Higher order spectra analysis
versus
Fourier transform (higher order)
on six-dimensional gyrokinetic data

Содержание

1 Report

1.1 Motivation

Higher order singular value decomposition (HOSVD) can be used as a tool for analyzing and compressing gyrokinetic data.
HOSVD is a high-dimensional variant of singular value decomposition (SVD) and SVD is a powerful and commonly used matrix decomposition.
These techniques are widely applied to hydrodynamic turbulence, fusion research, analysis of impurity transport, compression of magnetohydrodynamic simulation data, excitation of damped eigenmodes in gyrokinetic simulations.
The gyrokinetic model describes the time evolution of the distribution of particle gyro-centers in three spatial and two velocity dimensions.

1.2 Theory

The SVD decomposition of a matrix $M \in \mathbb{C}^{n_1 \times n_2}$

$$M = USV^\dagger$$

where $U \in \mathbb{C}^{n_1 \times \min(n_1,n_2)}$ and $V^\dagger \in \mathbb{C}^{\min(n_1,n_2) \times n_2}$ are unitary matrices and $S \in \mathbb{C}^{\min(n_1,n_2) \times \min(n_1,n_2)}$ is a positive definite diagonal matrix.

An alternative notation for the SVD

$$M_{ij} = \sum_{l=1}^{\min(n_1,n_2)} s_l u_i^{(l)} v_j^{(l)}$$

Let us now transition to tensor decompositions. Consider a three dimensional tensor $M \in \mathbb{C}^{n_1 \times n_2 \times n_3}$. The HOSVD is based on a decomposition of this form

$$M_{ijk} = \sum_{m_1=1}^{n_1} \sum_{m_2=1}^{n_2} \sum_{m_3=1}^{n_3} S_{m_1 m_2 m_3} u_i^{m_1} v_j^{m_2} w_k^{m_3}$$

In this expression, the data tensor, M_{ijk}, is reproduced as a superposition of tensors. The tensor S_{m1m2m3} is called the core tensor.

One drawback of the HOSVD is that there is no analogue to the SVD optimality theorem

$$\varepsilon^r = \|M - M_{\text{SVD}}^{(r)}\|_F^2 \quad \forall M^{(r)} \in \mathbb{C}^{n_1 \times n_2}$$

one must perform the calculation in order to see if it has good properties in terms of compression or extracting important features. However, the error bound gives an indication why the HOSVD can be useful for these purposes

$$\varepsilon_{\text{bound}}^{(r_1,r_2,r_3)} = \sum_{m_1=r_1+1}^{n_1} \sigma_{m_1}^2 + \sum_{m_2=r_2+1}^{n_2} \sigma_{m_2}^2 + \sum_{m_3=r_3+1}^{n_3} \sigma_{m_3}^2 \geq \varepsilon_{\text{HOSVD}}^{(r_1,r_2,r_3)}$$

1.3 Data

The gyrokinetic model describes the time evolution of the distribution of particle gyro-centers in three spatial and two velocity dimensions.

Example: gyrokinetic data describing the nonlinear turbulence produced by a variety of plasma microinstabilities thought to be important transport mechanisms in fusion plasmas.

A variety of simulations were performed with the GENE code. The physical parameters for these simulations are shown in Table 1, and the numerical parameters for both the original simulations and the HOSVD analyses are shown in Table 2.

Table 1

Physical parameters used in the GENE simulations. The parameters are safety factor q, magnetic shear $\hat{s}$, inverse aspect ratio r/R, ion (electron) density $n_{i(e)}$, ion (electron) temperature $T_{i(e)}$, ion (electron) temperature gradient scale length $R/L_{T_{i(e)}}$, density gradient scale length R/L_n, plasma β, and collision frequency ν. In all cases fourth order hyperdiffusion is applied in the z and $v_\parallel$ coordinates.

	q	$\hat{s}$	$\epsilon = r/R$	n_i/n_e	T_i/T_e	R/L_{T_i}	R/L_{T_e}	R/L_n	β	ν
ITG	1.4	0.8	0.18	1.0	1.0	6.9	n/a	2.2	0.0	0.0
ITG-coll	1.4	0.8	0.18	1.0	1.0	6.9	n/a	2.2	0.0	5.0×10^{-3}
ITG-KE	1.4	0.8	0.18	1.0	1.0	6.9	6.9	2.2	1.0×10^{-4}	0.0
ETG	1.4	0.35	0.18	1.0	1.0	n/a	6.9	2.2	0.0	0.0
TEM	1.4	0.8	0.16	1.0	1.0/3.0	0.0	6.0	3.0	1.0×10^{-3}	0.0

Table 2

Numerical parameters used in the GENE simulations and the HOSVD analysis. The parameters are box size in the x/y directions, $L_{x/y}$, number of $k_{x/y}$ modes, $N_{k_{x/y}}$, number of grid points in the $v_\parallel/z/\mu$ coordinate, $N_{z/v_\parallel/\mu}$, particle species (electron or ion), number of time steps used in the HOSVD analysis, $N_{t_{\mathrm{HOSVD}}}$, factor determining frequency of time step data output (e.g. the distribution function was output at every 100 time steps for the ITG simulation), $\Delta t_{\mathrm{HOSVD}}$, and size of the data set in gigabytes.

	L_x	L_y	N_{k_x}	N_{k_y}	N_z	$N_{v_\parallel}$	N_μ	Species	$N_{t_{\mathrm{HOSVD}}}$	$\Delta t_{\mathrm{HOSVD}}$	Size (GB)
ITG	125.6	125.6	64	16	16	32	8	i	240	100	8.1
ITG-coll	125.6	125.6	64	16	16	32	8	i	230	100	7.7
ITG-KE	125.6	104.7	128	24	16	32	8	i, e	220	400	22.5
ETG	114.3	125.6	64	16	16	32	8	e	280	200	11.7
TEM	94.2	78.5	128	24	16	40	8	i, e	250	400	31.5

1.4 Code analysis

1.5 Experiment