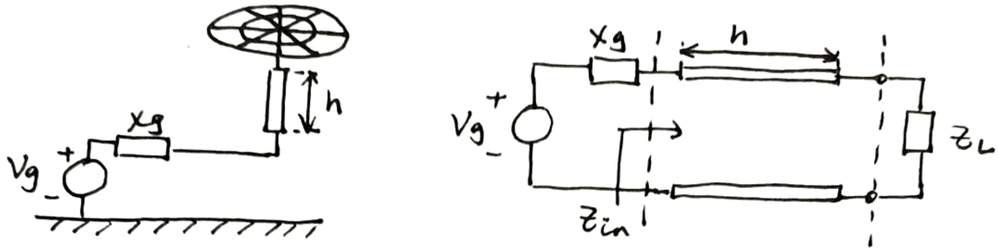
Homework No. 2 TONG Lin S287649

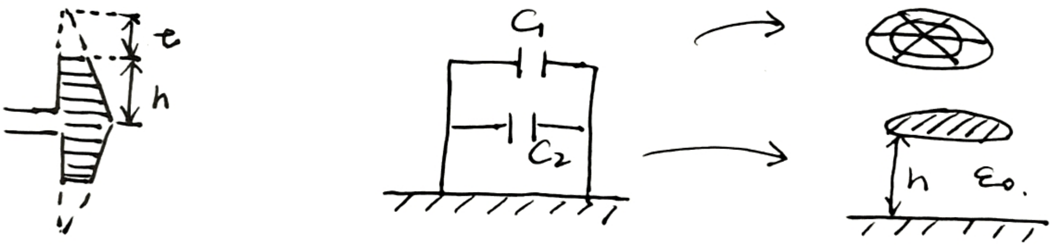
Problem 2.1



$$\lambda = \frac{c}{f} = \frac{3\times10^8}{2\times10^6} = 150\text{ m}$$

$$\frac{h}{\lambda} = \frac{4}{150} \approx 0.027 \ll 0.5$$

Assume that the current distribution of the dipole is a triangle distribution, where the equivalent capacitors have been placed.



$$C_1 = 8\varepsilon_0 a = 8\times 8.8542\times10^{-12}\times 1.5 \approx 1.06\times10^{-10}\text{F}$$

$$C_2 = \varepsilon_0\frac{A}{h} = 8.8542\times10^{-12}\times\frac{\pi\times1.5^2}{4} \approx 1.56\times10^{-11}\text{F}$$

$$C_{tot} = C_1 + C_2 = 12.16\times10^{-11}\text{ F}$$

With the total capacitance, the equivalent length of the segment line of t can be obtained,

$$t = \frac{\lambda}{2\pi}\cdot\tan(\omega C\cdot Z_{0m})$$

where the character impedance of the monopole is,

$$Z_{0m} = 60\left[\ln\left(\frac{2h}{a}\right)-1\right] \approx 341.08\,\Omega$$

Therefore, $t \approx 13.71$ m.

The equivalent full length of the dipole is then,

$$d = t + l = 17.71\ \text{m}.$$

The load impedance is,

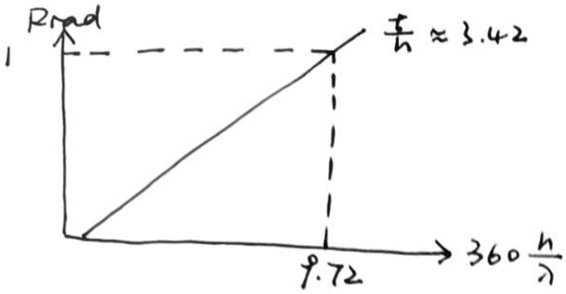
$$Z_L = \frac{1}{j\omega C_{tot}} \approx -6.54 \times 10^2 j$$

The input impedance is,

$$Z_{in} = Z_{om} \cdot \frac{Z_L + jZ_{om}\tan(kd)}{Z_{om} + jZ_L\tan(kd)} \approx j124\ \Omega$$

The analyzation of the matching network is required first the calculation of the radiation resistor, and having resonance, where $Im\{Z\} = 0$.

$$Z = Z_a + Z_{om}$$



$$360 \cdot \frac{h}{\lambda} = 9.72$$

$$\frac{t}{h} = \frac{13.71}{4} \approx 3.42$$

According to the figure above, $R_{rad} \approx 1\Omega$.

The imaginary of part of the resistor can be caculated as,

$$X_a = -Z_{om}\cot\frac{2\pi L}{\lambda}$$

where, $L = t + h = 17.71$

Therefore, $X_a \approx -372.17\ \Omega$

The radiation resistor is then $Z_a = 1 - j372.17\ [\Omega]$.

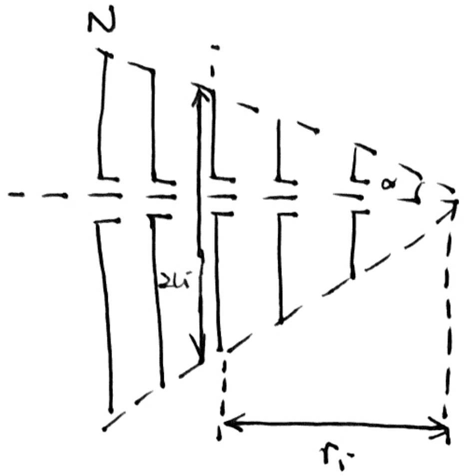
$$Z_{om} = -Im\{Z_a\} = 372.17\ \Omega.$$

The matching reactance shall therefore be an inductance with value of,

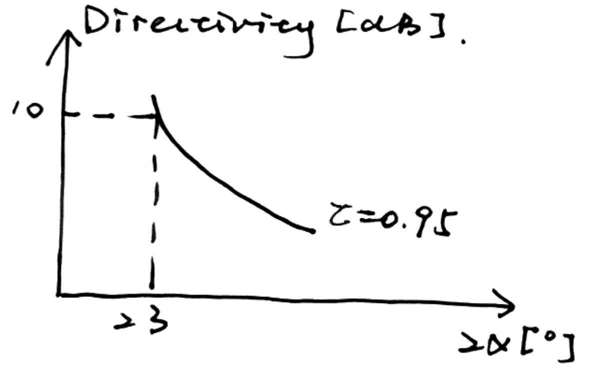
$$L = \frac{Z_{om}}{\omega} \approx 2.97 \times 10^{-5}\,\text{H}$$

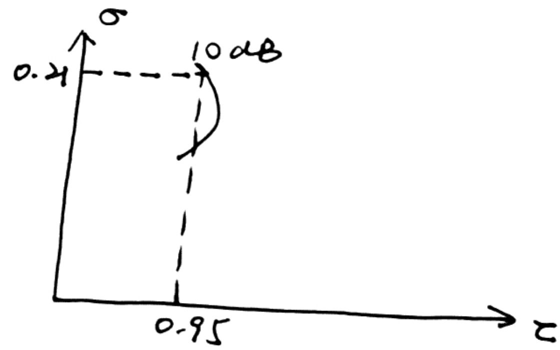
$$= 29.7\ \mu\text{H}$$

Problem 2.2



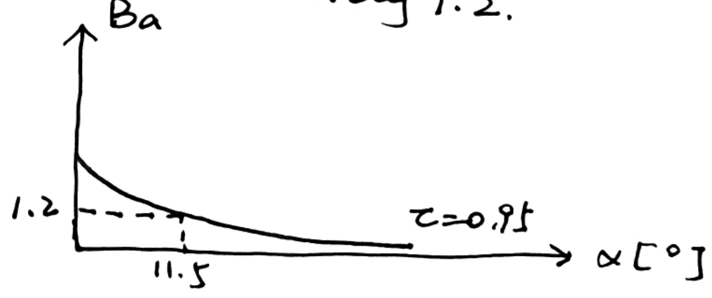
With the antenna gain of 10 dB, the values of angle α, scale factor τ and relative spacing σ can be obtained using the figures.





Therefore, $\alpha = 11.5°$, $\tau = 0.95$, $\sigma = 0.24$

The bandwidth factor Ba can be obtained also using figure, which is approximately 1.2.



The number of the elements can then be calculated,

$$N = 1 + \frac{\log \frac{f_{min}}{B_a f_{max}}}{\log \tau} = 1 + \frac{\log \frac{150\times10^6}{1.2\times 300\times 10^6}}{\log 0.95}$$

$$\simeq 18.07 \quad = \underline{19}$$

$$L_{max} = \frac{\lambda_{min}}{4} = \frac{c}{4 f_{min}} = \frac{3\times10^8}{4\times150\times10^6} = 0.5m.$$

$$L_{min} = \frac{\lambda_{max}}{4 B_a} = \frac{c}{4 f_{max} B_a} \simeq 0.21m.$$

Using the equations of

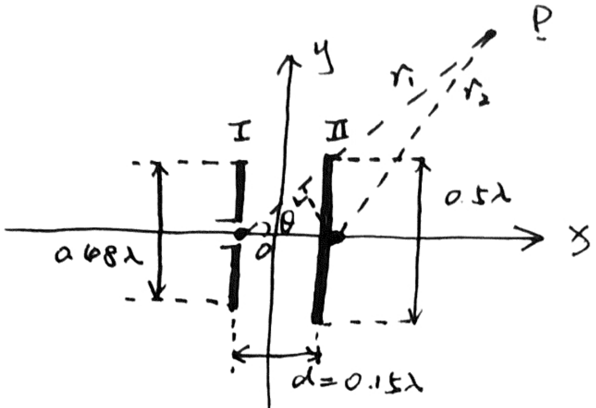
$$\frac{L_{i-1}}{L_i} = \frac{r_{i-1}}{r_i} = \tau = 0.95$$

$$\frac{L_i}{r_i} = \tan\alpha = \cancel{1.8114.}\ 0.2035$$

the legens and spacings can be easily obtained.

i	l_i [m]	r_i [m]
19	1.00	2.46
18	0.95	2.33
17	0.90	2.22
16	0.86	2.11
15	0.81	2.00
14	0.77	1.90
13	0.74	1.81
12	0.70	1.72
11	0.66	1.63
10	0.63	1.55
9	0.60	1.47
8	0.57	1.40
7	0.54	1.33
6	0.51	1.26
5	0.49	1.20
4	0.46	1.14
3	0.44	1.08
2	0.42	1.03
1	0.40	0.98

Problem 2.3



Assume that P is a point in the far field, the electric field of point P can be described as,

$$\underline{E}(P) = \underline{E}_1(P) + \underline{E}_2(P)$$

$$\begin{cases} \underline{E}_1(P) = -j\dfrac{Z_\infty}{2r_1\lambda}\cdot e^{-jkr_1}\cdot I_1\cdot e^{j\Phi_{I1}}\cdot \underline{h}_{eff1} \\ \underline{E}_2(P) = -j\dfrac{Z_\infty}{2r_2\lambda}\cdot e^{-jkr_2}\cdot I_2\cdot e^{j\Phi_{I2}}\cdot \underline{h}_{eff2} \end{cases}$$

Assuming both active and passive antennas have currents have a constant distribution, and as the point P is very far away from the dipoles, the value relations can be considered as satisfied as below,

$$\frac{|\underline{h}_{eff1}|}{|\underline{h}_{eff2}|} \approx \frac{|I_1|}{|I_2|} \approx \frac{0.48}{0.5} = 0.24$$

$$\Phi_{I1} \approx \Phi_{I2} = \Phi_I$$

$$\frac{1}{r} = \frac{1}{r_1} \approx \frac{1}{r_2} \quad (r_1 = r_2 + d\cos\theta)$$

Then the function of the electric field can be combined as,

$$\underline{E}(P) = -j\frac{Z_\infty}{2r\lambda}\left(\underline{h}_{eff1}\cdot e^{-jkr_1} I_1 e^{j\Phi_{I1}} + \underline{h}_{eff2}\cdot e^{-jkr_2} I_2 e^{j\Phi_{I2}}\right)$$

$$= -j\frac{Z_\infty}{2r\lambda}\ \underline{h}_{eff1}\cdot e^{-jkr_1} I_1 e^{j\Phi_{I1}}\left[1 + \frac{|\underline{h}_{eff2}|}{|\underline{h}_{eff1}|}\cdot\frac{|I_2|}{|I_1|}\cdot e^{-jk(r_2-r_1)}\cdot e^{j\overset{0}{(\Phi_{I2}-\Phi_{I1})}}\right] \quad 1$$

$$= \underline{E}_1(P)\cdot\left(1 + 0.0576\cdot e^{+j[k d\cos\theta + (\Phi_{I2}-\Phi_{I1})]}\right)$$

Therefore,

$$AF = 1 + 0.0576 \times e^{-j(kd\cos\theta + \Phi_{I2} - \Phi_{I1})} \qquad \theta\in[0,\pi]$$

for $\underline{E}_1(P)$,

$$\underline{E}_1(P) = -j\frac{Z_0}{2r_1\lambda}\cdot e^{-jkr_1}\cdot \underline{P}(\hat{r}).$$

$$\underline{P}(\hat{r}) = \underline{\underline{I}}_{tr}\hat{r}\cdot \mathcal{F}_3\{\underline{J}_e(\underline{r}')\}$$

$$= (\hat{\theta}\hat{\theta}+\hat{\varphi}\hat{\varphi})\cdot\hat{z}\int_{-\frac{L}{2}}^{\frac{L}{2}} I_1(z)\cdot e^{-jk\hat{r}_1\cdot\hat{z}\cdot z}\,dz.$$

$$= (\hat{\theta}\hat{\theta}+\hat{\varphi}\hat{\varphi})\cdot\hat{z}\cdot|I_1|\cdot \cancel{e^{j\varphi_1}}\, e^{j\Phi_1}\int_{-\frac{L}{2}}^{\frac{L}{2}} e^{jkz\cos\theta}\,dz.$$

$$= (\hat{\theta}\hat{\theta}+\hat{\varphi}\hat{\varphi})\cdot\hat{z}\cdot|I_1|\cdot e^{j\Phi_1}\cdot l_1\,\mathrm{sinc}(\tfrac{1}{2}kl\cos\theta).$$

$$= (-\sin\theta)\cdot\hat{\theta}\cdot|I_1|\cdot l_1\cdot\mathrm{sinc}(\tfrac{1}{2}kl\cos\theta)$$

for $l\to 0$ when $r \gg l$, therefore,

$$\underline{P}(\hat{r}) \approx (-\sin\theta)\,\hat{\theta}\cdot|I_1|\cdot l_1.$$

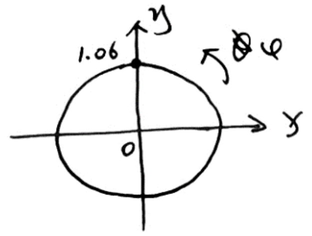
$$\underline{E}_1(P) \approx -j\frac{Z_0}{2r_1\lambda}\cdot e^{-jkr_1}\cdot(-\sin\theta)\,\hat{\theta}\cdot|I_1|\cdot l_1.$$

The analyzation of the radiation pattern is then,

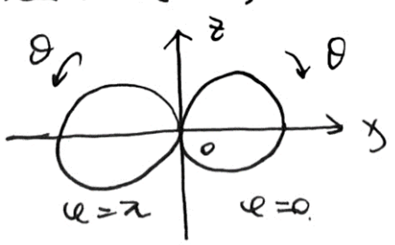
$$\frac{|\underline{E}(P)|}{\max[\underline{E}(P)]} = \frac{|\underline{E}_1(P)|}{\max|\underline{E}_1(P)|}\cdot|AF|$$

$$\approx 1.06\sin\theta$$

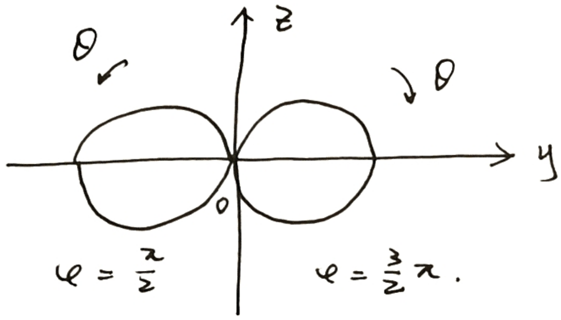
In (x,y) plane, where $\theta=\frac{\pi}{2}$, $\varphi\in[0,2\pi]$.



In (x,z) plane, where $\varphi=0$, $\theta\in[0,\pi]$.



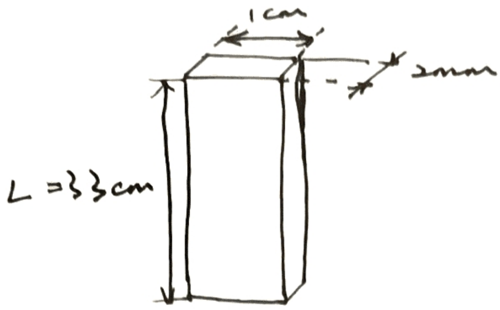
In (y, z) phane, where $\varphi = \frac{\pi}{2}$, $\theta \in [0, \pi]$, the pattern is simiiler with the one in (x, z) plane.

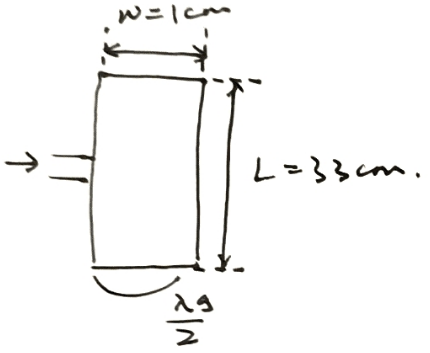


The front-to-back ratio is,

$$\left|\frac{AF(\theta=0)}{AF(\theta=\pi)}\right| = \left|\frac{1+0.0576\, e^{-j(1+\Phi_{I2}-\Phi_{I1})}}{1+0.0576\, e^{-j(-1+\Phi_{I2}-\Phi_{I1})}}\right|$$

Problem 2.4





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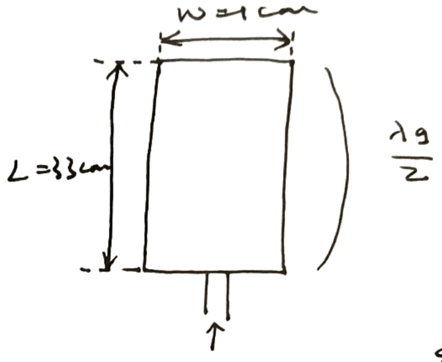

$\varepsilon_r = \varepsilon_0$

②

① $$\varepsilon_{r,eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12 \cdot \frac{L}{W}\right]^{-\frac{1}{2}} \approx 0.48$$

$$f_{res} = \frac{c}{\sqrt{\varepsilon_{r,eff}} \cdot \lambda_g} \approx \cancel{2.18}\ 21.8\ \text{GHz}$$

② $$\varepsilon_{r,eff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\,\frac{W}{L}\right]^{-\frac{1}{2}} \approx 0.07$$

$$f_{res} = \frac{c}{\sqrt{\varepsilon_{r,eff}} \cdot \lambda_g} \approx 1.70\ \text{GHz}.$$

Problem 2.5