180.633 Midterm
March 31, 2017. 1pm-3pm.
Note: this is a closed book exam.

1 (20 points) Consider two least-squares regressions

$$y = X_1\tilde{\beta}_1 + \tilde{e}$$

and

$$y = X_1\hat{\beta}_1 + X_2\hat{\beta}_2 + \hat{e}.$$

Let R_1^2 and R_2^2 be the R-squared from the two regressions. Show that $R_2^2 \geq R_1^2$. Is there a case (explain) when there is equality $R_2^2 = R_1^2$?

2 (20 points) In the classic homoskedastic linear regression model

$$Y = X\beta + e$$

(1) Show that the error variance estimator $\widehat{\sigma}^2$, i.e., average of squared residuals, is biased. And propose an unbiased estimator for σ^2.

(2) Is $\widehat{\sigma}^2$ consistent? Why? Propose a consistent estimator for σ and prove the consistency. Is your estimator for σ unbiased?

3 (30 points) Suppose $\sqrt{n}(\widehat{\mu} - \mu) \xrightarrow{d} N(0, v^2)$ and set $\beta = \mu^2$ and $\hat{\beta} = \hat{\mu}^2$.

(1) Use the Delta Method to obtain an asymptotic distribution for $\sqrt{n}(\hat{\beta} - \beta)$.

(2) Now suppose $\mu = 0$. Describe what happens to the asymptotic distribution from the previous part.

(3) Improve on the previous answer. Under the assumption $\mu = 0$, find the asymptotic distribution for $n\hat{\beta} = n\hat{\mu}^2$.

(4) Comment on the differences between the answers in parts 1 and 3.

4 (30 points) We consider a simple linear regression model for a consumption function

$$y = \alpha + \beta x^* + \eta$$

with measurement error

$$x = x^* + \varepsilon.$$

In a random sample, we only observe consumption and self-reported income $\{y, x\}$, where x^* is unobserved true income and ε is measurement error. For simplicity, we assume (x^*, η, ε) are mutually independent with $E\eta = E\varepsilon = 0$. Consider an estimator $\widehat{\beta} = 1/\widehat{\gamma}$ for β, where $\widehat{\gamma}$ is the OLS estimator for the slope in the regression of x on y with an intercept. Discuss the bias, variance, mean squared errors, and consistency of $\widehat{\beta}$. Do we know the direction of the bias if we know $\beta > 0$? Can you find some meaningful estimators to bound β, that is $\widehat{\beta}_L$ and $\widehat{\beta}_U$ such that

$$plim\widehat{\beta}_L \leq \beta \leq plim\widehat{\beta}_U.$$