# Systematic Staleness (JoE 2024)

Federico M. Bandi
Davide Pirino
Roberto Renò

Antônio Schwanke

São Paulo School of Economics – FGV

February 17, 2024

# Introduction

- Prices do not update as often as one would assume given traditional modeling in continuous time.
- Definition of *systematic* $\times$ *idiosyncratic* staleness.
- Systematic staleness is defined as lack of (large) price updates across many assets.
- Propose estimators for staleness and provide asymptotic results.
- Descriptive analysis and structural estimation of a market microstructure model.

# Some Notation

- Let $X_t^{(q)}$ be the $q$-th component of the vector $\mathbf{X}_t$.
- A smoother is an integrable function $\mathcal{S} : \mathbb{R}^+ \to (0, 1]$ with a bounded first derivative and such that $\mathcal{S}(0) = 1$.
- $\Theta_{t,n}^{(q,N)}$ is a threshold for price changes.

# Staleness Estimators

- The *univariate* staleness estimator of the $q$th process at frequency $\Delta_n$ is defined as:

$$\mathbb{U}_n^{(q)} := \Delta_n \sum_{j=1}^{n} \mathcal{S}\left( \frac{\left| X_{j\Delta_n}^{(q)} - X_{(j-1)\Delta_n}^{(q)} \right|}{\Theta_{\Delta_n,n}^{(q,N)}} \right)$$

- All of the price adjustments below the threshold (a case of extreme staleness), then $\mathbb{U}_n^{(q)}$ would be exactly 1.
- Intermediate cases of sluggish price adjustments: $\mathbb{U}_n^{(q)} \in (0, 1)$.

# Staleness Estimators

- The $N$-*multivariate* staleness estimator at frequency $\Delta_n$ is defined as:

$$\mathbb{M}_n^{(N)} := \Delta_n \sum_{j=1}^{n} \prod_{q=1}^{N} \mathcal{S} \left( \frac{\left| X_{j\Delta_n}^{(q)} - X_{(j-1)\Delta_n}^{(q)} \right|}{\Theta_{\Delta_n,n}^{(q,N)}} \right)^{1/N}$$

- The estimator measures the joint probability of stale price updates.
- $\mathbb{M}_n^{(N)}$ directly identifies systematic staleness.

# Asymptotics

- **Assumption 1:** There exist $N$ real-valued (logarithmic) efficient price processes each of which is a Brownian semimartingale

$$d\tilde{X}_t^{(q)} = \mu_t^{(q)} dt + \sigma_t^{(q)} dW_t^{(q)}$$

- + some technical conditions.

# Asymptotics

- $\mathcal{H}_0$: only idiosyncratic staleness is allowed. More formally, the collection
$$\left\{X^{(q)}_{j\Delta_n} : j = 0, \ldots, n; q = 1, \ldots, N\right\}$$
on the time grid $t_{j,n} = j\Delta_n$ is such that $X^{(q)}_0 = \tilde{X}^{(q)}_0$ and for $j = 1, \ldots, n$:
$$X^{(q)}_{j\Delta_n} = \tilde{X}^{(q)}_{j\Delta_n}\left(1 - B^{(q)}_{j,n}\right) + X^{(q)}_{(j-1)\Delta_n} B^{(q)}_{j,n}$$

- Where $B^{(q)}_{j,n}$ are $N$ triangular arrays of Bernoulli variables such that:[1]
$$p^{(q)}_n := \mathbb{P}\left[B^{(q)}_{j,n} = 1\right] \underset{n\to\infty}{\longrightarrow} p^{(q)}_\infty \in (0, 1).$$

[1] With $q = 1, \ldots, N$.

# Asymptotics

- $\mathcal{H}_A$: systematic staleness is also allowed. Prices may repeat themselves due to systematic effects or asset-specific idiosyncratic effects.

$$X_{j\Delta_n}^{(q)} = \left(1 - C_{j,n}^{(S)}\right)\left[\tilde{X}_{j\Delta_n}^{(q)}\left(1 - B_{j,n}^{(q)}\right) + X_{(j-1)\Delta_n}^{(q)} B_{j,n}^{(q)}\right] + C_{j,n}^{(S)} X_{(j-1)\Delta_n}^{(q)}$$

- Hierarchical structure $\rightarrow$ prices may repeat themselves due to systematic effects. Otherwise: asset-specific likelihood of repeated prices due to idiosyncratic effects.
- Denote by $p_n^{(S)}$ the probability of systematic staleness.

# Asymptotics

- Under **Assumption 1**:

$$\mathbb{U}_n^{(q)} \xrightarrow{p} \begin{cases} p_\infty^{(q)} & \text{under } \mathcal{H}_0 \\ p_\infty^{(S)} + (1 - p_\infty^{(S)})p_\infty^{(q)} & \text{under } \mathcal{H}_A \end{cases}$$

- Under some additional technical assumptions, we have:

$$\mathbb{M}_n^{(q)} \xrightarrow{p} \begin{cases} 0 & \text{under } \mathcal{H}_0 \\ p_\infty^{(S)} & \text{under } \mathcal{H}_A \end{cases}$$

- $\mathbb{M}_n^{(q)}$ is a consistent estimator of the probability of systematic staleness under both the null (in which it is zero) and the alternative.

# Descriptive Analysis

- Dataset: 250 most liquid NYSE-listed stocks. Trades from January 2006 to December 2014.
- Universe of stocks which should be affected by staleness the least $\rightarrow$ conservative implications.
- Idiosyncratic and systematic staleness are computed for each day.
  Details

# Descriptive Analysis

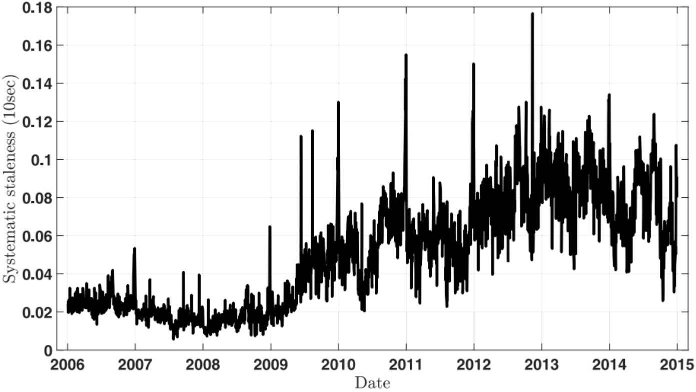


Figure: Systematic Staleness

# Descriptive Analysis

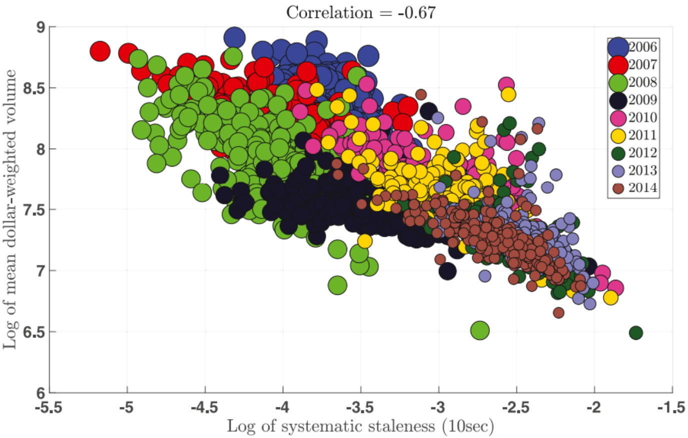


Figure: Systematic Staleness

# Descriptive Analysis

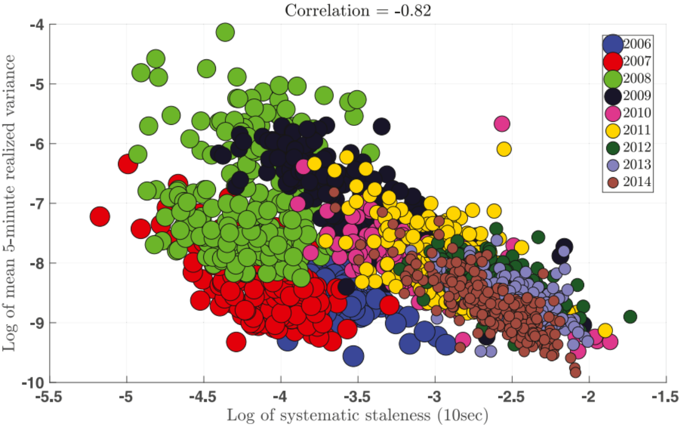


Figure: Systematic Staleness

# Micro-founding Staleness

- $N$-variate price formation process in which private information plays a key role in driving transaction prices.
- If the value of the information signal $>$ execution costs, informed traders will act on it and trade.
- Otherwise, they will choose not to trade, thereby leading to price staleness.
- Measured (systematic and idiosyncratic) staleness from the data can help identify key features of the assumed price formation process.

# The Model

- Three sets of $N$-variate prices: unobserved efficient prices, mid-quotes and transaction prices.
- Latent efficient prices evolve as

$$\mathfrak{e}_{q,t} = \mathfrak{e}_{q,t-1} + \beta_q \sigma_M \sqrt{\Delta} \varepsilon_{M,t} + \sigma_q^{(\mathfrak{e})} \sqrt{\Delta} \varepsilon_{q,t}^{(\mathfrak{e})}$$

- $\varepsilon_{q,t}^{(\mathfrak{e})}$ ($\varepsilon_{M,t}$) are zero mean, unit variance, white noise shocks independent (resp. common) across stocks.

# The Model

- Mid-quotes evolve as:

$$\mathfrak{m}_{q,t} = \mathfrak{m}_{q,t-1} + \delta_q(\mathfrak{e}_{q,t} - \mathfrak{m}_{q,t-1}) + (1 - \delta_q)\sigma_q^{\mathfrak{m}}\sqrt{\Delta}\varepsilon_{q,t}^{\mathfrak{m}}$$

- The market maker reacts to the order flow, with learning speed $\delta_q$.
- When $\delta_q = 0$, mid-quotes are random walks independent of the efficient prices.
- If $|\mathfrak{e}_{q,t} - \mathfrak{m}_{q,t}| \leqslant c_q := s_q + f$, informed traders do not trade and prices are stale.
- Cost of immediacy (half spread) $s_q$ and cost of funding $f$.

# Structural Estimation

- Estimate $\beta_q, \sigma_q^{(\mathfrak{e})}, \sigma_q^{(\mathfrak{m})}, \delta_q$ and $s_q$ for $q = 1, \ldots, N$ plus $\sigma_M, f$ and PAIT.

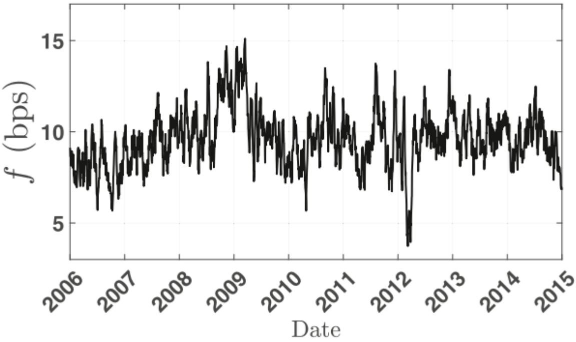


Figure: Systematic Component of Execution Costs

# Structural Estimation

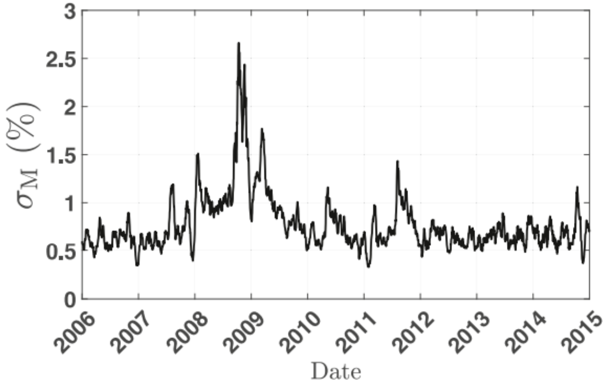


Figure: Systematic Component of Execution Costs

# Structural Estimation

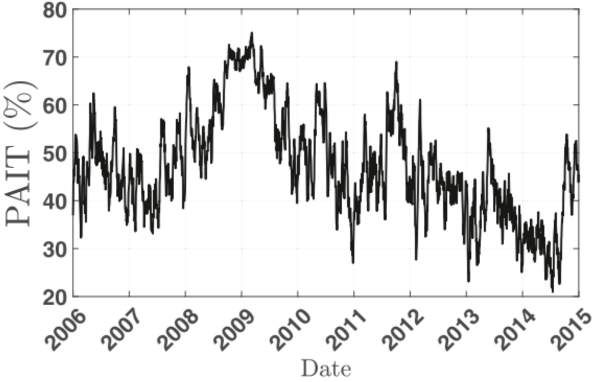


Figure: Probability of Arrival of Informed Traders

# Conclusion

- Define notions of (market-wide) systematic staleness and (asset-specific) idiosyncratic staleness.
- Theory of inference based on asymptotics.
- Descriptive analysis.
- Structural estimation of a microstructure model.

# More Details

- Compute staleness for each day using 10-second returns. The number $n$ of 10-second returns is 2340 and the number of days $N_D = 2265$. Thresholds are $\Theta_{t,n}^{(q,N)} = \Theta_n^{(q)} = \alpha\sigma^{(q)}/n^{1/2}$, where $\alpha = 0.1$ and $\sigma^{(q)}$ is the estimated volatility over the day for the stock indexed by $q$.[2]
- The smoother $\mathcal{S}(\cdot) = \exp(-|\cdot|)$ is employed.
- For each day $d = 1, \ldots, N_D$ in the sample, let $t_{j,n} = j\Delta_n = j/n$ be the 10-second partition.

[2] The square root of 5-minute daily realized variance

# More Details

- At each instant $t_{j,n}$, compute:

$$\zeta_{d,j} = \prod_{q=1}^{N_{d,j}} \exp\left(-\frac{|X_{j/n} - X_{(j-1)/n}|}{N_{d,j}\Theta_n^{(q)}}\right)$$

  where $N_{d,j}$ is the number of stocks available in the time interval $[t_{j-1,n}, t_{j,n}]$ of day $d$.
- Estimate systematic staleness for day $d$ by computing

$$\hat{p}_d^{(S)} := \frac{1}{n_d} \sum_{j=1}^{n_d} \zeta_{d,j} \mathbb{I}_{\{N_{d,j} \geqslant 60\}} \quad \text{and} \quad n_d := \sum_{j=1}^{n} \mathbb{I}_{\{N_{d,j} \geqslant 60\}}.$$