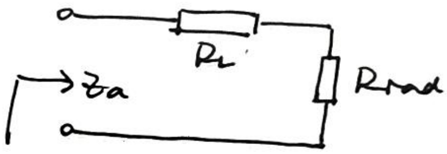
Homework No. 1 TONG LIN S287649

## Problem 1.1

The ohmic efficiency is defined as,

$$\eta_L = \frac{R_{rad}}{R_L + R_{rad}}$$




where $R_{rad} = 10\Omega$, and $R_{rad}$ is unknown.
According to other parameters,

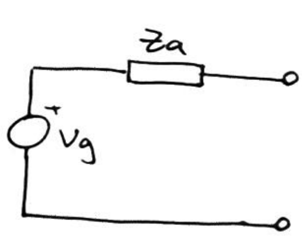
$$S = \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma|}{1-|\Gamma|} = 9.2$$

So that, $|\Gamma| = 0.8$, also with $\frac{d}{\lambda g} = 0.164$, the value of $R_{rad}$ can be obtained using Smith chart, and the characteristic impendance $Z_\infty = 100\Omega$.

## Problem 1.2

The voltage can be obtained using,

$$V_g = \underline{E}^{inc} \cdot \underline{h}_{eff}$$
$$= (3\hat{x} + 4\hat{y}) \cdot (\hat{x} + \hat{y})$$
$$= 3\hat{x}\cdot\hat{x} + 4\hat{y}\cdot\hat{y}$$
$$= 7 \text{ mV}.$$



## Problem 1.3

The maximum receiving power is related to the maximum voltage, due to the definition of the available power,

$$P_{av} = \frac{|V_g|^2}{4R_a}$$

where $V_g = \underline{h}_{eff} \cdot \underline{E}^{inc}$, to obtain $V_{max}$, the maximum effective height is needed, which can be assume as $\underline{h}_{eff} = \hat{x}\cos\varphi + \hat{y}\sin\varphi$, where $|h_{eff}| = 1$, therefore,

$$\begin{aligned} V_{max} &= \underline{h}_{eff} \cdot \underline{E}^{inc} \\ &= (\hat{x}\cos\varphi + \hat{y}\sin\varphi) \cdot (\hat{x} \cdot 3 + \hat{y} \cdot j6) \\ &= 3\cos\varphi + j \cdot 6\sin\varphi \end{aligned}$$

## Problem 1.4

The equivalent area is defined as the geometry area times the efficiency,

$$\begin{aligned} A_{eq} &= \nu \cdot A_{geo} \\ &= \nu \cdot \pi R^2 \\ &= 0.75\pi \times 1.5^2 \\ &\approx 5.30\ m^2 \end{aligned}$$

The gain can then be obtained as,

$$\begin{aligned} G &= \frac{4\pi}{\lambda^2} \cdot A_{eq} \\ &= \frac{4\pi f^2}{c^2} \cdot A_{eq} \\ &= \frac{4\pi \cdot (5\times10^9)^2}{(3\times10^8)^2} \times 5.30 \\ &\approx 18491.11 \end{aligned}$$

<u>Problem 1.5</u>

The received powers of the two antennas respectively with linear and circular polarizations can be written as,

Circular:

$$P_R^c = P_T \frac{G_R^c G_T}{\left(\frac{4\pi R}{\lambda}\right)^2} \left(1-|\Gamma_R^c|^2\right)\cdot\left(1-|\Gamma_T^c|^2\right)\cdot\left|\hat{P}_R^c\cdot\hat{P}_T^*\right|^2$$

Linear:

$$P_R^L = P_T \frac{G_R^L G_T}{\left(\frac{4\pi R}{\lambda}\right)^2} \left(1-|\Gamma_R^L|^2\right)\cdot\left(1-|\Gamma_T^L|^2\right)\cdot\left|\hat{P}_R^L\cdot\hat{P}_T^*\right|^2$$

where the differences exist in the parameters of reciving gain $G_R$, and transmitting and reciving transmission coefficients $\Gamma_R$ and $\Gamma_L$, and the direction of $\hat{P}_R$.

## Problem 1.6

The normalized equivalent area can be calculated using the definition function of G, where

$$G = \frac{4\pi}{\lambda^2} \cdot A_{eff}$$

so that, $\frac{A_{eq}}{\lambda^2} = \frac{4\pi}{G}$.

According to the definition,

$$G = \frac{\frac{dP}{d\Sigma}}{\left(\frac{dP}{d\Sigma}\right)_{iso}}$$

where,

$$\frac{dP}{d\Sigma} = \frac{|E|^2}{Z_0} = \frac{Z_t^2 \cdot I_a^2 \cdot |e^{-2jkr}|}{Z_0 \cdot r^2}$$

$$\left(\frac{dP}{d\Sigma}\right)_{iso} = \frac{P}{4\pi r^2} = \frac{|I_a|^2 \cdot R_a}{4\pi r^2}$$

Therefore,

$$G = \frac{\frac{Z_t^2 \cdot I_a^2 \cdot |e^{-2jkr}|}{Z_0 \cdot \cancel{r^2}}}{\frac{|I_a|^2 \cdot R_a}{4\pi \cancel{r^2}}}$$

$$= \frac{Z_t^2 \cdot 4\pi}{Z_0 \cdot R_a}$$

$$= \frac{100^2 \times 4\pi}{50 \times 75} \qquad = 33.49$$

So that,

$$\frac{A_{eq}}{\lambda^2} = \frac{4\pi}{G}$$

$$= \frac{4\pi}{33.49}$$

$$= 0.38$$

## Problem 1.7

According to the Transmission Equation, (assume $|\Gamma_T|=0, |\Gamma_R|=0$).

$$P_R = P_T \cdot \frac{G_T \cdot G_R}{\left(\frac{4\pi R}{\lambda}\right)^2} (1-|\Gamma_T|)^2 \cdot (1-|\Gamma_R|^2) \cdot |\hat{P}_r \cdot \hat{P}_T^*|$$

$$= EIRP \cdot \frac{\frac{4\pi}{\cancel{\lambda^2}} \cdot \pi\left(\frac{D}{2}\right)^2}{\left(\frac{4\pi R}{\cancel{\lambda}}\right)^2} \cdot 1 \cdot 1 \cdot 1$$

$$= \underbrace{10^{\frac{3.5}{10}}}_{2.24} \frac{0.5 \times 1.83^2}{4 \times (4000 \times 10^3)^2} \qquad = 5.86 \times 10^{-14} \quad = 0.05\ pW$$

# Problem 1.8

## 1. Polarization :

With the function of the electric field with $\theta \in [0, \frac{\pi}{2}]$,

$$\underline{E}(r,\theta,\varphi) = \frac{V_0}{r} e^{-jkr} \cdot \cos^4\theta \cdot [(\hat{p}+j\hat{q})\cos\frac{\theta}{2} + (\hat{p}-j\hat{q})\sin\frac{\theta}{2}].$$

Separating the real and imaginary part as. $\underline{E} = \underline{E}' + j\underline{E}''$

$$\underline{E}' = \frac{V_0}{r} \cdot e^{-jkr} \cdot \cos^4\theta \cdot (\cos\frac{\theta}{2} + \sin\frac{\theta}{2}) \cdot \hat{p}$$

$$\underline{E}'' = \frac{V_0}{r} \cdot e^{-jkr} \cdot \cos^4\theta \; (\cos\frac{\theta}{2} - \sin\frac{\theta}{2}) \cdot \hat{q}$$

Checking with the <u>circular</u> polarization condition,

$$|\underline{E}'| = k_0 \; (\cos\frac{\theta}{2} + \sin\frac{\theta}{2}) \cdot |\hat{p}|$$
$$= k_0 \, |\cos\frac{\theta}{2} + \sin\frac{\theta}{2}| \cdot |\cos\varphi\hat{\varphi} + \sin\varphi \cdot \hat{\theta}|.$$
$$= k_0 \, |\cos\frac{\theta}{2} + \sin\frac{\theta}{2}|$$

Similarly, where $k_0$ is the coefficient $k_0 = \frac{V_0}{r} \cdot e^{-jkr} \cdot \cos^4\theta$.

$$|\underline{E}''| = k_0 \, |\cos\frac{\theta}{2} - \sin\frac{\theta}{2}|$$

As in the circular polarization, $|\underline{E}'| = |\underline{E}''|$, where here the equation should satisfy with $|\cos\frac{\theta}{2} + \sin\frac{\theta}{2}| = |\cos\frac{\theta}{2} - \sin\frac{\theta}{2}|$, so that it should have $\theta = 0$.

To check the other condition of $\underline{E}' \perp \underline{E}''$, as $\underline{E}' \parallel \hat{p}$, and $\underline{E}'' \parallel \hat{q}$, this condition can simply check with the orthognal of $\hat{q}$ and $\hat{p}$,

$$\hat{p} \cdot \hat{q} = (\cos\varphi \cdot \hat{\varphi} + \sin\varphi \cdot \hat{\theta})(\sin\varphi \cdot \hat{\varphi} - \cos\varphi \cdot \hat{\theta})$$
$$= \cos\varphi \cdot \sin\varphi - \sin\varphi \cos\varphi$$
$$= 0$$

which can be easily tell that $\hat{p} \perp \hat{q}$, so that $\underline{E}' \perp \underline{E}''$.

Checking with the <u>linear</u> polarization condition, where the equations should either satisfy with $|\underline{E}'| = 0$ or $|\underline{E}''| = 0$ or $\underline{E}' \parallel \underline{E}''$.

Starting with $|\underline{E}'|=0$,

$$|\underline{E}'| = k_0 \left| \cos\frac{\theta}{2} + \sin\frac{\theta}{2} \right| = 0$$

which should satisfy $\cos\frac{\theta}{2} = -\sin\frac{\theta}{2}$, which is not able to satisify when $\theta \in [0, \frac{\pi}{2}]$.

Then with $|\underline{E}''|=0$,

$$|\underline{E}''| = k_0 \left| \cos\frac{\theta}{2} - \sin\frac{\theta}{2} \right| = 0$$

where $\cos\frac{\theta}{2} = \sin\frac{\theta}{2}$ can be obtained when $\theta = \frac{\pi}{2}$.

Finally with $\underline{E}' \parallel \underline{E}''$, it is impossible to be reached, as previously discussed that $\underline{E}' \perp \underline{E}''$ nomatter with each value of $\theta$ and $\varphi$, as $\hat{p} \perp \hat{q}$. when $\underline{E}' \parallel \hat{p}$, $\underline{E}'' \parallel \hat{q}$.

Overall, the antenna polarization is circular, when $\theta = 0$, $\varphi \in [0, 2\pi]$. The polarization is linear when $\theta = \frac{\pi}{2}$, $\varphi = [0, 2\pi]$.

As for $\theta \in [\frac{\pi}{2}, \pi]$, $|\underline{E}'| = |\underline{E}''| = 0$, the polarization is meaningless to be discussed.

2. Maximum Gain:

$$G = \frac{4\pi}{\int_\Omega g(\theta, \varphi)\, d\Omega}$$

where $g(\theta, \varphi) = \cos^8\theta$, as $\frac{dP}{d\Sigma} = \frac{|E|^2}{Z_0}$

$$\int_\Omega g(\theta,\varphi)\, d\Omega = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} \cos^8\theta \sin\theta \, d\theta \, d\varphi.$$

$$= 2\pi \cdot \frac{1}{9} \left( -\cos^9\theta \Big|_0^{\frac{\pi}{2}} \right)$$

$$= \frac{2}{9}\pi.$$

Therefore, the maximum gain.

$$G_{max} = \frac{4\pi}{\frac{2}{9}\pi} = 18.$$

## Problem 1.9

The relationship between antenna efficiency and antenna gain is shown as,

$$G = \nu \cdot D.$$

where $G$ can be calculated as,

$$G = G_0 \cdot \int_\Sigma g(\theta, \varphi)\, d\Sigma.$$

$$g(\theta, \varphi) = Q_1(\theta) \cos^2\varphi + Q_2(\theta) \cdot \sin^2\varphi$$

The integrate is,

$$\int_\Sigma g(\theta, \varphi)\, d\Sigma = \int_0^{2\pi}\int_0^{\theta_{B1}} Q_1 \cos^2\varphi\, d\Omega + \int_0^{2\pi}\int_{\theta_{B1}}^{5^\circ} Q_1 \cos^2\varphi\, d\Omega.$$

$$+ \int_0^{2\pi}\int_0^{\theta_{B2}} Q_2 \sin^2\varphi\, d\Omega + \int_0^{2\pi}\int_{\theta_{B2}}^{5^\circ} Q_2 \sin^2\varphi\, d\Omega.$$

$$= \int_0^{2\pi}\int_0^{0.302^\circ} \cos^2\varphi \cdot \sin\theta\, d\theta\, d\varphi + 10^{\frac{-28.5}{10}} \int_0^{2\pi}\int_{0.302^\circ}^{5^\circ} \cos^2\varphi \cdot \sin\theta\, d\theta\, d\varphi$$

$$+ \int_0^{2\pi}\int_0^{0.278^\circ} \sin^2\varphi\, \sin\theta\, d\theta\, d\varphi + 10^{\frac{-17.5}{10}} \int_0^{2\pi}\int_{0.278^\circ}^{5^\circ} \sin^2\varphi\, \sin\theta\, d\theta\, d\varphi.$$

$$= 3.09 \times 10^{-4}$$

Therefore, $G = G_0 \cdot 3.09 \times 10^{-4}$

Then the efficiency can be obtained,

$$\nu = \frac{G}{D}$$

$$= \frac{3.09 \times 10^{-4} \cdot G_0}{0.99}$$

$$\approx 3.13 \times 10^{-4}\, G_0$$