IT5005 Artificial Intelligence

Tutorial 3

1. Represent the following sentences in first order logic, using a consistent vocabulary defined as follows:

   ***Took*(*x,y,z*)** : is true if student *x* took subject *y* in semester *z*
   ***Score*(*x,y,z*)** : is true if student *x* obtains score *z* in subject *y*
   ***Passed*(*x,y*)** : is true if student *x* passed subject *y*
   ***Buys*(*x,p*)** : is true if person *x* buys policy *p*
   ***IsSmart*(*x*)** : is true if person *x* is smart
   ***IsExpensive*(*x*)** : is true if *x* is expensive
   ***Sells*(*x,y,p*)**: is true if person *x* sells policy *p* to person *y*
   ***IsInsured*(*x*)**: is true if person *x* is insured
   ***IsBarber*(*x*)**: is true if *x* is a barber
   ***Shaves*(*x,y*)**: is true if person *x* shaves person *y*

   (a) Some students took French in Spring 2001.
   (b) Every student who takes French passes it.
   (c) Only one student took Greek in Spring 2001.
   (d) The best score in Greek is always higher than the best score in French.
   (e) Everyone who buys a policy is smart. ∀*x,p* : *Buys*(*x*,*p*) ⇒ *IsSmart*(*x*) (f) No person buys an expensive policy.

   (g) There is an agent who sells policies only to those people who are not insured.

   (h) There is a barber who shaves all men in town who do not shave themselves.

   (i) There is a barber who shaves all men in town who does not shave himself.

2. Given the following logical statements, use model checking to show that $KB \models \alpha$. In other words, write down all possible true/false assignments to the variables, the ones for which KB is true and the one for which α is true, and see whether one is a subset of the other

   .

   (a)

   $$KB = (x_1 \lor x_2) \land (x_1 \Rightarrow x_3) \land \neg x_2 \ \alpha$$
   $$= x_3 \lor x_2$$

   (b)

   $$KB = (x_1 \lor x_3) \land (x_1 \Rightarrow \neg x_2) \ \alpha$$
   $$= \neg x_2$$

3. Here are two sentences in the language of first-order logic:
   (a) $\forall x : \exists y : (x \geq y)$
   (b) $\exists y : \forall x : (x \geq y)$

   (i) Assume that the variables range over all the natural numbers $0,1,2,\ldots$ and that the "$\geq$" predicate means "is greater than or equal to". Under this interpretation, translate (a) and (b) into English.

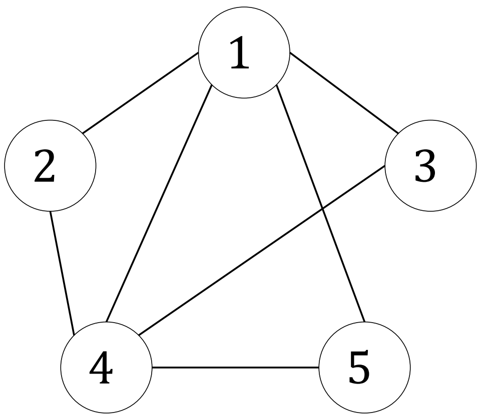
   (ii) Is (a) true under this interpretation? Is (b) true under this interpretation?

   (iii) Does (a) logically entail (b)? Does (b) logically entail (a)? Justify your answers.

4. Given a graph $G = \langle \mathrm{V}, \mathrm{E} \rangle$ we say that a subset of vertices $X \subseteq V$ is an *independent set* if no two vertices in X share an edge. Here we assume *pairwise* independent sets. I.e. each independent set has at most two vertices.

   a. Given a set of vertices $X \subseteq V$, write the constraint "no two vertices in X share an edge" in propositional logic. You may only use Boolean variables of the form $x_v \in \{\text{False, True}\}$ which indicate that x_v is part of the independent set. You may not refer to the set X in your solution, only the set V, and the edges in E. You can use basic arithmetic operators and basic logical operators.

   b. Write down the independent set constraints for the following graph in propositional logic. We assume that all independent sets have at most two vertices.



c. Using resolution show that vertex 1 is not in any independent set. We assume that the independent set size is 2, and this is equivalent to showing that if x_1 is True, then x_2, x_3, x_4 are all False.