

Advanced antenna engineering

Assignment 2. Patch

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Declaration:

References: Slides from the file 'Handouts'

1. patch design 3.5.pdf
2. patch_SRTLmodel_6x5.pdf

Discussed: with student Zhang Zhifan.

Problem 1

a) Design a rectangular patch antenna: width, length.

$$L \approx 0.5 \frac{\lambda_0}{\sqrt{\epsilon_r}} = 0.5 \frac{c}{f \sqrt{\epsilon_r}} \approx 0.0295 \text{ [m]}$$

① w_1

Using the formulas:

$$L = 0.0295 = \frac{1}{2} \lambda_g^p (f_R) - \Delta L$$

$$\lambda_g^p = \frac{\lambda_0}{\sqrt{\epsilon_{r,eff}}}$$

$$\Delta L = 0.412 \times h \cdot \frac{(\epsilon_{r,eff} + 0.3) \left(\frac{W}{h} + 0.264 \right)}{(\epsilon_{r,eff} - 0.258) \left(\frac{W}{h} + 0.8 \right)}$$

$$\epsilon_{r,eff} = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \cdot \frac{1}{\sqrt{1 + 12 \cdot \frac{h}{W}}}$$

Using Matlab, we can get $w_1 = 0.0263 \text{ [m]}$

However, using the Matlab function of Grad, 'grad_patch_polyapp.m' with the values L/λ_{muda0} , w_1/λ_{muda0} , we have $Z_{in}(f_0) = R_0 = 1366 \Omega$, which is much higher than 120Ω .

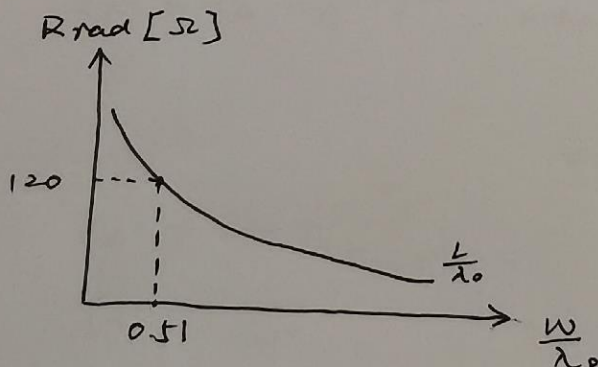
Therefore, w_1 is infeasible.

② w_2

Trying to satisfy to constrain $R_0 = Z_{in}(f_0) \leq 120 \Omega$,

we can firstly let $R_{rad} = 120$ to get an approximate value ~~from~~ ^{using} the relation graph between R_{rad} and

L/λ_0 , W/λ_0 . (Here, R_0 actually equals to $R_{rad} + \text{loss}$).



$$\begin{cases} R_{rad} = 120 \Omega \\ \frac{L}{\lambda_0} = 0.2411 \end{cases}$$

We can get, $w_2 = 0.0624 \text{ [m]}$

Accordingly, $R_0 = 129.8 \Omega > 120 \Omega$, which is also infeasible.

③ w3

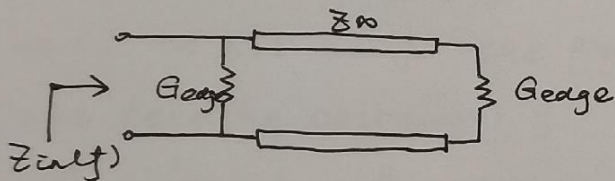
Try with a square patch this time, where $w = L = 0.0295 \text{ m}$, then $R_0 = Z_{in}(f_0) = 744.7 \Omega > 120 \Omega$, which is also infeasible.

Finally, to achieve the condition $Z_{in}(f_0) = R_0 \leq 120 \Omega$, from the plotted graph in problem 1, question b1, of $Z_{in}(f)$, a more suitable value can be found, which $W = 0.066 \text{ m}$, where $Z_{in}(f_0) = R_0 = 118.8 \Omega \leq 120 \Omega$.

Therefore, in this case:
$$\begin{cases} L = 0.0295 \text{ m} \\ W = 0.0660 \text{ m} \end{cases}$$

b) $Z_{in}(f)$

The transmission line model is shown as,



$$G_{gedg} = \sum G_{rad} \left(\frac{W}{\lambda_0}, \frac{L_{eff}}{\lambda_0} \right)$$

where $L_{eff} = L + 2 \cdot \Delta L$

$$\text{With } \frac{W}{h} \geq 1, \quad Z_0 = \frac{120\pi}{\sqrt{\epsilon_{r,eff}} \left[\frac{W}{h} + 1.393 + 0.667 \ln \left(\frac{W}{h} + 1.444 \right) \right]}$$

Using the transmission line formulas:

$$T_B^- = \frac{Z_s - Z_0}{Z_s + Z_0}, \quad \text{where } Z_s = \frac{1}{G_{gedg}}$$

$$T_A^+ = T_B^- \cdot e^{-2jkl}, \quad \text{where } k = \frac{2\pi f}{V_p} = \frac{2\pi f \sqrt{\epsilon_r}}{c}$$

$$L = 0.5 \frac{c}{f_0 \sqrt{\epsilon_r}}$$

$$\text{so, } kl = \pi \cdot \frac{f}{f_0}$$

$$Z_{A^+} = Z_0 \frac{1 + T_A^+}{1 - T_A^+}$$

$$Y_{A^+} = \frac{1}{Z_{A^+}}$$

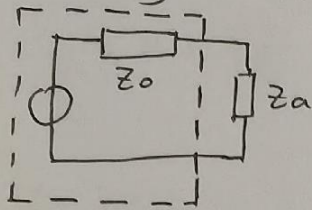
$$Y_{in} = Y_{A^+} + G_{gedg}$$

$$Z_{in} = \frac{1}{Y_{in}}$$

The figure of $\frac{f}{f_0} - Z_{in}(\frac{f}{f_0})$ can be plotted using Matlab.

b2) Return loss RL ; Bandwidth ($VSWR \leq 2$).

① Using the follow formulas to compute RL ,



$$Z_0 = R_0 = Z_{in}(f_0) = 120 \Omega$$

$$S_{11}(f) = \frac{Z_L(f) - Z_0}{Z_L(f) + Z_0}, \text{ where } Z_L(f) = Z_{in}(f).$$

$$RL(f) = |S_{11}(f)|^2$$

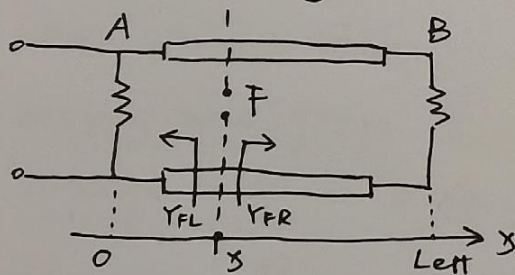
The figure of $\frac{f}{f_0} - RL(\frac{f}{f_0})$ can be plotted by Matlab. At centerband frequency, $RL = -46.25 \text{ dB}$.

② As $VSWR = \frac{1 + |S_{11}|}{1 - |S_{11}|} \leq 2$

Using Matlab to plot the figure $\frac{f}{f_0} - VSWR(\frac{f}{f_0})$, finding where $VSWR = 2$ in the figure, we can then get the bandwidth $[0.889 f_0, 1.11 f_0]$, which the absolute value is $[2.78.05 \text{ MHz}, 27.9.5 \text{ MHz}]$, which is approximately $\pm 1\%$.

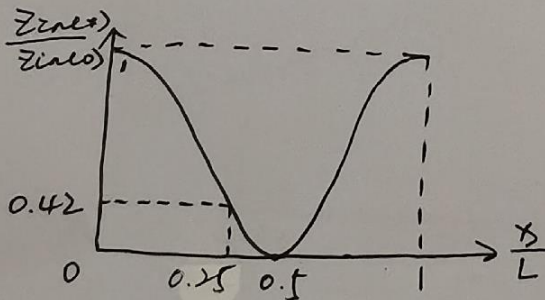
b3) Let $L = L$ to plot $Z_{in}(x)$, the figure scale is slightly smaller.

c) Probe feeding position: x .

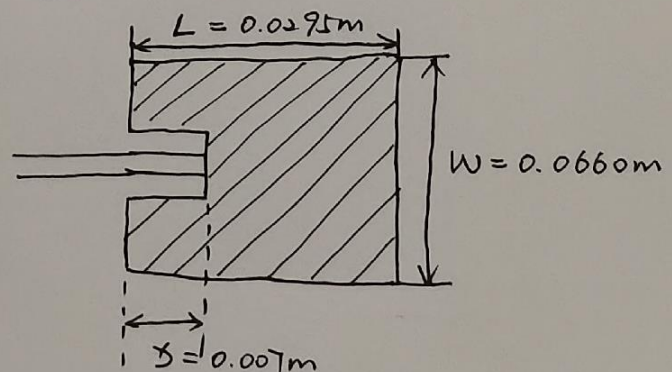


Using the graph $\frac{x}{L} - \frac{Z_{in}(x)}{Z_{in}(0)}$, where $Z_{in}(x) = 50 \Omega$,

$$Z_{in}(0) = R_0 = 118.8 \Omega, \text{ so } \frac{Z_{in}(x)}{Z_{in}(0)} = \frac{50 \Omega}{118.8 \Omega} \approx 0.42.$$



$$\frac{x}{L} = 0.25, \quad x = 0.0077 \text{ m}$$



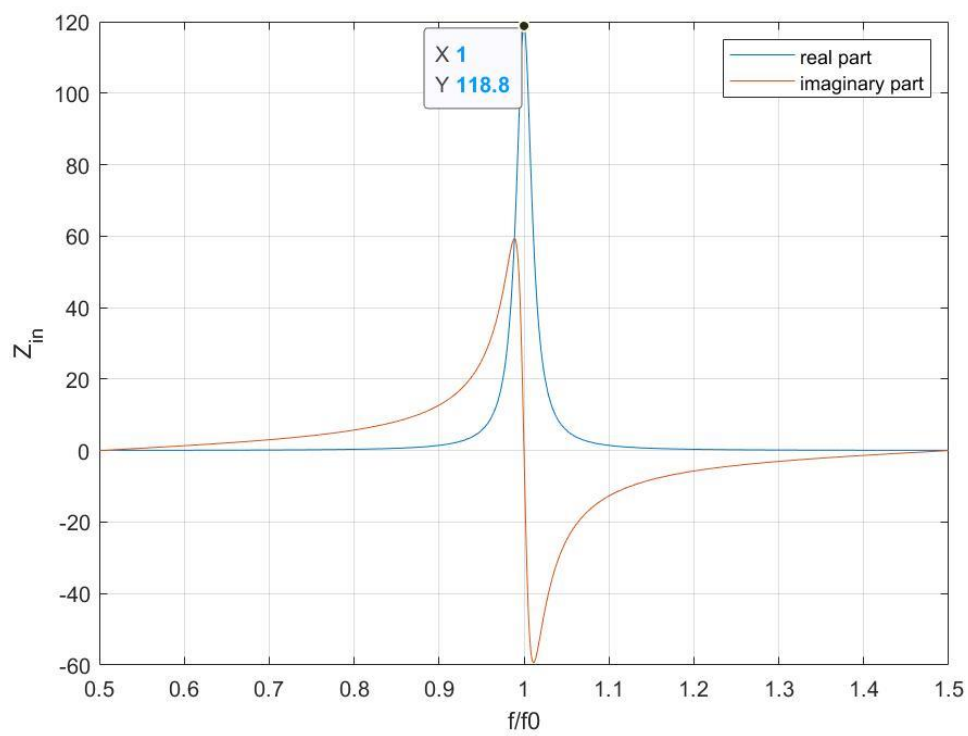


Figure 1. Problem1, Question b1) – Real and Imaginary part of input impedance

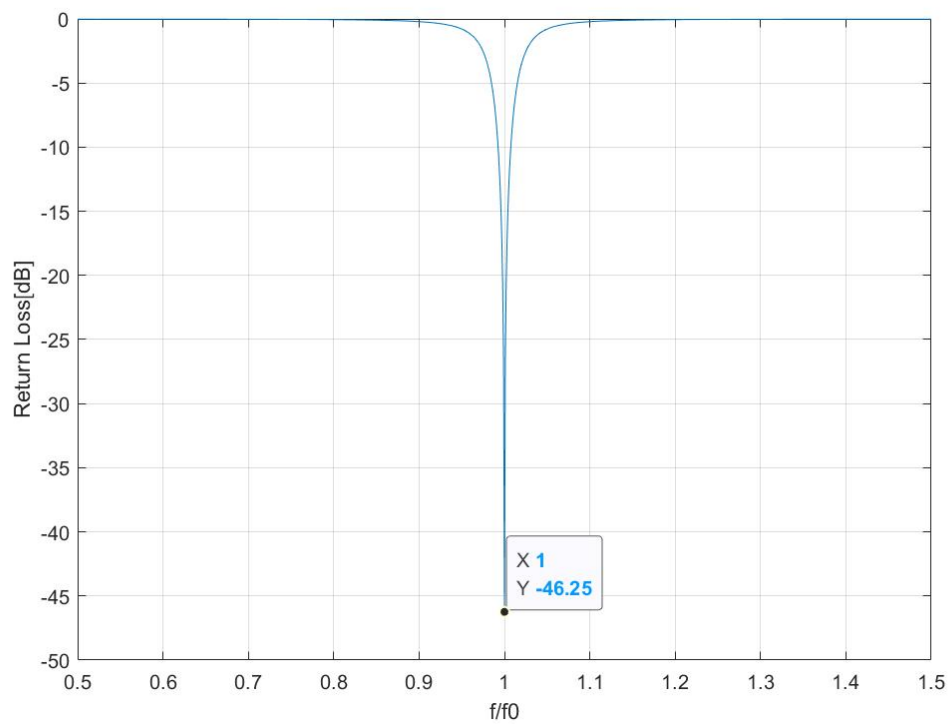


Figure 2. Problem 1, Question b2) – Return Loss

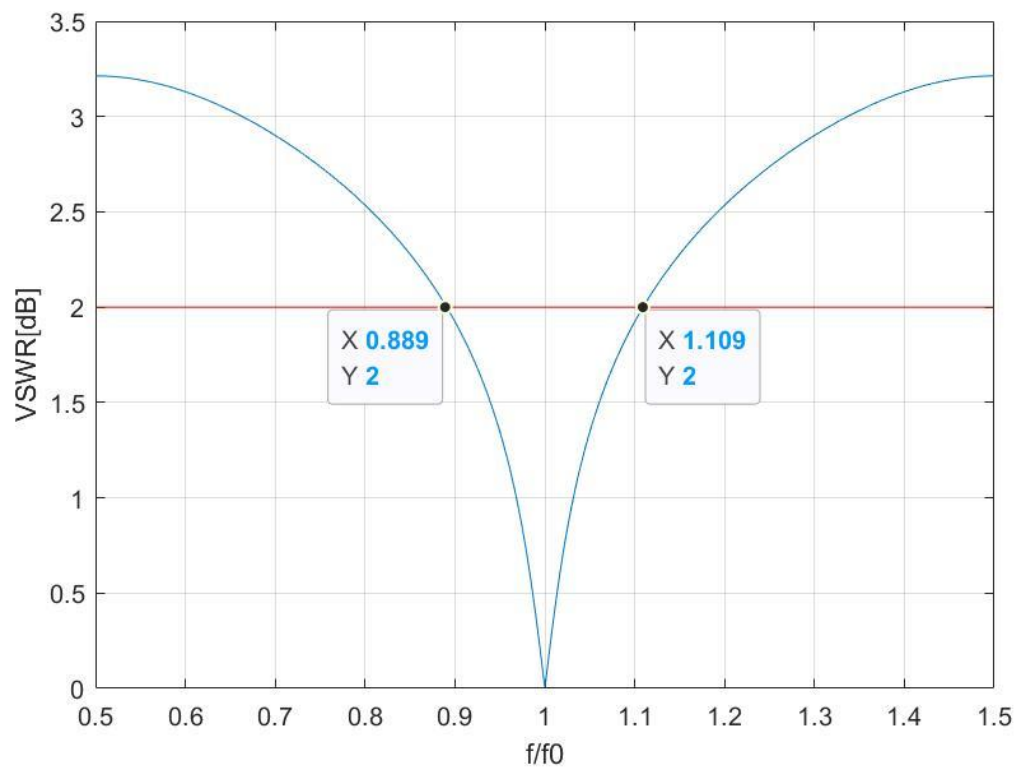


Figure 3. Problem 1, Question b2) – VSWR

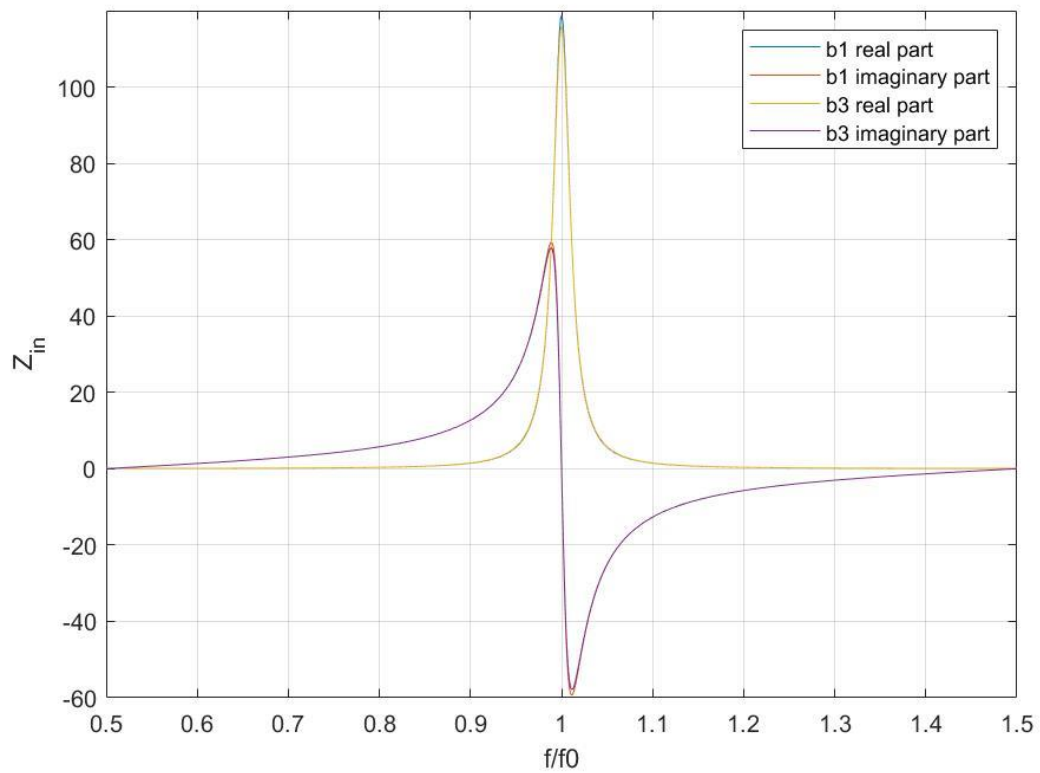


Figure 4. Problem 1, Question b3) – Input impedance with $L_{eff} = L$

Problem 2

a) Design: width & length

$$L \approx 0.5 \frac{\lambda_0}{\sqrt{\epsilon_r}}, \text{ where } \lambda_0 = \frac{c}{f_0}$$

$$\epsilon_{r, \text{air}} = 1$$

As this is a square patch, so $W = L$.

$$\text{We can then get } \begin{cases} L = 0.0612 \text{ m} \\ W = 0.0612 \pm 5\% \text{ m} \end{cases}$$

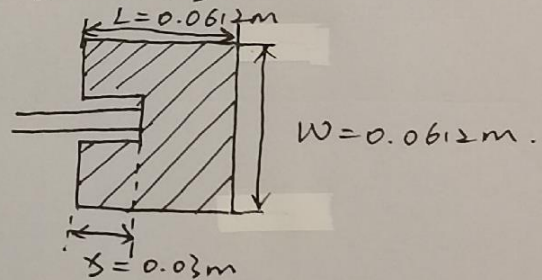
b) Probe Position: x

Using the relationship $\frac{R_{\text{rad}}(x)}{R_{\text{rad}(0)}} \approx \cos^4(\pi \frac{x}{L})$

where $R_{\text{rad}}(x) = 50 \Omega$

$$R_{\text{rad}(0)} = \frac{1}{\text{Grad}(\frac{W}{\lambda_0}, \frac{L_{\text{eff}}}{\lambda_0})} \approx 420.89 \Omega.$$

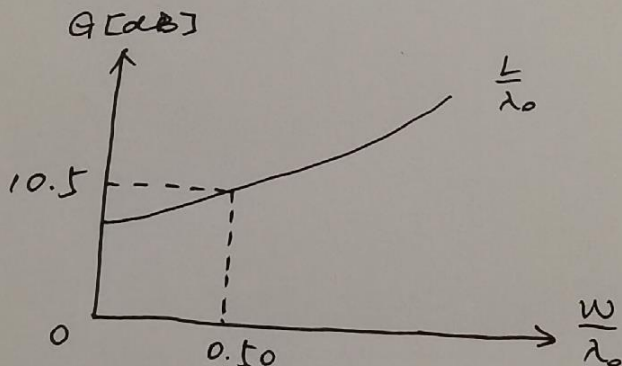
Therefore, $\frac{x}{L} \approx 0.3880$, $x = 0.03 \text{ m}$



c) Max gain: G_{max}

Using the graph $\frac{W}{\lambda_0} - G$ to find G_{max} ,

$$\text{where } \begin{cases} \frac{L}{\lambda_0} = 0.63 \\ \frac{W}{\lambda_0} = 0.50 \end{cases}$$



We can get an approximate value, $G_{\text{max}} \approx 10.5 \text{ dB}$.

Appendix: Matlab Codes

Matlab Code for Problem 1:

```
clear all;
close all;
clc

%P1-a: design
j = sqrt(-1);
c = 3e8;    %[m/s]
f0 = 2450e6;    %[Hz]
lamuda0 = c/f0;
ypr = 4.3;
L = 0.5*lamuda0/sqrt(ypr);
h = 1.55e-3;    %[m]

syms w_h1
ypr_eff = (ypr+1)/2+(((ypr-1)/2)*(1/sqrt((12/w_h1)+1)));
lamuda_g = lamuda0/sqrt(ypr_eff);
delta_L = 0.412*h*(ypr_eff+0.3)*(w_h1+0.264)/((ypr_eff-0.258)*(w_h1+0.8));
eq = L == (0.5*lamuda_g)-(2*delta_L);
x = solve(eq,w_h1);
w_h1 = double(x);
w1 = w_h1*h;

%find the optimum width
L_lamuda0 = L/lamuda0;
w2 = 0.51*lamuda0;    %0.51 -- from the graph
w3 = L;
W_L = [w1 w2 w3]/L;
W_lamuda0 = [w1 w2 w3]/lamuda0;    %try all 3 results to verify the condition of
R0<=120 Ohm
W = 0.066;    %opt: width=0.066 R0=118.8
W_h = W/h;

%P1-b1: plot Zin
ypr_eff0 = (ypr+1)/2+(((ypr-1)/2)*(1/sqrt((12*h/W)+1)));
delta_L = 0.412*h*(ypr_eff0+0.3)*(W/h+0.264)/((ypr_eff0-0.258)*(W/h+0.8));
L_eff1 = L+2*delta_L;
f_f0 = 0.5:0.001:1.5;
kl = pi*f_f0;
G_rad1 = grad_patch_polyapp(W/lamuda0,L_eff1/lamuda0);
G_edge1 = 0.5*G_rad1;
```



```

Z_C1 = 120*pi/(sqrt(ypr_eff0)*(W/h)+1.393+0.667*log(1.444+W/h));
Z_S1 = 2/G_rad1;
gamma_B1 = (Z_S1-Z_C1)/(Z_S1+Z_C1);
gamma_A1 = gamma_B1*exp(-2*j*kl);
Z_A1 = Z_C1*(1+gamma_A1)/(1-gamma_A1);
Y_A1 = 1./Z_A1;
Y_in1 = Y_A1 + G_edge1;
Z_in1 = 1./Y_in1;

```

```

figure
plot(f_f0,real(Z_in1));
hold on
plot(f_f0,imag(Z_in1))
xlabel('f/f0');
ylabel('Z_{in}');
legend('real part','imaginary part');
grid on

```

%P1-b3 plot the two Zin on the same plot

```

L_eff3 = L;
G_rad3 = grad_patch_polyapp(W/lamuda0,L_eff3/lamuda0);
G_edge3 = 0.5*G_rad3;
Z_C3 = 120*pi/(sqrt(ypr_eff0)*(W/h)+1.393+0.667*log(1.444+W/h));
Z_S3 = 2/G_rad3;
gamma_B3 = (Z_S3-Z_C3)/(Z_S3+Z_C3);
gamma_A3 = gamma_B3*exp(-2*j*kl);
Z_A3 = Z_C3*(1+gamma_A3)/(1-gamma_A3);
Y_A3 = 1./Z_A3;
Y_in3 = Y_A3 + G_edge3;
Z_in3 = 1./Y_in3;

```

```

figure
plot(f_f0,real(Z_in1));
hold on
plot(f_f0,imag(Z_in1))
hold on
plot(f_f0,real(Z_in3));
hold on
plot(f_f0,imag(Z_in3));
xlabel('f/f0');
ylabel('Z_{in}');
legend('b1 real part','b1 imaginary part','b3 real part','b3 imaginary part');
grid on

```

```

%P1-b2 Return loss
Z0 = 120;
S11 = (Z_in1-Z0)./(Z_in1+Z0);
RL = 10*log10((abs(S11)).^2);
figure
plot(f_f0,RL);
xlabel('f/f0');
ylabel('Return Loss[dB]');
grid on
% VSWR
VSWR = log10((1+abs(S11))./(1-abs(S11)));
level = 2*ones(1,1001);
figure
plot(f_f0,VSWR);
xlabel('f/f0');
ylabel('VSWR[dB]');
hold on
plot(f_f0,level,'r');
grid on

%P1-c: probe feeding position
R0 = Z_in1(f_f0==1);
position_int = 50/R0;      %find the position x in the graph
x = 0.25*L_eff1           %x/L = 0.25

```

Matlab Code for Problem 2:

```
clear all;  
close all;  
clc
```

```
%P2-a: design
```

```
j = sqrt(-1);  
c = 3e8;    %[m/s]  
f0 = 2.45e9;    %[Hz]  
lamuda0 = c/f0;  
ypr = 1;          %air  
L = 0.5*lamuda0/sqrt(ypr);  
h = 0.1*lamuda0;  
W = L;  
W_h = W/h;
```

```
%P2-b: probe feeding position
```

```
ypr_eff0 = (ypr+1)/2+(((ypr-1)/2)*(1/sqrt((12*h/W)+1)));  
delta_L = 0.412*h*(ypr_eff0+0.3)*(W/h+0.264)/((ypr_eff0-0.258)*(W/h+0.8));  
L_eff = L+2*delta_L;  
G_rad = grad_patch_polyapp(W/lamuda0,L_eff/lamuda0);  
R_rad = 1/G_rad;  
x_cos = 50/R_rad;  
x_L = acos(sqrt(x_cos))/pi;  
x = x_L*L_eff;
```

```
%P2-c: max gain
```

```
L_lamuda0 = L_eff/lamuda0;  
W_lamuda0 = W/lamuda0;          %Use the graph to find Gmax
```