Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

Course Title:	Low Power Digital Integrated Circuits
Course Number:	ELE 734
Semester/Year (e.g.F2016)	F2021

Instructor:	Andy Ye

Assignment/Lab Number:	2
Assignment/Lab Title:	CMOS Inverter

Submission Date:	October 12, 2021
Due Date:	October 12, 2021

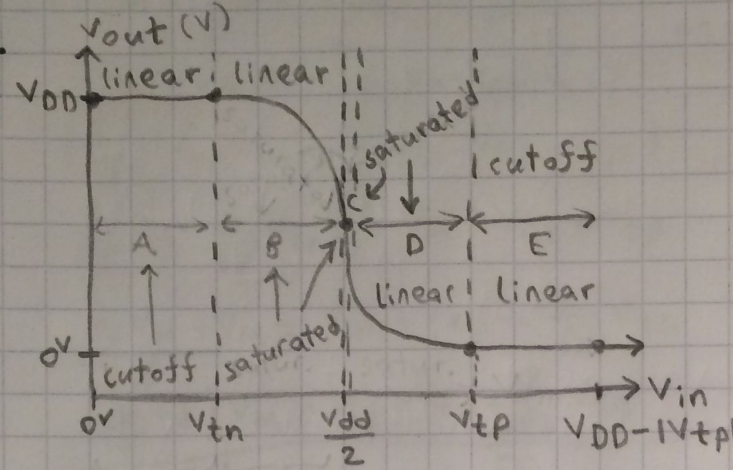
Student LAST Name	Student FIRST Name	Student Number	Section	Signature*
Pano	Toni	500822828	7	T.P.

*By signing above you attest that you have contributed to this written lab report and confirm that all work you have contributed to this lab report is your own work. Any suspicion of copying or plagiarism in this work will result in an investigation of Academic Misconduct and may result in a “0” on the work, an “F” in the course, or possibly more severe penalties, as well as a Disciplinary Notice on your academic record under the Student Code of Academic Conduct, which can be found online at: http://www.ryerson.ca/senate/current/pol60.pdf

Toni Pano #500822828 ELE 734 Section 7

Lab 2 Prelab

1.

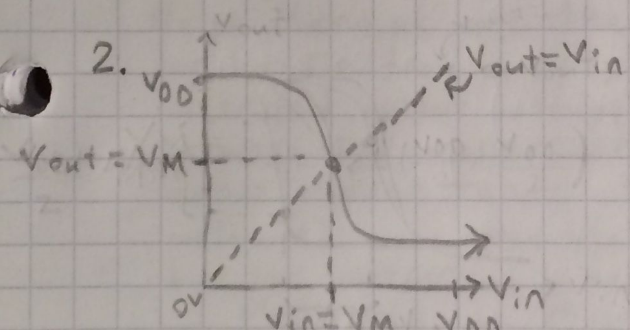


pMOS operating region

nMOS operating region

curve moves right as Beta Ratio decreases ← → curve moves left as Beta Ratio increases

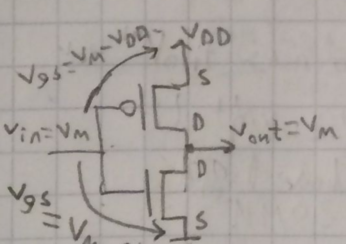
2.



Assume that:

$I_{dsn}=$

nMOS and pMOS are both in saturation operating region



$$I_{dsn}=I_{dsp}$$

$$\frac{\beta_n}{2}V_{GT_n}^2=\frac{\beta_p}{2}V_{GT_p}^2$$

$$\frac{\beta_n}{2}(V_{gs}-V_{tn})^2=\frac{\beta_p}{2}(V_{gs}-V_{tp})^2$$

$$(V_{gs}-V_{tn})^2=\frac{\beta_p}{\beta_n}(V_{gs}-V_{tp})^2$$

$$((V_M-0^V)-V_{tn})^2=\frac{\beta_p}{\beta_n}((V_M-V_{DD})-V_{tp})^2$$

$$(V_M-V_{tn})^2=\frac{\beta_p}{\beta_n}(V_M-V_{DD}-V_{tp})^2$$

$$\pm\sqrt{(V_M-V_{tn})^2}=\pm\sqrt{\frac{\beta_p}{\beta_n}(V_M-V_{DD}-V_{tp})^2}$$

$$(V_M-V_{tn})=\pm\sqrt{\frac{\beta_p}{\beta_n}}(V_M-V_{DD}-V_{tp})$$

Let $\frac{\beta_p}{\beta_n} = \gamma$

$$V_M - V_{tn} = \pm\sqrt{\gamma}\,(V_M - V_{DD} - V_{tp})$$

← negative root is easier to solve for V_M.

$$V_M - V_{tn} = -\sqrt{\gamma}\,(V_M - V_{DD} - V_{tp})$$

$$\left(\frac{1}{\sqrt{\gamma}}\right)(V_M - V_{tn}) = -(V_M - V_{DD} - V_{tp})$$

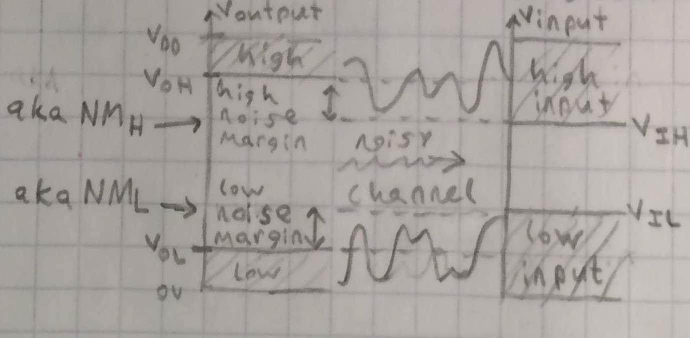
$$\frac{1}{\sqrt{\gamma}}V_M - \frac{1}{\sqrt{\gamma}}V_{tn} = -V_M + V_{DD} + V_{tp}$$

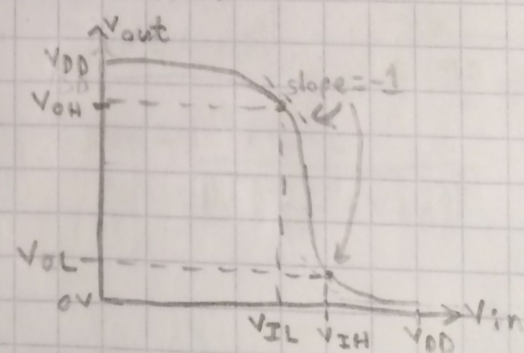
$$\frac{1}{\sqrt{\gamma}}V_M + V_M = V_{DD} + V_{tp} + \frac{1}{\sqrt{\gamma}}V_{tn}$$

$$\left(\frac{1}{\sqrt{\gamma}} + 1\right)V_M = V_{DD} + V_{tp} + \frac{1}{\sqrt{\gamma}}V_{tn}$$

$$\boxed{V_M = \frac{V_{DD} + V_{tp} + V_{tn}\sqrt{\frac{1}{\gamma}}}{\left(1 + \frac{1}{\sqrt{\gamma}}\right)}, \quad \gamma = \frac{\beta_p}{\beta_n}}$$

3. Noise Margins are the range of voltages that a circuit can handle from a noisy input voltage, without changing the intended output voltage. High noise margin is the difference between the smaller output voltage range and the bigger output voltage range when the output is a logical 1 (high voltage). Low noise margin is the same as high noise margin, but the output is a logical 0 (low voltage)



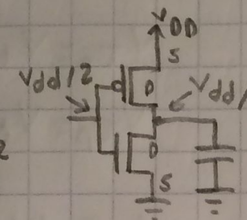


If the supply voltage decreases while the threshold voltage stays the same, the points with a slope of -1 on the Vout/Vin transfer function will move towards the threshold voltage point of the graph, so V_{IL} increases and V_{IH} decreases, and V_{OL} increases while V_{OH} decreases. If $NM_H = V_{OH} - V_{IH}$ and $NM_L = V_{IL} - V_{OL}$, then the noise may stay unchanged.

4.

$$R_{eq} = \frac{V_{DD}}{2 \cdot \ln(2) \cdot I_{Dsatn,p}}$$

$$R_{eqn} = \frac{V_{DD}}{2\ln(2) I_{Dsatn}}$$



$$I_{Dsatn} = \frac{\beta_n}{2} V_{GT}^2$$

$$I_{Dsatn} = \frac{\beta_n}{2}(V_{gs} - V_{tn})^2$$

$$I_{Dsatn} = \frac{\beta_n}{2}\left(\left(\frac{V_{DD}}{2} - 0V\right) - V_{tn}\right)^2$$

$$I_{Dsatn} = \frac{\beta_n}{2}\left(\frac{V_{DD}}{2} - V_{tn}\right)^2$$

$$R_{eqn} = \frac{V_{DD}}{2\ln(2)\left(\frac{\beta_n}{2}\left(\frac{V_{DD}}{2} - V_{tn}\right)^2\right)}$$

$$R_{eqn} = \frac{V_{DD}}{\beta_n \ln(2)\left(\frac{V_{DD}}{2} - V_{tn}\right)^2}$$

$$\tau_{pdf} = \ln(2) \cdot R_{eqn} \cdot C_L$$

$$= \cancel{\ln}(2) \cdot \left(\frac{V_{DD}}{\beta_n \cdot \cancel{\ln(2)} \cdot \left(\frac{V_{DD}}{2} - V_{tn}\right)^2}\right) \cdot C_L$$

$$\tau_{pdf} = \frac{V_{DD} C_L}{\beta_n\left(\frac{V_{DD}}{2} - V_{tn}\right)^2}$$

$$R_{eqp} = \frac{V_{DD}}{2\ln(2) I_{Dsatp}}$$

$$I_{Dsatp} = \frac{\beta_p}{2} V_{GT}^2$$

$$= \frac{\beta_p}{2}(V_{gs} - V_{tp})^2$$

$$= \frac{\beta_p}{2}\left(\left(\frac{V_{DD}}{2} - V_{DD}\right) - V_{tp}\right)^2$$

$$= \frac{\beta_p}{2}\left(-\frac{V_{DD}}{2} - V_{tp}\right)^2$$

$$I_{Dsatp} = \frac{\beta_p}{2}\left(\frac{V_{DD}}{2} + V_{tp}\right)^2$$

$$R_{eqp} = \frac{V_{DD}}{2\ln(2) \cdot \frac{\beta_p}{2}\left(\frac{V_{DD}}{2} + V_{tp}\right)^2}$$

$$R_{eqp} = \frac{V_{DD}}{\ln(2) \cdot \beta_p \cdot \left(\frac{V_{DD}}{2} + V_{tp}\right)^2}$$

$$\tau_{pdr} = \ln(2) \cdot R_{eqp} \cdot C_L$$

$$\tau_{pdr} = \cancel{\ln(2)}\left(\frac{V_{DD}}{\cancel{\ln(2)} \cdot \beta_p \cdot \left(\frac{V_{DD}}{2} + V_{tp}\right)^2}\right) C_L$$

$$\tau_{pdr} = \frac{V_{DD} C_L}{\beta_p\left(\frac{V_{DD}}{2} + V_{tp}\right)^2}$$

$$\tau_p = \frac{\tau_{pdf} + \tau_{pdr}}{2}$$

$$= \frac{\left(\frac{V_{DD} C_L}{\beta_n\left(\frac{V_{DD}}{2} - V_{tn}\right)^2}\right) + \left(\frac{V_{DD} C_L}{\beta_p\left(\frac{V_{DD}}{2} + V_{tp}\right)^2}\right)}{2}$$

$$\boxed{\tau_p = \frac{V_{DD} C_L}{2}\left(\frac{1}{\left(\mu_n C_{oxn} \frac{W_n}{L_n}\right)\left(\frac{V_{DD}}{2} - V_{th,n}\right)^2} + \frac{1}{\left(\mu_p C_{oxp} \frac{W_p}{L_p}\right)\left(\frac{V_{DD}}{2} + V_{tp}\right)^2}\right)}$$

assuming $V_{th,n} = V_{tn}$ and $V_{th,p} = V_{tp}$

5. Assuming:

$\tau_p = 100\ ps$ $\qquad$ $\mu_n = 459\ \frac{cm^2}{V \cdot s}$

$C_L = 50\ fF$ $\qquad$ $\mu_p = 109\ \frac{cm^2}{V \cdot s}$

$L = L_{min} = L_n = L_p = 180\ nm$

$t_{ox} = 4.08\ nm$ $\qquad$ $V_{Tn} = V_{th,n} = 0.445\ V$

$\varepsilon_o = 8.854 \cdot 10^{-12}\ F/m$ $\qquad$ $V_{Tp} = V_{th,p} = -0.438\ V$

$\varepsilon_{ox} = 3.9\ \varepsilon_o$

solve for relationship btwn W_n and W_p

$$\tau_p = \frac{V_{DD} C_L}{2} \left(\frac{1}{\mu_n \left(\frac{\varepsilon_{ox}}{t_{ox}}\right) \frac{W_n}{(L)} \left(\frac{V_{DD}}{2} - (V_{Tn})\right)^2} + \frac{1}{\mu_p \left(\frac{\varepsilon_{ox}}{t_{ox}}\right) \frac{W_p}{(L)} \left(\frac{V_{DD}}{2} + (V_{Tp})\right)^2} \right)$$

$$\tau_p = \frac{V_{DD} C_L}{2 \frac{\varepsilon_{ox}}{t_{ox} L}} \left(\frac{1}{\mu_n W_n \left(\frac{V_{DD}}{2} - V_{Tn}\right)^2} + \frac{1}{\mu_p W_p \left(\frac{V_{DD}}{2} + V_{Tp}\right)^2} \right)$$

$$\tau_p = \frac{V_{DD} C_L t_{ox} L}{2 \varepsilon_{ox}} \left(\frac{1}{\mu_n W_n \left(\frac{V_{DD}}{2} - V_{Tn}\right)^2} + \frac{1}{\mu_p W_p \left(\frac{V_{DD}}{2} + V_{Tp}\right)^2} \right)$$

$$\frac{2 \varepsilon_{ox} \tau_p}{V_{DD} C_L t_{ox} L} = \frac{1}{\mu_n W_n \left(\frac{V_{DD}}{2} - V_{Tn}\right)^2} + \frac{1}{\mu_p W_p \left(\frac{V_{DD}}{2} + V_{Tp}\right)^2}$$

$$\frac{2 (3.9 \varepsilon_o) \tau_p}{V_{DD} C_L t_{ox} L} - \frac{1}{\mu_n W_n \left(\frac{V_{DD}}{2} - V_{Tn}\right)^2} = \frac{1}{\mu_p W_p \left(\frac{V_{DD}}{2} + V_{Tp}\right)^2}$$

$$W_p = \frac{1}{\mu_p \left(\frac{V_{DD}}{2} + V_{Tp}\right)^2 \left(\frac{7.8 \varepsilon_o \tau_p}{V_{DD} C_L t_{ox} L} - \frac{1}{\mu_n W_n \left(\frac{V_{DD}}{2} - V_{Tn}\right)^2} \right)}$$

Assuming $L_{min} = 2\lambda$, and $W_{n,min} = 4\lambda$, (where $\lambda = 90 nm$), $V_{DD} = 5V$, then:

$$W_p = \frac{1}{\left(109 \frac{\left(\frac{m}{100}\right)^2}{V \cdot s}\right) \left(\frac{(5V)}{2} + (-0.438V)\right)^2 \left(\frac{7.8 (8.854 \cdot 10^{-12} \frac{F}{m})(100 \cdot 10^{-12} s)}{(5V)(50 \cdot 10^{-15} F)(4.08 \cdot 10^{-9} m)(180\ nm)} - \frac{1}{\left(459 \frac{\left(\frac{1m}{100}\right)^2}{V \cdot s}\right) (4 \cdot 90 \cdot 10^{-9} m) \left(\frac{(5V)}{2} - (0.445V)\right)^2} \right)}$$

$$W_p \simeq \frac{1\ s}{(0.046345)(37{,}615{,}032.68 - 14{,}330{,}493.99)}$$

$W_p \simeq 9.266773491 \cdot 10^{-7} \simeq 927\ nm$ for $W_n = 360\ nm$, $L = 180\ nm$, $V_{DD} = 5\ V$