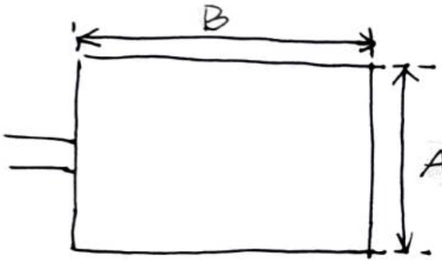
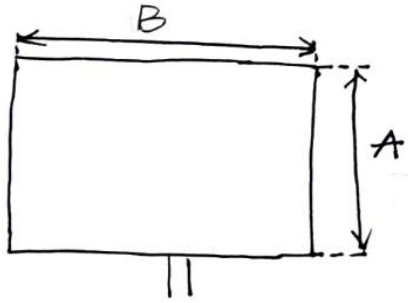
Homework 3 TONG Lin S287649

Problem 3.1

1. Resonance frequencies:



①



②

The function of the resonance frequency can be described as,

$$f_{res} = \frac{c}{\sqrt{\varepsilon_{r,eff}} \cdot \lambda_g}$$

where $\varepsilon_{r,eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{w}\right]^{-\frac{1}{2}}$

$\lambda_g = L + 2\Delta L.$

① $\varepsilon_{r,eff1} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{A}\right]^{-\frac{1}{2}} \approx 2.24$

$$\Delta L_1 = 0.412h \cdot \frac{\varepsilon_{r,eff1} + 0.3}{\varepsilon_{r,eff1} - 0.258} \cdot \frac{\frac{A}{h} + 0.262}{\frac{A}{h} + 0.813} \approx 0.0015\ m$$

$\lambda_{g1} = B + 2\Delta L_1 = 0.025\ m$

with $B = 2.2cm$, $h = 3mm$, $A = 2cm$

$$f_{res1} = \frac{c}{\sqrt{\varepsilon_{r,eff1}} \cdot \lambda_{g1}} \approx 8.04\ GHz.$$

② $\varepsilon_{r,eff2} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{B}\right]^{-\frac{1}{2}} \approx 2.25$

$$\Delta L_2 = 0.412h \cdot \frac{\varepsilon_{r,eff2} + 0.3}{\varepsilon_{r,eff2} - 0.258} \cdot \frac{\frac{B}{h} + 0.262}{\frac{B}{h} + 0.813} \approx 0.0015\ m$$

$\lambda_{g2} = A + 2\Delta L_2 = 0.023\ m$

with $A = 2cm$, $h = 3mm$, $B = 2.2cm$

$$f_{res2} = \frac{c}{\sqrt{\varepsilon_{r,eff2}} \cdot \lambda_{g2}} \approx 8.71\ GHz.$$

2. Equivalent circuit:

The function of conductance of a single slot is,

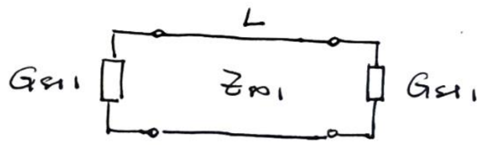
$$G_{sl} = \frac{W}{120\lambda_g}\left[1 - \frac{1}{24}\left(2\pi \frac{h}{\lambda_g}\right)^2\right]$$

The function of characteristic impedance is,

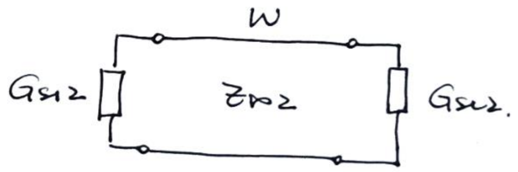
$$Z_{\infty} = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_{r,eff}}}\left(\ln\dfrac{8h}{W} + \dfrac{h}{W}\right) & \dfrac{W}{h} < 1 \\[2ex] \dfrac{120\pi}{\sqrt{\varepsilon_{r,eff}}\left(\dfrac{W}{h} + 1.393 + 0.667\cdot\ln\left(\dfrac{W}{h} + 1.444\right)\right)} & \dfrac{W}{h} > 1 \end{cases}$$

The results of case ① and case ② are respectively,

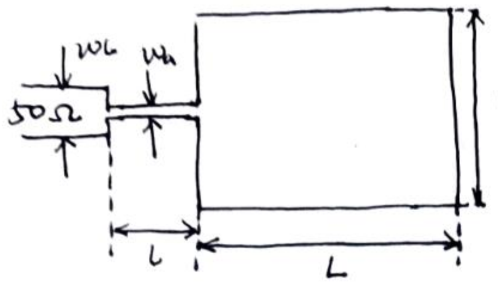
$G_{sl1} = 0.0065\ S$ $\quad Z_{\infty 1} = 26.65\ \Omega$



$G_{sl2} = 0.0078S$ $\quad Z_{\infty 2} = 24.69\ \Omega$.



Problem 3.2



The optimum width of the microstrip radiator is,

$$W_{opt} = \frac{c}{2f_r}\sqrt{\frac{2}{\varepsilon_r+1}} = \frac{3\times10^8}{2\times2.54\times10^9}\sqrt{\frac{2}{2.56+1}} \approx 0.044\,m$$

The patch length can be calculated with,

$$L + \Delta L = \frac{\lambda_g}{2}$$

Where ΔL is the extension length, and $\lambda_g = \frac{\lambda_0}{\sqrt{\varepsilon_{r,eff}}}$.

ΔL can be obtained by, $\quad (\lambda_0 = \frac{c}{f} \approx 0.12\,m)$

$$\frac{\Delta L}{h} = 0.412\,\frac{\varepsilon_{r,eff}+0.3}{\varepsilon_{r,eff}-0.258}\cdot\frac{\frac{W}{h}+0.262}{\frac{W}{h}+0.813}$$

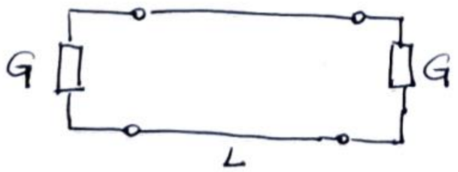
with $\varepsilon_{r,eff} = \frac{\varepsilon_r+1}{2} + \frac{\varepsilon_r-1}{2}\left[1+12\frac{h}{W}\right]^{-\frac{1}{2}} \approx 2.43$

Therefore $\Delta L = 8.06\times10^{-4}\,m$, $\lambda_g = 0.076\,m$

$$L = \frac{\lambda_g}{2} - \Delta L = 0.037\,m$$

The conductance of the patch can be equivalently obtained as,

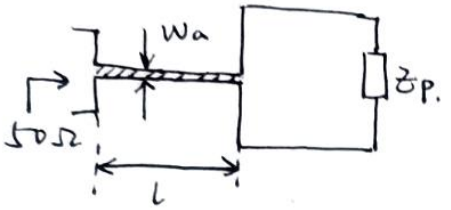
$$G = \frac{W}{120\lambda_0}\left[1-\frac{1}{24}\left(2\pi\frac{h}{\lambda_0}\right)^2\right] \approx 0.0031\,S$$

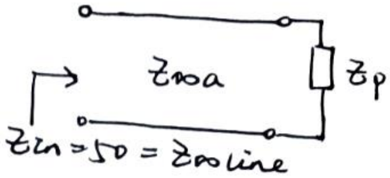


The equivalent impedance of the patch is then,

$$Z_p = \frac{1}{2G} \approx 160.15\,\Omega$$

The equivalent circuit of the middle section of the substrate can be shown as,





$$Z_{0a} = \sqrt{Z_{0line} \cdot Z_p} \approx 89.48\,\Omega$$

due to the transmission line theory, as with $\frac{\lambda}{4}$ matching.

With $Z_{0line} = 50\Omega$, $Z_{0a} = 89.48\Omega$

the width W_1 and W_a can be obtained using the functions,

$$\frac{W}{h} = \begin{cases} \dfrac{8e^A}{e^{2A} - 2} & \dfrac{W}{h} < 2 \\ \dfrac{2}{\pi}\left\{B - 1 - \ln(2B-1) + \dfrac{\varepsilon_r - 1}{2\varepsilon_r} C\right\} & \dfrac{W}{h} > 2 \end{cases}$$

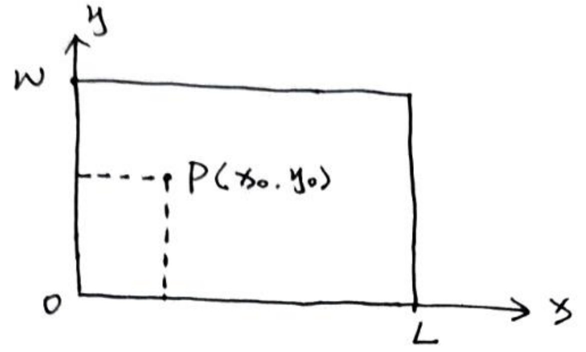
where
$$A = \frac{Z_0}{60}\sqrt{\frac{\varepsilon_r + 1}{2}} + \frac{\varepsilon_r - 1}{\varepsilon_r + 1}\left(0.23 + \frac{0.11}{\varepsilon_r}\right)$$

$$B = \frac{377\pi}{2 Z_0 \sqrt{\varepsilon_r}}$$

$$C = \ln(B-1) + 0.39 - \frac{0.61}{\varepsilon_r}$$

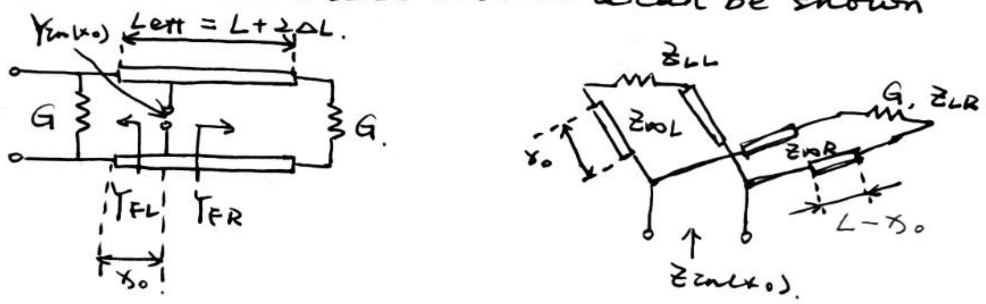
Finally,
$$W_1 = 0.0044 = 4.4\,\text{mm}$$
$$W_a = 0.0016 = 1.6\,\text{mm}$$
$$l_a = \frac{\lambda_g}{4} = 18.9\,\text{mm}.$$

Problem 3.3



Assuming the rectangular patch antenna is positioned in the coordinate system as above, with the feeding point at a random position of (x_0, y_0).

The equivalent transmission line model can be shown as.



$$Y_{in}(x_0) = Y_{FL}(x_0) + Y_{FR}(x_0).$$

and $G = \frac{W}{120\lambda_0}\left[1 - \frac{1}{24}\left(2\pi\frac{h}{\lambda_0}\right)^2\right] = \frac{1}{Z_{LR}} = \frac{1}{Z_{LL}}$

$$Y_{FR} = \frac{1}{Z_{0R}} \frac{Z_{0R} + jZ_{LR}\tan[k(L - x_0)]}{Z_{LR} + jZ_{0R}\tan[k(L-x_0)]}$$

$$Y_{FL} = \frac{1}{Z_{0L}} \frac{Z_{0L} + jZ_{LL}\tan(kx_0)}{Z_{LL} + jZ_{0L}\tan(kx_0)}$$

$Z_{0R} = Z_{0L}$, as Z_0 is only related to W, h and ε_r, with

$$Z_0 = \begin{cases} \frac{60}{\sqrt{\varepsilon_{r,eff}}}\ln\left(\frac{8h}{W} + \frac{h}{W}\right) & \frac{W}{h} < 1 \\ \frac{120\pi}{\sqrt{\varepsilon_{r,eff}}\left[\frac{W}{h} + 1.393 + 0.667\cdot\ln\left(\frac{W}{h} + 1.444\right)\right]} & \frac{W}{h} > 1 \end{cases}$$

where $\varepsilon_{r,eff} = \frac{\varepsilon_r + 1}{2} - \frac{\varepsilon_r - 1}{2}\left(1 + 12\frac{h}{W}\right)^{-\frac{1}{2}}$

Also, $Z_{LL} = Z_{LR} = \frac{1}{G}$

Therefore, the only difference of Y_{FR} and Y_{FL} is related on $\tan(kx_0)$ and $\tan[k(L-x_0)]$, where,

$$\tan[k(L-x_0)] = \frac{\tan kL - \tan kx_0}{1 + \tan kL \cdot \tan kx_0}$$

Then, Assume $Z_{\infty L} = Z_{\infty R} = Z_\infty$, $Z_{LL} = Z_{LR} = Z_L$.

Let $\tan kL = A$, $\tan kx_0 = B$

The combination function of $Y_{in}(x_0) = Y_{FR} + Y_{FL}$ could be simplified as,

$$Y_{in} = \frac{1}{Z_\infty} \frac{2Z_\infty Z_L(1-B^2) + j(Z_\infty^2 + Z_L^2)A(1+B^2)}{Z_L^2(1+AB) - Z_\infty^2 B(A-B) + jZ_L Z_\infty A(1+B^2)}.$$