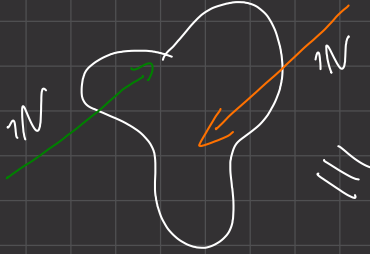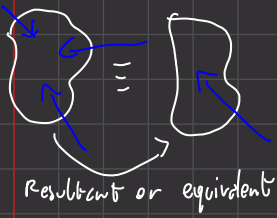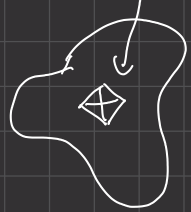26/04/23

There is no difference in the mechanical effect if we apply F along the line i

Resultant or equivalent

There is no force but there is moment

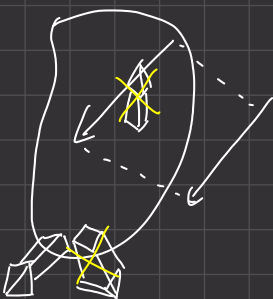| FORCE	MOMENT
↑ — side — ⬆	
⊙ — top — ◈	
⊗ — bottom — ▢	

F

F
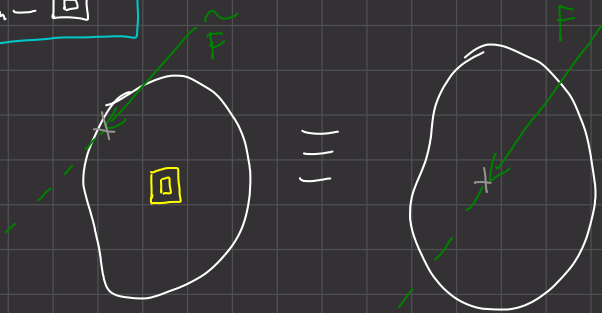
the **moment** does not allow the object to turn so the two systems are equal.
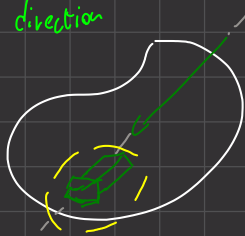
Let's see another example:
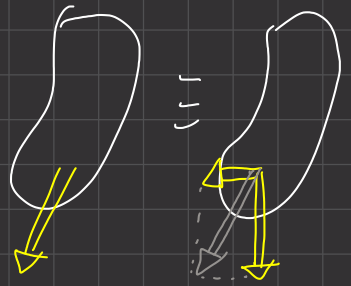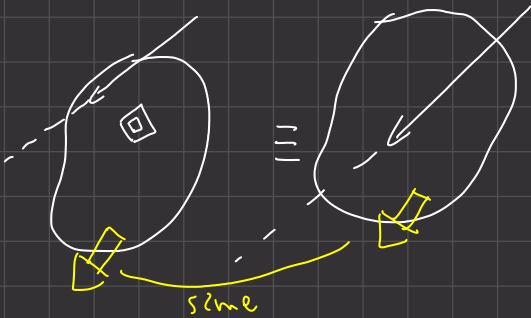
same

3D

Same value

It does not matter where you apply the torque.? BAH

We can Represent any system with ONE force and ONE
moment with the same direction

this is the canonical
← representation of
a class of equivalence

or

fundamental stat of statics

TODO check

The canonical representation is represented by ζ

$$\zeta = (\vec{f}, \vec{m}) \cong (\vec{F} | \vec{m})$$

$[N] \quad [Nm]$

N.B. this is invariant
with the reference
frame.

force   moment

this is also called WRENCH

You cannot change that moment with a displacement of the force. The moment must
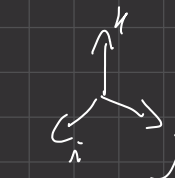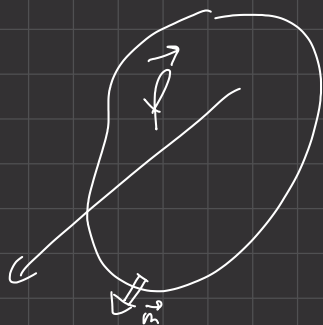be there to be in the class of equivalence
We also have:
   Special wrenches
   //   pure force $\varphi = (\vec{F} | \vec{m}) / \vec{m} \perp \vec{F}$ (When the moment is
                                                              or toyonal to the force)
   //   pure moment $\mu = (\vec{0} | \vec{m})$

$\vec{f} = (f_x, f_y, f_z)$ } this does
                            not codify
$\vec{m} = (m_x, m_y, f_z)$ where the
                            force is applied
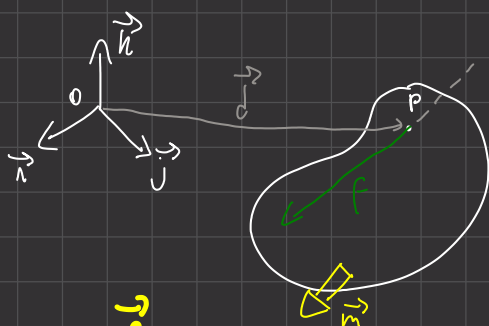
$\vec{d} = \vec{OP}$

$\vec{F} = (f_x, f_y, f_z)$

$\vec{m} = (m_x, m_y, m_z) + \vec{F} \times \vec{OP}$

$\vec{\varphi} = (\emptyset, -1, \emptyset | \emptyset, \emptyset, r)$

$\vec{\varphi} = (\emptyset, -1, \emptyset | \emptyset, \emptyset, \emptyset)$

Modulus of 1 N

φ $\odot$ β

2N

$$\varphi = (\emptyset, \emptyset, 2 \mid 2d \, c\beta, 2d \, s\beta, \emptyset)$$

2Nm

$$y = (3, \emptyset, \emptyset \mid (2, \emptyset, \emptyset) + (\emptyset, \emptyset, -3d)) =$$
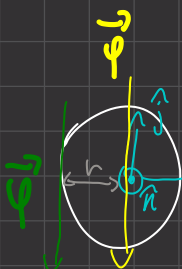
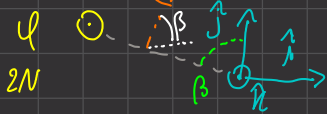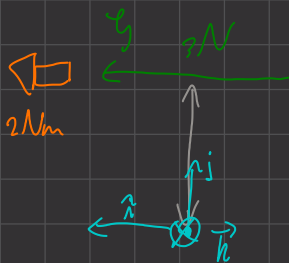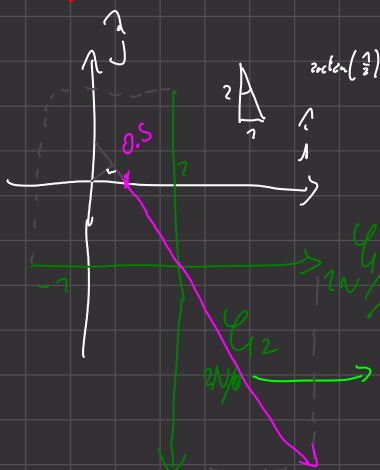$$= (3, \emptyset, \emptyset \mid 2, \emptyset, -3d)$$

Every kind of movement on a rigid body can be replaced with a wrench   N.B.

28/09/23

$\arctan\left(\frac{1}{2}\right)$

2
1

0.5

$$y_1 = (2, \emptyset, \emptyset \mid 0, 0, 7)$$ These are
$$y_2 = (\emptyset, -2, \emptyset \mid 0, 0, -2)$$ pure forces and can be summed

$$y_3 = (2, -2, \emptyset \mid 0, 0, \underline{d\sqrt{5}})$$
$$\sim 2$$

y_1
$2N/\emptyset$

y_2
$2N/\emptyset$
→ pitch N.B.

Another way of calculating y_3 is to sum

$y_1 + y_2$ and it will got the same result.

For a plane everything is good but in space lines could be skew. Summing up the components will also work in the space

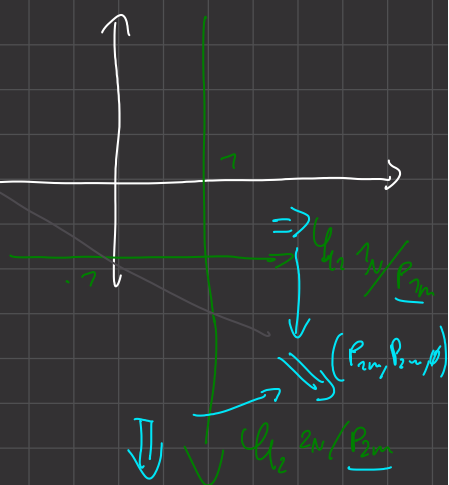Pitch is the amount of meters that a screw advances with a full turn.

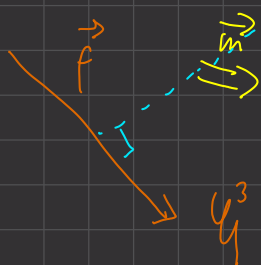let's now consider: $y_2 (2, \emptyset, \emptyset \mid P_1, \emptyset, 2)$

$$y_2 (\emptyset, -2, \emptyset \mid \emptyset, -2P_2, -2)$$

then $\quad \mathcal{G}_3 = (1,-2, \emptyset \mid p_2, -2p_2, -2)$

Same
thing

$\Rightarrow \mathcal{G}_1 \quad \frac{3N}{P_{2N}}$

$(P_{2N}, P_{2N}, \emptyset)$

$\mathcal{G}_2 \quad \frac{2N}{P_{2N}}$

Decomposing the moment
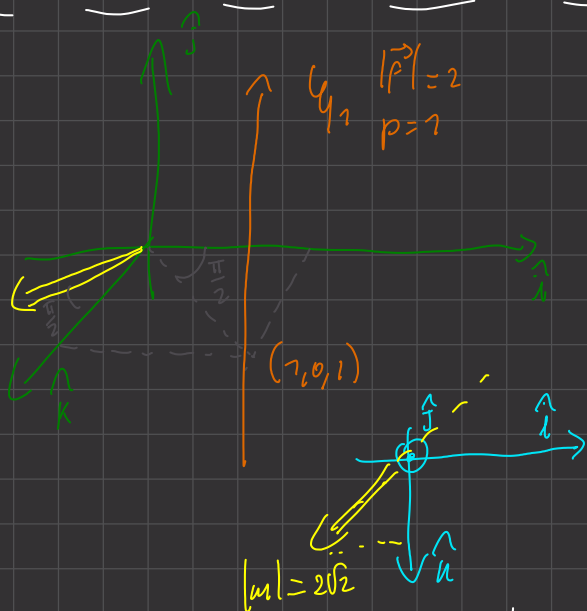
$\vec{f}$ $\vec{m}$

$\mathcal{G}^3$

The component that's on the direction of the force is:

$$\frac{\vec{f} \cdot \vec{m}}{|\vec{f}|}$$

So to get the moment that's orthogonal to the direction of the force you will do:

$$\vec{f} - \frac{\vec{f} \cdot \vec{m}}{|\vec{f}|}$$

$\mathcal{G} = (\vec{f} \mid \vec{m})$

$\rightsquigarrow \vec{f}_p + \vec{d} \times \vec{f}$

- - - - - - - - - - -

$\mathcal{G}_1 \quad |\vec{p}| = 2$
$\quad p = 1$

$(1,0,1)$

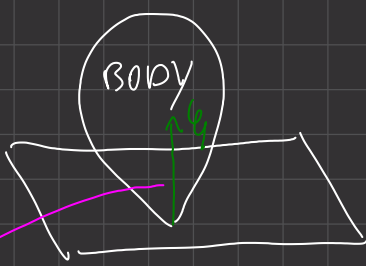$|m| = 2\sqrt{2}$

$\mathcal{G}_2 = (0, 2, 0 \mid -2, \emptyset, 2)$

$2 \cdot \frac{\sqrt{2} \cdot \sqrt{2}}{2}$

but we also have pitch = 1

$\mathcal{G}_2 = (0, 2, 0 \mid (-2, 0, 2) + \vec{f}_p)$

$(0,2,0) \cdot 1$

$\boxed{\mathcal{G}_2 = (0, 2, 0 \mid -2, 2, 2)}$

BODY

The body is constrained to stay in contact with the plane.
The plane is applying a force orthogonal to the direction of the plane. We cannot tell the intensity of the force.

Every interaction now should be described with a wrench.
We can immediately say that we have Ø pitch.

$$\mathcal{G} : \begin{cases} p = \emptyset \end{cases}$$

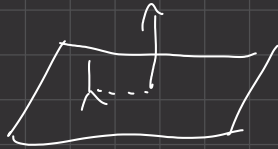Now we have an $\mathbb{R}^1$ subspace in $\mathbb{R}^6$ where there are many motions
so:
$$\mathcal{G} = (\emptyset, \emptyset, a \mid \emptyset, \emptyset, \emptyset)$$
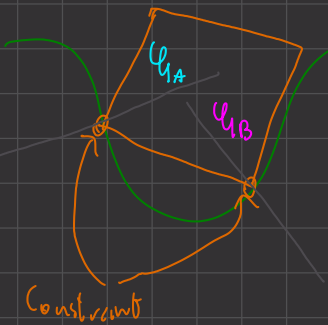
Base of the vector space

WRENCH SPACE = WRENCH SYSTEM

If the moment is orthogonal to the force then we have no spinning. It's only telling me where the reference frame is.

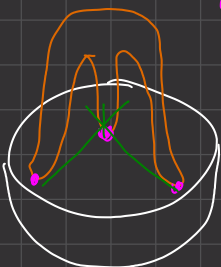$$\mathcal{G} = (\emptyset, \emptyset, a \mid b, c, \emptyset)$$

Linear example

$$\mathcal{G}_C = \lambda_1 \mathcal{G}_A + \lambda_2 \mathcal{G}_B$$

$\mathcal{G}_A$
$\mathcal{G}_B$

vector space of constraint

Constraint

Allowed motion is the one wich is orthogonal to the vector space.

Contact points to the bowl.

$$\mathcal{G}_A = (a, b, c \mid \emptyset, \emptyset, \emptyset)$$
$$\mathcal{G}_B = (d, e, f \mid \emptyset, \emptyset, \emptyset)$$
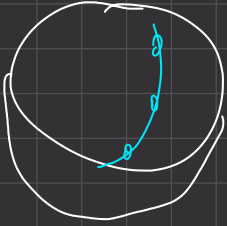$$\mathcal{G}_C = (g, h, i \mid \emptyset, \emptyset, \emptyset)$$

If we do a combination of these three we get:

$$\mathcal{G} = \lambda_A \mathcal{G}_A + \lambda_B \mathcal{G}_B + \lambda_C \mathcal{G}_C = (\sigma, p, q \mid \emptyset, \emptyset, \emptyset)$$
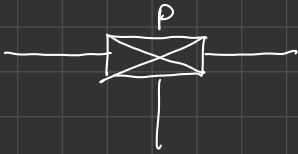
there is no possibility for this system (wich is a ball joint) to do any
translation because we have forces in any direction to block the movement.

If the three points of contact are on the same plane then they are
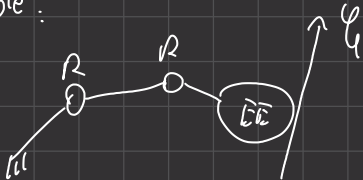linear dependent, so we don't have anymore a ball joint, because we have
rocking.

Constraint are ∅ pitch screws

When you apply a wrench you consider an external wrench to the body

For example :

In this case I am applying the wrench between the ee and the ground.
Everytime the wrench is applied between two bodies.

→ 𝒢 → this is applied between the base and the ee.

If we are now considering only static then now these wrenches are
only applying constraints. We need to act on body theory our constraints.

We want to understand how the prismatic joint constraint us. If we will
try to apply an ortogonal force to the direction of translation then if will be
rejected by another from the body.
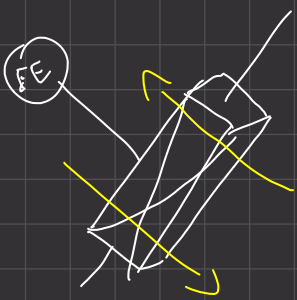
If you think of applying a force far away then you will have pure moment. So if the line is infinitely far away you get a infinitely strong moment. For this reason

We get infinite pitch.

If you fix the moment that you want and start applying the force further and further then the force will go to Ø when the distance will tend to infinity.

When we go really far the position of the force doesn't really change much.

The moment is always on the line between the force and Ø.

The point is that we should focus only on the moment and not on the force.

You can apply moments with any intensity but you cannot move the shuttle of the prismatic joint.
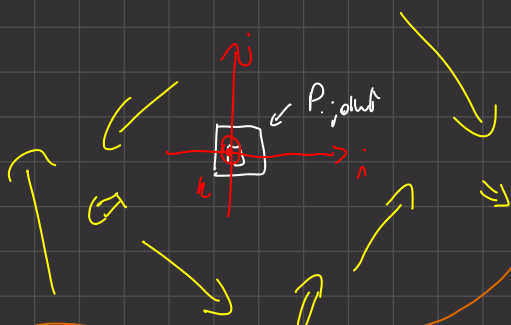
$$\mathcal{C}\left(a, b, \emptyset \,\middle|\, c, d, e\right)$$

this sphere represent the constraint the prismatic joint

It doesn't matter where you put the reference frame, the effect is the same.

N.B. In case of constraints we must consider $\mathcal{C}$ as the repelling force or moment, not the applying force.

→ The space in which this 5 dimensional wrench exists is:

-> $\mathbb{P}$ so we get: $\mathcal{C} \subset \mathbb{P}$ which is a 5-system.

Calligraphic P

If we want to find the base:

$$\mathbb{P} = \text{span}\left(\mathcal{C}_x, \mathcal{C}_y, M_x, M_y, M_z\right)$$

If we want to express all the vectors of the base of the constraint:

$\mathcal{C}_x = (1,0,0 \,|\, 0,0,0)$ $\mathcal{C}_y = (0,0,0 \,|\, 0,1,0)$

$\mathcal{C}_y = (0,1,0 \,|\, 0,0,0)$ $\mathcal{C}_z = (0,0,0 \,|\, 0,0,1)$

$\mathcal{C}_x = (0,0,0 \,|\, 1,0,0)$

It doesn't really matter the position of the reference frame but needs to be on the direction of the joint.

Revolut joint
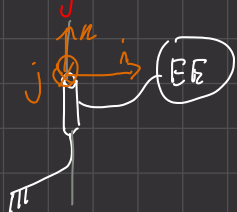
Applying these forces the door will not open.

The line in the revolut joint is called the INVARIANT of the joint.
Also the direction of the prismatic is called INVARIANT.

All the lines that intersect the invariant of the revolut do not move the hinge.

If we want to apply moments that are orthogonal to the direction of the hinge will not work.
Only moments that are parallel to the direction of the hinge or forces that do not intersect the hinge and they will move the revolut.

The best reference frame to take is along the line of the revolut and with one axes on the line.

For the screws we now need to make a distinction:
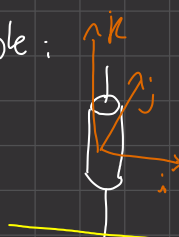the ones who are parallel to the direction of the joint:

$$\mathcal{Q}_{1} = (\emptyset, \emptyset, z \mid b, c, \emptyset)$$ ← this is a subset of the next one

And the ones that intersect the hinge of the door:

$$\mathcal{Q}_{2} = (d, e, f \mid l, m, \emptyset)$$

Note that the space is 5-dimensional, but it's not because we have 5 to values but because they are independent from each other

For example:

same direction of i
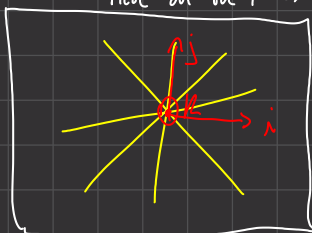
$$\mathcal{Q}(\eta, 0, 0 \mid 0, -\eta d, 0)$$

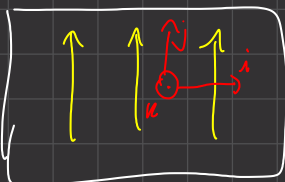Notice is not zd because every where you put the frame on the line of the joint does not matter.

Examples of systems dimension

Have all the forces directions go through the center.

$$\mathcal{G}(a, b, \phi \mid \phi, \phi, \phi)$$

this is a 2-system.

$$\mathcal{G} = (\phi, a, \phi \mid \phi, \phi, b)$$

2 - system.

N.B. look at the reference frame
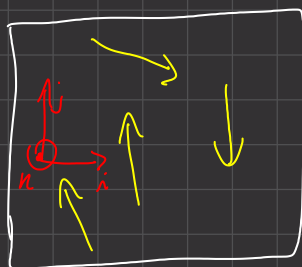
$$\mathcal{G} : (a, b, \phi \mid \phi, \phi, c)$$

3 - system

Spherical joint

$$\boxed{\text{AN INFINITE PITCH FORCE IS A MOMENT}}$$

E.E.
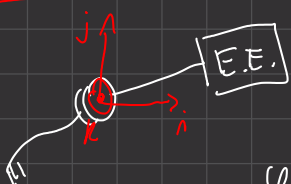
Any pure force going through the center will be rejected. So:

$$\mathcal{G} : (a, b, c \mid \phi, \phi, \phi)$$

3 - system

More than one joint in series

EE

You have to imagine:

Constraints of the revolute joint

$$\mathbb{R}^6$$

These are the constraints of the system

Constraints of the prismatic

R
$\mathbb{R}^5$
$\mathbb{R}^5$ P
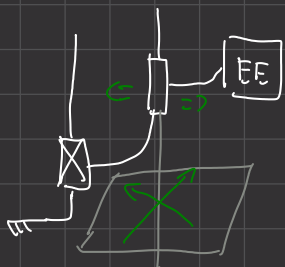
True by to analize the constraining forces we can see.
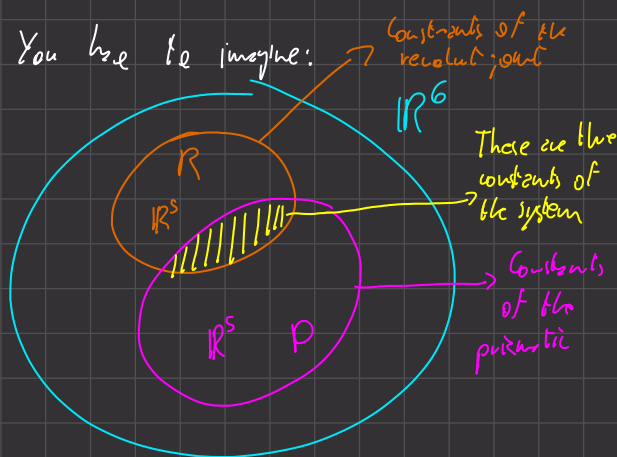
the structural constraint will be : $\mathcal{G} = (a, \phi, b \mid m, \phi, n)$

Planar mechanism is something that stays on a plane while moving. To do this you need constraints and so wrenches.
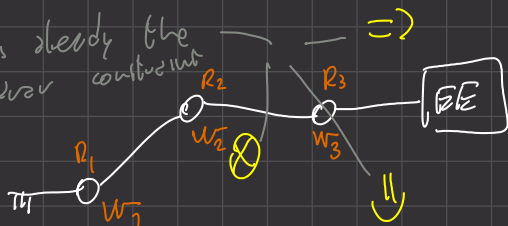
This constraint is called planar constraint.
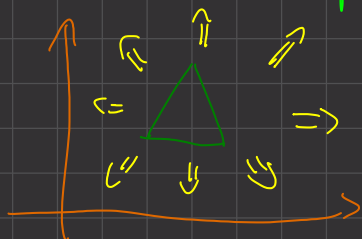
this moment span a 2 dimensian vector space.

We need to find the number of vectors that keep the object on the plane.

$$phi \to \varphi = (\phi, \phi, z \mid b, c, \phi) \to planar\ constraint$$

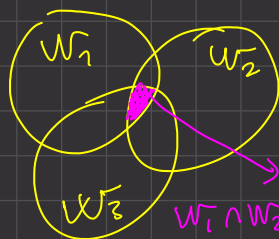It's already the planar constraint $\Rightarrow$

$$W_1 = (span\ (\varphi_1, \varphi_5) / \varphi_3\ not\ turning\ R_1)$$
S-system

$8|R|$
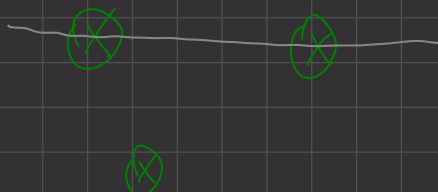
We have 3 inveraents

$W_1 \cap W_2 \cap W_3 \ T$

Planar constraint

We are in a planar constraint
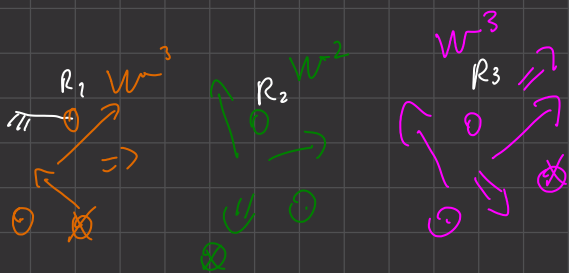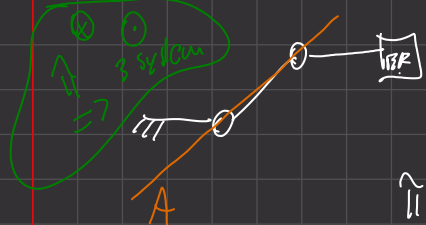

The common things are the forces on the plane

The vertical lines do not belong to the intersection because

Only two can be on the same plane.
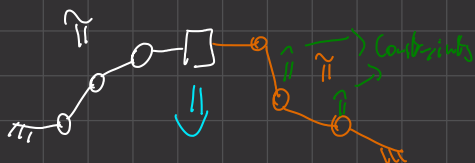
If we now consider

$2|R|$
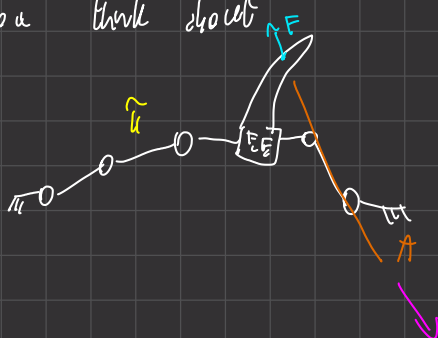
we also have / in the constraints so now it's a 4 system

$\hat{\Pi} \cap A = \emptyset \rightarrow \hat{\Pi} + A$ is a 4 system.

Let's now see this 2 (PPR) system → these two lines mean that what's inside has parallel invariant lines
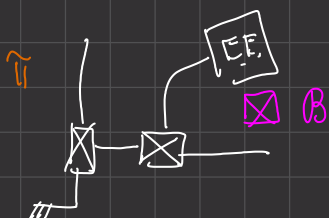

Constraints

$\hat{\Pi}$ and $\tilde{\Pi}$ are applied in parallel so at the end the system is behaving like there was only one $\hat{\Pi}$. So the constraint is the same but the forces are distributed in a different way.

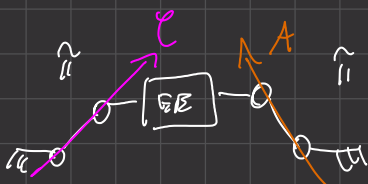Let's now think about

$\hat{u} \vee A$ is the constraint

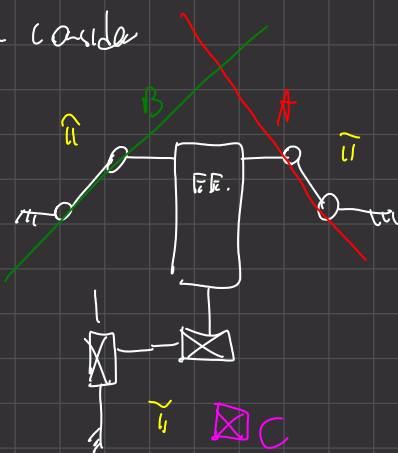the force will go to ground through A so the vector will be

---

$\hat{\Pi} + B$ is a 4-system.

E.E.

They don't need to be orthogonal but it's easier because the movements are decoupled

---
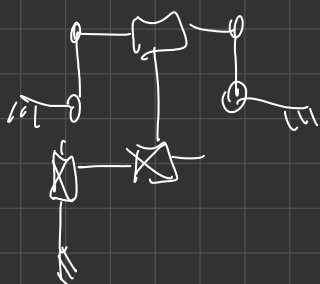
$\hat{\Pi} + A + C$ is a 5-system.

Let's now consider

$\hat{u}$ B   A $\widetilde{u}$

FF.

$\widetilde{u}$ C

$\hat{u} + A + B + C$ is a 6-system.

C cannot be produced by
A and B couple so it's another
constraint.
If they were parallel then this
would be 5-system.

---

if you consider

This is system and the
span of the intersection is C
so
$\hat{u} + C \wedge \hat{u} + A + B = span(c)$