Communication System Assignment 3

Tong Lin S287649

Exercise 1: 5G SSS Gold Set

1. Periodic Cross-Correlation between the First Two Sequences

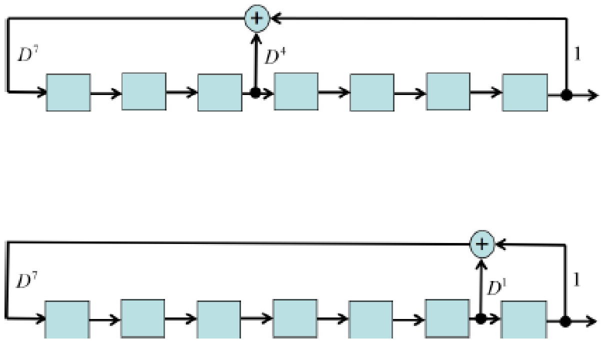

Figure 1.1: The Two 7-cell Linear Feedback Shift Register (LFSR)

In the first exercise, two m=7 cells LFSR were generated with the starting seed of [0 0 0 0 0 0 1], as shown in Figure 1.1, which the two polynomial functions can be described as below, in order to obtain the SSS sequences as the Gold sequence with the length of N=127 bits.

$$s_1(D) = D^7 + D^4 + 1$$
$$s_2(D) = D^7 + D + 1$$

Using the periodic autocorrelation function as follow, Figure 1.2 of the cross-correlation between the first 2 sequences has been plotted.

$$R_{12}(\tau) = \sum_{n=0}^{N-1} b_1(n) b_2(n-\tau) \ , \qquad -(N-1) \leq \tau \leq N-1$$

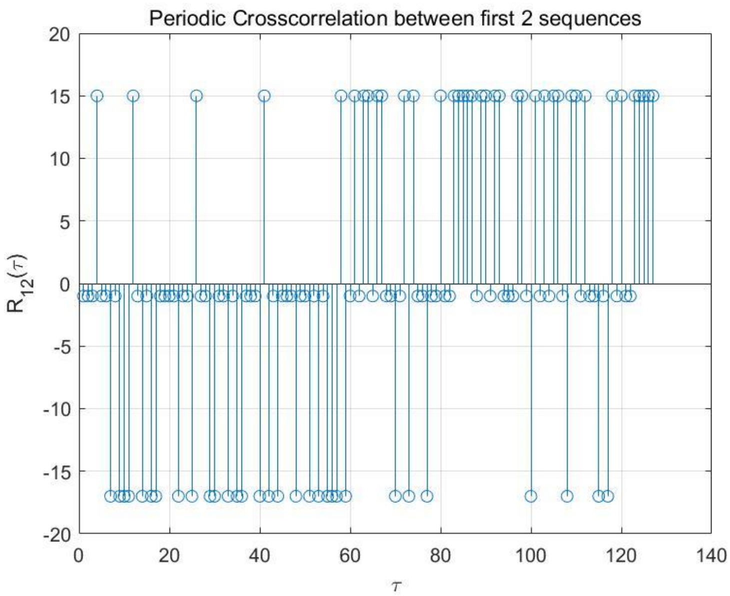

Figure 1.2: Periodic Cross-Correlation between the first 2 Sequences

As shown in Figure 1.2, there are only 3 results of the periodic cross-correlation, which are -1, -t, t-2, where t=17, which indicates a very good quality of the Gold sequence. The function of t is,

$$t = 2^{\frac{m+2}{2}} + 1$$

2. Periodic Cross-Correlation of the First Sequence against Other 1007 Sequences

The function of periodic cross-correlation between the first and all other 1007 SSS sequences is as follow.

$$R_{1n}(\tau) = \sum_{n=0}^{N-1} b_1(n) b_n(n-\tau) \ , \qquad -(N-1) \le \tau \le N-1$$

The maximum absolute values of the each periodic cross-correlations are shown in Figure 1.3.

The maximum values can be seen is not only |t|=17, but also in some particular and periodic circumstances equal to 127. To observe this problem, Figure1.4 has been further plotted as the periodic cross-correlation value between the first sequence and the No.33 sequence, who's maximum value is 127.

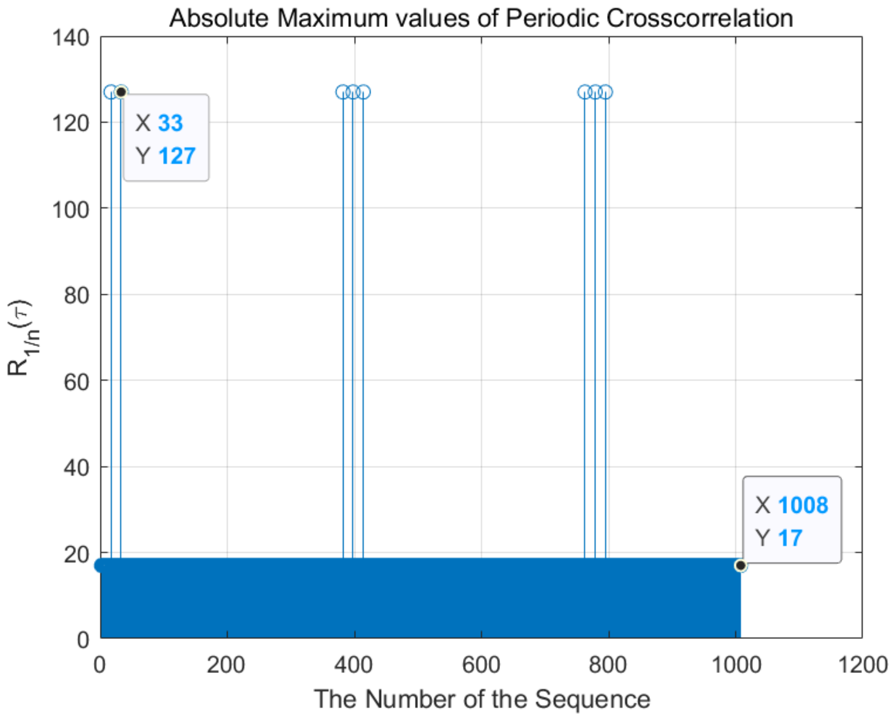

Figure 1.3: The Periodic Cross-Correlation of the first Sequence against all other 1007 Sequences

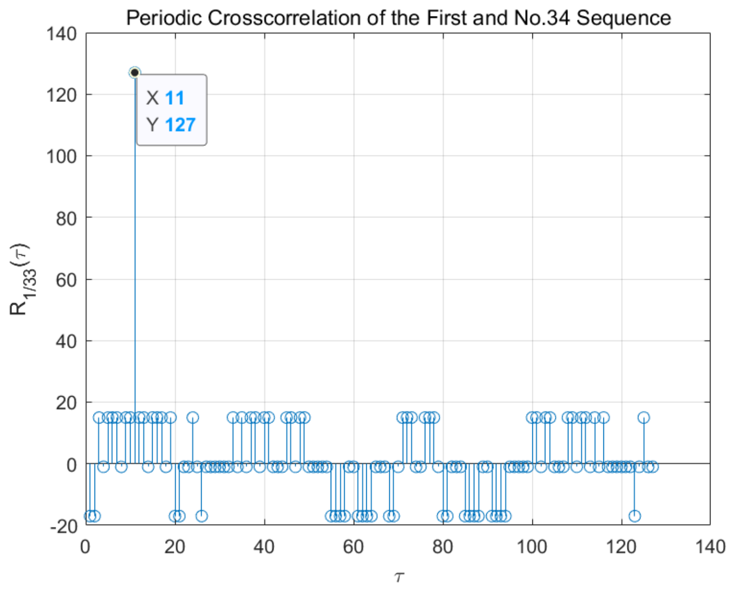

Figure 1.4: The Periodic Cross-Correlation between the first and No.33 Sequences

As show in figure 1.4, that the maximum value of the cross-correlation is 127, which appears in the 11th element. This phenomenon is similar as in the assignment 2 when we try to estimate the auto-correlation of a single PSS m-sequence on the position of τ=0.

According to this, after observing the cross-correlation between the first and first sequence, or to say the auto-correlation of the first sequence itself, as shown in Figure 1.5, it is obvious to see, that the maximum value of 127 appears on the first position of the τ.

As a consequence, simply right shift 10 bits of the No.33 sequence, a same sequence as the first sequence can be got.

Therefore, the maximum values of 127 are only appeared when the follow sequences can occasionally equal to the first sequence, with a certain bit of right shift.

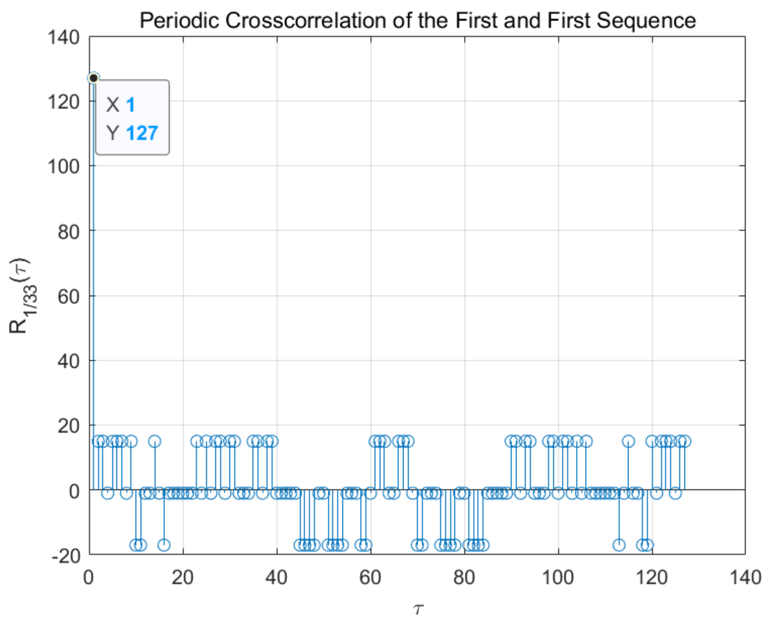

Figure 1.5: The Periodic Cross-Correlation between the first and First Sequence

Exercise 2: Welch Bound and Sidelnikov Bound

The Welch bound and Sidelnikov bound are used to estimate the whether the gold sequence is good enough by analyzing the tendency of maximum value of cyclic-correlation and auto-correlation magnitude according to the sequence number of K.

As shown in Figure 2.1, the functions of the plots are sequentially as in the follows.

The Sidelnikov binary function is,

$$r_M \geq \left\lceil \sqrt{\left[(2s+1)(N-s) + \frac{s(s+1)}{2} - \frac{2^s N^{2s+1}}{K(2s)!\binom{N}{s}}\right]} \right\rceil$$

$$\forall s \in \mathbb{N} = \{0, 1, 2, 3, ...\} \qquad 0 \leq s < \frac{2N}{5}$$

The Welch original function is,

$$r_M \geq N \sqrt[2s]{\frac{1}{KN-1}\left[\frac{KN}{\binom{N+s-1}{s}} - 1\right]} \qquad \forall s \in \mathbb{Z}^+ = \{1, 2, 3, ...\}$$

The Welch simplified formular is,

$$r_M \geq N\sqrt{\frac{K-1}{KN-1}}$$

The Welch square root of N formular when for a large K is,

$$\sqrt{N}$$

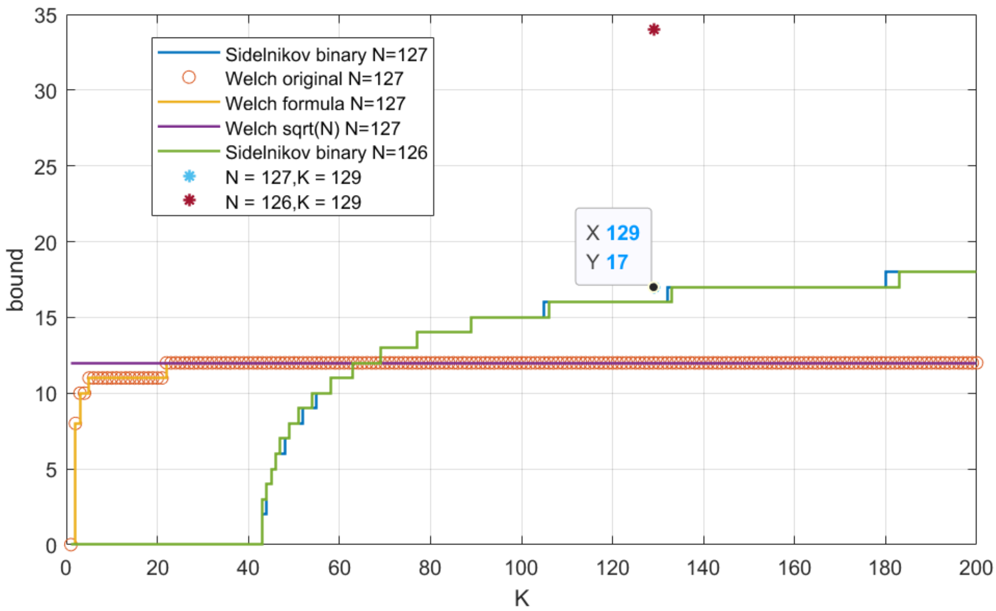

Figure 2.1: Welch Bound and Sidelnikov Bound

As shown in Figure 2.1, with the increasing of K, both the Sidelnikov bound and the Welch bounds show the tendency of increasing.

Using the Gold sequence in exercise 1, by adjusting the number of the sequences K and the sequence length N, the last two points can be plotted, separately with N=127, K=129, and N=126, K=129. The results of the both points are very different with 17 for N=127, which shows a good quality as it is near to the Sidelnikov bond of 16. The quality of the other point

with N=126 is worse, with the result of 34, which is far away from the Sidelnikov bound, due to the fact that there might have empty transmission bits.
The Welch bound functions on the other hand, shows less accurate when K is not large enough compare with the Sidelnikov bound.

Exercise 3: CRC

1. **Codebook Generation**

Firstly generate the CRC code using the code function of 'comm.CRCGenerator' in Matlab, with the polynomial function as below and let k=10.

$$g(D) = (D^{16} + D^{12} + D^5 + 1)$$

Then compute the Hamming weight of the non-zero codes, where all the codes are even and larger or equal to 4 and up to 18 in this simulation, and calculate the multiplicity values A_i for the later probability calculations.

With the codewords' weight and the multiplicity value, the probability of undetected error can be calculated with the function as below, where the number of the codewords $n=k+16=26$, the BSC transmission channel is with $p= [10^{-1}\ 10^{-2}\ 10^{-3}\ 10^{-4}]$.

$$\Pr(UE) = \sum_{i=1}^{n} A_i p^i (1-p)^i$$

Finally, plot the relationship between p and $Pr(UE)$ as shown in figure 3.1. It is obvious to see, that with the increasing of value p, the probability of undetected error is decreasing.

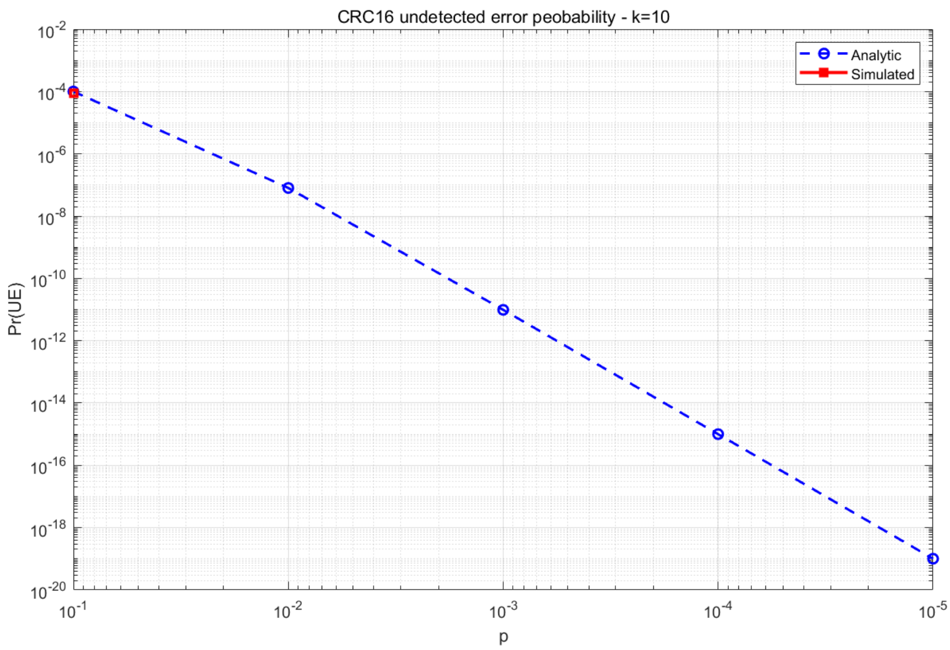

Figure 3.1: The Analytic and Simulated Probabilities of Undetected Errors

2. **Simulation**

The upon result can be verified with the simulation using the code function 'comm.CRCDetector', by generating a brand new serial of codewords. In this part, only one point has been simulated, which is $p=10^{-1}$.

As shown in figure 3.1, the red point is the simulated point, which is very slightly lower than the theoretical value using the probability function. This simulation has been repeated 1165380 times until 100 undetected error events were observed.