**Lab 3：Fourier Series Representation of Periodic Signals**

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| Introduction  In this lab, we will explore the Fourier Series representation of periodic signals. We have totally four problems in this lab:  In problem 3.5, we will first synthesize a periodic discrete-time signal, then examine the DTFS representation of several different square waves and finally write a function which computes the DTFS coefficients of a periodic signal.  In problem 3.8, we will examine the frequency responses of two different systems and also construct a periodic signal to use as input for these systems.  In problem 3.9, we will examine the response of a simple linear system to each of the harmonics that compose a periodic signal as well as to the periodic signal itself.  And in problem 3.10, we will analysis the algorithm.  Lab results & Analysis：  3.5 Synthesizing Signals with the Discrete-Time Fourier Series  Text  Description automatically generated      Text, letter  Description automatically generated    From the coefficient given, we found that is real, and and , and are conjecture with each other, so the imaginary part will offset each other, so is real.  Text  Description automatically generated  For N=5, we can derive that, , , so here we have  Text  Description automatically generated  We define and we get the plot, the real and imaginary part of signal, from the plot we can see that signal has only nonzero real part, so our prediction is verified.  Chart  Description automatically generated  Table  Description automatically generated  Text, letter  Description automatically generated  Here the plots of three signals are shown below.  Diagram  Description automatically generated with medium confidence  Text  Description automatically generated  The plots of DTFS coefficients of three signals are shown below. We can derive that , so for for for , and we can extract the same result from the plot, which also verify our predict.  A picture containing chart  Description automatically generatedText, letter  Description automatically generated  Chart, histogram  Description automatically generated  The signal which fewer coefficient synthesized are shown in the plot, we found that with more coefficient, the synthesized signal is more similar to the original signal .    Chart  Description automatically generated  Form the plot of real and imaginary part of the signal we can found that the imaginary part of is 0 so the signal is real.  Text  Description automatically generated  Diagram  Description automatically generated  The Gibb’s phenomenon is the peculiar manner in which the Fourier series of a piecewise continuously differentiable periodic function behaves at a jump discontinuity. From the plots we can find several significantly higher point before the signal jump from 1 to 0, and several significantly lower point after the jump, which is called the Gibb’s phenomenon. We can also find that the range of the phenomenon is much wider when we synthesized less coefficient, and if we let more coefficient involved the phenomenon is hard to identify due to the low accuracy of the plot.  Text, letter  Description automatically generated  function a=dtfs(x,n\_init);  a=[];  w=2\*pi/length(x); %fundamental frequency  **for** k=n\_init:n\_init+length(x)-1 %period from 0+n0 to N-1+n0  a\_k=0;  **for** n=1:length(x)  a\_k=a\_k+x(n)\*exp(-j\*k\*w\*(n+n\_init-1));  end  a=[a a\_k/length(x)];  end    **if** n\_init<0  **for** i=1:-n\_init  a=[a a(i)];  end  a=a(1-n\_init:length(a));  **else** **if** n\_init>0  **for** i=i:n\_init  a=[a(length(a)-i+1) a];  end  a=a(1:length(x));  end  end  3.8 First-Order Recursive Discrete-Time Filters  Text  Description automatically generated  Diagram, schematic  Description automatically generated    Text  Description automatically generated  Graphical user interface  Description automatically generated  The frequency response of two system is shown in the plot above. From the plots we can draw the conclusion that system I is a lowpass filter and system II is a highpass filter.  Text  Description automatically generated  Diagram  Description automatically generated  Form the plot we can see that the signal has nonzero coefficient , with system I will be attenuated and will be amplified, with system I will be amplified and will be attenuated.  Text  Description automatically generated  Chart  Description automatically generated  The plot of the signal is shown above.  Text, letter  Description automatically generated  A picture containing chart  Description automatically generated  We found that in , the signal is more smooth, which means the signals in high frequency is filtered and signal in low frequency is amplified, and this indicate that System I is a low pass filter. We found that in , the signal is more variance, which means the signals in high frequency is amplified and signals in low frequency domain are filtered, and this indicate that System I is a high pass filter.  Text  Description automatically generated  Chart  Description automatically generated  From the plot above we found that, after filtered by System I, the DTFS coefficient for low frequency domain increase and the DTFS coefficient for low frequency domain decrease, which indicated that System I is a lowpass filter, after filtered by System II, the DTFS coefficient for low frequency domain decrease and the DTFS coefficient for high frequency domain increase, which indicated that System II is a highpass filter. The result here meet the analysis in last part.  3.9 Frequency Response of a Continuous-Time System  Text, letter  Description automatically generated  Text  Description automatically generated with low confidence  Text  Description automatically generated  Chart, line chart  Description automatically generated  From the plot we found that after applying the system, the amplitude of the signal decrease and the phase of the signal is delayed. so and we can infer from these two parameters that the system will attenuate the amplitude and delay the phase of the input signal.  Text, letter  Description automatically generated  Chart  Description automatically generated  As shown in the image, the plot shown the square wave after being processed by the system. After processed by the system, the square wave become sawtooth wave.  Text  Description automatically generated  Chart, histogram  Description automatically generated  The wave form of s1 and the square wave are plot above. As shown in the plot, the sum of five signal is pretty close to the square wave, and we can see the vibration of the wave and the Gibb’s phenomenon in the plot.      Chart, histogram  Description automatically generated  The response of the ssum and the sum of the five signal is plot above. We can see that two plots are same, which proved that the response is also linear.    Chart, line chart  Description automatically generatedChart, histogram  Description automatically generated  In the first plot we can find that the frequency response of two signal are quite similar, in the second plot, we plot the magnitude of the CTFS coefficients of the signal, here the magnitude of the CTFS coefficients have direct relationship to the energy contained in that frequency, so we can infer that the first five pairs of coefficient have already contained most of the energy of the signal, so when we plot the sum of more than 5 pairs of CTFS coefficient, we can see they are pretty close. If we define the energy we will found that the function almost stop increase when n become larger.    Diagram  Description automatically generated  The plot in the left is the analytically determined signal of y1 to y5 and the right is simulated ones. We can see each pair is same. So signals y1, . . . , y5 are correct by constructing each signal from the system function H ( s ) and the CTFS for x2.  3.10 Computing the Discrete-Time Fourier Series    Text, letter  Description automatically generated  Normally we use formular to calculate each coefficient. In this formular, we need to calculate complex multiplication for the total number of calculation is N+1 here, then for the final result, we need to do add operation for N times for N+1 number, and the first number is real, so we need to do complex addition for N-1 times.  A picture containing text, newspaper, document  Description automatically generated    Text  Description automatically generated  MATLAB Code  % 3.5(a)通过变换可以得到x[n]=1+2cos(4pi/5 n+pi/4)+4cos(8pi/5 n+pi/3),所以是实的。  % 3.5(b)因为N=5,所以  % a0=1,a1=a-4=2\*exp(-j\*pi/3),a2=exp(j\*pi/4),a3=a-2=exp(-j\*pi/4),a4=2\*exp(j\*pi/3);  % 3.5(b)  a=[1 2\*exp(-j\*pi/3) exp(j\*pi/4) exp(-j\*pi/4) 2\*exp(j\*pi/3)];  % 3.5(c)  n=0:1:4;  x=5\*ifft(a);  subplot(2,1,1);  stem(n,real(x));  xlabel('n');  title('real x[n]');  subplot(2,1,2);  stem(n,imag(x));  xlabel('n');  title('image x[n]');  saveas(gcf, "plots/P3\_5\_c\_out.png");  close;  % 3.5(d)  n=0:1:63;  n1=0:1:7;  n2=0:1:15;  n3=0:1:31;  x1=ones(1,8);  x2=[ones(1,8) zeros(1,8)];  x3=[ones(1,8) zeros(1,24)];  x\_1=[x1 x1 x1 x1 x1 x1 x1 x1];  x\_2=[x2 x2 x2 x2];  x\_3=[x3 x3];  subplot(3,1,1);  stem(n,x\_1)  xlabel('n');  title('x\_1[n]');  subplot(3,1,2);  stem(n,x\_2)  xlabel('n');  title('x\_2[n]');  subplot(3,1,3);  stem(n,x\_3)  xlabel('n');  title('x\_3[n]');  saveas(gcf, "plots/P3\_5\_d\_out.png");  close;  % 3.5(e)  a1=1/8\*fft(x1);  a2=1/16\*fft(x2);  a3=1/32\*fft(x3);  subplot(3,1,1);  stem(n1,abs(a1));  xlabel('k');  title('a\_k1');  subplot(3,1,2);  stem(n2,abs(a2));  xlabel('k');  title('a\_k2');  subplot(3,1,3);  stem(n3,abs(a3));  xlabel('k');  title('a\_k3');  saveas(gcf, "plots/P3\_5\_e\_out.png");  close;  % 3.5(f)  m=0:1:31;  a3\_2=[a3(1:3) zeros(1,27) a3(31:32)];  a3\_8=[a3(1:9) zeros(1,15) a3(25:32)];  a3\_12=[a3(1:13) zeros(1,7) a3(21:32)];  a3\_all=a3;  x3\_2=32\*ifft(a3\_2);  x3\_8=32\*ifft(a3\_8);  x3\_12=32\*ifft(a3\_12);  x3\_all=32\*ifft(a3\_all);  subplot(2,2,1);  stem(m,x3\_2,'y');  xlabel('n');  title('x3 2[n]');  subplot(2,2,2);  stem(m,x3\_8,'m');  xlabel('n');  title('x3 8[n]');  subplot(2,2,3);  stem(m,x3\_12,'r');  xlabel('n');  title('x3 12[n]');  subplot(2,2,4);  stem(m,x3\_all,'b');  xlabel('n');  title('x3 all[n]');  saveas(gcf, "plots/P3\_5\_f\_out.png");  close;  % 3.5(g)  subplot(2,1,1);  stem(m,real(x3\_all),'r');  xlabel('n');  title('real(x3 all[n])');  subplot(2,1,2);  stem(m,imag(x3\_all),'g');  xlabel('n');  title('imag(x3 all[n])');  saveas(gcf, "plots/P3\_5\_g\_out.png");  close;  % 3,5(h)  a3\_5=[a3(1:3) zeros(1,27) a3(31:32)];  a3\_15=[a3(1:8) zeros(1,17) a3(26:32)];  a3\_25=[a3(1:13) zeros(1,7) a3(21:32)];  a3\_29=[a3(1:15) zeros(1,3) a3(19:32)];  a3\_32=a3;  x3\_5=32\*ifft(a3\_5);  x3\_15=32\*ifft(a3\_15);  x3\_25=32\*ifft(a3\_25);  x3\_29=32\*ifft(a3\_29);  x3\_32=32\*ifft(a3\_32);  subplot(3,2,1);  stem(m,x3\_5,'y');  xlabel('n');  title('use 5 series');  subplot(3,2,2);  stem(m,x3\_15,'m');  xlabel('n');  title('use 15 series');  subplot(3,2,3);  stem(m,x3\_25,'r');  xlabel('n');  title('use 25 series');  subplot(3,2,4);  stem(m,x3\_29,'b');  xlabel('n');  title('use 29 series');  subplot(3,2,5);  stem(m,x3\_32,'b');  xlabel('n');  title('use 32 series');  saveas(gcf, "plots/P3\_5\_h\_out.png");  close;  % 3.5最后一问  function a=dtfs(x,n\_init);  a=[];  w=2\*pi/length(x); %fundamental frequency  for k=n\_init:n\_init+length(x)-1 %period from 0+n0 to N-1+n0  a\_k=0;  for n=1:length(x)  a\_k=a\_k+x(n)\*exp(-j\*k\*w\*(n+n\_init-1));  end  a=[a a\_k/length(x)];  end    if n\_init<0  for i=1:-n\_init  a=[a a(i)];  end  a=a(1-n\_init:length(a));  else if n\_init>0  for i=i:n\_init  a=[a(length(a)-i+1) a];  end  a=a(1:length(x));  end  end  3.8  a1=[1 -0.8];  b1=1;  a2=[1 0.8];  b2=1;  x=0:1:19;  a\_x=[0 0.75 zeros(1,7) -0.5 0 -0.5 zeros(1,7) 0.75];  Wk=(pi/10).\*x;  subplot(3,1,1);  stem(Wk,a\_x);  title('DTFS coefficients of x[n]');  xlabel('w\_k=(2pi/20)k');  ylabel('a\_k');  %以下与b部分相同  a1=[1 -0.8];  b1=1;  a2=[1 0.8];  b2=1;  [H1 Omega] = freqz(b1, a1, 1024, 'whole');  [H2 Omega] = freqz(b2, a2, 1024, 'whole');  subplot(3,1,2);  %这是一个低通滤波器  plot(Omega, abs(H1),'r');  xlabel('\omega');  ylabel('H1(e^{j\omega} )');  title('frequency responses of Systems 1');  subplot(3,1,3);  %这是一个高通滤波器  plot(Omega, abs(H2),'g');  xlabel('\omega');  ylabel('H2(e^{j\omega} )');  title('frequency responses of Systems 2');  saveas(gcf, "plots/P3\_8\_c\_out.png");  close;  % d部分  x=0:1:19;  a\_x=[0 0.75 zeros(1,7) -0.5 0 -0.5 zeros(1,7) 0.75];  Wk=(pi/10).\*x;  N=20;  x\_20=N\*ifft(a\_x);  n=-20:1:99;  x\_6t=[x\_20 x\_20 x\_20 x\_20 x\_20 x\_20];  figure (1);  stem(n,x\_6t);  xlabel('n');  ylabel('x[n]');  saveas(gcf, "plots/P3\_8\_d\_out.png");  close;  % e部分  a1=[1 -0.8];  b1=1;  a2=[1 0.8];  b2=1;  y1=filter(b1,a1,x\_6t);  y2=filter(b2,a2,x\_6t);  figure (2);  subplot(2,1,1);  stem(n(21:120),y1(21:120));  xlabel('n');  ylabel('y\_1[n]');  subplot(2,1,2);  stem(n(21:120),y2(21:120));  xlabel('n');  ylabel('y\_2[n]');  saveas(gcf, "plots/P3\_8\_e\_out.png");  close;  % f部分  a\_y1=(1/20)\*fft(y1(1:20));  a\_y2=(1/20)\*fft(y2(1:20));  figure(3);  subplot(2,1,1);  stem(x,abs(a\_y1));  xlabel('n');  ylabel('a\_y1');  title('the DTFS coefficient of yl');  subplot(2,1,2);  stem(x,abs(a\_y2));  xlabel('n');  ylabel('a\_y2');  title('the DTFS coefficient of y2');  saveas(gcf, "plots/P3\_8\_f\_out.png");  close;  3.9  %3.9(a)  t=linspace(0,20,1000);  x=cos(t);  b=1;  a=[1 1]; % RC=1  y1=lsim(b,a,x,t);  plot(t,x,'r');  hold on;  plot(t,y1,'g');  xlabel('t');  ylabel('x(t) and y(t)');  xlim([10,20]);  legend('x(t)','y(t)');  saveas(gcf, "plots/P3\_9\_a\_out.png");  close;  %3.9(b)  x2=cos(t);  x2(x2>0)=ones(size(x2(x2>0)));  x2(x2<0)=-ones(size(x2(x2<0)));  y2=lsim(b,a,x2,t);  plot(t,y2,'g');  xlim([10,20]);  xlabel('t');  ylabel('y\_2(t)');  saveas(gcf, "plots/P3\_9\_b\_out.png");  close;  %3.9(c)  plot(t,x2,'g');  hold on;  apos\_k=[2/pi -2/(3\*pi) 2/(5\*pi) -2/(7\*pi) 2/(9\*pi)];  aneg\_k=[2/pi -2/(3\*pi) 2/(5\*pi) -2/(7\*pi) 2/(9\*pi)];  s1=apos\_k(1)\*exp(j\*t)+aneg\_k(1)\*exp(-j\*t);  s2=apos\_k(2)\*exp(3j\*t)+aneg\_k(2)\*exp(-3j\*t);  s3=apos\_k(3)\*exp(5j\*t)+aneg\_k(3)\*exp(-5j\*t);  s4=apos\_k(4)\*exp(7i\*t)+aneg\_k(4)\*exp(-7j\*t);  s5=apos\_k(5)\*exp(9j\*t)+aneg\_k(5)\*exp(-9j\*t);  sn=s1+s2+s3+s4+s5;  plot(t,sn,'r');  xlabel('t');  legend('x\_2(t)','sum');  saveas(gcf, "plots/P3\_9\_c\_out.png");  close;  %3.9(d)  ys1=lsim(b,a,s1,t);  ys2=lsim(b,a,s2,t);  ys3=lsim(b,a,s3,t);  ys4=lsim(b,a,s4,t);  ys5=lsim(b,a,s5,t);  ysn=lsim(b,a,sn,t);  ysum=ys1+ys2+ys3+ys4+ys5;  subplot(2,1,1);  plot(t,ysum);  xlabel('t');  title('response to sum of y1 to y5');  subplot(2,1,2);  plot(t,ysn);  xlabel('t');  title('response to ssum');  saveas(gcf, "plots/P3\_9\_d\_out.png");  close;  %3.9(e)  ysn=lsim(b,a,sn,t);  y2=lsim(b,a,x2,t);  plot(t,ysn,'r--');  hold on;  plot(t,y2,'g-');  xlabel('t');  title('response to ssum and x\_2(t)');  legend('y\_ssum(t)','y\_2(t)');  saveas(gcf, "plots/P3\_9\_e\_out.png");  close;  %3.9(f)  % y1  y1=(1/(1+j))\*apos\_k(1)\*exp(j\*t)+(1/(1+j))\*aneg\_k(1)\*exp(-j\*t);  ys1=lsim(b,a,s1,t);  subplot(5,2,1);  plot(t,y1)  xlim([10,20]);  xlabel('t');  title('analytically determined y1');  subplot(5,2,2);  plot(t,ys1)  xlim([10,20]);  xlabel('t');  title('simulated y1');  % y2  y2=(1/(1+3j))\*apos\_k(2)\*exp(3j\*t)+(1/(1-3j))\*aneg\_k(2)\*exp(-3j\*t);  ys2=lsim(b,a,s2,t);  subplot(5,2,3);  plot(t,y2)  xlim([10,20]);  xlabel('t');  title('analytically determined y2');  subplot(5,2,4);  plot(t,ys2)  xlim([10,20]);  xlabel('t');  title('simulated y2');  % y3  y3=(1/(1+5j))\*apos\_k(3)\*exp(5j\*t)+(1/(1-5j))\*aneg\_k(3)\*exp(-5j\*t);  ys3=lsim(b,a,s3,t);  subplot(5,2,5);  plot(t,y3)  xlim([10,20]);  xlabel('t');  title('analytically determined y3');  subplot(5,2,6);  plot(t,ys3)  xlim([10,20]);  xlabel('t');  title('simulated y3');  % y4  y4=(1/(1+7j))\*apos\_k(4)\*exp(7j\*t)+(1/(1-7j))\*aneg\_k(4)\*exp(-7j\*t);  ys4=lsim(b,a,s4,t);  subplot(5,2,7);  plot(t,y4)  xlim([10,20]);  xlabel('t');  title('analytically determined y4');  subplot(5,2,8);  plot(t,ys4)  xlim([10,20]);  xlabel('t');  title('simulated y4');  % y5  y5=(1/(1+9j))\*apos\_k(5)\*exp(9j\*t)+(1/(1-9j))\*aneg\_k(5)\*exp(-9j\*t);  ys5=lsim(b,a,s5,t);  subplot(5,2,9);  plot(t,y5)  xlim([10,20]);  xlabel('t');  title('analytically determined y5');  subplot(5,2,10);  plot(t,ys5)  xlim([10,20]);  xlabel('t');  title('simulated y5');  saveas(gcf, "plots/P3\_9\_f\_out.png");  close;  Note: Please indicate meaning of the symbols in all expressions. Please indicate the coordinate and unit in all figures. | |
| Experience  When we first analysis the plot of the frequency response of the filter, the instruction did not ask us to do the shift operation to the wave, however, in the past the frequency response of the filter is always shown in or so we have some difficulties in analysis the property of the filter shown in | |
| Score |  |

字体：英文Times new Roman；中文宋体，正文五号