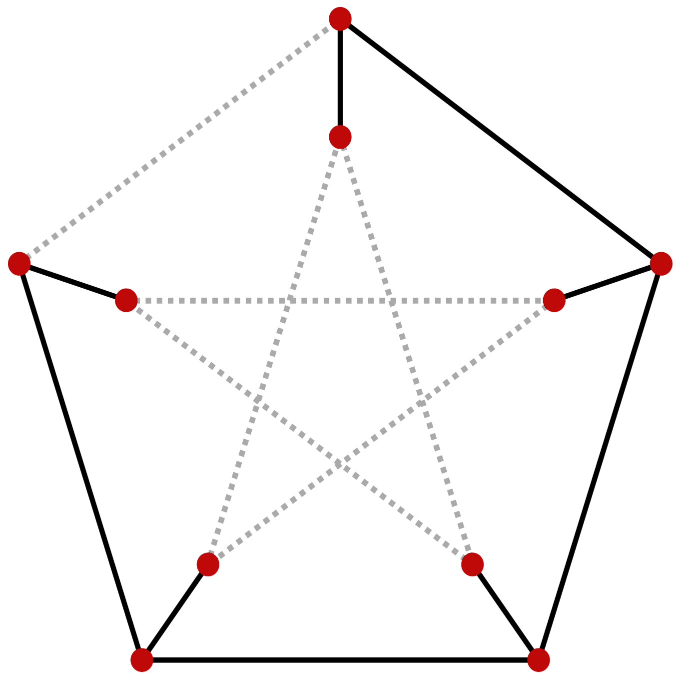
We want to show that the Petersen graph is homotopy equivalent to a bouquet of 6 1-spheres. We will give two arguments for this.

A visual argument: See the file `petersen_to_bouquet.mp4`, in the same directory as this pdf. The source code used to generate this file is available in this repository.

A formal argument: Consider removing six edges from the Petersen graph, as follows:



Notice that the remainder is a tree, which is contractible. Intuitively, it makes sense that there is an homotopy of the petersen graph contracting this tree to a point, such that the six remaining edges get dragged along, becoming six copies of S^1 identified at a single point.

We would like to show that this "intuitive homotopy" is actually a homotopy that exists. For convenience, we will generalize: Let G be a graph, and E a collection of r edges from this graph such that $T := G \setminus E$ is a tree (in particular, it is connected). This means that T is contractible. Importantly, T (when regarded as a CW-complex) is a subcomplex of G, meaning that G has the homotopy extension property with respect to T — from this it follows that G is homotopy equivalent to G/T. Name p the element corresponding to T in G/T, and notice that G/T is composed of the edges in E, joined together in the single point p — this is the wedge sum of r copies of S^1, which is homotopic to a bouquet of r 1-spheres.