NUS | Computing
National University of Singapore

CS5228: Knowledge Discovery and Data Mining

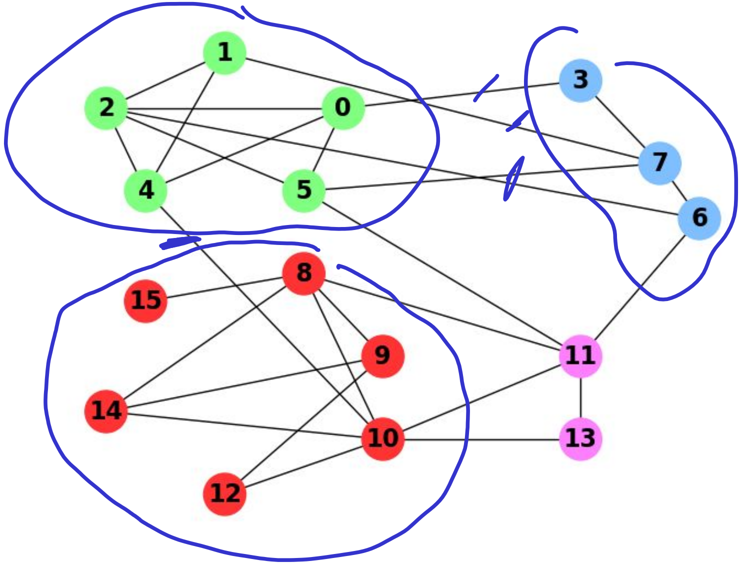
Lecture 11 — Data Stream Mining

Course Logistics

- Reminder for submission deadlines
  - A4: Nov 14, 11.59 pm
  - Project Report: Nov 4, 11.59 pm
- Project submission
  - Submission = project report (PDF, max 8 pages) + source code
  - Only 1 submission per team needed
  - Submission files should include team name
- Last Lecture Quiz
  - Friday, Nov 15, ~19:15 (30 min)
  - MCQs/MRQs, Lectures 7-11

Quick Recap — Graph Mining

- Community Detection
  - Identification of "interesting" subgraphs
    (≈ nodes in subgraph more tightly compared to other nodes)
  - Similar to the task of clustering
    (clustering algorithms can be adopted to find communities)



- No single definition of "community"
  ➜ Many algorithms for community detection
  - Similarity between nodes (e.g., AGNES)
  - Density-based (modularity + Louvain algorithm)
  - Split-based (Edge Betweenness, Min-Cut)

Girvan Newman algorithm → Edge Betweenness

Karger's algorithm → Min-Cut

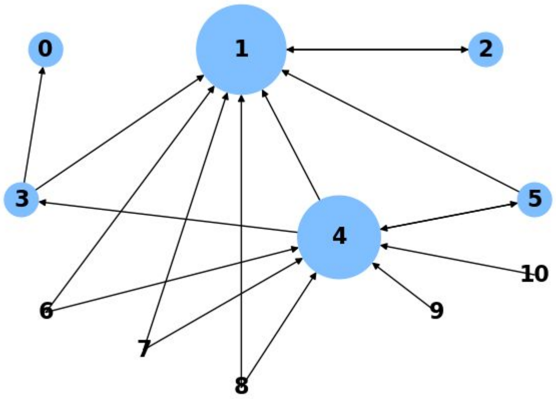
Shared focus: **connectedness**

Quick Recap — Graph Mining

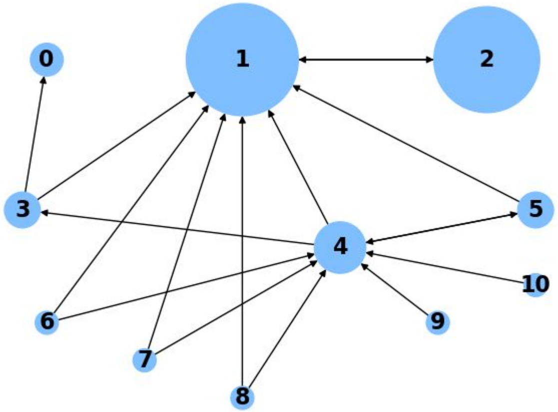
- Centrality = importance of a node
  - Based on a node's topological position in a graph
  - Different centrality measures focusing on different topological features
  - Not all measures applicable to all types of graphs

- Popular centrality measures covered
  - Local measures (Degree, InDegree, OutDegree)
  - Eigenvector-based measures (Eigenvector Centrality, PageRank)
  - Distance-based or path-based measures (Closeness, Betweenness)

InDegree



PageRank



Outline

- **Motivation**
  - Basic setup
  - Example Applications
- Core Techniques
  - Sampling
  - Filtering
  - Counting (distinct items)
- Summary

Data Stream Mining

- Data Mining so far
  - Access to complete dataset (at the same time)
  - Virtually unlimited storage and computing resources
    (also: runtime of algorithms typically not that important, compared to the results)
  - Support of arbitrary complex patterns

- Now: data items arrive one-by-one in real time...like a stream
  - All data never fully available
  - Often very high arrival speeds
  - Often limited amount of resources
  - Often time-critical decisions
    (common: execution in main memory only)

➜ Focus on simple patterns (but not too simple)

Quick Quiz

Given is a stream of temperature sensor values in °C.

What is the only **non-trivial** pattern to monitor?

A temp > 100 °C

B Average of all temperature values

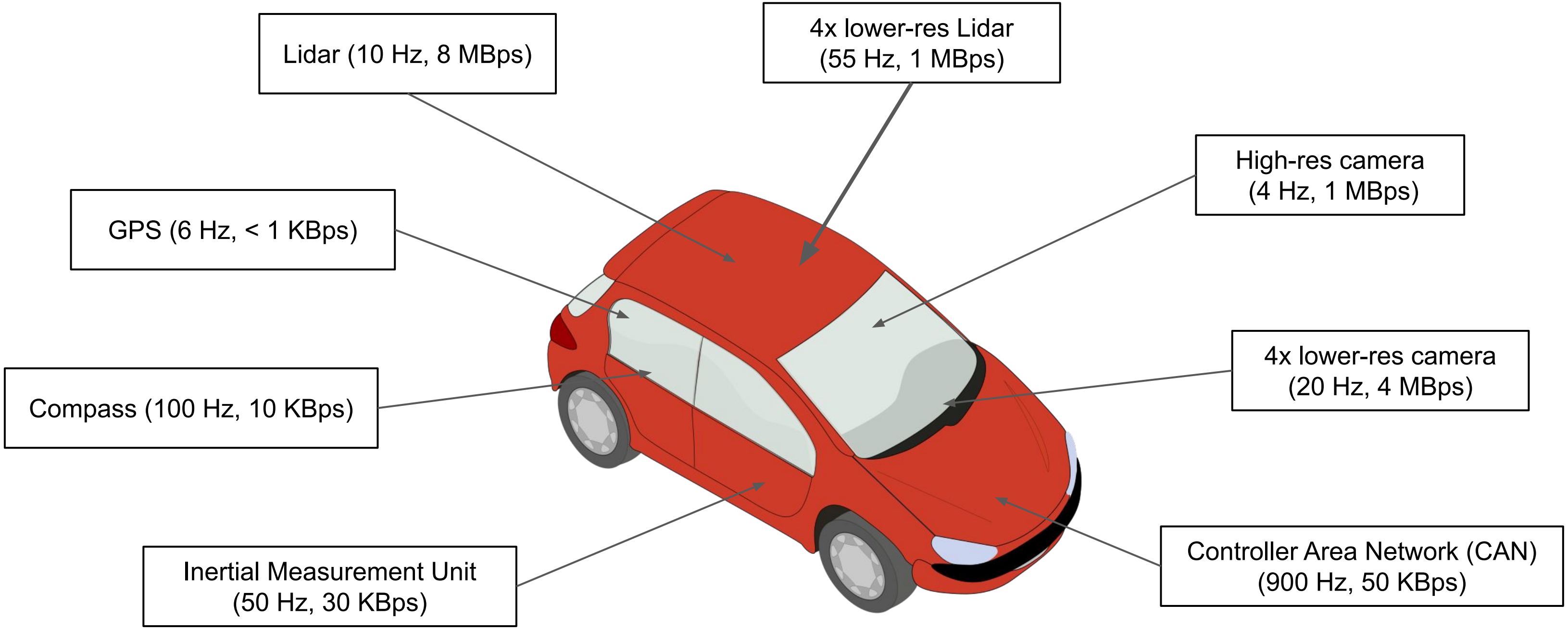
C Median of all temperature values

D Maximum range of all temperature values

(Self-Driving) Cars

- Sensor data generated during a 40-minutes trip: 30 GB (13 MBps)

Source: Exploring big volume sensor data with Vroom (Moll et al., 2017)



Smart Cities

- Example: Lamppost-as-a-Platform (LaaP)

  Source: https://www.developer.tech.gov.sg/technologies/sensor-platforms-and-internet-of-things/lamppost-as-a-platform

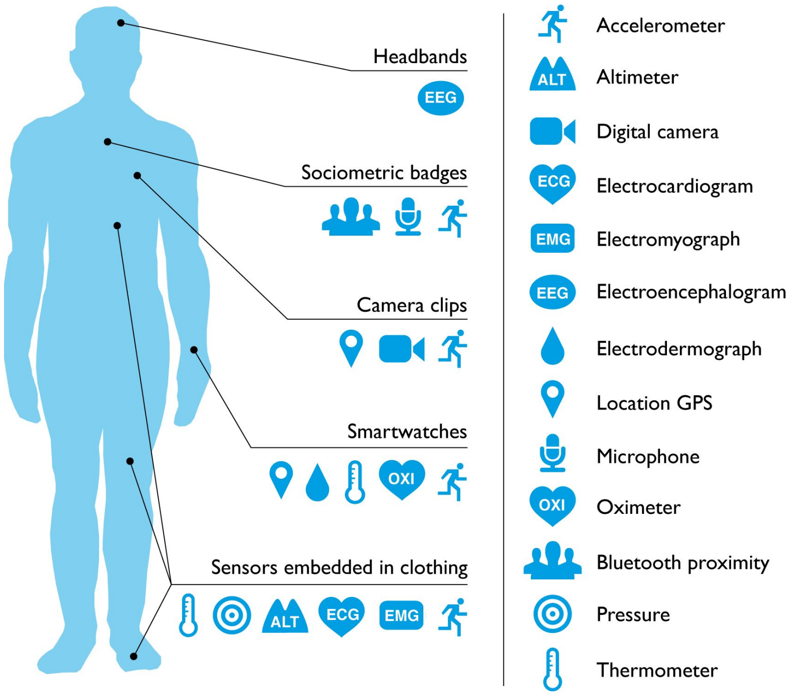
- Camera
- Temperature
- Humidity
- Gas (e.g., carbon monoxide)
- Air quality
- Rain

Health Monitoring

- ## Consumer Health Wearables

  Source: https://journals.plos.org/plosmedicine/article?id=10.1371/journal.pmed.1001953

  - Smartphones
  - Smartwatches
  - Fitness bands
  - Body cameras



Social Media

- Example: Daily data volume on Facebook

  Source: https://blog.wishpond.com/post/115675435109/40-up-to-date-facebook-facts-and-stats

  - 4.5 billion Likes
  - 17 billion location-tagged posts
  - 350 million photos uploaded
  - 4.75 billion items shared
  - 10 billion messages sent

Quick Sidenotes

- Covered techniques and algorithms not specific to streams
  - Applicable to many use cases involving large volumes of data

- Not of interest here: sliding window approaches
  - Keep the *k* most recent data items
  - Ignore all older items (delete from window)

  ➜ window = complete dataset (snapshot)

- Commonly used throughout the lecture: hash functions
  - We will assume "well-behaved" hash functions
    (same input ➜ same output, minimize duplication, avalanche effect)
  - No deeper discussion what this means here

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Sampling

- Sampling — basic definition
  - Process of selecting members of a population of interest (e.g., HDB residents)
  - Consideration of whole population typically impractical (e.g., too costly to survey all HDB residents)
  - Goal: statistical analysis of population (e.g., average happiness, most common complaints)

- Sampling data streams
  - Population = stream of incoming data items
  - Time and/or resource constraint generally make it impossible to consider all data items
  - Relevant patterns based on statistical analysis

➜ **Core challenge: How to get a representative sample?**

Problems with Naive Approach

- Toy example setup
  - Stream of search queries — items are tuples (user, query, time)
  - Goal: Fraction of queries issued more than once (here: twice) in the last 24h (by the average user)
  - Restriction only 10% of all tuples should be stored

- Naive approach: Store latest tuple with a probability of 1/10
  - Let *s* = number of time a user issued a query **once**
  - Let *d* = number of time a user issued a query **twice**

Correct estimate:
(assuming all tuples)

$$\frac{d}{s+d}$$

Estimate based on naive sample:

$$\frac{d}{10s+19d}$$

Problems with Naive Approach — Explanation

- The issue: of *d* queries issued twice
  - *d*/100 will be both in the sample $\left(\frac{1}{10}\right)\left(\frac{1}{10}\right) \cdot d = \frac{1}{100}d$
  - 18*d*/100 will be only once in the sample $2 \cdot \left(\frac{1}{10}\right)\left(\frac{9}{10}\right) \cdot d = \frac{18}{100}d$

↳ passa un filtr

- Fractions of queries issued twice in the sample:

$$\frac{\frac{1}{100}d}{\frac{1}{100}d + \frac{18}{100}d + \frac{1}{10}s} = \frac{d}{10s + 19d}$$

Sample with a Given Probability

- General Approach
  - Given: items/tuples with *n* components, e.g., (user, query, time)
  - Select subset of components as key on which the selection of the sample is based
    (for toy example: key = "user" ➜ consider only 10% of users instead of arbitrary tuples)
  - Choice of key depends on goal of analysis

- Question: How to obtain a sample consisting of any fraction a/b of keys?
  (in other words: How to decide whether to keep or discard a new tuple?)

- Common implementation via hash function
  - Define hash function h that maps *h*(*key*) in to *b* buckets 1..*b*
  - For each new tuple, if h(*key*) ≤ *a* (with a ≤ b), add tuples to sample

Sample with a Given Size Limit — Reservoir Sampling

- Goal: Maintain a uniform random sample of fixed size B
  - Uniform random = each item has the same probability to be sampled
  - Allows to approximate basic statistics such as mean, variance, median, etc.

- Basic algorithm
  - Input stream of items $\{a_1, a_2, ...\}$
  - Maximum reservoir size B
  - It can be shown that each item in the reservoir was sampled with the same probability ***B*/*t***
    (at any time t)

1) Add a_t with $t \leq B$ to reservoir
2) When receiving a_t with $t > B$:
   with probability ***B*/*t***, replace random item in reservoir with new item a_t

Reservoir Sampling — Example

- Initialization: t = 0, B = 3

Stream:

t = 1	**A**
t = 2	**B**
t = 3	**C**
t = 4	**A**
t = 5	**C**
t = 6	**B**
t = 7	**B**
t = 8	**A**
t = 9	**C**

Reservoir:

1) Add a_t with $t \le B$ to reservoir
2) When receiving a_t with $t > B$:

   with probability ***B*/*t***, replace random item in reservoir with new item a_t

Reservoir Sampling — Example

- $t = 3$

Stream:

t = 1	**A**
t = 2	**B**
t = 3	**C**
t = 4	**A**
t = 5	**C**
t = 6	**B**
t = 7	**B**
t = 8	**A**
t = 9	**C**

Reservoir:

A
B
C

1) Add a_t with $t \le B$ to reservoir
2) When receiving a_t with $t > B$:
   with probability ***B*/*t***, replace random item in reservoir with new item a_t

Just fill up the reservoir...

Reservoir Sampling — Example

- $t = 4$

Stream:

t = 1	**A**
t = 2	**B**
t = 3	**C**
t = 4	**A**
t = 5	**C**
t = 6	**B**
t = 7	**B**
t = 8	**A**
t = 9	**C**

Reservoir:

A
B
~~**C**~~ **A**

1) Add a_t with $t \leq B$ to reservoir
2) When receiving a_t with $t > B$:

   with probability $\boldsymbol{B/t}$, replace random item in reservoir with new item a_t

B=3

Probability B/t = 3/4

➜ Randomized decision: add a_4 = A to reservoir

➜ Replace random element — here B_3 = C with A

P(getting kicked out) = 1/B

Reservoir Sampling — Example

- $t = 5$

Stream:

t = 1	**A**
t = 2	**B**
t = 3	**C**
t = 4	**A**
t = 5	**C**
t = 6	**B**
t = 7	**B**
t = 8	**A**
t = 9	**C**

Reservoir:

A
~~**B**~~ **C**
A

1) Add a_t with $t \leq B$ to reservoir
2) When receiving a_t with $t > B$:
   with probability ***B*/*t***, replace random item in reservoir with new item a_t

Probability B/t = 3/5

➜ Randomized decision: add a_5 = C to reservoir

➜ Replace random element — here B_2 = B with C

Reservoir Sampling — Example

- $t = 6$

Stream:

t = 1	**A**
t = 2	**B**
t = 3	**C**
t = 4	**A**
t = 5	**C**
t = 6	**B**
t = 7	**B**
t = 8	**A**
t = 9	**C**

Reservoir:

A
C
A

1) Add a_t with $t \leq B$ to reservoir
2) When receiving a_t with $t > B$:
   with probability ***B*/*t***, replace random item in reservoir with new item a_t

Probability $B/t = 3/6$

➜ Randomized decision: discard $a_6 = \text{B}$

Reservoir Sampling — Example

- $t = 7$

Stream:

t = 1	**A**
t = 2	**B**
t = 3	**C**
t = 4	**A**
t = 5	**C**
t = 6	**B**
t = 7	**B**
t = 8	**A**
t = 9	**C**

Reservoir:

A
C
~~A~~ B

$\rightarrow p \ 3/7 \ ?$

$\rightarrow p = 3/7 \ ?$

$\rightarrow p = 3/7$ ✓

1) Add a_t with $t \leq B$ to reservoir

2) When receiving a_t with $t > B$:

   with probability ***B*/*t***, replace random item in reservoir with new item a_t

Probability $B/t = 3/7$

➜ Randomized decision: add a_7 = B to reservoir

➜ Replace random element — here B_3 = A with B

Proof Sketch — All Elements in *B* sampled with *B* / *t*

$t = 100$

- Obvious case: $i = t$
  - a_i was inserted into B with probability B/t (direct result from algorithm)

- Otherwise: $i < t$
  - Observation: in step i, an item gets replaced with probability $B/i * 1/B = 1/i$
  - Probability of an item in reservoir

e.g. 43

t

$$B/i \cdot \left(1 - \frac{1}{i+1}\right) \cdot \left(1 - \frac{1}{i+2}\right) \cdot \ldots \cdot \left(1 - \frac{1}{t}\right) = B/t$$

Probability that a_i was inserted in B

Probability that a_i was NOT replaced in B at step (i+1)

Probability that a_i was NOT replaced in B at step (i+2)

Probability that a_i was NOT replaced in B at step t

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Filtering Data Streams

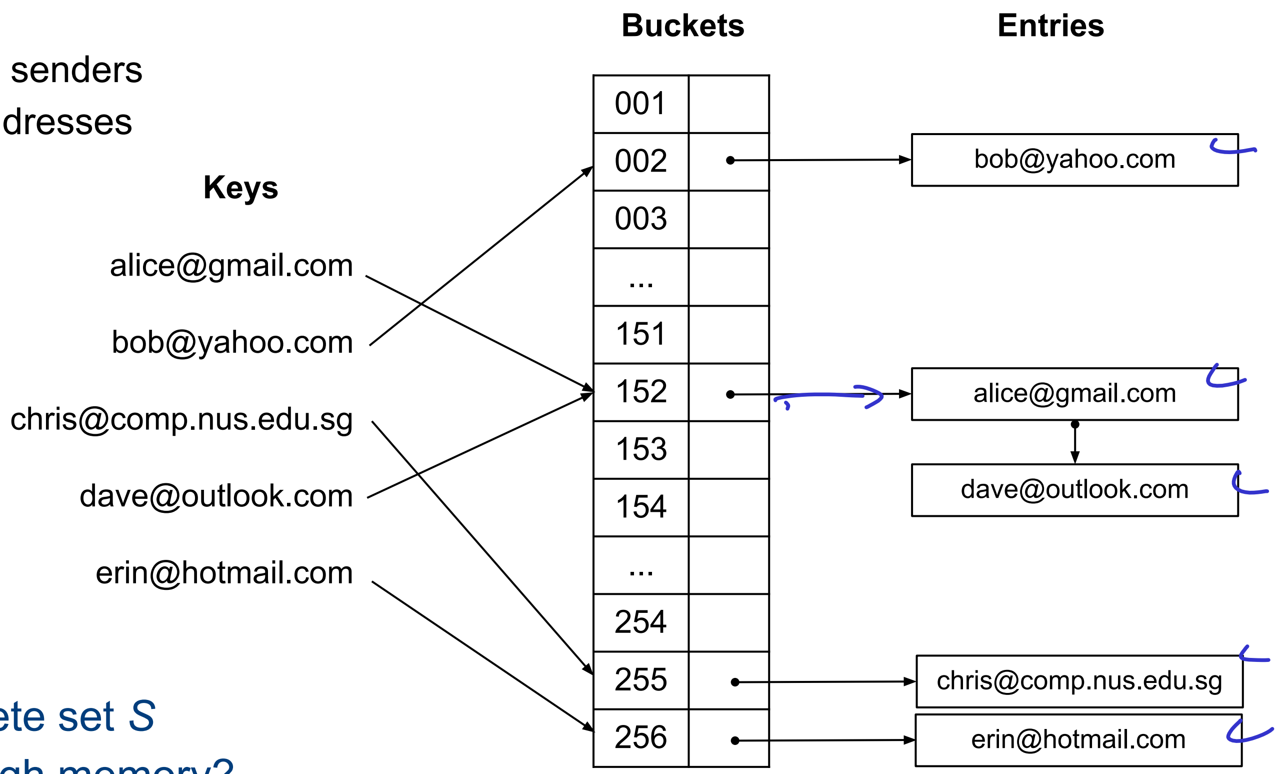
- Goal: Only accept items that meet certain criterion

- Simple: criterion is a property of item that can be (easily) calculated, e.g.:
  - Search queries with more than 2 keywords
  - Sensor values with valid status code

- Challenge: criterion involves lookup for membership in a (very large) set S, e.g.:
  - Emails with sender addresses that are whitelisted (spam filter)
  - Page visits to a predefined set of websites
  - Tweets from a selected group of users

Basic Solution — Create Hash Table for Set *S*

- Example: spam filter
  - Only accept emails from senders with whitelisted email addresses



➜ Requires to store complete set *S*

➜ What if there is not enough memory?

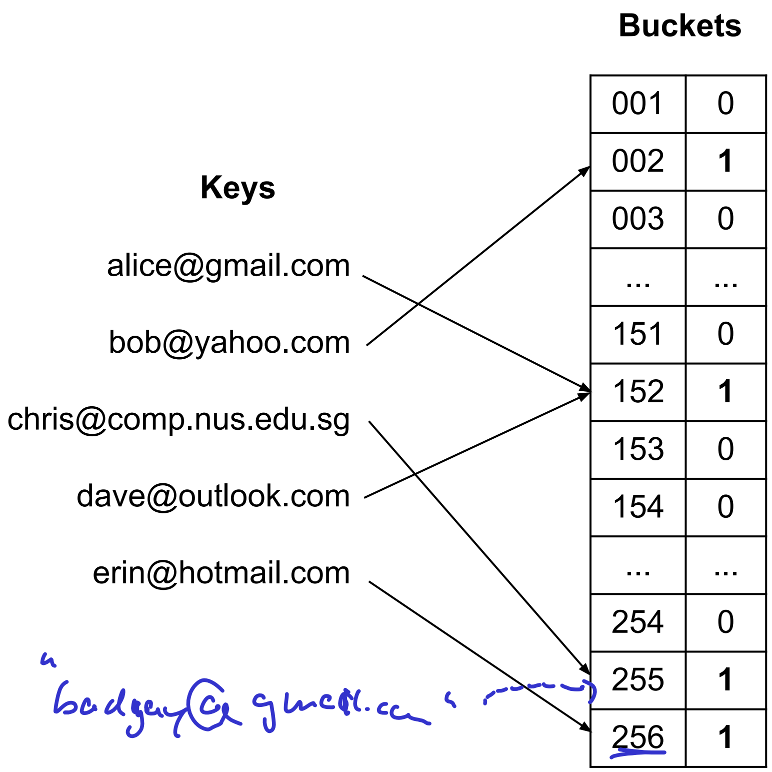
Hashing without Storing *S*

- Create lookup table
  - Create bit array B with 1..n bits and B[i] = 0
  - Choose hash function h(key) $\in$ [1, n]
  - For each key $s \in S$ set B[h(s)] = 1

- Filter step for new data item with key k
  - Accept if B[h(k)] = 1, discard otherwise

➜ Problem: **False Positives**
  - $a \in S$, $b \notin S$ and h(a) = h(b)

How bad is it?



Quick Quiz

Can **false negatives** occur?

A No

B Yes, but the probability is negligible

C Yes, but they do not matter for the application context.

D Yes, and that's why this is a bad approach.

False Positive Rate — Analysis

$$e^p = \left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n\right]^p$$

Probability of B[*i*] = 1 after inserting **one key** from *S*: $\frac{1}{|B|}$

Probability of B[*i*] = 0 after inserting **one key** from *S*: $1 - \frac{1}{|B|}$

Probability of B[*i*] = 0 after inserting **all keys** from *S*: $\left(1 - \frac{1}{|B|}\right)^{|S|} = \left(1 - \frac{1}{|B|}\right)^{|B|\frac{|S|}{|B|}} \approx e^{-\frac{|S|}{|B|}}$

Probability of B[*i*] = 1 after inserting **all keys** from *S*: $1 - e^{-\frac{|S|}{|B|}}$

Probability of B[h(*s*)] = 1 with key s ∉ *S*: $1 - e^{-\frac{|S|}{|B|}}$

(probability of a false positive)

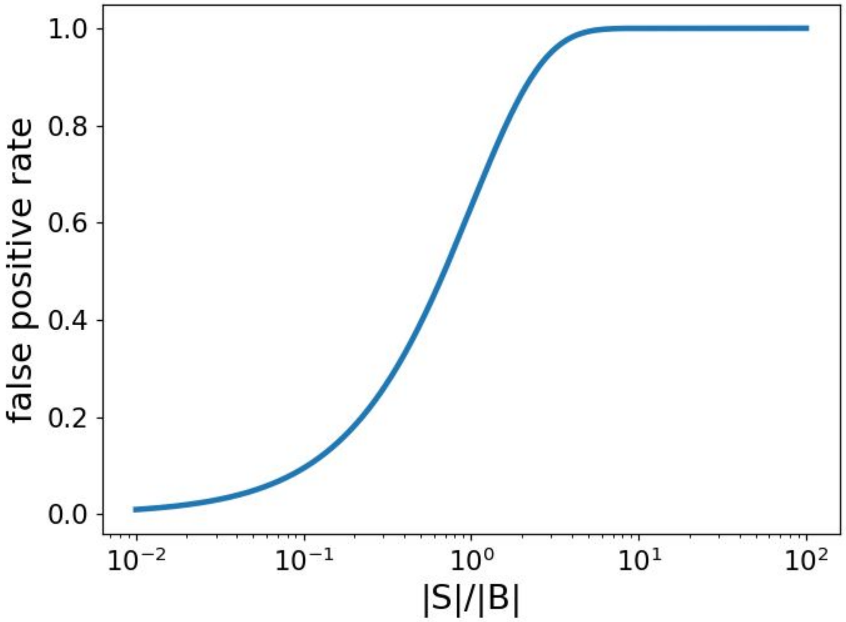
False Positive Rate — Visualization

- Effect of ratio |S|/|B| on false positive rate

- Example calculation
  - $|S| = 10^9$ whitelisted email addresses
  - $|B| = 8 \cdot 10^9$ (1 GB of main memory)

➜ $|S|/|B| = 1/8$

➜ False positive rate: $1 - e^{-1/8} =$ **11.75%**



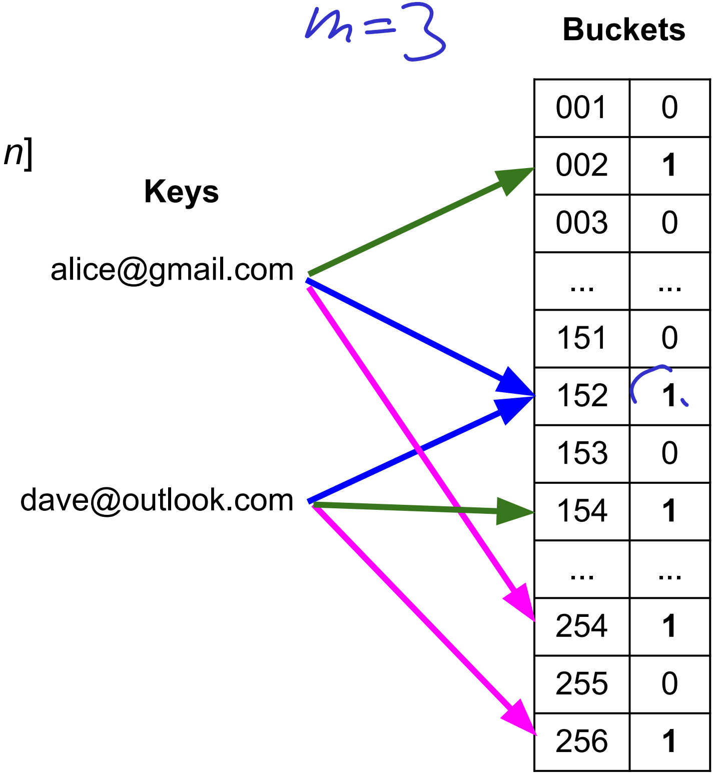
How to reduce false positive rate?

(without increasing the size of bit array B)

Bloom Filters

- Bloom Filters — basic idea
  - Create bit array B with 1..n bits and B[i] = 0
  - Choose ***m* independent hash functions** $h_i(key) \in [1, n]$
  - For each key $s \in S$ set $B[h_i(s)] = 1$, with $1 \le i \le m$
- Example
  - 3 hash functions: h_1, h_2, h_3
- Filter step for new data item with key k
  - Accept if $B[h_i(k)] = 1$ for all $1 \le i \le m$, discard otherwise

$m=3$



False Positive Rate — Analysis

Probability of B[h(*s*)] = 1 with key s ∉ *S*:

(probability of a false positive)

$$1 - e^{-\frac{|S|}{|B|}}$$

from 1 to *m* hash functions

Probability of B[h_i(*s*)] = 1 (1 ≤ *i* ≤ *m*) with key s ∉ *S*:

(probability of a false positive)

$$\left(1 - e^{-m\frac{|S|}{|B|}}\right)^{m}$$

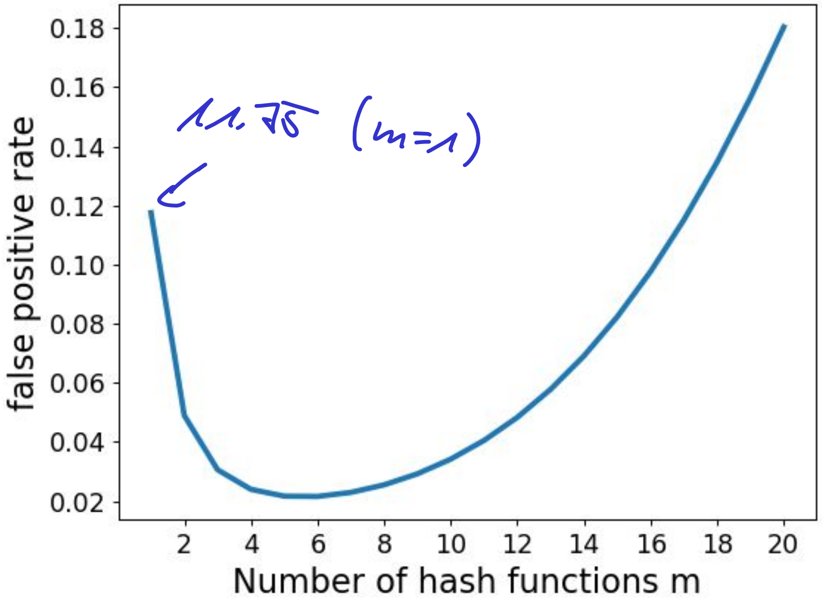
False Positive Rate — Visualization

- Effect of number of hash functions *m* on false positive rate
  - Here, |S|/|B| = 1/8 (see previous example)

- Global optimum — intuition: large *m*
  - More hash results need all be 1
  - But: also more 1s in bit array

- Optimal value for *m*

$$m_{best} = \frac{|B|}{|S|} ln2$$



$$m_{best} = \frac{8}{1} ln2 = 5.5 \approx 6$$

➜ False positive rate: **2.2%**

Bloom Filter — Discussion

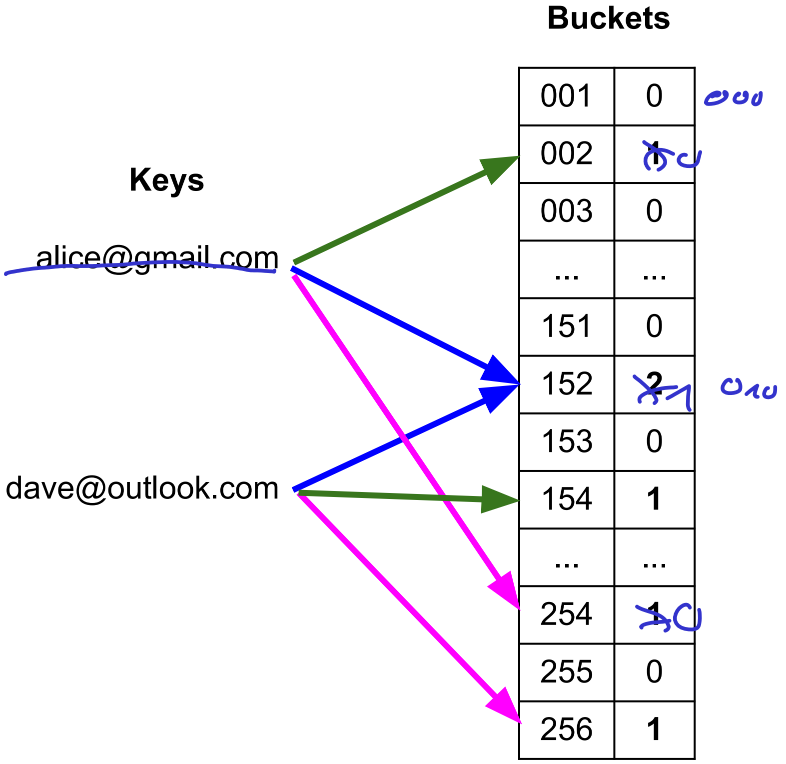
- Memory and space consumption
  - Time complexity: O(m)
  - Space complexity: O(|B|)

- Limitation: no support of removing keys from *S*
  - E.g.: not able to remove a whitelisted email address
  - Only workaround: rebuild lookup table (bit array) from scratch

Why?

Extension — Counting Bloom Filters

- Counting Bloom Filters 8-16
  - Replace bits for counters (typically 3-4 bits per counter)
  - Increased size of lookup table (3-4 times)
- Operations
  - Insert *s*: B[h_i(*s*)] **+= 1**, with 1 ≤ i ≤ m
  - Delete *s*: B[h_i(*s*)] **−= 1**, with 1 ≤ i ≤ m
  - Lookup k: Accept if B[h_i(*k*)] **> 0** for all 1 ≤ *i* ≤ *m*
- False positive rate
  - Same as normal Bloom Filter: $\left(1 - e^{-m\frac{|S|}{|B|}}\right)^{m}$



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Counting Unique/Distinct Elements

- Applications
  - Number of unique Facebook visitors (identified by user id)
  - Number of unique website visitors (identified by IP address)
  - Number of unique words in a (large) text document

- Straightforward solution
  - Maintain set *S* of items seen so far
  - Number distinct elements ➜ |*S*|

➜ **What if set *S* can grow very large?**

```python
S = set()

with open('data/ip-only-nasa-access.log') as file:
    for line in file:
        ip = line.strip()  # Get IP address
        S.add(ip)          # Add IP address to set

print('|S| = {}'.format(len(S)))
```

```
|S| = 7637
```

Flajolet-Martin Algorithm

$IP_1 \rightarrow \overbrace{0110 \ldots 11001000}^{32 \text{ bits}}$

$r(s) = 3$

$IP_2 \rightarrow \ldots 01110$

$r(s) = 1$

$IP_3 \rightarrow \ldots 010000$

$r(s) = 4$

- Approximation approach
  - Estimate distinct count in an unbiased way
  - Accept errors but minimize their probability

- Basic algorithm

(1) Choose hash function that maps each of the n the elements to at least $log_2 n$ bits

(2) For each element s in the stream
  - (a) Calculate hash h(s) ➜ bit string of length $log_2 n$
  - (b) Let r(s) = number of trailing 0s in h(s)
  - (c) Keep track of largest r(s) ➜ R

(3) Return estimate for distinct count as 2^R

Example: 2^{32} IPv4 addresses ➜ at least 32 bits

Flajolet-Martin Algorithm — Example

a: IPv4 address	h(a)
13.66.139.0	01010100110100100110111000001001
157.48.153.185	110010100100101001001011011111**00** r(s)
157.48.153.185	110010100100101001001011011111**00**
216.244.66.230	0001110001010101100111010001001**0**
54.36.148.92	010101010001010110010111000011**00**
92.101.35.224	10100011100001100000000100000001
73.166.162.225	00100100100111100100101101011**000**
73.166.162.225	00100100100111100100101101011**000**
54.36.148.108	00001100010100011000000001010001
54.36.148.1	011001011000101000101100100001**00**
162.158.203.24	1011100001001001010011100010111**0**
157.48.153.185	110010100100101001001011011011**00**
157.48.153.185	110010100100101001001011011011**00**
...	...

R = 3 (largest number of trailing 0s so far)

➜ Estimate for distinct count: $2^3 = 8$

Quick "Quiz"

$P($"at least 1× HHHH after k tries"$) =$

$= 1 - P($no HHHH after k tries$)$

$= 1 - \left(\frac{7}{8}\right)^k$

Someone is telling you that s/he flipped a fair coin **3 times** and got **3 Heads** after ***k* tries**?

Which **number of tries** is the most believable?

$k=1 \rightarrow 1/8$

$k=10 \rightarrow 0.737$

$k=100 \rightarrow 0.999$

$k=1000 \rightarrow$

A k = 1 $\left(\frac{1}{2}\right)^3$

B ✓ k = 10

C k = 100

D k =1,000

Flajolet-Martin Algorithm — Intuition & Proof Sketch

- Basic intuition
  - More distinct elements ➜ more different hash values ➜ "unusual" hash values more likely
  - "Unusual" hash value = hash value with rare bit pattern (e.g., number of trailing 0s)

Probability that h(*a*) ends in **at least** k trailing 0s

$$\overbrace{\frac{1}{2}\cdot\frac{1}{2}\cdot\ldots\cdot\frac{1}{2}}^{k \text{ factors}} = \frac{1}{2^k} = s^{-k}$$

Probability that h(*a*) ends in **less than** k trailing 0s

$$1 - \frac{1}{2^k}$$

Given *m* distinct elements, probability that **all** h(*a*) end **in less** than k trailing 0s

$$\left(1 - \frac{1}{2^k}\right)^m$$

Given *m* distinct elements, probability that **R ≥ k**
(i.e., at least one of the *m* elements has h(*a*) with at least k trailing 0s)

$$1 - \left(1 - \frac{1}{2^k}\right)^m$$

Flajolet-Martin Algorithm — Proof Sketch

$$e^p = \left[\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n\right]^p$$

Given *m* distinct elements, probability that **R ≥ k**
(i.e., at least one of the *m* elements has h(*a*) with at least k trailing 0s)

$$1-\left(1-\frac{1}{2^k}\right)^m = 1-\left(1-\frac{1}{2^k}\right)^{2^k\frac{m}{2^k}} \approx 1-e^{-\frac{m}{2^k}}$$

Case 1: $2^k \ll m \quad \rightarrow \quad 1-e^{-\frac{m}{2k}} \approx 1-0=1$

Case 2: $2^k \gg m \quad \rightarrow \quad 1-e^{-\frac{m}{2k}} \approx 1-\underbrace{\left(1-\frac{m}{2^k}\right)}_{\text{First 2 terms of the Taylor expansion of } e^x} \approx \frac{m}{2^k} \approx 0$

$$\boxed{e^x = 1+\frac{x}{1!},\ \text{if } x \ll 1}$$

$$e^x = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots$$

Flajolet-Martin Algorithm — Proof Sketch

2^R

- Interpretation

Case 1: $2^k \ll m$ ➜ $P(R \geq k) \approx 1$

The probability to get an h(*a*) with
enough trailing 0s is rather high

Case 2: $2^k \gg m$ ➜ $P(R \geq k) \approx 0$

The probability to get an h(*a*) with
too many trailing 0s is rather low

➜ R is typically in the right ballpark

Flajolet-Martin Algorithm — Problems & Extensions

- Obvious problem with basic Flajolet-Martin algorithm — given 2^R
  - An estimate is always a power of 2
  - If R is off by just 1, estimates doubles or halves

$p = 5$
$q = 4$ } → 20 hash function

- Practical solution: use multiple hash functions $h_i(a)$ — e.g.:
  - $p \cdot q$ hash functions ➜ $p \cdot q$ R values ➜ $p \cdot q$ estimates for the distinct counts
  - Put all estimates into p groups, each of size q
  - Calculate median of each group ➜ p medians
  - Calculate the mean over all p medians

Outline

- Motivation
  - Basic setup
  - Example Applications
- Core Techniques
  - Sampling
  - Filtering
  - Counting (distinct items)
- **Summary**

Summary

- Data streams — main challenges
  - Large data volumes + high arrival speeds
  - Limited resources + real-time requirements

  ➜ **patterns = statistical analysis**

- Common tasks on streams
  (or very large datasets in general)
  - Sampling
  - Filtering
  - Counting
    (distinct elements)

  ➜ trade-off: **speed / resource-efficiency** vs. **accuracy / errors**

Solutions to Quick Quizzes

- Slide 7: C
- Slide 30: A
- Slide 42: B