Activity 3: Analyzing Uncertainty in Digital Measurements

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Purpose

- To practice uncertainty analysis involving measurements made with digital measuring devices.
- To get hands-on experience working with simple digital measuring devices.

Problems

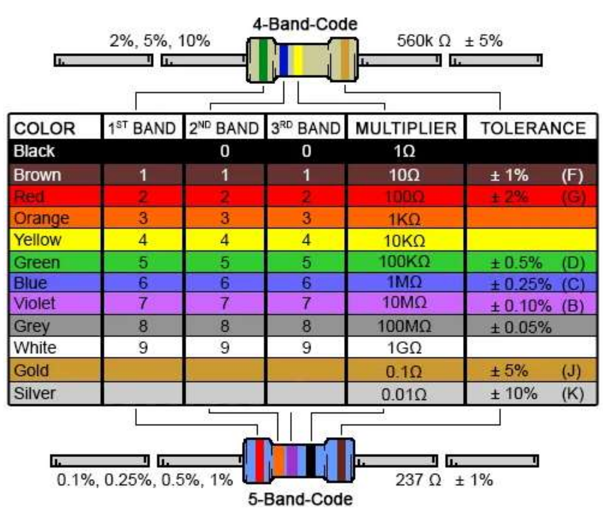
Note: You may need more room than is provided to answer the questions. If so, use additional paper.

For this activity, don't worry about taking every measurement several times, and don't worry about computing statistical (Type A) uncertainties for every quantity you measure. The focus of this lab should be on understanding how to determine (Type B) uncertainties due to inherent instrumentation limits.

Setup & Preparation

Problem 1.

You have been given five resistors of different values. Use the Resistor Color Code Table in Figure 3.1 to determine the values of each resistor you were given. Note that these should all have values of 1 $k\Omega$ or greater to limit the current drawn (and to "avoid letting the smoke out", i.e., to avoid blowing up the resistors).



COLOR	1ST BAND	2ND BAND	3RD BAND	MULTIPLIER	TOLERANCE
Black		0	0	1Ω	
Brown	1	1	1	10Ω	± 1% (F)
Red	2	2	2	100Ω	± 2% (G)
Orange	3	3	3	1KΩ	
Yellow	4	4	4	10KΩ	
Green	5	5	5	100KΩ	± 0.5% (D)
Blue	6	6	6	1MΩ	± 0.25% (C)
Violet	7	7	7	10MΩ	± 0.10% (B)
Grey	8	8	8	100MΩ	± 0.05%
White	9	9	9	1GΩ	
Gold				0.1Ω	± 5% (J)
Silver				0.01Ω	± 10% (K)

Figure 3.1: 4-Band and 5-Band Resistor Color Code Table

= 3330 + 1%

$$560 \cdot 0.1 = 56\,\Omega$$

On the following lines, record the stated value and tolerance as both a percent uncertainty and as an absolute uncertainty.

For example, $5\ k\Omega\ @\ 1\% = (5000 \pm 50)\ \Omega$

(a.) $330K \pm 1\% = (330{,}000 \pm 3300)\,\Omega$

(b.) $56\Omega \pm 1\% = (56 \pm 0.56)\,\Omega$

(c.) $1\Omega \pm 1\% = (1 \pm 0.01)\,\Omega$

(d.) $390\Omega \pm 1\% = (390 + 3.9)\,\Omega$

(e.) __________

Problem 2.

Record the model of the Digital Multi-Meter (DMM) you'll be using. Look up (on the internet) the *user's manual* for the DMM, find the accuracy of the DMM, and record the measurement uncertainty for the DMM for the various ranges and quantities listed; this information can usually be found in the `Technical Specifications` section of the device's user manual (which you should be able to find easily online).

<u>Note:</u> There will likely be different measurement accuracies specified for different types of measurements and for different ranges of values. Record all the accuracies in every range for the DMM measurements of all the quantities listed below. We will take this information as specifying the uncertainty in DMM measurements that we will make of these quantities in these ranges. If you are having trouble understanding the manual, see the appendix ***Common Digital Measurement Uncertainty Calculations*** in the back of the lab manual and ask for clarification/verification/help as needed. If you can't find the manual online after extensive searching, ask the instructor for suggestions.

(a.) DMM Model: Heathkit IM-1210

(b.) Resistance: ± 1.5% for all ranges (± 1 digit)

(c.) DC Voltage: ± 1% for all ranges (± 1 digit)

(d.) DC Current: ± 1.5% for full scale (± 1 digit)

Understanding Measurements

Problem 3.

Measure the resistance of each of your resistors using the DMM and record the value as well as the absolute uncertainty.

(a.) .34 MΩ @ $\pm$3 % = 340 $\pm$ 10 kΩ

(b.) 56 Ω @ $\pm$3 % = 56 $\pm$ 1 Ω

(c.) 1 Ω @ $\pm$3 % = 1 $\pm$ 0.03 Ω

(d.) 0.40 KΩ @ $\pm$3 % 400 $\pm$ 10 Ω

(e.) __________

Note: offset of +3 Ω found in the internal resistance and resistance of cables.

Problem 4.

Compare the resistance measurements you just made with the values and tolerances stated by the manufacturer from the beginning of this activity, by computing the discrepancy between your measured values (with uncertainties) and the manufacturer-reported values (with tolerances). Do the two values agree (i.e., is their discrepancy insignificant)? Show your work for each resistor!

> In other words, for each resistor, compare the manufacturer-reported resistance with its given tolerance as its uncertainty, $R_{\text{manufacturer}} \pm \delta R_{\text{manufacturer}}$, to the resistance and uncertainty measurement you made of that resistor with the DMM, $R_{\text{DMM}} \pm \delta R_{\text{DMM}}$, by computing their *discrepancy*,
>
> $$\Delta\left(R_{\text{manufacturer}}, R_{\text{DMM}}\right) = \left|R_{\text{manufacturer}} - R_{\text{DMM}}\right|,$$
>
> and determining whether the values agree or disagree. Recall that:
>
> $$\begin{cases} \text{Agreement:} & \Delta\left(R_{\text{manufacturer}}, R_{\text{DMM}}\right) \leq \delta R_{\text{manufacturer}} + \delta R_{\text{DMM}} \\ \text{Disagreement:} & \Delta\left(R_{\text{manufacturer}}, R_{\text{DMM}}\right) \geq \delta R_{\text{manufacturer}} + \delta R_{\text{DMM}} \end{cases}$$

(a.) $10000 \geq 6900$ (Disagrees)

(b.) $0 \leq 1.44$ (Agrees)

(c.) $0 \leq 6.11$ (Agrees)

(d.) $10 \geq 8.1$ (Disagrees)

(e.) ______

Problem 5.

Which is more precise, the factory tolerance or the DMM?

Factory tolerance

Problem 6. Series Resistors

Take two resistors and combine them in series using a breadboard.

(a.) Calculate the theoretical resistance of the circuit including the propagation of tolerances. Show your work!

In other words, use the manufacturer-reported resistances with their tolerances rewritten as absolute uncertainties,

$$R_1 \pm \delta R_1 \qquad \text{and} \qquad R_2 \pm \delta R_2,$$

to compute the equivalent series resistance,

$$R_{S,\ \text{Theoretical}} \pm \delta R_{S,\ \text{Theoretical}},$$

where $\delta R_{S,\ \text{Theoretical}}$ is determined via uncertainty propagation.

$R_{s,\text{Theoretical}} = R_1 + R_2$

$\delta R_{s,\text{Theoretical}} = \sqrt{\left(\frac{\partial R_{s,\text{Theoretical}}}{\partial R_1}\delta R_1\right)^2 + \left(\frac{\partial R_{s,\text{Theoretical}}}{\partial R_2}\delta R_2\right)^2}$

$= \sqrt{(.01)^2 + (.56)^2}$

$= 0.560089$

$R_{s,\text{Theoretical}} \pm \delta R_{s,\text{Theoretical}} = \boxed{57 \pm 1\,\Omega}$

(b.) Measure and record the resistance of the circuit (with uncertainty) using the DMM.

In other words, use the DMM to measure and report the resistance of the series combination with uncertainty:

$$R_{S,\ \text{DMM}} \pm \delta R_{S,\ \text{DMM}}.$$

Note: Internal Resistance is $5\,\Omega$

$R_{s,DMM} \pm \delta R_{s,DMM} \boxed{= 57 \pm 2\,\Omega}$

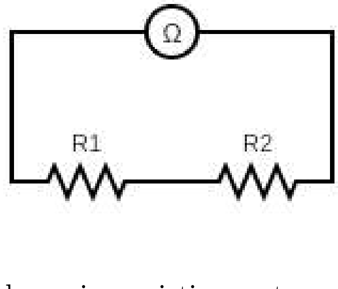


Figure 3.2: Measuring the series-resistive system with the DMM's ohmmeter.

(c.) Compare the calculated theoretical resistance of the circuit with what you measured, including uncertainties. Do they agree? Show your work!

In other words, compare the theoretical series resistance with uncertainty that you computed in Part (a.), $R_{S,\ \text{Theoretical}} \pm \delta R_{S,\ \text{Theoretical}}$, to the resistance and uncertainty measurement you made of the series resistive system with the DMM, $R_{S,\ \text{DMM}} \pm \delta R_{S,\ \text{DMM}}$, by computing their *discrepancy*,

$$\Delta\left(R_{S,\ \text{Theoretical}}, R_{S,\ \text{DMM}}\right).$$

Then determine from this computation whether the values *agree* or *disagree.*

$\Delta(R_{S,\text{Theoretical}}, R_{S,DMM}) \geq \text{or} \leq \delta R_{S,\text{Theoretical}} - \delta R_{S,DMM}$

$0 \leq 1$ $\boxed{\Rightarrow \text{Agree}}$

(d.) Combine the measured values and uncertainties of your resistors and compare them to your measurement of the circuit. Do they agree? Show your work!

In other words, for each resistor in your series resistive system, use their measured resistances with uncertainties that you found with the DMM in Problem 3,

$$R_{1,\ \text{DMM}} \pm \delta R_{1,\ \text{DMM}} \qquad \text{and} \qquad R_{2,\ \text{DMM}} \pm \delta R_{2,\ \text{DMM}},$$

to compute the equivalent series resistance with uncertainty,

$$R_S \pm \delta R_S,$$

where δR_S is determined via uncertainty propagation.

Then compare this computation, $R_S \pm \delta R_S$, to the measured series equivalent resistance with uncertainty you found with the DMM in Part (b.), $R_{S,\ \text{DMM}} \pm \delta R_{S,\ \text{DMM}}$, by computing their discrepancy. Determine whether the values *agree* or *disagree.*

$$R_1 = 1 \pm 0.03\ \Omega$$

$$R_2 = 56 \pm 2\ \Omega$$

$$\delta R_s = \sqrt{(0.03)^2 + (2)^2}$$

$$\boxed{\approx 2\ \Omega}$$

$$\delta R_s = \delta R_{s,DMM}$$

$$2\ \Omega = 2\ \Omega$$

$$\boxed{\Rightarrow \text{Agree}}$$

Problem 7. Parallel Resistors

Take two different resistors than you used in the previous problem and combine them in parallel using a breadboard.

(a.) Calculate the theoretical equivalent resistance of the circuit including the propagation of tolerances. Show your work!

In other words, use the manufacturer-reported resistances with their tolerances rewritten as absolute uncertainties,

$$R_1 \pm \delta R_1 \qquad \text{and} \qquad R_2 \pm \delta R_2,$$

to compute the equivalent parallel resistance,

$$R_{P,\ \text{Theoretical}} \pm \delta R_{P,\ \text{Theoretical}},$$

where $\delta R_{P,\ \text{Theoretical}}$ is determined via uncertainty propagation.

$$\frac{1}{R_p} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

$$= \left(\frac{1}{55} + \frac{1}{390}\right)^{-1}$$

$$= 48.97\,\Omega$$

$$\delta R_p = \sqrt{\left(\frac{\partial R_p}{\partial R_1}\delta R_1\right)^2 + \left(\frac{\partial R_p}{\partial R_2}\delta R_2\right)^2}$$

$$= \sqrt{\left(\left(\frac{1}{55}+\frac{1}{390}\right)^{-2}\left(\frac{1}{55}\right)^{2}(0.84)\right)^2 + \left(\left(\frac{1}{55}+\frac{1}{390}\right)^{-2}\left(\frac{1}{390}\right)^{2}(585)\right)^2}$$

$$= 0.5\ \Omega$$

$R_p \pm \delta R_p \boxed{= 48.97 \pm 1\,\Omega}$ (Theoretical)

(b.) Measure and record the resistance of the circuit (with uncertainty) using the DMM.

In other words, use the DMM to measure and report the resistance of the parallel combination with uncertainty:

$$R_{P,\ \text{DMM}} \pm \delta R_{P,\ \text{DMM}}.$$

Note: Internal resistance is $5\,\Omega$ $\quad R_{P,DMM} \pm \delta R_{P,DMM} \boxed{= 49 \pm 2\,\Omega}$

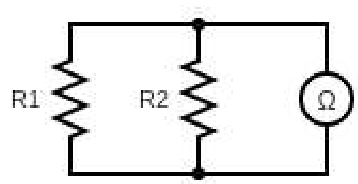


Figure 3.3: Measuring the parallel-resistive system with the DMM's ohmmeter.

(c.) Compare the calculated theoretical equivalent resistance of the circuit with what you measured, including uncertainties. Do they agree? Show your work!

In other words, compare the theoretical parallel resistance with uncertainty that you computed in Part (a.), $R_{P,\text{ Theoretical}} \pm \delta R_{P,\text{ Theoretical}}$, to the resistance and uncertainty measurement you made of the parallel resistive system with the DMM, $R_{P,\text{ DMM}} \pm \delta R_{P,\text{ DMM}}$, by computing their *discrepancy*,

$$\Delta\left(R_{P,\text{ Theoretical}}, R_{P,\text{ DMM}}\right).$$

Then determine from this computation whether the values *agree* or *disagree.*

$\Delta(R_P, R_{P,DMM}) \geq \text{or} \leq \delta R_P - \delta R_{P,DMM}$

$0 \leq 1 \quad \boxed{\Rightarrow \text{Agree}}$

(d.) Combine the measured values and uncertainties of your resistors and compare them to your measurement of the circuit. Do they agree? Show your work!

In other words, for each resistor in your parallel resistive system, use their measured resistances with uncertainties that you found with the DMM in Problem 3,

$$R_{1,\text{ DMM}} \pm \delta R_{1,\text{ DMM}} \qquad \text{and} \qquad R_{2,\text{ DMM}} \pm \delta R_{2,\text{ DMM}},$$

to compute the equivalent parallel resistance with uncertainty,

$$R_P \pm \delta R_P,$$

where δR_P is determined via uncertainty propagation.

Then compare this computation, $R_P \pm \delta R_P$, to the measured parallel equivalent resistance with uncertainty you found with the DMM in Part (b.), $R_{P,\text{ DMM}} \pm \delta R_{P,\text{ DMM}}$, by computing their discrepancy. Determine whether the values *agree* or *disagree.*

$$R_1 = 56 \pm 2 \ \Omega$$

$$R_2 = 400 \pm 10 \ \Omega$$

$$\delta R_p = \sqrt{\left(\left(\frac{1}{56}+\frac{1}{400}\right)^{-2}\left(\frac{1}{56}\right)^2 (2)\right)^2 + \left(\left(\frac{1}{56}+\frac{1}{400}\right)^{-2}\left(\frac{1}{400}\right)^2 (10)\right)^2}$$

$$\boxed{\approx 2\,\Omega}$$

$$\delta R_p = \delta R_{p,\mathrm{DMM}}$$

$$2\,\Omega = 2\,\Omega$$

$$\boxed{\Rightarrow \text{Agree}}$$

Problem 8.

Set the power supply to 5 V DC output using the built-in voltage meter. What is the estimated uncertainty of this meter? Go find the manual for the power supply and find within it the uncertainty associated with the digital voltmeter display on the power supply. This is the requested uncertainty. Write down the uncertainty formula from the power supply's manual and use it to report the voltage, with uncertainty, as read by the power supply's digital display.

Mastech GPS 1850D

Display: ±1% and ±1 digit

Measurement: 5 ± 0.05 V

Problem 9.

Measure the open-circuit voltage (nothing plugged into the output) of the power supply (i.e., the voltage across the power supply output terminals when nothing is plugged into the power supply output) and record this, including measurement uncertainty.

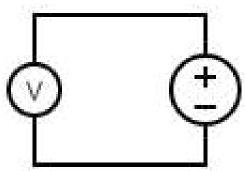


Figure 3.4: Measuring the open-circuit voltage of the power supply's voltage source with the DMM's voltmeter.

Open-Circuit Voltage: 4.8 ± 0.1 V

(a.) Compare the two measurements of the output voltage (i.e., compare the measurement you made in this problem to the measurement from the previous problem). Show your work!

$$\Delta(V_A, V_M) \geq \text{or} \leq \delta V_A - \delta V_M$$

$$0.2 \geq 0.05$$

$\boxed{\Rightarrow \text{Disagree}}$

(b.) Which meter is more precise? Explain.

The power supply since it has less internal resistance and smaller type B uncertainty

Problem 10.

Place each individual resistor in *series* with the power supply and the DMM. Record the current flowing through each resistor, including uncertainties. **Keep this information for a future activity.** (It's probably a good idea to record the information both here and elsewhere – like in your class notes – to ensure that you have it when you need it later. When we discuss plotting in Python, we'll be plotting this data.)

note internal resistance is 11 Ω

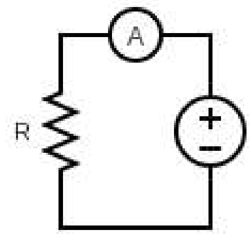


Figure 3.5: Measuring the current through the resistors connected to the power supply's voltage source with the DMM's ammeter.

(390Ω) (a.) 12 ± 0.4 mA

(56Ω) (b.) 56 ± 2 mA

(330kΩ) (c.) 0.02 ± 0.0006 mA

(33Ω) (d.) 101 ± 3 mA

(e.) ______