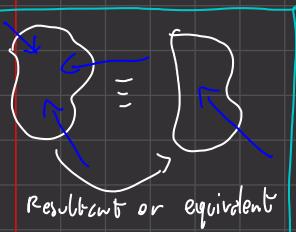


26/04/23

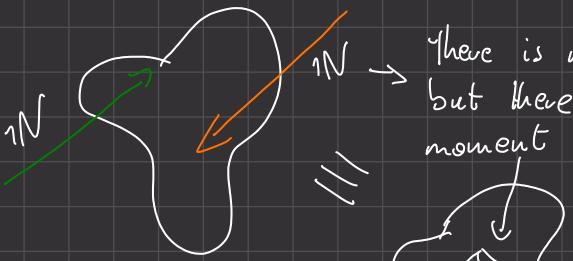


There is no difference in the mechanical effect if we apply P along the line:

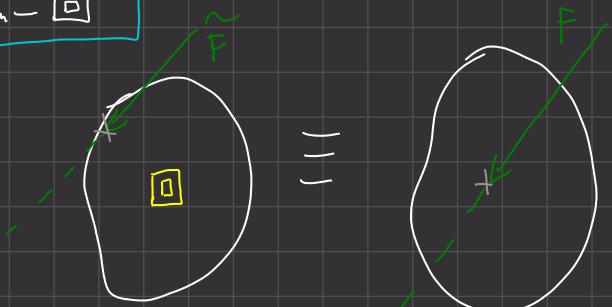


Resultant or equivalent

FORCE	MOMENT
— side —	↑
○ — top —	◇
⊗ — bottom —	□

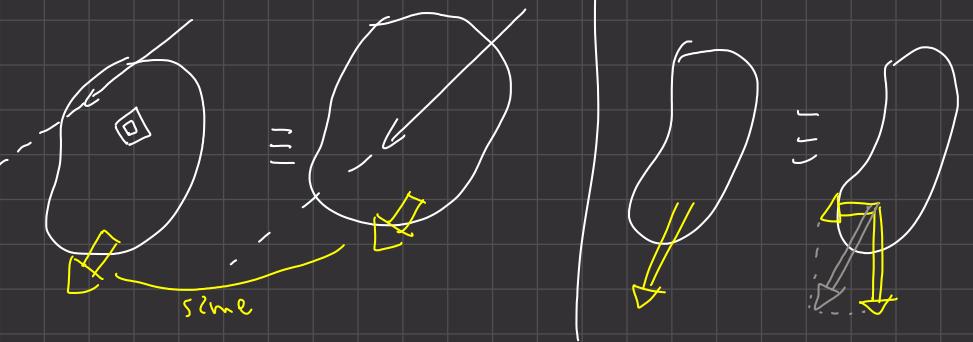


There is no force but there is moment

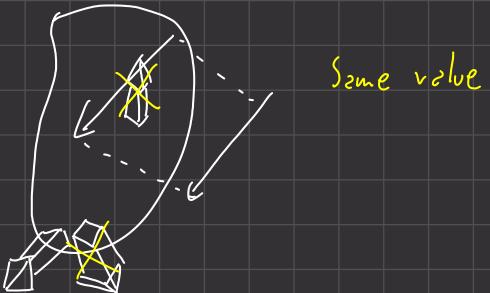


The moment does not allow the object to turn so the two systems are equal.

Let's see another example:

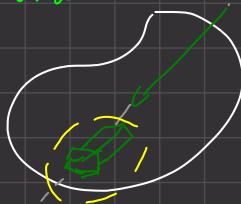


3D



It does not matter where you apply the torque. ? BAH

We can Represent any system with ONE force and ONE moment with the same direction



← this is the canonical representation of a class of equivalence
or fundamental stat of statics

TOPO dict

The canonical representation is represented by γ

$$\gamma = \begin{pmatrix} [N] \\ \vec{F}, \vec{m} \end{pmatrix} \equiv (\vec{F} | \vec{m})$$

↑ force ↑ moment

N.R. this is invariant with the reference frame.

This is also called WRENCH

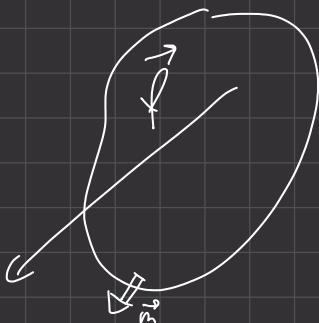
You cannot change that moment with a displacement of the force. The moment must be there to be in the class of equivalence

We also have:

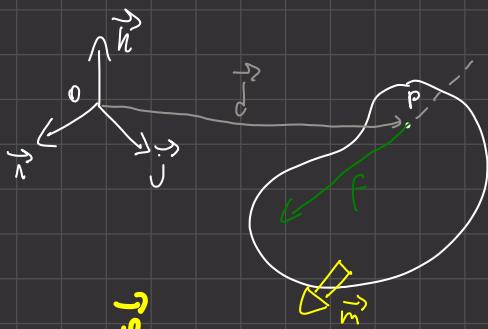
Special wrenches

// pure force $\varphi = (\vec{F} | \vec{m}) / \vec{m} \perp \vec{F}$ (When the moment is orthogonal to the force)

// pure moment $\mu = (\vec{o} | \vec{m})$



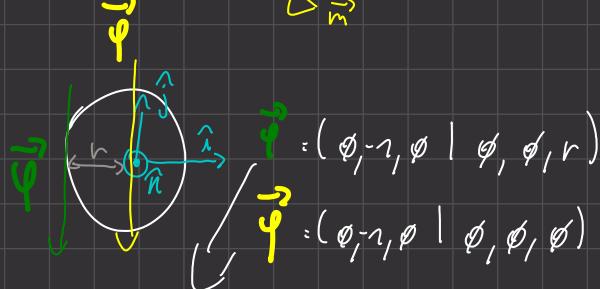
$$\begin{aligned} \vec{f} &= (f_x, f_y, f_z) \\ \vec{m} &= (m_x, m_y, m_z) \end{aligned} \quad \left. \begin{array}{l} \text{This does} \\ \text{not modify} \\ \text{where the} \\ \text{force is applied} \end{array} \right\}$$



$$\vec{d} = \vec{OP}$$

$$\vec{f} = (f_x, f_y, f_z)$$

$$\vec{m} = (m_x, m_y, m_z) + \vec{f} \times \vec{OP}$$

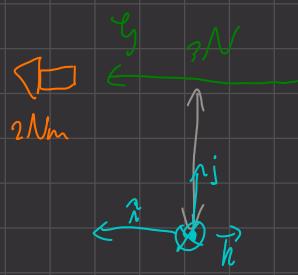


Modulus of γN

Example



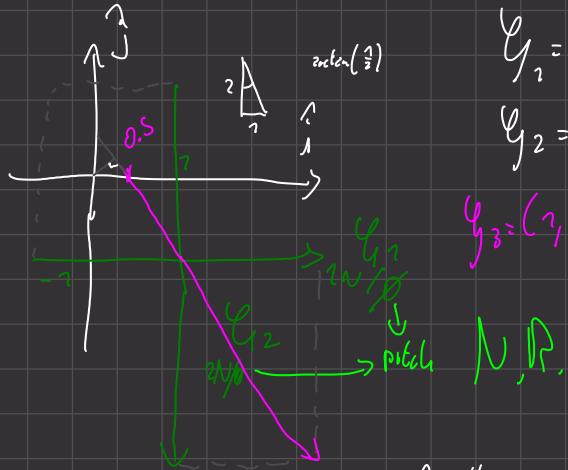
$$\Psi = (\phi, \psi, \theta | 2dc\beta, 2ds\beta, \phi)$$



$$\begin{aligned} \Psi &= (3, \phi, \psi | (2, \phi, \phi) + (\phi, \phi, -3d)) = \\ &= (3, \phi, \psi | 2, \phi, -3d) \end{aligned}$$

Every kind of movement on a rigid body can be replaced with 2 wrenches
N.B.

28/08/23



$$\begin{aligned} \Psi_1 &= (2, \phi, \psi | 0, 0, 1) \\ \Psi_2 &= (0, -2, \phi | 0, 0, -2) \\ \Psi_3 &= (1, -2, \phi | 0, 0, \underbrace{\text{d}\delta s}_{-1}) \end{aligned}$$

These are pure forces and can be summed

Another way of calculating Ψ_3 is to sum

$\Psi_1 + \Psi_2$ and it will get the same result.

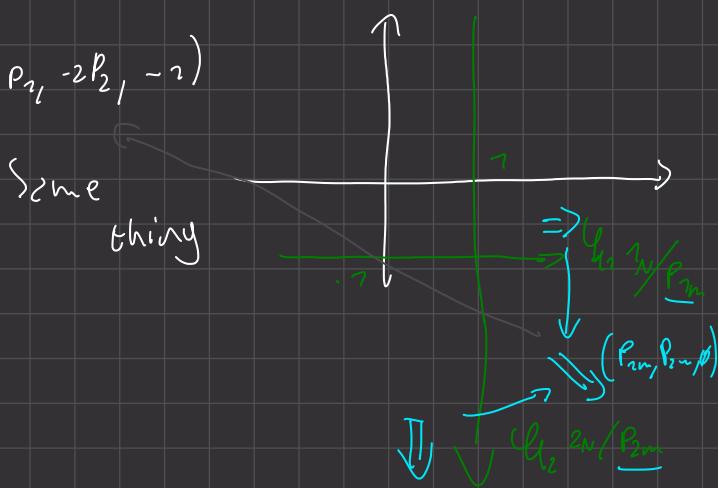
For a plane everything is good but in space lines could be skew. Summing up the components will also work in the space

Pitch is the amount of meters that a screw advances with a full turn.

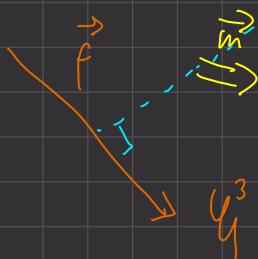
Let's now consider: $\Psi_1 (2, \phi, \psi | P_1, \phi, 1)$

$\Psi_2 (0, -2, \phi | \phi, -2P_2, -2)$

Then $\Psi_3 = (-1, -2, \emptyset | p_{-1}, -2p_2, -1)$



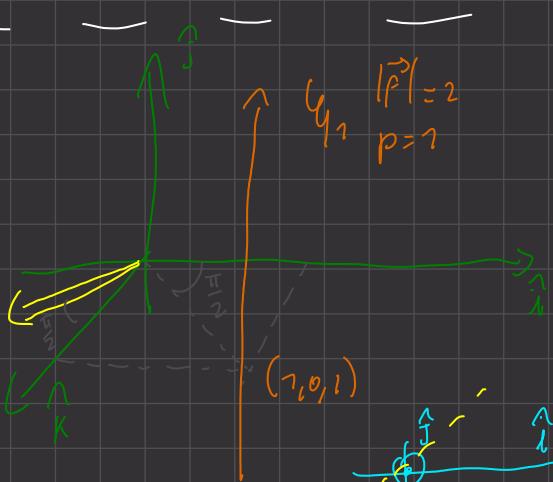
Decomposing the moment



The component along the direction of the force is: $\frac{\vec{f} \cdot \vec{m}}{|\vec{f}|}$

So to get the moment that's orthogonal to the direction of the force you will do:

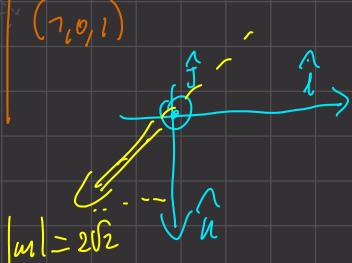
$$\Psi_3 = (\vec{f} \mid \vec{m}) \rightarrow \vec{f} \rho + \vec{d} \times \vec{f}$$



$$\Psi_1 = (0, 2, 0 | -2, \emptyset, \sqrt{2})$$

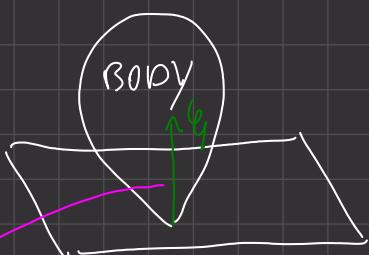
$$2 \cdot \frac{\sqrt{2} \cdot \sqrt{2}}{2}$$

but we also have pitch:



$$\Psi_1 = (0, 2, 0 | (-2, 0, 2) + \vec{f}_p) \\ (0, 2, 0)$$

$$\boxed{\Psi_1 = (0, 2, 0 | -2, 2, 2)}$$



The body is constrained to stay in contact with the plane. The plane is applying a force orthogonal to the direction of the plane. We cannot tell the intensity of the force.

Every interaction now should be described with a wrench. We can immediately say that we have 0 pitch.

$$\mathcal{W} : \left\{ \begin{array}{l} p = 0 \\ \end{array} \right.$$

Now we have an \mathbb{R}^3 subspace in \mathbb{R}^6 where there are many intensities so:

$$\mathcal{W} = (\phi, \theta, \varphi | \phi, \theta, \varphi)$$

Base of the vector space

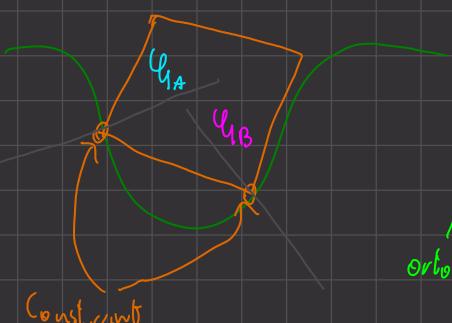
\mathcal{W} RENCH SPACE = WRENCH SYSTEM

If the moment is orthogonal to the force then we have no spinning. This only tells us where the reference frame is.



$$\mathcal{W} = (\phi, \theta, \varphi | b_1, c_1, \varphi)$$

Classic example

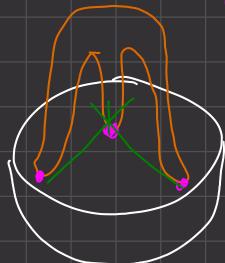


$$\mathcal{W}_C = \lambda_1 \mathcal{W}_A + \lambda_2 \mathcal{W}_B$$

Vector space of constant

Allowed motion is the one which is orthogonal to the vector space.

Contact points to the bowl.



$$\mathcal{W}_A = (a, b, c | 0, 0, 0)$$

$$\mathcal{W}_B = (d, e, f | 0, 0, 0)$$

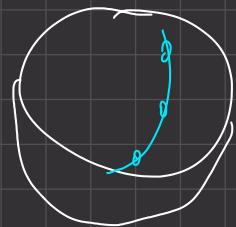
$$\mathcal{W}_C = (g, h, i | 0, 0, 0)$$

If we do a combination of these three we get:

$$\mathcal{W} = \lambda_A \mathcal{W}_A + \lambda_B \mathcal{W}_B + \lambda_C \mathcal{W}_C = (r, s, t | \phi, \theta, \varphi)$$

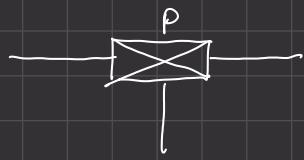
there is no possibility for this system (which is a ball joint) to do any translation because we have forces in any direction to block the movement.

If the three point of contact are on the same plane then they are linear dependent, so we don't have anymore a ball joint, because we have rocking.



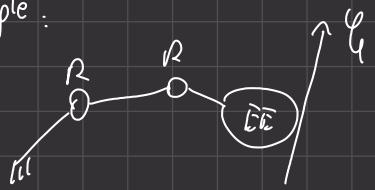
Constraint are of pitch screws

03/20/23

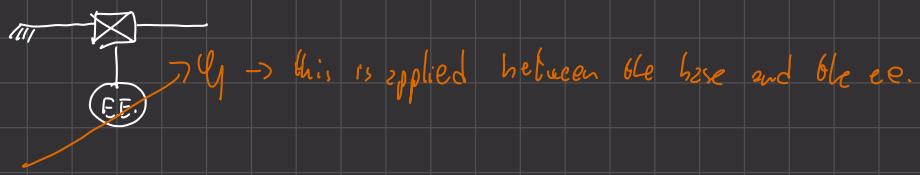


When you apply a wrench you consider an external wrench to the body

For example :



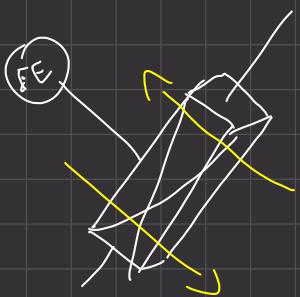
In this case I am applying the wrench between the ee and the ground.
Everytime the wrench is applied between two bodies.



If we are now considering only static then now these wrenches are only applying constraints. We need to set on body through our constraints.

We want to understand how the prismatic joint constraint is. If we will try to apply an orthogonal force to the direction of translation then it will be rejected by another from the body.

Constraints of the prismatic joint



If you think of applying a force for a very short time you will have pure moment. So if the line is infinitely far away you get an infinitely strong moment. For this reason we get infinite pitch.

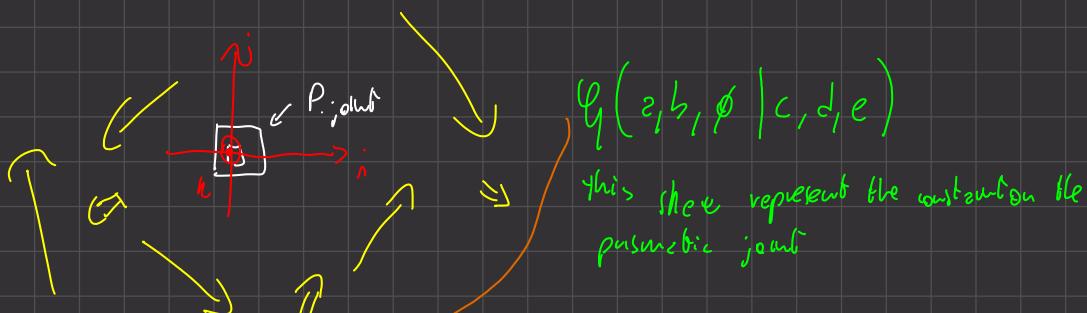
If you fix the moment that you want and start applying the force further and further then the force will go to 0 when the distance will tend to infinity.

When we go really far the position of the force doesn't really change much.

The moment is always on the line between the force and P.

The point is that we should focus only on the moment and not on the force.

You can apply moments with any intensity but you cannot move the sheaf of the prismatic joint.



It doesn't matter where you put the reference frame, the effect is the same.

N.R. In case of constraints we must consider φ as the repelling force or moment, not the applying force.

→ the space in which this 5-dimensional wrench exists is:

$\rightarrow \mathbb{P}$ So we get: $\varphi \subset \mathbb{P}$ which is a 5-system.
Geometric P

If we want to find the base:

$$\mathbb{P} = \text{span}(\varphi_x, \varphi_y, M_x, M_y, M_z)$$

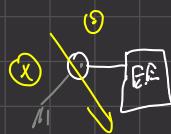
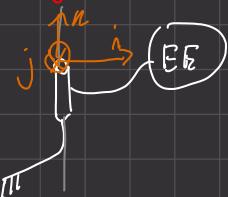
If we want to express all the vectors of the base of the constraint:

$$\begin{aligned}\varphi_x &= (1, 0, 0 | 0, 0, 0) & \varphi_y &= (0, 0, 0 | 0, 1, 0) \\ \varphi_y &= (0, 1, 0 | 0, 0, 0) & \varphi_2 &= (0, 0, 0 | 0, 0, 1) \\ \varphi_x &= (0, 0, 0 | 1, 0, 0)\end{aligned}$$

Constraints of the prismatic joint contain all the forces orthogonal to the direction of the shuttle of the prismatic and all the moments.

It doesn't really matter the position of the reference frame but needs to be on the direction of the joint.

Revolut joint



Applying here forces the door will not open.

The line in the revolut joint is called the invariant of the joint.
Also the direction of the prismatic is called invariant.

All the lines that intersect the invariant of the revolut do not move the hinge.

If we want to apply moments that are orthogonal to the direction of the hinge will not work.

Only moments that are parallel to the direction of the hinge or forces that do not intersect the hinge and they will move the revolut.

The best reference frame to take is along the line of the revolut and with one axes on the line.

For the wrenches we have need to make a distinction:
the ones who are parallel to the direction of the joint:

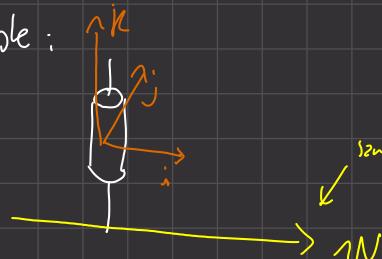
$$\Psi_1 = (\phi, \psi, \gamma | b, c, \theta) \quad \leftarrow \text{This is a subset of the next one}$$

And the ones that intersect the hinge of the door:

$$\Psi_2 = (d, e, f | l, m, \phi)$$

Note that the space is 5-dimensional, but it's not because we have 5 degrees of freedom but because they are independent from each other

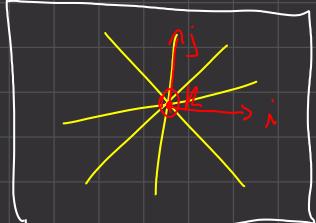
For example:



$$\Psi_1 (1, 0, 0 | 0, -\pi/2, 0)$$

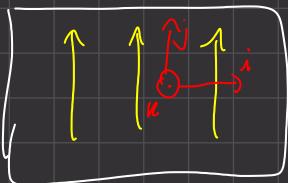
Examples of systems dimension

Have all the forces directions go through the center.



$$\mathcal{Q}_1 = (\alpha, b, \phi | \emptyset, \emptyset, \emptyset)$$

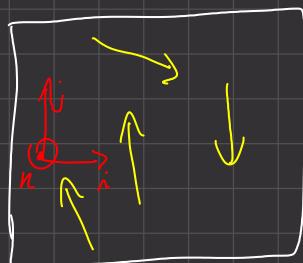
This is a 2-system.



$$\mathcal{Q}_1 = (\phi, \alpha, \beta | \emptyset, \emptyset, b)$$

2-system.

N.B. look at the reference

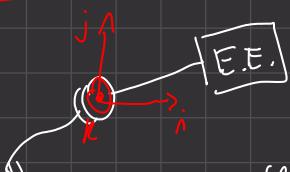


$$\mathcal{Q}_1 = (\alpha, b, \phi | \emptyset, \emptyset, c)$$

3-system

Spherical joint

AN INFINITE PITCH FORCE IS A MOMENT

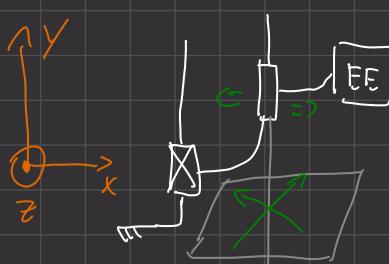


Any pure force going through the center will be rejected. So:

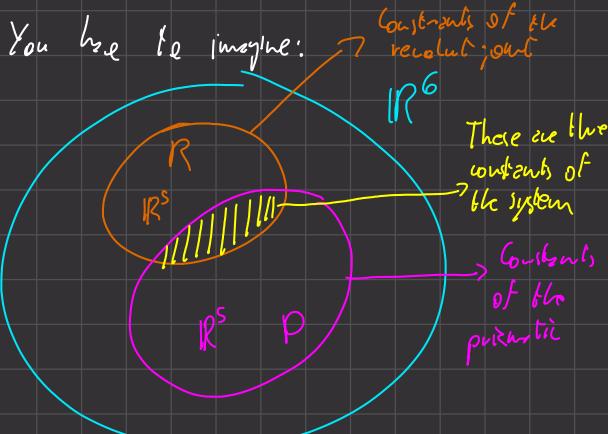
$$\mathcal{Q}_1 = (\alpha, b, c | \emptyset, \emptyset, \emptyset)$$

3-system

More than one joint in series



Try to visualize the constraint forces we can see.

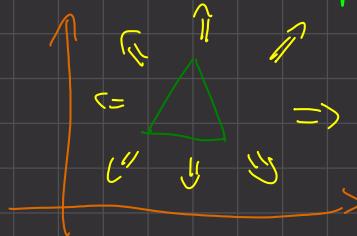


The structural constraint will be: $\mathcal{Q}_1 = (\alpha, \phi, b | m, \emptyset, n)$

OS/10/23

Planar mechanism is something that stays on a plane while moving. To do this you need constraints and so wrenches.

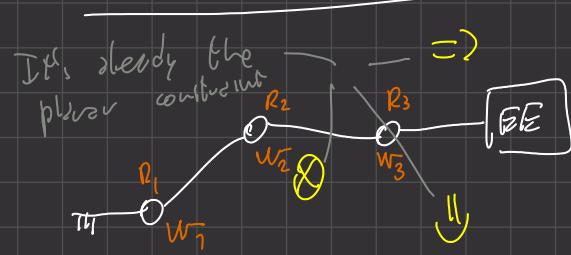
This constraint is called planar constraint.



this moment spans a 2 dimension vector space.

We need to find the number of vectors that keep the object on the plane.

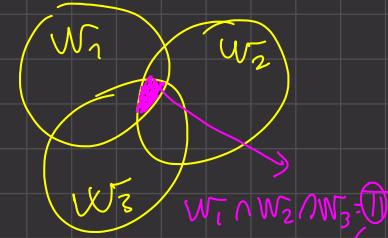
$$\phi \rightarrow \Psi = (\phi, \theta, z | b, c, \rho) \rightarrow \text{planar constraint}$$



$$3|R|$$

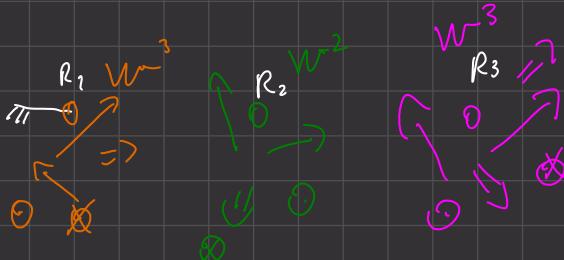
$$W_1 = \begin{cases} \text{span } (q_1, q_3) & (q_3 \text{ not curving } R_1) \\ \text{system} \end{cases}$$

We have 3 invariants



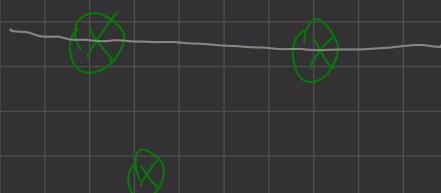
Planar constraint

We are in a planar constraint



The wrenches and the forces are by plane

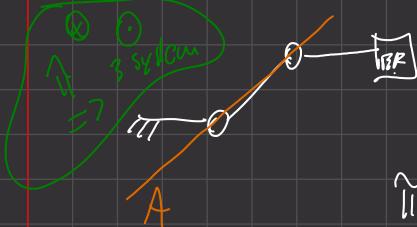
The vertical lines do not belong to the invariance because



Only two can be on the same place.

If we now consider

$$2|R|$$



we add here / in the constraints so now it's a system

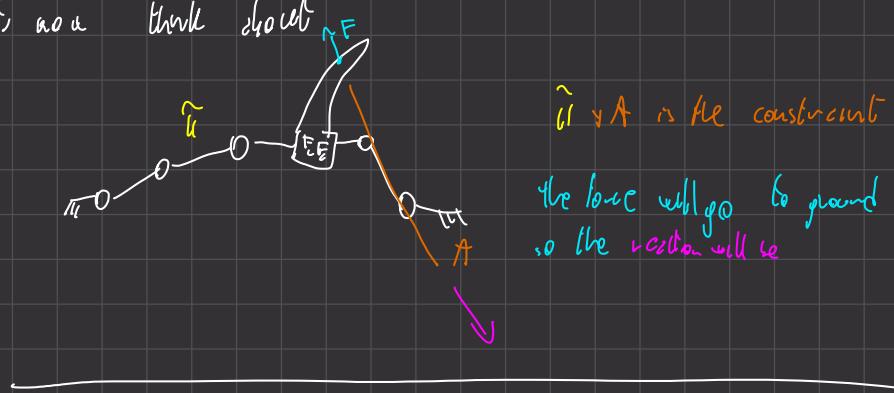
$$\tilde{\pi} \cap A = \emptyset \rightarrow \tilde{\pi} + A \text{ is a 2-q system.}$$

Let's now see this 2D parallel system 

These two (we mean both what's inside has parallel invariant lines)

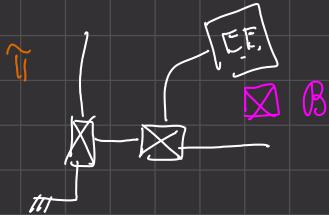
$\tilde{\pi}$ and $\tilde{\pi}$ are applied in parallel so at the end the system is behaving like there was only one $\tilde{\pi}$. So the constraint is the same but the forces are distributed in a different way.

Let's now think about



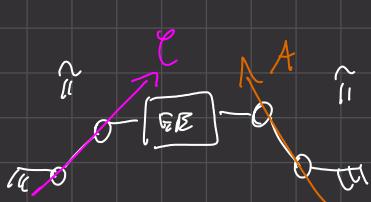
$\tilde{\pi} + A$ is the constraint

the force will go to ground through $\tilde{\pi}$ so the reaction will be



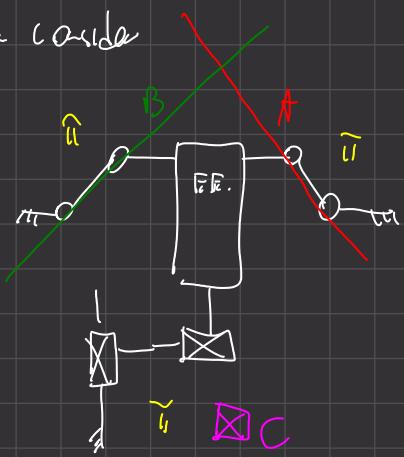
$\tilde{\pi} + B$ is a q-system.

They don't need to be orthogonal but it's easier because the moment are decoupled



$\tilde{\pi} + A + B$ is a S-system.

If you consider

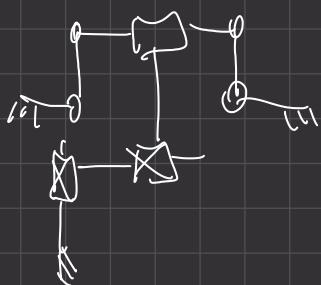


$\vec{F}_1 + \vec{F}_2 + \vec{F}_3$ is a 6-system.

C cannot be produced by A and B couple so it's another constraint.

If they were parallel then this would be a 5-system.

If you consider



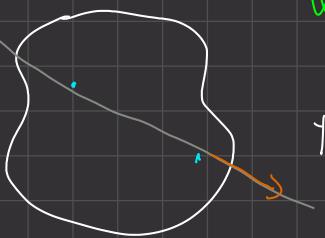
This is a system and the span of the intersection is C

so

$$\vec{F}_1 + \vec{C} \wedge \vec{F}_1 + \vec{A} + \vec{B} = \text{span}(C)$$

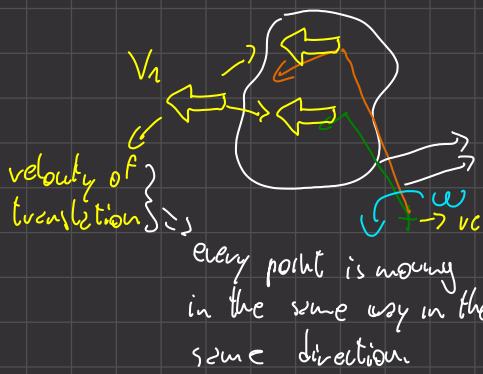
20/10/23

We are talking about instantaneous speeds



There can be only one speed in one direction in the body

Chasles realized that whatever is the distribution of velocities the body is spinning around a line in space and translating along the line.



every point is moving in the same direction.

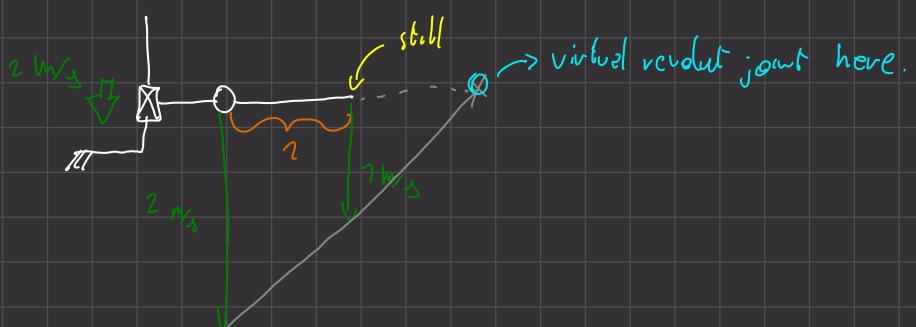
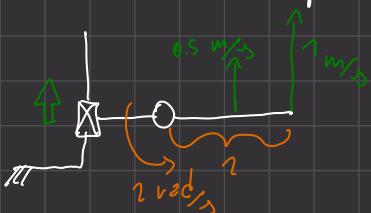
Obviously the further you are from the hinge the more speed you have. ω , angular velocity is the same.

$V_1 = (\omega_1, v_1)$ is the combination of the two velocities.

Let's now consider another hinge.

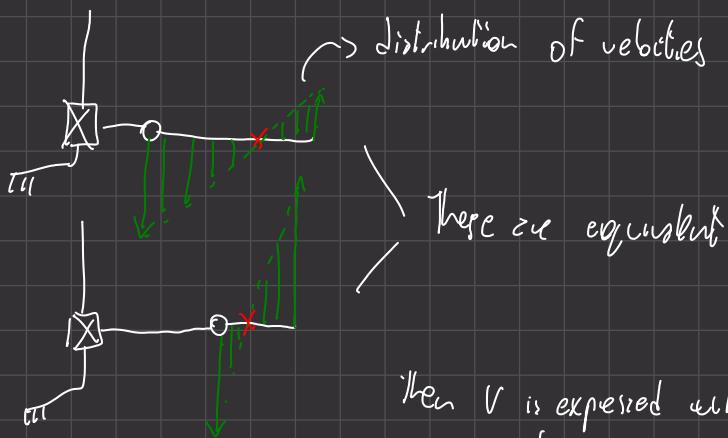
I can have the same motion by rotating around another hinge and changing the translational direction I will get the same movement.

Let's do another example:



If now I move the joint along the "arm" I can get different combinations of ω and v such that the point will have the same movement

$$(\omega_1, v_1) = (\omega_2, v_2) = (\omega_3, v_3) \rightarrow \text{class of equivalence}$$



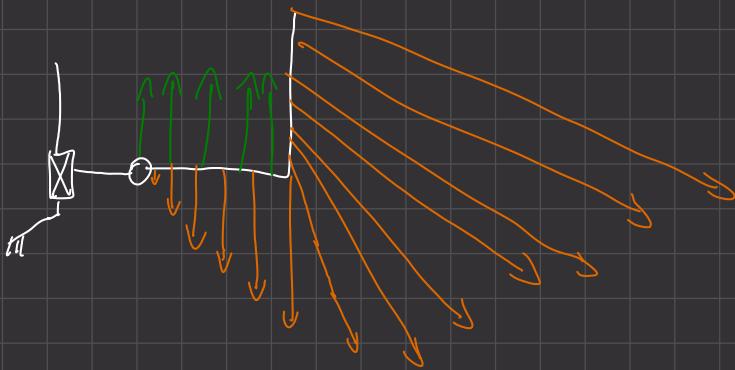
Then v is expressed with a parallelogram



We are building a class of equivalence \rightarrow economical representation

↪ twist: $\xi [v, \omega]$

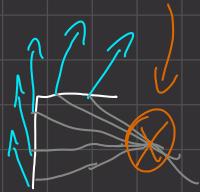
This is a representation of speed in the body and this distribution works for instantaneous movement.



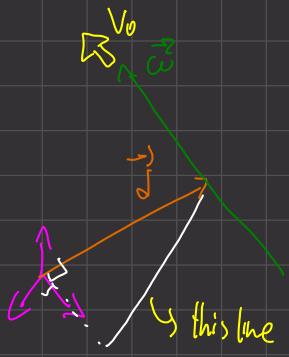
If you add the two then you get the actual speed.

the point in which the orthogonal of the resultant of the speeds meet is called:
IAR (Instantaneous Axis of Rotation)

For example:



$$\xi = (\vec{\omega} | \vec{v})$$

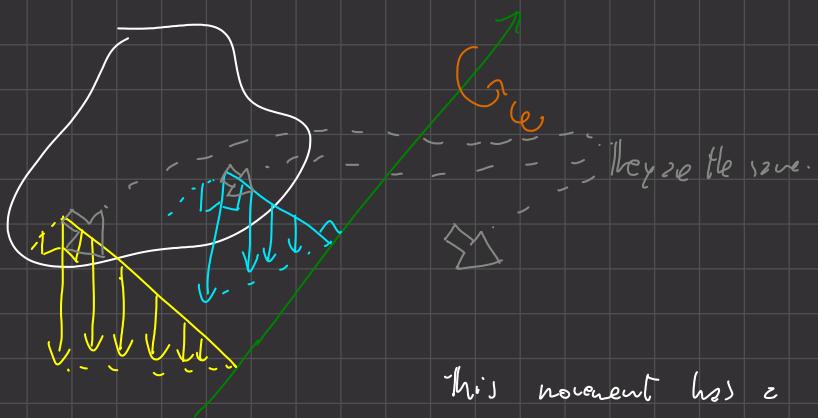


N.B.

$$\vec{v} = V_0 + \vec{d} \times \vec{\omega}$$

$$V_\theta = \rho \downarrow_{\text{pitch}} |\omega|$$

Speed is a free vector, the rotations are not. You can add translations with parallelism but not the rotations.



this movement has a pitch.

Now we can pose the question: what is the velocity of a body?

Now we have tools to describe this movement.

$$\begin{aligned} \text{Vectors in } \mathbb{R}^6 & \quad \vec{\omega}_l = (\vec{\omega} | \vec{v}) \quad P = \frac{|v|}{|\omega|} \\ & \quad \vec{\omega}_e = (\vec{\omega} | \vec{v}) \quad \text{line matters just direction matters} \\ & \quad \vec{\omega}_m = (\vec{\omega} | \vec{v}) \end{aligned}$$

72/20/23

$$\vec{\omega}_m = \vec{\omega}_{300} \quad / \quad \rho = 0 \quad / \quad \omega = \gamma \rightarrow$$

If you want to model the joint is convenient to give a twist with $\omega = \gamma$

Now if we take a motor and attach it to the joint with 300 rpm

$$\text{then } \vec{\omega}_m = \vec{\omega}_{300}$$

We are using a unit length vector like $(1, 0, 0)$ or $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0)$

So we are normalizing just the omega: $\vec{\omega} = (\vec{\omega} | \vec{v})$
 $\hookrightarrow \text{s.t. } |\vec{\omega}| = 1$

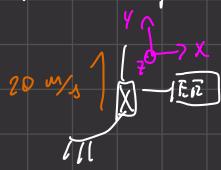
We use omega for direction and V for intensity.

$$\begin{aligned} \vec{\omega}_m &= (\vec{\omega} | \vec{v}) \quad \text{direction position of spinning of line.} \\ &= (\phi, \psi, \gamma | \phi, \theta, \phi) \end{aligned}$$

motorize the R joint
motor speed = 200 RPM

speed of the EE = 200 deg/min^6

Polar metric joint

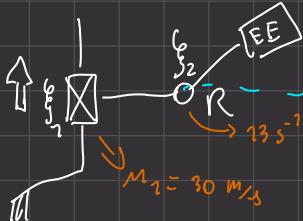


$$\dot{\xi}_1 = (\vec{\omega} | \vec{V}) \quad \rho = \infty$$

$$\dot{\xi}_m = 20 \dot{\xi}_1 = 20 (\vec{\omega} | \vec{V})$$

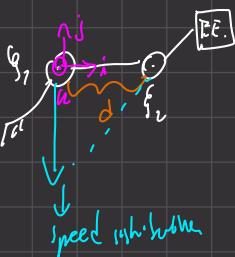
We prefer $|\vec{V}| = 2$

$$\dot{\xi} = 20 (\phi, \theta, \rho | \dot{\phi}, \dot{\theta}, \dot{\rho})$$



$$\dot{\xi} = 20 \dot{\xi}_1 + 73 \dot{\xi}_2$$

Movement is linear in velocity
TAR will be on this line



$$\dot{\xi}_{EE} = \omega_1 \dot{\xi}_1 + \omega_2 \dot{\xi}_2$$

$$\dot{\xi}_1 = (\dot{\phi}, \dot{\theta}, \dot{\rho} | 0, 0, 0)$$

$$\dot{\xi}_2 = (\dot{\theta}, \dot{\phi}, \dot{\gamma} | \dot{\phi}, -\dot{\rho}, \dot{\phi})$$

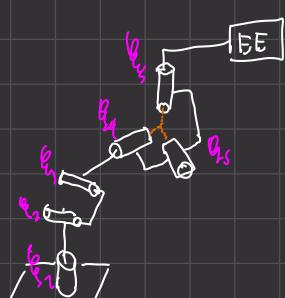
You get this value choosing to have v_2 fixed and moving a line above the origin of the axis.

Focused
kinematic
problem



$$\dot{\xi} = (0, 0, 1 | z_1 h, 0) \quad d = \sqrt{z_1^2 + h^2}$$

Let's now consider a 6R robot



$$\dot{\xi}_{EE} = 2 \dot{\xi}_1 - 5 \dot{\xi}_2 + 20 \dot{\xi}_3 + 72 \dot{\xi}_4 - 7 \dot{\xi}_5 + 3 \dot{\xi}_6$$

Motor	speed
1	2
2	-5
3	20
4	12
5	-7
6	3

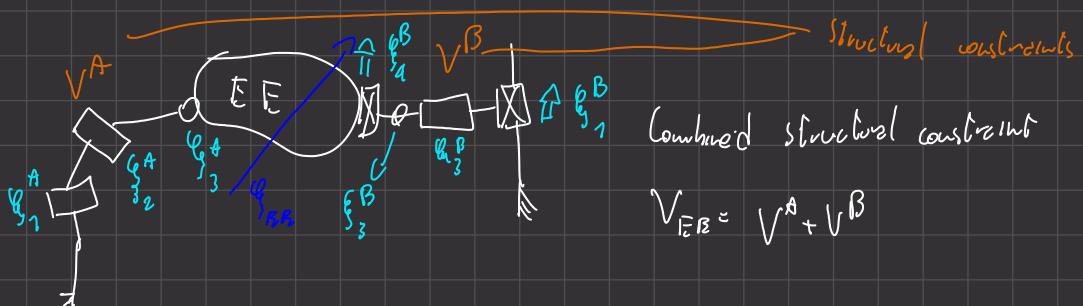
Geometric Jacobian

6×6 matrix

Joint velocities \rightarrow EEE velocities

$$\begin{bmatrix} \dot{\xi}_1 & \dot{\xi}_2 & \dot{\xi}_3 & \dot{\xi}_4 & \dot{\xi}_5 & \dot{\xi}_6 \end{bmatrix}_{\text{EEE}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \\ \omega_5 \\ \omega_6 \end{bmatrix}$$

depends on configuration parameters only



We can imagine an imaginary kinematic chain (mechanism)



Let's now make an analogy with velocity and discuss what happens.

The mechanism is consistent between the two chains so:

$$\dot{\xi}_{\text{EEE}} = \omega_1^A \dot{\xi}_1^A + \omega_2^A \dot{\xi}_2^A + \omega_3^A \dot{\xi}_3^A = V_1^B \dot{\xi}_1^B + V_2^B \dot{\xi}_2^B + V_3^B \dot{\xi}_3^B + V_{EE}^B$$

$M^A = \text{span}(\dot{\xi}_1^A, \dot{\xi}_2^A, \dot{\xi}_3^A) \rightarrow$ If they are linearly independent then it's a 3D system.

The $\dot{\xi}_{\text{EEE}}$ create = span of the feasible movements.

Any feasible or allowable $\dot{\xi}_{\text{EEE}} \in M^A$

Vector space in \mathbb{R}^6 .

$M^B = \text{span}(\dot{\xi}_1^B, \dot{\xi}_2^B, \dot{\xi}_3^B, \dot{\xi}_4^B) \rightarrow$ if they are linearly independent then we have a 4D system.

So Any feasible $\dot{\xi}_{\text{EEE}} \in M^A \cap M^B$

Concluding $\dot{\xi}_{\text{EEE}} \in M^A \cap M^B$

