Active Portfolio Management and Investment Policy

Source: Bodie, Kane and Marcus, Investments, 12 ed., McGraw-Hill, 2021

Overview

- Consider practical complexities in the process of constructing optimal portfolios
- Treynor-Black model
    - Show how to handle limited precision in the forecasts of alpha values and the extreme portfolio positions often prescribed by the model
- Black-Litterman model
    - Allows flexible views about the expected returns of major asset classes to improve asset allocation
- Investment Policy

Construction and Properties of the Optimal Risky Portfolio 1

- **Table 27.1** Construction and properties of the optimal risky portfolio

1.	Initial position of security *i* in the active portfolio	$$W_i^0 = \frac{\alpha_i}{\sigma^2(e_i)}$$
2.	Scaled initial positions	$$W_i = \frac{w_i^0}{\sum_{i=1}^{n} \frac{\alpha_i}{\sigma^2(e_i)}}$$
3.	Alpha of the active portfolio	$$\alpha_A = \sum_{i=1}^{n} W_i \alpha_i$$
4.	Residual variance of the active portfolio	$$\sigma^2(e_A) = \sum_{i=1}^{n} W_i^2 \sigma^2(e_i)$$
5.	Initial position in the active portfolio	$$W_A^0 = \frac{\frac{\alpha_A}{\sigma^2(e_A)}}{\frac{E(R_M)}{\sigma_M^2}}$$

Construction and Properties of the Optimal Risky Portfolio 2

- **Table 27.1** Construction and properties of the optimal risky portfolio

6. Beta of the active portfolio	$\beta_A = \sum_{i=1}^{n} W_i \beta_i$
7. Adjusted (for beta) position in the active portfolio	$W_A^* = \dfrac{W_A^0}{1+(1-\beta_A)W_A^0}$
8. Final weights in passive portfolio and in security *i*	$W_M^* = 1 - W_A^*; \qquad W_i^* = W_A^* W_i$
9. The beta of the optimal risky portfolio and its risk premium	$\beta_P = W_M^* + W_A^* \beta_A = 1 - W_A^*(1-\beta_A)$ $E(R_P) = \beta_P E(R_M) + W_A^* \alpha_A$
10. The variance of the optimal risky portfolio	$\sigma_P^2 = \beta_P^2 \sigma_M^2 + \left[W_A^* \sigma(e_A)\right]^2$
11. Sharpe ratio of the risky portfolio	$S_P^2 = S_M^2 + \sum_{i=1}^{n} \left(\dfrac{\alpha_i}{\sigma(e_i)}\right)^2$

Active Portfolio Management

	A	B	C	D	E	F	G	H	I	J
1										
2										
3	**Panel A: Risk Parameters of the Investable Universe (annualized)**									
4										
5		SD of Excess Return	Beta	SD of Systematic Component	SD of Residual	Correlation with the S&P 500				
6	**S&P 500**	0.1358	1.00	0.1358	0	1				
7	**HP**	0.3817	2.03	0.2762	0.2656	0.72				
8	**DELL**	0.2901	1.23	0.1672	0.2392	0.58				
9	**WMT**	0.1935	0.62	0.0841	0.1757	0.43				
10	**TARGET**	0.2611	1.27	0.1720	0.1981	0.66				
11	**BP**	0.1822	0.47	0.0634	0.1722	0.35				
12	**SHELL**	0.1988	0.67	0.0914	0.1780	0.46				
13										
14	**Panel B: The Index Model Covariance Matrix**									
15										
16			SP 500	*HP*	*DELL*	*WMT*	*TARGET*	*BP*	*SHELL*	
17		Beta	1.00	2.03	1.23	0.62	1.27	0.47	0.67	
18	**S&P 500**	1.00	0.0184	0.0375	0.0227	0.0114	0.0234	0.0086	0.0124	
19	**HP**	2.03	0.0375	0.1457	0.0462	0.0232	0.0475	0.0175	0.0253	
20	**DELL**	1.23	0.0227	0.0462	0.0842	0.0141	0.0288	0.0106	0.0153	
21	**WMT**	0.62	0.0114	0.0232	0.0141	0.0374	0.0145	0.0053	0.0077	
22	**TARGET**	1.27	0.0234	0.0475	0.0288	0.0145	0.0682	0.0109	0.0157	
23	**BP**	0.47	0.0086	0.0175	0.0106	0.0053	0.0109	0.0332	0.0058	
24	**SHELL**	0.67	0.0124	0.0253	0.0153	0.0077	0.0157	0.0058	0.0395	
25										
26	**Panel C: Macro Forecast (S&P 500) and Forecasts of Alpha Values**									
27										
28										
29		SP 500	*HP*	*DELL*	*WMT*	*TARGET*	*BP*	*SHELL*		
30	**Alpha**	0	0.0150	−0.0100	−0.0050	0.0075	0.012	0.0025		
31	**Risk premium**	0.0600	0.1371	0.0639	0.0322	0.0835	0.0400	0.0429		
32										
33	**Panel D: Computation of the Optimal Risky Portfolio**									
34										
35		**S&P 500**	***Active Pf A***		**HP**	**DELL**	**WMT**	**TARGET**	**BP**	**SHELL**
36				$\sigma^2(e)$	0.0705	0.0572	0.0309	0.0392	0.0297	0.0317
37			0.5505	$\alpha/\sigma^2(e)$	0.2126	−0.1748	−0.1619	0.1911	0.4045	0.0789
38			1.0000	$w_0(i)$	0.3863	−0.3176	−0.2941	0.3472	0.7349	0.1433
39				$[w_0(i)]^2$	0.1492	0.1009	0.0865	0.1205	0.5400	0.0205
40	α_A		0.0222							
41	$\sigma^2(e_A)$		0.0404							
42	w_0		0.1691	**Overall**						
43	w^*	0.8282	0.1718	**Portfolio**	0.0663	−0.0546	−0.0505	0.0596	0.1262	0.0246
44	Beta	1	1.0922	1.0158	0.0663	−0.0546	−0.0505	0.0596	0.1262	0.0246
45	Risk premium	0.06	0.0878	0.0648	0.0750	0.1121	0.0689	0.0447	0.0880	0.0305
46	SD	0.1358	0.2497	0.1422	0.3817	0.2901	0.1935	0.2611	0.1822	0.1988
47	Sharpe ratio	0.44	0.35	0.4556						
48	M-square	0	−0.0123	0.0019						
49	Benchmark risk			0.0346						

- An **active portfolio** of six stocks is added to the **passive market-index portfolio**
- Panel D shows:
  - Meager improvement in performance
  - M-square increases by only 19 basis points

The Optimal Risky Portfolio 1

- **Table 27.2** The optimal risky portfolio with the analysts' new forecasts of alpha

	S&P 500	***Active Portfolio A***		HP	DELL	WMT	TGT	BP	SHELL
			α	0.1471	0.1753	0.1932	0.2814	0.1797	0.0357
			$\sigma^2(e)$	0.0705	0.0572	0.0309	0.0392	0.0297	0.0317
		25.7562	$\alpha/\sigma^2(e)$	2.0855	3.0641	6.2544	7.1701	6.0566	1.1255
		1.0000	$w_0(i)$	0.0810	0.1190	0.2428	0.2784	0.2352	0.0437
			$[w_0(i)]^2$	0.0066	0.0142	0.0590	0.0775	0.0553	0.0019
α_A		0.2018							
$\sigma^2(e_A)$		0.0078							
w_0		7.9116							
w^*	−4.7937	5.7937		0.4691163	0.6892459	1.4069035	1.6128803	1.3624061	0.2531855

The Optimal Risky Portfolio 2

- **Table 27.2** The optimal risky portfolio with the analysts' new forecasts of alpha

			Overall Portfolio						
Beta	1	0.9538	0.7323	0.4691	0.6892	1.4069	1.6129	1.3624	0.2532
Risk premium	0.06	0.2590	1.2132	0.2692	0.2492	0.2304	0.3574	0.2077	0.0761
SD	0.1358	0.1568	0.5224	0.3817	0.2901	0.1935	0.2611	0.1822	0.1988
Sharpe ratio	0.44	1.65	2.3223						
M-square	0	0.1642	0.2553						
Benchmark risk			0.5146						

- The Sharpe ratio increases from .44 to 2.32, a huge risk-adjusted return advantage
- M-square of 25.53%

Results: Problems

- Potential major problem with Treynor-Black model
  - The optimal portfolio now calls for extreme long/short positions that may not be feasible for a real-world portfolio manager
  - Most of this risk is nonsystematic because the beta of the active portfolio, at .95, is less than 1.0, and the beta of the overall risky portfolio is even lower, at only .73, because of the short position in the passive portfolio
- A solution: Restrict extreme portfolio positions
  - This results in a lack of diversification

The Optimal Risky Portfolio, Constrained ($w_A \leq 1$) 1

- **Table 27.3** The optimal risky portfolio with constraint on the active portfolio ($w_A \leq 1$)

	S&P 500	*Active Portfolio A*		HP	DELL	WMT	TGT	BP	SHELL
			α	0.1471	0.1753	0.1932	0.2814	0.1797	0.0357
			$\sigma^2(e)$	0.0705	0.0572	0.0309	0.0392	0.0297	0.0317
		25.7562	$\alpha/\sigma^2(e)$	2.0855	3.0641	6.2544	7.1701	6.0566	1.1255
		1.0000	$w_0(i)$	0.0810	0.1190	0.2428	0.2784	0.2352	0.0437
			$[w_0(i)]^2$	0.0066	0.0142	0.0590	0.0775	0.0553	0.0019
α_A		0.2018							
$\sigma^2(e_A)$		0.0078							
w_0		7.9116							
w^*	0.0000	1.0000		0.0810	0.1190	0.2428	0.2784	0.2352	0.0437

The Optimal Risky Portfolio, Constrained (w_A ≤1) 2

- **Table 27.3** The optimal risky portfolio with constraint on the active portfolio ($w_A \leq 1$)

			Overall Portfolio						
Beta	1	0.9538	0.9538	0.0810	0.1190	0.2428	0.2784	0.2352	0.0437
Risk premium	0.06	0.2590	0.2590	0.2692	0.2492	0.2304	0.3574	0.2077	0.0761
SD	0.1358	0.1568	0.5224	0.3817	0.2901	0.1935	0.2611	0.1822	0.1988
Sharpe ratio	0.44	1.65	1.6515						
M-square	0	0.1642	0.1642						
Benchmark risk			0.0887						

- Volatility of the overall portfolio (0.53) is high
- The Sharpe ratio falls from 2.32 to 1.65
- M-square is now .1642

Tracking Error 1

- Most investment managers are judged on performance relative to a **benchmark portfolio**
- Such commitment raises the importance of **tracking error**

$$\text{Tracking Error} = T_E = R_P - R_M$$

where

$$R_P = w_A^* \alpha_A + \left[1 - w_A^* \times \left(1 - \beta_A\right)\right] \times R_M + w_A^* e_A$$

$$T_E = w_A^* \alpha_A - w_A^* \times \left(1 - \beta_A\right) \times R_M + w_A^* e_A$$

$$Var\left(T_E\right) = \left[w_A^*\left(1 - \beta_A\right)\right]^2 Var\left(R_M\right) + Var\left(w_A^* e_A\right) = \left[w_A^*\left(1 - \beta_A\right)\right]^2 \sigma_M^2 + \left[w_A^* \sigma\left(e_A\right)\right]^2$$

$$\text{Benchmark risk} = \sigma\left(T_E\right) = w_A^* \sqrt{\left(1 - \beta_A\right)^2 \sigma_M^2 + \left[\sigma\left(e_A\right)^2\right]}$$

Tracking Error 2

$$\text{Benchmark risk} = \sigma\left(T_E\right) = w_A^* \sqrt{\left(1-\beta_A\right)^2 \sigma_M^2 + \left[\sigma\left(e_A\right)^2\right]}$$

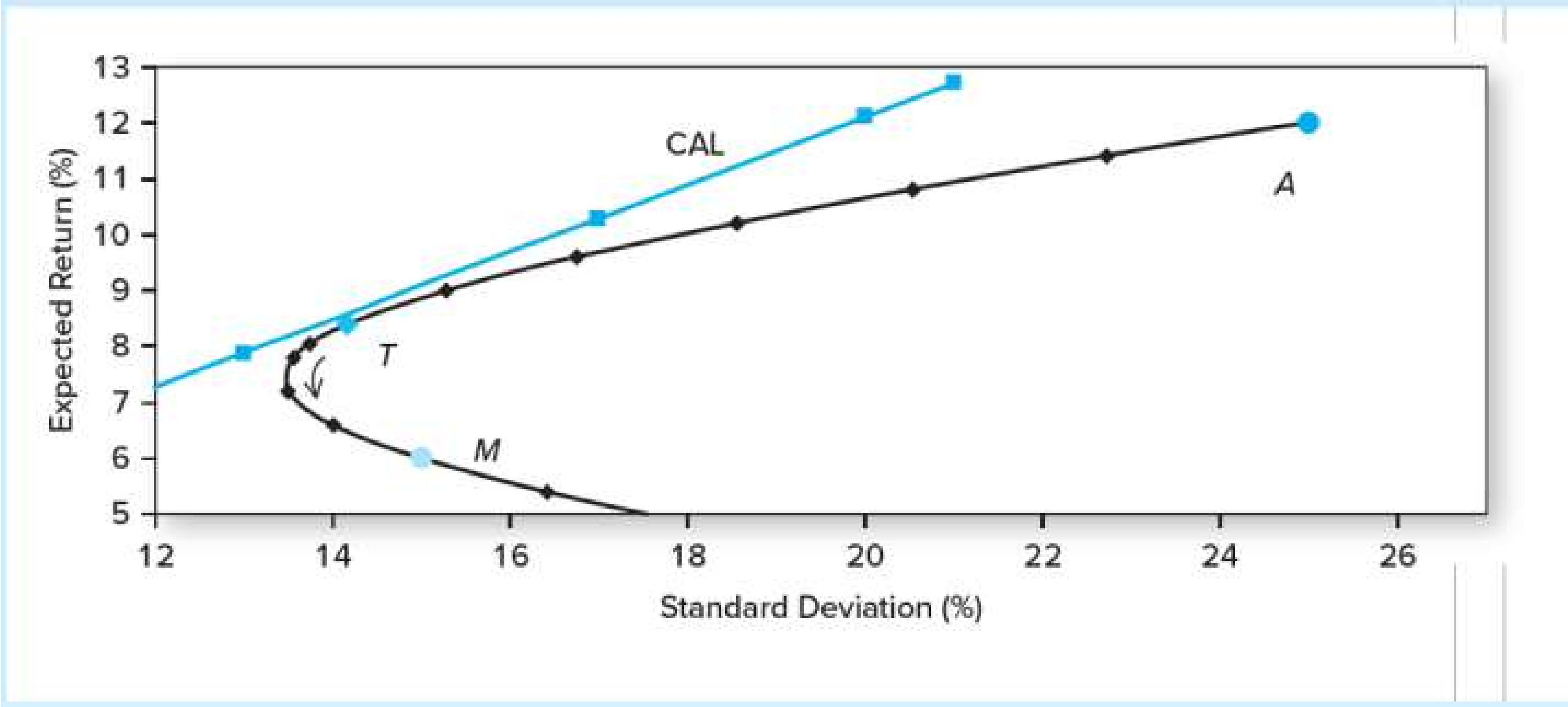
For a desired benhmark risk of $\sigma_0\left(T_E\right)$ and

$$\sigma_0\left(T_E; w_A^* = 1\right) = \sqrt{\left(1-\beta_A\right)^2 \sigma_M^2 + \left[\sigma\left(e_A\right)^2\right]}$$

we can restrict the weight of the active portfolio to:

$$w_A\left(T_E\right) = \frac{\sigma_0\left(T_E\right)}{\sigma\left(T_E; w_A^* = 1\right)}$$

Reduced Efficiency when Benchmark Risk is Lowered



- **Figure 27.1** Reduced efficiency when benchmark risk is lowered. Reducing tracking error risk results in a shift from the tangency portfolio, *T,* toward the benchmark portfolio, *M.*
- Benchmark risk is the standard deviation of the tracking error, $T_E = R_P - R_{M.}$ Control it by restricting W_A

The Optimal Risky Portfolio with the Analysts' New Forecasts
1

- **Table 27.4** The optimal risky portfolio with the analysts' new forecasts (benchmark risk constrained to 3.85%)

	S&P 500	*Active Portfolio A*		HP	DELL	WMT	TGT	BP	SHELL
			$\sigma^2(e)$	0.0705	0.0572	0.0309	0.0392	0.0297	0.0317
		25.7562	$\alpha/\sigma^2(e)$	2.0855	3.0641	6.2544	7.1701	6.0566	1.1255
		1.0000	$w_0(i)$	0.0810	0.1190	0.2428	0.2784	0.2352	0.0437
			$[w_0(i)]^2$	0.0066	0.0142	0.0590	0.0775	0.0553	0.0019
α_A		0.2018							
$\sigma^2(e_A)$		0.0078							
w_0		7.9116							
w^*	0.5661	0.4339		0.0351	0.0516	0.1054	0.1208	0.1020	0.0190

The Optimal Risky Portfolio with the Analysts' New Forecasts

- **Table 27.4** The optimal risky portfolio with the analysts' new forecasts (benchmark risk constrained to 3.85%)

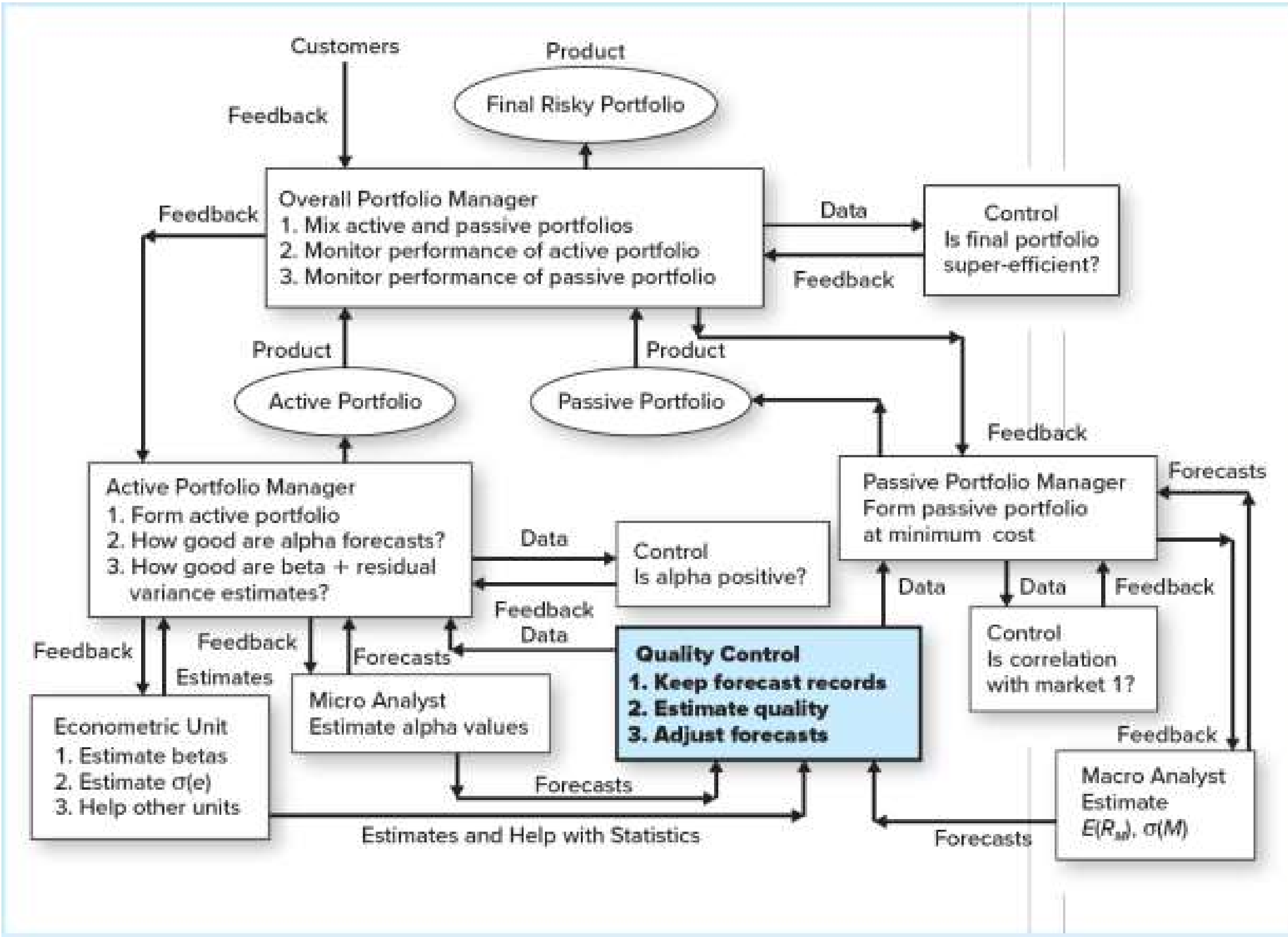
			Overall Portfolio						
Beta	1	0.9538	0.9800	0.0351	0.0516	0.1054	0.1208	0.1020	0.0190
Risk premium	0.06	0.2590	0.1464	0.0750	0.1121	0.0689	0.0447	0.0880	0.0305
Standard deviation	0.1358	0.1568	0.1385	0.3817	0.2901	0.1935	0.2611	0.1822	0.1988
Sharpe ratio	0.44	1.65	1.0569						
M-square	0	0.1642	0.0835						
Benchmark risk			0.0385						

  - The Sharpe ratio falls from 1.65 to 1.06
  - M-square is now 8.35%

Adjusting Forecasts for the Precision of Alpha

- Key questions
  - How accurate is your forecast?
  - How should you adjust your position to take account of forecast imprecision?
- Treynor and Black
  - Examine **forecasting record** of previous forecasts issued by same forecaster to quantify the uncertainty about the forecast
  - Adjust forecasts to account for imprecision by using **adjusted alpha**

Organizational Chart for Portfolio Management



- **Figure 27.3** Organizational chart for portfolio management

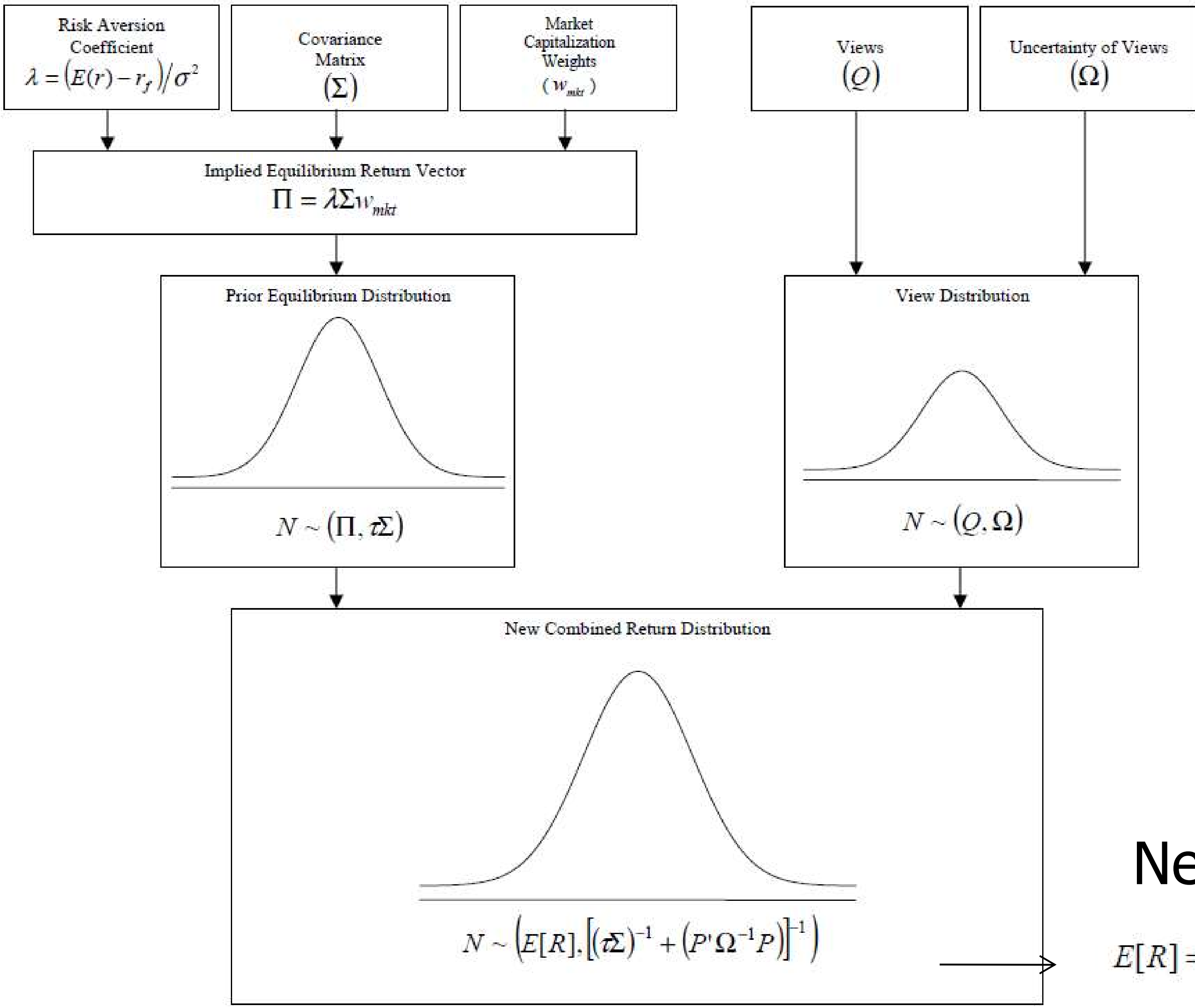
- Source: Adapted from Robert C. Merton, *Finance Theory,* Chapter 12, Harvard Business School.

Black Litterman approach

Black and Litterman (1990) proposes a mean-variance portfolio optimization model that includes investor's expectations:

- Creates different views that represent investor' s market expectations (investor's prior).
- Applies Bayes' rules to restrict sensitivity of optimal allocation function to model's inputs
- Market distribution (capitalization weighted market portfolio) is approximated to investor's prior
- Data hails from two sources: history and views
- Historical sample used to estimate covariance matrix and asset allocation to make **baseline forecasts**
- Views represent departure from baseline, establishing new alpha forecasts and optimal risky P

Black Litterman: combined return distribution



New combined return:

$$E[R] = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right]$$

Source: Idzorek (2004)

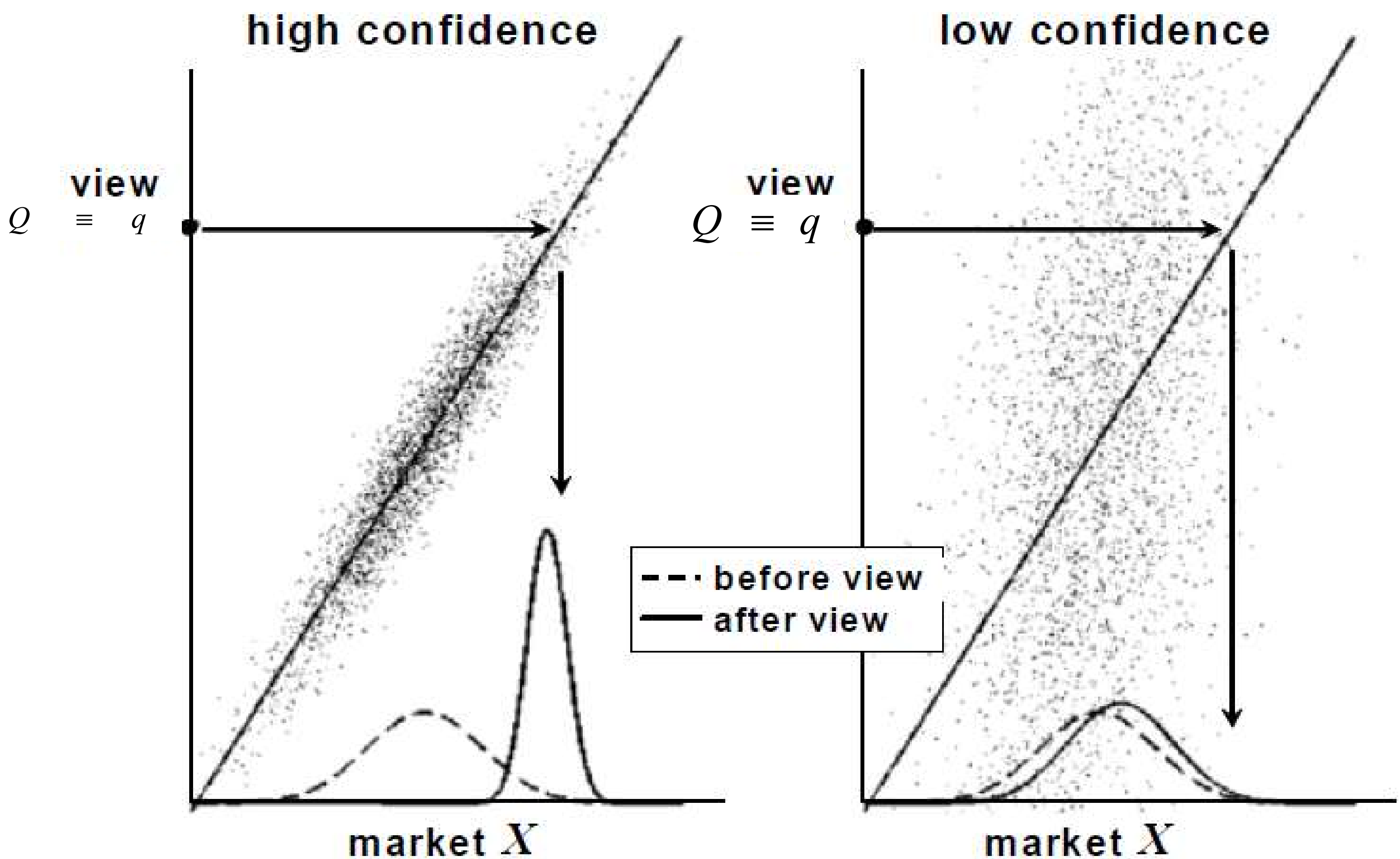
Black Litterman approach

Starts with market equilibrium return & investors' views and obtains excess return in relation to the risk free rate:

$$E[R] = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right]$$

P: investors' perspective vector
Omega: variance matrix of forecasts
II: implied equilibrium return vector
Tau: confidence parameter
Q: investors' views expected return vector
Sigma: covariance matrix of returns

Black Litterman: market estimation



Scenario is different from market   Scenario similar to market

Source: Meucci (2007)

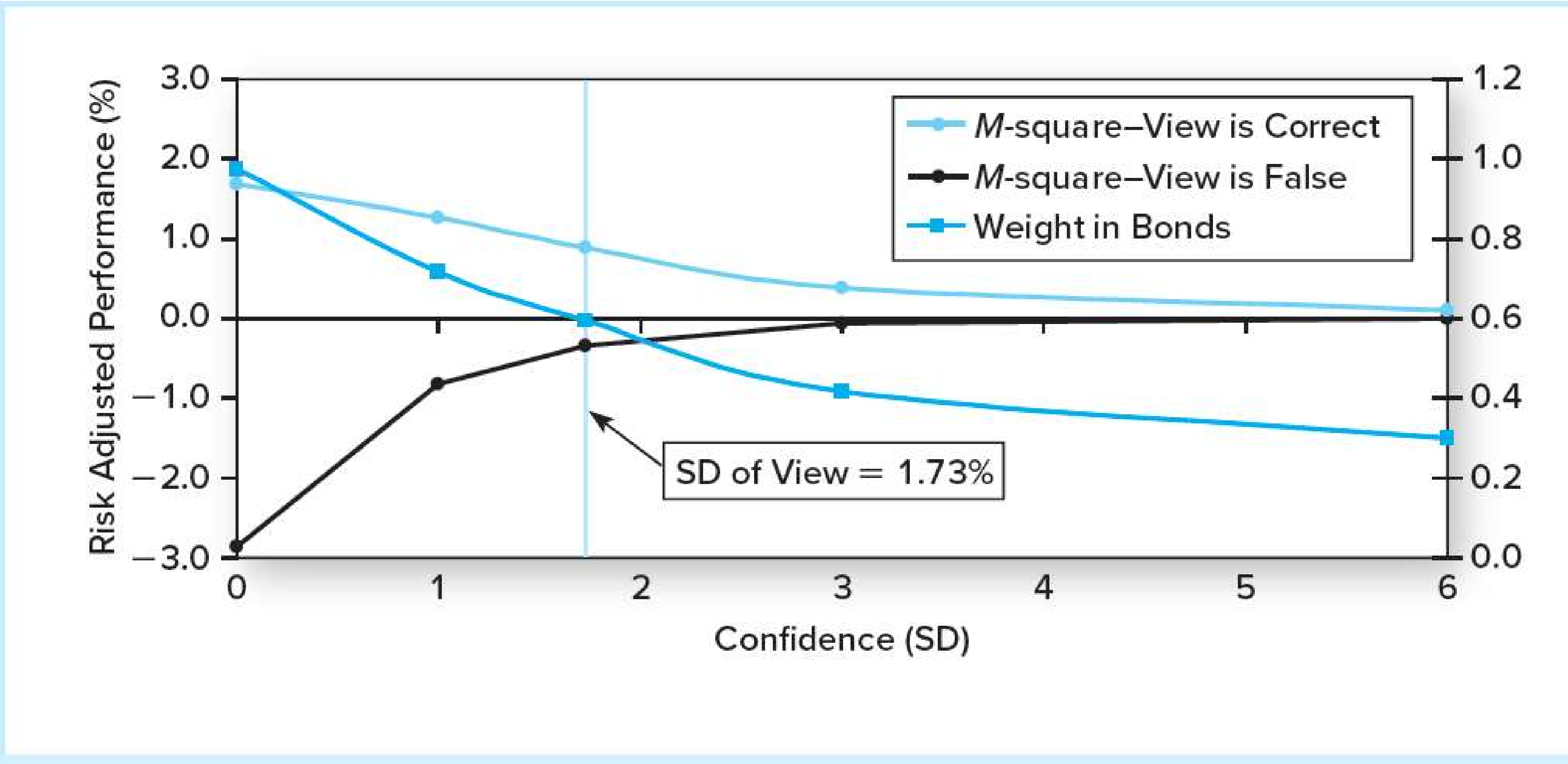
Steps in the BL Model

1. Estimate the covariance matrix from recent historical data
2. Determine a baseline forecast
3. Integrate the manager’s private views
4. Develop revised (posterior) expectations
5. Apply portfolio optimization

Sensitivity of Black-Litterman Portfolio to Confidence in Views 1

	A	B	C	D	E	F	G	H	I
1									
2									
3									
4	**Panel A: Bordered Covariance Matrix from Historical Excess Returns**								
5	**and Market-Value Weights and Calculation of Baseline Forecasts**								
6									
7			Bonds	Stocks					
8		Weights	**0.25**	**0.75**					
9	Bonds	**0.25**	64	40.8					
10	Stocks	**0.75**	40.8	289					
11		sumproduct	11.65	170.21					
12	Market portfolio variance V(M) = sum(c11,d11) =					181.86			
13	Coefficient of risk aversion of representative investor =					3			
14	Baseline market portfolio risk premium = 0.01A*V(M) =					5.46			
15	Covariance with R_M		46.6	226.95					
16	Baseline risk premiums		1.40	6.81		0.256237542			
17						1.247920819			
18	Proportion of covariance attributed to expected returns					0.01			
19	Covariance matrix of expected returns								
20			Bonds	Stocks					
21		Bonds	0.64	0.408					
22		Stocks	0.408	2.89					
23									
24	**Panel B: Views, Confidence, and Revised (Posterior) Expectations**								
25									
26	View: Difference between returns on bonds and stocks, Q =					0.5			
27	View embedded in baseline forecasts Q^E =					−5.41			
28	Variance of $Q^E = \mathrm{Var}(R_B - R_S)$					2.71			
29	$\mathrm{Var}[E(R_B)] - \mathrm{Cov}[E(R_B),E(R_S)] =$					0.23			
30	Cov[E(RB),E(RS)] − Var[E(RB)] =					−2.48			
31	Difference between view and baseline data, D =					5.91			
32	Confidence measured by standard deviation of view Q								
33	Possible SD	0	1	1.73	3.00	6.00			
34	Variance	0	1.5	3	9	36	Baseline		
35	$E(R_B\|P)$	1.90	1.72	1.64	1.52	1.43	1.40		
36	$E(R_S\|P)$	1.40	3.33	4.24	5.56	6.43	6.81		

Sensitivity of Black-Litterman Portfolio to Confidence in Views 2



- **Figure 27.4** Sensitivity of Black-Litterman portfolio performance to confidence level

T-B versus BL: Conclusions

- The BL and the Treynor-Black (TB) model are complements, not substitutes
- Once you reach the optimization stage, the models are identical
- BL can be viewed as generalization of TB
  - BL model allows you to adjust expected return from views about alpha values as in the TB model, but it also allows you to express views about *relative* performance that cannot be incorporated in the TB model

BL versus TB 1

Black-Litterman Model

- Optimal portfolio weights and performance are highly sensitive to the degree of confidence in the views
- The validity of the BL model rests largely upon the way in which the confidence about views is developed

Treynor-Black Model

- TB model is not applied in the field because it results in “wild” portfolio weights
- The extreme weights are a consequence of failing to adjust alpha values to reflect forecast precision

BL versus TB 2

Black-Litterman Model	Treynor-Black Model
• Use the BL model for asset allocation • Views about relative performance are useful even when the degree of confidence is inaccurately estimated	• Use the TB model for the management of security analysis with proper adjustment of alpha forecasts

Concluding Remarks

- The gap between theory and practice has been narrowing in recent years
- The CFA Institute has worked to transfer investment theory to the asset management industry
- The TB and BL models are not yet widely used in industry