Communication System Assignment 1

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Exercise 1: Bit Rate vs. Key Parameters

1. Values for 5G Base-Station

1.1 Maximum Achievable Bit Rate

The maximum achievable bit rate R_b for Additive White Gaussian Noise (AWGN) channel can be expressed by the function below.

$$R_b < C = Blog_2(1 + \frac{P_{RX}}{P_N})$$

In the above formular, B stands for the band, which is 100MHz for a typical 5G band. P_{RX} and P_N separately stands for the received power and noise power, which can be calculated as the functions below.

$$P_{RX} = P_{TX} \frac{G_{TX} G_{RX}}{(\frac{4\pi df}{c})^2} = 2.1988 \times 10^{-6}[W]$$

Where in this formular, P_{TX} stands for the transmitted power, which is 200W or 55dBm for a typical large 5G band. G_{TX} and G_{RX} are the transmitted and received antenna gains, which are used 10dBi and 0dBi as the 5G case. d is the distance between the transmitted and received antennas, which normally is 200m as a typical 5G cell radius. Frequency $f = 3.6GHz$ and light speed $c = 3 \times 10^8 m/s$ are used to calculate the wavelength λ. The final calculated value for P_{RX} is 2.1988×10^{-6}W, or -56.5781dBW.

$$P_N = kTBF = 2.4840 \times 10^{-12}\ [W]$$

In the function of noise power P_N as shown in the above. $k = 1.038 \times 10^{-23}$ is the Boltzmann constant. T is the operative temperature of the receiver in the unit of Kelvin degrees, which in theory usually use 300K as a normal indoor temperature. B and F separately stands for the band of 100MHz and noise figure as 6dB for a typical value. The calculated value P_N equals 2.4840×10^{-12}W, or -116.0485.

Using the mentioned formulars, the maximum achievable bit rate R_b can be calculated.

$$R_b = 1.9756 \times 10^9 bit/s \cong 1.98 Gbit/s$$

This value under this circumstance is conform to the international standard limitation up to 32 Gbit/s in the downlink mentioned in *Recommendation M.2150-0 (02/2021)* posted on 02/02/2021 of International Telecommunication Union (ITU), quote the paragraph in page 8 part 1.1.1 as, '*Transmission bandwidths up to 640 MHz are supported, yielding peak data rates up to roughly 32 Gbit/s in the downlink (DL) and 13.6 Gbit/s in the uplink (UL).*'

1.2 Free Space Attenuation

The free space attenuation A can be described as either the function below.

$$A = (\frac{4\pi df}{c})^2$$

$$A[dB] = 32 + 20\log(d) + 20\log(f[GHz])$$

Where in this formular, $d = 200m$ is the distance between the transmitted and received antennas as a typical 5G cell radius. Frequency $f = 3.6GHz$ and light speed $c =$

$3 \times 10^8 m/s$ are used to calculate the wavelength λ. The free space attenuation finally equals to 9.0958×10^8, or 89.5884dB.

2. Bit Rate Analyzations

Generally, the relationship between bit rate and other parameters can be described as the function below.

$$R_b = Blog_2(1 + \frac{P_{RX}}{P_N})$$

2.1 Bit rate vs. Band

As in the function above, the frequency band B has a linear relationship with the bit rate, as shown in figure 1.1, that with the increase of the frequency band, the bit rate will be increasing accordingly.

It is clear to figure out from the figure that when the frequency band is 100MHz, the bit rate approximately equals to 2000Mbit/s, which is near to the value calculated in section 1.1.

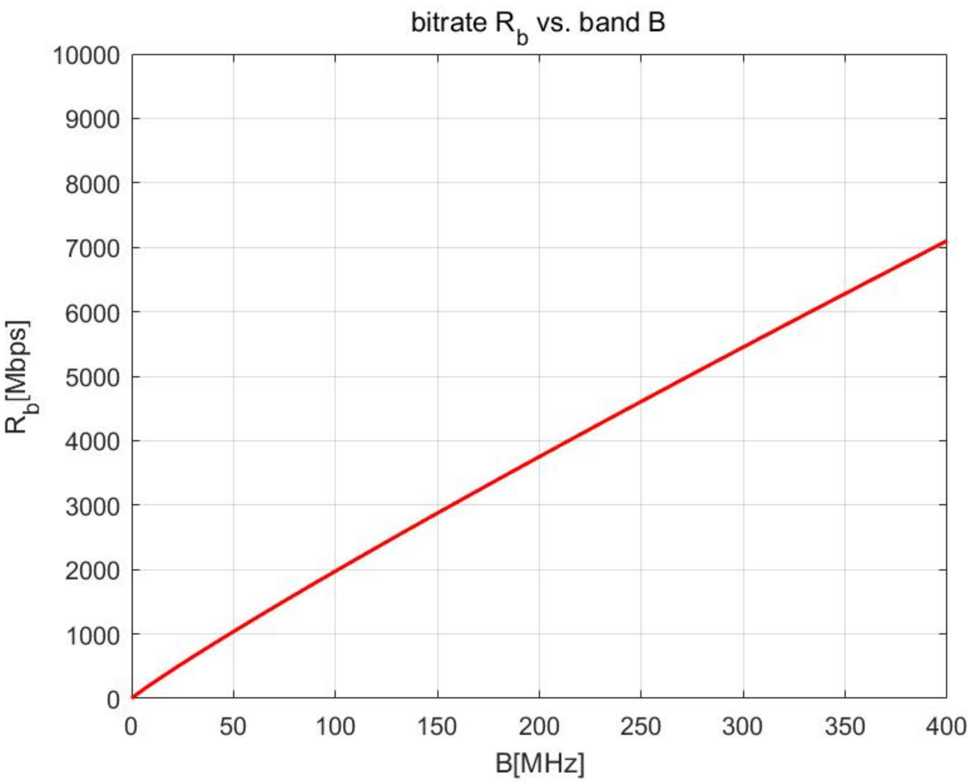

Figure 1.1: Bit-rate R_b vs. band B

2.2 Bit rate vs. Transmitted Power

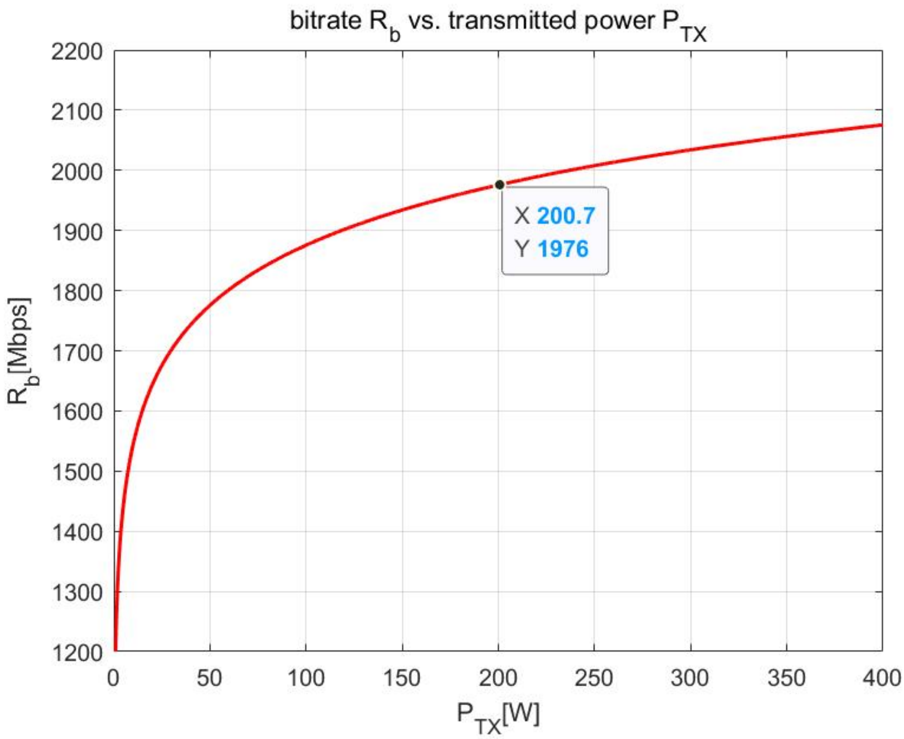

Figure 1.2: Bit-rate R_b vs. Transmitted Power P_{TX}

As the transmitted power has a linear relationship with the received power, expressed in the functions below. The bit rate has then the same but a scaled logarithm relationship with the transmitted power P_{TX} as the received power P_{RX}, as shown in figure 1.2.

$$P_{RX} = P_{TX}\frac{G_{TX}G_{RX}}{(\frac{4\pi df}{c})^2}$$

$$R_b = Blog_2(1 + \frac{P_{TX}\frac{G_{TX}G_{RX}}{(\frac{4\pi df}{c})^2}}{P_N})$$

2.3 Bit rate vs. Distance

$$A = (\frac{4\pi df}{c})^2$$

$$P_{RX} = P_{TX}\frac{G_{TX}G_{RX}}{A}$$

$$R_b = Blog_2(1 + \frac{P_{RX}}{P_N})$$

According to the functions above, that the free space attenuation A has a quadratic relationship to the distance d, and the received power P_{RX} has a reciprocal relationship to the attenuation A, and there's a logarithm relationship between bit rate R_b and received power P_{RX}. The bit rate can finally be described as the function below with distance d.

$$R_b = Blog_2\left(1 + \frac{P_{TX}\frac{G_{TX}G_{RX}}{\left(\frac{4\pi df}{c}\right)^2}}{P_N}\right)$$

Their relationship can be shown as in figure 1.3, with the increasing of the distance, the bit rate is decreasing and has a gradually slowing tendency.

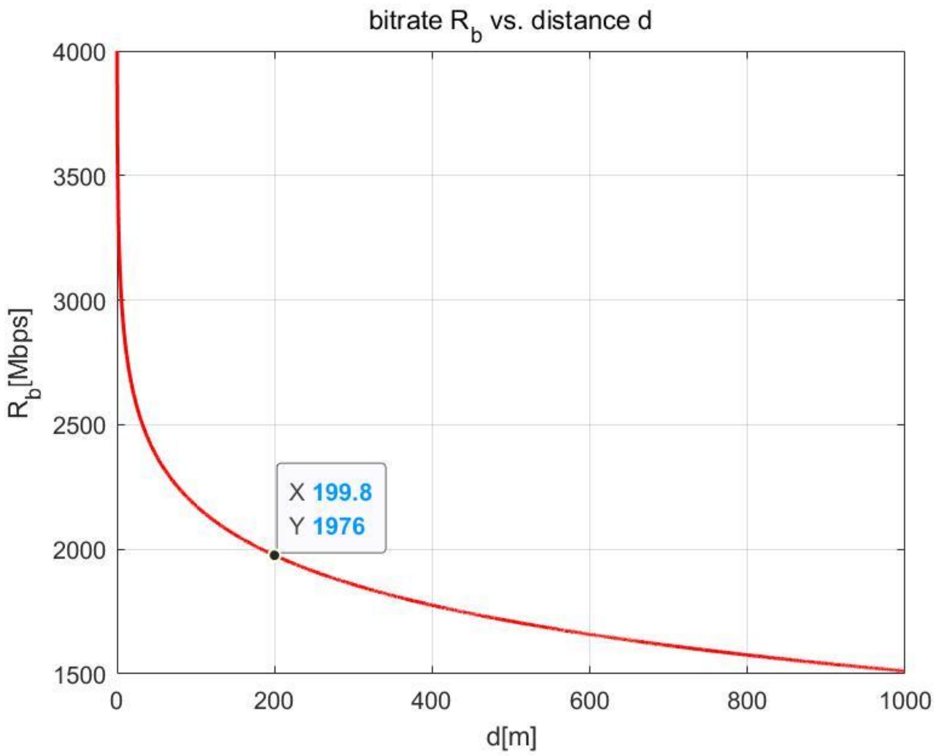

Figure 1.3: Bit-rate R_b vs. Distance d

The bit rate at d=200m equals to the maximum achievable bit rate as mentioned in section 1.1 as 1976Mbit/s, which the point is approximately marked in the figure above.

2.4 Bit rate vs. frequency

The detailed function between frequency f and bit rate R_b is once again according to the received power P_{RX} and free space attenuation A, as the function below.

$$R_b = Blog_2\left(1 + \frac{P_{TX}\frac{G_{TX}G_{RX}}{(\frac{4\pi df}{c})^2}}{P_N}\right)$$

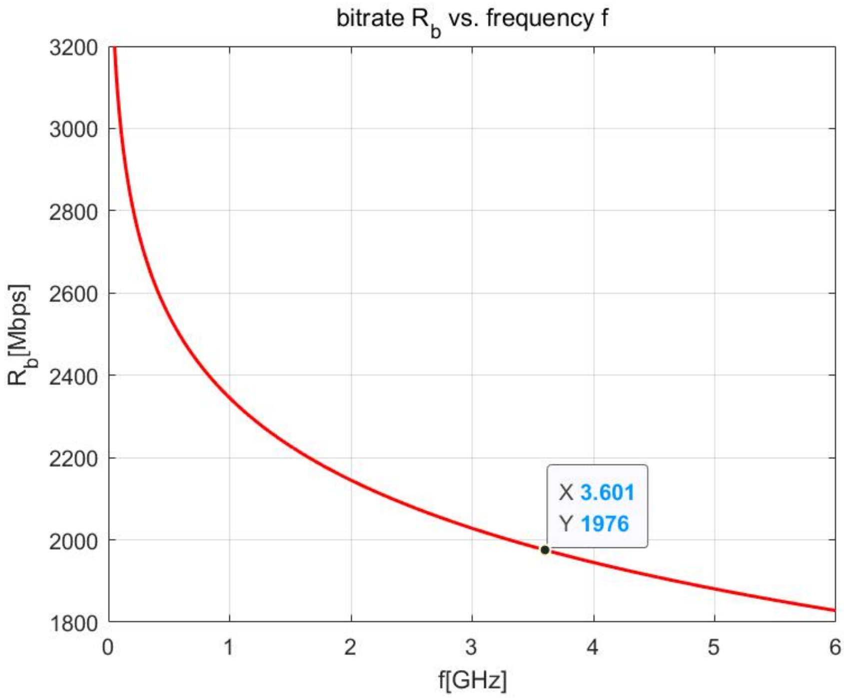

Figure 1.4: Bit-rate R_b vs. frequency f within the range of 0-6GHz

As shown in figure 1.4, with the frequency in range 0 to 6GHz, the frequency has a reciprocal logarithm relationship with the bit rate, which is more obvious when the scaled is amplified as shown in figure 1.5, that when the frequency is large enough, the decreasing of the bit rate will become rather slow.

The maximum achievable bit rate at f=3.6GHz equals to 1976Mbit/s, as calculated in section 1.1.

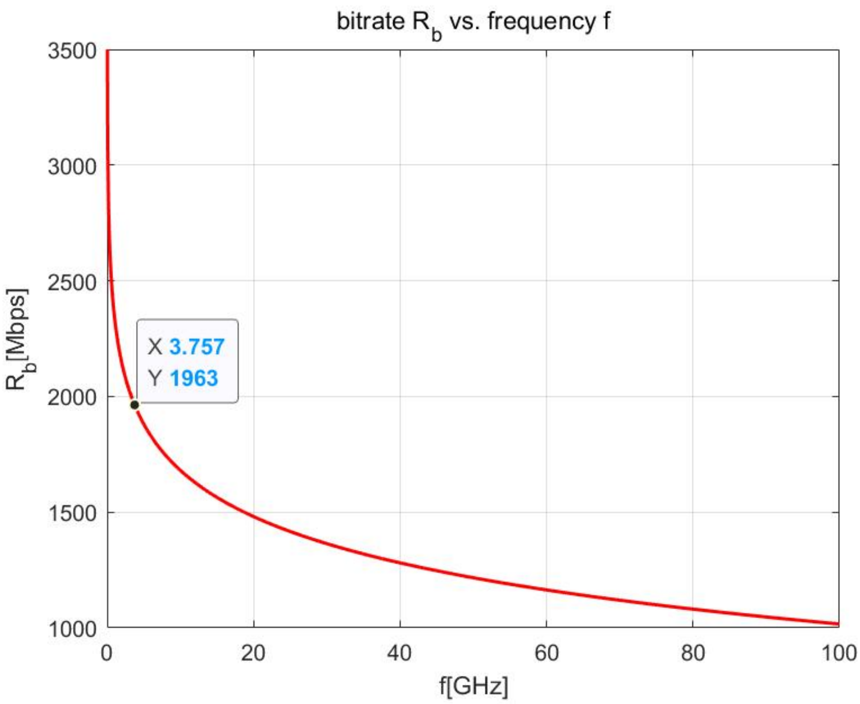

Figure 1.5: Bit-rate R_b vs. frequency f within the range of 0-100GHz

Exercise 2: Link Budget

1. Selected and Calculated Values

In this part's calculation, the slant range is using the distance between Earth and Moon, which is 384000km.

1.1 S/C TX power [dBw]

$$P[\text{dBw}] = 10log_{10}(P)$$

As the S/C power here equals to 3W, the P = 4.7712dBw.

1.2 The TX antenna gain of spacecraft in S band

S band indicates 2.110 to 2.120GHz for deep-space uplink or 2.290 to 2.300 GHz for deep-space downlink. Here the transmission antenna gain of the spacecraft can be 6dBi. The value here is from the COTS antenna produced by the company of IQ Spacecom, which is a single patch antenna of S band can operate at 1.980 to 2.500 GHz, used for Cubesats and other larger satellites.

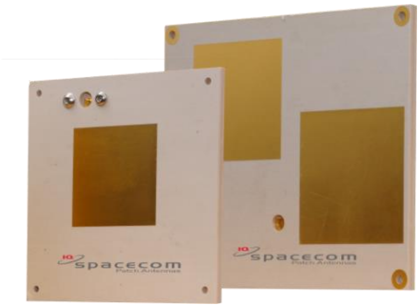

Figure 2.1: S Band Antenna

1.3 S/C EIRP of the transmitted

$$EIRP = P_{TX} + G_{TX} - L_{TX} - L_{pointing}$$

In this equation, $P_{TX} = 4.7712dBw$, $G_{TX} = 6dBi$, $L_{TX} = 1.5dB$, $L_{pointing} = 1dB$. Thus EIRP = 8.2712dBw.

1.4 Free space path loss

$$FSPL[dB] = 10log_{10}\left[(\frac{4\pi df}{c})^2\right]$$

$$FSPL[dB] = 32 + 20\log(d) + 20\log(f[GHz])$$

The free space path loss can be calculated as either of the functions above, which finally equals to 210.6264dB.

1.5 Total propagation loss

$$L_{tot}[dB] = FSPL + L_{atomsphere\&Ionospheric}$$

Where in the equation above, $FSPL = 210.6264dB$, $L_{atomsphere\&Ionospheric} = 2.5dB$, so L_{tot} is 213.1264dB.

1.6 Power at ground-station

$$P_{GS} = EIRP - L_{tot} = -204.8552dBw$$

1.7 RX power

$$P_{RX} = P_{GS} + G_{RX} - L_{pointing}$$

where $P_{GS} = -204.8552dBw$, $G_{RX} = 77dBi$, $L_{pointing} = 1dB$.

The RX power then equals to -128.8552dBw.

1.8 RX Noise power

$$P_N = 10log_{10}(T_{RX} * k * f_{BW})$$

In this function, $T_{RX} = 26K$, $k = 1.038 \times 10^{-23}$stands for the Boltzmann constant, $f_{BW} = 3MHz$ is the RF bandwidth. Therefore, the noise power P_N finally equals to -149.6803dBw.

1.9 RX SNR

$$SNR_{RX} = P_{RX} - P_N = 20.8251dB$$

1.10 RX Eb/N0

$$\frac{E_b}{N_0}_{RX} = SNR_{RX} - 10log_{10}(\frac{f_{BW}}{Rb_{Info}})$$

In this function, $Rb_{Info} = 1Mbit/s$ stands for the information bit rate, the other parameters are as mentioned before. $\frac{E_b}{N_0}_{RX}$ then equals to 16.0539dB.

1.11 Required Eb/N0 at FER=1e-6

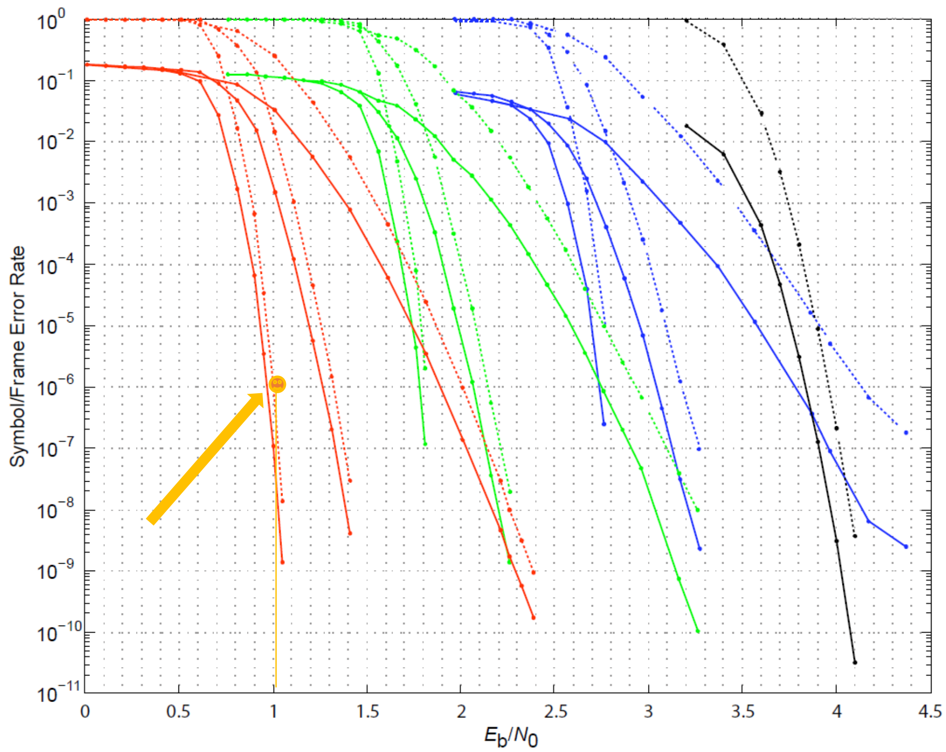

Figure 2.2: Frame Error Rate Performance of the CCSDS Punctured

The required Eb/N0 parameter is searched within the figure of FER, as shown in figure 2.2, which this figure is from the green book '*TM SYNCHRONIZATION AND CHANNEL CODING—SUMMARY OF CONCEPTAND RATIONALE'* published by CCSDS.

In this figure, the red lines stand for the 1/2 code rate, the dash lines stand for the FER parameters, and the right lines stands for the block length k=1024. In this case, the rightest red dash line is picked to search for the requires Eb/N0.

When FER=10^{-6}, the required Eb/N0 approximately equals to 1dB. (The point is pointed in the figure with a yellow smiling face.)

2. Margin of the Moon-to-Earth Communication

Using the results above, the calculated received Eb/N0 is 16.05dB, and the required one is 1dB. Therefore, the margin of the Moon-to-Earth communication is 15.05dB.

3. The Maximum Information Bit-Rate of the Mars-to-Earth Communication

Here calculated the mentioned parameters in section 1 again, using the distance between Earth and Mars as the slant range, which is 4×10^{12}m and the margin of 10dB. The calculated maximum information bit rate is then 3.389×10^{7}Mbit/s.

Exercise 3: Multipath fading and Rayleigh pdf

1. **Rayleigh and Exponential Probability Distribution Functions (pdf)**

As in the normal transmission, the average power received by the base-station is unstable, due to the unpredictable amount of path losses and the log-normal shading, which this phenomenon is called multipath fading. The probability distribution functions can be used to characterized this problem statistically.

1.1. Rayleigh Probability Distribution Function

The Rayleigh pdf is described as the function below, where $\sigma^2 = 1$.

$$f_R(r) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

The amplitude of the received signal is,

$$R = \sqrt{X_1^2 + X_2^2}$$

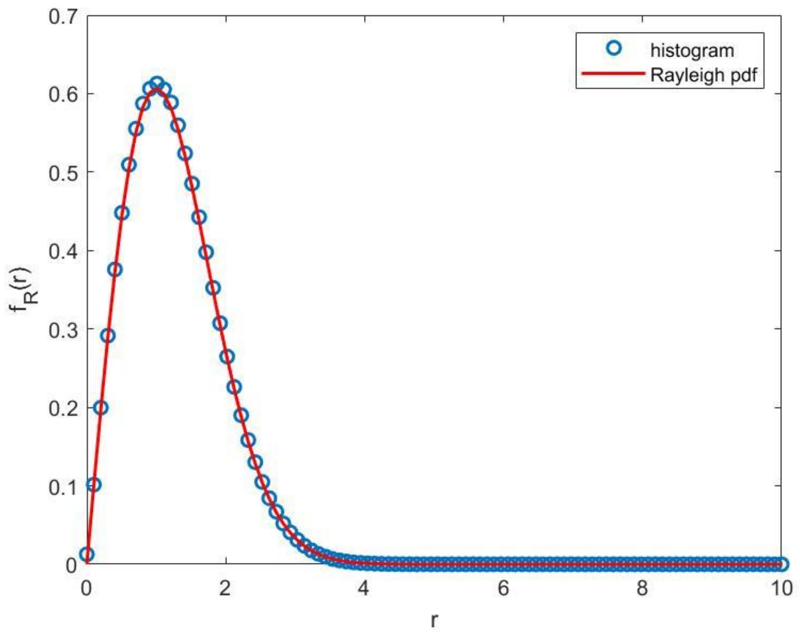

Figure 3.1: Rayleigh Probability Distribution Function

As shown in figure 3.1, the line of Rayleigh pdf and the histogram of the received signal can roughly coincide. Therefore, the Rayleigh pdf can be used to eliminate the multipath fading from algorithm for the signal.

1.2. Exponential Probability Distribution Function

The Exponential pdf is described as the function below, where $\sigma^2 = 1$.

$$f_P(p) = \frac{1}{\sigma^2} e^{-\frac{p}{\sigma^2}}$$

The power of the signal can be described as below, where R stands for the amplitude of the signal.

$$P = \frac{R^2}{2}$$

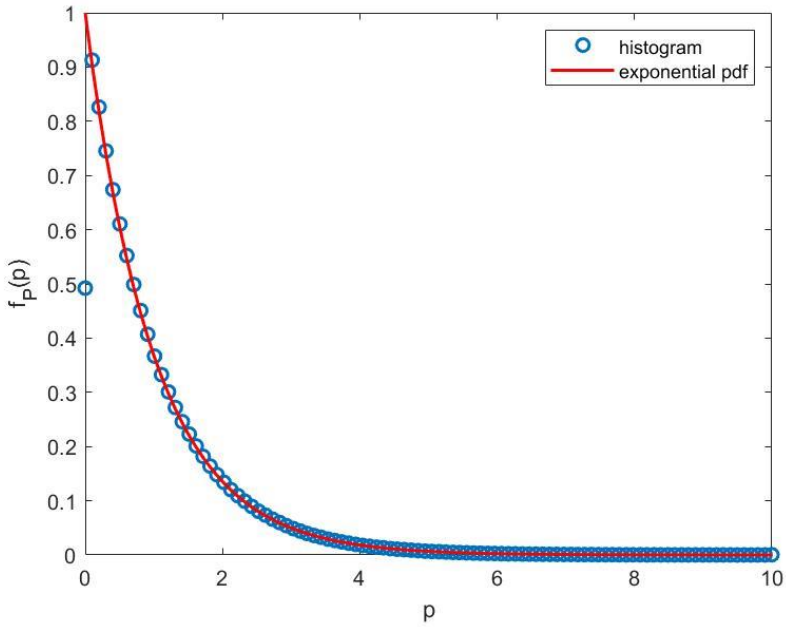

Figure 3.2: Exponential Probability Distribution Function

As shown in figure 3.2, that the lines of the exponential pdf and the histogram of the signal power can be nearly coincided, except when the power is 0, the histogram indicates 0.5 and the exponential pdf indicates 1. In general, the problem of power can be well present by the exponential pdf.

2. The probabilities of the received power

$$F_p(p) = P_r(P \leq p) = 1 - e^{-\frac{p}{\sigma^2}}$$

2.1 Probability when the received power is less than the mean value

$$F_p(\bar{P}) = P_r(P \leq \bar{P}) = 63.21\%$$

2.2 Probability when the received power is 10dB lower than the mean value

$$F_p(\bar{P} - 10dB) = P_r(P \leq \bar{P} - 10dB) = 9.52\%$$

2.3 Probability when the received power is 20dB lower than the mean value

$$F_p(\bar{P} - 20dB) = P_r(P \leq \bar{P} - 20dB) = 4.88\%$$

2.4 Probability when the received power is 30dB lower than the mean value

$$F_p(\bar{P} - 30dB) = P_r(P \leq \bar{P} - 30dB) = 3.28\%$$

From the probabilities above, when the power becomes lower, the probability is decreasing and with a slower decreasing tendency, which is just as shown in figure 3.2.

When the power is low enough as shown in the table below, when the power is -10, -20, -30dB, the probability becomes much smaller, which indicates that there'll be less amount of power with small value shown in the real transmission.

Power [dB]	Probability
-10	9.52%
-20	1.00%
-30	0.10%

Reference:

1. Notes from the class;
2. https://www.iq-spacecom.com/products/antenna-s-band, as the reference of the S band parameter;
3. TM Synchronization and Channel Coding—Summary of Concept and Rationale, from CCSDS website, www.CCSDS.org.