Article	Key Notes	Thoughts/Comments
Hu & Shum 2012 Nonparametric identification of Dynamic Models with Unobserved State Variables	**Primary:** This paper proposes a novel method for identifying a hidden Markov process, • Only 5 observations are needed in nonstationary cases, while only 4 are enough in stationary cases. • (W t , X t ∗) jointly evolves. • After the Markov kernel is identified, other relevant quantities can be recovered: Markov kernel = CCP*state law of motion • Application: dynamic optimization models with unobserved process. • Strength: ▪ Allow time-varying unobserved ▪ Evolve depending on past values of observables **Model:** • Observables: two components {action(decision), state} • Eq. 2 and 3 • Eq. 7 $$f_{X,Y,Z,S} = \int f_{X|X^*,S} f_{X^*,Z,S} f_{Y|X^*,Z} dx^*,$$ $$f_{W_{t+1},W_t,W_{t-1},W_{t-2}} = \int f_{W_{t+1}|W_t,X_t^*} f_{W_t|W_{t-1},X_t^*} f_{X_t^*,W_{t-1},W_{t-2}} dx_t^*$$ $$= \int f_{W_{t+1}|W_t,X_t^*} f_{W_t,W_{t-1},X_t^*} f_{W_{t-2}|X_t^*,W_{t-1}} dx_t^*. \quad (7)$$ **Assumptions:** • A1_1. First-order Markovian • A1_2. Limited Feedback	*How to identify other relevant quantities? Which formulae can illustrate? Equation (1)* *Why CCP and SLOM can be recovered??* *See Arellano Bonhemme 2017 Review paper, where more applications and examples are discussed.* *Why similar to Carrol 2010? In which S denotes a binary indicator, here no such binary sample.* *Why need the two equations?* ❖ *A1, A2 restrict the attention of models studied.* ❖ *A2 has an equivalent expression. See A&B 2017.* ❖ *Why transform? V_t exists.*

	• A2: Invertibility. Three injective linear operators. • Completeness • A3: Uniqueness of decomposition • A4: Monotonicity and Normalization. • X_t is scalar and continuous • $V_t \equiv g_t(W_t)$, • How to connect Carrol's assumptions with those in this paper?	❖ *It rules out X_t-1 has direct effects on W_t, or timing restriction.* ❖ *A4: imposed on eigenfunctions. Since they are identified up to an one-to-one transformation of X_t** ❖ *Deserves reading more about this high-level. Why high-level?*
	• Lemma_1: Representation of the observed density • Lemma_2: Representation of the Markov Law of Motion • Lemma_3: Identification of (Identification of f_V t +1 |W t ,X t ∗)	❖ Can prove Lemma_1 by direct definitions of Linear Operator? ❖ $L_{V_{t+1},w_t,w_{t-1},V_{t-2}}$ should be defined on V_t-2! Otherwise, equation 7 does not make sense. ❖ Use lemma 1 and lemma 3 to derive the spectrum decomposition, from which f_Vt+1|(W_t, X_t) can be identified. • *Stationary: 4 periods is enough, due to Lemma 2's equation.* *A1 ~ A4 ensures the uniqueness of the spectral decomposition.*
	Identification Strategy:	

	• Based on Hu&Schennach 2008 and Carrol 2010. • Unique spectral decomposition: A1~A4 • Two step identification: ▪ Step 1: By A1~A4, f_V t +1 |W t ,X t ∗ identified. ▪ Step 2: By Lemma_2, identify the Markov kernel. ▪ Step 3: Identify the joint distribution of the initial condition: f_W_(t −1)_X ∗ . • For stationary case, left as an exercise.	*Step 1 iterates two times, moving back by one period.*