

IT5005 Artificial Intelligence
Tutorial 1 – Propositional Logic

1. Are the following statements true or false?
 - a. Assuming that a is a real number, the negation of " $1 < a < 5$ " is " $1 \geq a \geq 5$ ".
 - b. In propositional logic, "he's welcome to come along only if he behaves himself" means "if he behaves himself then he's welcome to come along."
2. Simplify the propositions below using the laws given in **Theorem 2.1.1 (Slides 24 and 25 of the lecture notes)** and the **implication law** (if necessary) with only negation (\sim), conjunction (\wedge) and disjunction (\vee) in your final answers. Supply a justification for every step.

You should cite justification for every step. This is to ensure that you do not arrive at the answer by coincidence. Only after you have gained sufficient experience then would we relax this and allow you to skip obvious steps, or combine multiple steps in a line. a. $\sim a \wedge (\sim a \rightarrow (a \wedge b))$

Raflee worked out his answer as shown below. However, he skipped some steps and hence his answer will not be awarded full credit. Can you point out the omissions? (Note: To show that two logical statements are equivalent, we use \equiv , not $=$.)

$$\begin{aligned} &\sim a \wedge (\sim a \rightarrow (a \wedge b)) \\ &\equiv \sim a \wedge (a \vee (a \wedge b)) && \text{by the implication law (step 1)} \\ &\equiv \sim a \wedge a && \text{by the absorption law (step 2)} \\ &\equiv \text{false} && \text{by the negation law (step 3)} \end{aligned}$$

- b. $(p \vee \sim q) \rightarrow q$
- c. $\sim(p \vee \sim q) \vee (\sim p \wedge \sim q)$
- d. $(p \rightarrow q) \rightarrow r$

3. The SAFRA-DBS NS50 Lucky Draw 2017 is a lucky draw to win 100,000 AirAsia Miles.¹

The rule says that to qualify for the draw, SAFRA-DBS credit card holders must "charge a minimum of S\$50 nett to their card during the Qualifying Period", which is 1 July to 30 September 2017.

Let C = "Charge a minimum of S\$50 nett", P = "Charge during the Qualifying Period", and W = "Win 100,000 AirAsia Miles".

¹ <https://www.dbs.com.sg/iwov-resources/pdf/cards/promotions/safra-cards-taipei-tnc.pdf>

- a. Write a **conditional statement** using C , P and W that describes the rule above.
- b. Write the **converse**, **inverse**, **contrapositive** and **negation** forms of the statement in part (a).

4. The island of Wantuutrewan is inhabited by two types of people: **knight**s who always tell the truth and **knave**s who always lie. You visit the island and have the following encounters with the natives.



- a. Two natives A and B speak to you:

A says: Both of us are knights.

B says: A is a knave.

What are A and B ?

- b. Two natives C and D speak to you:

C says: D is a knave.

D says: C is a knave.

How many knights and knaves are there?

Part (a) has been solved for you (see below). Study the solution, and use the same format in answering part (b).

Answer for part (a):

Proof (by contradiction).

1. If A is a knight, then:
 - 1.1 What A says is true. (by definition of knight)
 - 1.2 $\rightarrow B$ is a knight too. (that's what A says)
 - 1.3 \rightarrow What B says is true. (by definition of knight)
 - 1.4 $\rightarrow A$ is a knave. (that's what B says)
 - 1.5 $\rightarrow A$ is not a knight. (since A is either a knight or a knave, but not both)
 - 1.6 \rightarrow Contradiction to 1.
2. $\rightarrow A$ is not a knight.
3. $\rightarrow A$ is a knave. (since A is either a knight or a knave, but not both)
4. \rightarrow What B says is true.
5. $\rightarrow B$ is a knight. (by definition of knight)
6. Conclusion: A is a knave and B is a knight.

Notes:

- It is tempting to say "Contradiction" right after line 1.4. However, this is not valid because contradiction requires $p \wedge \sim p$, but 'knave' is not the negation of 'knight'. Hence line 1.5 is required, before we arrive at the contradiction in 1.6.