

CS5228: Knowledge Discovery and Data Mining

Lecture 5 — Classification & Regression I

Course Logistics — Update

- Assignment 2

- Release: Fri, Sep 13
- Submission deadline: Thu, Oct 03 (11.59 pm)
- Reminder: Please adhere to naming and uploading guidelines

- Midterm exam preparation

- Install and check out Exemplify (see [student guide](#))
- Try practice exam (Non-Secure Block Internet)

■ Survey for lower laptops

Recap — Association Rules

- Association Rule Mining

- Input: database of transactions (i.e., sets of items)
- Output: rules predicting the occurrence of some items based on occurrence of other items

- Example: Market Basket Analysis

- Item = product in supermarket
- Transaction = products in basket at check-out
- Interesting rules = customers who by X als tend to Y

TID	Items
1	bread, yogurt
2	bread, milk, cereal, eggs
3	yogurt, milk, cereal, cheese
4	bread, yogurt, milk, cereal
5	bread, yogurt, milk, cheese

Recap — Association Rules

- **Interesting** association rules

- Different definitions that quantify usefulness of a rule:
support, confidence, lift, conviction, leverage, etc.

- **Important observation**

- Relationship between support and confidence

→ Decoupling Support and Confidence — 2 steps:

- 1) Frequent Itemset Generation
- 2) Association Rule Generation

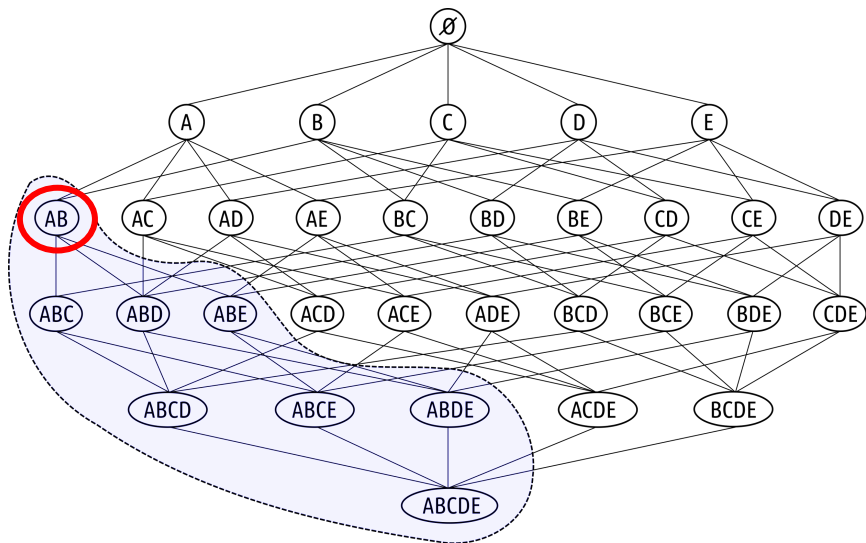
- **Anti-monotone** property of support and confidence

→ Apriori Algorithms for Frequent Itemset
and Association Rule Generation

$$C(X \rightarrow Y) = \frac{S(X \rightarrow Y)}{S(X)} = \frac{S(X \cup Y)}{S(X)}$$

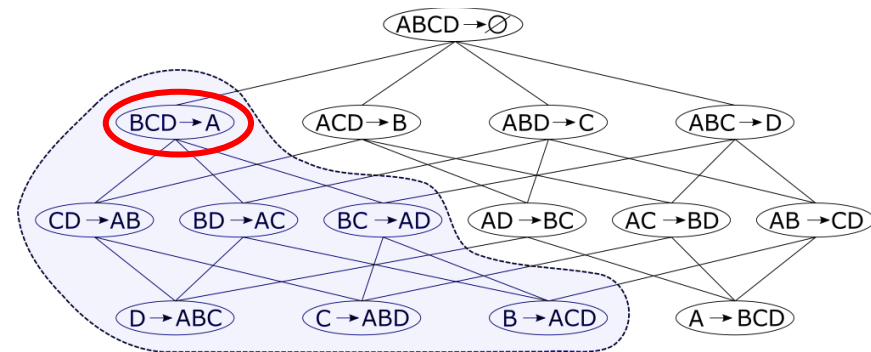
Recap — Association Rules

Itemset Lattice



If an itemset (e.g., $\{A,B\}$) is not frequent, then any superset (e.g., $\{A,B,D\}$, $\{A,B,C,D\}$) is also not frequent

Rule Lattice



If rule $BCD \rightarrow A$ has a insufficient confidence, so do all rules containing A in the consequent

Outline

- **Classification & Regression**
 - Overview & Examples
 - Basic Setup
 - Evaluation
- **Nearest Neighbor Methods**
 - KNN — K-Nearest Neighbor Algorithm
 - Pros, Cons & Caveats

Classification & Regression: Overview

- Classification **AND** Regression

- Core tasks of supervised machine learning
- Training: Find patterns in dataset between the values of **dependent variable(s)** given the values **independent variable(s) / features**
- Prediction: Use patterns to assign values to dependent variables new/unseen data

- Classification **VS.** Regression

- Classification: Dependent variable is categorical
- Regression: Dependent variable is continuous

Independent variables (features)

Age	Edu- cation	Marital Status	Income Level	Credit Approval	Credit Limit
23	Masters	Single	Mid	No	—
35	College	Married	High	Yes	\$S7,000
26	Masters	Single	High	No	—
41	PhD	Single	Mid	Yes	\$S5,000
18	Poly	Single	Low	No	—
55	Poly	Married	High	Yes	\$S10,000
30	College	Single	High	Yes	\$S8,000
35	PhD	Married	High	Yes	\$S10,000
28	Masters	Married	Mid	Yes	\$S5,000
45	Masters	Married	Mid	???	???

Dependent, categorical variable

Dependent, numerical variable

Application Examples

month	flat_type	block	street_name	floor_area_sqm	flat_model	lease_commence_date	latitude	longitude	subzone	planning_area	resale_price
2013-02	4 room	4B	boon tong road	91.0	model a	2005	1.286589	103.831950	tiong bahru	bukit merah	663888.0
1997-08	3 room	11	holland drive	65.0	improved	1975	1.309054	103.794063	holland drive	queenstown	228000.0
2007-08	executive	558	jurong west street 42	157.0	maisonette	1985	1.354198	103.717344	hong kah	jurong west	305000.0
1996-02	4 room	354	hougang avenue 7	105.0	new generation	1986	1.372408	103.899433	kangkar	hougang	235000.0
2015-06	3 room	99	old airport road	56.0	standard	1969	1.308590	103.888245	aljunied	geylang	320000.0

- Prediction of flat prices (regression task)
 - Help sellers and buyers to make better decisions
 - Understand the importance of attributes on flat prices

Application Examples

Has heart disease (1)
or not (0)

ID	age	sex	chest	rest_bp	chol	rest_ecg	max_hr	oldpeak	slope	num_vessels	thal	class
0	49.2	0	4.00	163.00	181.11	0	148.23	0.94	2	0	3	1
1	53.6	1	1.74	130.23	276.47	2	152.92	0.12	2	0	3	0
2	49.6	1	4.00	147.00	223.30	2	102.35	1.62	2	2	7	1
3	59.0	1	4.00	112.37	187.25	0	158.16	0.00	1	1	7	1
4	51.1	1	1.95	138.03	238.48	0	172.54	1.15	1	1	3	0

- Health analytics: prediction of heart disease
 - Diagnosis and therapy support systems
 - Identify most important risk factors

Application Examples

- Sentiment Analysis /
Opinion Mining (over text)

Regression



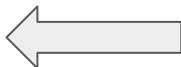
94%


TOMATOMETER



90%

AUDIENCE SCORE



 What you will be getting when you walk into an inevitably overstuffed movie theater is something singular that reflects our age in a way that none of the MCU films that preceded it have—indeed, very few Hollywood spectacles ever have.


May 1, 2019 | Rating: 3.5/4 | [Full Review...](#)



Oliver Jones

Observer

★ Top Critic

 What's missing from "Endgame" is the free play of imagination, the liberation of speculation, the meandering paths and loose ends that start in logic and lead to wonder.


April 28, 2019 | [Full Review...](#)



Richard Brody

New Yorker

★ Top Critic

 Endgame consists almost entirely of the downtime scenes that were always secretly everyone's favorite parts of these movies anyway.

April 26, 2019 | [Full Review...](#)



Dana Stevens

Slate

★ Top Critic

Classification



Fresh



Rotten



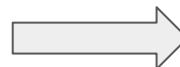
Fresh



Rotten

Tomatometer

Audience



Application Examples

- Self-driving vehicles
 - Input: image & sensor data
- Regression tasks, e.g.:
 - Acceleration
 - Steering angle
 - Event prediction (%)
- Classification tasks, e.g.:
 - Obstacle detection
 - Street sign identification



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Supervised Training/Learning — Basic Setup

Training Data

X_{train}

Age	Marital Status	Income Level	Credit Approval
23	Single	Mid	No
35	Married	High	Yes
26	Single	High	No
41	Single	Mid	Yes
18	Single	Low	No
55	Married	High	Yes
30	Single	High	Yes
35	Married	High	Yes
28	Married	Mid	Yes
45	Married	Mid	No

y_{train}

Test Data

X_{test}

Age	Marital Status	Income Level
25	Single	Low
28	Married	Mid
45	Married	High
41	Single	High
21	Single	Mid
26	Married	Mid

Predictions

\hat{y}_{test}

Credit Approval
No
Yes
No
Yes
No
Yes

Ground Truth

y_{test}

Credit Approval
No
Yes
Yes
Yes
No
No

Evaluation

Model

(classification / regression)

Supervised Training/Learning — Training & Test Data

- Given: Dataset D of pairs $\{(x, y)\}$
 - x — features (independent variables)
 - y — label (dependent variable)
- Split dataset D into:
 - D_{train} — data for training the model
 - D_{test} — data for evaluating the model
 - Important: $D_{\text{train}} \cap D_{\text{test}} = \emptyset$ (the test data is not allowed to be used during training)

		Age	Edu- cation	Marital Status	Income Level	Credit Approval
D_{train}		23	Masters	Single	Mid	No
		35	College	Married	High	Yes
		26	Masters	Single	High	No
		41	PhD	Single	Mid	Yes
		18	Poly	Single	Low	No
		55	Poly	Married	High	Yes
D_{test}		30	College	Single	High	Yes
		35	PhD	Married	High	Yes
		28	Masters	Married	Mid	Yes
		45	Masters	Married	Mid	No
features						labels

Supervised Training/Learning

- Model — Parameterized function $h(x, \Theta) = y$
 - Maps input space (features) to the output space (labels)
 - Θ — trainable/learnable parameters of the model
(note: not all models are parameterized)
- Model selection — Selection of a "family" of functions $h(x, \Theta) = y$
 - K-Nearest Neighbor, Decision Trees, Linear Models, Neural Networks, ...
- Training / Learning
 - Process of systematically finding the best values for Θ
 - $\Theta_{best} \Leftrightarrow$ best mapping between features and labels

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Classification & Regression — Evaluation

Regression

\hat{y}_{test}	y_{test}
Credit Limit	Credit Limit
\$S9,000	\$S10,000
\$S4,000	\$S4,000
\$S5,800	\$S6,000
\$S10,000	\$S12,000
\$S11,000	\$S10,000
\$S3,500	\$S3,000

Direct comparison of numerical values

Common: Root Mean Squared Error (RSME):

$$\underline{RMSE} = \sqrt{\frac{\sum_{i=1}^N (\hat{y}_i - y_i)^2}{N}}$$

prediction / estimate

Classification

\hat{y}_{test}	y_{test}
Credit Approval	Credit Approval
No	No
Yes	Yes
No	Yes
Yes	Yes
No	No
Yes	No

ground truth

Question: How to compare classification results?

Classification: Evaluation

- Base case: Binary classification (2 labels: 0 or 1)
- Example: COVID-19 test

test result

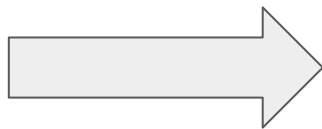
\hat{y}_{test}

0
0
0
1
0
1

infected

y_{test}

0
0
1
1
0
0



Confusion Matrix

ground truth label y

	1	0
predicted label \hat{y}	1 じ	1 じ
	1 じ	3 じ

Classification: Evaluation — Confusion Matrix

		ground truth label y	
predicted label \hat{y}		1	0
	1	True Positives (TP)	False Positives (FP)
	0	False Negatives (FN)	True Negatives (TN)

True Positives (TP): Number of positive classes that have been correctly predicted as positive

True Negatives (TN): Number of negative classes that have been correctly predicted as negative

False Positives (FP): Number of negative classes that have been incorrectly predicted as positive

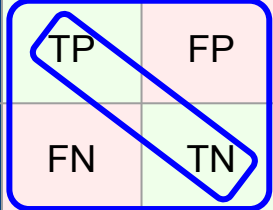
False Negatives (FN): Number of positive classes that have been incorrectly predicted as negative

Classification: Evaluation — Popular Metrics

- Accuracy

$$Accuracy = \frac{TP + TN}{TP + FP + TN + FN}$$

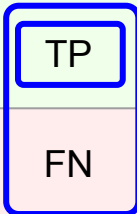
	y		
	1	0	
\hat{y}	1	TP	FP
	0	FN	TN



- Sensitivity, Specificity

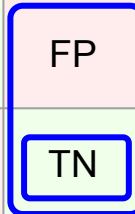
$$Sensitivity = \frac{TP}{TP + FN}$$

	y	
	1	0
\hat{y}	1	FP
	0	TN



$$Specificity = \frac{TN}{TN + FP}$$

	y	
	1	0
\hat{y}	1	TP
	0	FN



Classification: Evaluation — Popular Metrics

- Precision, Recall, F1 Score

$$F1 = \frac{P + R}{2} = 0.5$$

Harmonic Mean of
Precision and Recall

$$Precision = \frac{TP}{TP + FP}$$

	y	
	1	0
\hat{y}	1	TP ₀ FP ₀
	0	FN TN

$$Recall = \frac{TP}{TP + FN}$$

	y	
	1	0
\hat{y}	1	TP FN
	0	FP TN

↑

↑

healthy
patients

$$F1 = 2 \cdot \frac{Precision \cdot Recall}{Precision + Recall} = 0$$

	y	
	1	0
\hat{y}	1	TP FP
	0	FN TN

Classification: Evaluation — Why so Many Measures?

- Problem: (Highly) imbalanced datasets
- Example use case: COVID-19 test
 - Most people in a population are not infected
 - Assume a test that always(!) returns "negative"

$$\text{Precision} = \frac{0}{0} = ?$$

$$\text{Recall} = \frac{0}{0+200} = 0$$

		ground truth label y	
		1	0
predicted label \hat{y}	1	0	0
	0	200	10,000

Handwritten notes: A blue line connects the '0' in the (1,1) cell to the '200' in the (0,1) cell. Another blue line connects the '0' in the (1,0) cell to the '10,000' in the (0,0) cell. There is also a handwritten note in Arabic script next to the table.

$$\text{Accuracy} = \frac{0 + 10000}{0 + 0 + 10000 + 200} = \underline{98\%}$$

$$\text{Specificity} = \frac{10000}{10000 + 0} = 100\%$$

→ Very high values despite "useless" test

Classification: Evaluation — Why so Many Measures?

- Observation: FP and FN not always equally problematic

$$Precision = \frac{TP}{TP + FP}$$

$$Recall = \frac{TP}{TP + FN}$$

- Example: Heart disease prediction

- BAD: misclassifying a high-risk person
- OK-ish: misclassifying a healthy person



Recall > Precision



- Example: News article classification

(e.g., for search engines such as Google News)

- BAD: showing article of wrong category
- OK: missing a relevant article in result



Recall < Precision

Classification: Evaluation — Numerical Class Scores

- Assumption so far:

\hat{y} are **class labels**

- Output of many models:

\hat{y} are **class scores**

- e.g., probability of a class

→ Thresholding

- Use threshold to convert \hat{y} to a binary variable 0/1

*model
output*

0.45
0.30
0.55
0.25
0.35
0.55
0.30
0.60
0.40
0.45

class scores

$\hat{y}_{0.5}$
0.45 → 0
0.30 → 0
0.55 → 1
0.25 → 0
0.35 → 0
0.55 → 1
0.30 → 0
0.60 → 1
0.40 → 0
0.45 → 0

threshold=0.5

$\hat{y}_{0.43}$
0.45 → 1
0.30 → 0
0.55 → 1
0.25 → 0
0.35 → 0
0.55 → 1
0.30 → 0
0.60 → 1
0.40 → 0
0.45 → 1

threshold=0.43

y
1
0
0
0
1
1
0
1
0
1

ground truth

Classification: Evaluation — Numerical Class Scores

- Different threshold yield different results

y

	1	0
$\hat{y}_{0.5}$	2	1
	3	4

threshold=0.5

$$F1 = 0.66$$

y

	1	0
$\hat{y}_{0.43}$	4	1
	1	4

threshold=0.43

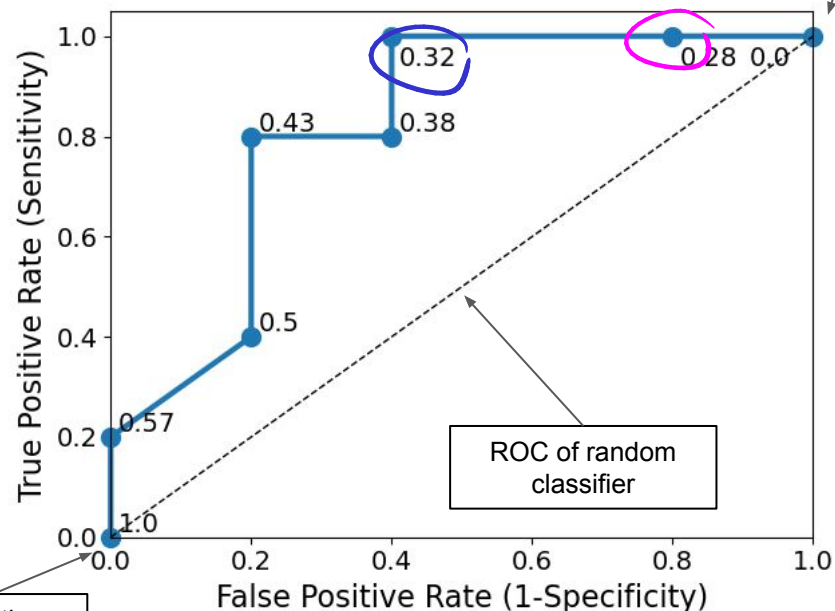
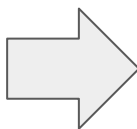
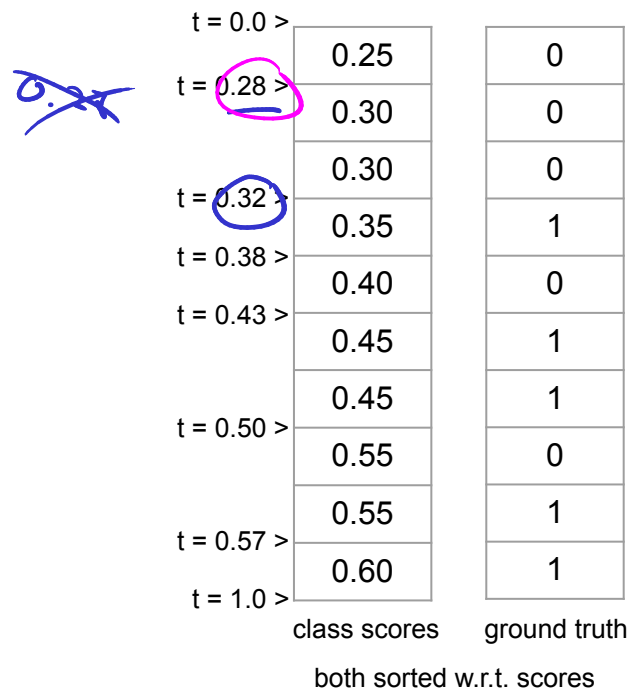
$$F1 = \underline{0.80}$$

→ Question: What threshold to use? — Answer: Try them all!

Receiver Operating Characteristic (ROC)

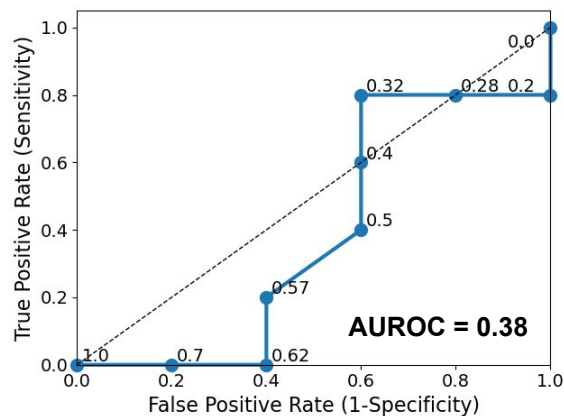
- Plot **True Positive Rate** (Sensitivity) against **False Positive Rate** (1-Specificity)

- Plot values for each meaningful threshold t (meaningful = has effect on result)

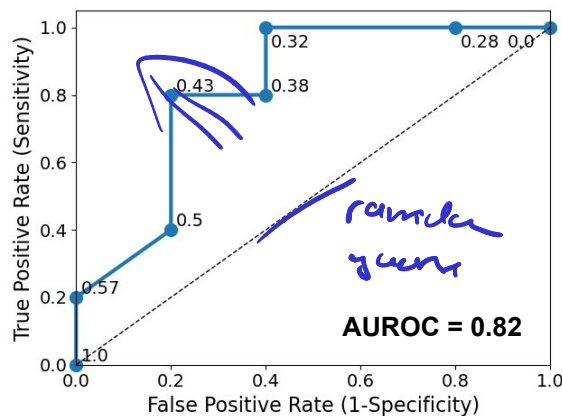


Receiver Operating Characteristic (ROC) — Comparison

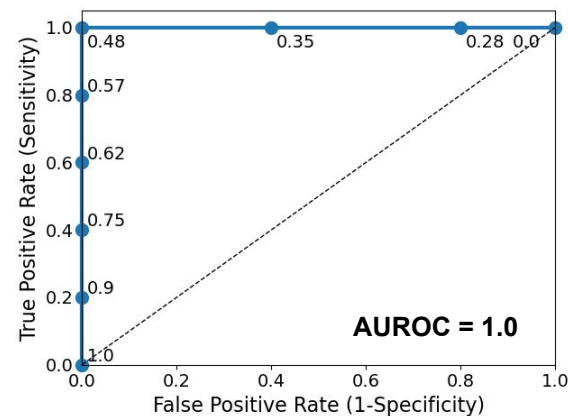
Class scores from 3 different classifiers



Poor classifier



"Decent" classifier



Perfect classifier

- Quantify quality of classifiers using **AUROC** (or AUC)

- Area Under Receiver Operating Characteristic
- AUROC of random classifier: 0.5

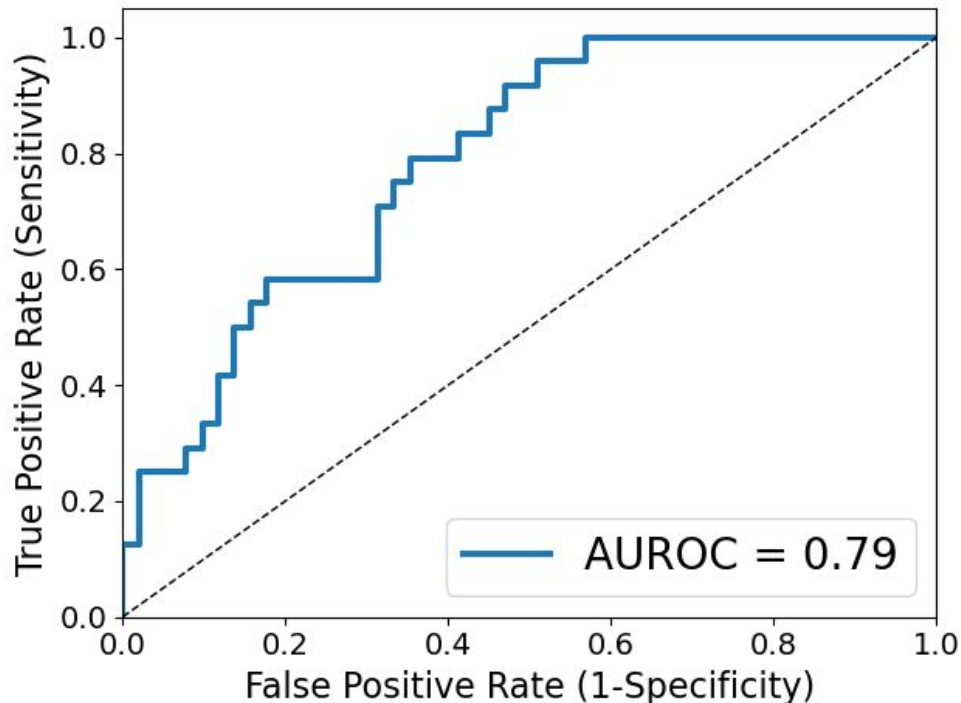
ROC / AUROC — Practical Example

- IRIS dataset

- 3 classes of Iris plants
- 50 samples per class
- 4 continuous features (sepal/petal length/width)

- ROC / AUROC

- More samples, more thresholds
- Smoother ROC
- Thresholds typically omitted



Quick Quiz

In what situation is **accuracy** a good measure?

A

The test data contains a sufficient amount of data samples

B

There are more than 2 class labels

C

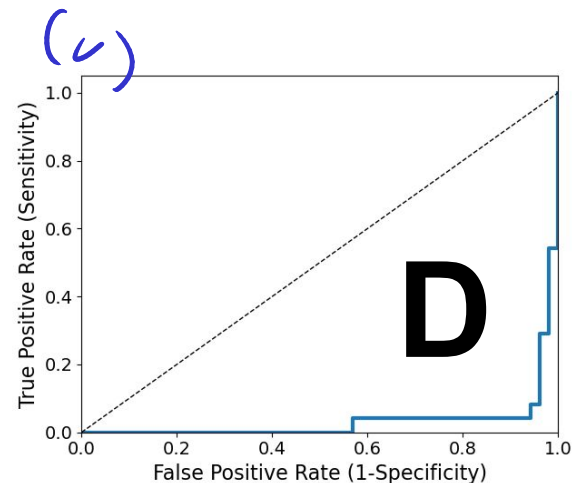
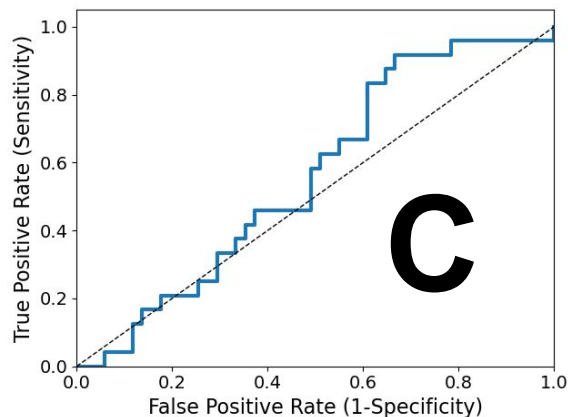
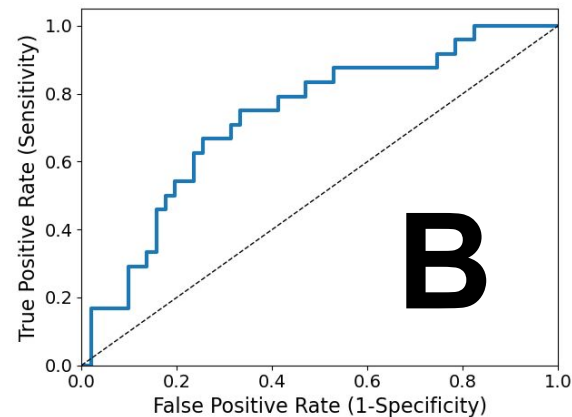
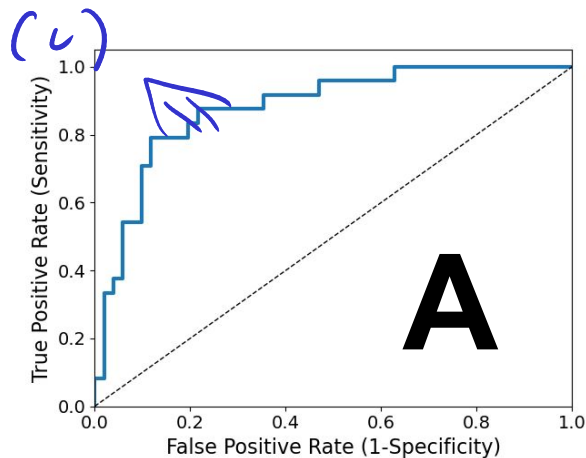
Accuracy is never a good measure

D



The dataset is reasonably balanced (similar number of class labels)

Quick Quiz



For a binary classification task, which of the 4 classifiers would you choose?

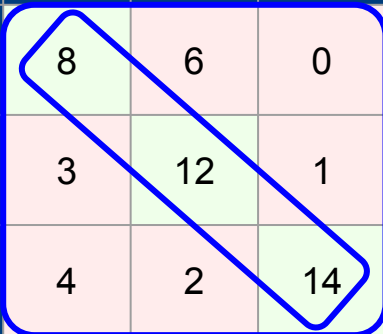
Classification: Evaluation — Beyond 2 Classes

- Example: 3 classes, 50 samples

ground truth label y

predicted label \hat{y}

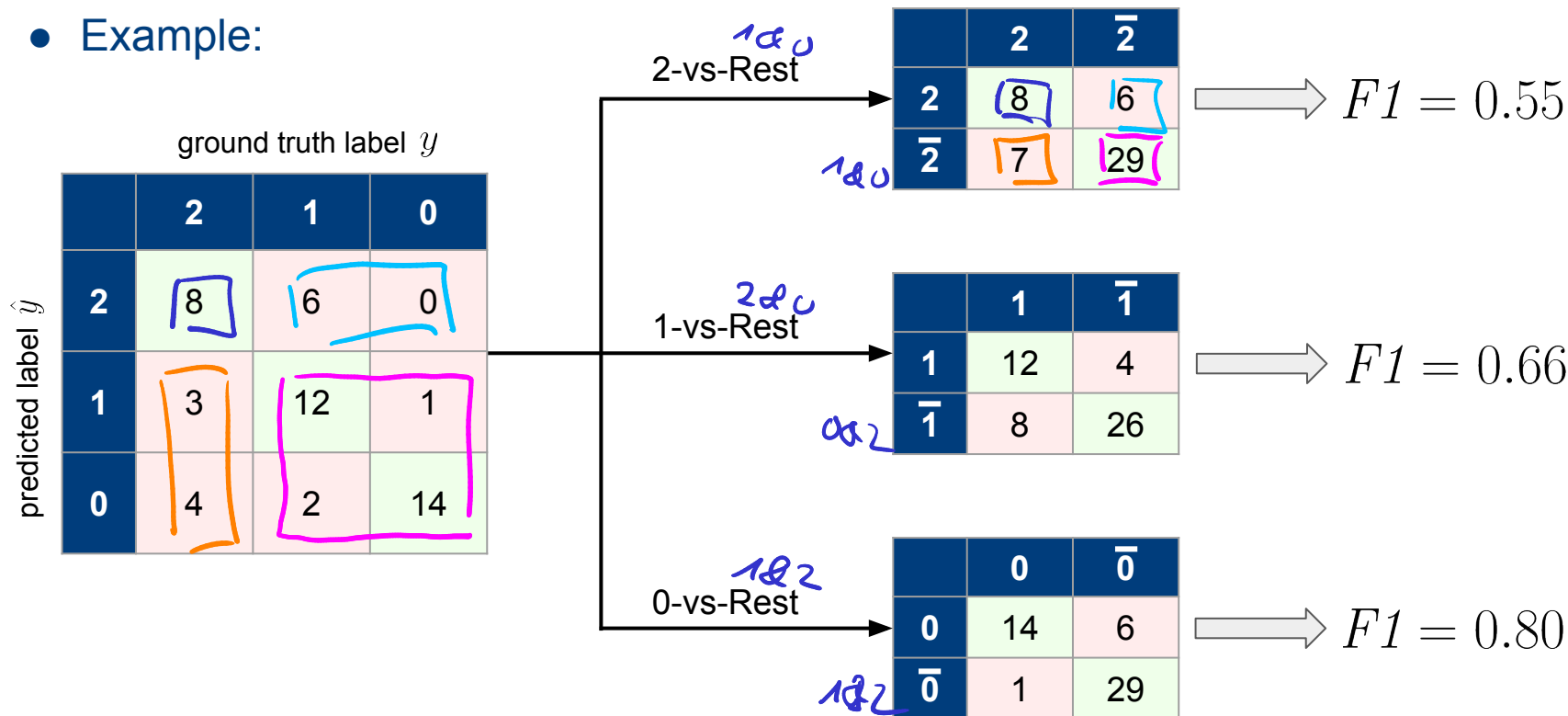
	2	1	0
2	8	6	0
1	3	12	1
0	4	2	14

A 3x3 confusion matrix for 3 classes. The columns represent ground truth labels (y) and the rows represent predicted labels (y-hat). The diagonal elements (8, 12, 14) are highlighted in green, while the off-diagonal elements (6, 0, 3, 1, 4, 2) are in pink. A blue line with rounded corners traces the diagonal from the top-left to the bottom-right.

$$Accuracy = \frac{8 + 12 + 14}{8 + 12 + 14 + 6 + 3 + 1 + 4 + 2} = 0.68$$

Multiclass Evaluation — One-vs-Rest Confusion Matrices

- Example:



One-vs-Rest — Micro Averaging

	2	$\bar{2}$
2	8	6
$\bar{2}$	7	29

	1	$\bar{1}$
1	12	4
$\bar{1}$	8	26

	0	$\bar{0}$
0	14	6
$\bar{0}$	1	29

Average over all
TP, FP, FN, TN



$$\frac{8 + 12 + 14}{3}$$

	c	\bar{c}
c	11.33	5.33
\bar{c}	5.33	28



$$F1 = 0.68$$

One-vs-Rest — Macro Averaging

	2	$\bar{2}$
2	8	6
$\bar{2}$	7	29

$\Rightarrow F1 = 0.55$

	1	$\bar{1}$
1	12	4
$\bar{1}$	8	26

$\Rightarrow F1 = 0.66$

	0	$\bar{0}$
0	14	1
$\bar{0}$	6	29

$\Rightarrow F1 = 0.80$

Average over
all metrics

$\Rightarrow F1 = 0.67$

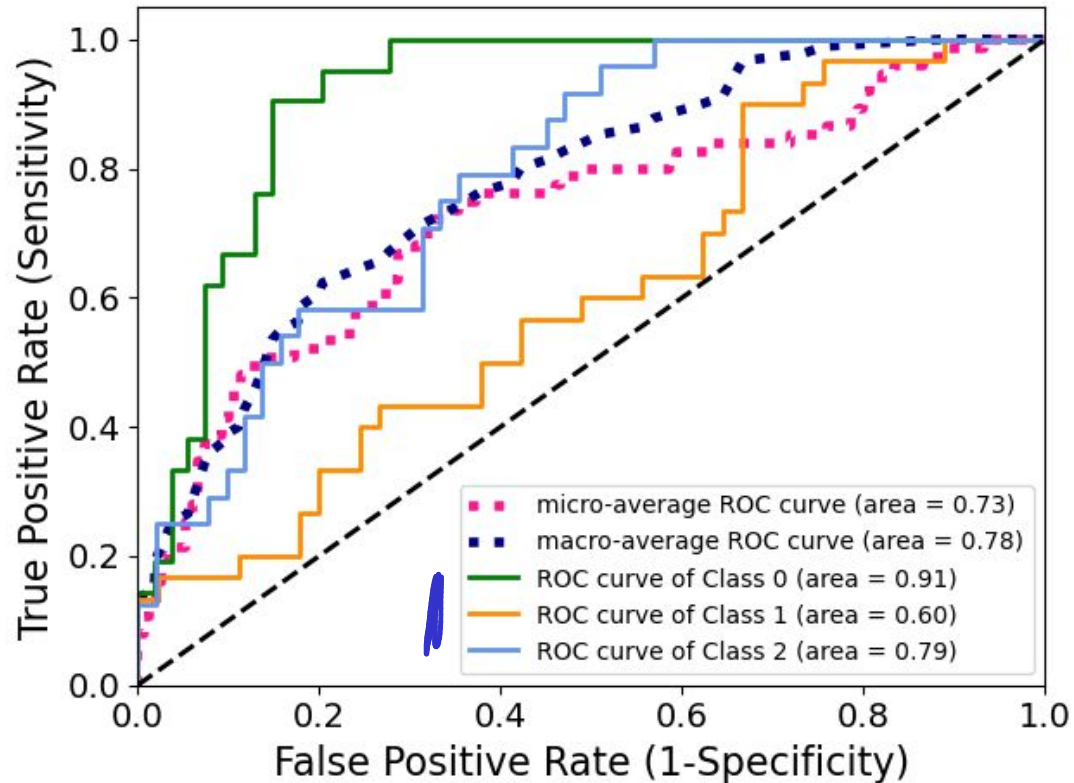
One-vs-Rest — Macro vs. Micro Averaging

- Both methods use One-vs-Rest confusion matrices
 - All introduced metrics applicable (incl. ROC/AUROC)
- Micro-averaging
 - Averaging over TP, FP, FN, TN values of all One-vs-Rest confusion matrices
 - Favors bigger classes (since average over counts)
- Macro-averaging
 - Averaging over metrics derived from each One-vs-Rest confusion matrix
 - Treats all class equally (since metrics are normalized)

ROC / AUROC — Practical Example (Multiclass)

- IRIS dataset

- 3 classes of Iris plants
- 50 samples per class
- 4 continuous features (sepal/petal length/width)



Quick Quiz

A **2-class** classifier and a **10-class** classifier have a f1-score of 0.6:
Which classifier does a **better** job?

A

The 2-class classifier

B ✓

The 10-class classifier

C

Both are equally good

D

Not comparable

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 - **KNN — K-Nearest Neighbor Algorithm**
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K-Nearest Neighbor Algorithm (KNN)

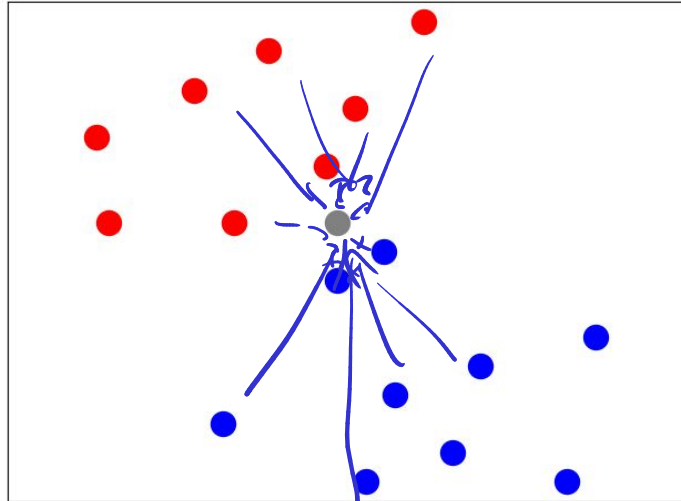
- Intuition behind KNN:

- Label of an unseen data point x derives from the labels of the k -nearest neighbors of x
- Similar data points \rightarrow similar labels

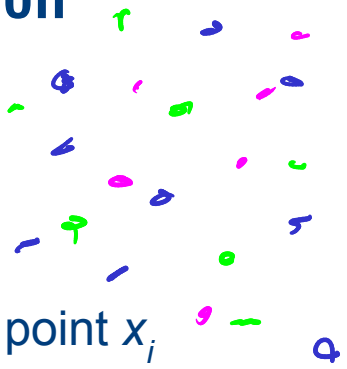
} Required: notion of similarity/distance

- Example

- What should be the label/color of the unseen (gray) data point?



KNN for Classification



- "Training"

- Remember training data

- Prediction for unseen point x_i

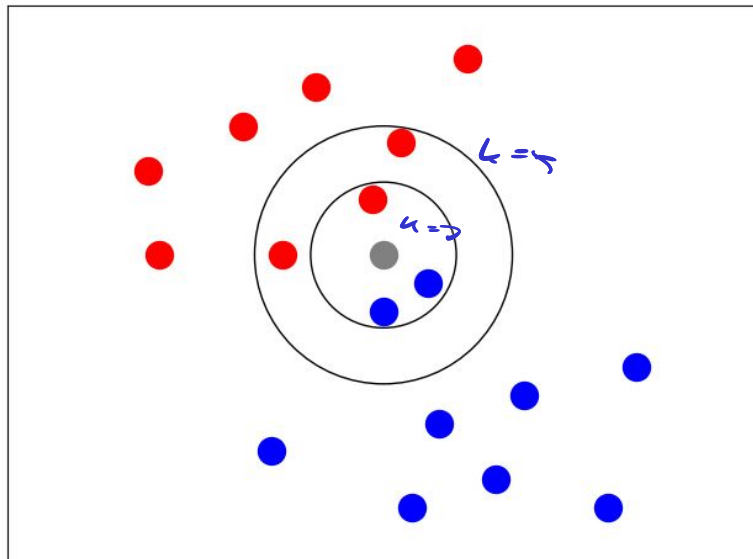
- Calculate distances to all training data points x_j , e.g.:

$$d(p, q) = \sqrt{\sum_i^d (q_i - p_i)^2}$$

Euclidean distance, $d = \text{\#features}$

- Get the k -nearest neighbors
- **Label of x_i = most frequent label among all k -nearest neighbors**

$k=3 \Rightarrow \text{Blue}$
 $k=5 \Rightarrow \text{Red}$



k is typically odd to minimize the chance of ties

Quick "Quiz"

Should k not better be a **prime**
and not just an odd number?

(e.g., to limit the chance of 3/3/3 ties for $k=9$)

A

Never set k to a prime

B ✓

No need but also no harm

C

Possible but there are risks

D

Setting k to a prime
is always preferable

KNN for Classification — Distance Metrics

- Example for different distance metrics

- **Euclidean distance**

- **Manhattan distance**

- **Chebyshev distance**

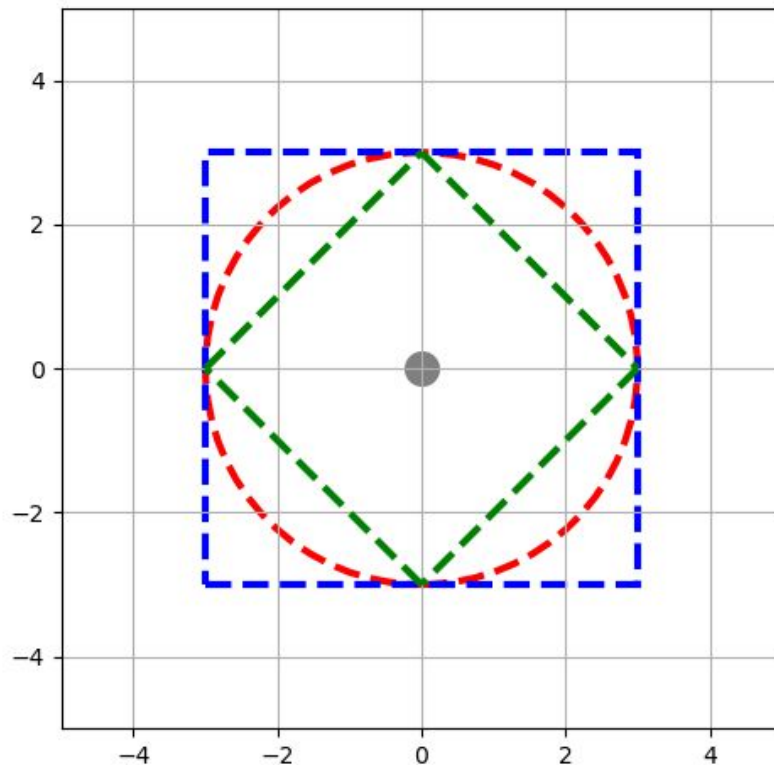
- Other metrics, e.g.:

- Cosine similarity

- Jaccard similarity

- ...

- User-defined metrics



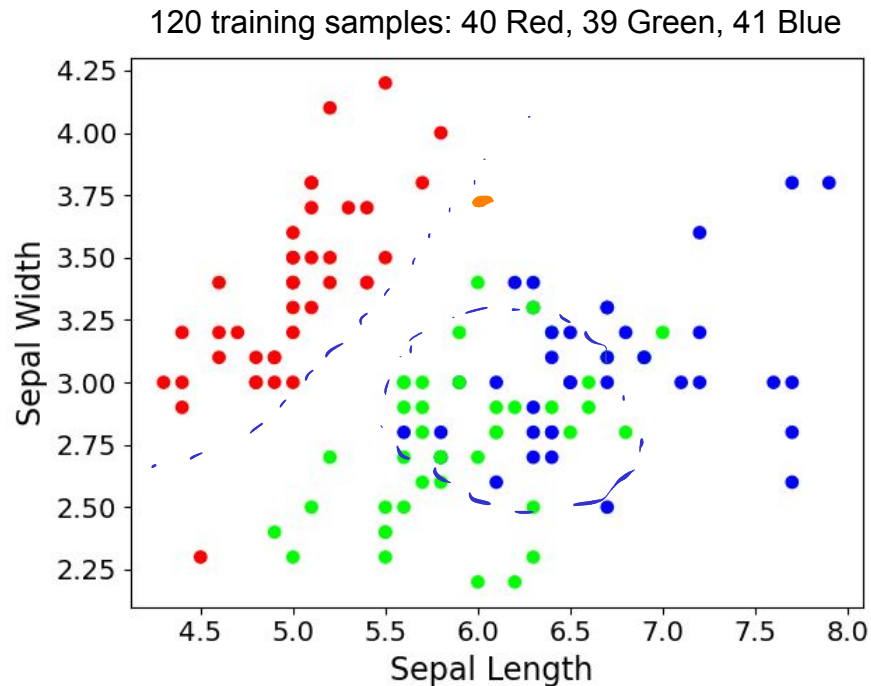
KNN for Classification — Example

- IRIS dataset

- 3 classes, 50 samples per class
- 4 continuous features, but only 2 used: (sepal length & width)

- Basic EDA

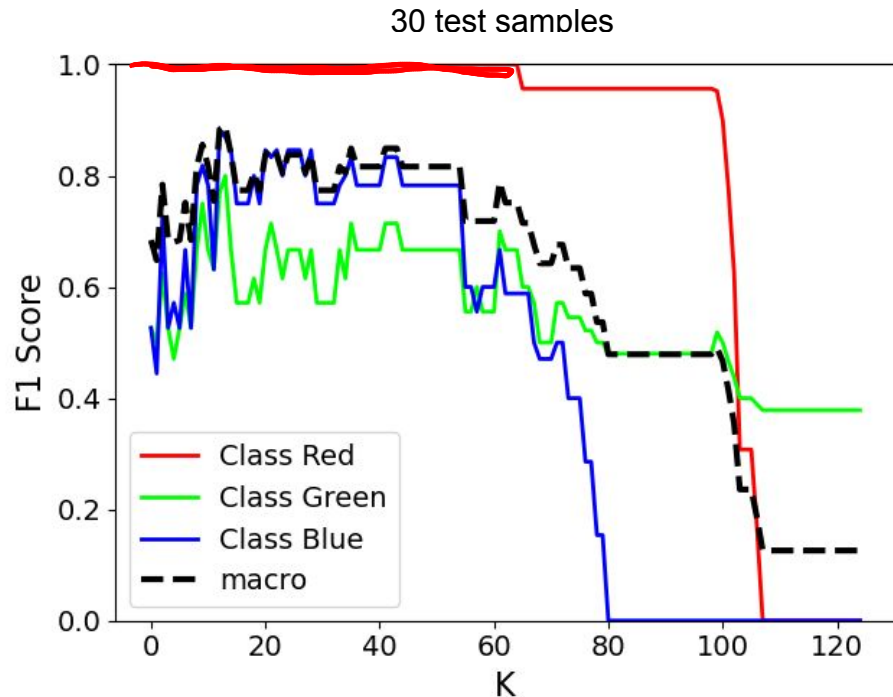
- Class "Red" well separated
- A lot of overlap between Classes "Green" and "



KNN for Classification — Example

- Common outcomes

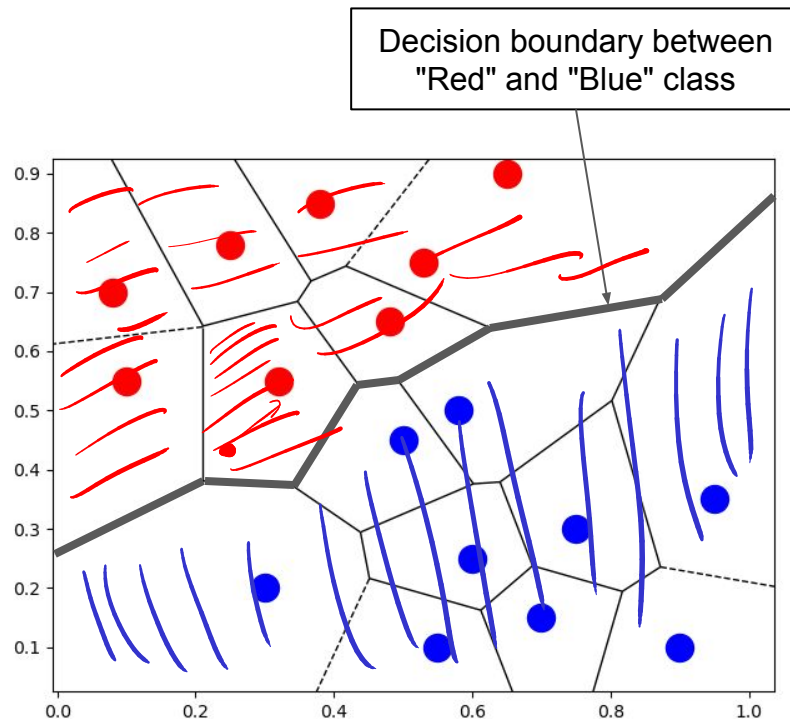
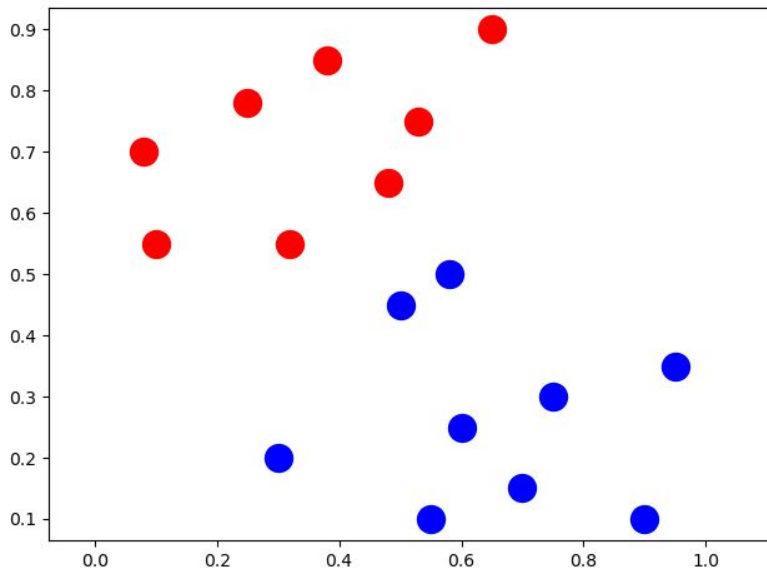
- Different k yield different results
- (Very) small k — results very sensitive to noise and outliers
- Large k — insufficient capacity to properly sep
- Very large k — most frequent class in training data starts to dominate



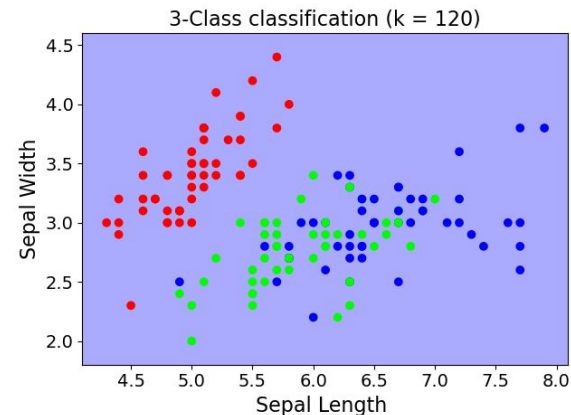
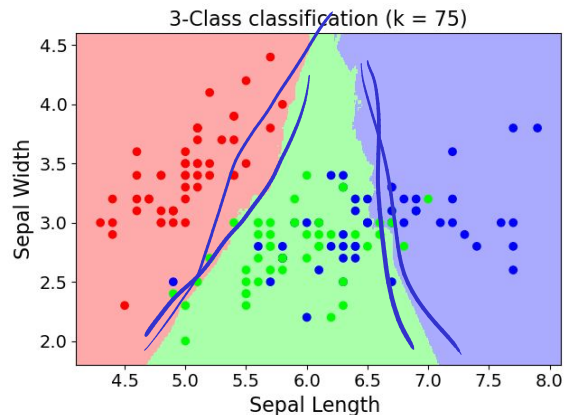
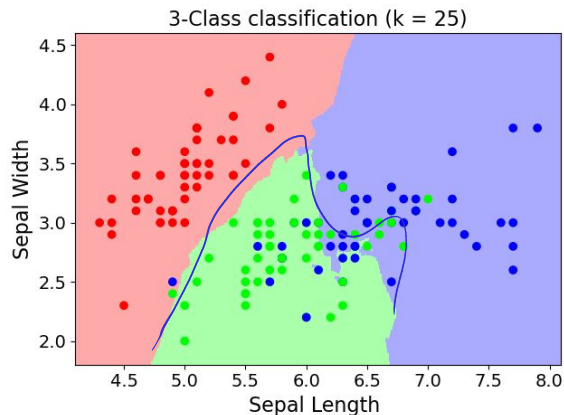
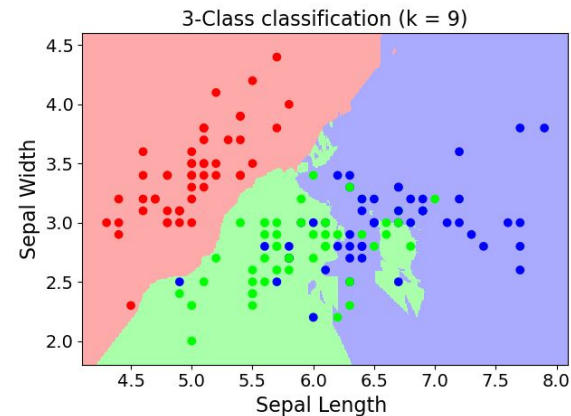
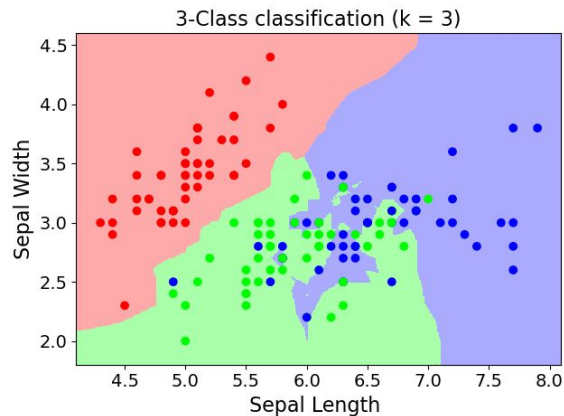
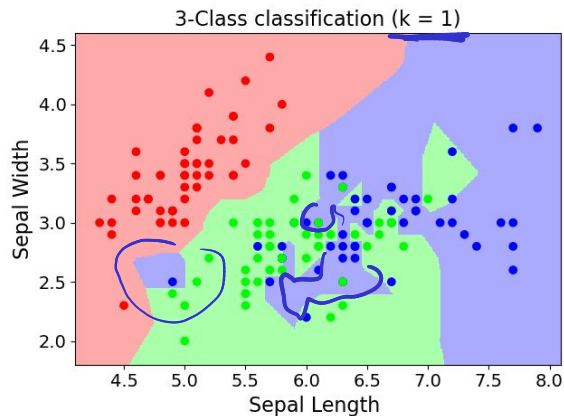
1-Nearest Neighbor: Voronoi Tessellation

- Decision boundaries for 1-NN = derived from Voronoi Tessellation

- Separation of space into cells
- Cell: set of points nearest to a single data point

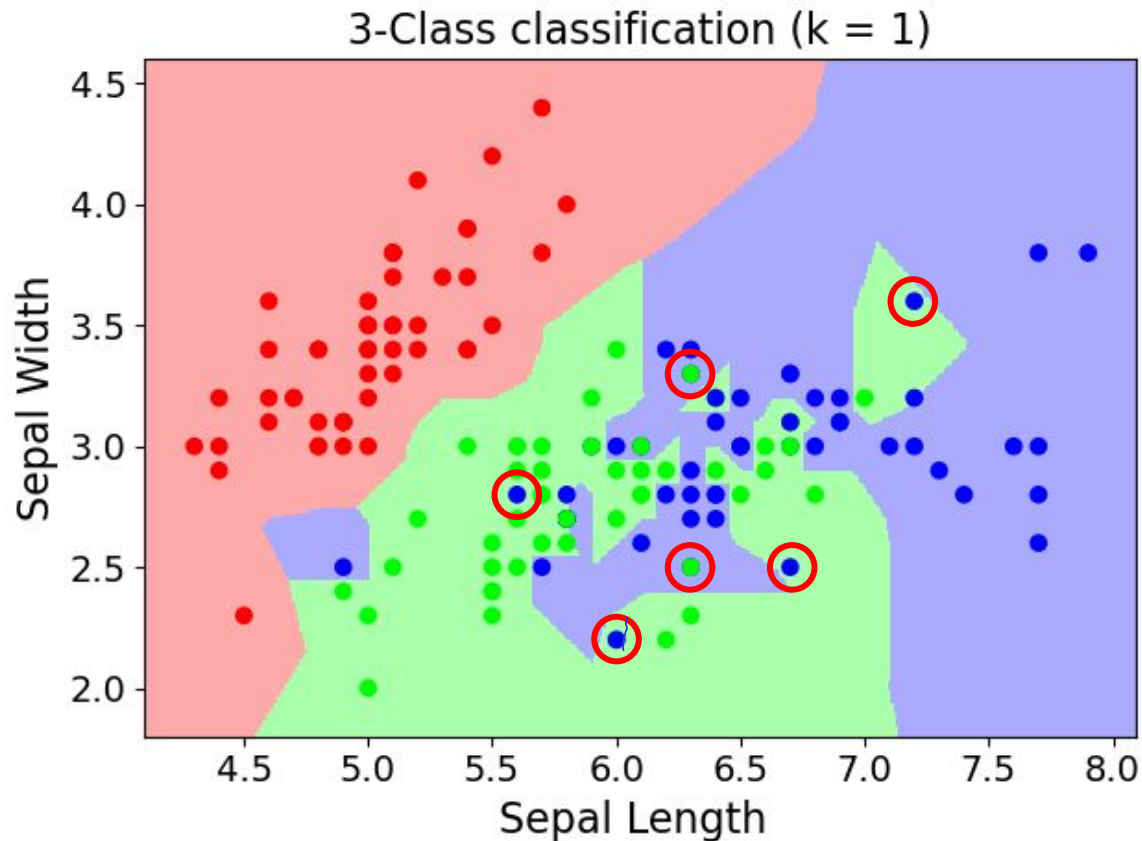


KNN for Classification — Example



Quick Quiz

All data points come from the **training data**:
What's going on there?



KNN for Regression

- "Training"

- Remember training data

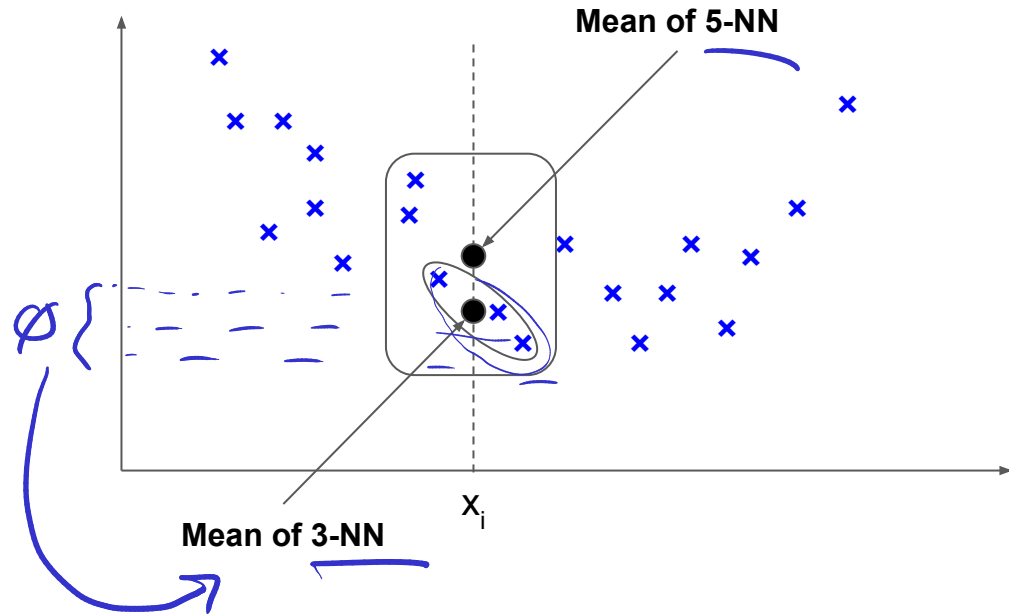
- Prediction for unseen point x_i

- Calculate distances to all training data points x_j , e.g.:

$$d(p, q) = \sqrt{\sum_i^d (q_i - p_i)^2}$$

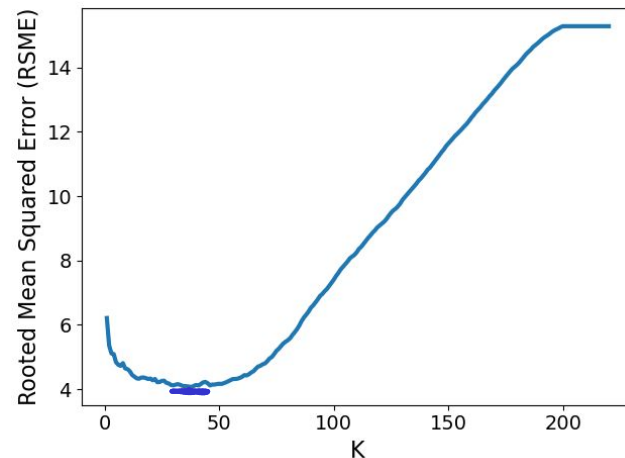
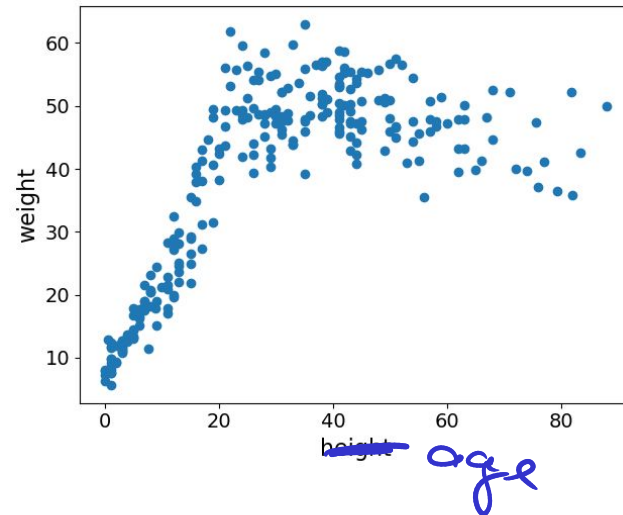
Euclidean distance, $d = \text{\#features}$

- Get the k-nearest neighbors
- **Label of x_i = mean of scores of all k-nearest neighbors**

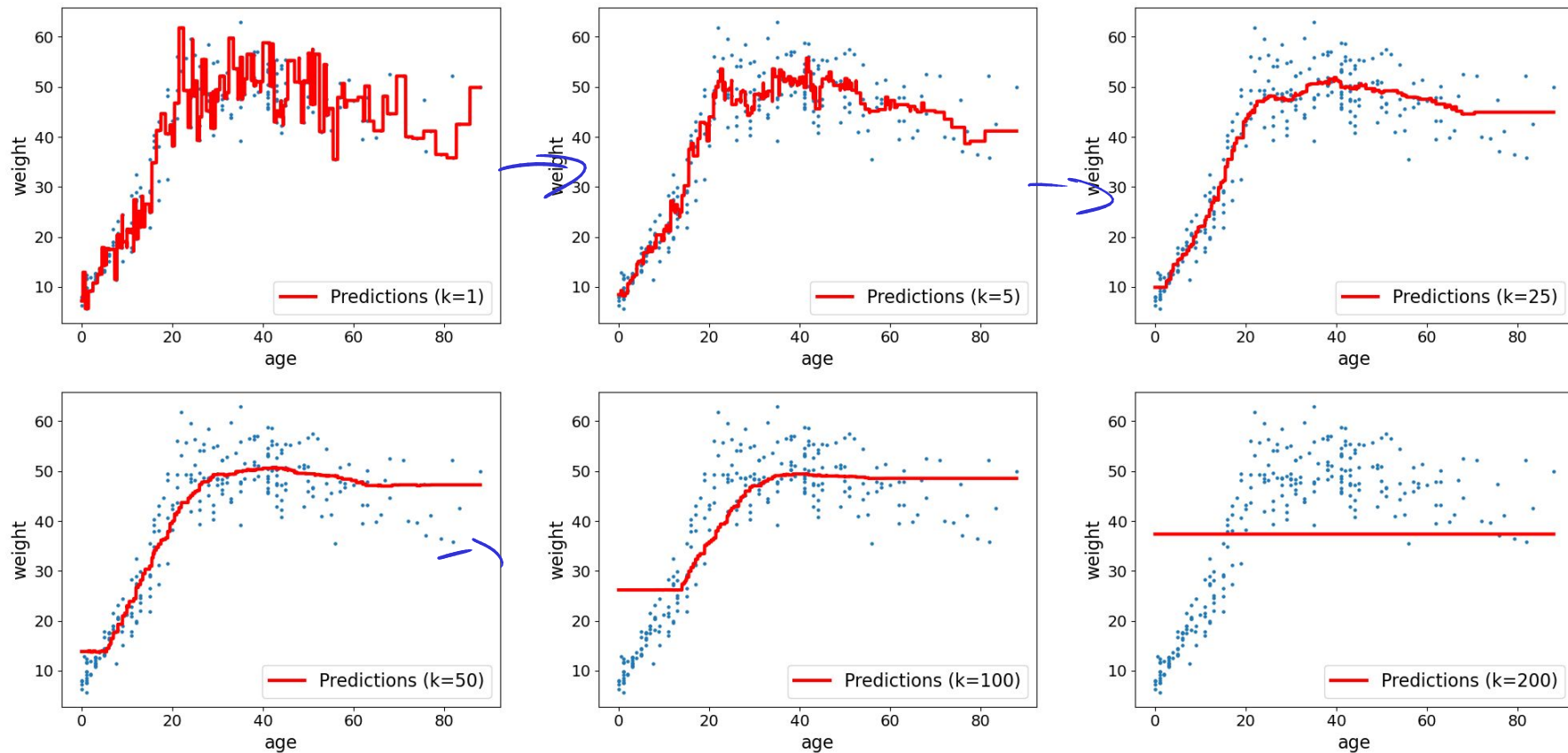


KNN for Regression — Example

- "age vs. weight" dataset
 - age and weight of 257 males
 - 200 training samples, 57 test samples
- Common outcomes (again)
 - Different k yield different scores
 - (Very) small k — predicted scores very sensitive to noise and outliers
 - Large k — mean over too many neighbors
 - Very large k — predicted scores converge to the mean over



KNN for Regression — Example



Choice of k — Summary

- k too small

- Predictions sensitive to noise/outliers
- Very uneven decision boundaries (or regression lines)



Risk of **overfitting**

- k too large

- Unable to capture local patterns
- Very smooth decision boundaries (or regression lines)



Risk of **underfitting**

Outline

- **Classification & Regression**
 - Overview & Examples
 - Basic Setup
 - Evaluation
- **Nearest Neighbor Methods**
 - KNN — K-Nearest Neighbor Algorithm
 - **Pros, Cons & Caveats**

Pros & Cons...and Caveats

• Pros

- Very simple and intuitive algorithm — but often surprisingly good performance!
- Generic algorithm — applicable as long as distance between points can be "meaningfully" measured
- Can produce arbitrarily shaped decision boundaries
- No training time

} Clustering

• Cons

- Finding neighbors at test time may be slow
(in practice: data structures and auxiliary algorithms to speed up k-NN search)
- All training data need to be stored

• Caveats

- *"...applicable as long as distance between points can be "meaningfully" measured"*

Caveat 1: Feature Values (Range, Magnitude)

- Euclidean distance sensitive to range and magnitude of attribute values

id	weight (kg)	height (cm)	sex
A	80	165	male
B	55	180	female
C	70	180	???

$$d(A, C) = \sqrt{(70 - 80)^2 + (180 - 165)^2} = 18.0$$

$$d(B, C) = \sqrt{(70 - 55)^2 + (180 - 180)^2} = 15.0$$

→ C classified as female

id	weight (kg)	height (m)	sex
A	80	1.65	male
B	55	1.80	female
C	70	1.80	???

$$d(A, C) = \sqrt{(70 - 80)^2 + (1.80 - 1.65)^2} \approx 10.0$$

$$d(B, C) = \sqrt{(70 - 55)^2 + (1.80 - 1.80)^2} = 15.0$$

→ C classified as male

Data Normalization

- Z-score normalisation, e.g.:

$$x_i^{weight} = \frac{x_i^{weight} - \mu^{weight}}{\sigma^{weight}}$$

- Min-max normalization, e.g.:

$$x_i^{weight} = \frac{x_i^{weight} - \min(x^{weight})}{\max(x^{weight}) - \min(x^{weight})}$$

id	weight (kg)	height (m)	sex
A	80	1.65	male
B	55	1.80	female
C	70	1.80	???



Z-score normalisation

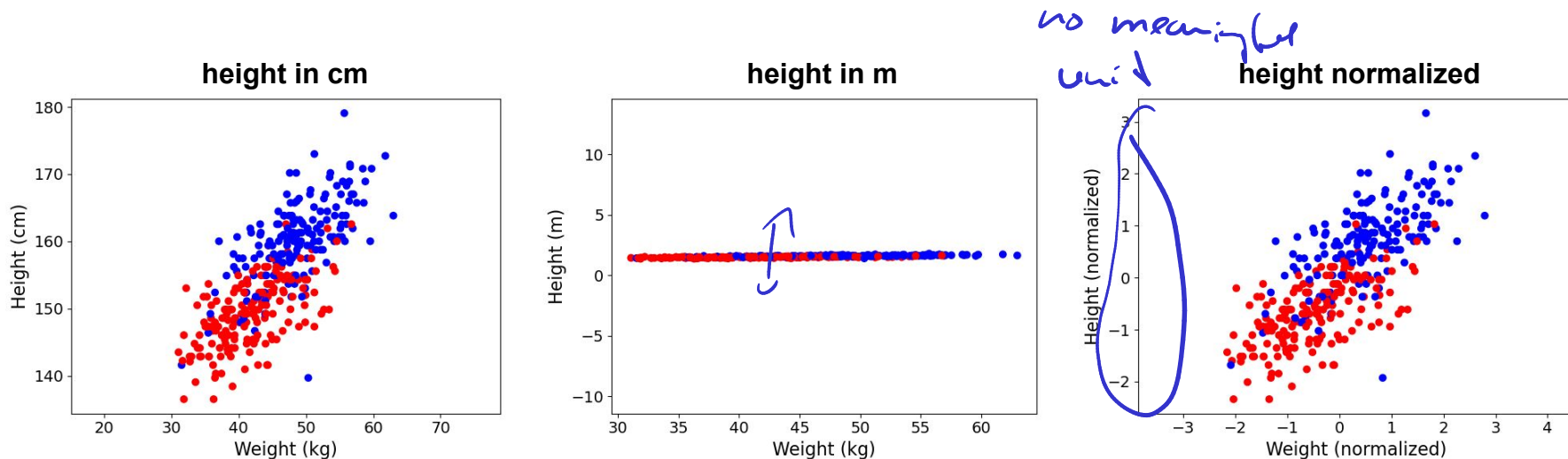
id	weight	height	sex
A	1.14	-1.41	male
B	-1.30	0.71	female
C	0.16	0.71	???

$$d(A, C) = 2.33, \quad d(B, C) = 1.46$$

→ C classified as female

Data Normalization — Visualized

- (weight, height) data points for 187 females (red) and 165 males (blue)



Note: Normalization assumes that all features are equally important. This may not always be true!

Caveat 2: Curse of Dimensionality

- Effect of high dimensions (i.e., many features)

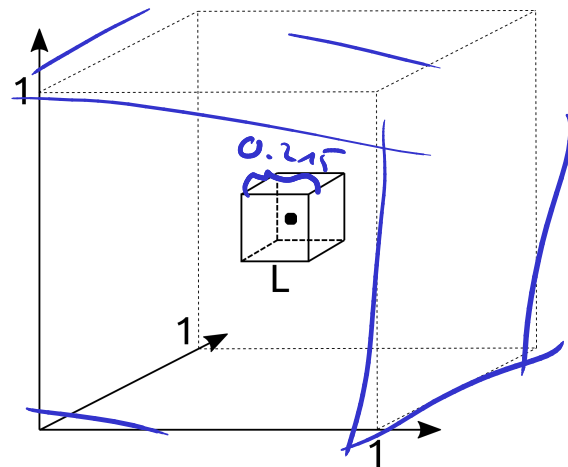
- Data points tend to never be close together
- Average distance between points converges

- Intuition

- Assume N data points uniformly distributed within a unit cube with d dimensions
- Let L be the length of the smallest cube containing the k -NN of a data point

$$L^{(2)} = \frac{10}{100}$$

$$L^d \approx \frac{k}{N} \Rightarrow L \approx \left(\frac{k}{N} \right)^{\frac{1}{d}}$$



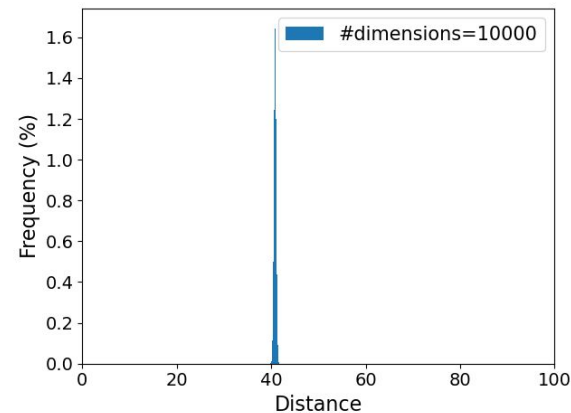
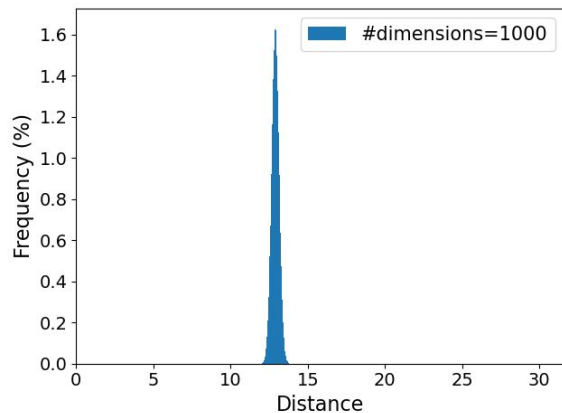
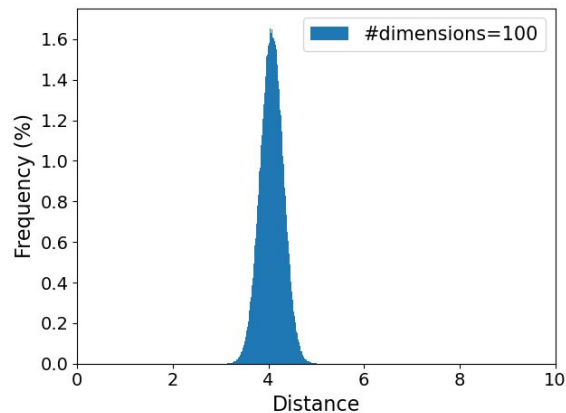
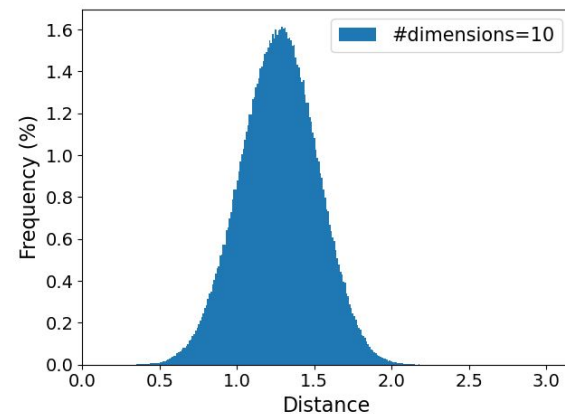
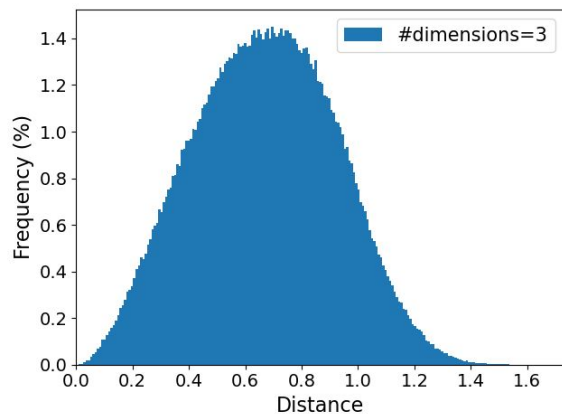
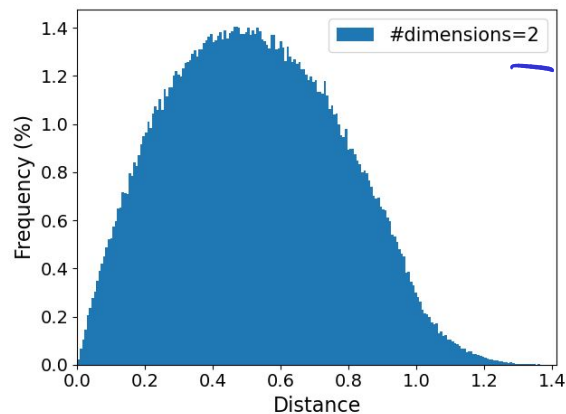
$N=1,000, k=10$

d	L
2	0.100
3	0.215
10	0.631
100	0.955
1,000	0.995
10,000	0.999

The cube with the k -NN is almost the whole unit cube!

Curse of Dimensionality

Distribution of pairwise distances between 1,000 random data points and different number of dimensions



Caveat 3: Non-Numerical Data

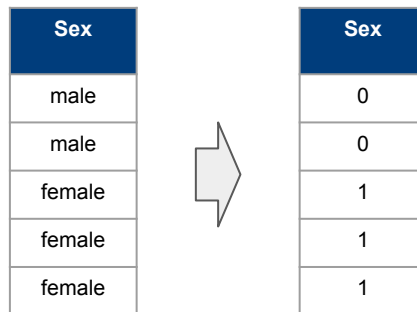
- In practice: Features often categorical
 - Ordinal — education level, grades, etc.
 - Nominal — sex, marital status, etc.
- Basic approach: Convert categorical features into numerical features
 - Allows direct use of common distance metrics
 - Does not automatically yield good results

Age	Sex	Edu- cation	Marital Status
23	male	Masters	Single
35	male	College	Married
26	female	Masters	Single
41	female	PhD	Single
18	female	Poly	Single
55	male	Poly	Divorced
30	female	College	Single
35	male	PhD	Married
28	male	Masters	Married
45	female	Masters	Married

Caveat 3: Non-Numerical Data

- Binary features

- Special case of categorical data
- Convert into binary variable 0/1 to indicate absence/presence



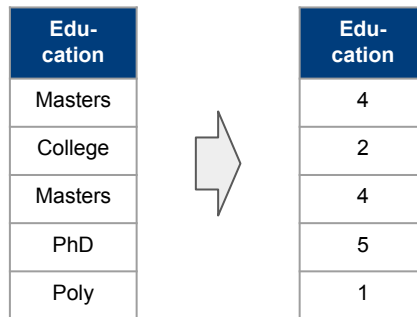
The diagram illustrates the conversion of categorical data to binary. On the left, a table with a dark blue header 'Sex' contains five rows of categorical values: 'male', 'male', 'female', 'female', and 'female'. A large grey arrow points to the right, where a second table with the same header 'Sex' shows the corresponding binary values: 0, 0, 1, 1, and 1.

Sex
male
male
female
female
female

Sex
0
0
1
1
1

- Ordinal features

- Utilize natural order of feature values
- Convert into numerical values while preserving their original order



The diagram illustrates the conversion of categorical data to numerical values. On the left, a table with a dark blue header 'Education' contains five rows of categorical values: 'Masters', 'College', 'Masters', 'PhD', and 'Poly'. A large grey arrow points to the right, where a second table with the same header 'Education' shows the corresponding numerical values: 4, 2, 4, 5, and 1.

Education
Masters
College
Masters
PhD
Poly

Education
4
2
4
5
1

Caveat 3: Non-Numerical Data

- **Nominal features** (with n different values)
 - **One-hot encoding**: convert nominal feature into N binary features
 - Convert each new binary feature into binary variable 0/1 to indicate absence/presence
 - Increases dimensionality!
(particularly if N is very large)

Marital Status
Single
Married
Single
Single
Single
Divorced



Single	Married	Divorced
1	0	0
0	1	0
1	0	0
1	0	0
1	0	0
0	0	1

Caveat 3: Non-Numerical Data

- Discussion / Limitations

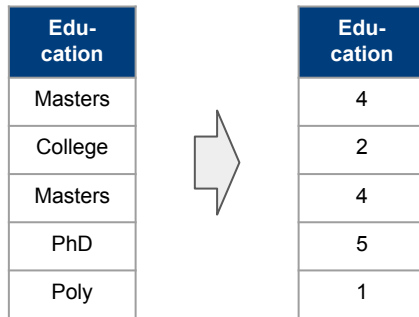
- One-hot encoding increases dimensionality of data
- Distance between ordinal values often not intuitive

$$5 \text{ (PhD)} - 4 \text{ (Masters)} \stackrel{?}{=} 2 \text{ (College)} - \text{Poly (1)}$$

- Typically requires data normalization
(particularly when combined with numerical features)

- Alternative approach: Custom distance metric

- Design metric that appropriately quantifies distance between categorical values
- Typically very specific to given dataset and application context.



Edu- cation
Masters
College
Masters
PhD
Poly


Edu- cation
4
2
4
5
1

Caveat 4: Semantic vs. Low-Level Similarity

- Unstructured data
(text, image, video, audio)
 - Object is often more than just the sum of its parts
- Example (dog food ad)
 - Pixelwise similarity large; easy to compute
 - Semantic similarity small



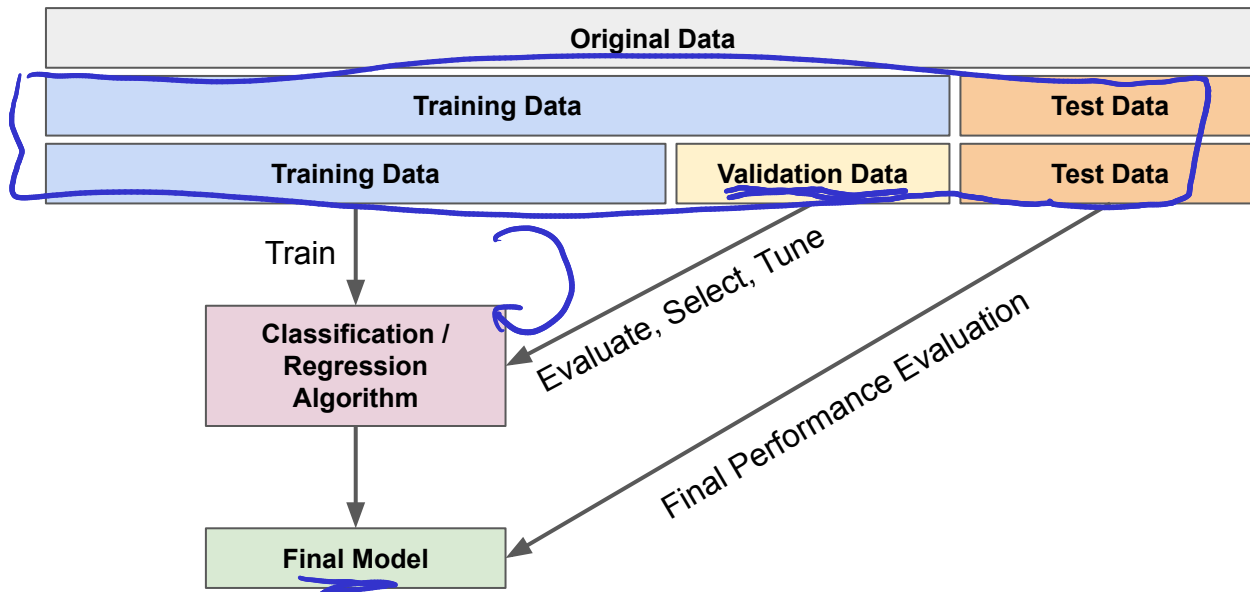
Supervised Training/Learning — Extended Setup

- Recall basic setup: training data + test data
- Building a good classifier or regressor
 - Find the best data preprocessing steps
 - Find the best model (model selection)
 - Find the best hyperparameter values

→ Iterative evaluation required
- **Important: Not allowed to use test data for this!**
 - Test data is supposed to contain truly unseen data
 - Test data no longer unseen when used to optimize/tune model
(for example, results might be different for a different training-test split)

Training & Evaluation Process Using Validation Data

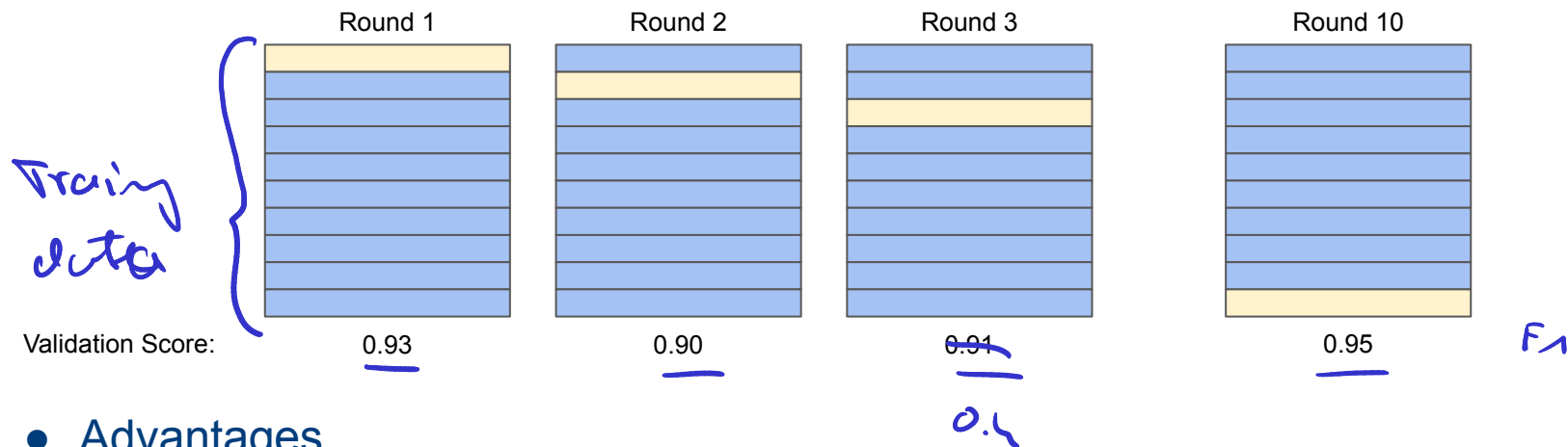
- Common data split to ensure generalizability of results
 - Use test data only at the very end to evaluate performance of final model
 - Use validation data for model selection and hyperparameter tuning



K-Fold Cross Validation (just one of many validation techniques)

- Core idea of k -fold cross validation

- Split training data into k blocks of equal size (e.g., $k=10$)
- Use $(k-1)$ blocks for training and remaining block for evaluation
- Repeat for k rounds with different permutations



- Advantages

- Averaged results more reliable
- Variance between results useful indicator

Information Leakage through Normalization

- Important guidelines

- Do not normalize before splitting dataset into training and test data!
- Normalize training and test data but only based on training data!

} Why?

- Example (pseudo) code using scikit-learn

```
# Split input data into training and test set
X_train, X_test = split(X, 0.2)

# Fit StandardScaler (i.e., calculate mean and variance)
scaler = preprocessing.StandardScaler().fit(X_train) # CORRECT!
scaler = preprocessing.StandardScaler().fit(X) # WRONG!!!

# Fit both training and test data
X_train_transformed = scaler.transform(X_train)
X_test_transformed = scaler.transform(X_test)
```

Summary

- **Evaluation of classifiers** (straightforward for regressors)
 - Different metric with different interpretations
 - Applicability of metrics depending on data and task
- **K-Nearest Neighbor (KNN) classifier/regressor**
 - Very intuitive supervised learning model
 - Limited applicability for (very) large datasets
(heavy lifting done during prediction time not the training time)
 - Choice of hyperparameter k important to address risk of overfitting and underfitting

Solutions to Quick Quizzes

- Slide 29: D
- Slide 30: Technically D, but A is the more practical result
- Slide 37: B
- Slide 41: B
- Slide 47: Duplicate data points
- Slide 67: Otherwise, risk of data leakage
 - Mean and standard deviation will be affected by test data