# Mean Variance Portfolio Optimization and the Black Litterman Model

Source: Germán Creamer (2015). “Can a Corporate Network and News Sentiment Improve Portfolio Optimization Using the Black Litterman Model?” Quantitative Finance 15 (8): 1405-1416.

# Portfolio Optimization: The General Linear Model

$$R_{p,t+1} = \sum_{i=1}^{n} w_i R_{i,t+1}$$

$$\sigma_P^2 = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_i \sigma_j \rho_{ij}$$

$$\sigma_P^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2\sum_{i=1}^{N} \sum_{j<1}^{N} w_i w_j \sigma_i \sigma_j \rho_{ij}$$

$$\sigma_P^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2\sum_{i=1}^{N} \sum_{j<i}^{N} w_i w_j \sigma_{ij}$$

where $\sigma_i$ is the volatility of variable $i$

$\sigma_P$ is the portfolio's standard deviation

$\sigma_{ij} = \sigma_i \cdot \sigma_j \cdot \rho_{ij}$ Covariance between i and j

## In matrix form

$$\sigma_p^2 = \left[w_1 \ldots w_N\right] \begin{bmatrix} \sigma_1^2 \ldots\ldots \sigma_{1N} \\ \vdots \\ \sigma_{N1} \ldots\ldots \sigma_N^2 \end{bmatrix} \begin{bmatrix} w_1 \\ \vdots \\ w_N \end{bmatrix} = w' \sum w$$

$$where \sum is\ the\ \mathrm{cov}\, ariance\ matrix$$

# Mean Variance Optimization

Mean variance optimization proposed by Markowitz (1952, 1959):

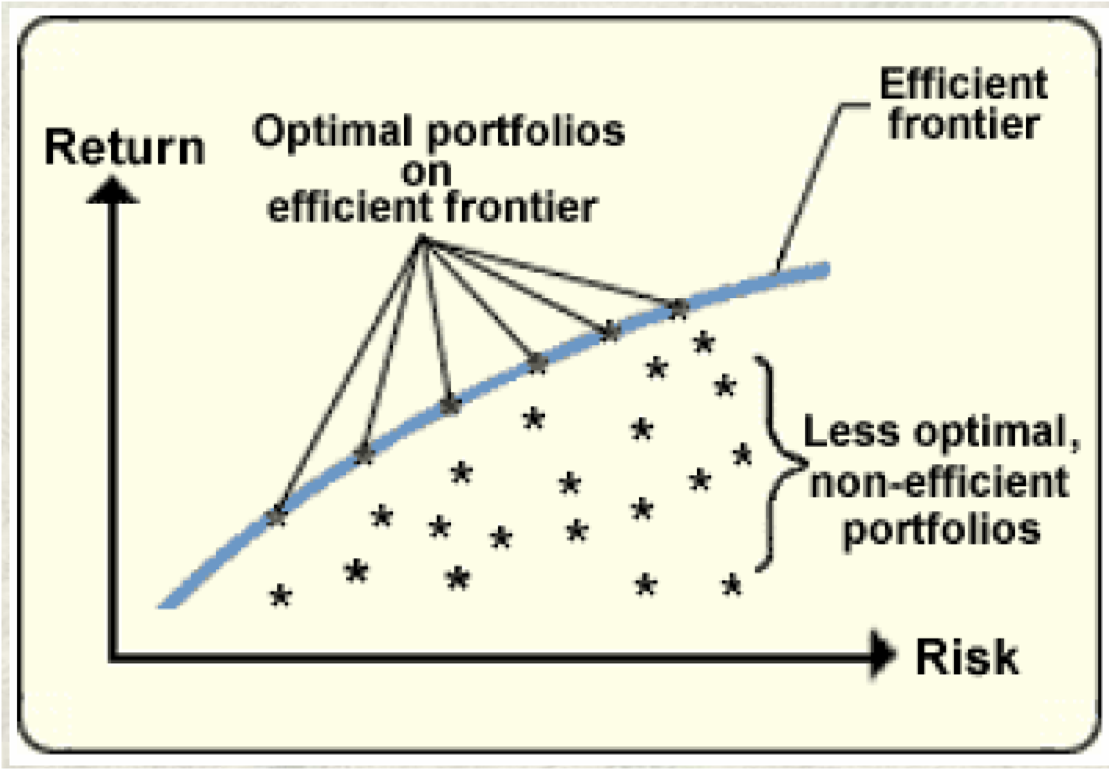
•Inputs: Mean of returns $(R_p)$
Variance covariance matrix of returns $(\Sigma)$

Result: Optimal portfolio without restrictions:
Find portfolio (w) that maximize return and minimize risk $(\Sigma)$ using a Lagrange multiplier ( $\lambda$ )

$$w_\lambda \equiv \arg\max_{w}\{w' R_p - \lambda w' \Sigma w\}$$

# Mean Variance Optimization

- Create efficient frontier
  - Select optimal portfolios from the efficient frontier



## Limitations of Mean Variance Optimization

- Porfolios are not intuitive and are very sensitive to small changes in inputs

- Biased towards the best assets: optimization allocates all its resource to the winner asset(s).

- Investors can not formally input their knowledge of the market.

## How to optimize portfolio including investor’s views?

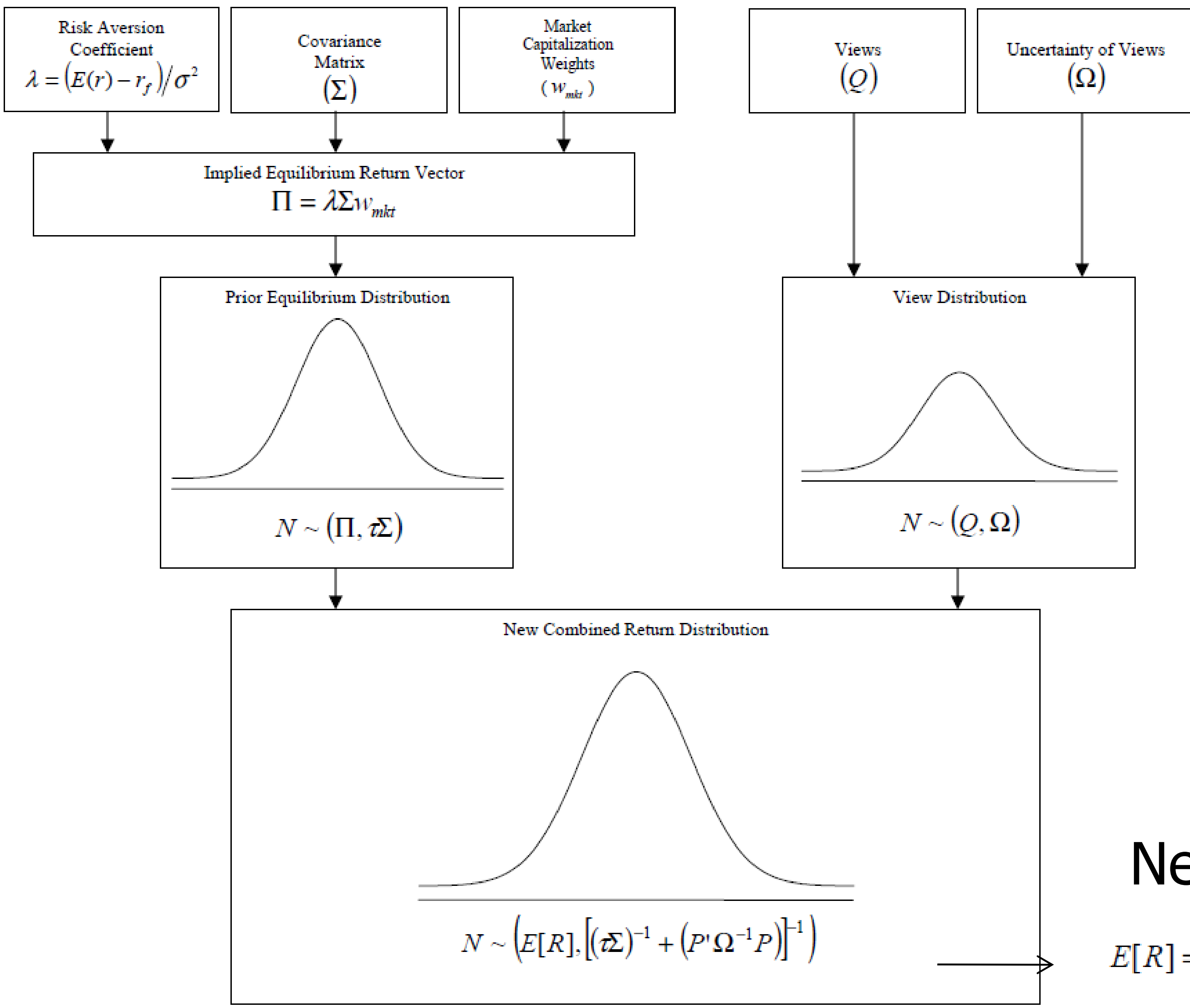
- Portfolio optimization may lead to selection of few top assets and it is based on past price history (backward-looking).
- Investors have market knowledge and views about market direction (forward-looking)
- Black Litterman portfolio optimization approach combines both dimensions.
- Problem: How can we incorporate qualitative factors that complement or substitute investors’ view?

Optimize portfolio including additional meaningful indicators such as
1) potential flow of information between directors and analysts and
2) news sentiment as “investor’s view.”

# Black Litterman approach

- Black and Litterman (1990) proposes a mean-variance portfolio optimization model that includes investor’s expectations.
- Creates different views that represent investor’ s market expectations (investor’s prior).
- Applies Bayes’ rules to restrict sensitivity of optimal allocation function to model’s inputs
- Market distribution (capitalization weighted market portfolio) is approximated to investor’s prior

# Black Litterman: combined return distribution




## New combined return:

$$E[R] = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right]$$

Source: Idzorek (2004)

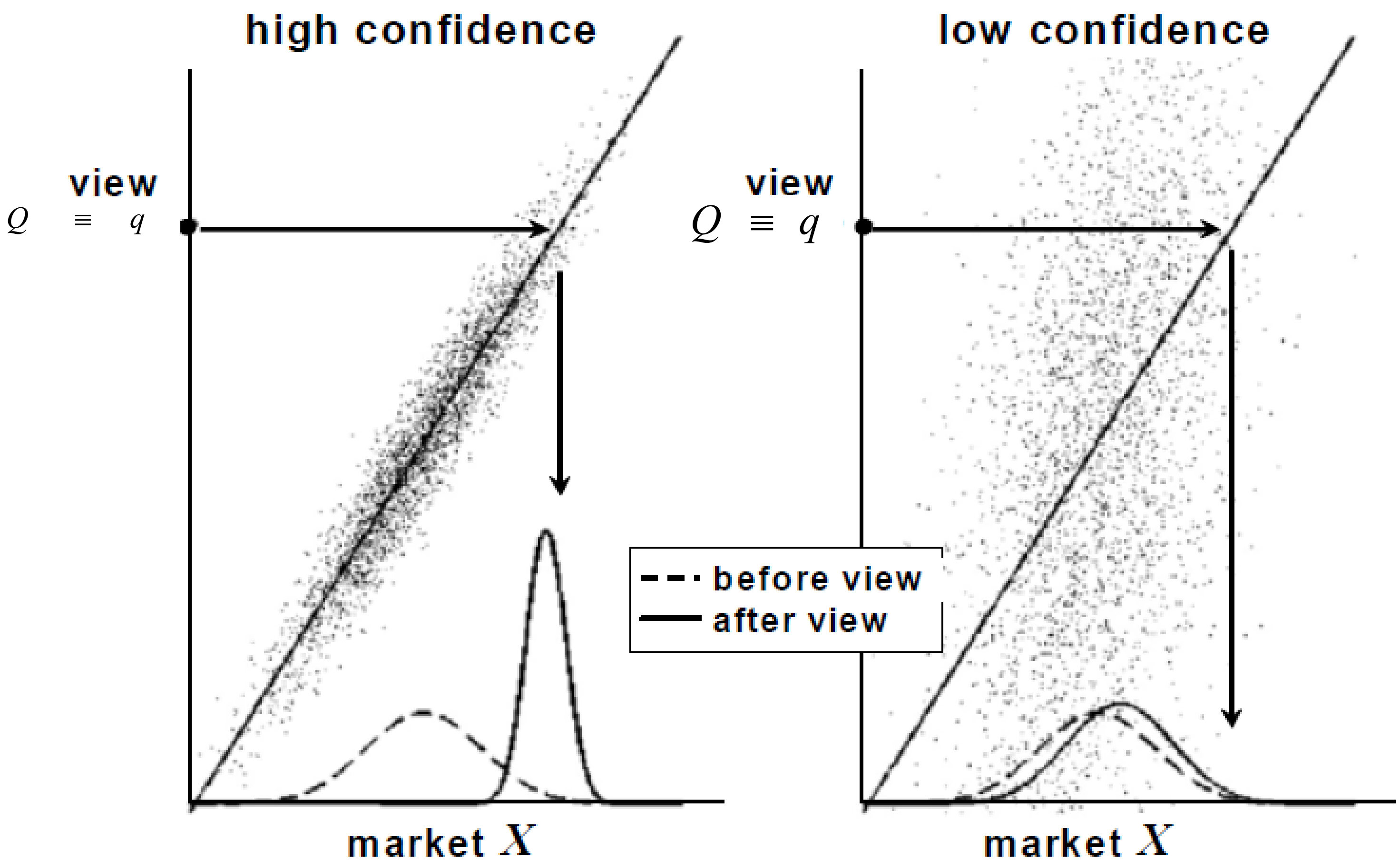
# Black Litterman approach

Starts with market equilibrium return & investors' views and obtains excess return in relation to the risk free rate:

$$E[R] = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right]$$

P: investors' perspective vector
Omega: variance matrix of forecasts
II: implied equilibrium return vector
Tau: confidence parameter
Q: investors' views expected return vector
Sigma: covariance matrix of returns

# Black Litterman: market estimation




Scenario is different from market

Scenario similar to market

Source: Meucci (2007)

# NLP methods to classify news

- Bag-of-words (BoW): tf and tf-idf of 1-, 2- and 3-grams for all words.
  - tf-idf: term frequency * inverse document frequency
- Dictionary of Affect in Language by Part of Speech (PDAL): pleasantness (PIs), activation (Act) and imagery (Img) scores for all words and part of speech
- Latent Semantic Analysis (LSA): computes similarities among documents
- Latent Dirilecht Analysis (LDA): discovers common topics among documents
- Part of speech (POS)
- Semantic frames (SF)

# Methodology for BL-News

- Restrict sample of test news to those directly associated with a STOXX 50 company that have at least two news present in the test sample: 125 news of 27 companies associated to the STOXX 50 index : 9/8/2005-12/30/2005
- Using Adaboost as the best classifier for top 30% returns and NLP indicators as features, predict news sentiments for the 3 hours horizon
- News sentiment as the investors�similar view  per stock of the BL model to optimize the European "long only" portfolio.
- Calculate the daily log return using the close prices of the day when the news arrives and the close prices of the next day. Rebalance portfolio with arrival of at least 1 news.
- Compare BL-news portfolio with the equally weighted portfolio, with the STOXX 50 index and with the market portfolio.

# News sentiment of STOXX50 market

| | 1 min. | 5 min. | 15 min. | 1 hr. | 2 hr. | 3 hr. | Mean |
|---|---|---|---|---|---|---|---|
| BoW-POS-SF-LDA | 0.43 | 0.35 | 0.44 | 0.48 | 0.46 | 0.50 | 0.44 |
| BoW-POS-SF | 0.38 | 0.34 | 0.45 | 0.47 | 0.47 | 0.50 | 0.43 |
| LDA | 0.33 | 0.39 | 0.42 | 0.41 | 0.47 | 0.46 | 0.41 |
| BoW-POS | 0.37 | 0.36 | 0.39 | 0.46 | 0.48 | 0.41 | 0.41 |
| BoW-SF | 0.30 | 0.39 | 0.43 | 0.39 | 0.46 | 0.51 | 0.41 |
| POS-SF | 0.35 | 0.31 | 0.37 | 0.41 | 0.52 | 0.49 | 0.41 |
| SF | 0.30 | 0.37 | 0.42 | 0.41 | 0.43 | 0.50 | 0.41 |
| POS | 0.33 | 0.34 | 0.36 | 0.46 | 0.45 | 0.48 | 0.41 |
| BoW | 0.35 | 0.37 | 0.39 | 0.42 | 0.44 | 0.45 | 0.40 |
| Mean | 0.35 | 0.36 | 0.41 | 0.43 | 0.46 | 0.48 | 0.42 |

Table 2.: F-score of asset return forecast using Adaboost by text analysis methods and time horizon in minutes (min.) and hours (hr.)

# Annual Sharpe ratio by portfolio

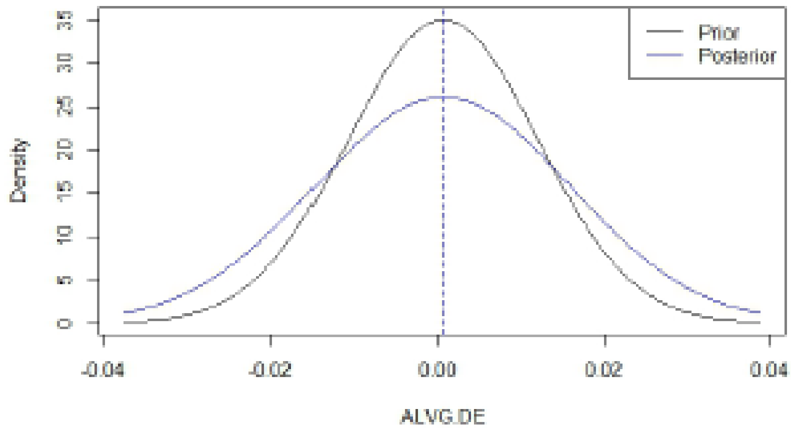
| Portolio/views | $\Omega$=0.0001 | $\Omega$=0.001 | $\Omega$=0.01 | $\Omega$=0.1 |
|---|---|---|---|---|
| BL-News, CI, $\tau$=1 | 8.892 ** | 8.818 ** | 8.806 ** | 8.805 ** |
| BL-News, CI, $\tau$=0.75 | 8.870 ** | 8.813 ** | 8.805 ** | 8.805 ** |
| BL-News, CI, $\tau$=0.5 | 8.842 ** | 8.809 ** | 8.805 ** | 8.804 ** |
| BL-News, CI, $\tau$=0.25 | 8.817 ** | 8.806 ** | 8.805 ** | 8.804 ** |
| BL-News, CI, $\tau$=0.1 | 8.807 ** | 8.805 ** | 8.804 ** | 8.804 ** |
| BL-News, CI, $\tau$=0.01 | 8.804 ** | 8.804 ** | 8.804 ** | 8.804 ** |
| Equally weighted | 8.194 ** | | | |
| Market capitalization | 4.461 | | | |
| STOXX 50 | 5.272 | | | |

Sharpe ratio declines when confidence on investors' views decreases or covariance of error term of the views increases

$\Omega$ : Covariance of the error term of the investors‘ views
$\tau$ :: Confidence indicator of a particular view
**:1% significance level of t-test mean difference between each scenario and market portfolio

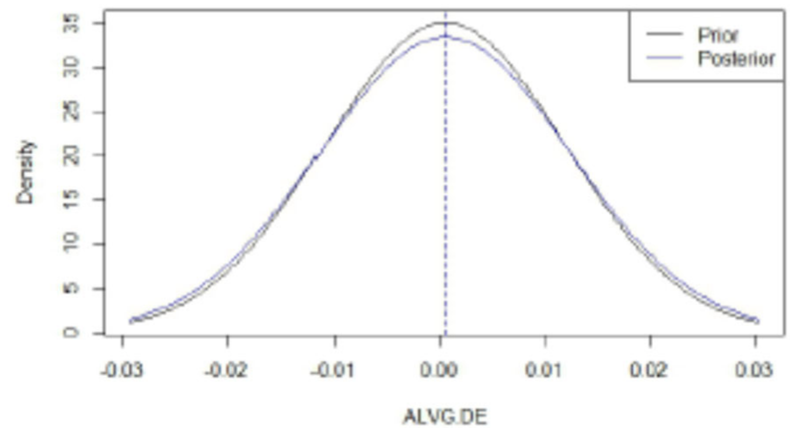
Scenario with large confidence (tao=1) on news sentiment and with low covariance of error term of the investors' views (omega=0.0001) leads to the largest average Sharpe ratio. The different versions of the BL-News portfolio outperform the market portfolio.

# Distribution of a sample asset's return according to the BL model




(a) Low covariance of error term ($\Omega$=0.001) and high confidence ($\tau$=1) of investors' view

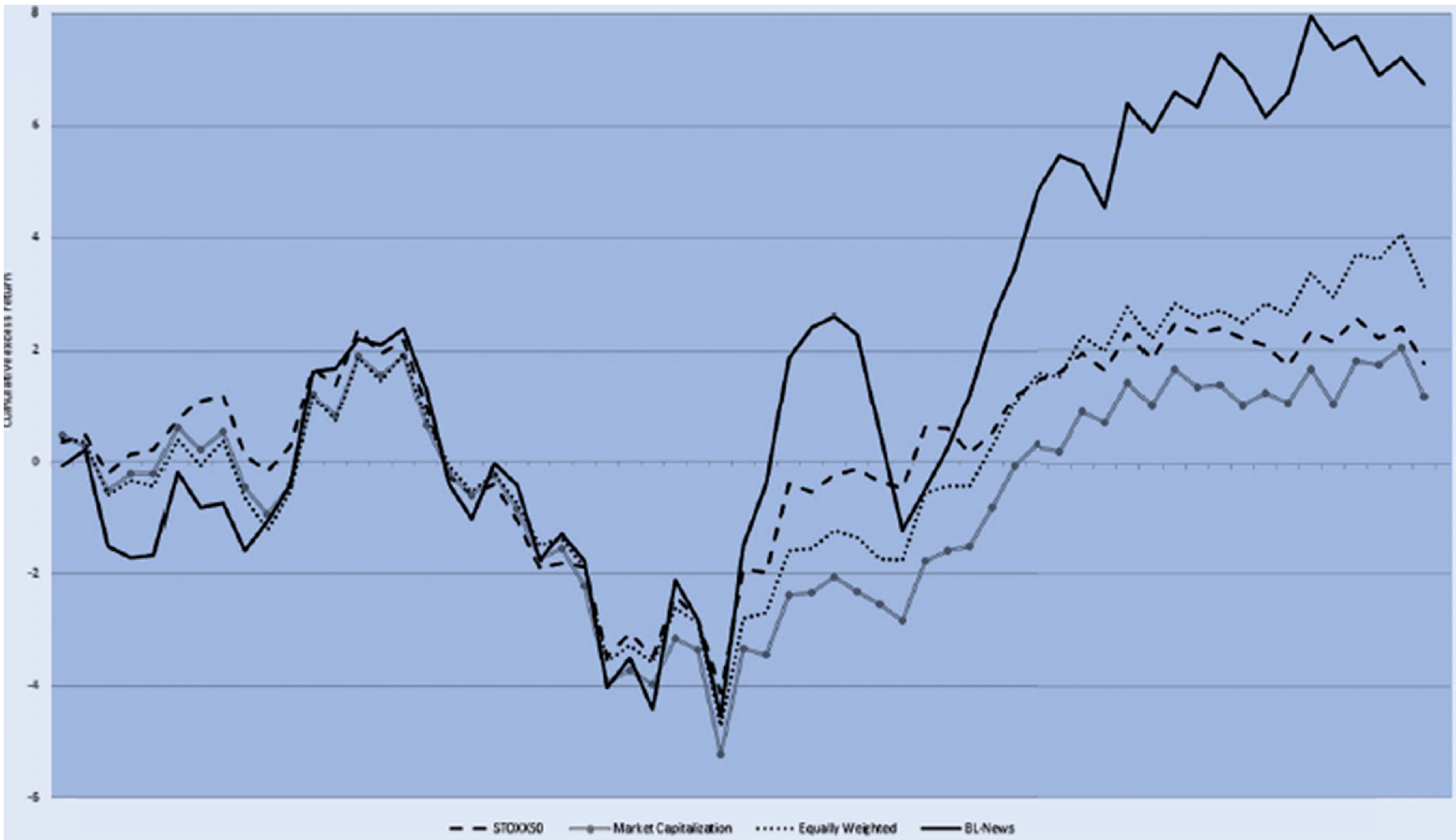
Different prior and posterior distributions of excess return




(b) Low covariance of error term ($\Omega$=0.001) and low confidences ($\tau$=0.1) of investors' view

Prior and posterior probability are practically identical: BL portfolio becomes the market portfolio

# Cumulative excess log return
# BL-news portfolio




Test sample: 9/8/2005-12/30/2005

# Conclusions and future research

- News sentiment has an important high frequency effect on return and they can be used as a proxy for investors' view on a Black Litterman (BL) portfolio. The simulations indicate that a news sentiment driven portfolio outperforms the market portfolio and the market index.
- In conclusion, the investors' subjective views of the BL model can be substituted or enriched by forecasts based on the optimal combination of social networks, news sentiment, accounting or other meaningful indicators.