

Lecture 7: Function Approximation

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RL Introduction: Schedule

- Lecture 1: Introduction to Reinforcement Learning
 - Lecture 2: Bandit Problem and MDP
 - Lecture 3: Model-based RL
 - Lecture 4: Model-free RL (Part I)
 - Lecture 5: Model-free RL (Part II)
 - Lecture 6: Eligibility Traces
 - **Lecture 7: Function Approximation (11/4)**
 - Lecture 8: Policy Gradient (11/11)
 - Lecture 9: Planning and Learning (11/18)
 - Lecture 10: Deep Reinforcement Learning (12/02)
 - Lecture 11: Advanced RL Topics (12/09)
 - **Final: 12/16**
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- Prof. Chong Li
- Prof. Chonggang Wang

Recap of Previous Lectures 1-6

- Model-based RL
 - Dynamic Programming (DP) – Bootstrapping
 - Policy Iteration: Policy Evaluation + Policy Improvement
 - Value Iteration
- Model-free RL
 - Monte-Carlo (MC) – No Bootstrapping (Unbiased)
 - Temporal-Difference (TD) – Bootstrapping
 - Policy Evaluation (State Value $v(s)$): TD(0), n-Step TD, TD(λ)
 - Policy Control (Action Value $q(s, a)$): Sarsa (On-Policy), Q-Learning (Off-Policy)
- All are **Tabular Methods**
 - Each update will only change the value of one *state* or one *state-action* pair, i.e., an entry in the *lookup table*
 - The lookup table may become unmanageable when the number of “*states*” or “*state-action*” pairs goes up
 - Good for episodic tasks, not for continuing tasks
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Outline – Function Approximation

- Introduction & Preliminaries
- RL Prediction with Function Approximation
- RL Control with Function Approximation
- Batch Method for RL Applications

*materials are modified from David Silver's RL lecture notes

Outline – Function Approximation

- Introduction & Preliminaries
- RL Prediction with Function Approximation
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Function Approximation

- **What** is Function Approximation?
 - To approximate value function $v(s)$ and action value function $q(s, a)$ in a parameterized functional format
 - $v(s) \rightarrow v(s, \mathbf{w})$ for all s ; $q(s, a) \rightarrow q(s, a, \mathbf{w})$ for all (s, a) pairs
- **Why** do we need Function Approximation?
 - To evaluate/predict $v(s)$ and $q(s, a)$ for large or high-dimension state space
 - To evaluate/predict $v(s)$ and $q(s, a)$ for continuing tasks (non-episodic)
 - To evaluate/predict $v(s)$ and $q(s, a)$ for partially observable problems
- **How** to achieve Function Approximation?
 - To use supervised learning based on experience data (i.e., training examples)
 - To define an **Objective Function**: Mean-Squared Value Error (VE)
 - To leverage **Stochastic Gradient Descent** to search for optimal parameters \mathbf{w} for estimated functions $v(s, \mathbf{w})$ and $q(s, a, \mathbf{w})$, in the fastest direction to minimize the Objective Function

Motivation

- How to solve large-scale reinforcement learning problems:
 - Backgammon: 10^{20} states
 - Computer GO: 10^{170} states
 - Autonomous driving: continuous state space
- Curse of dimensionality
 - How to leverage RL to achieve optimal control with the exponential growth of states and actions

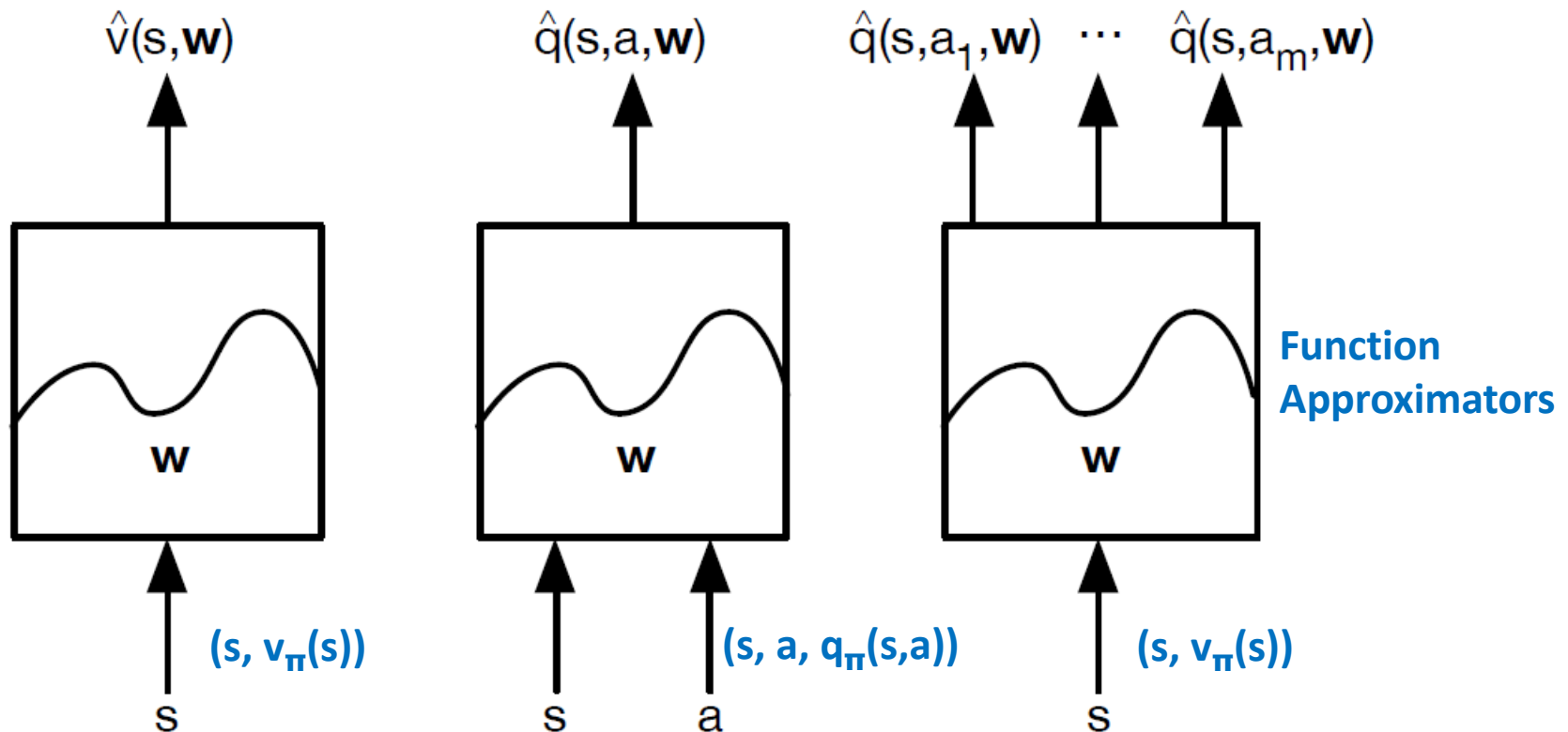
Value Function Approximation

- So far we have represented value function by a *lookup table*
 - Every state s has an entry $V(s)$
 - Or every state-action pair s, a has an entry $Q(s, a)$
- Problem with large MDPs:
 - There are too many states and/or actions to store in memory
 - It is too slow to learn the value of each state individually
- Solution for large MDPs:
 - Estimate value function with *function approximation*

$$\begin{array}{ccc} \text{Approximate} & \hat{v}(s, \mathbf{w}) \approx v_{\pi}(s) & \text{True} \\ \text{Value} & \text{or } \hat{q}(s, a, \mathbf{w}) \approx q_{\pi}(s, a) & \text{Value} \end{array}$$

Generalise from seen states to unseen states
Update parameter \mathbf{w} using MC or TD learning

Types of Approximation



- Typically, the number of weights (the dimensionality of w) is much less than the number of states $|S|$
- To update one weight will change the estimated value of many states

Which Function Approximator?

- There are many function approximators:
 - Linear: linear combinations of features
 - Non-linear: neural networks
 - Decision tree
 -
- We consider ***differentiable*** function approximators in this lecture:
 - Linear: linear combinations of features
 - Non-linear: neural networks
 - A static training set, uncorrelated data, stationary data, iid data

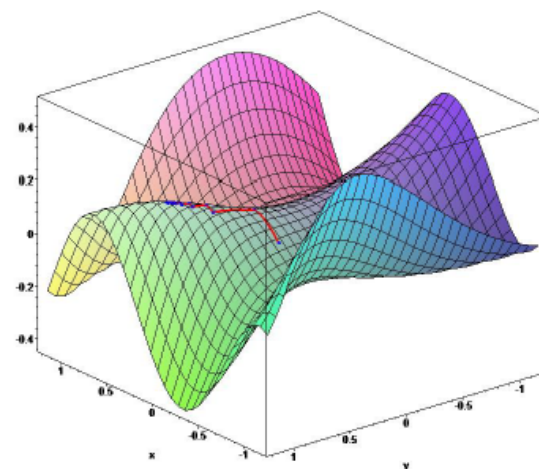
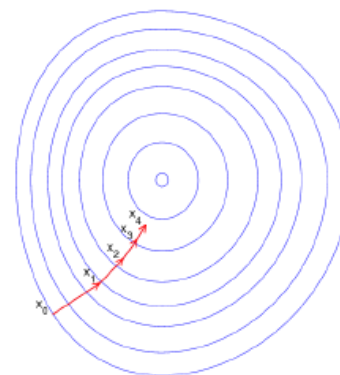
Gradient Descent

- Let $J(\mathbf{w})$ be a differentiable function of parameter vector \mathbf{w} , a column vector
- Define the *gradient* of $J(\mathbf{w})$ to be

$$\nabla_{\mathbf{w}} J(\mathbf{w}) = \begin{pmatrix} \frac{\partial J(\mathbf{w})}{\partial w_1} \\ \vdots \\ \frac{\partial J(\mathbf{w})}{\partial w_n} \end{pmatrix} \quad \begin{array}{l} \text{Partial} \\ \text{Derivatives} \\ \text{with} \\ \text{respect} \\ \text{to } \mathbf{w} \end{array}$$

- To find a local minimum of $J(\mathbf{w})$
- Adjust \mathbf{w} in direction of -ve gradient to reduce $J(\mathbf{w})$ (i.e., the Value Error (VE))

$$\Delta \mathbf{w} = -\frac{1}{2} \alpha \nabla_{\mathbf{w}} J(\mathbf{w})$$



Stochastic Gradient Descent

- Goal: find parameter vector \mathbf{w} minimising mean-squared error between approximate value fn $\hat{v}(s, \mathbf{w})$ and true value fn $v_\pi(s)$

$$J(\mathbf{w}) = \mathbb{E}_\pi [(v_\pi(S) - \hat{v}(S, \mathbf{w}))^2]$$

- Gradient descent finds a local minimum

$$\begin{aligned}\Delta \mathbf{w} &= -\frac{1}{2} \alpha \nabla_{\mathbf{w}} J(\mathbf{w}) \quad \rightarrow \text{Power Rule \& Chain Rule for Derivatives} \\ &= \alpha \mathbb{E}_\pi [(v_\pi(S) - \hat{v}(S, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w})] \\ &\quad \text{Assume it is irrespective of } w\end{aligned}$$

- Stochastic gradient descent *samples* the gradient

$$\Delta \mathbf{w} = \alpha (v_\pi(S) - \hat{v}(S, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w})$$

- Expected update is equal to full gradient update

Stochastic Gradient Descent - Example

- Suppose we want to fit a straight line

$$y = w_1 + w_2 x \quad \leftarrow \text{Approximate Function}$$

- Use data set

$$(x_1, y_1), \dots, (x_n, y_n) \quad \leftarrow \text{True Function/Data}$$

- Objective function: Mean-Squared Value Error

$$J(w) = \sum_{i=1}^n \underbrace{(w_1 + w_2 x_i)}_{\text{Estimated}} - \underbrace{y_i}_{\text{True Value}})^2$$


- SGD: sweep through the training set

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} := \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} - \eta \begin{bmatrix} 2(w_1 + w_2 x_i - y_i) \\ 2x_i(w_1 + w_2 x_i - y_i) \end{bmatrix}$$

Partial derivative of $Q(w)$ with respect to w_1

Partial derivative of $Q(w)$ with respect to w_2

Outline

- Introduction & Preliminaries
 - **RL Prediction with Function Approximation**
 - To find $v(s, \mathbf{w})$
 - RL Control with Function Approximation
 - To find $q(s, a, \mathbf{w})$
 - **Batch Method for RL Applications**
 - **Least-Square Method:** to find the best value function ($v(s, \mathbf{w})$) based on an experience data set by minimizing the sum of the errors/offsets.
 - Achieves a better utilization of samples
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- ```
graph LR; IM[Incremental Methods] --> RLP[RL Prediction with Function Approximation]; IM --> RLC[RL Control with Function Approximation];
```

# RL Prediction with Value Approximation

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- Have assumed true value function  $v_\pi(s)$  given by supervisor
- But in RL there is no supervisor, only rewards
- In practice, we substitute a *target* for  $v_\pi(s)$ 
  - For MC, the target is the return  $G_t$

$$\Delta \mathbf{w} = \alpha(\textcolor{red}{G}_t - \hat{v}(S_t, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w})$$

- For TD(0), the target is the TD target  $R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w})$

$$\Delta \mathbf{w} = \alpha(\textcolor{red}{R}_{t+1} + \gamma \hat{v}(\textcolor{red}{S}_{t+1}, \mathbf{w}) - \hat{v}(S_t, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w})$$

- For TD( $\lambda$ ), the target is the  $\lambda$ -return  $G_t^\lambda$

$$\Delta \mathbf{w} = \alpha(\textcolor{red}{G}_t^\lambda - \hat{v}(S_t, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w})$$

# How to compute the gradient?

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$$\Delta \mathbf{w} = \alpha (v_\pi(S) - \hat{v}(S, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w})$$

- We can compute it for
  - Linear: linear combinations of features
  - Non-linear: neural networks



# Feature Vectors

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- Represent state by a *feature vector*

$$\mathbf{x}(S) = \begin{pmatrix} \mathbf{x}_1(S) \\ \vdots \\ \mathbf{x}_n(S) \end{pmatrix}$$

- For example:
  - Distance of robot from landmarks
  - Trends in the stock market
  - Piece and pawn configurations in chess

- Ways to combine features
  - Polynomials
  - Fourier basis
  - Coding techniques
  - ....

# Linear Approximation

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- Represent value function by a linear combination of features

$$\hat{v}(S, \mathbf{w}) = \mathbf{x}(S)^\top \mathbf{w} = \sum_{j=1}^n \mathbf{x}_j(S) \mathbf{w}_j$$

- Objective function is quadratic in parameters  $\mathbf{w}$

$$J(\mathbf{w}) = \mathbb{E}_\pi \left[ (v_\pi(S) - \mathbf{x}(S)^\top \mathbf{w})^2 \right]$$

- Stochastic gradient descent converges on *global* optimum
- Update rule is particularly simple

$$\nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) = \mathbf{x}(S)$$

$$\Delta \mathbf{w} = \alpha (v_\pi(S) - \hat{v}(S, \mathbf{w})) \mathbf{x}(S)$$

Update = *step-size*  $\times$  *prediction error*  $\times$  *feature value*

# Table Lookup Features

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- Table lookup is a special case of linear value function approximation
- Using *table lookup features*

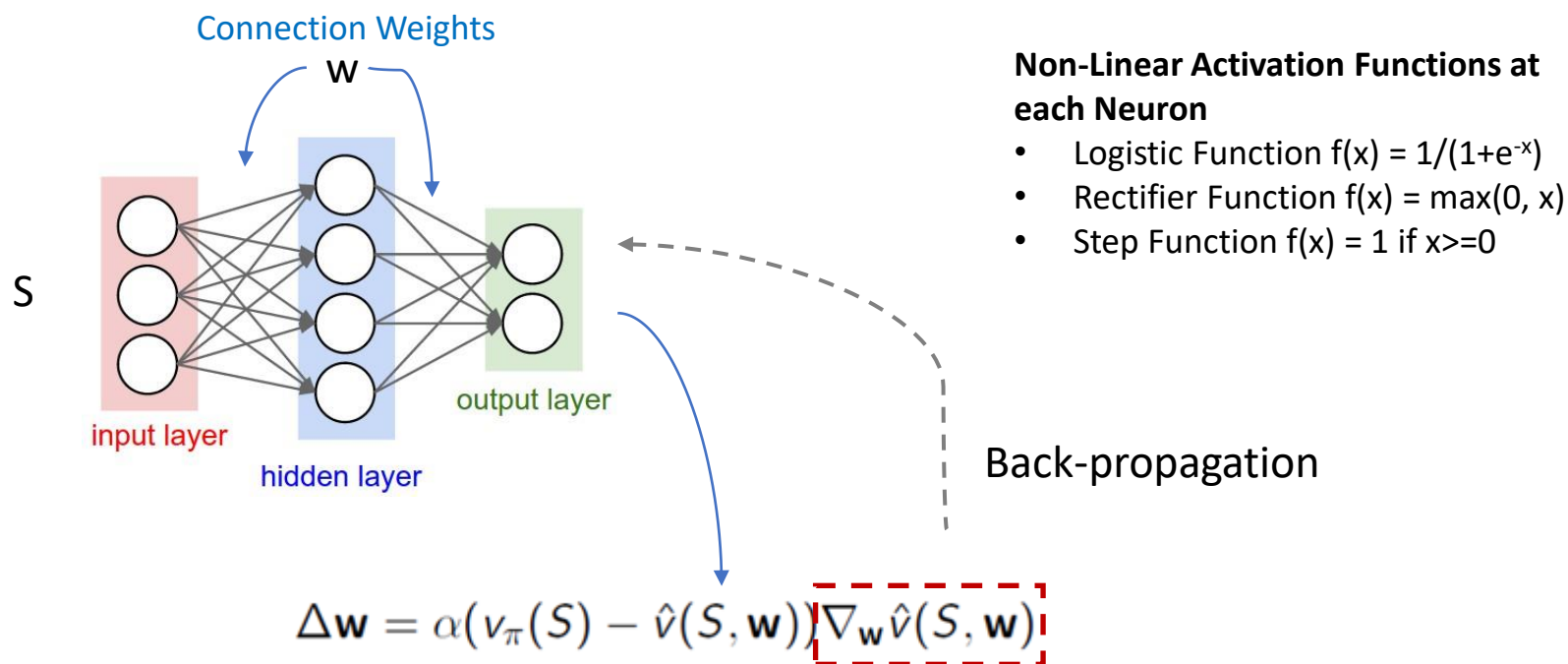
$$\mathbf{x}^{table}(S) = \begin{pmatrix} \mathbf{1}(S = s_1) \\ \vdots \\ \mathbf{1}(S = s_n) \end{pmatrix}$$

- Parameter vector  $\mathbf{w}$  gives value of each individual state

$$\hat{v}(S, \mathbf{w}) = \begin{pmatrix} \mathbf{1}(S = s_1) \\ \vdots \\ \mathbf{1}(S = s_n) \end{pmatrix} \cdot \begin{pmatrix} \mathbf{w}_1 \\ \vdots \\ \mathbf{w}_n \end{pmatrix}$$

# Non-linear Approximation

- Neural network (works fine for static training set, uncorrelated data, stationary/iid data)



# Monte-Carlo with Value Function Approximation

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- Return  $G_t$  is an unbiased, noisy sample of true value  $v_\pi(S_t)$
- Can therefore apply supervised learning to “training data”:

$$\langle S_1, G_1 \rangle, \langle S_2, G_2 \rangle, \dots, \langle S_T, G_T \rangle$$

- For example, using *linear Monte-Carlo policy evaluation*

$$\begin{aligned}\Delta \mathbf{w} &= \alpha(\textcolor{red}{G}_t - \hat{v}(S_t, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w}) \\ &= \alpha(G_t - \hat{v}(S_t, \mathbf{w})) \mathbf{x}(S_t)\end{aligned}$$

- Monte-Carlo evaluation converges to a local optimum
- Even when using non-linear value function approximation

# Monte-Carlo with Value Function Approximation

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- Input: The policy  $\pi$  to be evaluated; a differentiable function  $S \times R^d \rightarrow R$
- Algorithm Parameter: Step size  $\alpha > 0$  (e.g.,  $\alpha = 1/t$ )
- Initialize value-function weights  $\mathbf{w} \in R^d$  arbitrarily (e.g.,  $\mathbf{w} = 0$ )
- Loop forever (for each episode):
  - Generate an episode  $S_0, A_0, R_1, S_1, A_1, \dots, R_T, S_T$  using the policy  $\pi$
  - Loop for each step of episode,  $t = 0, 1, \dots, T - 1$ :
$$\mathbf{w} = \mathbf{w} + \alpha * [G_t - \hat{v}(S_t, \mathbf{w})] * \nabla \hat{v}(S_t, \mathbf{w})$$

# TD(0) with Value Function Approximation

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- The TD-target  $R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w})$  is a *biased* sample of true value  $v_\pi(S_t)$
- Can still apply supervised learning to “training data”:

$$\langle S_1, R_2 + \gamma \hat{v}(S_2, \mathbf{w}) \rangle, \langle S_2, R_3 + \gamma \hat{v}(S_3, \mathbf{w}) \rangle, \dots, \langle S_{T-1}, R_T \rangle$$

- For example, using *linear TD(0)*

$$\begin{aligned} \Delta \mathbf{w} &= \alpha(R + \gamma \hat{v}(S', \mathbf{w}) - \hat{v}(S, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w}) \\ &= \alpha \delta \mathbf{x}(S) \end{aligned}$$

- Linear TD(0) converges (close) to global optimum

# TD( $\lambda$ ) with Value Function Approximation

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- The  $\lambda$ -return  $G_t^\lambda$  is also a biased sample of true value  $v_\pi(s)$
- Can again apply supervised learning to “training data”:

$$\langle S_1, G_1^\lambda \rangle, \langle S_2, G_2^\lambda \rangle, \dots, \langle S_{T-1}, G_{T-1}^\lambda \rangle$$

- Forward view linear TD( $\lambda$ )

$$\begin{aligned}\Delta \mathbf{w} &= \alpha(\mathbf{G}_t^\lambda - \hat{v}(S_t, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(S_t, \mathbf{w}) \\ &= \alpha(\mathbf{G}_t^\lambda - \hat{v}(S_t, \mathbf{w})) \mathbf{x}(S_t)\end{aligned}$$

- Backward view linear TD( $\lambda$ )

$$\begin{aligned}\delta_t &= R_{t+1} + \gamma \hat{v}(S_{t+1}, \mathbf{w}) - \hat{v}(S_t, \mathbf{w}) \\ E_t &= \gamma \lambda E_{t-1} + \mathbf{x}(S_t) \\ \Delta \mathbf{w} &= \alpha \delta_t E_t\end{aligned}$$



# TD( $\lambda$ ) with Value Function Approximation

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```
Initialize \mathbf{w} as appropriate for the problem, e.g., $\mathbf{w} = \mathbf{0}$
Repeat (for each episode):
1 ---> $\mathbf{z} = \mathbf{0}$
2 ---> $S \leftarrow$ initial state of episode
3 ---> Repeat (for each step of episode):
4 ---> $A \leftarrow$ action given by π for S
5 ---> Take action A , observe reward, R , and next state, S'
6 ---> $\delta \leftarrow R + \gamma \hat{v}(S', \mathbf{w}) - \hat{v}(S, \mathbf{w})$
7 ---> $\mathbf{z} \leftarrow \gamma \lambda \mathbf{z} + \nabla_{\mathbf{w}} \hat{v}(S, \mathbf{w})$
8 ---> $\mathbf{w} \leftarrow \mathbf{w} + \alpha \delta \mathbf{z}$
9 ---> $S \leftarrow S'$
10 ---> until S' is terminal
```

# Convergence of Prediction Algorithms

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| On/Off-Policy | Algorithm       | Table Lookup | Linear | Non-Linear |
|---------------|-----------------|--------------|--------|------------|
| On-Policy     | MC              | ✓            | ✓      | ✓          |
|               | TD(0)           | ✓            | ✓      | ✗          |
|               | TD( $\lambda$ ) | ✓            | ✓      | ✗          |
| Off-Policy    | MC              | ✓            | ✓      | ✓          |
|               | TD(0)           | ✓            | ✗      | ✗          |
|               | TD( $\lambda$ ) | ✓            | ✗      | ✗          |

See Baird's counter-example (in the textbook) which shows the divergence of TD algorithm

Gradient TD algorithm resolved the divergence problem. See paper "Fast Gradient-Descent Methods for Temporal-Difference Learning with Linear Function Approximation" by Richard Sutton etc.

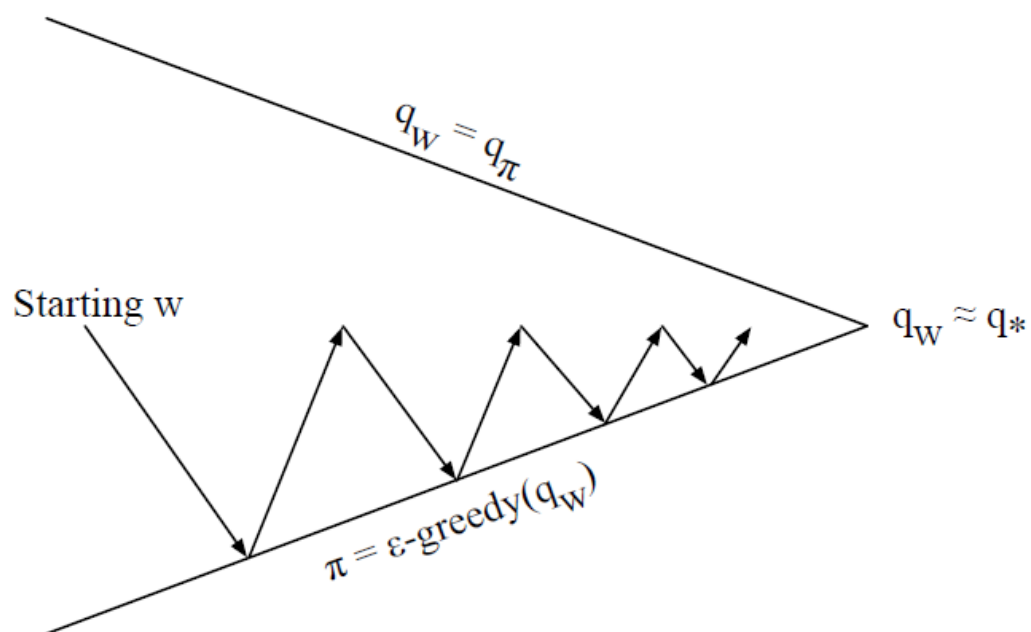
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# RL Control

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Policy evaluation **Approximate** policy evaluation,  $\hat{q}(\cdot, \cdot, \mathbf{w}) \approx q_\pi$

Policy improvement  $\epsilon$ -greedy policy improvement

# Action-value Function Approximation

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- Approximate the action-value function

$$\hat{q}(S, A, \mathbf{w}) \approx q_\pi(S, A)$$

- Minimise mean-squared error between approximate action-value fn  $\hat{q}(S, A, \mathbf{w})$  and true action-value fn  $q_\pi(S, A)$

$$J(\mathbf{w}) = \mathbb{E}_\pi [(q_\pi(S, A) - \hat{q}(S, A, \mathbf{w}))^2]$$

- Use stochastic gradient descent to find a local minimum

$$-\frac{1}{2} \nabla_{\mathbf{w}} J(\mathbf{w}) = (q_\pi(S, A) - \hat{q}(S, A, \mathbf{w})) \nabla_{\mathbf{w}} \hat{q}(S, A, \mathbf{w})$$

$$\Delta \mathbf{w} = \alpha (q_\pi(S, A) - \hat{q}(S, A, \mathbf{w})) \nabla_{\mathbf{w}} \hat{q}(S, A, \mathbf{w})$$

# Linear Action-value Function Approximation

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- Represent state *and* action by a *feature vector*

$$\mathbf{x}(S, A) = \begin{pmatrix} \mathbf{x}_1(S, A) \\ \vdots \\ \mathbf{x}_n(S, A) \end{pmatrix}$$

- Represent action-value fn by linear combination of features

$$\hat{q}(S, A, \mathbf{w}) = \mathbf{x}(S, A)^\top \mathbf{w} = \sum_{j=1}^n \mathbf{x}_j(S, A) \mathbf{w}_j$$

- Stochastic gradient descent update

$$\begin{aligned} \nabla_{\mathbf{w}} \hat{q}(S, A, \mathbf{w}) &= \mathbf{x}(S, A) \\ \Delta \mathbf{w} &= \alpha (\underbrace{q_\pi(S, A)}_{\text{Update Target}} - \hat{q}(S, A, \mathbf{w})) \mathbf{x}(S, A) \end{aligned}$$

# RL Control with Value Approximation

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- For MC, the target is the return  $G_t$

$$\Delta \mathbf{w} = \alpha(G_t - \hat{q}(S_t, A_t, \mathbf{w})) \nabla_{\mathbf{w}} \hat{q}(S_t, A_t, \mathbf{w})$$

- For TD(0), the target is the TD target  $R_{t+1} + \gamma Q(S_{t+1}, A_{t+1})$

$$\Delta \mathbf{w} = \alpha(R_{t+1} + \gamma \hat{q}(S_{t+1}, A_{t+1}, \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w})) \nabla_{\mathbf{w}} \hat{q}(S_t, A_t, \mathbf{w})$$

- For forward-view TD( $\lambda$ ), target is the action-value  $\lambda$ -return

$$\Delta \mathbf{w} = \alpha(q_t^\lambda - \hat{q}(S_t, A_t, \mathbf{w})) \nabla_{\mathbf{w}} \hat{q}(S_t, A_t, \mathbf{w})$$

- For backward-view TD( $\lambda$ ), equivalent update is

$$\delta_t = R_{t+1} + \gamma \hat{q}(S_{t+1}, A_{t+1}, \mathbf{w}) - \hat{q}(S_t, A_t, \mathbf{w})$$

$$E_t = \gamma \lambda E_{t-1} + \nabla_{\mathbf{w}} \hat{q}(S_t, A_t, \mathbf{w})$$

$$\Delta \mathbf{w} = \alpha \delta_t E_t$$

# Convergence of Control Algorithms

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| Algorithm           | Table Lookup | Linear | Non-Linear |
|---------------------|--------------|--------|------------|
| Monte-Carlo Control | ✓            | (✓)    | ✗          |
| Sarsa               | ✓            | (✓)    | ✗          |
| Q-learning          | ✓            | ✗      | ✗          |

(✓) = chatters around near-optimal value function



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# Motivation

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- Gradient descent is simple and appealing
- But it is *not* sample efficient
- Batch methods seek to find the best fitting value function
- Given the agent's experience ( “training data” )

# Least Square Prediction

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- Given value function approximation  $\hat{v}(s, \mathbf{w}) \approx v_\pi(s)$
- And *experience*  $\mathcal{D}$  consisting of  $\langle \text{state}, \text{value} \rangle$  pairs

$$\mathcal{D} = \{ \langle s_1, v_1^\pi \rangle, \langle s_2, v_2^\pi \rangle, \dots, \langle s_T, v_T^\pi \rangle \}$$

- Which parameters  $\mathbf{w}$  give the *best fitting* value fn  $\hat{v}(s, \mathbf{w})$ ?
- **Least squares** algorithms find parameter vector  $\mathbf{w}$  minimising sum-squared error between  $\hat{v}(s_t, \mathbf{w})$  and target values  $v_t^\pi$ ,

$$\begin{aligned} LS(\mathbf{w}) &= \sum_{t=1}^T (v_t^\pi - \hat{v}(s_t, \mathbf{w}))^2 \\ &= T \mathbb{E}_{\mathcal{D}} [(v^\pi - \hat{v}(s, \mathbf{w}))^2] \end{aligned}$$

# Experience Replay

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- Given experience consisting of  $\langle state, value \rangle$  pairs

$$\mathcal{D} = \{ \langle s_1, v_1^\pi \rangle, \langle s_2, v_2^\pi \rangle, \dots, \langle s_T, v_T^\pi \rangle \}$$

- Repeat:
  - Sample state, value from experience

$$\langle s, v^\pi \rangle \sim \mathcal{D}$$

- Apply stochastic gradient descent update

$$\Delta \mathbf{w} = \alpha (v^\pi - \hat{v}(s, \mathbf{w})) \nabla_{\mathbf{w}} \hat{v}(s, \mathbf{w})$$

- Converges to least squares solution

$$\mathbf{w}^\pi = \underset{\mathbf{w}}{\operatorname{argmin}} LS(\mathbf{w})$$

# Experience Replay in Deep Q-Network (DQN)

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DQN uses **experience replay** and **fixed Q-targets**

→ Remove correlations between consecutive observations in experience data

→ Better convergence

- Take action  $a_t$  according to  $\epsilon$ -greedy policy
- Store transition  $(s_t, a_t, r_{t+1}, s_{t+1})$  in replay memory  $\mathcal{D}$
- Sample random mini-batch of transitions  $(s, a, r, s')$  from  $\mathcal{D}$
- Compute Q-learning targets w.r.t. old, fixed parameters  $w^-$
- Optimise MSE between Q-network and Q-learning targets

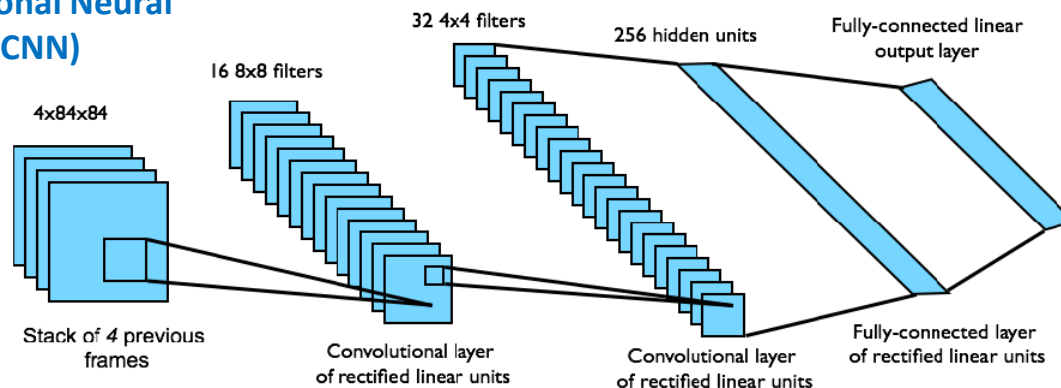
$$\mathcal{L}_i(w_i) = \mathbb{E}_{s,a,r,s' \sim \mathcal{D}_i} \left[ \left( \underbrace{r + \gamma \max_{a'} Q(s', a'; w_i^-)}_{\text{Q-learning targets}} - \underbrace{Q(s, a; w_i)}_{\text{Q-network}} \right)^2 \right]$$

- Using variant of stochastic gradient descent

# DQN in Atari Games

- End-to-end learning of values  $Q(s, a)$  from pixels  $s$
- Input state  $s$  is stack of raw pixels from last 4 frames
- Output is  $Q(s, a)$  for 18 joystick/button positions
- Reward is change in score for that step

## Convolutional Neural Network (CNN)



Network **architecture** and **hyperparameters** do not change across games

See paper “Human-level control through deep reinforcement learning” Nature, 2015