Algorithm 1 NorSand model algorithm

Assembling the tangent matrix D_{ijkl}
$\delta\sigma_{ij} = D_{ijkl}\delta\epsilon_{kl}$
$\sigma_{trial} = \sigma + \delta\sigma$
$J_2 = \frac{1}{6}((\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{22})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{33}^2))$
$p_{trial} = \sigma_i j\delta_{ij}/3,\ q_{trial} = \sqrt{3J_2}$
Then we going to calculating the yield surface
$M_i = M_{tc} \cdot (1 - \frac{\chi N|\psi|}{M_{tc}})$
$p_{max} = p_i \exp\left(\frac{\chi\psi}{M_{tc}}\right)$
$F = q_{trial} - p_{trial}M_i(1 - \ln(\frac{p_{trial}}{p_i})),\ F_2 = p_{trial} - p_{max}$

Algorithm 2 Procesures to generate the random loading path via gaussian process and principal rotation

Given: Given: The ampltitude for principle strains

Algorithm 3 Summary of the procedure used to calculate the total stress with regard to the give strain increment

Given: The total stress $\boldsymbol{\sigma}$, the total strain $\boldsymbol{\epsilon}$ at current step, and the accumulated plastic strain ϵ (used as the internal variable), $\Delta\boldsymbol{\epsilon}$ is to be applied to the configuration; Yield function f

Ensure: $f(\boldsymbol{\sigma}, H) \leq 0$

$\Delta\hat{\boldsymbol{\sigma}} = \mathbf{D}_{\mathrm{e}} \cdot \Delta\boldsymbol{\epsilon}$ ▷ Calculate the stress increment assuming elastic behavior

$\hat{\boldsymbol{\sigma}} = \boldsymbol{\sigma} + \Delta\hat{\sigma}$ ▷ Calculate the trial stress

$f(\hat{\boldsymbol{\sigma}})$ ▷ Calculate the value of the yield function, with $\hat{\sigma}$ as the state of stress,

if $f(\hat{\boldsymbol{\sigma}}) \leq 0$ **then**

    $\boldsymbol{\sigma} = \hat{\boldsymbol{\sigma}}$ ▷ The strain increment is elastic. In this case, the trial stress is correct; we return.

    return $\boldsymbol{\sigma}$

else

    if The previous stress is plastic **then**

        $r = 0$ ▷ r is the portion of incremental strain taken elastically.

        Call ELASTOPLASTIC RETURN MAPPING($\Delta\epsilon^{ep}$)

    else ▷ There is a transition from elstic to plastic

        Determine r via $F(\boldsymbol{\sigma} + r \cdot \Delta\boldsymbol{\epsilon}) = 0$

        $\boldsymbol{\sigma} = \mathbf{D}_{\mathrm{e}} \cdot r\Delta\boldsymbol{\epsilon}$

        $\Delta\boldsymbol{\epsilon}^{ep} = (1 - r) \cdot \Delta\boldsymbol{\epsilon}$ ▷ $\Delta\boldsymbol{\epsilon}^{ep}$ is the elastoplastic strain increment

        Call ELASTOPLASTIC RETURN MAPPING($\Delta\boldsymbol{\epsilon}^{ep}$)

    end if

end if

return $\boldsymbol{\sigma}$

Algorithm 4 Elastoplastic return mapping (Convergence of this algorithm needs verification)

Given: The strain increment $\Delta\epsilon$, the hardening function $H(\bar{\epsilon}^p)$, and the yield function $f(\boldsymbol{\sigma}, H)$

Ensure: $f(\boldsymbol{\sigma}, H) = 0$

$f_{tol} = 1\mathrm{e}-2$

$k = 1$, $\sigma_k^{(n+1)} = \sigma^{(n)} + \mathbf{D}_\mathrm{e}\Delta\epsilon$ ▷ Calculate the trial stress as the begining of the return mapping iteration

while $f(\sigma_k^{(n+1)}, H(\bar{\epsilon}_k^p)) > f_{tol}$ **do** ▷ $f = g$ in Association Flow assumption

  $\mathbf{a}_k = \mathbf{b}_k = (\partial f/\partial\boldsymbol{\sigma})_k$

  $B_k = \delta\bar{\epsilon}^p/\delta\lambda = \sqrt{2/3}\|\mathbf{a}_k\|$

  $A_k = -\delta f_k/\delta\bar{\epsilon}_k^p \cdot B_k$

  $\delta\lambda = f_k/(\mathbf{a}_k^T\mathbf{D}_\mathrm{e}\mathbf{a}_k)$

  $\delta\boldsymbol{\sigma} = -f_k\mathbf{D}_\mathrm{e}\mathbf{b}_k/\left(A_k + \mathbf{a}_k^\mathrm{T}\mathbf{D}_\mathrm{e}\mathbf{b}_k\right)$

  $\delta\bar{\epsilon}^p = f_k B_k/\left(A_k + \mathbf{a}_k^\mathrm{T}\mathbf{D}_\mathrm{e}\mathbf{b}_k\right)$

  $\boldsymbol{\sigma}_{k+1}^{(n+1)} = \boldsymbol{\sigma}_k^{(n+1)} + \delta\boldsymbol{\sigma}$

  $\bar{\epsilon}_{k+1}^p = \bar{\epsilon}_k^p + \delta\bar{\epsilon}^p$

  $\boldsymbol{\epsilon}_{k+1}^p = \boldsymbol{\epsilon}_k^p + \mathbf{a}_k\delta\lambda$

  $k = k + 1$

end while

return $\boldsymbol{\sigma}$, $\boldsymbol{\epsilon}_{k+1}^p$, $\bar{\epsilon}_{k+1}^p$