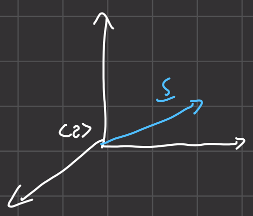
Quadratic programming for task priority

26/09/24

Time derivative of vectors

Time derivative of a projected vector on a given frame.



$${}^a\dot{s}(t) \triangleq \frac{d_a}{dt} s(t) \triangleq D_a\, s(t)$$

notations

The differentiation happens after the projection

$${}^a s(t) = \int_0^t {}^a\dot{s}(\tau)\, d\tau + {}^a s(0)$$

$${}^a s(t+dt) = {}^a s(t) + {}^a\dot{s}(t)\, dt + O(dt)$$

Higher order derivatives

Approximation of the state.

Note that you can't do the diff with an observer in b and then integrate on another frame.

Change of differentiation frame.

$${}^b\dot{s}(t) \neq {}^b_aR\; {}^a\dot{s}(t) \qquad \text{N.B.}$$

$${}^b\left({}^a\dot{s}(t)\right) = {}^b_aR\; {}^a\dot{s}(t)$$

The mother of all the formulas

$${}^a\dot{s}(t) = D\, {}^a s(t) = D\left({}^a_bR\; {}^b s(t)\right) =$$

$$= D\left({}^a_bR\right) {}^b s(t) + {}^a_bR\, D\left({}^b s(t)\right) =$$

$$= {}^a_b\dot{R}\; {}^b s(t) + {}^a_bR\; {}^b\dot{s}(t) =$$

$${}^a\dot{s}(t) = \begin{cases} {}^a_bR\; {}^b\dot{s}(t) + {}^a_bR\left[{}^b\omega_{b/a} \times\right] {}^b s(t) \\ {}^a_bR\; {}^b\dot{s}(t) + \left[{}^a\omega_{b/a} \times\right] {}^a_bR\; {}^b s(t) \end{cases}$$

Algebraic formula of diff frame

$$\boxed{D_a\, s = D_b\, s + \omega_{b/a} \times s}$$

Time derivative of constant module vectors

$|s| = \sigma > 0$ and $\dot{\sigma} = \emptyset$

We want to differentiate w.r.t. $\langle a \rangle$

Consider another frame $\langle h \rangle$ such that:

$$ {}^h\dot{s} = \emptyset $$

This means that I have attached a frame to the vector s which is rotating but not changing in modulus.

If now apply the mother of all the formulas:

$$ D_a s = \underbrace{\omega_{h/a}}_{} \times s \quad \forall \langle h \rangle \text{ s.t. } D_h s = \emptyset $$

let's express this like $\omega_{h/a}$ where

$$ \omega_{h/a} = \omega_{s/a} + \hat{n} z \quad \forall z \in \mathbb{R} $$

↳ unity vector of s

The minimum norm solution is:

$$ D_a s = \omega_{s/a} \times s $$

Note without nz which creates the span of other vectors.

admittance / impedance control

Time derivative of generic vector.

constant modulus vector

$s = n\sigma \qquad |\underline{n}| = 1 \qquad \sigma > \emptyset$

$$ D_a s = D_a(n\sigma) = \underbrace{n\dot{\sigma}}_{\text{aligned to } s} + \underbrace{\sigma D_a(n)}_{\text{ortogonal to } s} $$

$$ D_a \underline{s} = n \underbrace{(n\cdot)\, D_a s}_{\text{this scalar must be } \dot{\sigma}} + (\mathbb{1} - n(n\cdot))\, D_a s $$

↳ this states: $\sigma D_a n = (\mathbb{1} - n(n\cdot))\, D_a s$

$$ \begin{cases} \dot{\sigma} = n \cdot D_a s \\ \sigma D_a n = (\mathbb{1} - n(n\cdot))\, D_a s \end{cases} $$

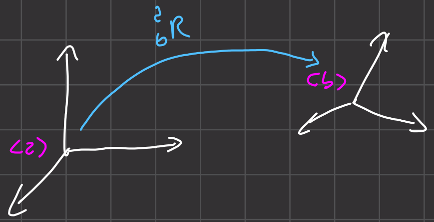
Recalling that:

$$D_a(n) = \omega_{b/a} \times n$$

then substituting

$$\boxed{D_a(\underline{s}) = n\dot{\sigma} + \sigma\,\omega_{b/a} \times \underline{n} = n\dot{\sigma} + \omega_{b/a} \times \underline{n}}$$

Derivative of rotation vector



$\rho \triangleq \rho_{b/a}$ $\qquad \rho \triangleq \underline{n}\,\theta$

$\omega_{b/a}$

$$D_a \rho = \underline{n}\dot{\theta} + \theta\, D_a \underline{n}$$

wrt <a> and not <b> because the rotation vector is the axis of rotation, so common

$$\boxed{{}^aD_a \underline{\rho} = {}^bD_b \underline{\rho}}$$

$$D_a \rho = [\overbrace{n(n\cdot)}^{\text{aligned}} + \overbrace{N_a(\theta)}^{\text{ortogonal}}]\,\omega_{b/a}$$

$$N_a(\theta) \triangleq \frac{\theta}{2}\left[\frac{1}{\tan(\frac{\theta}{2})} - [n\times]\right]\underline{[I - n(n\cdot)]}$$

↳ because of this necessarily ortogonal to n

$\dot{\rho} = -\lambda\rho$ → desired behaviour to reach the goal

ω has to be aligned with ρ why (125:38)

If you take an ω with elements not aligned with n you also make the rotation vector rotate.