

4.

$$f(x) = 3 - 17x^3, \text{ at } x=2.5$$

$$p_2(x) = f(2.5) + \frac{f'(2.5)(2.5-2.5)}{1} + \frac{f''(2.5)(2.5-2.5)}{2!}$$

$$+ \frac{f'''(2.5)(2.5-2.5)}{3!}$$

$$R_2(2.5) = f(2.5) - p_2(2.5) = 3$$

5

$$f(x) = 16x^5 - 73x^2 - 133$$

$$x_0 = 3, x_1 = 2.5$$

$$\epsilon = 0.001$$

$$x_{n+1} = x_n - \frac{f(x_n) * (x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

$$x_2 = \frac{2.5 - \frac{f(2.5) * (2.5 - 3)}{f(2.5) - f(3)}}{f(2.5) - f(3)} = 2.5 - \frac{973.25 * (-0.5)}{973.25 - 3098}$$

$$= 2.270973$$

$$|x_2 - x_1| = 0.229027 > 0.001$$

$$x_3 = \frac{2.270973 - \frac{f(2.270973) * (2.270973 - 2.5)}{f(2.270973) - f(2.5)}}{f(2.270973) - f(2.5)} = 2.068259$$

$$|x_3 - x_2| = 0.2027138 > 0.001$$

$$x_4 = \frac{2.068259 - \frac{f(2.068259) * (2.068259 - 2.270973)}{f(2.068259) - f(2.270973)}}{f(2.068259) - f(2.270973)} = \frac{1.958756}{1.958756 - 1.958756} = 1.958756$$

$$|x_4 - x_3| = 0.0471791 > 0.001$$

$$x_5 = x_4 - \frac{f(x_4) * (x_4 - x_3)}{f(x_4) - f(x_3)} = 1.911577$$

$$|x_5 - x_4| = 0.0471791 > 0.001$$

$$x_6 = x_5 - \frac{f(x_5) * (x_5 - x_4)}{f(x_5) - f(x_4)} = 1.901285$$



$$x_7 = x_6 - \frac{f(x_6) + (x_6 - x_5)}{f(x_6) - f(x_5)} = 1.900475$$

$$x_7 - x_6 = 0.0008104 < 0.001$$

$x_7$  is a root of  $f(x)$  accurate to within  $\epsilon$

6.  $f(x) = x^6 - x - 1$ ,  $\epsilon = 0.001$ ,  $x_0 = 1.5$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = 1.5 - \frac{f(1.5)}{f'(1.5)} = 1.5 - \frac{8.890625}{44.5625} = 1.300491$$

$$\left| \frac{x_1 - x_0}{x_0} \right| = 0.199509 > \epsilon$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.181480$$

$$\left| \frac{x_2 - x_1}{x_1} \right| = \frac{0.091512}{1.300491} = 0.070365 > \epsilon$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.139456$$

$$\left| \frac{x_3 - x_2}{x_2} \right| = \frac{0.035570}{1.181480} = 0.030105 > \epsilon$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.134778$$

$$\left| \frac{x_4 - x_3}{x_3} \right| = \frac{0.004105}{1.139456} = 0.003598 > \epsilon$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 1.134724$$

$$\left| \frac{x_5 - x_4}{x_4} \right| = \frac{0.000054}{1.134778} = 0.000047 < \epsilon$$

$x_5 = 1.134724$  is root within acceptable error  $\epsilon$

$$x_4 = 1.134778$$

$$A = \left[ \begin{array}{ccc|ccc} 0 & -3 & -2 & 1 & 0 & 0 \\ 1 & -4 & -2 & 0 & 1 & 0 \\ -3 & 4 & 1 & 0 & 0 & 1 \end{array} \right]$$

~~add~~ ~~aug~~  
write augmented matrix

$$\left[ \begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & -3 & -2 & 1 & 0 & 0 \\ 0 & -8 & -5 & 0 & 3 & 1 \end{array} \right] \begin{array}{l} \textcircled{1} \text{ Swap } r_1, r_2 \\ \textcircled{2} r_3 \leftarrow 3 * \text{new } r_1 \end{array}$$

$$\left[ \begin{array}{ccc|ccc} 1 & -4 & -2 & 0 & 1 & 0 \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & -8 & -5 & 0 & 3 & 1 \end{array} \right] r_2 \div -3 \quad (r_2 = r_2 \div -3)$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & \frac{2}{3} & -\frac{4}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & -\frac{8}{3} & 3 & 1 \end{array} \right] \begin{array}{l} \textcircled{1} r_1 + = 4 * r_2 \\ \textcircled{2} r_3 + = 8 * r_2 \end{array}$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & \frac{2}{3} & -\frac{4}{3} & 1 & 0 \\ 0 & 1 & \frac{2}{3} & -\frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & -8 & 9 & 3 \end{array} \right] r_3 * = 3$$

$$\left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & -5 & 2 \\ 0 & 1 & 0 & 5 & -6 & -2 \\ 0 & 0 & 1 & -8 & 9 & 3 \end{array} \right] \begin{array}{l} \textcircled{1} r_1 - = \frac{2}{3} * r_3 \\ \textcircled{2} r_2 - = \frac{2}{3} * r_3 \end{array}$$

$$\text{Inverse} = \left[ \begin{array}{ccc} 4 & -5 & -2 \\ 5 & -6 & -2 \\ -8 & 9 & 3 \end{array} \right]$$



$$\begin{aligned}
 12x_1 + 7x_2 + 3x_3 &= 2 \\
 x_1 + 5x_2 + x_3 &= -5 \\
 2x_1 + 7x_2 - 11x_3 &= 6
 \end{aligned}$$

$$\begin{aligned}
 x_1^{(0)} &= 1 \\
 x_2^{(0)} &= 3 \\
 x_3^{(0)} &= 5
 \end{aligned}$$

$$x_1 = \frac{2 - 7x_2 - 3x_3}{12} \quad x_2 = \frac{-5 - x_1 - x_3}{5} \quad x_3 = \frac{6 - 2x_1 - 7x_2}{-11}$$

$$x_1^{(1)} = \frac{2 - 7(3) - 3(5)}{12} = \left[ \frac{-17}{6} = x_1^{(1)} \right]$$

$$x_2^{(1)} = \frac{-5 - (1) - (5)}{5} = \left[ \frac{-11}{5} = x_2^{(1)} \right]$$

$$x_3^{(1)} = \frac{6 - 2(1) - 7(3)}{-11} = \left[ \frac{17}{11} = x_3^{(1)} \right]$$

$$x_1^{(2)} = \frac{2 - 7(-\frac{11}{5}) - 3(\frac{17}{11})}{12} = \frac{117}{110} = x_1^{(2)}$$

$$x_2^{(2)} = \frac{-5 - (-\frac{17}{16}) - (\frac{17}{11})}{5} = -\frac{193}{176} = x_2^{(2)}$$

$$x_3^{(2)} = \frac{6 - 2(-\frac{17}{6}) - 7(-\frac{11}{5})}{-11} = -\frac{406}{165} = x_3^{(2)}$$

$$x_1^{(3)} = \frac{2 - 7(-\frac{193}{176}) - 3(-\frac{406}{165})}{12} = \frac{71}{440} = 0.161363 = x_1^{(3)}$$

$$x_2^{(3)} = \frac{-5 - (\frac{117}{110}) - (-\frac{406}{165})}{5} = -0.720606 = x_2^{(3)}$$

$$x_3^{(3)} = \frac{6 - 2(\frac{117}{110}) - 7(-\frac{193}{176})}{-11} = -1.049867 = x_3^{(3)}$$

10.

$$\begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix}$$

$$\rightarrow A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 8 & 14 \\ 2 & 6 & 13 \end{bmatrix}$$

$$A = LU$$

$$B = \begin{bmatrix} 3 \\ 13 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & \cancel{2} & 2 \\ 2 & 6 & 13 \end{bmatrix}$$

$$r_2 = r_2 + (-3r_1)$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & \cancel{1} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & \cancel{2} & 2 \\ 0 & 2 & 3 \end{bmatrix}$$

$$r_3 = r_3 + (-2r_1)$$

$$U = \begin{bmatrix} 1 & 2 & 4 \\ 0 & \cancel{2} & 2 \\ 0 & 0 & \cancel{5} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & \cancel{2} & 2 \\ 0 & 0 & \cancel{5} \end{bmatrix}$$

$$r_3 = r_3 + (\cancel{2r_1}) \quad r_3 = r_3 + (-1r_2)$$

$$L \cdot Y = B$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & \cancel{1} & 1 \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 3 \\ \cancel{13} \\ 4 \end{bmatrix}$$

$$L \cdot Y = B$$

$$y_1 = 3$$

$$3y_1 + y_2 = 13$$

$$2y_1 - \frac{1}{2}y_2 + y_3 = 4$$

$$3(3) + y_2 = 13$$

$$y_2 = 4$$

$$2(3) - \frac{1}{2}(4) + y_3 = 4$$

$$y_3 = \frac{10}{2} - 2 = 3$$

10.

cont'd

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & \cancel{1} & 2 \\ 0 & 0 & \cancel{\frac{2}{7}} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ \frac{10}{7} \end{bmatrix}$$

$$U \cdot X = Y$$

$$x_1 + 2x_2 + 4x_3 = 3$$

$$-\cancel{\frac{2}{7}}x_2 + 2x_3 = 4$$

$$\frac{2}{7}x_3 = \frac{10}{7} \Rightarrow -2$$

$$x_3 = -2$$

$$-\cancel{\frac{2}{7}}x_2 + 2(-2) = 4$$

$$x_2 = 4$$

$$x_1 + 2(4) + 4(-2) = 3$$

$$x_1 = 3$$