REMS Assignment 4 TONG LIN S2287649

Problem 4.1

1. Design dimensions A and B:

According to the expression in H plane, with $\varphi=0°$,

$$20\log_{10}\left|\frac{\underline{E}(\theta_0=55°, \varphi=0°)}{\underline{E}(\theta=0°, \varphi=0°)}\right| = -10\,dB.$$

where $\underline{E} = \frac{jZ_0}{2r\lambda_0} e^{-jkr} \cdot \underline{Pe}(\theta, \varphi)$,

and $\underline{Pe}(\theta,\varphi) = -2H_0AB\frac{2}{\pi}[\hat{\varphi}\cos\varphi + \hat{\theta}\sin\varphi]\cos^2\frac{\theta}{2}$

$\cdot\ \tilde{F}_H(\frac{A}{\lambda}\sin\theta\cos\varphi)\cdot \tilde{F}_E(\frac{B}{\lambda}\sin\theta\sin\varphi)$.

with $\varphi=0$, then,

$$\underline{Pe}(\theta, 0°) = -2H_0AB\frac{2}{\pi}\cdot\hat{\varphi}\cdot\cos^2\frac{\theta}{2}\cdot\tilde{F}_H(\frac{A}{\lambda}\cdot\sin\theta)\cdot\tilde{F}_E(0)$$

Therefore,

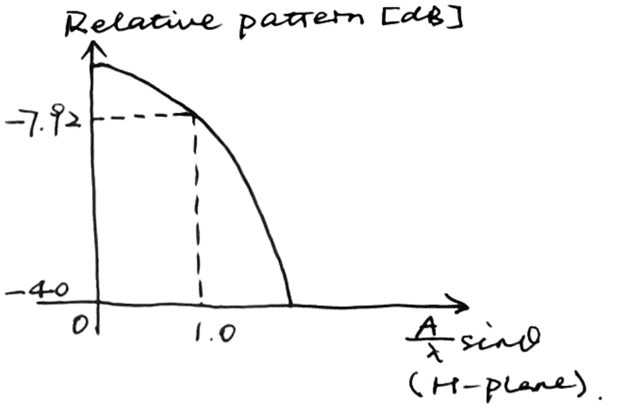
$$\frac{\underline{E}(\theta_0=55°)}{\underline{E}(\theta=0°)} = \frac{\cos^2(27.5°)\cdot\tilde{F}_H(\frac{A}{\lambda}\cdot\sin55°)}{\tilde{F}_H(0)}.$$

$$20\log_{10}\cos^2\frac{\theta}{2} + \tilde{F}_H(\frac{A}{\lambda}\cdot\sin55°)|_{dB} - \tilde{F}_H(0)|_{dB} = -10\,dB.$$

Assuming that there's zero phase error, which $\tilde{F}_H(0) = 0\,dB$. Also, $20\log_{10}\cos^2(27.5°) \simeq -2.08\,dB$.
The above expression can be then written as,

$$\tilde{F}_H(\frac{A}{\lambda}\cdot\sin55°)|_{dB} = -7.92\,dB$$

According to the figure with the line of F_H, with $t=0$,



$\frac{A}{\lambda}\cdot\sin55° \simeq 1.0$

with $\lambda = \frac{c}{f} = \frac{3\times10^8 m/s}{22\times10^9 Hz} \simeq 0.014m$

$\sin55° \simeq 0.82$

Thus, $A = 0.017\,m = 17\,mm$

According to the expression on the E-plane, with $\varphi = 90°$,

$$20\log_{10}\left|\frac{\underline{E}(\theta_0=55°, \varphi=90°)}{\underline{E}(\theta=0°, \varphi=90°)}\right| = -10\,dB.$$

where $\underline{E} = \frac{j z_0}{2\lambda r} e^{-jkr} \underline{P_e}(\theta, \varphi)$,

$$\underline{P_e}(\theta, \varphi=90°) = -2H_0AB\frac{2}{\pi}\hat{\theta}\cdot\cos^2\frac{\theta}{2}\cdot\tilde{F}_H(0)\cdot\tilde{F}_E\left(\frac{B}{\lambda}\cdot\sin\theta\right).$$

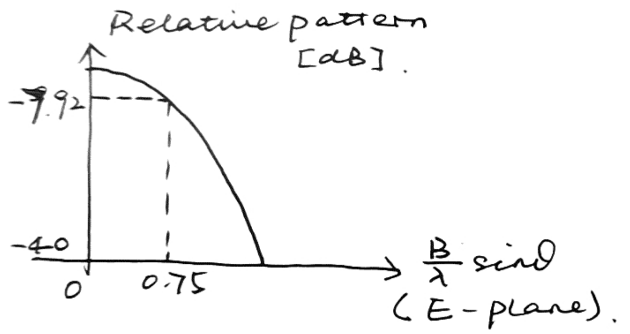
Therefore,

$$20\log_{10}\left|\frac{\cos^2(27.5°)\cdot\tilde{F}_E\left[\frac{B}{\lambda}\cdot\sin(55°)\right]}{\tilde{F}_E(0)}\right| = -10\,dB,$$

with zero phase error $\tilde{F}_E(0) = 0\,dB$, and $20\log_{10}(\cos^2 27.5°) \simeq -2.08\,dB$

$$\tilde{F}_E\left(\frac{B}{\lambda}\cdot\sin 55°\right) = -7.92\,dB.$$

According to the figure with the line of F_E, with $s=0$,



$$\frac{B}{\lambda}\cdot\sin 55° \simeq 0.75$$

Thus, $B = 12.8$ mm.

2. Gain of the antenna:

The gain of the antenna can be expressed as,

$$G = \frac{4\pi}{\lambda^2}\cdot\nu_A\cdot A_{geo}$$

where for TE_{10} $\nu_A = \frac{8}{\pi^2}$, and according to question 1, $A_{geo} = A\cdot B = 17\,mm \times 12.8\,mm = 2.176\times10^{-4}\,m^2$.

Therefore,

$$G \simeq 11.31 = 10.53\,[dB].$$

3. The distance r from the aperture:

According to the definition function of the gain,

$$G = \frac{\frac{d\Gamma}{d\Sigma}}{\left(\frac{dP}{d\Sigma}\right)_{\mathrm{iso}}}$$

where $\frac{d\Gamma}{d\Sigma} = \frac{|E|^2}{Z_0} = Z_0 \cdot |H|^2$

$$\left(\frac{dP}{d\Sigma}\right)_{\mathrm{iso}} = \frac{P_{\mathrm{feed}}}{4\pi r^2}$$

Therefore,

$$G = \frac{Z_0 \cdot |H|^2 \cdot 4\pi r^2}{P_{\mathrm{feed}}}.$$

$$r = \sqrt{\frac{G \cdot P_{\mathrm{feed}}}{Z_0 \cdot |H|^2 \cdot 4\pi}}$$

where $G = 11.31$, $P_{\mathrm{feed}} = 50 \times 10^{-3}\,W$, $Z_0 = 50\,\Omega$,

$|H| \leq 2 \times 10^{-3}\, A/m$, thus,

$$r \geq 15\ m.$$

Problem 4.2

Similar as in Promblem 4.1, on the H plane with $\varphi = 0°$, the expression can be written as,

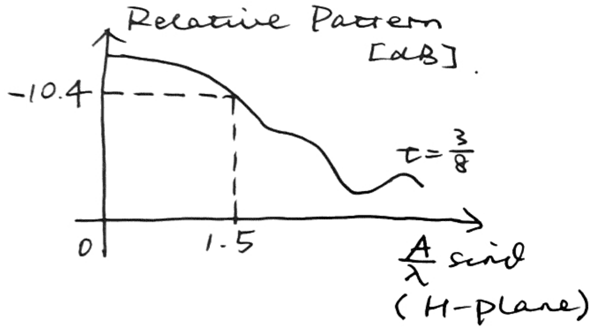
$$20\log_{10}\cos^2\frac{\theta_{-10dB}}{2} + \tilde{F}_H\left(\frac{A}{\lambda}\cdot\sin\theta_{-10dB}\right) - \tilde{F}_H(0) = -10dB.$$

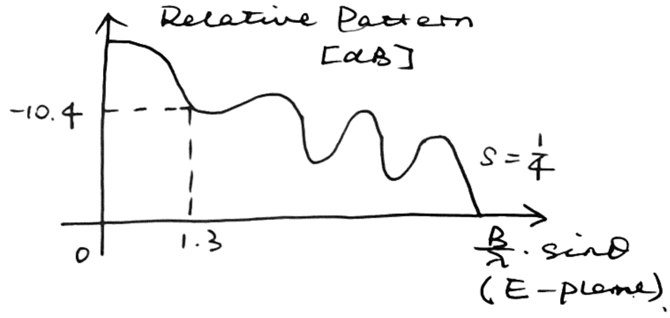
where $\theta_0 = 30°$, with the optimum load, $\tilde{F}_H(0) = -1dB$.

$$\tilde{F}_H\left(\frac{A}{\lambda}\cdot\frac{1}{2}\right) \simeq -10.40\ dB$$

Similarly, $\tilde{F}_E\left(\frac{B}{\lambda}\cdot\frac{1}{2}\right) \simeq -10.40\ dB$.

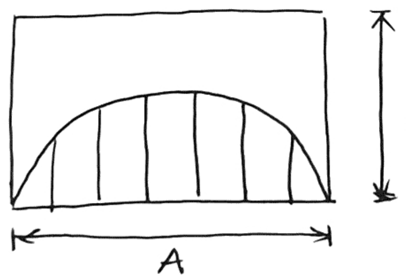
According to the figures of H-plane and E-plane below, with the optimum values $t = \frac{3}{8}$, $s = \frac{1}{4}$.

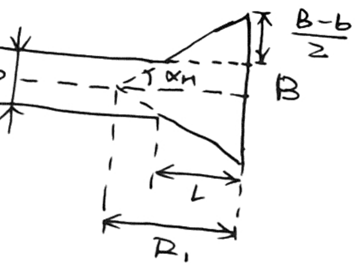


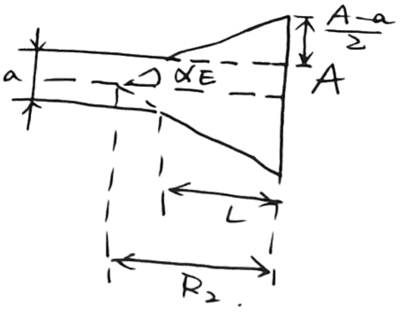


$$\frac{A}{\lambda} = 3, \quad A \simeq 42mm$$

$$\frac{B}{\lambda} = 2.6, \quad B \simeq 36.2mm.$$







For $t = \frac{1}{8}\left(\frac{A}{\lambda}\right)^2 \cdot \frac{1}{R_2/\lambda}$, $R_2 = 8.23 \times 10^{-6}\ m$

$s = \frac{1}{8}\left(\frac{B}{\lambda}\right)^2 \cdot \frac{1}{R_1/\lambda}$, $R_1 = 9.17 \times 10^{-6}\ m$.

Geometrically, the following relationships should be satisified.

$$\tan \alpha_E = \frac{\frac{A}{2}}{R_2} = \frac{\frac{A-a}{2}}{L}$$

$$\tan \alpha_H = \frac{\frac{B}{2}}{R_1} = \frac{\frac{B-b}{2}}{L}$$

thus, $$L = R_2 \cdot \frac{A-a}{A} = R_1 \cdot \frac{B-b}{B}.$$

$$\frac{R_1}{R_2} = \frac{1 - \frac{a}{A}}{1 - \frac{b}{B}}.$$

The above condition should be satisified during design for the feasibility of the antenna.

Problem 4.3

Rectangular horn antenna aperture efficiency $\nu_A = \frac{8}{\pi^2}$:

From the point of view of gain G,

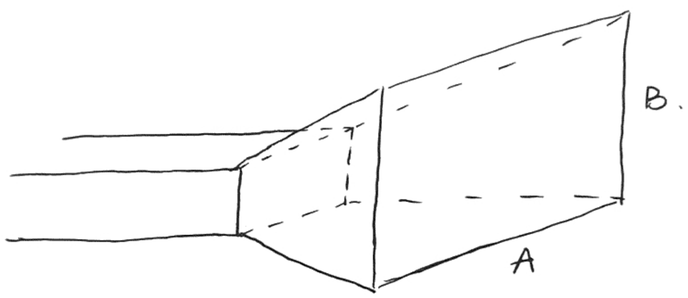
$$G = \frac{\frac{dP}{d\Sigma}}{\left(\frac{dP}{d\Sigma}\right)_{Iso}} = \frac{4\pi}{\lambda^2} \frac{\left| \int_A \underline{E}_A \, dA \right|^2}{\int_A |\underline{E}_A|^2 \, dA}$$

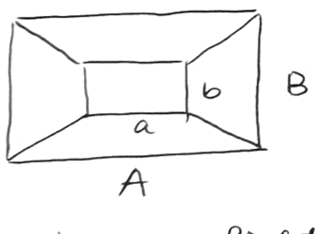
$$G = \frac{4\pi}{\lambda^2} \nu_A A_{geo}.$$

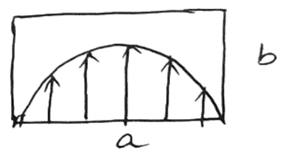
Therefore,

$$\nu_A = \frac{1}{A_{geo}} \frac{\left| \int_A \underline{E}_A \, dA \right|^2}{\int_A |\underline{E}_A|^2 \, dA}$$

where $\underline{E}_A = \hat{x} E_x + \hat{y} E_y$.







Assuming the field distribution is same as for TE_{10}.

$$E = E_0 \cos\left(x \frac{\pi}{a}\right) \hat{y}$$