# Linear

### Ex4.5

```r
data4.5<-read.table("CH01PR22.txt",header = FALSE);colnames(data4.5)<-c("Y","X")
lm4.5<-lm(Y~X,data = data4.5)
confint(lm4.5,level=0.95)
```

```
##                2.5 %    97.5 %
## (Intercept) 162.9013 174.29875
## X             1.8405   2.22825
```

**a)** The Bonferroni joint confidence interval for $\beta_0$ and $\beta_1$ is [162.9013,174.29875] and [1.8405,2.22825]. According to Bonferroni procedure, the family confidence coefficient is at least 0.90.

**b)** According to the statement (4.5) in the textbook, $\sigma(b_0, b_1) = -\bar{X}\sigma^2(b_1)$, as the mean of X is positve, $b_0$ and $b_1$ is negatively correlated.

**c)** We have at least 0.90 confidence that both conclusion for $\beta_0$ and $\beta_0$ are correct when each confidence intervals are as given in a).

### Ex4.10

```r
data4.10<-read.table("CH01PR27.txt",header=FALSE);colnames(data4.10)<-c("Y","X")
lm4.10<-lm(Y~X,data = data4.10)
newdata<-data.frame(X=c(45,55,65))
std4.10<-predict(lm4.10,newdata,se.fit=TRUE,interval="confidence")
W<-sqrt(2*pf(0.95,2,58))
CI4.10L<-std4.10$fit[,1]-W*std4.10$se.fit
CI4.10U<-std4.10$fit[,1]+W*std4.10$se.fit
CI4.10L;CI4.10U
```

```
##         1         2         3
## 100.90711  89.63280  77.73154
```

```
##         1         2         3
## 104.68642  92.16082  80.26217
```

**a)** From Working-Hotelling procedure, the interval for age = 45, 55, 65 are [100.90711, 104.68642], [89.63280, 92.16082], [77.73154, 80.26217].

**b)** Let's calculate the Bonferroni joint confidence interval.

```r
predict(lm4.10,newdata,interval="confidence",level = (1-0.05/3))
```

```
##         fit      lwr       upr
## 1 102.79677 98.56965 107.02388
## 2  90.89681 88.06924  93.72438
## 3  78.99686 76.16637  81.82734
```

In comparison, Working Hotelling is the better one here.

**c)** The joint confidence interval level is 0.98333. By Bonferroni method, the point estimates and confidence intervals for age 48, 59, 74 are

99.22678, [78.73541, 11971815]

86.13683, [65.81829, 106.45537]

68.28690, [47.73184, 88.84195]

```r
newdata<-data.frame(X=c(48,59,74))
predict(lm4.10,newdata,interval = "prediction",level = (1-0.05/3))
```

```
##        fit      lwr       upr
## 1 99.22678 78.73541 119.71815
## 2 86.13683 65.81829 106.45537
## 3 68.28690 47.73184  88.84195
```

**d)** The joint intervals have to be re-calculated, for the change in number of intervals will affect confidence level in Bonferroni method. Scheffe method will also need modification, according to statement (4.8), $S^2 = gF(1-\alpha; g, n-2)$, where g is the number of points. Here g changed from 3 to 4, so Scheffe also need modification.

## Ex4.16

```r
data4.16<-read.table("CH01PR20.txt",header=FALSE);colnames(data4.16)<-c("Y","X")
lm4.16<-lm(Y~ -1 + X,data=data4.16)
```

**a)** The estimated regression function is Y = 14.9472296X.

```r
confint(lm4.16,"X",level=0.9)
```

```
##        5 %     95 %
## X 14.56678 15.32767
```

**b)** The 0.90 confidence interval for b is [14.56678, 15.32767].

```r
newdata<-data.frame(X=c(6))
predict(lm4.16,newdata,interval="prediction",level=0.9)
```

```
##        fit      lwr      upr
## 1 89.68338 74.69559 104.6712
```

**c)** The point estimate is 89.68338, the confidence interval is [74.69559,104.6712].

## Ex4.20

```r
Xnew<-(238-lm4.5$coefficients[1])/lm4.5$coefficients[2]
ANOVA<-anova(lm4.5)
```

**a)** The 0.99 confidence interval for elapsed time is [30.84652,37.38082].

```r
ANOVA<-anova(lm4.5)
numerator<-qt(0.995,43)^2*ANOVA$`Mean Sq`[2]
denu<-ANOVA$`Sum Sq`[1]
numerator/denu
```

```
## [1] 0.01434054
```

**b)** The criterion(4.33) has a value of 0.0143405, so our prediction on elapsed time is appropriate.

### Ex5.7

```r
data5.7<-read.table("CH01PR22.txt",header=FALSE);colnames(data5.7)<-c("Y","X")
sum(data5.7$Y^2)
```

```
## [1] 819499
```

**a)** $Y^tY = 819499$.

```r
X<-cbind(rep(1,16),data5.7$X)
t(X)%*%X
```

```
##      [,1]  [,2]
## [1,]   16   448
## [2,]  448 13824
```

```r
t(X)%*%data5.7$Y
```

```
##        [,1]
## [1,]   3609
## [2,] 103656
```

**b)** $X^tX = \begin{pmatrix} 16 & 448 \\ 448 & 13824 \end{pmatrix}$

**c)** $X^tY = \begin{pmatrix} 3609 \\ 103656 \end{pmatrix}$

### Ex5.26

```r
solve(t(X)%*%X)
```

```
##           [,1]        [,2]
## [1,]  0.675000 -0.02187500
## [2,] -0.021875  0.00078125
```

```r
lm5.26<-lm(Y~X,data=data5.7)
```

**a)**

- (1) $(X^tX)^{-1} = \begin{pmatrix} 0.675000 & -0.02187500 \\ -0.021875 & 0.00078125 \end{pmatrix}$
- (2) b $= \begin{pmatrix} 168.6 \\ 2.03 \end{pmatrix}$
- (3) $\hat{Y}^t$ = [-2.15, 3.85, -5.15, -1.15, 0.575, 2.575, -2.425, 5.575, 3.3, 0.3, 1.3, -3.7, 0.025, -1.975, 3.025, -3.975]
- (4) H

```r
X%*%solve(t(X)%*%X)%*%t(X)
```

```
##        [,1]  [,2]  [,3]  [,4]  [,5]  [,6]  [,7]  [,8]  [,9] [,10]
##  [1,] 0.175 0.175 0.175 0.175 0.100 0.100 0.100 0.100 0.025 0.025
##  [2,] 0.175 0.175 0.175 0.175 0.100 0.100 0.100 0.100 0.025 0.025
##  [3,] 0.175 0.175 0.175 0.175 0.100 0.100 0.100 0.100 0.025 0.025
##  [4,] 0.175 0.175 0.175 0.175 0.100 0.100 0.100 0.100 0.025 0.025
##  [5,] 0.100 0.100 0.100 0.100 0.075 0.075 0.075 0.075 0.050 0.050
```

```
##  [6,]  0.100  0.100  0.100  0.100 0.075 0.075 0.075 0.075 0.050 0.050
##  [7,]  0.100  0.100  0.100  0.100 0.075 0.075 0.075 0.075 0.050 0.050
##  [8,]  0.100  0.100  0.100  0.100 0.075 0.075 0.075 0.075 0.050 0.050
##  [9,]  0.025  0.025  0.025  0.025 0.050 0.050 0.050 0.050 0.075 0.075
## [10,]  0.025  0.025  0.025  0.025 0.050 0.050 0.050 0.050 0.075 0.075
## [11,]  0.025  0.025  0.025  0.025 0.050 0.050 0.050 0.050 0.075 0.075
## [12,]  0.025  0.025  0.025  0.025 0.050 0.050 0.050 0.050 0.075 0.075
## [13,] -0.050 -0.050 -0.050 -0.050 0.025 0.025 0.025 0.025 0.100 0.100
## [14,] -0.050 -0.050 -0.050 -0.050 0.025 0.025 0.025 0.025 0.100 0.100
## [15,] -0.050 -0.050 -0.050 -0.050 0.025 0.025 0.025 0.025 0.100 0.100
## [16,] -0.050 -0.050 -0.050 -0.050 0.025 0.025 0.025 0.025 0.100 0.100
##       [,11] [,12]  [,13]  [,14]  [,15]  [,16]
##  [1,] 0.025 0.025 -0.050 -0.050 -0.050 -0.050
##  [2,] 0.025 0.025 -0.050 -0.050 -0.050 -0.050
##  [3,] 0.025 0.025 -0.050 -0.050 -0.050 -0.050
##  [4,] 0.025 0.025 -0.050 -0.050 -0.050 -0.050
##  [5,] 0.050 0.050  0.025  0.025  0.025  0.025
##  [6,] 0.050 0.050  0.025  0.025  0.025  0.025
##  [7,] 0.050 0.050  0.025  0.025  0.025  0.025
##  [8,] 0.050 0.050  0.025  0.025  0.025  0.025
##  [9,] 0.075 0.075  0.100  0.100  0.100  0.100
## [10,] 0.075 0.075  0.100  0.100  0.100  0.100
## [11,] 0.075 0.075  0.100  0.100  0.100  0.100
## [12,] 0.075 0.075  0.100  0.100  0.100  0.100
## [13,] 0.100 0.100  0.175  0.175  0.175  0.175
## [14,] 0.100 0.100  0.175  0.175  0.175  0.175
## [15,] 0.100 0.100  0.175  0.175  0.175  0.175
## [16,] 0.100 0.100  0.175  0.175  0.175  0.175
```

- (5) SSE $= 146.425$.

```r
solve(t(X)%*%X)*anova(lm5.26)$'Mean Sq'[2]
```

```
##            [,1]         [,2]
## [1,]  7.0597768 -0.228789063
## [2,] -0.2287891  0.008171038
```

- (6) $s^2(b) = \begin{pmatrix} 7.0597768 & -0.228789063 \\ -0.2287891 & 0.008171038 \end{pmatrix}$

```r
newdata<-data.frame(X=c(30))
predict(lm5.26,newdata,se.fit = TRUE,interval = "prediction")$se.fit^2
```

```
## [1] 0.6863672
```

- (7) $s^2(predict)$ when $X_h = 30$ is 0.6863672.