Supplementary Information of "Distributed AC-DC Optimal Power Dispatch of VSC-Based Energy Routers in Smart Microgrids"

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NOMENCLATURE

Sets

$\mathcal{N}$	Set of all buses.
$\mathcal{N}^{ac}$	Set of AC buses that are not connected to the energy router.
$\mathcal{N}_k^{er-ac}$	Set of set of AC buses that are connected to the kth energy router.
$\mathcal{N}_{er_k}^{ac}$	Set of AC buses in the kth energy router.
$\mathcal{N}_{er_k}^{dc}$	Set of DC buses in the kth energy router.
$\mathcal{N}_{er_k}^{dc}$	Set of all DC buses.
$\mathcal{N}_{er_k}$	Set of all the buses of the kth energy router.
$\mathcal{E}$	Set of edges.
$\mathcal{R}_r$	Set of buses in Region r.
$\mathcal{B}_r$	Joint set including the buses in $\mathcal{R}_r$ and the buses duplicated from neighboring regions that are directly connected to the buses in $\mathcal{R}_r$.
$\mathcal{O}_r$	Set of inter-region lines connected to region r.
$\mathcal{N}_G$	Set of generator buses.
$\mathcal{X}_r$	Feasible set of (P_{G_i}, Q_{G_i}, V_i) in $\mathcal{R}_r$.

Abbreviation

ER	Energy router.
ER-OPF	ERs based AC-DC optimal power flow
SDP	Semidefinite program.
SQP	Sequential quadratic programming.
ADMM	Alternating direction method of multipliers.

Variables

P_{G_i}	Active power by generators at bus i.
Q_{G_i}	Reactive power by generators at bus i.
V_i	Complex voltage at bus i.
$Q_{A_{k-i}}$	Reactive power by VSCs at bus A_{k-i}.
$V_{A_{k-i}}$	Complex voltage at bus A_{k-i}.
$Q_{a_{k-i}}$	Reactive power by VSCs at bus a_{k-i}.
$V_{a_{k-i}}$	Complex voltage at bus a_{k-i}.
V_{D_k}	Voltage magnitude at DC bus D_k.
V_{d_k}	Voltage magnitude at DC bus d_k.
S_{ij}	Apparent power flow of line (i,j)
x_r	Variables (P_{G_i}, Q_{G_i}, V_i) in $\mathcal{R}_r$.
λ_r	Multiplier in $\mathcal{R}_r$.
$\mathbf{v}$	The vector of complex bus voltages.
$\mathbf{x}$	The vector of the real and imaginary parts of $\mathbf{v}$.
$\mathbf{W}$	The square matrix of $\mathbf{x}$.
β_i	Auxiliary variables for objective functions.
$\vartheta_{r\mapsto 0}^i$	Auxiliary variables for the linear parts of boundary objective functions.
$\zeta_{r\mapsto 0}^i$	Auxiliary variables for the quadratic parts of boundary objective functions.

Constants

$\bar{a}_j, \bar{b}_j, \bar{c}_j$	Coefficients of the quadratic production cost function of unit j
G_{sw}	A conductance with a degree of power behavior.
$P_{loss}^{D_k}$	Power losses of VSCs at bus D_k.
P_{D_i}	Active power demand at bus D_i.
P_{d_i}	Active power demand at bus d_i.
P_i	Active power demand at bus i.
Q_i	Reactive power demand at bus i.
Y	Nodal admittance matrix.
y_{ij}	Series admittance of line (i,j).
Q_A^{min}	Lower bound of $Q_{A_{k-i}}$.
Q_A^{max}	Upper bound of $Q_{A_{k-i}}$.
Q_a^{min}	Lower bound of $Q_{a_{k-i}}$.
Q_a^{max}	Upper bound of $Q_{a_{k-i}}$.
$P_{G_i}^{min}$	Lower bound of active generation P_{G_i}.
$P_{G_i}^{max}$	Upper bound of active generation P_{G_i}.
$Q_{G_i}^{min}$	Lower bound of reactive generation Q_{G_i}.
$Q_{G_i}^{max}$	Upper bound of reactive generation Q_{G_i}.
V_i^{min}	Lower bound of voltage magnitude $\lvert V_i\rvert$.
V_i^{max}	Upper bound of voltage magnitude $\lvert V_i\rvert$.
z_r^{ac}	All the AC messages to region r.
z_r^{dc}	All the DC messages to region r.
$z_{r\mapsto 0}^{ac}$	The AC messages from the master region to region r.
$z_{0\mapsto r}^{ac}$	The AC messages from region r to the master region.
$z_{r\mapsto l}^{dc}$	The AC messages from region r to region l.

I. PROOF OF PROPOSITION 1

If there is a sequence $(V_i^v, P_{G_i}^v, Q_{G_i}^v, Q_{A_{k-i}}^v, Q_{a_{k-i}}^v, S_{ij}^v; \lambda_r^v)$ in buses $\mathcal{B}_r$, $\forall r = 0, \cdots, R$ of solutions to problems (33)-(34), which, as $v \mapsto \infty$, makes residues Γ_r^v, $r = 0, \cdots, R$ converge to zero, then this sequence converges to a feasible solution to the original problem (20). We consider the following two scenarios.

1) Subproblems without DC links. When $\|z_{0\mapsto r}^{ac} - z_{r\mapsto 0}^{ac}\|_{\rho_0}^2$ and $\|z_{r\mapsto 0}^{ac} - z_{0\mapsto r}^{ac}\|_{\rho_r}^2$ approach zeros for the master problem and subproblems, Problem (33) and Problem (34) are equivalent to Problem (26) and Problem (27), respectively. As such, we can obtain $S_{\ell(0,r),0} = S_{\ell(0,r),r}$,

$|V_{i(0,r),0}|^2 = |V_{i(0,r),r}|^2$, and $|V_{j(0,r),0}|^2 = |V_{j(0,r),r}|^2$. We can deduce the following relation:

$$\theta_{i(0,r),0} - \theta_{j(0,r),0} = \theta_{i(0,r),r} - \theta_{j(0,r),r}. \qquad (1)$$

For the tree structure of the regions' connection in Fig. 3, we do not need to enforce $\theta_{i(0,r),0} = \theta_{i(0,r),r}$ and $\theta_{j(0,r),0} = \theta_{j(0,r),r}$, and Eq. (1) is enough to recover a feasible solution. When we obtain $\theta_{i,0}$ and $\theta_{j,0}$ from the master problem and $\theta_{i,r}$ and $\theta_{j,r}$ from subproblem r, we have a gap $gap_{i,0\leftrightarrow r} = \theta_{i(0,r),0} - \theta_{i(0,r),r}$ and $gap_{j,0\leftrightarrow r} = \theta_{j(0,r),0} - \theta_{j(0,r),r}$. According to Eq. (1), $gap_{i,0\leftrightarrow r} = gap_{j,0\leftrightarrow r}$. Therefore, with $\theta_{i(0,r),0}$ and $\theta_{j(0,r),0}$ fixed, we add this gap to $\theta_{i(0,r),r}$ and $\theta_{j(0,r),r}$ simultaneously without changing power flows. We conduct this process for all the subproblems. As such, this solution is feasible for the original problem if there is no cycle with AC connections.

2) Subproblems with DC links. Similarly to above, boundary constraints between subregions and the master region are satisfied if $\|z^{ac}_{0\mapsto r} - z^{ac}_{r\mapsto 0}\|^2_{\rho_0}$ and $\|z^{ac}_{r\mapsto 0} - z^{ac}_{0\mapsto r}\|^2_{\rho_r}$ are zeros. In addition, when $\left\|z^{dc}_{r\mapsto l} - z^{dc}_{l\mapsto r}\right\|^2_{\rho_r}$ and $\left\|z^{dc}_{l\mapsto r} - z^{dc}_{r\mapsto l}\right\|^2_{\rho_r}$ are zeros for subproblems r and l, the boundary constraint between subregions r and l associated with their DC connection is satisfied. For DC connection, we only need to enforce equal voltage magnitudes across boundaries, whereas no equality constraint is needed for phase angles. The tree structure of AC connections is thus well retained, which plays an important role in exactness of the SDP relaxation we will apply below.