Universidad Pontificia Bolivariana

80 AÑOS

Vigilada Mineducación

AERONAUTICAL ENGINEERING FACULTY

AERODYNAMICS

1st Mid-term exam – Professor: Juan Pablo Alvarado P.

Due: February 12th

Directions: This mid-term exam is to be completed without help from any other individual. This includes help from other students (whether or not they are taking Aerodynamics), the monitor (Teaching Assistant), other faculty staff, people on the Internet, etc. You are NOT permitted to use your books, class notes, library resources, etc. You only need a pen, pencil, eraser and your calculator. You must provide the numerical or text answer as asked in each problem or question. The extra sheets given to you will be used as a calculation aid. **The procedure will be graded, as well as the answer, by the grade breakdown table below**. Answers in the English language are compulsory for this exam. Answers with wrong or incorrect units will be penalized. The error percentage of the numerical answers is 5.0%. The exam will be in a maximum of one hour and fifty minutes. Absolutely no late exams will be accepted.

Please sign this cover sheet and turn it in with your solution.

I understand and have followed the directions for this exam. The solutions I have turned in represent my understanding:

Name: Andrés Zuleta Signature: ____________

Grade breakdown:

Problem No.	Percentage [%]	Value	Grade
1 procedure	2.0	0.1	0.1
1	8.0	0.4	0.4
2 procedure	8.0	0.4	0.4
2	32.0	1.6	1.6
3 procedure	2.0	0.1	0,1
3	8.0	0.4	0.4
4 procedure	3.0	0.15	0.15
4	12.0	0.6	0.6
5 procedure	3.0	0.15	0.15
5	12.0	0.6	0.6
6 procedure	2.0	0.1	0.1
6	8.0	0.4	0.4
TOTAL GRADE			5.0

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For points 1 to 6, use the following information:

The local velocity (in *km/h*) distribution over a section of a two-dimensional wing inserted in the test section of a wind tunnel is measured as follows:

- Upper surface: V increases from the leading edge to a value of 396 at 2% of the chord distance and then decreases to 387.9, 365.4, and 340.2 at 9%, 12%, and 50% of the chord distance, respectively. At the trailing edge, the local velocity decreases to 240.3.
- Lower surface: V increases from the leading edge to 245.25 and 266.4 at 5% and 70% of the chord distance, respectively.

It is also known that the atmospheric pressure value is 96.92% of the local static pressure at the airfoil's leading edge. The wing section's main aerodynamic airfoil characteristics are a lift slope of 6.436 1/rad and a zero-lift angle of attack of -1.45 [*deg*]. This section has a chord distance of 43 [*cm*]. The test is made at a 1.7 [*km*] height over the sea level. The graph of its drag coefficient versus the angle of attack (Re = 1.0 x 10^6) can be plotted from the following functions:

If the airfoil is a conventional cambered airfoil type (i.e., Clark Y):

- $c_d = 3.05 \times 10^{-5}\alpha^2 - 2.1 \times 10^{-5}\alpha + 0.00695$; for: $-10.15\,[deg] \le \alpha \le -3.03[deg]$
- $c_d = 8.5 \times 10^{-7}\alpha^3 + 4.74 \times 10^{-5}\alpha^2 + 8.07 \times 10^{-5}\alpha + 0.00713$; for: $-3.03\,[deg] \le \alpha \le 2.31[deg]$
- $c_d = 6.23 \times 10^{-5}\alpha^2 + 1.89 \times 10^{-6}\alpha + 0.00725$; for: $2.31\,[deg] \le \alpha \le 9.43[deg]$

If the airfoil is a laminar airfoil type (i.e., NASA/LANGLEY NLF(1)-0215F):

- $c_d = 3.05 \times 10^{-5}\alpha^2 - 8.77 \times 10^{-6}\alpha + 0.00494$; for: $-10.35\,[deg] \le \alpha \le -3.23[deg]$
- $c_d = 1.72 \times 10^{-5}\alpha^3 + 3.25 \times 10^{-4}\alpha^2 + 2.78 \times 10^{-4}\alpha + 0.00338$; for: $-3.23\,[deg] \le \alpha \le 2.11[deg]$
- $c_d = 6.23 \times 10^{-5}\alpha^2 + 2.68 \times 10^{-5}\alpha + 0.00525$; for: $2.11\,[deg] \le \alpha \le 9.23[deg]$

You are asked to find:

1. The value of the air density for this flight condition ρ_∞ = 1.08725 [kg/m^3]
2. The value of the angle of attack at which this test is performed α = 4.514° [*deg*]
3. The value of the lift force per unit of span L' = 755.1499 [*N*]
4. The value of the aerodynamic efficiency at an AoA of 1.3 *degrees*: *L/D* = 42.2072
5. Moment coefficient with respect to the leading edge if $c_{m\_c/4}$ is equal to -0.04 and the airfoil is at the same angle of attack as point four: c_{m_LE} = -0.11720675
6. If the airfoil is inverted, calculate the angle of attack to obtain the same value of L' of point three: α = 7.4153 [*deg*]

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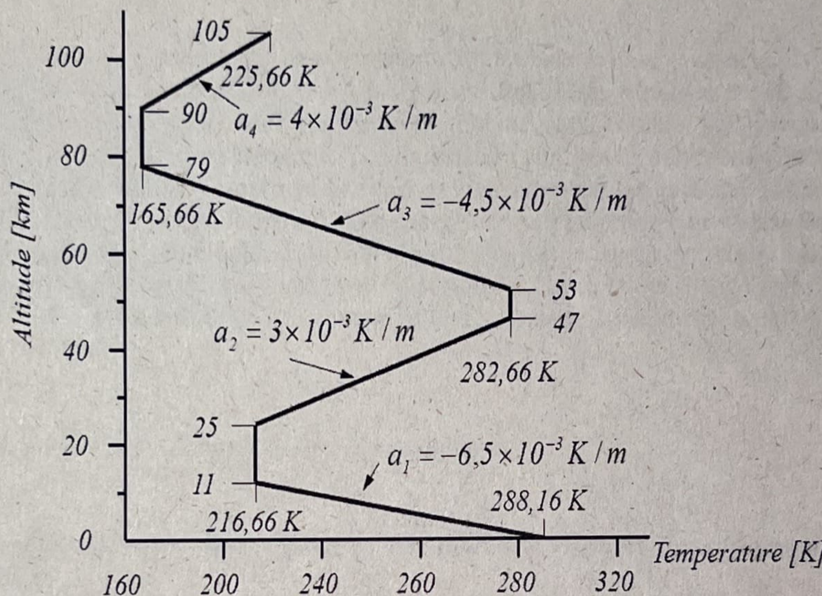
Sheet of useful formulas:



*(Hints: $\rho_{@SL}$ = 1.225 [kg/m³]; $p_{@SL}$ = 101325.0 [Pa]; R = 287.0 [J/(kg*K)]; $\mu_{@SL}$ = 1.789 x 10^{-5} [kg/(m*seg)]; g_0 = 9.81 [m/s²])*

Power law relation: $\mu = \mu_0\left(\dfrac{T}{T_0}\right)^{\left(\frac{2}{3}\right)}$

Isothermal:

- $\dfrac{p}{p_1} = e^{-\left(\frac{g_0}{R\cdot T}\right)(h-h_1)}$
- $\dfrac{\rho}{\rho_1} = e^{-\left(\frac{g_0}{R\cdot T}\right)(h-h_1)}$

Gradient:

- $T = T_1 + a\,(h-h_1)$
- $\rho = \rho_1\left(\dfrac{T}{T_1}\right)^{-\left(\frac{g_0}{aR}+1\right)}$
- $\dfrac{p}{p_1} = \left(\dfrac{T}{T_1}\right)^{-\frac{g_0}{a\cdot R}}$

$$q_\infty = \frac{\rho_\infty \cdot V_\infty^2}{2}$$

$$L_{airfoil} = q_\infty S c_l$$

$$D_{airfoil} = q_\infty S c_d$$

$$C_p = \frac{p - p_\infty}{q_\infty}$$

$$p + \frac{1}{2}\rho_\infty V_\infty^2 = p_1 + \frac{1}{2}\rho_\infty V_1^2 = const.$$

Nombre: Andrés Zuleta Cardona   Fecha: 12 02 24

Profesor: Alvarado   Materia: Aerodinámica

Institución: PARCIAL 1   Curso: Nota:

Upper

x/c	V (Km/h)	V (m/s)
0	0	0
0.02	396	110
0.09	387.9	107.75
0.12	365.4	101.5
0.5	340.2	94.5
1	240.3	66.75

$P_\infty = 0.9692\, P_{LE}$

$a_0 = 6.436 \frac{1}{rad} = 0.1123 \frac{1}{°}$

$\alpha_{L=0} = -1.45°$

$c = 0.43\,m$

$h = 1.7\,Km$

$P_1 = 101325\,Pa$

$\rho_1 = 1.225\,kg/m^3$

$T = 288.16\,K$

Lower

x/c	V (Km/h)	V (m/s)
0	0	0
0.05	245.25	68.125
0.7	266.4	74
1	240.3	66.75

$$\rho = \frac{P}{RT}$$

1)

$$T_{1.7} = T_1 + a(h - h_1) = 288.16K - 6.5\cdot10^{-3}\frac{K}{m}(1700m) = 277.11K$$

$$P = P_1\left(\frac{T}{T_1}\right)^{\frac{-g}{aR}} = 101325\,Pa\left(\frac{277.11}{288.16}\right)^{\frac{-9.81}{-6.5\cdot10^{-3}\cdot287}} = 82492.975\,Pa$$

$$\rho = \frac{82492.975\,Pa}{287\cdot 277.11\,K} = 1.03725\,kg/m^3$$

2)

$$C_p = \frac{P - P_\infty}{q_\infty} \qquad q_\infty = \frac{1}{2}\rho V^2$$

In order to find V_∞

$$q = \frac{P - P_\infty}{C_p} \qquad \text{En Leading Edge} \quad q = P - P_\infty$$

$$P = \frac{82492.975\,Pa}{0.9692} = 85114.50165\,Pa$$

$$\frac{1}{2}\rho V^2 = P - P_\infty \qquad V = \sqrt{\frac{2(P - P_\infty)}{\rho}}$$

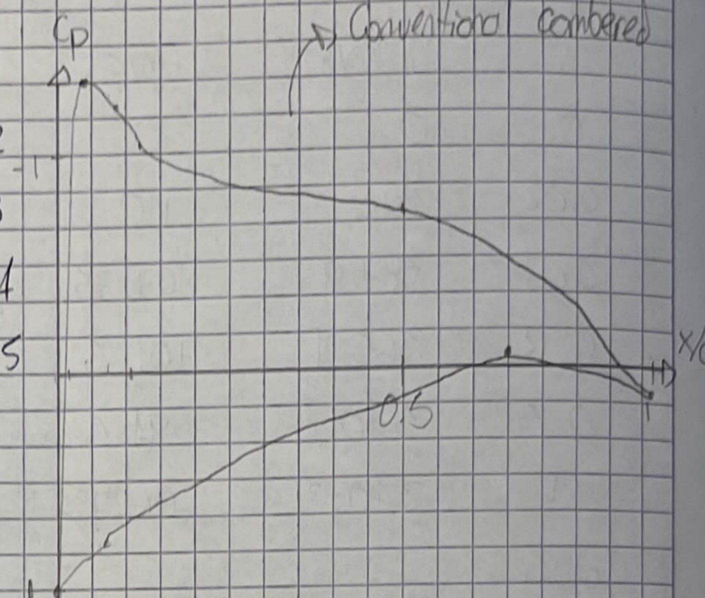
$$V_\infty = \sqrt{\frac{2(85114.30165 - 82492.975)}{1.03725\ kg/m^3}} = 71.0969\ m/s$$

But also equalling C_p with bernouli:

$$C_p = 1 - \left(\frac{V}{V_\infty}\right)^2 \rightarrow \text{Calculating}$$

Upper

x	V (m/s)	C_p	
0	0	1	
0.02	110	-1.3938	1
0.09	107.75	-1.2969	2
0.12	101.5	-1.0381	3
0.5	94.5	-0.7667	4
1	66.75	0.1185	5



Lower

X/C	V(m/s)	C_p	
0	0	1	
0.05	68.125	0.0818	1
0.7	74	-0.0833	2
1	66.75	0.1185	3

Upper

① 0 - 0.02 $\quad y - 1 = \frac{-1.3938 - 1}{0.02}(x - 0) \quad y = -119.69x + 1$ ✓

② 0.02 - 0.09 $\quad y + 1.3938 = \frac{-1.2969 + 1.3938}{0.09 - 0.02}(x - 0.02) \quad y = 1.3843x - 1.4215$ ✓

③ 0.09 - 0.12 $\quad y + 1.2969 = \frac{-1.0381 + 1.2969}{0.12 - 0.09}(x - 0.09) \quad y = \frac{647}{75}x - 2.0733$ ✓

④ 0.12 - 0.5 $\quad y + 1.0381 = \frac{-0.7667 + 1.0381}{0.5 - 0.12}(x - 0.12) \quad y = \frac{1357}{1900}x - 1.1238$ ✓

⑤ 0.5 - 1 $\quad y + 0.7667 = \frac{0.1186 + 0.7667}{1 - 0.5}(x - 0.5) \quad y = 1.7704x - 1.6519$ ✓

Lower

① 0 - 0.05 $\quad y - 1 = \frac{0.0818 - 1}{0.05}(x - 0) \quad y = -\frac{4591}{250}x + 1$ ✓

② 0.05 - 0.7 $\quad y - 0.0818 = \frac{-0.0833 - 0.0818}{0.7 - 0.05}(x - 0.05) \quad y = -0.254x + 0.0945$ ✓

③ 0.7 - 1 $\quad y + 0.0833 = \frac{0.1186 + 0.0833}{1 - 0.7}(x - 0.7) \quad y = \frac{1009}{1500}x - \frac{133}{240}$ ✓

$$C_\ell = \int C_{p_L} - C_{p_U}\,dx$$

$$C_\ell = \int_0^{0.05} -\frac{4591}{250}x + 1\,dx + \int_{0.05}^{0.7} -0.254x + 0.0945\,dx + \int_{0.7}^{1} \frac{1009}{1500}x - \frac{133}{240}\,dx$$

$$-\left[\int_{0}^{0.02} -119.69x + 1\,dx + \int_{0.02}^{0.09} 1.3843x - 1.4215\,dx + \int_{0.09}^{0.12} \frac{647}{75}x - 2.0733 + \int_{0.12}^{0.5} \frac{1357}{1900}x - 1.1238 + \int_{0.5}^{1} 1.7704x - 1.6519\right]$$

$$C_l = 0.0318375 - [-0.638098145]$$

$$C_l = 0.6699$$

$$a_0 = \frac{C_l - 0}{\alpha - \alpha_{L=0}}$$

$$\alpha - \alpha_{L=0} = \frac{C_l}{a_0}$$

$$\alpha = \frac{C_l}{a_0} + \alpha_{L=0}$$

$$\alpha = \frac{0.6699}{0.1123\frac{1}{°}} - 1.45$$

$$\alpha = 4.514°$$

③

$$L = q_\infty S\, c_l$$

$$L = \frac{1}{2} \cdot 1.03725\,\frac{kg}{m^3} \cdot \left(71.0969\,\frac{m}{s}\right)^2 \cdot 0.43m \cdot 1m \cdot 0.6699$$

$$L' = 755.1499\ N$$

④ @ 1.3°

$$a_0 = \frac{C_l - 0}{\alpha - \alpha_{L=0}}$$

$$C_l = a_0(\alpha - \alpha_{L=0})$$

$$C_l = 0.1123\frac{1}{°}\,(1.3° - (-1.45))$$

$$C_l = 0.308825$$

Considering Conventional Cambered airfoil

$$C_d = 8.5 \cdot 10^{-7}\alpha^3 + 4.74 \cdot 10^{-5}\alpha^2 + 8.07 \cdot 10^{-5}\alpha + 0.00713$$

$$C_d = 7.31688 \cdot 10^{-3}$$

$$\frac{L}{D} = \frac{q_\infty S\, C_l}{q_\infty S\, C_D} = \frac{C_l}{C_D}$$

$$\frac{L}{D} = \frac{0.308825}{7.31688 \cdot 10^{-3}}$$

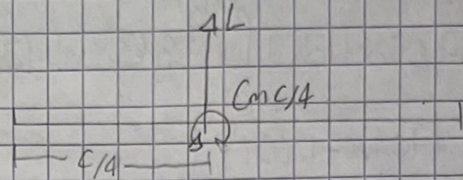
$$\frac{L}{D} = 42.2072$$

(5) $\alpha = 1.3°$

$C_{m_{c/4}} = -0.04$

$C_l = 0.308825$

$$C_{m_{LE}} = C_{m_{c/4}} - C_l\,(h_{c/4})$$



$$C_{m_{LE}} = -C_{m_{c/4}} - C_l\,h_{c/4}$$

$$C_{m_{LE}} = -0.04 - 0.25 \cdot 0.308825$$

$$C_{m_{LE}} = -0.11720625 \,//$$

(6) If airfoil is inverted: $\alpha_{L=0} = 1.45°$

$$L' = \frac{1}{2}\rho V^2 S\, C_l$$

$$C_l = \frac{L'}{\frac{1}{2}\rho V^2 S} = \frac{755.1499\,N}{\frac{1}{2} \cdot 1.03725\,\frac{kg}{m^3} \cdot (71.0969\,\frac{m}{s})^2 \cdot 0.43\,m}$$

$C_l = 0.6699$ → Same as in regular condition

$$a_0 = \frac{0 - C_l}{\alpha_{L=0} - \alpha}$$

$$\alpha_{L=0} - \alpha = \frac{-C_l}{a_0}$$

$$\alpha = \frac{C_l}{a_0} + \alpha_{L=0}$$

$$\alpha = \frac{0.6699}{0.1123\,\frac{1}{°}} + 1.45$$

$$\alpha = 7.4153° \,//$$

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AERONAUTICAL ENGINEERING FACULTY

NRC: 20493 AERODYNAMICS

2nd Mid-term exam – Professor: Juan Pablo Alvarado P.

Due: March 4th at 10:00 am

Directions: This mid-term exam is to be completed without help from any other individual. This includes help from other students (whether or not they are taking Aerodynamics), the monitor (Teaching Assistant), other faculty staff, people on the Internet, etc. You are NOT permitted to use your books, class notes, library resources, etc. You only need a pen, pencil, eraser, and your calculator. You must provide the numerical or text answer as asked in each problem or question. The extra sheets given to you will be used as a calculation aid. **The procedure will be graded, as well as the answer, by the grade breakdown table below**. Answers in the English language is compulsory for this exam. Answers with wrong or incorrect units will be penalized. The error percentage of the numerical answers is 5.0%. The exam will be in a maximum of one hour and fifty minutes. Absolutely no late exams will be accepted.

Please sign this cover sheet and turn it in with your solution.

I understand and have followed the directions for this exam. The solutions I have turned in represent my understanding:

Name: Andrés Zuleta Signature:

Grade breakdown:

Problem No.	Percentage [%]	Value	Grade
1 procedure	5	0.25	0.25
1	12.5	0.625	0.625
2 procedure	5	0.25	0.25
2	12.5	0.625	0.625
3 procedure	5	0.25	0.25
3	12.5	0.625	0.73
4	15	0.75	0.08
5 procedure	5	0.25	0
5	12.5	0.625	0
6	15	0.75	0.75
TOTAL GRADE			3.16

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For points 1, and 2, use the following information:

A group of aeronautical engineers are designing a new airplane with a gross take-off weight of 2000 [kg] and a wing area of 20 [m²]. The operational cruise-flight speed required is 62.5 [m/s], and the stall speed needs to be 48% of that established for the cruise-flight. You have the job of studying the selected airfoil that will be used in the wing, whose mean-camber-line section is described by the following functions:

- Front part: $\frac{z}{c} = 0.08\left(\frac{x}{c}\right) - 0.08\left(\frac{x}{c}\right)^2$; $for\ 0 \leq \frac{x}{c} \leq 0.5$
- Rear part: $\frac{z}{c} = 0.08\left(\frac{x}{c}\right) - 0.08\left(\frac{x}{c}\right)^2$; $for\ 0.5 \leq \frac{x}{c} \leq 1.0$

It is known that the airfoil's lift coefficient is twenty two percent higher than the wing's lift coefficient. Assuming that the airplane is operating at sea level conditions ($\rho_\infty = 1.225$ [kg/m³]), find:

1. The airfoil's angle of attack, which, according to the design requirements, must be less than 1.0 degrees:

$\alpha_{airfoil\_cruise\text{-}flight} =$ 2.2697° [deg] ✓

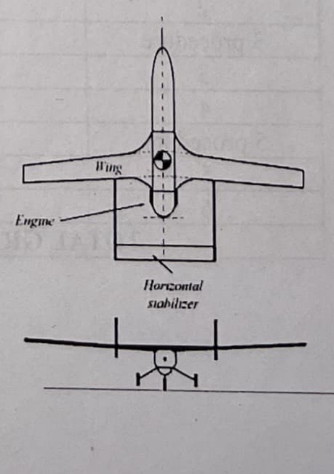
2. A plain flap will be installed on the wing's trailing edge (at 80.0 % from the leading edge) for low-speed (stall) flight conditions such as landing (assume that the airfoil/flap combination lift coefficient is twenty two percent higher than the wing/flap lift coefficient for this flight condition). You are asked to calculate the deflection angle if the airfoil is at the same angle of attack as point 1. to accomplish the stall requirements:

$\delta_f =$ -20.3758 [deg] ✓

For points 3, 4, 5 and 6, use the following information:

An unmanned aerial vehicle (UAV) is used for surveillance missions to inspect rural highways. During service, the operators discovered a strange movement of the airplane when they tried to change the flight path causing a sudden change in pitch-up attitude. They suspect the problem is related to the horizontal stabilizer (HS) because this behavior occurs when they apply the elevator control.

You are hired to correct this undesirable condition by conducting a study and analysis in the wind tunnel (**sea level cond.**). The horizontal stabilizer uses a symmetrical airfoil at the aircraft's back part (empennage). This surface is attached between the two vertical tails (VTs); therefore, you can assume that it behaves as an infinite wing surface, as shown in the following figure:



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The HS has a fixed incidence angle of -2.0 [deg]. The maximum deflection of the elevator surface is ±5.0 [deg]. The HS has a chord distance of 28.0 [cm], and the elevator extends 100% of the span (0.98 [m]) with a chord equal to 7.0 [cm]. The study inside the wind tunnel is performed at a cruise speed flight condition of 90 [m/s], giving the following tendency: $\cdot 10^{-6}$

$$D_{total} = 3.222 \times 10^{-12} Re_{x_{crit}}^2 - 3.709 Re_{x_{crit}} + 20.7$$

3. With the previous information, calculate and complete the following table:

$\delta_{elevator}$ [deg]	Re_{x_crit}	x_{crit} [cm]	D_f [N]	D_f [%]	D_p [N]	D_p [%]
+5	1'657,000	26.9 ✓	2.7174 ✗	11.62 ✗	20.6826 ✗	88.38 ✗
0	720,000	11.69 ✓	2.0852 ✗	10.58 ✗	17.6146 ✗	89.42 ✗
-5	1'295,000	21.03 ✓	2.5052 ✗	11.762 ✗	18.7949 ✗	88.238 ✗

4. Analyzing the drag behavior obtained in point 3., determine the possible causes of the undesirable pitching movements of the airplane during the three cases:

~~The vast majority of the undesired pitching moment is due to the force of lift created by the HS and the drag force since the HS is higher than the center of gravity and this contributes to the additional pitch up moment. You can obtain the least moment produced by the HS if the elevator is NOT deflected. When the elevator is deflected down, the pilot won't feel the pitch up moment since he is making the aircraft pitch down. When it's deflected up, the aircraft will generate more pitch up momentum than the desired.~~

The main cause is that the Horizontal stabilizer is not at the same height of the center of gravity.

5. A member of your team suggested taping a thin sandpaper strip (0.98 [m] x 0.01 [m]) along the span on one of the HS surfaces. By doing so, choose the surface (mark with an x) and the location along the chord at which the strip must be bonded.

CG L D

L & D generate pitch up movements

Surface: Upper ~~X~~ X
Lower ~~X~~ X X

$x_{sandpaper\_strip}$ = 0. ______ [m] → As close to the Leading Edge as possible

CG L D

L generates pitch down and D generates pitch up

6. Justify the answers given in point 5.:

~~Since the pitch up moment is experienced when the elevator is deflected up, I consider the sandpaper plate should go at ___ For the elevator deflected at 5° I want to force the turbulence before the transition point in order to reduce the pressure drag, which is the one that contributes the vast majority of drag, thus moment.~~ As close to the LE

CG D

D generates pitch up

The undesired pitch up moment on all 3 conditions are due to the Drag Force since the horizontal stabilizer is higher than the center of gravity. Therefore we want to reduce the drag force as much as possible. To do so, we have to induce turbulence in the flow as soon as it comes in contact with the airfoil to reduce as much as possible the pressure drag that currently represents >88% of the total drag. I believe putting the strips on both surfaces will reduce drag as much as possible in any of the 3 conditions.

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Sheet of useful formulas:

Fourier sine coefficients (airfoil):

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz}{dx} d\theta$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx} \cos n\,\theta d\theta$$

$$c_{m,le} = -\frac{\pi}{2}\left(A_0 + A_1 - \frac{A_2}{2}\right)$$

$$= -\left[\frac{c_l}{4} + \frac{\pi}{4}(A_1 - A_2)\right]$$

$$c_{m,c/4} = \frac{\pi}{4}(A_2 - A_1)$$

TAT generalized equations:

$$c_l = \pi(2A_0 + A_1) = a_0(\alpha - \alpha_{L=0});\ \alpha[rad]$$

$$a_0 = 2\pi\left[{}^1/_{rad}\right]$$

$$\alpha_{L=0} = -\frac{1}{\pi}\int_0^\pi \frac{dz}{dx}(\cos\theta - 1)d\theta$$

Airfoil transformation x to θ:

$$x = \frac{c}{2}(1 - \cos\theta)$$

Fourier sine coefficients (airfoil):

$$A_0 = \alpha - \frac{1}{\pi}\int_0^\pi \frac{dz}{dx} d\theta$$

$$A_n = \frac{2}{\pi}\int_0^\pi \frac{dz}{dx} \cos n\,\theta d\theta$$

Fourier sine coefficients (airfoil with flap):

$$A_0 = \alpha - \frac{1}{\pi}\left(\int_0^\phi \frac{dz}{dx} d\theta + \int_\phi^\pi -\delta_f d\theta\right)$$

$$A_n = \frac{2}{\pi}\left(\int_0^\phi \frac{dz}{dx} \cos n\,\theta d\theta + \int_\phi^\pi -\delta_f \cos n\,\theta d\theta\right)$$

Flapped surface transformation F to ϕ:

$$(1 - F) = \frac{1}{2}(1 - \cos\phi);\ \delta_f = -\frac{h}{F}$$

Reynolds number:

$$Re_x = \frac{\rho_\infty \cdot V_\infty \cdot x}{\mu_\infty}$$

For laminar boundary layer:

$$\delta_{lam} = \frac{5x}{\sqrt{Re_x}};\ C_{f,lam} = \frac{1.328}{\sqrt{Re_x}}$$

For turbulent boundary layer:

$$\delta_{tur} = \frac{0.37x}{Re_x^{1/5}};\ C_{f,tur} = \frac{0.074}{Re_x^{1/5}}$$

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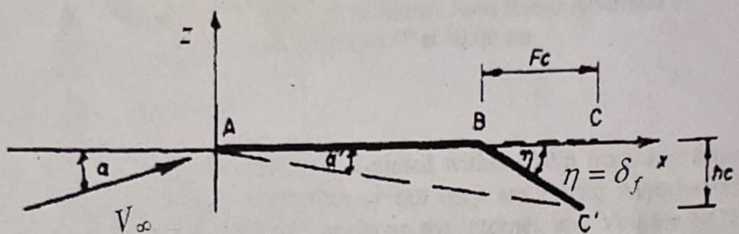
- Nomenclature



Integrals solution:

$$\int \cos\theta \, d\theta = \sin\theta$$

$$\int \cos^2\theta \, d\theta = \frac{1}{2}\sin\theta\cos\theta + \frac{\theta}{2}$$

$$\int \cos^3\theta \, d\theta = \frac{1}{3}\sin\theta\left(\cos^2\theta + 2\right)$$

$$\int \cos^4\theta \, d\theta = \frac{1}{4}\cos^3\theta\sin\theta + \frac{3}{8}\left(\sin\theta\cos\theta + \theta\right)$$

Nombre: Andrés Zuleta Cardona

Fecha: 4 03 2024

Profesor: Alvarado

Materia: Aerodinámica

Institución: PARCIAL 2

Curso:

Nota:

1.

$W = 2000\,kg$

$S_{wing} = 20\,m^2$

$V_{cruise} = 62.5\,\frac{m}{s}$

$V_{stall} = 0.48\,V_{cruise} = 30\,\frac{m}{s}$

$\rho = 1.225\,\frac{kg}{m^3}$

$C_{l_{airfoil}} = 1.22\,C_{L_{wing}}$

$C_{l_{airfoil}} = 2\pi(\alpha - \alpha_{L=0}) \rightarrow \alpha = \frac{1.22\,C_{L_{wing}}}{2\pi} + \alpha_{L=0}$

$L = W$

$$C_{L_{wing}} = \frac{L}{\frac{1}{2}\rho V^2_{cruise} S_{wing}} = \frac{(2000\,kg \cdot 9.81)\,N}{\frac{1}{2} \cdot 1.225\frac{kg}{m^3} \cdot (62.5\frac{m}{s})^2 \cdot 20\,m^2}$$

$C_{L_{wing}} = 0.41002$

For $\alpha_{L=0}$:

$$\alpha_{L=0} = \frac{-1}{\pi}\int_0^{\pi} \frac{dz}{dx}(\cos\theta - 1)\,d\theta \qquad \theta = \cos^{-1}(1 - 2X_{max})$$

$$\frac{dz}{dc} = 0.08 - 2 \cdot 0.08\left(\frac{x}{c}\right) \qquad 0 < \frac{x}{c} < 1$$

$$\frac{dz}{dx} = 0.08 - 0.08(1 - \cos\theta) \qquad 0 < \theta < \pi$$

$$\alpha_{L=0} = \frac{-1}{\pi}\int_0^{\pi} (0.08 - 0.08(1 - \cos\theta))(\cos\theta - 1)\,d\theta$$

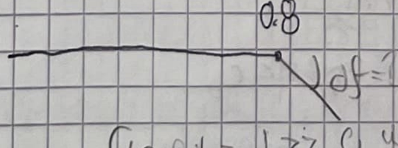
$\alpha_{L=0} = -0.04$ rad

$C_{l_{flap}} = C_{l_{clean}} + \Delta C_l$

$$\alpha = \frac{1.22 \cdot 0.41002}{2\pi} - 4 \cdot 10^{-3}\,rad = 0.0396\,rad = 2.2697^{0}$$

2.

$\alpha_{clean} = 0.0396\,rad$



$\phi = \cos^{-1}(1 - 2(1-F)) = \cos^{-1}(1 - 2 \cdot 0.8)$

$\phi = 2.2142$

$C_{l_{airfoil\,flap}} = 1.22\,C_{L_{wing\,flap}}$

$C_{l_{airfoil}} = C_{l_{clean}} + \Delta C_l$

$\Delta C_l = 1.22\,C_{L_{wing\,flap}} - C_{l_{clean}}$

$$C_{L_{wing\,flap}} = \frac{L}{0.5\,\rho\,V_{stall}^2\cdot S} = \frac{(2000\cdot 9.81)\,N}{0.5\cdot 1.225\frac{kg}{m^3}\cdot\left(30\frac{m}{s}\right)^2\cdot 20\,m^2} = 1.7796$$

$$\alpha_{real} = \alpha_{clean} + \tan^{-1}\left(\frac{F\sin|\delta_f|}{1+F(\cos\delta_f-1)}\right)$$

$$\Delta C_l = 1.22\cdot 1.7796 - 1.22\cdot 0.41002 = 1.67088$$

$$\Delta C_l = C_{l\,flat\,plate\,flapped} - C_{l\,flat\,plate}$$

$$\Delta C_l = 2\pi(\alpha_{real} - \alpha_{L=0}) - 2\pi(\alpha_{clean} - \alpha_{L=0})$$

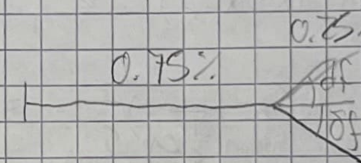
$$\Delta C_l = 2\pi\left(\alpha_{clean} + \tan^{-1}\left(\frac{F\sin\delta_f}{1+F(\cos\delta_f-1)}\right) + \frac{1}{\pi}\int_0^{\phi}\frac{dz}{dx}(\cos\theta-1)\,d\theta + \int_{\phi}^{\pi}\delta_f(\cos\theta-1)\,d\theta\right) - 2\pi\alpha_{clean}$$

$$\Delta C_l = 2\pi\left[\tan^{-1}\left(\frac{F\sin\delta_f}{1+F(\cos\delta_f-1)}\right) + \frac{\delta_f}{\pi}(-\sin\phi - \pi + \phi)\right]$$

$$1.67088 = 2\pi\left[\tan^{-1}\left(\frac{0.2\sin|\delta_f|}{1+0.2(\cos|\delta_f|-1)}\right) + \frac{\delta_f}{\pi}(-\sin(2.21429) - \pi + 2.21429)\right] \rightarrow \text{Solving}$$

$$\delta_f = -0.3556\ \text{rads} = -20.3757^\circ$$

(3)



$\alpha_{HS} = -2^\circ$

$c_{HS} = 0.28\,m = 1$

$c_f = 0.07\,m = 0.25$

$V = 90\,\frac{m}{s}$ $\quad \rho = 1.225\,\frac{kg}{m^3}$ $\quad \mu = 1.79\cdot 10^{-5}\,\frac{kg}{m\,s}$

x_{crit}: $\quad Re_{x_{crit}} = \dfrac{\rho_\infty V_\infty x_{crit}}{\mu_\infty}$

$$x_{crit} = \frac{Re_{x_{crit}}\cdot\mu_\infty}{\rho_\infty V_\infty}$$

For $\delta_f = 5^\circ$:

$Re_{x,crit} = 1657000$

$$x_{crit} = \frac{1657000\cdot 1.79\cdot 10^{-5}\,\frac{kg}{m\,s}}{1.225\frac{kg}{m^3}\cdot 90\frac{m}{s}} = 0.269\,m = 26.9\,cm$$

For $\delta_f = 0$

$Re_{x,crit} = 720000$

$x_{crit} = 0.11689\,m = 11.69\,cm$

For $\delta_f = -5^\circ$

$Re_{x,crit} = 1295000$

$x_{crit} = 0.2103\,m = 21.03\,cm$

D_f

$D = q\,S\,C_D$

$D_f = D_{f\,lam,x} + D_{f\,turb,c} - D_{f\,turb,x}$

$$D_f = qS\left(C_{f,lam,x} + C_{f,turb,c} - C_{f,turb,x}\right) = qS\left(\frac{1.328}{\sqrt{Re_x}} + \frac{0.074}{Re_c^{1/5}} - \frac{0.074}{Re_x^{1/5}}\right)$$

$$q = \frac{1}{2}\cdot 1.225\frac{kg}{m^3}\left(90\frac{m}{s}\right)^2 = 4961.25\,Pa$$

$$Re_c = \frac{\rho_\infty V c}{\mu} = \frac{1.225\frac{kg}{m^3}\cdot 90\frac{m}{s}\cdot 0.28\,m}{1.79\cdot 10^{-5}\frac{kg}{m\,s}} = 1724581$$

For $df = 5°$

$$D_f = 496125\,Pa \cdot 0.28\,m \cdot 0.98\,m \left(\frac{1.328}{\sqrt{1657000}} + \frac{0.074}{1724581^{1/5}} - \frac{0.074}{(1657000)^{1/5}} \right)$$

$D_f = 1.3587\,N \cdot 2 = 2.7174\,N$

For $df = 0°$

$D_f = 1.04259\,N \cdot 2 = 2.08518\,N$

For $df = -5°$

$D_f = 1.2526\,N = 2.5052\,N$

For D_f %

$$D_{Total} = 3.222 \cdot 10^{-12} Re_x^2 - 3.709 \cdot 10^{-6} Re_x + 20.7$$

$$D_f\% = \frac{D_f}{D_T}$$

For $df = 5°$

$$D_{Total} = 3.222 \cdot 10^{-12} (1657000)^2 - 3.709 \cdot 10^{-6} (1657000) + 20.7$$

$D_{Total} = 23.4\,N$

$$D_f(\%) = \frac{2.7174\,N}{23.4\,N} = 0.1161 = 11.62\,\%$$

For $df = 0°$

$D_{Total} = 19.6998\,N$

$$D_f(\%) = \frac{1.04259 \cdot 2\,N}{19.6998\,N} = 0.1058 = 10.58\,\%$$

For $df = -5°$

$D_{Total} = 21.3\,N$

$$D_f(\%) = \frac{1.2526 \cdot 2\,N}{21.3\,N} = 0.1176 = 11.76\%$$

For D_p and $D_p(\%)$

$D_p = D_T - D_f$

$$D_p(\%) = \frac{D_p}{D_T}$$

For $Df = 5°$

$D_p = 23.4 - 1.3587 \cdot 2 = 20.6826$

$$D_p(\%) = \frac{20.6826}{23.4}$$

$D_p(\%) = 88.38\%$

for $\delta f = 0$

$$D_p = 17.6146\ N \qquad D_p(\%) = 89.42\%$$

for $\delta f = -5°$

$$D_p = 18.7948\ N \qquad D_p(\%) = 88.238\%$$

X sandpaper

$$C_l = C_{l\,perfil} + \Delta C_l$$

$$\Delta C_l = 2.17112 - 0.5$$

$$\Delta C_l = 1.67088$$

$$\Delta C_l = C_{l\,flat\ plate,\ flapped} - C_{l\,flat\ plate}$$

$$\Delta C_l = 2\pi(\alpha_{real} - \alpha_{L=0}) - 2\pi(\alpha_{clean} - \cancelto{0}{\alpha_{L=0}}) \quad \text{porque es flat plate}$$

$$\Delta C_l = 2\pi\left[\alpha_{clean} + \tan^{-1}\left(\frac{0.25\,sen\,|\delta f|}{1+0.2(\cos|\delta f|-1)}\right) + \frac{1}{\pi}\left[\int_0^{\phi} \cancelto{0}{\frac{dz}{dx}}(\cos\theta-1)\,d\theta + \int_{\phi}^{\pi} \delta f(\cos\theta-1)\,d\theta\right]\right] - 2\pi\alpha_{clean}$$

$$\Delta C_l = 2\pi\left[\alpha_{clean} + \tan^{-1}\left(\frac{0.25\,sen|\delta f|}{1+0.2(\cos|\delta f|-1)}\right) + \frac{\delta f}{\pi}(-\pi - sen\,\phi + \phi)\right] - 2\pi\alpha_{clean}$$

$$\Delta C_l = 2\pi\left[\tan^{-1}\left(\frac{0.25\,sen|\delta f|}{1+0.2(\cos|\delta f|-1)}\right) + \frac{\delta f}{\pi}(\phi - \pi - sen\,\phi)\right]$$

$$1.67088 = 2\pi\left[\tan^{-1}\left(\frac{0.25\,sen|\delta f|}{1+0.2(\cos|\delta f|-1)}\right) + \frac{\delta f}{\pi}(2.21429 - \pi - sen\,2.21429)\right]$$

$$\delta f = -0.3556\ rad = -20.3756$$

5.

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80 AÑOS

Vigilada Mineducación

AERONAUTICAL ENGINEERING FACULTY

NRC: 20493 AERODYNAMICS

3rd Mid-term exam – Professor: Juan Pablo Alvarado P.

Due: April 29th at 10:00 am

Directions: This mid-term exam is to be completed without help from any other individual. This includes help from other students (whether or not they are taking Aerodynamics), the monitor (Teaching Assistant), other faculty staff, people on the Internet, etc. You are NOT permitted to use your books, class notes, library resources, etc. You only need a pen, pencil, eraser, and your calculator. You must provide the numerical or text answer as asked in each problem or question. The extra sheets given to you will be used as a calculation aid. **The procedure will be graded, as well as the answer, by the grade breakdown table below**. Answers in the English language is compulsory for this exam. Answers with wrong or incorrect units will be penalized. The error percentage of the numerical answers is 5.0%. The exam will be in a maximum of one hour and fifty minutes. Absolutely no late exams will be accepted.

Please sign this cover sheet and turn it in with your solution.

I understand and have followed the directions for this exam. The solutions I have turned in represent my understanding:

Name: Andrés Zuleta Signature:

Name: Signature:

Grade breakdown:

Problem No.	Percentage [%]	Value	Grade
1	10	0.5	0.5
Procedure	8	0.4	0.4
2	32	1.6	0.58
Procedure	8	0.4	0.28
3	12	0.6	0.6
4	10	0.5	0.5
Procedure	8	0.4	0.4
5	12	0.6	0.6
TOTAL GRADE			3.86

Universidad Pontificia Bolivariana 80 AÑOS

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For points 1, 2 and 3, use the following information:

As part of a group of aerodynamicists, you are assigned a wing design for a general aviation airplane. This aircraft is powered by a single engine-propeller-driven combination located at the front of the vehicle. The aircraft design has a conventional configuration with a horizontal stabilizer at the tail. The MTOW of this airplane is 2041 [kg]. The wingspan is 12.8 meters, with a chord at the root of 1.3 meters. The horizontal tail span is 4.68 meters, the chord at the tip is 0.8 meters long, and the taper ratio is 1.0. The following table gives some useful information about the wing and stabilizers:

Surface	Airfoil type	a_0 [1/deg]	$\alpha_{L=0}$ [deg]	$c_{d,0}$
Wing	Asymmetrical	0.1123	-4.1	$5.032 \times 10^{-3} c_l^2 - 3.307 \times 10^{-3} c_l + 5.389 \times 10^{-3}$
Horizontal tail	Symmetrical	0.1118	0	$4.680 \times 10^{-3} c_l^2 - 4.225 \times 10^{-5} c_l + 5.040 \times 10^{-3}$

The cruise speed of this aircraft is 82.0 [m/s] at sea level conditions ($\rho_\infty = 1.225$ [kg/m^3]). So far, the research group knows that the horizontal stabilizer needs to provide a down lift force (L_{ht}) of -5820 [N] ($e_{ht} = 0.92$).

You must decide which of the six wing platforms best suits the design. The researchers already developed an *LLT* study that tested the wings at a geometric angle of attack of 4 degrees, giving the following results:

AR	S (m²)	Wing No.	λ_w	e_w	β_w [deg]	A(1)	CL@α0	a (deg⁻¹)
9.84615	16.64	1	1.0	0.92	0	0.023379	0.7232	0.08928
		2			-1.0	0.022037	0.68166	0.08415
11.25274	14.56	3	0.75	0.95	0	0.021164	0.74818	0.09237
		4			-1.0	0.020110	0.71092	0.08777
11.93417	13.728	5	0.65	0.96	0	0.020221	0.75817	0.0936
		6			-1.0	0.019288	0.72318	0.08928

Calculate for a cruise flight condition the following:

1. The wing's net lift force required for the cruise condition: L_w = 25 842.21 [N] ✓
2. Complete the following table:

Wing No.	λ_w	$C_{L,w\_req.}$	$\alpha_{geo,w}$ [deg]	$\alpha_{i,w}$ [deg]	L/D*	w [m/s]	i_w [deg]	
1	1.0	0.37709	0.1237 ✓	0.0137 ✗	22.0[illegible]	0.0188[illegible]	0.1237	✓
2			0.3812 ✓	0.0132 ✗	21.94[illegible]	0.0189[illegible]	~~0.1737~~ 0.6312 ✗	
3	0.75	0.43096	0.5666 ✓	0.0128 ✗	23.2[illegible]	0.018[illegible]	0.5666	✓
4			0.8101 ✓	0.0128 ✗	23.16[illegible]	0.01[illegible]	1.0817	✗
✗ 5	0.65	0.45708	0.7833 ✓	0.0127 ✗	23.7[illegible]	0.018[illegible]	0.7833	✓
6			1.0196 ✓	0.0127 ✗	23.[illegible]	0.019[illegible]	1.2519	✗

*both lifting surfaces

3. Analyzing your results, which of the six wings is optimized and why? Explain and justify your answer, giving technical reasons and engineering criteria for this aircraft's operational behavior.

For points 4 and 5, use the following information:

The *Short SC.7 Skyvan* airplane is a twin-turboprop engine freighter with a fixed landing gear designed to carry heavy payloads and possibly fly to difficult areas. The followings are the specifications and flying characteristics of this aircraft: total Take-off weight of 5670 [kg], cruise speed of 317 kilometers per hour, a service ceiling of 22500 [ft] (ρ_∞ = 0.6 [kg/m^3]), the cabin height/width is 1.97 [m], the wings extend 8.905 [m] from the fuselage on each side and the root chord is 1.47 [m] ($e = 0.82$).

According to the drag coefficient study, this airplane has the following sources of parasite drag:

No.	Component	C_{D_0}	Parasite drag [%]
1	Fuselage	0.0057	15.00%
2	Spinner	0.00057	1.50%
3	Wing	0.00874	23.00%
4	Horizontal tail	0.0024396	6.42%
5	Vertical tail	0.001976	5.20%
6	Landing gear	0.00684	18.00%
7	Wing struts	0.0038	10.00%
8	External appendages	0.000076	0.20%
9	Interference	0.00076	2.00%
10	Engine cooling	0.00304	8.00%
11	Antennas	0.0000304	0.08%
12	Pitot	0.000038	0.10%
13	Miscellaneous	0.0019	5.00%
14	Leakage	0.00057	1.50%
15	Rivets and screws	0.00152	4.00%

Use the correction factor K_c = 1.2. With the previous information, calculate:

4. The total aircraft drag force - D_{total} = 4403.879 [N]

5. Which component will you recommend modifying, why, and what would be your optimization proposal? (Explain your answer).

The 3 views of this aircraft are shown in the following figure:

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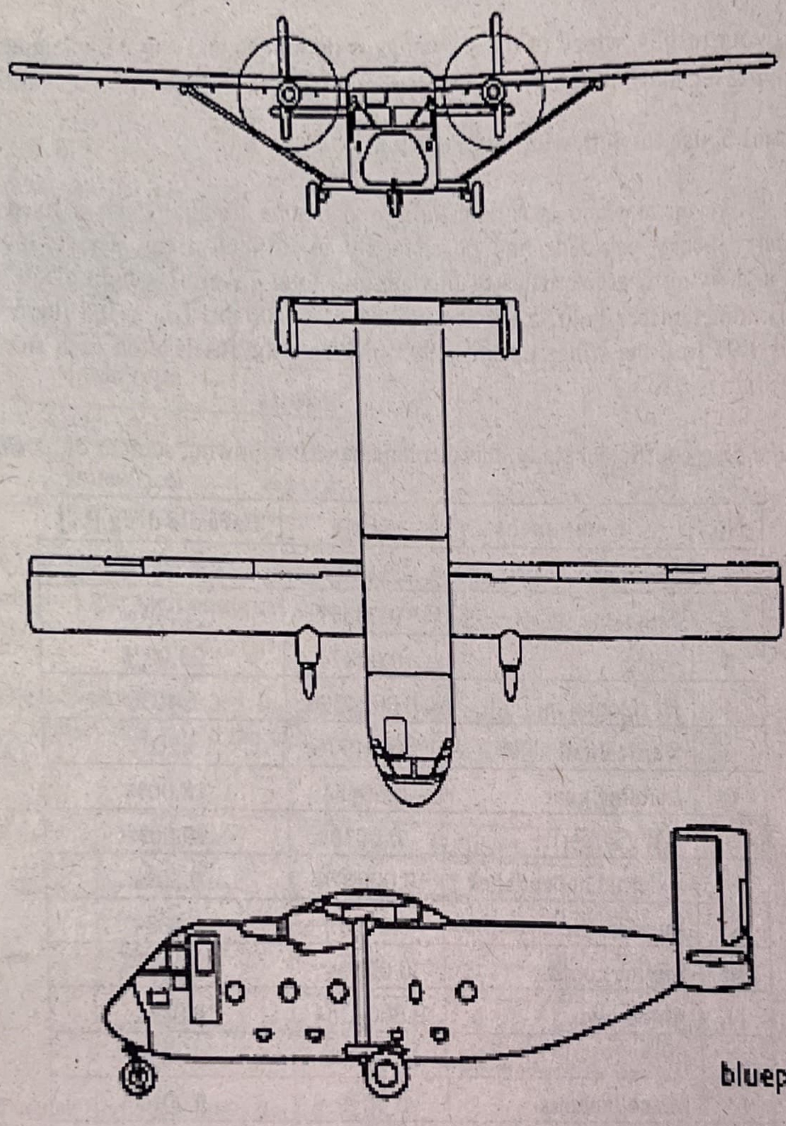

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Sheet of useful formulas:

- $\bar{c} = \frac{2}{3} c_{root} \frac{(1+\lambda+\lambda^2)}{(1+\lambda)}$
- $\bar{Y} = \frac{b}{6}\left[\frac{(1+2\lambda)}{(1+\lambda)}\right]$

Reynolds number:

- $Re_x = \frac{\rho_\infty \cdot V_\infty \cdot x}{\mu_\infty}$

Aerodynamic characteristics of a wing:

- $\alpha_i = \sin^{-1}\left(\frac{D_{w\_i}}{L_w}\right) = C_{L\_w} \cdot K; \quad K = \frac{1}{\pi \cdot e \cdot AR}$
- $a = \frac{a_0}{1 + \frac{57.3 \cdot a_0}{\pi \cdot e \cdot AR}}$; where: a_0 [$1/deg$]
- $C_{M,ac\_wf} = c_{m,ac} \frac{AR\cos^2\Lambda}{AR + 2\cos\Lambda} + 0.01\beta_w$

From ***LLT*****:**

- $D(k) = \alpha_{geo} - \alpha_{L=0}(k) + \beta(k)$
- $C(k,n) = \left[\left(\frac{4b_w}{a_0(k)c(k)} + \frac{2n-1}{\sin(\theta(k))}\right)\sin((2n-1)\theta(k))\right]$
- $\sum_{n=1}^{N} C(k,n)A(n) = D(k)$, for $k = 1, 2, 3, \ldots, N$
- $C(k,n)A(n) = D(k) \Rightarrow A(n) = [C(k,n)]^{-1} \cdot D(k)$
- $C_{L\_w} = \pi \cdot AR \cdot A(1)$
- $e = (1+\delta)^{-1}$; where: $\delta = \sum_{n=2}^{N} n\left(\frac{A(n)}{A(1)}\right)^2$
- $C_{L\_w}(y) = \frac{2\Gamma(y)}{V_\infty c(y)} \Rightarrow \frac{C_{L\_w}(y)}{C_{L\_w}} = \frac{1}{C_{L\_w}}\left(\frac{2\Gamma(y)}{V_\infty c(y)}\right)$
- $L(y) = \frac{1}{2}\rho_\infty V_\infty^2 c(y) \frac{C_{L\_w}(y)}{C_{L\_w}}$

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Fecha: 02 05 2024
Profesor: Juan Pablo Alvarado
Materia: Aerodinámica
Institución: PARCIAL 3
Curso:
Nota:

1) Wing

$W = 2041\ kg = 20022.21\ N$

$b = 12.8\ m$

$C_{root} = 1.3\ m$

$a_0 = 0.1123\ deg^{-1}$

$\alpha_{L=0} = -4.1°$

$V = 82\ m/s$ $\rho = 1.225\ kg/m^3$

HS:

$b = 4.68\ m$ $e = 0.92$

$C_{tip} = 0.8\ m = C_{root}$

$\lambda = 1$

$a_0 = 0.1118\ deg^{-1}$

$\alpha_{L=0} = 0$

$L_{HT} = -5820\ N$

$L_T = L_w + L_{HS} \rightarrow L_T = W$

$L_w = L_T - L_{HS} = 20022.21\ N - (-5820) = 25842.21\ N$

$C_{L,w}$ req:

$C_L\ req = \frac{L_w}{q\, S_w}$

$q = \frac{1}{2}\rho V^2 = \frac{1}{2} \cdot 1.225 \frac{kg}{m^3} (82 \frac{m}{s})^2$

$q = 4118.45\ Pa$

Como $\lambda = 1$, $C_{root} = C_{tip}$

$S_1 = S_2$

$S_1 = b \cdot c = 12.8 \cdot 1.3\ m = 16.64\ m^2$

$\lambda = 0.75$

$\lambda = \frac{C_{tip}}{C_{root}}$ $C_{tip} = 0.75 \cdot 1.3\ m = 0.975\ m$

$S_3 = S_4$

$S_3 = \frac{C_{tip} + C_{root}}{2} \cdot b = \frac{(1.3 + 0.975)\ m}{2} \cdot 12.8\ m = 14.56\ m^2$

$\lambda = 0.65$

$S_5 = S_6$

$C_{tip} = 0.65 \cdot 1.3\ m = 0.845\ m$

$S_5 = \frac{(0.845 + 1.3)\ m}{2} \cdot 12.8\ m = 13.728\ m^2$

$C_{L_{1,2}req} = \frac{25842.21\ N}{4118.45\ Pa \cdot 16.64\ m^2} = 0.37709$

$C_{L_{3,4}} = 0.43096$ $C_{L_{5,6}} = 0.45708$

$\alpha_{geo,w}$

$$C_L = a(\alpha_{geo} - \alpha_{L=0})$$

$$\alpha_{geo} = \frac{C_L}{a} + \alpha_{L=0}$$

Thanks to LLT, $a = \frac{C_{L_T} - 0}{\alpha - \alpha_{L=0}}$

$$C_{L@4^\circ} = \pi AR\, A(1)$$

$$AR_{1,2} = \frac{b^2}{S_1} = \frac{(12.8)^2}{16.64} = 9.84615$$

$AR_{3,4} = 11.2527$

$AR_{5,6} = 11.9347$

$C_{L_1@4^\circ} = \pi \cdot 9.8461 \cdot 0.023379 = 0.7232$

$C_{L_2@4^\circ} = 0.68166$

$C_{L_3@4^\circ} = 0.748177$

$C_{L_4@4^\circ} = 0.71092$

$C_{L_5@4^\circ} = 0.75817$

$C_{L_6@4^\circ} = 0.72318$

$a_1 = \frac{0.7232 - 0}{4^\circ - (-4.1)^\circ} = 0.08928 \frac{1}{°}$ $\quad a_2 = 0.08415 \frac{1}{°}$ $\quad a_3 = 0.09237 \frac{1}{°}$

$a_4 = 0.08777 \frac{1}{°}$ $\quad a_5 = 0.0936 \frac{1}{°}$ $\quad a_6 = 0.08928 \frac{1}{°}$

$$\alpha_{geo_1} = \frac{0.37709}{0.08928} - 4.1^\circ = 0.123^\circ$$

$\alpha_{geo_2} = 0.3812^\circ$ $\quad \alpha_{geo_3} = 0.5656^\circ$ $\quad \alpha_{geo_4} = 0.8101$

$\alpha_{geo_5} = 0.7833$ $\quad \alpha_{geo_6} = 1.0196$

$\alpha_{i,w}$:

$$\alpha_{i_1} = \alpha_{i_2} = \frac{C_{L_{req}}}{\pi e AR} = \frac{0.37709}{\pi \cdot 0.92 \cdot 9.8461} = 0.0132^\circ$$

$\alpha_{i_3} = \alpha_{i_4} = 0.0128^\circ$ $\quad \alpha_{i_5} = \alpha_{i_6} = 0.0127^\circ$

$\frac{L}{D}$: since its for both surfaces $\quad \frac{L}{D} = \frac{L_T}{D_w + D_{HS}}$

$$D_w = qS(C_{d_0} + C_{D_i})$$

$$C_{D_i} = \frac{C_L^2}{\pi e AR} \qquad C_{d,0,w} = 5.032 \cdot 10^{-3} c_l^2 - 3.307 \cdot 10^{-3} c_l + 5.389 \cdot 10^{-3}$$

$$c_l = a_0(\alpha_{eff} - \alpha_{L=0})$$

$$\alpha_{eff} = \alpha_{geo} - \alpha_i$$

For wing 1:

$$C_{Di_1} = \frac{0.37709^2}{\pi \cdot 0.92 \cdot 9.8461} = 4.9967 \cdot 10^{-3}$$

$\alpha_{eff_1} = 0.1237 - 0.0132 = 0.1105^\circ$

$C_{l_1} = 0.1123\,deg^{-1}(0.1105^\circ - (-4.1)) = 0.4728$

$C_{d_{0,w_1}} = 5.032\cdot10^{-3}(0.4728)^2 - 3.307\cdot10^{-3}(0.4728) + 5.389\cdot10^{-3} = 4.9503\cdot10^{-3}$

$D_{w_1} = 4118.45\,Pa\cdot 16.64\,m^2(4.9967\cdot10^{-3} + 4.9503\cdot10^{-3})$

$D_{w_1} = 681.6779\,N$

	α_{eff} (°)	C_l	$C_{d,0w}\cdot10^{-3}$	$C_{Di}\cdot10^{-3}$	D_w (N)
1	0.1105	0.4728	4.9503	4.9967	681.6779
2	0.368	0.5018	4.9966	4.9967	684.8509
3	0.5528	0.5225	5.0349	5.5302	633.5323
4	0.7973	0.5499	5.0921	5.5302	636.9623
5	0.7706	0.5469	5.0855	5.8043	615.6884
6	1.0069	0.5735	5.1475	5.8043	619.1938

For HS:

$D_{HS} = qS(C_{d0} + C_{D,i})$

$C_{D,i} = \frac{C_L^2}{\pi e AR}$ $\qquad AR = \frac{b^2}{S} = \frac{b^2}{bc} = \frac{b}{c} = \frac{4.68}{0.8} = 5.85$

$C_L = \frac{L_{HS}}{qS} = \frac{-5820\,N}{4118.45\,Pa\cdot4.68\,m\cdot0.8\,m} = -0.3774$

$C_{D,i\,HS} = \frac{(-0.3774)^2}{\pi\cdot0.92\cdot\left(\frac{4.68\,m}{0.8\,m}\right)} = 8.4258\cdot10^{-3}$

$C_{d,0HS} = 4.68\cdot10^{-3}C_l^2 - 4.225\cdot10^{-5}C_l + 5.040\cdot10^{-3}$

$C_l = a_0(\alpha_{eff} - \cancel{\alpha_{L=0}})$ 0

$\alpha_{eff} = \alpha_{geo} - \alpha_i$

$\alpha_{geo} = \frac{C_L}{a}$ $\qquad \alpha_i = \frac{C_L}{\pi e AR} = \frac{-0.3774}{\pi\cdot0.92\cdot\left(\frac{4.68\,m}{0.8\,m}\right)} = -0.0223^\circ$

$a = \frac{a_0}{1 + \frac{57.3 a_0}{\pi e AR}} = \frac{0.1118}{1 + \frac{57.3\cdot0.1118}{\pi\cdot0.92\cdot5.85}} = 0.08108\,\frac{1}{deg}$

$\alpha_{geo} = \frac{-0.3774}{0.08109} = -4.6546^\circ$ $\qquad \alpha_{eff} = -4.6546^\circ - (-0.0223^\circ)$

$\alpha_{eff} = -4.6323^\circ$ $\qquad C_l = 0.1118\frac{1}{deg}(-4.6323^\circ) = -0.5179$

$C_{d_0} = 4.68 \cdot 10^{-3}(-0.5179)^3 - 4.226 \cdot 10^{-5}(-0.5179) + 5.04 \cdot 10^{-3}$

$C_{d_0} = 6.3171 \cdot 10^{-3}$

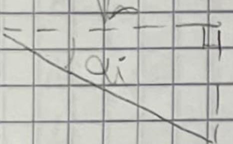
$D_{HS} = 4118.45\,Pa \cdot 4.68\,m \cdot 0.8\,m\,(8.4258 \cdot 10^{-3} + 6.3171 \cdot 10^{-3}) = 227.3278\,N$ //

$\frac{L}{D_1} = \frac{20\,022.21\,N}{(681.6779 + 227.3278)\,N} = 22.0265$ //

$L/D_2 = 21.9499$ // $L/D_3 = 23.2584$ // $L/D_4 = 23.1661$ //

$L/D_5 = 23.7507$ // $L/D_6 = 23.7365$ //

w:



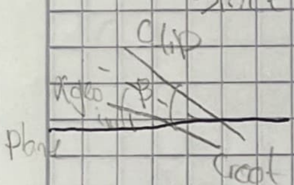
$\tan \alpha_i = \frac{w}{V}$ $\rightarrow$ $w = V \tan(\alpha_i)$

$w_2 = w_1 = 82\,m/s \tan(0.0132°) = 0.01889\,m/s$ //

$w_3 = w_4 = 0.01832\,m/s$ // $w_5 = w_6 = 0.01818\,m/s$ //

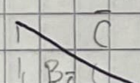
i_w:

Since α_{geo} is according to $\bar{c}$

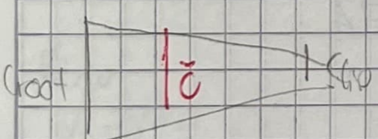


$i_w = \alpha_{geo} - \beta_{\bar{c}}$

$\beta_{\bar{c}} = \frac{\bar{y}}{b} \cdot \beta_{tip}$



$\bar{c} = \frac{2}{3} C_{root} \left(\frac{1+\lambda+\lambda^2}{1+\lambda}\right)$ $\quad \bar{y} = \frac{b}{6}\left(\frac{1+2\lambda}{1+\lambda}\right)$



	$\bar{c}$	$\bar{y}$	$\beta_{\bar{c}}$	i_w
2	1.3	3.2m	-0.25	0.6312
4	1.1452	3.0476	-0.2716	
6	1.0886	2.9737	-0.2323	

For 2:

$\bar{c} = \frac{2}{3} 1.3 \left(\frac{1+1+1}{1+1}\right) = 1.3\,m$

$\bar{y} = \frac{12.8}{6}\left(\frac{1+2\cdot 1}{1+1}\right) = 3.2\,m$

$\beta_{\bar{c}} = \frac{3.2}{12.8}(-1°) = -0.25$

$i_w = 0.3812 - (-0.25) = 0.6312°$

(3) The wing that is optimized for this aircraft and flight condition is #5.
This is because it has the best aerodynamic efficiency compared to the others. Eventhough the wing needs a higher AoA compared to others, it produces the least drag.
Also, an important factor is the manufacture process, which is easier since it doesn't have a twist angle and has a smaller surface area, thus less material, less weight and a lower cost.

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Nota:

4) $W = 5670\ kg = 55622.7\ N$ $\qquad K_c = 1.2$

$V = 317\ \frac{km}{h} = 88.05\ m/s$ $\qquad C_{d0} = 0.038$

$\rho = 0.6\ \frac{kg}{m^3}$ $\qquad e = 0.82$ $\qquad q = \frac{1}{2}\rho V^2 = 2326.134\ Pa$

$S = (2 \cdot 8.905 + 1.97) \cdot 1.47 = 29.0766\ m^2$

$$D_{Total} = qS\,(C_{D,i} + C_{d,0})$$

$$C_{d,0} = K_c C_{d,0} = 1.2 \cdot 0.038 = 0.0456$$

$$C_{D,i} = \frac{C_L^2}{\pi e AR} \qquad AR = \frac{b}{c} = \frac{2 \cdot 8.905\,m + 1.97\,m}{1.47\,m}$$

$$AR = 13.4558$$

$$C_L = \frac{W}{qS} = \frac{55622.7\,N}{2326.134\,Pa \cdot 29.0766\,m^2}$$

$$C_L = 0.82238$$

$$C_{D,i} = \frac{0.8223^2}{\pi \cdot 0.82 \cdot 13.4558} = 0.019515$$

$$D_{Total} = 2326.134\,Pa \cdot 29.0766\,m^2\,(0.0456 + 0.019515)$$

$$D_{Total} = 4403.8291\,N$$

5) I would recommend to add fairings to the landing gear since for this application I can't make them retractible. This will reduce drag in the landing gear by half. This will be the easiest thing to modify and improve the aircraft efficiency the most since currently it contributes to 18% of total $C_{d,0}$.

$$C_{D,0\ Lg\ new} = \frac{0.00684}{2} = 0.00342$$

$$C_{d,0\ new} = C_{d0} - C_{d0\ LG\ new} = 0.038 - 0.00342 = 0.03458$$

$$C_{d0\ corrected} = 0.03458 \cdot 1.2 = 0.041496$$

$$\%\ C_{d0}\ reduction = 1 - \frac{0.041496}{0.0456} = 9\%\ reduction$$

$$D_{new} = 7326.134\,Pa \cdot 29.0766\,m^2 (0.04496 - 0.01951) = 4126.2$$

$$\%\ \text{reduction } D = 1 - \frac{D_{new}}{D} = 1 - \frac{4126.2}{4403} = 6.3\%$$

This will reduce total drag by 6% which is significant and will make the aircraft more efficient.

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Vigilada Mineducación

AERONAUTICAL ENGINEERING FACULTY

NRC: 20493 AERODYNAMICS

4th Mid-term exam – Professor: Juan Pablo Alvarado P.

Due: May 17th at 10:00 am

Directions: This mid-term exam is to be completed without help from any other individual. This includes help from other students (whether or not they are taking Aerodynamics), the monitor (Teaching Assistant), other faculty staff, people on the Internet, etc. You are NOT permitted to use your books, class notes, library resources, etc. You only need a pen, pencil, eraser, and your calculator. You must provide the numerical or text answer as asked in each problem or question. The extra sheets given to you will be used as a calculation aid. **The procedure will be graded, as well as the answer, by the grade breakdown table below**. Answers in the English language are compulsory for this exam. Answers with wrong or incorrect units will be penalized. The error percentage of the numerical answers is 5.0%. The exam will be in a maximum of one hour and fifty minutes. Absolutely no late exams will be accepted.

Please sign this cover sheet and turn it in with your solution.

I understand and have followed the directions for this exam. The solutions I have turned in represent my understanding:

Name: Andrés Zuleta Signature:

Grade breakdown:

Problem No.	Percentage [%]	Value	Grade
1	8	0.4	0.4
Procedure	2	0.1	0.1
2	8	0.4	0.4
Procedure	2	0.1	0.1
3	8	0.4	0.4
Procedure	2	0.1	0.1
4	8	0.4	0.4
Procedure	2	0.1	0.1
5	8	0.4	0.4
Procedure	2	0.1	0.1
6	8	0.4	0.4
Procedure	2	0.1	0.1
7	8	0.4	0.4
Procedure	2	0.1	0.1
8	8	0.4	0.4
Procedure	2	0.1	0.1
9	8	0.4	0.4
Procedure	2	0.1	0.1
10	10	0.5	0.5
TOTAL GRADE			5.0

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1. A high-speed subsonic Boeing 777 airliner is flying at an altitude where the pressure has a value of 1.94×10^{4} Pa. A Pitot tube on the vertical tail measures a pressure of 2.96×10^{4} N/m^2. At what Mach number is the airplane flying?

   M_1 = 0.80094 ✓

2. Consider a wing mounted in the test section of a subsonic wind tunnel. The velocity of the airflow is 160 ft/s, and the air temperature is 510°R. In those conditions, the velocity at a point on the wing is 195 ft/s. Now, if the flow velocity is increased to a value of 700 ft/s, what is the pressure coefficient at the same point? (for the air: $\gamma = 1.4$; R = 1716 [(ft lb)/(slug °R)])

   C_p = −0.62654 ✓

3. Consider a NACA 1412 airfoil at an angle of attack of 4° corresponding to a lift coefficient of 0.58 at subsonic conditions. If the free-stream Mach number is increased to a value of 0.8, calculate the compressive value of the lift coefficient.

   c_l = 0.96667 ✓

4. Consider an airfoil at a given angle of attack, say α. At low speeds, the minimum pressure coefficient on the top surface of the airfoil is −0.90. What is the critical Mach number of the airfoil?

   M_{crit} = 0.67235 ✓

5. Consider a uniform flow with a Mach number of 2. What cone angle does a Mach wave make with respect to the flow direction?

   Mach cone angle = 30° ✓

6. Consider the high hypersonic *Darkstar* airplane flying at Mach 6.5 at an altitude of 10 km. Assume that the angle of the shock wave from the nose is approximated by the Mach angle (this is a very weak shock). How far behind the nose of the vehicle will the shock wave impinge upon the ground? (Ignore the fact that the speed of sound, and hence the Mach angle, changes with altitude.)

   d = 64.2261 [km] ✓

7. The wing area of the *Lockheed F-104* straight-wing supersonic fighter is approximately 210 ft^2, and a wingspan of 22.0 ft. If the airplane weighs 16000 lb and is flying in level flight at Mach 2.2 at a standard altitude of 36000 ft (T_∞ = 390.5 °R; $\rho_\infty = 7.1\times10^{-4}$ slug/ft^3), estimate the wave drag on the wings.

   D_w = 329.4778 [lb] ✓

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8. An airplane with a straight wing has a critical Mach number of 0.72. If you, as an aeronautical engineer, are assigned the task of increasing this number to 0.935, what leading-edge swept angle is needed to accomplish this goal?

Λ_{LE} = 39.6413 [deg] ✓

9. Captain Pete Mitchell (call sign "*Maverik*") from the Navy performed a test flight with the *Darkstar* airplane (wing area of 147.2 m^2) at an altitude of 10.0 km (T_∞ = 223.2 K; ρ_∞ = 0.413 kg/m^3), which ended in the complete destruction of the aircraft. The maximum load factor for the structure was designed to withstand a value of 15 g's. So far, it is known that the captain reached a top speed of Mach 10.3 when suddenly he pushed the control stick forward, changing the attitude of the airplane by 3.2 deg. Calculate the value of the actual load factor this aircraft had at the moment of the sudden attitude change. W=18770 kg

n = 134.224 g's ✓

10. When supersonic flow is reached, you will find different types of shock waves, which are:

Normal waves ✓

Expansion waves ✓

Oblique waves ✓

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Curso:

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(1) 747 $P_\infty = 1.94 \cdot 10^4$ Pa $P_0 = 2.96 \cdot 10^4$ N/m² $M = ?$

$$M_1^2 = \frac{2}{\gamma-1}\left[\left(\frac{P_0}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

$$M = \sqrt{\frac{2}{1.4-1}\left[\left(\frac{2.96 \cdot 10^4 \text{ Pa}}{1.94 \cdot 10^4}\right)^{\frac{0.4}{1.4}} - 1\right]} = 0.80094$$

(2) 1. $\begin{cases} V = 160 \text{ ft/s} \\ V_1 = 195 \text{ ft/s} \end{cases}$ $T = 510°R$

↳ inviscid flow

2. $\begin{cases} V = 700 \text{ ft/s} \\ C_{P1} = ? \end{cases}$ $R = 1716 \frac{\text{ft·lb}}{\text{slug R}}$

↳ viscid flow

$$C_{P1} = 1 - \left(\frac{V_1}{V_\infty}\right)^2$$

$$C_{P0,1} = 1 - \left(\frac{195}{160}\right)^2 = -0.485351$$

For viscid flow

$$C_P = \frac{C_{P,0}}{\sqrt{1-M_\infty^2}}$$

$$M = \frac{V}{a} = \frac{V}{\sqrt{\gamma R T}} = \frac{700 \text{ ft/s}}{\sqrt{1.4 \cdot 1716 \cdot 510}} = 0.63239$$

$$C_{P1} = \frac{-0.485351}{\sqrt{1-0.63239^2}} = -0.62654$$

(3) $\alpha = 4° \rightarrow C_{l,0} = 0.58$ Si $M = 0.8$ $C_l = ?$

$$C_l = \frac{C_{l,0}}{\sqrt{1-M^2}} = \frac{0.58}{\sqrt{1-0.8^2}} = 0.9667$$

(4) $C_{P0,min} = -0.9$ $M_{cr} = ?$

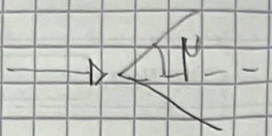
↳ since C_{P0} is the minimum value $C_{P0,min} = C_{Pr}$

$$C_{Pr} = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{2 + (\gamma-1)M_\infty^2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

$$-0.9 = \frac{2}{1.4 M_\infty^2}\left[\left(\frac{2 + 0.4 M_\infty^2}{2.4}\right)^{\frac{1.4}{0.4}} - 1\right]$$

↳ using solve: $M_{cr} = 0.67235$

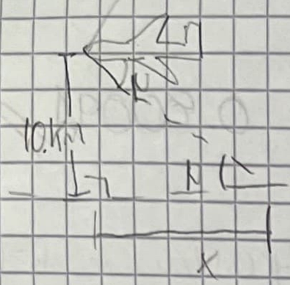
(5) $M=2$ $\qquad \mu=?$



$$\mu = \operatorname{sen}^{-1}\left(\frac{1}{M}\right) = \operatorname{sen}^{-1}\left(\frac{1}{2}\right)$$

$$\mu = 30°$$

(6) $M=6.5$ $\qquad h=10\,km$



$$\mu = \operatorname{sen}^{-1}\left(\frac{1}{M}\right) = \operatorname{sen}^{-1}\left(\frac{1}{6.5}\right) = 8.8499°$$

$$\tan\mu = \frac{10\,km}{x}$$

$$x = \frac{10\,km}{\tan\mu} = \frac{10\,km}{\tan 8.8499°} = 64.2261\,km$$

(7) $S = 210\,ft^2$ $\quad b = 22\,ft$ $\quad w = 16000\,lb$ $\quad M = 2.2$

$T_\infty = 390.5°R$, $\rho_\infty = 7.1\cdot10^{-4}\,slug/ft^3$ $\quad D_w = ?$

$$D_w = q S C_{D,w}$$

$$q = \frac{1}{2}\rho V^2$$

$$V = M a$$

$$a = \sqrt{\gamma R T} = \sqrt{1.4\cdot1716\cdot390.5} = 968.5748\,\frac{ft}{s}$$

$$V = 2.2\cdot968.57\,\frac{ft}{s} = 2130.8646\,\frac{ft}{s}$$

$$q = \frac{1}{2}\cdot7.1\cdot10^{-4}\,\frac{slug}{ft^3}\cdot\left(2130.8646\,\frac{ft}{s}\right)^2 = 1611.907\,\frac{lb}{ft^2}$$

$$C_{D,w} = \frac{4\alpha^2}{\sqrt{M_\infty^2-1}}\left[1-\frac{1}{2\left(AR\sqrt{M_\infty^2-1}\right)}\right]$$

$$C_L = \frac{4\alpha}{\sqrt{M_\infty^2-1}} \qquad C_L = \frac{L}{qS} = \frac{16000\,lb}{1611.907\,\frac{lb}{ft^2}\cdot210\,ft^2}$$

$$C_L = 0.047267$$

$$\alpha = \frac{0.047267}{4}\sqrt{2.2^2-1}$$

$$\alpha = 0.023156\,rad$$

$$AR = \frac{b^2}{S} = \frac{(22\,ft)^2}{210\,ft^2} = 2.3048$$

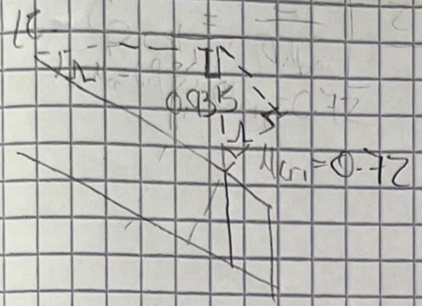
$$C_{D,w} = \frac{4\cdot(0.023156\,rad)^2}{\sqrt{2.2^2-1}}\left[1-\frac{1}{2\cdot2.3048\sqrt{2.2^2-1}}\right]$$

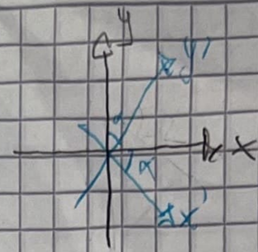
$$C_{D,w} = 9.73345\cdot10^{-4}$$

$$D_w = 1611.907\,\frac{lb}{ft^2}\cdot210\,ft^2\cdot9.73345\cdot10^{-4} = 329.4778\,lb$$

8) $M_{cr_1} = 0.72$ → increase to $M_{cr_{new}} = 0.935$

$\Lambda_{LE} = ?$





$$\cos\Lambda = \frac{0.72}{0.935}$$

$$\Lambda = \cos^{-1}\left(\frac{0.72}{0.935}\right) = 39.6413^\circ$$

9) $S = 147.2\,m^2$ $h = 10\,km$ $T_\infty = 223.2\,K$ $\rho_\infty = 0.413\,kg/m^3$

load factor max 15g $M = 10.3$ $\alpha = -3.2^\circ = 0.05585$ rad

$$\frac{L}{W}$$

$$L = qS\,c_l \qquad c_l = \frac{4\alpha}{\sqrt{M^2-1}} = \frac{4\cdot(-0.05585\,rad)}{\sqrt{10.3^2-1}}$$

$$c_l = -0.02179$$

$$q = \frac{1}{2}\rho V^2$$

$$V = Ma = M\sqrt{\gamma R T} = 10.3\sqrt{1.4\cdot 287\cdot 223.2} = 3084.532\,\frac{m}{s}$$

$$q = \frac{1}{2}\cdot 0.413\,\frac{kg}{m^3}\cdot\left(3084.532\,\frac{m}{s}\right)^2 = 1964710.78\,Pa$$

$$L = 1964710.78\,Pa\cdot 147.2\,m^2\cdot(-0.02179) = +6301786.251\,N$$

$$n = \frac{+6301786.251\,N}{(18770\cdot 9.81)\,N} = +34.22397\ g's$$