Diffeomorphic 3D Image Registration via Geodesic Shooting

Project of Géométrie et Espaces de Formes

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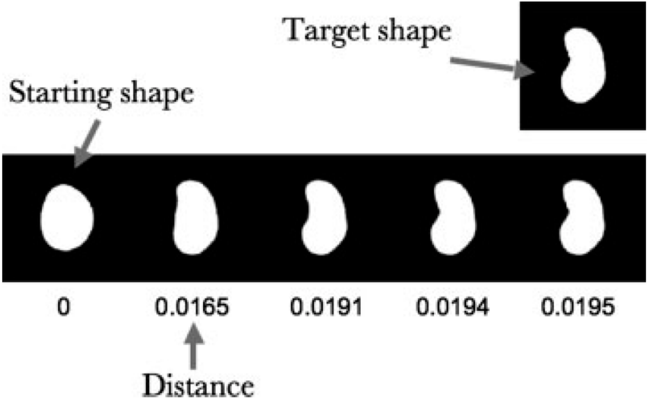
Overview

Introduction

Theory

Task

Given an input image and a target image, how to find a dense one-to-one point correspondence?



LDDMM

Large Deformation Diffeomorphic Metric Mapping (LDDMM) is a framework which models the mapping of one from the other via a dynamic flow of diffeomorphisms $t \in [0, 1]$ of the ambient space $\mathbb{R}^d$.

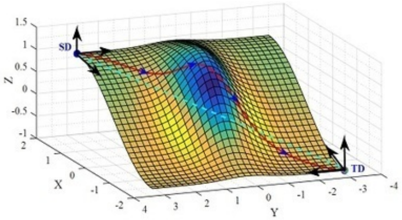


Illustration of the main idea of geodesic flow for domain adaptation.

LDDMM

Given an input image I_0 and a target image J, the dynamic flow is encoded completely by a time-dependent velocity field $v \in L^2([0,1], V)$.

So what we need to do is to find a minimizer v^* for the following energy funtion:

$$S(v) = \frac{\lambda}{2} \int_0^1 \|v(t)\|_V^2 dt + \frac{1}{2} \| I_0 \circ \Phi_{0,1}^{-1}, J \|_{L^2}^2$$

So we have:

$$\boxed{v^* = \text{argmin}_v S(v)}$$

LDDMM

The diffeomorphisms $\phi_{0,t}$ is then calculated through the ODE:

$$\dot{\phi}_{0,t}(x) = v(t) \circ \phi_{0,t}$$

with initial condition:

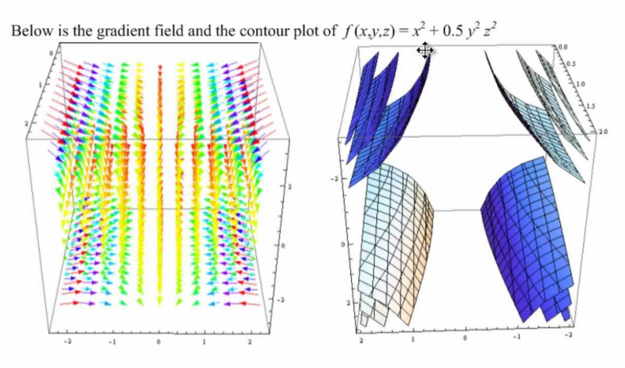
$$\phi_{0,0} = Id$$

If the path is geodesic, the whole vector field can be encoded completely by the initial vector v_0.

LDDMM in image space?

In a geodesic case, the momentum vector field v is in $L^2([0,1], V)$ given a Banach space.

However, it is not the case in images. For a pixel x in the 3D voxel image, the deplacement in each discrete time step is not continuous.



Forward: Geodesic Equation

Given the momentum P and the image I which are both scalar functions, the geodesic equations read:

$$\begin{cases} \partial_t I + \nabla I \cdot v = 0 \\ \partial_t P + \nabla \cdot (Pv) = 0 \\ v + K \star \nabla IP = 0 \end{cases}$$

subject to initial conditions at time 0, $I(0), P(0) \in H^1(\Omega)$

Backward: Adjoint Equation

The gradient of S is given by:

$$\nabla_{P(0)} S = \lambda \delta I(0) \cdot K \star (P(0) \nabla I(0)) - \hat{P}(0)$$

where $\hat{P}(0)$ is given by the solution of the following PDE solved backward in time:

$$\begin{cases} \partial_t \hat{I} + \nabla \cdot (v \hat{I}) + \nabla \cdot (P \hat{v}) = 0 \\ \partial_t \hat{P} + v \cdot \nabla \hat{P} - \nabla I \cdot \hat{v} = 0 \\ \hat{v} + K \star (\hat{I} \nabla I - P \nabla \hat{P}) = 0 \end{cases}$$

subject to the initial conditions: $\hat{I}(1) = J - I(1)$, $\hat{P}(1) = 0$.