Algorithm 1 NorSand model algorithm

Assembling the tangent matrix D_{ijkl}

$\delta\sigma_{ij} = D_{ijkl}\delta\epsilon_{kl}$

$\sigma_{trial} = \sigma + \delta\sigma$

$J_2 = \frac{1}{6}((\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{22})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{33}^2))$

$p_{trial} = \sigma_i j\delta_{ij}/3,\ q_{trial} = \sqrt{3J_2}$

Then we going to calculating the yield surface

$M_i = M_{tc} \cdot (1 - \frac{\chi N|\psi|}{M_{tc}})$

$p_{max} = p_i \exp{(\frac{\chi\psi}{M_{tc}})}$

$F = q_{trial} - p_{trial}M_i(1 - \ln(\frac{p_{trial}}{p_i})),\ F_2 = p_{trial} - p_{max}$

Algorithm 2 Summary of the procedure used to calculate the total stress with regard to the give strain increment

Require: Given: the total stress σ and the total strain ϵ at current step, $\Delta\epsilon$ is to be applied to the configuration; Yield function F

Ensure: $F(\sigma) \leq 0$

$\Delta\hat{\sigma} = C_E \cdot \Delta\epsilon$ ▷ Calculate the stress increment assuming elastic behavior
$\hat{\sigma} = \sigma + \Delta\hat{\sigma}$ ▷ Calculate the trial stress
$F(\hat{\sigma})$ ▷ Calculate the value of the yield function, with $\hat{\sigma}$ as the state of stress,
if $F(\hat{\sigma}) \leq 0$ **then**
    $\sigma = \hat{\sigma}$ ▷ The strain increment is elastic. In this case, the trial stress is correct; we return.
    return σ
else
    if The previous stress is plastic **then**
        $r = 0$ ▷ r is the portion of incremental strain taken elastically.
    else ▷ There is a transition from elstic to plastic
        Determine r via $F(\sigma + r \cdot \Delta\epsilon) = 0$
        $\sigma = C_E \cdot r\Delta\epsilon$
        $\Delta\epsilon^p = (1 - r) \cdot \Delta\epsilon$ ▷ $\Delta\epsilon^p$ is the elastoplastic strain increment
        $\sigma \leftarrow \sigma + C_{EP} \cdot \Delta\epsilon_i^p$ ▷ Divide $\Delta\epsilon^p$ into $\Delta\epsilon_{0\sim i}^p$ subincrements and iterate
    end if
end if
return σ

Algorithm 3 Procesures to generate the random loading path via gaussian process and principal rotation

Require: Given:
Ensure: $F(\sigma) \leq 0$

$\Delta\hat{\sigma} = C_E \cdot \Delta\epsilon$ ▷ Calculate the stress increment assuming elastic behavior
$\hat{\sigma} = \sigma + \Delta\hat{\sigma}$ ▷ Calculate the trial stress
$F(\hat{\sigma})$ ▷ Calculate the value of the yield function, with $\hat{\sigma}$ as the state of stress,
if $F(\hat{\sigma}) \leq 0$ **then**
    $\sigma = \hat{\sigma}$ ▷ The strain increment is elastic. In this case, the trial stress is correct; we return.
else
    if The previous stress is plastic **then**
        $r = 0$ ▷ r is the portion of incremental strain taken elastically.
    else ▷ There is a transition from elstic to plastic
        Determine r via $F(\sigma + r \cdot \Delta\epsilon) = 0$
        $\sigma = C_E \cdot r\Delta\epsilon$
        $\Delta\epsilon^p = (1 - r) \cdot \Delta\epsilon$ ▷ $\Delta\epsilon^p$ is the elastoplastic strain increment
        $\sigma \leftarrow \sigma + C_{EP} \cdot \Delta\epsilon_i^p$ ▷ Divide $\Delta\epsilon^p$ into $\Delta\epsilon_{0\sim i}^p$ subincrements and iterate
    end if
end if