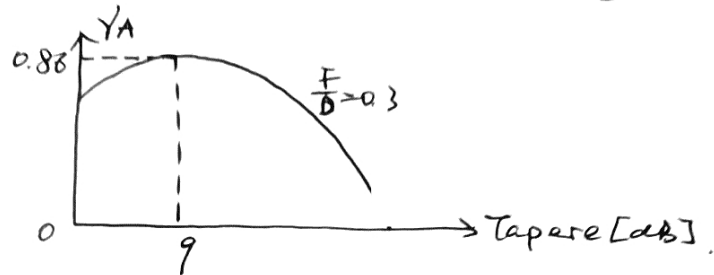


Problem 5.1

In order to obtain the optimum value of the diameter (D), the highest paraboloid efficiency should be selected, as 0.86, according to the figure, with the case of $\frac{F}{D} = 0.3$.



Considering the additional reduction of the aperture efficiency of 1 dB, the final efficiency is,

$$Y = Y_A |_{dB} - 1 \text{ dB} = 10 \log_{10} 0.86 - 1 \approx -1.66 \text{ dB}.$$

The diameter D can be then obtained with the function of gain, which is,

$$G = \frac{4\pi}{\lambda^2} \cdot Y \cdot A_{\text{geo}} \\ = \frac{4\pi}{\lambda^2} \cdot Y \cdot \pi \left(\frac{D}{2}\right)^2$$

The value of gain can be analyzed as follows.

Starting with the receiving power,

$$P_R = P_T \cdot \frac{G_R G_T}{\left(\frac{4\pi R}{\lambda}\right)^2} (1 - |T_R|^2)(1 - |T_T|^2) \cdot |\hat{P}_R \cdot \hat{P}_T^*|^2$$

where in the condition of matched antenna, $T_R = 0$, $T_T = 0$, also with the same polarization of transmitter and the receiver, $|\hat{P}_R \cdot \hat{P}_T^*| = 1$.

The expression can then written in the unit of dB as,

$$P_R [\text{dB}] = P_T [\text{dB}] + G_R [\text{dB}] + G_T [\text{dB}] - 10 \log_{10} \left(\frac{4\pi R}{\lambda}\right)^2$$

where in the expression, the known values are,

$$P_R \geq -70 \text{ dB}, P_T = 10 \text{ kW} = 40 \text{ dB}, G_R = 25 \text{ dB}, R = 36000 \text{ km},$$

$$f = 18 \text{ GHz}, 10 \log_{10} \left(\frac{4\pi R}{\lambda}\right)^2 \approx 208.67 \text{ dB}.$$

Therefore,

$$G_T = P_R - P_T - G_R + 10 \log_{10} \left(\frac{4\pi R}{\lambda}\right)^2 \\ \geq -70 - 40 - 25 + 208.67 \\ = 73.67 \text{ dB}.$$

Then, $D \approx 31 \text{ m}$.

Problem 5.2

The diameter of the reflector can be obtained with the value of gain and aperture efficiency, with the expression,

$$G = \frac{4\pi}{\lambda^2} \cdot \eta_A \cdot A_{\text{geo}}$$

$$\text{where } \lambda = \frac{c}{f} = \frac{3 \times 10^8}{2.54 \times 10^9} = 0.12 \text{ m}$$

$$\text{and } A_{\text{geo}} = \pi \left(\frac{D}{2} \right)^2.$$

The gain G can be calculated using the expression,

$$G = \frac{\frac{dP}{d\Sigma}}{\left(\frac{dP}{d\Sigma} \right)_{\text{iso}}}$$

$$\text{where } \frac{dP}{d\Sigma} = \frac{|E|^2}{Z_0} = \frac{(150 \times 10^{-3} \text{ V/m})^2}{120 \pi \Omega} \approx 5.97 \times 10^{-5}$$

$$\left(\frac{dP}{d\Sigma} \right)_{\text{iso}} = \frac{P_{\text{feed}}}{4\pi R^2} = \frac{55}{4\pi (15 \times 10^3)^2} \approx 1.946 \times 10^{-8}$$

$$G = \frac{5.97 \times 10^{-5}}{1.95 \times 10^{-8}} \approx 3062$$

Assuming that the fields are symmetrical,

$$\text{HPBW} = \sqrt{\frac{3 \times 10^4}{G}} \approx 3.13^\circ$$

For the first side-lobe,

$$\frac{\theta_{1L}}{\text{HPBW}} = 1.5, \quad \theta_{1L} = 3.13^\circ \times 1.5 \approx 4.70^\circ$$

$$\frac{g_{\text{SL}}(\theta_{1L})}{G} \text{ dB} = (-20.6 - 1.2 \times 4.70^\circ) = -26.24 \text{ dB}$$

For the maximum angle,

$$\frac{\theta_{\text{max}}}{\text{HPBW}} = \frac{20^\circ}{3.13^\circ} \approx 6.39$$

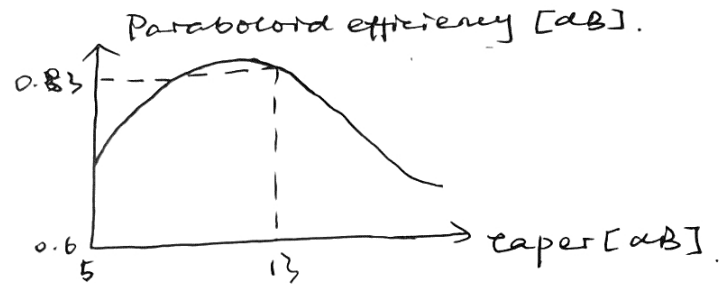
$$\frac{g_{\text{SL}}(\theta_{\text{max}})}{G} \text{ dB} = (-20.6 - 1.2 \times 20^\circ) = -44.6 \text{ dB}$$

Checking the figure with 'Side lobes envelope vs. θ/HPBW '



$$t \approx -13 \text{ dB}$$

With $\tau = -13 \text{ dB}$, $\frac{F}{D} = 0.4$, according to the figure, $V_A = 0.83$.



Considering the reduction of 1 dB of the aperture efficiency,

$$\begin{aligned} V &= V_A /_{\text{dB}} - 1 \\ &= 10 \log_{10} 0.83 - 1 \approx -1.8092 \text{ dB}. \end{aligned}$$

Therefore,

$$D = \sqrt{\frac{G \cdot \lambda^2}{4\pi \cdot V}} \cdot 2 \approx 2.6 \text{ m}.$$

The field intensity can be expressed as the ratio between the field along the maximum angle $\theta_{\max} = 20^\circ$ and the maximum field along $\theta = 0$,

$$\epsilon_0 = \frac{|E(\theta_{\max} = 20^\circ, \varphi)|}{|E(\theta_0 = 0^\circ, \varphi)|}$$

$$\text{As } \underline{E}(\omega, \varphi) = V_0 \cdot \frac{e^{-jkr}}{r} \cdot F(\omega, \varphi) \cdot \hat{P}(\omega, \varphi).$$

$$\begin{aligned} \epsilon_0 &= \frac{|F(\theta_{\max}, \varphi)|}{|F(\theta_0, \varphi)|} \cdot \frac{r_0}{r_{\theta_{\max}}} \\ &= \cos^2\left(\frac{\theta_{\max}}{2}\right) \cdot \frac{|F(\theta_{\max}, \varphi)|}{|F(\theta_0, \varphi)|} \\ &= \alpha_{\text{space}} + \alpha_{\text{feed}}. \end{aligned}$$

$$\text{Where } \alpha_{\text{space}} = \cos^2\left(\frac{\theta_m}{2}\right).$$

$$\begin{aligned} \theta_m &= 2 \arctan\left(\frac{1}{4} \cdot \frac{D}{f}\right), \text{ thus } \alpha_{\text{space}} = 0.99^\circ \\ &= 56.73 \text{ rad.} \\ &= -0.04 \text{ dB} \end{aligned}$$

$$\begin{aligned} \alpha_{\text{feed}} &= \epsilon_0 - \alpha_{\text{space}} \\ &= (-13 \text{ dB}) - (-0.04 \text{ dB}) = -13.04 \text{ dB}. \end{aligned}$$