Electromagnetic fields and biological tissues: effects and medical applications

Please **initialize** individual items of the declaration, and **sign** it at bottom.

Upon my word of honor, and aware of the consequences of a false declaration under the Italian law, as well as those deriving from unfair conduct at Politecnico,

I, the undersigned Tong Lin

ID n. (matricola) S287649

Hereby declare (*dichiarazione sostitutiva di atto notorio*) that the home assignment n. 3

Has been carried out in a strictly individual manner from beginning to end; in particular,

TL I have not obtained help from any classmate or external person to carry out in part or whole the assignment;

TL I have not employed any paper or electronic material directly related to the assignment; (note: textbooks are indirectly related only)

TL I have not employed scripts, computer programs or any other such procedures that have not been entirely developed by myself, or provided as course material (by the Instructor and/or the Teaching Assistant), and that are not commercial, or cannot be referenced in the open literature or internet; please note that *all employed software not personally and individually developed must be referenced in the submitted papers*. In particular, I have not employed any script, programs etc. developed by my classmates, and that the employed scripts, programs etc. have not been developed in cooperation with my classmates.

TL I have discussed this assignment with the following persons: (enter "none" if appropriate):

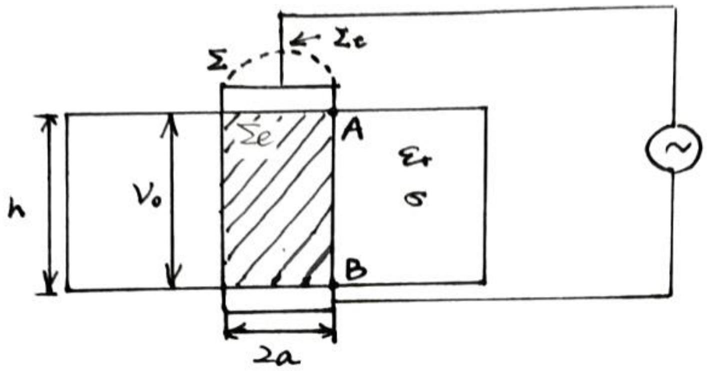
Tong Lin
(Complete name, please print)

Tong Lin
signature

Torino, 2022/5/4 (date)

Note: Use of commercial software, of free-ware or shareware, or otherwise publicly available software (e.g. via Internet) is allowed, but usage of all software not developed personally and individually by the student, or provided as course material, MUST be clearly stated and precisely referenced in the submitted paper.

Problem No. 1

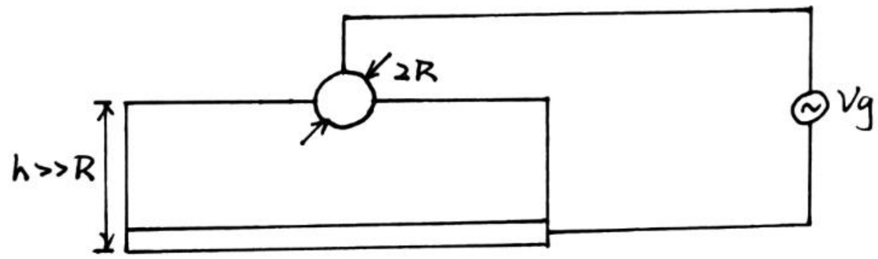


The relationship between the electric field E_0 and the applied voltage V_0 using the "voltage" law can be expressed as,

$$V_0 = \int_A^B E_0 \, ds$$

$$V_0 = E_0 \cdot h$$

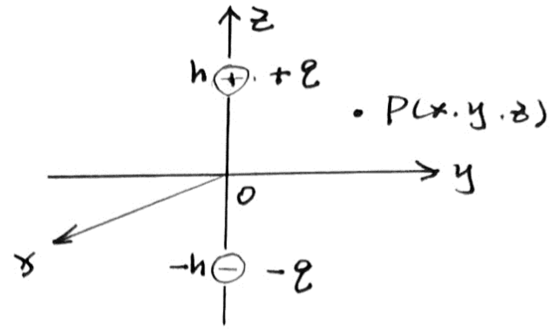
Problem No. 2



1. Expressions of $\Phi(P)$ and $E(P)$:

The magnetic field and electric field of the spherical electrode can be seen as the field generated by two opposite point charges placed respectively upon and under the flat return electrode with the same distant of h, as the distant h is much larger than the radius R, thus the flat electrode can played as a "mirror".

The equivalent structure can be shown as the schematic below according to the 'Image Theorem'.



Assuming the point P is placed at a random point (x.y.z).

According to the previous assignment 1, the electric field can be expressed as.

$$\underline{E}(\underline{P}) = k\cdot q\left\{\frac{x\hat{x}+y\cdot\hat{y}+(z-h)\hat{z}}{[x^2+y^2+(z-h)^2]^{\frac{3}{2}}}-\frac{x\hat{x}+y\hat{y}+(z+h)\hat{z}}{[x^2+y^2+(z+h)^2]^{\frac{3}{2}}}\right\}$$

The potential on the other hand can be analyzed using the function of the potential generated by point charges.

$$\Phi(\underline{P}) = \sum_n \Phi_n(\underline{P})$$

$$\Phi_n(\underline{P}) = \frac{kq_n}{|\underline{P}-\underline{C_n}|}$$

where $C_1 = (0.0.h)$, $C_2 = (0.0.-h)$, therefore.

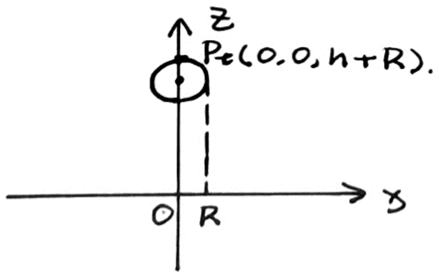
$$\Phi_1(\underline{P}) = \frac{kq}{\sqrt{x^2+y^2+(z-h)^2}}$$

$$\Phi_2(\underline{P}) = \frac{-kq}{\sqrt{x^2+y^2+(z+h)^2}}$$

Thus.

$$\Phi(\underline{P}) = kq\left[\frac{1}{\sqrt{x^2+y^2+(z-h)^2}}-\frac{1}{\sqrt{x^2+y^2+(z+h)^2}}\right]$$

2. Determine the charge kq :



The applied voltage Vg have the same value as the potential at the top of the spherical electrode at point Pt(0.0.h+R).

$$V_g = \Phi(\underline{P_t})$$

The potential at P_t is,

$$\Phi(\underline{P_t}) = kq\left[\frac{1}{R} - \frac{1}{(2h+R)}\right]$$
$$= kq \cdot \frac{2h}{R(2h+R)}$$

Applying $V_g = \Phi(\underline{P_t})$, therefore,

$$kq = \frac{V_g R(2h+R)}{2h}$$

As $R \ll h$, the equation can be approximately simplified as,

$$kq = V_g \cdot R.$$

3. The plot of $A(\underline{P}) = |\underline{E}(\underline{P})|^2$:

The electric field is,

$$\underline{E}(\underline{P}) = kq\left\{\frac{x\hat{x} + y\hat{y} + (z-h)\hat{z}}{[x^2+y^2+(z-h)^2]^{\frac{3}{2}}} - \frac{x\hat{x} + y\hat{y} + (z+h)\hat{z}}{[x^2+y^2+(z+h)^2]^{\frac{3}{2}}}\right\}$$

The following analyzation and plots are displayed at (x, z) plane.

The seperate components along x axis and z axis are expressed as,

$$|E_x| = kq\left\{\frac{x}{[x^2+(z-h)^2]^{\frac{3}{2}}} - \frac{x}{[x^2+(z+h)^2]^{\frac{3}{2}}}\right\}$$
$$|E_z| = kq\left\{\frac{z-h}{[x^2+(z-h)^2]^{\frac{3}{2}}} - \frac{z+h}{[x^2+(z+h)^2]^{\frac{3}{2}}}\right\}$$

where $kq = \dfrac{V_g \cdot R(2h+R)}{2h} = V_g \cdot R.$

The amplitude is finally,

$$A(\underline{P}) = |E_x|^2 + |E_z|^2$$

The final results plotted using MATLAB are as shown in Figure 2.1. and Figure 2.3, which indicate respectively the cases of $R = 5\,mm$ and $R = 0.05\,mm$.

Figure 2.2 and 2.3 are the 'surf' plots of the amplitude.

As shown in the figures, the electric field are concentrated around the surface of the spherical electrode, and are constant in space between the two electrodes.

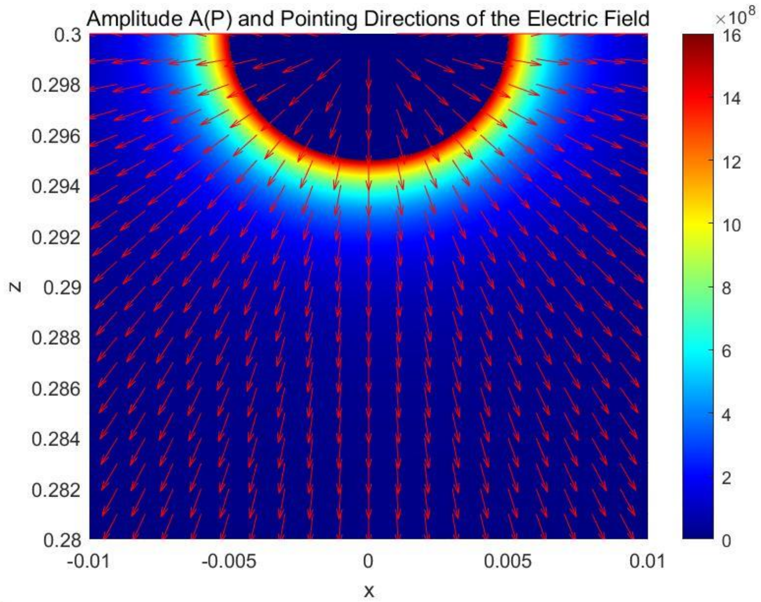


Figure 2.1: Electric field and pointing direction with R = 5mm

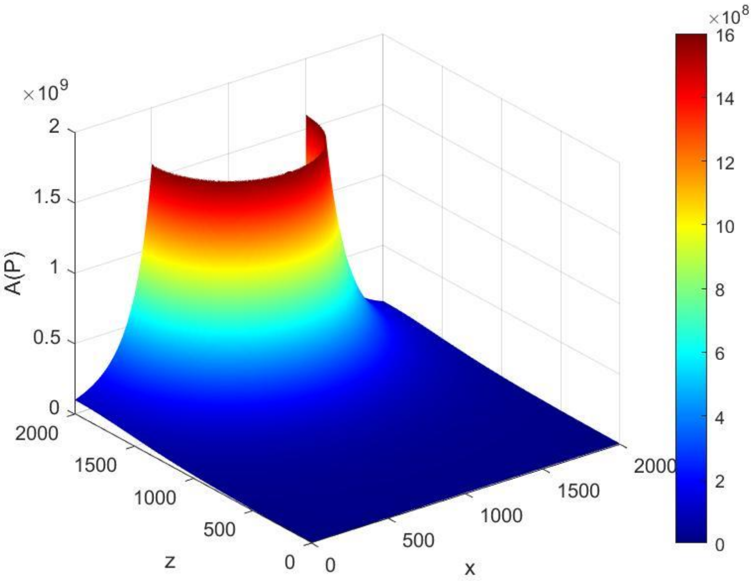


Figure 2.2: 3D plot of the electric field with R = 5mm

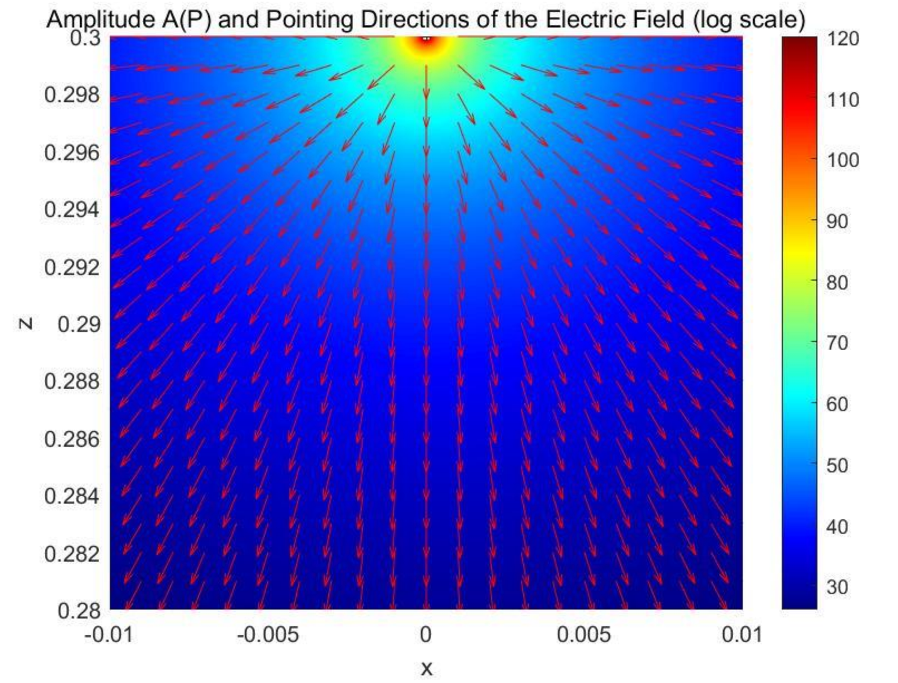


Figure 2.3: Electric field and pointing direction with R = 0.05mm

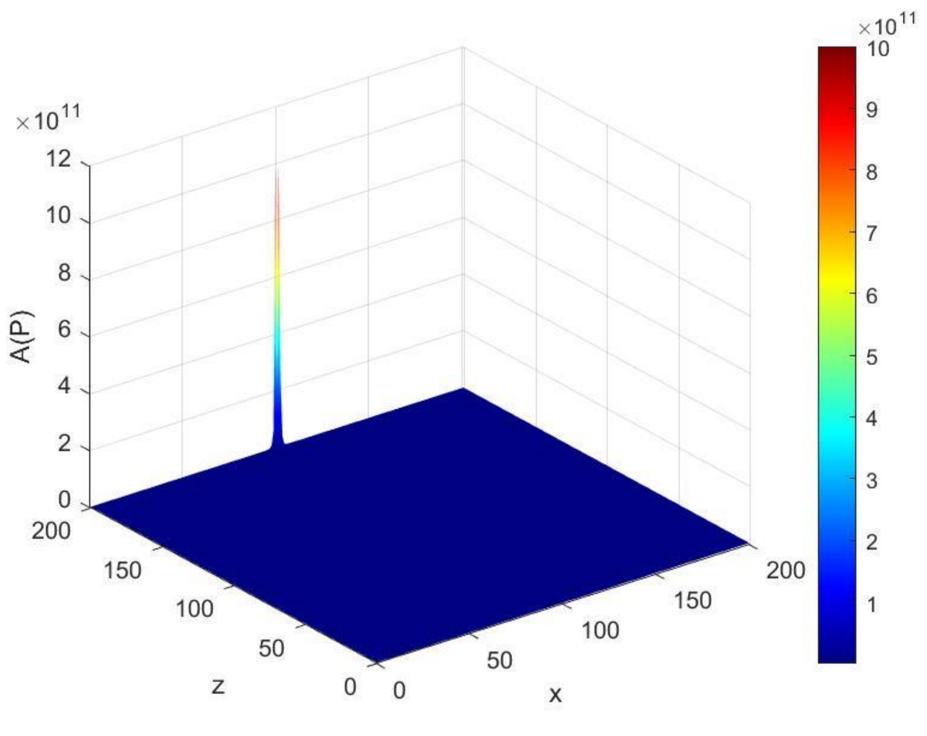


Figure 2.4: 3D plot of the electric field with R = 0.05mm

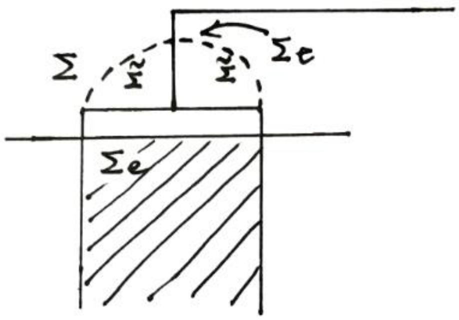
Problem No.3

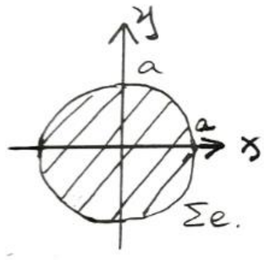
a. Expressions of current I and admittance Y:

The current flows through the electrode Σ_e has been expressed as,

$$I_{e,tot} = \int_{\Sigma_e} (\hat{n} \cdot \underline{J}_{tot})\, d\Sigma$$

where, $\underline{J}_{tot} = (\sigma + j\omega\varepsilon)\underline{E}$.





As the electric field is constant along z axis,

$$\underline{E} = \frac{V_0}{h} \cdot \hat{z}$$

$$\Sigma_e = \pi \cdot a^2$$

Therefore,

$$I_{e,tot} = \int_{\Sigma_e} [\hat{n} \cdot (\sigma + j\omega\varepsilon) \cdot \frac{V_0}{h} \cdot \hat{z}]\, d\Sigma$$

$$I = (\sigma + j\omega\varepsilon)\frac{V_0}{h} \cdot \pi a^2.$$

The admittance is,

$$Y = \frac{I}{V_0} = (\sigma + j\omega\varepsilon)\frac{\pi a^2}{h}.$$

b. The meaning of real and imaginary part of Y:

$$Y = (\sigma + j\omega\varepsilon)\frac{\pi a^2}{h}.$$

$$Y_{-real} = \frac{\sigma \pi a^2}{h}.$$

$$Y_{-imag} = \frac{\omega\varepsilon\pi a^2}{h}.$$

From the circuit theory aspect, the real part of the admittance indicates the admittance of the conduction current, and the imaginary part indicates the admittance of the displacement current.

From the aspect of the physical meaning, the real part indicates the conductance, and the imaginary part indicates the susceptance.

C. The displacement and polarization current in the Y expression are allowed to be neglected with the values of permittivity σ and frequency f (or ω), due to the expression of the displacement current as below.

$$I_{\Sigma disp} = j\omega \oiint_{\Sigma} \varepsilon \underline{E} \cdot \hat{n} \, d\Sigma .$$

Problem No. 4

1. Conduction current I_1:

The analyzation of the inward following current is similar as in Problem 3,

$$I_1 = \int_{\Sigma_t} (-\hat{n} \cdot \underline{J}_{cond})\, d\Sigma$$
$$= (\sigma + j\omega\varepsilon) \cdot |\underline{E}| \cdot S_e.$$

where $S_e = \frac{1}{2} \cdot 4\pi r^2 = 2\pi r^2$

The electric field in the function is expressed as,

$$\underline{E} = kq \left\{ \frac{x\hat{x} + y\hat{y} + (z-h)\hat{z}}{[x^2+y^2+(z-h)^2]^{\frac{3}{2}}} - \frac{x\hat{x} + y\hat{y} + (z+h)\hat{z}}{[x^2+y^2+(z+h)^2]^{\frac{3}{2}}} \right\}$$

The components along each axises are used for the calculation of the amplitude of the electric field,

$$|E_x| = kq \left\{ \frac{x}{[x^2+y^2+(z-h)^2]^{\frac{3}{2}}} - \frac{x}{[x^2+y^2+(z+h)^2]^{\frac{3}{2}}} \right\}$$
$$|E_y| = kq \left\{ \frac{y}{[x^2+y^2+(z-h)^2]^{\frac{3}{2}}} - \frac{y}{[x^2+y^2+(z+h)^2]^{\frac{3}{2}}} \right\}$$
$$|E_z| = kq \left\{ \frac{z-h}{[x^2+y^2+(z-h)^2]^{\frac{3}{2}}} - \frac{z+h}{[x^2+y^2+(z+h)^2]^{\frac{3}{2}}} \right\}$$
$$|\underline{E}| = \sqrt{|E_x|^2 + |E_y|^2 + |E_z|^2}$$

where $kq = \dfrac{V_0 \cdot R(2h+R)}{2h} \simeq V_0 \cdot R$

2. Analytic expression of Admittance and Impedance:

The expression of the admittance is,

$$Y = \frac{I_1}{V_0} = \frac{(\sigma + j\omega\varepsilon) \cdot 2\pi r^2 \cdot |\underline{E}|}{V_0}$$

where V_0 can be eliminated in the above expression as analyzed in the above.

The impedance can be calculated with the expression of the admittance,

$$Z = \frac{1}{Y}$$

3. Plot of the impedance:

Pick a random point between the two electrodes, for calculating the value of electric field, as the field is constant in this space, for example at point (0, 0, 0.2).

Then following the functions of analyzation of part 1 and 2 of problem No.4, figure 4.1 and 4.2 are plotted using MATLAB, which respectively stands for the case of $R = 5mm$ and $R = 0.05mm$.

As shown in the figures, the impedance maintains a rather high value between the frequency of 10Hz to 100MHz, and with the increasing of the frequency, the impedance decreases. The imaginary part of the impedance on the other hand shows a tendency of increase between the frequency from 10Hz to 0.1MHz, and the decreases after 0.1MHz, ~~as~~ the value is always negative during ~~the~~ all the cases, which indicates a negative reactance, that is considered as capacitive.

With the comparison between $R = 5mm$ and $R = 0.05mm$, the overall impedance increase, which is due to a lower electric field with $R = 0.05mm$ ($E = 1.04 V/m$, while $E = 104 V/m$ with $R = 5mm$).

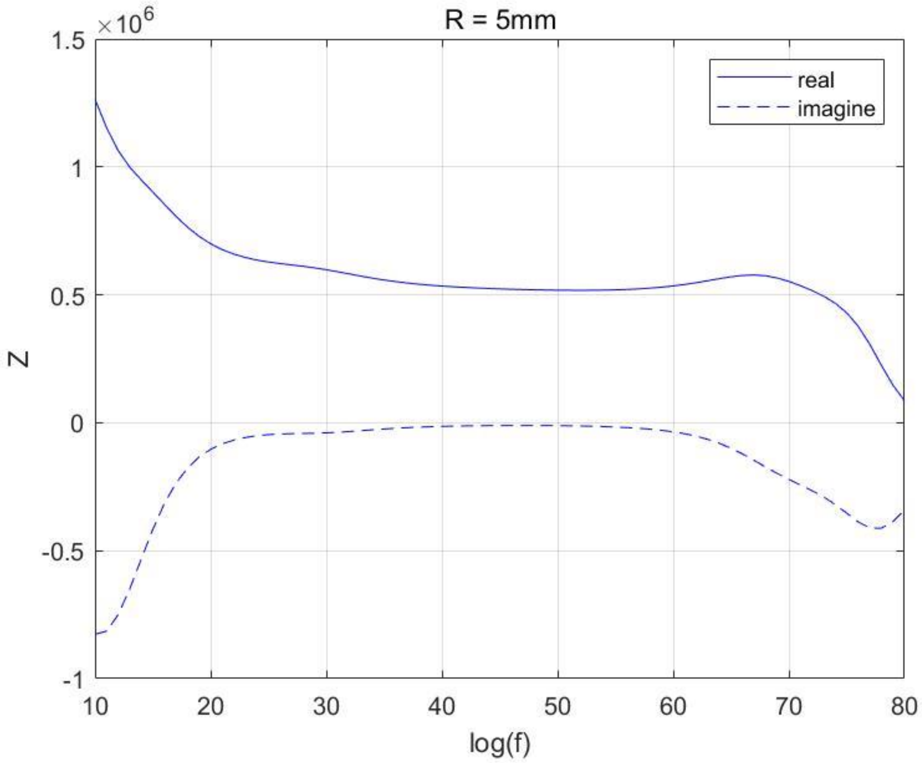


Figure 4.1: The real and imaginary part of the impedance, R = 5mm

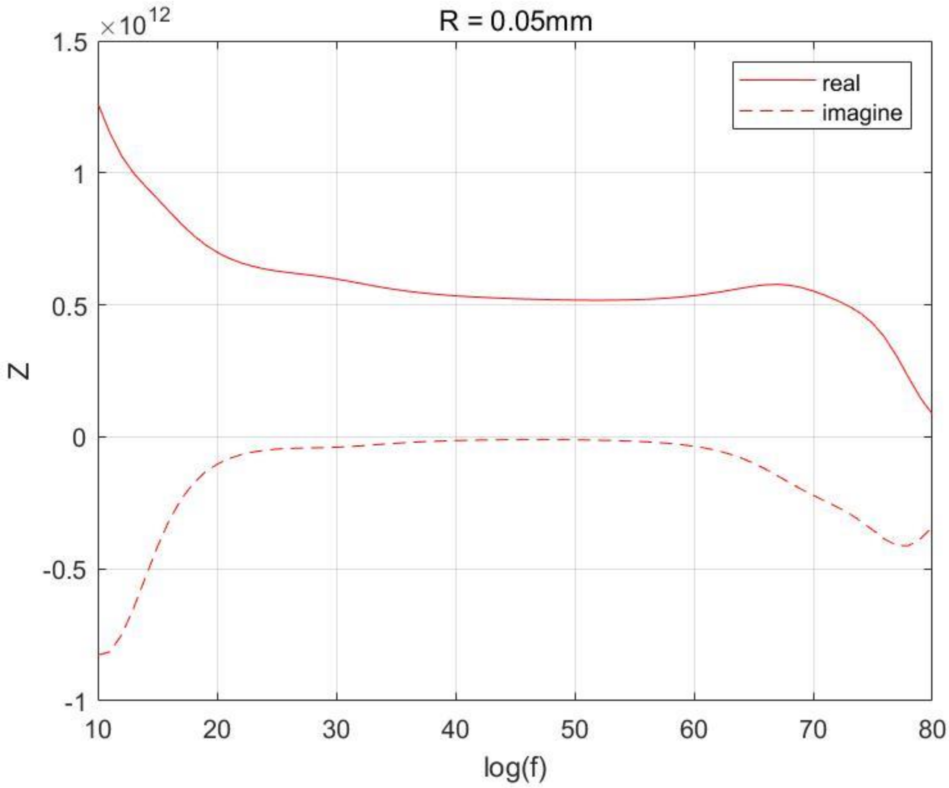


Figure 4.2: The real and imaginary part of the impedance, R=0.05mm

Appendix: MATLAB codes

<u>**Problem 2, R = 5mm:**</u>

```
clear all;
close all;
clc

j = sqrt(-1);
h = 30e-2;  %m
Vg = 200;   %V
R = 5e-3;   %m
kq = Vg*R;

x = -0.01:0.00001:0.01;
z = 0.28:0.00001:0.3;     %(x,z)
[x,z] = meshgrid(x,z);

A0 = sqrt(x.^2+(z-h).^2);
A0(A0<R) = 0;
A0(A0>0) = 1;

sc = (x.^2+(z-h).^2).^1.5;
ic = (x.^2+(z+h).^2).^1.5;
Ex = kq*(x./sc - x./ic);
Ez = kq*((z-h)./sc-(z+h)./ic);
A = (Ex.^2+Ez.^2);
A = A.*A0;

figure
contour(x,z,A,666);
axis equal
xlabel('x');
ylabel('z');

figure
surf(A);
shading interp;
colorbar;
colormap(jet);
xlabel('x');
ylabel('z');
zlabel('A(P)');
```

```
figure
pcolor(x,z,A);
shading interp;
colorbar;
colormap(jet);
xlabel('x');
ylabel('z');
hold on
x = -0.01:0.001:0.01;
z = 0.28:0.001:0.3;     %(x,z)
[x,z] = meshgrid(x,z);
sc = (x.^2+(z-h).^2).^1.5;
ic = (x.^2+(z+h).^2).^1.5;
Ex = kq*(x./sc - x./ic);
Ez = kq*((z-h)./sc-(z+h)./ic);
A = (Ex.^2+Ez.^2);

quiver(x,z,Ex./sqrt(A),Ez./sqrt(A),'r');
xlabel('x');
ylabel('z');
title('Amplitude A(P) and Pointing Directions of the Electric
Field');
```

<u>Problem 2, R = 0.05mm:</u>

```
clear all;
close all;
clc

j = sqrt(-1);
h = 30e-2;  %m
Vg = 200;   %V
R = 0.05e-3;   %m
kq = Vg*R;

x = -0.01:0.0001:0.01;
z = 0.28:0.0001:0.3;     %(x,z)
[x,z] = meshgrid(x,z);

A0 = sqrt(x.^2+(z-h).^2);
A0(A0<R) = 0;
A0(A0>0) = 1;
```

```
sc = (x.^2+(z-h).^2).^1.5;
ic = (x.^2+(z+h).^2).^1.5;
Ex = kq*(x./sc - x./ic);
Ez = kq*((z-h)./sc-(z+h)./ic);
A = (Ex.^2+Ez.^2);
A = A.*A0;

figure
contour(x,z,A,666);
axis equal
xlabel('x');
ylabel('z');

figure
surf(A);
shading interp;
colorbar;
colormap(jet);
xlabel('x');
ylabel('z');
zlabel('A(P)');

figure
pcolor(x,z,10*log10(A));
shading interp;
colorbar;
colormap(jet);
xlabel('x');
ylabel('z');
title('Amplitude A(P)[dB]');
hold on

x = -0.01:0.001:0.01;
z = 0.28:0.001:0.3;     %(x,z)
[x,z] = meshgrid(x,z);
sc = (x.^2+(z-h).^2).^1.5;
ic = (x.^2+(z+h).^2).^1.5;
Ex = kq*(x./sc - x./ic);
Ez = kq*((z-h)./sc-(z+h)./ic);
A = (Ex.^2+Ez.^2);

quiver(x,z,Ex./sqrt(A),Ez./sqrt(A),'r');
xlabel('x');
```

```
ylabel('z');
title('Amplitude A(P) and Pointing Directions of the Electric
Field (log scale)');
```

Problem 4:

```
clear all;
close all;
clc

j = sqrt(-1);
fats =
textread('D:\EE\Electromagnetic_Fields_and_Biological_Tissues\Ass
ignment\A3\fat_S.txt');
fats = sortrows(fats,1);
f = fats(:,1);
omg = 2*pi*f;
sigma = fats(:,2);   %[S/m]
yp1 = fats(:,3);
yp0 = 8.85418782e-12;
tg_loss = atan(fats(:,4));
yp2 = (tg_loss.*omg.*yp1-sigma)./omg;
%yp2 = 0;
yp = yp1*yp0+j*yp2*yp0;

h = 30e-2;  %m
Vg = 200;   %V
R = [5e-3 0.05e-3];   %m
kq = Vg*R;

x = 0;  y = 0;  z = 0.2;

sc = (x.^2+y.^2+(z-h).^2).^1.5;
ic = (x.^2+y.^2+(z+h).^2).^1.5;
Ex = kq.*(x./sc - x./ic);
Ey = kq.*(y./sc - y./ic);
Ez = kq.*((z-h)./sc-(z+h)./ic);
E = sqrt(Ex.^2+Ey.^2+Ez.^2);

S = 2*pi*R.^2;
I = (sigma+j*omg.*yp).*E.*S;
Z = Vg./I;
logf = 10*log10(f);
```

```
figure
plot(logf, real(Z(:,1)),'b-',logf,real(-j*Z(:,1)),'b--');
xlabel('log(f)');
ylabel('Z');
title('R = 5mm');
legend('real','imagine')
grid on
figure
plot(logf, real(Z(:,2)),'r-',logf,real(-j*Z(:,2)),'r--');
xlabel('log(f)');
ylabel('Z');
title('R = 0.05mm')
legend('real','imagine')
grid on
```