Machine Intelligence 1 | **Exercise Sheet 1**
WS 2021/22, Obermayer/Kashef | due: 25.10.2021 at 23:55

Maths primer

Exercises prefixed with **T** are explained in the tutorial. The **H** prefix is for the homework questions.

The maths primer slides (file `maths_primer.pdf` on ISIS) could be helpful for solving the questions in this sheet.

Exercise T1.1: Learning paradigms (tutorial)

(a) Describe the difference between *supervised, unsupervised,* and *reinforcement learning*.

(b) Which of the above learning techniques would be most appropriate in the following cases and what would be the corresponding *observations*, *labels* and/or *rewards*?

- To identify groups of users with the same taste of music.
- To read hand written addresses from letters.
- To teach a robot to walk through a labyrinth.

Exercise T1.2: Additional maths background (optional) (tutorial)

More topics of the maths primer (than what is covered in the homework) can be discussed on-demand.

Exercise H1.1: Distributions and expected values (homework, 2 points)

Let X be a random variable with probability density $p : \mathbb{R} \to \mathbb{R}$ with:

$$p(x) = \begin{cases} c \cdot \sin(x), & x \in [0, \pi] \\ 0, & \text{elsewhere} \end{cases}$$

(a) Determine the parameter value $c \in \mathbb{R}$ such that $p(x)$ is indeed a probability density.

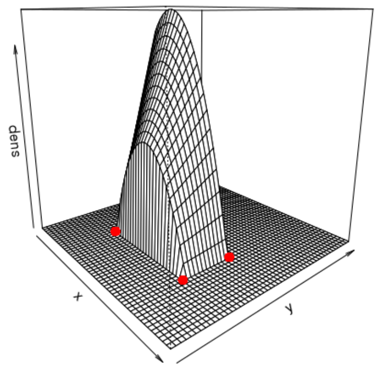
(b) Determine the expected value $\langle X \rangle_p$

(c) Determine the variance of X.
Hint: Use the identity $\mathrm{var}(X) = \left\langle X^2 \right\rangle_p - \langle X \rangle_p^2$ for simplicity.

Exercise H1.2: Marginal densities (homework, 2 points)

Assume the joint probability density of a two-dimensional random vector $\boldsymbol{Z} = (X, Y)^\top$ is

$$p_{\boldsymbol{Z}}(\boldsymbol{z}) = p_{X,Y}(x, y) = \begin{cases} \frac{3}{7}(2-x)(x+y), & x \in [0,2], y \in [0,1] \\ 0, & \text{elsewhere} \end{cases}$$



(a) Write down the marginal densities $p_X(x)$ and $p_Y(y)$ of the variables X and Y.

(b) Determine if the two variables are independent or uncorrelated.

Exercise H1.3: Taylor expansion (homework, 1 point)

For the function $\sqrt{1+x}$, write down the Taylor series around $x_0 = 0$ up to 3rd order.

Exercise H1.4: Determinant of a matrix (homework, 1 point)

Consider the 3×3 matrix

$$\underline{\mathbf{A}} = \begin{pmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{pmatrix}$$

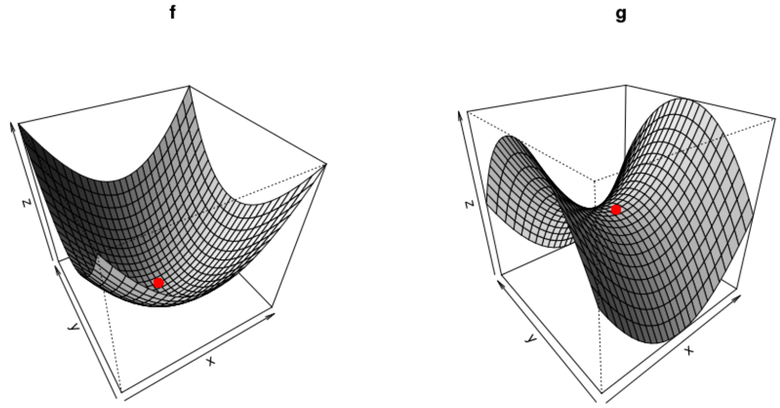
Calculate the determinant and the trace of $\underline{\mathbf{A}}$ (directly, not via eigenvalues).

Exercise H1.5: Critical points (homework, 2 points)

Consider the two functions

$$f(x,y) := c + x^2 + y^2$$
$$g(x,y) := c + x^2 - y^2,$$

where $c \in \mathbb{R}$ is a constant.



(a) Show that $\underline{\mathbf{a}} = (0,0)$ is a critical point of both functions.

(b) Check for f and for g whether $\underline{\mathbf{a}}$ is a minimum, maximum, or no extremum by calculating the Hessian matrix. Make use of the fact that a matrix is positive (negative) definite if and only if all its eigenvalues are positive (negative).

Exercise H1.6: Bayes rule (homework, 2 points)

Assume it is known that 1% of the population suffer from a certain disease. A company has developed a test for diagnosing the disease, which comes up either positive ("+", disease found) or negative ("-", disease not found). People suffering from the disease (D) are diagnosed positive with probability 0.95, and healthy people ($\bar{D}$) are diagnosed negative with probability 0.999.

Apply Bayes' rule to find

(a) the probabilities that a person for which the test yielded a positive result is indeed suffering from the disease $P(D|+)$, respectively is healthy $P(\bar{D}|+)$.

(b) the probabilities that a person for which the test yielded a negative result is indeed healthy $P(\bar{D}|-)$, respectively is suffering from the disease $P(D|-)$.

Total 10 points.