Machine Learning HW2

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Problem 2

Let's set derivative of the Lagrangian function with respect to each of the parameters w, b and ξ_i to zero.

$$\frac{\partial L}{\partial w} = w - \sum_{i=1}^{m} \alpha_i y_i x_i = 0 \Rightarrow w = \sum_{i=1}^{m} \alpha_i y_i x_i$$

This gives equation (2)

$$\frac{\partial L}{\partial b} = -\sum_{i=1}^{m} \alpha_i y_i = 0 \Rightarrow \sum_{i=1}^{m} \alpha_i y_i = 0$$

This gives equation (3)

$$\frac{\partial L}{\partial \xi_i} = C - \alpha_i - \beta_i = 0 \Rightarrow \alpha_i + \beta_i = C$$

This gives equation (4). And then, for slackness conditions, we have:

$$\alpha_i(y_i(w^T x_i + b) - 1 + \xi_i) = 0$$

$$\Rightarrow \alpha_i = 0 \; or \; y_i(w^T x_i + b) = 1 - \xi_i$$

This gives equation (5)

$$\beta_i \xi_i = 0 \Rightarrow \beta_i = 0 \; or \; \xi_i = 0$$

This gives equation (6)

Problem 3

When $d = 3$, we have:

$$K(x, x^{'}) = (\langle x, x^{'} \rangle + c)^{3} = (x_1 x_1^{'} + x_2 x_2^{'} + c)^3$$

$$= x_1^3 x_1^{'3} + 3x_1^2 x_1^{'2} x_2 x_2^{'} + 3x_1^2 x_1^{'2} c + 3x_1 x_1^{'} x_2^2 x_2^{'2} + 6x_1 x_1^{'} x_2 x_2^{'} c + 3x_1 x_1^{'} c^2 + x_2^3 x_2^{'3} + 3x_2^2 x_2^{'2} c + 3x_2 x_2^{'} c^2 + c^3$$

$$= \left[x_1^3, \sqrt{3}x_1^2 x_2, \sqrt{3c}x_1^2, \sqrt{3}x_1 x_2^2, \sqrt{6}x_1 x_2 c, \sqrt{3}x_1 c, x_2^3, \sqrt{3c}x_2^2, \sqrt{3}x_2 c, c^{\frac{3}{2}}\right] \begin{bmatrix} x_1^{'3} \\ \sqrt{3}x_1^{'2} {x_2}' \\ \sqrt{3c}x_1^{'2} \\ \sqrt{3}{x_1}' x_2^{'2} \\ \sqrt{6}{x_1}'{x_2}'c \\ \sqrt{3}x_1^{'}c \\ x_2^{'3} \\ \sqrt{3c}x_2^{'2} \\ \sqrt{3}x_2^{'}c \\ c^{\frac{3}{2}} \end{bmatrix}$$

Hence, when $d = 3$,
$\Phi(x) = \left[x_1^3, \sqrt{3}x_1^2 x_2, \sqrt{3c}x_1^2, \sqrt{3}x_1 x_2^2, \sqrt{6}x_1 x_2 c, \sqrt{3}x_1 c, x_2^3, \sqrt{3c}x_2^2, \sqrt{3}x_2 c, c^{\frac{3}{2}}\right]^T$ in this case.