Department of Electrical, Computer, & Biomedical Engineering
Faculty of Engineering & Architectural Science

Course Title:	Low-Power Digital Integrated Circuits
Course Number:	ELE 734
Semester/Year (e.g.F2016)	F2021

Instructor:	Dr. Andy Ye

Assignment/Lab Number:	3
Assignment/Lab Title:	CMOS Logic Families Prelab

Submission Date:	November 19th, 2021
Due Date:	November 16th, 2021

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Toni Pano #500822828 Section 07

ELE 734: Prelab 3

Q1. assume $\tau_{pdf} = k \cdot \tau_{pdr} \longleftarrow \frac{\tau_{pdf}}{\tau_{pdr}} = k$

i) high skewed $\rightarrow \tau_{pdf} = 1.5 \cdot \tau_{pdr}$

$$\frac{\tau_{pdf}}{\tau_{pdr}} = 1.5$$

$$\boxed{k = 1.5} = 3/2$$

ii) unskewed $\longrightarrow \tau_{pdf} = \tau_{pdr}$

$$\frac{\tau_{pdf}}{\tau_{pdr}} = 1$$

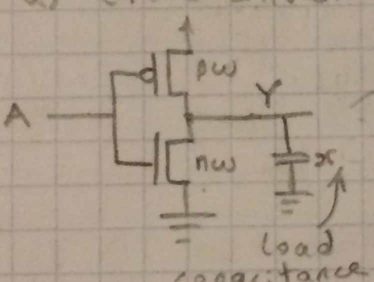
$$\boxed{k = 1}$$

iii) low skewed $\longrightarrow 1.5\tau_{pdf} = \tau_{pdr}$

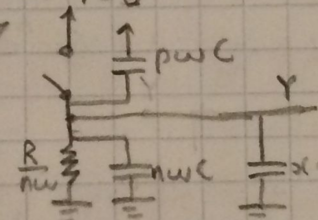
$$\frac{\tau_{pdf}}{\tau_{pdr}} = \frac{1}{1.5} = \frac{2}{3}$$

$$k = \frac{2}{3}$$

a) CMOS Inverter sizing



when Y=0



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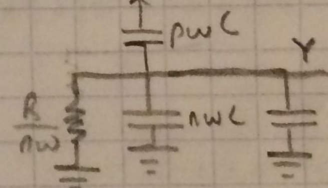
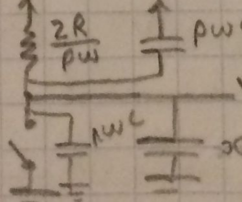


$$\tau_{pdf} = \frac{R}{nw}(nwC + pwC + x)$$

when Y=1



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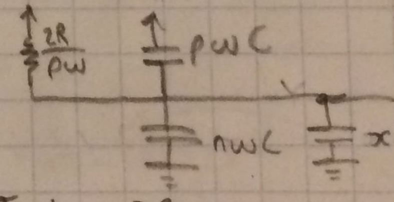


$$\tau_{pdr} = \frac{2R}{pw}(nwC + pwC + x)$$

$$\tau_{pdf} = k \cdot \tau_{pdr}$$

$$\frac{R}{n\cancel{w}}\cancel{(nwC + pwC + x)} = k \cdot \frac{2R}{p\cancel{w}}\cancel{(nwC + pwC + x)}$$

$$\frac{1}{n} = k \cdot \frac{2}{p}$$

$$\frac{p}{n} = \frac{2k}{1}$$

$$\therefore \boxed{\begin{array}{l} p = 2k \\ n = 1 \end{array}}$$

assuming $x = 50fF$ when $\tau_p = 100ps$. and $k=1$

$$\tau_p = \frac{\tau_{pdf} + \tau_{pdr}}{2} = \frac{\tau_{pdr} + \tau_{pdr}}{2} = \tau_{pdr}$$

$$\tau_p = \frac{2R}{pw}(nwC + pwC + x)$$

$$\tau_p = \frac{2R}{(2)w}((1)wC + (2)wC + x)$$

$$\tau_p = \frac{R}{w}(3wC + x) \rightarrow w\left(\frac{\tau_p}{R} - 3C\right) = x$$

$$\frac{w\tau_p}{R} = 3wC + x$$

$$\frac{w\tau_p}{R} - 3wC = x$$

$$w = \frac{x}{\frac{\tau_p}{R} - 3C} = \frac{(50fF)}{\frac{(100ps)}{((1.2)10k\Omega)} - 3(1fF)}$$

$$w = 4.375620635 \simeq 4.376$$

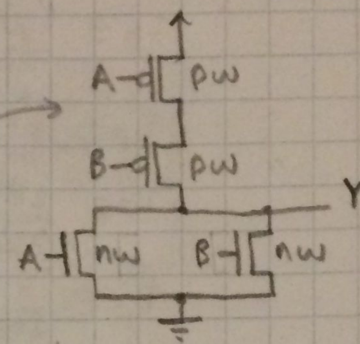
assume $C \simeq C_g \times W = C_g \cdot n \simeq (1fF/\mu m)(1\mu m) \simeq 1fF$, $R = 10k\Omega$

W_n for unit nmos

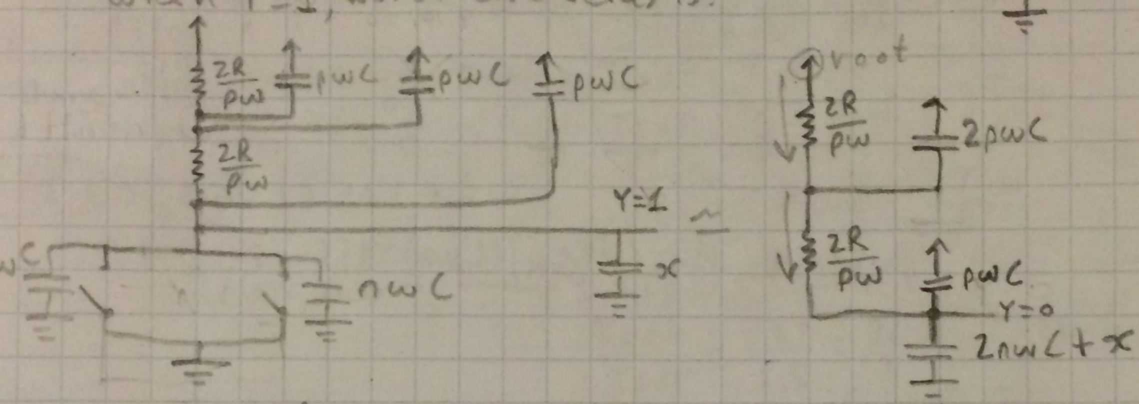
b) NOR2 gate sizing

$Y = \overline{A+B} \rightarrow \overline{Y} = \overline{\overline{(A+B)}}$

$\underbrace{Y = \overline{A} \cdot \overline{B}}_{pmos} \qquad \underbrace{\overline{Y} = A+B}_{nmos}$



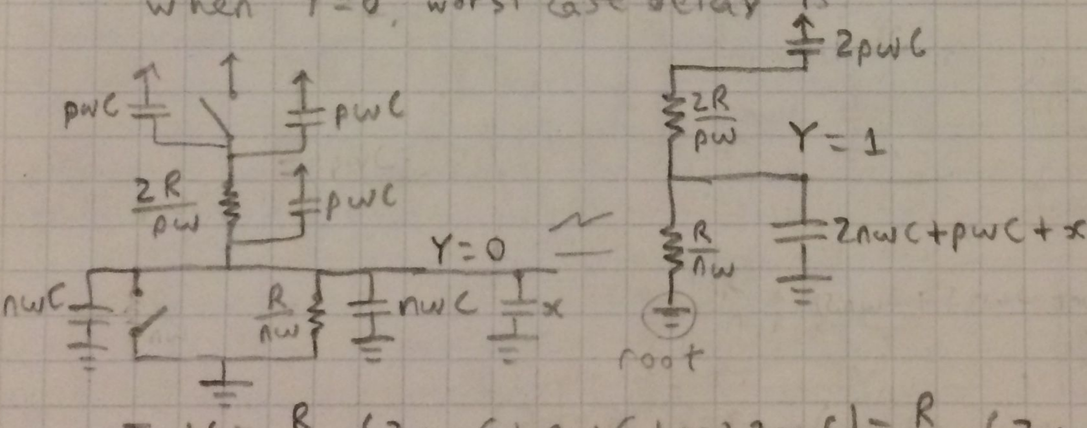
when Y=1, worst case delay is:



$$\tau_{pdr} = \frac{2R}{pw}(2pwC + pwC + 2nwC + x) + \frac{2R}{pw}(pwC + 2nwC + x)$$

$$\tau_{pdr} = \frac{2R}{pw}(4nwC + 4pwC + 2x) = \frac{4R}{pw}(2nwC + 2pwC + x)$$

when Y=0, worst case delay is:



$$\tau_{pdf} = \frac{R}{nw}(2nwC + pwC + x + 2pwC) = \frac{R}{nw}(2nwC + 3pwC + x)$$

$$\tau_{pdf} = k \cdot \tau_{pdr}$$

$$\underbrace{\frac{R}{nw}}\cancel{(2nwC + 3pwC + x)} = k \cdot \underbrace{\frac{4R}{pw}}\cancel{(2nwC + 2pwC + x)}$$

$$\frac{1}{n} = k \cdot \frac{4}{p}$$

$$\frac{p}{n} = \frac{4k}{1} = \frac{w_p}{w_n}$$

$\therefore\ p = 4k\ \mu m$

$n = 1\ \mu m \leftarrow$ assumes W_n for unit nmos is $1\mu m$

when $\tau_p = 100ps$, assume:

unskewed gate ($k=1$)

$x = 50fF$

$C = C_g \times W_n = C_g \times n = (1 fF/\mu m)(1\mu m) = 1fF$

$W_n = 1\mu m$

$R = 10k\Omega \cdot \ln(2)$

$\frac{p}{n} = \frac{4k}{1} = \frac{4(1)}{1} = \frac{4}{1}$

$$\tau_p = \frac{\tau_{pdf} + \tau_{pdr}}{2} = \frac{\left[\frac{R}{nw}(2nwC + 3pwC + x)\right] + \left[\frac{4R}{pw}(2nwC + 2pwC + x)\right]}{2}$$

$$\tau_p = \frac{\left[\frac{R}{(1)w}(2(1)wC + 3(4)wC + x)\right] + \left[\frac{4R}{(4)w}(2(1)wC + 2(4)wC + x)\right]}{2}$$

$$\tau_p = \frac{\frac{R}{w}(2wC + 12wC + x) + \frac{R}{w}(2wC + 8wC + x)}{2}$$

$$\tau_p = \frac{R}{2w}(24wC + 2x)$$

$$\tau_p = \frac{R}{w}(12wC + x)$$

$$w\frac{\tau_p}{R} = 12wC + x$$

$$w\frac{\tau_p}{R} - 12wC = x$$

$$w\left(\frac{\tau_p}{R} - 12C\right) = x$$

$$w = \frac{x}{\frac{\tau_p}{R} - 12C}$$

$$w = \frac{(50fF)}{\frac{(100ps)}{(10k\Omega \cdot \ln 2)} - 12(1fF)} = \frac{50}{\frac{10}{\ln(2)} - 12}$$

$w \simeq 20.60198668 \simeq 20.602$

assume, $W_p = p \cdot w = 4\mu m \cdot k \cdot w$, $W_n = n \cdot w = 1\mu m \cdot w$

i) high skewed nor2 gate ($k = 1.5$)

$W_p = 4\mu m(1.5)(20.602) \simeq 123.6119201\mu m \simeq 123.612\mu m$

$W_n = 1\mu m(20.602) \simeq 20.602\mu m$

ii) unskewed nor2 gate ($k=1$)

$W_p = 4\mu m(1)(20.602) \simeq 82.40794672\mu m \simeq 82.408\mu m$

$W_n = 1\mu m(20.602) \simeq 20.602\mu m$

iii) low skewed nor2 gate ($k = \frac{2}{3}$)

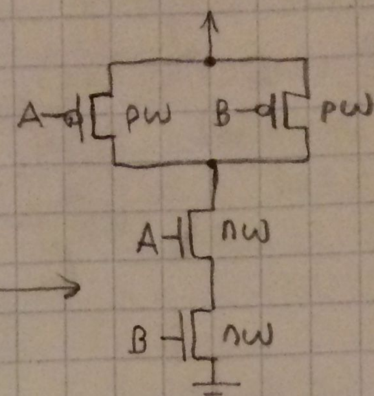
$W_p = 4\mu m(\frac{2}{3})(20.602) \simeq 54.93863115\mu m \simeq 54.939\mu m$

$W_n = 1\mu m(20.602) \simeq 20.602\mu m$

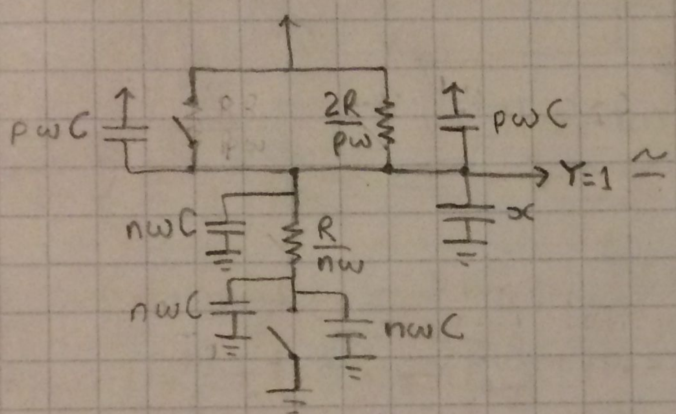
c) NAND2 gate sizing

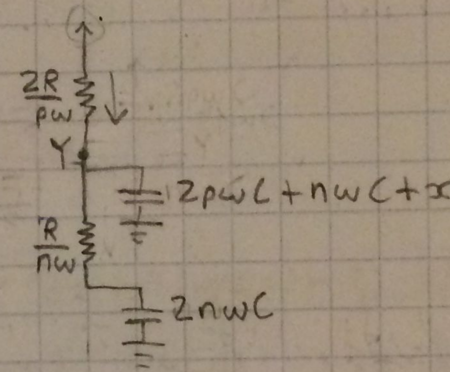
$Y = \overline{A \cdot B} \rightarrow \overline{Y} = \overline{(\overline{A \cdot B})}$

$\underbrace{Y = \overline{A} + \overline{B}}_{\text{pmos}} \qquad \underbrace{\overline{Y} = A \cdot B}_{\text{nmos}}$



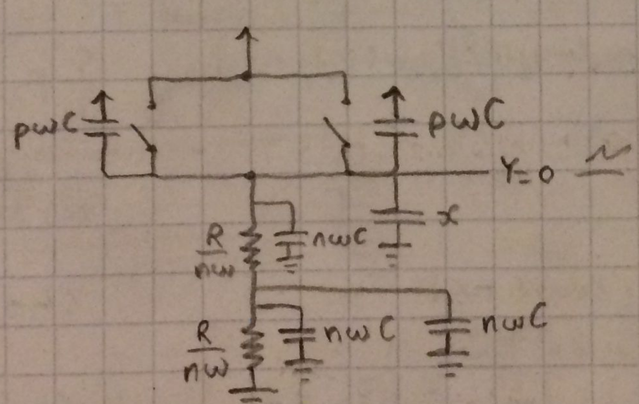
when Y=1, worst case delay is:

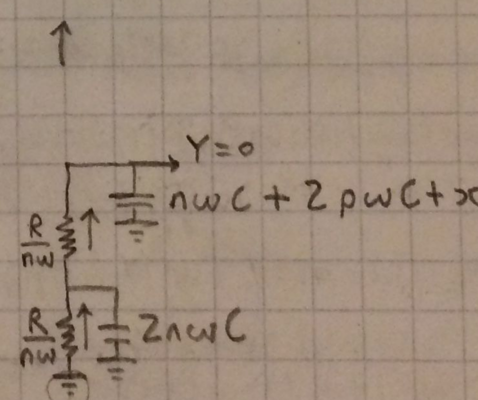




$$T_{pdr} = \frac{2R}{pw}(2pwC + nwC + x + 2nwC) = \frac{2R}{pw}(3nwC + 2pwC + x)$$

when Y=0, worst case delay is:





$$T_{pdf} = \frac{R}{nw}(2nwC + nwC + 2pwC + x) + \frac{R}{nw}(nwC + 2pwC + x)$$

$$= \frac{R}{nw}(4nwC + 4pwC + 2x) = \frac{2R}{nw}(2nwC + 2pwC + x)$$

$$T_{pdf} = k \cdot T_{pdr}$$

$$\frac{2R}{nw}(2nwC + 2pwC + x) = k \cdot \frac{2R}{pw}(3nwC + 2pwC + x)$$

The capacitances on each side don't cancel out! Approximate transistor sizing by assuming that resistances are more significant

$$\frac{2R}{nw} = k \cdot \frac{2R}{pw}$$

$$\frac{1}{n} = k \cdot \frac{1}{p}$$

$$\frac{p}{n} = \frac{k}{1} \xrightarrow[\text{assume } n = 1\mu m]{} \frac{p}{1\mu m} = k$$

$$p = 1\mu m \cdot k$$

when $\tau_p = 100ps$, assume:

unskewed gate ($k=1$)

load capacitance $x = 50fF$

$C = C_g \times (W_n \text{ for unit nmos}) = C_g \times n = (1fF/\mu m)(1\mu m) = 1fF$

$R = 10k\Omega \cdot \ln(2)$

$n = 1\mu m$

$p = 1\mu m \cdot k = 1\mu m(1) = 1\mu m$

$$\tau_p = \frac{\tau_{pdf} + \tau_{pdr}}{2} = \frac{\left[\frac{2R}{nw}(2nwC + 2pwC + x)\right] + \left[\frac{2R}{pw}(3nwC + 2pwC + x)\right]}{2}$$

$$\tau_p = \frac{2R}{2w} \cdot \left[\frac{1}{n}(2nwC + 2pwC + x) + \frac{1}{p}(3nwC + 2pwC + x)\right]$$

$$\frac{\tau_p}{R} w = \left[2wC + 2\frac{p}{n}wC + \frac{x}{n} + 3\frac{n}{p}wC + 2wC + \frac{x}{p}\right]$$

$$= 4wC + \left(2\frac{p}{n} + 3\frac{n}{p}\right)wC + \left(\frac{1}{n} + \frac{1}{p}\right)x$$

$$= 4wC + \left(2\frac{(1)}{(1)} + 3\frac{(1)}{(1)}\right)wC + \left(\frac{1}{(1)} + \frac{1}{(1)}\right)x$$

$$= 4wC + 5wC + 2x$$

$$\frac{\tau_p}{R} w = 9wC + 2x$$

$$\frac{\tau_p}{R} w - 9wC = 2x$$

$$w\left(\frac{\tau_p}{R} - 9C\right) = 2x$$

$$w = \frac{2x}{\frac{\tau_p}{R} - 9C} = \frac{2(50fF)}{\frac{100ps}{10k\Omega \cdot \ln(2)} - 9(1fF)}$$

$$w = \frac{100}{\frac{10}{\ln(2)} - 9} = 18.42655496$$

$$w \simeq 18.427$$

For all 3 circuits, $W_n = n \cdot w = (1\mu m)(18.427) = 18.427\mu m$

i) high skewed circuit W_p ($k = 1.5$)

$W_p = p \cdot w = (1\mu m \cdot k)w = 1\mu m(1.5)(18.427) = 27.63983245\mu m \simeq 27.640\mu m$

ii) unskewed circuit ($k=1$) W_p

$W_p = p \cdot w = (1\mu m \cdot k) \cdot w = 1\mu m(1)(18.427) = 18.427\mu m$

iii) low skewed circuit ($k = \frac{1}{1.5}$) W_p

$W_p = p \cdot w = (1\mu m \cdot k)w = 1\mu m\left(\frac{1}{1.5}\right)(18.427) = 12.28436998\mu m \simeq 12.284\mu m$

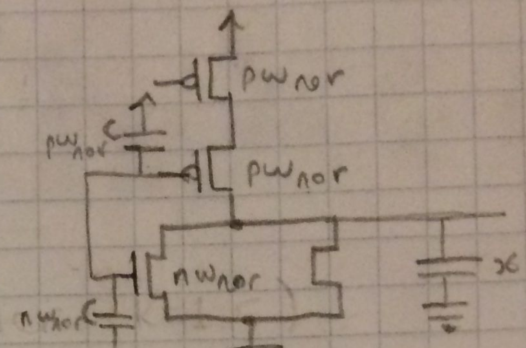
Q2. Rising, falling, average logic effort

a) NOR2 gates

For nor 2: $w = 20.602 = w_{nor}$

For inverter: $w = 4.376 = w_{inv}$

$$\frac{w_{nor}}{w_{inv}} = \frac{20.602}{4.376} \approx 4.708$$



$$h = \frac{\text{load cap}}{\text{nor input cap}} = \frac{x}{nw_{nor}C + pw_{nor}C} = \frac{x}{nw_{nor}C + 4knw_{nor}C}$$

$$\frac{p}{n} = \frac{4k}{1}, \; p = 4kn$$

$$\therefore h = \frac{x}{nw_{nor}C(1+4k)} \quad \text{for nor2}$$

t_{pd}' = Normalized delay of inverter: no load capacitance

$$t_{pd}' = t_{pdf} = t_{pdr} = \frac{R}{nw_{inv}}(nw_{inv}C + pw_{inv}C + x) \quad x = 0$$

$$t_{pd}' = \frac{R}{nw_{inv}}(nw_{inv}C + (2kn)w_{inv}C + (0))$$

$$t_{pd}' = \frac{R}{\cancel{nw_{inv}}} \cdot (2k+1)\cancel{nw_{inv}}C = \underbrace{RC(2k+1)}_{} = t_{pdr}$$

i) NOR2 rising delay:

$$t_{pdr} = \frac{4R}{pw_{nor}}(2nw_{nor}C + 2pw_{nor}C + x)$$

$$p = 4kn \leftarrow \frac{p}{n} = \frac{4k}{1}$$

$$= \frac{\cancel{4}R}{(\cancel{4}kn)w_{nor}}(2nw_{nor}C + 2(4kn)w_{nor}C + x)$$

$$t_{pdr} = \frac{R}{knw_{nor}}((16k+2)nw_{nor}C + x)$$

$$t_{pdr} = 2\left(\frac{8k+1}{k}\right)RC + \frac{R}{k} \cdot \frac{x}{nw_{nor}}$$

$$d_r = \frac{t_{pdr}}{t_{pd}'} = \frac{2\left(\frac{8k+1}{k}\right)RC + \frac{R}{k} \cdot \frac{x}{nw_{nor}}}{RC(2k+1)}$$

$$d_r = 2\left(\frac{8k+1}{k \cdot (2k+1)}\right) + \frac{1}{k(2k+1)} \cdot \frac{x}{nw_{nor}C}$$

$$d_r = 2\left(\frac{8k+1}{k(2k+1)}\right) + \frac{(1+4k)}{k(2k+1)} \cdot \frac{x}{nw_{nor}C(1+4k)}$$

$$d_f = \frac{2}{k}\left(\frac{3k+1}{2k+1}\right) + \frac{(4k+1)}{k(2k+1)} \cdot h$$

~~rising logic effort for nor is:~~ $g = \frac{4k+1}{k(2k+1)}$

low skewed: $g_r = \frac{4(\frac{2}{3})+1}{(\frac{2}{3})(2(\frac{2}{3})+1)} = \frac{\frac{8}{3}+\frac{3}{3}}{(\frac{2}{3})(\frac{4}{3}+\frac{3}{3})} = \frac{(\frac{11}{3})}{(\frac{2}{3})(\frac{7}{3})} = \frac{11}{3}\cdot\frac{3}{2}\cdot\frac{3}{7} = \frac{33}{14}$

unskewed: $g_r = \frac{4(1)+1}{(1)(2(1)+1)} = \frac{4+1}{2+1} = \frac{5}{3}$

high skewed: $g_r = \frac{4(\frac{3}{2})+1}{(\frac{3}{2})(2(\frac{3}{2})+1)} = \frac{6+1}{\frac{3}{2}(3+1)} = \frac{7}{\frac{3}{2}(4)} = \frac{7}{6}$

ii) NOR2 falling delay:

$$t_{pdf} = \frac{R}{nw}(2nwC + 3pwC + x) = \frac{R}{nw}(2nwC + 3(4kn)wC + x)$$

$$\frac{p}{n} = \frac{4k}{1} \rightarrow p = 4kn$$

$$t_{pdf} = \frac{R}{nw}(2nwC + 12knwC + x) = 2RC + 12kRC + \frac{Rx}{nw}$$

$$t_{pdf} = 2RC(1+6k) + \frac{Rx}{nw}$$

$$d_f = \frac{t_{pdf}}{t_{pd}'} = \frac{2RC(1+6k) + \frac{Rx}{nw}}{RC(2k+1)} = 2\left(\frac{6k+1}{2k+1}\right) + \frac{x}{nwC(2k+1)}$$

$$d_f = 2\left(\frac{6k+1}{2k+1}\right) + \underbrace{\frac{(4k+1)}{(2k+1)}}\cdot\underbrace{\frac{x}{nwC(4k+1)}}$$

falling logic effort for NOR is: $g_f = \frac{4k+1}{2k+1}$

low skewed: $g_f = \frac{4(\frac{2}{3})+1}{2(\frac{2}{3})+1} = \frac{\frac{8}{3}+\frac{3}{3}}{\frac{4}{3}+\frac{3}{3}} = \frac{(\frac{11}{3})}{(\frac{7}{3})} = \frac{11}{3}\cdot\frac{3}{7} = \frac{11}{7}$

unskewed: $g_f = \frac{4(1)+1}{2(1)+1} = \frac{4+1}{2+1} = \frac{5}{3}$

high skewed: $g_f = \frac{4(\frac{3}{2})+1}{2(\frac{3}{2})+1} = \frac{6+1}{3+1} = \frac{7}{4}$

iii) NOR2 average delay:

$$t_p = \frac{t_{pdf}+t_{pdr}}{2} \rightarrow d_{avg} = \frac{t_p}{t_{pd}'} = \frac{\left(\frac{t_{pdf}+t_{pdr}}{2}\right)}{t_{pd}'} = \frac{1}{2}\left(\frac{t_{pdf}+t_{pdr}}{t_{pd}'}\right)$$

$$d_{avg} = \frac{\left(\frac{t_{pdf}}{t_{pd}'} + \frac{t_{pdr}}{t_{pd}'}\right)}{2} = \frac{d_f + d_r}{2} = \frac{p_f + g_f h + p_r + g_r h}{2}$$

$$d_{avg} = \frac{p_f + p_r}{2} + \frac{(g_f + g_r)}{2}\cdot h \rightarrow \therefore \boxed{g_{avg} = \frac{g_f + g_r}{2}}$$

low skewed: $g_{avg} = \frac{\left(\frac{11}{7}\right) + \left(\frac{33}{14}\right)}{2} = \frac{\frac{22}{14} + \frac{33}{14}}{2} = \frac{\left(\frac{55}{14}\right)}{2} = \frac{55}{28}$

unskewed: $g_{avg} = \frac{\left(\frac{5}{3}\right) + \left(\frac{5}{3}\right)}{2} = \frac{5}{3}$

high skewed: $g_{avg} = \frac{\left(\frac{7}{4}\right) + \left(\frac{7}{6}\right)}{2} = \frac{7}{2 \cdot 2}\left(\frac{1}{3} + \frac{1}{2}\right) = \frac{7}{4}\left(\frac{2}{6} + \frac{3}{6}\right) = \frac{7}{4}\left(\frac{5}{6}\right) =$

$g_{avg} = \frac{35}{24}$

b) NAND2 gates:

$h = \frac{\text{load cap}}{\text{input cap}} = \frac{x}{nwC + pwC} = \frac{x}{nwC + (kn)wC} = \frac{x}{nwC(k+1)}$

$\frac{p}{n} = \frac{k}{1} \rightarrow \boxed{p = kn}$

i) NAND2 rising delay:

$t_{pdr} = \frac{2R}{pw}(3nwC + 2pwC + x) = \frac{2R}{(kn)w}(3nwC + 2(kn)wC + x)$

$t_{pdr} = \frac{2R}{knw}((3+2k)nwC + x) = 2RC\left(\frac{3+2k}{k}\right) + \frac{2Rx}{knw}$

$d_r = \frac{t_{pdr}}{t_{pd}'} = \frac{2RC\left(\frac{3+2k}{k}\right) + \frac{2Rx}{knw}}{RC(2k+1)} = \frac{2}{k}\left(\frac{3+2k}{1+2k}\right) + \frac{2x}{nwC}$

$d_r = \frac{2}{k}\left(\frac{2k+3}{2k+1}\right) + \underbrace{2(k+1)} \cdot \frac{x}{nwC(k+1)} \leftarrow h$

rising logic effort for NAND2: $g_r = 2(k+1)$

low skewed: $g_r = 2\left(\left(\frac{2}{3}\right) + 1\right) = 2\left(\frac{2}{3} + \frac{3}{3}\right) = 2\left(\frac{5}{3}\right) = \frac{10}{3}$

unskewed: $g_r = 2((1)+1) = 2(2) = 4$

high skewed: $g_r = 2\left(\left(\frac{3}{2}\right) + 1\right) = 2\left(\frac{3}{2} + \frac{2}{2}\right) = 2\left(\frac{5}{2}\right) = 5$

ii) NAND2 falling delay:

$t_{pdf} = \frac{2R}{nw}(2nwC + 2pwC + x) = \frac{2R}{nw}(2nwC + 2(kn)wC + x)$

$t_{pdf} = \frac{2R}{nw}(2(1+k)nwC + x) = 4RC(1+k) + \frac{2Rx}{nw}$

$d_f = \frac{t_{pdf}}{t_{pd}'} = \frac{4RC(1+k) + \frac{2Rx}{nw}}{RC(2k+1)} = 4\left(\frac{1+k}{2k+1}\right) + \frac{2x}{nwC(2k+1)}$

$d_f = 4\left(\frac{k+1}{2k+1}\right) + \underbrace{2\frac{(k+1)}{(2k+1)}} \cdot \frac{x}{nwC(k+1)} \leftarrow h$

falling logic effort for NAND2: $g_f = 2\frac{(k+1)}{(2k+1)}$

low skewed: $g_f = 2\frac{\left(\left(\frac{2}{3}\right)+1\right)}{\left(2\left(\frac{2}{3}\right)+1\right)} = 2\frac{\left(\frac{2}{3}+\frac{3}{3}\right)}{\left(\frac{4}{3}+\frac{3}{3}\right)} = 2\frac{\left(5/3\right)}{\left(7/3\right)} = 2\left(\frac{5}{3}\right)\left(\frac{3}{7}\right)$

$g_f = 10/7$

unskewed: $g_f = 2\frac{((1)+1)}{(2(1)+1)} = 2\frac{(1+1)}{(2+1)} = 2\left(\frac{2}{3}\right) = \frac{4}{3}$

high skewed: $g_f = 2\frac{\left(\left(\frac{3}{2}\right)+1\right)}{\left(2\left(3/2\right)+1\right)} = 2\frac{\left(\frac{3}{2}+\frac{2}{2}\right)}{(3+1)} = \cancel{2}\frac{\left(5/\cancel{2}\right)}{4} = \frac{5}{4}$

iii) NAND2 average logic effort

$g_{avg} = \frac{g_f + g_r}{2} = \frac{1}{2}(g_f + g_r)$

low skewed: $g_{avg} = \frac{1}{2}\left(\left(\frac{10}{7}\right)+\left(\frac{10}{3}\right)\right) = \frac{10}{2}\left(\frac{1}{7}+\frac{1}{3}\right) = 5\left(\frac{3}{21}+\frac{7}{21}\right)$

$g_{avg} = 5\left(\frac{10}{21}\right) = \frac{50}{21}$

unskewed: $g_{avg} = \frac{1}{2}\left((4)+\left(\frac{4}{3}\right)\right) = \frac{1}{2}\left(\frac{12}{3}+\frac{4}{3}\right) = \frac{1}{2}\left(\frac{16}{3}\right) = \frac{8}{3}$

high skewed: $g_{avg} = \frac{1}{2}\left((5)+\left(\frac{5}{4}\right)\right) = \frac{1}{2}\left(\frac{20}{4}+\frac{5}{4}\right) = \frac{1}{2}\left(\frac{25}{4}\right) = \frac{25}{8}$