

## Advanced Antenna Engineering

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ID n. (matricola) 287649

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## Advanced Antenna Engineering

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TONG LIN

(Complete name, please print)

全霖. Tong Lin

signature<sup>6</sup>

Torino, \_\_\_\_\_ (date)

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<sup>6</sup> If your National language is not written in Latin characters (e.g. Chinese, Arabic), you must sign both with your name in Latin characters and as you sign official documents in your National language  
HW declaration, rev. 3, June 2021

# 1. Design a BS AF of Dolph-Chebyshev.

To achieve min complexity and an optimum solution, the system should satisfy the conditions,

$$f_{BW}(N') \leq \frac{d}{\lambda} \leq f_{GL}(N')$$

where  $N' = N - 1$ .

To be specified,

$$\textcircled{1} f_{GL}(N') = \frac{\frac{1}{\pi} \cos^{-1} \left[ \frac{-1}{\gamma_0(N')} \right]}{1 + |\cos \alpha_{\max}|}$$

where,

- $\gamma_0(N') = \cosh \left( \frac{\cosh^{-1} R_0}{N'} \right)$ ,  $R_0 = \frac{1}{SLL}$
- $\cos \alpha_{\max} = -\frac{\Phi}{kd} = 0$ , according to the Broadside design,  $\Phi = 0$ .

$$\begin{aligned} \textcircled{2} f_{BW}(N') &= \frac{\psi_{BW}^{(N)} / 2}{\sin \Delta \beta} \\ &= \frac{1}{\pi \sin \Delta \beta} \cos^{-1} \left[ \frac{\gamma_{BW}(N')}{\gamma_0(N', R_0)} \right]. \end{aligned}$$

where,

- $\gamma_{BW}(N') = \gamma_{HP}(N', R_0)$   
 $= \cosh \left[ \frac{1}{N'} \cosh^{-1} \left( \frac{R_0}{\sqrt{2}} \right) \right]$ ,  $R_0 = \frac{1}{SLL}$
- $\gamma_0(N', R_0) = \cosh \left( \frac{1}{N'} \cosh^{-1} R_0 \right)$ .
- $\Delta \beta = \frac{1}{2} BW = \frac{1}{2} HPBW \leq 5^\circ$

Applying the conditions from the question, that  $SLL \leq -20dB$  and  $HPBW \leq 10^\circ$ , the results can be computed by using Matlab.

$$N = 10, \quad \frac{d}{\lambda} = 0.9430.$$

To verify this results,  $SLL$  and  $HPBW$  have been calculated again using Matlab, where  $SLL = -20dB$ ,  $HPBW = 7.69^\circ$ .

Here also, 'x to  $\psi$  mapping' and 'Chebyshev Polynomials' have been plotted using Matlab, to verify the design, as shown in figure 1.1 and figure 1.2.

The normalized amplitude (excitation coefficients)  $a_n = \frac{I_n}{I_{\max}}$ , has been ~~plotted~~ plotted in figure 1.3.

The plot of AF vs. observable angle  $\alpha$  comparing with the uniform array is as shown in figure 2.1.

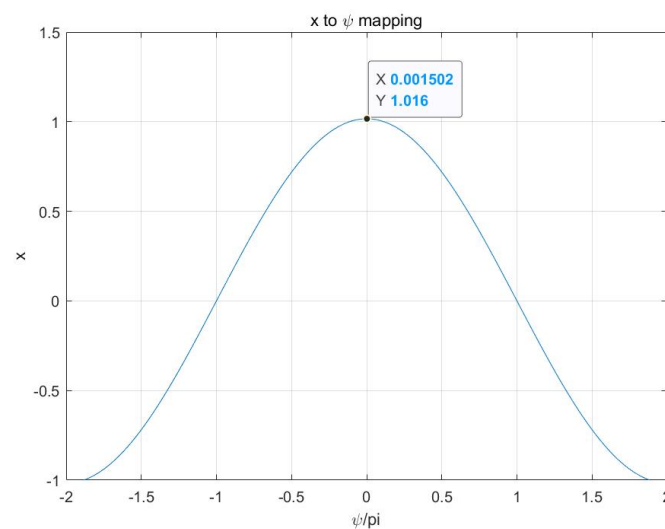


Figure 1.1: x to  $\psi$  Mapping

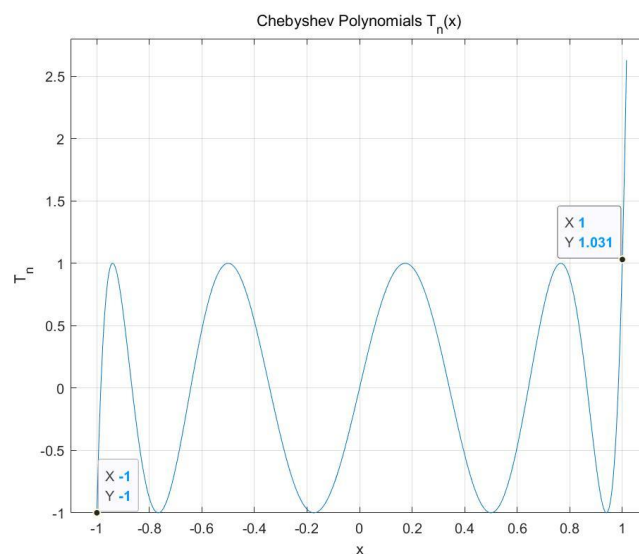


Figure 1.2: Chebyshev Polynomials  $T_n(x)$

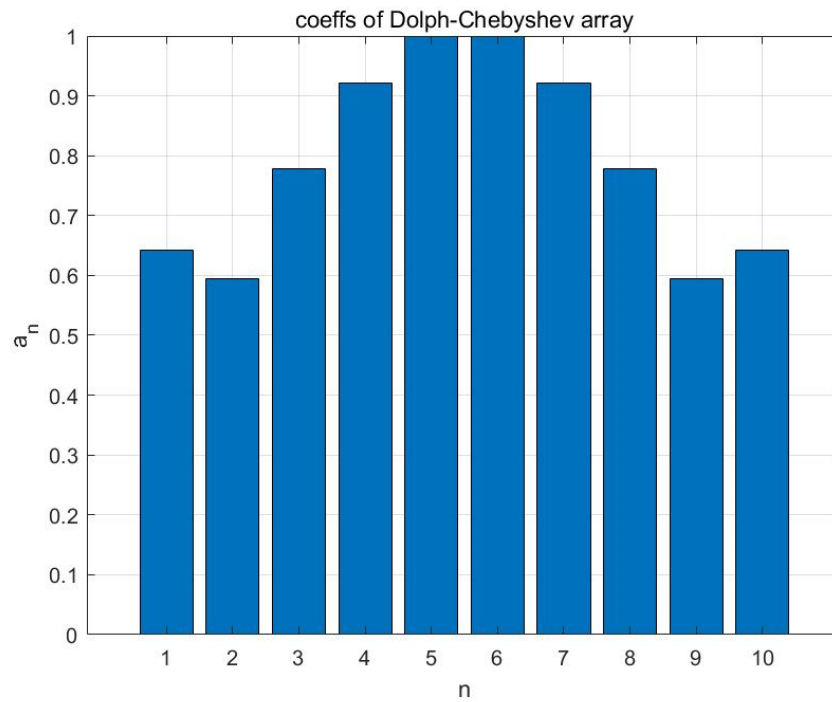


Figure 1.3: Excitation Coefficients

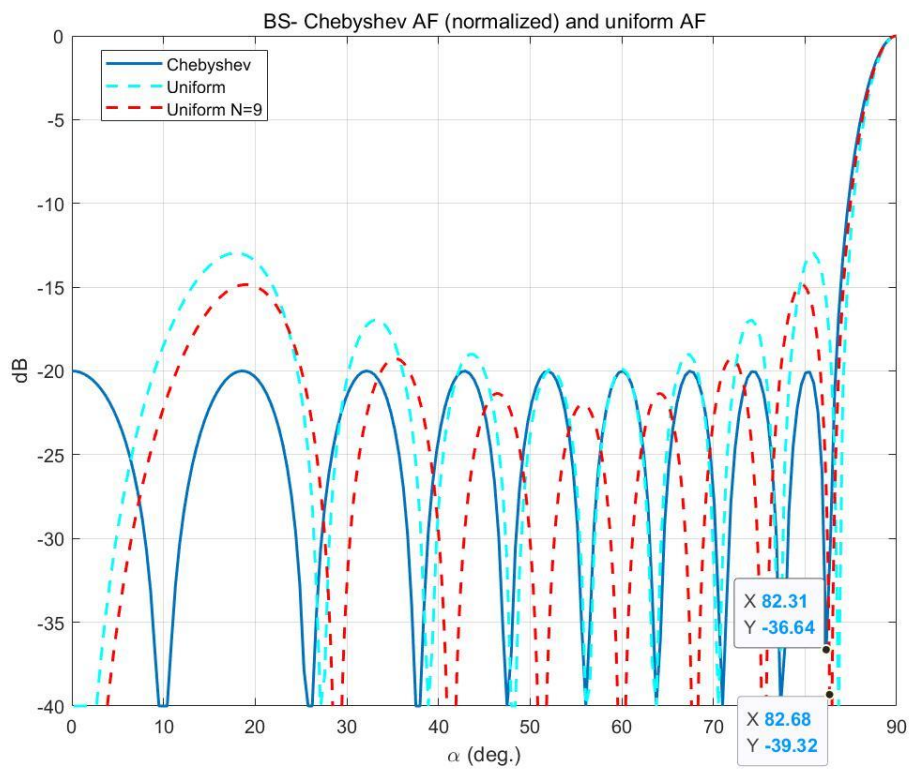


Figure 2.1: Chebyshev vs. Uniform AFs

## 2. Uniform Array

Applying the same specifications, that  $SLL \leq -20 \text{ dB}$ , and  $HPBW \leq 10^\circ$ , and using the conditions,

$$f_{BW}(N_x) \leq \frac{d}{\lambda} \leq f_{GL}(N_x).$$

$$\text{where, } f_{BW}(N_x) = \frac{\gamma_{BW}}{\sin \frac{1}{2} BW} \cdot \frac{1}{N_x}$$

- $\gamma_{BW} = 0.45$
- $BW = HPBW \leq 10^\circ$

$$f_{GL}(N_x) = 1 - \frac{1}{N_x}$$

$$N = N_x + 1.$$

The results can be calculated using Matlab.

$$\begin{cases} N = 9 \\ \frac{d}{\lambda} = 0.8889. \end{cases}$$

Plot the both AF  $f$  satisfies in the same figure as shown in figure 2.1.

The line of 'Chebyshev' and 'Uniform' are plotted with the same ~~can~~ design, with  $N=10$ ,  $\frac{d}{\lambda} = 0.9430$ .

The line of 'Uniform  $N=9$ ' is plotted with  $N=9$ ,  $\frac{d}{\lambda} = 0.8889$  according to the same specifications as Chebyshev array.

It can be easily called by the figure, that the bandwidth of the uniform design is slightly smaller than the Chebyshev design, whereas the number of elements with uniform design is smaller than the Chebyshev design when achieve the same specifications. However, according to the parameter of side lobe level, Chebyshev design shows much better behavior compares with the uniform design, and also better behavior according to the max directivity.



## Appendix: Matlab codes

### Problem 1: Design of question 1

```
close all;
clear all;
clc

SLL = -20;      % -20dB
HPBW = 10;      % [degree]
%R0 = real(10*log10(1./SLL));
R0 = 1/SLL;
delta_beta = HPBW*pi/360; % [rad]
N1 = 1:1:100;

x0 = cosh(acos(R0)./N1);
xBW = cosh(acosh(R0/sqrt(2))./N1);
fGL = real(acos(-1./x0)/pi);
fBW = real(acos(xBW./x0)./(pi*sin(delta_beta)));

a = [fBW;fGL];      % ---N1=9; N=10

%verify
N1 = 9;
N = N1+1;
x0 = cosh(acos(R0)./N1);
xBW = cosh(acosh(R0/sqrt(2))./N1);
fGL = real(acos(-1./x0)/pi);
d_lamuda = fGL;
alp = 0:0.001:pi;
phi = 2*pi*d_lamuda*cos(alp);
T0 = cosh(N1*acos(x0));
R0 = T0;
SLL = 1/R0;
    %HPBW accroding to the figures

figure
x = x0*cos(phi/2);
plot(phi/pi,x);
xlabel('\psi/pi');
ylabel('x');
title('x to \psi mapping');
grid on
figure
TN1 = cos(N1*acos(x));
```

```

plot(x,TN1);
xlabel('x');
ylabel('T_n');
title('Chebyshev Polynomials T_n(x)');
axis([-1.1 1.1 -1 2.8]);
grid on

```

### Problem 2: Design of question 2

```

clear all;
close all;
clc

N1 = 0:1:15;
xBW = 0.45;
BW = 10*pi/180; %[rad]

fBW = (1./N1).*xBW./sin(0.5*BW);
fGL = (1-1./N1);

A = [fBW;fGL];

N = 9;
d_lamuda = 1-1/N;

%yield N, d/lamuda, Phi
j = sqrt(-1);
alp = -pi/2:0.001:0;
PHI = 0

phi = 2*pi*d_lamuda*cos(alp)+PHI;
F_N = (1/N)*abs(sin(N*phi/2)./sin(phi/2));
F_Ndb = 10*log(F_N);
figure
plot(alp*180/pi,F_Ndb);
axis([-90 0 -40 0]);
grid on

```

### Problem 1: Analyzations

```

% Dolph_Chebyshev array coefficients
clear all;
close all;
clc

% input pars
N=10; % number of elements

```



```

R0dB=-20; %SLL in dB
nsamp=200; %n. of samples for plot in psi
bottom=-40; %dB, bottom of scale          ????????

%----- start -----
clear p
clear xz
clear psiz
clear psizu

P=N-1;
R0=10^(abs(R0dB)/20);
x0=0.5*( (R0+sqrt(R0^2-1))^(1/P)+(R0-sqrt(R0^2-1))^(1/P));

%zeros in x and zeros in psi
if parity(P)==0
    PP=P/2;
    p=(1:1:PP);
    xzp=cos(pi*(2*p-1)/(2*P));
    psizp=2*acos(xzp/x0);
    psiz(1:PP)=psizp;
    psiz(PP+1:P)=-fliplr(psizp);
    % zeri della schiera uniforme
    %psizup=p*(2*pi/P);
    psizup=p*(2*pi/N);
    psizu(1:PP)=psizup;
    psizu(PP+1:P)=-fliplr(psizup);
    % fine
else
    PP=fix(P/2);
    p=(1:1:PP);
    xzp=cos(pi*(2*p-1)/(2*P));
    psizp=2*acos(xzp/x0);
    psiz(1:PP)=psizp;
    psiz(PP+1)=pi;
    psiz(PP+2:P)=-fliplr(psizp);
    % zeri dell'uniforme
    %psizup=p*(2*pi/P);
    psizup=p*(2*pi/N);
    psizu(1:PP)=psizup;
    psizu(PP+1)=pi;
    psizu(PP+2:P)=-fliplr(psizup);
end
% zeros in w

```

```

clear wz
wz=exp(j*psiz);
clear a;
clear an
a(1)=-wz(1);
a(2)=1;
for k=2:P;
    wzk=wz(k);
    an(1)=-wzk*a(1);
    an(2:k)=a(1:(k-1))-wzk*a(2:k);
    an(k+1)=a(k);
    a=an;
end
a=real(a); %clean up num. noise
% schelkunoff's circle
figure(1);
polar(psiz,ones(size(psiz)),'bo');
hold on;
polar(psizu,ones(size(psiz)),'r+');
hold off;
title('zeros; o=Chebyshev, +=uniform');

aa=a/max(a);
%figure(2);bar(real(aa),0.1) % (a should be real except for num. roundoff)
figure(2);bar(real(aa)) % (a should be real except for num. roundoff)
title('coeffs of Dolph-Chebyshev array')
xlabel('n');
ylabel('a_n');
grid on
% AF in psi plane
psi=linspace(0,pi,nsamp);
% uniform array, normalized to 1
afu=af(psi,N);
% Cheb. array
afc=af_nu(psi,a);
aaa=max(abs(afc));
afc=afc/aaa;
figure(3);plot(psi/pi,abs(afc),psi/pi,abs(afu),'--');
title('Chebyshev AF (normalized) and uniform AF');
xlabel('\psi/\pi')
legend('Chebyshev','Uniform');
grid on
cdb=db(abs(afc).^2,bottom);
udb=db(abs(afu).^2,bottom);

```

```

figure(4);plot(psi/pi, cdb, psi/pi, udb, '--');
title('Chebyshev AF (normalized) and uniform AF');
xlabel('\psi/\pi')
ylabel('dB')
legend('Chebyshev', 'Uniform');
grid

% BS case, optimal d/lambda and plot in obs. angle
dlam_u=1-1/N;
dlam_c=(acos(-1 ./x0))/pi

%dlam_c=0.8

beta=linspace(0,pi/2,200);
psiu=2*pi*dlam_u*sin(beta);
psic=2*pi*dlam_c*sin(beta);
% uniform array, normalized to 1
afu=af(psiu,N);
% Cheb. array
afc=af_nu(psic,a);
aaa=max(abs(afc));
afc=afc/aaa;

bottom=-40; %dB, bottom of scale
cdb=db(abs(afc).^2,bottom);
udb=db(abs(afu).^2,bottom);
figure(5);plot((pi/2-beta)*180/pi, cdb, (pi/2-beta)*180/pi, udb, 'c--',
'LineWidth',1.5);
hold on
%uniform
N = 9;
d_lamuda = 1-1/N;
j = sqrt(-1);
alp = 0:0.001:pi/2;
PHI = 0
phi = 2*pi*d_lamuda*cos(alp)+PHI;
F_N = (1/N)*abs(sin(N*phi/2)./sin(phi/2));
F_Ndb = 10*log(F_N);
plot(alp*180/pi, F_Ndb, 'r--', 'LineWidth',1.5);
axis([0 90 -40 0]);
grid on
title('BS- Chebyshev AF (normalized) and uniform AF');
xlabel('\alpha (deg.)')
ylabel('dB')

```

```

legend('Chebyshev','Uniform','Uniform N=9');
grid on

```

## Appendix: Additional plotted figures of Problem 1

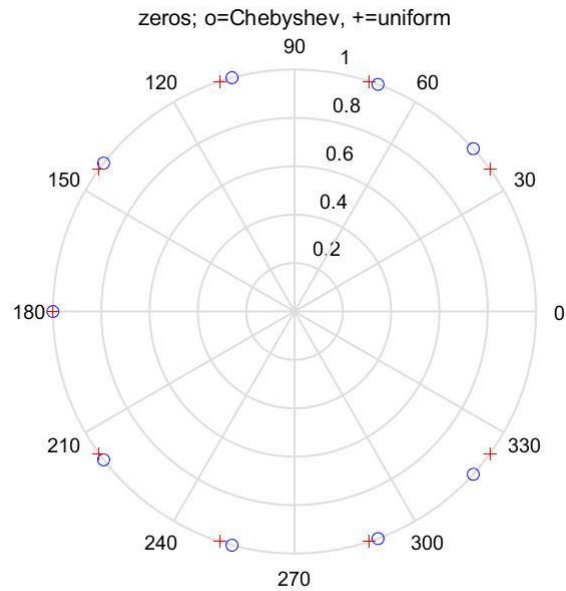


Figure 1.4: Zeros of Chebyshev and Uniform design with same  $N$  and  $d/\lambda$  plot in polar coordinates

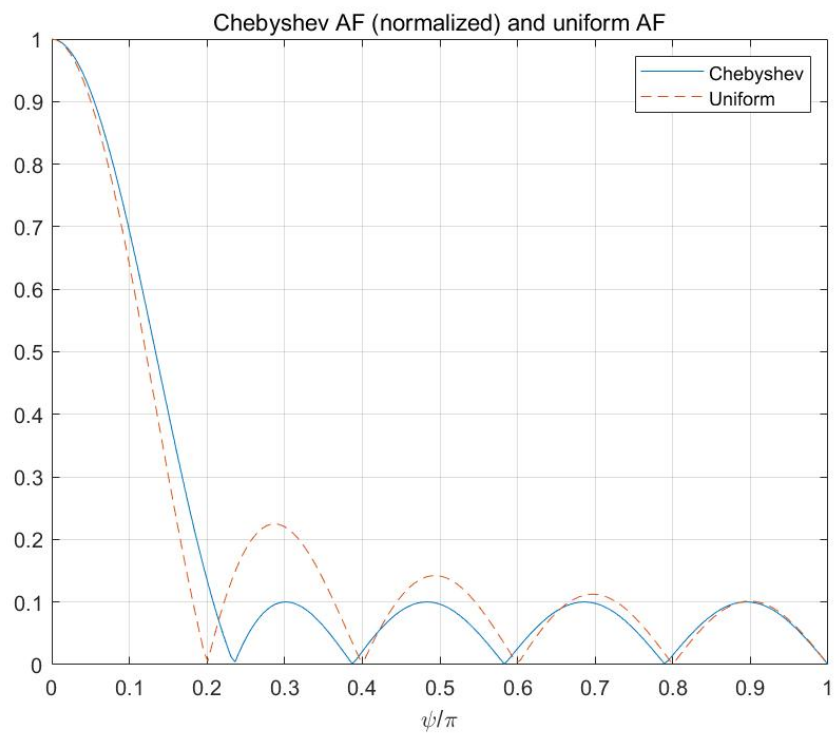


Figure 1.5: Array Factors of Chebyshev and Uniform design with same  $N$  and  $d/\lambda$

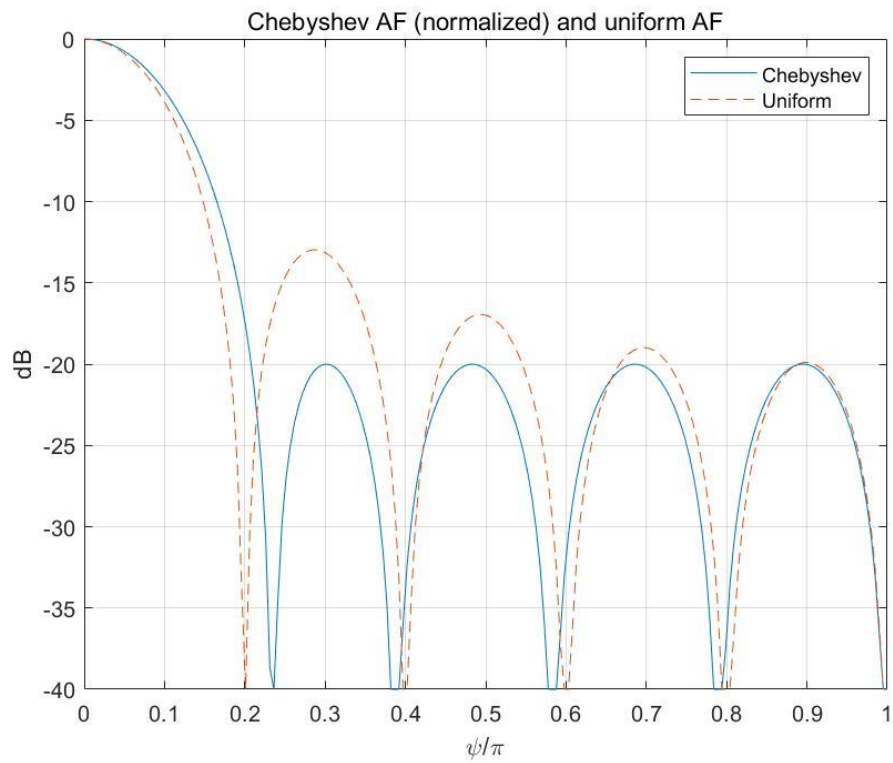


Figure 1.6: Array Factors [dB] of Chebyshev and Uniform design with same  $N$  and  $d/\lambda$

### Reference:

1. linear\_AF\_7x3.pdf, in section 'Handouts';
2. Cheb\_design\_v6x0.pdf, in section 'Handouts';
3. Matlab codes in 'chebyshev\_matlab\_v5x1'.