thickness dz with distance z below the neutral plane is given by:

$$\varepsilon_x = \frac{z}{\rho_x}$$

$$\varepsilon_y = \frac{z}{\rho_y}$$

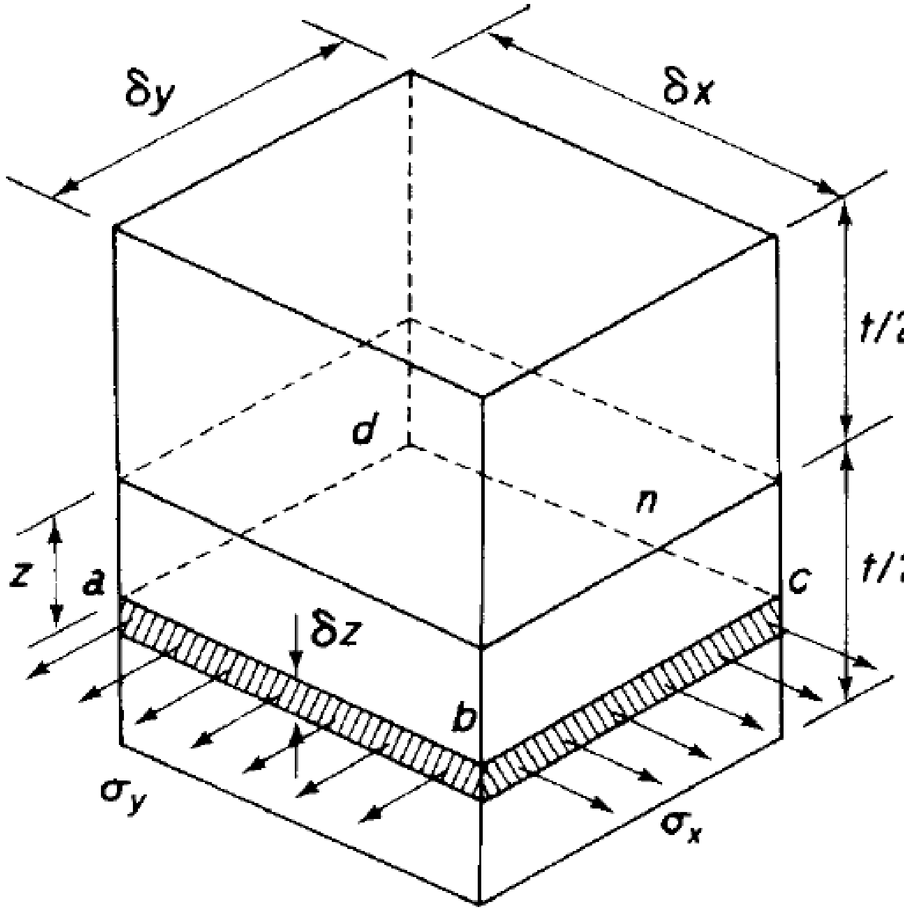


Pure bending of thin plates

From the multiaxial loading generalized Hooke's Law:

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\sigma_y\right)$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\sigma_x\right)$$



Pure bending of thin plates

Substituting equations for ε_x and ε_y:

$$\sigma_x = \frac{Ez}{1-\nu^2}\left(\frac{1}{\rho_x} + \frac{\nu}{\rho_y}\right)$$

$$\sigma_y = \frac{Ez}{1-\nu^2}\left(\frac{1}{\rho_y} + \frac{\nu}{\rho_x}\right)$$

The direct stresses vary linearly across the thickness of the plate

Pure bending of thin plates

The internal stress distribution must be in equilibrium with the applied bending moment, thus:

$$M_x \partial y = \int_{-t/2}^{t/2} \sigma_x z \, \delta y \partial z$$

$$M_y \partial x = \int_{-t/2}^{t/2} \sigma_y z \, \delta x \partial z$$

Pure bending of thin plates

Substituting equations for σ_x and σ_y:

$$M_x = \int_{-t/2}^{t/2} \frac{Ez^2}{1-\nu^2}\left(\frac{1}{\rho_x} + \frac{\nu}{\rho_y}\right) z \, \mathrm{dz}$$

$$M_y = \int_{-t/2}^{t/2} \frac{Ez}{1-\nu^2}\left(\frac{1}{\rho_y} + \frac{\nu}{\rho_x}\right) z \, dz$$

Pure bending of thin plates

Let D (flexural rigidity):

$$D = \int_{-t/2}^{t/2} \frac{Ez^2}{1 - \nu^2} dz \quad \rightarrow$$

Then:

$$M_x = \int_{-t/2}^{t/2} \frac{Et^3}{12(1 - \nu^2)} \left(\frac{1}{\rho_x} + \frac{\nu}{\rho_y} \right) z \, \mathrm{dz} \rightarrow M_x = D \left(\frac{1}{\rho_x} + \frac{\nu}{\rho_y} \right)$$

$$M_y = \int_{-t/2}^{t/2} \frac{Et^3}{12(1 - \nu^2)} \left(\frac{1}{\rho_y} + \frac{\nu}{\rho_x} \right) z \, dz \rightarrow M_y = D \left(\frac{1}{\rho_y} + \frac{\nu}{\rho_x} \right)$$

Pure bending of thin plates

If w is the deflection of any point on the plate in the z direction, then we may relate w to the curvature of the plate in the same manner as the well-known mathematical expression for beam curvature:

$$\frac{1}{\rho_x} = -\frac{\partial^2 w}{\partial x^2}$$

$$\frac{1}{\rho_y} = -\frac{\partial^2 w}{\partial y^2}$$

Pure bending of thin plates

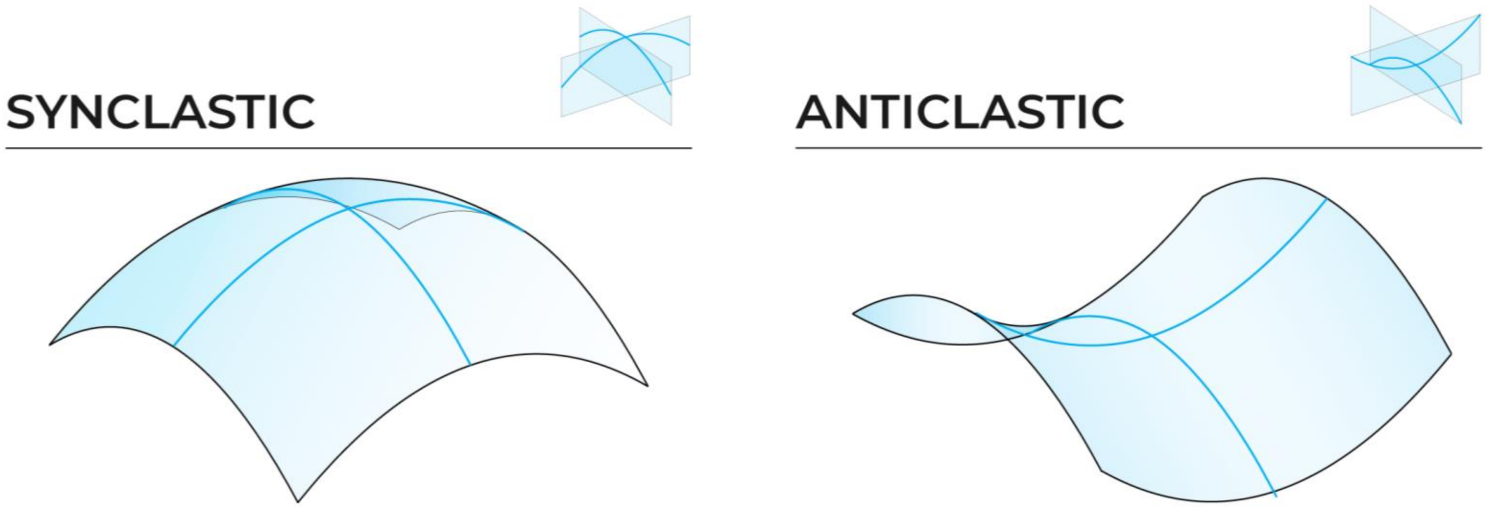
Thus:

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2}\right)$$

$$M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + v\frac{\partial^2 w}{\partial x^2}\right)$$

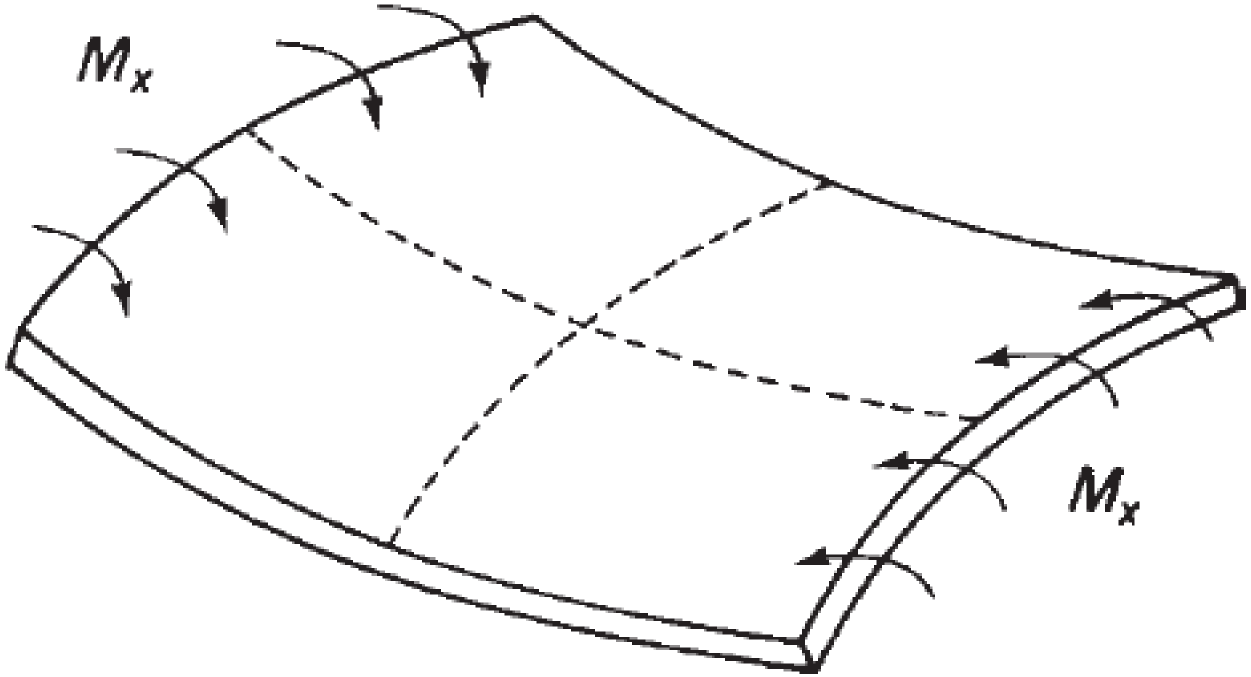
Pure bending of thin plates

- A surface possessing two curvatures of opposite sign is known as an **anticlastic surface**
- As opposed to a **synclastic surface**, which has curvatures of the same sign

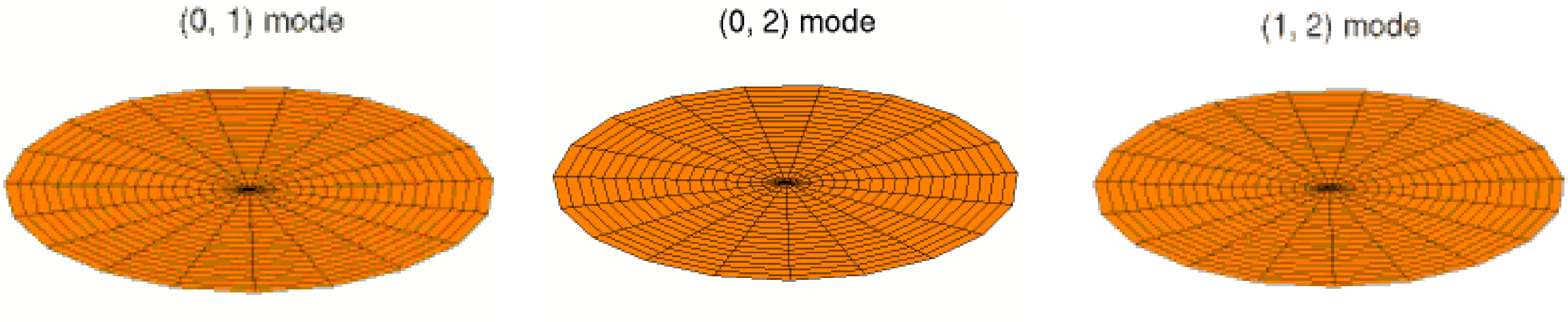


Pure bending of thin plates

When $M_y = 0$



Pure bending of thin plates



Plates subjected to bending and twisting

The bending moments applied to the plate will not be in planes perpendicular to its edges

In this case, such bending moments may be resolved in the normal manner into tangential $\left(M_x \ and \ M_y\right)$ and perpendicular components $\left(M_{xy} \ and \ M_{yx}\right)$.

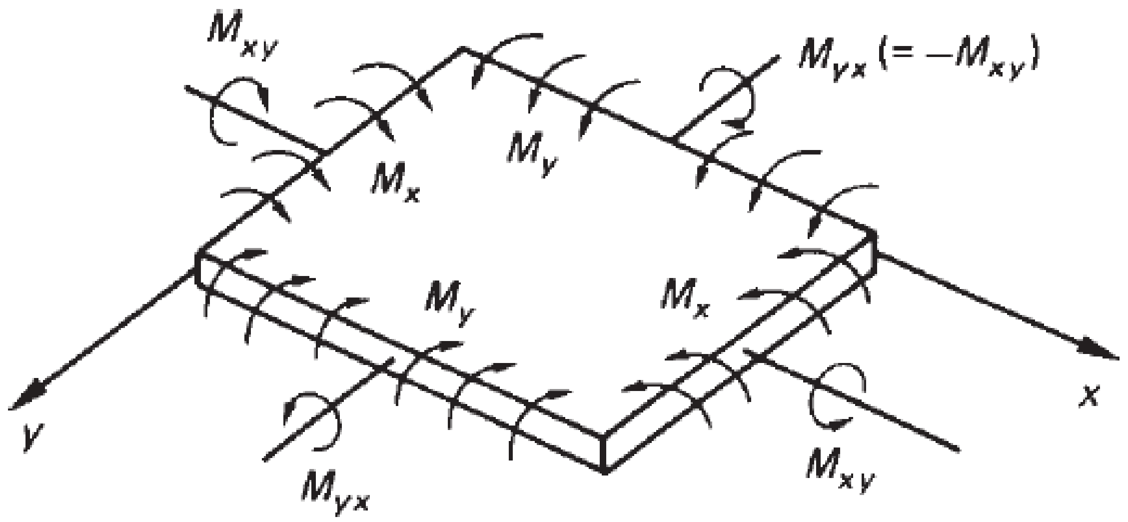
Plates subjected to bending and twisting

Bending moments: M_x, M_y

Twisting moments: M_{xy}, M_{yx}

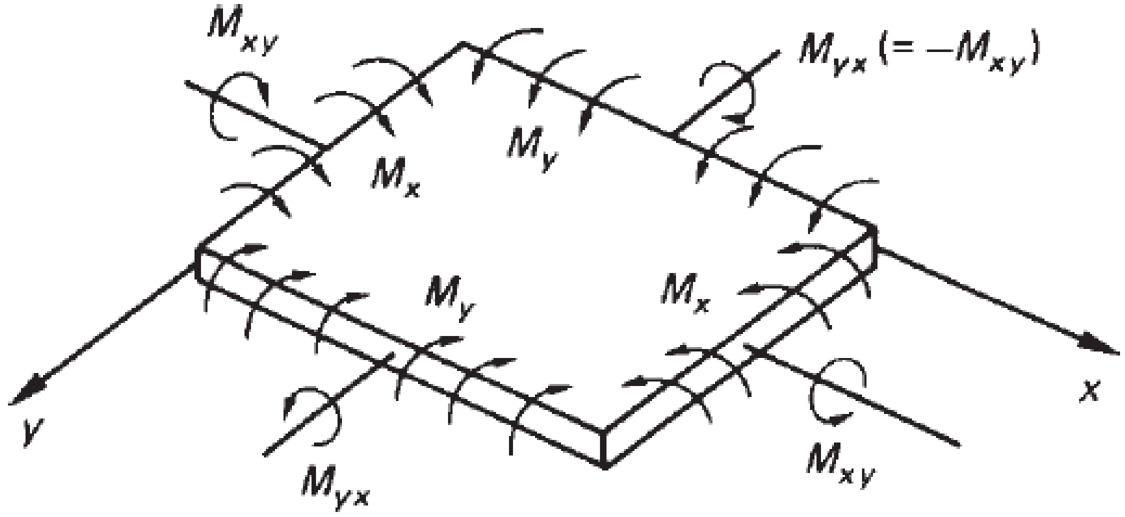
These moments produce shear stresses τ_{xy}.

Clockwise moments are positive.



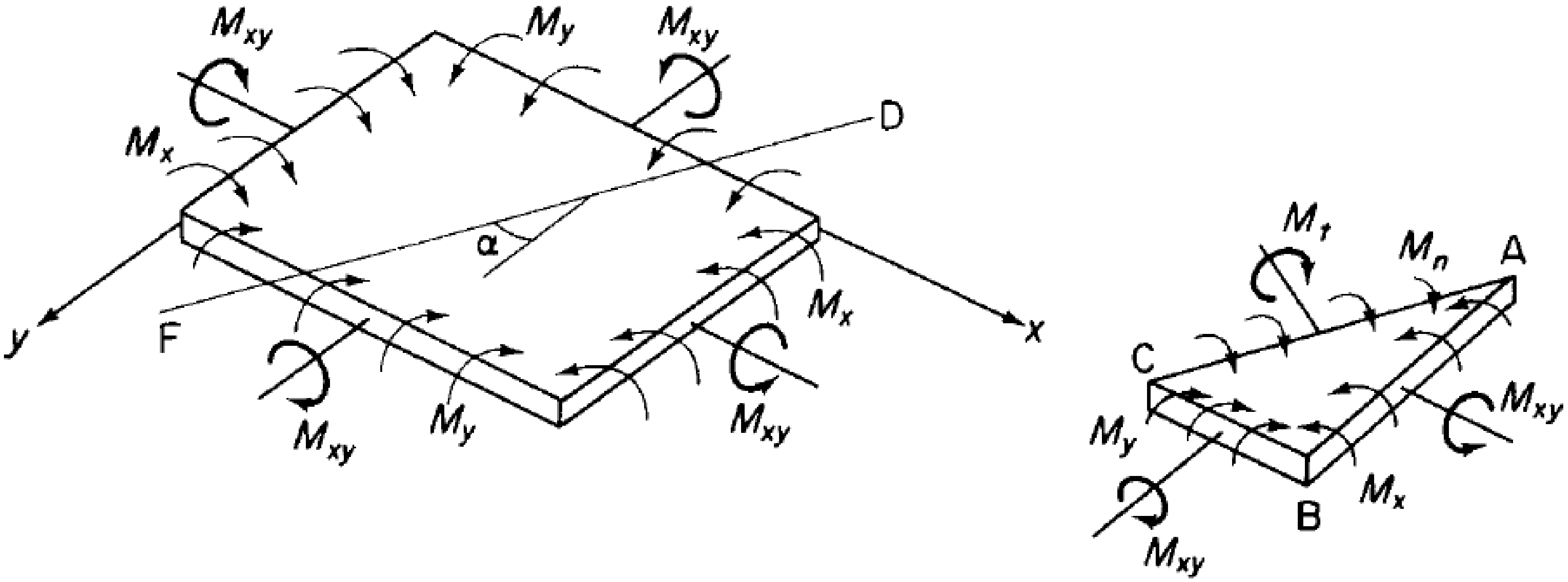
Plates subjected to bending and twisting

We represent the general moment application to the plate in terms of M_x , M_y , M_{xy} due to complementary shear stresses.



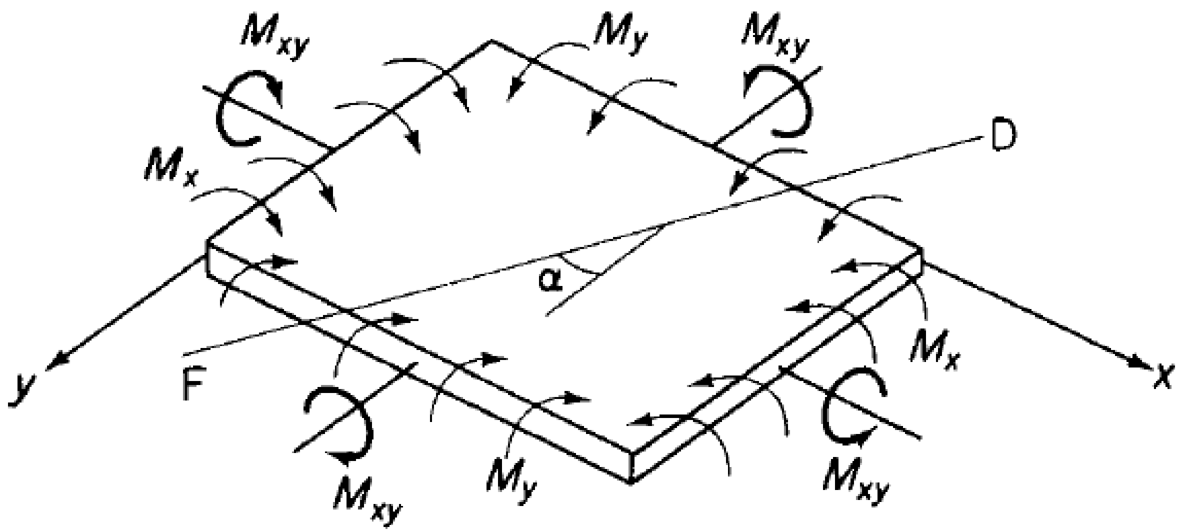
Plates subjected to bending and twisting

Let’s consider an arbitrary plane:



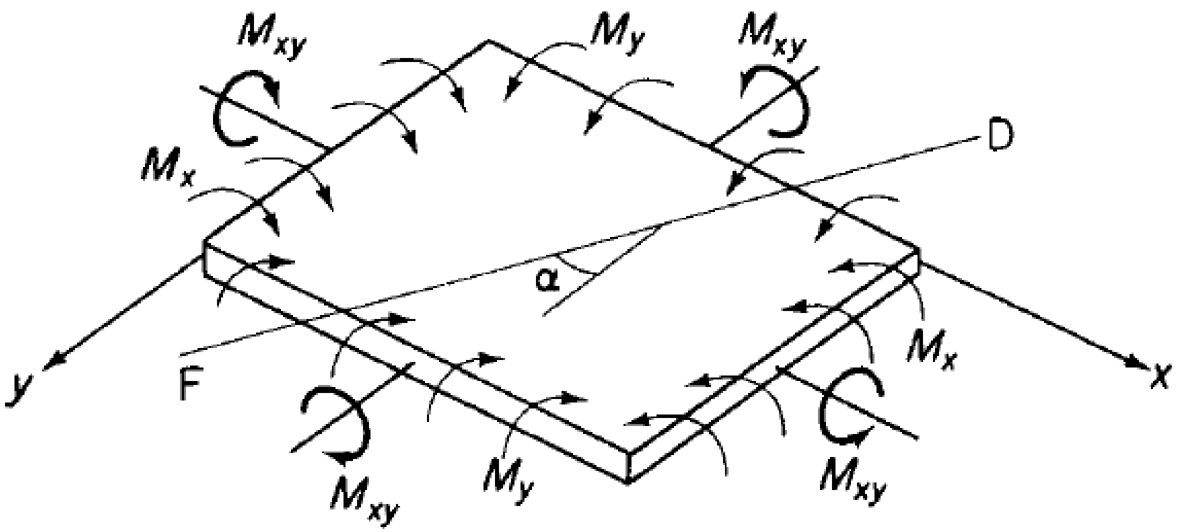
Plates subjected to bending and twisting

In the arbitrary plane FD M_x, M_y, M_{xy} will produce tangential and normal moments M_t and M_n.



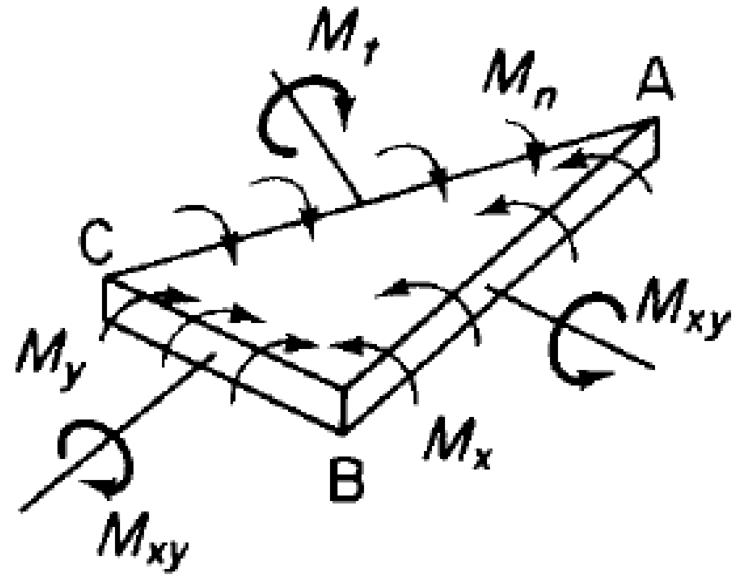
Plates subjected to bending and twisting

We may express these moments intensities in terms of M_x, M_y, M_{xy}.



Plates subjected to bending and twisting

Thus, for equilibrium of the triangular element ABC, in a plane perpendicular to AC:

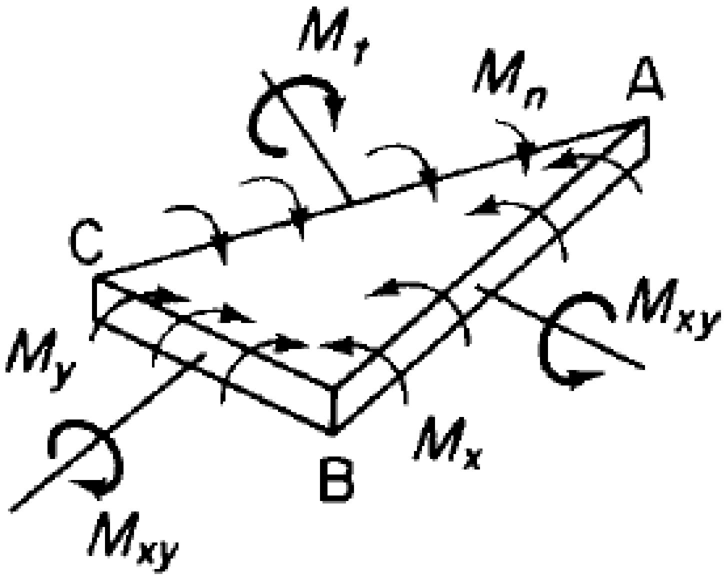


$$M_n AC = M_x AB cos\alpha + M_y BC sin\alpha - M_{xy} AB sin\alpha - M_{xy} BC cos\alpha$$

$$M_n = M_x \cos^2\alpha + M_y \sin^2\alpha - M_{xy}\, sin\, 2\alpha$$

Plates subjected to bending and twisting

Similarly for equilibrium in a plane parallel to AC:



$$M_t AC = M_x AB sin\alpha - M_y BC cos\alpha + M_{xy} AB cos\alpha - M_{xy} BC sin\alpha$$

$$M_t = \frac{M_x - M_y}{2} \sin 2\alpha + M_{xy} \cos 2\alpha$$

Plates subjected to bending and twisting

We observe that there are two values of α, differing by 90° and given by:

$$tan\,2\alpha = -\frac{2M_{xy}}{M_x - M_y}$$

For which $M_t = 0$, leaving **normal moments** of intensity M_n on **two mutually perpendicular planes**.

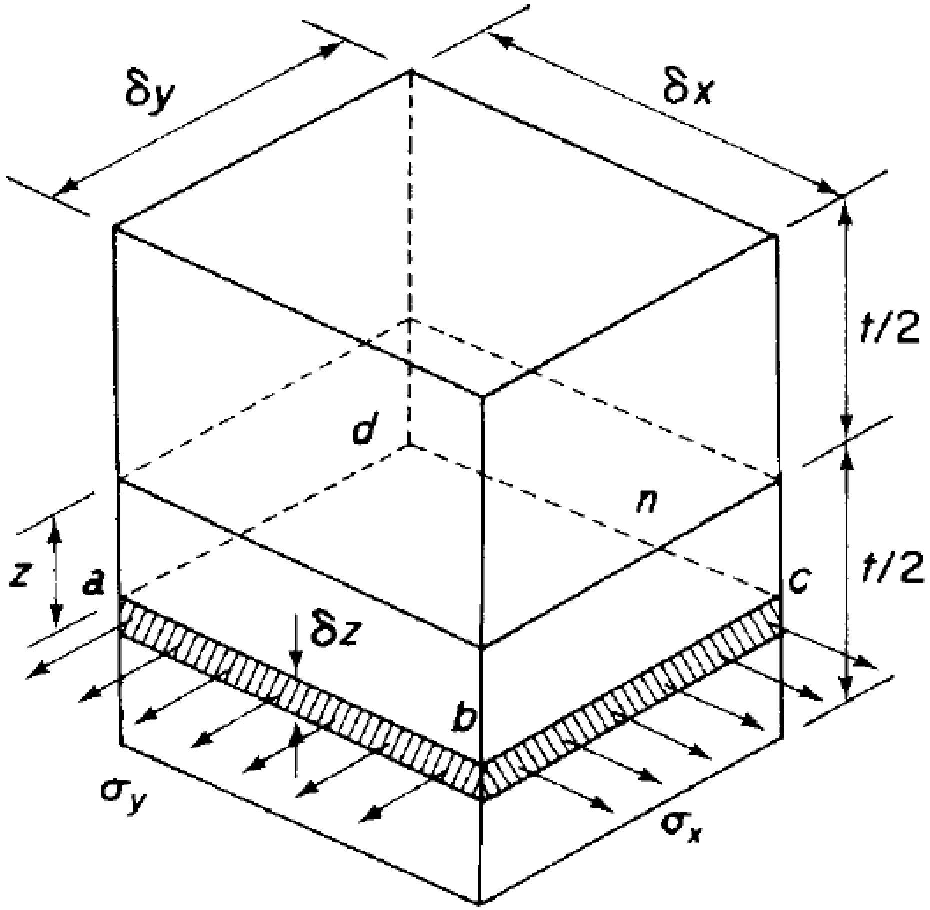
These moments are named **principal moments** and their corresponding curvatures, **principal curvatures**.

Plates subjected to bending and twisting

- Let relate the twisting moment M_{xy} to w

- M_{xy} is resisted by a system of horizontal complementary shear stresses.

- Consider an element of the plate, the shear stresses on a lamina of the element are τ_{xy}



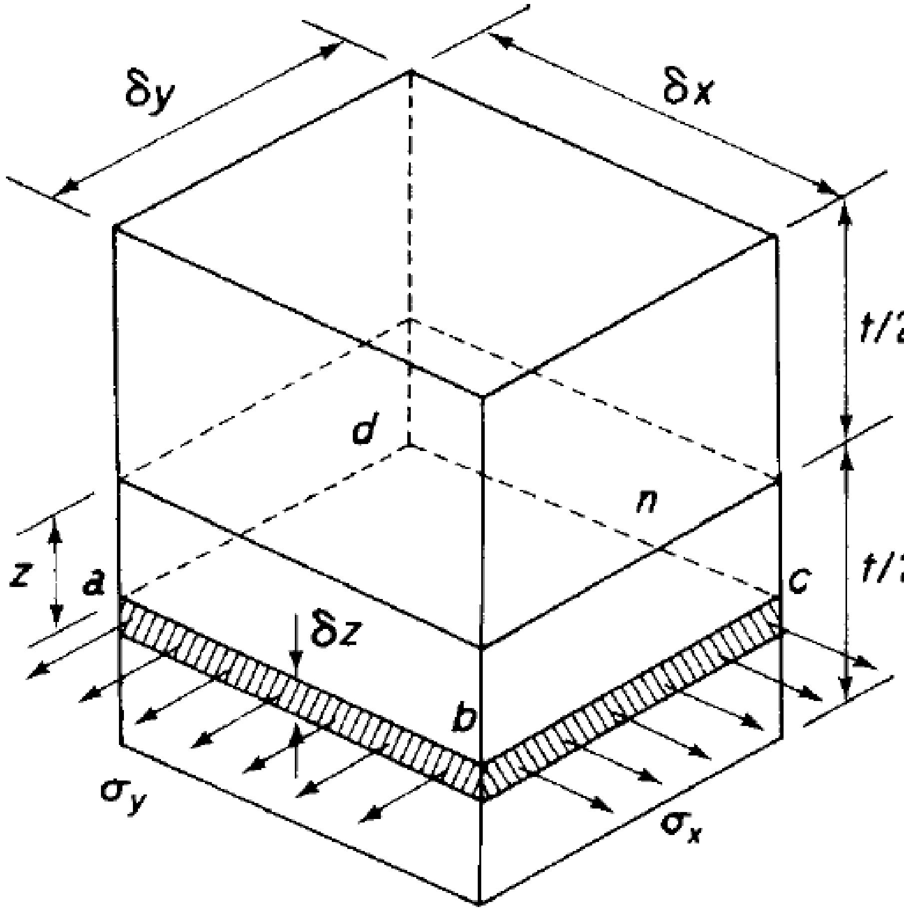
Plates subjected to bending and twisting

On the face ABCD:

$$M_{xy}\delta y = -\int_{-t/2}^{t/2} \tau_{xy}\delta y z\, dz$$

On the face ADFE:

$$M_{xy}\delta x = -\int_{-t/2}^{t/2} \tau_{xy}\delta x z\, dz$$



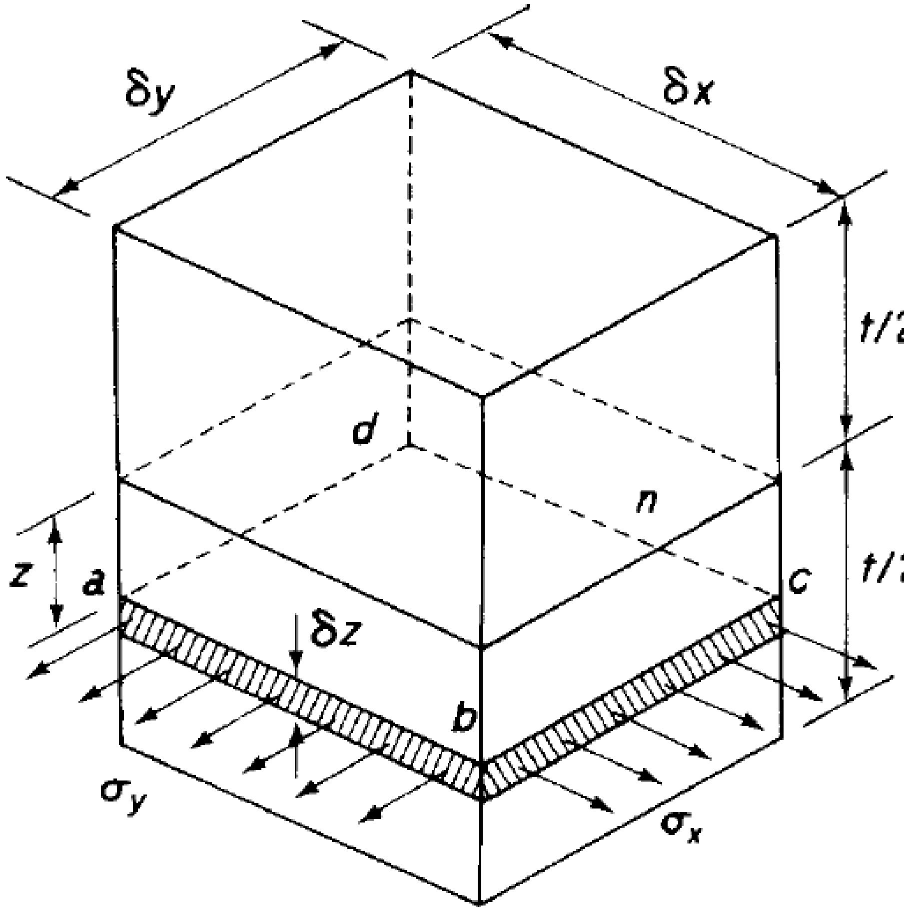
Plates subjected to bending and twisting

Giving:

$$M_{xy} = -\int_{-t/2}^{t/2} \tau_{xy} z \, dz$$

That, in terms of shear strain γ_{xy} and modulus of rigidity G :

$$M_{xy} = -G \int_{-t/2}^{t/2} \gamma_{xy} z \, dz$$



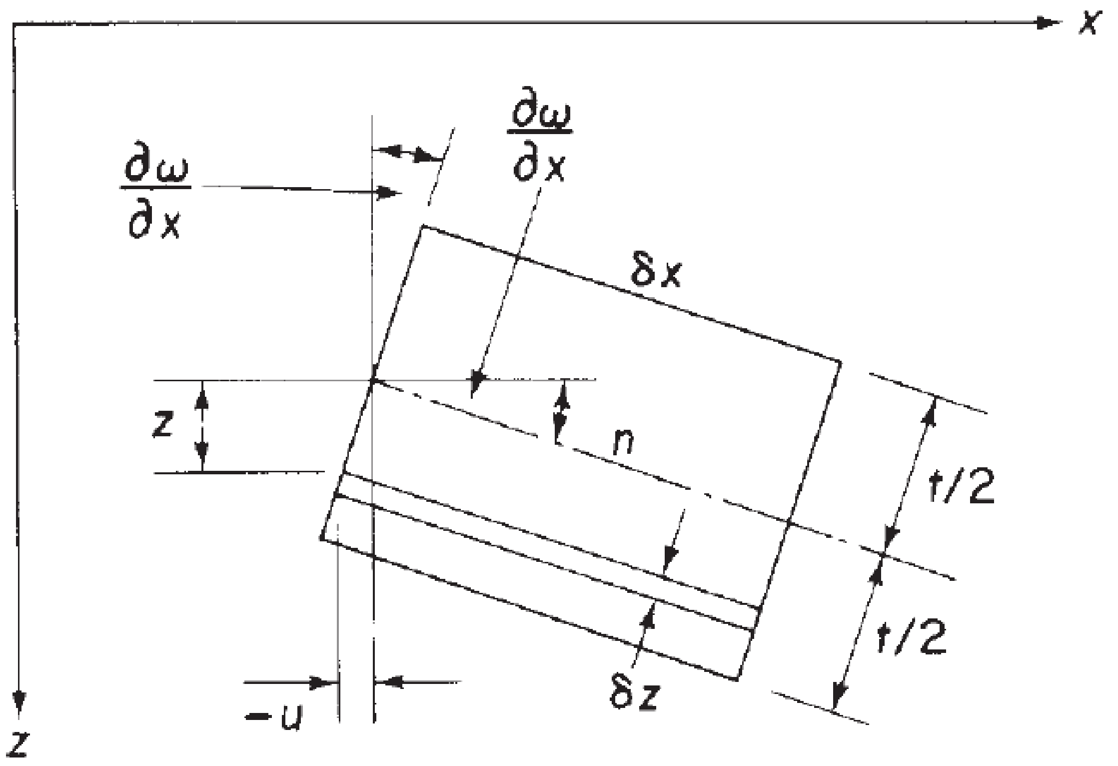
Plates subjected to bending and twisting

Then the shear strain is defined by:

$$\gamma_{xy} = -2z\frac{\partial^2 w}{\partial x \partial y}$$

Therefore:

$$M_{xy} = G\int_{-t/2}^{t/2} 2z^2 \frac{\partial^2 w}{\partial x \partial y} dz$$



Plates subjected to bending and twisting

Twisting moment results in the following:

$$M_{xy} = \frac{Gt^3}{6}\frac{\partial^2 w}{\partial x \partial y}$$

Replacing $G = \frac{E}{2(1-v^2)}$:

$$M_{xy} = \frac{Et^3}{12(1-v^2)}\frac{\partial^2 w}{\partial x \partial y}$$

Plates subjected to bending and twisting

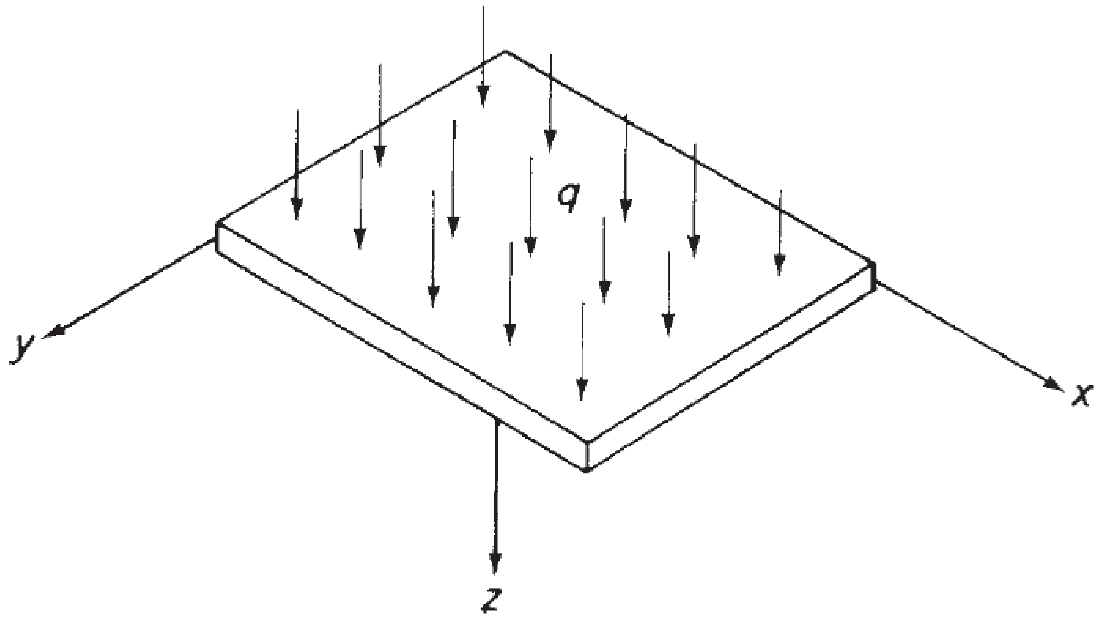
Replacing D and multiplying by $(1 - v)$, thus:

$$\boldsymbol{M_{xy} = D(1 - v)\frac{\partial^2 w}{\partial x \partial y}}$$

Plates subjected to a distributed transverse load

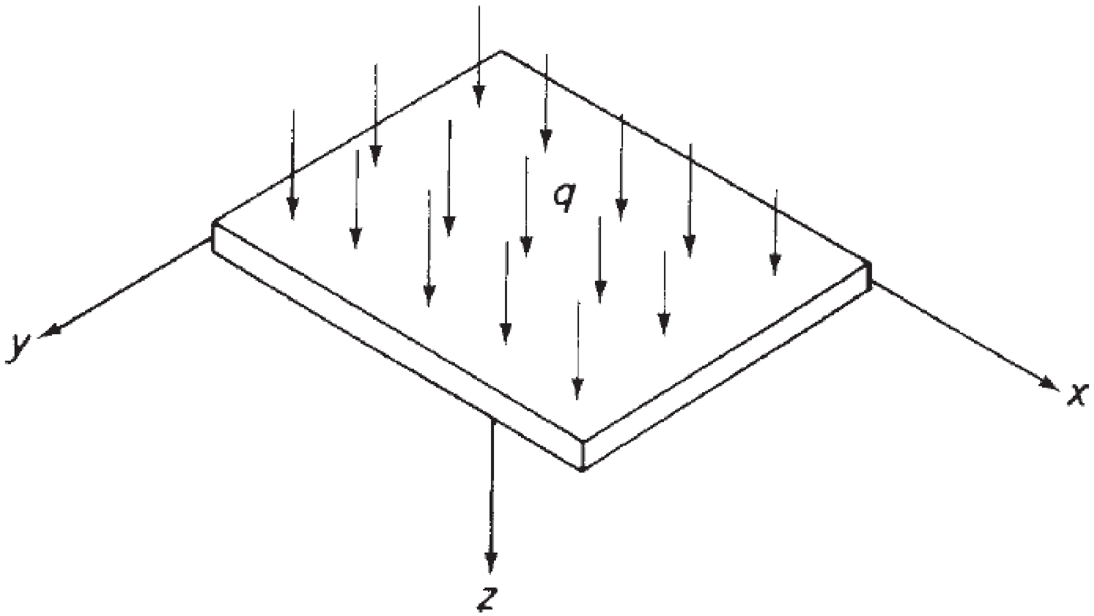
The relationship between bending and twisting moments and plate deflection will be used **in order to solve a thin plate supporting a distributed load of intensity q per unit area**.

The distributed load of intensity q per unit of area will vary over the surface, then is function of x and y.

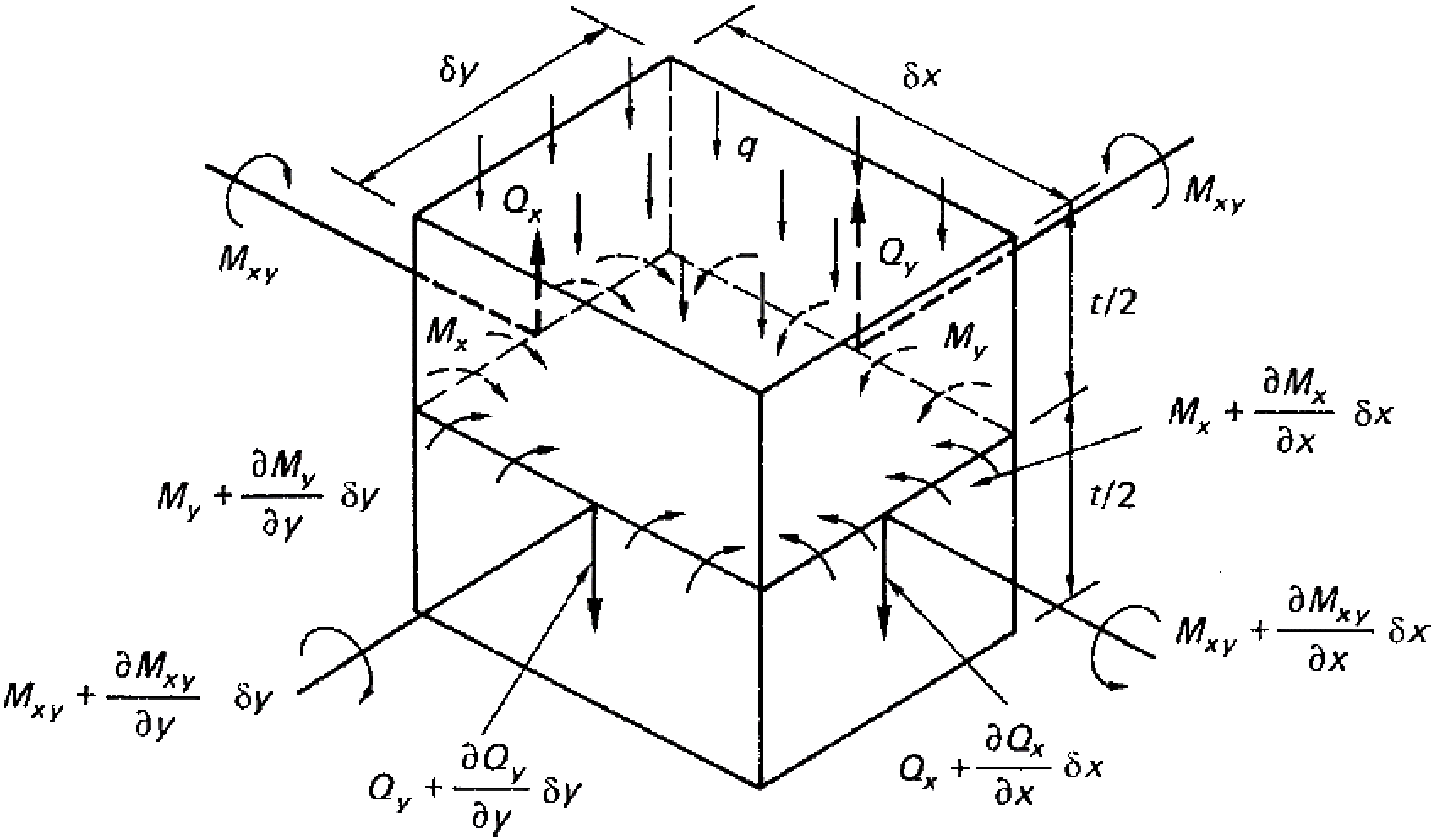


Plates subjected to a distributed transverse load

The element of plate supports bending and twisting moments, vertical shear forces Q_x and Q_y per unit of length on faces perpendicular to the axis and y.



Plates subjected to a distributed transverse load



Plates subjected to a distributed transverse load

Shear forces $Q_x \delta y \; and \; Q_y \delta x$ are assumed to act through the centroid of the faces of the element:

$$Q_x = \int_{-t/2}^{t/2} \tau_{xz} dz$$

$$Q_y = \int_{-t/2}^{t/2} \tau_{yz} dz$$

Plates subjected to a distributed transverse load

Shear forces $Q_x \delta y \; and \; Q_y \delta x$ are assumed to act through the centroid of the faces of the element:

$$Q_x = \frac{\partial M_x}{\partial x} - \frac{\partial M_{xy}}{\partial y} = -D \frac{\partial}{\partial x}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

$$Q_y = \frac{\partial M_y}{\partial y} - \frac{\partial M_{xy}}{\partial x} = -D \frac{\partial}{\partial y}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

Plates subjected to a distributed transverse load

After relating the moments and shear forces, we can express the general transverse distribution in terms of w as:

$$\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) = \frac{q}{D}$$

or

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^2 w = \frac{q}{D}$$

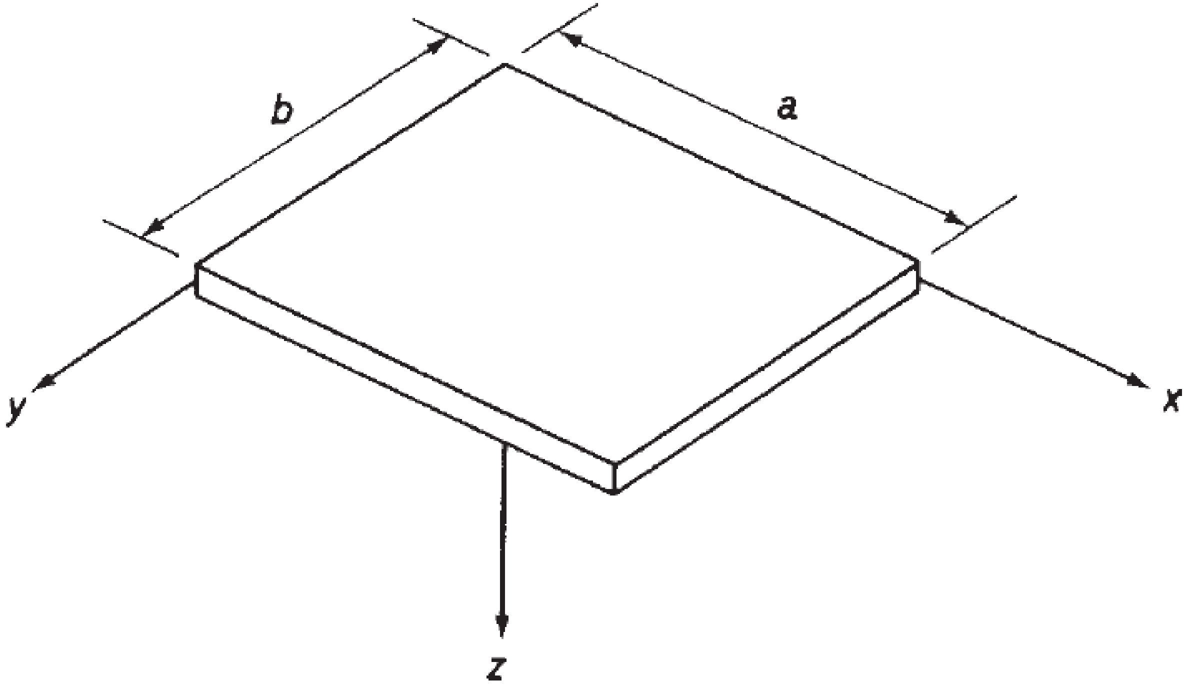
The partial derivative inside the parenthesis is known as Laplace operator:

$$(\nabla^2)^2 w = \frac{q}{D}$$

Boundary conditions

Before the solution of the general transverse load distribution equation, we can define three boundary conditions:

- The simply supported edge
- The built-in edge
- The free edge



The simply supported edge

The edge $x = 0$, is free to rotate but not to deflect and the bending moment along this edge is zero:

$$\left(M_{xy}\right)_{x=0} = -D\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)_{x=0}$$

The deflection along this edge is zero:

$$(w)_{x=0} = 0$$

Thus,

$$(w)_{x=0} = 0, \qquad \left(\frac{\partial^2 w}{\partial x^2}\right)_{x=0} = 0$$

The built-in edge

If the edge $x = 0$ is firmly clamped so that it can neither rotate nor deflect, then:

$$(w)_{x=0} = 0, \qquad \left(\frac{\partial w}{\partial x}\right)_{x=0} = 0$$

The free edge

Along the free edge:

$$(M_x)_{x=0} = 0, \qquad \left(M_{xy}\right)_{x=0} = 0, \qquad (Q_x)_{x=0} = 0$$

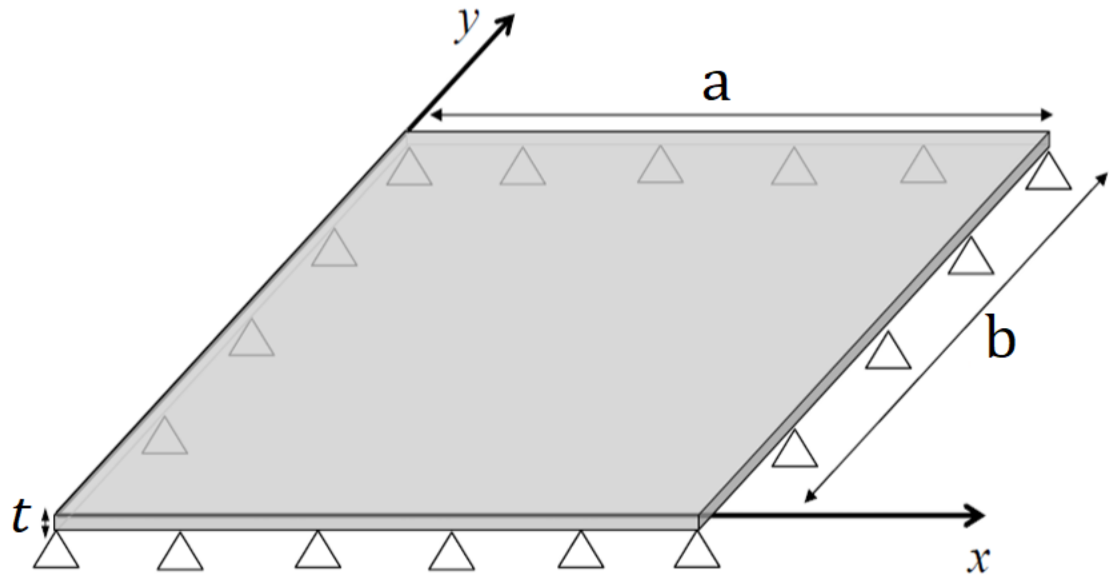
In this case:

$$\left(Q_x - \frac{\partial M_{xy}}{\partial y}\right)_{x=0} = 0$$

No shear forces

Solution for a simply supported plate

Having discussed various types of boundary conditions we shall proceed to obtain the solution for the relatively simple case of a thin rectangular plate of dimensions $a \times b$, **simply supported** along each of its four edges and carrying a distributed load $q(x, y)$



Solution for a simply supported plate

The following equation is satisfied by representing the **deflection *w*** as an infinite trigonometrical or Fourier series. This is the general solution for a thin rectangular plane under a transverse load:

$$w = \frac{1}{\pi^4 D} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{a_{mn}}{\left[\left(\frac{m^2}{a^2}\right) + \left(\frac{n^2}{b^2}\right)\right]^2} \sin\frac{m\pi x}{a} \sin\frac{n\pi x}{b}$$

m represents the number of half waves in x direction and n the corresponding number in the y direction.

A_{mn} are unknown coefficients which must satisfy the differential equations.

Example

The deflection function

$$w = x^2y^2 - bx^2y - axy^2 + abxy$$

is valid for a rectangular plate of sides a and b, built in on all four edges and subjected to a uniformly distributed load of intensity q. If the material of the plate has a Young's modulus E and is of thickness t, **determine the distributions of bending moment along the edges of the plate**. (Matlab)

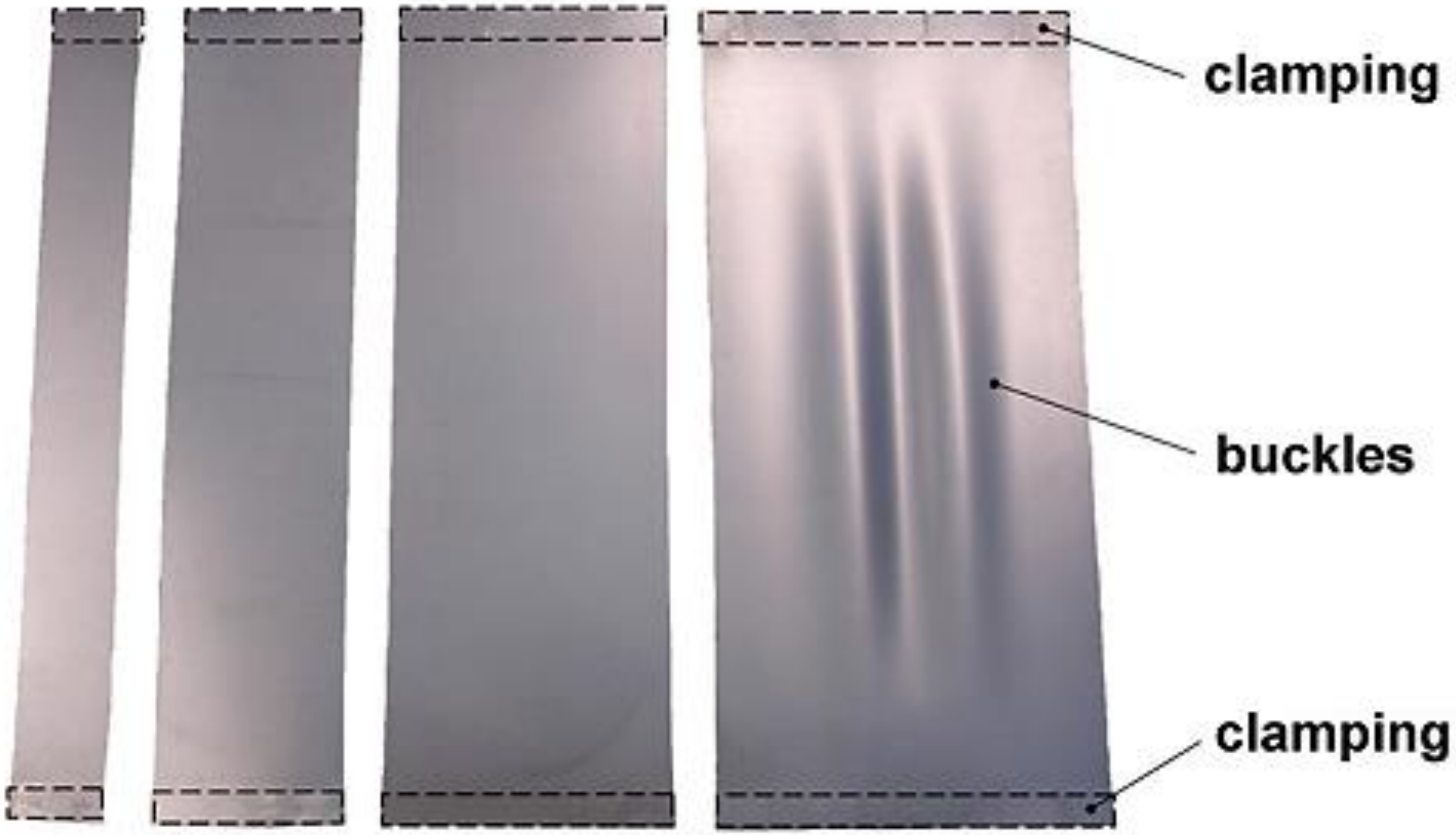
Buckling of thin plates

A thin plate may buckle in a variety of modes depending upon its dimensions, the loading and the method of support.

Usually buckling loads are much lower than those likely to cause failure in the material of the plate.

Buckling of thin plates

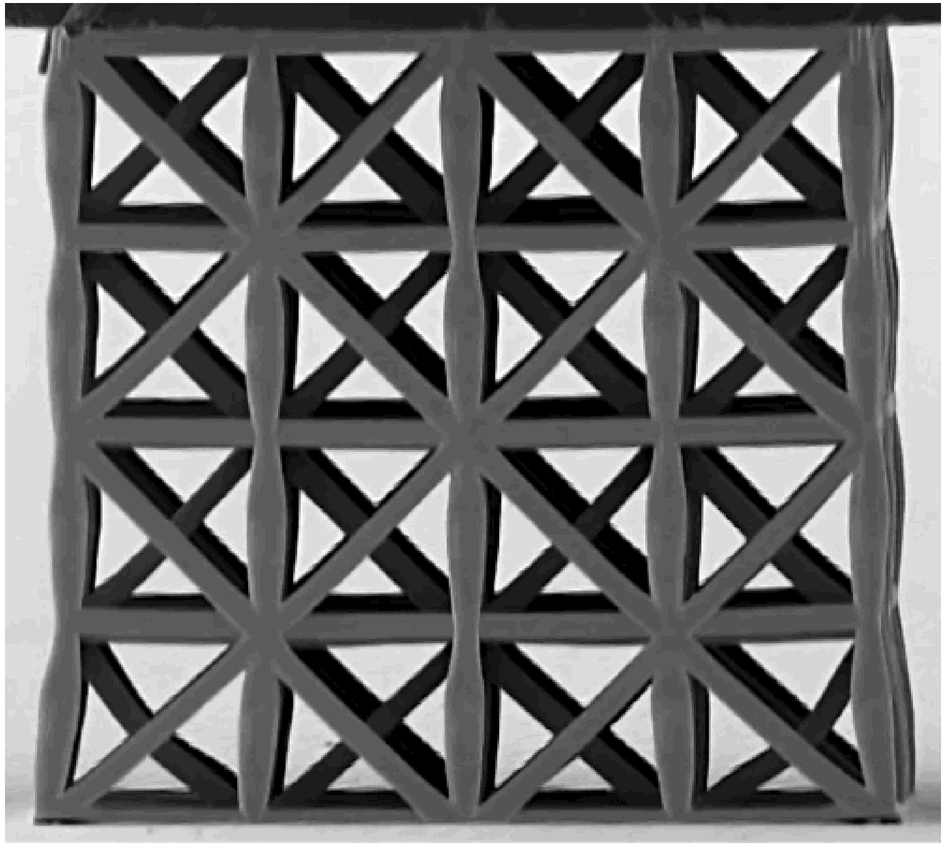
Buckling of thin plates



Buckling of thin plates

The simplest form of buckling arises when compressive loads are applied to simply supported opposite edges and the unloaded edges are free.

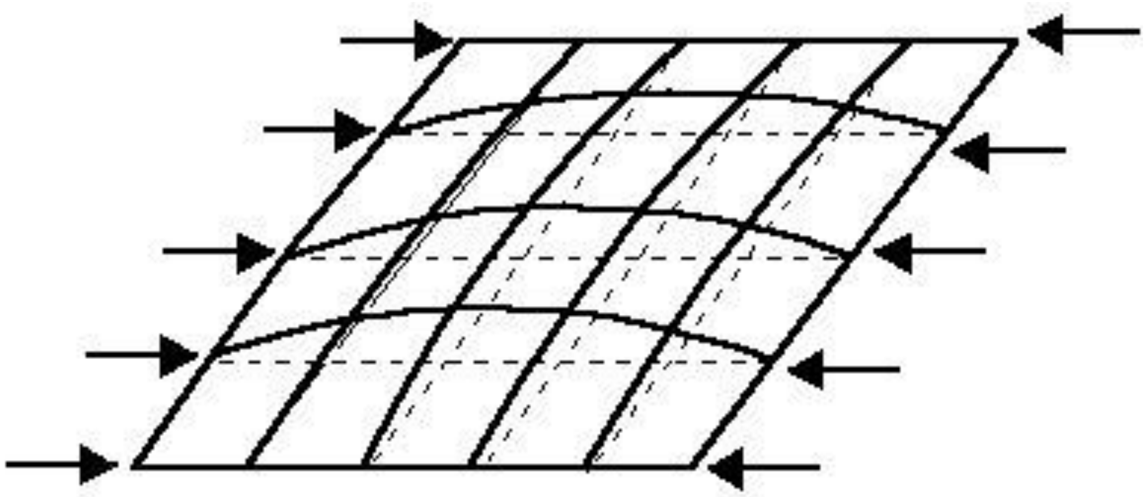
A thin plate in this configuration behaves in exactly the same way as a **pin-ended column** so that the **critical load** is that predicted by the Euler theory.



Buckling of thin plates

Once this critical load is reached the plate is **incapable of supporting** any further load.

Buckling, for such plates, takes the form of a **bulging displacement of the central region of the plate** while the parts adjacent to the supported **edges remain straight**.

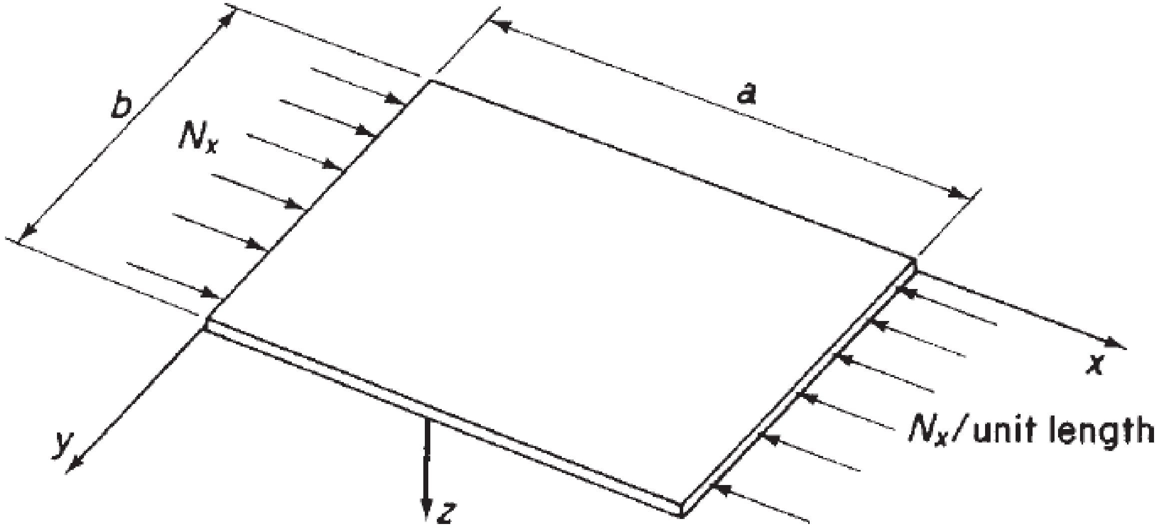


Buckling of thin plates

In general, a nontrivial solution for the in plane direct force for a simply loaded plate is given by:

$$N_{x,CR} = \pi^2 a^2 D \frac{1}{m^2}\left(\frac{m^2}{a^2} + \frac{1}{b^2}\right)^2$$

Where D is the flexural rigidity and m is the number of half waves of the plate in the x direction.



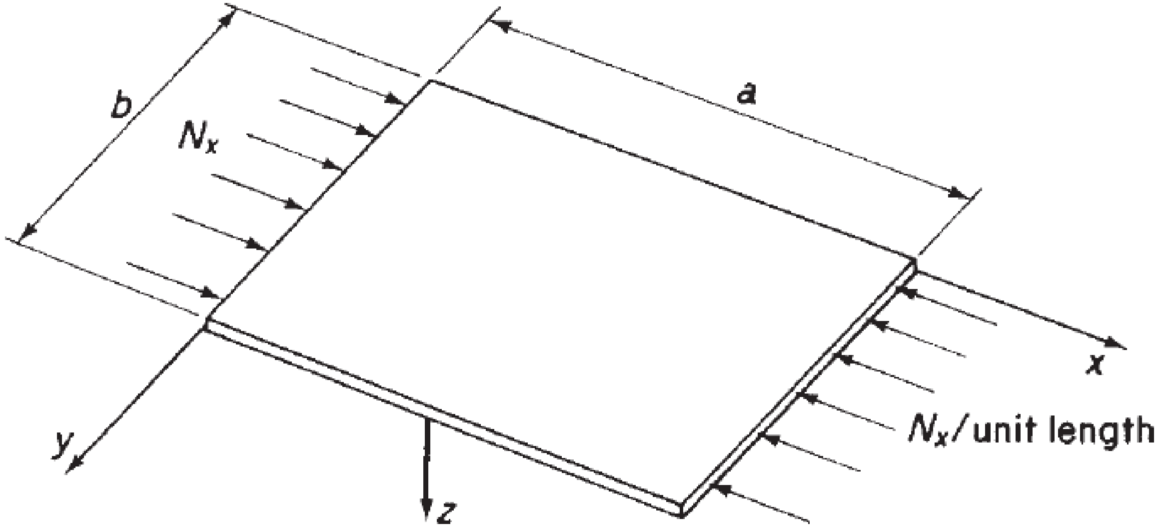
Buckling of thin plates

The previous equation can be also expressed as:

$$N_{x,CR} = \frac{k\pi^2 D}{b^2}$$

k is the simply supported plate buckling coefficient given by:

$$k = \left(\frac{mb}{a} + \frac{a}{mb}\right)^2$$

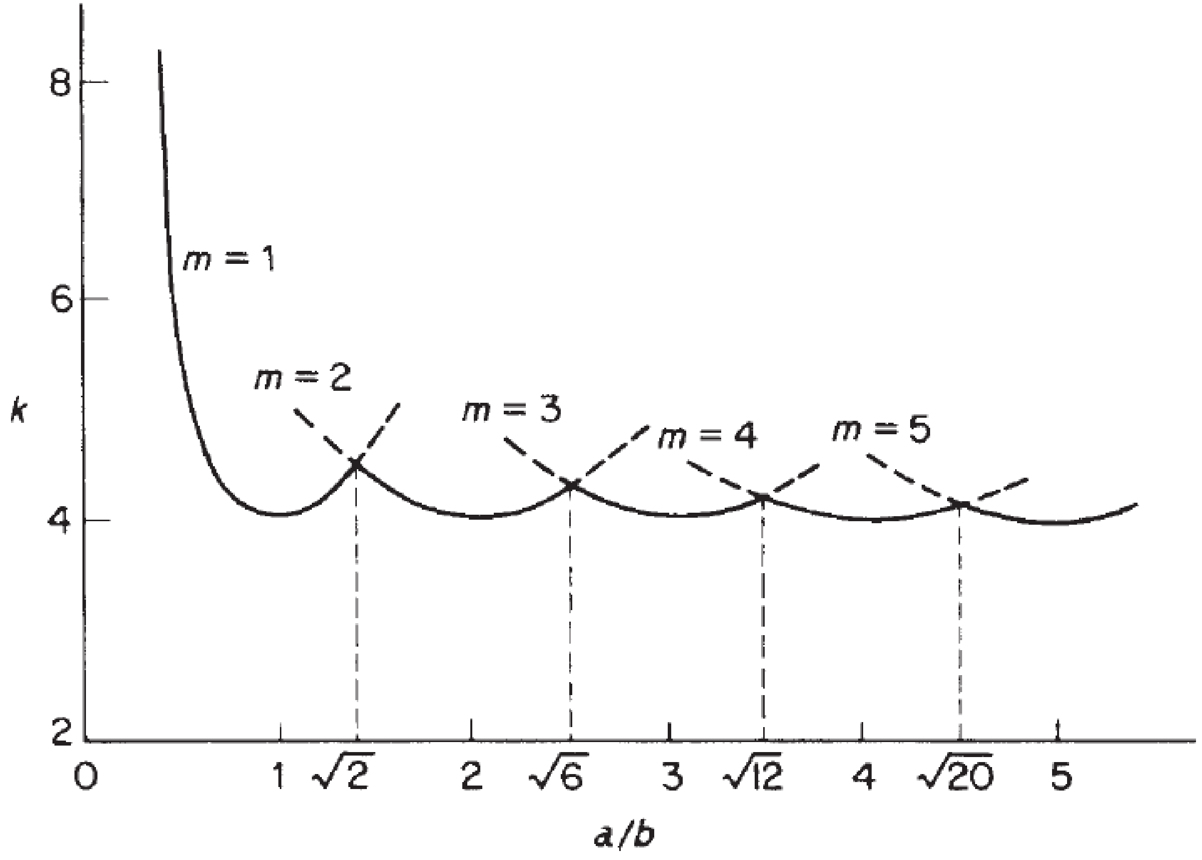


Buckling of thin plates

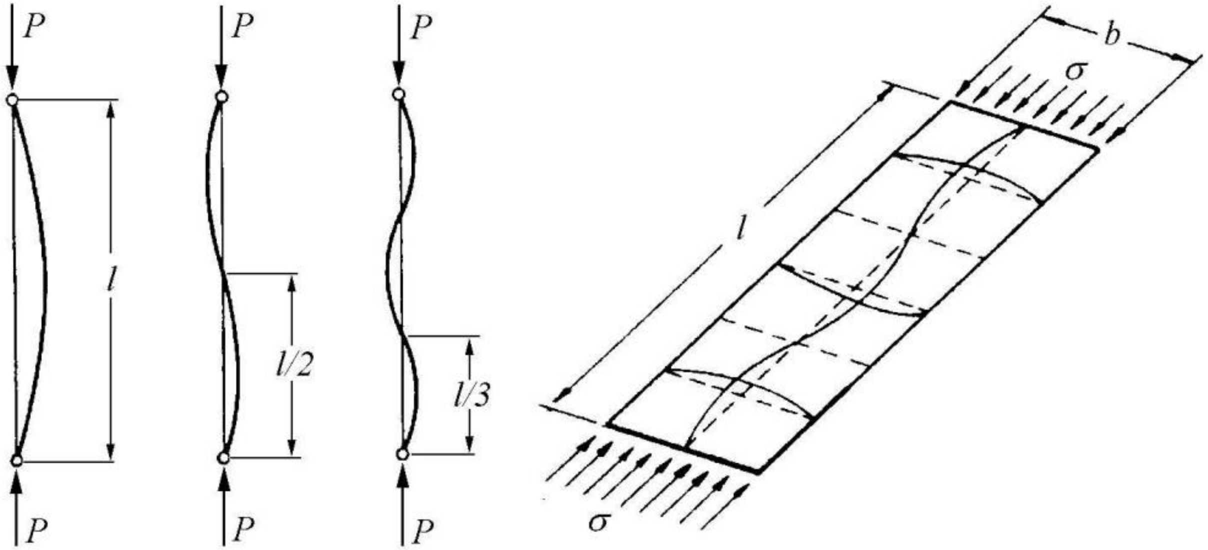
For a given value of a/b a k value can be determined for different values of m

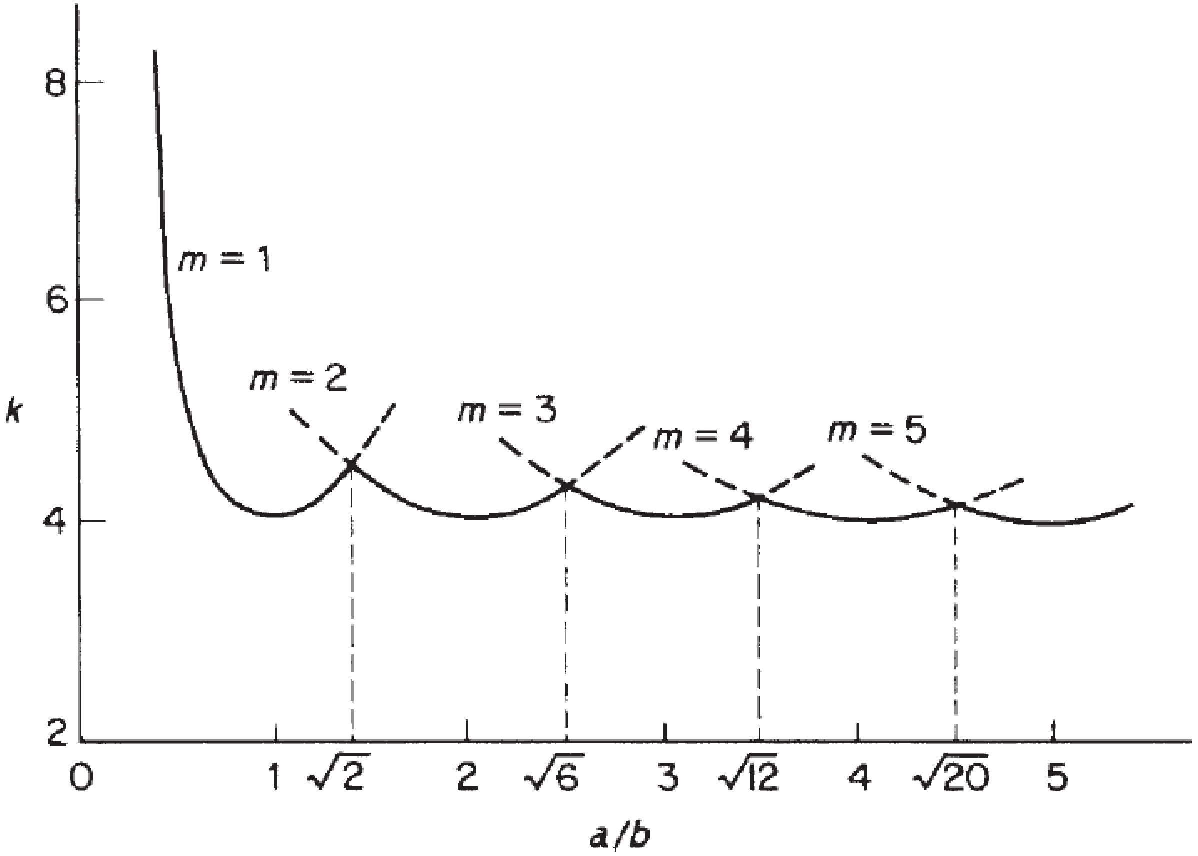
REMEMBER

m is the number of half waves of the plate in the x direction



Buckling of thin plates

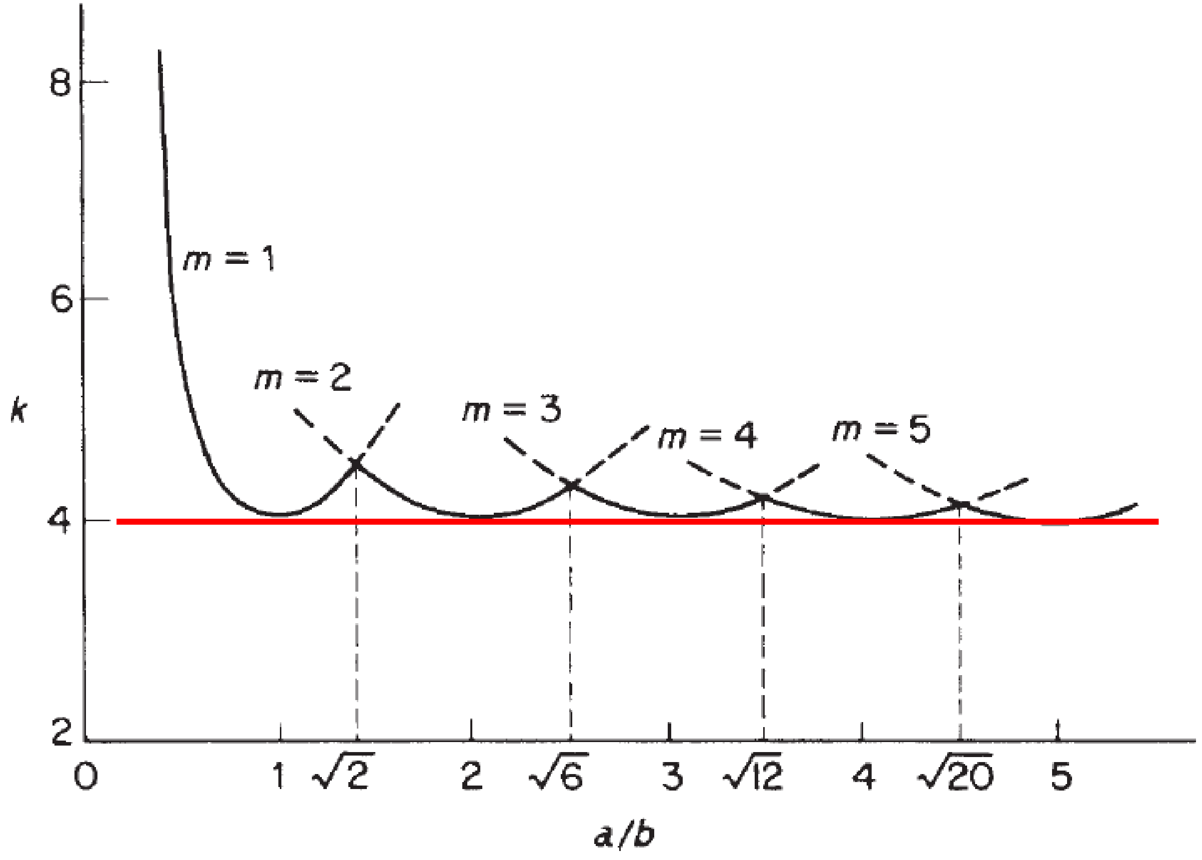




Buckling of thin plates

It can be seen that m varies with the ratio of a/b and that $\boldsymbol{k}$ **and the buckling load are minimum when** $\boldsymbol{k = 4}$.

The length becomes "irrelevant" if a/b is higher than 1.



Buckling of thin plates

If:

$$\sigma = \frac{N_{x,CR}}{t} = \frac{\frac{N}{m}}{m}$$

For a given value of a/b the critical stress σ_{cr} is found from:

$$\sigma_{CR} = \frac{k\pi^2 E}{12(1-v^2)}\left(\frac{t}{b}\right)^2$$

Where, b/t is the plate slenderness and k the plate buckling coefficient.

Buckling of thin plates

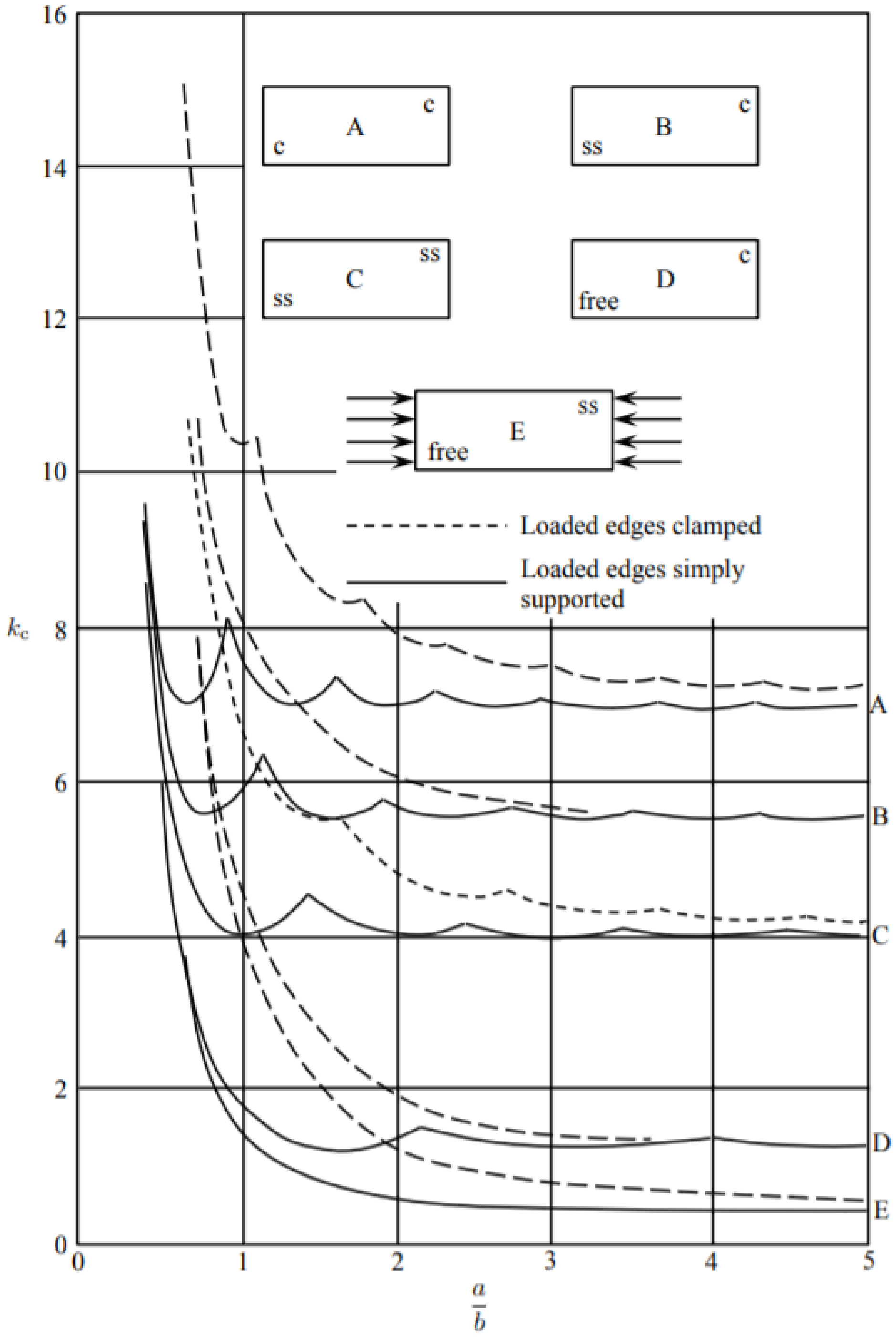
From

$$\sigma_{CR} = \frac{k\pi^2 E}{12(1-v^2)}\left(\frac{t}{b}\right)^2$$

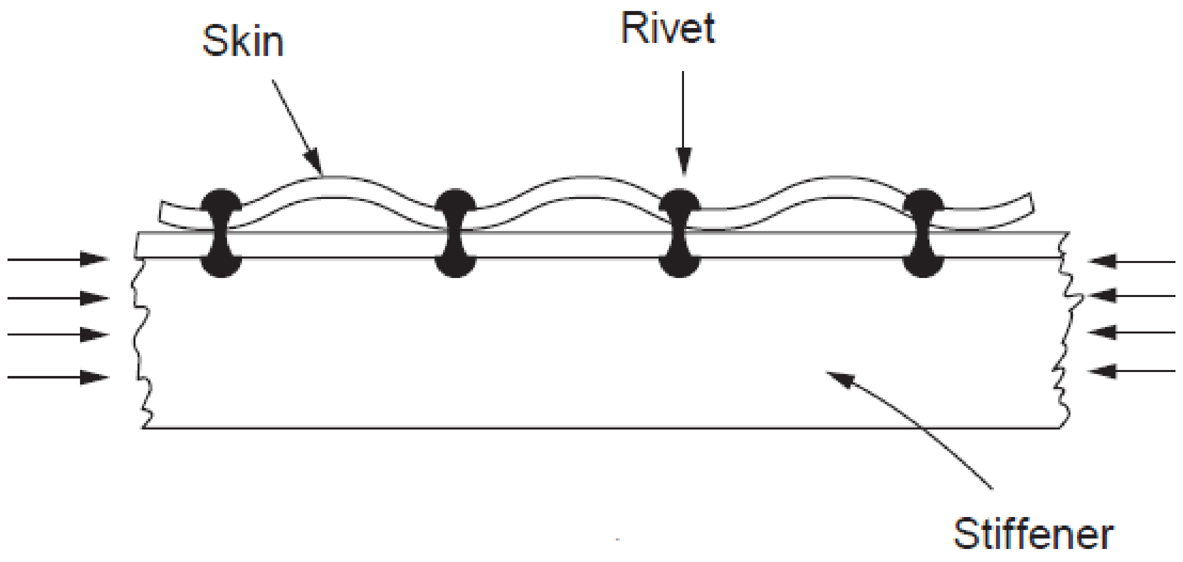
it can be done a good approximation given by:

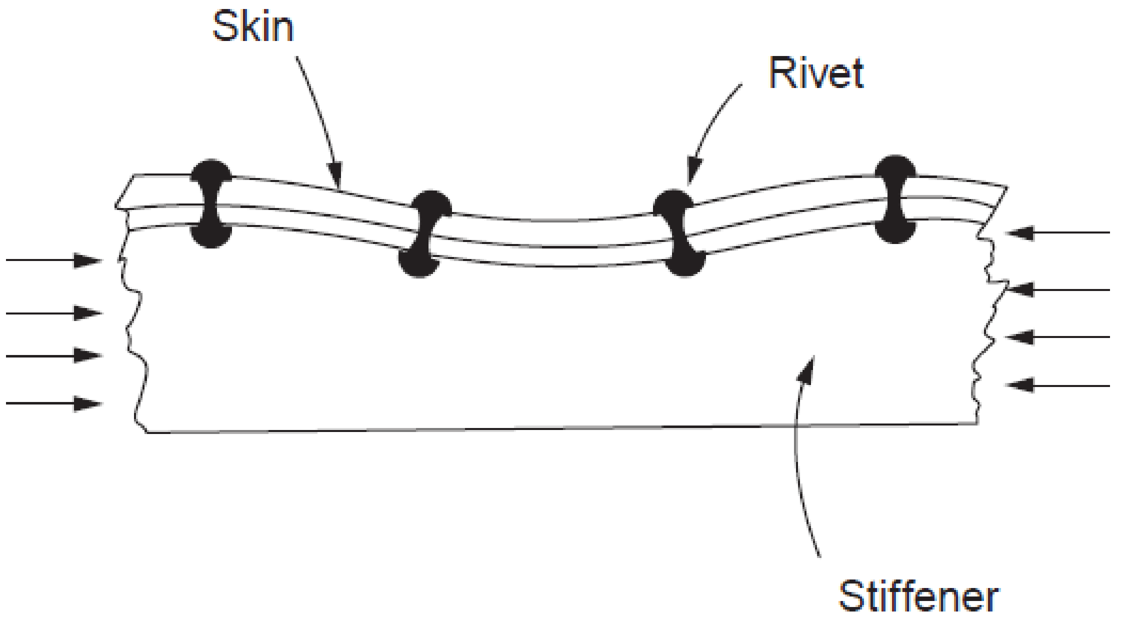
$$\sigma_{CR} = KE\left(\frac{t}{b}\right)^2$$

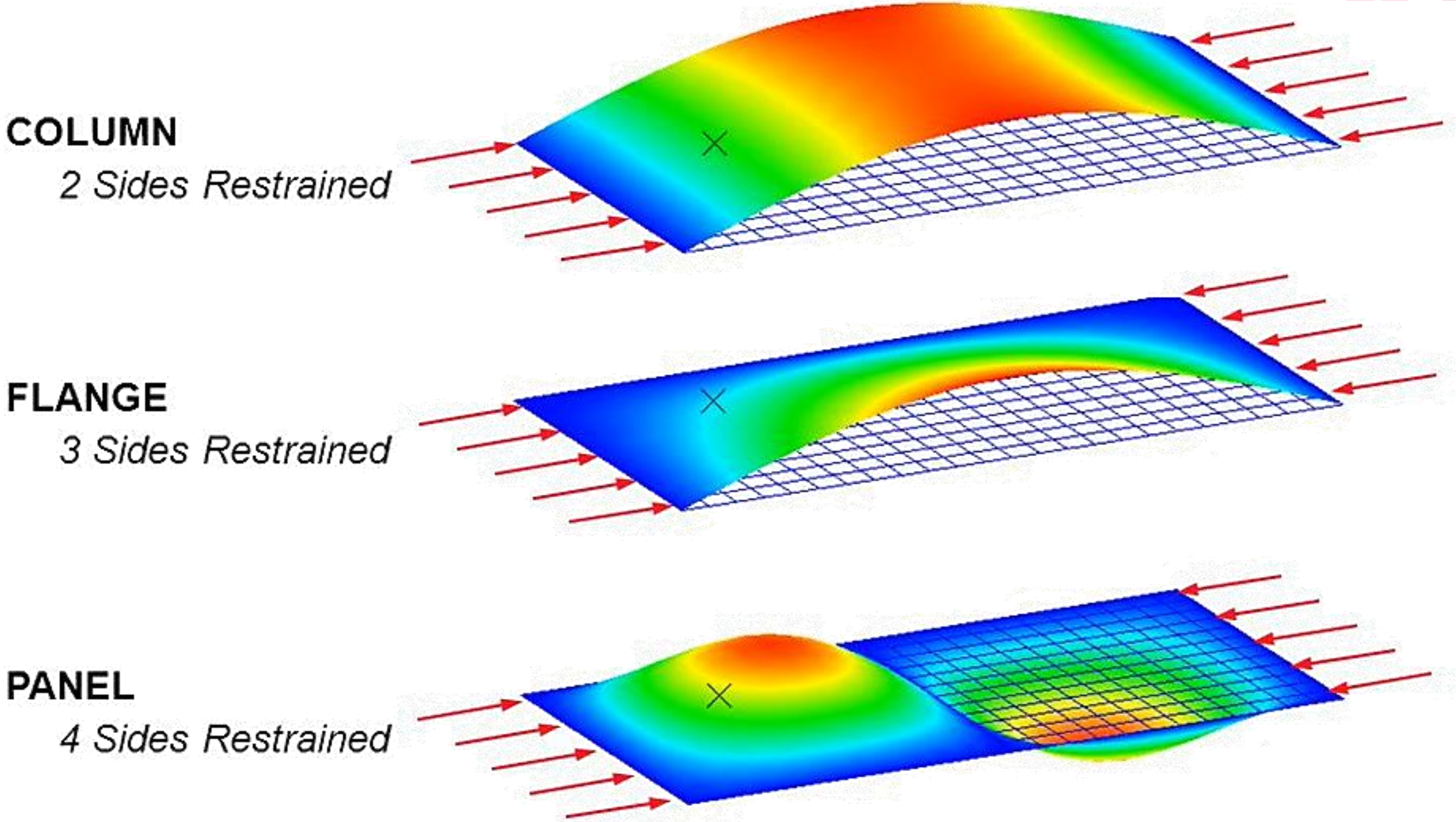
- Remember, b is always the perpendicular direction of the edge of the load
- Where k is a non dimensional coefficient constant that depend upon conditions of edge restrain and shape of the plate, k_c compression, k_s shear.



16
14
12
10
8
6
4
2
0
k_c
0
1
2
3
4
5
$\frac{a}{b}$
A
c
c
B
c
ss
C
ss
ss
D
c
free
E
ss
free
Loaded edges clamped
Loaded edges simply supported
A
B
C
D
E

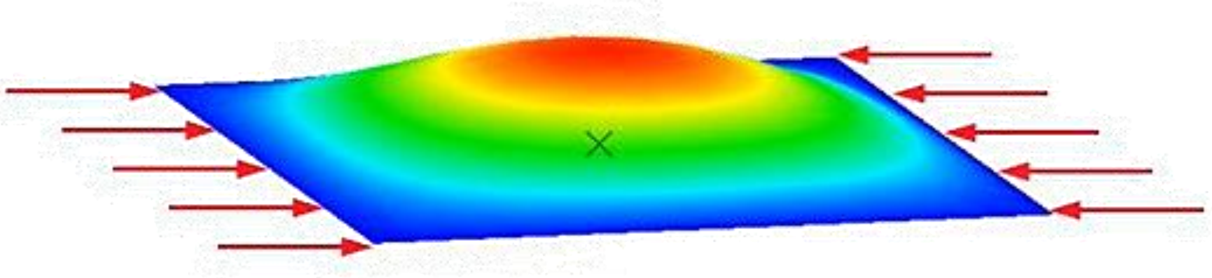

Skin
Rivet
Stiffener


Skin
Rivet
Stiffener


COLUMN
2 Sides Restrained
FLANGE
3 Sides Restrained
PANEL
4 Sides Restrained

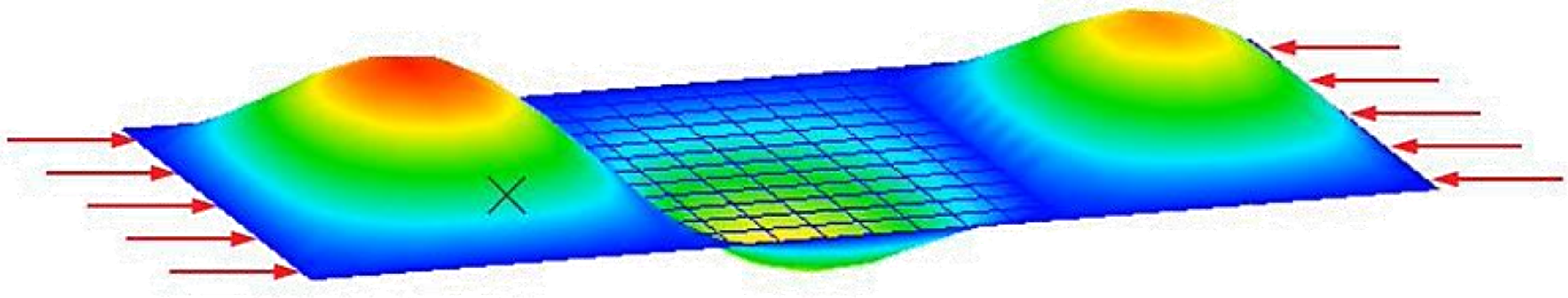
LOW ASPECT RATIO

Single Buckling Wave



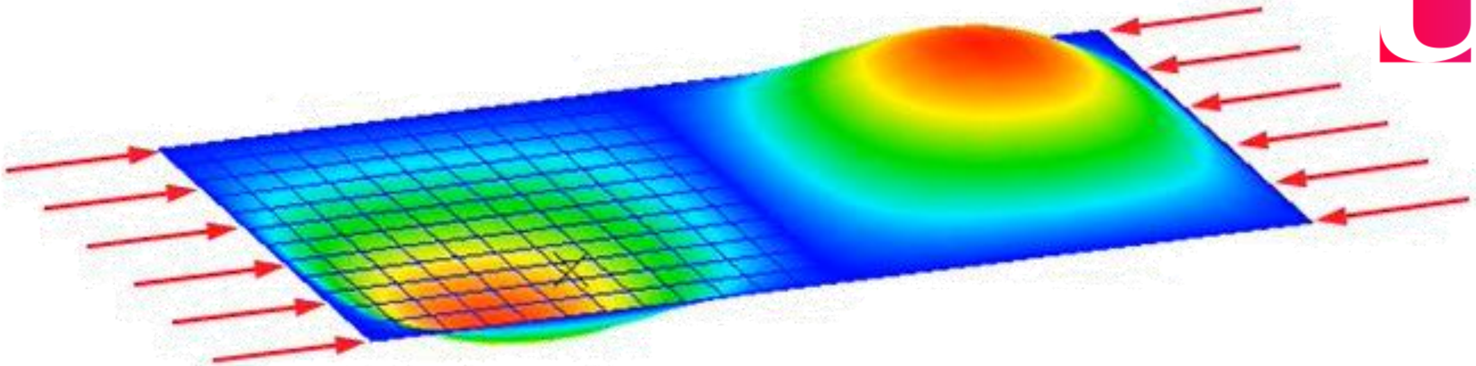
HIGH ASPECT RATIO

Multiple Buckling Waves



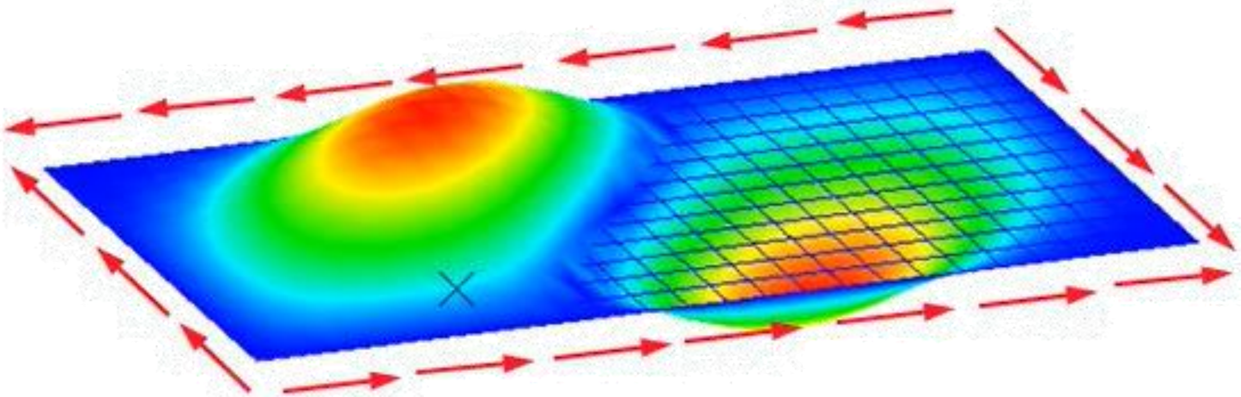
COMPRESSION

Symmetric Buckling Waves



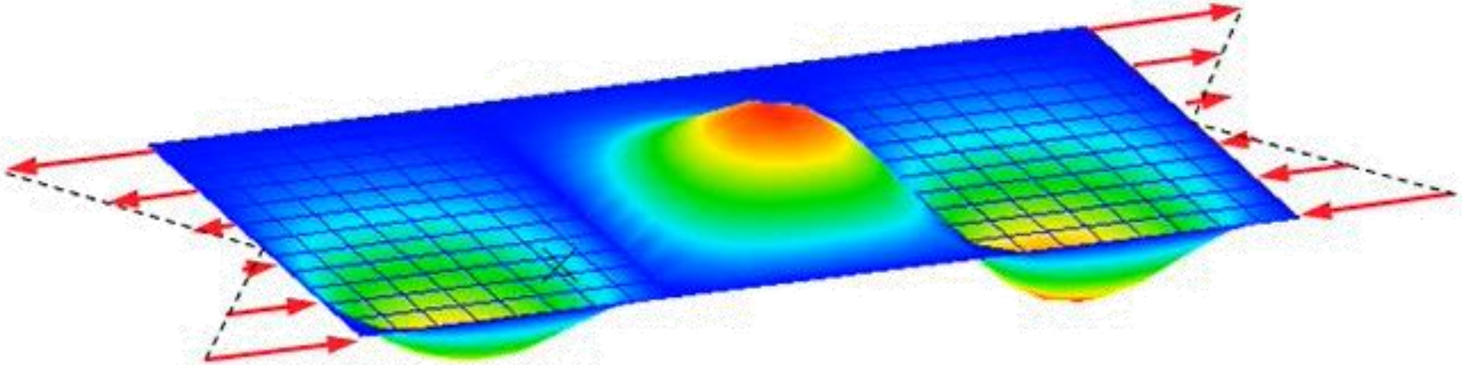
SHEAR

Skewed Buckling Waves



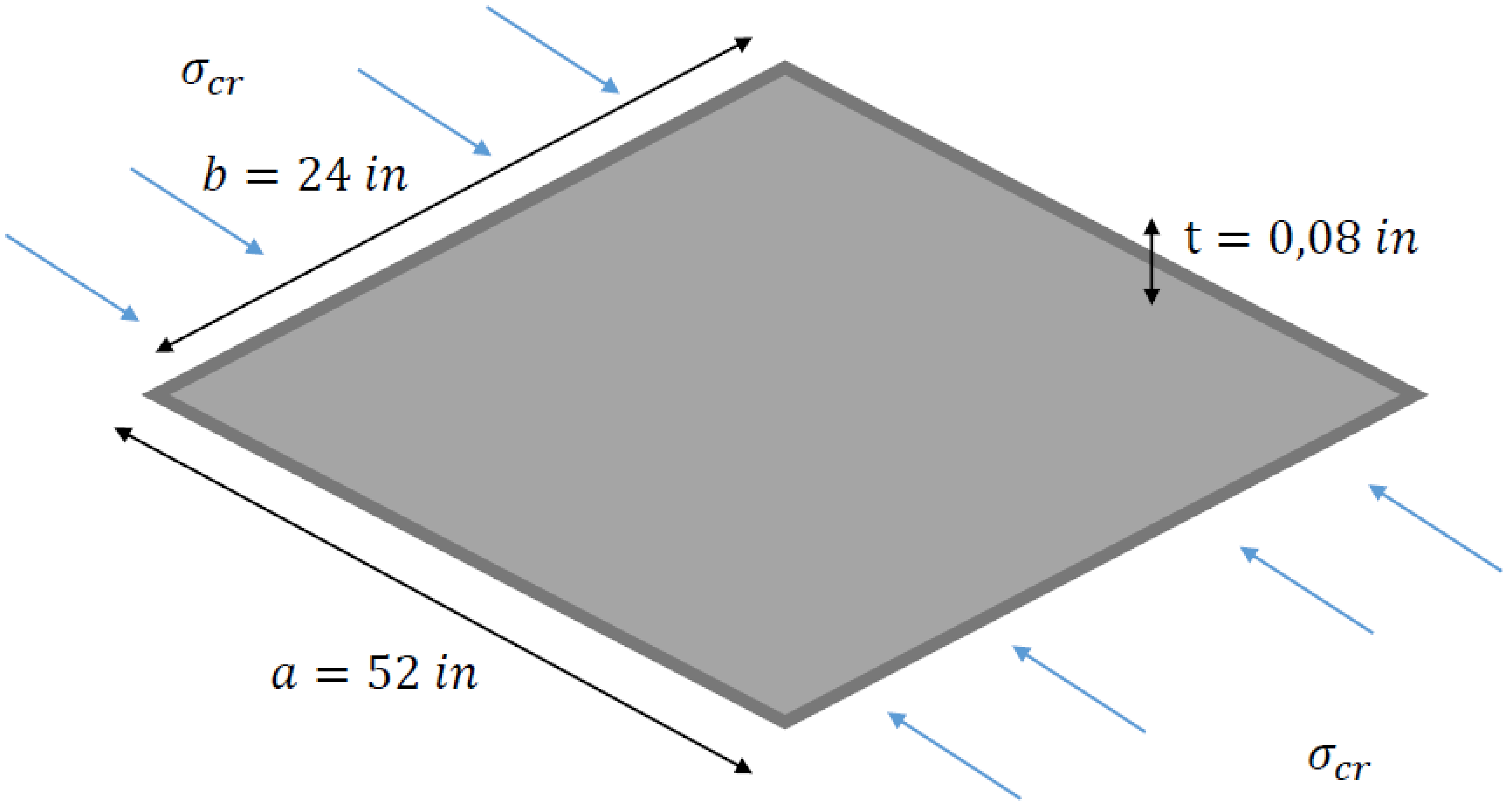
“BENDING”

Offset Buckling Waves



Example

Find σ_{CR} of the following simply supported plate



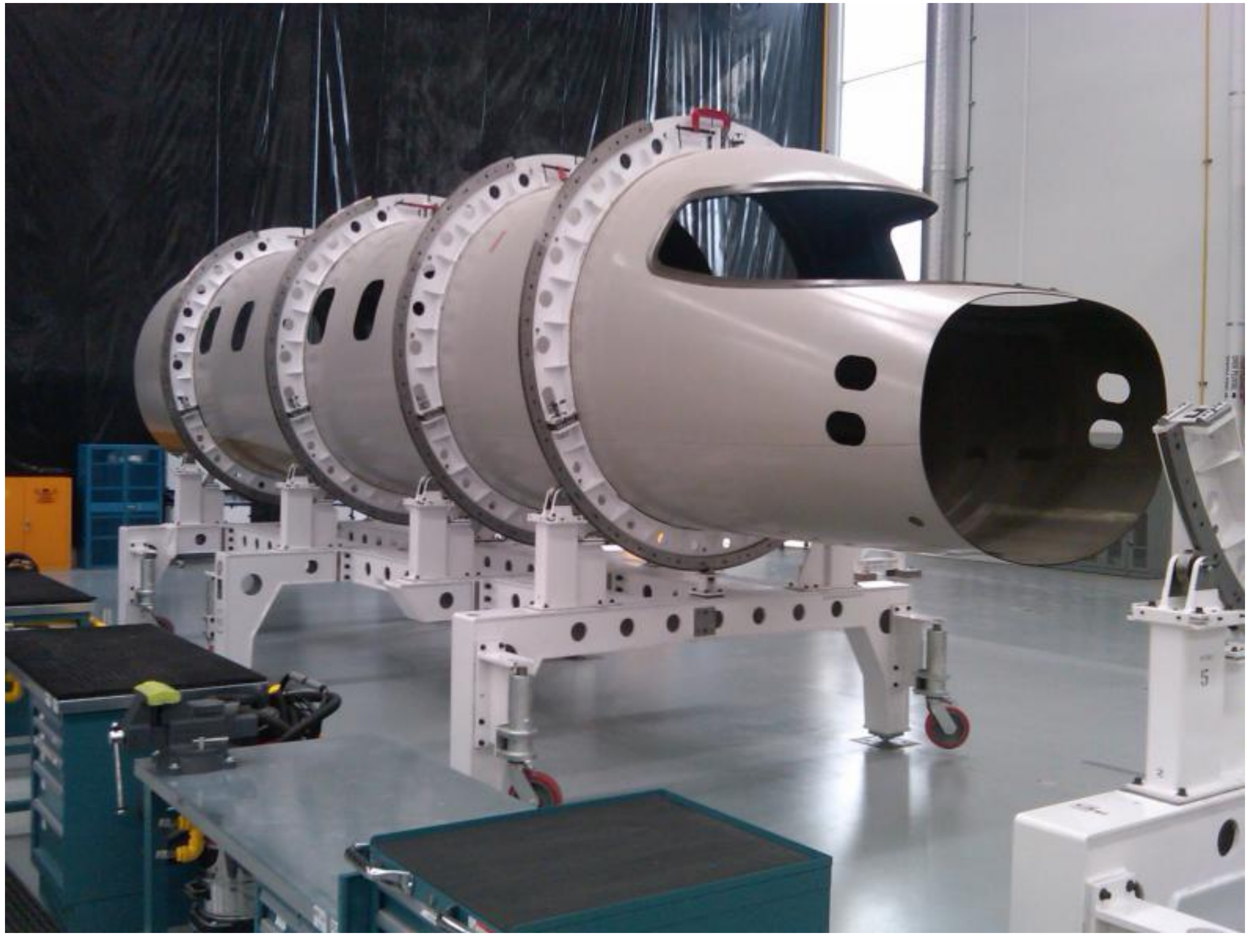
$v = 0.33$

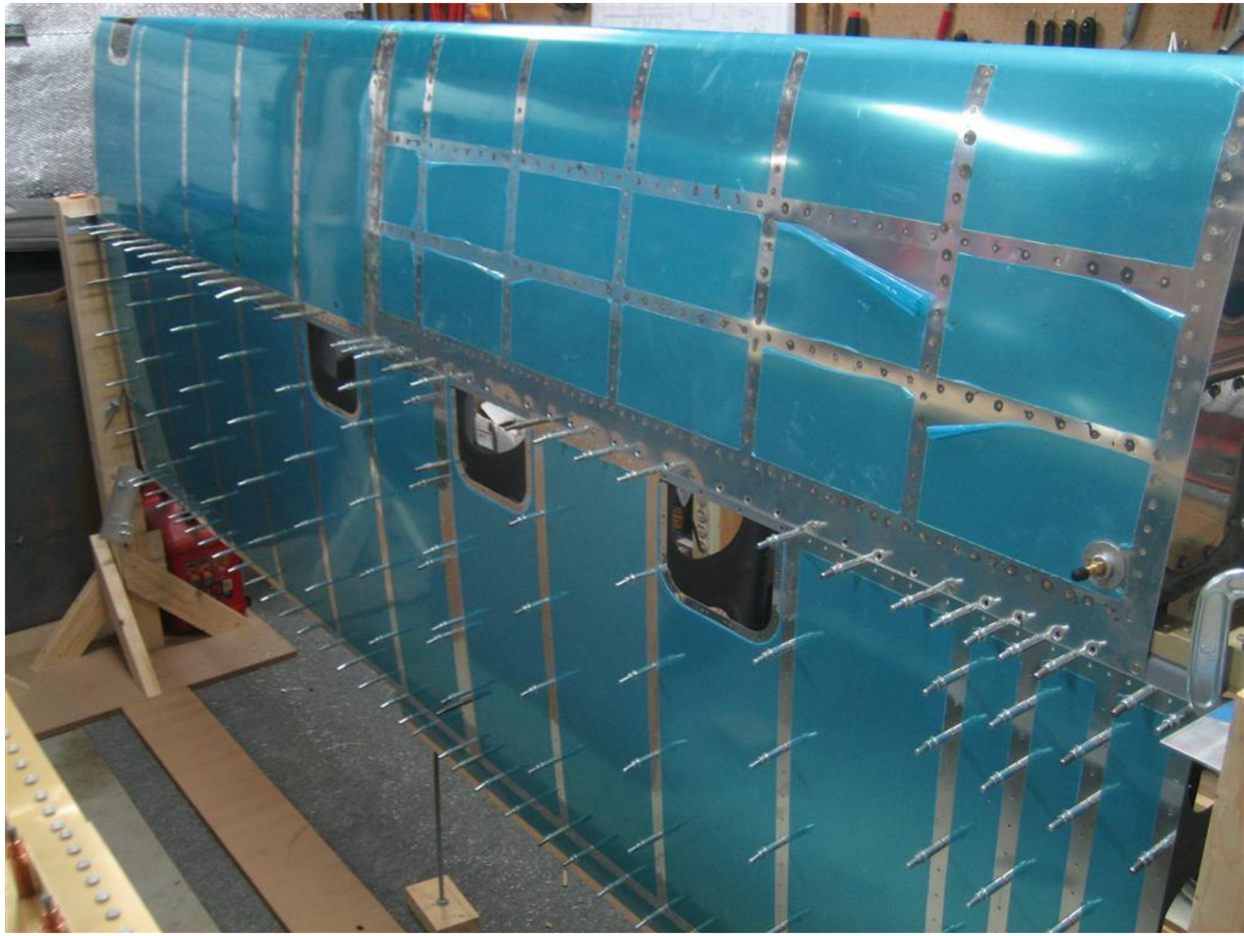
$E = 10e10^6\ PSI$

$$\boldsymbol{\sigma_{CR} = 410.21\ PSI}$$

Curved panels

Curved panels





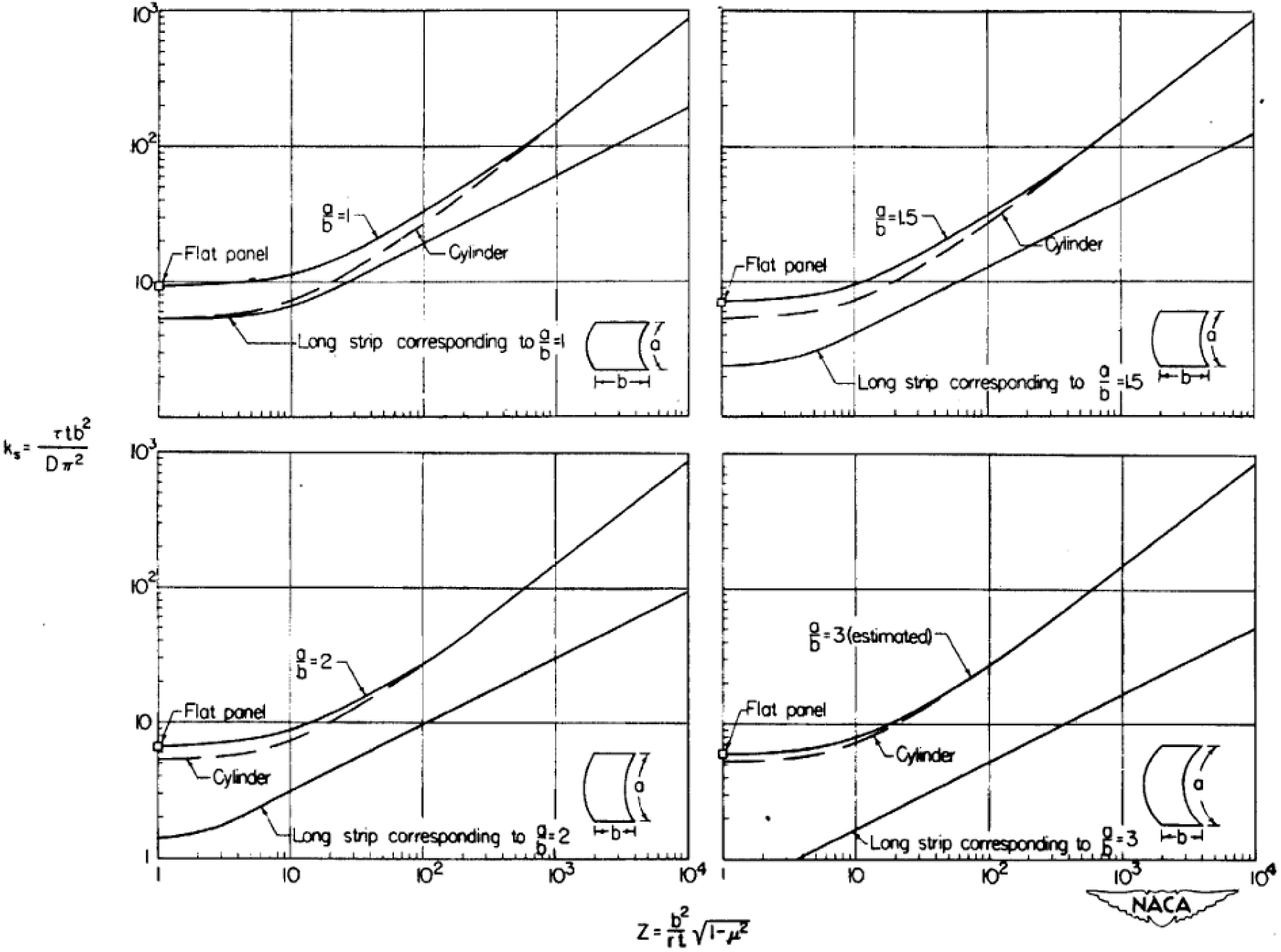
Buckling under shear loads (curved panels)

$$\tau_{CR} = \frac{k_s \pi^2 E}{12(1-v^2)} \left(\frac{t}{b}\right)^2$$

τ_{CR} is the buckling shear stress
k_s is the shear-buckling coefficient

$$Z = \frac{b^2}{rt}\sqrt{1-v^2}$$

Z is the curvature parameter
r is the radius of curvature of panel



Flat panel
a/b = 1
Cylinder
Long strip corresponding to a/b = 1
Flat panel
a/b = 1.5
Cylinder
Long strip corresponding to a/b = 1.5
$k_s = \frac{\tau t b^2}{D \pi^2}$
Flat panel
a/b = 2
Cylinder
Long strip corresponding to a/b = 2
a/b = 3 (estimated)
Flat panel
Cylinder
Long strip corresponding to a/b = 3
a
b
NACA
$Z = \frac{b^2}{rt}\sqrt{1-\mu^2}$

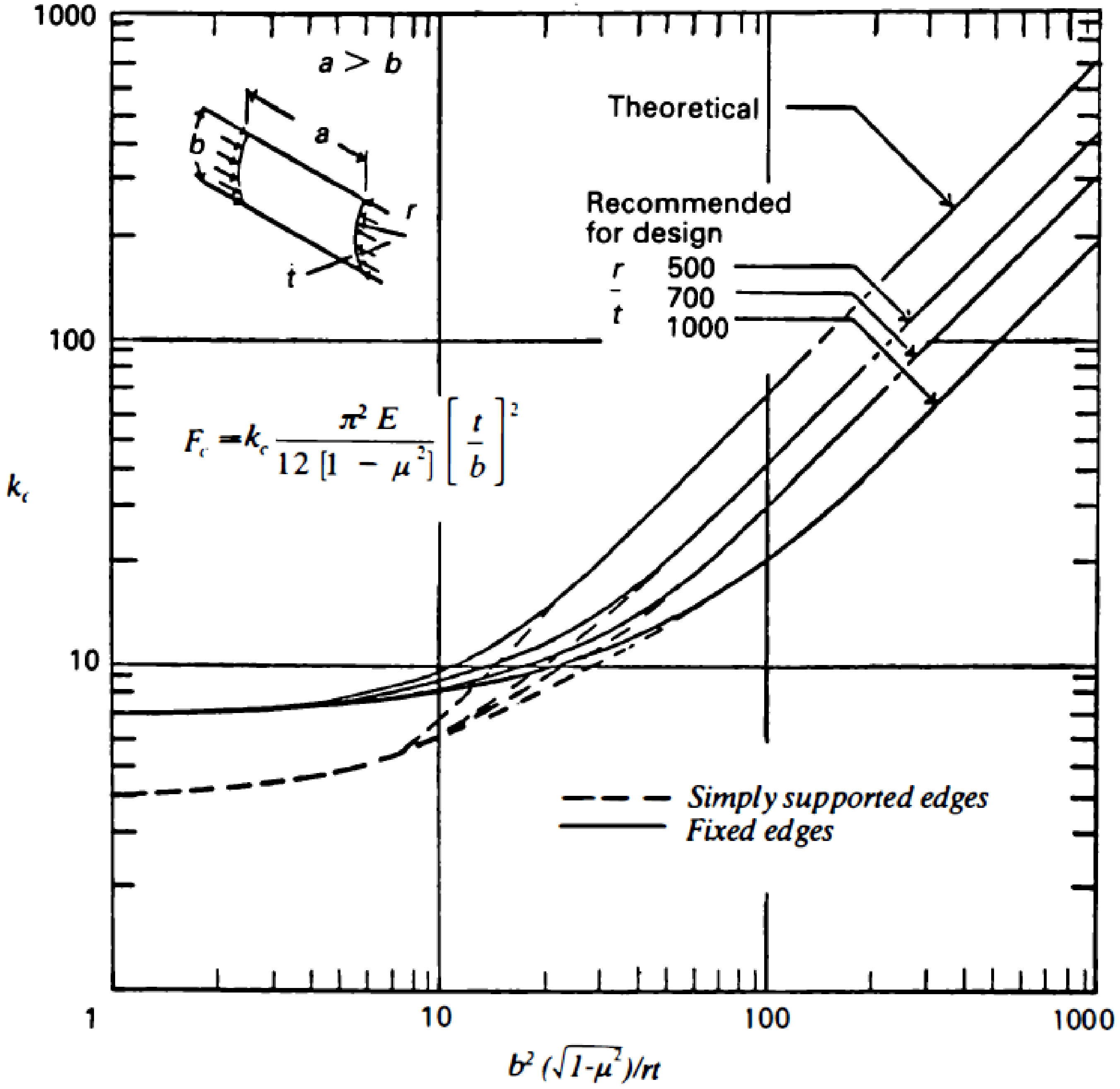


Fig. 5.4.4 Compression buckling coefficients k_c (curved plates).

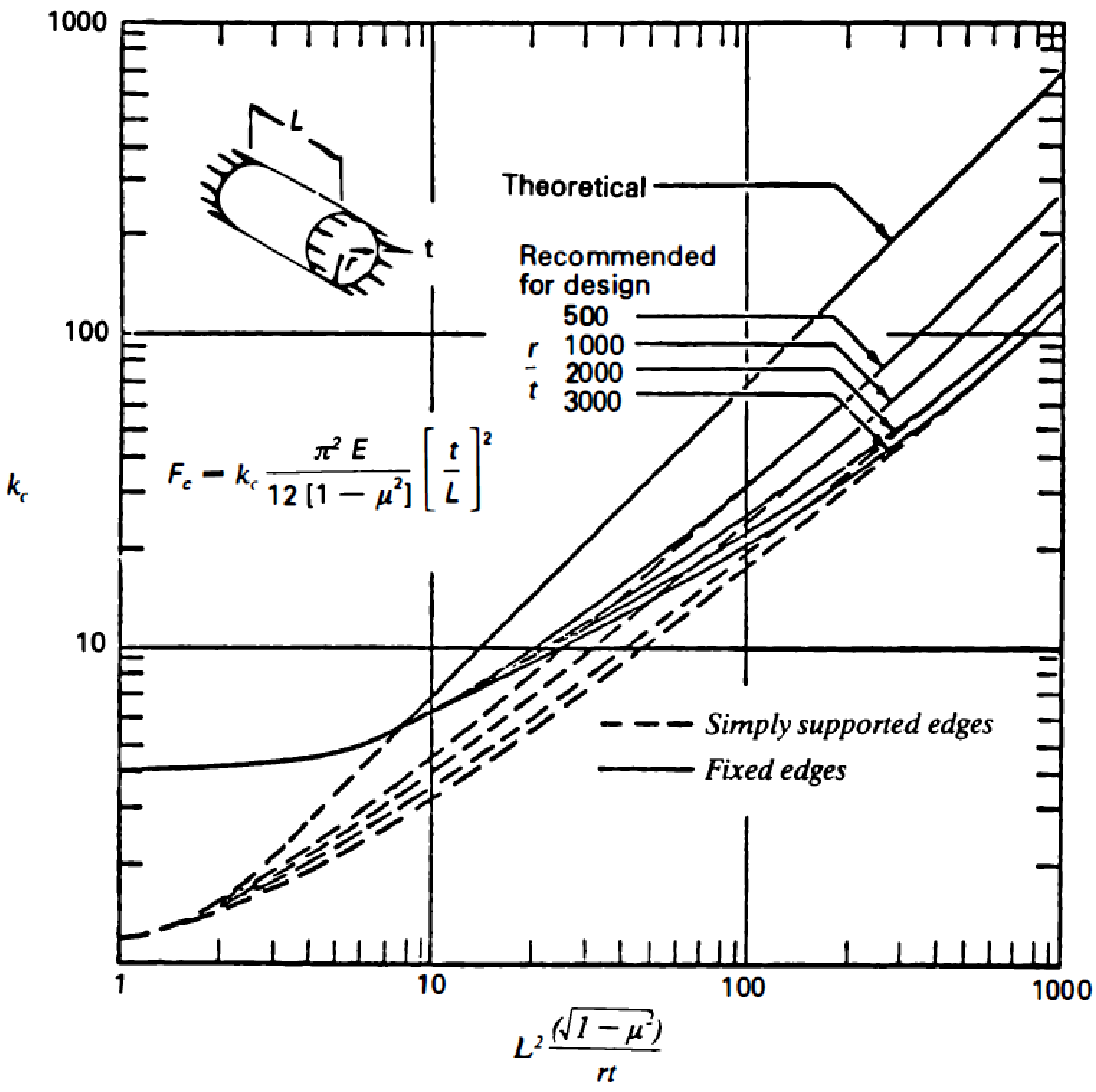


Fig. 5.4.5 Compression buckling coefficients k_c (circular cylinders).

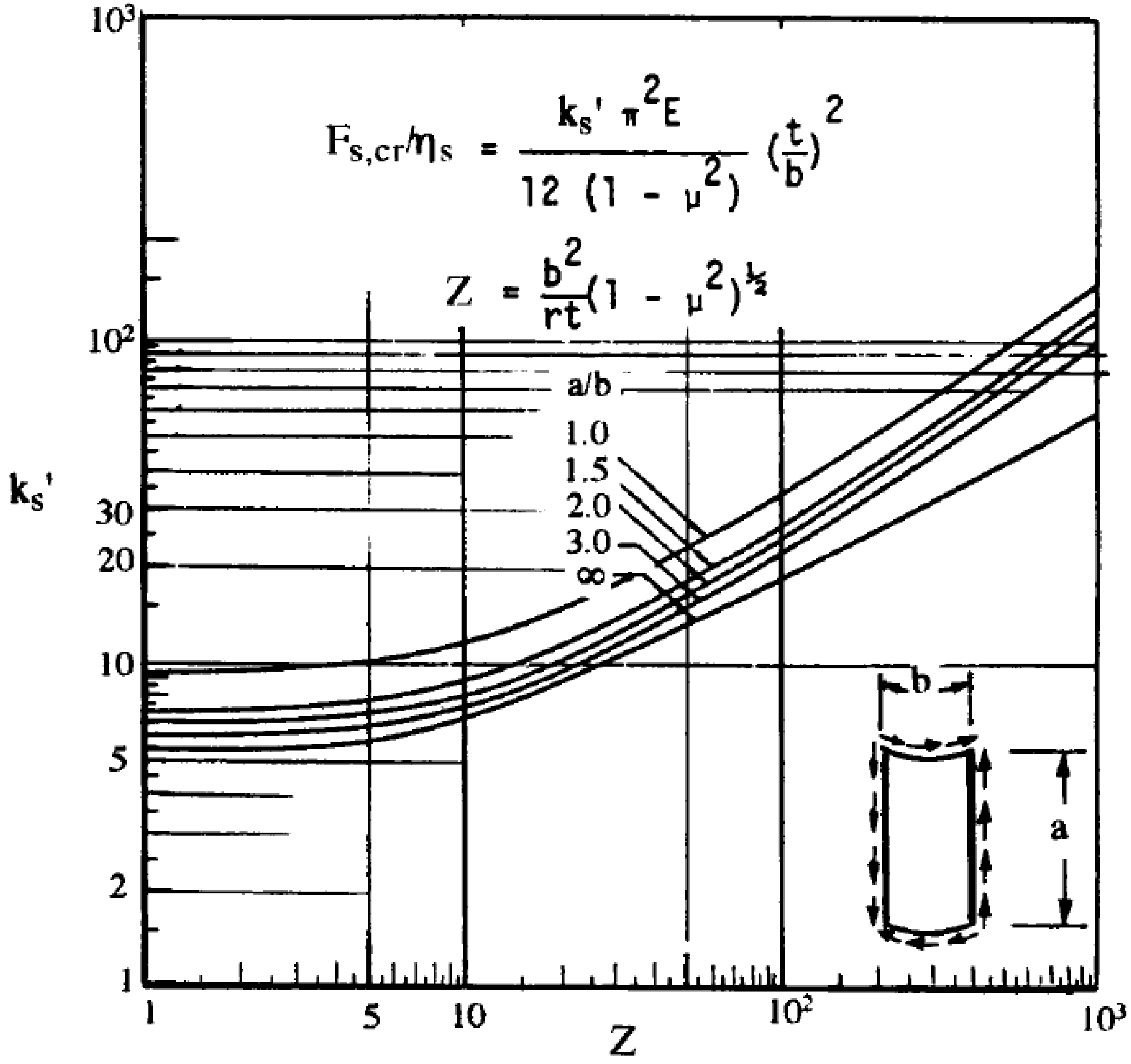


Fig. 11.4.2 Long Curved Plate Coefficient k_s' (Shear) – Four Edges are Hinged (Ref. 11.14)

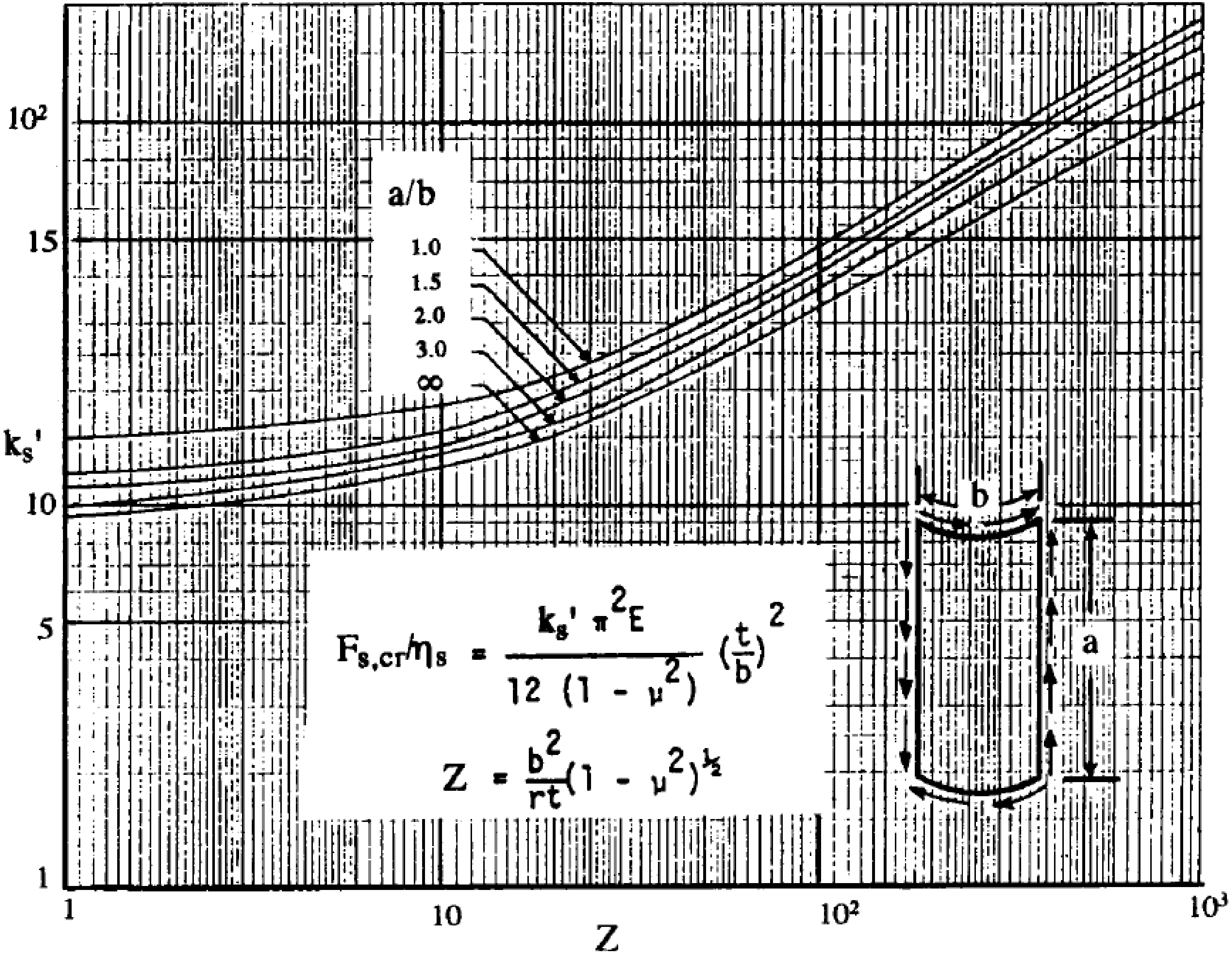


Fig. 11.4.4 Long Curved Plate Coefficient k_s' (Shear) – Four Edges are Clamped (Ref. 11.14)

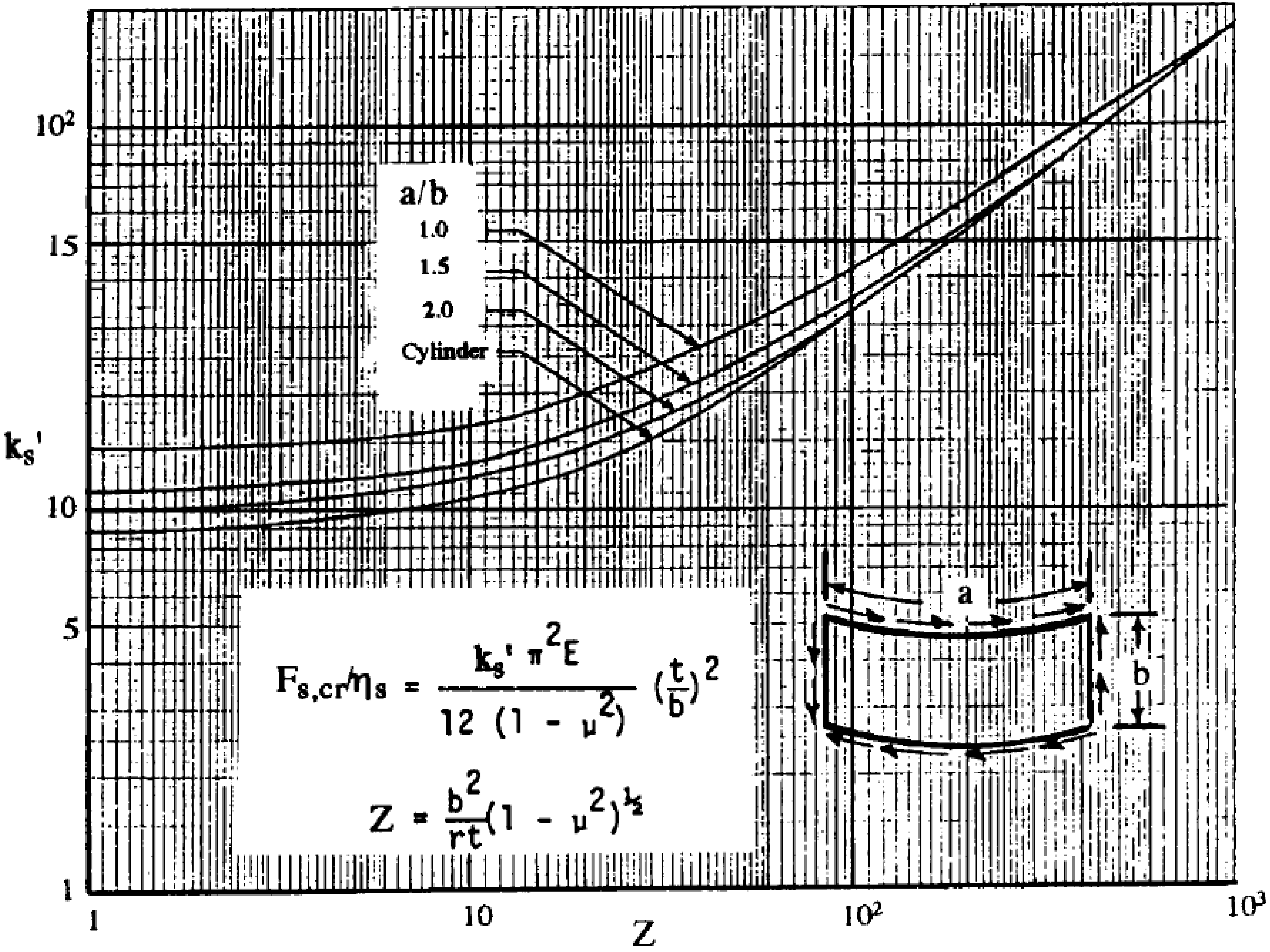


Fig. 11.4.5 Wide Curved Plate Coefficient k_s' (Shear) – Four Edges are Clamped (Ref. 11.14)

Cladding effect

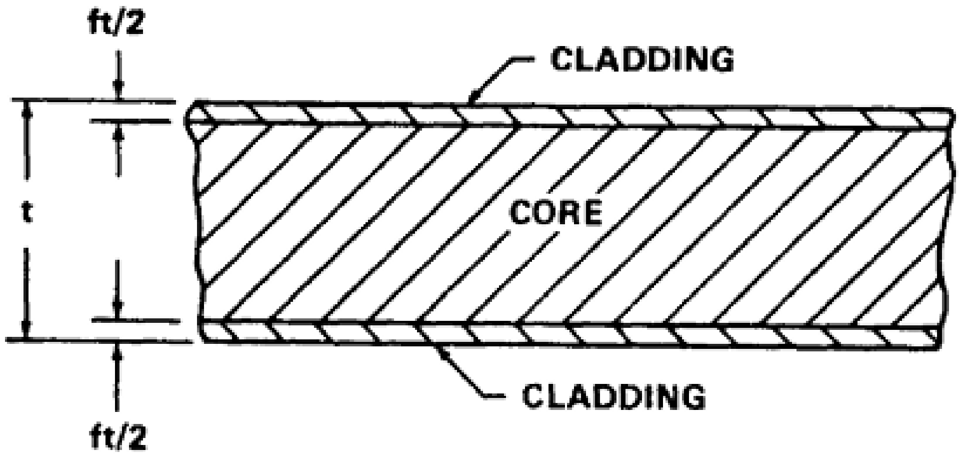


FIGURE C2-1. ALUMINUM CLADDING

Material	Cladding	Plate thickness (in.)	Reduction facotr (λ)
2014	6053	t < 0.04	0.8
		t > 0.04	0.9
2024	1230	t < 0.064	0.9
		t > 0.064	0.95
7075	7072	All thickness	0.92

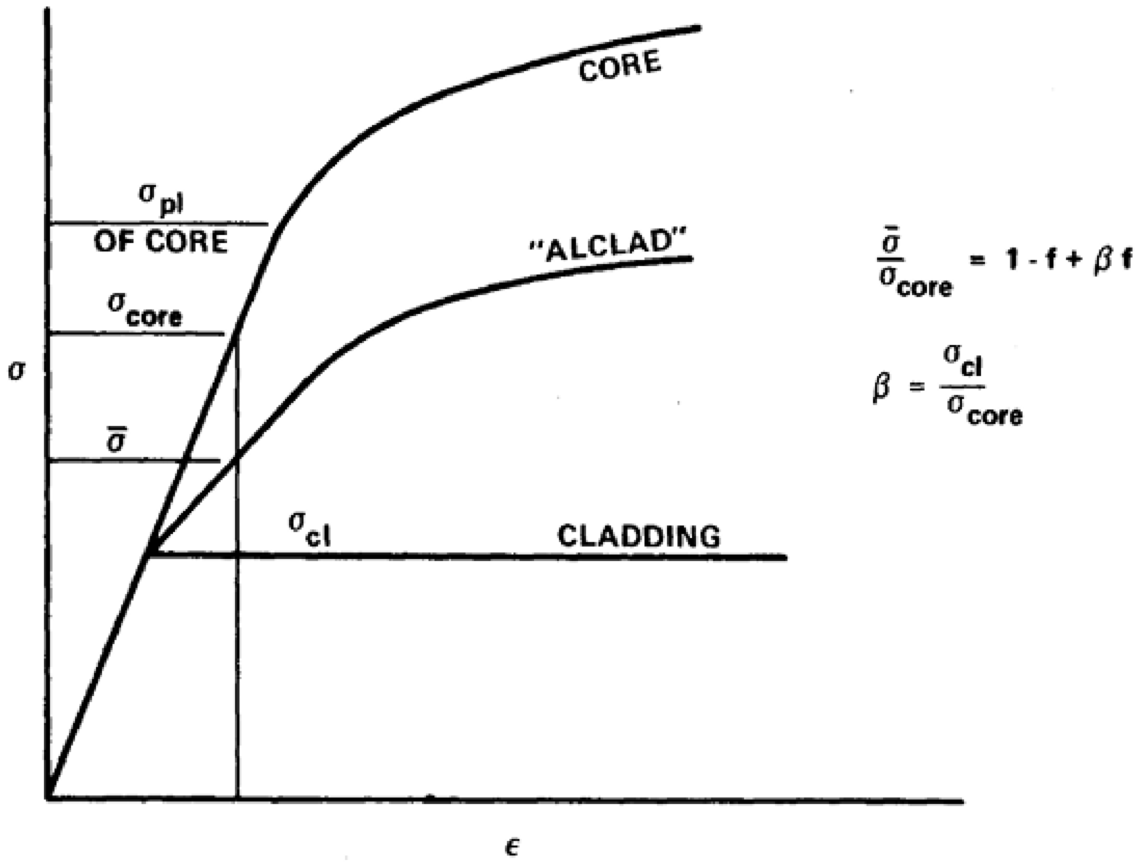


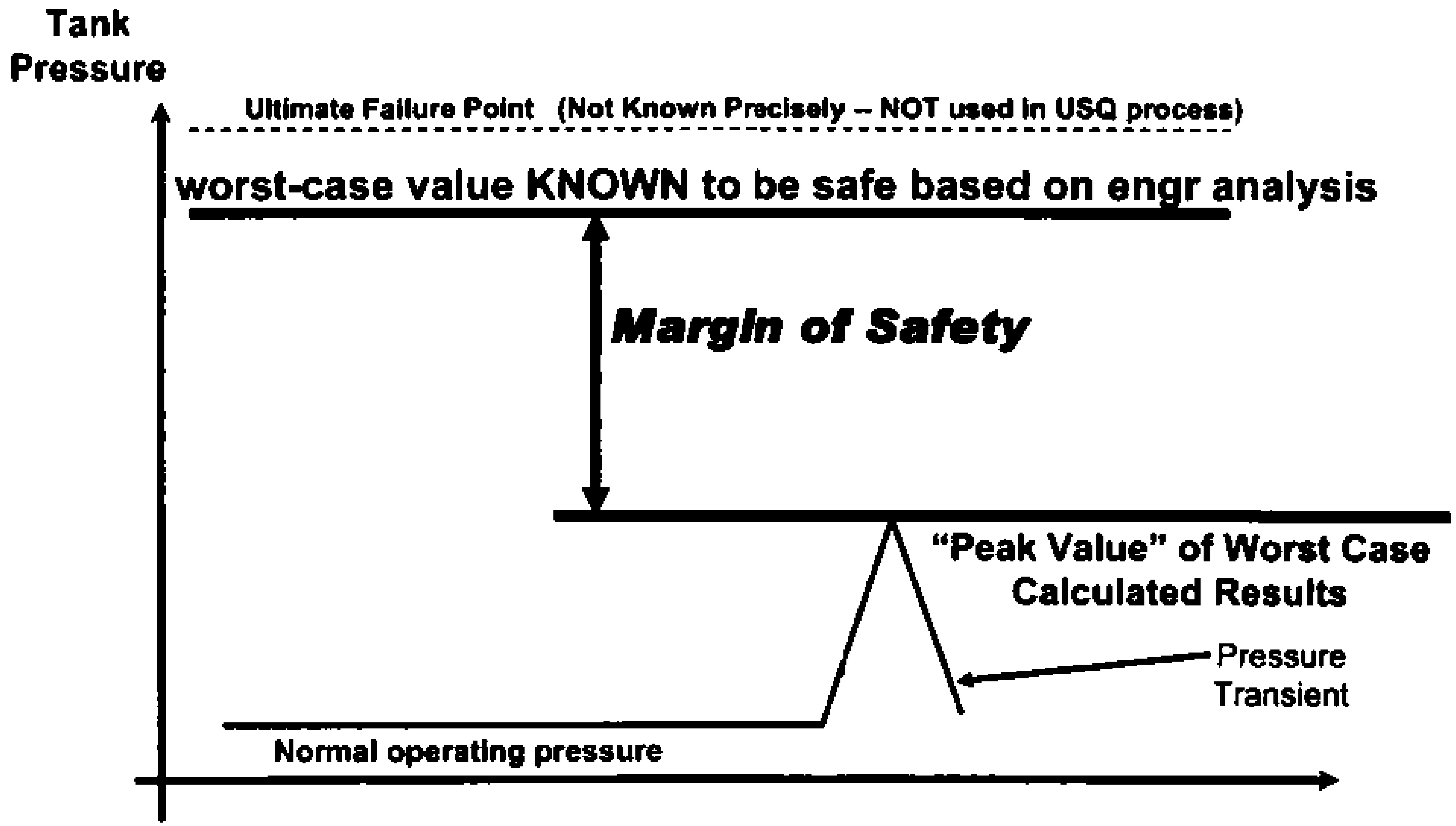
FIGURE C2-2. STRESS-STRAIN CURVES FOR CLADDING, CORE, AND "ALCAD" COMBINATIONS

Margin of safety

$$Factor\ of\ safety = FS = \frac{ultimate\ stress}{allowable\ stress}$$

$$\boldsymbol{Margin\ of\ safety = MS = FS - 1.0}$$

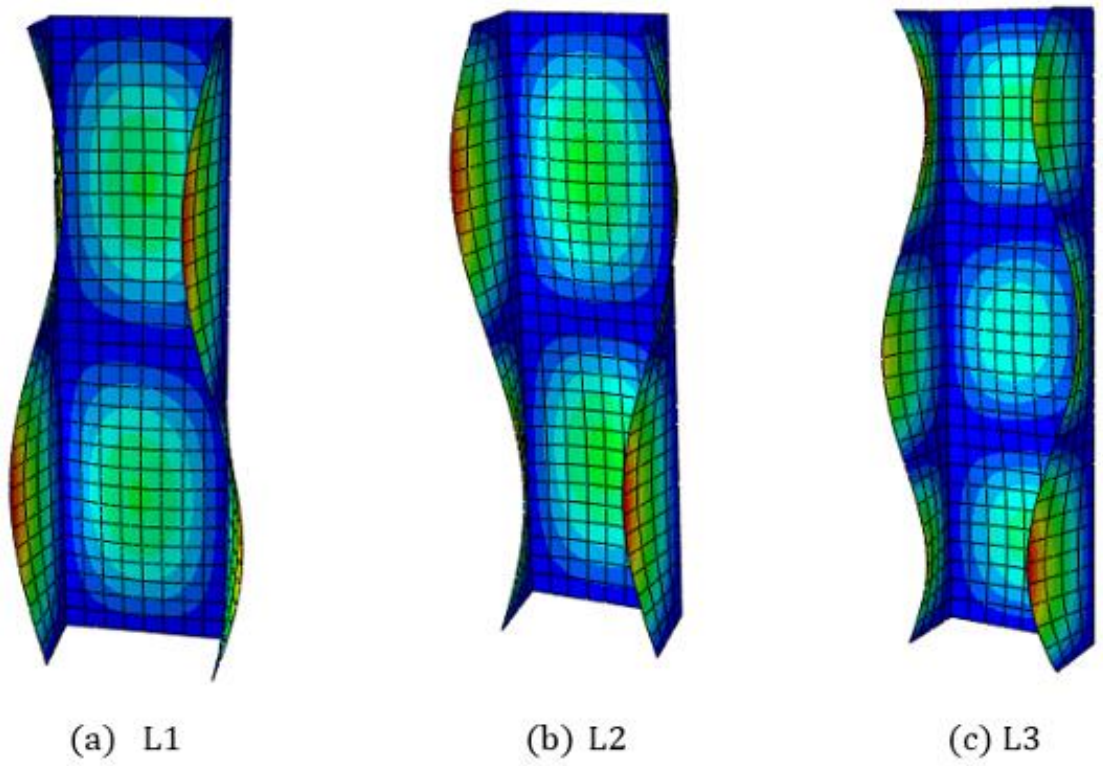
Definition of Margin of Safety



Stiffener local buckling

Stiffeners are prone to local instability if $L_e/r < 20$,

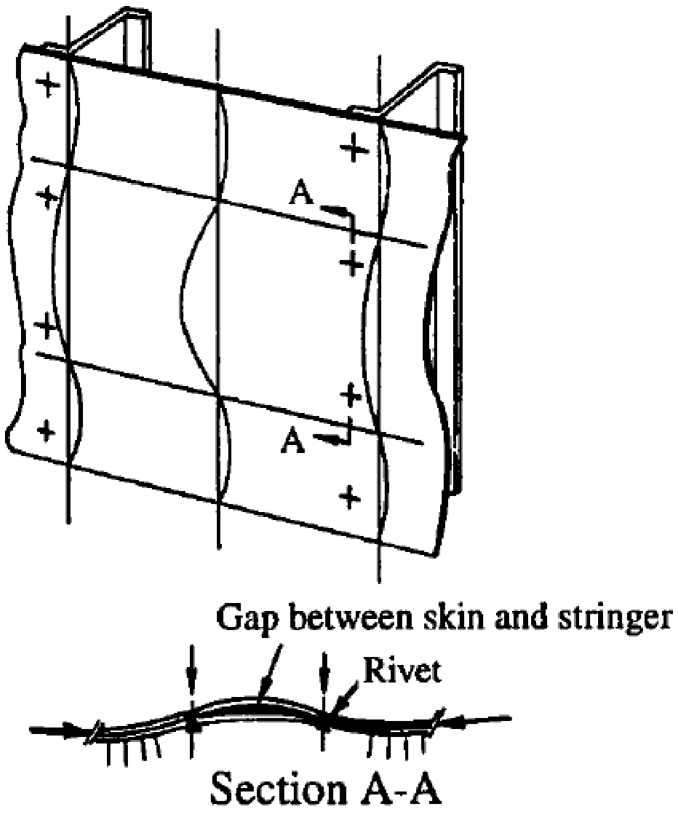
Analysis treats each flange like a separate panel for panel buckling calculation



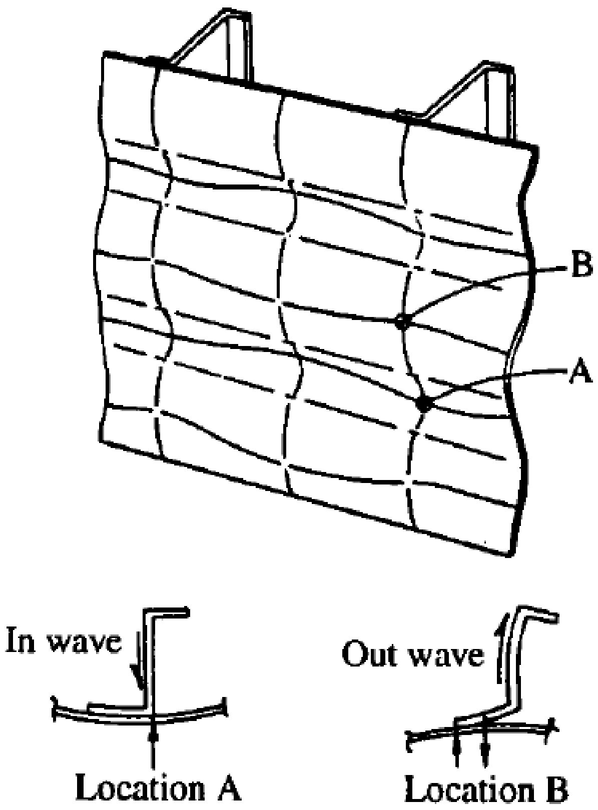
Local buckling and post-critical behavior of thin-walled composite channel section columns

Inter-rivet buckling

Failure Modes of a Skin-Stringer Panel



(e) Inter-Rivet Buckling



(f) Skin Wrinkling

Inter-rivet buckling (plate method)

$$\sigma_{CR} = \frac{e\,\pi^2\,\eta\,\bar{\eta}\,E}{12(1-\nu^2)}\left(\frac{t}{p}\right)^2$$

e End-fixity coefficient

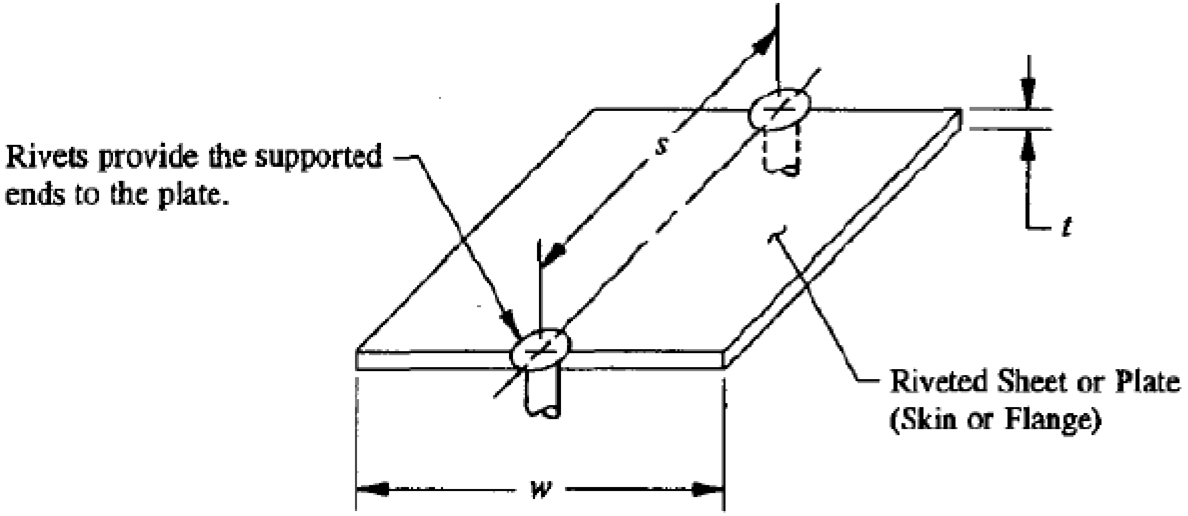
η Plastic reduction factor

$\bar{\eta}$ Cladding reduction factor

p Fastener pitch

END-FIXITY COEFFICIENTS FOR INTERRIVET BUCKLING

Fastener type	e	Reference
Flathead rivet	4	8
Spotwelds	3.5	10
Brazier-head rivet	3	10
Countersunk rivet	1	11



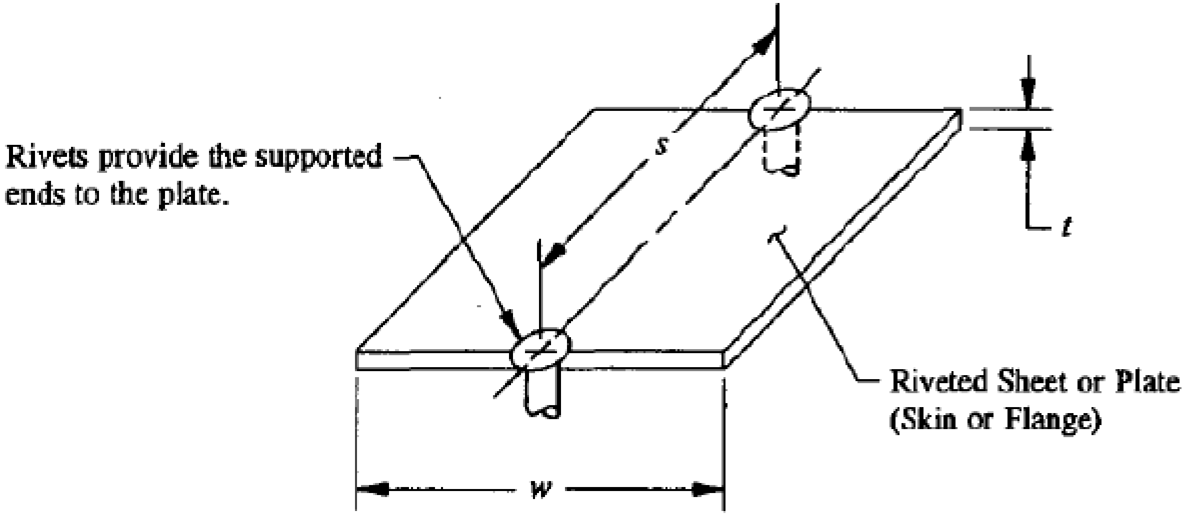
Inter-rivet buckling (column method)

$$\sigma_{CR} = \frac{\pi^2 E}{\left(\frac{2\sqrt{3}}{\sqrt{e}} \frac{p}{t}\right)^2}$$

END-FIXITY COEFFICIENTS FOR INTERRIVET BUCKLING

Fastener type	e	Reference
Flathead rivet	4	8
Spotwelds	3.5	10
Brazier-head rivet	3	10
Countersunk rivet	1	11

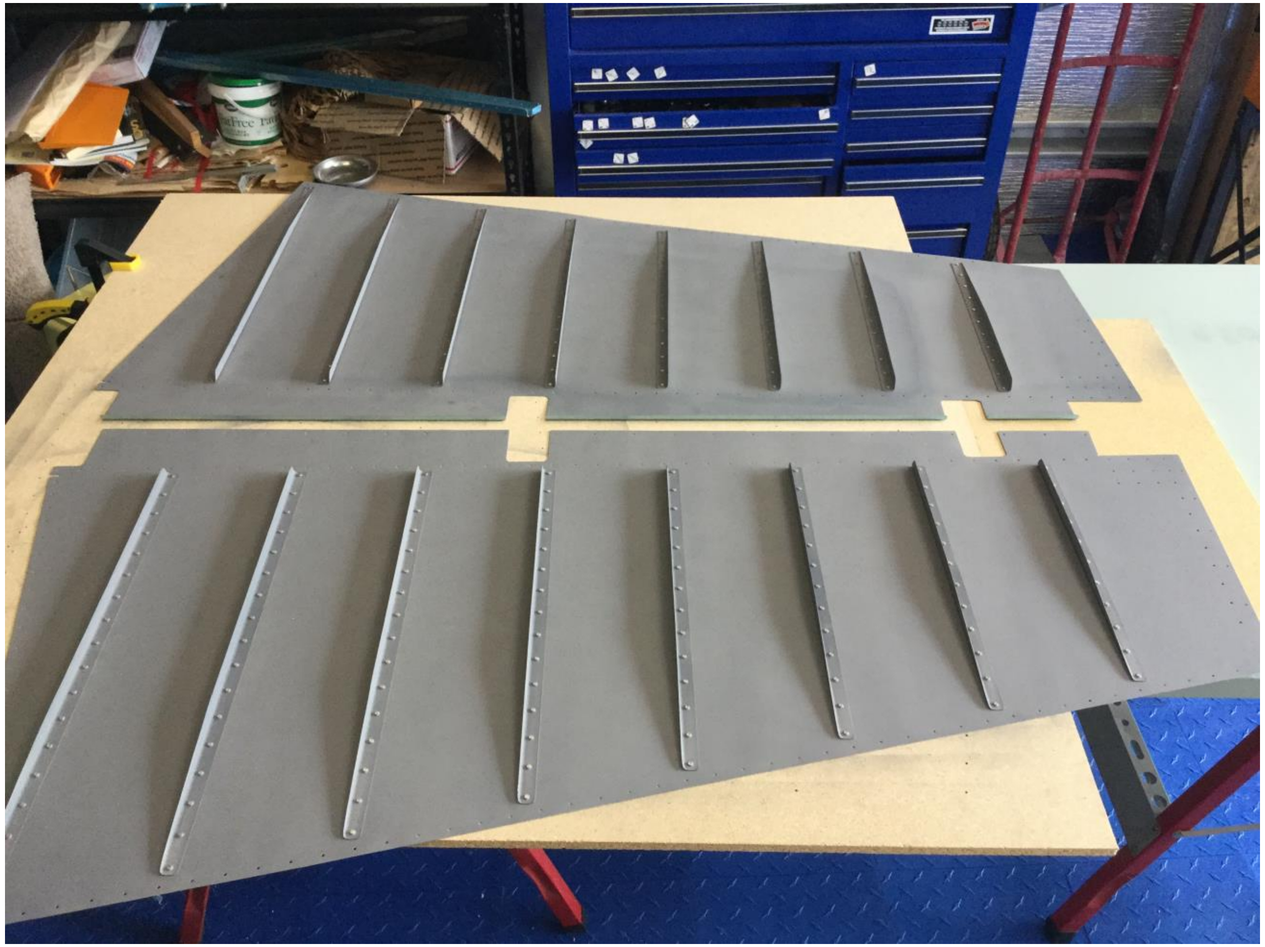
This method is preferred over the plate one.

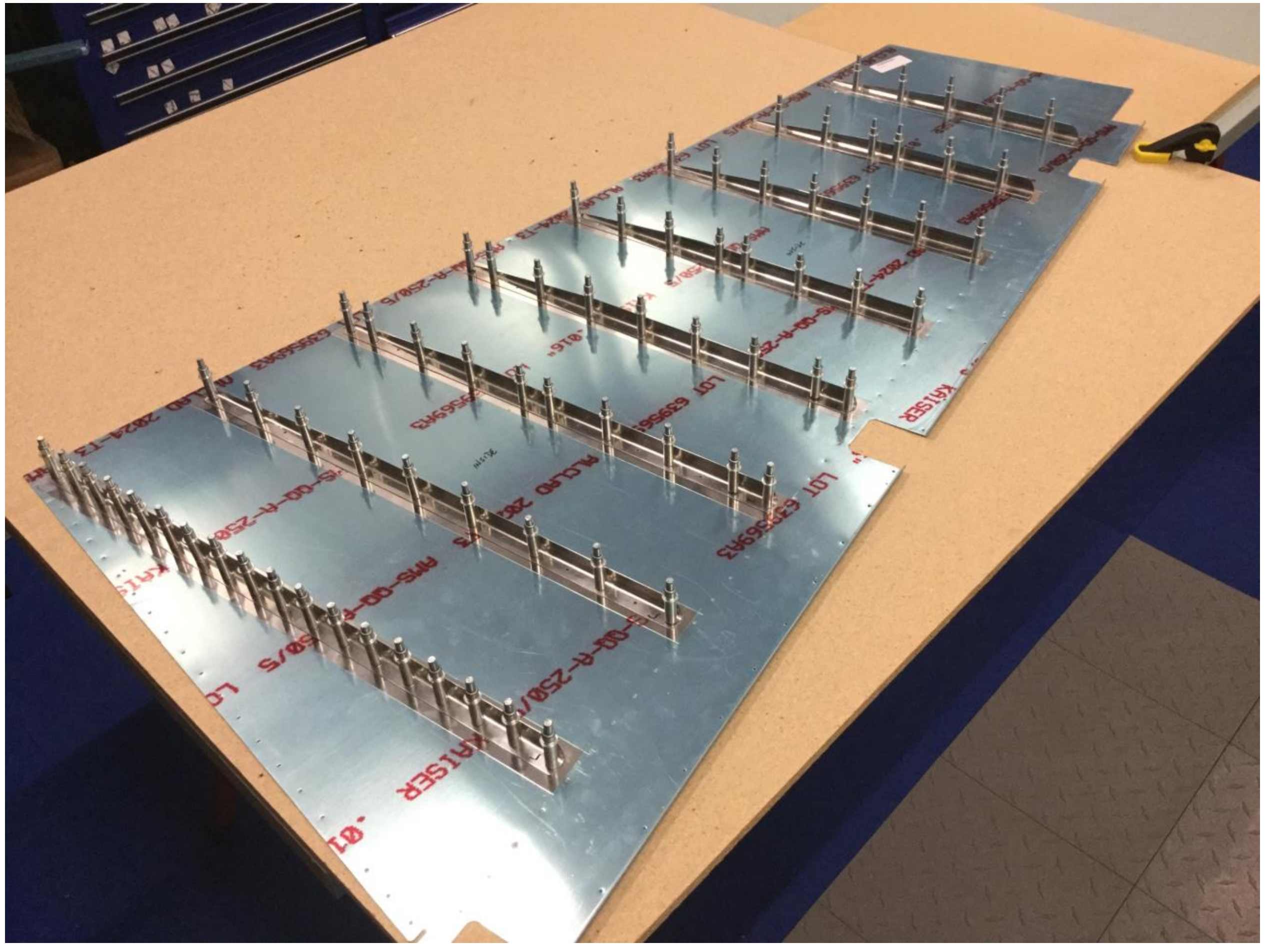


Example

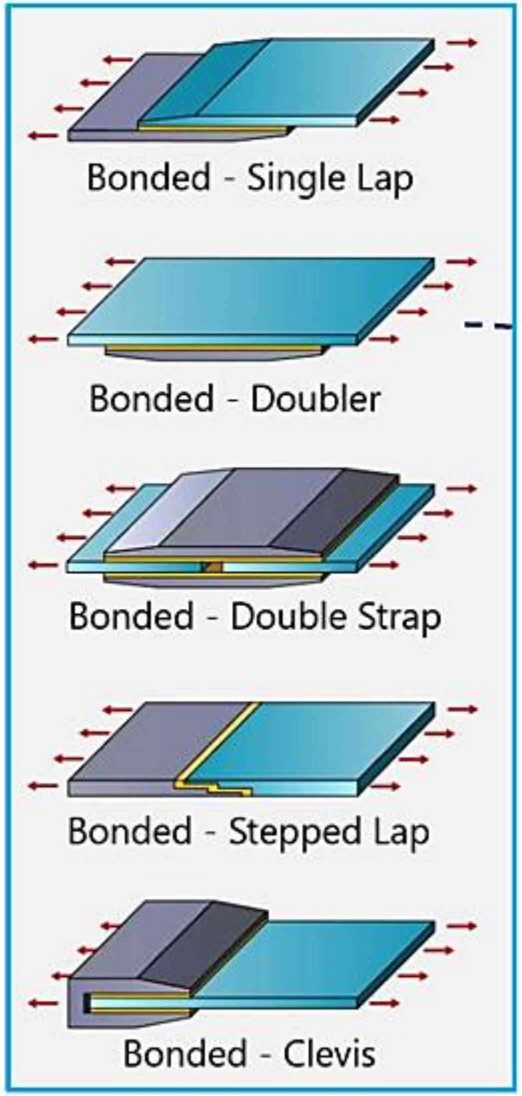
- Assume a skin-stringer panel made up of AL 2024 T3 for both universal rivets and plates.
- The plate is $t = 2\, mm$, $E = 70\, GPa$ and $\nu = 0.3$
- Find the maximum stiffener pitch p_{max} if $\sigma_{CR} < 100\, MPa$

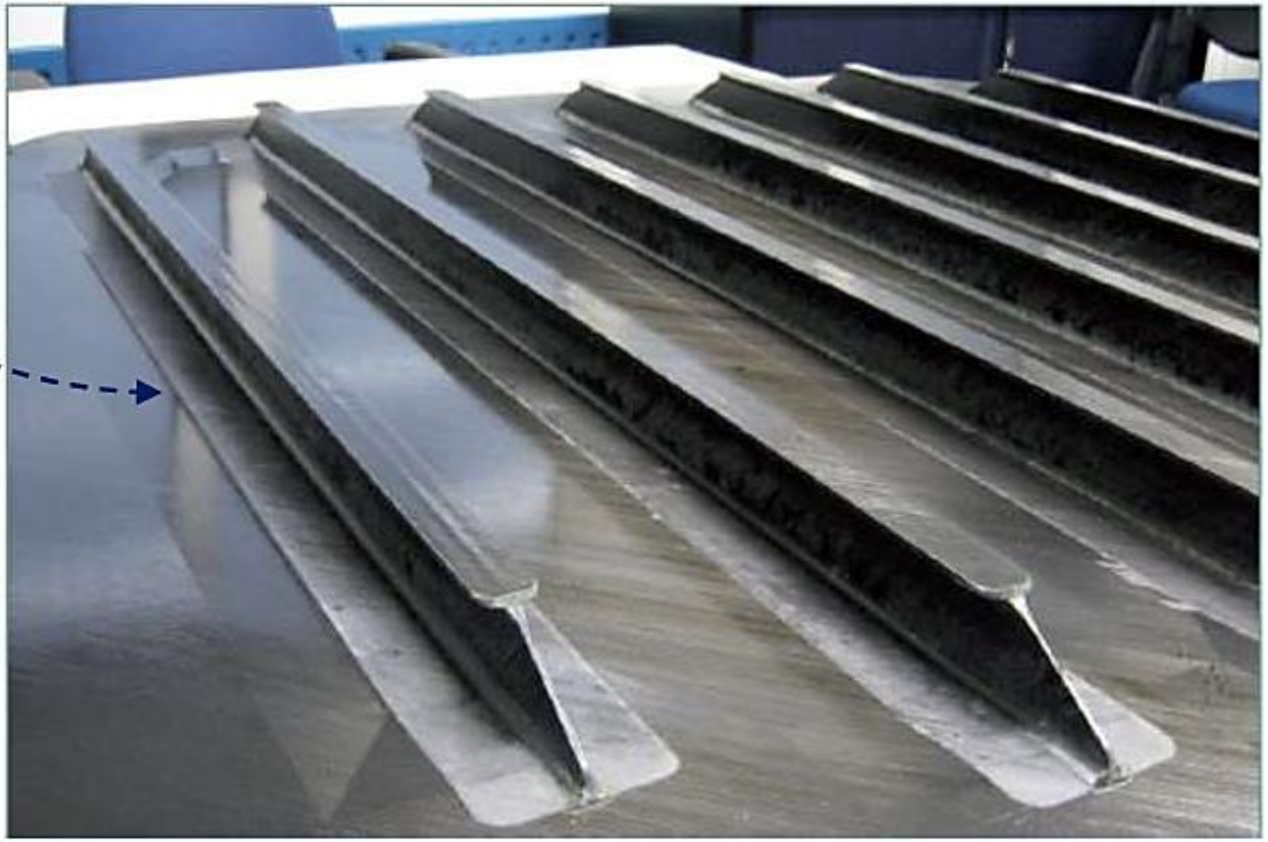
Instability of stiffened panels



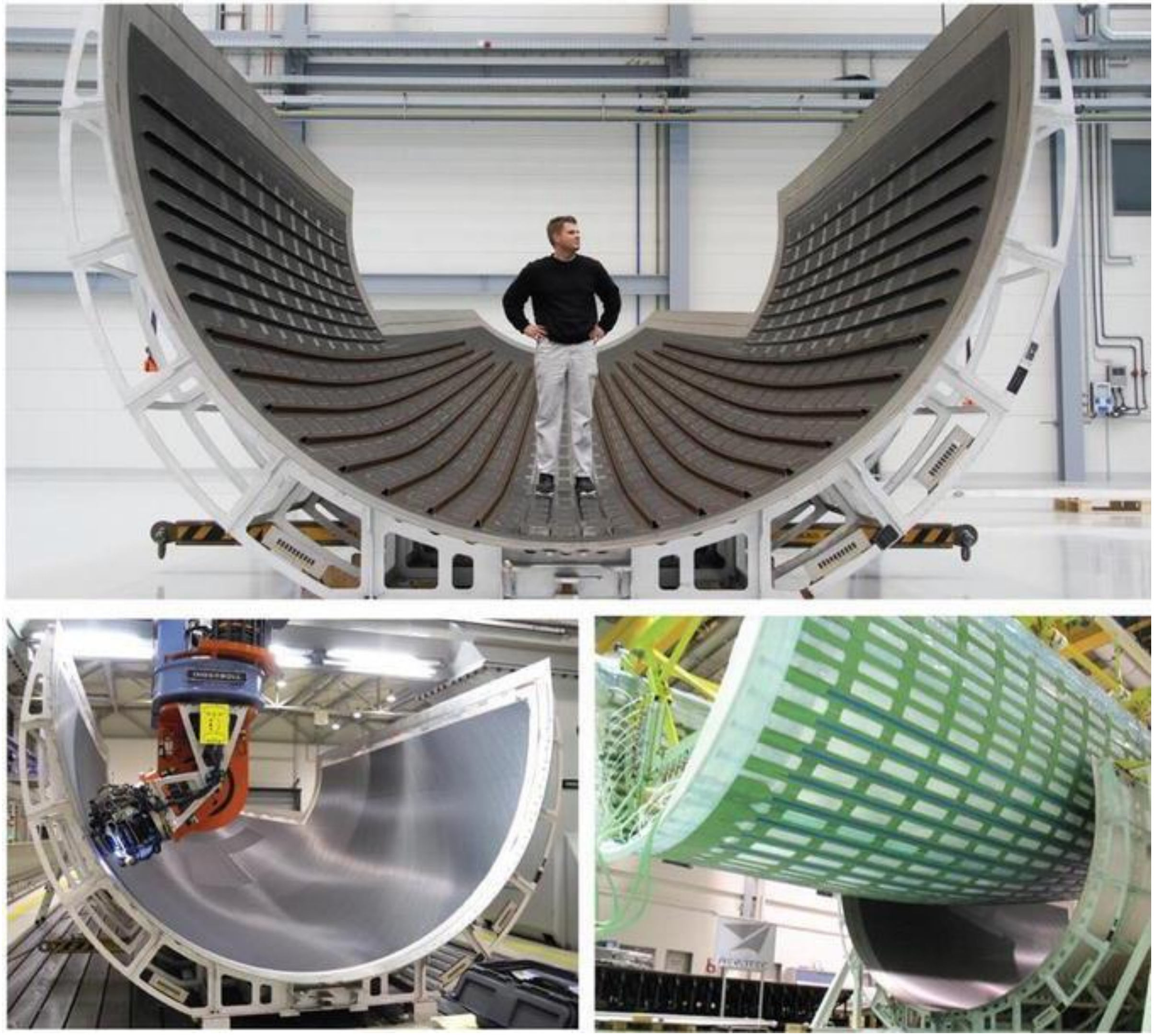


Instability of stiffened panels

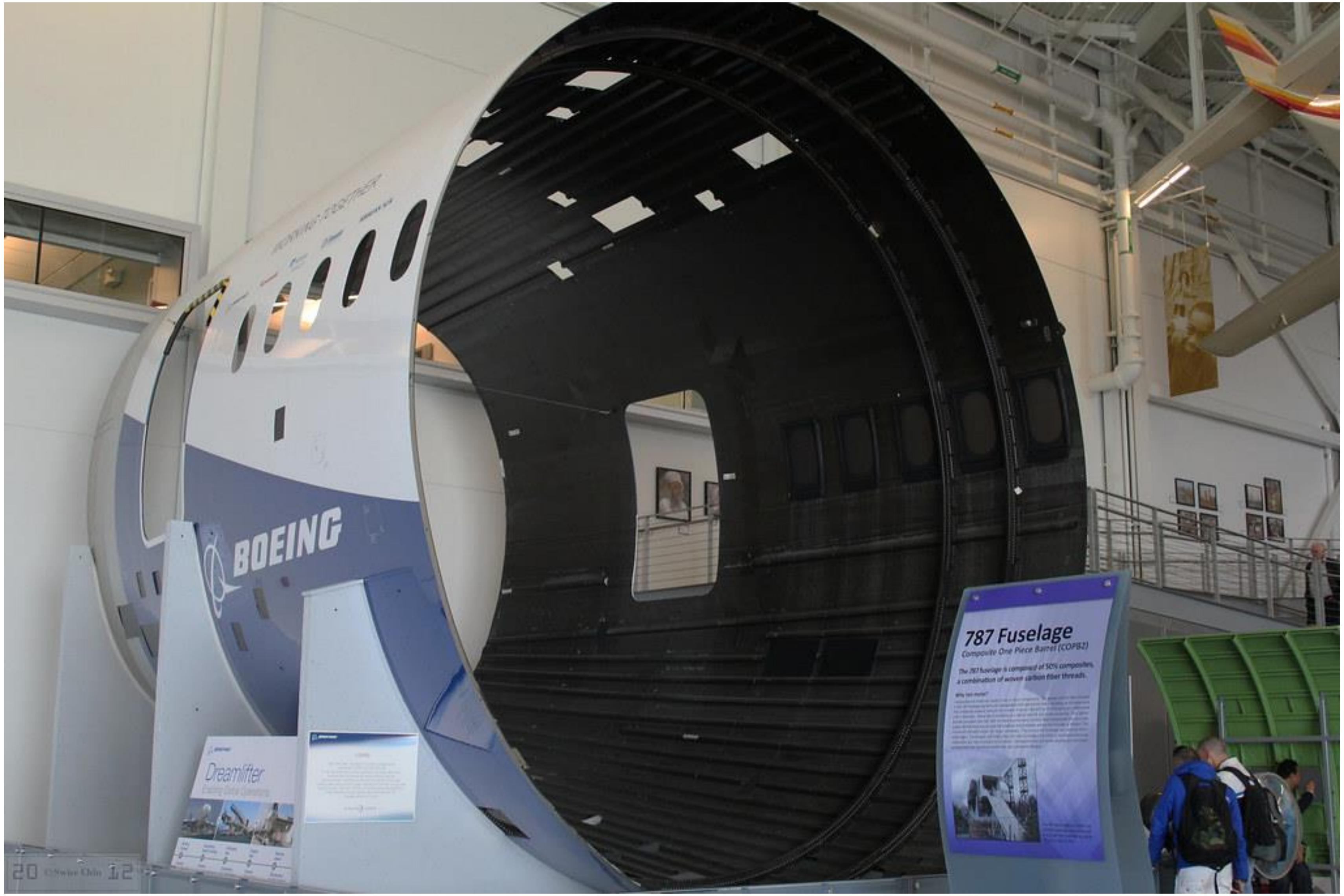




Instability of stiffened panels

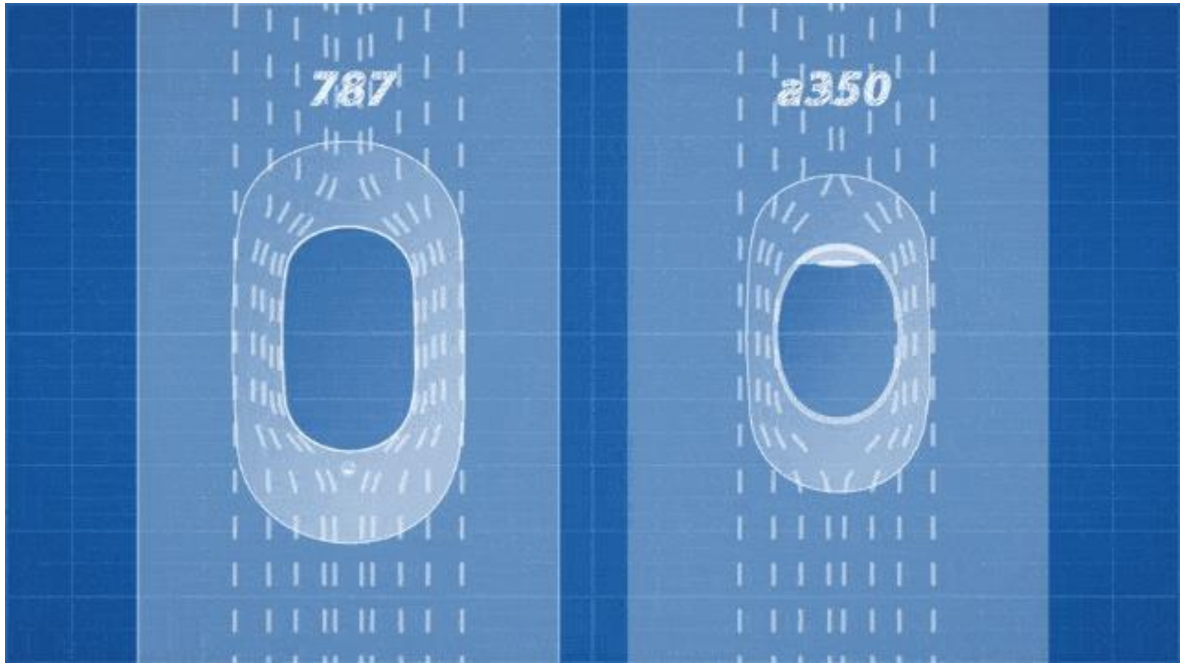


Instability of stiffened panels



Instability of stiffened panels

Instability of stiffened panels



https://www.youtube.com/watch?v=7-I20Ru9BwM&ab_channel=RealEngineering

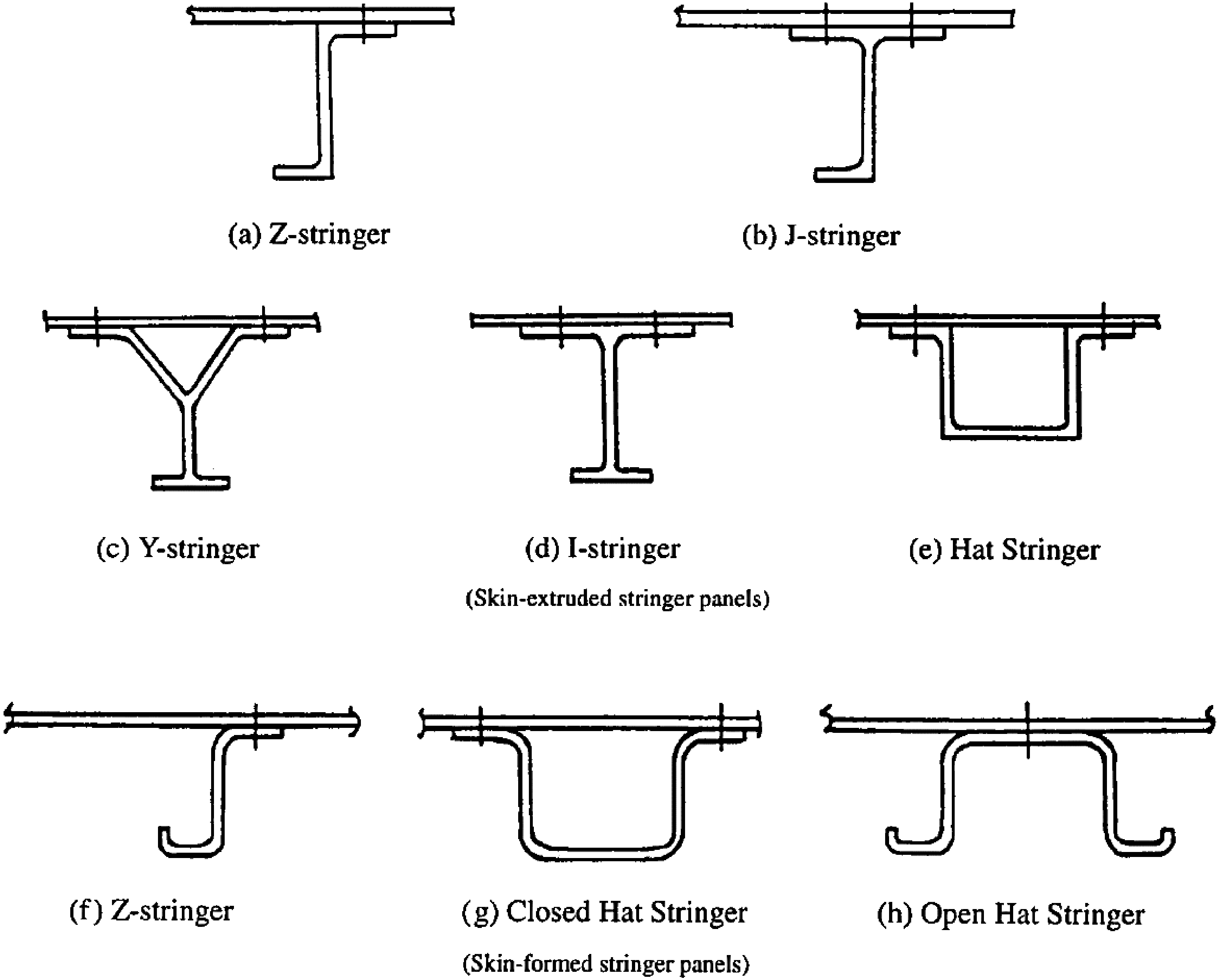
Instability of stiffened panels

- Plates having a large value of b/t buckle at low values of critical stress.

- An effective method of reducing this parameter is to **introduce stiffeners along the length** of the plate.

- An efficient structure is obtained by adjusting the stiffeners so that buckling occurs in **both stiffeners and plate about the same stress**.

Typical stringer skin panels



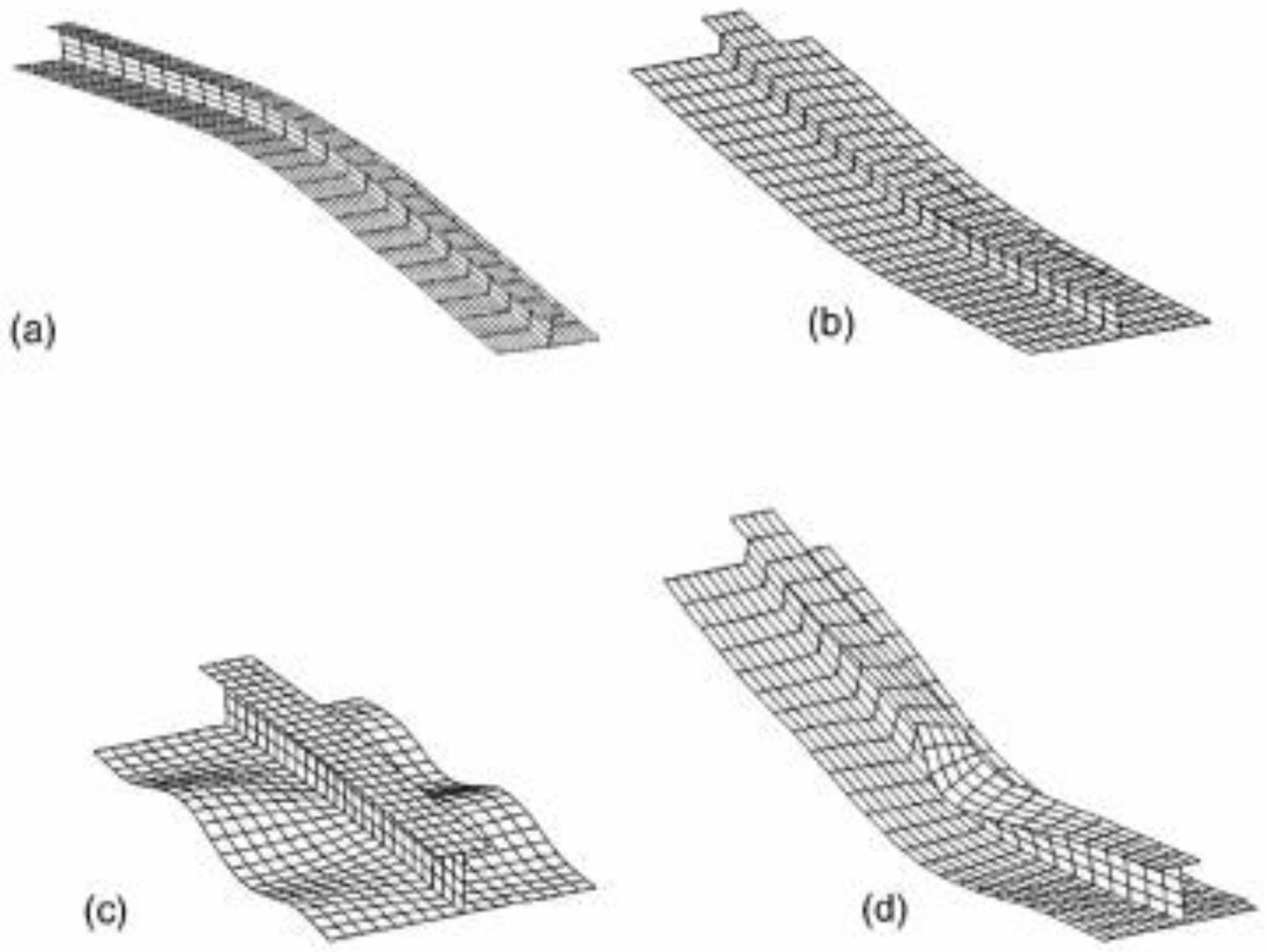
Typical buckling modes:

(a) overall buckling (plate induced)

(b) overall buckling (stiffener induced)

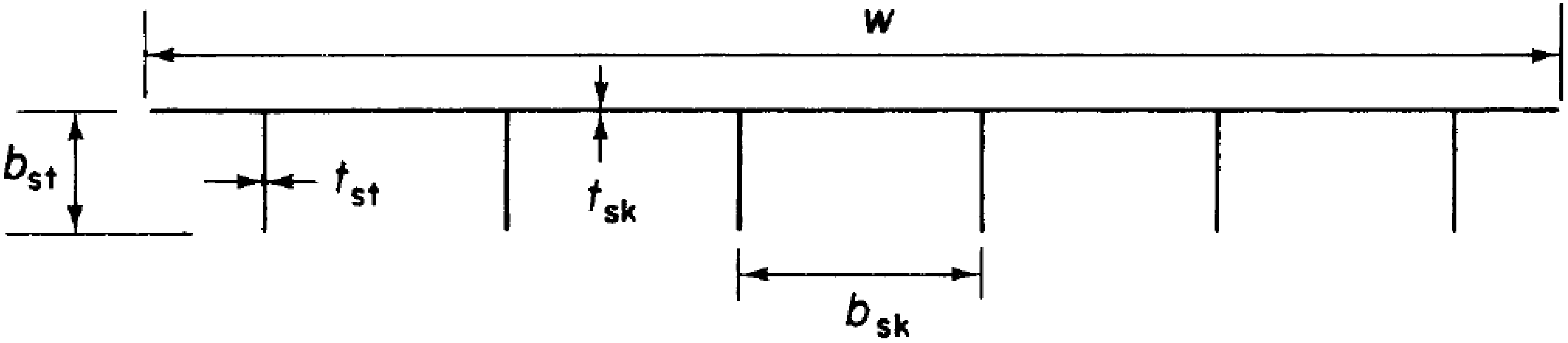
(c) plate buckling

(d) stiffener tripping



Instability of stiffened panels

A relatively simple approach for instability in panels is presented by Gerard and Becker. Assume a panel with width w.



Instability of stiffened panels

Longitudinal members which may be flat, z, I, channel or top-hat sections, may behave as Euler column:

$$\sigma_{CR} = \frac{\pi^2 E}{(L_e/r)^2}$$

Instability of stiffened panels

In addition to the column buckling mode, individual plate elements comprising the panel cross section may buckle as long plates:

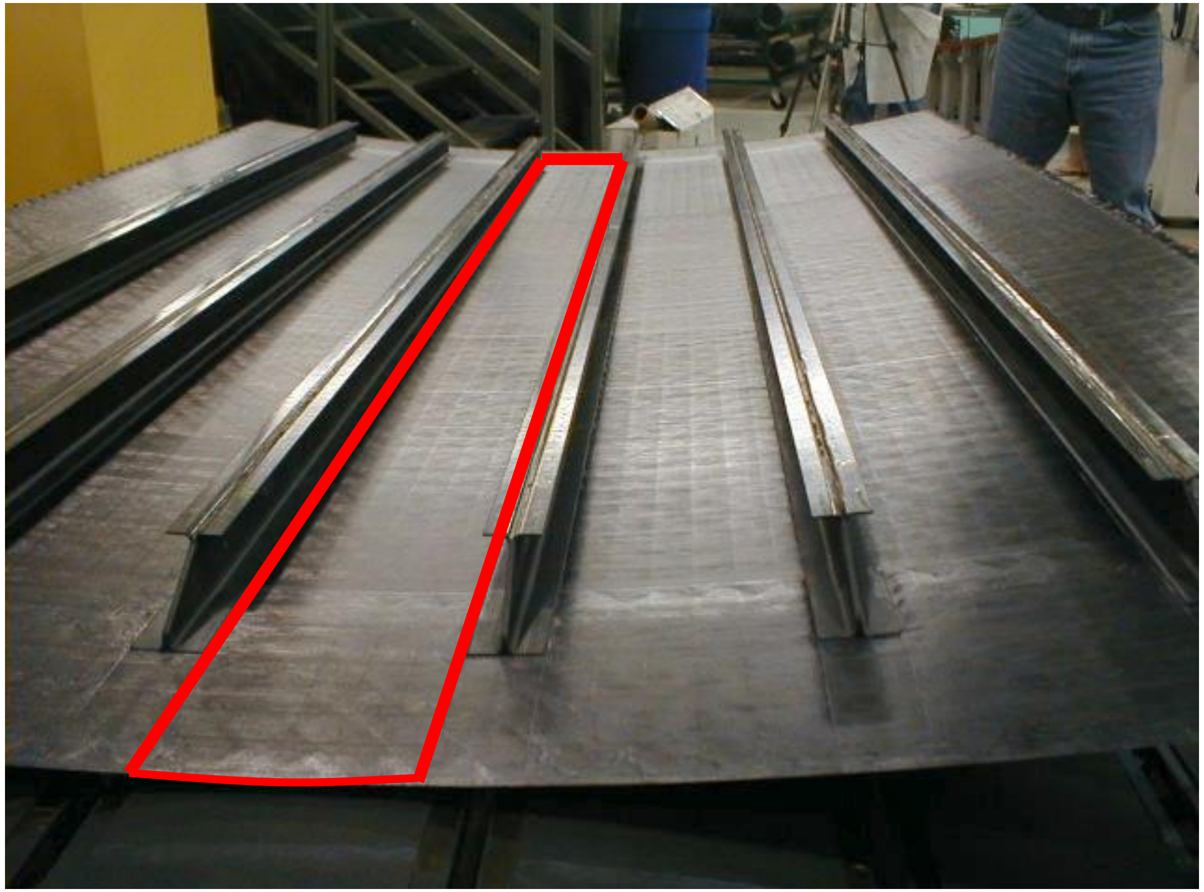
$$\sigma_{CR} = \frac{k\pi^2 E}{12(1-v^2)}\left(\frac{t}{b}\right)^2$$

k , t and b depend upon a particular portion of the panel being investigated.

Instability of stiffened panels

The portions of skin between stiffeners may buckle as a plate simply supported on all four sides:

$$\sigma_{CR} = \frac{4\pi^2 E}{12(1-v^2)}\left(\frac{t_{sk}}{b_{sk}}\right)^2$$



Instability of stiffened panels

A further possibility is that the stiffeners may buckle as long plates simply supported on three sides with one edge free.

$$\sigma_{CR} = \frac{0.43\pi^2 E}{12(1-v^2)}\left(\frac{t_{st}}{b_{st}}\right)^2$$

Instability of stiffened panels

$$\sigma_{CR} = \frac{\pi^2 E}{(L_e/r)^2}$$

$$\sigma_{CR} = \frac{4\pi^2 E}{12(1-v^2)}\left(\frac{t_{sk}}{b_{sk}}\right)^2$$

$$\sigma_{CR} = \frac{0.43\pi^2 E}{12(1-v^2)}\left(\frac{t_{st}}{b_{st}}\right)^2$$

Clearly, the minimum value of these critical stresses is the critical stress for the panel taken as a whole.

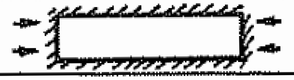
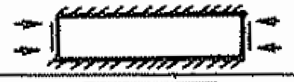
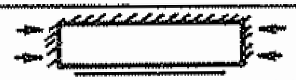
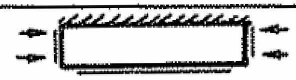
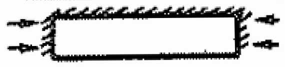
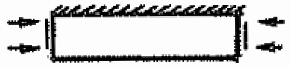
Instability of stiffened panels

The compressive load is applied to the panel over its complete cross section. To relate this load to an applied compressive stress σ_A acting on each element of the cross section we divide the load per unit with, say N_x by an equivalent skin $\bar{t}$ thickness, hence:

$$\sigma_A = \frac{N_x}{\bar{t}}$$

$$\bar{t} = \frac{A_{st}}{b_{sk}} + t_{sk}$$

Instability of stiffened panels

	Support conditions at edges		McDonnell-Douglas	Lockheed	NASA	Convair	Rockwell Int'l	Northrop	Republic	Boeing
			Sources							
Plate panel all edges supported	①		3.62	3.62	4.0	4.0	3.62	3.62	3.62	3.62
	②		6.3	6.3	6.98	6.98	6.98	6.28	6.45	6.3
	③		6.25	6.3	—	6.98	6.98	6.28	6.3	6.3
	④		3.7	6.32	—	4.0	4.0	3.62	3.7	6.32
	⑤		5.1	5.02	—	5.4	—	4.9	5.0	5.0
	⑥		5.0	5.02	—	5.4	5.41	4.9	4.9	5.0
Plate panel one edge free	⑦		1.2	1.16	1.28	1.28	—	1.2	1.05	1.12
	⑧		1.17	1.16	—	1.28	1.28	1.2	1.14	1.12
	⑨		0.45	0.378	—	0.429	—	0.367	0.4	0,4
	⑩		0.429	0.378	0.43	0.429	0.429	0.367	0.388	0.4

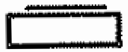
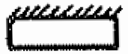
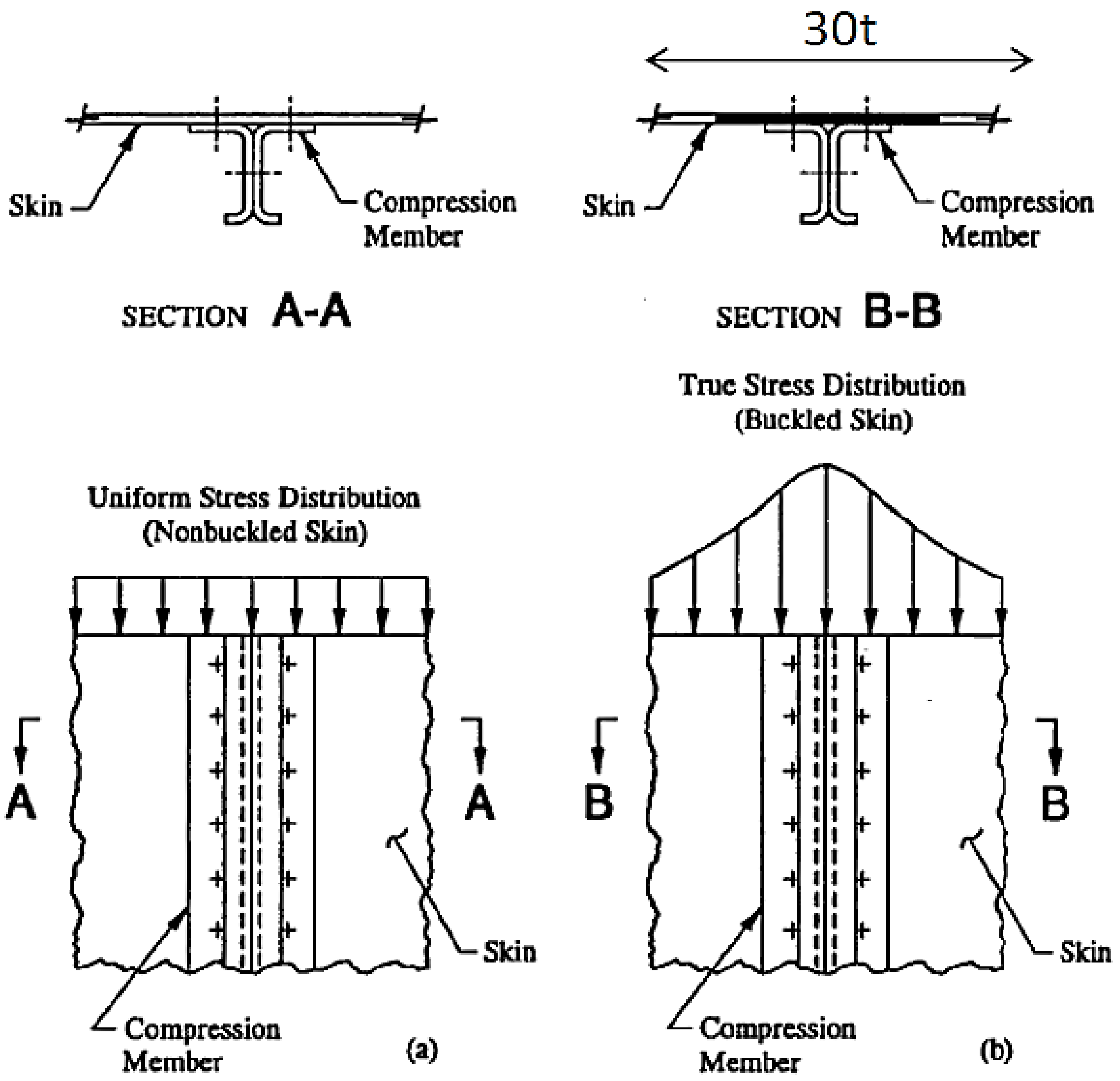
 *Simply supported edge*  *Fixed edge*

Fig. 5.4.3 Compression buckling coefficients K_c(flat plates).


30t
Skin
Compression Member
SECTION A-A
Skin
Compression Member
SECTION B-B
True Stress Distribution (Buckled Skin)
Uniform Stress Distribution (Nonbuckled Skin)
A
A
B
B
Skin
Skin
Compression Member
(a)
Compression Member
(b)

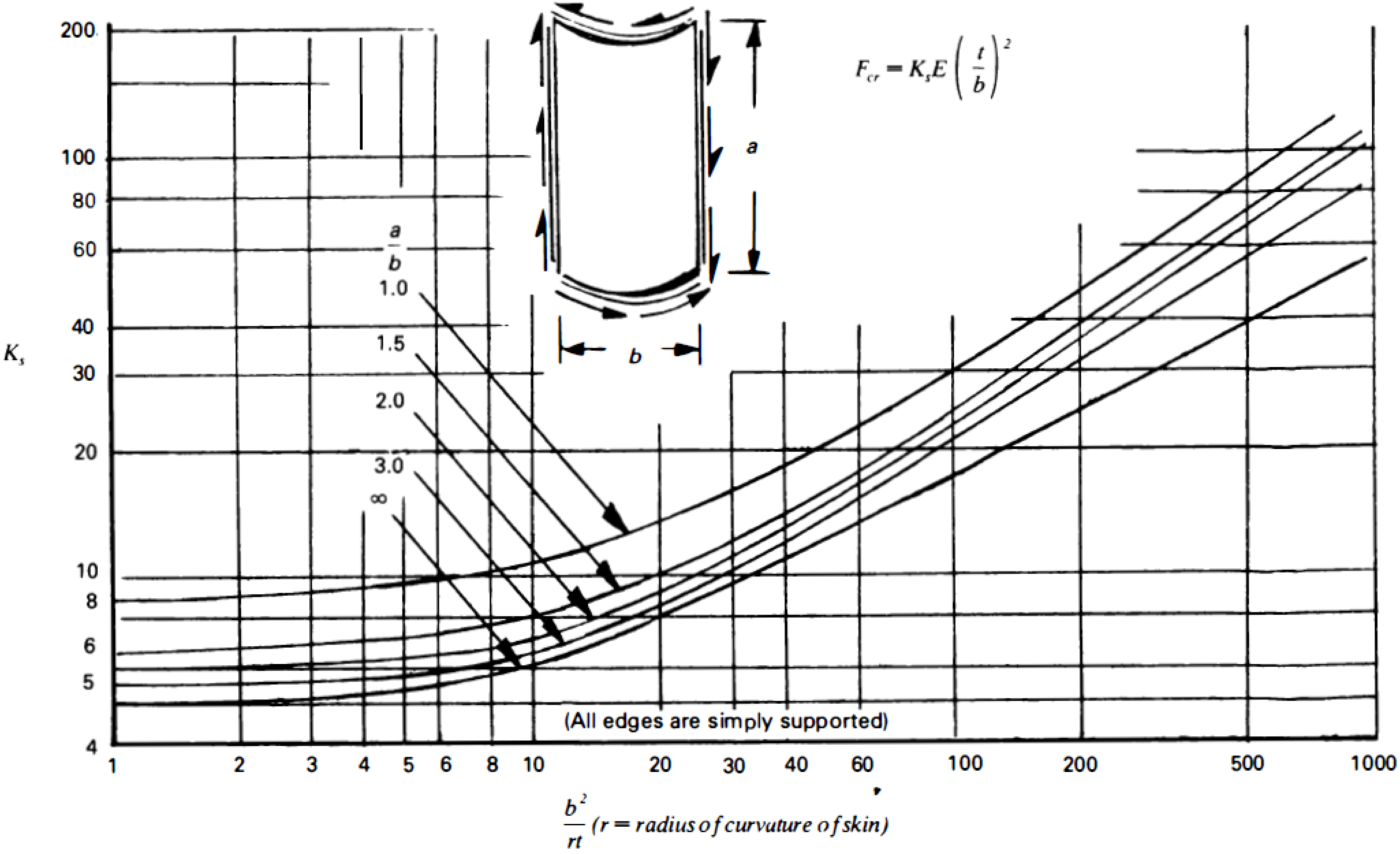


Fig. 5.4.7 Shear buckling coefficients K_s (curved plates, b at curved side).

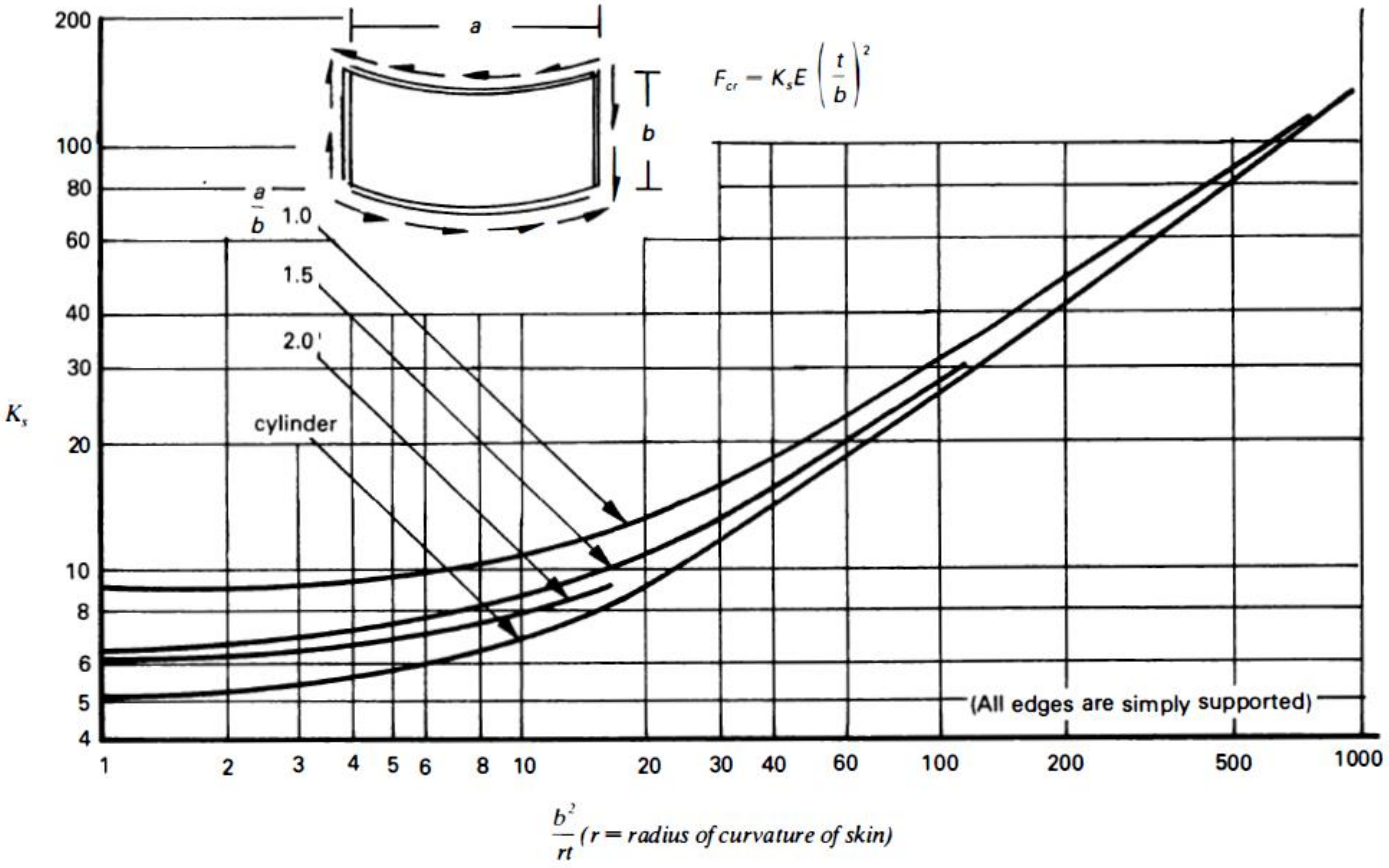


Fig. 5.4.8 Shear buckling coefficients K_s (curved plates, b at straight side).

Example

- A plate $10\,mm$ thick is subjected to bending moments $Mx = 10\,Nm/mm$ and $My = 5\,Nm/mm$. Calculate the maximum direct strain stresses in the plate.

- Find the maximum twisting moment per unit length in the plate $\alpha = 45°$

- If $Mxy = 5\,Nm/mm$ find the principal moments in the plate, the planes on which they act and the corresponding principal stresses.