Communication System Assignment 2

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Exercise 1.1 Properties of 5G PSS m-sequence

In the first exercise, a primary synchronization signal (PSS) using m-sequence has been generated by a linear feedback shift register with 7 and the starting seed is [1 1 1 0 1 1 0], the polynomial description is,

$$p_1(D) = D^7 + D^4 + 1$$

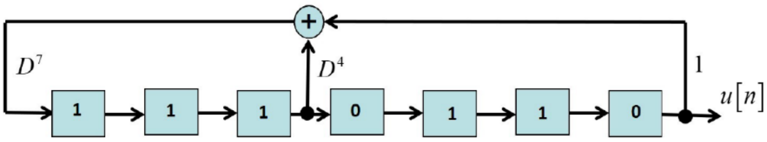

Figure 1: The block of LFSR

After transferring all the 0 in the generated sequence u(n) into -1, we can get the generated 127 symbols of PSS m-sequence b(n) as presents in Figure 2.

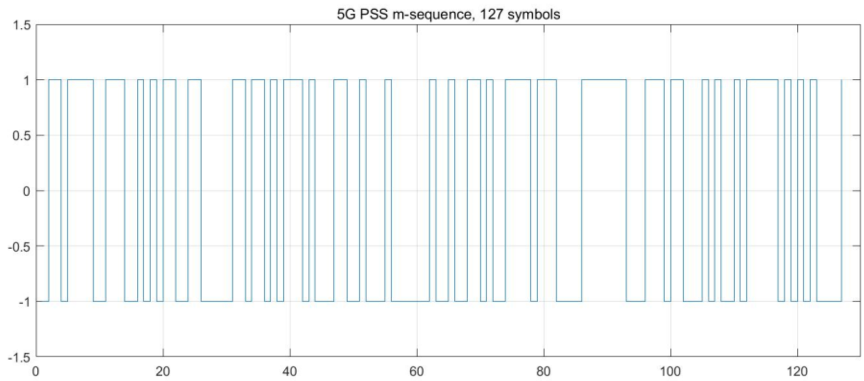

Figure 2: The plot of 127-symbol of bipolar sequence b(n)

The numbers of 0s and 1s in the originally generated sequence u(n), N_0 and N_1, is shown in table 1. Meanwhile the numbers of no-transition and transition, N_{NT} and N_T, has also been counted and shown in table 2, where no-transition indicates the element in the same position has been change from 0 to 0 or 1 to 1, and transition indicates that the change is from 1 to 0 or 0 to 1.

Table 1: Numbers of 0 and 1 in u(n)

N_0	63
N_1	64

Table 2: Numbers of no-transition and transition

N_{NT}	63
N_T	64

Both the two properties in Table 1 and 2 are well verified as the property of m-sequence, which the relationships are shown as,

$$N_1 = N_0 + 1$$

$$N_T = N_{NT} + 1$$

The numbers of length-i run of 0 and 1, $NR_0(i)$ and $NR_1(i)$, are present in Table 3. For example, number of i=2 of 0-runs is 8, indicates that consecutively repeated 0 of twice has been seen for 8 times. The same property is shown in Figure 4.

Table 3: $NR_0(i)$ and $NR_1(i)$

LENGTH	0-RUNS	1-RUNS
1	16	16
2	8	8
3	4	4
4	2	2
5	1	1
6	1	0
7	0	1
SUM	32	32

The statistics are well fitted with the standard table shown in Figure 3, with m=7.

length	0-runs	1-runs
1	2^{m-3}	2^{m-3}
2	2^{m-4}	2^{m-4}
$\vdots$	$\vdots$	$\vdots$
r	2^{m-r-2}	2^{m-r-2}
$\vdots$	$\vdots$	$\vdots$
$m-2$	1	1
$m-1$	1	0
m	0	1
Totals:	2^{m-2}	2^{m-2}

Figure 3: Standard table of $NR_0(i)$ and $NR_1(i)$

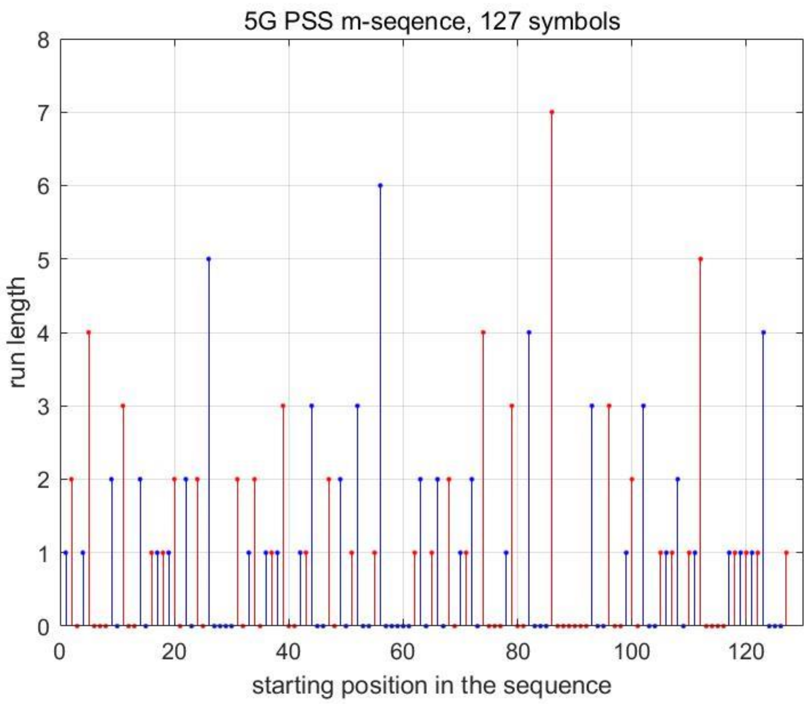

Figure 4: Plot of the run lengths vs. their starting points in the sequence

Using the periodic autocorrelation function, Figure 5 has been plotted, where N=127.

$$R(\tau) = \sum_{n=0}^{N-1} b(n)b(n-\tau) \ , \qquad -(N-1) \leq \tau \leq N-1$$

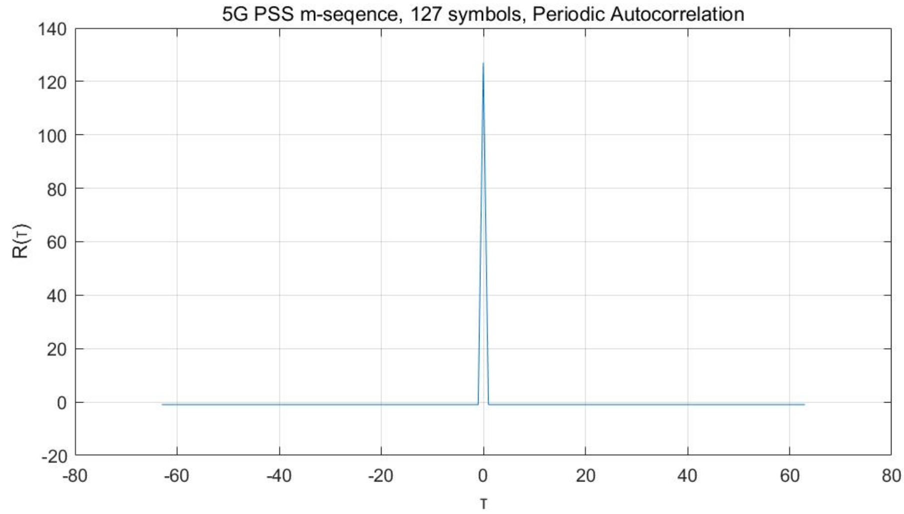

Figure 5: Periodical autocorrelation

It can be easy to verify that, when τ equals to 0, R(τ)=N, and as long as τ is not equals to 0, R(τ)=-1.

Exercise 1.2 Properties of truncated m-sequence

The property MPSL is discussed here in exercise 1.2, with the follow function,

$$MPSL = max_{\tau \neq 0}|R(\tau)|$$

When the last 10 bits of the sequence have been truncated, MPSL=13.

However, when changing the starting seed into [0 0 1 1 1 0 1], MPSL=17.

As we can see, the maximum value of periodic autocorrelation (except for the time τ equals to 0) is remain unchanged only if the statistics are complete, and it fits the following function,

$$MPSL = 2^{\frac{m+1}{2}} + 1$$

Exercise 1.3 Cross-correlation of m-sequence

In this exercise, the primitive polynomial has been changed into p2 and p3, where the functions are presented as follow,

$$p_2(D) = D^7 + D^3 + D^2 + D + 1$$

$$p_3(D) = D^7 + D + 1$$

The function periodic cross-correlation is as follow, where b1 stands for the sequence generated by p1, and b2 stands for the sequence generated by p2. Same approach is also applied with p3.

$$R_{12}(\tau) = \sum_{n=0}^{N-1} b_1(n)b_2(n-\tau) \ , \qquad -(N-1) \leq \tau \leq N-1$$

The cross-correlations are shown in Figure 6 and 7.

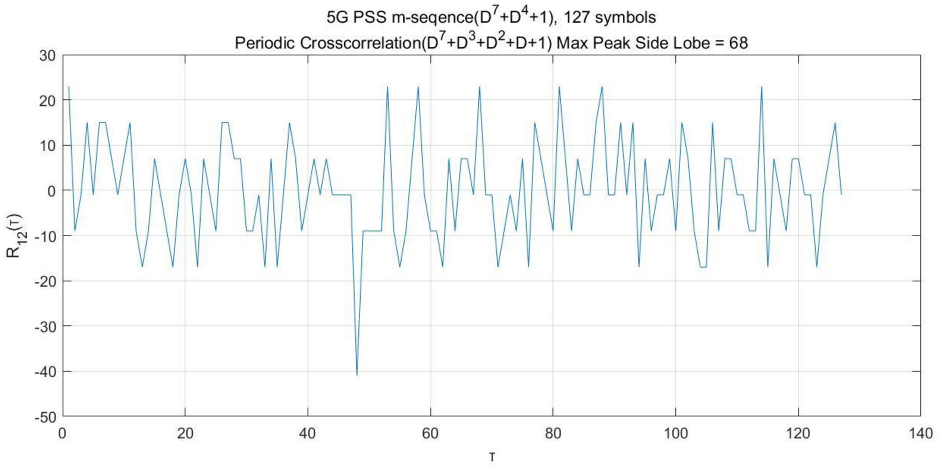

Figure 6: The cross-correlations between p1 and p2

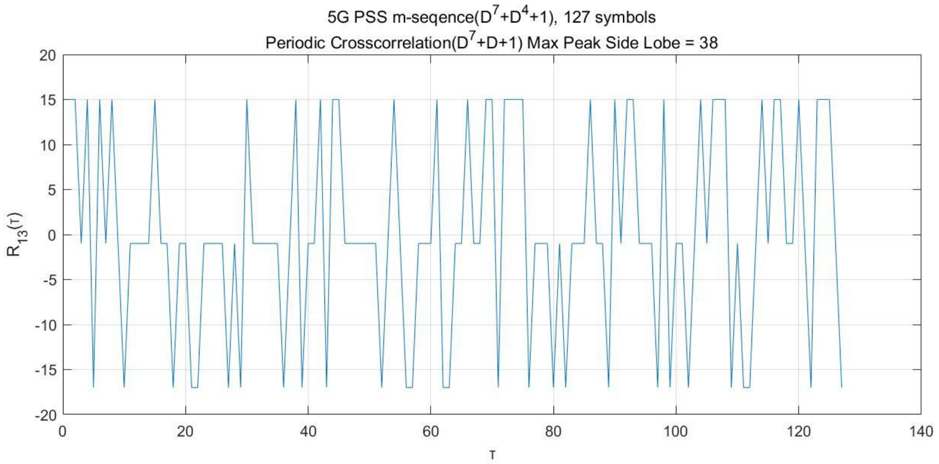

Figure 7: The cross-correlation between p1 and p3

We can conclude that, when the polynomial descriptions are changed, the signals have bad cross correlations.

Exercise 2: PSS detection

In this exercise, we first simulate a approximate real signal with inserting a PSS sequence and Additive White Gaussian Noise, then evaluate the cross correlation between the shifted signal and the original PSS sequence. As shown in Figure 8, the relationship has been well presented.

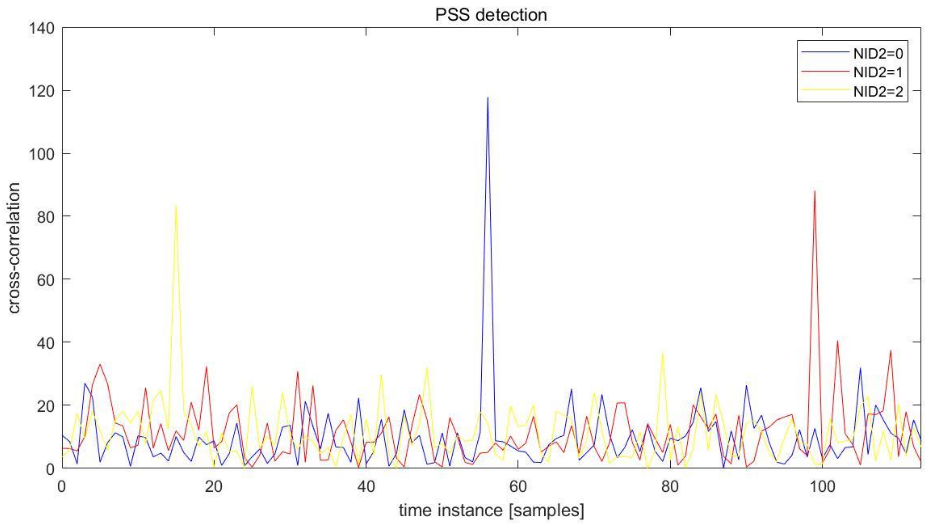

Figure 8: PSS detection