Aerospace Structures

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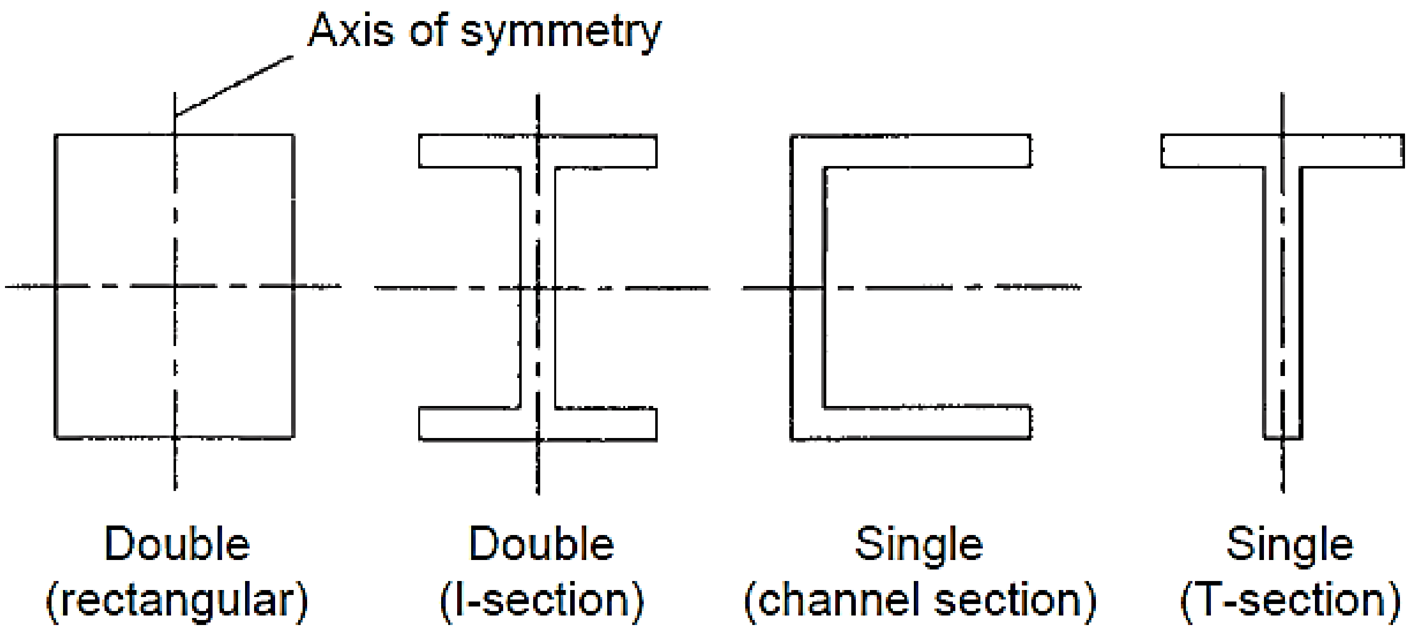
SHEAR, BENDING AND TORSION LOADS IN THIN-WALLED BEAMS

Symmetrical bending

Symmetrical bending produces beams that have a single or doubly symmetrical cross section.



Symmetrical bending

The general equation for a symmetrical bending is defined by the equation:

$$\sigma_z = \frac{M_x y}{I_{xx}} \quad or \quad \sigma_z = \frac{M_y x}{I_{yy}}$$

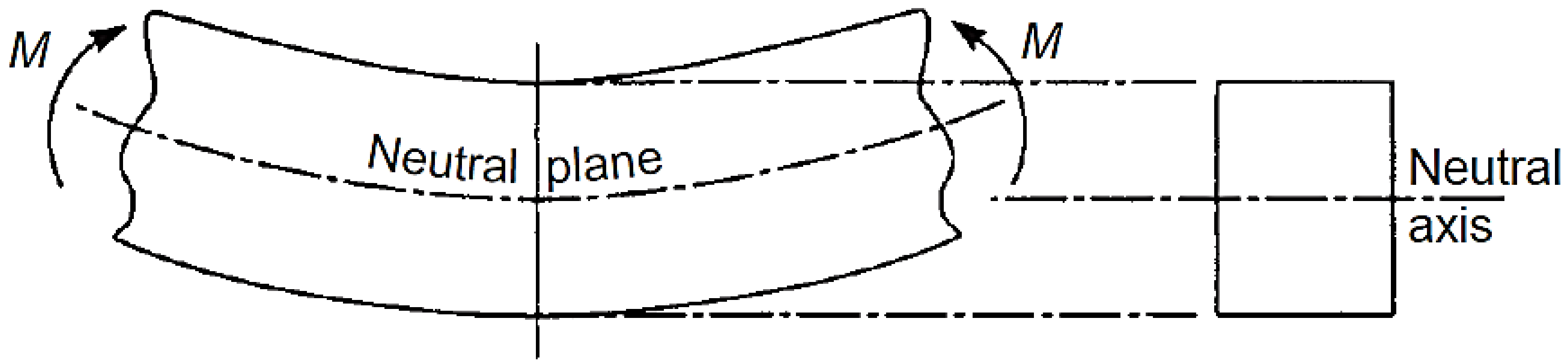
If the bending is unsymmetrical the stress is calculated by:

$$\sigma_z = \frac{M_x y}{I_{xx}} + \frac{M_y x}{I_{yy}}$$

Symmetrical bending

The neutral plane can be defined by the following equation:

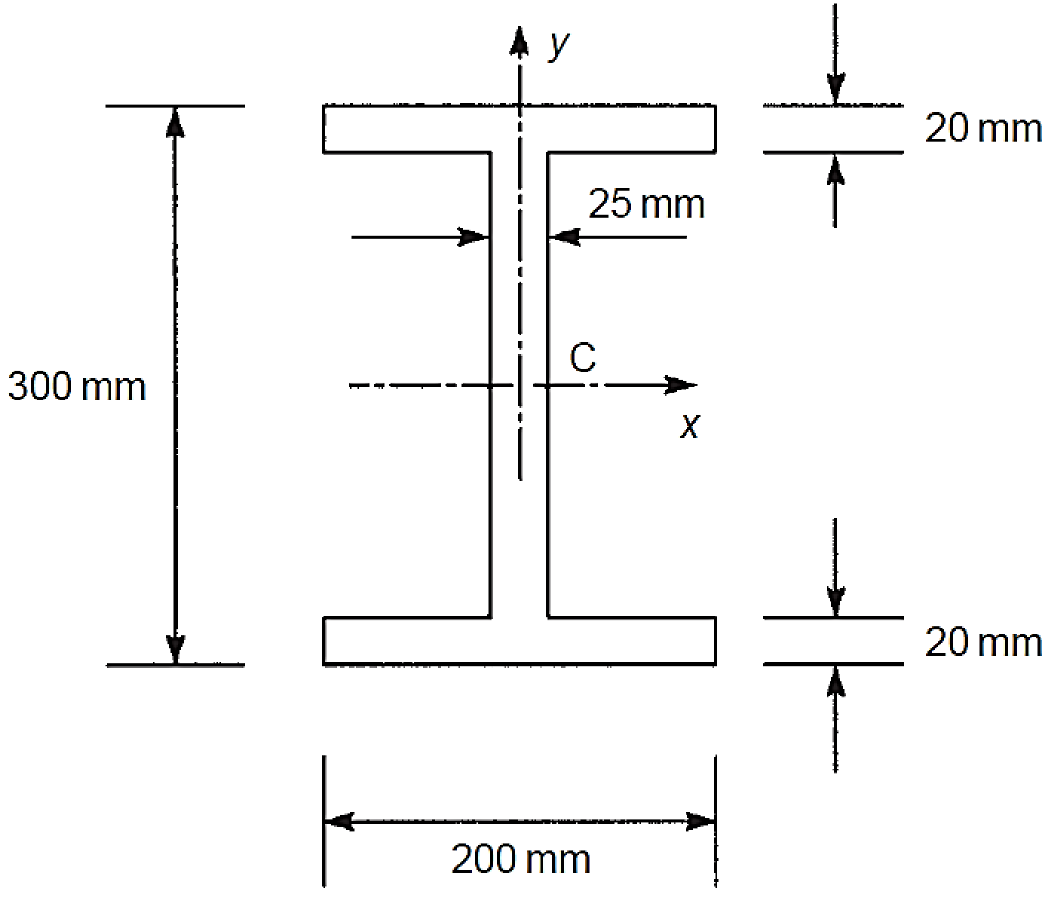
$$tan\alpha = \frac{M_y I_{xx}}{M_x I_{yy}}$$



Example

The cross-section of a beam has the dimensions shown. If the beam is subjected to a negative bending moment of $100\ kNm$ applied in a vertical plane, determine the distribution of direct stress through the depth of the section.

$$\sigma_z = \pm 78\ N/mm^2$$



Assignment

The I beam is subjected to bending moment of 100 KNm applied in a plane parallel to the longitudinal axis of the beam but inclined 30° to the left of vertical. Determine the distribution of direct stress.

Example 16.3

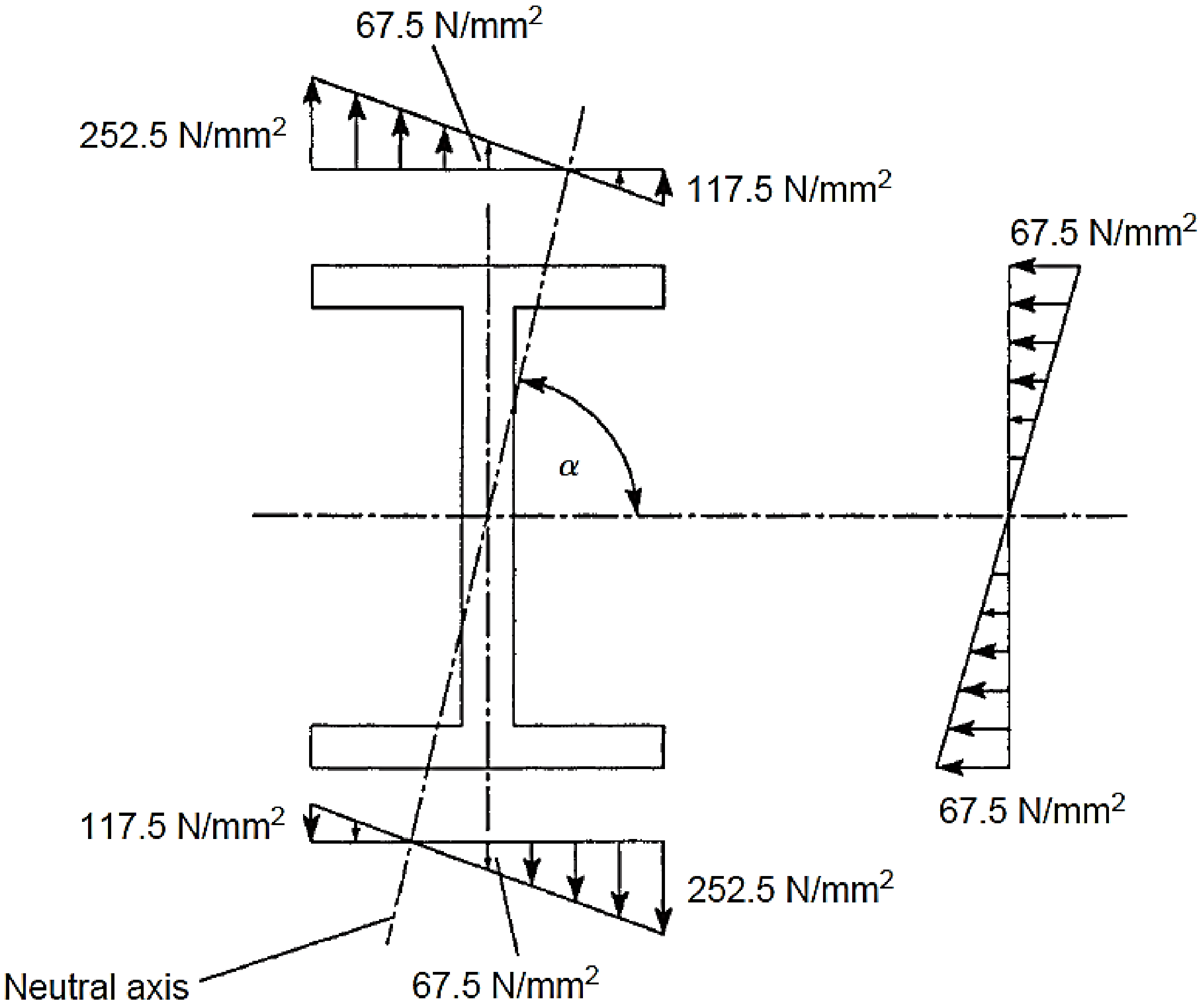
Aircraft structures of engineering students – Megson 2021

The I beam is subjected to bending moment of 100 KNm applied in a plane parallel to the longitudinal axis of the beam but inclined 30° to the left of vertical. Determine the distribution of direct stress.

Example 16.3

Aircraft structures of engineering students – Megson 2021

Assignment



67.5 N/mm^2
252.5 N/mm^2
117.5 N/mm^2
67.5 N/mm^2
α
117.5 N/mm^2
252.5 N/mm^2
67.5 N/mm^2
Neutral axis
67.5 N/mm^2

Direct stress distribution due to bending (unsymmetrical bending)

The stress due to bending moment of an arbitrary cross section about some axis in its cross section which is the neutral axis (zero stress) is given by:

$$\sigma_z = \left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) x + \left(\frac{M_x I_{yy} - M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) y$$

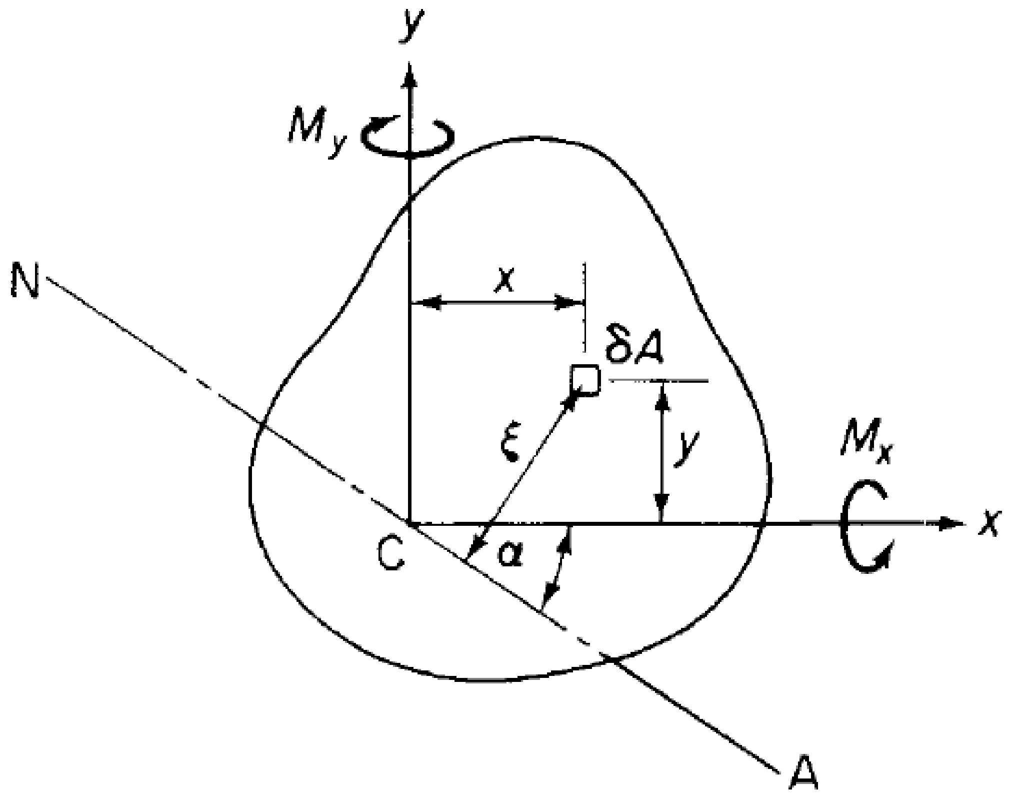
Or

$$\sigma_z = \frac{M_x\left(I_{yy} y - I_{xy} x\right)}{I_{xx} I_{yy} - I_{xy}^2} + \frac{M_y\left(I_{xx} x - I_{xy} y\right)}{I_{xx} I_{yy} - I_{xy}^2}$$

Direct stress distribution due to bending

The position of the neutral axis always passes through the centroid of area of a beam’s cross section but its inclination α to the x axis depends on the form of the applied loading and the geometrical properties of the beam’s cross-section:

$$tan\alpha = \frac{M_yI_{xx} - M_xI_{xy}}{M_yI_{yy} - M_yI_{xy}}$$



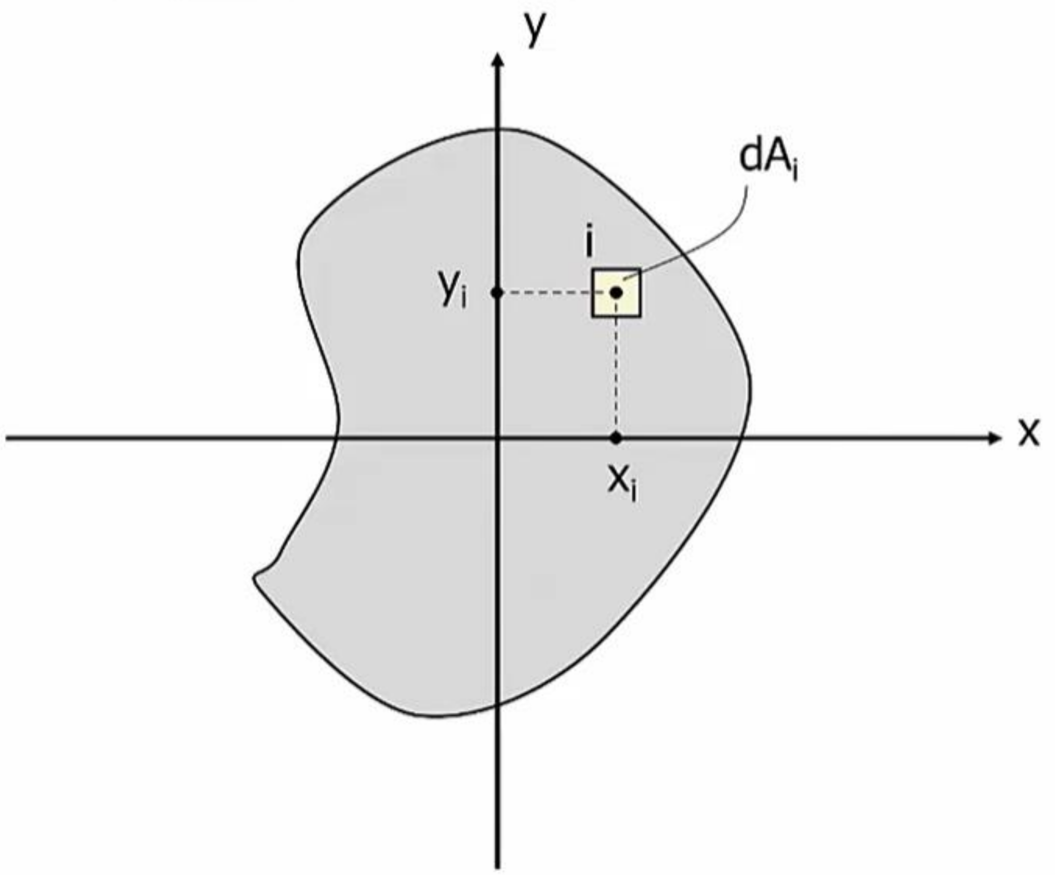
Product of second moment of area

The product of second moment of area I_{xy} of a beam section with respect to x and y axes is defined by:

$$I_{xy} = xyA$$

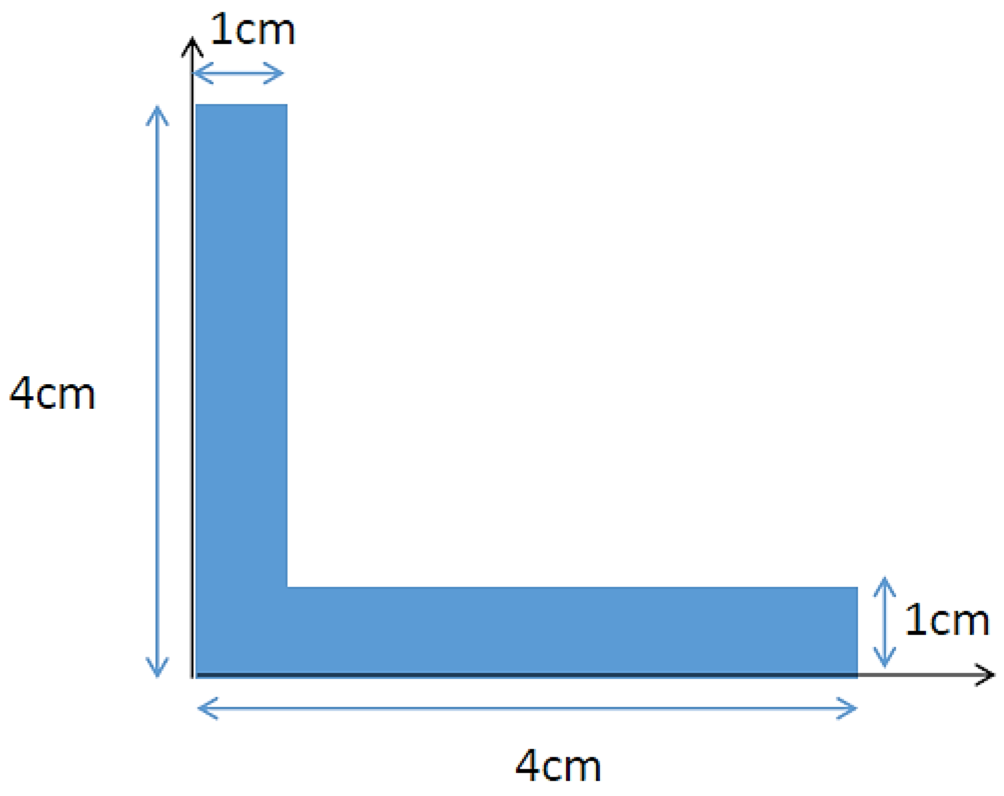
x and y are the distance from the centroid of a segment to the centroid of the section, and A the area of the section.

If the mass is not distributed symmetrically, the result is a nonzero POI causing an imbalance when the object is rotated.



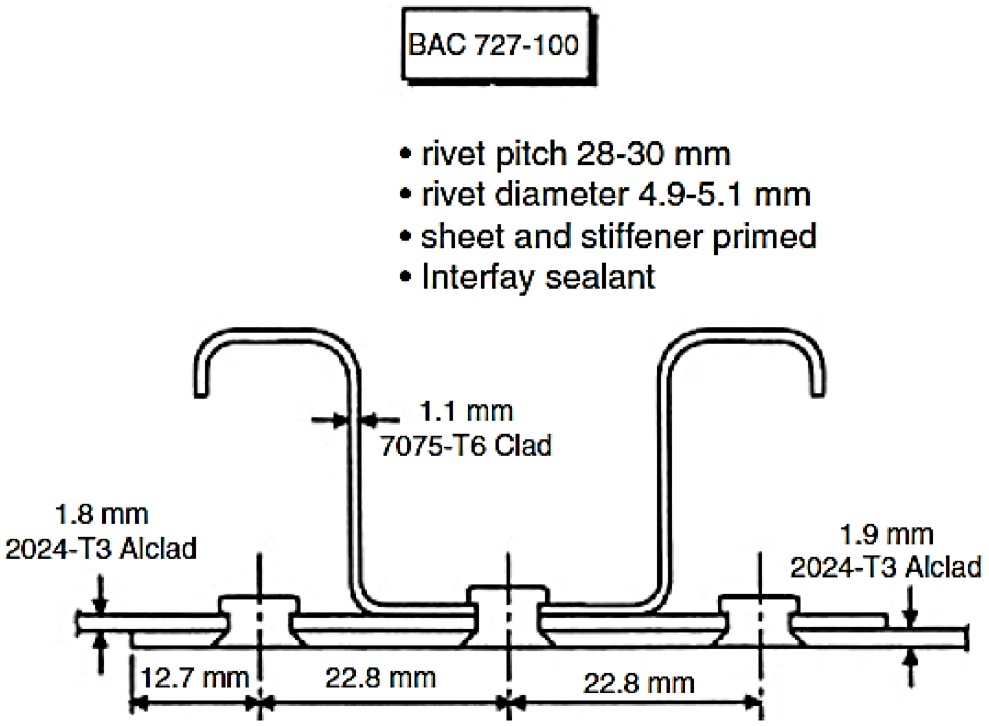
Example

Determine the product of inertia of the following section:

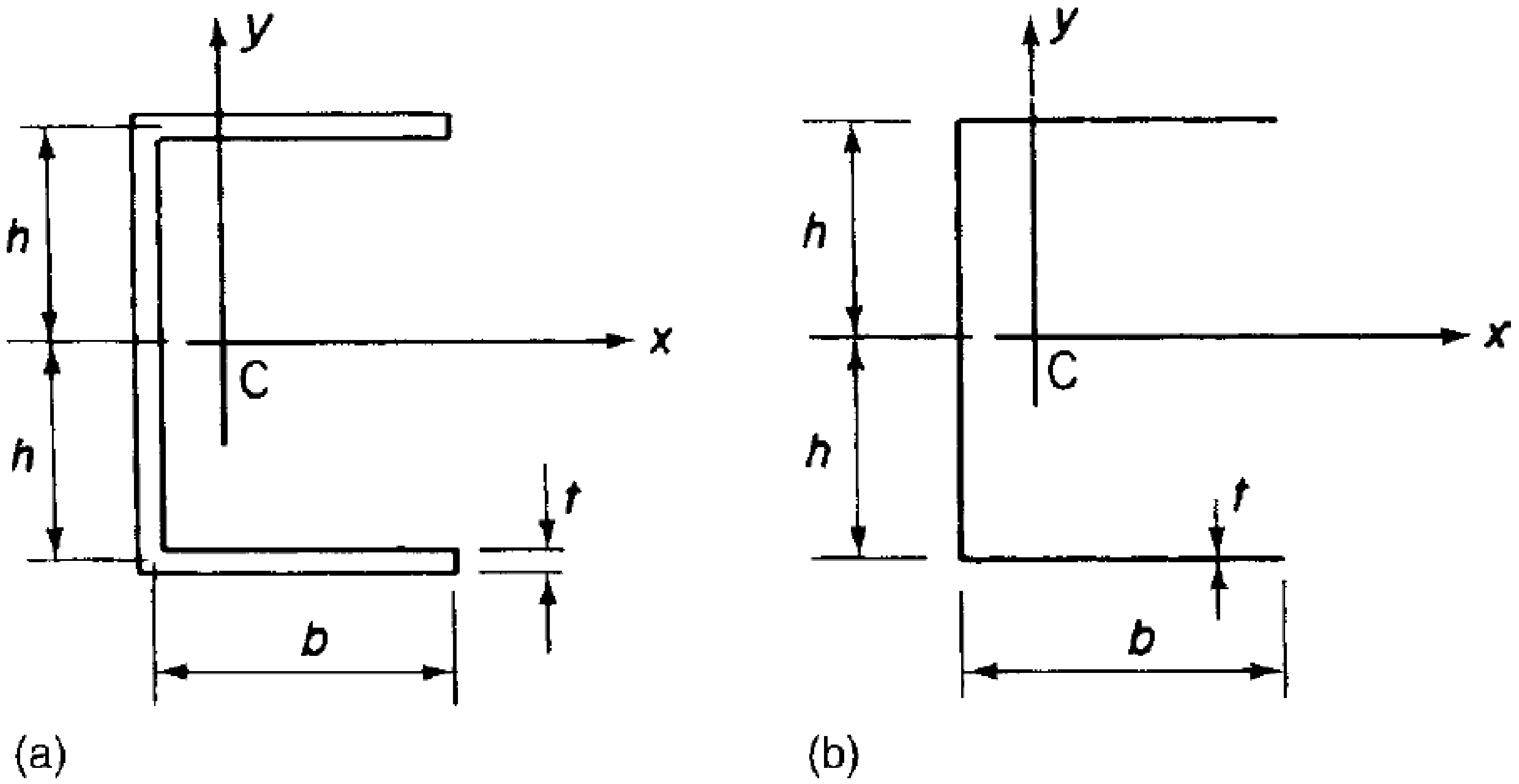


Approximation of a thin-walled section

- It can be performed an approximation of a thin-walled section due to the nature of aircraft structures.
- The thickness of a thin-walled section can be assumed small compared with the cross section.
- **The squares and higher powers of t in the calculation of the section properties can be neglected.**



Approximation of a thin-walled section



(a) Actual Thin-Walled Channel Section; (b) Approximate Representation of Section

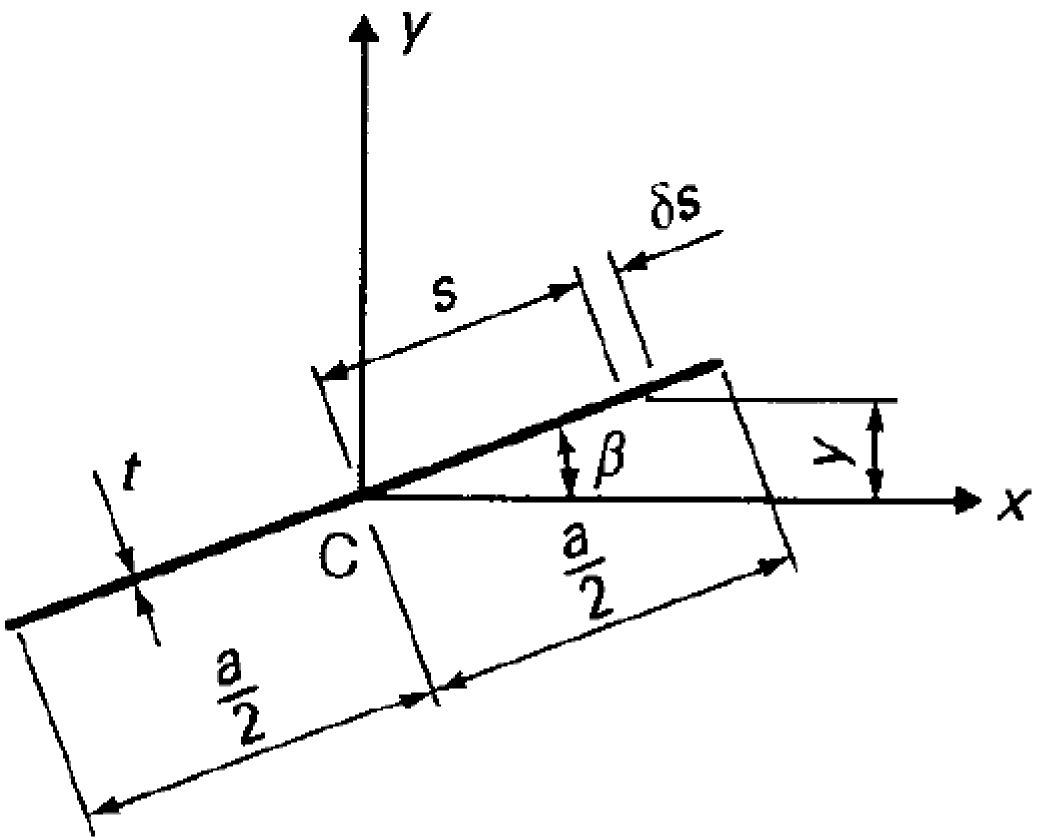
Approximation of a thin-walled section

For an inclined curves:

$$I_{xx} = \frac{a^3\, t\, sin^2\beta}{12}$$

$$I_{yy} = \frac{a^3\, t\, cos^2\beta}{12}$$

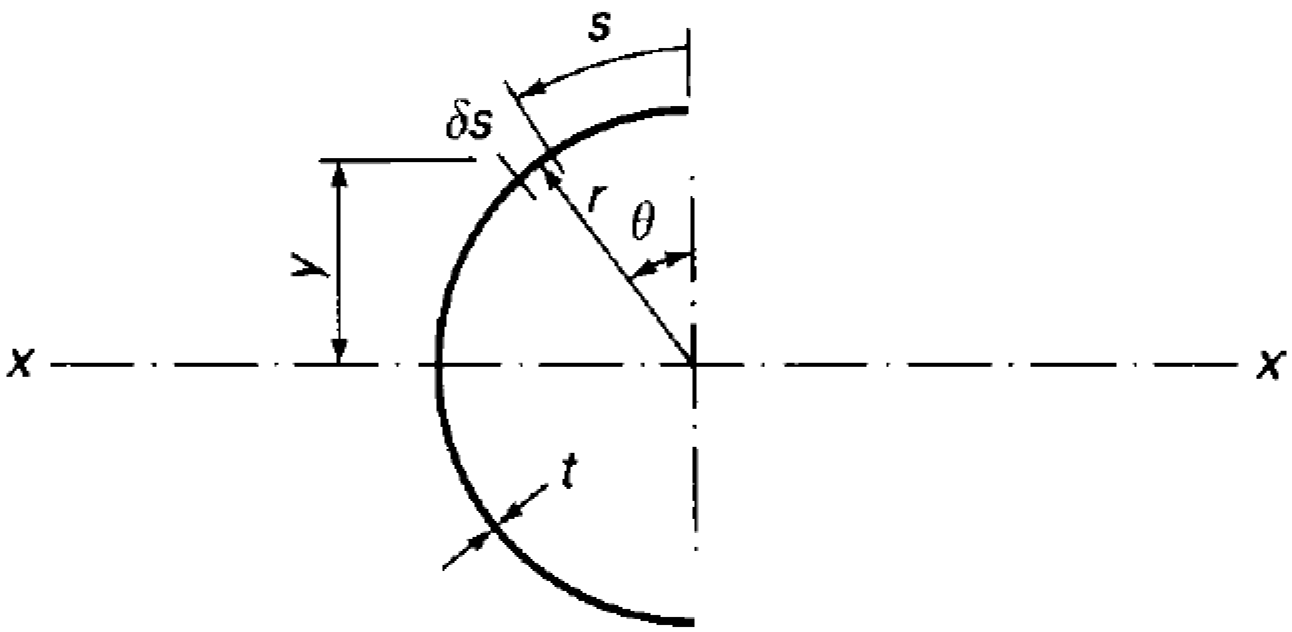
$$I_{xy} = \frac{a^3\, t\, sin2\beta}{12}$$



Approximation of a thin-walled section

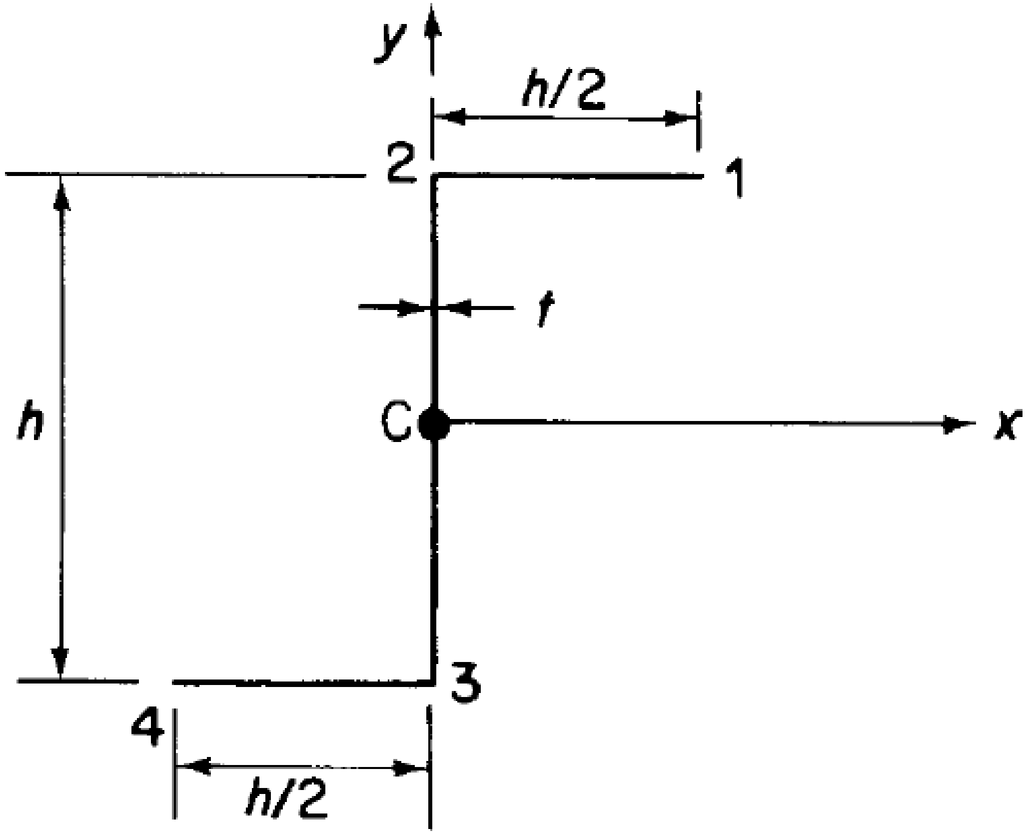
For curved sections:

$$I_{xx} = I_{yy} = \frac{\pi r^3 t}{2}$$



Example

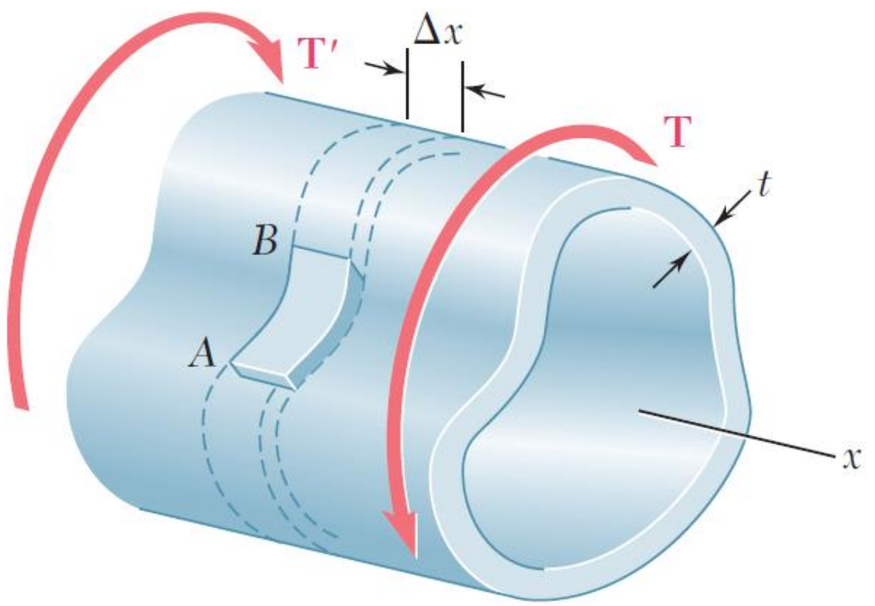
Determine the direct stress distribution general form, in the thin-walled Z section, shown in the figure, produced by a positive bending moment M_x

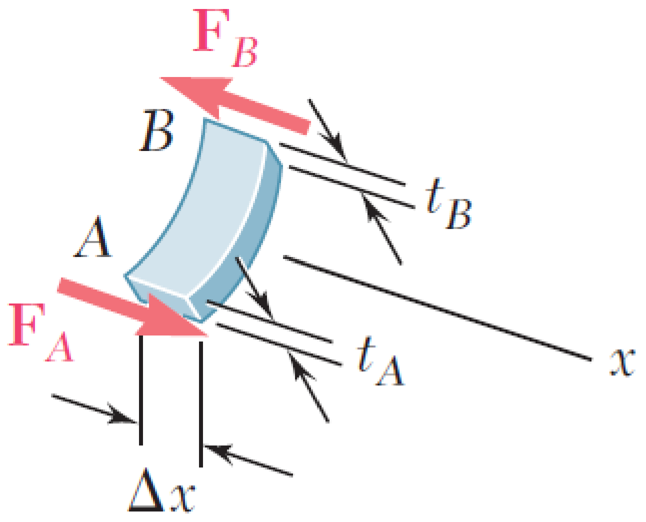


Shear flow

- Shear stress per unit length.
- A good approximation of the distribution stresses in a thin wall element can be obtained by a simple computation.
- Let's detach from the marked portion of wall AB two transverse planes at a distance Δx from each other. Since the portion AB is in equilibrium:

$$F_A - F_B = 0$$





Shear flow

$$F_A - F_B = 0$$

If

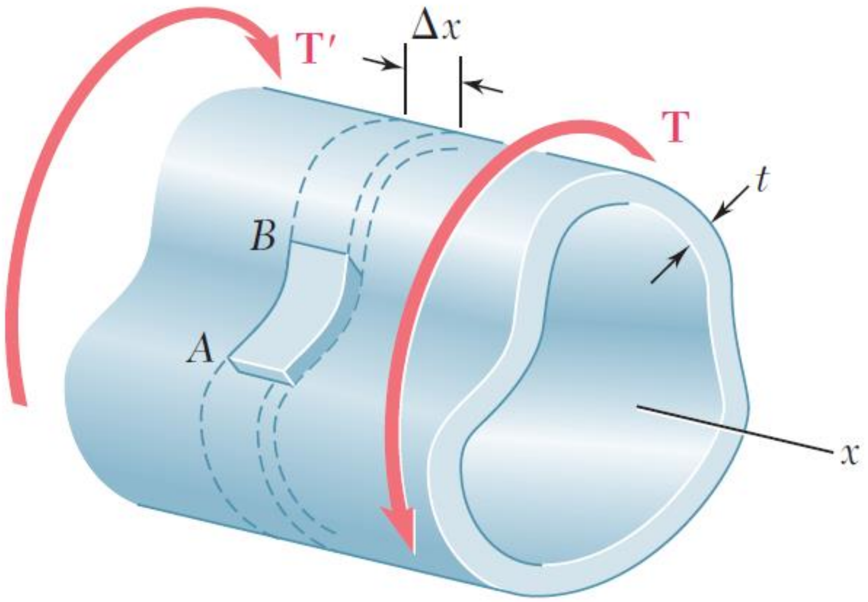
$$F_A = \tau_A(t_A \Delta x)$$

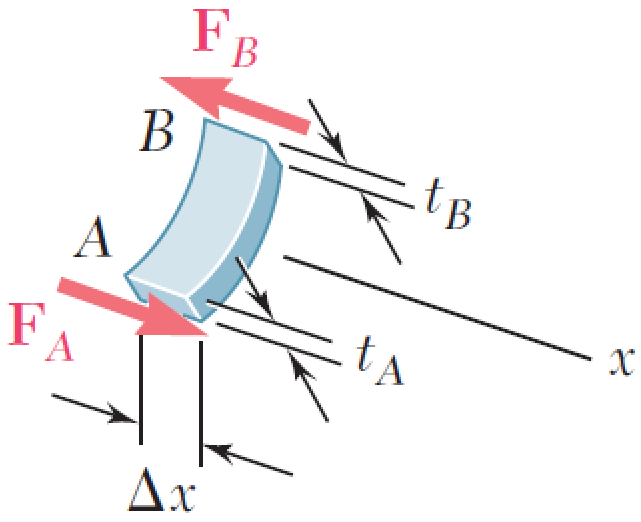
$$\tau_A(t_A \Delta x) - \tau_B(t_B \Delta x) = 0$$

$$\tau_A t_A = \tau_B t_B$$

This expression can be denoted as:

$$q = \tau t$$

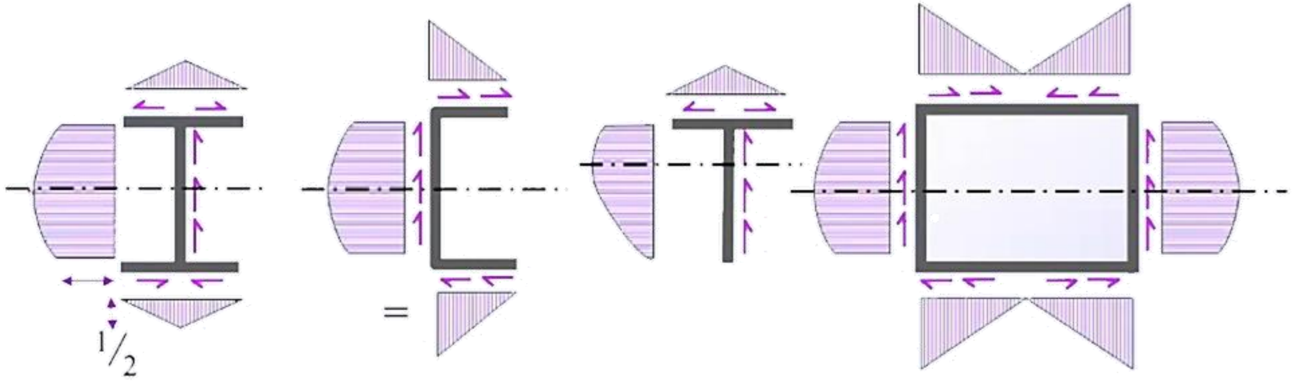


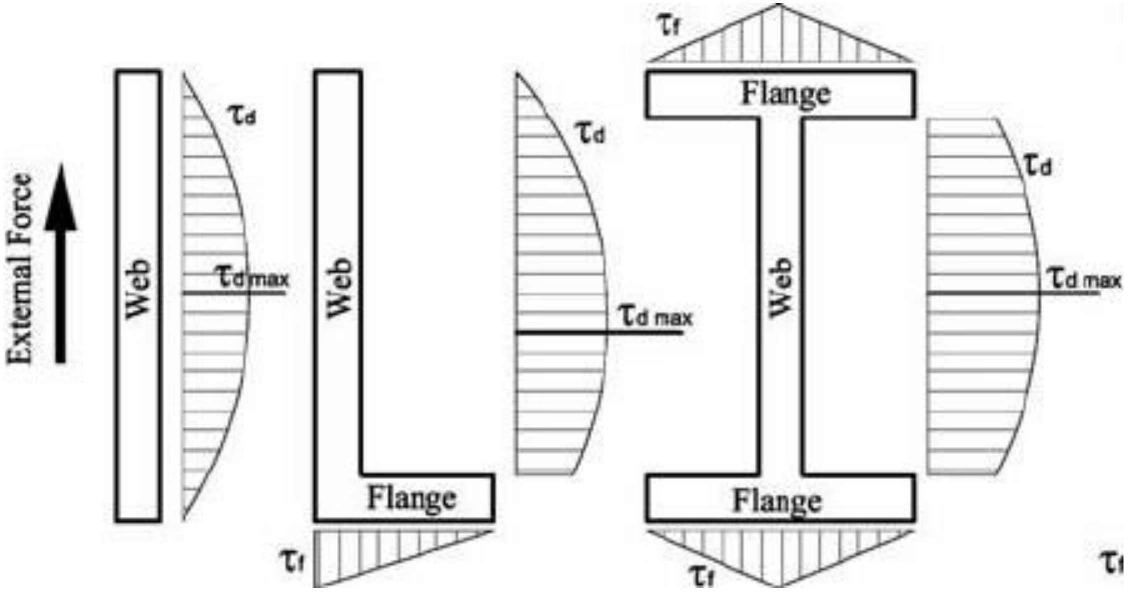


Shear of thin-walled beams

Shear flow is defined by the internal shear stress per unit of length:

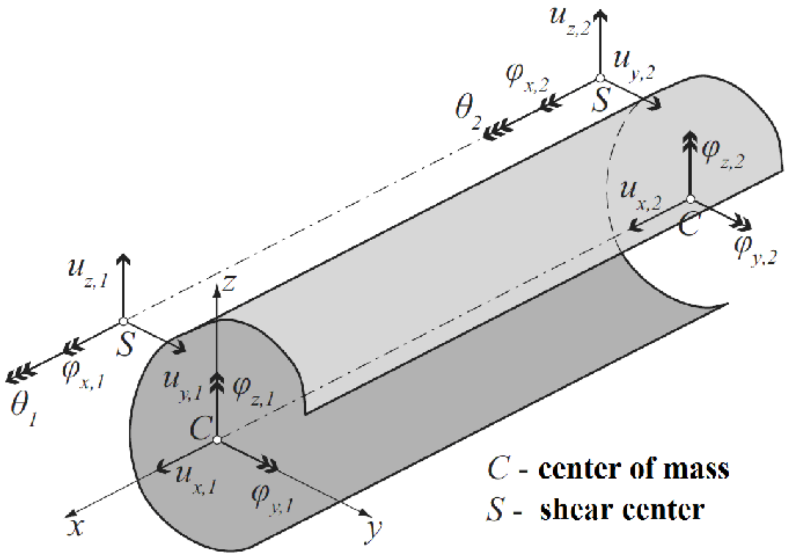
$$q = \tau t \ \ [N/m \ or \ lb/in]$$



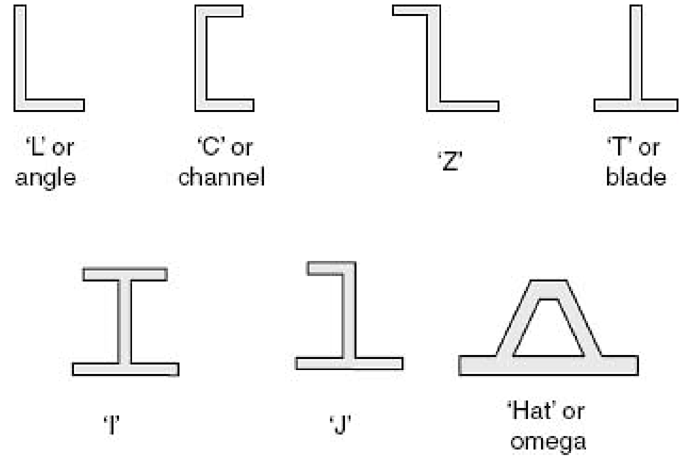


Shear flow distribution on thin-walled open section beams

An open section beam of an arbitrary section supports **shear loads S_x** and S_y such that there is no twisting of the beam cross section.



Further, the shear loads must both past through a particular point in the cross section known as the **shear center**.



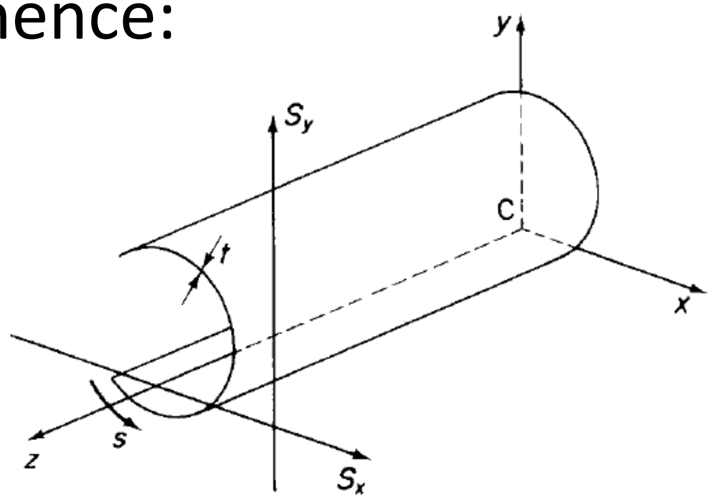
Shear flow distribution on thin-walled open section beams

The shear distribution can be defined by:

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s tx\, ds - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s ty\, ds$$

If the section has an axis of symmetry, then $I_{xy} = 0$, hence:

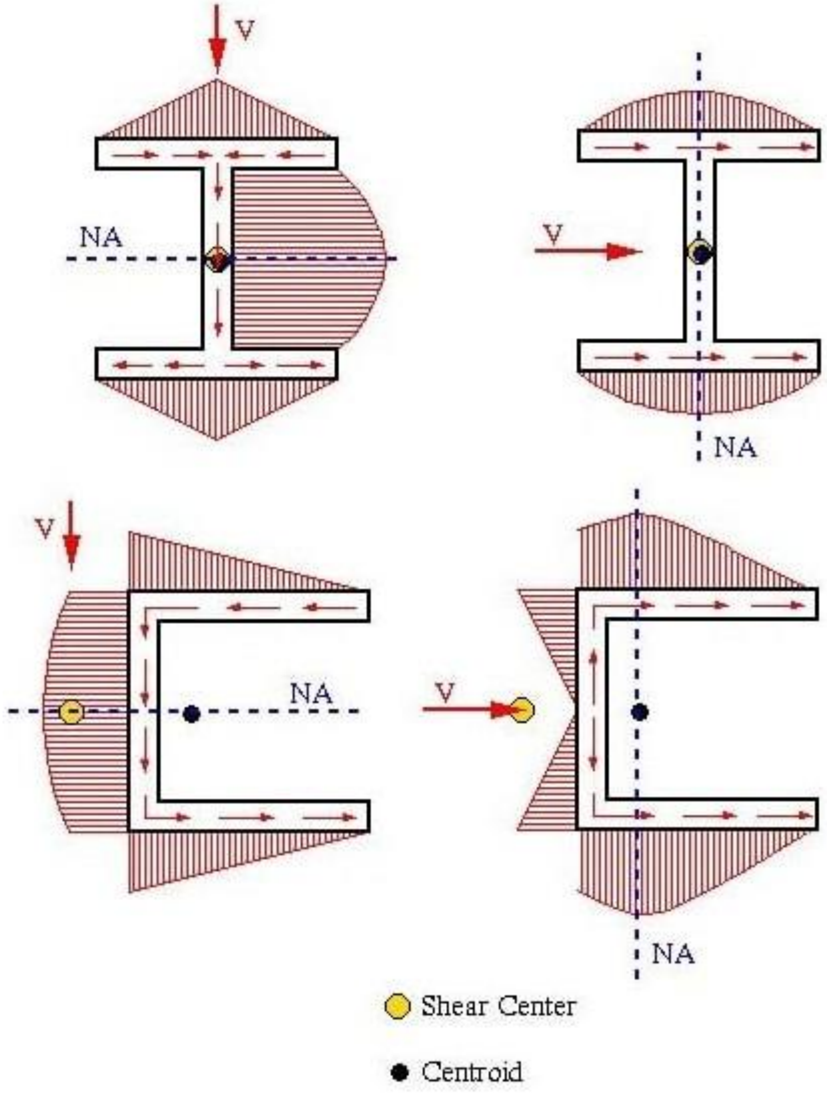
$$q_s = -\frac{S_x}{I_{yy}} \int_0^s tx\, ds - \frac{S_y}{I_{xx}} \int_0^s ty\, ds$$



Shear center

The shear center is defined as the point in the cross section in which shear loads produce no twisting, thus, only bending occurs.

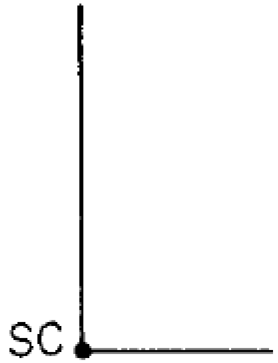
Pure shear is produced.

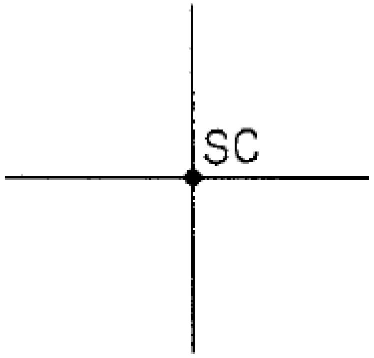


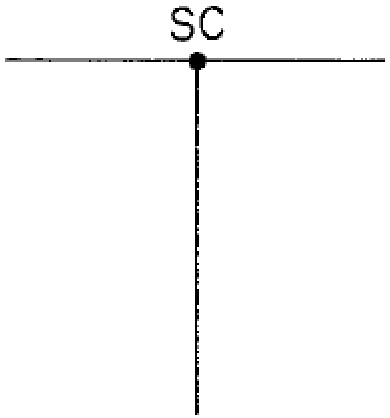
Shear center

If a beam has two axes of symmetry, shear center coincides with the centroid.

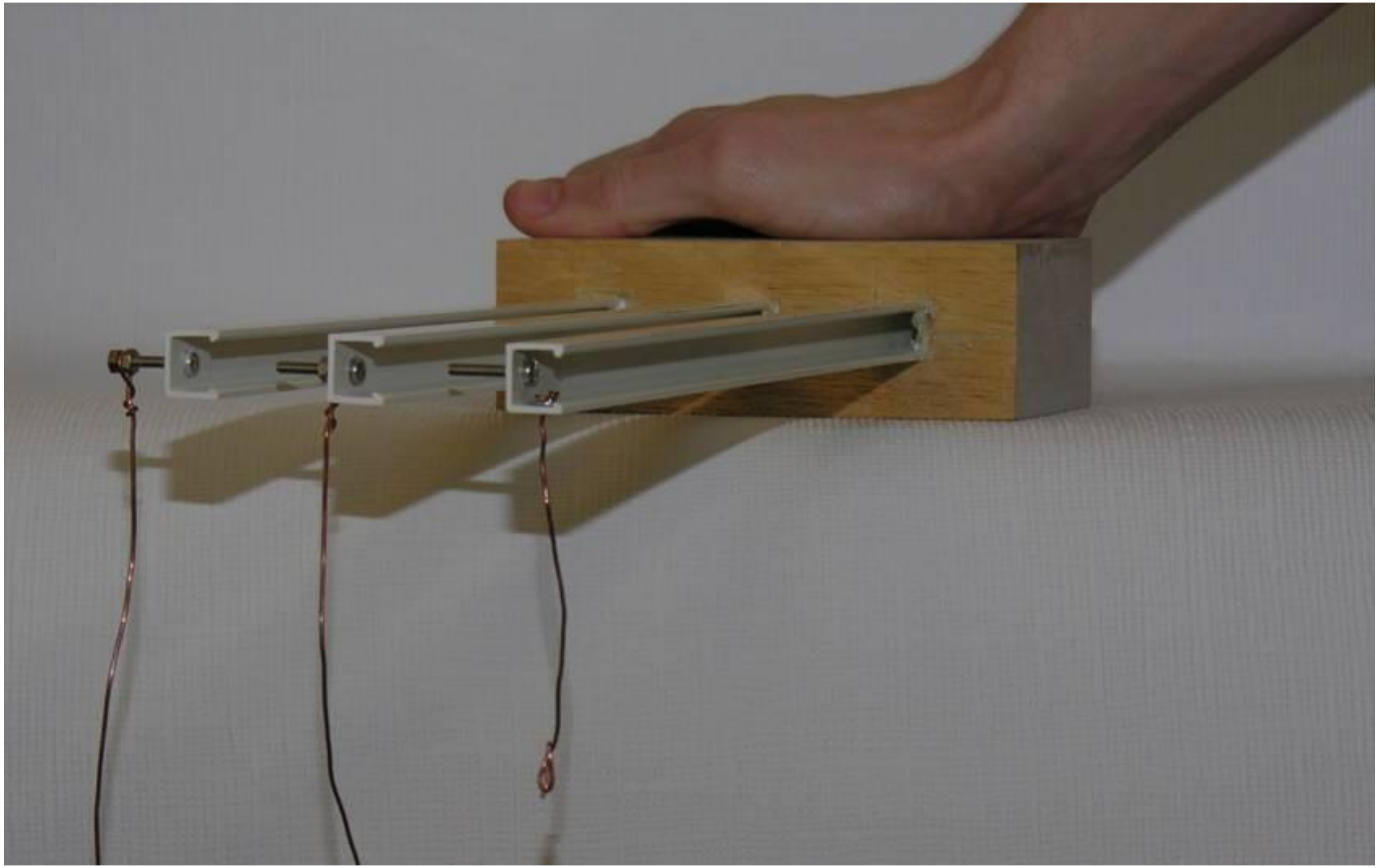
With one axis of symmetry the shear center does not coincide with the centroid but lies on the axis of symmetry.







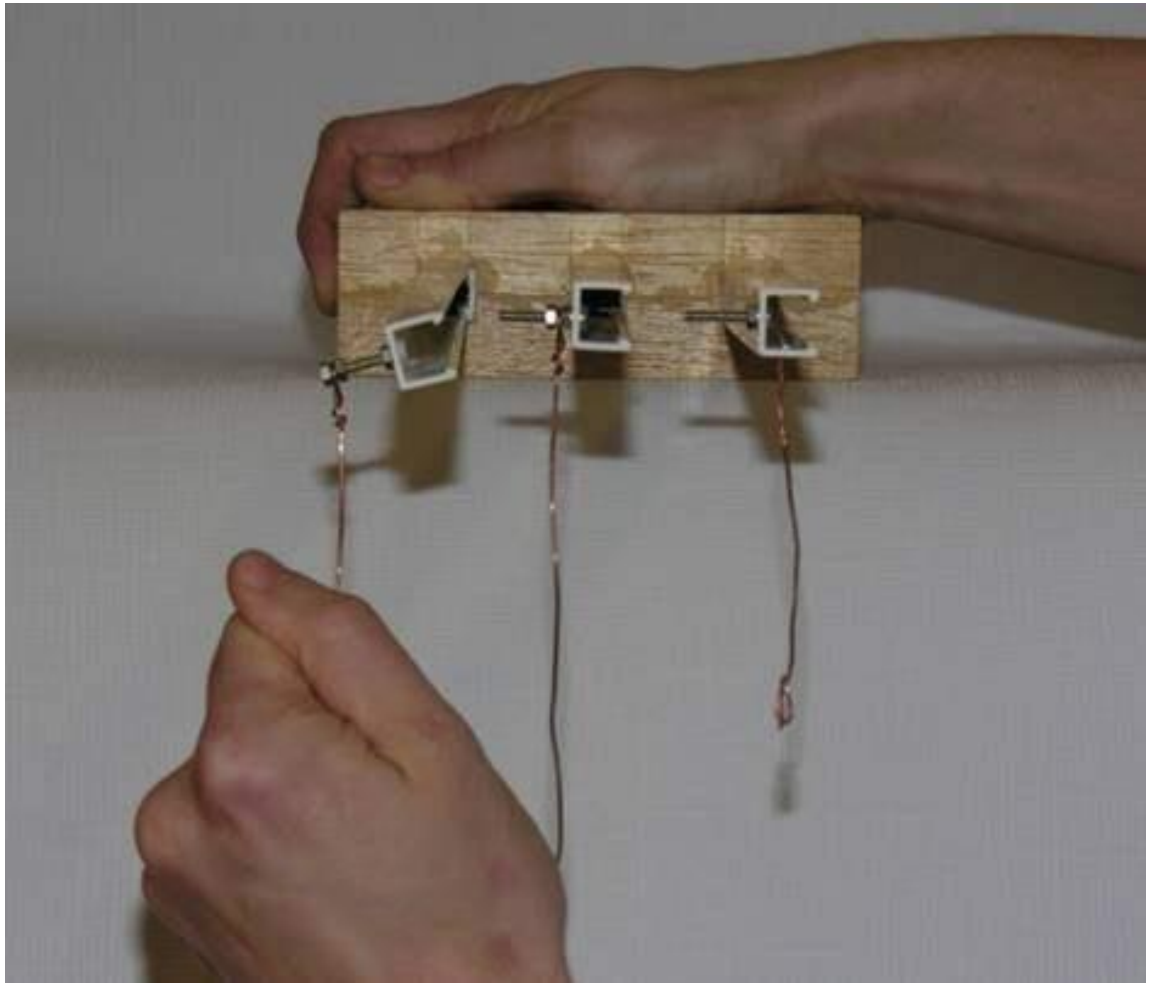
Shear center



https://epsassets.manchester.ac.uk/structural-concepts/Statics/shear/shear_mod6.php

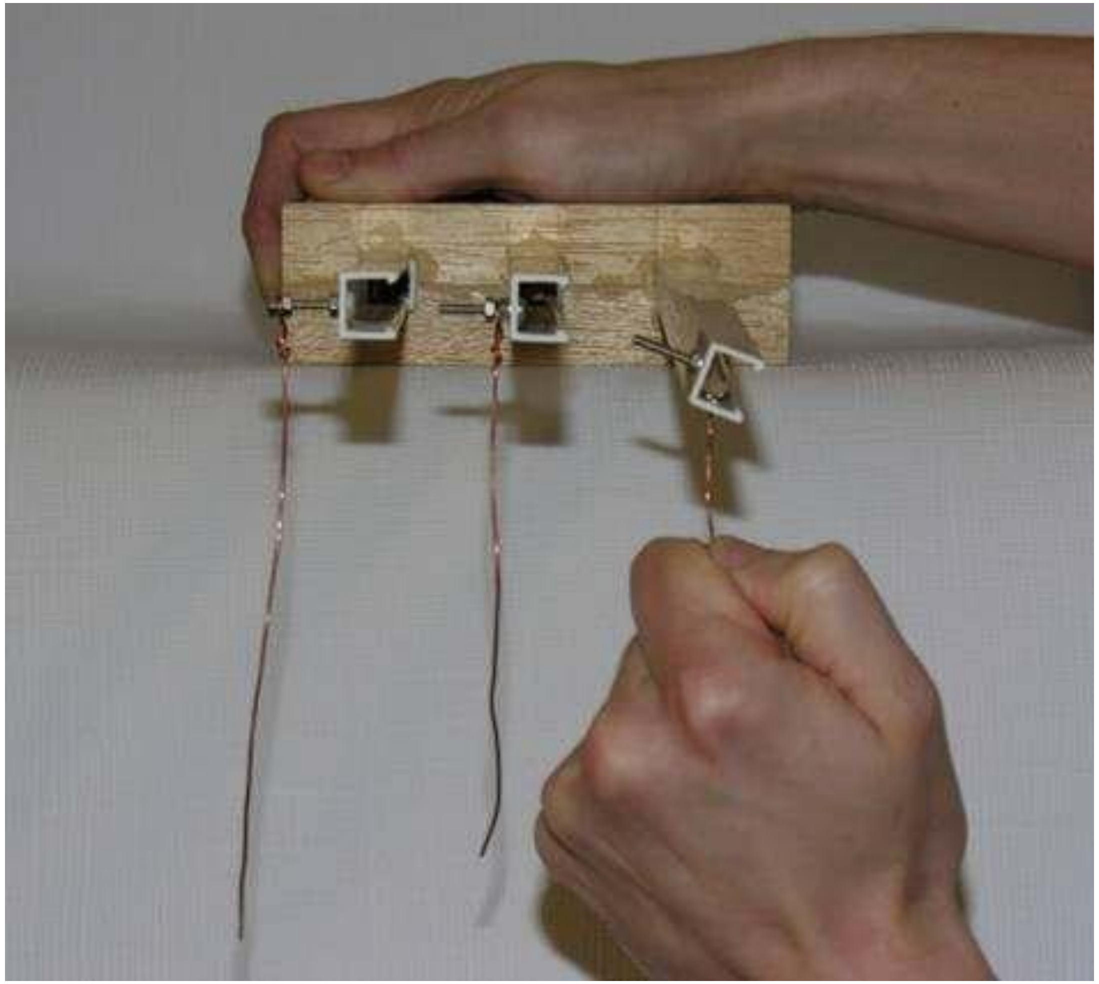
Shear center

Torsion due to shear load being applied at a distance eccentric to the shear center



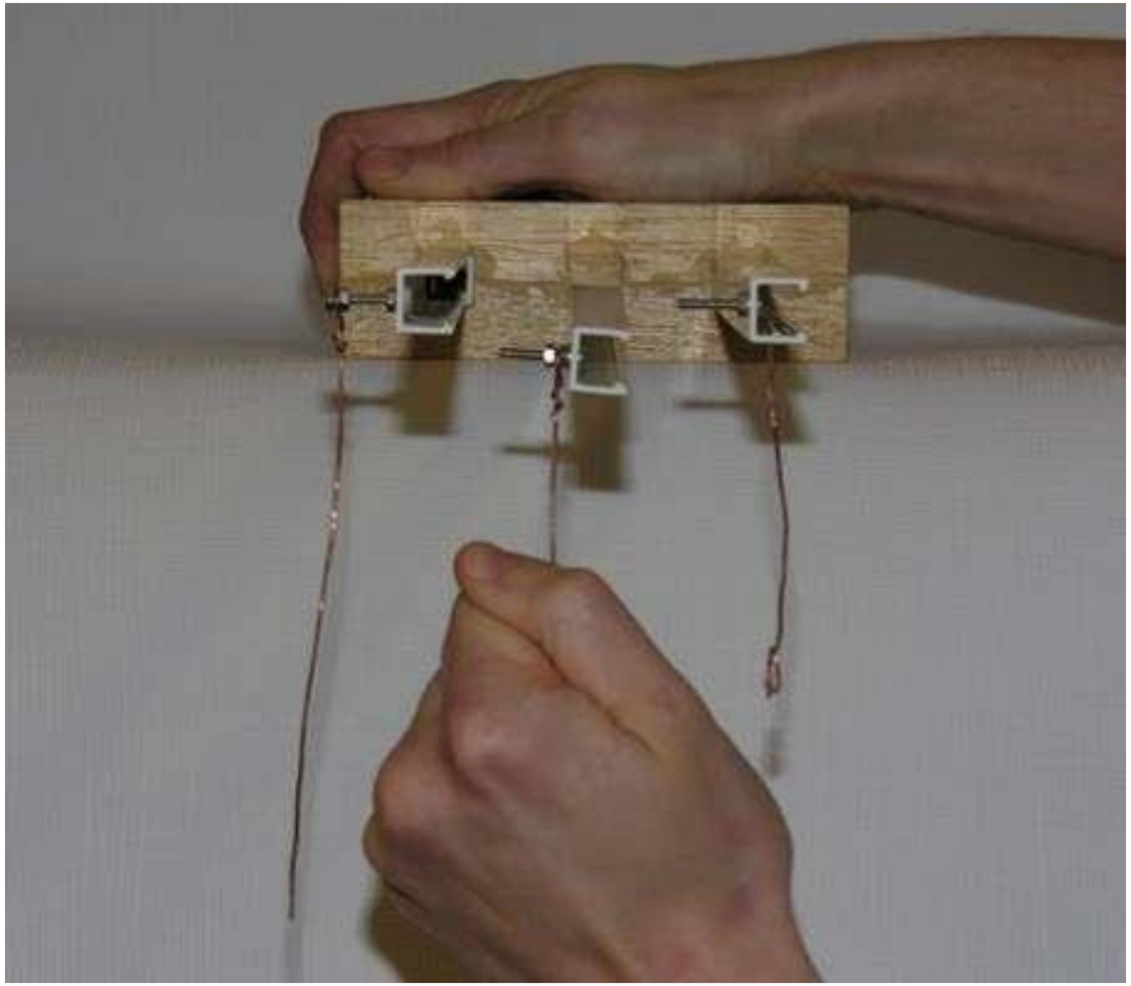
Shear center

Torsion due to bending load applied a distance eccentric to the shear center



Shear center

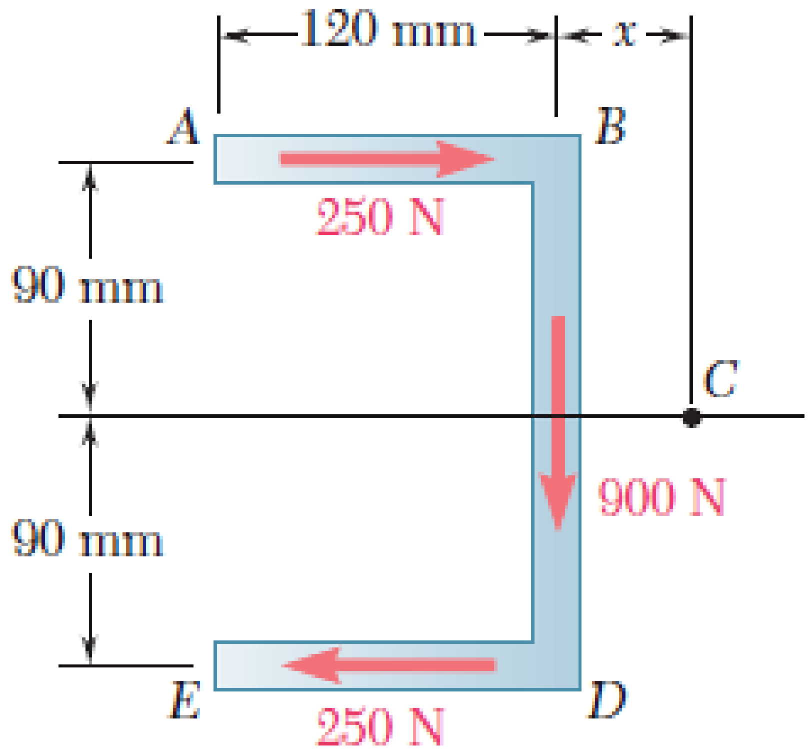
No torsion, only bending due to load being applied at the shear center



Shear center

Replace the system of forces shown with an equivalent force and couple system applied at C.

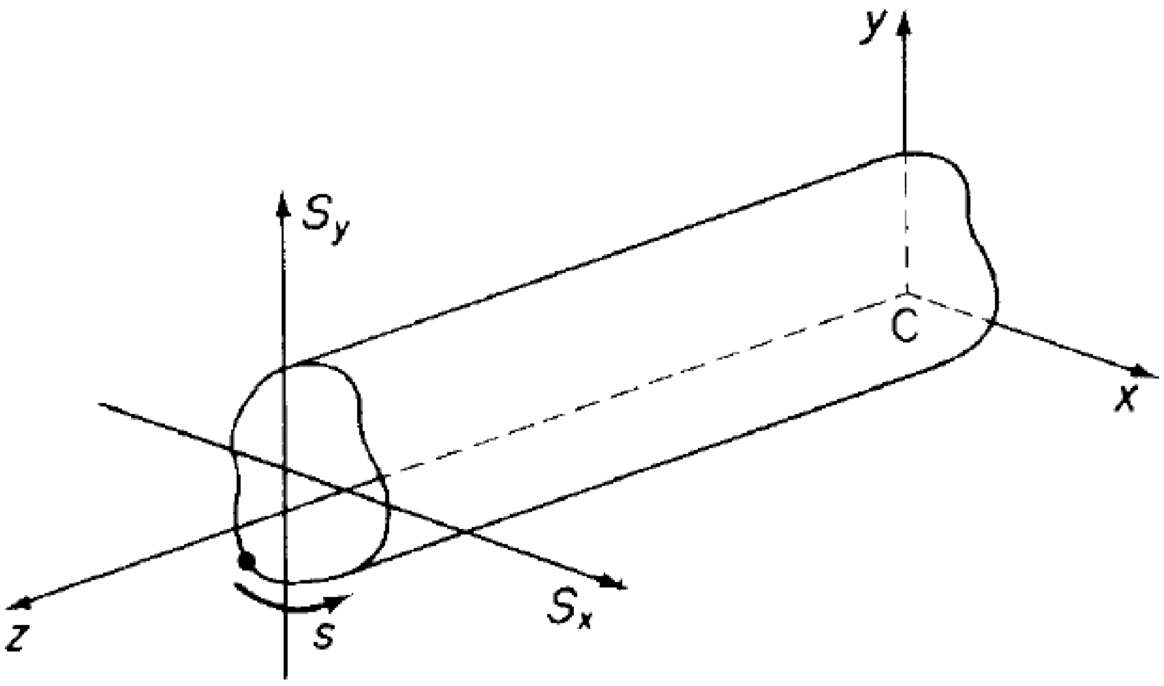
What is x such that $\sum M_c = 0$



Shear flow distribution on thin-walled closed section beams

Shear flow over closed sections beams follows a similar resolution as the one presented for open sections.

Shear loads may be applied through points other than shear center thus, torsion and shear effects may be included.



Shear flow distribution on thin-walled closed section beams

$$q_s = -\left(\frac{S_x I_{xx} - S_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s tx\, ds - \left(\frac{S_y I_{yy} - S_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) \int_0^s ty\, ds + q_{S,0}$$

$$q_s = q_b + q_{S,0}$$

Where

q_b is the representation of the open section shear flow,

$q_{S,0}$ is an unknown value of shear flow defined by the closed section.

Shear flow distribution on thin-walled closed section beams

$q_{S,0}$ is an unknown value of shear flow defined by the closed section and is defined by:

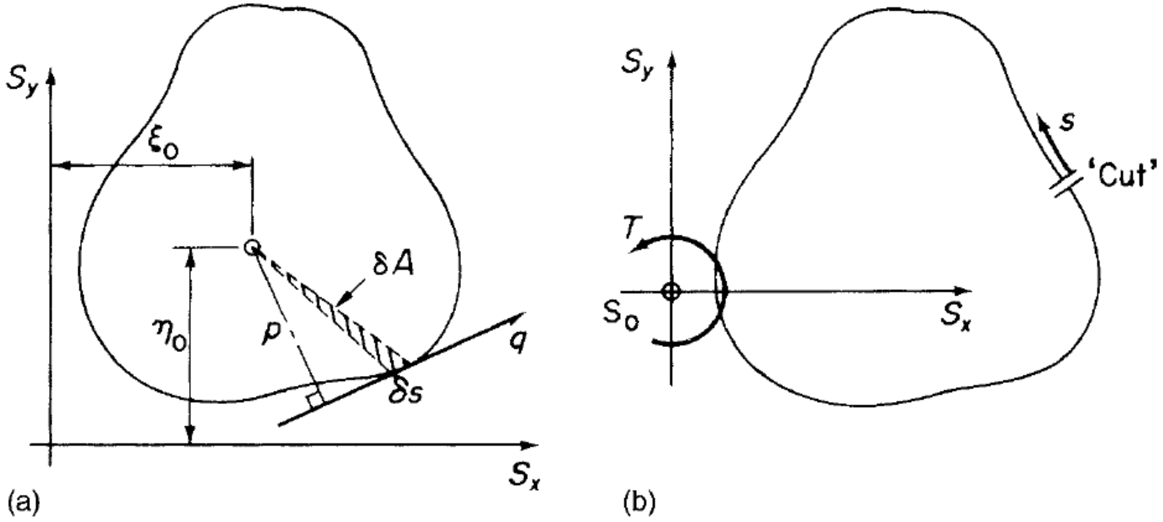
$$S_x\eta_0 - S_y\varepsilon_0 = \oint pq_b ds + 2Aq_{S,0}$$

p is a normal distance regarding to ds

η_0 and ε_0 are the distances to the moment center

A is considered the area bounded

by the center line



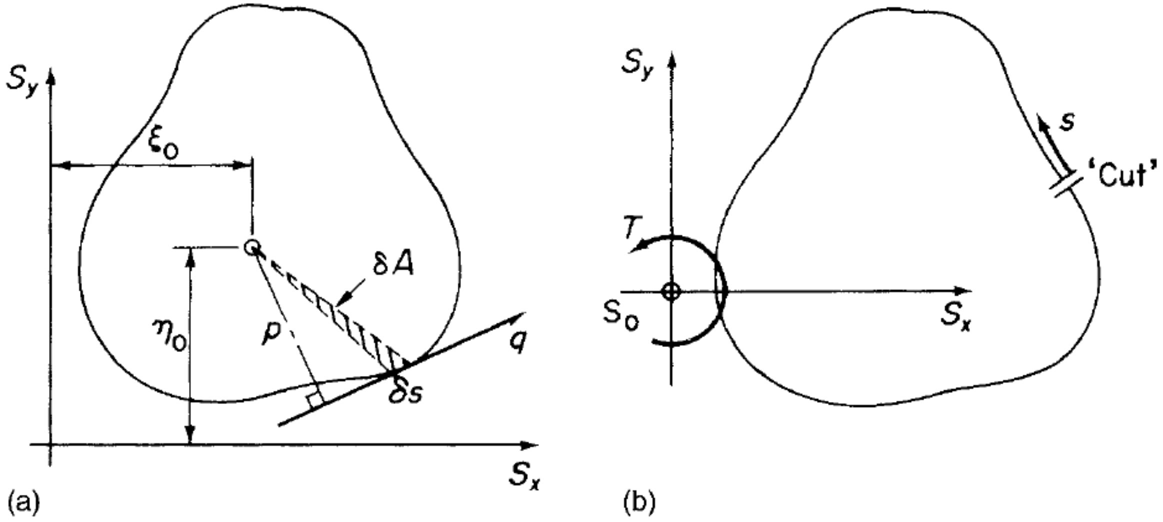
Shear flow distribution on thin-walled closed section beams

If the moment center is chosen to coincide with the lines of action of S_x and S_y then the previous equation reduces to:

$$0 = \oint p q_b ds + 2A q_{S,0}$$

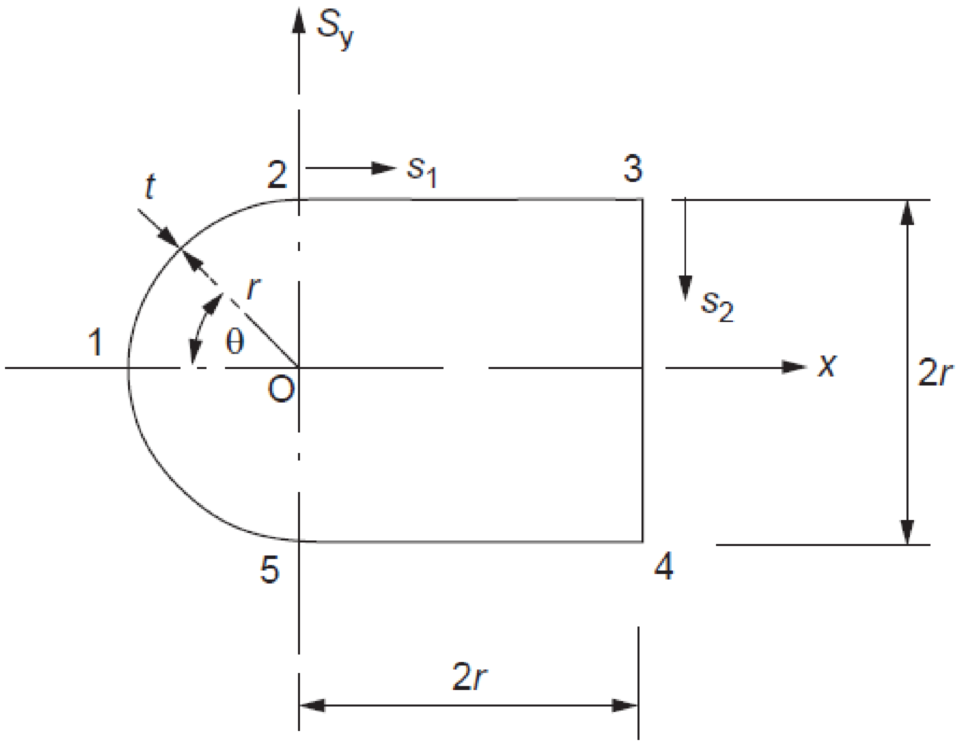
If also the section is symmetrical:

$$q_{S,0} = 0$$



Example

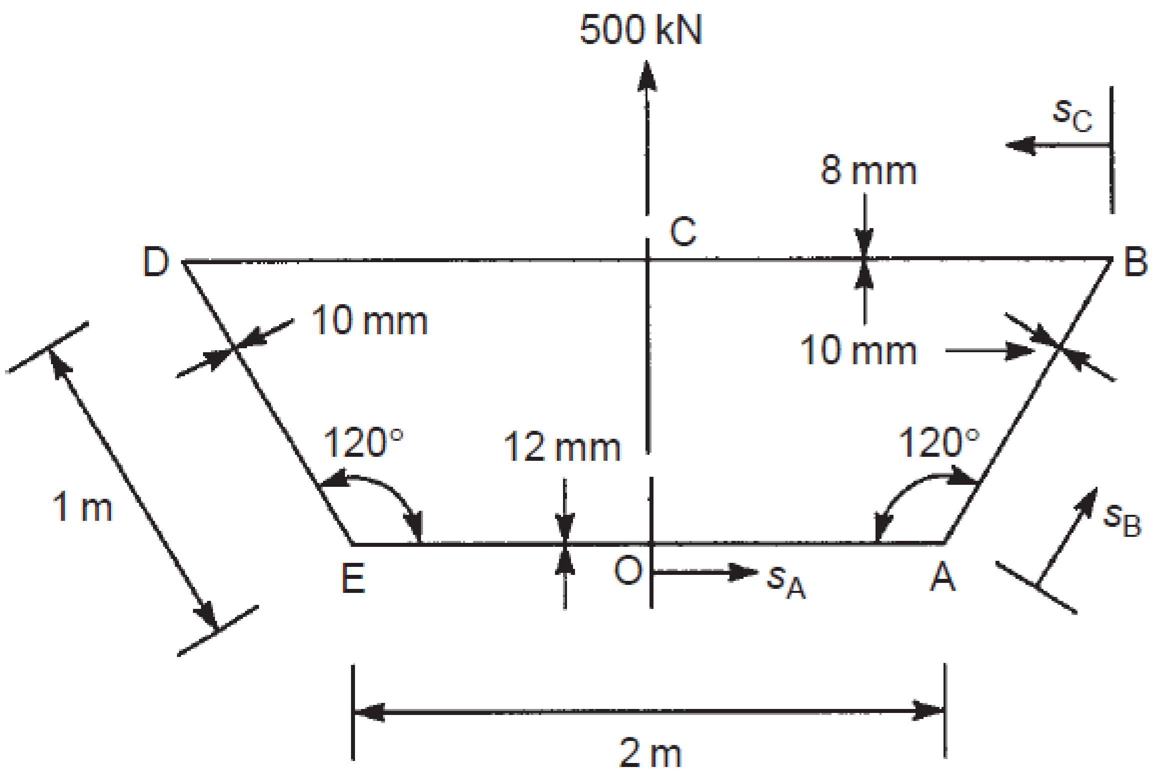
Determine the shear flow distribution in the walls of the thin-walled closed section beam. The wall thickness, t, is constant throughout.



Example

A box girder has the singly symmetrical trapezoidal cross-section. It supports a vertical shear load of $500\ kN$ applied through its shear center and in a direction perpendicular to its parallel sides.

Calculate the shear flow distribution and the maximum shear stress in the section.

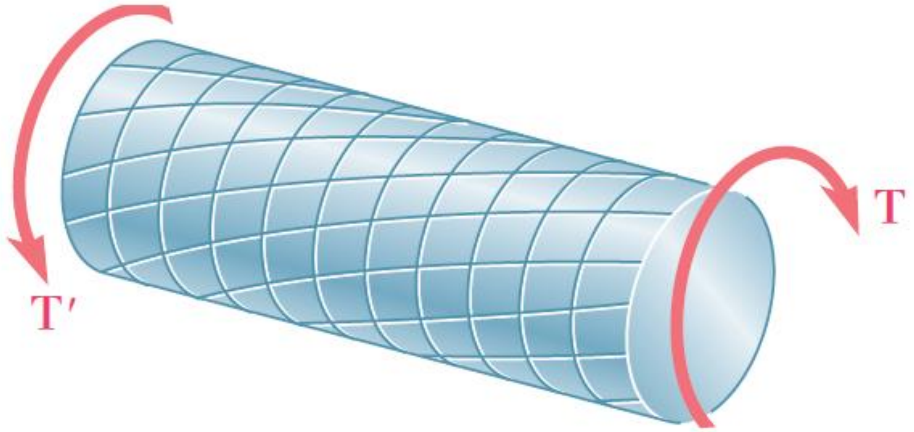


Torsion of non-circular members

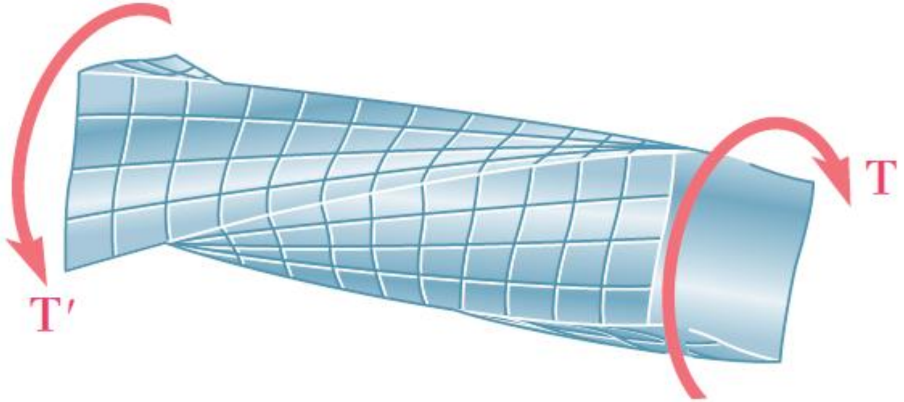
The plane of a circular member under torsional load will always remain plain (axisymmetry).

A square bar, retains the same appearance only when it is rotated through 90° or 180°

Because the lack of axisymmetry, the cross section itself will be warped out of its original plane.



(a)



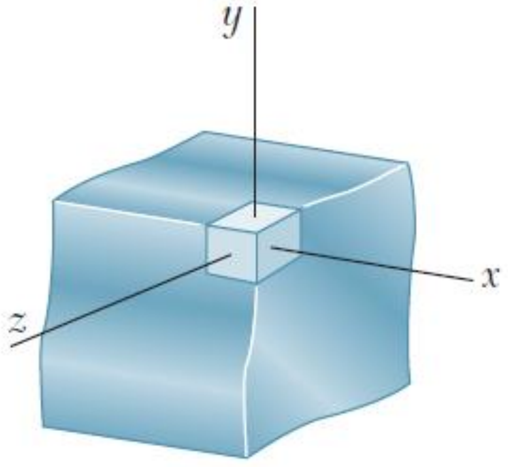
(b)

Torsion of non-circular members

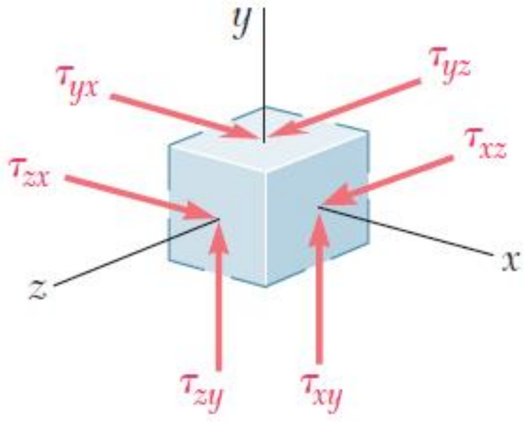
Consider an element located at a corner of the section, in the surfaces perpendicular to y axis the shear stress must be cero, therefore:

$$\tau_{yx} = \tau_{yz} = \tau_{zx} = \tau_{zy} = \tau_{xy} = \tau_{xz} = 0$$

There is no shearing stress at the corners of the cross section of the bar.



(a)



(b)

Torsion of non-circular members

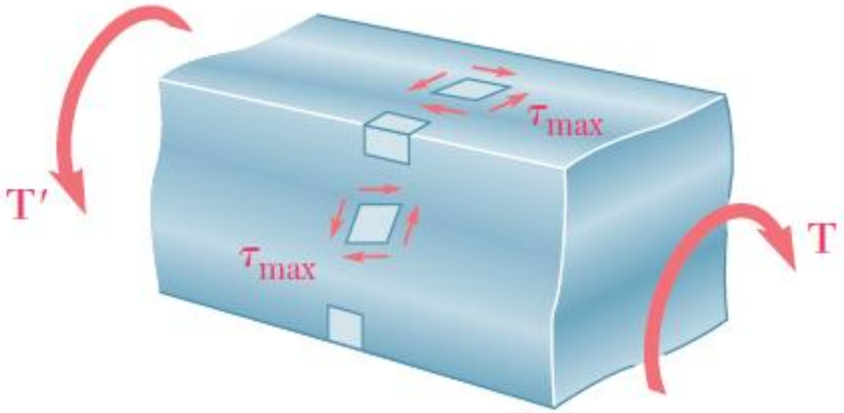
The maximum shear stress for non-circular member occurs along the center of the wider face of the bar and is defined by:

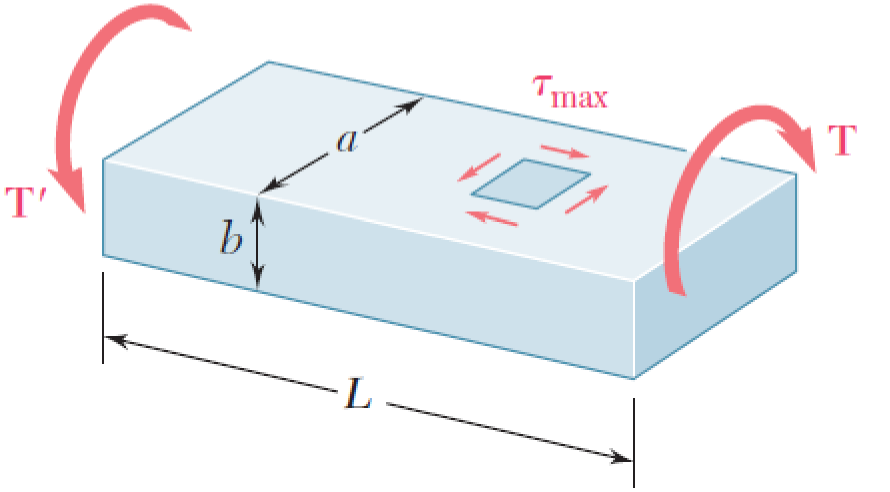
$$\tau_{max} = \frac{T}{c_1 a b^2}$$

T is the magnitude of the torque

a and b are the cross-section lengths

c_1 is the parameter that depends on the ratio a/b



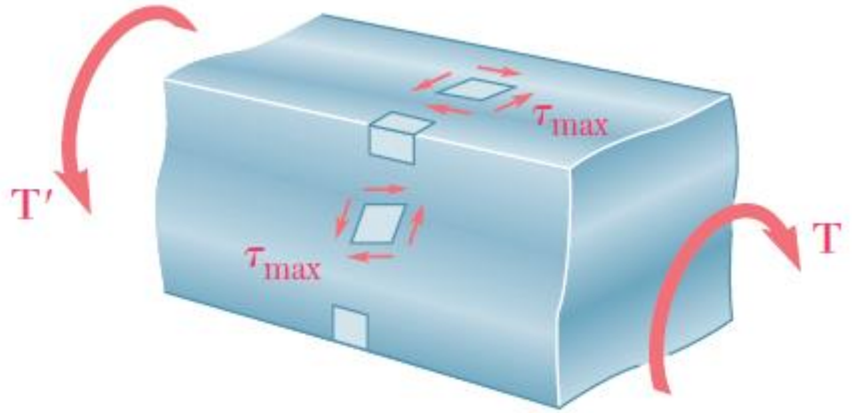


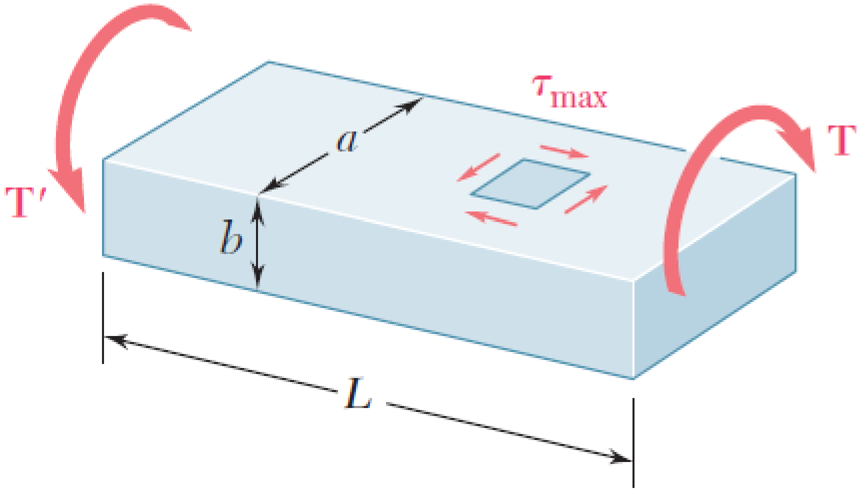
Torsion of non-circular members

The shear angle for non-circular member is defined by:

$$\theta = \frac{TL}{c_2 a b^3 G}$$

Where G is the shear modulus





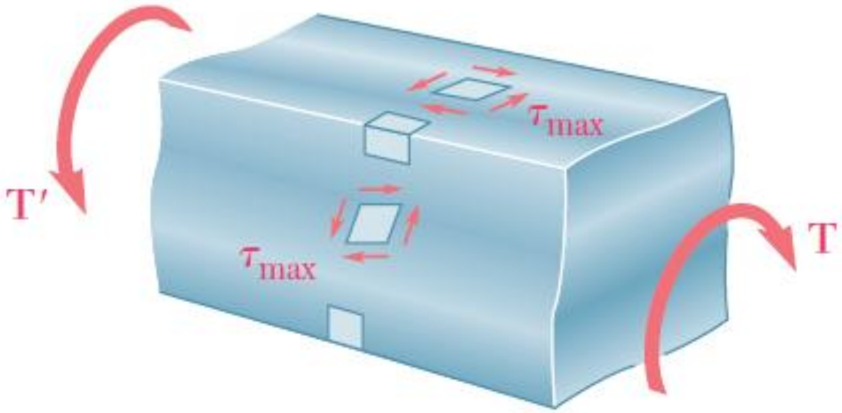
Coefficients of rectangular bars in torsion

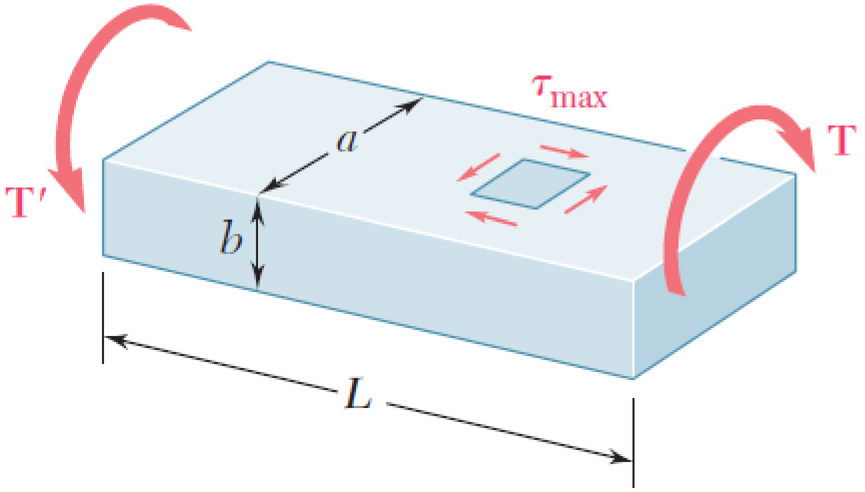
a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

Torsion of non-circular members

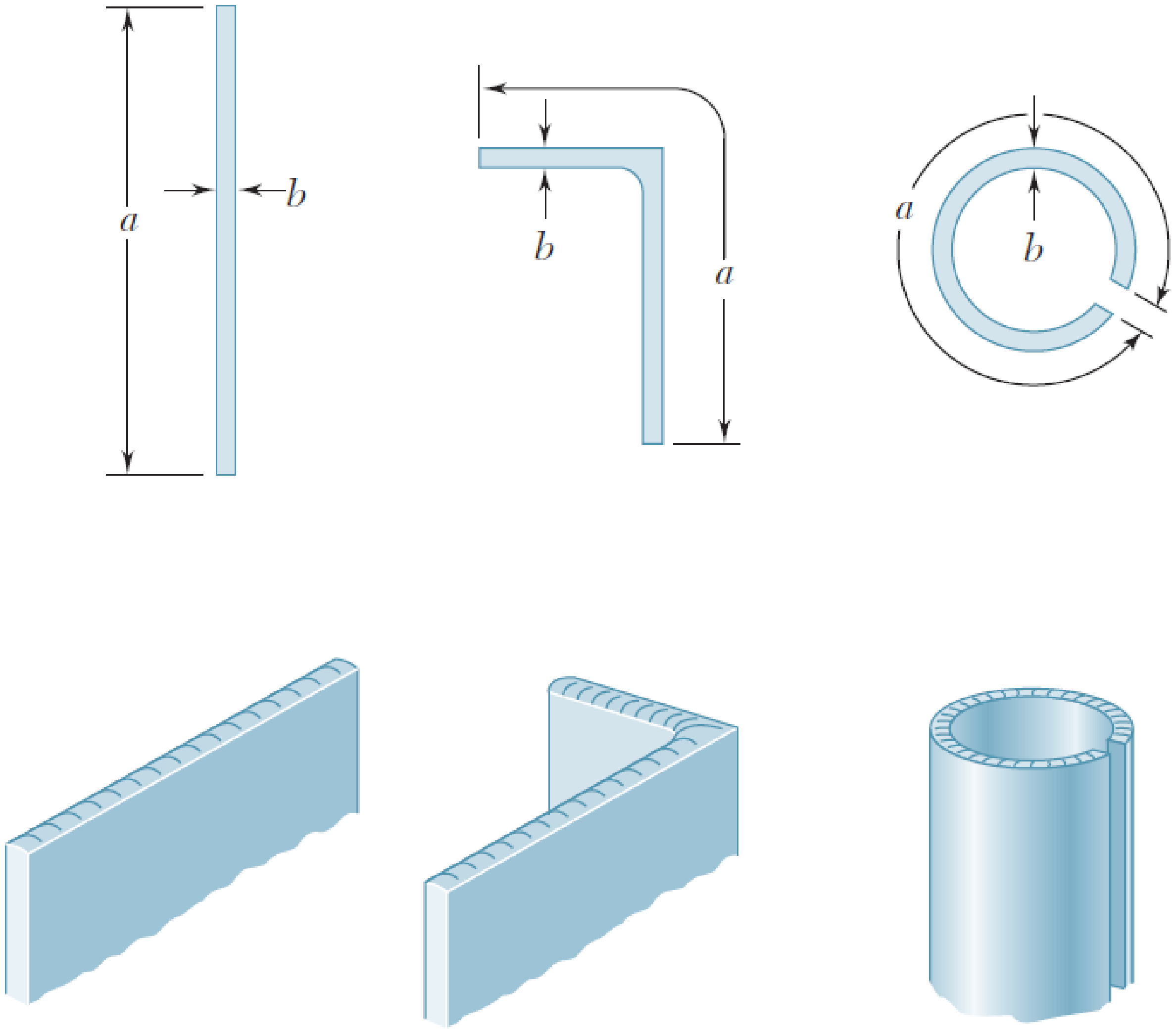
Parameters c_1 and c_2 for $a/b \geq 5$ are defined by:

$$c_1 = c_2 = \frac{1}{3}\left(1 - \frac{0.630b}{a}\right)$$





Torsion of non-circular members



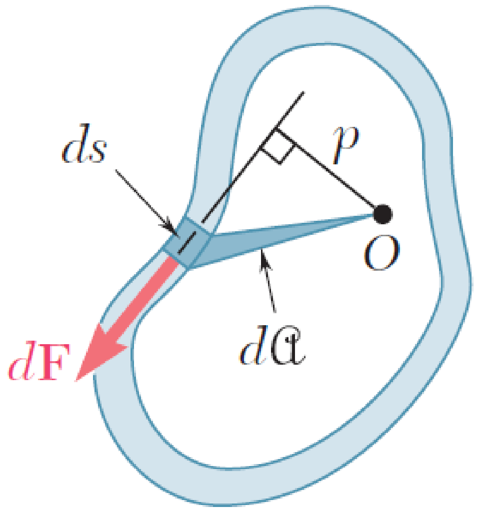
Torsion of non-circular members

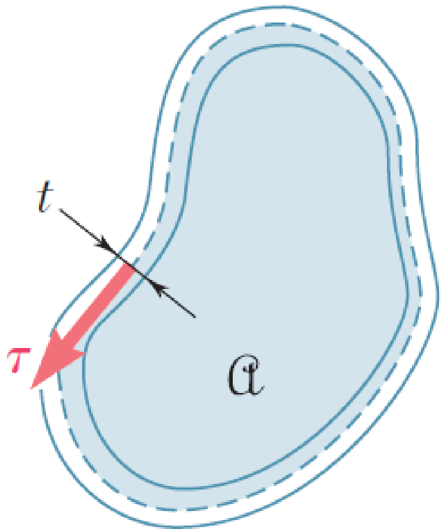
The average shearing stress is defined by:

$$\tau = \frac{T}{2tA}$$

A is considered the area bounded by the center line

t is the thickness

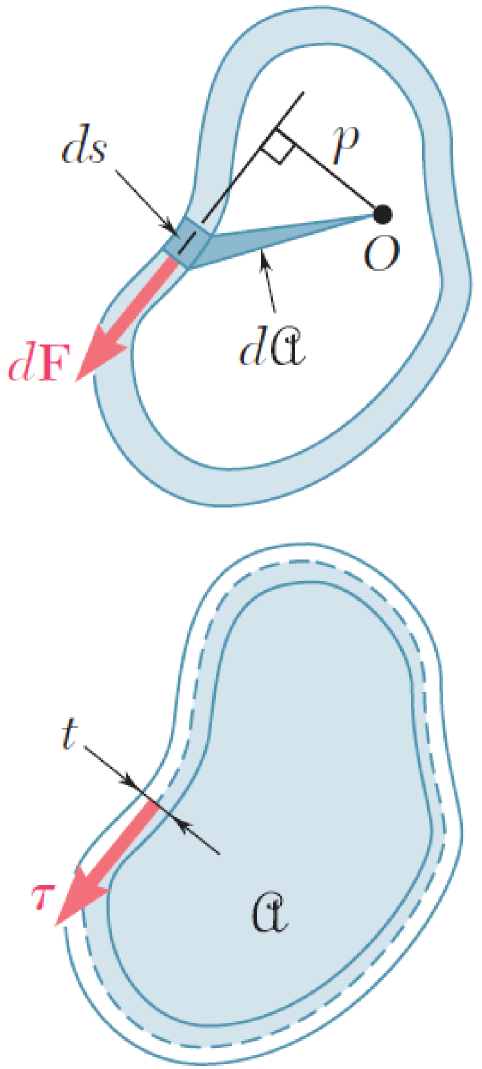




Torsion of non-circular members

The angle of twist of a thin-walled hollow shaft may be obtained using the energy method:

$$\phi = \frac{TL}{4A^2G}\oint \frac{ds}{t}$$



Example

- Structural aluminum tubing of $2.5 \times 4\ in.$ rectangular cross section was fabricated by extrusion.
- Determine the shearing stress in each of the four walls of a portion of such tubing when it is subjected to a torque of $24\ kip\ in.$ ($1\ kip = 1000\ lbf$)
- Assume a uniform thickness $t = 0.16\ in$
- What if by a defective fabrication, $t = 0{,}12\ in$ for the sections AB and AC, and $t = 0{,}2\ in$ for the sections BD and CD