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**Algorithm 1** NorSand model algorithm

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- 1: Assembling the tangent matrix  $D_{ijkl}$
  - 2:  $\delta\sigma_{ij} = D_{ijkl}\delta\epsilon_{kl}$
  - 3:  $\sigma_{trial} = \sigma + \delta\sigma$
  - 4:  $J_2 = \frac{1}{6}((\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{22})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{33}^2))$
  - 5:  $p_{trial} = \sigma_{ij}\delta_{ij}/3$ ,  $q_{trial} = \sqrt{3J_2}$
  - 6: Then we going to calculating the yield surface
  - 7:  $M_i = M_{tc} \cdot (1 - \frac{\chi N|\psi|}{M_{tc}})$
  - 8:  $p_{max} = p_i \exp(\frac{\chi\psi}{M_{tc}})$
  - 9:  $F = q_{trial} - p_{trial}M_i(1 - \ln(\frac{p_{trial}}{p_i}))$ ,  $F_2 = p_{trial} - p_{max}$
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**Algorithm 2** Procesures to generate the random loading path via gaussian process and principal rotation

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**Require:** Given: The amplititude for principle strains

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**Algorithm 3** Summary of the procedure used to calculate the total stress with regard to the give strain increment

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**Require:** Given: the total stress  $\sigma$  and the total strain  $\epsilon$  at current step,  $\Delta\epsilon$  is to be applied to the configuration; Yield function  $F$

**Ensure:**  $F(\sigma) \leq 0$

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1:  $\Delta\hat{\sigma} = C_E \cdot \Delta\epsilon$   $\triangleright$  Calculate the stress increment assuming elastic behavior
2:  $\hat{\sigma} = \sigma + \Delta\hat{\sigma}$   $\triangleright$  Calculate the trial stress
3:  $F(\hat{\sigma})$   $\triangleright$  Calculate the value of the yield function, with  $\hat{\sigma}$  as the state of stress,
4: if  $F(\hat{\sigma}) \leq 0$  then
5:    $\sigma = \hat{\sigma}$   $\triangleright$  The strain increment is elastic. In this case, the trial stress is correct; we return.
6:   return  $\sigma$ 
7: else
8:   if The previous stress is plastic then
9:      $r = 0$   $\triangleright$   $r$  is the portion of incremental strain taken elastically.
10:  else  $\triangleright$  There is a transition from elstic to plastic
11:    Determine  $r$  via  $F(\sigma + r \cdot \Delta\epsilon) = 0$ 
12:     $\sigma = C_E \cdot r \Delta\epsilon$ 
13:     $\Delta\epsilon^p = (1 - r) \cdot \Delta\epsilon$   $\triangleright$   $\Delta\epsilon^p$  is the elastoplastic strain increment
14:    Call ELASTOPLASTIC RETURN MAPPING( $\Delta\epsilon^{ep}$ )
15:  end if
16: end if
17: return  $\sigma$ 

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**Algorithm 4** Elastoplastic return mapping (Convergence of this algorithm needs verification)

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**Require:** Given:  $d\epsilon$ ,  $H(\bar{\epsilon}^p)$

**Ensure:**  $f(\sigma, H) = 0$

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1:  $k = 1$ ,  $\sigma^{(n+1,k)} = \sigma^{(n)} + Dd\epsilon$   $\triangleright$  Calculate the trial stress as the beginning of
   the return mapping iteration
2: while  $f(\sigma^{(n+1,k)}, H(\bar{\epsilon}^p)) > 0$  do  $\triangleright f = g$  in Association Flow
   assumption
3:    $h = -(\frac{\partial f}{\partial \int |d\bar{\epsilon}^p|} \|\frac{\partial f}{\partial \sigma}\|)^{(k)}$ 
4:    $H = h + \frac{\partial f}{\partial \sigma} D \frac{\partial g}{\partial \sigma}$ 
5:    $d\lambda = \frac{1}{H} \frac{\partial f}{\partial \sigma} D d\epsilon$ 
6:    $d\epsilon^p = d\lambda \frac{\partial g}{\partial \sigma}^{(k)}$ 
7:    $\bar{\epsilon}^{p(n+1,k)} = \bar{\epsilon}^{p(n+1,k-1)} + \|d\epsilon^p\|$ 
8:    $\sigma^{(n+1,k)} = \sigma^{(n+1,k-1)} - Dd\epsilon^p$ 
9:    $k++$ 
10: end while
11: if  $f(\sigma^{(n+1,k)}, H(\bar{\epsilon}^p)) < -f_{tolerance}$  then  $\triangleright$  Avoiding overestimating
   plasticity
12:   SEARCHING FOR THE RIGHT SCALAR OF PLASTIC STRAIN VIA DI-
   CHOTOMY( $d\lambda$ )
13: end if
14:  $d\epsilon^p = d\lambda \cdot \frac{\partial g}{\partial \sigma}^{(k)}$ 
15:  $\bar{\epsilon}^p = \bar{\epsilon}^p + \|d\epsilon^p\|$ 
16:  $\sigma = \sigma + D \cdot (d\epsilon - d\epsilon^p)$ 
17: return  $\sigma$ 

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**Algorithm 5** Searching for the right scalar of plastic strain via dichotomy

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**Require:**  $d\lambda$

**Ensure:**  $f_{tolerance} < f(\sigma, H) < 0$

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1:  $r_{min} = 0, r_{max} = 1$ 
2: while  $f(\sigma, H) < -f_{tolerance}$  or  $f(\sigma, H) > 0$  do
3:    $r_{mid} = (r_{min} + r_{max})/2$ 
4:    $d\epsilon_{mid}^p = r_{mid}d\epsilon^p$ 
5:    $\bar{\epsilon}_{mid}^p = \bar{\epsilon}^p + \|d\epsilon_{mid}^p\|$ 
6:    $H_{mid} = \hat{H}(\bar{\epsilon}_{mid}^p)$ 
7:    $\sigma_{mid} = \sigma_{trial} - Dd\epsilon_{mid}^p$ 
8:    $f_{mid} = \hat{f}(\sigma_{mid}, H_{mid})$ 
9:   if  $f_{mid} > 0$  then
10:     $r_{min} = r_{mid}$ 
11:   else
12:     $r_{min} = r_{mid}$ 
13:   end if
14: end while
15: return  $d\epsilon_{mid}^p$ 

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