Algorithm 1 NorSand model algorithm

Assembling the tangent matrix D_{ijkl}
$\delta\sigma_{ij} = D_{ijkl}\delta\epsilon_{kl}$
$\sigma_{trial} = \sigma + \delta\sigma$
$J_2 = \frac{1}{6}((\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{22})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{33}^2))$
$p_{trial} = \sigma_i j\delta_{ij}/3,\ q_{trial} = \sqrt{3J_2}$
Then we going to calculating the yield surface
$M_i = M_{tc} \cdot (1 - \frac{\chi N|\psi|}{M_{tc}})$
$p_{max} = p_i \exp(\frac{\chi\psi}{M_{tc}})$
$F = q_{trial} - p_{trial}M_i(1 - \ln(\frac{p_{trial}}{p_i})),\ F_2 = p_{trial} - p_{max}$

Algorithm 2 Summary of the procedure used to calculate the total stress with regard to the give strain increment

Require: Given: the total stress σ and the total strain ϵ at current step, $\Delta\epsilon$ is to be applied to the configuration; Yield function F

Ensure: $F(\sigma) \leq 0$

$\Delta\hat{\sigma} = C_E \cdot \Delta\epsilon$ ▷ Calculate the stress increment assuming elastic behavior

$\hat{\sigma} = \sigma + \Delta\hat{\sigma}$ ▷ Calculate the trial stress

$F(\hat{\sigma})$ ▷ Calculate the value of the yield function, with $\hat{\sigma}$ as the state of stress,

if $F(\hat{\sigma}) \leq 0$ **then**

  $\sigma = \hat{\sigma}$ ▷ The strain increment is elastic. In this case, the trial stress is correct; we return.

  return σ

else

  if The previous stress is plastic **then**

    $r = 0$ ▷ r is the portion of incremental strain taken elastically.

  else ▷ There is a transition from elstic to plastic

    Determine r via $F(\sigma + r \cdot \Delta\epsilon) = 0$

    $\sigma = C_E \cdot r\Delta\epsilon$

    $\Delta\epsilon^p = (1 - r) \cdot \Delta\epsilon$ ▷ $\Delta\epsilon^p$ is the elastoplastic strain increment

    $\sigma \leftarrow \sigma + C_{EP} \cdot \Delta\epsilon^p_i$ ▷ Divide $\Delta\epsilon^p$ into $\Delta\epsilon^p_{0\sim i}$ subincrements and iterate

  end if

end if

return σ

Algorithm 3 Procesures to generate the random loading path via gaussian process and principal rotation

Require: Given: The ampltitude for principle strains
Ensure: $F(\sigma) \leq 0$

$\Delta\hat{\sigma} = C_E \cdot \Delta\epsilon$ $\triangleright$ Calculate the stress increment assuming elastic behavior
$\hat{\sigma} = \sigma + \Delta\hat{\sigma}$ $\triangleright$ Calculate the trial stress
$F(\hat{\sigma})$ $\triangleright$ Calculate the value of the yield function, with $\hat{\sigma}$ as the state of stress,
if $F(\hat{\sigma}) \leq 0$ **then**
    $\sigma = \hat{\sigma}$ $\triangleright$ The strain increment is elastic. In this case, the trial stress is correct; we return.
else
    if The previous stress is plastic **then**
        $r = 0$ $\triangleright$ r is the portion of incremental strain taken elastically.
    else $\triangleright$ There is a transition from elstic to plastic
        Determine r via $F(\sigma + r \cdot \Delta\epsilon) = 0$
        $\sigma = C_E \cdot r\Delta\epsilon$
        $\Delta\epsilon^{ep} = (1 - r) \cdot \Delta\epsilon$ $\triangleright$ $\Delta\epsilon^{ep}$ is the elastoplastic strain increment
        Call Algorithm for elastoplastic remapping($\Delta\epsilon^{ep}$)
    end if
end if

Algorithm 4 Elastoplastic return mapping (Convergence of this algorithm needs verification)

Require: Given: $\mathrm{d}\epsilon$, $H(\bar{\epsilon^p})$, and $f(\sigma, H) = 0$

$k = 1$, $\sigma^{(n+1,k)} = \sigma^{(n)} + D\mathrm{d}\epsilon$ $\triangleright$ Calculate the trial stress as the begining of the return mapping iteration
while $f(\sigma^{(n+1,k)}, H(\bar{\epsilon^p}^{(k)})) > 0$ **do** $\triangleright$ $f = g$ in Association Flow assumption
    $h = -\left(\frac{\partial f}{\partial \int |\mathrm{d}\bar{\epsilon^p}|} \| \frac{\partial f}{\partial \sigma} \|\right)^{(k)}$
    $\mathrm{d}\epsilon^p = \left(\frac{\frac{\partial f}{\partial \sigma} D \frac{\partial g}{\partial \sigma}}{h + \frac{\partial f}{\partial \sigma} D \frac{\partial g}{\partial \sigma}}\right)^{(k)} \mathrm{d}\epsilon$
    $\mathrm{d}\epsilon^p = \mathrm{d}\lambda \frac{\partial g}{\partial \sigma}^{(k)}$
    $\bar{\epsilon^p}^{(n+1,k)} = \bar{\epsilon^p}^{(n+1,k-1)} + \|\mathrm{d}\epsilon^p\|$
    $\sigma^{(n+1,k)} = \sigma^{(n+1,k-1)} - D\mathrm{d}\epsilon^p$
    $k{+}{+}$
end while