Numeric Experiment

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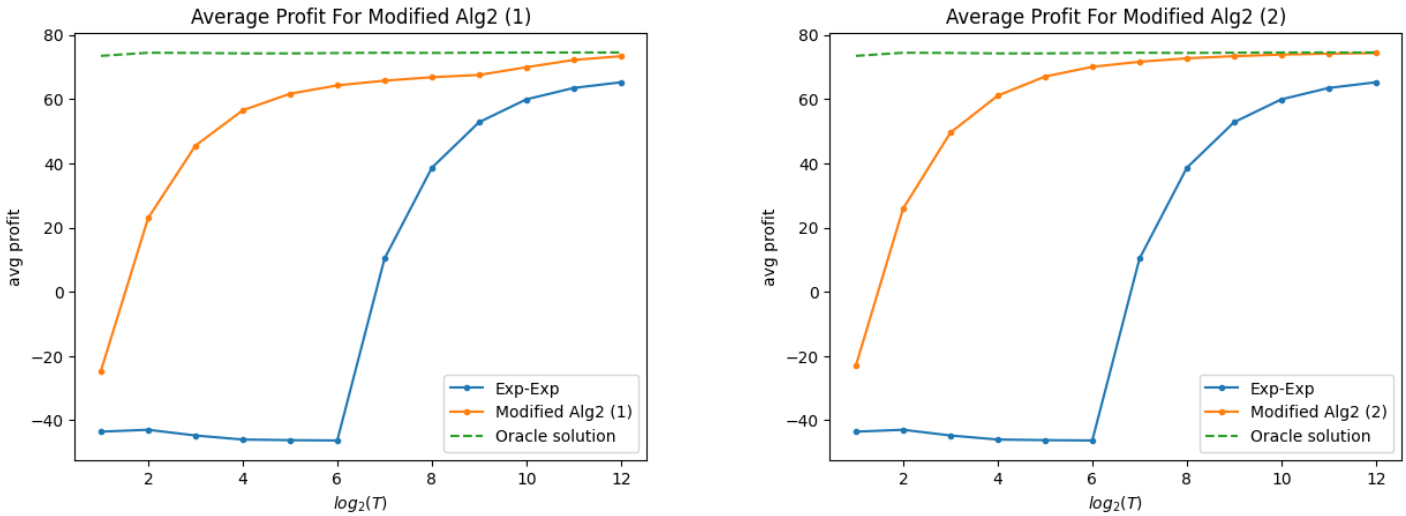
1 Compare Alg 2 with the modified Alg 2

1. Compare the modified Algorithm 2 using (1) with Explore-then-exploit algorithm.



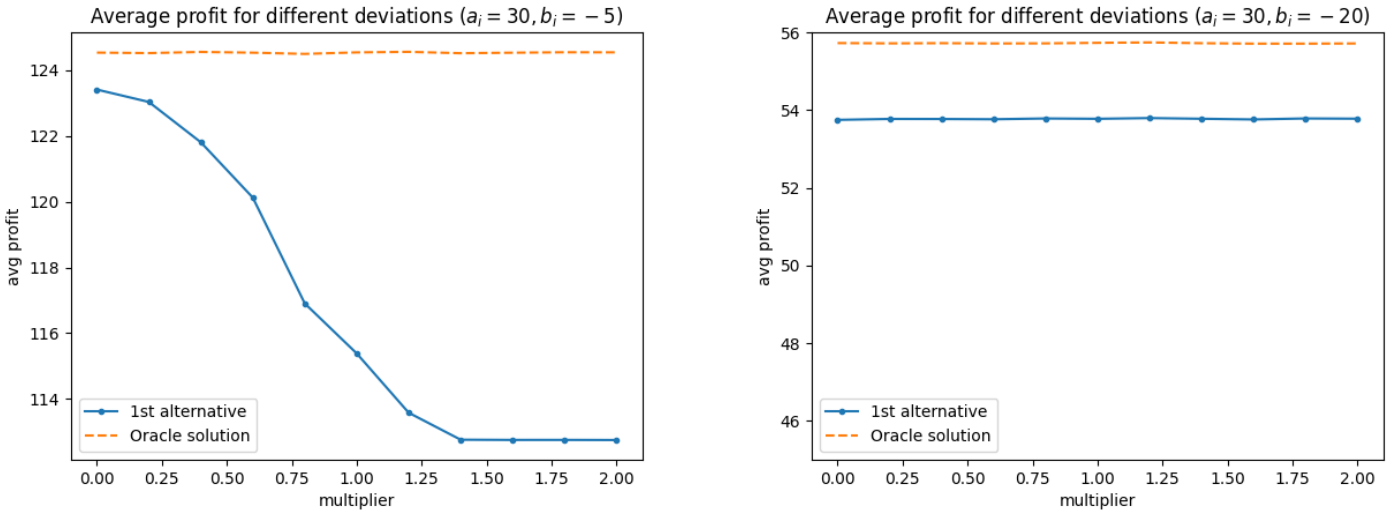
(a) Mod Alg (1)

(b) Mod Alg (2)

Figure 1: Comparison of Modified Alg2 with Exp-Exp

Parameters: m = 3, H = 5, a = 20, b = -5, Optimal Allocation.

2. Try different values of lower bounds di and see how the average profit changes for the modified algorithm 2.



(a)

(b)

Figure 2: Modified (1)

$\underline{d}_i < -\underline{d}_i - shift \times sigma$, shift = 0,0.2,...,1.0. Parameters: m = 3, H = 5, a = 20, b = -5, Optimal Allocation.

3. Compare the two modified algorithm 2 with original Algorithm 2

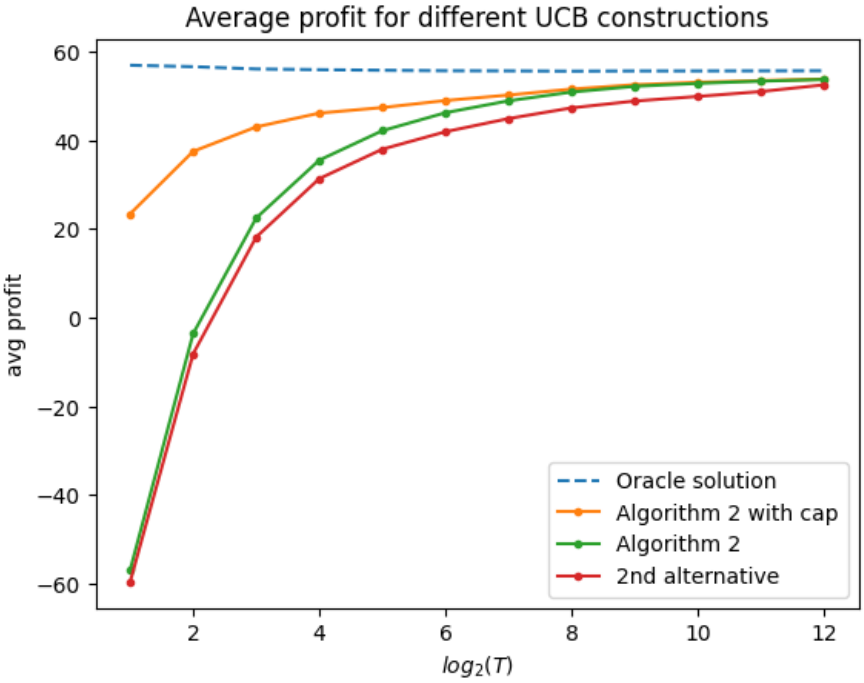
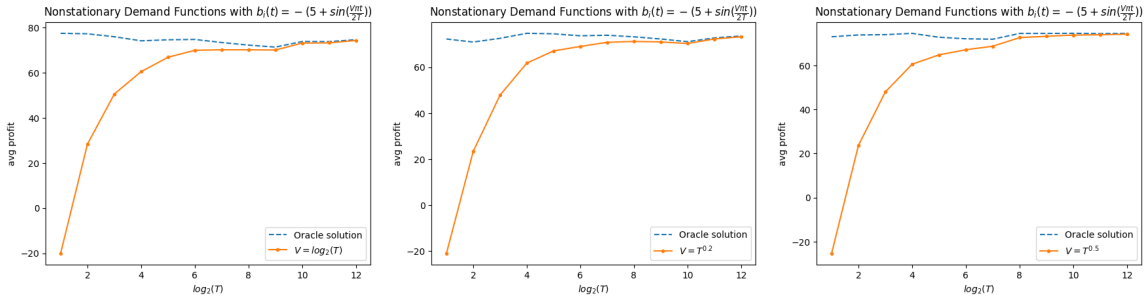


Figure 3: Alg2 and Mod Alg2

Parameters: m = 3, H = 5, a = 30, b = -20, Optimal Allocation.

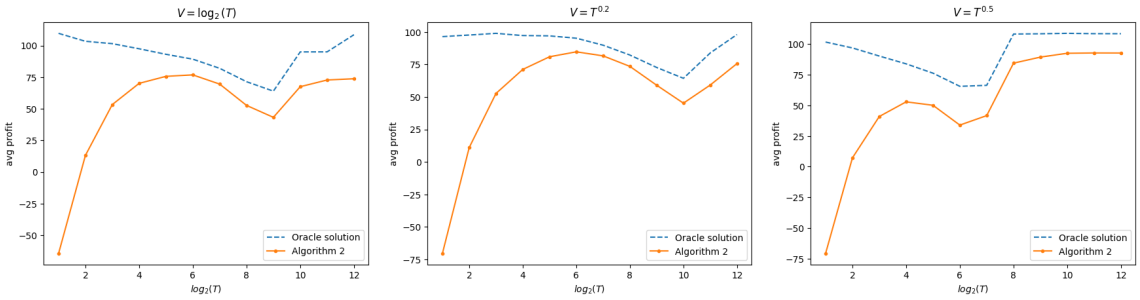
2 Nonstationary demand functions over epochs



(a) Log (b) $T^{0.2}$ (c) $T^{0.5}$

Figure 4: Nonstationary demand functions over epochs

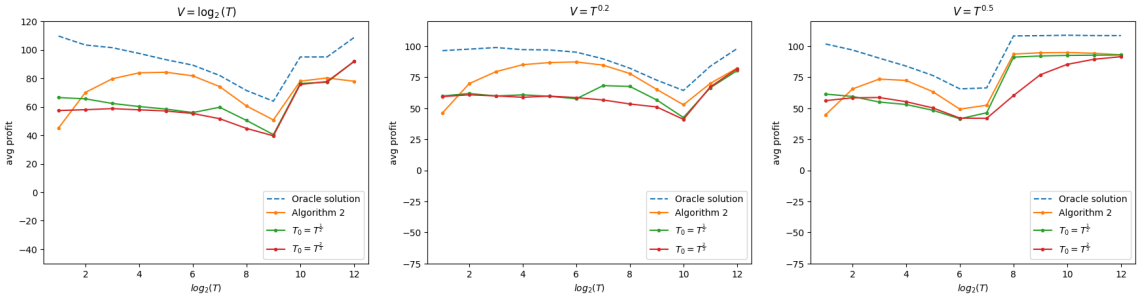
Parameters: m = 3, H = 5, a = 20, b = -5, $T = 2^{12}$, amplitude=1, Optimal Allocation.



(a) Log (b) $T^{0.2}$ (c) $T^{0.5}$

Figure 5: Nonstationary demand functions over epochs

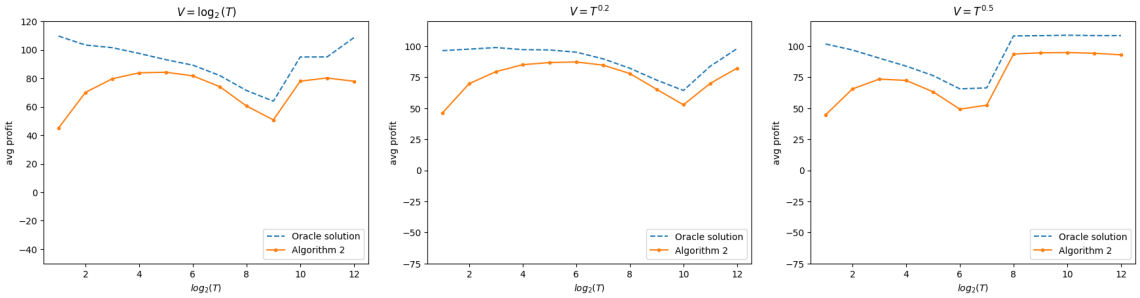
Parameters: m = 10, H = 20, a = 10, b = -5, $T = 2^{12}$, amplitude=4, Optimal Allocation, Modified Alg 2(2).



(a) Log (b) $T^{0.2}$ (c) $T^{0.5}$

Figure 6: Nonstationary demand functions over epochs

Parameters: m = 10, H = 20, a = 10, b = -5, $T = 2^{12}$, amplitude=4, Optimal Allocation.



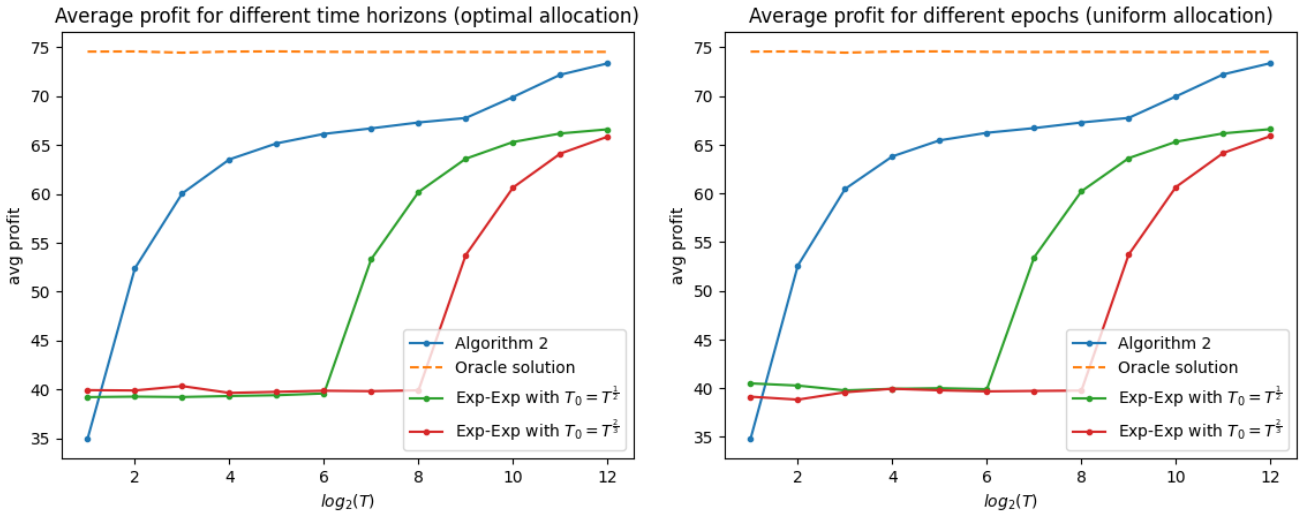
(a) Log (b) $T^{0.2}$ (c) $T^{0.5}$

Figure 7: Nonstationary demand functions over epochs

Parameters: m = 10, H = 20, a = 10, b = -5, $T = 2^{12}$, amplitude=4, Optimal Allocation.

3 Compare with Explore-then-exploit Heuristic

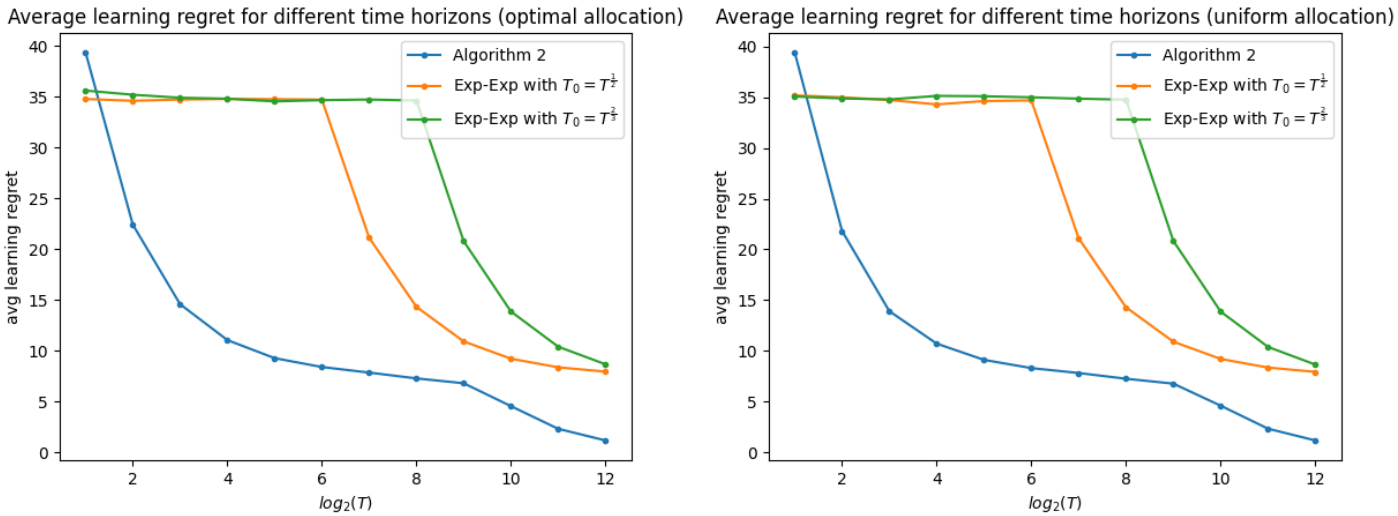
Parameters: m = 3, H = 5, a = 20, b = -5.



(a) Optimal Allocation

(b) Uniform Allocation

Figure 8: Average profits of two allocation policies



(a) Optimal Allocation

(b) Uniform Allocation

Figure 9: Average learning regret of two allocation policies

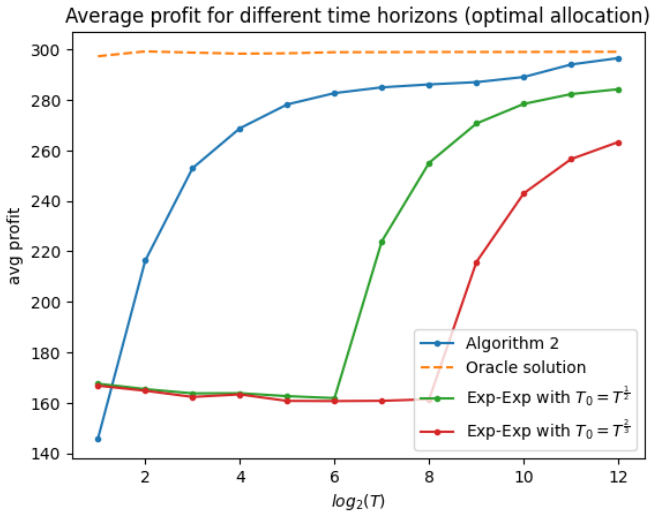
T	Algorithm 2	Exp-Exp with $T_0 = T^{\frac{1}{2}}$	Exp-Exp with $T_0 = T^{\frac{2}{3}}$
1024	6.2%	12.4%	18.6%
2048	3.2%	11.2%	13.9%

Table 1: Relative profit loss (Optimal Allocation)

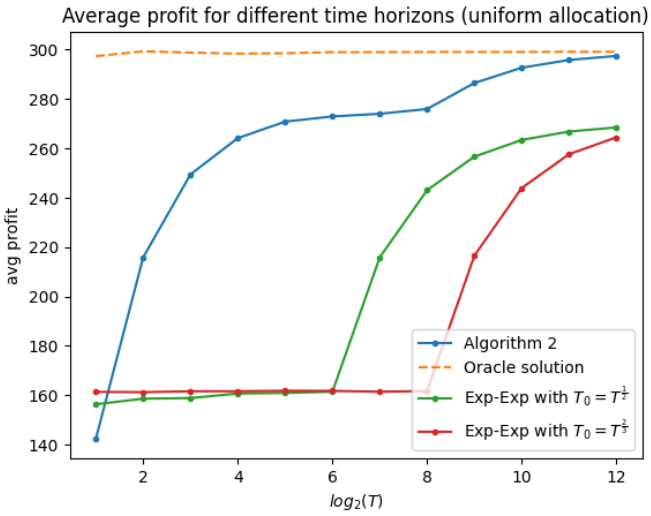
T	Algorithm 2	Exp-Exp with $T_0 = T^{\frac{1}{2}}$	Exp-Exp with $T_0 = T^{\frac{2}{3}}$
1024	6.1%	12.4%	18.6%
2048	3.1%	11.2%	13.9%

Table 2: Relative profit loss (Uniform Allocation)

Parameters: m = 5, H = 20, a = 20, b = -5.

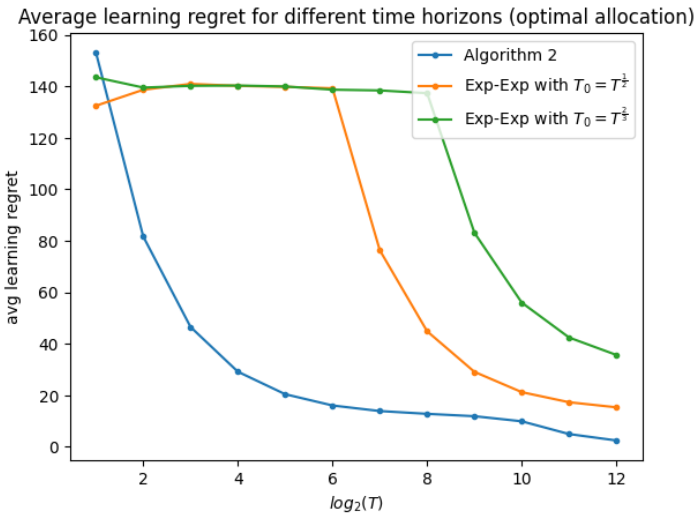


(a) Optimal Allocation

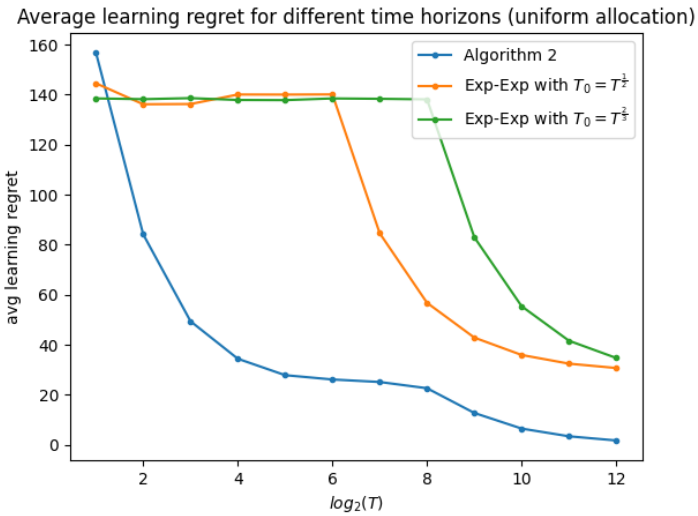


(b) Uniform Allocation

Figure 10: Average profits of two allocation policies



(a) Optimal Allocation



(b) Uniform Allocation

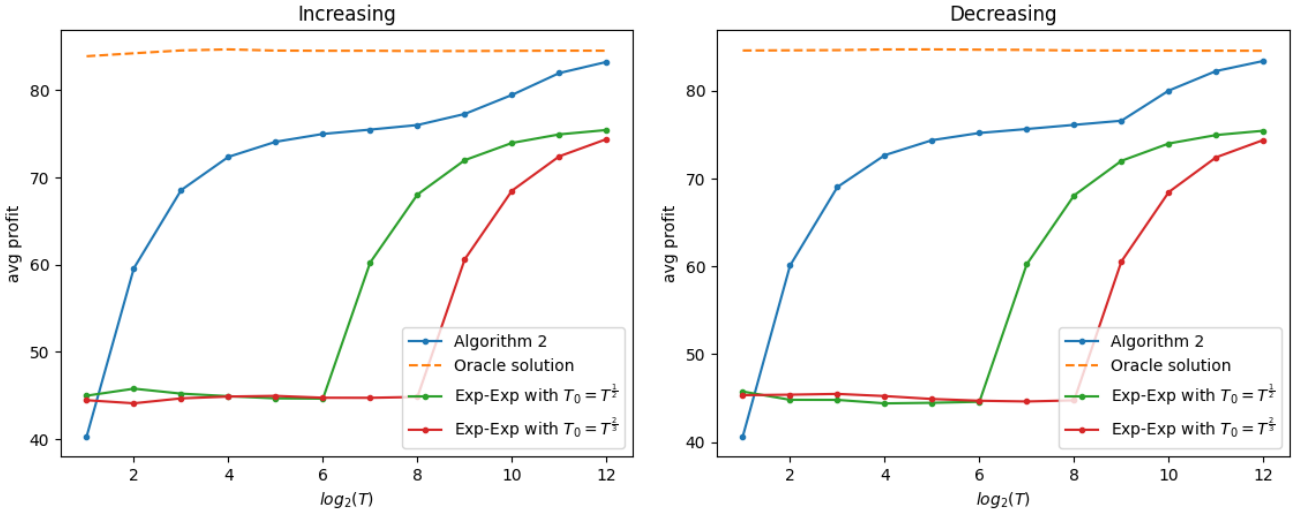
Figure 11: Average learning regret of two allocation policies

T	Algorithm 2	Exp-Exp with $T_0 = T^{\frac{1}{2}}$	Exp-Exp with $T_0 = T^{\frac{2}{3}}$
1024	3.3%	6.9%	18.8%
2048	1.7%	5.6%	14.2%

Table 3: Relative profit loss (Optimal Allocation)

T	Algorithm 2	Exp-Exp with $T_0 = T^{\frac{1}{2}}$	Exp-Exp with $T_0 = T^{\frac{2}{3}}$
1024	2.1%	11.9%	18.4%
2048	1.1%	10.8%	13.9%

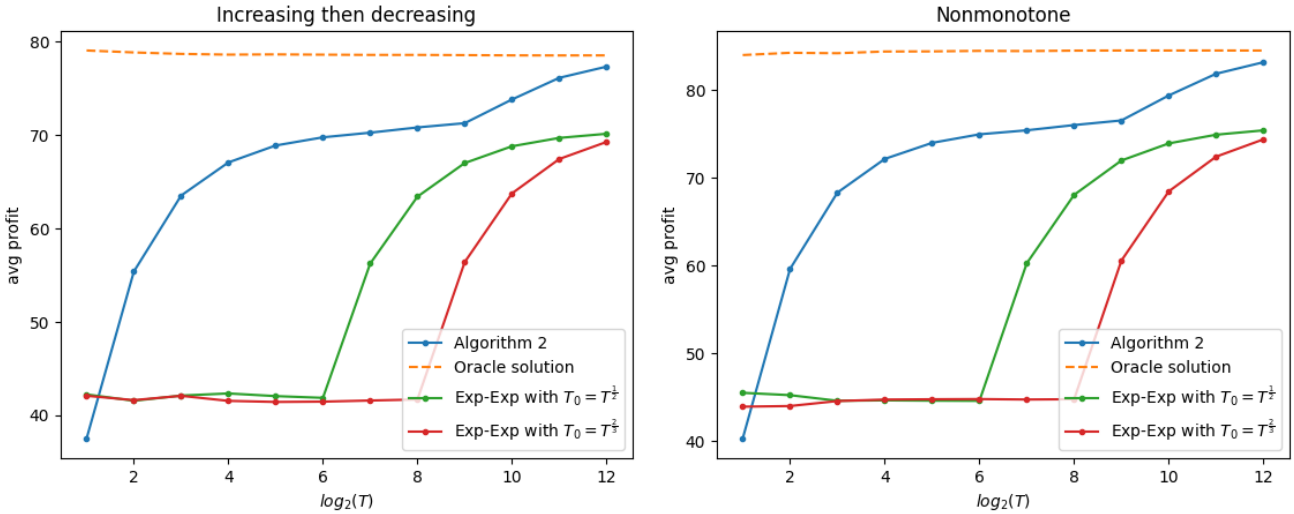
Table 4: Relative profit loss (Uniform Allocation)



(a) Increasing

(b) Decreasing

Figure 12: Average profits of two allocation policies

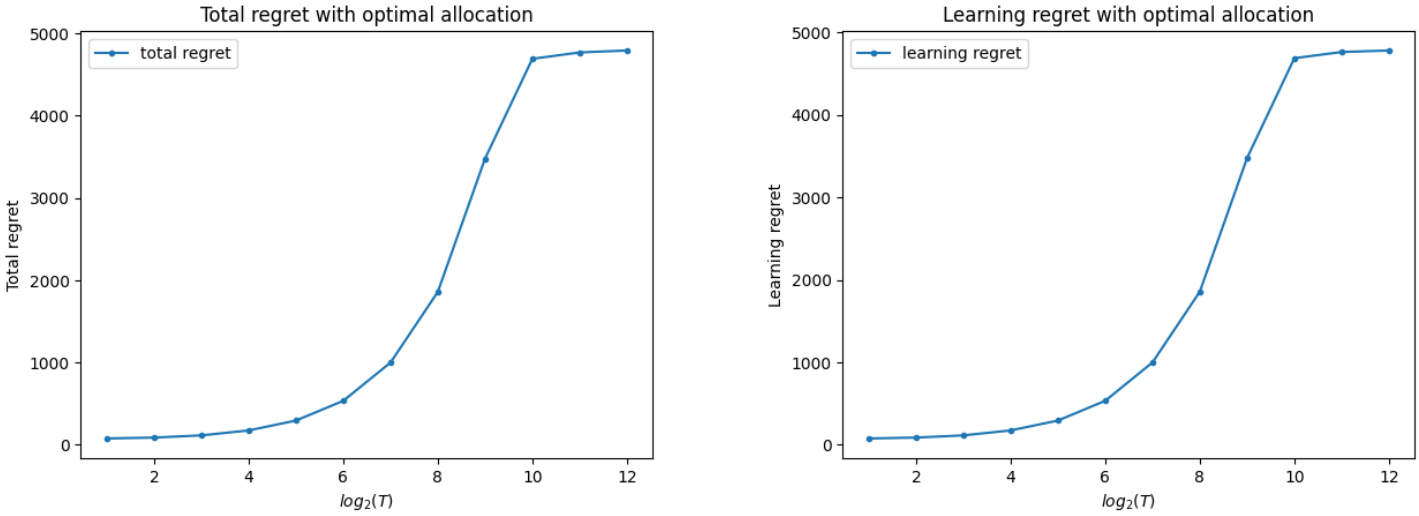


(a) Increasing then decreasing

(b) Nonmonotone

Figure 13: Average profits of two allocation policies

4 More figures

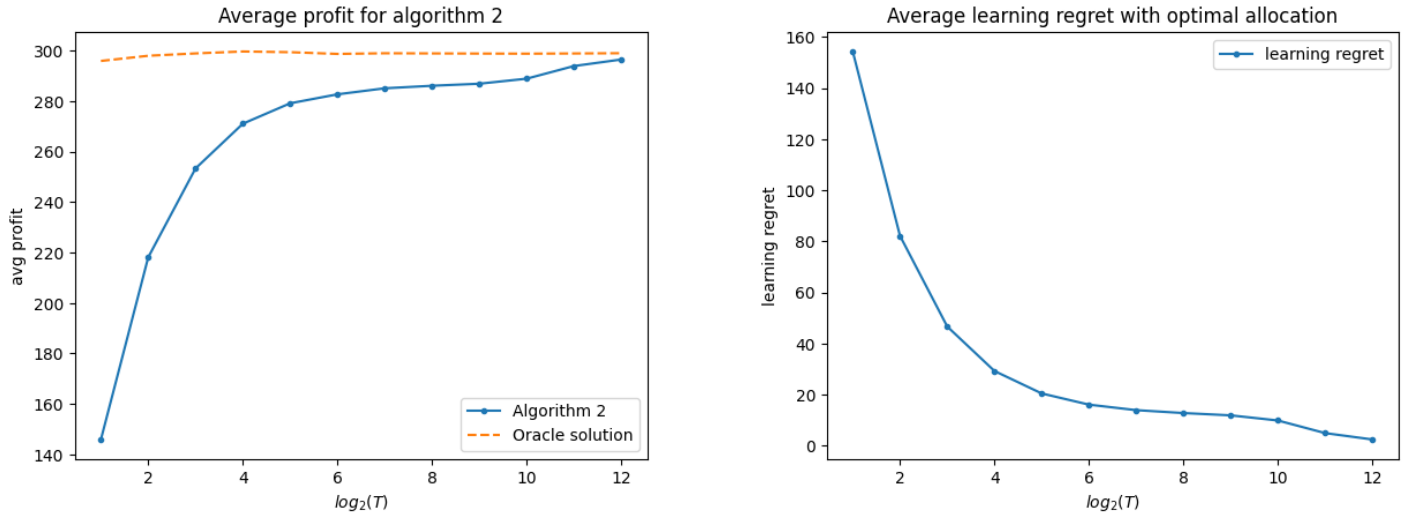


(a) Total regret

(b) Learning regret

Figure 14: Comparison of Regret

Parameters: m = 3, H = 5, a = 20, b = -5, Optimal Allocation.



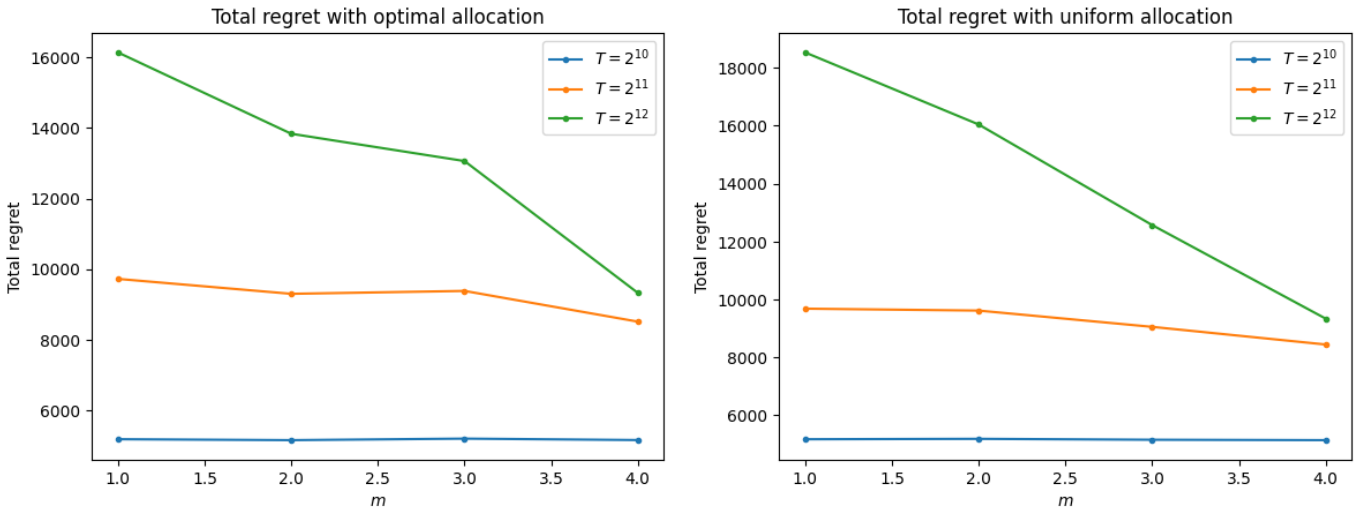
(a) Average Profit

(b) Average Learning regret

Figure 15: Comparison of Regret

Parameters: m = 5, H = 20, a = 20, b = -5, Optimal Allocation.

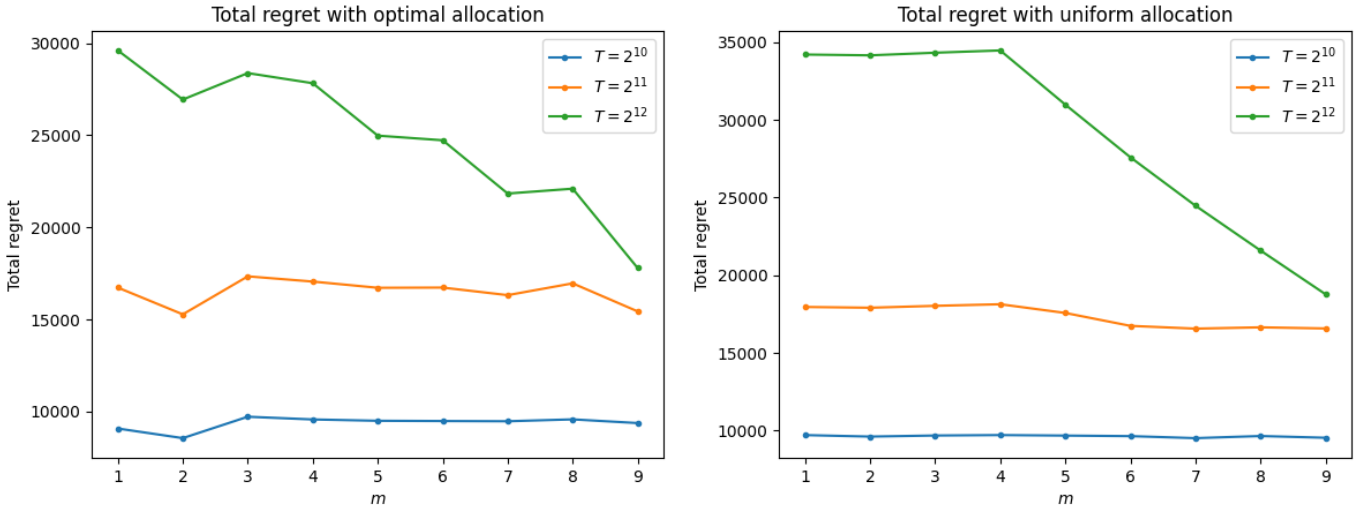
5 Phase Transition



(a) Optimal Allocation (b) Uniform Allocation

Figure 16: Phase Transition

Parameters: m = 3, H = 5, $a_1 = 20, b_1 = -5, a_2 = 30, b_2 = -20$.

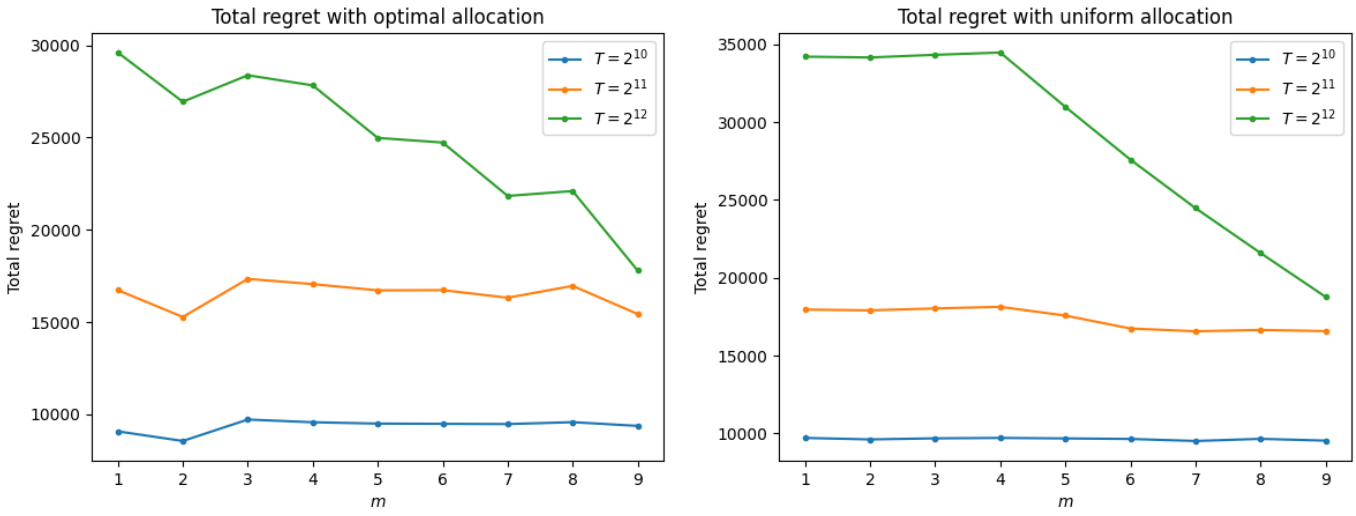


(a) Optimal Allocation (b) Uniform Allocation

Figure 17: Phase Transition

Parameters: m = 5, H = 10, $a_1 = 20, b_1 = -5, a_2 = 30, b_2 = -20$.

6 Compare uniform allocation and optimal allocation



(a) Learning regret

(b) Total regret

Figure 18: Two allocations

Parameters: T = 128, H = 15, a = 20, b = -5.

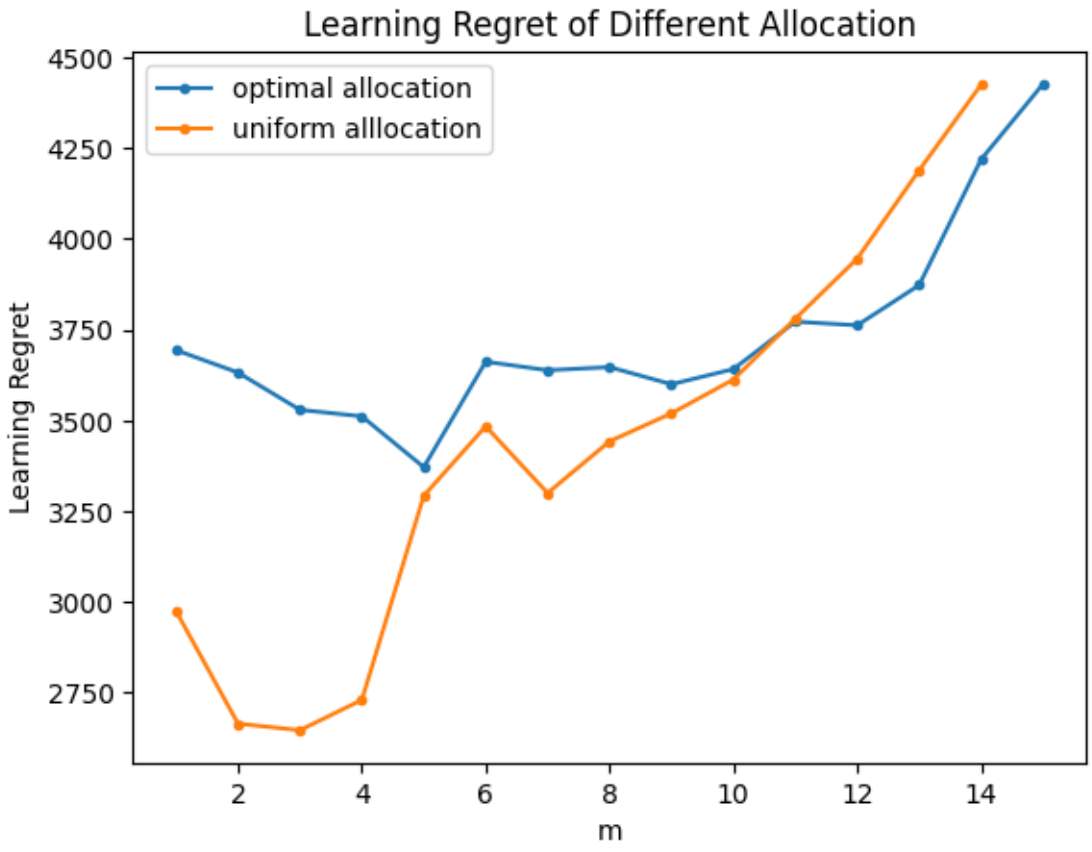


Figure 19: Two allocations

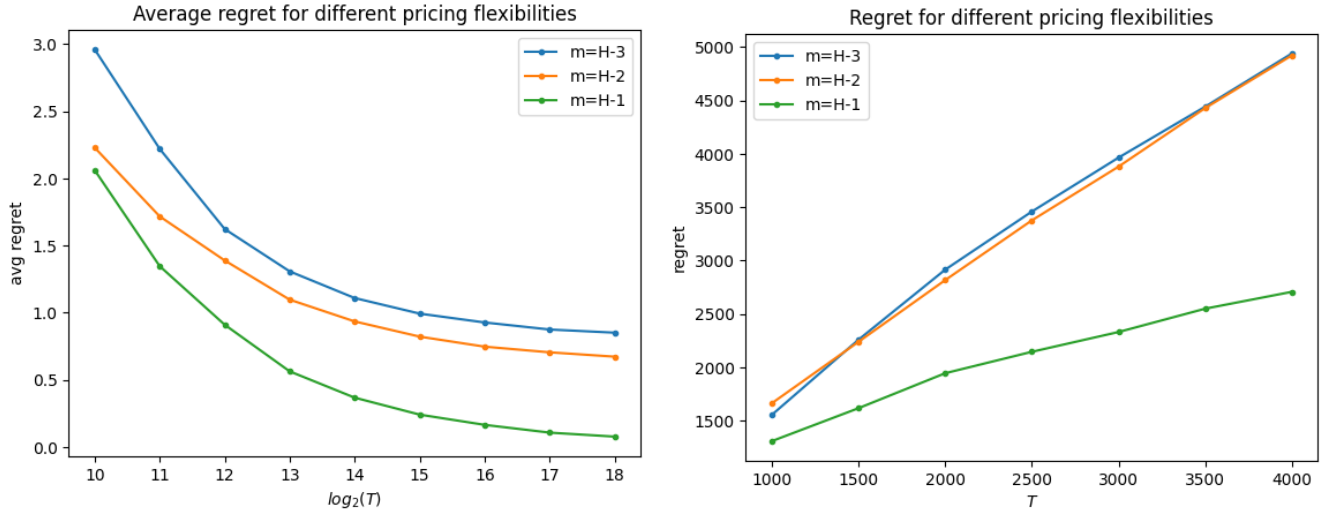
Parameters: $T = 2^{12}$, H = 15, a = 20, b = -5, Alg2 Modified 2.

7 Compare average regret for $m = H-3, H-2, H-1$

Parameters: H=5, $a_1 = 10, b_1 = -5, a_2 = 5, b_2 = -5$.

Plot a figure for optimal allocaiton and uniform allocation each. The figure has x-aixs the $\log(T)$ <= 12, y-axis is the average total regret (i.e., totoal regret / T). There are three curves in one figure, each for $m = H-3, H-2, H-1$

Expected outcome: the curves for $m = H-3, H-2$ are flat, the curve for $m = H-1$ is convex decreasing.



(a) Average Total Regret

(b) Total Regret

Figure 20: Regret