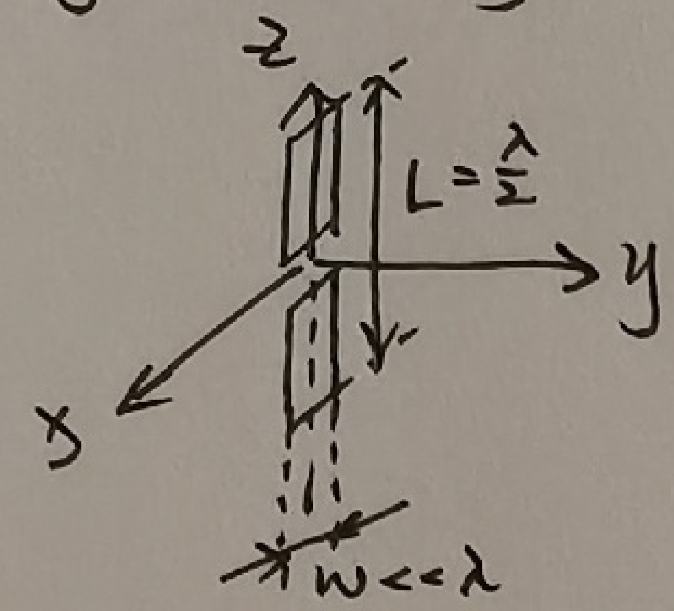
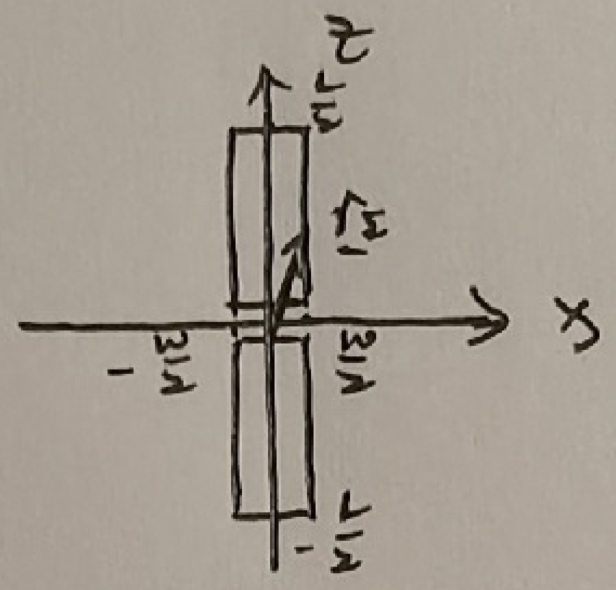
Assignment 1. Slot and dipole TONG LIN S2876491

Problem 1

1. a. geometry





b. position vector $\underline{r}_\Sigma = x\cdot\hat{x} + z\cdot\hat{z}$

c. exp term ~~of the~~ in the radiation integral

spherical coordinate:

$$\exp(jk\hat{r}\cdot\underline{r}_\Sigma) = \exp[jk\hat{r}(x\cdot\hat{x}+z\cdot\hat{z})]$$

where, $x = \rho\sin\theta\cos\varphi$

$\hat{x} = \hat{\rho}\sin\theta\cos\varphi + \hat{\theta}\cos\theta\cos\varphi - \hat{\varphi}\sin\varphi$

$z = \rho\cos\theta$

$\hat{z} = \hat{\rho}\cos\theta - \hat{\theta}\sin\theta$

Also in this case, $\varphi = 0, \pi$ so $\begin{cases}\sin\varphi = 0 \\ \cos\varphi = \pm 1\end{cases}$

We can then get, $x\cdot\hat{x} + z\cdot\hat{z} = \hat{\rho}\cdot\rho$

Also $\hat{r} = \hat{\rho}$, $r = \rho$ (the symbols are different, however with the same meaning).

Then, $\exp[jk\hat{r}(x\cdot\hat{x}+z\cdot\hat{z})] = \exp(jk\rho)$

source coordinate:

$$\exp(jk\hat{r}\cdot\underline{r}_\Sigma) = \exp[jk\hat{r}\cdot(x\cdot\hat{x}+z\cdot\hat{z})]$$

where, $\hat{r} = \hat{\rho} = \hat{x}\sin\theta\cos\varphi + \hat{y}\sin\theta\sin\varphi + \hat{z}\cos\theta$

So, $\hat{r}(x\cdot\hat{x}+z\cdot\hat{z}) = x\cdot\sin\theta\cos\varphi + z\cdot\cos\theta$

Then, $\exp[jk\hat{r}\cdot(x\cdot\hat{x}+z\cdot\hat{z})] = \exp[jk(x\cdot\sin\theta\cos\varphi + z\cdot\cos\theta)]$

d. specific expression of $\tilde{\underline{J}}(\theta,\varphi)$

$$\tilde{\underline{J}}(\theta,\varphi) \equiv \underline{N}(\theta,\varphi) = \iiint_V \underline{J}(\underline{\rho}')\exp(jk\hat{r}\cdot\underline{r}')\,dV(\underline{\rho}')$$

$$= \iint_\Sigma \underline{J}(\theta,\varphi)\cdot\exp(jk\rho)\,d\Sigma$$

e. surface integral

$$\underline{\tilde{J}_s}(\theta,\varphi) = \iint_\Sigma \underline{J_s}(\underline{r}'_\Sigma)\exp(jk\hat{r}\cdot\underline{r}'_\Sigma)\,d\Sigma$$

where $\underline{J_s}(\underline{r}'_\Sigma) = \underline{J_s}(x,z)$

$$= \hat{z}\frac{I(z)}{w}\Pi(x)$$

$$= \hat{z}\frac{I_0\cos(\pi\frac{z}{L})}{w}$$

$$\exp(jk\hat{r}\cdot\underline{r}'_\Sigma) = \exp[jk(x\cdot\sin\theta\cos\varphi + z\cdot\cos\theta)]$$

$$\approx \exp(jk\cdot z\cdot\cos\theta)$$

with $\theta = \arccos(\frac{z}{\rho})$

so $\cos\theta = \cos[\arccos(\frac{z}{\rho})]$

$$= \frac{z}{\rho}$$

$$= \frac{z}{\sqrt{x^2+z^2}}$$

as $x \ll z$, then $\cos\theta \approx 1$

in this case, $\exp(jk\hat{r}\cdot r_\Sigma) \approx \exp(jkz)$

Finally, $\underline{\tilde{J}_s}(x,z) = 4\int_0^{\frac{w}{2}}dx\int_0^{\frac{L}{2}}dz\left[\hat{z}I_0\frac{\cos(\pi\frac{z}{L})}{w}\exp(jkz)\right]$

$$= 4\hat{z}\frac{I_0}{w}\int_0^{\frac{w}{2}}dx\int_0^{\frac{L}{2}}dz\left[\cos(\pi\frac{z}{L})\exp(jkz)\right]$$

f. radiated far field $e_\theta(\theta,\varphi)$, $e_\varphi(\theta,\varphi)$

$$\begin{cases} e_\theta(\theta,\varphi) = \underline{\tilde{J}_s}(\theta,\varphi)\cdot\hat{\theta} \\ e_\varphi(\theta,\varphi) = \underline{\tilde{J}_s}(\theta,\varphi)\cdot\hat{\varphi} \end{cases}$$

Recall $\underline{\tilde{J}_s}(x,z) = 4\hat{z}\frac{I_0}{w}\int_0^{\frac{w}{2}}dx\int_0^{\frac{L}{2}}dz\left[\cos(\pi\frac{z}{L})\cdot\exp(jkz)\right]$

$$= 4\hat{z}\frac{I_0}{w}\int_0^{\frac{w}{2}}dx\left[\frac{\frac{\pi}{L}\cdot\exp(jkz)\cdot\sin(\frac{\pi}{L}\cdot z) + jk\exp(jkz)\cos(\frac{\pi}{L}\cdot z)}{(\frac{\pi}{L})^2+(jk)^2}\Bigg|_0^{\frac{L}{2}}\right]$$

$$= 4\hat{z}\frac{I_0}{w}\cdot\frac{w}{2}\cdot\frac{\frac{\pi}{L}-jk}{(\frac{\pi}{L})^2-k^2}\cdot\exp(jkz)$$

$$= 8I_0\cdot\hat{z}\cdot\frac{\pi L - jkL^2}{\pi^2-k^2L^2}\exp(jkz)$$

where, $\hat{z} = \hat{\rho}\cos\theta - \hat{\theta}\sin\theta$, $z = \rho\cos\theta$, $\rho = \sqrt{x^2+z^2} \approx z$

so, $\underline{\tilde{J}_s}(\theta,\varphi) = 8I_0(\hat{\rho}\cos\theta - \hat{\theta}\sin\theta)\frac{\pi L - jkL^2}{\pi^2-k^2L^2}\cdot\exp(jk\rho\cancel{\cos\theta})$

Finally, $e_\theta(\theta,\varphi) = -8I_0\sin\theta\frac{\pi L - jkL^2}{\pi^2-k^2L^2}\cdot\exp(jk\rho)$

$$e_\varphi(\theta,\varphi) = 0$$

2. equivalent length $h_e(\theta, \varphi)$

From $\underline{e}(\theta,\varphi) = -j\frac{Z_0}{2\lambda}\cdot 4\pi \cdot I_0 \cdot h_e(\theta,\varphi)$

$$= -j\frac{Z_0}{\lambda}\cdot 2\pi I_0\, h_e(\theta,\varphi)$$

where $\underline{e}(\theta,\varphi) = -8\hat{\theta} I_0 \sin\theta \cdot L \frac{\pi - jkL}{\pi^2 - k^2L^2} \exp(jk\rho)$

So, $h_e(\theta,\varphi) = \dfrac{\underline{e}(\theta,\varphi)}{-j\frac{Z_0}{\lambda}\cdot 2\pi I_0}$

$$= \frac{\cancel{-}\overset{4}{\cancel{8}}\hat{\theta}\cancel{I_0}\sin\theta \cdot L\frac{\pi - jkL}{\pi^2-k^2L^2}\exp(jk\rho)}{\cancel{-}j\frac{Z_0}{\lambda}\cdot \cancel{2}\pi \cancel{I_0}}$$

$$= -j\cdot 4\frac{\lambda L \cdot \hat{\theta}}{Z_0 \cdot \pi}\cdot \sin\theta \cdot \frac{\pi - jkL}{\pi^2 - k^2L^2}\cdot \exp(jk\rho)$$

3. a. radiated power P_{rad}.

Firstly, $e_\theta(\frac{\pi}{2}) = -8 I_0 L \frac{\pi - jkL}{\pi^2 - k^2L^2}\cdot \exp(jk\rho)$

$\tilde{e}_\theta(\frac{\pi}{2}) = C$

so, $C = -8 I_0 L \frac{\pi - jkL}{\pi^2 - k^2L^2}\cdot \exp(jk\rho)$

Then, $\underline{e}_\theta(\theta,\varphi) = -8 I_0 L \frac{\pi - jkL}{\pi^2-k^2L^2}\cdot \exp(jk\rho)\cdot (\sin\theta)^{\frac{3}{2}}\cdot \hat{\theta}$

$P_{rad} = \frac{1}{2Z_0}\cdot \frac{1}{(4\pi\rho)^2}\iint_\Sigma |\underline{e}(\theta,\varphi)|^2 d\Sigma$

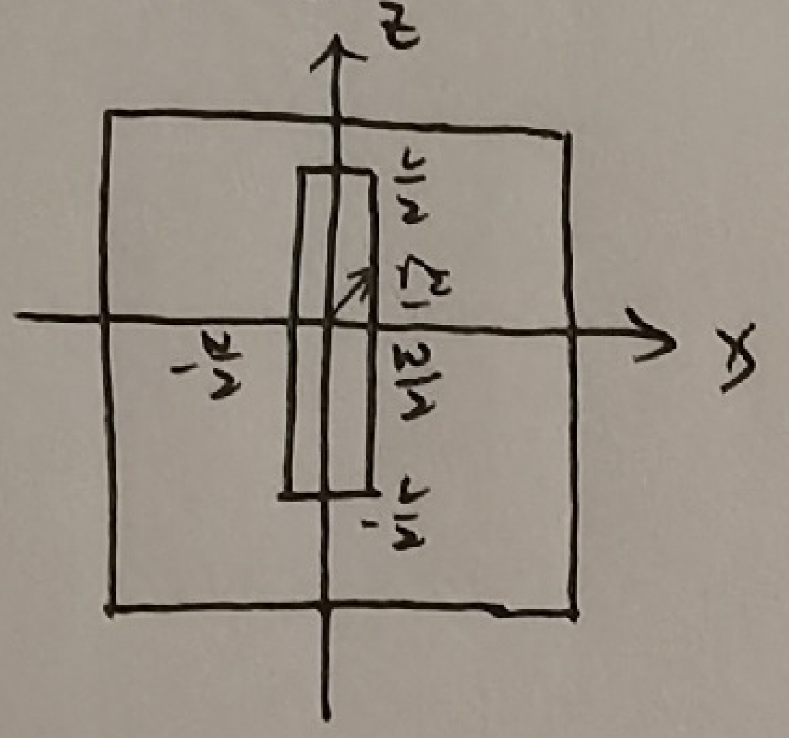
$$= \frac{1}{2Z_0(4\pi\rho)^2}\cdot 64 \cdot I_0^2 \cdot L^2 \left(\frac{\pi - jkL}{\pi^2 - k^2L^2}\right)^2 \iint_\Sigma \exp(2jk\rho)\cdot (\sin\theta)^3 d\Sigma$$

b. radiated resistance R_{rad}.
with $P_{rad} = \frac{1}{2}R_{rad}|I_0|^2$

so, $R_{rad} = \frac{1}{I_0^2 \cdot Z_0 \cdot (4\pi)^2}\int_0^{2\pi} d\varphi \int_0^{\pi} d\theta\, |\underline{e}(\theta,\varphi)|^2$

Problem 2

1. Magnetic Current $\underline{M}_s^s$



$$\underline{E}_{slot}(x,z) = \hat{x}\,\frac{V(z)}{W}\,\Pi(x)$$

$$= \hat{x}\,\frac{V_0 \cdot \cos(\pi \frac{z}{L})}{W}$$

$$\underline{M}_s^s = -\hat{n} \times \underline{E}\big|_{\Sigma_t}$$

$$= -\hat{y} \times \underline{E}_{slot}(x,z)$$

$$= -\frac{V_0}{W}\cos\left(\frac{\pi}{L}\cdot z\right)$$

2. a. Radiated E and H field.

Following the same steps as for dipole:

the position vector $\underline{r}_\Sigma = x\cdot\hat{x} + z\cdot\hat{z}$

$$\underline{\tilde{E}}(\theta,\varphi) = \iint_\Sigma \underline{E}(\theta,\varphi)\exp(jk\hat{r}\cdot\underline{r}_\Sigma)\,d\Sigma$$

$$\underline{\tilde{E}}(x,z) = \iint_\Sigma \hat{x}\,\frac{V_0\cos(\frac{\pi}{L}\cdot z)}{W}\cdot\exp[jk(x\sin\theta\cos\varphi + z\cdot\cos\theta)]$$

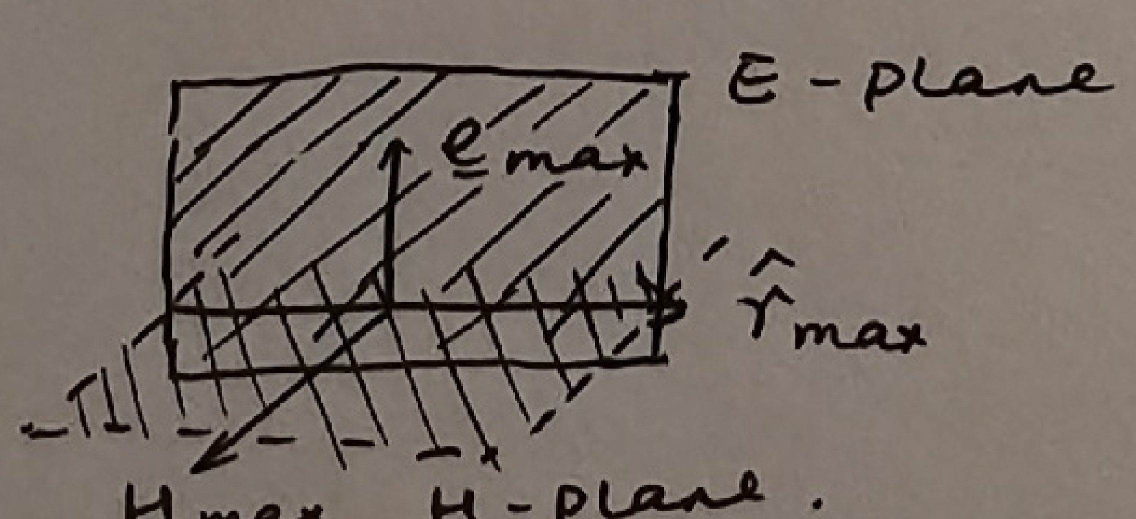
$$= \hat{x}\,\frac{V_0}{W}\cdot 4\int_{\frac{W}{2}}^{+\infty}dx\int_{\frac{L}{2}}^{+\infty}dz\left[\cos\left(\frac{\pi}{L}\cdot z\right)\cdot\exp(jkz)\right]$$

$$= 4\hat{x}\,\frac{V_0}{W}\int_{\frac{W}{2}}^{+\infty}\left[\cos\left(\frac{\pi}{L}\cdot z\right)\cdot\exp(jkz)\right]_{\frac{L}{2}}^{+\infty}dx.$$

For far field, $\underline{H}(\underline{P}) = -\frac{1}{Z_0}\hat{P}\times E(\underline{P})$

$$\underline{\tilde{H}}(\theta,\varphi) = -\frac{1}{Z_0}(\hat{\theta}\cos\theta\cos\varphi - \hat{\varphi}\sin\varphi)\cdot 4\frac{V_0}{W}\int_{\frac{W}{2}}^{+\infty}\left[\cos\left(\frac{\pi}{L}\cdot z\right)\cdot\exp(jkz)\right]_{\frac{L}{2}}^{+\infty}dx$$

b. E and H planes of the antenna.



c. Radiated Power Prad.

$$P_{rad} = \iint_{sphere} S(\rho, \theta, \varphi)\, d\Sigma$$

$$= \frac{1}{2 Z_0 (4\pi\rho)^2} \iint_\Sigma |\underline{e}(\theta, \varphi)|^2 d\Sigma$$

where $\underline{e}(\theta, \varphi) = e_\theta + e_\varphi$

$$e_\theta = jk\hat{\varphi} \cdot \underline{M}_s^s = -jk\hat{\varphi} \cdot \frac{V_0}{W} \cos(\frac{\pi}{L} \cdot z)$$

$$e_\varphi = -jk\hat{\theta} \cdot \underline{M}_s^s = jk\hat{\theta} \cdot \frac{V_0}{W} \cos(\frac{\pi}{L} \cdot z).$$

So. $\underline{e}(\theta, \varphi) = (-\hat{\varphi} + \hat{\theta}) \cdot jk \cdot \frac{V_0}{W} \cdot \cos(\frac{\pi}{L} \cdot z).$

$$|\underline{e}(\theta, \varphi)|^2 = -2k^2 \frac{V_0^2}{W^2} \cos^2(\frac{\pi}{L} \cdot z).$$

$$P_{rad} = \frac{1}{2 Z_0 (4\pi\rho)^2} \int_{\frac{W}{2}}^{+\infty} dx \int_{\frac{L}{2}}^{+\infty} dz \left[-2k^2 \frac{V_0^2}{W^2} \cos^2(\frac{\pi}{L} \cdot z)\right]$$

d. Power Density $S(r, \theta, \varphi)$.

$$\underline{S}(\underline{P}) = \frac{1}{2} \frac{1}{Z_0} (\underline{E} \cdot \underline{E}^*) \hat{r}$$

f. Radiated Resistance. Rrad.

$$R_{rad} = \frac{1}{I_0^2 \cdot Z_0 \cdot (4\pi)^2} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta\, |\underline{e}(\theta, \varphi)|^2$$

$$= \frac{Z_0}{V_0^2 \cdot (4\pi)^2} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \left[-2k^2 \frac{V_0^2}{W^2} \cos^2(\frac{\pi}{L} \cdot z)\right]$$

$$= \frac{-2 Z_0 k^2}{(4\pi)^2 \cdot W^2} \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \cos^2(\frac{\pi}{L} \cdot \rho \cdot \cos\theta)$$