SVEUČILIŠTE U ZAGREBU
FAKULTET STROJARSTVA I BRODOGRADNJE
Katedra za strojarsku automatiku

Seminarski rad iz kolegija

Modeliranje i simulacija mehatroničkih sustava

Inverzni klin

Zadatak zadao:
Prof. dr. sc. Željko Šitum

Ocjena:

Student:
Antonio Ćuk
0035239603

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Sadržaj

Popis oznaka

m	masa klizača [kg]
M	masa okvira [kg]
c	udaljenost težišta od središta rotacije [m]
d	udaljenost klizne plohe pd središta rotacije [m]
b_1	koeficijent viskoznog trenja okvira [Nms]
b_2	koeficijent viskoznog trenja klizača [Ns/m]
g	ubrzanje sile [m/s^2]
θ	kut zakreta okvira [rad]
$\dot{\theta}$	kutna brzina okvira [rad/s]
$\ddot{\theta}$	kutno ubrzanje okvira [rad/s^2]
x	pozicija klizača na okviru [m]
$\dot{x}$	brzina klizača [m/s]
$\ddot{x}$	ubrzanje klizača [m/s^2]

1. Raspis Euler-Lagrange-ove metode za nelinearni sustav

1 Općenito Euler-Lagrange jednadžbe

$$\frac{d}{dt}(\frac{\partial K_j}{\partial \dot{q}_i}) - \frac{\partial K_j}{\partial q_i} + \frac{\partial P_j}{\partial q_i} = T_{ij} \tag{1}$$

T_{ij} - sila/moment u i-toj upravljanoj koordinati
K_j - kinetička energija j-te mase
P_j - potencijalna energija j-te mase
q_i - i-ta upravljana koordinata
$\dot{q}_i$ - brzina i-te upravljane koordinate

Figure 1: Skica kosine

Iz slike je vidljivo da vrijedi: $\begin{aligned} h_1 &= x\sin\theta \\ h_2 &= d\cos\theta \end{aligned}$

2 Kinetička i potencijalna energija te brzina za klizač

$$K = \frac{1}{2}\int_{(m)} v^2 dm = \frac{1}{2}mv^2 \tag{2}$$

$$P = mgh \tag{3}$$

$$v = \frac{d\theta}{dt} \tag{4}$$

3 Okvir

$$v = r\omega = c\dot{\theta} \tag{5}$$

$$K_1 = \frac{1}{2}Mc^2\dot{\theta}^2 \tag{6}$$

$$P_1 = Mgc\cos\theta \tag{7}$$

4 Klizač

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, \quad \begin{matrix} x_1 = x \\ x_2 = d \end{matrix} \tag{8}$$

$$p_2 = \begin{bmatrix} x\cos\theta & -d\sin\theta \\ x\sin\theta & d\cos\theta \end{bmatrix} \tag{9}$$

$$v_2 = \frac{dp_2}{dt} = \begin{bmatrix} \dot{x}\cos\theta - x\dot{\theta}\sin\theta & -d\dot{\theta}\cos\theta \\ \dot{x}\sin\theta + x\dot{\theta}\cos\theta & -d\dot{\theta}\sin\theta \end{bmatrix} \tag{10}$$

$$\begin{aligned}
v_2^2 &= [(x - d\dot{\theta})\cos\theta - x\dot{\theta}\sin\theta]^2 + [(x - d\dot{\theta})\sin\theta + x\dot{\theta}\cos\theta]^2 \\
&= [(\dot{x} - d\dot{\theta})^2\cos^2\theta - 2(\dot{x} - d\dot{\theta})\cos\theta x\dot{\theta}\sin\theta + x^2\dot{\theta}^2\sin^2\theta] \\
&\quad + [(\dot{x} - d\dot{\theta})^2\sin^2\theta + 2(\dot{x} - d\dot{\theta})\sin\theta x\dot{\theta}\cos\theta + x^2\dot{\theta}^2\cos^2\theta] \\
&= [(\dot{x} - d\dot{\theta})^2 + x^2\dot{\theta}^2] \\
&= \dot{x}^2 - 2d\dot{x}\dot{\theta} + d^2\theta^2 + x^2\dot{\theta}^2
\end{aligned} \tag{11}$$

$$K_2 = \frac{1}{2}m(\dot{x}^2 - 2d\dot{x}\dot{\theta} + d^2\dot{\theta}^2 + x^2\dot{\theta}^2) \tag{12}$$

$$P_2 = mgx\sin\theta + mgd\cos\theta \tag{13}$$

$$K = K_1 + K_2 = \frac{1}{2}(\dot{x}^2 - 2d\dot{x}\dot{\theta} + d^2\dot{\theta}^2 + x^2\dot{\theta}^2) + \frac{1}{2}Mc^2\dot{\theta}^2 \tag{14}$$

$$P = P_1 + P_2 = mgx\sin\theta + mdg\cos\theta - Mgc\cos\theta \tag{15}$$

5 Raspis Euler-Lagrange formula

$q_1 = \theta \quad , \quad T_1 = Fd$
$q_2 = x \quad , \quad T_2 = F$

$$\frac{\partial K}{\partial \dot{x}} = \frac{1}{2}m(2\dot{x} - 2d\dot{\theta}) \tag{16}$$

$$\frac{d}{dt}(\frac{\partial K}{\partial \dot{x}}) = m\ddot{x} - md\ddot{\theta} \tag{17}$$

$$\frac{\partial K}{\partial x} = \frac{1}{2}m(2x\dot{\theta}^2) = mx(\dot{\theta})^2 \tag{18}$$

$$\frac{\partial P}{\partial x} = mg\sin\theta \tag{19}$$

$$m\ddot{x} - md\ddot{\theta} - mx\dot{\theta}^2 + mg\sin\theta = F \tag{20}$$

$$\frac{\partial K}{\partial \dot{\theta}} = -md\ddot{x} + md^2\theta^2 + 2mx\dot{x}\dot{\theta} + mx^2\ddot{\theta} + Mc^2\ddot{\theta} \tag{21}$$

$$\frac{\partial K}{\partial \theta} = 0 \tag{22}$$

$$\frac{\partial P}{\partial \theta} = mgx\cos\theta - mgd\sin\theta - Mgc\sin\theta \tag{23}$$

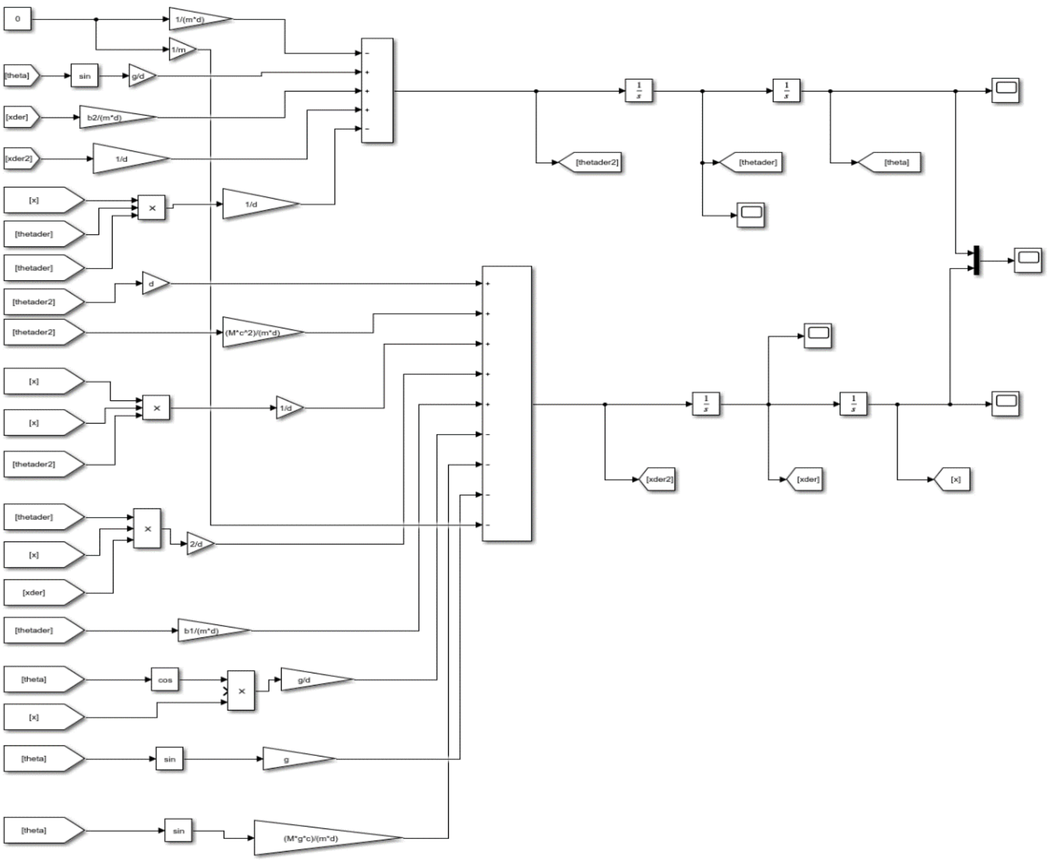
$$md^2\ddot{\theta} + mx^2\ddot{\theta} + Mc^2\ddot{\theta} - md\ddot{x} + 2mx\dot{x}\dot{\theta} + mgx\cos\theta - mgd\sin(\theta) - Mgc\sin\theta = Fd \tag{24}$$

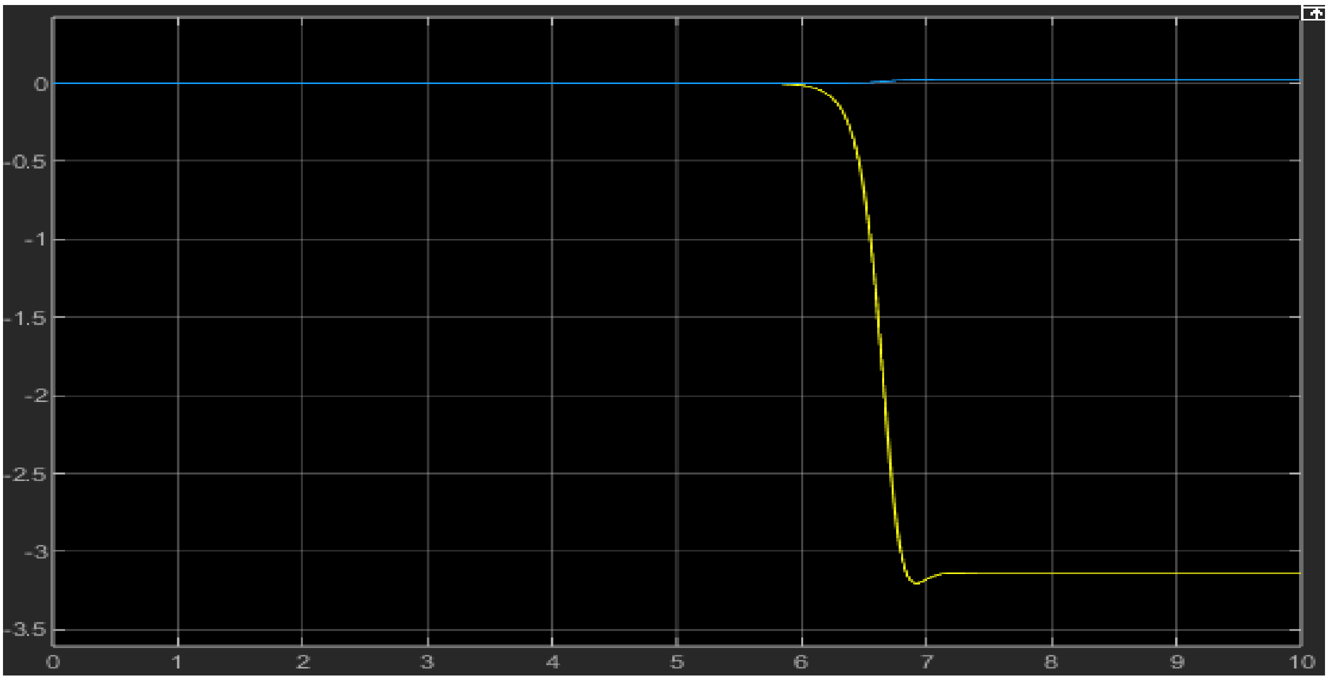
U jednadžbe (20) i (24) dodajemo faktore $b_1\dot{\theta}$ i $b_2\dot{x}$ koji predstavljaju trenje u sustavu

$$m\ddot{x} + b_2\dot{x} - md\ddot{\theta} - mx\dot{\theta}^2 + mg\sin\theta = F \tag{25}$$

$$md^2\ddot{\theta} + mx^2\ddot{\theta} + Mc^2\ddot{\theta} - md\ddot{x} + 2mx\dot{x}\dot{\theta} + b_1\dot{\theta} + mgx\cos\theta - mgd\sin(\theta) - Mgc\sin\theta = Fd \tag{26}$$

1.1. Simulacijski model nelinearnog sustava i njegov odziv





Slika 1 Odziv nelinearnog sustava

2.Linearizacija sustava i prostor stanja

6 Linearizacija sustava

$\cos\theta \approx 1 \quad , \quad \sin\theta \approx \theta \quad , \quad \dot{\theta}^2 \approx x^2 \approx \dot{x}\dot{\theta} = 0$

$$m\ddot{x} + b_2\dot{x} - md\ddot{\theta} + mg\theta = F \tag{27}$$

$$md^2\ddot{\theta} + Mc^2\ddot{\theta} - md\ddot{x} + b_1\dot{\theta} + mgx - mdg\theta - Mgc\theta = Fd \tag{28}$$

$$\ddot{x} = -\frac{b_2}{m}\dot{x} + d\ddot{\theta} - g\theta + \frac{1}{m}F \tag{29}$$

$$\ddot{\theta} = \frac{g}{c}\theta - \frac{b_1}{Mc^2}\dot{\theta} - \frac{mg}{Mc^2}x - \frac{b_2 d}{Mc^2}\dot{x} + \frac{2d}{Mc^2}F \tag{30}$$

$$\ddot{x} = (\frac{gd}{c} - g)\theta - \frac{b_1 d}{Mc^2}\dot{\theta} - \frac{mgd}{Mc^2}x - (\frac{b_2 d^2}{Mc^2} + \frac{b_2}{m})\dot{x} + (\frac{2d^2}{Mc^2} + \frac{1}{m})F \tag{31}$$

7 Prostor stanja

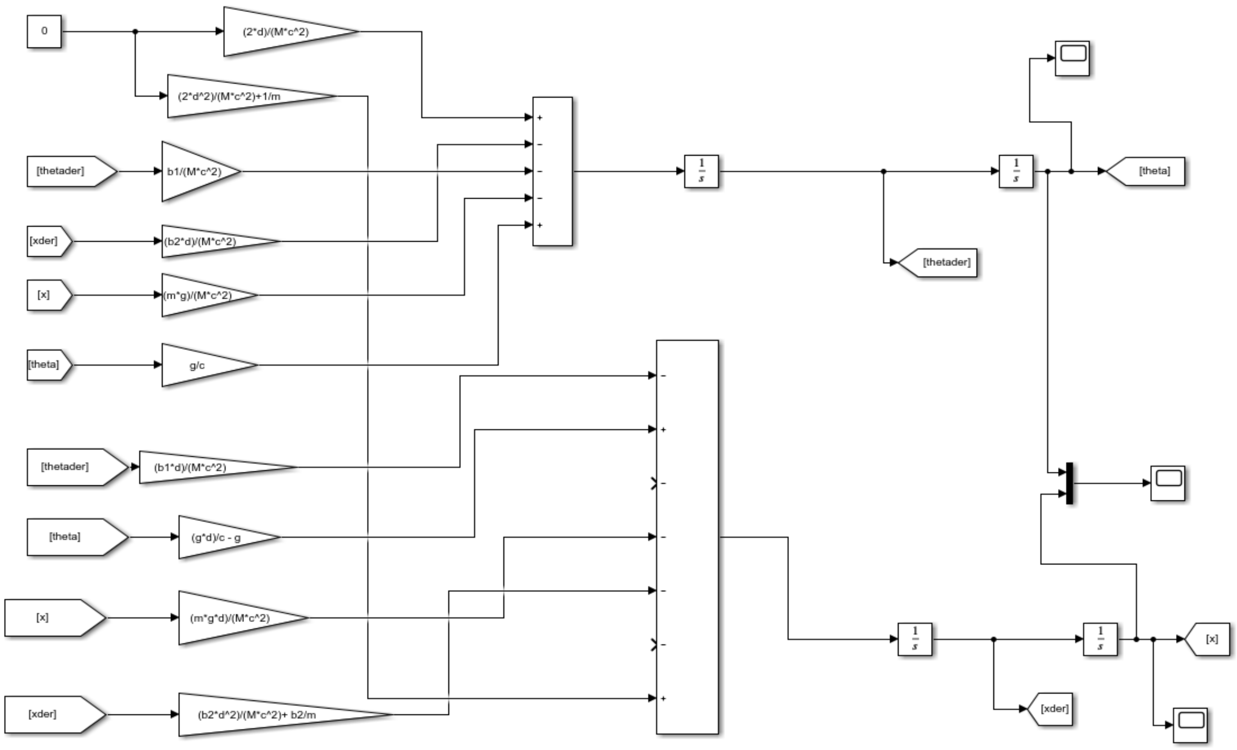
$\dot{x} = \varLambda x + Bu$
$y = Cx + Du$

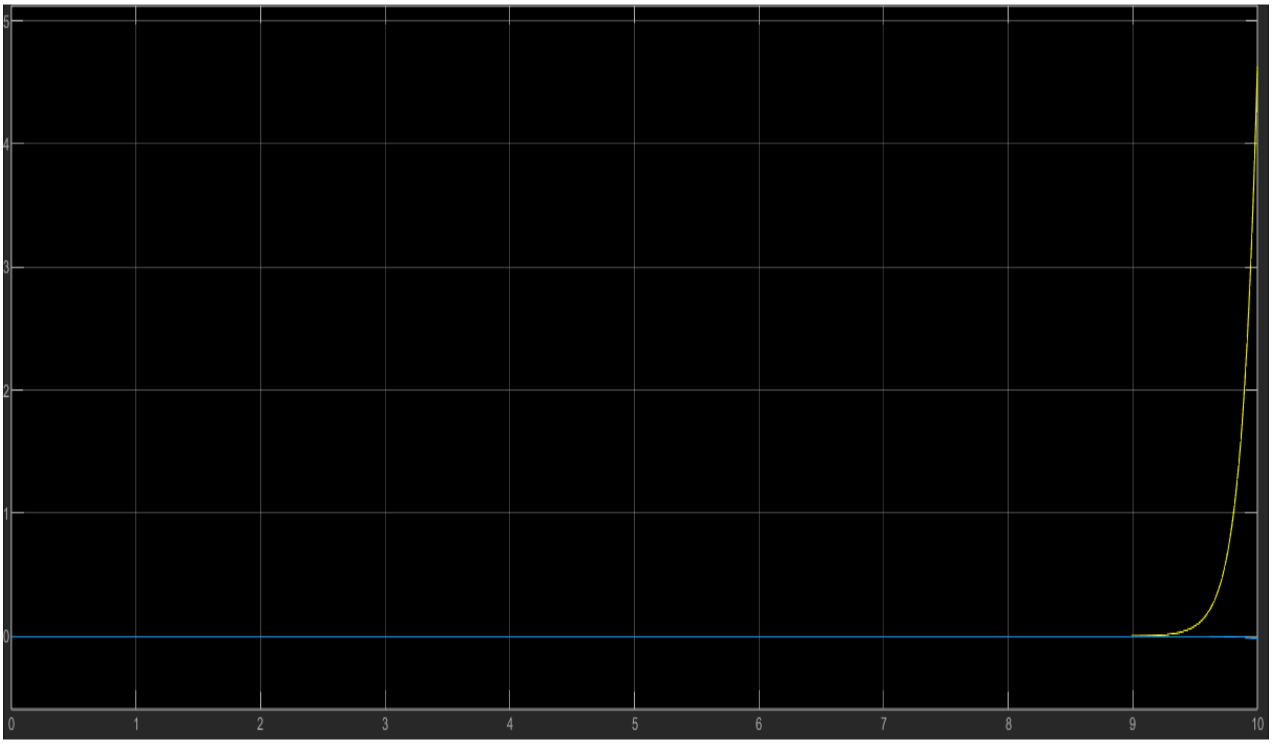
$$x = \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} \tag{32}$$

$$\begin{bmatrix} \dot{\theta} \\ \ddot{\theta} \\ \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{g}{c} & -\frac{b_1}{Mc^2} & -\frac{mg}{Mc^2} & -\frac{b_2 d}{Mc^2} \\ 0 & 0 & 0 & 1 \\ \frac{gd}{c} - g & -\frac{b_1 d}{Mc^2} & -\frac{mgd}{Mc^2} & -(\frac{b_2 d^2}{Mc^2} + \frac{b_2}{m}) \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2d}{Mc^2} \\ 0 \\ (\frac{2d}{Mc^2} + \frac{1}{m}) \end{bmatrix} F \tag{33}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{34}$$

2.2. Simulacijski model linearnog sustava i njegov odziv





Slika 2 Odziv linearnog sustava

3. Sinteza regulatora

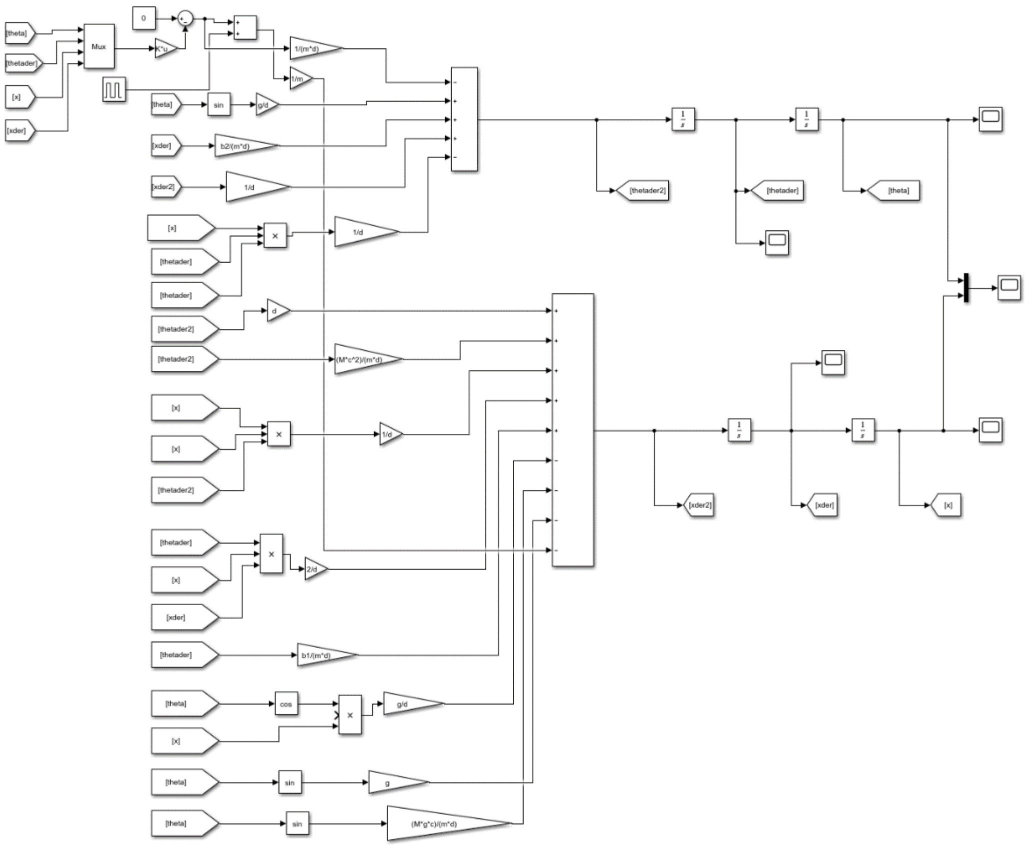
Pošto su odzivi u oba sustava; linearnog i nelinearnom nestabilni bez regulatora, u sljedećim poglavljima ćemo izmodelirati optimalan LQR regulator za naš linearni sustav.

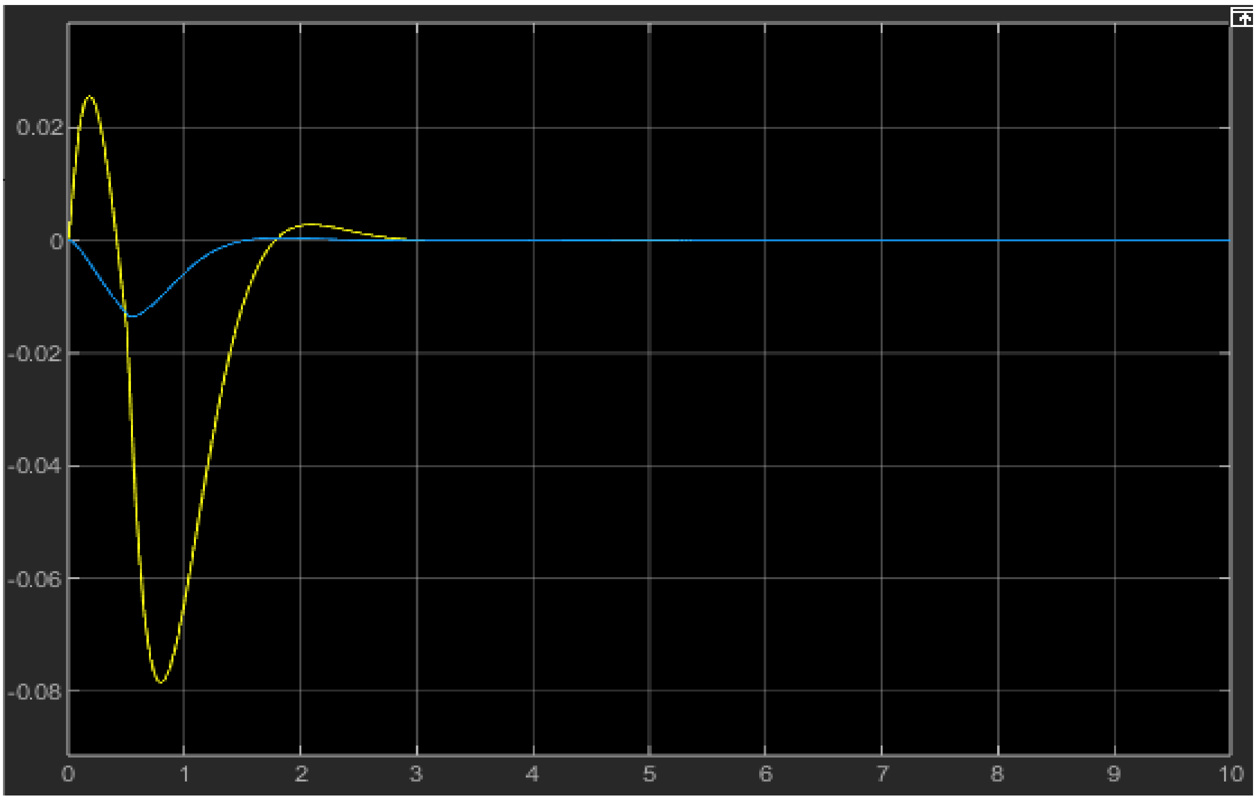
Za pronalaženje optimalne matrice pojačanja koristimo kod naveden dolje. On pomoću naredbe „lqr" pronalazi optimalne vrijednosti s obzirom na trenutni prostor stanja.

```
m = 0.3; %masa klizaca
M = 1.2; %masa okvira
b1 = 0.05; %koef. viskoznog trenja okvira
b2 = 45; %koef. viskoznog trenja klizača
g = 9.81; %gravitacijsko ubrzanje
d = 0.062; %visina težišta klizača od centra rotacije
c = 0.03;
A = [0 1 0 0;
g/c -b1/(M*c*c) -(m*g)/(M*c*c) -(b2*d)/(M*c*c);
0 0 0 1;
(g*d/c - g) -(b1*d)/(M*c*c) -(m*g*d)/(M*c*c) -((b2*d*d)/(M*c*c) + b2/m) ];
B = [0; 2*d/(M*c*c); 0; ((2*d*d)/(M*c*c)+1/m)];
C = [1 0 0 0; 0 0 1 0];

D = [0; 0];
q1=(1/0.5)^2;
q2=(1/0.5)^2;
q3=(1/0.3)^2;
q4=(1/0.3)^2;
% Matrice Q i R
Q=[q1 0 0 0 ;
0 q2 0 0 ;
0 0 q3 0 ;
0 0 0 q4 ];
R=1;
K=lqr(A,B,Q,R)

Ac=[(A-B*K)];
Bc=[B];
Cc=[C];

Cc=[C];
Dc=[D];
x0=[0; 0; 0.1; 0];
T=0:0.01:10;
U=0*ones(size(T));
[Y,X]=lsim(Ac,Bc,Cc,Dc,U,T,x0);
plot(T,Y)
legend('okvir','klizac')
xlabel('t,s'), ylabel('x, \theta')
```

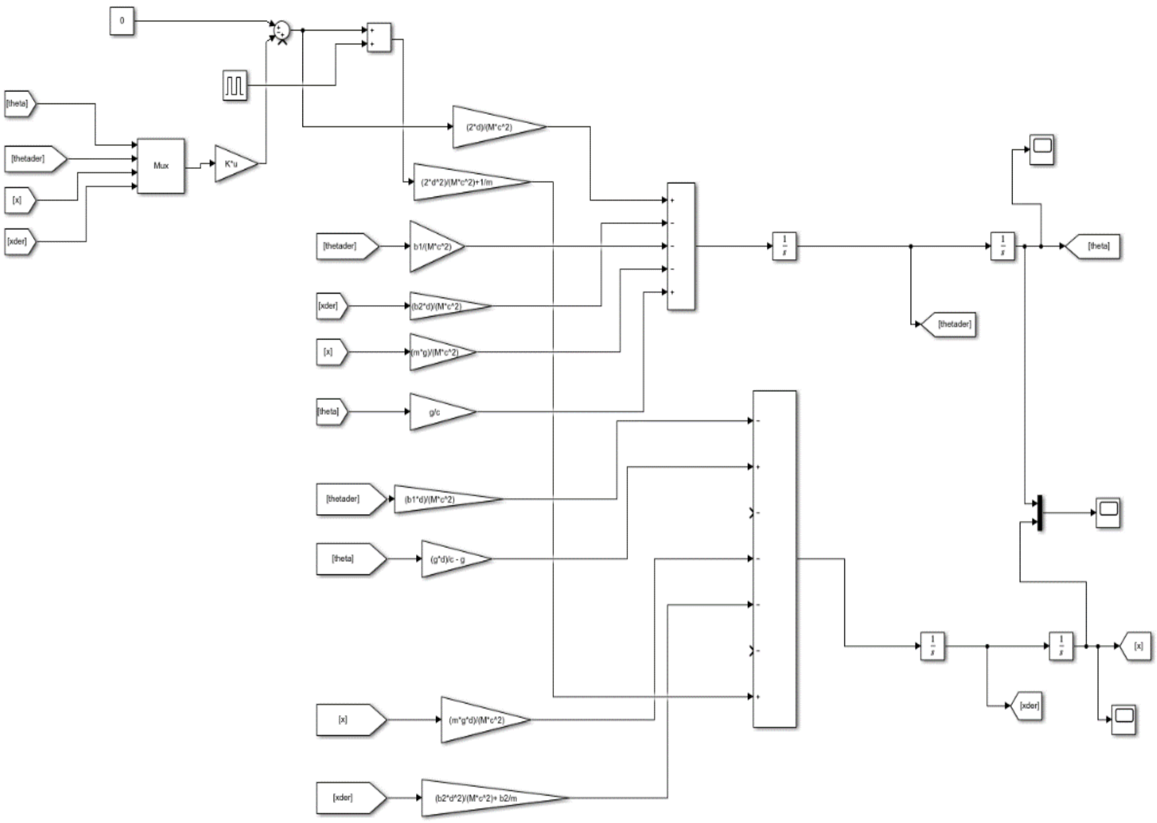
Slika 3 Matlab kod LQR regulatora

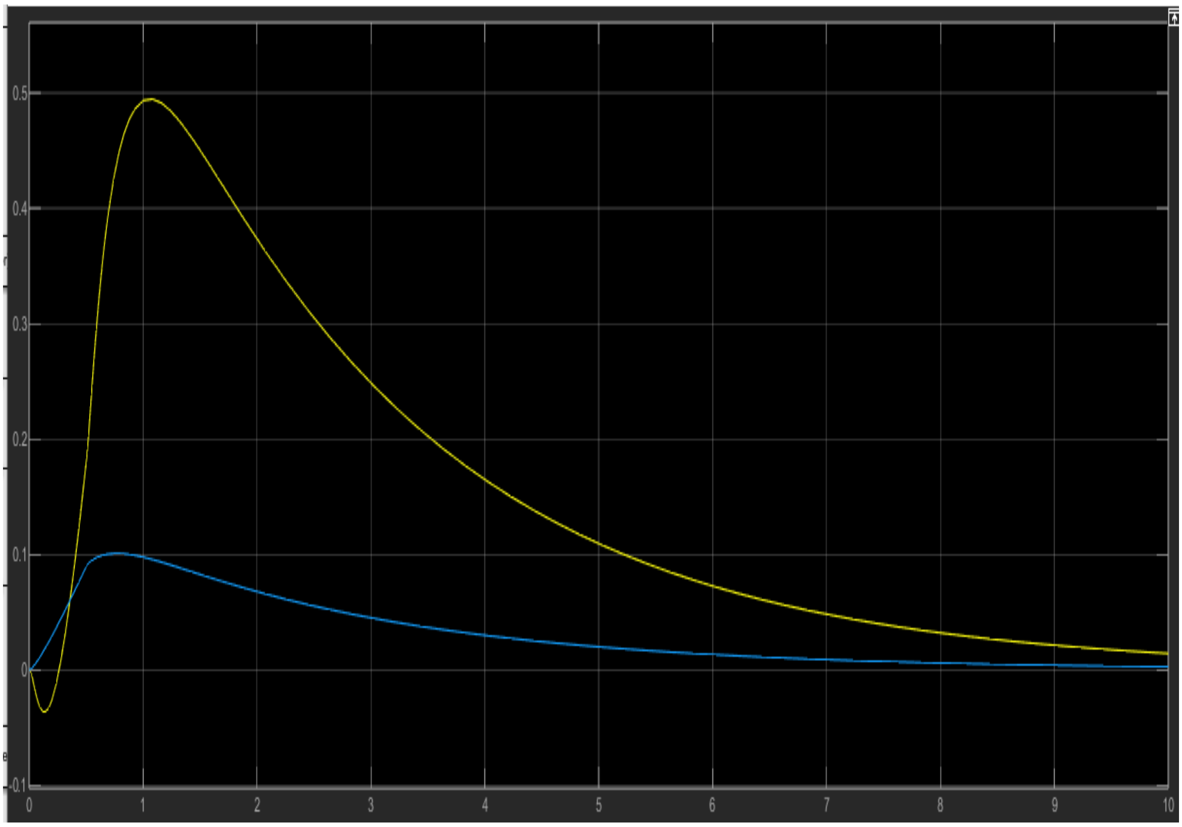
3.1. Odzivi sustava sa uključenim regulatorom





Slika 4 Odziv nelinearnog sustava sa uključenim regulatorom





Slika 5 Odziv linearnog sustava sa uključenim regulatorom

4. Zaključak

Iz rezultata možemo zaključiti da je sustav bez regulatora nestabilan i u stvarnosti bi klizač samo pao na jednu stranu okvira. Sa uključenim regulatorom cijelo vrijeme se kompenzira greška i sustav se s vremenom dovede u stanje ravnoteže. Možemo primijetiti da je linearni sustav jednostavniji za analizu i simulaciju, ali nelinearni sustav je precizniji i točniji.