Electromagnetic fields and biological tissues: effects and medical applications

Please **initialize** individual items of the declaration, and **sign** it at bottom.

Upon my word of honor, and aware of the consequences of a false declaration under the Italian law, as well as those deriving from unfair conduct at Politecnico,

I, the undersigned Tong Lin

ID n. (matricola) S287649

Hereby declare (*dichiarazione sostitutiva di atto notorio*) that the home assignment n. 2

Has been carried out in a strictly individual manner from beginning to end; in particular,

TL I have not obtained help from any classmate or external person to carry out in part or whole the assignment;

TL I have not employed any paper or electronic material directly related to the assignment; (note: textbooks are indirectly related only)

TL I have not employed scripts, computer programs or any other such procedures that have not been entirely developed by myself, or provided as course material (by the Instructor and/or the Teaching Assistant), and that are not commercial, or cannot be referenced in the open literature or internet; please note that *all employed software not personally and individually developed must be referenced in the submitted papers*. In particular, I have not employed any script, programs etc. developed by my classmates, and that the employed scripts, programs etc. have not been developed in cooperation with my classmates.

TL I have discussed this assignment with the following persons: (enter "none" if appropriate):

Tong Lin
(Complete name, please print)

Tong Lin
signature

Torino, 2022/4/9 (date)

Note: Use of commercial software, of free-ware or shareware, or otherwise publicly available software (e.g. via Internet) is allowed, but usage of all software not developed personally and individually by the student, or provided as course material, MUST be clearly stated and precisely referenced in the submitted paper.

Problem 0.

a) With $\underline{H} = \underline{H}' + j\underline{H}''$

$$a^2 = |\underline{H}|^2 = \underline{H}^* \cdot \underline{H}$$

$$= (\underline{H}' - j\underline{H}'')(\underline{H}' + j\underline{H}'')$$

$$= \underline{H}'^2 + \underbrace{j\underline{H}'\underline{H}'' - j\underline{H}''\underline{H}'}_{=0} + \underline{H}''^2$$

$$= \underline{H}'^2 + \underline{H}''^2$$

Assuming that $\underline{H}' = |\underline{H}'|\,\hat{x}$

$\underline{H}'' = |\underline{H}''|\,\hat{y}$

$$\underline{H}'^2 = |\underline{H}'|^2 \cdot \hat{x}^2 = |\underline{H}'|^2$$

$$\underline{H}''^2 = |\underline{H}''|^2 \cdot \hat{y}^2 = |\underline{H}''|^2$$

Therefore,

$$a^2 = |\underline{H}|^2 = \underline{H}'^2 + \underline{H}''^2 = |\underline{H}'|^2 + |\underline{H}''|^2.$$

b) With $\underline{H} = H_x\hat{x} + H_y\hat{y}$

~~$a^2 =$~~ Assuming that $\underline{H} = |\underline{H}|\,\hat{n}$

$$\underline{H}^2 = |\underline{H}|^2\hat{n}^2 = |\underline{H}|^2$$

So, $a^2 = |\underline{H}|^2 = \underline{H}^2$

$$= (H_x\hat{x} + H_y\hat{y})^2$$

$$= H_x^2\hat{x}^2 + 2H_xH_y\underbrace{\hat{x}\hat{y}}_{=0} + H_y^2\hat{y}^2$$

$$= H_x^2 + H_y^2 = H_x^* \cdot H_x + H_y^* \cdot H_y$$

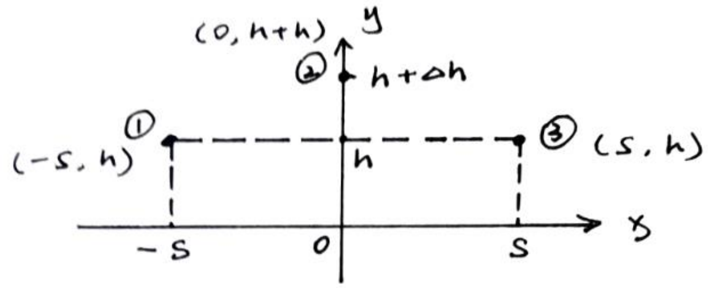
In conclusion,

When $\underline{H} = \underline{H}' + j\underline{H}''$, $a^2 = |\underline{H}|^2 = |\underline{H}'|^2 + |\underline{H}''|^2$

When $\underline{H} = H_x\hat{x} + H_y\hat{y}$, $a^2 = |\underline{H}|^2 = H_x^2 + H_y^2$

$$= H_x^* \cdot H_x + H_y^* \cdot H_y$$

Problem No. 1



1. Analytic expressions of field $H(x, 0)$ and induction $B(x, 0)$:

The expression of the magnetic field of a DC-current carring conductor has been expressed as,

$$\underline{H}(P) = \frac{1}{2\pi R} I \hat{z} \times \hat{R}$$

where the total magnetic field can be expressed as a sum,

$$\underline{H}(P) = \underline{H}(P_1) + \underline{H}(P_2) + \underline{H}(P_3)$$

The differences of each field relys on R and I.

The positions and directions of each point are, $(P(x, 0))$

$$\underline{R}_1 = P - C_1 = (x+s)\hat{x} - h\hat{y}$$

$$\hat{R}_1 = \frac{(x+s)\hat{x} - h\hat{y}}{\sqrt{(x+s)^2 + h^2}}, \quad R_1 = \sqrt{(x+s)^2 + h^2}$$

$$\underline{R}_2 = P - C_2 = x\hat{x} - (h+\Delta h)\hat{y}$$

$$\hat{R}_2 = \frac{x\hat{x} - (h+\Delta h)\hat{y}}{\sqrt{x^2 + (h+\Delta h)^2}}, \quad R_2 = \sqrt{x^2 + (h+\Delta h)^2}$$

$$\underline{R}_3 = P - C_3 = (x-s)\hat{x} - h\hat{y}$$

$$\hat{R}_3 = \frac{(x-s)\hat{x} - h\hat{y}}{\sqrt{(x-s)^2 + h^2}}, \quad R_3 = \sqrt{(x-s)^2 + h^2}$$

The currents are,

$$I_1 = I\exp(j\psi_1) = I$$

$$I_2 = I\exp(j\psi_2) = I\exp(j\tfrac{2}{3}\pi)$$

$$I_3 = I\exp(j\psi_3) = I\exp(j\tfrac{4}{3}\pi).$$

The magnetic fields are separately,

$$\underline{H}(P_1) = \frac{1}{2\pi R_1} I_1\, \hat{z} \times \hat{R}_1$$

$$= \frac{1}{2\pi\sqrt{(x+s)^2+h^2}}\, I\, \frac{\hat{y}(x+s) + \hat{x}h}{\sqrt{(x+s)^2+h^2}}$$

$$= \frac{I[h\hat{x} + (s+x)\hat{y}]}{2\pi[(x+s)^2+h^2]}$$

Similarly,

$$\underline{H}(P_2) = \frac{I\exp(j\frac{2}{3}\pi)[(h+\Delta h)\hat{x} + x\hat{y}]}{2\pi[x^2+(h+\Delta h)^2]}.$$

$$\underline{H}(P_3) = \frac{I\cdot\exp(j\frac{4}{3}\pi)\cdot[h\hat{x} + (x-s)\hat{y}]}{2\pi[(x-s)^2+h^2]}$$

The total magnetic field can be written as,

$$\underline{H}(P) = \underline{H}(P_1) + \underline{H}(P_2) + \underline{H}(P_3)$$

$$= \frac{I}{2\pi}\Bigg[\frac{h\hat{x}+(s+x)\hat{y}}{(x+s)^2+h^2} + \exp(j\tfrac{2}{3}\pi)\frac{(h+\Delta h)\hat{x}+x\hat{y}}{(h+\Delta h)^2+x^2} + \exp(j\tfrac{4}{3}\pi)\frac{h\hat{x}+(x-s)\hat{y}}{h^2+(x-s)^2}\Bigg]$$

The induction can then be expressed as,

$$\underline{B}(P) = \mu\underline{H}$$

$$= \frac{\mu I}{2\pi}\Bigg[\frac{h\hat{x}+(s+x)\hat{y}}{h^2+(s+x)^2} + \exp(j\tfrac{2}{3}\pi)\frac{(h+\Delta h)\hat{x}+x\hat{y}}{(h+\Delta h)^2+x^2} + \exp(j\tfrac{4}{3}\pi)\frac{h\hat{x}+(x-s)\hat{y}}{h^2+(x-s)^2}\Bigg]$$

2. Magnitude of magnetic field and induction at (0.0):

Accrodding to the expressions at $P(x,0)$, now setting also $x=0$, comes,

$$\underline{H}(0.0) = \frac{I}{2\pi}\left[\frac{h\hat{x}+s\hat{y}}{s^2+h^2} + \exp(j\tfrac{2}{3}\pi)\frac{(h+\Delta h)\hat{x}}{(h+\Delta h)^2} + \exp(j\tfrac{4}{3}\pi)\frac{h\hat{x}-s\hat{y}}{h^2+s^2}\right]$$

Applying the Euler's formular,

$$\exp(j\tfrac{2}{3}\pi) = \cos\tfrac{2}{3}\pi + j\sin\tfrac{2}{3}\pi = -\tfrac{1}{2} + j\tfrac{\sqrt{3}}{2}$$

$$\exp(j\tfrac{4}{3}\pi) = \cos\tfrac{4}{3}\pi + j\sin\tfrac{4}{3}\pi = -\tfrac{1}{2} - j\tfrac{\sqrt{3}}{2}$$

Therefore,

$$\underline{H}_x(0.0) = \hat{x}\frac{I}{2\pi}\left[\frac{h+(-\frac{1}{2}+j\frac{\sqrt{3}}{2})h}{h^2+s^2} + \frac{-\frac{1}{2}-j\frac{\sqrt{3}}{2}}{h+\Delta h}\right]$$

$$= \hat{x}\frac{I}{2\pi}\left(\frac{\frac{1}{2}h + j\frac{\sqrt{3}}{2}h}{h^2+s^2} - \frac{\frac{1}{2}+j\frac{\sqrt{3}}{2}}{h+\Delta h}\right) = \hat{x}\,h_x$$

$$\underline{H}_y(0.0) = \hat{y}\frac{I}{2\pi}\cdot\frac{s-s(-\frac{1}{2}-j\frac{\sqrt{3}}{2})}{h^2+s^2}$$

$$= \hat{y}\frac{I}{2\pi}\,\frac{\frac{3}{2}+j\frac{\sqrt{3}}{2}}{h^2+s^2}\cdot s. \quad = \hat{y}\,h_y$$

The magnitudes can be calculated as,

$$|\underline{H}| = \sqrt{h_x^2+h_y^2} = \sqrt{h_x^*\cdot h_x + h_y^*\cdot h_y}$$

$$|\underline{B}| = \mu|\underline{H}|$$

Where $\mu \approx 1.26\times10^{-6}$ H/m in air.

Using Matlab, the results can be obtained as follows,

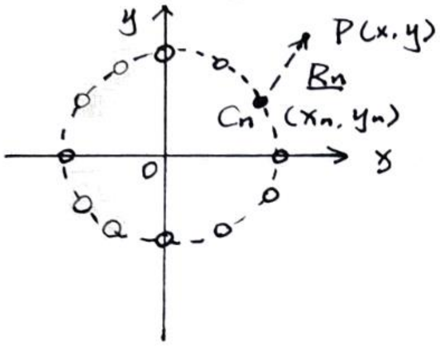
$$|\underline{H}| = 227.75 \quad [A/m].$$

$$|\underline{B}| = 286.20 \quad [\mu T]$$

Problem No.3

A) Generic Expression of the magnetic field:

Assuming a random point $P(x, y)$.



The pointing vector $\underline{R_n}$, which points from the conductor n to the random point $P(x, y)$, can be expressed as,

$$\underline{R_n} = P - C = (x - x_n)\hat{x} + (y - y_n)\hat{y}$$

Therefore,

$$R_n = \sqrt{(x-x_n)^2 + (y-y_n)^2}$$

$$\hat{R}_n = \frac{\hat{x}(x-x_n) + \hat{y}(y-y_n)}{\sqrt{(x-x_n)^2 + (y-y_n)^2}}$$

The generic magnetic field generated by conductor n is then,

$$\underline{H_n}(P) = \frac{1}{2\pi R_n} I_n \hat{z} \times \hat{R}_n$$

where $I_n = I \exp[j(n-1)\frac{2\pi}{N}]$.

$$\hat{z} \times \hat{R}_n = \frac{\hat{y}(x-x_n) - \hat{x}(y-y_n)}{\sqrt{(x-x_n)^2 + (y-y_n)^2}}$$

Therefore,

$$\underline{H_n}(P) = \frac{I \exp[j(n-1)\frac{2\pi}{N}]}{2\pi\sqrt{(x-x_n)^2 + (y-y_n)^2}} \cdot \frac{\hat{y}(x-x_n) - \hat{x}(y-y_n)}{\sqrt{(x-x_n)^2 + (y-y_n)^2}}$$

$$= \frac{I \exp[j(n-1)\frac{2\pi}{N}]}{2\pi[(x-x_n)^2 + (y-y_n)^2]} [\hat{y}(x-x_n) - \hat{x}(y-y_n)].$$

The total generic magnetic field is,

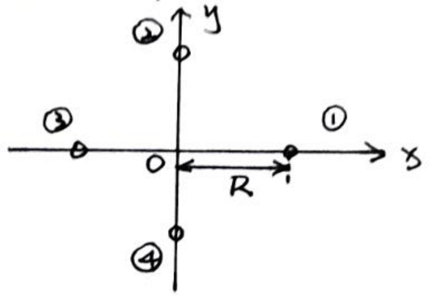
$$\underline{H}(P) = \sum_{n=1}^{N} \underline{H_n}(P)$$

B) The magnetic field at center, with $N=4$,

Firstly, the magnetic field at center generated by the conductor n is,

$$\underline{H}_n(0) = \frac{I\exp[j(n-1)\frac{2\pi}{N}]}{2\pi(x_n^2+y_n^2)}\cdot(\hat{x}y_n - \hat{y}x_n)$$

Assuming that the first conductor is placed along x-axis, and the 4 conductors placed as in the figure below,



The positions of the conductors $C_n(x_n, y_n)$ can then be expressed as below, where R is the distance from the conductors to the center,

$$x_n = R\cdot\cos\frac{2\pi(n-1)}{N}$$

$$y_n = R\cdot\sin\frac{2\pi(n-1)}{N}$$

Also applying the Euler's formular,

$$\exp[j(n-1)\frac{2\pi}{N}] = \cos\frac{2\pi(n-1)}{N} + j\sin\frac{2\pi(n-1)}{N}$$

Attaching the above functions into the $\underline{H}_n(0)$,

$$\underline{H}_n(0) = \frac{I[\cos\frac{2\pi(n-1)}{N} + j\sin\frac{2\pi(n-1)}{N}]}{2\pi[R^2\cos^2\frac{2\pi(n-1)}{N} + R^2\sin^2\frac{2\pi(n-1)}{N}]}\cdot\left[\hat{x}\cdot R\sin\frac{2\pi(n-1)}{N} - \hat{y}\cdot R\cos\frac{2\pi(n-1)}{N}\right]$$

$$= \frac{I}{2\pi R}\cdot\left\{\cos\frac{2\pi(n-1)}{N}\left[\hat{x}\cdot\sin\frac{2\pi(n-1)}{N} - \hat{y}\cos\frac{2\pi(n-1)}{N}\right] + j\sin\frac{2\pi(n-1)}{N}\left[\hat{x}\sin\frac{2\pi(n-1)}{N} - \hat{y}\cos\frac{2\pi(n-1)}{N}\right]\right\}$$

Seperating the real and imaginary parts of $\underline{H}_n(0)$,

with $\underline{H}_n(0) = \underline{H}'_n(0) + j\,\underline{H}''_n(0)$,

$$\underline{H}'_n(0) = \frac{I}{2\pi R} \cdot \cos\frac{2\pi(n-1)}{N}\left[\hat{x}\cdot \sin\frac{2\pi(n-1)}{N} - \hat{y}\cos\frac{2\pi(n-1)}{N}\right].$$

$$\underline{H}''_n(0) = \frac{I}{2\pi R} \cdot \sin\frac{2\pi(n-1)}{N}\left[\hat{x}\cdot \sin\frac{2\pi(n-1)}{N} - \hat{y}\cos\frac{2\pi(n-1)}{N}\right].$$

As the total magnetic field $\underline{H}(P) = \sum_{n=1}^{4} \underline{H}_n(P)$

$$= \sum_{n=1}^{4} \underline{H}'_n(P) + j\sum_{n=1}^{4} \underline{H}''_n(P),$$

The two sums $\sum_{n=1}^{4} \underline{H}'_n(0)$, $\sum_{n=1}^{4} \underline{H}''_n(0)$ are needed to be discussed when study polarization.

The items of $n = 1, 2, 3, 4$ are discussed seperately as below,

When $n=1$, $\cos 0 = 1$, $\sin 0 = 0$.

$\underline{H}'_1(0) = -\frac{I}{2\pi R}\hat{y}$ $\quad\quad$ $\underline{H}''_1(0) = 0$

When $n=2$, $\cos\frac{\pi}{2} = 0$, $\sin\frac{\pi}{2} = 1$

$\underline{H}'_2(0) = 0$ $\quad\quad$ $\underline{H}''_2(0) = \frac{I}{2\pi R}\hat{x}$

When $n=3$, $\cos\pi = -1$, $\sin\pi = 0$

$\underline{H}'_3(0) = -\frac{I}{2\pi R}\hat{y}$ $\quad\quad$ $\underline{H}''_3(0) = 0$

When $n=4$, $\cos\frac{3}{2}\pi = 0$, $\sin\frac{3}{2}\pi = -1$

$\underline{H}'_4(0) = 0$ $\quad\quad$ $\underline{H}''_4(0) = \frac{I}{2\pi R}\hat{x}$

Therefore,

$$\sum_{n=1}^{4} \underline{H}'_n(0) = -\frac{I}{\pi R}\hat{y}$$

$$\sum_{n=1}^{4} \underline{H}''_n(0) = \frac{I}{\pi R}\hat{x}$$

The explicit function of the magnetic field is finally,

$$\underline{H}(0) = -\frac{I}{\pi R}\hat{y} + j\frac{I}{\pi R}\hat{x}$$

As for $\underline{H}'(0) = -\frac{I}{\pi R}\hat{y}$, $\underline{H}''(0) = \frac{I}{\pi R}\hat{x}$ are satisfying the relationships of,

$$\underline{H}' \perp \underline{H}''$$

$$|\underline{H}'| = |\underline{H}''|$$

Thus, the polarization of the magnetic field at the center is circular.

c) Polarization near a conductor:

Assuming the random point $P(x, y)$ is near conductor 1, which can be expressed as,

$$x = x_1 + \Delta x$$
$$y = y_1 + \Delta y$$

where Δx and Δy are extremely small.

The general expression of the magnetic field generated by the conductor is,

$$\underline{H}_n(P) = \frac{I\exp[j(n-1)\frac{2\pi}{N}]}{2\pi[(x-x_n)^2+(y-y_n)^2]}[\hat{y}(x-x_n)-\hat{x}(y-y_n)].$$

As x_n and y_n can be expressed as, $x_n = R\cos\frac{2\pi(n-1)}{N}$ and $y_n = R\sin\frac{2\pi(n-1)}{N}$ when assuming the first conductor placed along x axis with a distance R.

According to the Euler's formular,

$$\exp[j(n-1)\frac{2\pi}{N}] = \cos\frac{2\pi(n-1)}{N} + j\sin\frac{2\pi(n-1)}{N}$$

Therefore, $\exp[j(n-1)\frac{2\pi}{N}] = \frac{1}{R}(x_n + jy_n)$

Then the general $\underline{H}_n(P)$ can be written as.

$$\underline{H}_n(P) = \frac{I(x_n + j y_n)[\hat{y}(x_1+\Delta x - x_n) - \hat{x}(y_1+\Delta y - y_n)]}{2\pi R[(x_1+\Delta x - x_n)^2 + (y_1+\Delta y - y_n)^2]}$$

The real and imaginary parts can be written seperately.

$$\underline{H}_n'(P) = \frac{I\cdot x_n[\hat{y}(x_1+\Delta x - x_n) - \hat{x}(y_1+\Delta y - y_n)]}{2\pi R[(x_1+\Delta x - x_n)^2 + (y_1+\Delta y - y_n)^2]}$$

$$\underline{H}_n''(P) = \frac{I\cdot y_n[\hat{y}(x_1+\Delta x - x_n) - \hat{x}(y_1+\Delta y - y_n)]}{2\pi R[(x_1+\Delta x - x_n)^2 + (y_1+\Delta y - y_n)^2]}$$

where, $\underline{H}_n(P) = \underline{H}_n'(P) + j\,\underline{H}_n''(P)$.

With $x_n = R\cos\frac{2\pi(n-1)}{N}$ and $y_n = R\sin\frac{2\pi(n-1)}{N}$, $N=6$

The results are analyzed as ~~the~~ in the table below.

n	x_n	y_n	$\underline{H}_n'(P)$	$\underline{H}_n''(P)$
1	R	0	$\frac{I(-\Delta y\hat{x}+\Delta x\hat{y})}{2\pi(\Delta x^2+\Delta y^2)}$	0
2	$\frac{R}{2}$	$\frac{\sqrt{3}}{2}R$	$\simeq \frac{I}{8\pi R}(\sqrt{3}\hat{x}+\hat{y})$	$\simeq \frac{\sqrt{3}I}{8\pi R}(\sqrt{3}\hat{x}+\hat{y})$
3	$-\frac{R}{2}$	$\frac{\sqrt{3}}{2}R$	$\simeq -\frac{\sqrt{3}I}{24\pi R}(\hat{x}+\sqrt{3}\hat{y})$	$\simeq \frac{I}{8\pi R}(\hat{x}+\sqrt{3}\hat{y})$
4	$-R$	0	$\simeq -\frac{I}{4\pi R}\hat{y}$	$\simeq 0$
5	$-\frac{R}{2}$	$-\frac{\sqrt{3}}{2}R$	$\simeq -\frac{\sqrt{3}I}{24\pi R}(-\hat{x}+\sqrt{3}\hat{y})$	$\simeq \frac{I}{8\pi R}(\hat{x}-\sqrt{3}\hat{y})$
6	$\frac{R}{2}$	$-\frac{\sqrt{3}}{2}R$	$\simeq \frac{I}{8\pi R}(-\sqrt{3}\hat{x}+\hat{y})$	$\simeq \frac{\sqrt{3}I}{8\pi R}(\sqrt{3}\hat{x}-\hat{y})$

For further analyzation of $\underline{H}_1'(P)$, as $y_1=0$,

$$\lim_{\Delta y\to 0}\underline{H}_1'(P) = \frac{I\cdot\Delta x\,\hat{y}}{2\pi\,\Delta x^2} = \frac{I}{2\pi\Delta x}\hat{y}$$

As Δx is very close to 0, $\underline{H}_1'(P)$ is tended to be infinite along $\hat{y}$. Therefore,

$$\sum_{n=1}^{6}\underline{H}_n'(P) = \frac{I}{2\pi\Delta x}\hat{y} - \frac{3}{4}\frac{I}{\pi R}\hat{y} \approx \infty\cdot\hat{y}$$

$$\sum_{n=1}^{6}\underline{H}_n''(P) = \frac{I}{\pi R}\hat{x}$$

which is obvious to be observed, that $\sum_{n=1}^{6} \underline{H}_n'(P) >> \sum_{n=1}^{6} \underline{H}_n''(P)$, so that, in comparision, $\sum_{n=1}^{N} \underline{H}_n''(P)$ can be seen as tend to be zero, thus $\underline{H}'(P) \parallel \underline{H}''(P)$ could be approximately obtained. Therefore, the polarization near the conductors can be seen as linear.

D) Graphs of the magnitude of the magnetic field within the region of interest:

The general function of the magnetic field has been,

$$\underline{H}_n(P) = \frac{I(x_n + j y_n)[-\hat{x}(y-y_n) + \hat{y}(x-x_n)]}{2\pi R[(x-x_n)^2 + (y-y_n)^2]}.$$

Seperating the functions along axises,

$$\underline{H}_{nx}(P) = -\hat{x}\frac{I(x_n + j y_n)(y-y_n)}{2\pi R[(x-x_n)^2 + (y-y_n)^2]} = h_{nx}\hat{x}$$

$$\underline{H}_{ny}(P) = \hat{y}\frac{I(x_n + j y_n)(x-x_n)}{2\pi R[(x-x_n)^2 + (y-y_n)^2]} = h_{ny}\hat{y}$$

The magnitude can be expressed as,

$$\underline{H}(P) = \sqrt{\underline{H}_x^2 + \underline{H}_y^2}$$

$$= \sqrt{\left|\sum_{n=1}^{N} h_{nx}\right|^2 + \left|\sum_{n=1}^{N} h_{ny}\right|^2}$$

where $\left|\sum_{n=1}^{N} h_{nx}\right|^2 = \left(\sum_{n=1}^{N} h_{nx}\right)^* \cdot \sum_{n=1}^{N} h_{nx}$

$\left|\sum_{n=1}^{N} h_{ny}\right|^2 = \left(\sum_{n=1}^{N} h_{ny}\right)^* \cdot \sum_{n=1}^{N} h_{ny}$.

and $N = 6$. $x_n = R \cdot \cos\frac{2\pi(n-1)}{N}$

$y_n = R \cdot \sin\frac{2\pi(n-1)}{N}$

The graphs are plotted using MATLAB as follows.

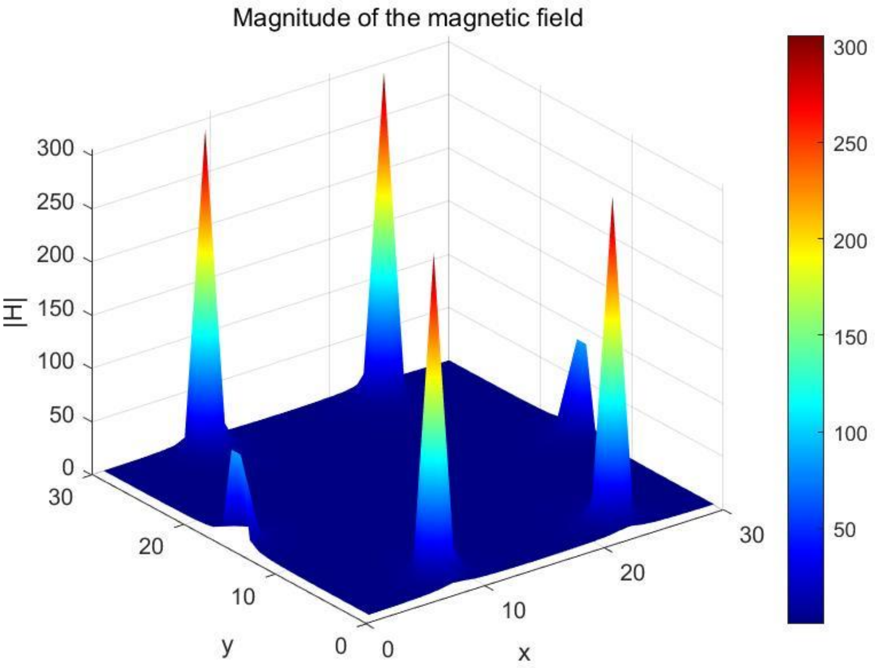


Figure 2d.1: The 3D Plot of the Magnitude of the Magnetic Field

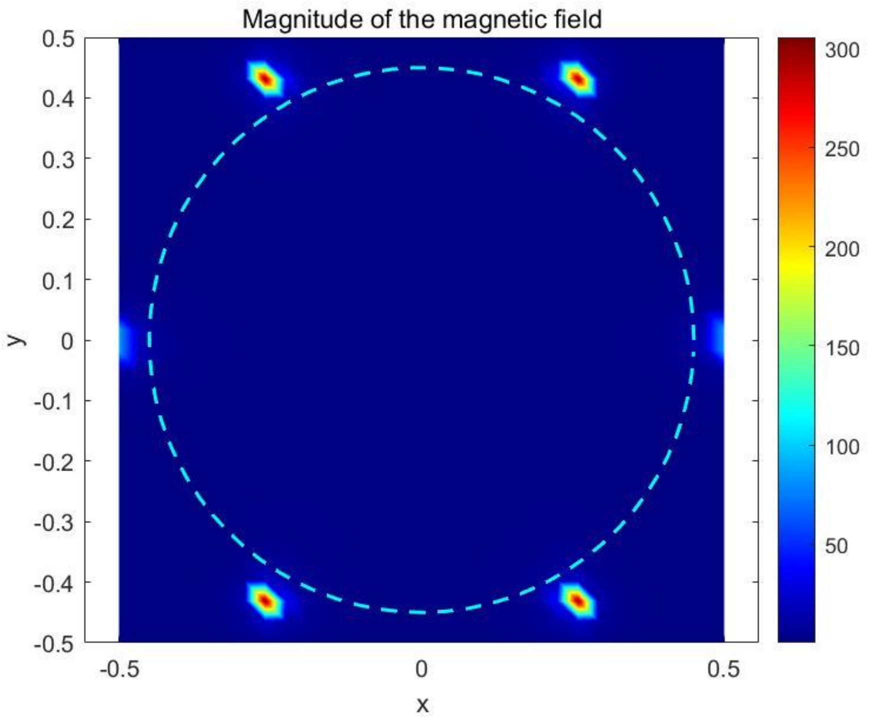


Figure 2d.2: The 2D Plot of the Magnitude of the Magnetic Field

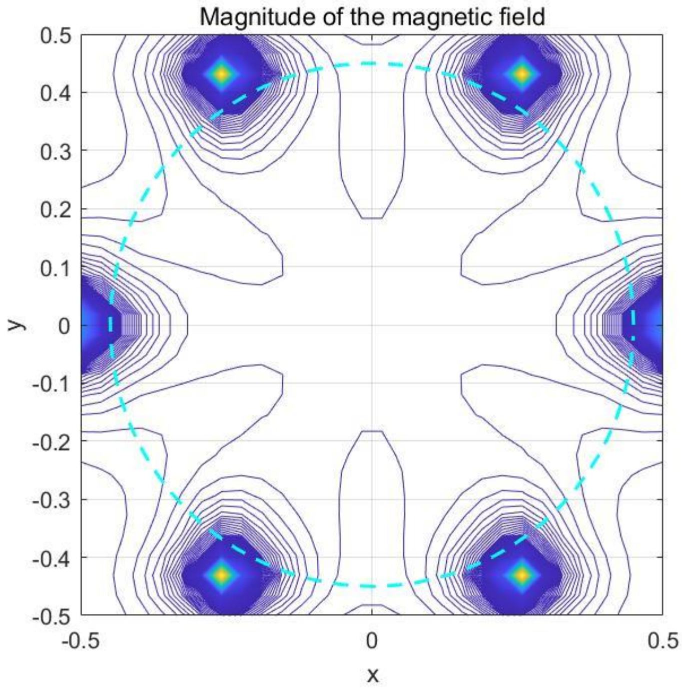


Figure 2d.3: The Contour Line Plot of the Magnitude of the Magnetic Field

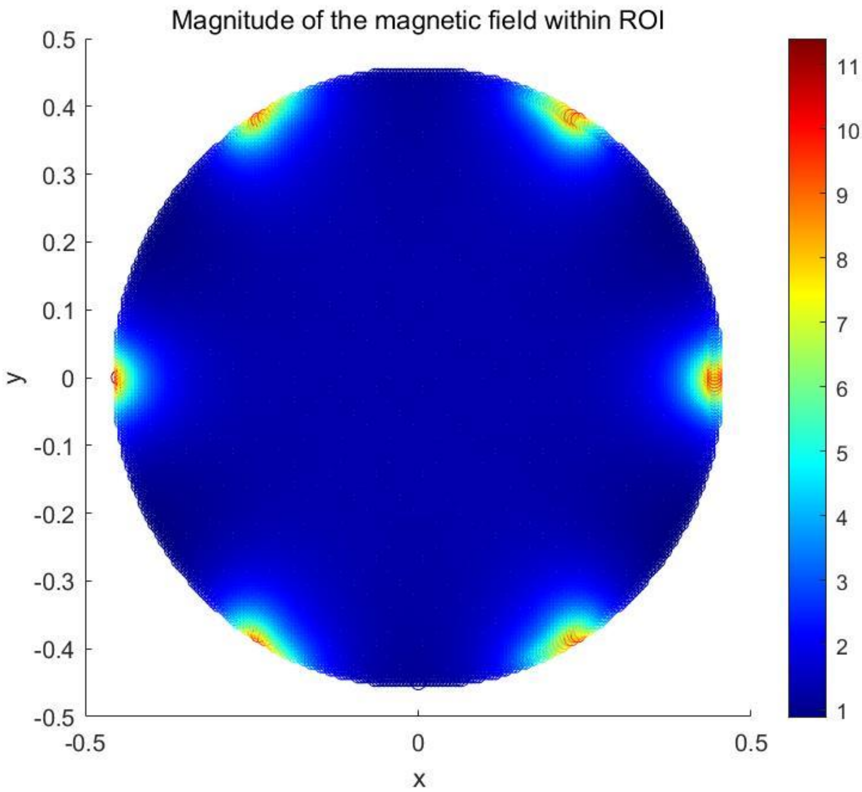


Figure 2d.4: The Scatter Plot of the Magnitude of the Magnetic Field only within the Region of Interest (ROI)

E) Graphs of vector field:

The complex unit vector is expressed as,

$$\hat{h}(x, y) = \frac{\underline{H}(x, y)}{|\underline{H}(x, y)|}$$

so that,

$$\hat{h}_x = \frac{\underline{H}_x}{|\underline{H}(x, y)|} = \frac{\sum_{n=1}^{N} \underline{H}_{nx}}{|\underline{H}(x, y)|}$$

$$\hat{h}_y = \frac{\underline{H}_y}{|\underline{H}(x, y)|} = \frac{\sum_{n=1}^{N} \underline{H}_{ny}}{|\underline{H}(x, y)|}$$

where,

$$\underline{H}_{nx} = -\frac{I(y - y_n)(x_n + j y_n)}{2\pi R[(x - x_n)^2 + (y - y_n)^2]} \cdot \hat{x} = h_{nx} \cdot \hat{x}$$

$$\underline{H}_{ny} = \frac{I(x - x_n)(x_n + j y_n)}{2\pi R[(x - x_n)^2 + (y - y_n)^2]} \cdot \hat{y} = h_{ny} \cdot \hat{y}$$

$$\underline{H}(x, y) = \sqrt{\left(\sum_{n=1}^{N} h_{nx}\right)^* \cdot \left(\sum_{n=1}^{N} h_{nx}\right) + \left(\sum_{n=1}^{N} h_{ny}\right)^* \cdot \left(\sum_{n=1}^{N} h_{ny}\right)}$$

Applying the time variations,

$$\hat{h}_{xt} = \mathrm{Real}\{\hat{h}_x \cdot \exp(j\omega t)\}$$

$$\hat{h}_{yt} = \mathrm{Real}\{\hat{h}_y \cdot \exp(j\omega t)\}$$

Using MATLAB, with the instruction 'quiver(x, y, $\hat{h}_{xt}$, $\hat{h}_{yt}$)', the graphs are plotted as follows, with seperately $t=0$ and $t=\frac{T}{4}$.

From the figure 'The vector field of the Magnetic field', it is clear to be observed that the polarization is circuler in the center of the region of interest, and become lineer when near to the conductors.

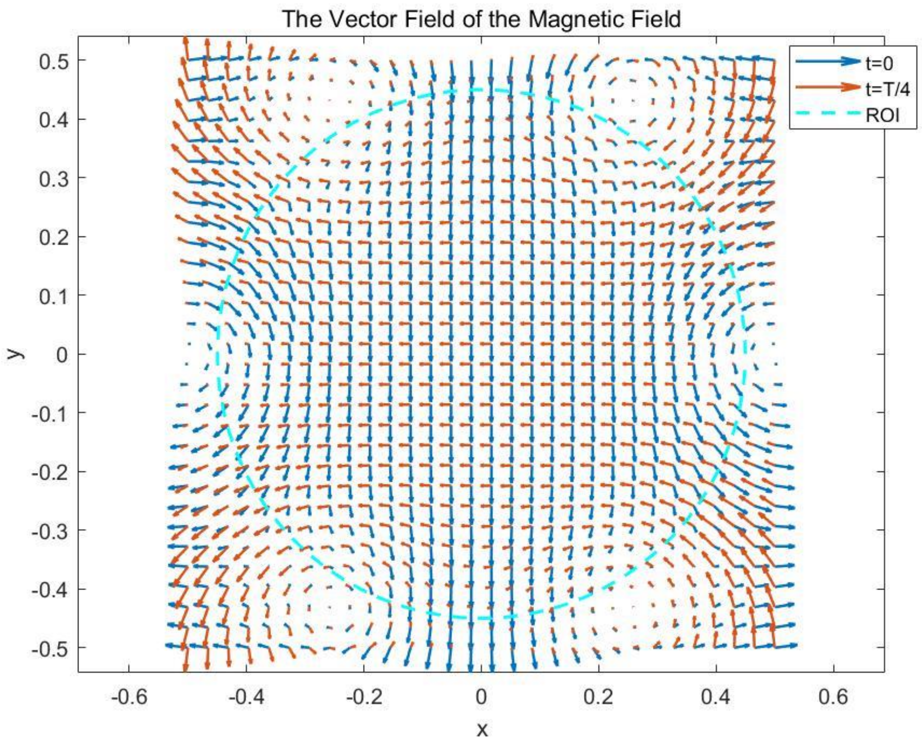


Figure 2e: The Plot of the Vector Field of the Magnetic Field

Appendix: MATLAB codes

Problem 1:

```
clear all;
close all;
clc

j = sqrt(-1);
h = 11.34;      %[m]
delta = 1;      %[m]
s = 7.4;      %[m]
I = 1500;      %[A]

%H(0,0) midpoint of the ground
Hx = (I/(2*pi))*((h*(0.5+j*sqrt(3)/2))/(s^2+h^2))-((0.5+j*sqrt(3)/2)/(h+delta));
Hy = (I/(2*pi))*s*(1/5+j*sqrt(3)/2)/(s^2+h^2);
Hx = conj(Hx)*Hx;
Hy = conj(Hy)*Hy;

H = sqrt(Hx^2+Hy^2) %[A/m]

%B(0,0)
miu = 1.25663753e-6;      %approximately,[H/m]
B = miu*H    %[T]
```

Problem 2d:

```
clear all;
close all;
clc

j = sqrt(-1);
R = 0.5;      %[m]
R_ROI = 0.9*R;      %[m]
I = 1;      %[A]
N = 6;
x = linspace(-R,R,30);
y = linspace(-R,R,30);
[x,y] = meshgrid(x,y);
%%%x^2+y^2 <= R_ROI;

hx = 0;
hy = 0;
coe = I/(2*pi*R);
for n = 1:1:N
    xn = cos(2*pi*(n-1)/N)*R;
    yn = sin(2*pi*(n-1)/N)*R;
    Hnx = -coe*(xn+j*yn).*(y-yn)./((x-xn).^2+(y-yn).^2);
    hx = hx+Hnx;
    Hny = coe*(xn+j*yn).*(x-xn)./((x-xn).^2+(y-yn).^2);
    hy = hy+Hny;
end
Hxab = conj(hx).*hx;
Hyab = conj(hy).*hy;
H = sqrt(Hxab.^2+Hyab.^2);    %[A/m]

figure
surf(H);
shading interp;
colorbar;
colormap(jet);
xlabel('x');
ylabel('y');
zlabel('|H|');
title('Magnitude of the magnetic field');

figure
pcolor(x,y,H);
shading interp;
colorbar;
```

```
colormap(jet);
xlabel('x');
ylabel('y');
title('Magnitude of the magnetic field');
hold on
theta = 0:0.08:2*pi;
plot(R_ROI*cos(theta),R_ROI*sin(theta),'c--','LineWidth',1.5)
axis equal

figure
contour(x,y,H,1000);
axis equal
xlabel('x');
ylabel('y');
title('Magnitude of the magnetic field');
hold on
plot(R_ROI*cos(theta),R_ROI*sin(theta),'c--','LineWidth',1.5)
grid on

figure
for x = -R:0.005:R
    for y = -R:0.005:R
        hx = 0;
        hy = 0;
        for n = 1:1:N
            xn = cos(2*pi*(n-1)/N)*R;
            yn = sin(2*pi*(n-1)/N)*R;
            Hnx = -coe*(xn+j*yn).*(y-yn)./((x-xn).^2+(y-yn).^2);
            hx = hx+Hnx;
            Hny = coe*(xn+j*yn).*(x-xn)./((x-xn).^2+(y-yn).^2);
            hy = hy+Hny;
          end
          Hxab = conj(hx).*hx;
          Hyab = conj(hy).*hy;
          H = sqrt(Hxab.^2+Hyab.^2);    %[nA/m]
        if x.^2+y.^2 <= R_ROI^2
            scatter(x,y,[],H);hold on
        end
    end
end
%shading interp;
colorbar;
colormap(jet);
xlabel('x');
```

```
ylabel('y');
title('Magnitude of the magnetic field within ROI');
```

Problem 2e:

```
close all;
clear all;
clc

j = sqrt(-1);
R = 0.5;      %[m]
R_ROI = 0.9*R;      %[m]
I = 1;      %[A]
N = 6;
x = linspace(-R,R,30);
y = linspace(-R,R,30);
[x,y] = meshgrid(x,y);

H = 0;
Hx = 0;
Hy = 0;
hx = 0;
hy = 0;
coe = I/(2*pi*R);
H = 0;
for n = 1:1:N
            xn = cos(2*pi*(n-1)/N)*R;
            yn = sin(2*pi*(n-1)/N)*R;
            Hxn = -coe*(xn+j*yn).*(y-yn)./((x-xn).^2+(y-yn).^2);
            Hx = Hx+Hxn;
            Hyn = coe*(xn+j*yn).*(x-xn)./((x-xn).^2+(y-yn).^2);
            Hy = Hy+Hyn;
end
Hxab = conj(Hx).*Hx;
Hyab = conj(Hy).*Hy;
H = sqrt(Hxab.^2+Hyab.^2);
hx = Hx./H;
hy = Hy./H;
figure
quiver(x,y,abs(hx),abs(hy));
axis equal;
xlabel('x')
ylabel('y')
title('The Vector of the Magnetic Field')

figure
hxt1 = real(hx*exp(j*0));
hyt1 = real(hy*exp(j*0));
```

```
quiver(x,y,hxt1,hyt1,'LineWidth',1.25);
axis equal;
xlabel('x')
ylabel('y')
hold on
hxt2 = real(hx*exp(j*pi/2));
hyt2 = real(hy*exp(j*pi/2));
quiver(x,y,hxt2,hyt2,'LineWidth',1.25);
title('The Vector Field of the Magnetic Field')
hold on
theta = 0:0.08:2*pi;
plot(R_ROI*cos(theta),R_ROI*sin(theta),'c--','LineWidth',1.5)
legend('t=0','t=T/4','ROI')
```