ORLM: Training Large Language Models for Optimization Modeling

Zhengyang Tang^{1,5} Chenyu Huang² Xin Zheng² Shixi Hu³ Dongdong Ge⁴ Zizhuo Wang¹ Benyou Wang^{1,5} ¹The Chinese University of Hong Kong, Shenzhen ²Shanghai University of Finance and Economics ³ Cardinal Operations ⁴Shanghai Jiao Tong University ⁵Shenzhen Research Institute of Big Data {zhengyangtang}@link.cuhk.edu.cn {wangzizhuo, wangbenyou}@cuhk.edu.cn {chenyuhuang}@stu.sufe.edu.cn {xin.zheng}@duke.edu {ddge}@sjtu.edu.cn

Abstract

Large Language Models (LLMs) have emerged as powerful tools for tackling complex Operations Research (OR) problem by providing the capacity in automating optimization modeling. However, current methodologies heavily rely on prompt engineering (e.g., multi-agent cooperation) with proprietary LLMs, raising data privacy concerns that could be prohibitive in industry applications. To tackle this issue, we propose training open-source LLMs for optimization modeling. We identify four critical requirements for the training dataset of OR LLMs, design and implement OR-INSTRUCT, a semi-automated process for creating synthetic data tailored to specific requirements. We also introduce the IndustryOR benchmark, the first industrial benchmark for testing LLMs on solving real-world OR problems. We apply the data from OR-INSTRUCT to various open-source LLMs of 7b size (termed as ORLMs), resulting in a significantly improved capability for optimization modeling. Our best-performing ORLM achieves state-of-the-art performance on the NL4OPT, MAMO, and IndustryOR benchmarks. Our code and data are available at https://github.com/Cardinal-Operations/ORLM.

1 Introduction

Large language models (LLMs) have emerged as powerful tools for tackling complex operations research (OR) problem by providing the capacity in automating optimization modeling Xiao et al. [2023], AhmadiTeshnizi et al. [2024], Li et al. [2023a]. These models excel at interpreting problem descriptions and generating mathematical models and programs with unprecedented accuracy and efficiency. By applying LLMs to the challenges in logistics, healthcare, finance, and beyond Singh [2012], industries can achieve faster, more accurate decision-making outcomes. This integration of LLMs also enhances the capability to handle dynamic and uncertain scenarios where traditional approaches might falter. As these technologies evolve, the synergy between optimization modeling and LLMs is expected to yield breakthroughs that could transform industries, enabling more automated and versatile decision-making across various scenarios.

^{*} Corresponding author.

Previous research has focused on applying pre-trained language models (PLMs)[Liu et al., 2019, Lewis et al., 2019] in formulating mathematical models for optimization problems. NL4OPT Ramamonjison et al. [2023] decomposes the task into multiple steps, where semantic entities are first recognized upon which mathematical models are formulated subsequently. However these approaches often result in accumulated errors in the pipeline and have limited generalization capabilities because of relatively-small parameter scale. With the advent of LLMs like ChatGPT, researchers have begun to generate mathematical models by directly prompting these models. To extend to a complete solution that includes a program using mature solvers, prompting engineering methods such as Chain-of-Experts Xiao et al. [2023], OptiMUS AhmadiTeshnizi et al. [2024] and OptiGuide Li et al. [2023a] employ multi-agent collaboration based on proprietary LLMs. These agents collaborate using complex reasoning chains to refine both the mathematical models and programs. Nevertheless, these approaches heavily rely on proprietary LLMs which require users to submit sensitive data. This can be prohibitive in industry applications since data privacy is always a top concern.

To address the limitations mentioned above, we propose training open-source LLMs for optimization modeling. To ensure our model is effective, robust, and applicable to real-world scenarios, we pinpoint four critical requirements that the training dataset of the model should satisfy, based on academic research Alzubaidi et al. [2023] and industry experience. First, the dataset must cover various scenarios, question types and difficulties, which are essential for our model's robustness. Second, the objectives and constraints may frequently change due to shifts in business goals, market conditions, or resource availability. The dataset should reflect these changes to prepare the model for such adaptability. Furthermore, different customers may describe the same problem using different terms, and the dataset should accommodate this linguistic diversity. Lastly, there are usually multiple modeling techniques for the same problem. Including these in the dataset would enable the model to learn various modeling skills. However, we must admit that collecting data at scale that meets these requirements is challenging, as it primarily exists in private industrial cases and no public data ever comes close to meeting these requirements.

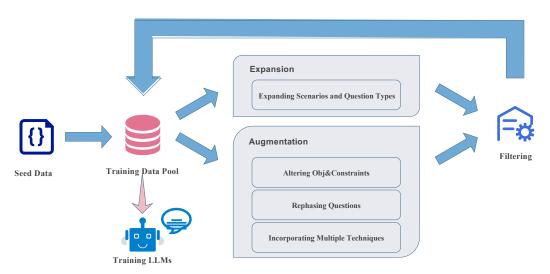


Figure 1: Overview of OR-INSTRUCT.

Therefore, we design and implement OR-INSTRUCT, a semi-automated process for creating synthetic data tailored to specific requirements. The process uses an iterative bootstrapping algorithm (see Figure 1). Initially, we collect a set of seed industry cases (e.g., 686 cases in our study)and add them to the training data pool. Following this, we use two strategies. One is expansion, which employs GPT-4 to generate data covering a wider range of scenarios and question types, expanding to thousands of scenarios and tens of question types. Although this strategy partially addresses comprehensive coverage, we observe that it falls short of meeting the other requirements, as these exceed GPT-4's capabilities. We then take a step back and seek to augment the training data pool. The augmentation include altering objectives and constraints, rephrasing questions, and incorporating multiple modeling techniques. Essentially, we break down the remaining requirements into subtasks, allowing GPT-4 to generate the examples of higher problem-solution diversity within its limitations.

Finally, heuristics are used to automatically filter out obviously low-quality data. This process can be repeated through many iterations until reaching the desired data size. Notably, the process also supports easy customization by simply adding new seed data and scenarios.

To evaluate the effectiveness of OR-INSTRUCT, we introduce IndustryOR, the first industrial benchmark for optimization modeling. This benchmark uses data sourced from 13 different industries, covering 5 types of questions across 3 levels of difficulty. For comprehensive evaluation, we also include NL4OPT Ramamonjison et al. [2023] and MAMO Huang et al. [2024] benchmarks.

We apply the data from OR-INSTRUCT to train open-source LLMs of approximately 7b size, including Mistral-7B Jiang et al. [2023], Deepseek-Math-7B-Base Shao et al. [2024], and LLaMA-3-8B AI@Meta [2024]. We call the resulting models as ORLMs and observe significantly improved capability in optimization modeling. Our experimental results show that our best-performing ORLM achieves state-of-the-art performance on the NL4OPT, MAMO, and IndustryOR benchmarks.

In summary, our contributions are: (1) To our knowledge, we are the first to train open-source LLMs for optimization modeling applicable to real-world industries; (2) We pinpoint four critical requirements for the training dataset of OR LLMs and design OR-INSTRUCT to semi-automatically generate synthetic data tailored to these requirements; (3) We introduce the IndustryOR benchmark, the first industrial benchmark testing LLMs on solving real-world OR problems; (4) Our best-performing ORLM achieves state-of-the-art performance on the NL4OPT, MAMO, and IndustryOR benchmarks.

2 Background and Desiderata

In this section, we provide the formal definition of the optimization modeling task and discuss the critical requirements (called 'desiderata') for effectively training an open-source LLM for the task.

2.1 Definition of Optimization Modeling

Given an OR problem p described in natural language, optimization modeling Berry and Houston [1995], AhmadiTeshnizi et al. [2024] involves constructing a mathematical model m that conceptize the real-world problem into formal objectives and constraints. Completing the solution also entails translating the mathematical model into a program c that leverages highly mature solvers. The primary goal is to find the optimal solution from a set of feasible options, subject to certain constraints.

Hence, an expected training example for this task is usually required in the form of the triplet (p,m,c), as illustrated in Figure 2. Training an OR model f for this task fundamentally involves learning a mapping $f:p\to(m,c)$. Note that in the example presented in Figure 2, we use COPT as the default solver. COPT Ge et al. [2022] stands for Cardinal Optimizer and is commonly used for tackling large-scale optimization challenges. Nonetheless, any other mature solver can be employed when constructing the training dataset.

2.2 Desiderata of Training LLMs for Optimization Modeling

We then discuss the *Desiderata* for effectively training open-source LLMs to excel in the task. Ideally, the trained OR LLM should be effective, robust, and applicable to real-world scenarios. As a result, there are critical requirements that the training dataset of the model should satisfy Alzubaidi et al. [2023]. To improve generalization, the first desideratum is defined as follows:

Desideratum 1 *Comprehensive Coverage:* The dataset should cover: 1) diverse scenarios such as supply chain optimization, scheduling, inventory management, and transportation logistics; 2) different problem types like linear programming, integer programming, and mixed integer programming; and 3) varying difficulty levels (easy, medium, hard as judged by criteria in Appendix A.1).

With a comprehensive coverage, one would expect diversity at the *problem-solution* level, encompassing environmental dynamics, linguistic diversity, and solution variability, as explained in Desiderata 2, 3, and 4.

Desideratum 2 Environmental Adaptability: In real-world industrial settings, the objectives and constraints of problems often change due to shifts in business goals, market conditions, or resource

An Expected Training Example

Input - Natural Language Problem:

```
A company has three transportation options to choose from to transport 25 tons of cargo, namely trucks,
airplanes, and ships with costs $100, $120, $80 per ton and capacities of 10, 20, 30 tons respectively.
The company can't choose trucks and ships together. How should the company optimize the selection
and allocation of these methods to minimize overall costs?
Target - Mathematical Model and Program:
## Mathematical Model:
### Variables:
-x_1, x_2, x_3: 0-1 variables indicating whether trucks, airplanes, and ships are selected, respectively.
-y_1, y_2, y_3: Non-negative continuous variables indicating the volume of cargo transported by trucks,
airplanes, and ships.
### Objectives and Constraints:
  Minimize<sub>x_i \in \{0,1\}, y_i \ge 0</sub> 100y_1 + 120y_2 + 80y_3
               subject to
                         x_1 + x_2 + x_3 \ge 1
                                                                   (At least one transportation)
                        y_1 \le 10x_1, \quad y_2 \le 20x_2, \quad y_3 \le 30x_3
                                                                              (Capacity limits)
                        x_1 + x_3 < 1
                                                                  (Mutual exclusion constraint)
                        y_1 + y_2 + y_3 \ge 25
                                                                  (Transportation requirements)
## Program using COPT solver:
import coptpy as cp
env = cp.Envr()
model = env.createModel("TransportationOptimization")
model.setObjective(cp.quicksum(costs[mode] * y[mode] for mode in costs),
     → sense=cp.COPT.MINIMIZE)
model.addConstr(x['trucks'] + x['ships'] <= 1, name="ModeExclusivity")</pre>
model.addConstr(cp.quicksum(x[mode] for mode in costs) >= 1, name="
     → AtLeastOneMode")
for mode in costs:
    model.addConstr(y[mode] <= capacities[mode] * x[mode], name=f"Capacity_{</pre>
         \hookrightarrow mode}")
model.solve()
```

Figure 2: An expected training example for optimization modeling task.

availability. This mirrors the concept of sensitivity analysis Ward and Wendell [1990] in linear programming, which explores how changes in parameters affect the optimal solution of a model. It's vital that the dataset includes cases reflecting these dynamic changes.

Desideratum 3 *Linguistic Diversity:* Problems described in natural language often show different syntax, ambiguities, and complexities. For example, one problem might mention "inventory overflow" while another refers to "excess stock." Including this linguistic diversity in the dataset could improve the model's ability to understand varied descriptions.

Desideratum 4 *Solution Variability:* For some challenging problems, there may be multiple modeling techniques, such as linearizing a nonlinear problem by introducing auxiliary variables. Including this variety in the dataset allows the model to learn different modeling techniques and approaches.

3 OR-INSTRUCT: Towards Training Effective OR LLMs

As highlighted in the *Desiderata* above, the key recipe for training effective OR LLMs is the training dataset. However, collecting such data on a large scale that meets the *Desiderata* is challenging because: (1) it mainly exists in private industrial cases and no public dataset closely matches the

Desiderata, and (2) general synthesis methods Wang et al. [2022] face difficulties in creating such data, as we will discuss in Section 3.1. Therefore, we design and implement OR-INSTRUCT, a semi-automated process for creating synthetic data tailored to these requirements. The pipeline is depicted in Figure 1, which iteratively applies two strategies, namely expansion and augmentation, to the training data pool, followed by the filtering of obviously low-quality data.

3.1 Expansion for Comprehensive Coverage (Desideratum 1)

Initially, OR-INSTRUCT attempts to generate new data by expanding scenarios and question types from the training data pool in a bootstrapping fashion with GPT-4. We starts with 686 real-world industry cases. For each generation, we sample 3 entries from this pool as in-context examples, keeping the input token length suitable for GPT-4. Of the 3 entries, 2 are from real-world entries and 1 is from the model-generated entries in previous iterations, if available, to promote diversity. The prompting template is shown in Appendix A.3.

This approach is similar to general synthesis methods Wang et al. [2022], but OR-INSTRUCT focuses on expanding scenarios rather than tasks. We observe that this step partially addresses *Comprehensive Coverage* in terms of scenarios and question types. However, it falls short of meeting the other requirements of the *Desiderata*, especially concerning varying levels of difficulty. In manually reviewing the difficulty of generated example, of 50 cases, 87% are deemed easy, 13% medium, and none hard, as judged by criteria in Appendix A.1. We also provide examples in Appendix A.9 for comparing generated easy entries with real-world hard entries. Fortunately, the original seed data already shows a diverse range of difficulties. Thus, we can naturally enhance the difficulty diversity by augmenting them described in the next Section, thereby further addressing *Comprehensive Coverage* (*Desideratum 1*).

3.2 Augmentation for Problem-Solution Diversity (Desideratum 2, 3, and 4)

To fulfill the remaining requirements of the *Desiderata*, OR-INSTRUCT takes a step back and seeks to augment the training data pool. The augmentations aim to address all possible changes, such as alterations in problem descriptions, model modifications, or simultaneous changes. These correspond to rephrasing questions, altering objectives and constraints, and incorporating various modeling techniques. Overall, the augmentation aims to enhance problem-solution diversity.

Altering Objectives and Constraints: The first augmentation involves adding, removing, or replacing objectives and constraints in the problem, along with making necessary adjustments to the mathematical models and programs. Specifically, we start by providing GPT-4 with the original example and ask it to list five potential changes to the objectives and constraints. These suggested changes are then fed back to GPT-4 using a prepared few-shot prompt to modify the problem, model, and programs accordingly. This augmentation is designed to enhance *Environmental Adaptability* (*Desideratum 2*). An example is provided in Figure 3, and the prompting template in Appendix A.4.

```
Altering Objectives and Constraints for Desideratum 2

Original:
Q: ... The company can't choose trucks and ships together. Denote the cost ...

Augmented:
Q: ... The company can't choose trucks and ships together. Due to the special nature of the goods, the company has decided that if trucks are chosen, ships must also be selected for transportation. Denote the cost ...
A: ... New dependency constraint (choosing trucks necessitates choosing ships): x_1 \le x_3 ...

... model.addConstr(x['trucks'] <= x['ships'], name="New constraint")
...
```

Figure 3: An example illustrating Altering Objectives and Constraints. Maroon denotes augmentation.

Rephrasing Questions: The second augmentation modifies the formulation of the question, simulating the expression habits of different customers. This process involves instructing GPT-4 to rewrite the OR problem, either simplifying or complicating it, while ensuring the core logic aligns with the solution, including the mathematical model and programs. This augmentation is designed to enhance *Linguistic Diversity* (*Desideratum 3*). An example is provided in Figure 4, and the prompting template in Appendix A.5.

Original: Q: A company has three transportation options to choose from to transport 25 tons of cargo, namely trucks, airplanes, and ships with costs \$100, \$120, \$80 per ton and capacities of 10, 20, 30 tons respectively. The company can't choose trucks and ships together. How should the company optimize the selection and allocation of these methods to minimize overall costs? Augmented: Q: A corporation wants to transport 25 tons of cargo with least cost, and must choose from three transportation modes: trucks, airplanes, and ships. These options cost \$100, \$120, and \$80 per ton, respectively, with capacities of 10, 20, and 30 tons. However, trucks and ships cannot be used together.

Figure 4: An example illustrating Rephrasing Questions. Maroon denotes augmentation.

Incorporating Multiple Modeling Techniques: The third augmentation explores the use of different modeling techniques. We identify five potential techniques from engineers' experiences, such as introducing auxiliary variables or using the Big M method, for GPT-4 to choose from in modifying an objective or constraint in the original mathematical model. This augmentation is designed to enhance *Solution Variability (Desideratum 4)*. An example is provided in Figure 5, and the prompting template in Appendix A.6.

```
Incorporating Multiple Modeling Techniques for Desideratum 4

Original:

A: Mutual exclusion constraint (trucks and ships cannot be selected simultaneously): x_1 + x_3 \le 1

Augmented:

A: Mutual exclusion constraint (Using big M method): x_1 \le (1 - x_3)M, where M is a large number

...

model.addConstr(x['trucks'] <= (1-x['ships'])*M, name="New constraint")

...
```

Figure 5: An example illustrating Incorporating Multiple Modeling Techniques.

3.3 Postprocessing and Filtering

Finally, OR-INSTRUCT applies several heuristics to the generated examples. First, we remove examples where questions duplicate each other or match questions in any of our evaluation benchmarks. Next, we manually correct minor grammatical errors in the programs, sometimes caused by GPT-4's unfamiliarity with the COPT API (see Appendix A.2). We also discard examples whose programs cannot be executed successfully, as these are considered obviously low-quality data. After a manual review of the remaining data, which shows acceptable accuracy of correctness (70% for expansion data and 75% for augmentation data), we decide to forgo additional filtering in favor of developing a fully automatic filtering mechanism. This filtering step will remove about 39% of the generated examples, and the remaining examples are then added to the training data pool.

4 Experiments

4.1 OR-INSTRUCT Data from GPT-4

Data Generation: We start with 686 industry cases as seed data, adding to the training data pool. We employ gpt-4-0613 as the default proprietary LLM throughout the OR-INSTRUCT process. For each iteration, we apply expansion 20K times and apply each augmentation operation 6K times to the training data pool, and then we automatically filter out obviously low-quality entries. We perform 2 iterations of this process, ultimately resulting in 30K training examples.

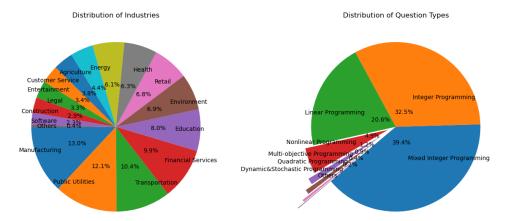


Figure 6: Industry and question type statistics of the OR-INSTRUCT Data.

Statistics: Figure 6 presents the statistics of the OR-INSTRUCT Data. Of the data, 57% is generated by the expansion operation, 17.2% by the augmentation of altering objectives and constraints, 15.3% by the augmentation of rephrasing questions, and 10.5% by the augmentation of incorporating multiple modeling techniques. For scenario diversity, we have abstracted 16 industries from a total of 1,556 expanded scenarios to facilitate their representation in a plot. Regarding question type diversity, it expands to 8 question types. In terms of data quality, as described in Section 3.3, the accuracy of correctness are 70% for expansion data and 75% for augmentation data. We find these accuracy acceptable in exchange for automatic filtering and more cost-effective data synthesis. Additionally, our preliminary experiments have demonstrated the benefits of incorporating such data.

4.2 Model Training and Inference

We borrow a widely used training framework from open-instruct Wang et al. [2023], where we take a natural language OR problem wrapped in an Alpaca-like template Taori et al. [2023] as the input prompt (see Appendix A.7), and treat a complete solution including mathematical models and programs as the target completion. During training, we compute the loss only over the target completion. We apply OR-INSTRUCT Data to train several open-source LLMs of approximately 7b size, including Mistral-7B Jiang et al. [2023], Deepseek-Math-7B-Base Shao et al. [2024], and LLaMA-3-8B AI@Meta [2024], as the backbones. We list the hyper-parameters for each backbone in Appendix A.8. We call the resulting models as ORLMs. For inference during evaluation, we always use greedy decoding under 0-shot setting to eliminate randomness, selecting the top-1 completion as the final solution. We extract the program from the solution and execute it to obtain the predicted optimal value.

4.3 Evaluation and Baselines

Evaluation Benchmarks and Metrics: We use NL4OPT Ramamonjison et al. [2023], MAMO Huang et al. [2024], and IndustryOR as evaluation benchmarks. NL4OPT is the most widely used benchmark for operations research and includes 289 linear programming problems in its test set. However, NL4OPT only provides mathematical models as targets, which complicates the verification of execution accuracy due to the absence of optimal solutions. To address this, we convert these mathematical models into programs using GPT-4, calculate and check the optimal solutions, and use these as ground truth. MAMO, a concurrent project, evaluates the mathematical modeling

capabilities in LLMs. It includes 652 easy and 211 complex linear programming problems, each paired with its corresponding optimal solution, sourced from various academic materials. IndustryOR, the first industrial benchmark, consists of 100 real-world OR problems from eight industries. It covers 5 types of questions—linear programming, integer programming, mixed integer programming, non-linear programming, and others—across 3 levels of difficulty. We measure performance using execution accuracy, where an executed optimal value that matches any provided ground truth optimal value is considered correct. Compared to NL4OPT, this metric enables a fully automated evaluation and provides greater flexibility for mathematical modeling approaches.

Baselines: To ensure a comprehensive evaluation, we select a diverse set of models from previous methods for comparison. We include tag-BART Kani and Gangwar [2022], which secured first place in the NeurIPS competition Ramamonjison et al. [2022]. Additionally, we consider methods that utilize proprietary LLMs. The Standard prompting method involves prompting a proprietary LLM to produce mathematical programs, serving as a fundamental baseline. We also incorporate complex prompt engineering methods such as Reflexion Shinn et al. [2023], Chain-of-Experts Xiao et al. [2023], and OptiMUS AhmadiTeshnizi et al. [2024]. These methods employ agents to refine both mathematical models and programs, achieving exceptional performances on NL4OPT. We report their performance based on GPT-3.5 and GPT-4, respectively. Note that we implement the standard prompting on IndustryOR using the toolkit released by Chain-of-Experts Xiao et al. [2023].

4.4 Main Results

Table 1: Comparison of performance on the NL4OPT, MAMO, and IndustryOR benchmarks. Values marked with a * are directly copied from original papers, with blanks where data were not reported. The highest results are highlighted in bold.

Method	NL4OPT	MAMO EasyLP	MAMO ComplexLP	IndustryOR	Micro Avg	Macro Avg		
Methods based on PLMs								
tag-BART	$47.9\%^*$	-	-	-	-	-		
Methods based on GPT-3.5								
Standard	$42.4\%^{*}$	-	-	-	-	-		
Reflexion	$50.7\%^*$	-	-	-	-	-		
Chain-of-Experts	$58.9\%^*$	-	-	-	-	-		
Methods based on GPT-4								
Standard	$47.3\%^{*}$	$66.5\%^{*}$	14.6%*	28.0%	50.2%	39.1%		
Reflexion	$53.0\%^*$	-	-	-	-	-		
Chain-of-Experts	$64.2\%^*$	-	-	-	-	-		
OptiMUS	$78.8\%^*$	-	-	-	-	-		
ORLMs based on open-source LLMs								
ORLM-Mistral-7B	84.4%	81.4%	32.0%	27.0%	68.8%	56.2%		
ORLM-Deepseek-Math-7B-Base	86.5%	82.2%	37.9%	33.0%	71.2%	59.9%		
ORLM-LLaMA-3-8B	85.7%	82.3%	37.4%	38.0%	71.4%	60.8%		

The results are presented in Table 1. First, it is clear that methods based on LLMs generally outperform the PLM-based best method (tag-BART) in the NL4OPT test. This suggests that PLMs have limited generalization capabilities. For proprietary LLMs, as the mathematical reasoning capability increases from GPT-3.5 to GPT-4, we observe that performance has obviously advanced across all prompt engineering methods. Finally, ORLMs based on various open-source LLMs have demonstrated significantly improved optimization modeling capabilities, compared to vanilla open-source LLMs, which score 0 for all benchmarks in our preliminary experiments due to failing to output executable programs. This underscores the effectiveness of the OR-INSTRUCT data. Our best-performing ORLM, trained on LLaMA-3-8B, achieves state-of-the-art performance on NL4OPT, MAMO (including both easy and complex linear programming), and IndustryOR benchmarks, surpassing Standard prompting based on GPT-4 by 42.2% in micro average and 55.4% in macro average. We also want to highlight that, unlike tag-BART, we ensure no training and validation data from NL4OPT was used. Thus, the performances of the ORLMs across all benchmarks reflect out-of-domain generalization.

5 Analysis and Discussion

5.1 Detailed Comparison of ORLM vs GPT-4 on IndustryOR

Table 2: Comparison of ORLM and GPT-4 on IndustryOR across different difficulty levels and question types.

Method	Difficulty				Question Types			
	Easy	Medium	Hard	LP	NLP	IP	MIP	Others
Standard-GPT-4	45.0%	17.5%	15.0%	33.3%	0.0%	38.7%	12.9%	0.0%
ORLM-LLaMA-3-8B	57.5%	20.0%	35.0%	36.1%	0.0%	61.2%	19.3%	0.0%

To assess the optimization modeling capabilities across different levels of difficulty and question types, we compare ORLM-LLaMA-3-8B and Standard-GPT-4 on IndustryOR as shown in Table 2. ORLM-LLaMA-3-8B shows superior performance over Standard-GPT-4 across all difficulty levels, especially in the hard category. Regarding different question types, ORLM-LLaMA-3-8B outperforms Standard-GPT-4 in linear programming, integer programming, and mixed-integer programming. Both models perform poorly in non-linear programming and other rare question types. We hypothesize this is due to the inherent complexity and scarcity of these types of questions. Overall, the OR-INSTRUCT data proves effective in enhancing *Comprehensive Coverage* across various question types and difficulty levels.

5.2 Ablation Study on OR-INSTRUCT Augmentations

Table 3: Ablation study on OR-INSTRUCT Augmentations.

Method	NL4OPT	MAMO EasyLP	MAMO ComplexLP	IndustryOR	Micro Avg	Macro Avg
Full Augmentations	78.3%	80.6%	43.1%	21.0%	68.6%	55.7%
w/o Altering Obj&Const w/o Rephrasing Questions w/o Multiple Modeling	77.5% 74.2% 78.3%	79.2% 77.3% 78.0%	36.4% 41.1% 38.8%	20.0% 15.0% 18.0%	66.4% 65.1% 66.2%	53.2% 51.9% 53.2%

To further verify the effectiveness of the augmentations in OR-INSTRUCT, we conducted detailed ablation experiments to study the effects of altering objectives and constraints, rephrasing questions, and incorporating multiple modeling techniques. Specifically, we selected a controlled sample size of 3K with certain augmentations from the OR-INSTRUCT data, and then used these to train the LLaMA-3-8B model with the same hyper-parameters across all experiments. The results are presented in Table 4. Training data with all three augmentations (denoted as Full Augmentations) achieves a base performance of 68.6% in micro average and 55.7% in macro average. Removing any of the three augmentations from the base setting leads to a performance drop across all benchmarks, both in micro and macro averages. Rephrasing questions seems slightly more important than the other two. Overall, the results show that all three augmentations contribute to general performance, proving their effectiveness in enhancing the *Problem-solution Diversity*.

6 Related Work

NL4OPT Competition focuses on formulating mathematical models for optimization problems Ramamonjison et al. [2023]. This competition introduces a two-step framework using PLMs and offers a widely used benchmark for the OR community He et al. [2022], Ning et al. [2023], Prasath and Karande [2023], Li et al. [2023b], Xiao et al. [2023], AhmadiTeshnizi et al. [2024]. However, this framework often results in accumulated errors in the pipeline and has limited generalization capabilities due to the relatively-small parameter scale. In this paper, we demonstrate that it is feasible to train open-source LLMs to generate both mathematical models and programs in a single step.

Prompt Engineering uses proprietary LLMs to generate mathematical models and programs for OR problems. For instance, Chain-of-Experts Xiao et al. [2023], OptiMUS AhmadiTeshnizi et al. [2024], and OptiGuide Li et al. [2023a] use multi-agent collaboration with ChatGPT to refine both mathematical models and programs through a complex reasoning chain and execution feedbacks, achieving outstanding performance on NL4OPT. Techniques such as Chain-of-Thought Wei et al.

[2022] and Reflexion Shinn et al. [2023] are also commonly used to boost performance. Unlike these methods, our approach involves training open-source LLMs and delivering a complete solution using direct prompting.

Synthetic Data is an emerging area within the OR community. Prasath and Karande [2023] suggest augmenting the NL4OPT dataset by mutating variables and parameters and generate new word problems through back translation. Li et al. [2023b] aim to synthesize MILP mathematical models, again for NL4OPT problems. In this paper, we design a semi-automated process for creating synthetic data significantly expanding the diversity and complexity beyond augmentations of the NL4OPT dataset.

7 Conclusion

In this paper, we propose training open-source LLMs for optimization modeling. We identify four critical requirements for the training dataset of OR LLMs, design and implement OR-INSTRUCT, a semi-automated process for creating synthetic data tailored to specific requirements. We also introduce the IndustryOR benchmark, the first industrial benchmark. We apply the OR-INSTRUCT data to open-source LLMs of 7b size, resulting in a significantly improved capability for optimization modeling. Looking ahead, we anticipate that extending the OR-INSTRUCT to train open-source agents will lead to further advancements.

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Broader Impact

Our study did not explore all potential biases in ORLMs. It's crucial to mitigate these biases and align the models with societal values, underscoring the importance of thorough evaluations that consider both technical performance and ethical factors.

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A Appendix

A.1 Criteria for judge difficulty level of an OR problem

To assess the difficulty of translating a natural language problem description into an operations research model, the following four criteria can be considered:

- 1. Problem Size: This includes the number of constraints, objectives, and variables described in the natural language problem. Generally, the larger the problem, the higher the complexity of the model.
- 2. Complex and Logical Relationships: The complexity of relationships between variables and the logical structure of objective functions and constraints significantly affect modeling difficulty. Problems involving intricate logical conditions, such as "if-else" statements, "or" conditions, and nonlinear functions, increase the challenge of formulating precise mathematical models.
- 3. Ambiguity and Complexity of Natural Language Description: Natural language descriptions often contain ambiguous or polysemous expressions, which complicate the task of translating them into precisely defined mathematical models. Ambiguities in wording, implicit constraints, and unclear priorities require additional clarification and assumptions for accurate modeling.
- 4. Requirement for Interdisciplinary Knowledge: Certain natural language problems may encompass specific domain knowledge (e.g., economics, engineering, biology), necessitating the application of expertise from these fields for accurate modeling. This requires not only operations research skills but also an interdisciplinary understanding, thereby increasing the complexity of the modeling process.

These four criteria collectively provide a comprehensive framework for evaluating the difficulty of translating natural language descriptions into mathematical models in operations research.

A.2 Correcting Grammatical Errors of Programs

```
params = match.group(2)
           name_type = match.group(3)
           original_name = match.group(4).strip()
9
           # Determine the correct attribute type by checking if the last character of
               \hookrightarrow the method name is 's'
           correct_name_type = 'nameprefix' if method[-1] == 's' else 'name'
13
           # Replace if the current type is incorrect
14
           if name_type != correct_name_type:
15
              # If it is a dynamic string, retain the original expression
16
              return f"model.{method}({params}, {correct_name_type} = {original_name})"
17
           else:
18
19
              return match.group(0)
20
21
       # Correct the regular expressions and replacement logic for model status checks
       pattern_status = re.compile(r"if\s+.*?\s*==\s*COPT\.OPTIMAL:", re.DOTALL)
       replacement_status = r"if model.Status == COPT.OPTIMAL:"
23
24
25
       # Correct the optimize method call
       pattern_optimize = re.compile(r"\.optimize\(\\)")
26
       replacement_optimize = ".solve()"
27
28
       # Perform regex replacements on the entire code
30
       corrected_code = pattern_var_constr.sub(replacement_var_constr, code)
       corrected_code = pattern_status.sub(replacement_status, corrected_code)
31
       corrected_code = pattern_optimize.sub(replacement_optimize, corrected_code)
32
33
34
       return corrected_code
35
   file_corrected = []
36
37
   with open(data_path + 'train_data_version2.json', 'r') as f:
38
       for file in f.readlines():
39
           file = json.loads(file)
40
           code = file['completion']
41
42
           corrected_code = correct_code(code)
43
           # Save the corrected code
          file_corrected.append({'prompt': file['prompt'], 'completion':
44
               → corrected_code})
```

A.3 Prompt Template for Expansion

```
Act as an Operations Research Teacher and create problems based on their scenarios and question types.

# Example:
#Scenario#:
Retailing

#Question Type#:
Integer Programming

#Problem#:
A leather shoe store employs 5 full-time salespersons and 4 part-time salespersons. In order to optimize the working environment and consider employee health, the store decides to limit the overtime hours of each full-time employee. The following table shows the working hours and wage information for the employees:

| | Monthly Working Hours | Sales (pairs/hour) | Wage (dollars/hour) | Overtime Pay (dollars/hour) |
| :--: | :--: | :--: | :--: | :--: |
| Full-time | 160 | 5 | 1 | 1.5 |
```

```
| Part-time | 80 | 2 | 0.6 | 0.7 |
The profit per pair of shoes sold is 0.5 dollars. The store has set the following goals:
p_1: Achieve a monthly sales volume of at least 5500 pairs.
p_2: Limit the overtime hours of each full-time salesperson to no more than 20 hours.
p_3: Ensure full employment for all salespersons and give extra consideration to full-time employees.
p_4: Minimize overtime hours as much as possible.
Please develop an objective programming model for this problem.
#Completion Solution#:
## Mathematical Model:
To achieve the goals of the leather shoe store, we will use an objective programming model to balance the
achievement levels of each goal. The model is as follows:
# Example:
#Scenario#:
Agriculture
#Question Type#:
Non-Linear Programming
#Problem#:
An agricultural company wants to optimize the climate conditions inside their greenhouse to improve the
yield and quality of specific crops. The company grows two main crops: tomatoes and cucumbers. To
achieve the optimal growth conditions, precise control of temperature, humidity, and CO2 concentration
inside the greenhouse is required. Each crop has different requirements for these environmental factors,
and the company wants to adjust the greenhouse's environmental parameters to meet the growth needs of both
crops and maximize the total yield.
#Completion Solution#:
# Mathematical Model:
### Decision Variables:
- T: Temperature inside the greenhouse (°C)
- H: Humidity inside the greenhouse (%)
- C: CO2 concentration inside the greenhouse (ppm)
...one more example...
```

A.4 Prompt Template for Altering Objective and Constraints

If you are an Operations Research Algorithm Engineer, here is a problem description and its mathematical model, COPT code. Please follow the steps below for your output:

1. Real-World Scenario Consideration:

Based on the problem description, please consider how this problem would change in a real-world scenario, listing a specific as well as feasible change.

2. Modify Problem Description:

Example:

Considering the variation you listed in the previous step, make changes to the problem description.

3. Model and Code Modification:

You are asked to modify the original operations research model based on the original operations research model, taking into account the changed problem situation, and modify the corresponding COPT code.

```
4. Complete Output:
After modifying the model, you should output the complete problem description, mathematical model, and COPT
code for the new scenario in the original format.
Follow the steps outlined to adapt the problem for a real-world scenario, and provide a complete solution
in the original format.
Original Problem Description:
                              (Provide the original problem description here)
Original Mathematical Model:
                              (Provide the original mathematical model here)
Original COPT Code:
                                   (Provide the original COPT code here)
#Completion Solution#:
## Modified Problem Description:
To address the real-world scenario identified, the problem description has been modified as follows:
(Provide the modified problem description)
## Modified Mathematical Model:
Based on the changes in problem scenario, the mathematical model has been adapted as follows:
(Provide the modified mathematical model)
## Modified COPT Code:
The COPT code has been updated to reflect the changes in the mathematical model:
(Provide the modified COPT code)
A.5 Prompt Template for Rephrasing Questions
Rewrite the given problem description to improve clarity and readability, ensuring that the corresponding
mathematical model remains unchanged. Your output should replace the problem description of the original
problem with the rewritten problem description, leaving the rest of the formatting strictly unchanged.
Given the original problem description provided below, follow the instructions to enhance the clarity and
readability. Provide a detailed explanation of the changes made and the rationale behind each change.
Original Problem Description:
                              (Provide the original problem description here)
#Completion Solution#:
## Rewritten Problem Description:
To enhance the clarity and readability of the original problem description, the following changes have
```

(Provide the rewritten problem description)

-- Original Mathematical Model:

A.6 Prompt Template for Incorporating Multiple Modeling Techniques

(Provide the specific model details here)

#Question Type#: Model Modification and Enhancement Incorporate multiple modeling techniques into the provided mathematical model. Follow these steps: 1. Techniques Instruction: The following shows the different modeling techniques and the conditions under which they are applied: - Auxiliary Variables: Suitable for simplifying complex relationships or non-linearities in the model. - Big M Method: Appropriate for models with conditional constraints within a linear programming framework. - Penalty Functions: Useful for converting hard constraints into an unconstrained optimization problem. 2. Identify Modification Needs: Analyze the original model and identify areas where modifications could be used based on Techniques Instruction. Apply the selected technique(s) to modify either the objective function or the constraints of the original mathematical model. 4. Modify the Code: Based on the modified model, the corresponding code is modified following the code format of COPT. 5. Organize the results: Organize the problem description, modified model and modified code strictly against the format of the original problem. Task: Given the original mathematical model provided below, follow the steps above to enhance the model by incorporating multiple modeling techniques. Provide detailed explanations for each modification and the rationale behind selecting each technique. Original Mathematical Model: (Provide the specific model details here) #Completion Solution#: ## Mathematical Model: To enhance the original mathematical model, the following techniques have been applied: (Provide the modified model details with corresponding techniques) Based on the enhanced mathematical model, the corresponding code in COPT format is provided below: (Provide the modified code)

A.7 Alpaca-like Template for ORLMs Training

Below is an operations research question. Build a mathematical model and corresponding python code using

```
'coptpy' that appropriately addresses the question.
```

Question: {Question}

Response:

A.8 Hyper-parameters for Training ORLMs

All experiments are conducted on a single GPU server equipped with eight A100 GPUs, each with 80GB of memory.

Table 4: Hyper-parameters for Training ORLMs.	Table 4:	Hyper-parameters	for Training	ORLMs.
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Backbone	BatchSize	LearningRate	Epochs
Mistral-7B	512	3e-6	2
Deepseek-Math-7B-Base	128	2e-5	2
LLaMA-3-8B	64	5e-6	2

A.9 Comparing Easy and Hard Examples

A.9.1 Easy Example

A company sells custom scooters and bikes for customers. The profit per scooter is \$200 and the profit per bike is \$300. Each product requires time with the design team and engineering team. Each scooter needs 2 hours with the design team and 3 hours with the engineering team. Each bike needs 4 hours with the design team and 5 hours with the engineering team. Per month, there are 5000 hours available on the design team and 6000 hours available on the engineering team. How many of each should the company make per month to maximize profit?

A.9.2 Hard Example

The scheduling of hot coil transportation involves vehicle dispatch between warehouses and between warehouses and docks. Transportation tasks between warehouses are called *transfer tasks*, while those between warehouses and docks are called *dock tasks*.

Before the start of each shift, schedulers need to assign vehicles to the existing steel coil transportation tasks, determine the execution time for each task, and ensure all tasks are assigned. Tasks have merging rules: tasks with the same starting and ending points can be executed by the same vehicle, but the total weight and the total number of steel coils must not exceed the vehicle's limits. Vehicles do not pick up new tasks while en route; they can only pick up a new task after completing the current one.

Task Format: Steel coil ID, steel coil weight, starting warehouse, destination warehouse (dock), ship ID, task priority

- 1. Minimize the number of vehicles used
- 2. Ensure that all tasks are completed as early as possible
- 3. Prioritize tasks with high priority

Constraints is listed as follows:

- 1. Vehicles have an initial parking spot and must start from this spot when executing the first task of the shift
- 2. The number of steel coils loaded on a vehicle must not exceed the vehicle's limit
- 3. The weight of steel coils loaded on a vehicle must not exceed the vehicle's limit
- 4. The vehicle's transportation speed must be within the maximum and minimum speed limits

- 5. Different ships at the same dock must be loaded sequentially; the next ship's loading can only start after the previous ship's loading is completed
- 6. There is a limit to the number of vehicles simultaneously executing dock tasks
- 7. There is a limit to the number of vehicles simultaneously executing transfer tasks
- 8. The number of vehicles operating simultaneously in the warehouse area has an upper limit
- 9. No new tasks should be assigned to vehicles in the last half hour of the current shift

The sequence of tasks and estimated time nodes for all vehicles within this shift.