Lecture 9: Planning and Learning

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Outline

- Monte Carlo Tree Search
- Model-based Planning
- Integrating Learning and Planning

Note: Chapter 8 & Chapter 2 (2.7 on UCB) of
The textbook "Reinforcement Learning: An Introduction"

*materials are modified from David Silver's RL lecture notes

Outline

- Monte Carlo Tree Search
    - Motivation: Computer GO
    - Upper Confidence Bound
    - MC Tree Search (MCTS)
- Model-based Planning
- Integrating Learning and Planning

Model-Based RL Revisited

- Model is known, but curse of dimensionality
  - Each sweeping over states using DP is computationally costly.
- Game “GO” has high move and state complexity:
  - States: 10^171
- Studied for decades

- What can we do ….

Rules of Go

- Usually played on 19x19, also 13x13 or 9x9 board
- Simple rules, complex strategy
- Black and white place down stones alternatively
- Surrounded stones are captured and removed
- The player with more territory wins the game

Position Evaluation in GO

- How good is a position *s*?
- Reward function (undiscounted):

$$R_t = 0 \text{ for all non-terminal steps } t < T$$

$$R_T = \begin{cases} 1, & \text{if Black wins} \\ 0, & \text{if White wins} \end{cases}$$

- Policy $\pi = \langle \pi_B, \pi_W \rangle$ selects moves for both players
- Value function (how good is position *s*):

$$v_\pi(s) = \mathrm{E}_\pi[R_T \mid S = s] = \mathrm{P}[\text{Black wins} \mid S = s]$$

$$v_*(s) = \max_{\pi_B} \min_{\pi_W} (v_\pi(s))$$

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Upper Confidence Bounds (UCB)

- One-step bandit problem – a row of slot machine, each with different payout probabilities and amounts.
- Fundamental tradeoff of exploration and exploitation
- What is the optimal exploration?
  - We have learnt non-optimal Ɛ-greedy and Softmax.

Definition of “Optimal”

μ_*: The average (or mean or expected) reward of the best action
μ_j: The average (or mean or expected) reward of any other action *j*
K: The total number of possible actions
n: The total number of tries
n_j: The number of times that the action *j* has been tried during *n* tries

$$reget(n) = \mu^* n - \sum_{j=1}^{K} \mu_j * E[n_j]$$

- The regret is the loss due to the fact that the globally optimal policy is not followed all the times
- Our goal is to minimize the regret
- Lai and Robbins showed that the regret for the multi-armed bandit problem has to grow at least logarithmically w.r.t. the number of plays *n*
    - "Asymptotically Efficient Adaptive Allocation Rules" (1985)
- UCB is proved to grow the regret logarithmically -> optimal policy

Upper Confidence Bounds 1 (UCB1)

- The simplest UCB policy is called UCB1 *

exploit

explore

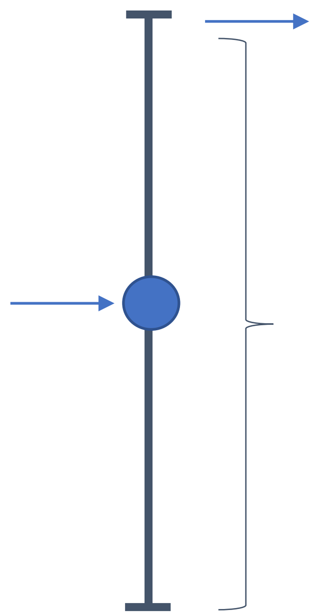
$$UCB1 = \bar{X}_j + C\sqrt{\frac{\ln n}{n_j}}$$

$\bar{X}_j$ is the estimated reward of choice *j*
n is the number of all plays done so far
n_j Is the number of times choice *j* has been tried
C is a constant (e.g., $\sqrt{2} = 1.414$)

*P. Auer, et al. "Finite-time Analysis of the Multiarmed Bandit Problem", 2002

Upper Confidence Bounds 1 (UCB1)

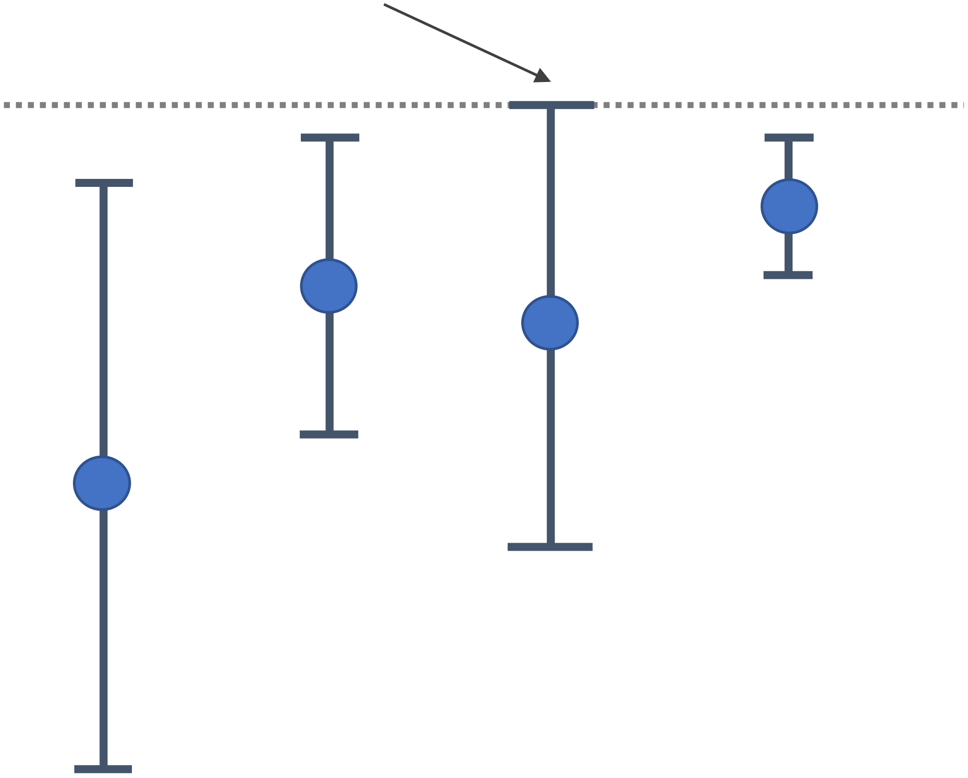
Confidence in the Reward's Accuracy



More visits -> large n_j -> tighter bound

Upper Confidence Bounds 1 (UCB1)

Most urgent node has **the highest UCB** (Not highest reward & Not widest spread)



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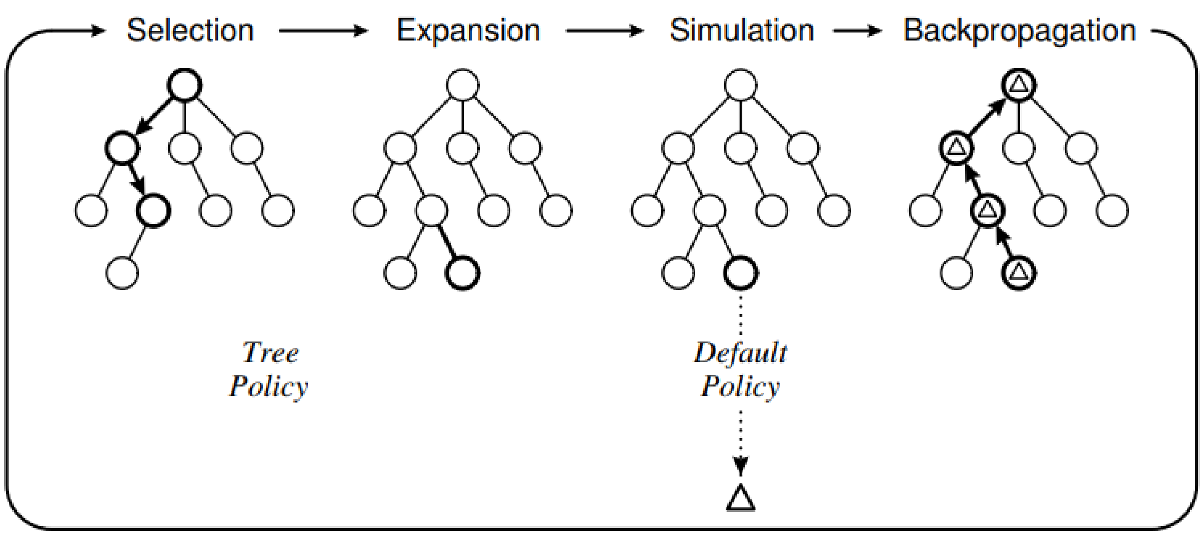
Monte Carlo Tree Search (MCTS)

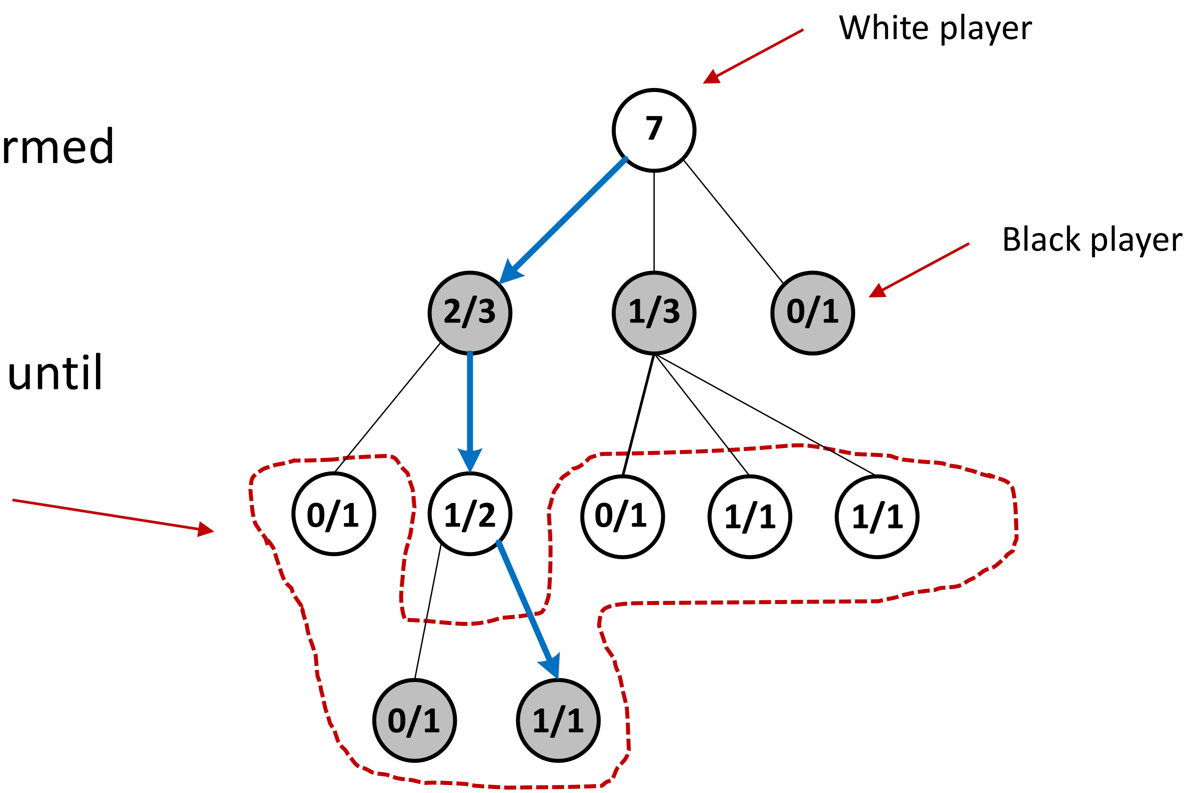
- A method for finding optimal decisions by taking random samples and building a search tree according to the results
- Profound impact on AI
- MCTS includes four steps:



Browne et al (2012)

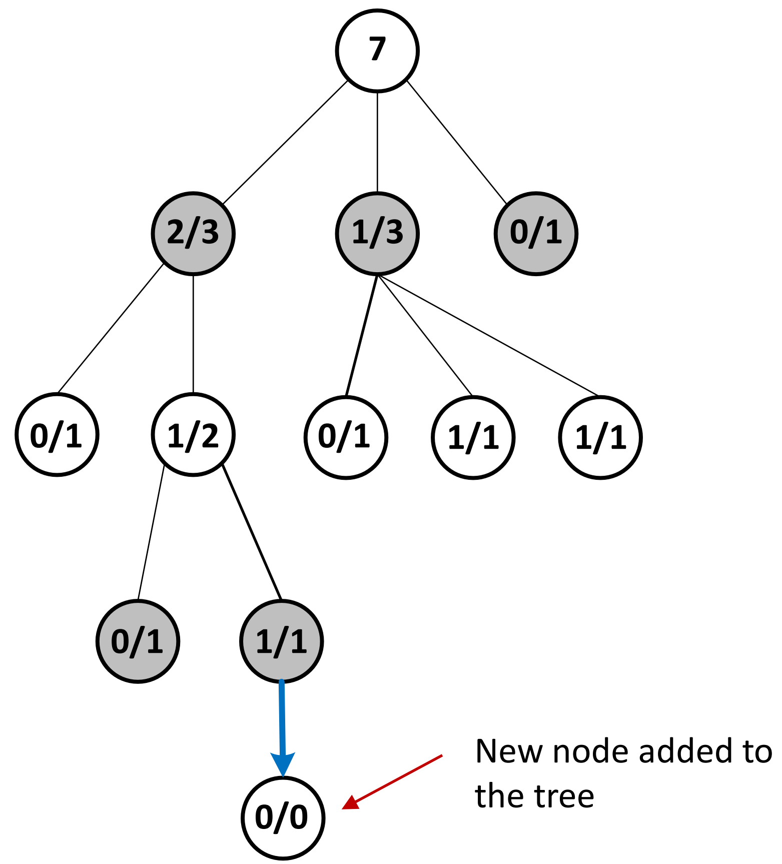
Step 1: Selection (Tree Descent)

- Start at Root
- Select a child by an informed policy, e.g. UCB1
- Move to the child
- Repeat above step 2&3 until hitting tree boundary



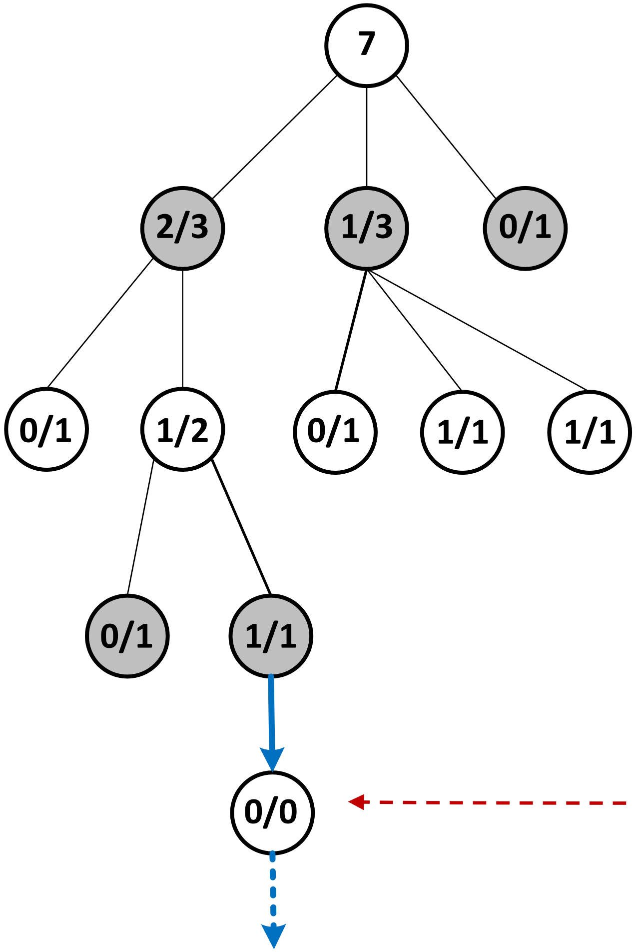
Step 2: Expansion

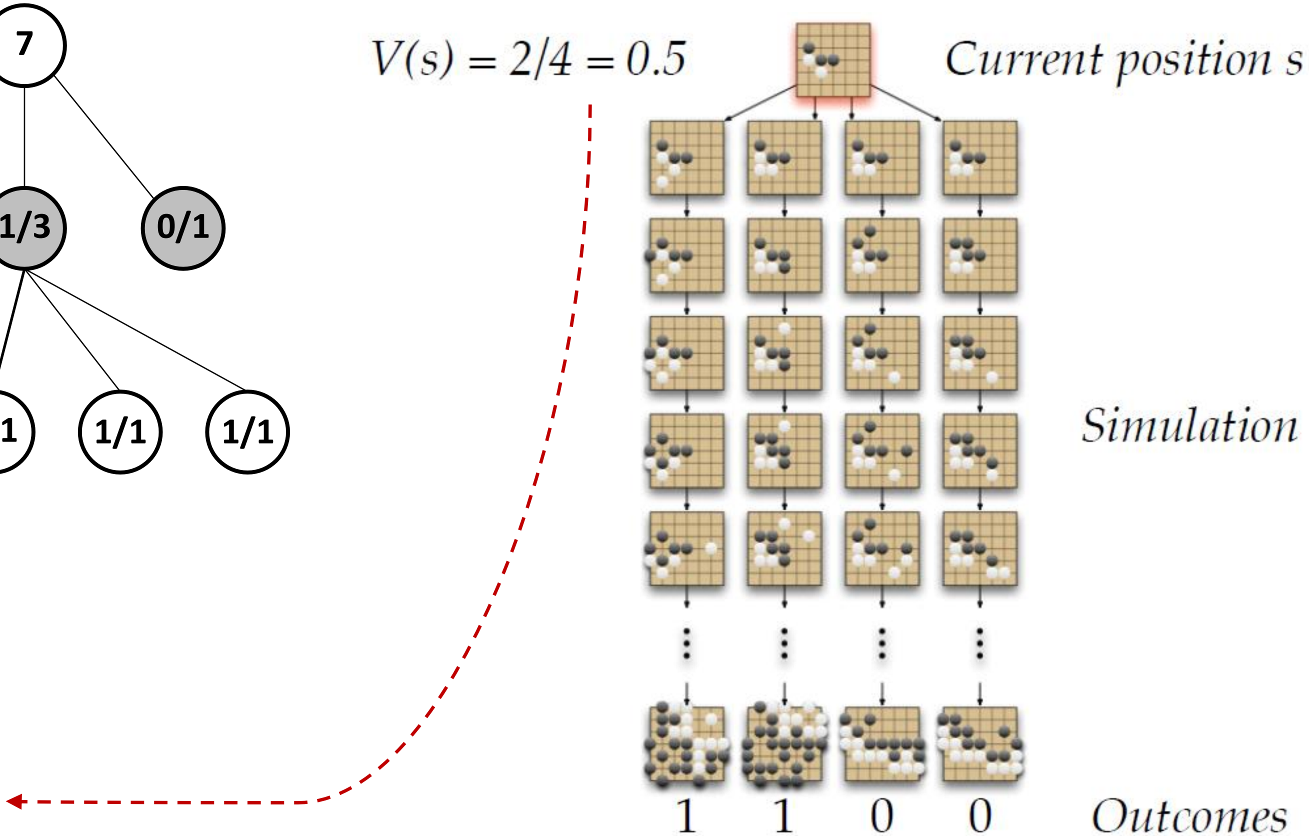
- At boundary, no longer apply UCB1
- Then, an unvisited child position is randomly chosen
- A new record node is added to the tree



Step 3: Simulation

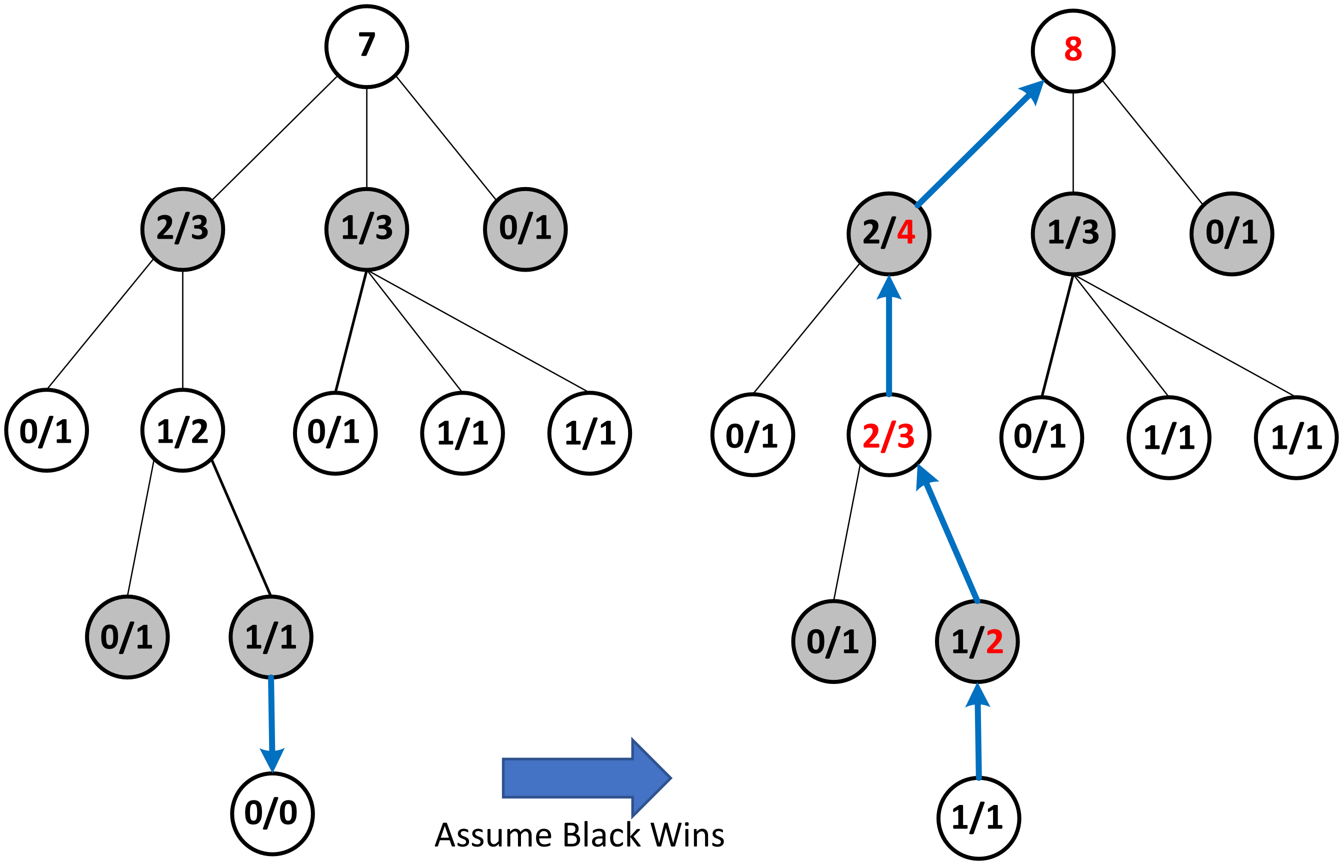
- Run K times typical MC simulation





Step 4: Back-Propagation

- All of the records of nodes in the path/branch taken are updated
- Each has its play count incremented by one
- Each that matches the winner has its win count increased by one



Pro and Cons

- Pros:
  - Tree growth focuses on more promising areas
  - Can stop algorithm anytime to get search results
  - Avoid the problem of globally approximating an action-value function
  - Convergence to minimax solution

- Cons:
  - Memory intensive: entire tree in memory
  - For complex problems, enhancement needed for good performance

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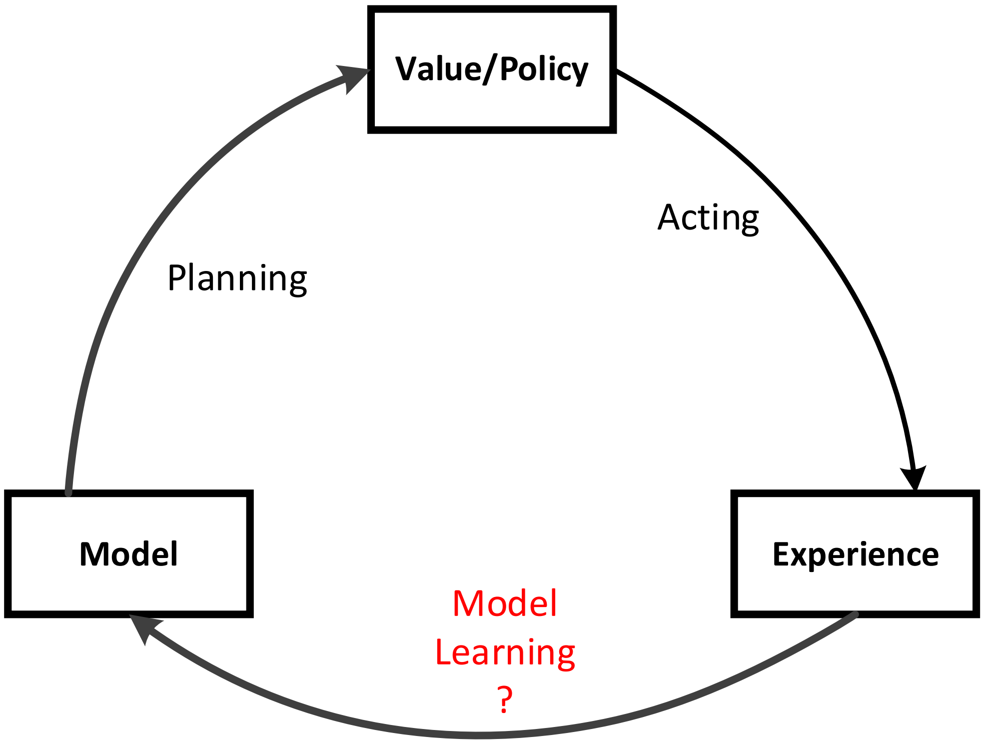
We have learnt…

- Model-Free RL
  - No Model
  - Learn value functions or policy from experience
- Model-based RL
  - Model is known
  - Plan value function or policy from model: DP algorithms, MC tree search

Question:

- If model is unknown, can we first learn model and then use the model to plan?



What is a Model?

- A *model* $\mathcal{M}$ is a representation of an MDP $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R} \rangle$, parametrized by η
- We will assume state space $\mathcal{S}$ and action space $\mathcal{A}$ are known
- So a model $\mathcal{M} = \langle \mathcal{P}_\eta, \mathcal{R}_\eta \rangle$ represents state transitions $\mathcal{P}_\eta \approx \mathcal{P}$ and rewards $\mathcal{R}_\eta \approx \mathcal{R}$

$$S_{t+1} \sim \mathcal{P}_\eta(S_{t+1} \mid S_t, A_t)$$
$$R_{t+1} = \mathcal{R}_\eta(R_{t+1} \mid S_t, A_t)$$

- Typically assume conditional independence between state transitions and rewards

$$\mathbb{P}\left[S_{t+1}, R_{t+1} \mid S_t, A_t\right] = \mathbb{P}\left[S_{t+1} \mid S_t, A_t\right] \mathbb{P}\left[R_{t+1} \mid S_t, A_t\right]$$

Model Learning

- Goal: estimate model $\mathcal{M}_\eta$ from experience $\{S_1, A_1, R_2, ..., S_T\}$
- This is a supervised learning problem

$$
\begin{aligned}
S_1, A_1 &\rightarrow R_2, S_2 \\
S_2, A_2 &\rightarrow R_3, S_3 \\
&\vdots \\
S_{T-1}, A_{T-1} &\rightarrow R_T, S_T
\end{aligned}
$$

- Learning $s, a \rightarrow r$ is a *regression* problem
- Learning $s, a \rightarrow s'$ is a *density estimation* problem
- Pick loss function, e.g. mean-squared error, KL divergence, ...
- Find parameters η that minimise empirical loss

Example: Table Lookup Model

- Model is an explicit MDP, $\hat{\mathcal{P}}, \hat{\mathcal{R}}$
- Count visits $N(s, a)$ to each state action pair

$$\hat{\mathcal{P}}^a_{s,s'} = \frac{1}{N(s,a)} \sum_{t=1}^{T} \mathbf{1}(S_t, A_t, S_{t+1} = s, a, s')$$

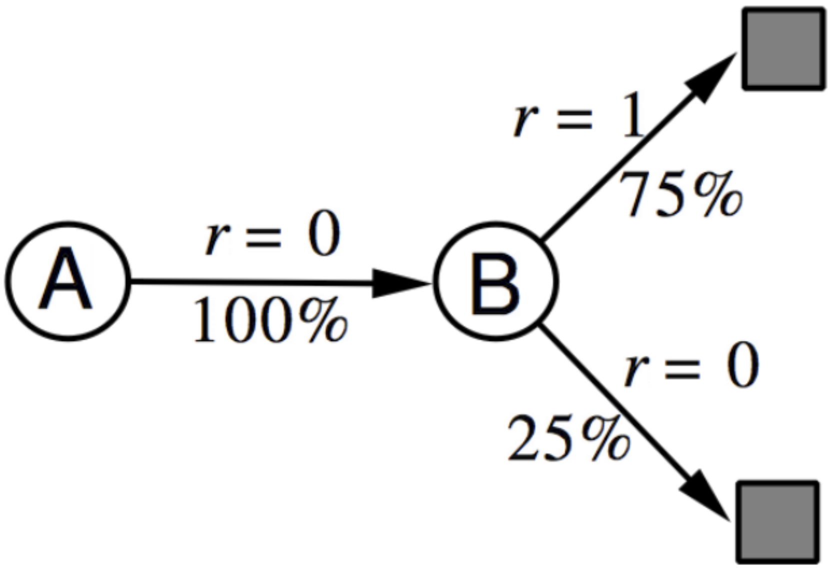
$$\hat{\mathcal{R}}^a_s = \frac{1}{N(s,a)} \sum_{t=1}^{T} \mathbf{1}(S_t, A_t = s, a) R_t$$

- Alternatively
  - At each time-step t, record experience tuple $\langle S_t, A_t, R_{t+1}, S_{t+1} \rangle$
  - To sample model, randomly pick tuple matching $\langle s, a, \cdot, \cdot \rangle$

A-B Example

- Two states A, B; no discounting; 8 episodes of experience

$A, 0, B, 0$
$B, 1$
$B, 1$
$B, 1$
$B, 1$
$B, 1$
$B, 1$
$B, 0$



- We have constructed a table lookup model from the experience

Planning with a Model

- Model is known
- Need to solve MDP
- Using planning algorithms we have learnt:
  - Value iteration
  - Policy iteration
  - Monte Carlo tree search
  - …
- Background Planning vs Decision-Time Planning
  - Background Planning: Not focus on the current state
    - Good for low-latency applications
  - Decision-Time Planning: Begin with the current state
    - Applications not requesting fast responses

Sample-based Planning

- A simple but powerful approach to planning
- Use the model only to generate samples
- Sample experience from model

$$S_{t+1} \sim \mathcal{P}_\eta(S_{t+1} \mid S_t, A_t)$$
$$R_{t+1} = \mathcal{R}_\eta(R_{t+1} \mid S_t, A_t)$$

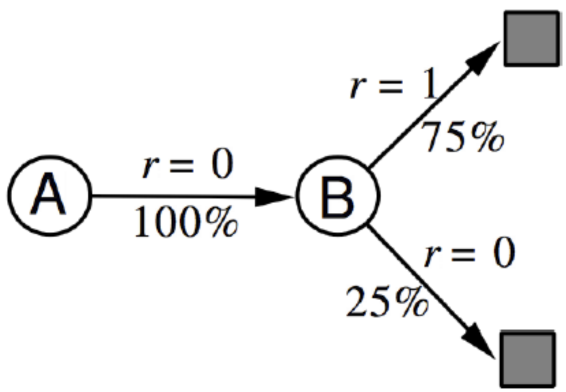
- Apply model-free RL to samples, e.g.:

  Monte-Carlo control
  Sarsa
  Q-learning
- Sample-based planning methods are often more efficient

AB Example

- Construct a table-lookup model from real experience
- Apply model-free RL to sampled experience

Real experience			
A,	0,	B,	0
B,	1		
B,	1		
B,	1		
B,	1		
B,	1		
B,	1		
B,	0		



Sampled experience			
B,	1		
B,	0		
B,	1		
A,	0,	B,	1
B,	1		
A,	0,	B,	1
B,	1		
B,	0		

e.g. Monte-Carlo learning: $V(A) = 1, V(B) = 0.75$

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Real and Simulated Experience

- We consider two sources of experience
- Real experience Sampled from environment (true MDP)

$$S' \sim \mathcal{P}^a_{s,s'}$$
$$R = \mathcal{R}^a_s$$

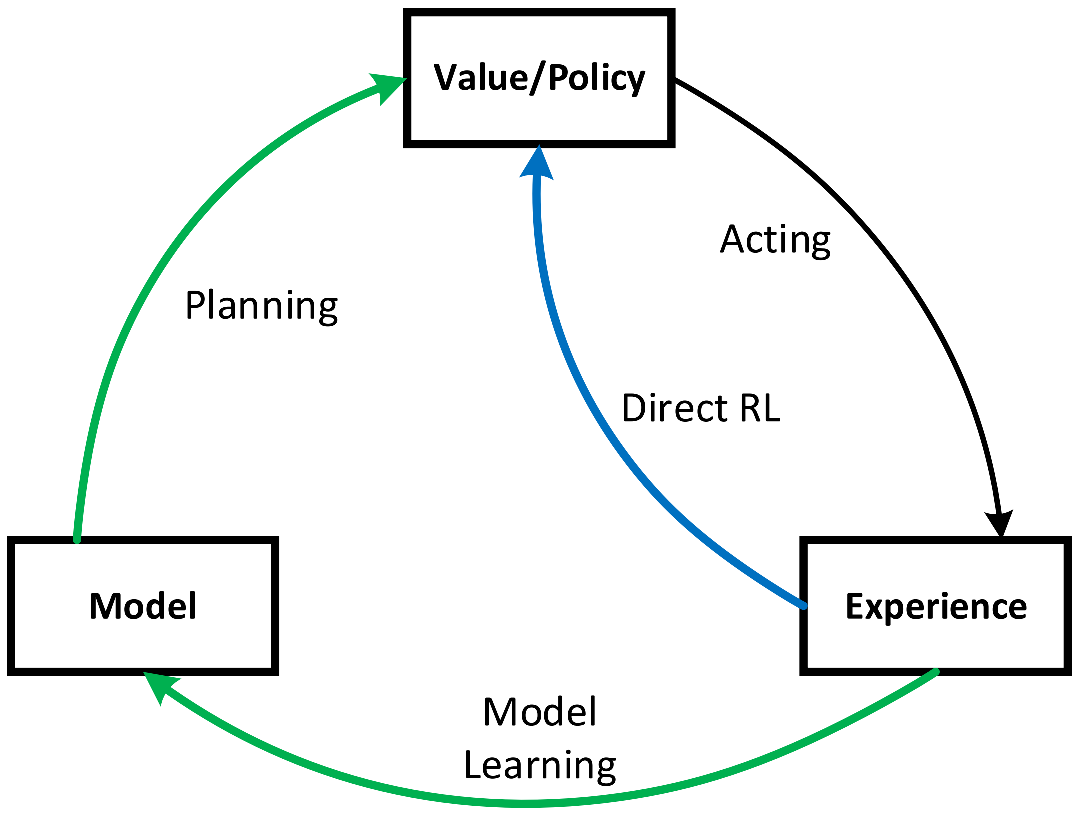
- Simulated experience Sampled from model (approximate MDP)

$$S' \sim \mathcal{P}_\eta(S' \mid S, A)$$
$$R = \mathcal{R}_\eta(R \mid S, A)$$

Integrating Learning and Planning

- Model Free RL
  - No model
  - Learn value function and/or policy from real experience
- Model based RL (using sample-based planning)
  - Learn a model from real experience
  - Plan value function and/or policy from simulated experience
- Dyna
  - Learn a model from real experience
  - Learn and plan value function and/or policy from real and simulated experience

Dyna Architecture

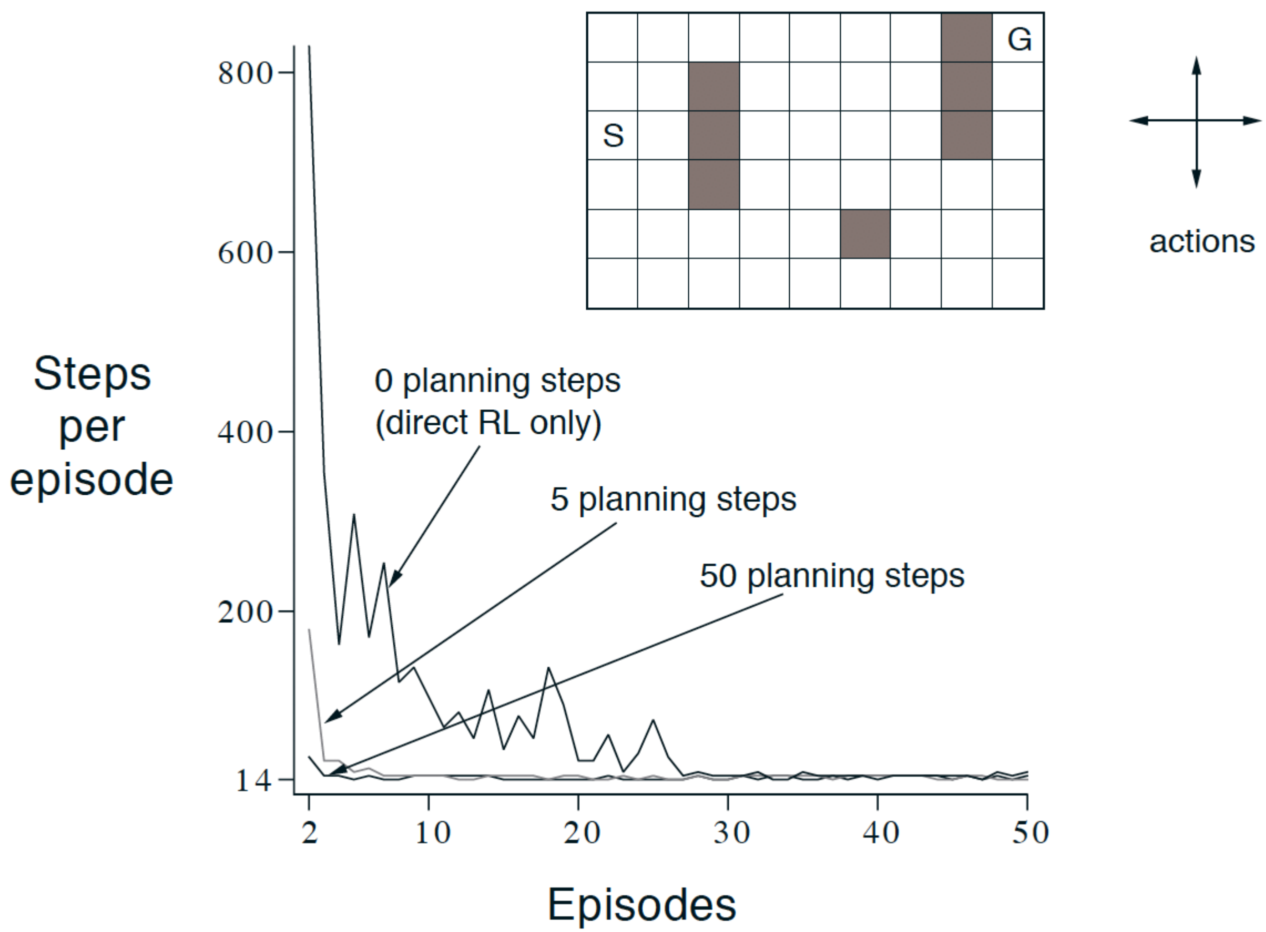


Dyna-Q Algorithm

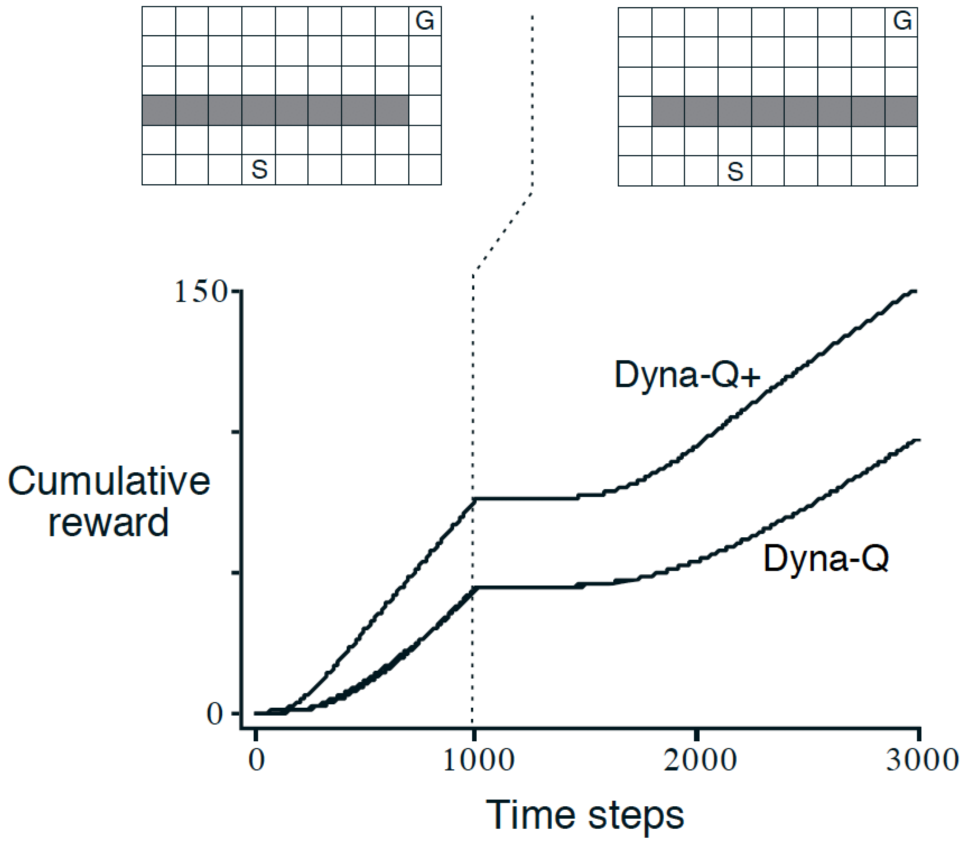
Initialize $Q(s, a)$ and $Model(s, a)$ for all $s \in \mathcal{S}$ and $a \in \mathcal{A}(s)$
Do forever:
- (a) $S \leftarrow$ current (nonterminal) state
- (b) $A \leftarrow \varepsilon$-greedy(S, Q)
- (c) Execute action A; observe resultant reward, R, and state, S'
- (d) $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$
- (e) $Model(S, A) \leftarrow R, S'$ (assuming deterministic environment)
- (f) Repeat n times:
  - $S \leftarrow$ random previously observed state
  - $A \leftarrow$ random action previously taken in S
  - $R, S' \leftarrow Model(S, A)$
  - $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma \max_a Q(S', a) - Q(S, A)\big]$

Model-based planning

Dyna-Q on a Simple Maze



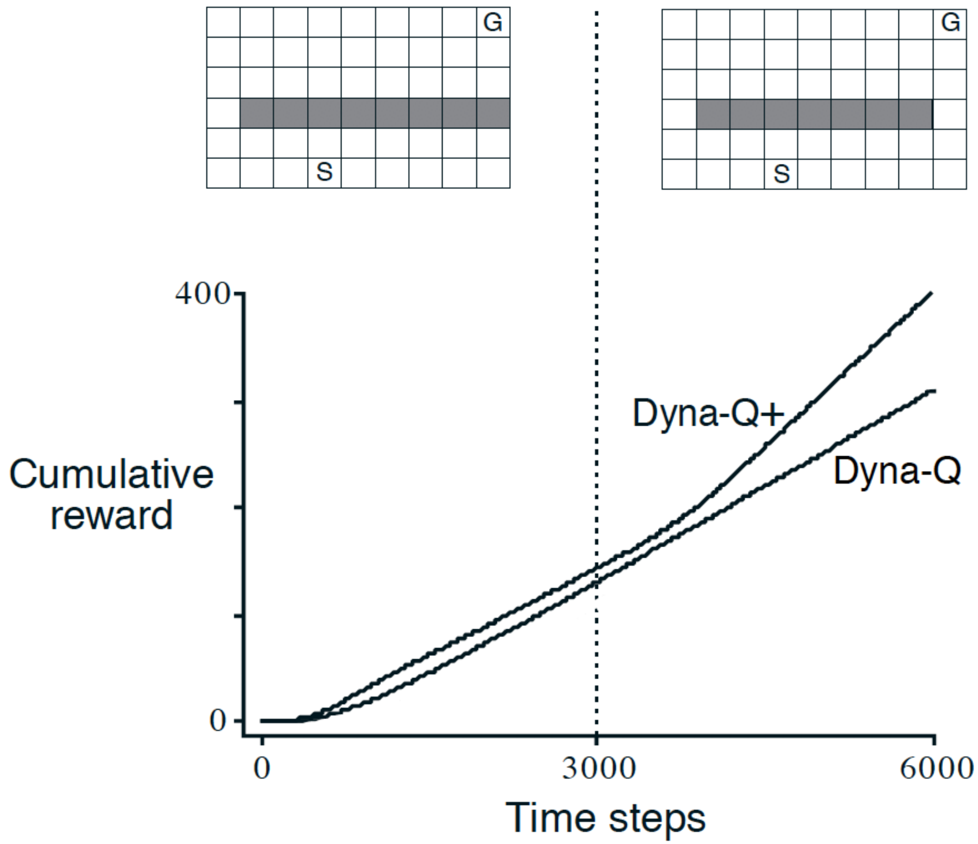
Dyna-Q on Blocking Maze



The left environment was used for the first 1000 steps, the right environment for the rest.
Dyna-Q+ is Dyna-Q with an exploration bonus that encourages exploration

$r + k\sqrt{\tau}$ (τ: the number of times being not tried)

Dyna-Q on Shortcut Maze



The left environment was used for the first 3000 steps, the right environment for the rest