Communication System Assignment 5

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Exercise 5.1: Spatial Multiplexing

1. Transmitter Side

   Here a 2x2 MIMO system is used to simulate the spatial multiplexing with an known channel matrix *H*, which is generated as the sample of independent Raylegh pdf with randem real and imaginary part generated as independent Gaussian pdfs. In this case, *H* matrix is,

$$H = \begin{bmatrix} -0.4311 - j0.1220 & -0.6131 + j0.0303 \\ -0.6385 - j1.0531 & 0.2149 + j0.1660 \end{bmatrix}$$

   The matrix of transmitted two 4-QAM symbols *X* is,

$$X = \begin{bmatrix} 1 + j \\ -1 - j \end{bmatrix}$$

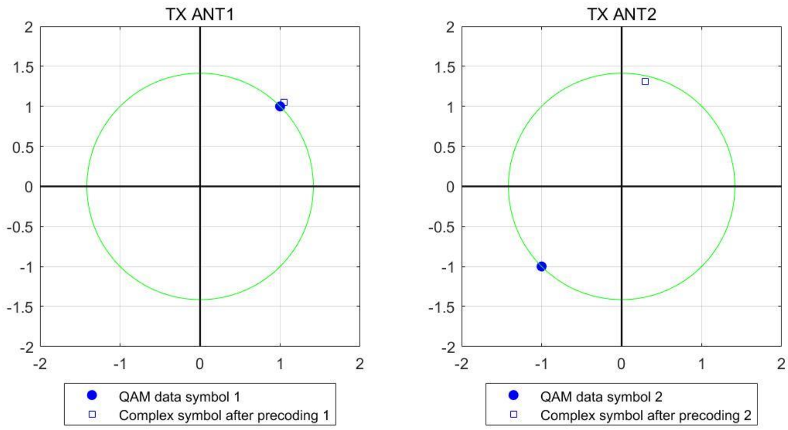

Figure 1.1: Original and After-Precoding Transmitted Symbols

   The original and after-precoding symbols of antenna 1 and 2 are shown in figure 1.1, where the precoding is used to transmit the 2 QAM symbols to the different users (here is the 2 receiver antennas) at the same time on the same band according to the channel the symbols transmitted. With the precoder operation, the transmitted symbols matrix now is *X'*.

$$X' = \begin{bmatrix} 1.0467 + j1.0467 \\ 0.2965 + j1.311 \end{bmatrix}$$

2. Receiver Side

   To the receiver side, the initial received vector is *Y* according to *X'*.

$$Y = \begin{bmatrix} -0.5451 - j1.3743 \\ 0.2800 - j1.4394 \end{bmatrix}$$

   By applying the postcoding, the received vector is then matrix *Z*. The received symbols of different antennas are shown in figure 1.2.

$$Z = \begin{bmatrix} 1.3116 + j1.3116 \\ -0.6691 - j0.6691 \end{bmatrix}$$

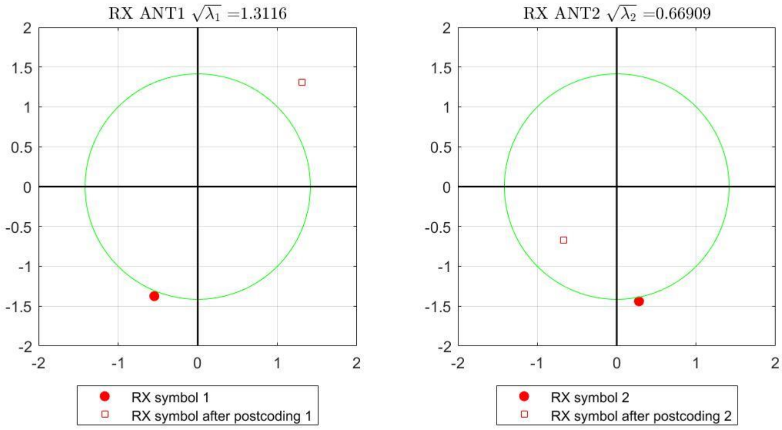

Figure 1.2: Original and After-Postcoding Received Symbols

To verify the rightness of the final result, Z1 and Z2 of Z matrix can be computed again.

$$Z_1 = \sqrt{\lambda_1}X_1 = 1.3116 \times (1 + j) = 1.3116 + j1.3116$$
$$Z_2 = \sqrt{\lambda_2}X_2 = 0.66909 \times (-1 - j) = -0.6691 - j0.6691$$

3. Another Case

Here attached another case's figure as below, where H and X varies.

$$H = \begin{bmatrix} 0.0515 - j3.4210 & 0.7332 + j0.0614 \\ 0.4419 + j0.1107 & 0.0552 - j0.2855 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 + j \\ 1 + j \end{bmatrix}$$

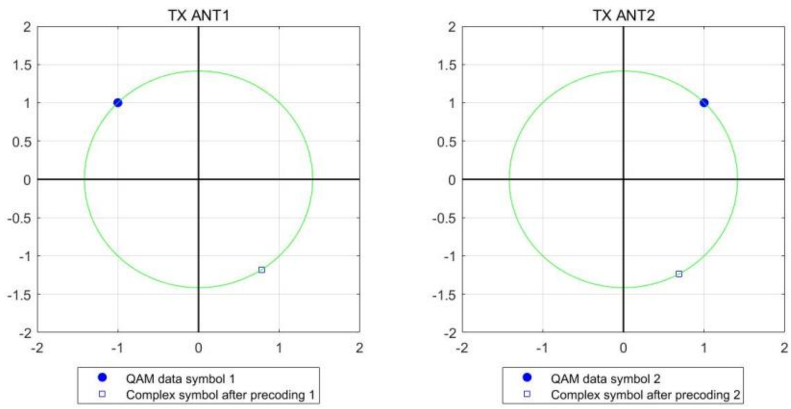

Figure 1.3: Original and After-Precoding Transmitted Symbols of case 2

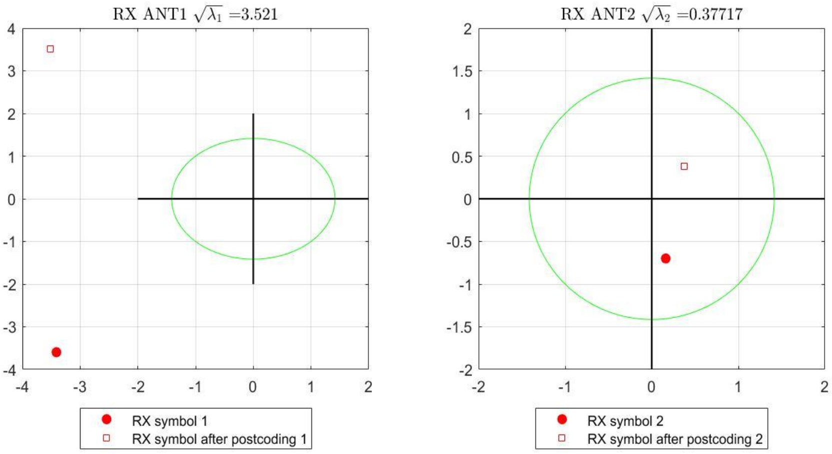

Figure 1.4: Original and After-Postcoding Received Symbols of case 2

Exercise 5.2: Antenna Array

1. Uniform Linear Array

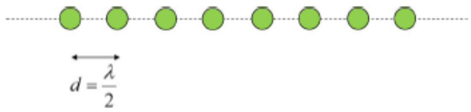

Figure 2.1: Uniform Linear Array

The uniform linear antenna array is an array with M elements setting in a line with an interval of a certain distance d between each element, shown as figure 2.1. Here M=8 and d=λ/2. By supposing that the receiver is very far away from the array, so that the signals transmitted can be seem as parallel for different antenna in the linear array, which means that the visible angles are same for each element.

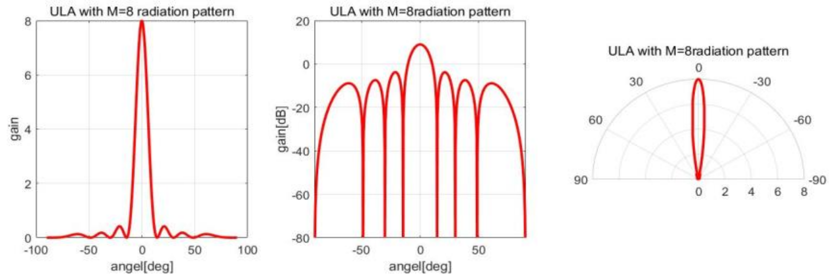

Figure 2.2: Radiation Patterns with M=8

The radiation patterns of M=8 elements are shown in figure 2.2. For a uniform linear array, the beamformer can be generated as a uniform 1xM vector, which can be consider as the receiver is in the right top of the antenna array and the transmission orientation is vertical with the array. Therefore, the gain has a maximum value at 0°, at about 8dB.

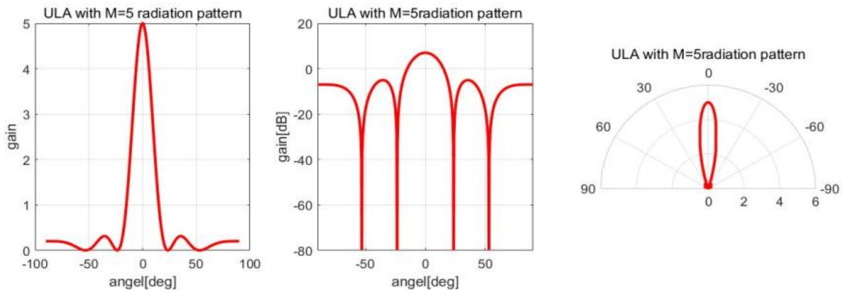

Figure 2.3: Radiation Patterns with M=5

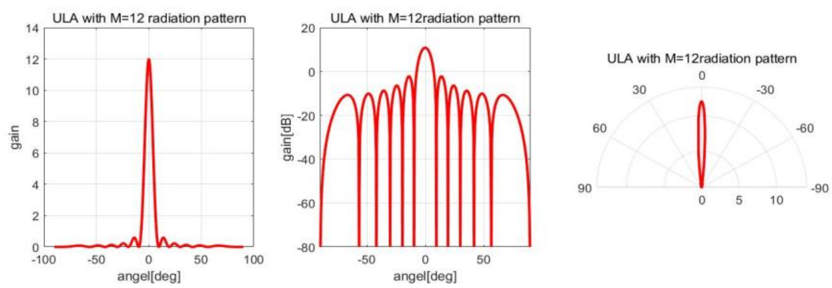

Figure 2.4: Radiation Patterns with M=12

As shown in figure 2.3 and 2.4, where the M changes into 5 and 12. Still the maximum gain is along 0°, as the position of the receiver has not change. However, with the change of the numbers of the elements, it is obvious that the maximum gain has been increasing with the increasing of the antenna numbers in the array.

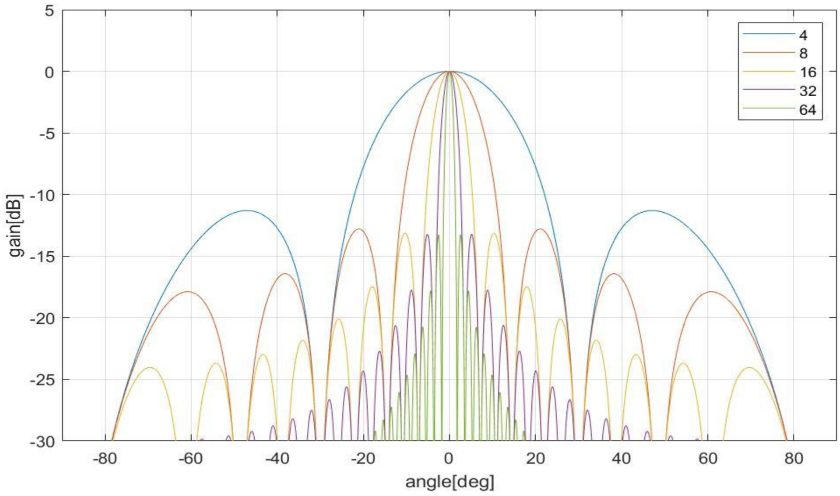

Figure 2.5: The Normalized Gain for M=4,8,16,32

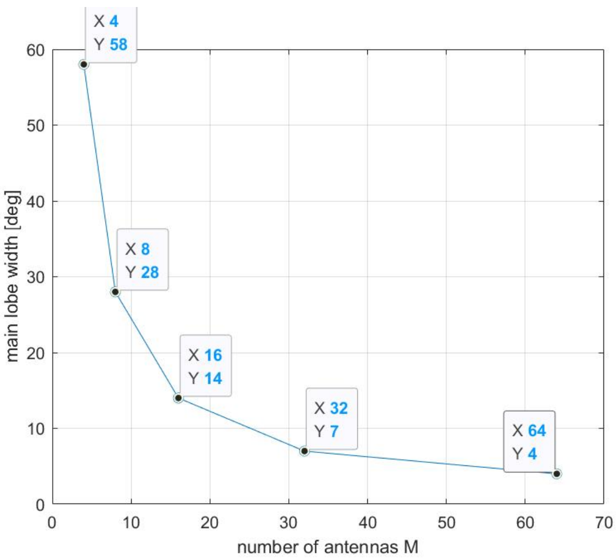

Figure 2.6: The Main Lobe Width for M= M=4,8,16,32

By plotting the normalized gain with the antenna numbers from 4 to 64, as shown in figure 2.5, the main lobe width can be easier to observe. As shown in figure 2.6, the main lobe width, which stands for the first null-to-null angles values, is decreasing with the increase of the number of the antennas. With the increasing of the numbers of the antennas we can also get smaller main lobes, which stands for more directive radiation patterns.

In addition, the first and second lobes peaks have almost the same amplitudes for different numbers of the antennas, as the antenna gain for uniform linear array is equal to the number of the antennas when the angle equals to 0, proved as the equation below.

$$g(\sigma = 0, M) = \frac{P(\sigma, M)}{P(\sigma, M = 1)} = M$$

2. Conventional Beamforming

For a conventional beamforming, where the angles are random, the radiation patterns are in figure 2.7, 2.8 and 2.9, which separately represents the arrays of M=8, ϑ_0=30°; M=5, ϑ_0=30°; M=8, ϑ_0=60°.

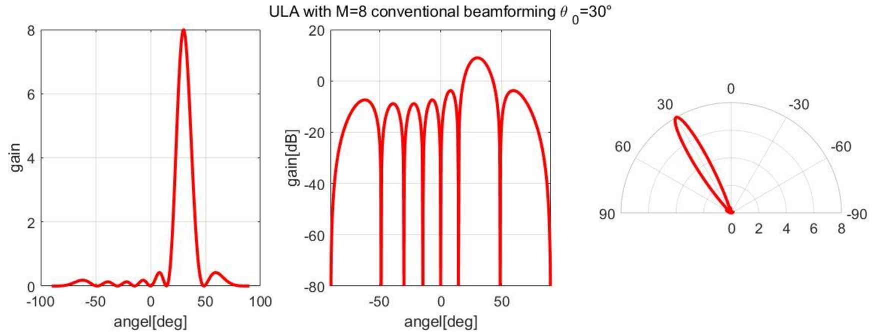

Figure 2.7: Gain of Uniform Linear Array for M=8, ϑ_0=30°

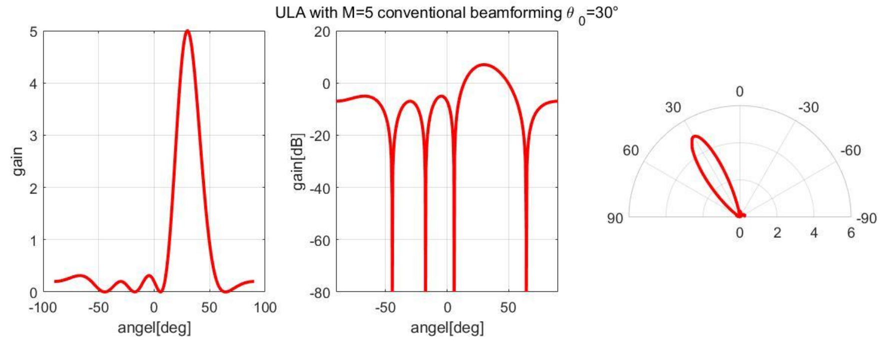

Figure 2.8: Gain of Uniform Linear Array for M=5, ϑ_0=30°

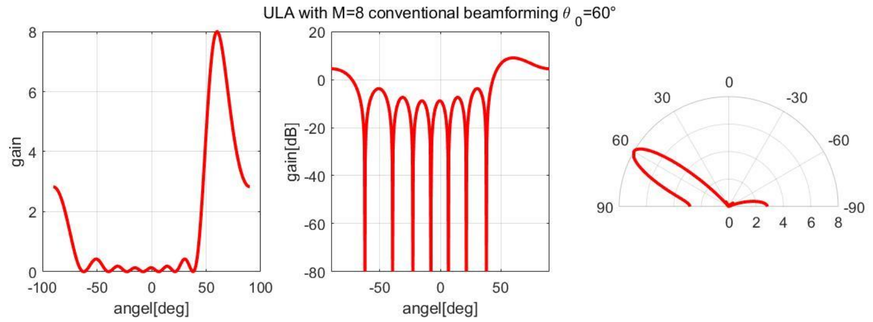

Figure 2.9: Gain of Uniform Linear Array for M=8, ϑ_0=60°

It can be observed that, the maximum gain is still increasing with the increase number of the antennas and equals to the number of the antenna. The angles of the maximum gains is changing with the receivers' position, which equals to ϑ_0.

3. Vandermolde DFT Matrix

By applying the conventional beamforming, the radiations can only point in the specific direction of ϑ_0. However, this is only valid when there's only one receiver. If there's multiple receivers with multiple RF chains, the Vandermolde DFT matrix needed to be used for the beamformer, also to reduce the interference on other selected directions produced by the other beams.

In figure 2.10, the number of the antennas in the array is 8. Here the transmissions can be operated along different angles with the same amplitude of the gain.

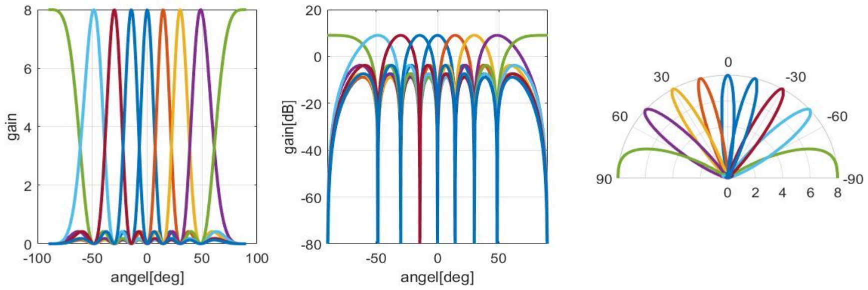

Figure 2.10: Gain of Vandermolde matrix for M=8

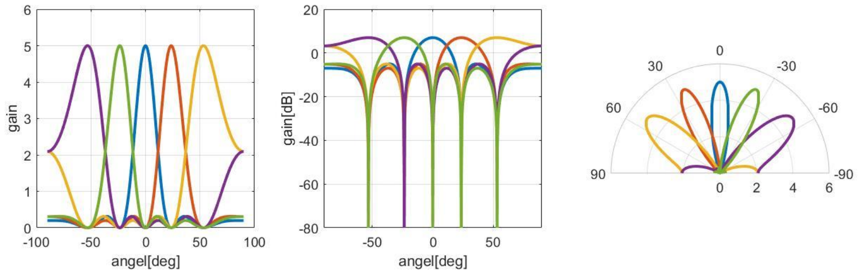

Figure 2.11: Gain of Vandermolde matrix for M=5

As shown in figure 2.11, when changing the number of the antennas, it has the same phenomenon as the conventional beamforming.

4. Butler Matrix

However, the application of the Vandermolde matric only allows to generates a beam pointing at $\pi/2$ and $-\pi/2$. To improve this, the Butler matrix can be used to rotating the beams as shown in figure 2.12 and 3.13 separately with number of the antennas equals to 8 and 5.

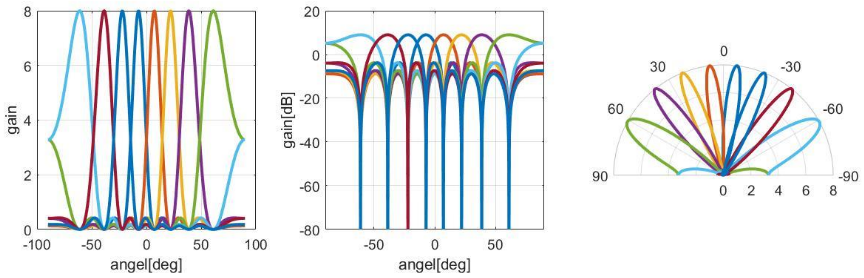

Figure 2.12: Gain of Butler matrix for M=8

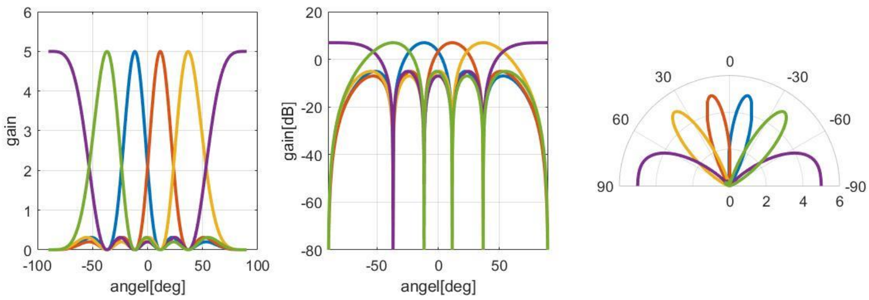

Figure 2.13: Gain of Butler matrix for M=5