IT5005 Artificial Intelligence

Tutorial #1: Propositional Logic

Answers

Tutorial Questions

1. Are the following statements true or false?

   a. Assuming that a is a real number, the negation of "$1 < a < 5$" is "$1 \geq a \geq 5$".

   b. In propositional logic, "he's welcome to come along only if he behaves himself" means "if he behaves himself then he's welcome to come along."

   Answers:

   a. The statement of the form "$1 < a < 5$" is short-form for "$(1 < a)$ and $(a < 5)$". By De Morgan's law, its negation is "$(1 \geq a)$ or $(a \geq 5)$". Whereas "$1 \geq a \geq 5$" means "$(1 \geq a)$ and $(a \geq 5)$". Therefore, (a) is false.

   b. Let p be "he's welcome to come along" and q be "he behaves himself". "p only if q" means that "p can hold only in the situations where q holds". In other words, if q does not hold, then p does not hold, which is equivalent to "$\sim q \rightarrow \sim p$", which is in turn logically equivalent to its contrapositive "$p \rightarrow q$".

      Hence, the statement "he's welcome to come along only if he behaves himself" is equivalent to "if he's welcome to come along, then he behaves himself". Therefore, (b) is false.

      Recap: "p only if q" is logically equivalent to "if p then q" (or "$p \rightarrow q$").

2. Simplify the propositions below using the laws given in **Theorem 2.1.1 (Epp)** and the **implication law** (if necessary) with only negation ($\sim$), conjunction ($\wedge$) and disjunction ($\vee$) in your final answers. Supply a justification for every step.

   For the first half of the module, we want students to cite justification <u>for every step</u>. This is to ensure that you do not arrive at the answer by coincidence. Only after you have gained sufficient experience then would we relax this and allow you to skip obvious steps, or combine multiple steps in a line.

   a. $\sim a \wedge (\sim a \rightarrow (a \wedge b))$

      Raflee worked out his answer as shown below. However, he skipped some steps and hence his answer will not be awarded full credit. Can you point out the omissions? (Note: To show that two logical statements are equivalent, we use $\equiv$, not $=$.)

      $\sim a \wedge (\sim a \rightarrow (a \wedge b))$

      $\equiv \sim a \wedge (a \vee (a \wedge b))$ by the implication law (step 1)

      $\equiv \sim a \wedge a$ by the absorption law (step 2)

$\equiv \mathbf{false}$ by the negation law (step 3)

b. $(p \vee \sim q) \rightarrow q$
c. $\sim(p \vee \sim q) \vee (\sim p \wedge \sim q)$
d. $(p \rightarrow q) \rightarrow r$

Answers

a. $\sim a \wedge (\sim a \rightarrow (a \wedge b))$

$\equiv \sim a \wedge (\sim(\sim a) \vee (a \wedge b))$ by the implication law
$\equiv \sim a \wedge (a \vee (a \wedge b))$ by the ~~implication law~~ double neg law (step 1)
$\equiv \sim a \wedge a$ by the absorption law (step 2)
$\equiv a \wedge \sim a$ by the commutative law
$\equiv \mathbf{false}$ by the negation law (step 3)

b. $(p \vee \sim q) \rightarrow q$

$\equiv \sim(p \vee \sim q) \vee q$ by the implication law (step 1)
$\equiv (\sim p \wedge \sim(\sim q)) \vee q$ by De Morgan's law (step 2)
$\equiv (\sim p \wedge q) \vee q$ by the double negative law (step 3)
$\equiv q \vee (\sim p \wedge q)$ by the commutative law (step 4)
$\equiv q \vee (q \wedge \sim p)$ by the commutative law (step 5)
$\equiv q$ by the absorption law (step 6)

Check:

- Did you jump from step 1 straight to step 3 by citing only De Morgan's law but omitting double negative law?
- Did you jump from step 3 straight to step 6 by skipping the two steps involving commutative law?

Also, remember to add appropriate parenthesis to avoid ambiguous statements. For example, from $\sim(p \vee \sim q) \vee q$ (step 1) to $(\sim p \wedge \sim(\sim q)) \vee q$ (step 2), if step 2 were written as $\sim p \wedge \sim(\sim q) \vee q$, it would become an ambiguous statement since $\wedge$ and $\vee$ are coequal in precedence.

c. $\sim(p \vee \sim q) \vee (\sim p \wedge \sim q)$

$\equiv (\sim p \wedge \sim(\sim q)) \vee (\sim p \wedge \sim q)$ by De Morgan's law
$\equiv (\sim p \wedge q) \vee (\sim p \wedge \sim q)$ by the double negative law
$\equiv \sim p \wedge (q \vee \sim q)$ by the distributive law
$\equiv \sim p \wedge \mathbf{true}$ by the negation law
$\equiv \sim p$ by the identity law

d. $(p \rightarrow q) \rightarrow r$

$\equiv (\sim p \vee q) \to r$ by the implication law

$\equiv (\sim(\sim p \vee q)) \vee r$ by the implication law

$\equiv (\sim(\sim p) \wedge \sim q) \vee r$ by De Morgan's law

$\equiv (p \wedge \sim q) \vee r$ by the double negative law

3. The SAFRA-DBS NS50 Lucky Draw 2017 is a lucky draw to win 100,000 AirAsia Miles.[1]

   The rule says that to qualify for the draw, SAFRA-DBS credit card holders must "charge a minimum of S$50 nett to their card during the Qualifying Period", which is 1 July to 30 September 2017.

   Let *C* = "Charge a minimum of S$50 nett", *P* = "Charge during the Qualifying Period", and *W* = "Win 100,000 AirAsia Miles".

   a. Write a **conditional statement** using *C*, *P* and *W* that describes the rule above.

   b. Write the **converse**, **inverse**, **contrapositive** and **negation** forms of the statement in part (a).

   Answers

   a. The qualifying conditions are necessary but not sufficient conditions. They are needed to participate in the draw, but do not guarantee winning the prizes. Thus, *C* and *P* are necessary conditions for *W*, which translates to:

      if W then $(C \wedge P)$ or $W \to (C \wedge P)$

   b. Statement: $W \to (C \wedge P)$

      Converse: $(C \wedge P) \to W$

      Inverse: $\sim W \to \sim(C \wedge P)$ or $\sim W \to (\sim C \vee \sim P)$

      Contrapositive: $\sim(C \wedge P) \to \sim W$ or $(\sim C \vee \sim P) \to \sim W$

      Negation: $W \wedge \sim(C \wedge P)$

[1] https://www.dbs.com.sg/iwov-resources/pdf/cards/promotions/safra-cards-taipei-tnc.pdf

4. The island of Wantuutrewan is inhabited by two types of people: **knights** who always tell the truth and **knaves** who always lie. You visit the island and have the following encounters with the natives.

a. Two natives *A* and *B* speak to you:

A says: Both of us are knights.
B says: *A* is a knave.

What are *A* and *B*?

b. Two natives *C* and *D* speak to you:

C says: *D* is a knave.
D says: *C* is a knave.

How many knights and knaves are there?

Part (a) has been solved for you (see below). Study the solution, and use the same format in answering part (b).

Answer for part (a):

Proof (by contradiction).

1. If *A* is a knight, then:

1.1	What *A* says is true.	(by definition of knight)
1.2	$\therefore$ *B* is a knight too.	(that's what *A* says)
1.3	$\therefore$ What *B* says is true.	(by definition of knight)
1.4	$\therefore$ *A* is a knave.	(that's what *B* says)
1.5	$\therefore$ *A* is not a knight.	(since *A* is either a knight or a knave, but not both)
1.6	$\therefore$ Contradiction to 1.	

2. $\therefore$ *A* is not a knight.
3. $\therefore$ *A* is a knave. (since *A* is either a knight or a knave, but not both)
4. $\therefore$ What *B* says is true.
5. $\therefore$ *B* is a knight. (by definition of knight)
6. Conclusion: *A* is a knave and *B* is a knight.

Notes:
- It is tempting to say "Contradiction" right after line 1.4. However, this is not valid because contradiction requires $p \wedge \sim p$, but 'knave' is not the negation of 'knight'. Hence line 1.5 is required before we arrive at the contradiction in 1.6.

Answer

b. Proof (by division into cases)

1. If *C* is a knight:
   - 1.1 What *C* says is true. (by definition of knight)
   - 1.2 $\therefore$ *D* is a knave. (that's what *C* says)
2. If *C* is not a knight:
   - 2.1 Then *C* is a knave. (one is either a knight or a knave)
   - 2.2 $\therefore$ what *C* says is false. (by definition of knave)
   - 2.3 $\therefore$ *D* is not a knave. (*C* says *D* is a knave, but what *C* says is false)
   - 2.4 $\therefore$ *D* is a knight. (one is either a knight or a knave)
3. In both cases, there is one knight and one knave.