Paper

Nonparametric Identification of Dynamic Models with Unobserved State Variables

Primary

This paper proposes a novel method for identifying a hidden Markov process:

- Only 5 observations are needed in non-stationary cases, while only 4 are enough in station cases.
- (W_t, X_t^*) jointly evolves.
- After the Markov kernel is identified, other relevant quantities can be recovered:

  Markov Kernel = CCP*State Law of Motion

- Application: dynamic optimization models with unobserved process.
- Strength:
  - Allow time-varying unobserved
  - Evolve depending on past values of observables.

Comments

How to identify other relevant quantities?

Which formulae can ilustrate eqaution (1)

Why CCP and SLOM can be recovered?

See Arellano Bonhemme 2017 Review Paper, where more applications and examples are discussed.

Model

- Observables: two components: action(decision) and state.
- Eq. 2 and 3.
- Eq. 7.

$$f_{X,Y,Z,S} = \int f_{X|X^*,S} f_{X^*,Z,S} f_{Y|X^*,Z} \, \mathrm{d}x^*,$$

$$\begin{aligned} f_{W_{t+1},W_t,W_{t-1},W_{t-2}} &= \int f_{W_{t+1}|W_t,X_t^*} f_{W_t|W_{t-1},X_t^*} f_{X_t^*,W_{t-1},W_{t-2}} \, \mathrm{d}x_t^* \\ &= \int f_{W_{t+1}|W_t,X_t^*} f_{W_t,W_{t-1},X_t^*} f_{W_{t-2}|X_t^*,W_{t-1}} \, \mathrm{d}x_t^* \end{aligned}$$

Comments

Assumptions

A1.1. First-order Markovian
A1.2. Limited feedback
A2. Invertibility. Three injective linear operators.
A3. Uniqueness of decomposition.
A4. Monotonicity and Normalization.
Other assumptions: 1. X_t is scalar and continuous.
2. $V_t \equiv g_t(W_t)$.
3. How to connect Carrol's assumptions with those in this paper?

Comments

Lemmas

Comments

* Lemma 1: Representation of the observed density. * Lemma 2: Representation of the Markov Law of Motion. * Lemma 3: Identification of $f_{V_{t+1}\|W_t,X_t^*}$.	

Identification Strategy	**Comments**
1. Based on *Hu & Schennach 2008* and *Carrol 2010*. 2. Unique spectral decomposition: A1 $\rightarrow$ A4. 3. Two step Identification: (a) By A1 $\rightarrow$ A4, $f_{V_{t+1}\|W_t,X_t^*}$ identified. (b) By Lemma 2, the Markov kernel is identified. (c) Identify the joint distribution of the initial condition: $f_{W_{t-1}}, X_t^*$.	