Electromagnetic fields and biological tissues: effects and medical applications

Please **initialize** individual items of the declaration, and **sign** it at bottom.

Upon my word of honor, and aware of the consequences of a false declaration under the Italian law, as well as those deriving from unfair conduct at Politecnico,

I, the undersigned Tong Lin

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Hereby declare (*dichiarazione sostitutiva di atto notorio*) that the home assignment n. 4

Has been carried out in a strictly individual manner from beginning to end; in particular,

TL I have not obtained help from any classmate or external person to carry out in part or whole the assignment;

TL I have not employed any paper or electronic material directly related to the assignment; (note: textbooks are indirectly related only)

TL I have not employed scripts, computer programs or any other such procedures that have not been entirely developed by myself, or provided as course material (by the Instructor and/or the Teaching Assistant), and that are not commercial, or cannot be referenced in the open literature or internet; please note that *all employed software not personally and individually developed must be referenced in the submitted papers*. In particular, I have not employed any script, programs etc. developed by my classmates, and that the employed scripts, programs etc. have not been developed in cooperation with my classmates.

TL I have discussed this assignment with the following persons: (enter "none" if appropriate):

Tong Lin — Tong Lin

(Complete name, please print) — signature

Torino, 2022 / 5 / 23 (date)

Note: Use of commercial software, of free-ware or shareware, or otherwise publicly available software (e.g. via Internet) is allowed, but usage of all software not developed personally and individually by the student, or provided as course material, MUST be clearly stated and precisely referenced in the submitted paper.

Problem No. 1

The specific absorption rate (SAR) is defined as in the function as below.

$$\Delta T_{in} = \frac{SAR}{C} \cdot \Delta t$$

Where C stands for the specific heat of different situation.

Therefore, $SAR = \frac{\Delta T_{in}}{\Delta t} \cdot C$

where $\Delta T_{in} = 1\,°C$, Δt respectively equals to 1s and 1 min. Also, $\Delta T_{in} = 1°C = 274.15\,K$.

The SAR of different tissues with $\Delta t = 1s$ and $\Delta t = 1\,min$ are listed in the table as below.

The uniformation of the units can be expressed as,

$$[W/kg] = \frac{[K]}{[s]} \left[\frac{kJ}{kg \cdot K}\right] \times 10^3$$

Tissue	**specific heat**	**SAR (Δt = 1s)**	**SAR (Δt = 60s)**
	[J/(kg*K)]	**[kW/kg]**	**[kW/kg]**
Brain	3600	987	16.4
Muscle	3700	1010	16.9
Liver	3600	987	16.4
Eye liquid (water)	4200	1,150	19.2

	units	brain	eye liquid	muscle	liver	blood	
Steady State (with perfusion)							
SAR	W/kg	34.4	0.0	2.7	48.1		
Initial Transient							
SAR, 1s	W/kg	3650.0	4200.0	3700.0	3600.0		
SAR, 60s	W/kg	60.8	70.0	61.7	60.0		
SAR, 10m	W/kg	6.1	7.0	6.2	6.0		
Initial Transient (electrosurgery)							
Delta T	C	62.5					
t	s	0.10					
SAR, t	W/kg			2,312,500		W/g	2312.5

sigma=3.956e-1 %muscle, CNR 250kHz
eps_r=5.763e+3 %muscle, CNR 250kHz

Problem No. 2

The function of SAR can be described as below.

$$SAR = \frac{q_{EM}}{\rho}.$$

According to the Pennes Model,

$$\rho C \frac{\partial T_{tn}}{\partial t} = \nabla(k \cdot \nabla T_{tn}) + q_s$$

As the heat pump/sink prevails over the thermal gradients. $q_{EM} >> |\nabla \cdot (k \cdot \nabla T_{ss})|$

Also, $q_s = q_{EM} + q_{BP}$ in the stationary state.

When $\frac{\partial T}{\partial t} = 0$,

$$0 = q_{EM} + q_{BP}$$

Therefore, $q_{EM} = -q_{BP} = \rho_B C_B w \rho (T - T_a)$.

$$SAR = \rho_B C_B w (T - T_a)$$

where ρ_B is the blood mass density, C_B is blood specific heat, $w = \frac{w_B}{\rho}$, where w_B is the blood perfusion rate per unit volume of tissue and ρ is the mass density of different tissue. $\Delta T = T - T_a = 1°C$, and $T_a = 37.5°C$, T stands for the local temperature.

The values of SAR of different tissues are listed as follows, with $\rho_B = 1.06 \times 10^3 [kg/m^3]$, $C_B = 3.89 \times 10^3 [J/(kg \cdot K)]$. where $\Delta T = 1°C = 274.15K$. The uniformation of the units is expressed as,

$$[W/kg] = \left[\frac{10^3 kg}{m^3}\right] \cdot \left[\frac{10^3 J}{kg \cdot k}\right] \cdot \left[\frac{m^3}{kg \cdot s}\right] \cdot \frac{10^{-6}}{6} \cdot [k]$$

Tissue	perfusion, w	SAR
	[m3/(kg*s)]	[kW/kg]
Brain	$8.33*10^{-6}$	9.42
Muscle	$6.67*10^{-7}$	0.754
Liver (portal vein)	$1.17*10^{-5}$	13.2

In the situation in d), in the vitreous humor, where there is no blood perfusion ($w_b = 0$). However, the electric conductivity σ is not negligible, the value of SAR can be directly calculated by q_{EM}.

$$SAR = \frac{q_{EM}}{\rho} = \frac{1}{2\rho}\sigma|\underline{E}|^2$$

which corresponding to the heat diffusion term.

Problem 3

1) Admittance and impedance at the electrode:

Using the MATLAB code of the Assignment 3, the admittance and impedance are calculated applying the satistics of muscle,

$$Y = 8.98 \times 10^{-5} + j2.52 \times 10^{-5}$$
$$Z = 1.03 \times 10^{4} - j2.90 \times 10^{3}$$

2) Voltage and power for raising temperature from 37.5°C to 100°C within $\Delta t = \frac{1}{10}$ s at the active electrode:

From the functions of SAR,

$$SAR = \frac{1}{2}\sigma \cdot |E|^2 \cdot \frac{1}{\rho} = \frac{\Delta T_{in}}{\Delta t} \cdot c$$

The amplitude of the electric field can be then obtained,

$$|E| = \sqrt{\frac{\Delta T_{in} \cdot c \cdot 2 \cdot \rho}{\Delta t \cdot \sigma}}$$

where $\Delta T_{in} = (100°C - 37.5°C) + 273.15 = 335.65\ [K]$
$\Delta t = 0.1 [s]$, $c = 3.7 \left[\frac{J}{g \cdot K}\right]$, $\rho = 1.03 \times 10^{6} \left[\frac{g}{m^3}\right]$,
$\sigma = 0.396\ [S/m]$.

The voltage of at the electrode can therefore be calculated, for at the bottom of the electrode the electric field can be approximated of constant.

$$V_g = E \cdot (h - R).$$

The power is then,

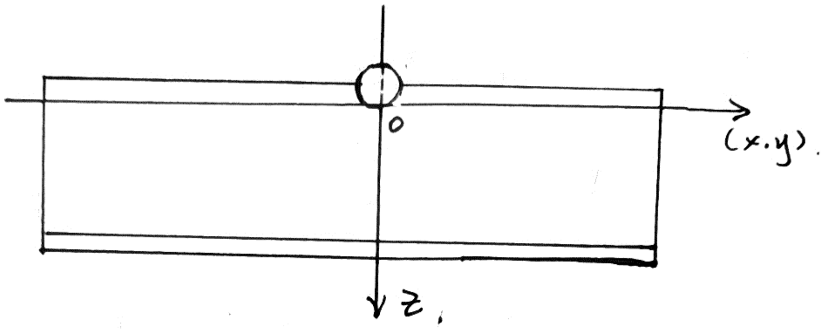
$$P = \frac{V_g^2}{Z}$$

Using MATLAB, the results are,

$$V_g = 3.83\times10^5\ [V]$$

$$P = 1.32\times10^7 + j3.70\times10^6\ [W].$$

3) SAR(P) and $\Delta T(P)$ along z-axis:



$$SAR(z) = \frac{1}{2}\sigma|E(z)|^2\cdot\frac{1}{\rho}$$

where $|E(z)| = \frac{V_g}{z}$, $z \in [0, h-R]$

$$\Delta T(z) = \frac{SAR(z)\cdot\Delta t}{C}$$

where Δt is still assuming to be 0.1s.

The plots of SAR(z) and $\Delta T(z)$ are shown as in figures below,

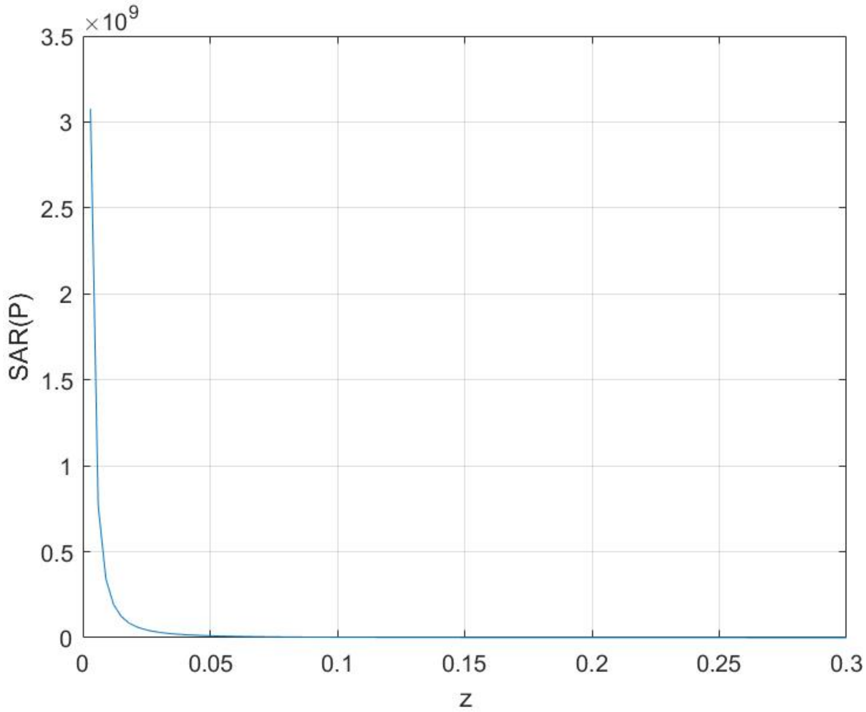


Figure 3.1: The plot of SAR(P) in linear scale

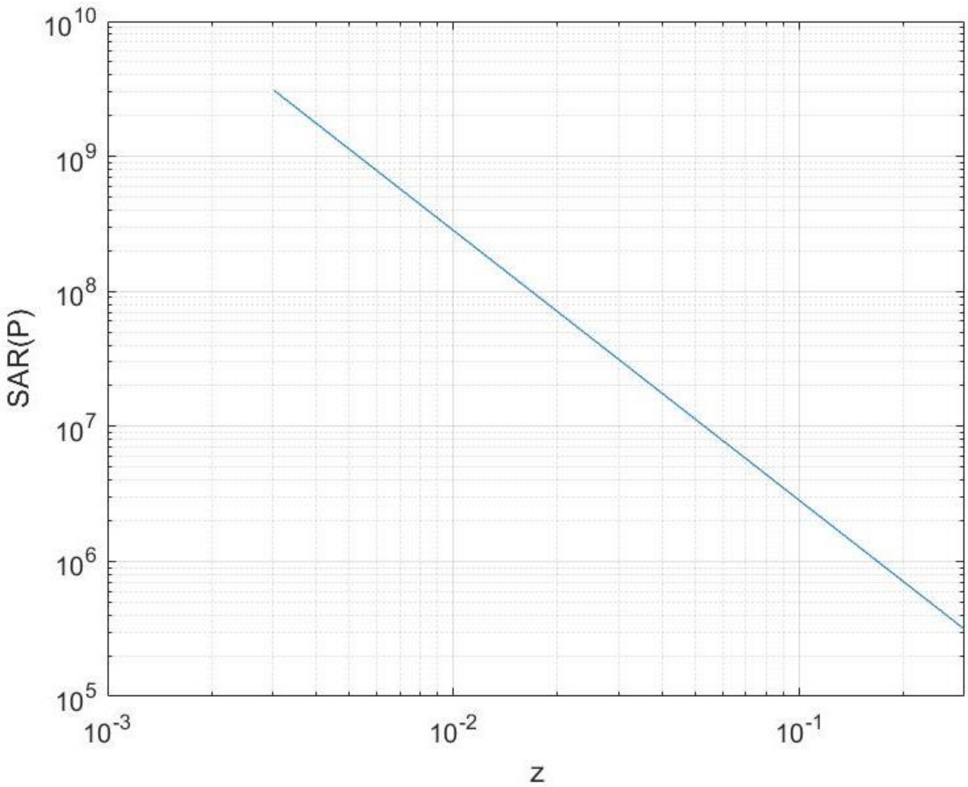


Figure 3.2: The plot of SAR(P) in log scale

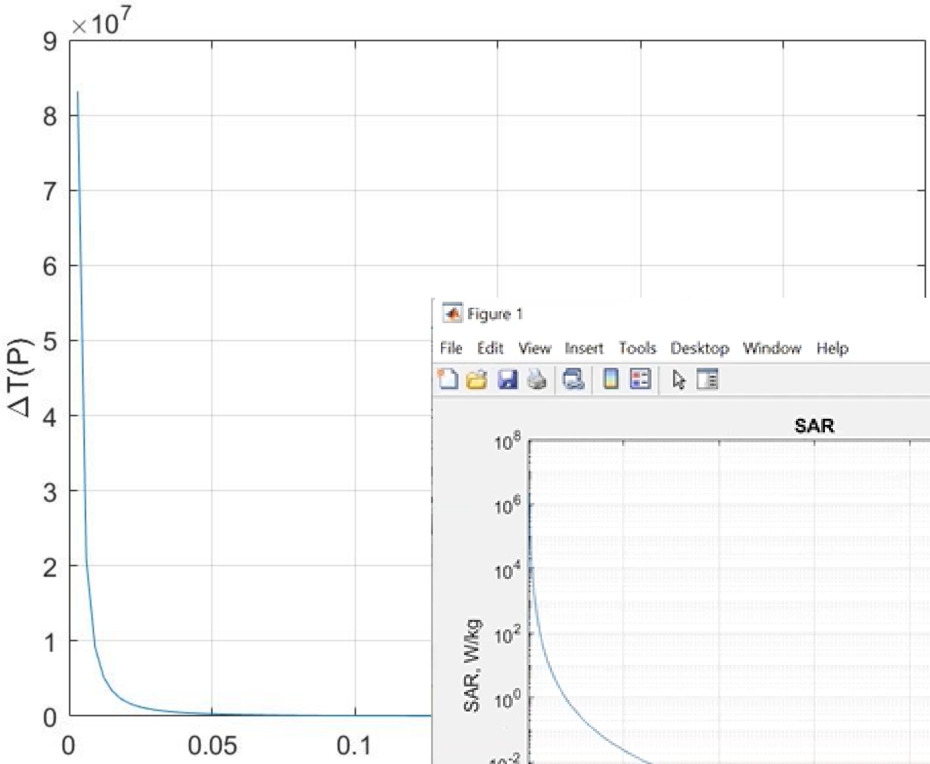
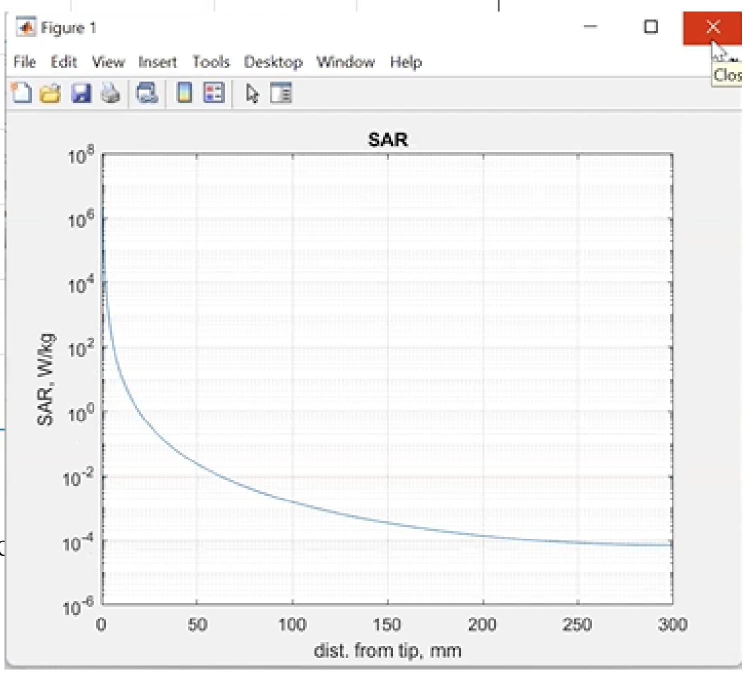


Figure 3.3: The plo



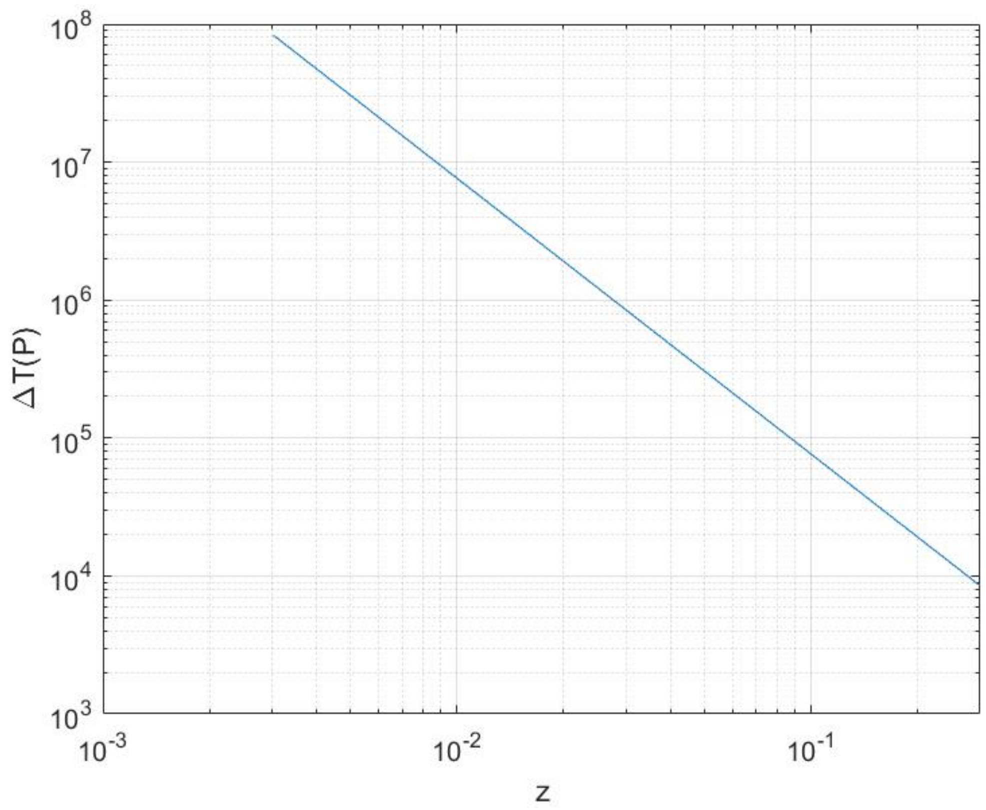
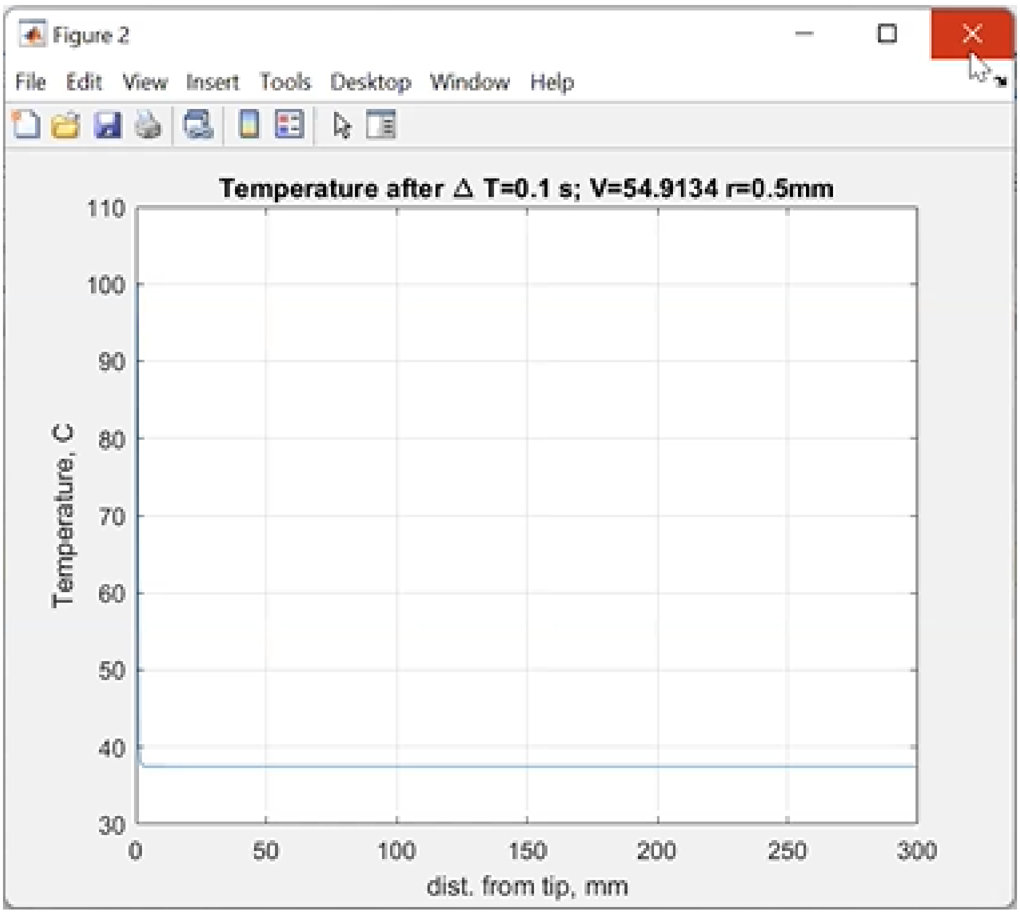


Figure 3.4: The plot of ΔT(P) in log scale



4) Average SAR:

Using the function of the averaging SAR,

$$\overline{SAR}(L) = \frac{1}{L}\int_{z}^{H} SAR(z)\,dz$$

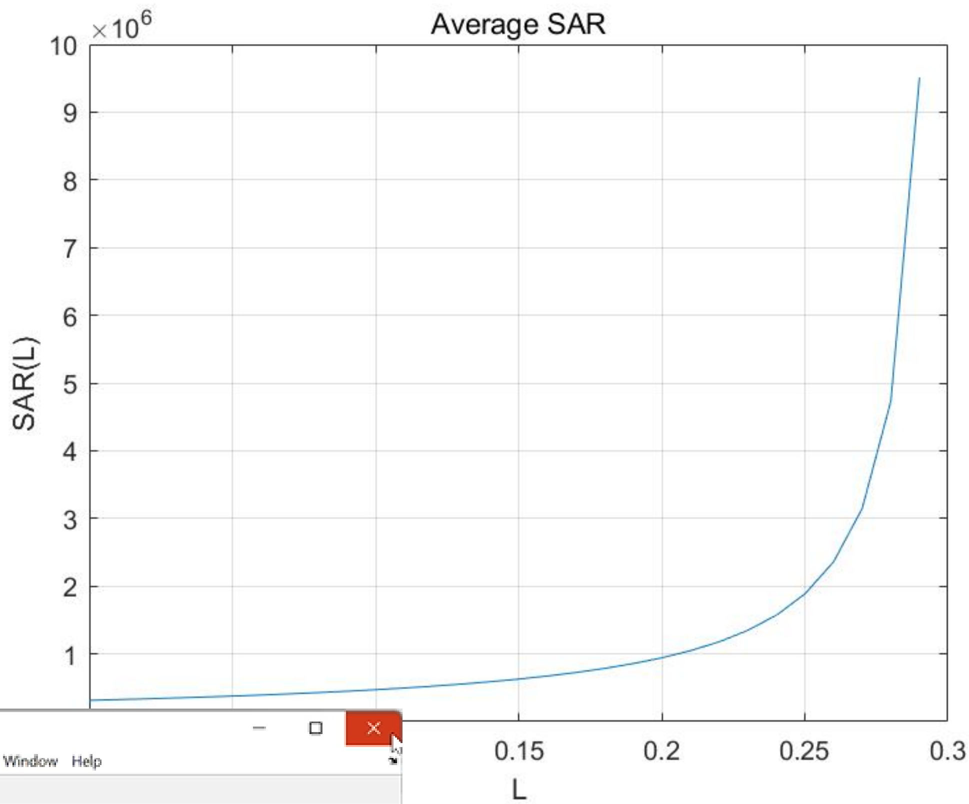
where, $SAR(z) = \frac{1}{2}\sigma\left|\frac{V_g}{z}\right|^2 \cdot \frac{1}{\rho}$ as in the previous excercise,

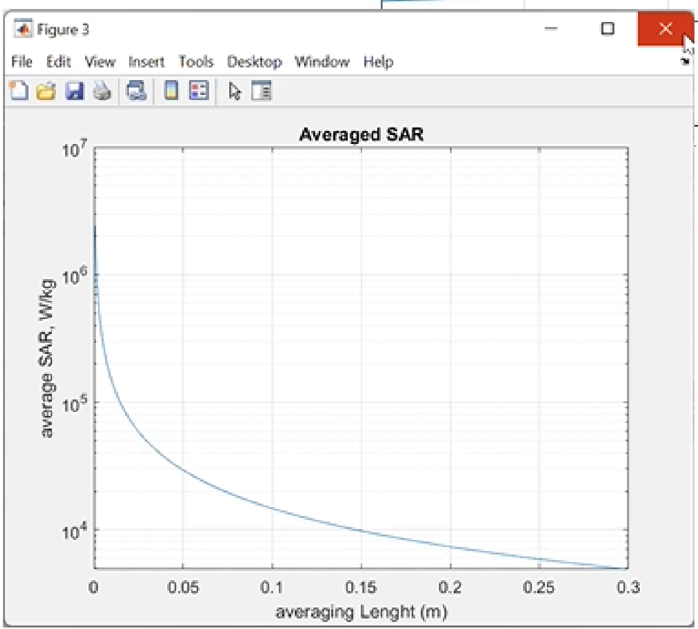
$L \in [R, H]$,

$H = h - R$,

$z = H - L$.

The graph is plotted using MATLAB as in Figure 3.5.





The plot of the average SAR

5) a) Power for desiccating $1 mm^3$ water after reaching 100°C :

With the given value of the latent vaporization heat, the energy for desiccation is,

$$W = \Delta H \cdot m$$

where the mass $m = V \cdot \rho_{water} = 1 mm^3 \times 1 g/cm^3 = 10^{-3} g$.

Therefore, $W = 22.6 J$.

b) Time for desiccation :

Using the power for raising the tempreture to 100°C, where the active power $P = 1.32 \times 10^7 W$, for the applying voltage is still constant, the time for desiccation can be obtained using the relation between power and energy,

$$t = \frac{W}{P} \simeq 1.59 \times 10^{-6} s.$$

6) Voltage for producing a discharge arc :

With the value of breakdown field, $E_b = 2MV/m$, the electrode (RF) voltage V_b can be obtained.

$$V_b = E_b \cdot h = 2 \times 10^6 [V/m] \times 0.3 [m] = 6 \times 10^5 V$$

which is larger than the applying voltage for raising temperature from 37.5°C to 100°C, of $V_g = 3.83 \times 10^5 V$.

Appendix: MATLAB codes

Problem 3:

```
clear all;
close all;
clc

j = sqrt(-1);
f = 250e3;   %[Hz]
omg = 2*pi*f;
sigma = 0.39563;   %Conductivity, [S/m]
yp1 = 5762.7;   %Relative Permittivity
yp0 = 8.85418782e-12;
tg_loss = atan(4.9364);   %Loss Tangent
yp2 = (tg_loss.*omg.*yp1-sigma)./omg;
%yp2 = 0;
yp = yp1*yp0+j*yp2*yp0;

h = 30e-2;  %m
Vg = 200;   %V
R = 0.05e-3;   %m
kq = Vg*R;

x = 0;  y = 0;  z = h-R;

sc = (x.^2+y.^2+(z-h).^2).^1.5;
ic = (x.^2+y.^2+(z+h).^2).^1.5;
Ex = kq.*(x./sc - x./ic);
Ey = kq.*(y./sc - y./ic);
Ez = kq.*((z-h)./sc-(z+h)./ic);
E = sqrt(Ex.^2+Ey.^2+Ez.^2);

%% Q1
S = 2*pi*R.^2;
I = (sigma+j*omg.*yp).*E.*S;
Z = Vg./I
Y = 1/Z

%% Q2
dTin = 100-37.5+273.15;   %[degree C] --> [K]
C = 3.7;   %Specific Heat of Muscle, [kJ/(kg*K) = J/(g*K)]
rou = 1030e3;   %Mass Density of Muscle, [g/m3]
```

```
dt = 0.1;
E = sqrt(dTin*C*rou*2)/(dt*sigma);
Vg = E*(h-R)
P = Vg^2/Z

%% Q3
z = linspace(0,(h-R));
Ep = Vg./z;
SARp = sigma*(Ep.^2)/(2*rou);
dTp = SARp*dt/C;
figure
plot(z,SARp);
xlabel('z');
ylabel('SAR(P)');
grid on
figure
loglog(z,SARp);
xlabel('z');
ylabel('SAR(P)');
grid on

figure
plot(z,dTp)
grid on
xlabel('z');
ylabel('\DeltaT(P)');
figure
loglog(z,dTp)
grid on
xlabel('z');
ylabel('\DeltaT(P)');

%% Q4
H = h-R;
SARL = [];
for L = R:0.01:H
    Z = H-L;
    syms z
    SARz = 0.5*sigma*(Vg./z).^2/rou;
    SARint = int(SARz,z,Z,H);
    SARL = [SARL SARint./L];
end
L = R:0.01:H;
figure
```

```
plot(L,SARL)
grid on
xlabel('L');
ylabel('SAR(L)');
title('Average SAR');

%% Q5
dH = 2260;   %[kJ/kg] --> [J/g]
m = 10e-3*1;   %[g], density of water 1g/cm3
W = dH*m   %[J]
t = W/P   %[s]

%% Q6
Eb = 2e6;   %[V/m]
Vb = Eb*h
```