Final Project: Optimizing Crop Rotation

Presented to

Professor Javad Nasiry

Presented by

Joshua Poozhikala - 261184574
Vinay Govias- 261143063
Valentin Najean – 261134857
Michelle Barabasz- 261152119
Tomy Pelletier - 261165447

MGSC 662

McGill University - Desautels Faculty of Management

1. Introduction

This project unfolds in Saskatchewan, the heartland of Canadian agriculture. This province, renowned for its expansive and fertile farmlands, is a significant contributor to the country's grain production, accounting for over 40% of it. Traditionally, farming practices have predominantly employed monoculture techniques, where a single type of crop is cultivated extensively over numerous growing seasons on the same plot of land. However, the discovery of crop rotation has introduced more sophisticated planting and harvesting methods that are economically efficient, sustainable, and environmentally friendly. Crop rotation is an agricultural practice involving the planting of different crops in the same area in sequential seasons to maximize yield by minimizing soil nutrient depletion and pest damage (Ball et al., 2005). While offering numerous benefits, crop rotation is a complex task that requires careful consideration of various variables.

In line with the Canadian government's recent initiatives of investing $469.5 million dollars into climate solutions for agriculture to provide support to farmers in adopting beneficial management practices such as nitrogen management and cover cropping, this project aims to optimize crop rotation practices (Government of Canada, 2022). The model will focus on four summer crops - wheat, canola, oat, and lentils - and two winter varieties, chosen for their diversity in yield, botanical family, and varying nutrient requirements and contributions. The goal is to develop a model that promotes sustainable farming practices.

The project's significance extends far beyond the boundaries of Saskatchewan. It addresses pressing global issues by adapting traditional farming techniques to contemporary environmental and economic contexts. The insights gleaned from the output of the model are expected to be beneficial not only to local farmers but also to the larger discourse on sustainable agriculture. The objective is to forge a model that is not only effective in Saskatchewan's diverse agricultural landscape but is also adaptable and scalable to various farming environments worldwide.

2. Problem Definition

To formulate an effective cropping sequence, several critical aspects must be evaluated, including soil nutrient characteristics, budget constraints, and considerations related to botanical family and plot adjacency. The challenge lies in balancing the fulfillment of nutrient requirements and optimizing the overall profitability of the farm, as outlined in the objective function later, which seeks to maximize revenue while minimizing costs. The model integrates the following constraints to address these diverse requirements effectively:

- Budget availability: Ensures that the total costs, encompassing labor, equipment, purchase of seeds, fertilizer, plant protection, and other expenses, do not exceed the pre-set budget limit.
- Minimum nutrient requirements: Guarantees that each crop receives adequate nutrients, by ensuring that the combination of nutrients applied and already present in the soil meets or exceeds the crop's consumption needs.
- Botanical family and adjacency: Addresses the risk of disease and pest spread by preventing the planting of crops from the same botanical family in adjacent plots simultaneously.
- Succession of crops on same family: Promotes crop rotation and soil health by preventing the successive planting of crops from the same botanical family on the same plot.
- Logical growing constraints: Includes constraints for ensuring crops are planted only in their allowed months, crops have sufficient growth duration within the planning horizon, and plots are not left fallow during critical growing months.

2.1. The Land

To initialize the model, data regarding land constraints and monetary availability was required. Research conducted in 2016 by Statistics Canada indicates that the average landholding of a farmer in Saskatchewan is approximately 1,784 square acres (Statistics Canada, 2016). A determination of 4 plots, each of 401 square acres, was made to ensure that constraints involved in the model creation were adequately used and crops were selected with diversity. The success of crop rotation in Canada is significantly influenced by seasonal variations, making it essential to implement a modeling approach that spans 36 months, commencing in May. A budget of $4.5 million dollars for the 3 cropping seasons was also used to provide a limiting factor that would force the model to come up with an effective spending strategy.

To ensure optimal crop growth, it is crucial to initiate the soil with specific nutrient requirements. These requirements include the application of 15 lbs. of nitrogen per acre, 100 lbs. of phosphorus per acre, and 28 lbs. of potassium per acre to create an adequate initial state for planting.

2.2. The Crops

For the purpose of this model, a selection of six crops commonly grown in Saskatchewan have been carefully chosen, as detailed in Appendix 9. The chosen crops include wheat, canola, lentils, and oats, all of which are primarily planted during the summer season. These crops represent a diverse range of characteristics and growth requirements, making them ideal candidates for studying the effects of crop rotation in a typical agricultural cycle. The summer season, often regarded as the 'regular' planting season, is when the majority of agricultural activity takes place. During this time, crops like wheat and canola are planted for their high economic value and nutritional content (Onunkwo & Holston, 1997). Lentils, being leguminous, contribute to soil health by fixing nitrogen, thus enhancing soil fertility for subsequent crops (Mesfin et al., 2023). Oats are included for their adaptability and resilience, offering a stable yield under various environmental conditions (Ponisio & Kremen, 2016).

In contrast to the summer crops, hairy vetch and winter rye were selected for the winter planting season. These crops serve a dual purpose. Firstly, as cover crops, they play a crucial role in preserving soil nutrients and preventing erosion during the off-season. Cover crops like hairy vetch and winter rye are known for their ability to enhance soil structure and fertility ("Hairy Vetch as a Crop Cover," (n.d.)), with hairy vetch primarily being planted because of its above average nitrogen contribution to the soil compared to other crops. They protect the soil from nutrient leaching and erosion, ensuring that the soil remains fertile and ready for the next planting season ("Hairy Vetch as a Crop Cover," (n.d.)). Secondly, these winter crops provide economic benefits by maximizing land utilization throughout the year. By planting hairy vetch and winter rye, farmers can ensure continuous production, which is essential for maintaining a steady income stream and efficient land use.

2.3 Costs

The model's costs can be categorized into various components, including labor, equipment, seeds, protection, and miscellaneous expenses. Miscellaneous expenses include insurance expenses, expenses associated with land (covering lease or rent payments, property taxes, and costs related to land enhancements), and general overhead costs. The second table in Appendix 9 provides detailed pricing information for each crop with respect to these cost categories.

3. Modeling

For the evaluation of the proposed agricultural optimization model, the Gurobi optimizer was selected, interfaced with Python. Gurobi is renowned for its efficiency in solving large-scale linear and mixed integer programming problems, making it an ideal choice for this complex model. The model, implemented in Python using the Gurobi Python API, combines various problem-solving techniques, including linear programming (LP) and mixed-integer programming (MIP) to account for the linear and non-linear constraints needed, which was particularly effective for handling the model's dynamic requirements, such as tracking crop families, managing nutrient levels across different periods, and ensuring compliance with the predefined budget.

3.1. Decision Variables

To optimize this problem, there are three sets of constraints that need to be introduced to the model.

- The planting decision, x_{cml}: A binary variable indicating whether a crop is cultivated in a particular plot during a specific month.
- The nutrient amendment decision variable, Z_{nml}: A continuous variable specifying the amount of nutrient n to be applied in plot l during month m.
- The stored nutrient variable, S_{nml}: A continuous variable which quantifies the stored amount of nutrient m in plot l at the end of month m.

3.2. Revenue Maximization Model

To better understand the proposed model, the set of parameters and variables are defined in Table 1.

Table 1: List of Parameters and Variables

Parameter/Variable	**Meaning**
M	Number of periods (in months), set to 36
L	Number of plots, set to 4
C	Set of main crops (cash crops), includes wheat, canola, oat, lentils, hairy vetch, and winter rye
F	Botanical families and their associated crops
$duration$	Duration of each crop in months
$Yield\_max$	Maximum yield per crop (in pounds per acre)
$Yield\_min$	Minimum yield per crop (in pounds per acre), calculated as a percentage of $Yield\_max$
$Revenue$	Revenue per crop (in dollars per pound)
$nutrients$	Set of nutrients, includes Nitrogen, Phosphorus, and Potassium
$nutrient\_consumption$	Nutrient consumption per crop for each nutrient (in pounds per acre)
$nitrogen\_production$	The amount of nitrogen produced by each crop (in pounds per acre)
$Fertilizer\_cost$	Cost per unit of each type of fertilizer (in dollars per pound)
$acre\_per\_plot$	Area of each plot in acres, set to 401
$allowed\_months$	Specific months for growing certain crops
$residual\_percentage$	Residual percentage for each crop
A	Adjacency list for plots, representing which plots are adjacent to each other
B	Total budget for the production cycle
Z	Decision variables for nutrient amendment
S	Decision variables for nutrient stored
$CropLabor$	Cost of labor per acre for each crop (in dollars per acre)
L_cost	Labor cost for each crop
$TotalLabor$	Total labor cost
$CropEquip$	Cost of equipment per acre for each crop (in dollars per acre)

E_cost	Equipment cost for each crop (in dollars per acre)
$TotalEquip\_operating$	Total operating cost for equipment
big_M	Large constant for the big-M method in optimization
$Seed\_cost$	Cost of seeds per acre for each crop (in dollars per acre)
$TotalSeed$	Total cost for seeds
$TotalFertilizer$	Total cost for fertilizers
$ProtectionCost$	Cost for plant protection per acre for each crop (in dollars per acre)
P_cost	Plant protection cost for each crop (in dollars per acre)
$TotalProtection$	Total cost for plant protection
$OtherCost$	Other expenses (insurance, land, overhead, etc.) (in dollars per acre)
$TotalOther$	Total other expenses
$plant$	A binary variable to indicate whether a particular crop family is planted in a specific plot during a given month
is_planted	A binary variable to indicate if any crop is planted on that plot during that month

3.3. Logical Constraints

Before any crop rotation can be implemented, it's essential to understand the logical constraints that dictate how farming operates. These constraints are critical in ensuring that crop production is both feasible and efficient. The logical constraints are:

- Once a crop is planted on a plot of land, no other crop can be planted on the same plot for the entire growth period of the initially planted crop.
$$\sum_{i\in C} x_{ijk}\left(\sum_{c\in C}\sum_{t=j}^{\min(j+\text{duration}_i,M)-1} x_{ctk}\right)\leq 1,\quad \forall j\in\{1,\dots,M\},\forall k\in\{1,\dots,L\}$$
Where x_{ijk} is a binary decision variable indicating whether crop i is planted in month j on plot k. This is implemented by iterating over each month for each plot and imposing a constraint that the sum of the planting decisions across all crops and all months within the growth duration of each crop must be less than or equal to one.
- Crops are only planted if they can complete their entire growth cycle before the end of the planting period.
$$x_{ijk}=0,\quad \forall i\in C,\forall j\in\{M-\text{duration}_i+1,\dots,M-1\},\forall k\in L$$
For each crop in the set of crops, it iterates through the months towards the end of the planting period and subtracts the growth duration of that crop. If the remaining time is less than the crop's growth duration, the code enforces that the crop cannot be planted on any plot. This prevents the scheduling of crops that would otherwise extend beyond the end of the planning timeframe, ensuring that all crops have sufficient time to grow within the specified period.
- The various costs associated with crop production cannot exceed the available budget.

$$\sum_{c\in C}\sum_{j=1}^{M}\sum_{l=1}^{L}\Big(\text{CropLabor}_c \cdot x_{cjl} \cdot \text{total\_acre\_per\_plot} + \text{CropEquip}_c \cdot x_{cjl} \cdot \text{total\_acre\_per\_plot} + \text{Seed}_c \cdot x_{cjl} \cdot \text{total\_acre\_per\_plot} + \text{ProtectionCost}_c \cdot x_{cjl} \cdot \text{total\_acre\_per\_plot}\Big) + \sum_{n\in\text{nutrients}}\sum_{j=1}^{M}\sum_{l=1}^{L}\text{Fertilizer\_cost}_n \cdot Z_{njl} + \text{OtherCost} \cdot \sum_{c\in C}\sum_{j=1}^{M}\sum_{l=1}^{L} x_{cjl} \cdot \text{total\_acre\_per\_plot} \le B$$

The calculations of the above costs can be found in Appendix 9.

- Cover crops must be planted during the winter season.

$$\sum_{c\in C} x_{cjl} \ge 1, \quad \forall j \in \{5,17,29\}, \forall l \in \{1,\dots,L\}$$

- Crops can only be planted during their optimal growing months.

$$x_{cjk} = 0, \quad \forall c \in C, \forall j \in M \setminus \text{allowed\_months}_c, \forall k \in L$$

3.4. Nutrient Amendment Constraints

The determination of a crop rotation plan for a set of plots can be limited by the nutrients required by each crop for each plot for each month, therefore it is critical to ensure that every crop receives the necessary amount of nutrients needed for sustainability for each plot for each month. The nutrient amendment constraints are:

- Each plot must start with a defined quantity of nitrogen, potassium, and phosphorus.

$$S_{\text{Nitrogen},0,l} = 15 \times \text{acre\_per\_plot}, \quad \forall l \in \{1,\dots,L\}$$
$$S_{\text{Phosphorus},0,l} = 100 \times \text{acre\_per\_plot}, \quad \forall l \in \{1,\dots,L\}$$
$$S_{\text{Potassium},0,l} = 28 \times \text{acre\_per\_plot}, \quad \forall l \in \{1,\dots,L\}$$

Where $S_{n,0,l}$ denotes the storage level of nutrient n on plot l at the beginning of the period.

- The amount of nutrients applied is at least as much as the crop needs.

$$\text{nutrient\_consumption}_{cn} \times x_{cjl} \times \text{acre\_per\_plot} \le Z_{njl} + S_{njl}, \quad \forall c \in C, \forall n \in \text{nutrients}, \forall j \in M, \forall l \in L$$

This constraint addresses the minimum nutrient requirements for each crop at the time of planting. It multiplies the nutrient consumption requirement of each crop by its planting decision variable and the acreage per plot and ensures this value does not exceed the sum of applied and stored nutrients for that nutrient type, in that plot, during that month.

- Update of nutrient storage levels in soil.

For each nutrient n, each month $j \in \{2,\dots,M\}$, and each plot $l \in \{1,\dots,L\}$:

$$\text{nutrient\_req\_sum}_{njl} = \sum_{c\in C}\big(\text{nutrient\_consumption}_{cn} \times x_{c,j-1,l} \times \text{acre\_per\_plot}\big),$$

If the nutrient is either potassium or phosphorus:

$$S_{njl} = S_{n,j-1,l} - \text{nutrient\_req\_sum}_{njl} + Z_{n,j-1,l} \times \sum_{c\in C} x_{c,j-1,l},$$

If the nutrient is nitrogen:

$$\text{residual\_nutrient\_sum}_{njl} = \sum_{c\in C}\big(\text{acre\_per\_plot} \times \text{nitrogen\_production}_c \times x_{cjl}\big),$$

$$S_{njl} = S_{n,j-1,l} - \text{nutrient\_req\_sum}_{njl} + \text{residual\_nutrient\_sum}_{njl} + Z_{n,j-1,l} \times \sum_{c\in C} x_{c,j-1,l}.$$

This constraint works by updating nutrient storage levels for each plot monthly. This constraint, applicable from the second month onward, calculates and adjusts the available nutrient levels, factoring in both the consumption by the crops planted and the nutrients added through fertilization. Specifically, for each nutrient type, it computes the total nutrient requirements based on previous month's crop consumption and adjusts the nutrient storage accordingly. Uniquely for Nitrogen, the model also accounts for residual nutrients, incorporating an additional layer of realism by considering the Nitrogen produced by certain crops.

3.5. Pest and Disease Control Constraints

These measures address concerns like soil depletion and heightened susceptibility to pests and diseases associated. The pest and disease control constraints are:

- Crops from the same botanical family cannot be planted successively on the same plot.

$$x_{cml} = 1 \Rightarrow \text{plant}_{f_c, m-1, l} = 0, \quad \forall c \in C, \forall m \in \{2, \dots, M\}, \forall l \in \{1, \dots, L\}$$

  Starting from the second month (as the first month is indexed as 0), it iterates through each plot for each crop. For each crop, it identifies its botanical family. The constraint then enforces a condition: if a crop is to be planted in a given month on a plot, no crop from the same family must have been planted on that plot in the preceding month. This ensures that there is a rotation in crop families on each plot from month to month, which can be crucial for soil health and pest control in agricultural planning. The arrow operator in Gurobi creates a general constraint, which acts as a logical implication between planting a crop and the absence of its family in the previous month on the same plot.

- Track which botanical family of crops is planted on each plot for every month to ensure crops from the same botanical family are not planted simultaneously on the same plot.
  For each month $m \in \{1, \dots, M\}$, and each plot $l \in \{1, \dots, L\}$:
  is_planted$_{ml}$ = Binary variable indicating if any crop is planted on plot l in month m,
  For each family $f \in F$:
  If $m = 1$:

$$\text{plant}_{fml} = \sum_{c \in C, c \in F[f]} x_{cml},$$

  Else:

$$\text{prev\_plant}_{fl} = \text{plant}_{f,m-1,l},$$
$$\text{plant}_{fml} \leq \sum_{c \in C, c \in F[f]} x_{cml} + \text{big\_M} \times (1 - \text{is\_planted}_{ml}),$$
$$\text{plant}_{fml} \geq \sum_{c \in C, c \in F[f]} x_{cml} - \text{big\_M} \times (1 - \text{is\_planted}_{ml}),$$
$$\text{plant}_{fml} \leq \text{prev\_plant}_{fl} + \text{big\_M} \times \text{is\_planted}_{ml},$$
$$\text{plant}_{fml} \geq \text{prev\_plant}_{fl} - \text{big\_M} \times \text{is\_planted}_{ml}.$$

  This constraint is a further extension of the previous one. The model ensures that *is_planted* reflects the actual planting activity, updating based on the planting decisions of all crops. The $plant$ variable is then updated to show whether crops from a specific family are planted. In the first month, it's initialized based on the current planting decisions. For subsequent months, the model uses the big-M method to either update the $plant$ variable according to the new planting decisions or carry forward the previous month's value if no new planting occurs.

- Crops from the same botanical family cannot be planted on adjacent plots simultaneously.

$$x_{cml} + x_{c'tp'} \leq 1, \quad \forall c, c' \in C, \text{with } c' \in F[f_c], \forall m \in \{1, \ldots, M\}, \forall l, p' \in L, p' \in A[l], \forall t \in T_{cm}$$
$$T_{cm} = \{t: \max(0, m - \text{duration}_{c'} + 1) \leq t \leq \min(m + \text{duration}_c - 1, M)\}$$

The code iterates over every month and plot, using an adjacency list that identifies which plots are neighboring. For each crop, it determines its family and then checks against all adjacent plots to ensure that no crop from the same family is currently growing or will be growing during the overlapping period (Refer to Appendix 1).

After establishing the necessary parameters and constraints for the crop rotation model, the next step involves the introduction of the objective function. This function is crucial for calculating the optimal balance between revenue and cost, aiming to ensure the most efficient and profitable crop rotation plan. The objective function is defined as follows:

$$\text{Yield\_expected}_c = \text{Yield\_max}_c \times \text{acre\_per\_plot} \quad \forall c \in C$$

$$\text{revenue} = \sum_{c \in C} \sum_{t=1}^{M} \sum_{l=1}^{L} \text{Revenue}_c \times x_{ctl} \times \text{Yield\_expected}_c$$

$$\text{cost} = \text{TotalFertilizer} + \text{TotalLabor} + \text{TotalEquip\_operating} + \text{TotalSeed} + \text{TotalProtection} + \text{TotalOther}$$

$$Maximize \quad \text{revenue} - \text{cost}$$

The model calculates the expected yield for each crop based on its maximum yield per acre, multiplied by the acreage of each plot. The total revenue is determined by summing the product of the expected yield and revenue for each crop across all crops, months, and plots. The model computes the total cost, which includes factors such as fertilizer, labor, equipment operation, seed, protection measures, and other miscellaneous expenses. The objective is to maximize the difference between total revenue and total cost, thereby optimizing the profitability of the crop rotation plan.

With the objective function in place, the model is run to obtain results. These results will reveal the most effective crop rotation strategies. The subsequent section will present and analyze these results, offering a detailed perspective on the potential outcomes of the proposed crop rotation plan.

4. Solution

The optimization model yielded a strategic planting schedule spanning 36 months. This solution aligns with the defined constraints, optimizing for profitability while adhering to agricultural best practices. The generated schedule proposes a rotation system for various crops, including canola, lentils, hairy vetch, and winter wheat, distributed across different plots and seasons.

The recommended planting strategy initiates with canola and lentils in adjacent plots during the spring planting season. This choice likely reflects their compatibility in terms of soil nutrients and growth cycles. As the colder months approach, the model suggests transitioning to hairy vetch and winter wheat, crops better suited to winter conditions. This cycle repeats over the course of the 3 crop seasons, with the same summer crops being planted in the same plots and the same winter crops being planted. In the final winter season, the model reverts to hairy vetch and winter wheat but in opposite plots from the previous winter seasons. Appendix 2 provides a visual representation of the optimized crop planting schedule over a 36-month period. Such a rotation aids in maintaining soil health and managing pest and disease cycles.

The model provides a detailed breakdown of the financial implications of the planting schedule. Approximately $372472.44 is needed to invest in machinery necessary for efficient farming operations, $222,777 for the labor costs required over the 36-month period, $840,397.80 for seed costs, $375646.18 for additional nutrient fertilizer, $471,056.28 for plant protection insurance, and $2,128,062.24 for other costs, encompassing insurance, land costs (including lease or rent payments for land use, property taxes and expenses related to land improvement), and overheads (including utility bills (electricity, water), maintenance and repairs of buildings and non-farming equipment, and administrative expenses). The cumulative cost of the operation comes to $4,410,411.94.

Against these costs, the final projected revenue stands at $1095982.83. This figure, while significant, suggests that the primary objective of the model may have been more focused on sustainable farming practices rather than short-term profit maximization.

In addition to the profit maximization that the code returned, there is also information about the nutrient values for each plot at every given season. Let's take an example to further understand the output of this aspect, specifically plot 4. Starting with phosphorus, the initial state of the soil is 40,140 lbs/plot. The first crop planted in this plot (months 1-6) is canola. Over that time frame, canola consumes 16,457lbs of phosphorus, leaving 23,683lbs in the soil for the following crop which is hairy vetch in the first winter season. At the time of planting hairy vetch, 42,548lbs of phosphorus is amended to the soil. The large amount of addition of phosphorus is able to provide phosphorus for the rest of the crops throughout the 36-month period. In the final period (third winter cycle), winter rye consumes the remaining 6,824lbs of phosphorus, leaving the soil depleted of phosphorus at the end of the 3-year cycle (Refer to Appendix 3). A similar process happens for potassium (Refer to Appendix 4) as well although the process with nitrogen is slightly different based on the crops ability to leave residuals of nitrogen in the soil after harvest. Let's provide another example to understand how this works, using plot 4 (Refer to Appendix 2). The soil starts with 6,021lbs of nitrogen in the initial state of the soil. 24,485lbs of nitrogen are amended to the soil to ensure the effective growth of canola over the first 5-month period of growth. At the end of this first period, there is no more nitrogen in the soil. However, hairy vetch, a cover crop planted at the end of the summer season, fixes its required nitrogen from the atmosphere, and actually puts residual nitrogen back into the soil. Over its course of growth (7 months), it puts 40,140lbs of nitrogen back into the soil. For the second summer and winter seasons, an identical pattern to the one described above is seen. However, in the third summer season, canola is planted again but this time, leaves an excess of 23,121lbs of nitrogen in the soil. Following its course, winter rye is planted for the final season of growth, leaving behind 7,065lbs of nitrogen in the soil after its season is done.

The final output of the model is consistent with its established logic and constraints. The decision-making process of the model seems to comply effectively with the key constraints identified earlier. However, the resulting profit of $1,095,982.83 over three seasons, while reasonable, may be due to the assumptions made and specific costs like irrigation and fluctuating demand not accounted for or assumed static, which may suggest that some costs associated with farming and crop rotation have not been fully accounted for.

5. Model Extensions

Further enhancements can be integrated into the model to broaden its scope and refine the optimal solution. The initial model overlooked certain aspects of crop rotation. The subsequent section will explain two extensions that were applied to the model to address these omissions and improve the approach.

5.1. Introduction of an Additional Cover Crop

In the initial model, the cover crops hairy vetch and winter rye were integrated to preserve soil health during colder seasons. Recognizing the limitations of having only two cover crop options, the model was expanded to include winter wheat as an additional winter cover crop option. This inclusion aimed to diversify the model's choices during winter months. To integrate winter wheat, new data was gathered, similar to what was collected for the existing crops. The introduction of winter wheat added more decision variables related to its planting and harvesting (x_{cml}). Winter wheat was subjected to the same constraints as the other crops, ensuring a consistent approach across all crop types. Furthermore, the existing cover crop constraint was applied, restricting the planting of winter wheat to winter months only.

5.2. Maximize Minimum Expected Yield

In the initial model, the expected yield for each crop was derived from a fixed value, representing an average yield based on data for a specific crop. However, this approach does not reflect the inherent uncertainty associated with crop yields, which can be influenced by a variety of factors. It is relevant to note that the yield values taken initially were average yields calculated over a period of 5 years ("Crop Planning Guide and crop planner," (2022)). These include technological aspects like managerial decisions and agricultural practices, biological challenges such as diseases, insects, pests, and weeds, as well as environmental variables like climate conditions, soil fertility, and water quality (Tudi et al., 2021). While modeling all these variables could add significant complexity and additional constraints, it's essential to incorporate some level of uncertainty to represent real-world conditions more accurately.

Recognizing this, modifications were introduced to the model to better handle yield uncertainty. Instead of relying on a single fixed yield value, the model now considers three scenarios for each crop: an optimistic yield, a pessimistic yield, and an expected yield (assuming 50% probability of maximum yield and 50% probability of minimum yield). The optimistic yield, associated with conditions favorable to high production, is assigned a probability of 50% to reflect more favorable outcomes. Conversely, the pessimistic yield, which represents lower production scenarios possibly due to adverse conditions, is given a 50% probability of occurrence. The new modified objective function is to maximize the minimum profit.

6. Results of Extensions

6.1. Main Discrepancies Identified in the Updated Model: Crop Scheduling and Profit Optimization

The analysis of these possible extensions revealed two primary discrepancies in the extended model: variations in the crop plotting schedule over three seasons and differences in the optimal profit calculation. Initially, the model projected an optimal profit of $1095982.82. However, after incorporating the uncertainties of yield and introducing the winter wheat extension, the recalculated optimal profit significantly dropped to $62,568.02. This notable reduction in profit can primarily be attributed to the adjustments made in the crop yield forecasts (modelled to maximize the profit for worst case yield scenario), specifically the optimistic, pessimistic, and expected yield estimates for each crop type. These modifications in yield predictions, alongside the inclusion of winter wheat, substantially influenced the overall profitability and plotting schedule in the enhanced model. This is because this change provides an adjustment to the model's objective function. Instead of maximizing profit based on a fixed yield, the objective now seeks to maximize the minimum profit across these two yield scenarios. The model calculates separate profits under both optimistic and pessimistic conditions, and the objective function is reformulated to maximize the lower of these two profit values. This approach ensures that the model's strategy is robust against yield uncertainties, aiming for the best outcome in the least favorable conditions.

6.2. Revised Model vs. Initial Model: Differences in Crop Planting Schedule

Another notable distinction between the original and extended models is evident in the crop planting schedules, a difference primarily driven by the implementation of the model extensions. As can be seen from the models in Appendices 2 and 5, within the revised model it can be observed that winter rye is being replaced with winter wheat during winter periods. This shift is attributed to winter wheat's higher revenue potential, yielding $0.16 per pound, compared to winter rye's $0.15 per pound. As a result of this, the model favors winter wheat for increased profitability.

Another prominent difference is the absence of crop planting during months 1-6, 13-18 and months 25-32 (Refer to Appendices 5, 6 and 7). This change is a direct result of introducing yield uncertainty. Unlike the initial model, which aimed to maximize profit under fixed crop yields, the revised model's objective is to maximize the minimum achievable profit. This approach, combined with the extended constraints, led the model to determine that leaving certain plots empty during these periods was more profitable than planting any crops. This strategic adjustment in planting schedules reflects the model's adaptation to uncertainty and its focus on optimizing long-term profitability and yield stability. Here, the yield is not always sufficient to offset the large number of fixed and variable costs. This is true for many farming practices as situations exist where the farmer experience crop loss while still having land expenses and other overhead costs. The model accounted for the variations in fertilizer application when planting different crops, recognizing this as a key factor in optimizing profitability. By minimizing fertilizer use, it effectively reduced costs, contributing to an increase in profit margins. This aspect was particularly crucial in scenarios with uncertain crop yields, where managing fertilizer costs became essential. Additionally, the model's inclusion of winter wheat as a cover crop brought into consideration its specific fertilization needs. These requirements were factored into the model, significantly influencing the overall results and outputs.

7. Recommendations and Conclusions

In conclusion, this paper presents a comprehensive model for crop rotation, integrating critical factors such as plot adjacency, botanical family constraints, and the use of mineral fertilizers. The primary goal was to optimize farmer incomes within established biophysical, structural, and organizational limits. The model, though robust in various aspects, could be enhanced by more thoroughly considering all farming costs and accurately representing crop yield uncertainties and price fluctuations. These improvements are essential for increasing the model's accuracy and practical application in real-world farming scenarios. Additionally, this model underscores the environmental and sustainability benefits of crop rotation. By optimizing crop placement and sequencing, the model contributes to soil health preservation, reduced pest and disease incidence, and decreased reliance on chemical inputs. This ecological aspect highlights the model's potential in promoting sustainable agricultural practices.

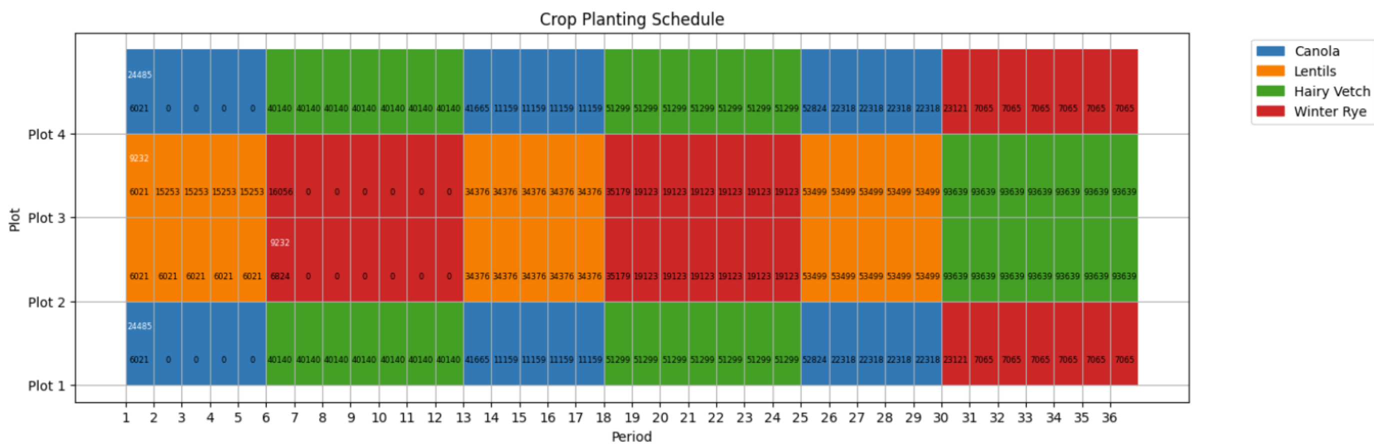
Currently, the model's applicability is primarily limited to farming practices in Saskatchewan. Enhancing its flexibility to accommodate different climates and farming requirements would expand its usability and relevance. Future developments will focus on these improvements, aiming to create a tool that not only ensures economic viability but also supports sustainable and environmentally friendly farming practices. Future developments of this model can be extended in numerous different ways including modeling for degradation of nutrients amended into the soil over time due to soil erosion, adding water required for irrigation as a constraint, taking into account uncertainty in the weather, and modeling dynamic prices and demand of the crops. Overall, this holistic approach aspires to contribute significantly to advanced agricultural modeling and more effective farming methodologies.

Appendices

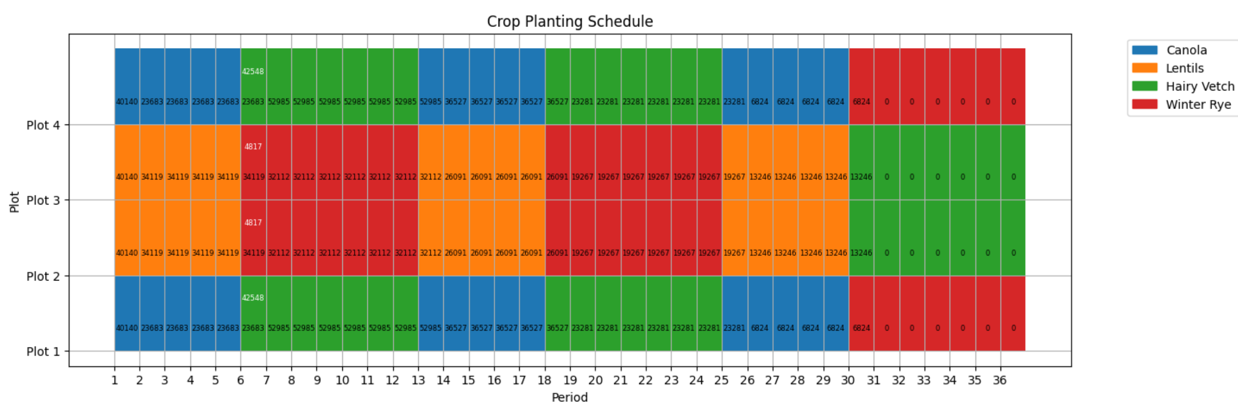
Appendix 1: Adjacency matrix corresponding to the farm, with blue cells indicating adjacent plots and black cells indicating non-adjacent plots.

	Plot 1	**Plot 2**	**Plot 3**	**Plot 4**
Plot 1				
Plot 2				
Plot 3				
Plot 4				

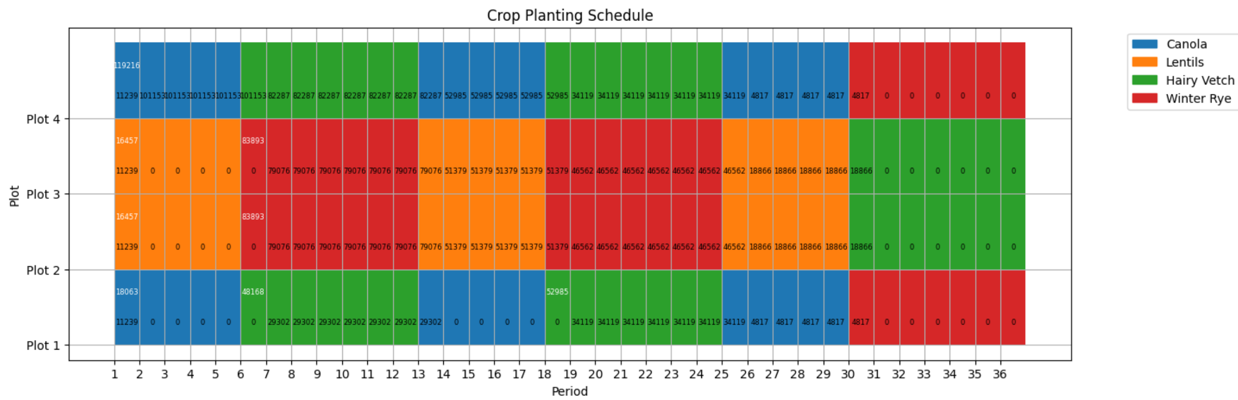
Appendix 2: Initial Model Final Output including Nitrogen Storage and Amendment Values



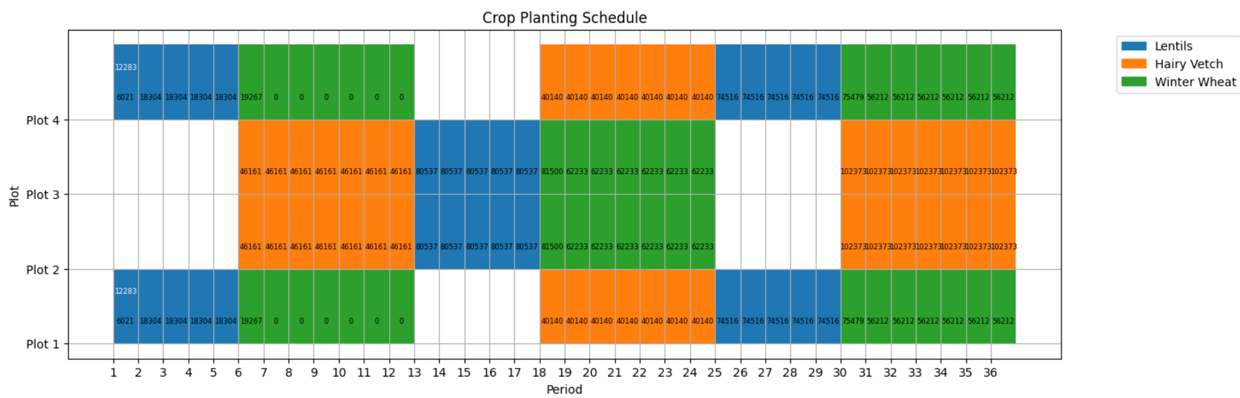
Appendix 3: Initial Model Final Output including Phosphorus Storage and Amendment Values



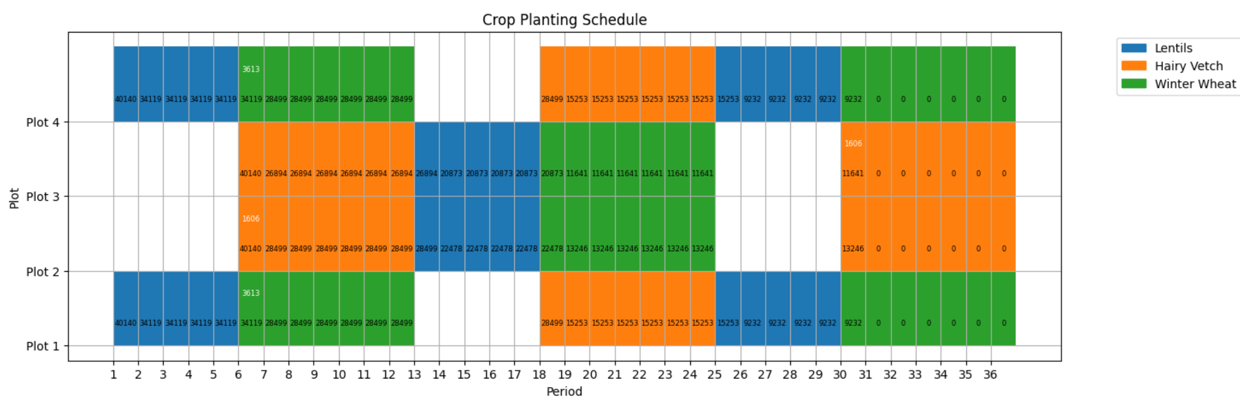
Appendix 4: Initial Model Final Output Including Potassium Storage and Amendment Values



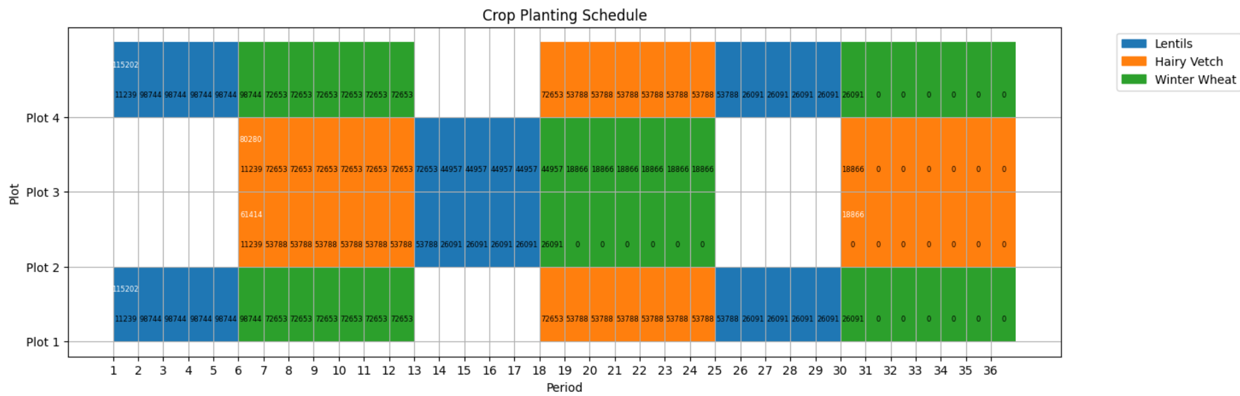
Appendix 5: Extension Model Final Output Including Nitrogen Storage and Amendment Values



Appendix 6: Extension Model Final Output Including Phosphorus Storage and Amendment Values



Appendix 7: Extension Model Final Output Including Potassium Storage and Amendment Values



Appendix 8: Table of the Number Mapping for Months

Month	Number
May	0
June	1
July	2
Aug	3
Sep	4
Oct	5
Nov	6
Dec	7
Jan	8
Feb	9
Mar	10
Apr	11

Appendix 9: Tables of Crops Information

*Crop	Botanical Family	Duration (months)	Bushels per Acre	Weight per Bushel (lbs)	Seed Cost per Acre ($)	Price per Bushel ($)	Nutrient Consumption (N, P, K)
Wheat	Poaceae	5	44,1	60	24,96	10,15	Nitrogen: 68, Phosphorus: 48, Potassium: 65
Canola	Brassicaceae	5	36,1	50	79,75	17,61	Nitrogen: 76, Phosphorus: 41, Potassium: 73
Oat	Poaceae	5	53,8	34	27,2	5,19	Nitrogen: 37, Phosphorus: 15, Potassium: 131
Lentils	Papilionaceae	5	1344,8	1	60,06	0,49	Nitrogen: 0, Phosphorus: 15, Potassium: 69
Hairy Vetch	Vicia Villosa	7	6	60	126,99	2	Nitrogen: 0, Phosphorus: 33, Potassium: 47
Winter Rye	Poaceae	7	34,3	56	47,25	7,92	Nitrogen: 40, Phosphorus: 17, Potassium: 12
Winter Wheat	Poaceae	7	41,9	60	31,35	9,91	Nitrogen: 48, Phosphorus: 23, Potassium: 65

*Crop	Labor Cost per Acre ($)	Equipment Cost per Acre ($)	Seed Cost per Acre ($)	Protection Cost per Acre ($)	Other Cost per Acre ($)
Wheat	22,75	29,35	24,96	56,1	198,81
Canola	21,5	30,47	79,75	66,93	198,81
Oat	22,25	31,59	27,2	25,25	198,81
Lentils	20,25	41,13	60,06	56,16	198,81
Hairy Vetch	20,25	38,24	126,99	25,1	198,81
Winter Rye	21,25	29,35	47,25	27,84	198,81
Winter Wheat	22,75	29,35	31,35	16,47	198,81

*Refer to the following sources for values for the tables above: Covers & Co. (n.d.) & Government of Saskatchewan. (2022)

Appendix 10: Initialization of the Model

Assumptions	**Value**
Total Budget	4500000
Plot Size (acres)	446
Land used for Planting	446*.9 = 401.4

References

Fendji, J. L. E. K.; Kenmogne, C. T.; Fotsa-Mbogne, D. J.; Förster, A. "Improving Farmers' Revenue in Crop Rotation Systems with Plot Adjacency Constraints in Organic Farms with Nutrient Amendments." Appl. Sci. 2021, 11, 6775. DOI: 10.3390/app11156775.

Crop Planning Guide and crop planner. Government of Saskatchewan. (2022). https://www.saskatchewan.ca/business/agriculture-natural-resources-and-industry/agribusiness-farmers-and-ranchers/farm-business-management/crop-planning-guide-and-crop-planner

Saskatchewan Ministry of Agriculture. (2022). Crop planning guide 2022. Saskatchewan Ministry of Agriculture. URL: https://www.saskatchewan.ca/business/agriculture-natural-resources-and-industry/agribusiness-farmers-and-ranchers/market-and-trade-statistics/crops-statistics/crop-report

Government of Canada. (2022, April 11). Investing in Sustainable Agriculture to Fight Climate Change. https://www.canada.ca/en/office-infrastructure/news/2022/04/investing-in-sustainable-agriculture-to-fight-climate-change.html

Covers & Co. (n.d.). Fall Seeded Cover Grain. https://www.coversandco.ca/fall-seeded-cover-grain

Statistics Canada. (2016, February 16). Agriculture in Canada. https://www150.statcan.gc.ca/n1/pub/95-640-x/2016001/article/14807-eng.htm

Ponisio, L. C., & Kremen, C. (2016). System-level approach needed to evaluate the transition to more sustainable agriculture. Agronomy for Sustainable Development, 36(1), 1-8. https://link.springer.com/article/10.1007/s13593-016-0407-5

United States Department of Agriculture (2022). Farms and Land in Farms. https://www.nass.usda.gov/Publications/Todays_Reports/reports/fnlo0222.pdf

Hairy vetch as a crop cover. Penn State Extension. (n.d.). https://extension.psu.edu/hairy-vetch-as-a-crop-cover

Tudi, M., Daniel Ruan, H., Wang, L., Lyu, J., Sadler, R., Connell, D., Chu, C., & Phung, D. T. (2021). Agriculture Development, Pesticide Application and Its Impact on the Environment. *International journal of environmental research and public health*, *18*(3), 1112. https://doi.org/10.3390/ijerph18031112

B. C. Ball, I. Bingham, R. M. Rees, C. A. Watson, and A. Litterick. 2005. The role of crop rotations in determining soil structure and crop growth conditions. *Canadian Journal of Soil Science*. **85**(5): 557-577. https://doi.org/10.4141/S04-078

Onunkwo, E. N., & Holston, N. W. (1997). The economics of canola production by limited-resource farmers in South Carolina. *Journal of Sustainable Agriculture*, *10*(4), 49–74. https://doi.org/10.1300/j064v10n04_06

Mesfin, S., Gebresamuel, G., Haile, M., & Zenebe, A. (2023). Potentials of legumes rotation on yield and nitrogen uptake of subsequent wheat crop in northern Ethiopia. *Heliyon*, *9*(6), e16126. https://doi.org/10.1016/j.heliyon.2023.e16126