REMS Assignment 6 TONG LIN S287649

Problem 6.1

a) Radiation Pattern:

The electric field of the array antenna can be expressed as. Where the center frequency at 97MHz.

$$\underline{E}(\theta,\varphi) = -j\frac{Z_0}{2\lambda}\frac{e^{-jkr}}{r}\cdot \underline{P}(\hat{r}).$$

$$= \underline{E}_1(\theta,\varphi)\cdot AF$$

Where the electric field radiated by a single dipole can be expressed as below, whose radiation pattern is same as a dipole.

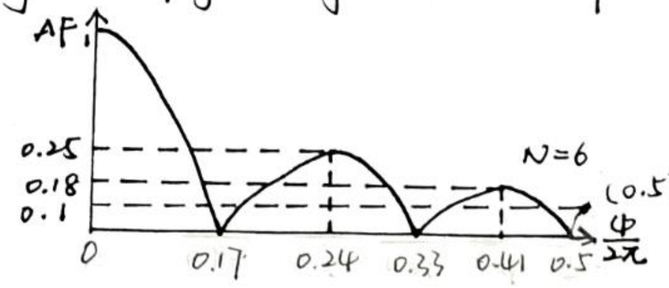
$$\underline{E}_1(\theta,\varphi) = -j\frac{Z_0}{2\lambda}\frac{1}{r_1}\underline{\hat{P}}_1(\theta,\varphi)\cdot h_{eff}\cdot e^{jkr_1}\cdot \underline{I}_1$$

The normalized area factor is,

$$AF = \frac{1}{N}\left|\frac{\sin\frac{N\psi}{2}}{\sin\frac{\psi}{2}}\right|$$

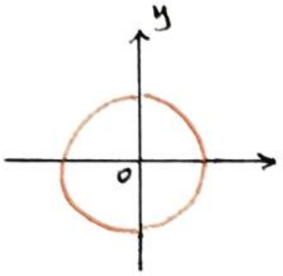
where $\psi = kd\cos\theta + \Phi$, Φ is assumed equal for each element in the uniform array. The visible range $\theta\in[0,\pi]$, where $\psi/2\pi \in [-0.53, 0.53]$

Checking the figure of the area factor with $N=6$,

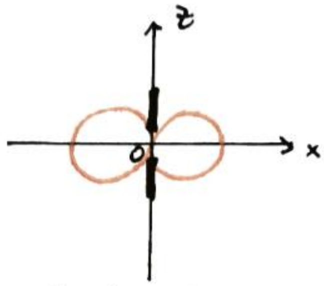


The total radiation pattern can be plotted as the dipole radiation pattern times the AF radiation pattern.

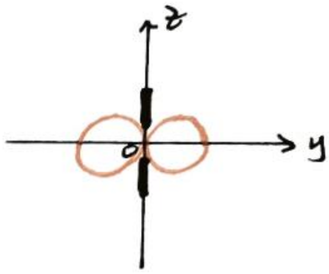
The radiation pattern of the dipole at each planes are,



(x.y) plane

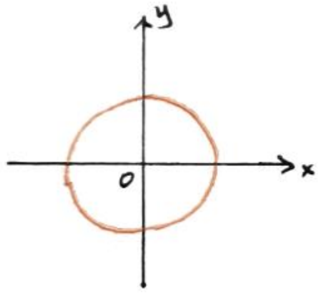


(x. z) plane

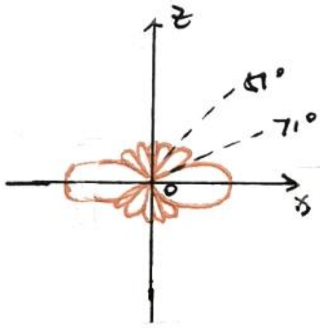


(y. z) plane

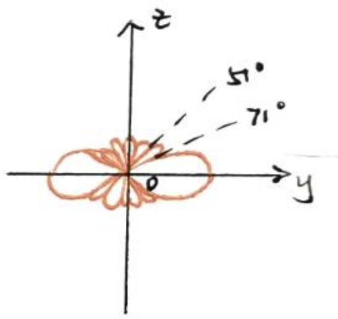
The radiation patterns of the area factor at each plane are.



(x.y) plane

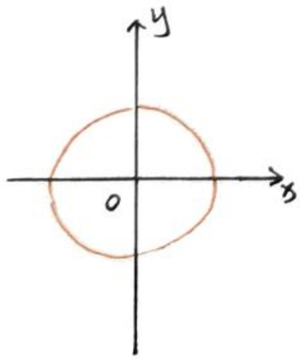


(x. z) plane

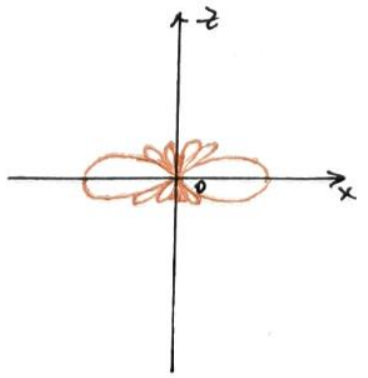


(y. z) plane

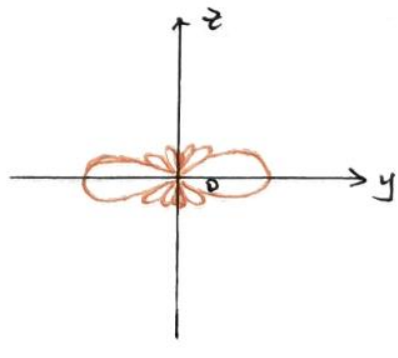
The total radiation patterns at each plane are,



(x.y) plane



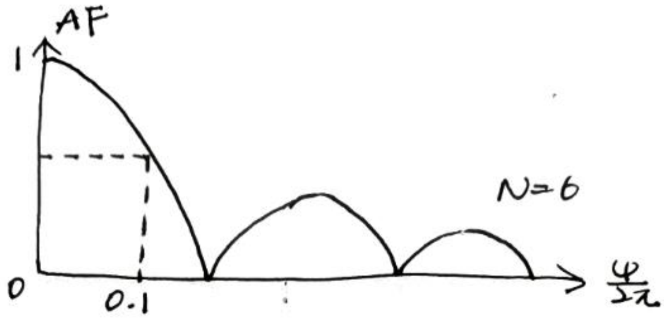
(x. z) plane



(y. z) plane

b) HPBW and aperture of the main beam:

According to the figure of the Area Factor with $\frac{\psi}{2\pi}$,



With $\frac{\psi}{2\pi} = \frac{kd}{2\pi}\cos\theta = 0.1$, $0.53\cos\theta = 0.1$,

therefore $\theta = 1.3810$ [rad] $\simeq 79.12°$

$$HPBW = 2 \times (90° - 79.12°) = 21.76°$$

c) Maximum gain:

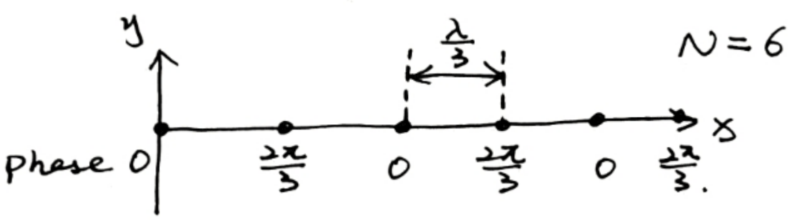
The gain of the antenna can be calculated from,

$$G = \frac{\frac{dP}{d\Sigma}}{\left(\frac{dP}{d\Sigma}\right)_{ISO}}$$

$$= G_0 \cdot \sum_{i=1}^{N} g_i(\theta, \varphi).$$

Problem 6.2

1. Qualitative radiation pattern:



The electric-field is expressed as,

$$\underline{\underline{E}}(\theta,\varphi) = \underline{\underline{E}}_1(\theta,\varphi)\cdot AF$$

where, $\underline{E}_1(\theta,\varphi) = -j\frac{Z_0}{2\lambda}\frac{1}{r_1}\hat{P}_1(\theta,\varphi)h_{eff}\cdot e^{jkr_1}\cdot I_1$

$$AF = \sum_{i=1}^{N} e^{-jk(r_i-r_1)}\cdot e^{(\varphi_i-\varphi_1)j}.$$

As the spaces between the elements are equal,

$$r_i - r_1 = -(i-1)d\cos\theta.$$

However, the phases of the elements are different.

$$\varphi_2-\varphi_1 = \frac{2\pi}{3} \qquad \varphi_5-\varphi_1=0$$

$$\varphi_3-\varphi_1 = 0 \qquad \varphi_6-\varphi_1=\frac{2\pi}{3}.$$

$$\varphi_4-\varphi_1 = \frac{2\pi}{3}$$

Therefore,

$$AF = \sum_{i=1}^{N} e^{j[k(i-1)d\cos\theta + (\varphi_i-\varphi_1)]}.$$

Notating $x_i = k(i-1)d\cos\theta + (\varphi_i-\varphi_1)$.

When, $i=1,3,5$, $x_i = k(i-1)d\cos\theta$

$i=2,4,6$, $x_i = k(i-1)d\cos\theta + \frac{2\pi}{3}$.

Then, the sum can be calculated respectivity as two parts.

$$AF_1 = \sum_{i=1,3,5} e^{jk(i-1)d\cos\theta} = \sum_{n=0,2,4} e^{jknd\cos\theta}$$

$$= e^{j2kd\cos\theta}\cdot\frac{\sin(jkd\cos\theta\cdot 3)}{\sin(jkd\cos\theta)}$$

$$AF_2 = \sum_{i=2,4,6} e^{jk(i-1)d\cos\theta+\frac{2\pi}{3}} = e^{\frac{2\pi}{3}}\cdot\sum_{i=2,4,6} e^{jk(i-1)d\cos\theta}.$$

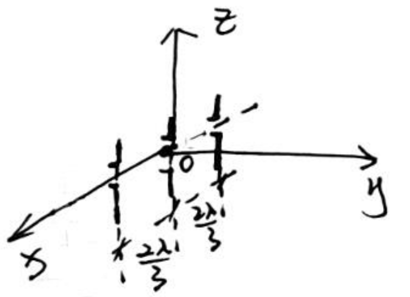
$$= e^{j3kd\cos\theta+\frac{2\pi}{3}}\cdot\frac{\sin(jkd\cos\theta\cdot 3)}{\sin(jkd\cos\theta)}$$

Then, the total area factor is,

$$AF = AF_1 + AF_2$$
$$= e^{2jkd\cos\theta} \frac{\sin(jkd\cos\theta \cdot 3)}{\sin(jkd\cos\theta)} (1 + e^{jkd\cos\theta + \frac{2\pi}{3}})$$

OR, the full array antenna can be separated as two equi-spaced and constant-phased array antenna, with both $N=3$. Setting the original point at third element.

The first array antenna is like,

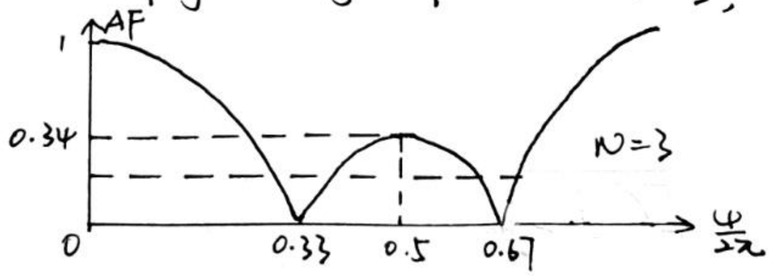


$$AF_1 = \sum_{i=1}^{3} e^{j(i-1)\psi}$$
$$\psi = kd\cos\theta + \Phi$$
where, $d = \frac{2\lambda}{3}$, $\Phi = 0$

Thus, $\psi = \frac{2\pi}{\lambda} \cdot \frac{2\lambda}{3} \cos\theta = 2\pi \cdot \frac{2}{3}\cos\theta$.

$$\frac{\psi}{2\pi} = \frac{2}{3}\cos\theta.$$

The visible range with $\theta \in [0, \pi]$, with $\frac{\psi}{2\pi} \in [-0.67, 0.67]$.
According to the figure of AF with $N=3$,



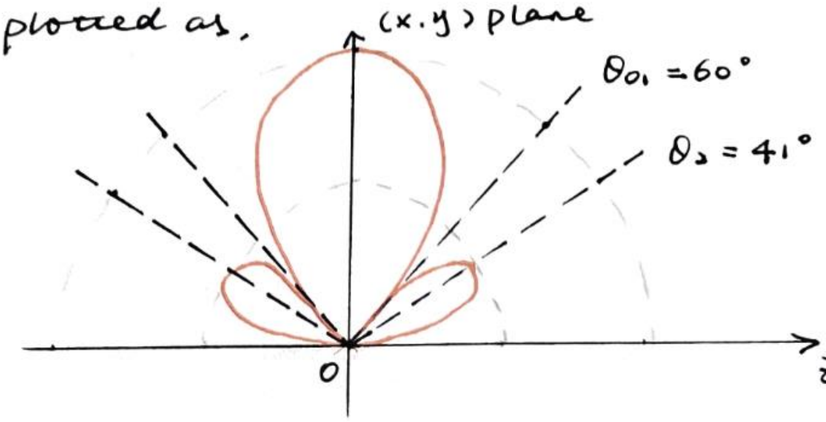
The first-zero is at, $\frac{\psi_{01}}{2\pi} = 0.33 = \frac{2}{3}\cos\theta_{01} \to \theta_{01} \simeq 60°$

The first-maximum value is at, $\frac{\psi_1}{2\pi} = 0 = \frac{2}{3}\cos\theta_1$,
$\theta_1 = 90°$

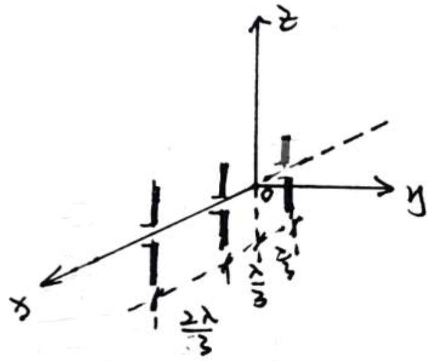
The second-maximum value is at, $\frac{\psi_2}{2\pi} = 0.5 = \frac{2}{3}\cos\theta_2$,
$\theta_2 \simeq 41°$.

In addition, the second-zero is exactly at $\theta = 0$ or π.

The radiation pattern of the first array antenna can be roughly plotted as,



The second antenna array is,



$$AF_2 = \sum_{i=1}^{3} e^{j(i-1)\psi}$$

$$\psi = kd\cos\theta + \Phi$$

where, $d = \frac{2\lambda}{3}$, $\Phi = \frac{2\pi}{3}$

Thus, $\psi = \frac{2\pi}{\lambda} \cdot \frac{2\lambda}{3}\cos\theta + \frac{2\pi}{3}$

$$\frac{\psi}{2\pi} = \frac{2}{3}\cos\theta + \frac{1}{3}$$

The visible range $\theta \in [0, \pi]$, is $\frac{\psi}{2\pi} \in [-0.33, 1]$

With the ~~first~~ zeros at $\frac{\psi_{01}}{2\pi} = \frac{2}{3}\cos\theta_{01} + \frac{1}{3} = 0.33$, $\frac{\psi_{02}}{2\pi} = 0.67$,

$\theta_{01} = 1.5758\,[rad] \simeq 90°$, $\theta_{02} = 1.0414\,[rad] \simeq 60°$

The first-maximum value at $\frac{\psi_{11}}{2\pi} = \frac{2}{3}\cos\theta_1 + \frac{1}{3} = 0$, $\frac{\psi_{12}}{2\pi} = 1$,

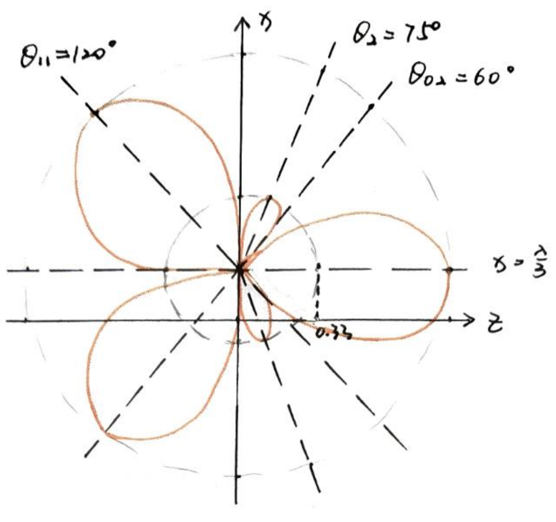
$\theta_{11} = 2.0944\,[rad] \simeq 120°$, $\theta_{12} = \cancel{180°}\ 0°$.

The second-maximum value at $\frac{\psi_2}{2\pi} = \frac{2}{3}\cos\theta_2 + \frac{1}{3} = 0.5$,

$\theta_2 = 1.3181\,[rad] \simeq 75°$

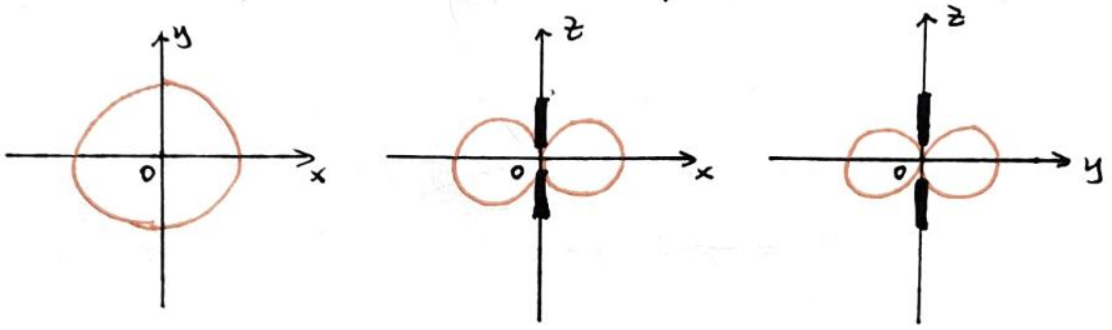
At $\theta = 0$, $AF = 1$, At $\theta = \pi$, $AF = 0$.

The radiation pattern of the second array antenna can be roughly plotted as.

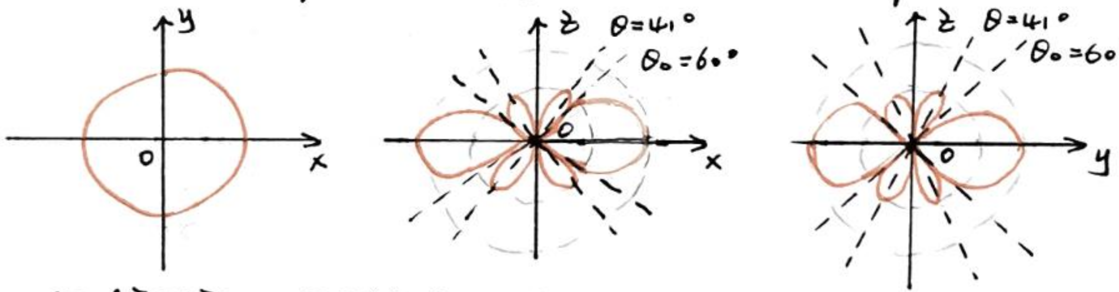


The total radiation pattern is the radiation pattern of the dipole times the sum of the two area factors.

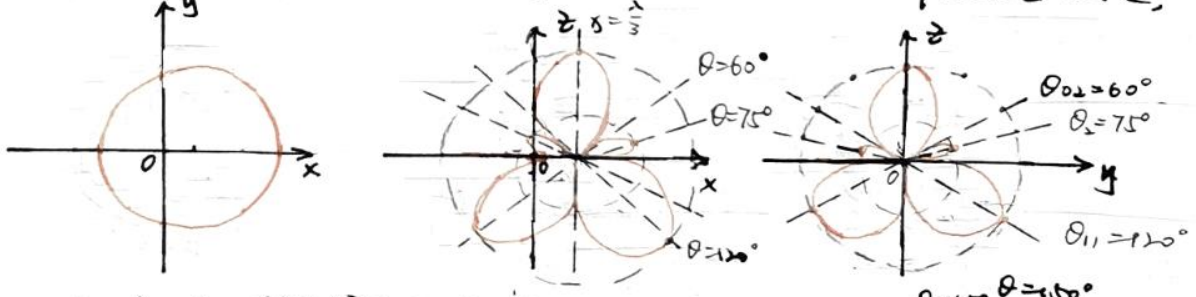
The radiation pattern of the dipole at each plane are,



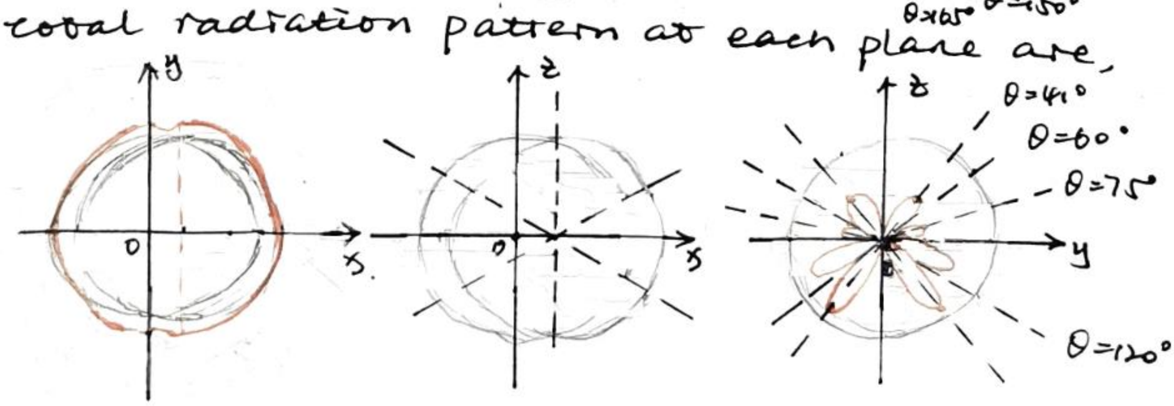
The radiation pattern of AF1 at each plane are.



The radiation pattern of AF2 at each plane are,



The total radiation pattern at each plane are,



Problem 6.3

With HPBW < 7°, the first-zeros at right and left sides are.

$$\theta_{\frac{1}{2}L} = 90° - \frac{7°}{2} = 86.5°$$

$$\theta_{\frac{1}{2}R} = 90° + \frac{7°}{2} = 93.5°$$

At the position of HPBW, AF[dB] = -3 dB.
Assumming that $\Phi = 0$, and the array is symmetry.

$$AF[dB] = 10 \log_{10}\left[\left| \frac{\sin(\frac{N\psi}{2})}{\sin(\frac{\psi}{2})} \right| \right]$$

where $\psi = kd\cos\theta_{\frac{1}{2}R} = \frac{2\pi}{\lambda} \cdot d \cdot \cos\theta_{\frac{1}{2}R} = 2\pi \cdot 0.6 \cdot \cos 93.5°$
$\simeq -0.2301°$

The value of N can be then solve as $N = 13.1525$ or $N = 0.4979$.

As HPBW < 7°, thus $\theta_{\frac{1}{2}R}$ shall be smaller, and $N \neq 0$.

Therefore, $N = 13$