180.633 Econometrics
Midterm, Spring 2019 Name:

1 (20 points) Consider the short and long projections

$$y = x\gamma_1 + e$$

$$y = x\beta_1 + x^2\beta_2 + u$$

(a) Under what condition does $\gamma_1 = \beta_1$?
(b) Now suppose the long projection is

$$y = x\theta_1 + x^3\theta_2 + v$$

Is there a similar condition under which $\gamma_1 = \theta_1$?

2 (20 points) Consider the least-squares regression estimators

$$y_i = x_{1i}\hat{\beta}_1 + x_{2i}\hat{\beta}_2 + \hat{e}_i$$

and the "one regressor at a time" regression estimators

$$y_i = \tilde{\beta}_1 x_{1i} + \tilde{e}_{1i} \quad y_i = \tilde{\beta}_2 x_{2i} + \tilde{e}_{2i}$$

Under what condition does $\tilde{\beta}_1 = \hat{\beta}_1$ and $\tilde{\beta}_2 = \hat{\beta}_2$?

3 (20 points) Take the linear regression model with $E(y|X) = X\beta$. Define the ridge regression estimator

$$\hat{\beta} = (X'X + I_k\lambda)^{-1}X'y$$

where $\lambda > 0$ is a fixed constant.
(a) Find $E(\hat{\beta}|X)$. Is $\hat{\beta}$ biased for β?
(b) Find the probability limit of $\hat{\beta}$ as $n \to \infty$. Is $\hat{\beta}$ consistent for β?

4 (20 points) The data $\{y_i, x_i, w_i\}$ is from a random sample, $i = 1, 2, ..., n$. The parameter β is estimated by minimizing the criterion function

$$S(\beta) = \sum_{i=1}^{n} w_i(y_i - x_i'\beta)^2$$

That is $\hat{\beta} = argmin_\beta S(\beta)$
(a) Find an explicit expression for $\hat{\beta}$.
(b) Find the probability limit for $\hat{\beta}$ as $n \to \infty$.
(c) Find the asymptotic distribution of $\sqrt{n}(\hat{\beta} - \beta)$ as $n \to \infty$.

5 (20 points) Suppose $\sqrt{n}(\hat{\mu} - \mu) \xrightarrow{d} N(0, v^2)$ and set $\beta = \mu^2$ and $\hat{\beta} = \hat{\mu}^2$
(a) Use the Delta Method to obtain an asymptotic distribution for $\sqrt{n}(\hat{\beta} - \beta)$

(b) Now suppose $\mu = 0$. Describe what happens to the asymptotic distribution from the previous part.
(c) Improve on the previous answer. Under the assumption $\mu = 0$, find the asymptotic distribution of $n\hat{\beta} = n\hat{\mu}^2$.
(d) Comment on the differences between the answers in parts (a) and (c).

6 (extra credit, 10 points) We consider a simple linear regression model for a consumption function

$$y = \beta x^* + \eta$$

with measurement error

$$x = x^* + \varepsilon.$$

In a random sample, we only observe consumption and self-reported income $\{y, x\}$, where x^* is unobserved true income and ε is measurement error. For simplicity, we assume (x^*, η, ε) are mutually independent with mean zero. Consider an OLS estimator $\widehat{\beta}$ of β by regressing y on x. Discuss the bias, variance, mean squared errors, and consistency of $\widehat{\beta}$. Do we know the direction of the bias if we know $\beta > 0$? Can you find some meaningful estimators to bound β, that is $\widehat{\beta}_L$ and $\widehat{\beta}_U$ such that

$$\text{plim}\widehat{\beta}_L \leq \beta \leq \text{plim}\widehat{\beta}_U.$$