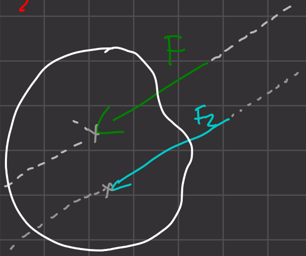
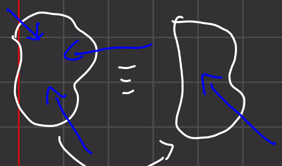
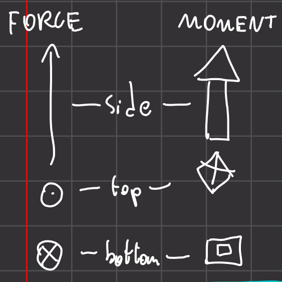
26/04/23

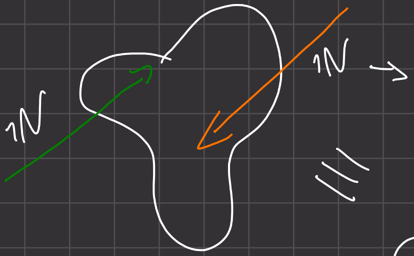


There is no difference in the mechanical effect if we apply F along the line:

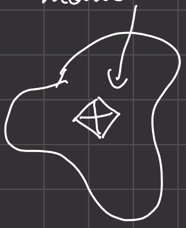


Resultant or equivalent



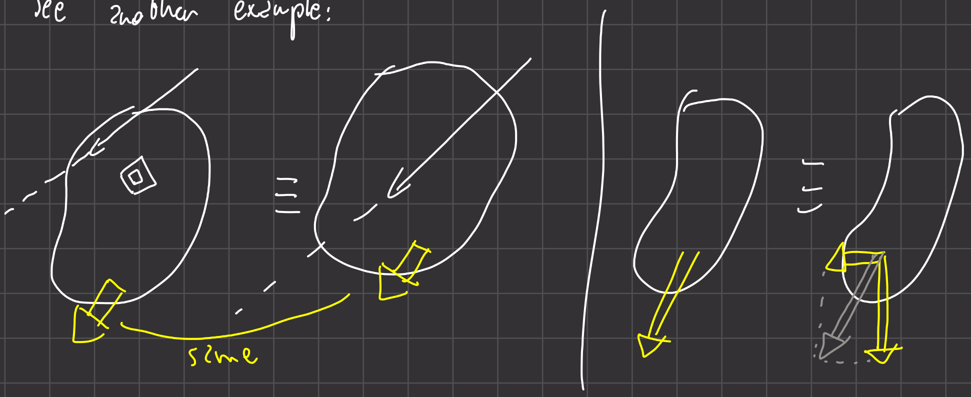


There is no force but there is moment

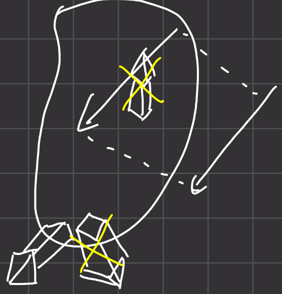


The moment does not allow the object to turn so the two systems are equal.

Let's see another example:



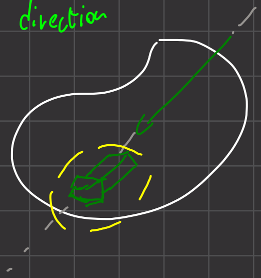
3D



Same value

It does not matter where you apply the torque. ? BAH

We can Represent any system with ONE force and ONE moment with the same direction



← this is the canonical representation of a class of equivalence
or
fundamental stat of statics

$\boxed{\text{TODO check}}$

The canonical representation is represented by $\mathcal{C}$

$$\mathcal{C} = (\underset{[N]}{\vec{f}}, \underset{[Nm]}{\vec{m}}) \equiv (\vec{F} | \vec{m})$$

force   moment

→ N.B. this is invariant with the reference frame.

This is also called WRENCH

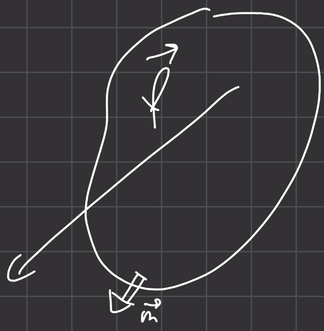
You cannot change that moment with a displacement of the force. The moment must be there to be in the class of equivalence

We also have:

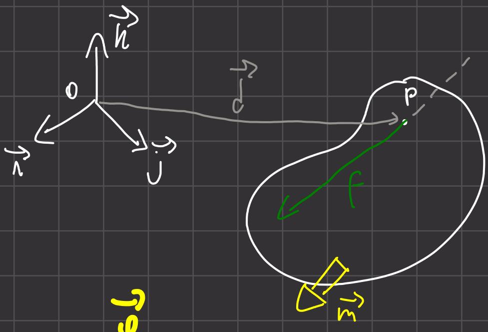
Special wrenches

" pure force $\varphi = (\vec{f} \mid \vec{m}) / \vec{m} \perp \vec{f}$ (When the moment is ortogonal to the force)

" pure moment $\mu = (\vec{0} \mid \vec{m})$



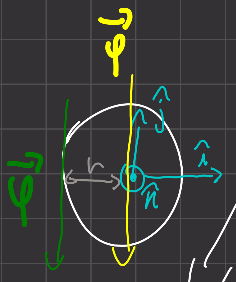
$\vec{f} = (f_x, f_y, f_z)$
$\vec{m} = (m_x, m_y, f_z)$ } this does not codify where the force is applied



$\vec{d} = \vec{OP}$

$\vec{F} = (F_x, F_y, F_z)$

$\vec{m} = (m_x, m_y, m_z) + \vec{F} \times \vec{OP}$

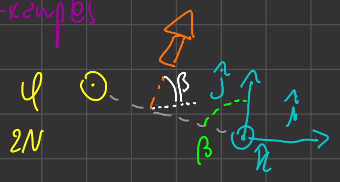


$\vec{\varphi} : (\phi, -1, \phi \mid \phi, \phi, r)$

$\vec{\varphi} : (\phi, -1, \phi \mid \phi, \phi, \phi)$

Modulus of 1N

Examples



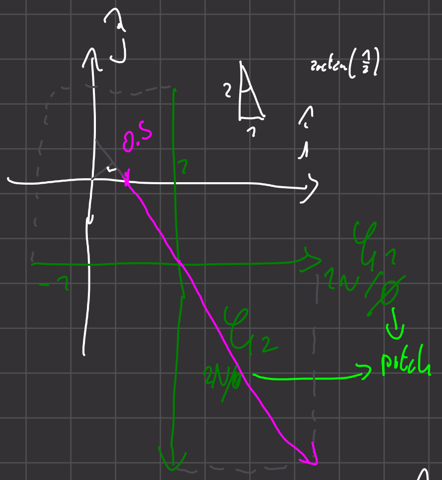
$$\psi = (0,0,2 \,|\, 2d\,c\beta,\, 2d\,s\beta,\, 0)$$

$$\psi = (3,0,0 \,|\, (2,0,0) + (0,0,-3d)) =$$

$$= (3,0,0 \,|\, 2,0,-3d)$$

Every kind of movement on a rigid body can be replaced with a wrench N.B.

28/09/23



$$\psi_1 = (1,0,0 \,|\, 0,0,1)$$
$$\psi_2 = (0,-2,0 \,|\, 0,0,-2)$$

these are pure forces and can be summed

$$\psi_3 = (1,-2,0 \,|\, 0,0,\underbrace{d\sqrt{5}}_{2})$$

N.B.

Another way of calculating ψ_3 is to sum $\psi_1 + \psi_2$ and it will get the same result.

For a plane everything is good but in space lines could be skew. Summing up the components will also work in the space

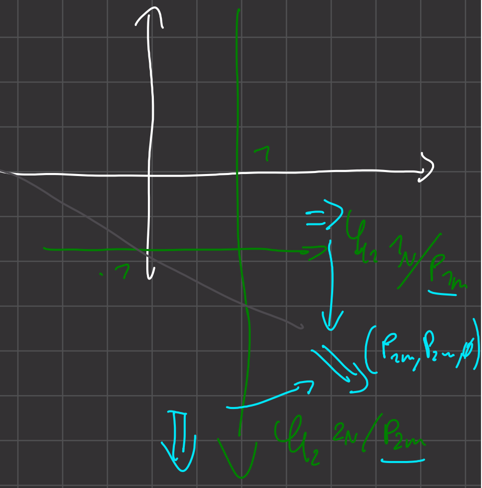
Pitch is the amount of meters that a screw advances with a full turn.

Let's now consider: $\psi_1(1,0,0 \,|\, P_1, 0, 1)$

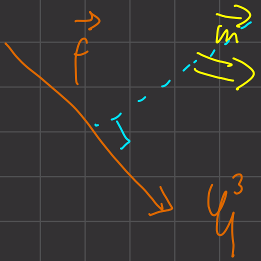
$\psi_2(0,-2,0 \,|\, 0, -2P_2, -2)$

Then $\psi_3 = (1,-2,\emptyset \mid p_{1},-2p_2,-1)$

Same thing



Decomposing the moment



The component that's on the direction of the force is:

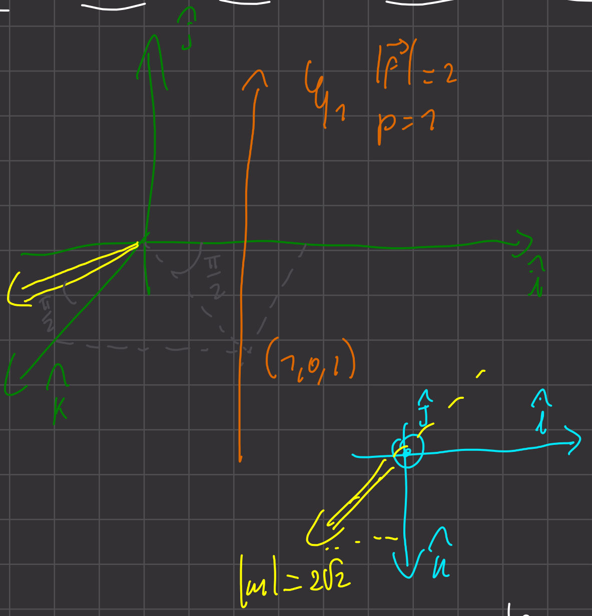
$$\frac{\vec f \cdot \vec m}{|\vec f|}$$

So to get the moment that's orthogonal to the direction of the force you will do:

$$\vec f - \frac{\vec f \cdot \vec m}{|\vec f|}$$

$\psi = (\vec f \mid \vec m)$

$\hookrightarrow \vec f p + \vec d \times \vec f$



$\psi_1 = (0,2,0 \mid -2,\emptyset,2)$

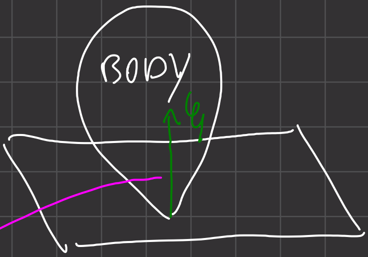
$2 \cdot \frac{\sqrt2 \cdot \sqrt2}{2}$

but we also have pitch: 1

$\psi_1 = (0,2,0 \mid (-2,0,2) + \vec f p)$

$(0,2,0)\cdot 1$

$$\boxed{\psi_2 = (0,2,0 \mid -2,2,2)}$$



The body is constrained to stay in contact with the plane.
the plane is applying a force orthogonal to the direction of the plane. We cannot tell the intensity of the force.

Every interaction now should be described with a wrench.
We can immediately say that we have ∅ pitch.

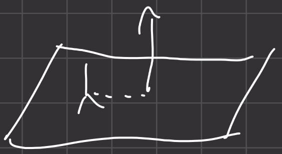
$$\psi : \left\{ p = \emptyset \right.$$

Now we have an $\mathbb{R}^1$ subspace in $\mathbb{R}^6$ where there are many intensities so:

$$\underbrace{\psi = (\emptyset, \emptyset, a \mid \emptyset, \emptyset, \emptyset)} \rightarrow \text{Base of the vector space}$$

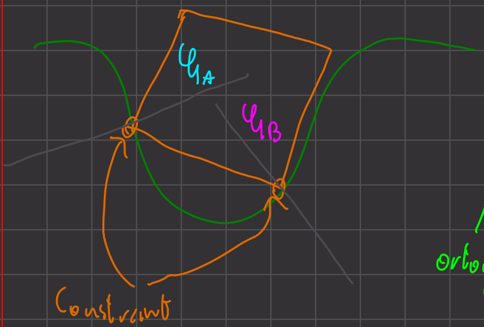
WRENCH SPACE = WRENCH SYSTEM

If the moment is orthogonal to the force then we have no spinning. It's only telling me where the reference frame is.



$$\psi = (\emptyset, \emptyset, a \mid b, c, \emptyset)$$

Chain example



$$\psi_C = \lambda_1 \psi_A + \lambda_2 \psi_B$$

↳ vector space of constraint

Allowed motion is the one wich is orthogonal to the vector space.

Contact points to the bowl.

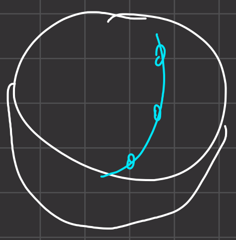
$$\psi_A = (a, b, c \mid 0, 0, 0)$$
$$\psi_B = (d, e, f \mid 0, 0, 0)$$
$$\psi_C = (g, h, i \mid 0, 0, 0)$$

If we do a combination of these three we get:

$$\psi = \lambda_A \psi_A + \lambda_B \psi_B + \lambda_C \psi_C = (o, p, q \mid \emptyset, \emptyset, \emptyset)$$

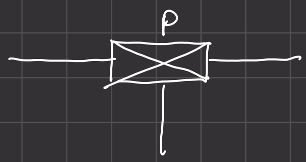
there is no possibility for this system (wich is a ball joint) to do any translation because we have forces in any direction to block the movement.

If the three point of contact are on the same plane then they are linear dependent, so we don't have anymore a ball joint, because we have rocking.



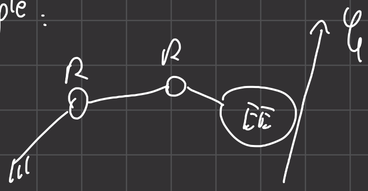
Constraint are ø pitch screws

03/20/23



When you apply a wrench you consider an external wrench to the body

For example:



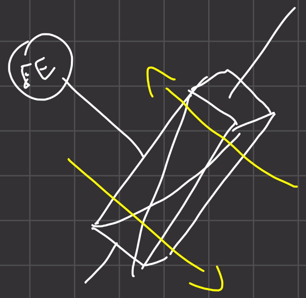
In this case I am applying the wrench between the ee and the ground. Everytime the wrench is applied between two bodies.

-> this is applied between the base and the ee.

If we are now considering only static then now these wrenches are only applying constraints. We need to act on body throug our constraints.

We want to understand how the prismatic joint constraint us. If we will try to apply an ortogonal force to the direction of translation then it will be rejected by another from the body.

Constraints of the prismatic joint



If you think of applying a force far away then you will have pure moment. So if the line is infinitely far away you get a infinitely strong moment. For this reason we get infinite pitch.

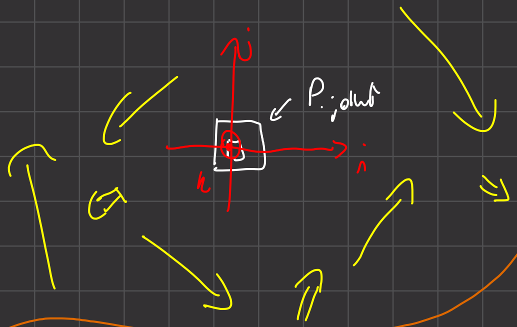
If you fix the moment that you want and start applying the force further and further then the force will go to $\emptyset$ when the distance will tend to infinity.

When we go really far the position of the force doesn't really change much.

The moment is always on the line between the force and $\emptyset$.

The point is that we should focus only on the moment and not on the force.

You can apply moments with any intensity but you cannot move the shuttle of the prismatic joint.



$\varphi(a, b, \emptyset \mid c, d, e)$

this shee represent the constraint on the prismatic joint

It doesn't matter where you put the reference frame, the effect is the same.

N.B. In case of constraints we must consider φ as the repelling force or moment, not the applying force.

→ the space in which this 5 dimensional wrench exists is:

→ $\mathbb{P}$ (calligraphic P) So we get: $\varphi \subset \mathbb{P}$ which is a 5-system.

If we want to find the base:

$$\mathbb{P} = span(\varphi_x, \varphi_y, \mu_x, \mu_y, \mu_z)$$

If we want to express all the vectors of the base of the constraint:

$\varphi_x = (1,0,0 \mid 0,0,0)$ $\varphi_y = (0,0,0 \mid 0,1,0)$

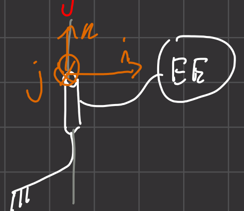
$\varphi_y = (0,1,0 \mid 0,0,0)$ $\varphi_z = (0,0,0 \mid 0,0,1)$

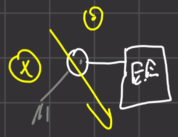
$\varphi_x = (0,0,0 \mid 1,0,0)$

Constraints of the prismatic joint contain all the forces ortogonal to the direction of the shuttle of the prismatic and all the moments.

It doesn't really matter the position of the reference frame but needs to be on the direction of the joint.

Revolut joint





Applying these forces the door will not open.

The line in the revolut joint is called the INVARIANT of the joint.
Also the direction of the prismatic is called INVARIANT.

All the lines that intersect the invariant of the revolut do not move the hinge.

If we want to apply moments that are ortogonal to the direction of the hinge will not work.
Only moments that are parallel to the direction of the hinge or forces that do not intersect the hinge and they will move the revolut.

The best reference frame to take is along the line of the revolut and with one axes on the line.

For the wrenches we now need to make a distinction:
the ones who are parallel to the direction of the joint:

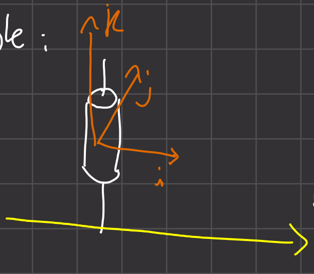
$$\varphi_1 = (\emptyset, \emptyset, a \mid b, c, \emptyset)$$ ← This is a subset of the next one

And the ones that intersect the hinge of the door:

$$\varphi_2 = (d, e, f \mid l, m, \emptyset)$$

Note that the space is 5-dimensional, but it's not because we have 5 to values but because they are indipendent from each other

For example:

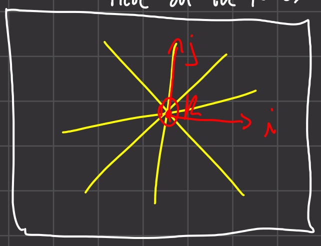


same direction of i

$$\varphi_1 (1, 0, 0 \mid 0, -1d, 0)$$

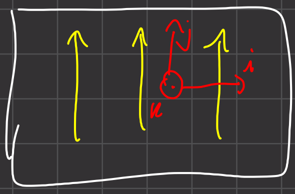
Examples of systems dimension

Here all the forces directions go through the center.



$\Psi(a,b,\phi \mid \phi,\phi,\phi)$

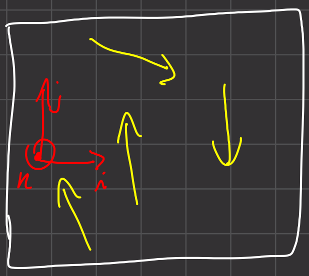
this is a 2-system.



$\Psi := (\phi, a, \phi \mid \phi, \phi, b)$

2 - system.

N.B. look at the reference frame

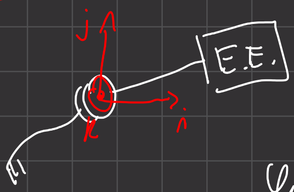


$\Psi : (a, b, \phi \mid \phi, \phi, c)$

3 - system

Spherical joint

AN INFINITE PITCH FORCE IS A MOMENT

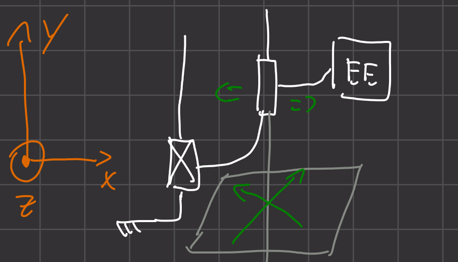


Any pure force going through the center will be rejected. So:

$\Psi : (a, b, c \mid \phi, \phi, \phi)$

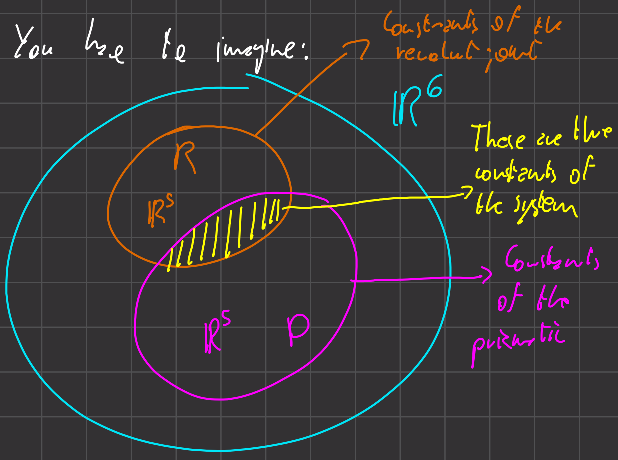
3 - system

More than one joint in series



I we try to analize the constraining forces we can see.

You have to imagine:

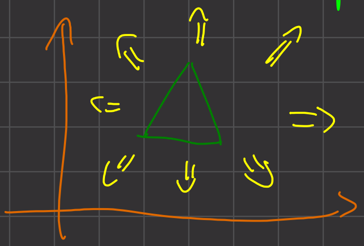


The structural constant will be: $\Psi := (a, \phi, b \mid m, \phi, n)$

05/20/23

Planar mechanism is something that stays on a plane while moving. To do this you need constraints and so wrenches.

This constraint is called planar constraint.



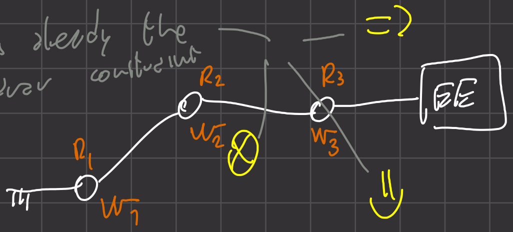
This moment span a 2 dimension vector space.

We need to find the number of vectors that keep the object on the plane.

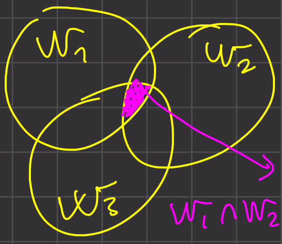
phi → $\varphi = (\phi, \phi, a \mid b, c, \phi)$ → planar constraint

$3|R|$

It's already the planar constraint



We have 3 invariants

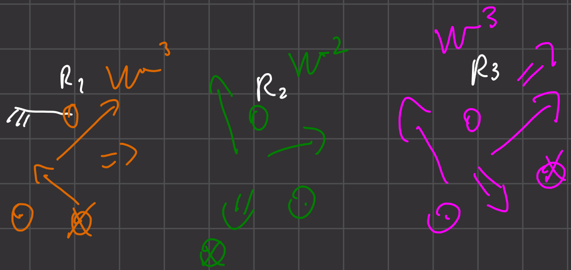


$W_1 \cap W_2 \cap W_3 = \Pi$

Planar constraint

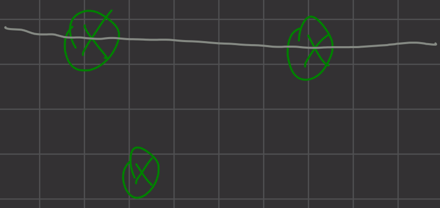
$W_1 = (\text{span}(\varphi_1, \varphi_5) / \varphi_3$ not turning $R_1)$
s-system

We are in a planar constraint



The common things are the forces on the plane

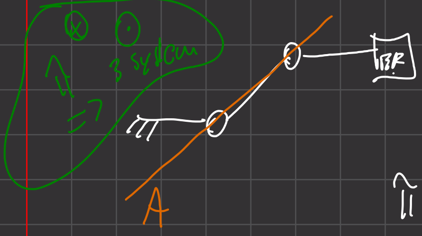
The vertical lines do not belong to the intersection because



Only two can be on the same plane.

If we now consider

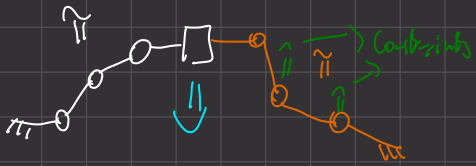
$2|R|$



we add a line in the constraints so now it's a 4 system

$\tilde{\Pi} \cap A = \emptyset \to \tilde{\Pi} + A$ is a 4 system.

Let's now see this 2(RRR) system → these two lines mean that what's inside has parallel invariant lines



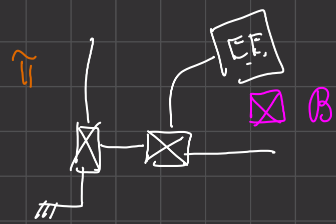
$\hat{\Pi}$ and $\tilde{\Pi}$ are applied in parallel so at the end the system is behaving like there was only one $\hat{\Pi}$. So the constraint is the same but the forces are distributed in a different way.

Let's now think about

$\tilde{\Pi} + A$ is the constraint

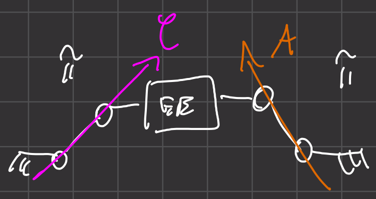
the force will go to ground through A so the reaction will be

$\tilde{\Pi} + B$ is a 4-system.

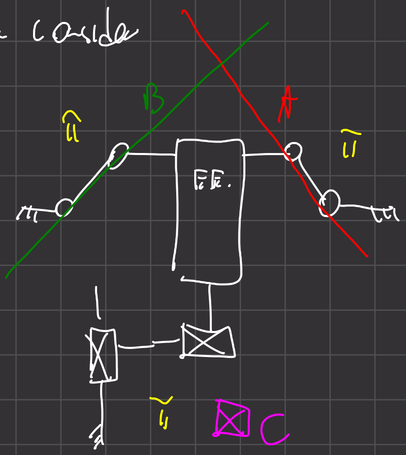


They don't need to be orthogonal but it's easier because the movements are decoupled

$\tilde{\Pi} + A + C$ is a 5-system.



Let's now consider

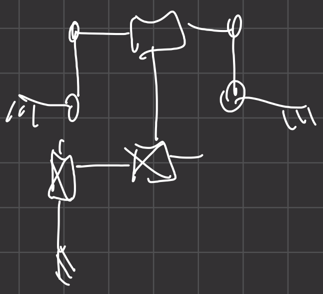


$\hat{\Pi} + A + B + C$ is a 6-system.

C cannot be produced by A and B couple to the revolute constraint.

If they were parallel then this would be 5-system.

if you consider

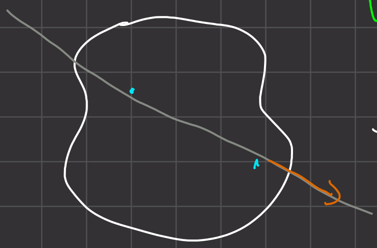


It's a 5 system and the span of the intersection is C

so

$\hat{\Pi} + C \cap \tilde{\Pi} + A + B = \text{span}(C)$

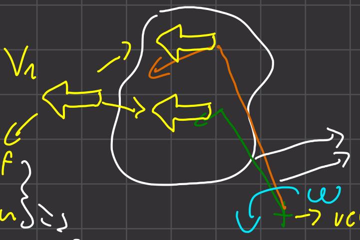
20/10/23



We are talking about instantaneous speeds

There can be only one speed in one direction in the body

Chasles realized that whatever is the distribution of velocities the body is spinning around a line in space and translating along the line.



velocity of translation $\Big\}$ ⇒ every point is moving in the same way in the same direction.

ω → revolute joint around which we have the velocity of rotation

Obviously the farther you are from the hinge the more speed you have.

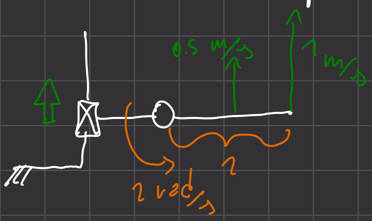
ω, angular velocity is the same.

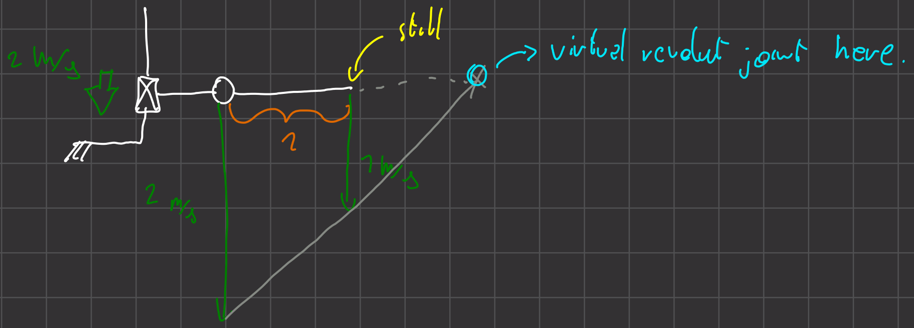
$V_1 = (\omega_1, v_1)$ is the combination of the two velocities.

Let's now consider another hinge.

I can have the same motion by rotating around another hinge and changing the translating direction I will get the same movement.

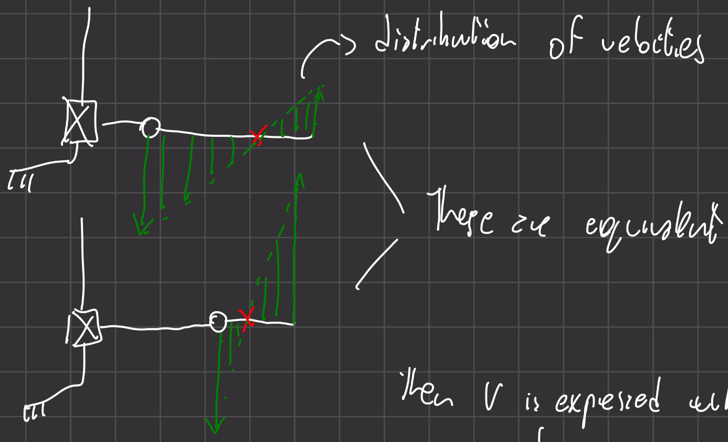
Let's do another example:

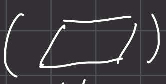




If now I move the r joint along the "arm" I can get different combinations of ω and v such that the point will have the same movement

$$(\omega_1, v_1) = (\omega_2, v_2) = (\omega_3, v_3) \rightarrow \text{class of equivalence}$$



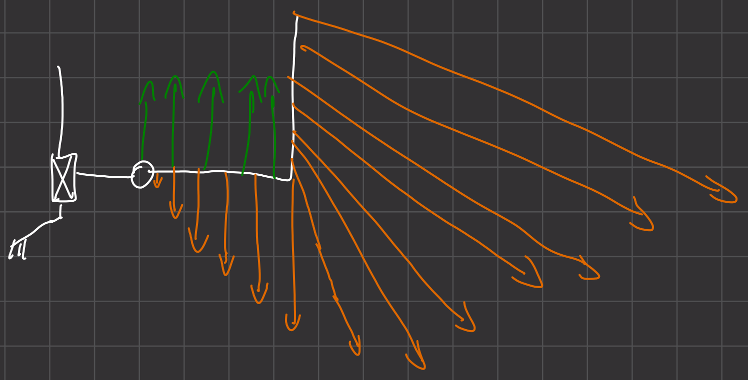


Then V is expressed with a parallelogram

We are building a class of equivalence → canonical representation

↳ twist: $\xi\,[x,]$

This is a representation of speed in the body and this distribution works for instantaneous movement.

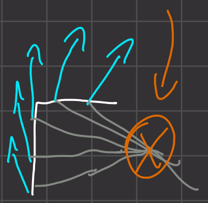


If you add the two then you get the actual speed.

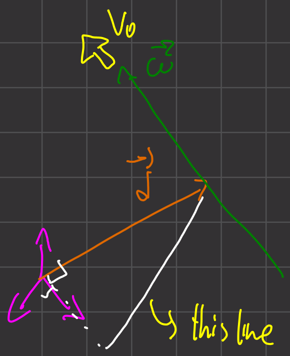
The point in which the orthogonals of the resultant of the speeds meet is called:

IAR (Istantaneous Axis of Rotation)

For example:



$\xi = (\vec{\omega} \,|\, \vec{v})$



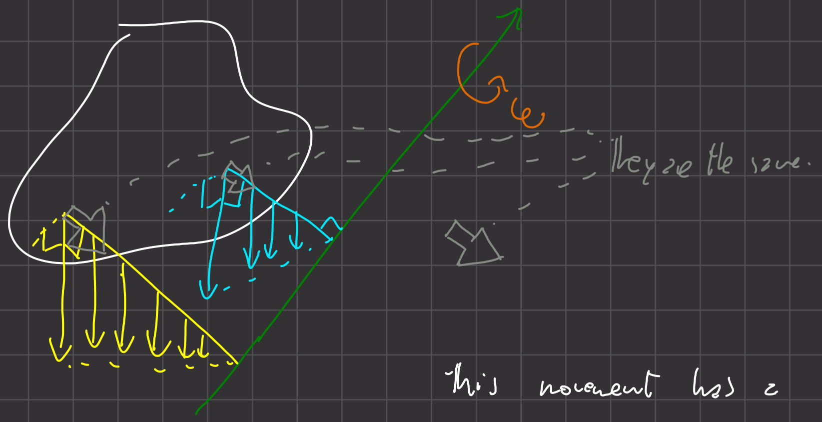
↳ this line is coming out of the screen. N.B.

$$V = V_0 + \vec{d} \times \vec{\omega}$$

$$V_0 = p\,|\omega|$$

↘ pitch

Speed is a free vector, the rotations are not. You can add translations with parallelogram but not the rotations.



this movement has a pitch.

Now we can pose the question: what is the velocity of a body?

Now we have tools to describe this movement.

Vectors in $\mathbb{R}^6$

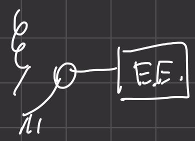
$$\zeta_\ell = (\vec{\ell} \,|\, \vec{m}) \qquad p = \frac{|m|}{|\ell|}$$

$$\xi = (\vec{\omega} \,|\, \vec{v}) \qquad p = \frac{|v|}{|\omega|}$$

line matters — just direction matters

NB

12/10/23



ξ $p=0$ / $\omega = 1 \to$

If you want to model the joint is convenient to give a twist with $\omega = 1$

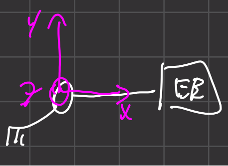
Now if we take a motor and attach it to the joint with 300RPM

then $\xi_m = \xi\, 300$

We are using a unit lenght vector like $(1,0,0)$ or $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$

So we are normalizing just the omega: $\xi(\vec{\omega} \,|\, \vec{v})$
↳ s.t. $|\vec{\omega}| = 1$

We use omega for direction and V for intensity.

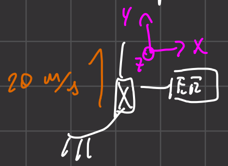


$$\xi = (\overbrace{0,0,1}^{\text{direction}} \,|\, \overbrace{0,0,0}^{\text{position of a point of line.}})$$

motorize the R joint
motor speed $= 200$ RPM

speed of the EE $= 200\ \xi \in \mathbb{R}^6$

Prismatic joint

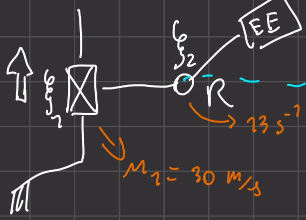


$\xi = (\vec{0} \mid \vec{v})$ $\quad p = \infty$

We prefer $|\vec{v}| = 1$

$\xi_m = 20\, \xi = 20(\vec{0} \mid \vec{v})$

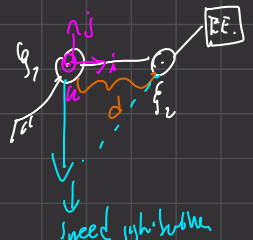
$\xi = 20(\emptyset, \emptyset, \emptyset \mid \emptyset, 1, \emptyset)$



Movement is linear in velocity

$$\boxed{\xi = 20\, \xi_1 + 23\, \xi_2}$$

IAR will be on this line



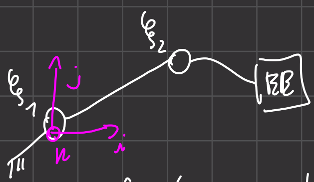
$\xi_{EE} = \omega_1 \xi_1 + \omega_2 \xi_2$

$\xi_1 = (0, 0, 1 \mid 0, 0, 0)$

$\xi_2 = (\emptyset, \emptyset, 1 \mid \emptyset, -d, \emptyset)$

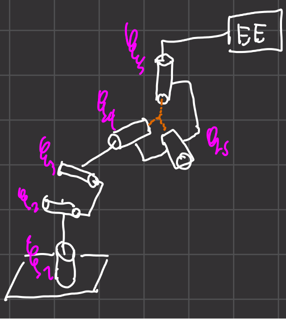
Forward Kinematic Problem

you get this value imagining to have r_2 fixed and moving a line above the origin of the axis.



$\xi = (0, 0, 1 \mid a, b, 0) \qquad d = \sqrt{a^2 + b^2}$

Let's now consider a 6R robot



$\xi_{EE} = 2\xi_1 - 5\xi_2 + 10\xi_3 + 12\xi_4 - 7\xi_5 + 3\xi_6$

MOTOR	Speed
1	2
2	-5
3	10
4	12
5	-7
6	3

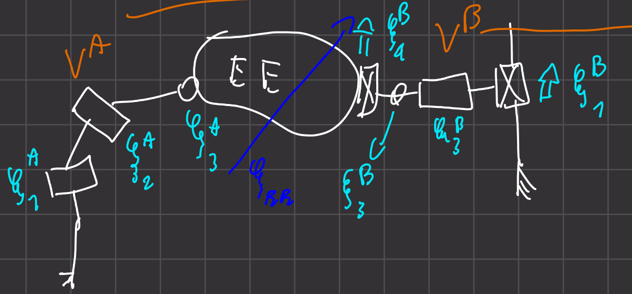
Geometric Jacobian

6 x 6 matrix

Joint velocities <-> E.E. velocities

$$\xi_{EE} = \underbrace{\begin{bmatrix} | & | & | & | & | & | \\ \xi_1 & \xi_2 & \xi_3 & \xi_4 & \xi_5 & \xi_6 \\ | & | & | & | & | & | \end{bmatrix}}_{\text{depends on configuration parameters only}} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \\ \omega_6 \end{bmatrix}$$

Structural constraints



Combined structural constraint

$$V_{EE} = V^A + V^B$$

We can imagine an imaginary kinematic chain (mechanism)

Let's now make an analogy with velocities and discuss what happens.

The mechanism is consistent between the two chains so:

$$\xi_{EE} = \omega_1^A \xi_1^A + \omega_2^A \xi_2^A + \omega_3^A + \xi_3^A = V_1^B \xi_1^B + \omega_2^B \xi_2^B + \omega_3^B \xi_3^B + V_4^B \xi_4^B$$

$\mathcal{M}_0^A = \text{span}(\xi_1^A, \xi_2^A, \xi_3^A)$ -> If they are linear independent then it's a 3 system.

The ξ_i create a span of the feasible movements.

Any feasible or allowable $\xi_{EE} \in \mathcal{M}_0^A$

Vector space in $\mathbb{R}^6$.

$\mathcal{M}_0^B = \text{span}(\xi_1^B, \xi_2^B, \xi_3^B, \xi_4^B)$ -> if they are linear independent then we have a 4- system.

So Any feasible $\xi_{EE} \in \mathcal{M}_0^B$

Concluding $\xi_{EE} \in \mathcal{M}_0^A \cap \mathcal{M}_0^B$