SOUTHERN UNIVERSITY OF SCIENCE AND TECHNOLOGY
SCHOOL OF MICROELECTRONIC

The title of your LAB Report

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Course: *LAB Session I* – Professor: *YU Yajun*
Date: *April 2, 2021*

Introduction

A thorough understanding of image filtering is impossible without having at least a working knowledge of how the Fourier transform and the frequency domain can be used for image filtering. Briefly, the frequency domain is a space which is defined by **Fourier Transform** – a very wide application in image processing, **frequency domain analysis** is used to indicate how signal energy can be distributed in a range of frequency. By filtering in frquency domain, we can manipulate signal with specific frquency contianed in the image.

In this LAB, we will first introduce the procedure by which we used to apply Discrete Fourier Transform to image and different kinds of operator, then we will apply **soble**, **Gaussian** and **Butterworth** notch filters to differnt image and analysis their performance.

Gengrally, this serises of methods can be used to extract specific imformation contained in the image, can sharp or unshape the image, pre-process the image for neural network training, and also can add special effect to the iamge. As the most widely used technic for image processing, Fourier Transform still benifits many areas.

Fourier Transform and Filtering in Frquency Domain

Fourier Transform Basic.

In 1D case, we demostrate that all converage continous signal can be writen in the form of

$$f(t) = \sum_{n=-\infty}^{+\infty}$$

with

$$c_n = \frac{1}{T}\int_{-T/2}^{T/2} f(t)e^{j\frac{2\pi n}{T}t}$$

and we define fourier transform as

$$F(\mu) = \Im\{f(t)\} = \int_{-\infty}^{\infty} f(t)e^{j2\pi\mu t}$$

One important property of fourier transform is that the Fourier transform of the convolution of two functions in the spatial domain is equal to the product in the frequency domain of the Fourier transforms of the two functions, and can be expressed as

$$f(t)\bigstar h(t) = H(\mu)F(\mu)$$

or

$$f(t)h(t) = H(\mu)\bigstar F(\mu)$$

which will be the basic idea in Frquency domain filtering.

For arbitery discrete signal, we have

$$F(u) = \sum_{n=0}^{M-1} f_n e^{-j2\pi un/M}$$

and

$$f(x) = \frac{1}{M}\sum_{u=0}^{M-1} F(u)e^{j2\pi ux/M}$$

with property

$$F(u) = F(u + kM)$$

Fourier Transform For Image.

To apply Fourier tranform to image, firstly we need to expand the Fourier Transform to 2D form. The 2D fourier transform can be expressed as

$$F(\mu,\nu) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(t,z)e^{-j2\pi(\mu t+\nu z)}dtdz$$

and

$$f(u,v) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(\mu,\nu)e^{j2\pi(\mu t+\nu z)}d\mu d\nu$$

And in decrete form we have

$$F(u,v) = \sum_{x=0}^{M-1}\sum_{y=0}^{N-1} f(x,y)e^{-j2\pi(ux/M+vy/N)}$$

and

$$f(x,y) = \frac{1}{MN}\sum_{x=0}^{M-1}\sum_{y=0}^{N-1} F(u,v)e^{-j2\pi(ux/M+vy/N)}$$

Image Filtering in Frquency domain.

Property. Several important properties of Fourier Transform will be introduced here so we can design the procedure of the image processing steps.

Firstly, is the relationship that

$$f(t)\bigstar h(t) = H(\mu)F(\mu)$$

or

$$f(t)h(t) = H(\mu)\bigstar F(\mu)$$

which we have introduced before, according to this property we can first do the Fourier Transform to both image and the operator, multiple them together, and than do the reverse Fourier Transform to the product, the result is the image filtered in frquency domain.

Secondly is about the periodicity, we have

$$f(x)e^{j2\pi(u_0 x/M)} \Leftrightarrow F(u - u_0)$$

we assign $u_0 = \frac{M}{2}$ to the fomular and we obtain

$$f(x)(-1)^x \Leftrightarrow F(u - M/2)$$

and for 2D we have

$$f(x,y)(-1)^{x+y} \Leftrightarrow F(u - M/2, v - N/2)$$

the result of such transform are shown in Figure 1 The periodicities of the transform and its inverse are important issues in the implementation of DFT-based algorithms. the transform data in the interval from 0 to M - 1 consists of two back-to-back half periods meeting at point M/2. Shift the zero-frequency component to the center of the spectrum **For display and filtering purposes**, it is more convenient to **have in this interval a complete period of the transform** in which the data are contiguous. If we do not perform the shift operation, we will get the result shown in Figure 2b

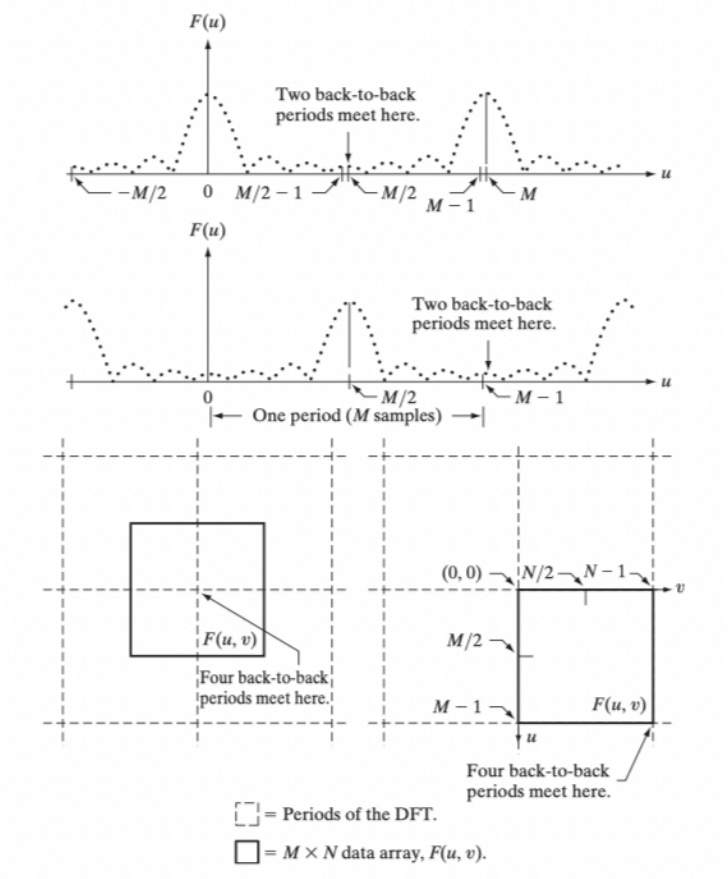


Figure 1: Shifting of the signal

Thirdly, and most importantly, is about the Symmetry Properties, we define decrete function that satisfy $f(x) = f(N - x)$ even function and those satisfy $f(x) = -f(N - x)$ odd function.

Zero padding. Before we apply the Fourier Transform to the image, we will first pad the image as shown in Figure 2c. Because we are dealing here with discrete quantities, computation of the Fourier transforms is carried out with a DFT algorithm. If we elect to compute the spatial convolution using the IDFT of the product of the two transforms, then the periodicity issues must be taken into account. when convolving two periodic functions, the convolution itself is periodic, so we may meet wraparound error, so to avoid this we must pad the image with the

$$P > A + C - 1$$

and

$$Q > B + D - 1$$

with

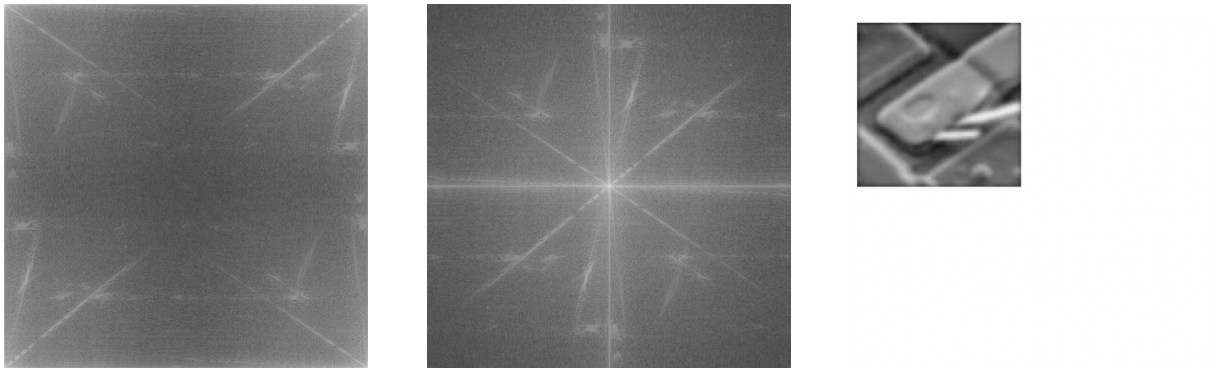
$$P > 2M - 1$$

and

$$Q > 2N - 1$$

In the meanwhile, Zero-padding before an FFT gives you a higher density interpolation, which can increase graphic or plot resolution, but it's only an interpolation, not

added information. Any closer spectral peak pairs or finer "wrinkles" in the spectrum won't appear, and you can get almost the same effect by using an appropriate smooth curve-fitting algorithm (such as splines or Sinc kernel interpolation) before graphing the non-zero-padded FFT result.



(a) without shift (b) with shift (c) Padding

Figure 2: Why we need padding and shift

Procedure. Here we summrize the steps to filtering in frquency domain

1. Given an input image $f(x, y)$ of size $M \times N$, obtain the padding parameters P and Q. Typically, P = 2M and Q = 2N.
2. Form a padded image, $f_p(x, y)$ of size $P \times Q$ by appending the necessary number of zeros to $f(x, y)$
3. Multiply $f_p(x, y)$ by $(-1)^{x+y}$ to center its transform note: as mentioned in property 2, centering helps in visualizing the filtering process and in generating the filter functions themselves, but centering is not a fundamental requirement.
4. Compute the DFT, F(u,v) of the image from step 3
5. Generate a real, symmetric filter function, $H(u, v)$, of size $P \times Q$ with center at coordinates $(P/2, Q/2)$
6. Form the product $G(u, v) = H(u, v)F(u, v)$ using array multiplication
7. Obtain the processed image

$$g_p(x, y) = \{real[\Im^{-}1[G(u, v)]]\}(-1)^{x+y}$$

8. Obtain the final processed result, $g(x, y)$, by extracting the $M \times N$ region from the top, left quadrant of $g_p(x, y)$

Task 1: Sobel filter

Implement the Sobel filter to the input images Q5_1.tif in both spatial domain and frequency domain. Compare the results. Refer to slides 78 to 81 of Lecture 4.

Figure 3: Q5_1.tif

Analysis. The Sobel operator is used in image processing and computer vision, particularly within **edge detection** algorithms where it creates an image emphasising edges. The application of Sobel opeartor can be write as

$$\mathbf{G}_x = \begin{bmatrix} +1 & 0 & -1 \\ +2 & 0 & -2 \\ +1 & 0 & -1 \end{bmatrix} * A$$

and

$$\mathbf{G}_y = \begin{bmatrix} +1 & +2 & +1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix} * A$$

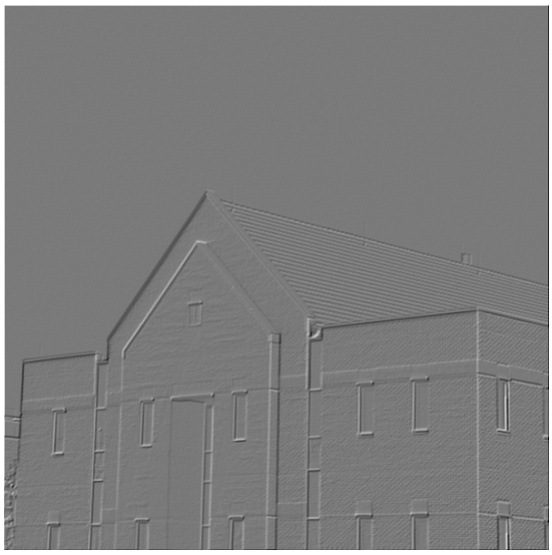
In this task we will apply both vertal and horizontal filter to the given image to detect the eage contained in the image on both spatal domain and frquency domain.

And to apply a spatal filter to frquency domain, we need to do the step 2 to 4 in the basic steps of image processing in frquency domain, then multiply them together, finally we do IDFT to the product and we can get the result.

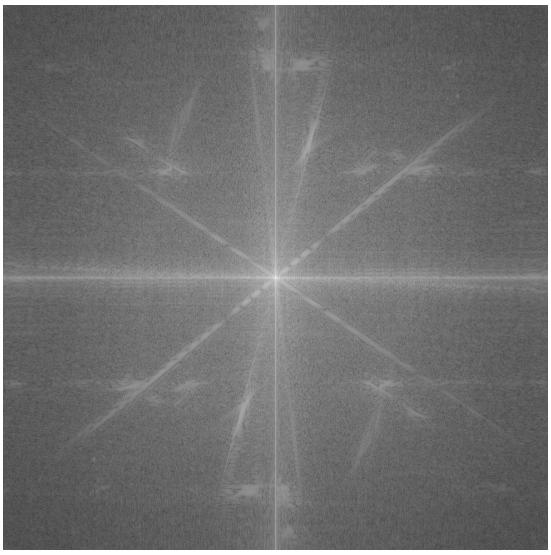
Result.

In this task we first apply the sobel filter to the image in spatal domain, and we get the result shown in Figure 4a.

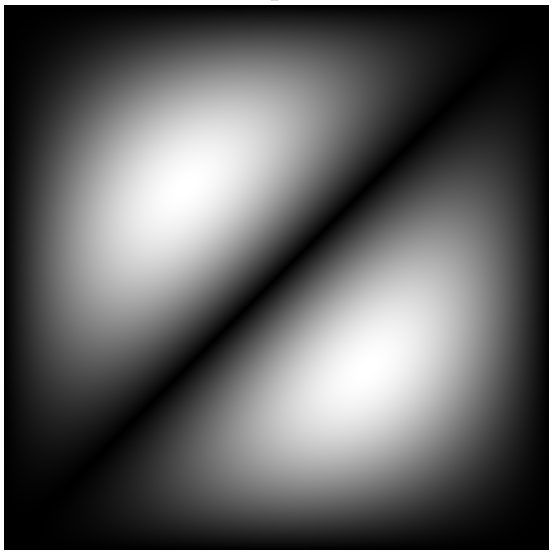
Then we apply the sobel operator on frquency domain, first convert the image by shift and DFT to frquency domain as shown in Figure 4b, second, we apply same procedure to the sobel filter after padding it and we obtain Figure 4c, finally we multiply then together and get Figure 4d, compare Figure 4a and Figure 4d, we found that they are almost the same, which means here filter in spatal domain and filter in frequence domain are equal.



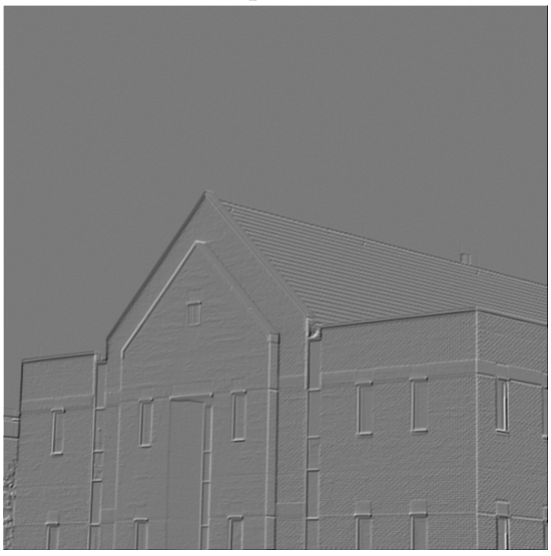
(a) spatal



(b) spectrum



(c) Filter



(d) frquency

Figure 4: Image filtered by sobel filter in spatal domain

Discussion. In this task we apply a shift at the step 4, and the reason has been discussed at last session.

And there is another rules that in both procedure, if you filp the sobel operator we can found the output image also "flipped", the "direction" of the gridant in both image will change.

Task 2: Gaussian low pass and high pass

Implement the Gaussian low pass and high pass to the input image Q5_2.tif. results for D_0 0 = 30 , 60 , and 160, respectively.

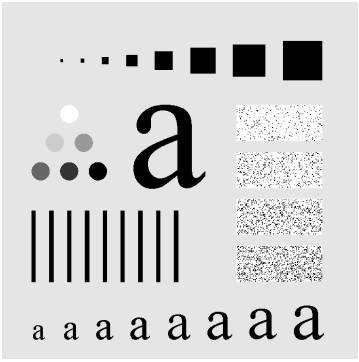


Figure 5: Q5_2.tif

Analysis.

In this task, our goal is to reduce the signal intensity contianed in high or low frquency domain by Gaussian filter. The Gaussian filter can be described as

$$H(u,v) = e^{-D^2(u,v)/2\sigma^2}$$

as shown in Figure 6, bigger n is, less signal in high frquency will be filtered.

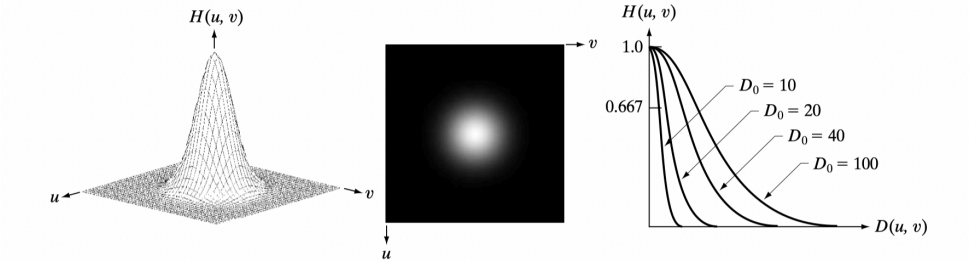
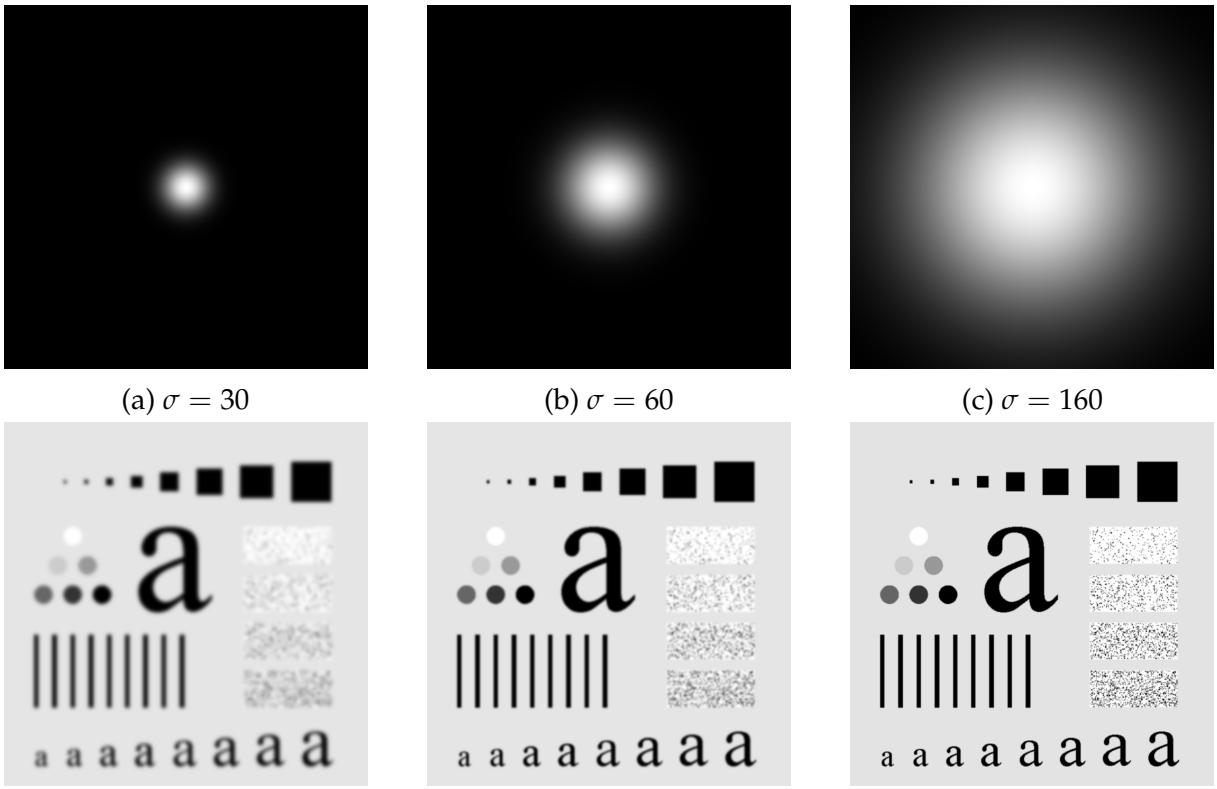


Figure 6: Gaussian lowpass filter

Result.

In this task, we apply Gaussian highpass and lowpass filter with σ equal to 30, 60, 160, the Gaussian lowpass filters and the filtered images are shown in Figure 7



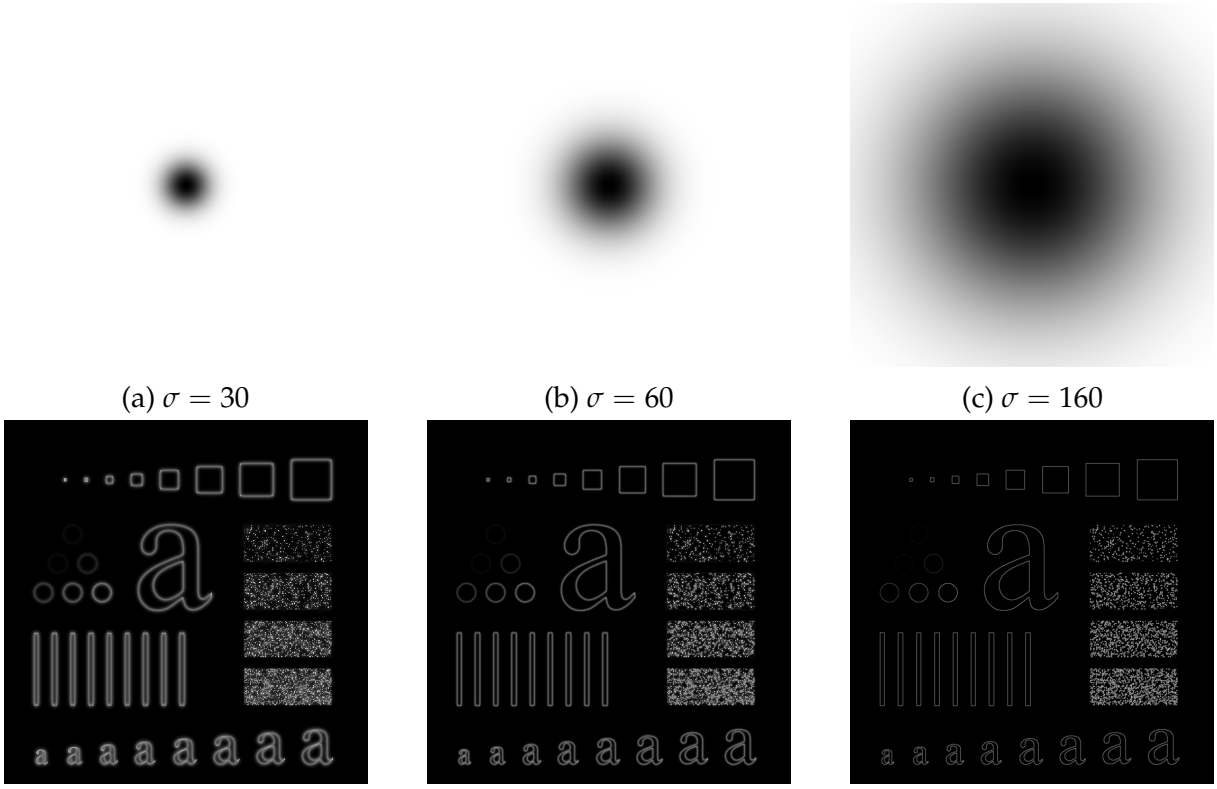
(a) $\sigma = 30$ (b) $\sigma = 60$ (c) $\sigma = 160$

(d) $\sigma = 30$ (e) $\sigma = 60$ (f) $\sigma = 160$

Figure 7: The Gaussian low filter and output images

Here we can found that, as σ increase, the image become clearer, which means that more infromation in high frequency domain is reserved, and the image is sharper.

Then we convert Gaussian lowpass filter to Gaussian highpass filter via the fomular $max(H) - H$, and we apply the filter to the image to the frequency domain.



(a) $\sigma = 30$ (b) $\sigma = 60$ (c) $\sigma = 160$

(d) $\sigma = 30$ (e) $\sigma = 60$ (f) $\sigma = 160$

Figure 8: The Gaussian highpass filter and output images

With the increase of σ more features in low frquency will be filtered, and finally only edge can be reserved.

Discussion.

Interestingly, we found that while the addition of highpass filter and lowpass filter with same σ produce an all-white image, the addition of the filtered image is equal to the original image. When $\sigma = 1$, almost no feature remained after filtered by lowpass filter, which means no feature contained in the extreme low frquency, with more infromation in relatively high frquency remained, almost all the fertures remained, for highpass filter, only edge remained after most of the information in low frquency is removed. So we can draw a conclusion that, most of the information, except th edge, contained in low frquency domain.

We can also conclude the application of differnet kind of Gaussian filters. For Gaussian lowpass filter, with lower σ we can remove the details in the image. For Gaussian highpass filter, with higher σ we can remove most of the pixels except the edge.

Task 3: Butterworth notch filters

Implement the Butterworth notch filters to the input images Q5_3.tif. Refer to slides 110 to 114 of Lecture 4.

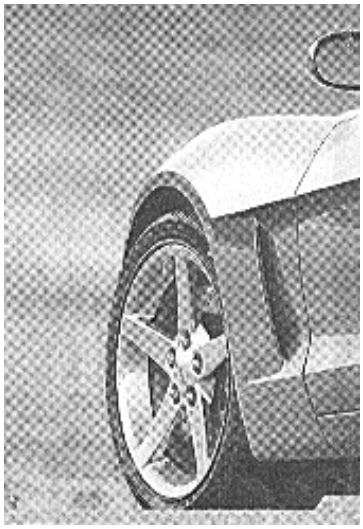


Figure 9: Q5_3.tif

Analysis. In this task, our goal is quite similar to that in Task 2, we want to reduce signal with speicfic frquency to , the Butterworth notch filters can be described as

$$H(u,v) = \frac{1}{1 + [D(u,v)/D_0]^{2n}}$$

as shown in Figure 10 bigger n is, more signal in high frquency will be filtered, and compared with Gaussian filter, we found that with bigger n, wider range of signal in low frquency will be presevered.

Different from Gaussian filter, two papameter in butterworth filter can be adjusted, σ, the cutoff frquency and the n, the order, with bigger n, as shown in Figure 10, the information in high frquency domain will be absoultely filtered, with smaller n, some information in high frquency domain will be reserved and soem information in low frquency will be filtered.

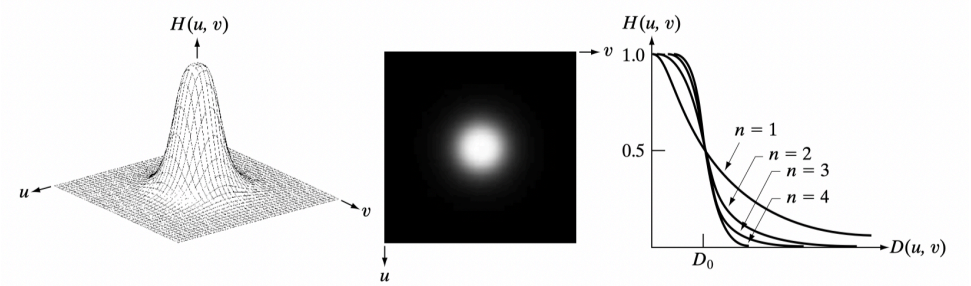
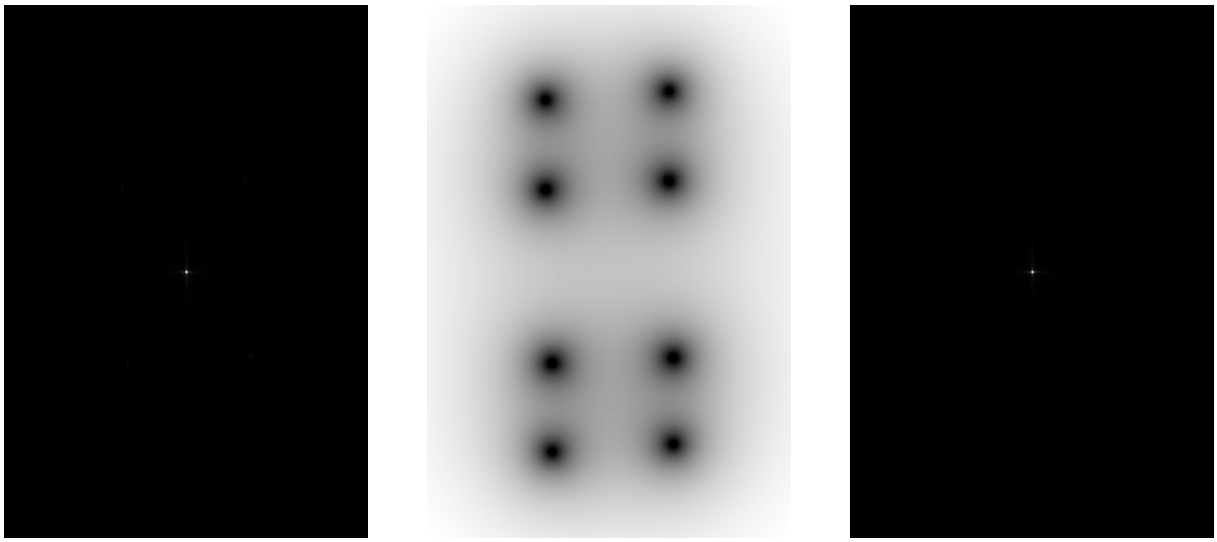


Figure 10: Butterworth lowpass filter

For the given image we first apply the DFT to it, and we found that the noise is contained in 8 points, and our task is to remove them. We generate 8 Butterworth

highpass filter, and add them together to obtain the filter shown in Fugure 11b, then we apply it to the spectrum of the spectrum and get Figure 11c, then we will compare which parameters are better.

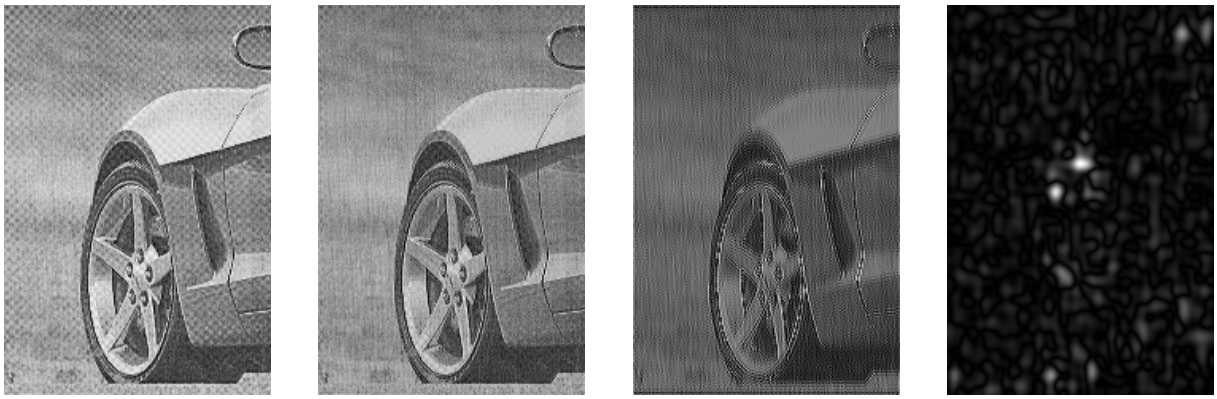


(a) Original image after DFT (b) Butterworth filter (c) Specturm after filtering

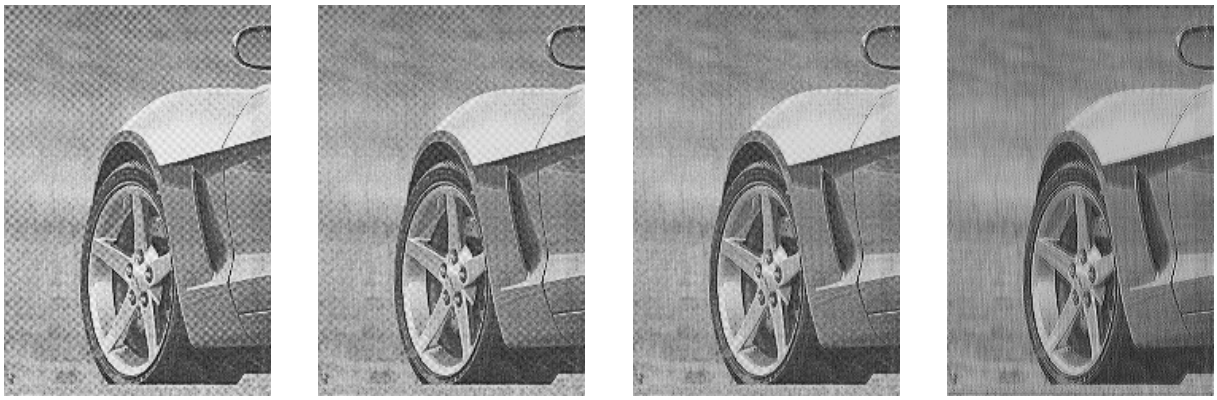
Figure 11: Butterworth filter

Result.

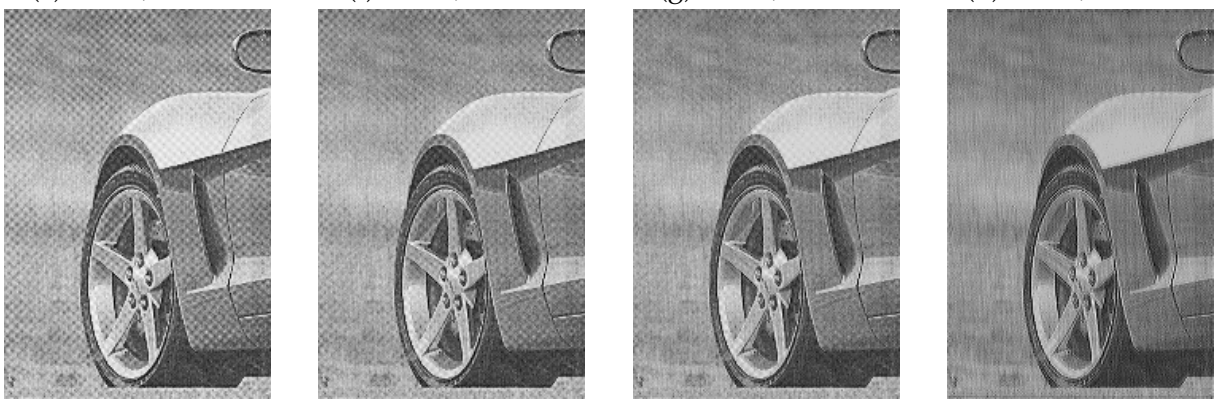
Here we apply Butterworth filter with differnet n and σ to image.



(a) $n = 1, \sigma = 10$ (b) $n = 1, \sigma = 30$ (c) $n = 1, \sigma = 60$ (d) $n = 1, \sigma = 120$



(e) $n = 2, \sigma = 10$ (f) $n = 2, \sigma = 30$ (g) $n = 1, \sigma = 60$ (h) $n = 2, \sigma = 120$



(i) $n = 3, \sigma = 10$ (j) $n = 3, \sigma = 30$ (k) $n = 3, \sigma = 60$ (l) $n = 3, \sigma = 120$

Figure 12: Apply butterworth filter

Discussion.

We found that, with n = 1, we increase σ, when $\sigma = 30$, most of the blemish disappeared, if we keep increase σ, the image begin to be damaged. For n = 2, blemish disappeared at $\sigma = 60$, then image begin to be damaged, and for n=3, this number is bigger. We can find from Figure 12 that with n=1 the buttereorth filter is similar to Gaussian filter, and for bigger n it is similar to ideal lowpass filter. So both increase n and σ can let more signal in low frequency remain, and we also find that when most of the blemish is removed, smaller n can led to less damage to image.

So generally, if using butterworth filter to reduce the blemish in the figure, we can use small n (Control the sparpness of the filter) so only a small area will be totally moved and the image will not be damaged, and we need to use relative high σ (control

the range of the filter) to make sure the blemish is removed.

Conclusion

In this lab we applied sobel filter on both spatal and frequency domain, we use Gaussian and Butterworth lowpass and lowpass filter in frquency domain.

Filtering in spatal domain focus on the relationship between pixels within a specific domain, and filtering in frquency domain focus on the speed pixels are chaning in all image and adjust the relationship.

In my opinion, for remove unwanted signal from a image, we can use both spatal and frquency filtering, while filtering in frquency often have better performance, filtering in spatal domain require less computing, and for edge detection, I recommend to use spatal filter due to the error in digital calculating may cause bad disturb in the result.

Source code

```python
'''
Library USE for EE326 2021
'''

import numpy as np
from skimage import io, data
import math
from scipy import interpolate
import matplotlib.pyplot as plt


# General
def format_image(input_image):
    output_image = input_image
    output_image -= np.min(output_image)
    output_image = (output_image/np.max(output_image))*255
    return output_image

# LAB 4


def convolution_3x3(input_image, operator_3x3):
    col, row = input_image.shape
    output_image = np.zeros([col, row])
    input_image = np.pad(input_image, 1)
    for i in range(0, col):
        for j in range(0, row):
            for i2 in range(3):
                for j2 in range(3):
                    output_image[i, j] += input_image[i+i2, j+j2] *
                     ↪  operator_3x3[i2, j2]

    return output_image


def sobel_filter(input_image):

    operator1 = np.array([[-1, -2, -1], [0, 0, 0], [1, 2, 1]])
    operator2 = np.array([[-1, 0, 1], [-2, 0, 2], [-1, 0, 1]])

    output_image1 = convolution_3x3(input_image, operator1)
    output_image2 = convolution_3x3(input_image, operator2)
    #
    # output_image1 = np.clip(output_image1, 0, 255)
    # output_image2 = np.clip(output_image2, 0, 255)
```

```
45
46      output_image = output_image1 + output_image2 # + input_image
47      # output_image = np.clip(output_image, 0, 255)
48
49      output_image = output_image.astype(np.uint8)
50
51      return output_image
52
53
54  def zero_padding(input_image, P, Q):
55      output_image = np.zeros([P, Q])
56
57      return output_image
58
59  # LAB 5
60
61
62  def extract_result_eastsouth(input_image):
63      x, y = input_image.shape
64      output_image = input_image[int(x/2):x, int(y/2):y]
65
66      return output_image
67
68
69  def extract_result_westnorth(input_image):
70      x, y = input_image.shape
71      output_image = input_image[0:int(x/2), 0:int(y/2)]
72
73      return output_image
74
75
76  def zero_padding_DFT(input_image, P, Q):
77      m, n = input_image.shape
78
79      output_image = np.zeros([P, Q])
80      output_image[0:m, 0:n] = input_image
81
82      return output_image
83
84
85  def zero_padding_DFT(input_image):
86      m,n = input_image.shape
87
88      P = 2*m
89      Q = 2*n
90
91      output_image = np.zeros([P, Q])
```

```
92      output_image[0:m, 0:n] = input_image
93
94      return output_image
95
96
97  def centering(size):
98      m, n = size
99      centering_matrix = np.ones(size)
100     mul1 = 1
101     for i in range(m):
102         mul2 = mul1
103         for j in range(n):
104             centering_matrix[i, j] = centering_matrix[i, j] * mul2
105             mul2 *= -1
106         mul1 *= -1
107     return centering_matrix
108
109
110 def generating_from_spatial_filter(input_filter, P, Q):
111     output_filter = np.zeros(P, Q)
112
113     return output_filter
114
115
116 def gaussian_filter(a, b, sigma):
117     x, y = np.meshgrid(np.linspace(0, a-1, a), np.linspace(0, b-1, b))
118     x = x - a/2
119     y = y - b/2
120     d = x * x + y * y
121     g = np.exp(-(d / (2.0 * sigma ** 2)))
122     # g = g/np.sum(g)
123     return g
124
125
126 def butterworth_filter(b, a, center, n, sigma):
127     cx, cy = center
128     x, y = np.meshgrid(np.linspace(0, a - 1, a), np.linspace(0, b - 1,
        ↪  b))
129     x = x - cx
130     y = y - cy
131     d = np.sqrt(x * x + y * y)
132     h = 1/((1+(d/sigma))**(2*n))
133     return h
134
135 if __name__ == '__main__':
136
137     test_input = np.array([[1, 2, 3, 4], [1, 2, 3, 4], [1, 2, 3, 4], [1,
        ↪  2, 3, 4]])
```

```
138     print(transform_centering(test_input))
```

```
1   import numpy as np
2   import numpy.fft
3   from skimage import io, data
4   import math
5   from scipy import interpolate
6   import matplotlib.pyplot as plt
7   from matplotlib import cm
8   import matplotlib.image as mplimg
9   import EE326_SUSTech
10
11
12  def sobel_filter_11810818(input_image):
13      m, n = input_image.shape
14      kernel1 = np.array([[-1, 0, 1], [-2, 0, 2], [-1, 0, 1]])
15      kernel2 = np.array([[-1, -2, -1], [0, 0, 0], [1, 2, 1]])
16      kernel = kernel1 + kernel2
17
18      # Filtering in the spatial Domain
19      # output_spatial = EE326_SUSTech.convolution_3x3(input_image,
        ↪  kernel)
20      # plt.imsave("Q5_1_spatial.png",
21      #            output_spatial,
22      #            cmap=cm.gray)
23
24      # Filtering in the Frequency Domain
25      kernel = np.flip(kernel)
26      input_image = np.pad(input_image, ((0, m), (0, n)))
27
28      input_image = np.multiply(input_image,
        ↪  EE326_SUSTech.centering(input_image.shape))
29      input_image = np.fft.fft2(input_image)
30
31      mplimg.imsave("Q5_1_spectrum.png",
32                    np.log(np.abs(input_image)),
33                    cmap=cm.gray)
34      sz = (input_image.shape[0] - kernel1.shape[0],
35            input_image.shape[1] - kernel1.shape[1])
36      kernel = np.pad(kernel,
37                      (((sz[0] + 1) // 2, sz[0] // 2),
38                       ((sz[1] + 1) // 2, sz[1] // 2)))
39
40      kernel = np.multiply(kernel, EE326_SUSTech.centering(kernel.shape))
41      kernel_fft = np.fft.fft2(kernel)
42
```

```
    mplimg.imsave("Q5_1_filter.png",
                  (np.abs(kernel_fft)),
                  cmap=cm.gray)

    filtered = np.multiply(input_image, kernel_fft)

    mplimg.imsave("Q5_1_spectrum_filtered.png",
                  (np.abs(filtered)),
                  cmap=cm.gray)
    filtered = np.fft.ifft2(filtered)
    filtered = np.multiply(filtered,
     ↪  EE326_SUSTech.centering(filtered.shape))

    output_image = EE326_SUSTech.extract_result_eastsouth(filtered)
    output_image = np.real(output_image)

    mplimg.imsave("Q5_1_frequency.png",
                  (output_image),
                  cmap=cm.gray)


if __name__ == '__main__':
    sobel_filter_11810818(io.imread("Q5_1.tif"))
```

```
import numpy as np
import numpy.fft
from skimage import io, data
import math
from scipy import interpolate
import matplotlib.pyplot as plt
from matplotlib import cm
import matplotlib.image as mplimg
import EE326_SUSTech


def gaussian_pass_11810818(input_image):

    # input_image = EE326_SUSTech.zero_padding_DFT(input_image)
    x, y = input_image.shape

    input_image = np.fft.fft2(input_image)
    input_image = np.fft.fftshift(input_image)

    mplimg.imsave("Q5_2_spectrum.png",
                  (np.abs(input_image)),
                  cmap=cm.gray)
```

```

    for sigma in [1, 10, 30, 60, 160]:
        filter_lowpass = EE326_SUSTech.gaussian_filter(x, y, sigma)
        output_image_frquency = np.multiply(input_image, filter_lowpass)
        output_image = np.fft.fftshift(output_image_frquency)
        output_image = (np.abs(np.fft.ifft2(output_image)))

        mplimg.imsave("Q5_2_lowpass_filter_"+str(sigma)+".png",
                      (np.abs(filter_lowpass)),
                      cmap=cm.gray)
        mplimg.imsave("Q5_2_lowpass_"+str(sigma)+".png",
                      (np.abs(output_image)),
                      cmap=cm.gray)

        filter_highpass = np.ones((x, y)) - filter_lowpass
        output_image_frquency = np.multiply(input_image,
         ↪  filter_highpass)
        output_image = np.fft.fftshift(output_image_frquency)
        output_image = np.real(np.fft.ifft2(output_image))

        mplimg.imsave("Q5_2_highpass_filter_" + str(sigma) + ".png",
                      (np.abs(filter_highpass)),
                      cmap=cm.gray)
        mplimg.imsave("Q5_2_highpass_" + str(sigma) + ".png",
                      (np.abs(output_image)),
                      cmap=cm.gray)


if __name__ == '__main__':
    gaussian_pass_11810818(io.imread("Q5_2.tif"))
```

```
import numpy as np
import numpy.fft
from skimage import io, data
import math
from scipy import interpolate
import matplotlib.pyplot as plt
from matplotlib import cm
import matplotlib.image as mplimg
import EE326_SUSTech


def butterworth_notch_filters_11810818(input_image):
    input_image = EE326_SUSTech.zero_padding_DFT(input_image)
    x, y = input_image.shape

```

```
    input_image = np.fft.fft2(input_image)
    input_image = np.fft.fftshift(input_image)

    show_image = np.abs(input_image)
    mplimg.imsave("Q5_2_spectrum.png",
                  show_image,
                  cmap=cm.gray)

    for sigma in [10, 30, 60, 90, 120, 160]:
        for n in [1, 2, 3]:
            centers = [
                [109, 87],
                [109, 170],
                [115, 330],
                [115, 412],
                [227, 405],
                [227, 325],
                [223, 162],
                [223, 79]
            ]

            filter_lowpass = np.zeros([x, y])

            for center in centers:
                filter_lowpass += EE326_SUSTech.butterworth_filter(x, y,
                 ↪  center, n, sigma)

            filter_highpass = np.ones([x, y]) - np.clip(filter_lowpass,
             ↪  0.00001, 0.99999)
            output_image = np.multiply(input_image, filter_highpass)

            mplimg.imsave("Q5_3_spectrum_filtered_" + str(n) + "_" +
             ↪  str(sigma) + ".png",
                          (np.abs(output_image)),
                          cmap=cm.gray)

            output_image = np.fft.fftshift(output_image)
            output_image = np.abs(np.fft.ifft2(output_image))
            output_image =
             ↪  EE326_SUSTech.extract_result_westnorth(output_image)

            mplimg.imsave("Q5_3_filter_" + str(n) + "_" + str(sigma) +
             ↪  ".png",
                          (np.abs(filter_highpass)),
                          cmap=cm.gray)

            mplimg.imsave("Q5_3_" + str(n) + "_" + str(sigma) + ".png",
```

```
                            (np.abs(output_image)),
                            cmap=cm.gray)


if __name__ == '__main__':
    butterworth_notch_filters_11810818(io.imread("Q5_3.tif"))
```