### Electromagnetic fields and biological tissues: effects and medical applications

Please initialize individual items of the declaration,	and <b>sign</b> it at bottom.	
Upon my word of honor, and aware of the consequer Italian law, as well as those deriving from unfair con I, the undersigned	orio) that the home assignment	
Has been carried out in a strictly individual manner f	rom beginning to end; in particular,	
74 I have <u>not</u> obtained help from any cla out in part or whole the assignment;	assmate or external person to carry	
TL I have <u>not</u> employed any paper or ele the assignment; (note: textbooks are indir		
I have <u>not</u> employed scripts, computer programs or any other such procedures that have not been entirely developed by myself, or provided as course material (by the Instructor and/or the Teaching Assistant), and that are not commercial, or cannot be referenced in the open literature or internet; please note that <i>all employed software not personally and individually developed must be referenced in the submitted papers</i> . In particular, I have no employed any script, programs etc. developed by my classmates, and that the employed scripts, programs etc. have not been developed in cooperation with my classmates.		
TLI have discussed this assignment wit "none" if appropriate):	th the following persons: (enter	
Tong Lin	Tong Lin	
(Complete name, please print)	signature	

Note: Use of commercial software, of free-ware or shareware, or otherwise publicly available software (e.g. via Internet) is allowed, but usage of all software not developed personally and individually by the student, or provided as course material, MUST be clearly stated and precisely referenced in the submitted paper.

Torino, 2021/6/8\_ (date)

## Problem 1

1) Amplitude of the wave along the propagation direction:

Along the propagation, where  $K = K.\tilde{T}$ ,  $K.\tilde{T} = K.T$ , therefore,  $e^{-jkT}$ 

$$E = \frac{e^{-jH}}{4\pi I} \in (0, e)$$

$$= E_0 \cdot \exp(-jH).$$

The amplitude, |E|= |E0| = (0,0)

a. Expression of the wavevector:

As  $\xi$  lies along the propagation vector,  $\hat{E} = \hat{\xi}$ 

Thus, the wavevector can be expressed as,

6. Values of the wavevectors:

According to the expression of the wavevector,  $K = \sqrt{W^2 \cdot \tilde{\xi} \cdot \mu}$ 

where, w - related with the frequencies,

E - complex permittivity, where the imaginary part can be obtained with the tangent loss,

u - permeability, which can simply use the value of us.

The parameters of each cirsue for each frequency, along with the results are listed in the cable below.

Tissue	Frequency	Conductivity [S/m]	Relative permittivity	Loss tangent	Wavevector (k)
	435MHz	0.80536	56.859	0.5853	71.4 - 19.4i
Muscle	2.45GHz	1.7388	52.729	0.24194	375.5 - 44.8i
	26GHz	31.595	25.848	0.84508	2976.9 - 1089.4i
Skin Dry	435MHz	0.70272	46.039	0.63073	64.6 - 18.7i
	2.45GHz	1.464	38.007	0.28262	319.6 - 44.3i
ыу	26GHz	24.408	17.709	0.95287	2502.2 - 1001.2i

# 3) Depens of Le and Lo:

According to the expression of SAR,

The general function for calculating the depths can be then expressed as,

$$\frac{SAR(L)}{SAR(0)} = \frac{\frac{1}{2}\sigma[E(L)]\frac{1}{p}}{\frac{1}{2}\sigma[E(D)]\frac{1}{p}} = \frac{|E(L)|^{2}}{|E(D)|^{2}}$$

$$= [exp(-jk\cdot L)]^{2}$$

Where K is complex, K = K' + j E'', which is already obtained in question 2.

The values of Le and Lio can therefore be obtained by,

Le: | exp(-2jk.Le) | = exp(-2)

LIO: |exp(-2jk.LIO)| = -10dB = 0.1

The resulted calculated using MATLAB are Wited as follows,

Tissue	Frequency	L <sub>e</sub> [m]	L <sub>10</sub> [m]
Muscle	435MHz	0.0516	0.0595
	2.45GHz	0.0223	0.0257
	26GHz	0.0009	0.0011
Skin Dry	435MHz	0.0535	0.0616
	2.45GHz	0.0226	0.0260
	26GHz	0.0010	0.0011

# Problem 2

a. Wavevectors for incidents and cransmitted waves:

The inerdent wave in the (x, 2) plane is expressed as,

where is a real value, is = wolfing

The cransmitted wavevector in the (x, t) plane can be expressed as,

where to indicates a complex value, to = w TE, 1,

A relationship exist between the thetdent and cransmitted values lays on, kxt = kisindi, that 75,

b. 4000) and Lew) for the ewo incidences:

Assuming that the propagation direction is along  $\tilde{z}$ -axis as in the previous problem, where  $\underline{r} = L \cdot \hat{z}$ 

As discussed in Problem 1, L1010) and Leld) can be obtained from,

The results calculated using MATLAB are as Shown in the cable below.

		9=0°		9=45°		
Tissue	Frequency	L <sub>e</sub> [m]	L <sub>10</sub> [m]	L <sub>e</sub> [m]	L <sub>10</sub> [m]	
Muscle	435MHz	0.0516	0.0595	0.0514	0.0592	
	2.45GHz	0.0223	0.0257	0.0222	0.0256	
	26GHz	0.0009	0.0011	0.0009	0.0010	
Skin Dry	435MHz	0.0535	0.0616	0.0533	0.0614	
	2.45GHz	0.0226	0.0260	0.0224	0.0258	
	26GHz	0.0010	0.0011	0.0010	0.0011	

## 2) Power cransmission:

a. The ratio between the inerdent power density and the robal dissipated power per unit area is has been discribed as,

where, Sairs =  $\frac{1}{2} \cdot \frac{1}{2} \cdot \left| \frac{1}$ 

Therefore,
$$T(0) = \frac{2.05}{2.0} \frac{|E0(0)|^2}{|E'(0)|^2}$$

where,  $z_0 = \sqrt{\frac{1}{80}}$ ,  $\sigma_s$  is the conductivity of medium 2,  $\alpha_s$  indicates the imaginary part of kt.

$$\frac{\left| \text{EvTM}(\omega) \right|^{2}}{\left| \text{EvTM}(\omega) \right|^{2}} = \frac{\mathcal{E}_{1}^{2}}{\tilde{\mathcal{E}}_{2}^{2}} \frac{\left| \text{Kx2} \right|^{2} + \left| \text{Kx2} \right|^{2}}{\left| \text{Kx2} \right|^{2} + \left| \text{Kx2} \right|^{2}} \cdot \left| 1 - \text{TTM} \right|^{2}}{\left| \text{Kx2} \right|^{2} + \left| \text{Kx2} \right|^{2}} \cdot \left| \frac{\text{Kx2}}{\left| \text{Kx2} \right|^{2}} \cdot \left| \frac{\text{K$$

where, 
$$K_{21} = K_1 \cdot \cos \theta_1$$
 $K_{22} = K_2 \cdot \cos \theta_2$ 
 $K_{31} = K_1 \cdot \sin \theta_1$ 
 $K_{42} = K_2 \cdot \sin \theta_2$ 

The ploos of Two) respectively with TE and TM are as shown in the figures as follows.

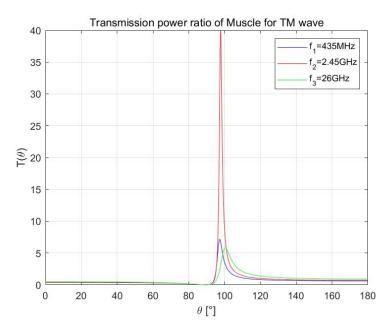


Figure 2.1: Power transmission for Muscle of TM polarization

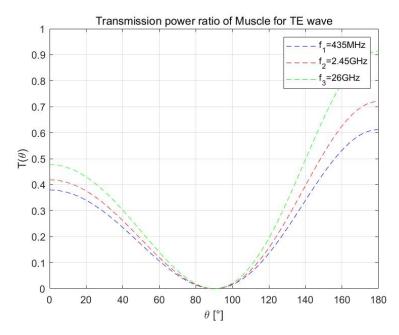


Figure 2.2: Power transmission for Muscle of TE polarization

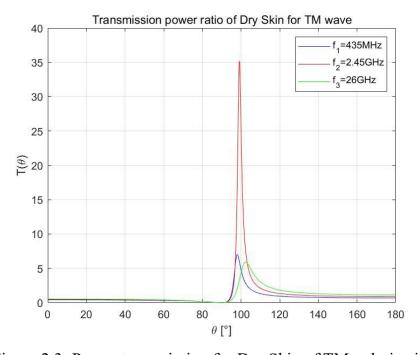


Figure 2.3: Power transmission for Dry Skin of TM polarization

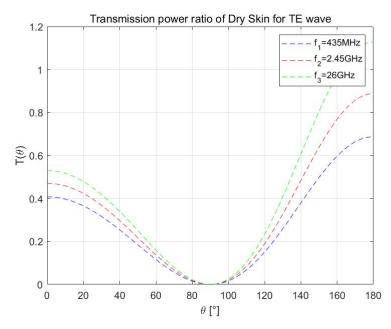


Figure 2.4: Power transmission for Dry Skin of TE polarization

3)

a. As shown in Figure 2.1, the worst case for penetration appears in the TM poterization with the angle approximately as 97.79° and 99.14° respectively for the medium of musue and for the dry skin, where the frequency lies on f=2.45 GHz, and with Two) equal to 39.92 and 35.21.

### b. E-freed:

Taking the worst case of Muscle with  $f=2.45\,\mathrm{GHz}$  as an example, where  $0i=97.79^\circ$ .

The value of E-field along z-axis can be calculated using the function.  $E = |E_0| \cdot \exp(-j \cdot E \cdot E)$ 

respectively for the TM and TE polarization.

where the functions of the ratto 75 the same as in question 26. with specific values in this condition.

The plot of the E-freed 7s as shown in figure below, which 7s obvious to obtain, that the maximum E-freed stays at beginning of the interface, where Z=0.

The result 76 same with the medium of dry skin.

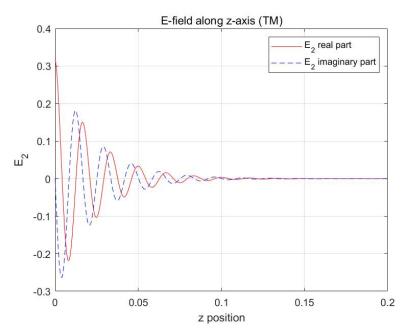


Figure 2.5: E-field of TM polarization

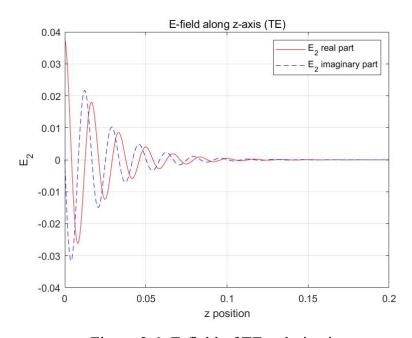


Figure 2.6: E-field of TE polarization

# 4) Calculation of E-field with given SAR:

According to the expression of SAR,

The cransmission E-field can be obtained. The value of the tresdent E-field can be calculated with the ratio of the E-freeds.

The results are listed as follows. When SAR = IN/kg,

For musice, TM: |Ei| = 108.89 [V/m]

TE: |Ei| = 809.9. [V/m]

For dry skin, Tan: | Ei = 99.13 [V/m]

TE: [E] = 771.4 [V/m]

When SAR = 0.08 W/69,

For musile, TM: |Ei| = 21.78 [V/m] TE: |Ei| = 181.98 [V/m]

For dry skin, TM: |Ei| = 19.82 [V/m]

TE: |E1 = 142.24 [V/m]

### 5) Maximum SAR:

With the given value of the Entrolent freed, the chansmitted E-freed can be then calculated, along with the function of SAR.

where K. I = K2. Z.

The plots of SAR are shown in the figures below, (Figure 2.5.1  $\sim$  2.5.4), where is easy to observe the the maximum value of SAR lays on the points of  $\overline{t}=0$ , thus the maximum values are,

For dry skin, TM: SARmex = 0.0037 [W/kg]
TE: SARmex = 7.12 x 10-5 [W/kg]

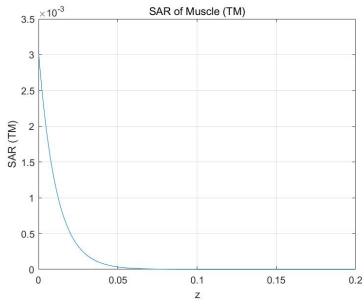


Figure 2.7: SAR for muscle of TM polarization

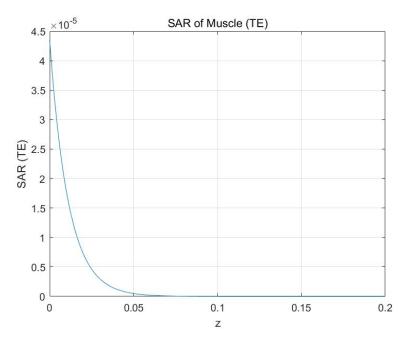


Figure 2.8: SAR for muscle of TE polarization

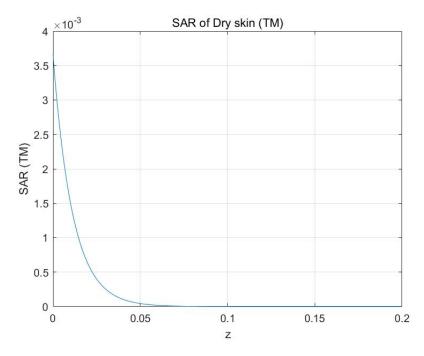


Figure 2.9: SAR for dry skin of TM polarization

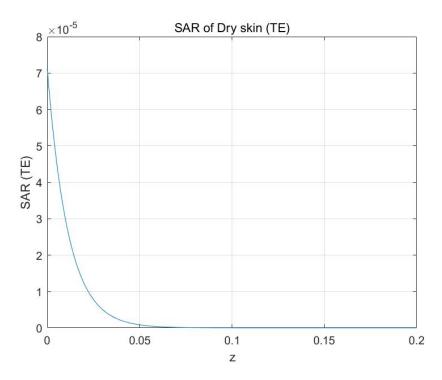


Figure 2.10: SAR for dry skin of TE polarization

## **Appendix: MATLAB codes**

### Problem 1:

```
clear all;
close all;
clc
sts =
textread('D:\EE\Electromagnetic Fields and Biological Tissues\Assignm
ent\A5\statistics.txt');
j = sqrt(-1);
%% P1 2
f = sts(:,1); %[Hz]
omg = 2*pi*f;
miu0 = pi*4e-7; %[H/m]
sigma = sts(:,2); %Conductivity, [S/m]
yp0 = 8.85418782e-12; %Perimittivity free space, [F/m]
yp1 = sts(:,3)*yp0; %Permittivity, real part
yp2 = sts(:,4).*yp1; %Permittivity, imaginary part
yp = yp1-j*yp2;
k = omg.*sqrt(yp.*miu0)
%% P1 3
Le = [];
L10 = [];
for i = 1:1:length(k)
   syms Lei L10i
   eq1 = abs(exp(-2*j*k(i)*Lei)) == exp(-2);
   eq2 = 10*log10(abs(exp(-2*j*k(i)*L10i))) == -10;
   Lei = solve(eq1,Lei); Le = [Le double(Lei)];
   L10i = solve(eq2, L10i); L10 = [L10 double(L10i)];
end
Le
L10
```

#### **Problem 2:**

```
clear all;
close all;
clc
sts =
textread('D:\EE\Electromagnetic_Fields_and_Biological_Tissues\Assignm
ent\A5\statistics.txt');
j = sqrt(-1);
%% P2 1b
f = sts(:,1); %[Hz]
omg = 2*pi*f;
miu0 = pi*4e-7; %[H/m]
sigma = sts(:,2); %Conductivity, [S/m]
yp0 = 8.85418782e-12; %Perimittivity free space, [F/m]
yp1 = sts(:,3)*yp0; %Permittivity, real part
yp2 = sts(:,4).*yp1; %Permittivity, imaginary part
yp_2 = yp1-j*yp2;
yp 1 = yp0;
k1 = omg.*sqrt(1*yp0*miu0);
k2 = omg.*sqrt(yp 2.*miu0);
%% Theta1 = 0;\tilde{a}
theta1 = 0;
theta2 = k1*sin(theta1)./k2;
Le 0 = [];
L10 0 = [];
for i = 1:1:length(k2)
   syms Lei L10i
   eq1 = abs(exp(-2*j*k2(i)*cos(theta2(i))*Lei)) == exp(-2);
   eq2 = 10*log10(abs(exp(-2*j*k2(i)*cos(theta2(i))*L10i))) == -10;
   Lei = solve(eq1,Lei); Le 0 = [Le 0 double(Lei)];
   L10i = solve(eq2, L10i); L10 0 = [L10 0 double(L10i)];
end
%% Theta1 = 45;ã
theta1 = 45*pi/180;
theta2 = k1*sin(theta1)./k2;
Le 45 = [];
L10 45 = [];
for i = 1:1:length(k2)
   syms Lei L10i
   eq1 = abs (\exp(-2*j*k2(i)*\cos(theta2(i))*Lei)) == \exp(-2);
```

```
eq2 = 10*log10(abs(exp(-2*j*k2(i)*cos(theta2(i))*L10i))) == -10;
   Lei = solve(eq1,Lei); Le 45 = [Le 45 double(Lei)];
   L10i = solve(eq2, L10i); L10 45 = [L10 45 double(L10i)];
end
%% P2 2
theta1 = linspace(0,pi,10000);
theta2 = asin(k1.*sin(theta1)./k2);
alp2 = -imag(k2);
kx1 = k1.*sin(theta1);
kz1 = k1.*cos(theta1);
kx2 = k2.*sin(theta2);
kz2 = k2.*cos(theta2);
gammaTM = (kz2./yp 2-kz1./yp 1)./(kz2./yp 2+kz1./yp 1);
gammaTE = (kz1-kz2)./(kz1+kz2);
yp 2ab = sqrt(conj(yp 2).*yp 2);
gammaTMab = sqrt(conj(1-gammaTM).*(1-gammaTM)); % |1-gammaTM|
kx2ab = sqrt(conj(kx2).*kx2);
kz2ab = sqrt(conj(kz2).*kz2);
kx1ab = sqrt(conj(kx1).*kx1);
kzlab = sqrt(conj(kz1).*kz1);
Eratio TM =
(gammaTMab.^2.*((kx2ab.^2+kz2ab.^2)./(kx1ab.^2+kz1ab.^2))).*(yp 1^2./
yp 2ab.^2);
gammaTEab = sqrt(conj(1+gammaTE).*(1+gammaTE)); % |1+gammaTE|
Eratio TE = gammaTEab.^2;
Z0 = sqrt(miu0/yp0);
const = Z0*sigma./(2*alp2);
T TM = const.*Eratio TM;
T TE = const.*Eratio TE;
theta1 = theta1*180/pi;
figure %Muscle TM
plot(theta1, T TM(1,:), 'b', theta1, T TM(2,:), 'r', theta1, T TM(3,:), 'g')
grid on
xlabel('\theta [;\tilde{a}]');
ylabel('T(\theta)')
title('Transmission power ratio of Muscle for TM wave');
legend('f 1=435MHz','f 2=2.45GHz','f 3=26GHz')
figure %Muscle TE
```

```
plot(theta1, T TE(1,:), 'b--', theta1, T TE(2,:), 'r--
', theta1, T TE(3,:), 'g--')
grid on
xlabel('\theta [;ã]');
ylabel('T(\theta)')
title('Transmission power ratio of Muscle for TE wave');
legend('f 1=435MHz','f 2=2.45GHz','f 3=26GHz')
figure %Skin TM
\texttt{plot}(\texttt{theta1}, \texttt{T}_\texttt{TM}(4,:), \texttt{'b'}, \texttt{theta1}, \texttt{T}_\texttt{TM}(5,:), \texttt{'r'}, \texttt{theta1}, \texttt{T}_\texttt{TM}(6,:), \texttt{'g'})
grid on
xlabel('\theta [;ã]');
ylabel('T(\theta)')
title('Transmission power ratio of Dry Skin for TM wave');
legend('f 1=435MHz','f 2=2.45GHz','f 3=26GHz')
figure %Skin TE
plot(theta1, T TE(4,:), 'b--', theta1, T TE(5,:), 'r--
',theta1,T TE(6,:),'g--')
grid on
xlabel('\theta [;ã]');
ylabel('T(\theta)')
title('Transmission power ratio of Dry Skin for TE wave');
legend('f 1=435MHz','f 2=2.45GHz','f 3=26GHz')
%% Check
figure
plot(theta1,gammaTMab(5,:))
hold on
plot(theta1, abs(gammaTM(5,:)))
axis([0 90 0 21)
grid on
xlabel('\theta');
ylabel('gammaTM');
legend('|1-\Gamma {TM}|','|\Gamma {TM}|')
title('Reflection coefficient')
figure
plot(theta1, abs(gammaTM(5,:)))
hold on
grid on
plot(theta1, abs(gammaTE(5,:)))
hold on
plot(theta1, sqrt(Eratio TM(5,:)))
hold on
```

```
plot(theta1, sqrt(Eratio TE(5,:)))
axis([0 90 0 1])
xlabel('\theta');
title('Dry skin, f=2.45GHz')
legend('|\Gamma {TM}|','|\Gamma {TE}|','Eratio {TM}','Eratio {TE}')
%% P2 3
% a. power dissipation along theta in medium 2 -- as shown in figures
above
% b. E-field along z -- worst case when f=2.45[GHz] in Muscle (2,;)
%-----
theta wi = 97.79*pi/180;
kxi = k1(2)*sin(theta wi); kxiab = sqrt(conj(kxi)*kxi);
kzi = k1(2)*cos(theta wi);
theta wt = k1(2)*sin(theta wi)/k2(2);
kxt = k2(2)*sin(theta wt); kxtab = sqrt(conj(kxt)*kxt);
kzt = k2(2)*cos(theta wt);
yp 2ab = sqrt(conj(yp 2(2))*yp 2(2));
gammaTM = (kzt/yp 2(2)-kzi/yp 1)/(kzt/yp 2(2)+kzi/yp 1);
gammaTMab = sqrt(conj(1-gammaTM) * (1-gammaTM)); % | 1-gammaTM|
Eratio TMsq = gammaTMab^2*(kxtab^2/kxiab^2)*(yp 1^2/yp 2ab^2);
gammaTE = (kzi-kzt)./(kzi+kzt);
gammaTEab = sqrt(conj(1+gammaTE).*(1+gammaTE)); % |1+gammaTE|
Eratio TEsq = gammaTEab.^2;
\}-----
E0 1 = 1; %normalized
z = linspace(0, 0.2, 1000);
E0_2TM = sqrt(Eratio_TMsq)*E0_1;
E2TM = E0 2TM*exp(-j*k2(2)*z);
E0_2TE = sqrt(Eratio_TEsq)*E0_1;
E2TE = E0 2TE*exp(-j*k2(2)*z);
figure %TM
plot(z,real(E2TM),'r',z,imag(E2TM),'b--');
xlabel('z position');
ylabel('E 2');
legend('E 2 real part','E 2 imaginary part')
grid on
title('E-field along z-axis (TM)')
figure %TE
plot(z, real(E2TE), 'r', z, imag(E2TE), 'b--');
xlabel('z position');
```

```
ylabel('E 2');
legend('E 2 real part','E 2 imaginary part')
grid on
title('E-field along z-axis (TE)')
%% P2 4/P2 5
%Muscle
rou m = 1.03e3; %Mass density of muscle, [kg/m3]
Et 2 = \operatorname{sqrt}(2*\operatorname{rou} \, m/\operatorname{sigma}(2)); \, %[V/m]
Ei 2TM m = Et 2/sqrt(Eratio TMsq)
Ei 2TE m = Et 2/sqrt(Eratio TEsq)
Et 008 = sqrt(0.08*rou m/sigma(2));
Ei 008TM m = Et 008/sqrt(Eratio TMsq)
Ei 008TE m = Et 008/sqrt(Eratio TEsq)
Ei0 = 6; %[V/m]
Et0 TM = sqrt(Eratio TMsq)*Ei0;
Et TM = Et0 TM*exp(-j*k2(2)*z); EtTMab = sqrt(conj(Et TM).*Et TM);
SAR TM = 0.5*sigma(2)*EtTMab.^2/rou m;
Et0 TE = sqrt(Eratio TEsq)*Ei0;
Et TE = Et0 TE*exp(-j*k2(2)*z); EtTEab = sqrt(conj(Et TE).*Et TE);
SAR TE = 0.5*sigma(2)*EtTEab.^2/rou m;
SAR TMmax m = 0.5*sigma(2)*Et0 TM^2/rou m
SAR TEmax m = 0.5*sigma(2)*Et0 TE^2/rou m
figure
plot(z,SAR TM)
xlabel('z');
ylabel('SAR (TM)')
title('SAR of Muscle (TM)')
grid on
figure
plot(z,SAR TE)
xlabel('z');
ylabel('SAR (TE)')
title('SAR of Muscle (TE)')
grid on
%Dry skin
theta wi = 99.14*pi/180;
kxi = k1(5)*sin(theta wi); kxiab = sqrt(conj(kxi)*kxi);
kzi = k1(5)*cos(theta wi);
theta wt = k1(5)*sin(theta wi)/k2(5);
```

```
kxt = k2(5)*sin(theta wt); kxtab = sqrt(conj(kxt)*kxt);
kzt = k2(5)*cos(theta wt);
yp \ 2ab = sqrt(conj(yp \ 2(5))*yp \ 2(5));
gammaTM = (kzt/yp 2(5)-kzi/yp 1)/(kzt/yp_2(5)+kzi/yp_1);
gammaTMab = sqrt(conj(1-gammaTM) * (1-gammaTM)); % | 1-gammaTM|
gammaTE = (kzi-kzt)./(kzi+kzt);
gammaTEab = sqrt(conj(1+gammaTE).*(1+gammaTE)); % |1+gammaTE|
Eratio TMsq s = gammaTMab^2*(kxtab^2/kxiab^2)*(yp 1^2/yp 2ab^2);
Eratio TEsq s = gammaTEab.^2;
%_____
rou s = 1.02e3; %Mass density of muscle, [kg/m3]
Et 2 s = sqrt(2*rou s/sigma(5)); %[V/m]
Ei_2TM_s = Et_2_s/sqrt(Eratio_TMsq_s)
Ei 2TE s = Et 2 s/sqrt(Eratio TEsq s)
Et 008 s = sqrt(0.08*rou s/sigma(5));
Ei 008TM s = Et 008 s/sqrt(Eratio TMsq s)
Ei 008TE s = Et 008 s/sqrt(Eratio TEsq s)
Ei0 = 6; %[V/m]
Et0_TM_s = sqrt(Eratio_TMsq_s)*Ei0;
Et TM s = Et0 TM s*exp(-j*k2(5)*z); EtTMab s =
sqrt(conj(Et TM s).*Et TM s);
SAR TM s = 0.5*sigma(5)*EtTMab s.^2/rou s;
Et0_TE_s = sqrt(Eratio_TEsq_s)*Ei0;
Et TE s = Et0 TE s*exp(-j*k2(5)*z); EtTEab s =
sqrt(conj(Et TE s).*Et TE s);
SAR TE s = 0.5*sigma(5)*EtTEab s.^2/rou s;
SAR TMmax s = 0.5*sigma(5)*Et0 TM s^2/rou s
SAR TEmax s = 0.5*sigma(5)*Et0_TE_s^2/rou_s
figure
plot(z,SAR TM s)
xlabel('z');
ylabel('SAR (TM)')
title('SAR of Dry skin (TM)')
grid on
figure
plot(z,SAR TE s)
xlabel('z');
ylabel('SAR (TE)')
grid on
title('SAR of Dry skin (TE)')
```

### **Plots of check:**

