

# Lab 6 Frequency Domain Filtering

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Course: *LAB Session I* – Professor: *YU Yajun*

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## Introduction

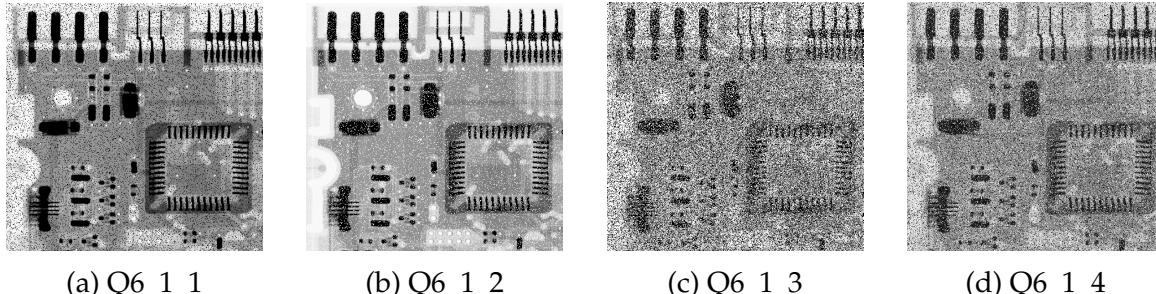
The objective of restoration is to improve a given image in some predefined sense. Compare with image enhancement which is largely a subjective process, image restoration is for the most part an objective process. When we apply image restoring to a degraded image, we will first construct a model about how the image was degraded and then we apply the degrade filter to the restoring filter. After that we could get the reconstructed image, however, we can't build the degrade model precisely and there may be some overlap in frequency domain after the image was degrade, so we can not always restore that image like what it used to be.

In this lab, there are three tasks will be performed.

1. Apply adaptive filter to the image with high noise intensity
2. Apply full inverse filtering, radially limited inverse filtering and Wiener filtering to a image degraded by atmosphere turbulence. Discuss how the parameters, if any, are determined, and the different effects by using the different algorithms.
3. Restore a image degraded by motion blur and noise.

### Task I: Adaptive Filter for Image Denoising

Remove the noise from the input images Q6\_1\_1.tif, Q6\_1\_2.tif, Q6\_1\_3.tif and Q6\_1\_4.tif. Explain your observation and the method used to each of the images, and why such methods are used.



(a) Q6\_1\_1

(b) Q6\_1\_2

(c) Q6\_1\_3

(d) Q6\_1\_4

Figure 1: Task I, Figures with noise

**Analysis.** For traditional noise filter in spatial domain, the size of the filter is limited, here comes the problem: if the size of the filter is too small, for areas with high noise intensity, the filter can't reduce all the noise, for example, when apply a median filter to a 3x3 square with more than 5 pixels are noise with 255, the median will always be the 255, for average filter, the number of the result will be greatly affected by the noise.

**To solve this problem, two different methods are designed.**

**Smart filter.** The first method is called "smart filter". The filter will analyze the noise situation of the square being filtered, if the noise intensity in the square is high, some pixels in extremely high or low intensity will be removed in future operation. The procedure of the algorithm is shown below.

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#### Algorithm 1: Smart denoise filter

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**input :** The image with noise

**output:** The denoised image

**forall** Pixels in the given image **do**

**if** Pixels with intensity between (5, 250) is contained in the image **then**

Find the normal pixel most close to the middle

**else**

Calculate the average value

Assign the value to the related pixels in the output image

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**Adaptive filter.** In this method, if the given square has high noise intensity, we will expand the size of the filter until the noise intensity in the domain fit our requirement. The procedure is shown below.

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**Algorithm 2:** Smart denoise filter

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**input :** The image with noise, the initial size, the max size

**output:** The denoised image

$S_{xy}$ : The domain we apply the filter

$z_{min}$ : The minimum intensity value in  $S_{xy}$

$z_{max}$ : The maximum intensity value in  $S_{xy}$

$z_{med}$ : median intensity value in  $S_{xy}$

$z_{xy}$ : intensity value at coordinates (x, y)

$S_{max}$ : maximum allowed size of  $S_{xy}$

**forall** Pixels in the given image **do**

**repeat**

$A_1 = z_{med} - z_{min}$

$A_2 = z_{med} - z_{max}$

**if**  $A_1 > 0$  and  $A_2 < 0$  **then**

$B_1 = z_{xy} - z_{min}$

$B_2 = z_{xy} - z_{max}$

**if**  $B_1 > 0$  and  $B_2 < 0$  **then**

└ output  $z_{xy}$

**else**

└ output  $z_{med}$

**else**

Increase the size of  $S_{xy}$  **if** size <  $S_{max}$  **then**

└ repeat the loop

**else**

└ output  $z_{med}$

**until** output is generated

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**Result.** After applying both filter to the given given image, we obtain the following results

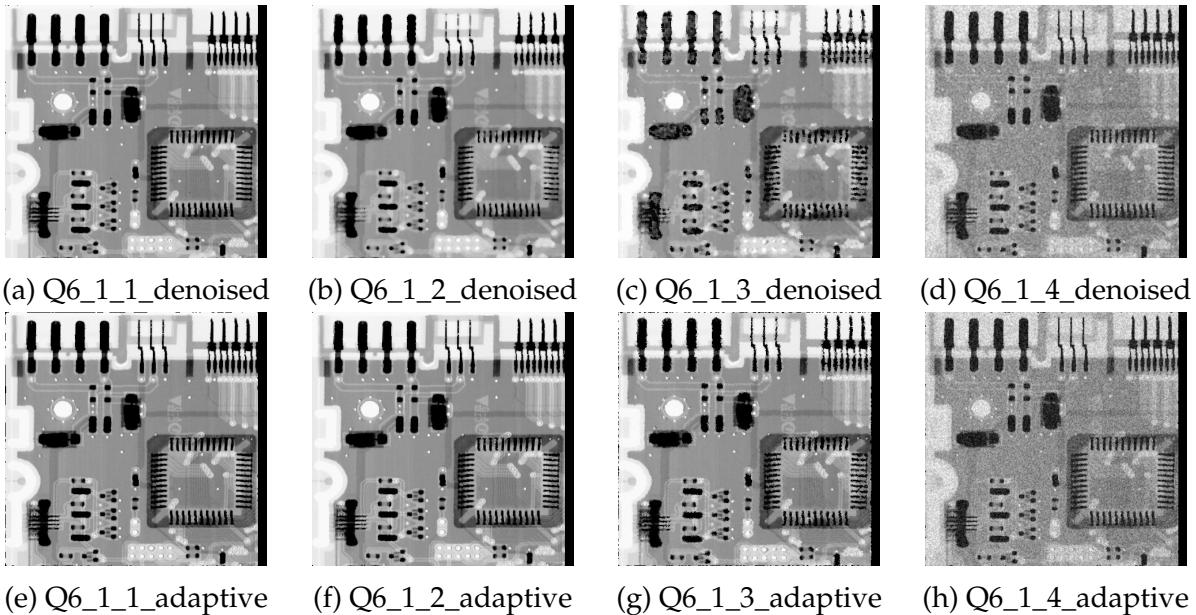


Figure 2: The first line is denoised by smart denoise filter, the second line is denoised by adaptive filter.

**Discussion.** From the result we found that, as the intensity of the noise increase, some noise in relatively low frequency domain will remain in the image, that is because the noise have cover most of the information in the image, so some information cannot be restored. And compare two different method, we found that the adaptive filter cause less damage to the details.

### Task II: Image Restoration

Image Q6\_2.tif was degraded from an original image due to the atmosphere turbulence given on slide 65 with  $k = 0.0025$ . Restore the original image from the input Q6\_2.tif by using full inverse filtering, radially limited inverse filtering and Wiener filtering. Discuss how the parameters, if any, are determined, and the different effects by using the different algorithms.

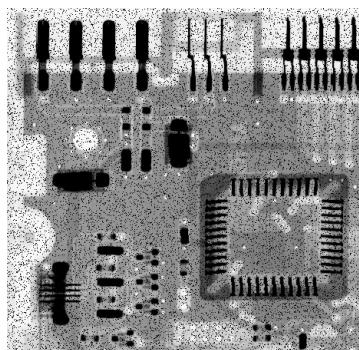


Figure 3: Q6\_1\_1

**Analysis.** In this task we will apply three different kinds of inverse filters including full inverse filtering, radially limited inverse filtering and Wiener filtering. Then we

will compare the performance of each filter.

**full inverse filtering.** The image degrade process can be modeled as image filtering process with degrade filter

$$H(u, v)$$

and get result  $G(u, v)$ , so most directly, the process can be inversed by constructing the inverse filter

$$\frac{1}{H(u, v)}$$

Then the inverse filter will be applied to the degraed image. Here we have

$$\hat{F}(u, v) = \frac{G(u, v)}{H(u, v)}$$

In this task we will apply the filters to a image degraded by atmosphere turbulence blur

$$H(u, v) = e^{-k(u-M/2)^2 + (v-N/2)^2}$$

with  $k = 0.0025$ .

**Radially limited inverse filtering.** The edge of most of the lowpass is close to 0 so when we apply the inverse filter, due to the error in digital computing and image storing, the noise in high frequency will be greatly enhanced, so to avoid the situation, we will add a cutoff filter, here a Butterworth lowpass function of order 10. This provided a sharp (but smooth) transition at the desired radius.

**Wiener filtering.** Here we discuss an approach that incorporates both the degradation function and statistical characteristics of noise into the restoration process. So that the image degrading during image storing and computing process can be take into consideration.

The filter can be obtained by the expression

$$\hat{F}(u, v) = \left[ \frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + S_\eta(u, v)/S_f(u, v)} \right] G(u, v)$$

However for most of the case we can not get the exact value of  $S_\eta(u, v)/S_f(u, v)$  which represent the signal noise ratio (SNR) of the image, so we will represent it by  $K$  so the expression become

$$\hat{F}(u, v) = \left[ \frac{1}{H(u, v)} \frac{|H(u, v)|^2}{|H(u, v)|^2 + K} \right] G(u, v)$$

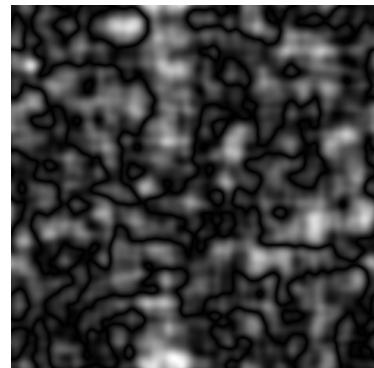


Figure 4: Full inverse filtering

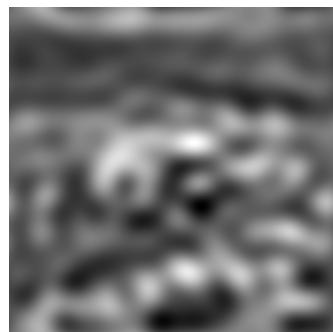
(a)  $\sigma = 10, n = 10$ (b)  $\sigma = 30, n = 10$ (c)  $\sigma = 50, n = 10$ (d)  $\sigma = 60, n = 10$ (e)  $\sigma = 70, n = 10$ (f)  $\sigma = 85, n = 10$ 

Figure 5: Filtered by redially limited inverse filter

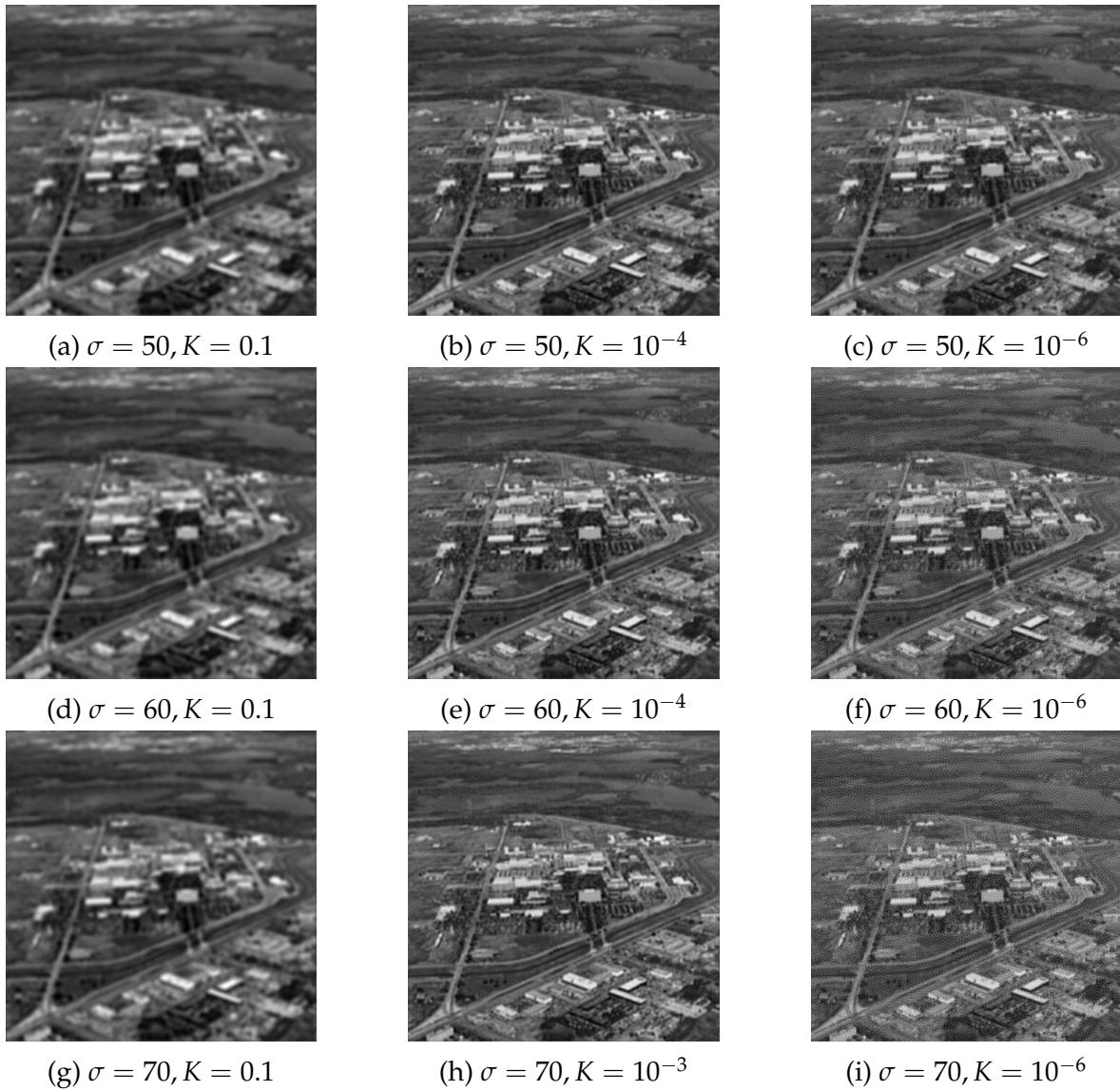


Figure 6: Filtered by wiener filter

## Result.

**Discussion.** The result shows that full inverse filter is not capable for most of the case due to the error that may occur that we discussed in the Analysis section.

Fig 5 shows that  $\sigma = 60$  can be a proper cut off frequency for the filter, when  $\sigma$  keeps increase, the high frequency noise will appear again, when  $\sigma$  keeps decrease, the details in the image will also be filtered.

Fig 6 shows that  $\sigma = 60, K = 10^{-6}$  can be a proper pair to restore the image by wiener filter, comparing between each image in the image set, when  $K$  is decreasing, more details in the image will be restored while the noise increase, that because the detail of the image and noise are similar somehow, so when less detail is filtered, less noise will be filtered. The adjustment of  $\sigma$  has been discussed in last paragraph.

### Task III: Motion Deblurring

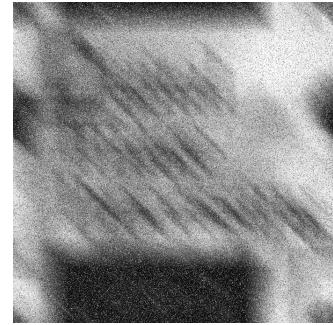
Restore the original images from the inputs Q6\_3\_1.tif, Q6\_3\_2.tif and Q6\_3\_3. Explain your observation and the method used.



(a) Q6\_3\_1



(b) Q6\_3\_2



(c) Q6\_3\_3

Figure 7: Task III, Figures with motion blur

**Analysis.** In this task we will reconstruct the image degraded by motion blur and noise. The motion blur can be expressed as filter

$$\begin{aligned}
 H(u, v) &= \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]} dt \\
 * &\quad = \int_0^T e^{-j2\pi[ua+vb]t/T} dt \\
 &= \frac{T}{\pi(ua + vb)} \sin[\pi(ua + vb)] e^{-j\pi(ua + vb)}
 \end{aligned} \tag{1}$$

And we will apply radially limited inverse filter and wieber filter to the degreaded image.

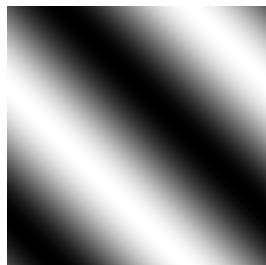
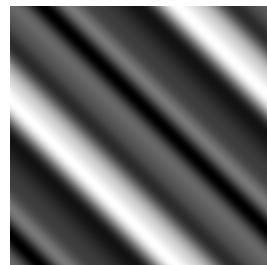
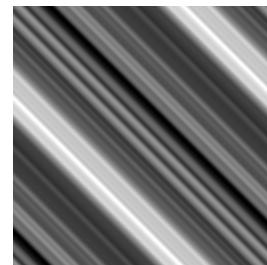
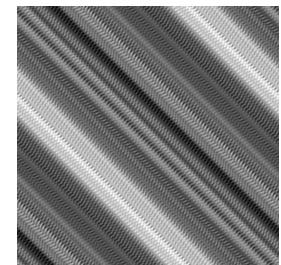
(a)  $\sigma = 1$ (b)  $\sigma = 10$ (c)  $\sigma = 50$ (d)  $\sigma = 100$ 

Figure 8: Filtered by radially limited inverse filter

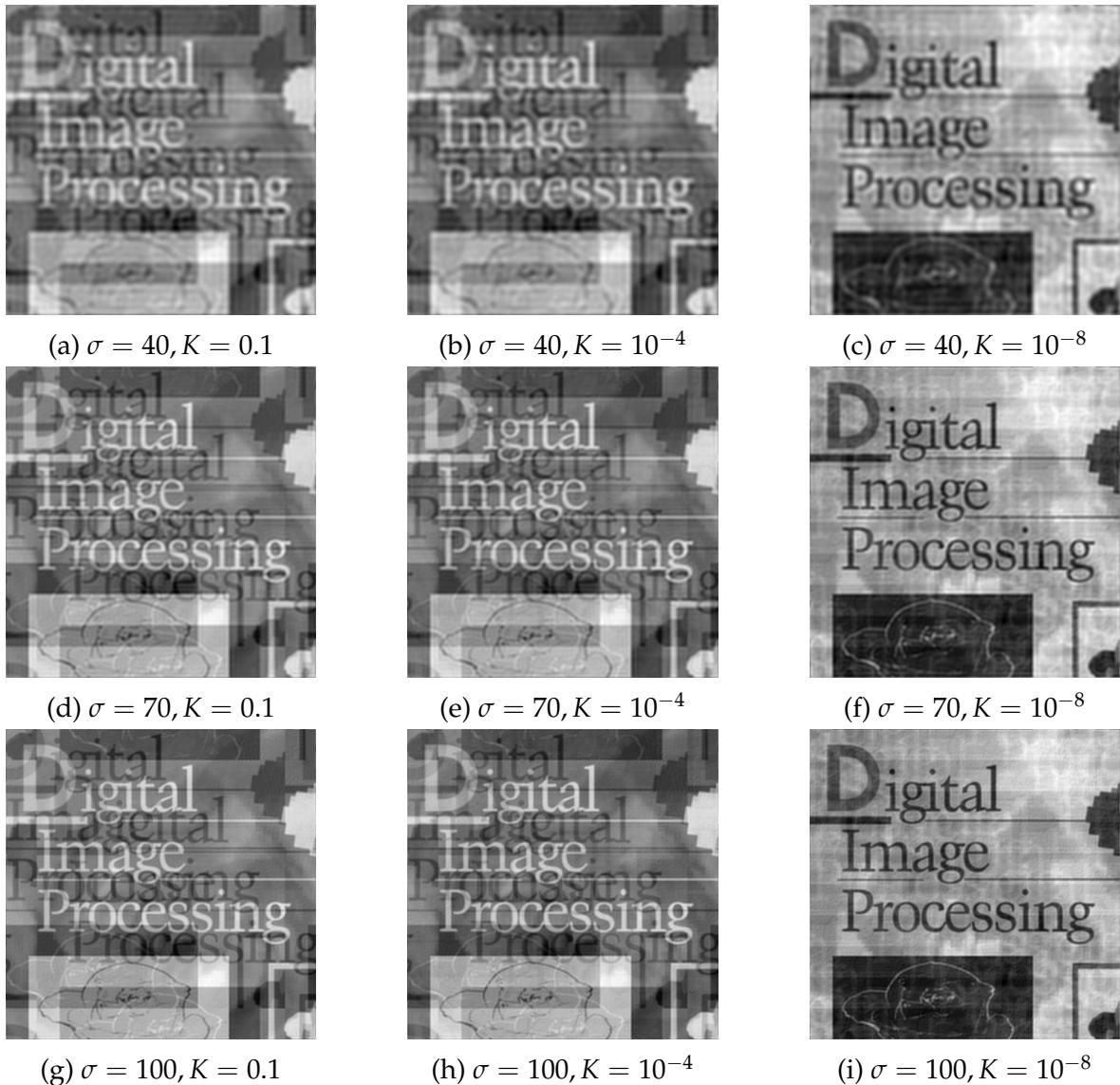


Figure 9: Filtered by wiener filter

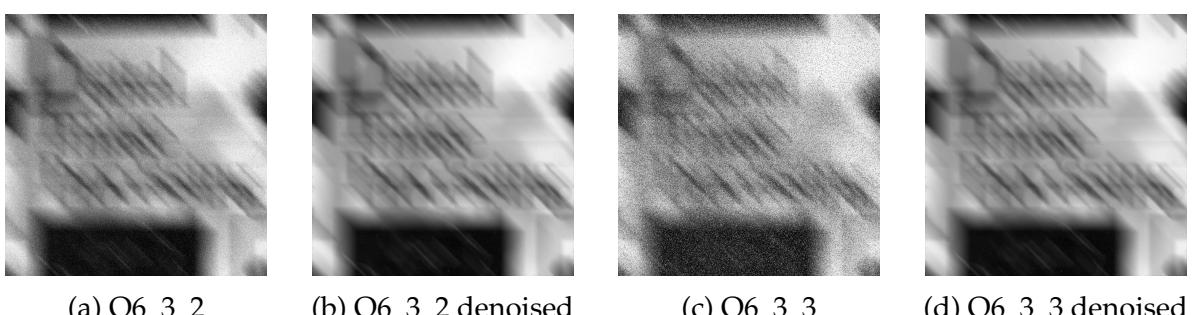


Figure 10: original image and denoised image

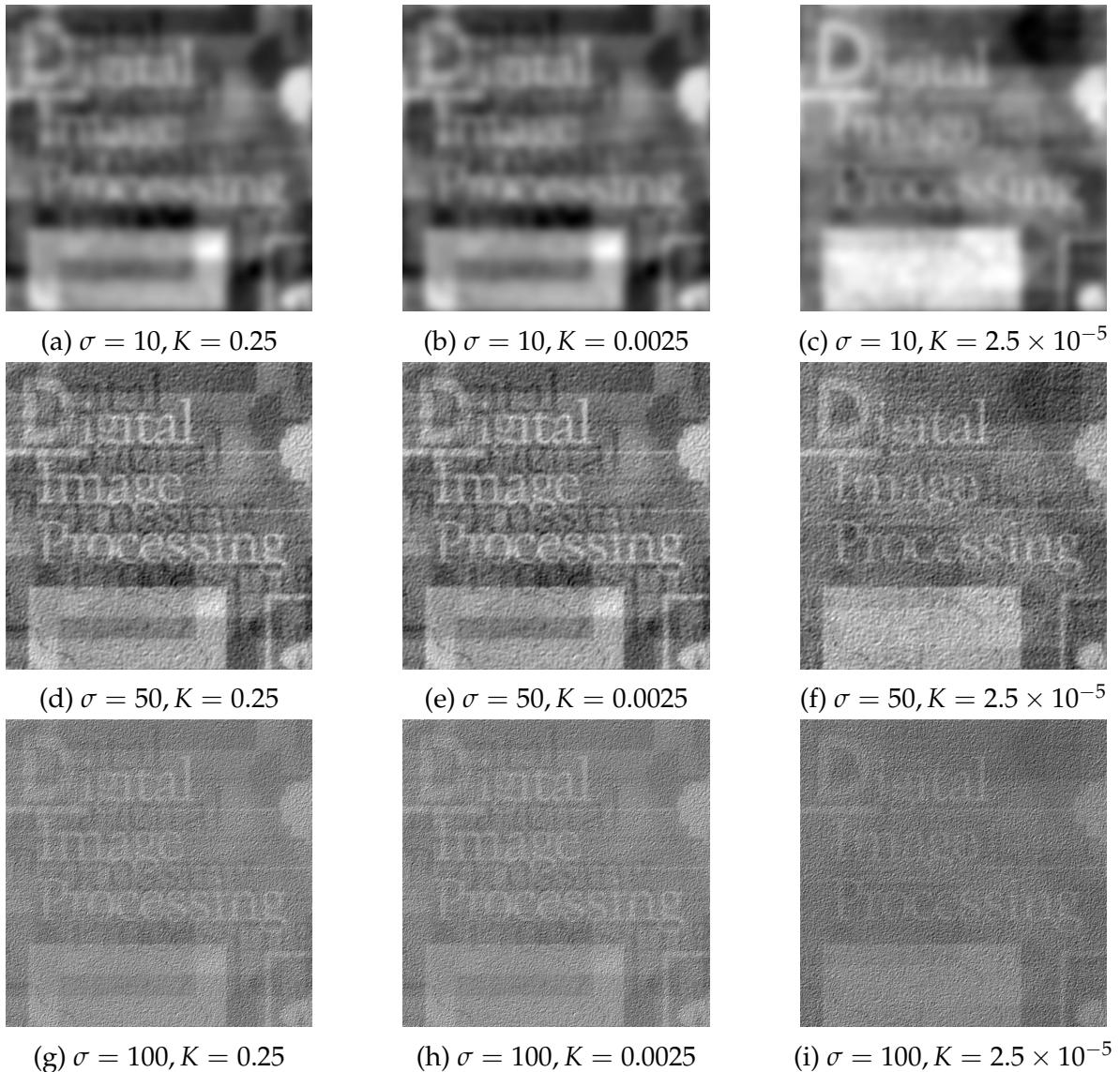


Figure 11: motion deblur of the figure

## Result.

**Discussion.** Fig 8 indicate that we can't use radially limited inverse filter to restore the image. Here a unit impulse is used to estimate the behavior of the filters

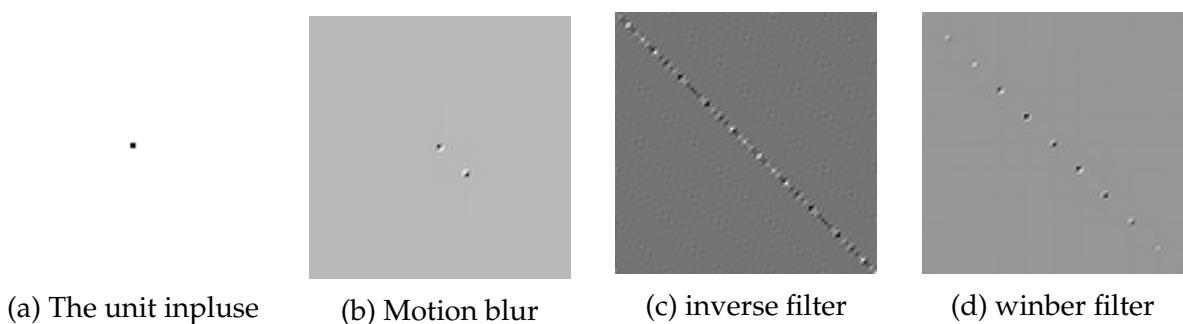


Figure 12: The behavier of the system to unit impulse

After the system is applied to the unit impulse response, we can see a motion blur appeared, then two inverse filters are applied to the degraded image. By inverse filter, the motion blur become more severe, for wiener filter, the impulse appear periodically and their intensity decrease when their distance to the center of the image increase.

However, in previous process, we will save the image and read it again before we apply the inverse filter to it, to save the image, we need to apply `abs()` function to convert it from complex number to pure real number, so during this process some information is lost. Which create a special kind of noise, then, in further process, the noise is enhanced by the inverse filter. That's why we get useless result in Fig 8.

Fig 9 shows the result after restored by wiener filter, here  $\sigma = 100, K = 10^{-4}$  is a proper pair to restore the image. Compare between image, if the details of the image is not clear enough, we just increase  $\sigma$ , if the shadow is to outstanding, we increase the  $K$  and if the noise appeared, we increase  $K$ .

In addition, the reason of the shadow have been analysed when we apply the system to the unit impulse.

Then we try to apply the filter to image with noise, first we denoise the image by adaptive filter we introduced in section 1, after filtering 3-5 times, we get the filtered image with most of the noise removed, then the wiener filter is applied to the image. In the result we found that  $\sigma = 50K = 0.25$ , is a proper pair to restore the image, however, there is still a layer of noise in the result, that is because, the denoise process also create lot of noise with different motion to the image, so the noise is also enhanced during the filtering.

## Conclusion

In this lab, we studied adaptive image denoise and image reconstruction method, we found that the adaptive filter is a better method for image denoising due to its auto adjusting feature.

When we use image restoring technic, we have one or multiple parameters to adjust. For example,  $\sigma$  in both radially limited inverse filter and wieber filter is used to reduce the high frequency noise, and  $K$  in wiener filter can adjust the reduced noise, when we adjust the parameters, one demision of image quality will increase while another decrease, so we need to find a balance between each demision of image quality.