МИНИСТЕРСТВО НАУКИ И ВЫСШЕГО ОБРАЗОВАНИЯ
РОССИЙСКОЙ ФЕДЕРАЦИИ

Федеральное государственное автономное образовательное учреждение
высшего образования

**«Дальневосточный федеральный университет»**
**(ДВФУ)**

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**ИНСТИТУТ МАТЕМАТИКИ И КОМПЬЮТЕРНЫХ ТЕХНОЛОГИЙ**

**Кафедра математического и компьютерного моделирования**

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# ИНДИВИДУАЛЬНОЕ ДОМАШНЕЕ ЗАДАНИЕ №1 ПО ТЕОРИИ ФУНКЦИЙ КОМПЛЕКСНОГО ПЕРЕМЕННОГО

Направление подготовки 02.03.01сцт Сквозные цифровые технологии,
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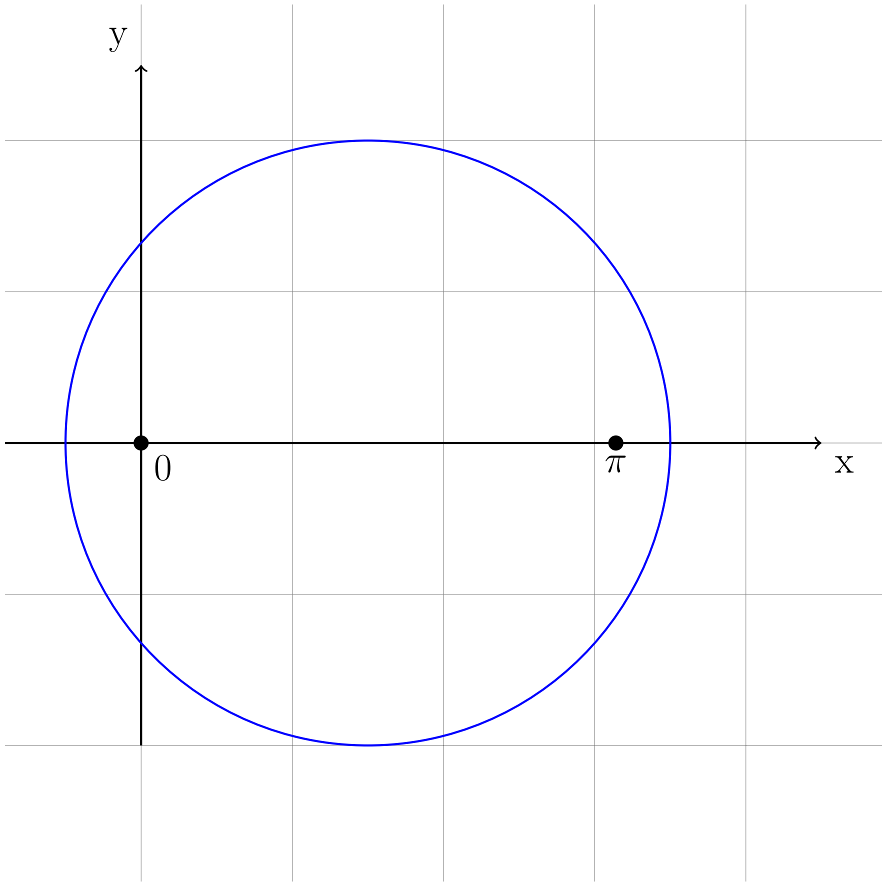
# Содержание

# 1 Вычислить интеграл:

$$\oint\limits_{\left|z-\frac{3}{2}\right|=2} \frac{2z(z-1)}{\sin z}\,dz =$$




$\sin z = 0$

$z = \pi n, n \in \mathbb{Z}$ – и.о.т.о.х.

$$= 2\pi i \left( \operatorname{Res}_{z=0} \frac{2z(z-1)}{\sin z} + \operatorname{Res}_{z=\pi} \frac{2z(z-1)}{\sin z} \right) =$$

$$\lim_{z\to 0} \frac{2z(z-1)}{\sin z} = \lim_{z\to 0} \frac{2z(z-1)}{z - \frac{z^3}{3!} + \frac{z^5}{5!} - \ldots} = \lim_{z\to 0} \frac{2(z-1)}{1 - \frac{z^2}{3!} + \frac{z^4}{5!} - \ldots} = -2 \Rightarrow z =$$

$0$ – устранимая особая точка $\Rightarrow \operatorname{Res}_{z=0} \dfrac{2z(z-1)}{\sin z} = 0$

$$\lim_{z\rightarrowtail \pi+0} \frac{2z(z-1)}{\sin z} = \lim_{z\rightarrowtail \pi+0} \frac{2\pi(\pi-1)}{\sin(\pi+0)} = \lim_{z\rightarrowtail \pi+0} \frac{2\pi(\pi-1)}{-0} = -\infty$$

$$\lim_{z\rightarrowtail \pi-0} \frac{2z(z-1)}{\sin z} = \lim_{z\rightarrowtail \pi-0} \frac{2\pi(\pi-1)}{\sin(\pi-0)} = \lim_{z\rightarrowtail \pi-0} \frac{2\pi(\pi-1)}{+0} = +\infty$$

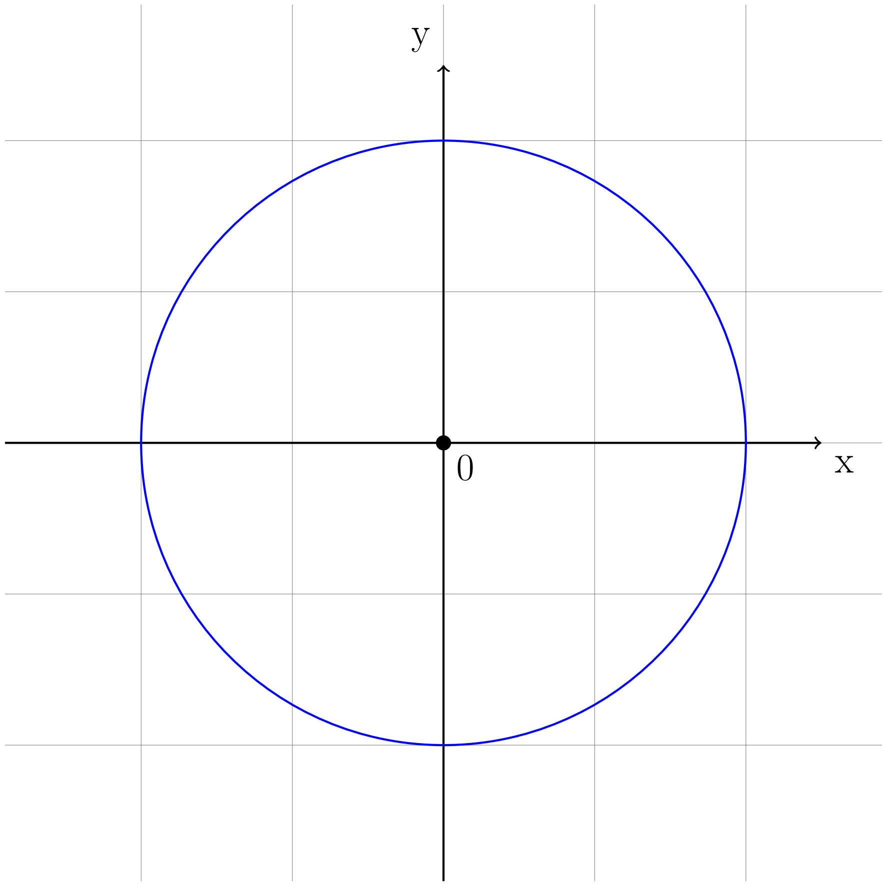
$$\lim_{z\rightarrowtail \pi+0} \frac{2z(z-1)}{\sin z} \neq \lim_{z\rightarrowtail \pi-0} \frac{2z(z-1)}{\sin z} \Rightarrow \nexists \lim_{z\rightarrowtail \pi} \frac{2z(z-1)}{\sin z} \Rightarrow z = \pi$$ – существенная

особая точка $\Rightarrow \operatorname{Res}_{z=\pi} \dfrac{2z(z-1)}{\sin z} = c_{-1}$

$$2z = 2(z - \pi + \pi) = \frac{\pi}{2} + 2(z - \pi)$$
$$z - 1 = z - \pi + \pi - 1 = (\pi - 1) + (z - \pi)$$
$$2z(z-1) = \left(\frac{\pi}{2} + 2(z-\pi)\right)((\pi - 1) + (z - \pi)) = \frac{\pi}{2}(\pi - 1) + \frac{\pi}{2}(z - \pi) +$$
$$2(\pi - 1)(z - \pi) + 2(z - \pi)^2 = \frac{\pi}{2}(\pi - 1) + \left(\frac{\pi}{2} + 2(\pi - 1)\right)(z - \pi) + 2(z - \pi)^2$$
$$\sin z = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n-1)!}(z - \pi)^{2n-1}$$
$$\frac{2z(z-1)}{\sin z} = \frac{\frac{\pi}{2}(\pi - 1) + \left(\frac{\pi}{2} + 2(\pi - 1)\right)(z - \pi) + 2(z - \pi)^2}{\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n-1)!}(z - \pi)^{2n-1}}$$
$$c_{-1} = -\frac{\pi}{2}(\pi - 1) + 2 \cdot 3! = \frac{\pi}{2} - \frac{\pi^2}{2} + 12$$
$$= 2\pi i \left(0 + \frac{\pi}{2} - \frac{\pi^2}{2} + 12\right) = \pi^2 i - \pi^3 i + 24\pi i$$

## 2 Вычислить интеграл:

$$\oint\limits_{|z|=2} \frac{1-\cos z^2}{z^2}\,dz =$$




$z = 0$ – и.о.т.о.х.

$$= 2\pi i \operatorname{Res}_{z=0} \frac{1-\cos z^2}{z^2} =$$

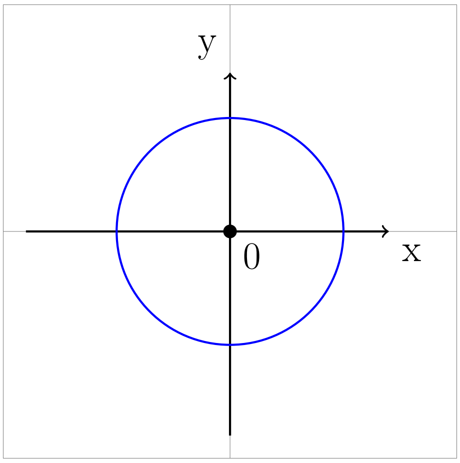
$$\lim_{z\to 0} \frac{1-\cos z^2}{z^2} = \lim_{z\to 0} \frac{1-1+\frac{z^4}{2!}-\frac{z^8}{4!}+\ldots}{z^2} = \lim_{z\to 0}\left(\frac{z^2}{2!}-\frac{z^6}{4!}+\ldots\right) = 0 \Rightarrow$$

$z = 0$ – устранимая особая точка

$$= 2\pi i \cdot 0 = 0$$

# 3 Вычислить интеграл:

$$\oint\limits_{|z|=0.5} \frac{e^{2z}-1-2z}{z\operatorname{sh}^2 4iz}\,dz \circledequal$$




$z\operatorname{sh}^2 4iz = 0$

$\operatorname{sh}^2 4iz = 0$

$\operatorname{sh} 4iz = 0$

$i\sin 4z = 0$

$\sin 4z = 0$

$4z = \pi n, n \in \mathbb{Z}$

$z = \frac{\pi}{4}n, n \in \mathbb{Z}$

$\frac{\pi}{4} \approx 0.8 > 0.5$

$z = 0$ – и.о.т.о.х.

$$\circledequal 2\pi i \operatorname{Res}_{z=0} \frac{e^{2z}-1-2z}{z\operatorname{sh}^2 4iz} \circledequal$$

$$\lim_{z\to 0} \frac{e^{2z}-1-2z}{z\operatorname{sh}^2 4iz} = \lim_{z\to 0} \frac{1+2z+\frac{(2z)^2}{2!}+\ldots-1-2z}{z\operatorname{sh}^2 4iz} = \lim_{z\to 0} \frac{\frac{2^2}{2!}z+\frac{2^3}{3!}z^2+\ldots}{\operatorname{sh}^2 4iz} =$$

$$\lim_{z\to 0} \frac{4(\frac{2^2}{2!}z+\frac{2^3}{3!}z^2+\ldots)}{(e^{4iz}-e^{-4iz})^2} = 4\lim_{z\to 0} \frac{\frac{2^2}{2!}z+\frac{2^3}{3!}z^2+\ldots}{e^{8iz}-2+e^{-8iz}} =$$

$$4\lim_{z\to 0} \frac{\frac{2^2}{2!}z+\frac{2^3}{3!}z^2+\ldots}{\cos 8z+i\sin 8z+\cos 8z-i\sin 8z-2} = 4\lim_{z\to 0} \frac{\frac{2^2}{2!}z+\frac{2^3}{3!}z^2+\ldots}{2(\cos 8z-1)} =$$

$$2\lim_{z\to 0} \frac{\frac{2^2}{2!}z+\frac{2^3}{3!}z^2+\ldots}{1-\frac{8^2}{2!}z^2+\frac{8^4}{4!}z^4-\ldots-1} = 2\lim_{z\to 0} \frac{\frac{2^2}{2!}z+\frac{2^3}{3!}z^2+\ldots}{-\frac{8z^2}{2!}z^2+\frac{8^4}{4!}z^4-\ldots} =$$

$$2\lim_{z\to 0} \frac{\frac{2^2}{2!}+\frac{2^3}{3!}z+\ldots}{-\frac{8z^2}{2!}z+\frac{8^4}{4!}z^3-\ldots} = \infty \Rightarrow z=0 - \text{полюс}$$

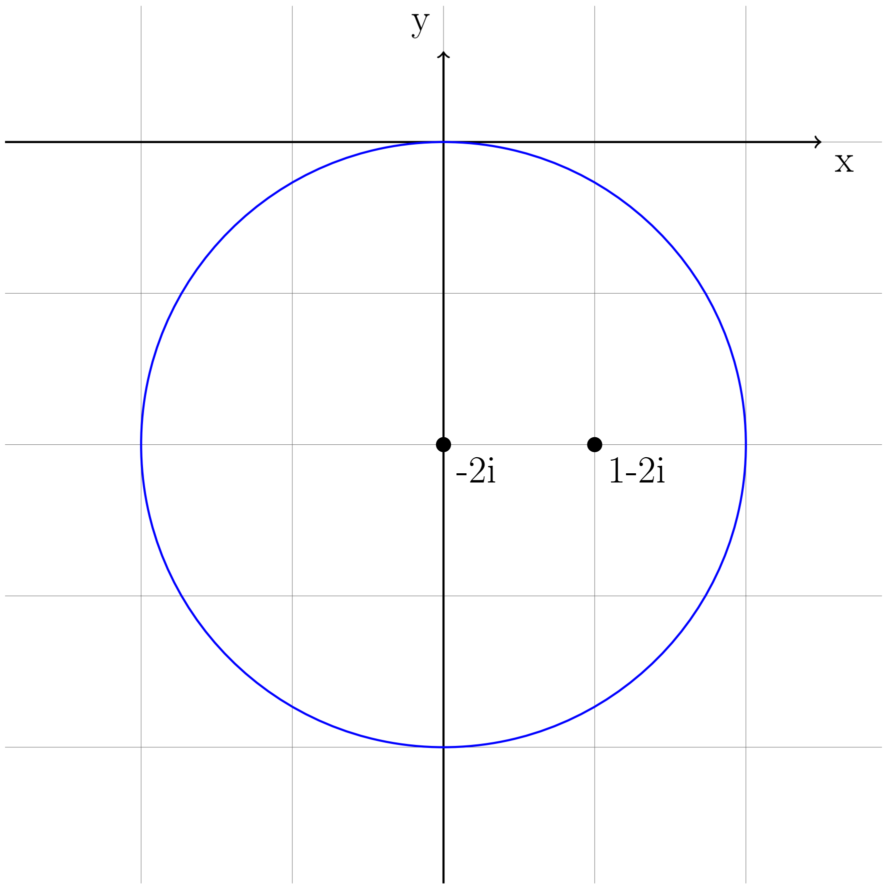
$$\lim_{z\to 0}\frac{e^{2z}-1-2z}{z\,\mathrm{sh}^2\,4iz}z^n = 2\lim_{z\to 0}\frac{\frac{2^2}{2!}+\frac{2^3}{3!}z+\ldots}{-\frac{8z^2}{2!}z+\frac{8^4}{4!}z^3-\ldots}z^n =$$

$$2\lim_{z\to 0}\frac{\frac{2^2}{2!}+\frac{2^3}{3!}z+\ldots}{-\frac{8z^2}{2!}+\frac{8^4}{4!}z^2-\ldots}z^{n-1} = \begin{cases} -\dfrac{1}{16}, n=1 \\ 0, n>1 \end{cases} \Rightarrow z=0 \text{—полюс 1-го порядка}$$

$$= 2\pi i\lim_{z\to 0}\frac{e^{2z}-1-2z}{z\,\mathrm{sh}^2\,4iz}z = 2\pi i\left(-\frac{1}{16}\right) = -\frac{\pi i}{8}$$

# 4 Вычислить интеграл:

$$\oint\limits_{|z+2i|=2}\left(\frac{\pi}{e^{\frac{\pi z}{2}}+1}+\frac{4\cos\frac{\pi z}{1-2i}}{(z-1+2i)^2(z-3+2i)}\right)dz =$$




$e^{\frac{\pi z}{2}}+1=0$

$e^{\frac{\pi z}{2}}=-1=\cos(\pi+2\pi n)=\cos(\pi+2\pi n)+i\sin(\pi+2\pi n)=e^{i(\pi+2\pi n)}, n\in\mathbb{Z}$

$\dfrac{\pi z}{2}=i\pi(1+2n)$

$\dfrac{z}{2}=i(1+2n)$

$z=2i+4in, n\in\mathbb{Z}$

$z=-2i$ − и.о.т.о.х.

$z-1+2i=0$

$z=1-2i\in\{z:|z+2i|=2\}$ − и.о.т.о.х.

$z-3+2i=0$

$z=3-2i\notin\{z:|z+2i|=2\}$

⊜ $2\pi i\left(\mathrm{Res}_{z=-2i}\dfrac{pi}{e^{\frac{\pi z}{2}}+1}+\mathrm{Res}_{z=1-2i}\dfrac{4\cos\frac{\pi z}{1-2i}}{(z-1+2i)^2(z-3+2i)}\right)$ ⊜

$\lim\limits_{z\to-2i}\dfrac{\pi}{e^{\frac{\pi z}{2}}+1}=\lim\limits_{z\to-2i}\dfrac{\pi}{e^{-\pi i}+1}=\infty\Rightarrow z=-2i$ − полюс

$\lim\limits_{z\to-2i}\dfrac{\pi}{e^{\frac{\pi z}{2}}+1}(z+2i)^n=$

$\lim\limits_{z\to-2i}\dfrac{\pi}{1-1-\frac{\pi}{2}(z+2i)-\left(\frac{\pi}{2}\right)^2\frac{1}{2!}(z+2i)^2-...}(z+2i)^n=$

$\lim\limits_{z\to-2i}\dfrac{1}{-\frac{1}{2}-(\frac{\pi}{2^2\cdot2!}(z+2i)-...}(z+2i)^{n-1}=\begin{cases}-2, n=1\\0, n>1\end{cases}\Rightarrow z=-2i$ −
полюс 1-го порядка

$\lim\limits_{z\to1-2i}\dfrac{4\cos\frac{\pi z}{1-2i}}{(z-1+2i)^2(z-3+2i)}=\infty\Rightarrow z=1-2i$ − полюс

$\lim\limits_{z\to1-2i}\dfrac{4\cos\frac{\pi z}{1-2i}}{(z-1+2i)^2(z-3+2i)}(z-1+2i)^n=$

$\lim\limits_{z\to1-2i}\dfrac{4\cos\frac{\pi z}{1-2i}}{z-3+2i}(z-1+2i)^{n-2}=\begin{cases}2, n=2\\0, n>2\\\infty, n=1\end{cases}\Rightarrow z=1-2i$−полюс 2-го порядка

⊜ $2\pi i\left(\lim\limits_{z\to-2i}\dfrac{\pi}{e^{\frac{\pi z}{2}}+1}(z+2i)+\lim\limits_{z\to1-2i}\left(\dfrac{4\cos\frac{\pi z}{1-2i}}{(z-1+2i)^2(z-3+2i)}(z-1+2i)^2\right)'\right)$ ⊜

$\lim\limits_{z\to-2i}\dfrac{\pi}{e^{\frac{\pi z}{2}}+1}(z+2i)=-2$

$\lim\limits_{z\to1-2i}\left(\dfrac{4\cos\frac{\pi z}{1-2i}}{(z-1+2i)^2(z-3+2i)}(z-1+2i)^2\right)'=\lim\limits_{z\to1-2i}\left(\dfrac{4\cos\frac{\pi z}{1-2i}}{z-3+2i}\right)'=$

$\lim\limits_{z\to1-2i}\dfrac{-4\frac{\pi}{1-2i}\sin\frac{\pi z}{1-2i}(z-3+2i)-4\cos\frac{\pi z}{1-2i}}{(z-3+2i)^2}=\dfrac{4}{4}=1$

⊜ $2\pi i(-2+1)=-2\pi i$

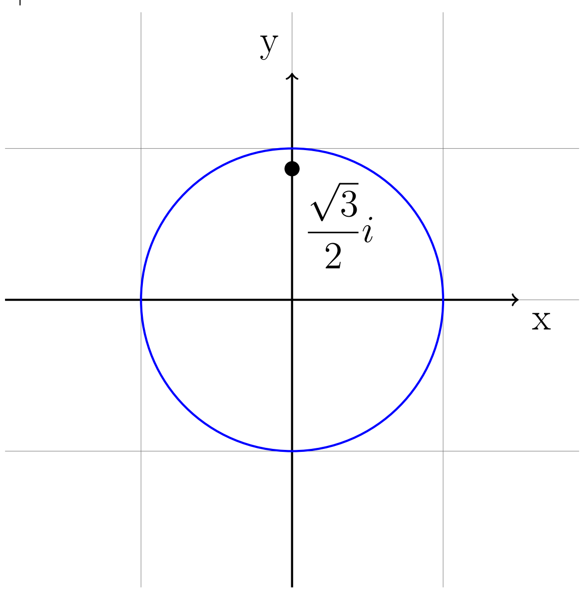
# 5 Вычислить интеграл:

$$\int_0^{2\pi} \frac{dt}{4\sqrt{3}\sin t - 7} =$$

$$\left| \begin{matrix} z = e^{it} & dt = \dfrac{dz}{zi} & \sin t = \dfrac{e^{it} - e^{-it}}{2i} = \dfrac{z - \frac{1}{z}}{2i} & 0 \le t \le \pi \mapsto |z| = 1 \\ dz = ie^{it}dt & & & \end{matrix} \right| =$$

$$\oint\limits_{|z|=1} \frac{dz}{zi\left(\dfrac{2\sqrt{3}}{i}\left(z - \dfrac{1}{z}\right) - 7\right)} = \oint\limits_{|z|=1} \frac{dz}{2\sqrt{3}(z^2 - 1) - 7zi} =$$

$$\oint\limits_{|z|=1} \frac{dz}{2\sqrt{3}z^2 - 7zi - 2\sqrt{3}} =$$




$$2\sqrt{3}z^2 - 7iz - 2\sqrt{3} = 0$$

$$D = -494 \cdot 4 \cdot 3 = -49 + 48 = -1$$

$$z = \frac{7i + i}{4\sqrt{3}} = \frac{2i}{\sqrt{3}} \notin \{z : |z| = 1\}$$

$$z = \frac{7i - i}{4\sqrt{3}} = \frac{3i}{2\sqrt{3}} = \frac{\sqrt{3}}{2}i \in \{z : |z| = 1\} - \text{и.о.т.о.х.}$$

$$= 2\pi i \operatorname{Res}_{z=\frac{\sqrt{3}}{2}i} \frac{1}{2\sqrt{3}z^2 - 7iz - 2\sqrt{3}} =$$

$$\lim_{z \to \frac{\sqrt{3}}{2}i} \frac{1}{2\sqrt{3}z^2 - 7iz - 2\sqrt{3}} = \infty \Rightarrow z = \frac{\sqrt{3}}{2}i - \text{полюс}$$

$$\lim_{z \to \frac{\sqrt{3}}{2}i} \frac{1}{2\sqrt{3}z^2 - 7iz - 2\sqrt{3}} \left(z - \frac{\sqrt{3}}{2}i\right)^n =$$

$$\lim_{z\to\frac{\sqrt{3}}{2}i} \frac{1}{2\sqrt{3}(z-\frac{\sqrt{3}}{2}i)(z-\frac{2}{\sqrt{3}}i)} \left(z-\frac{\sqrt{3}}{2}i\right)^n$$

$$\lim_{z\to\frac{\sqrt{3}}{2}i} \frac{1}{2\sqrt{3}(z-\frac{2}{\sqrt{3}}i)} \left(z-\frac{\sqrt{3}}{2}i\right)^{n-1} = \begin{cases} -\frac{1}{i} = i, n=1 \\ 0, n>1 \end{cases} \Rightarrow z = \frac{\sqrt{3}}{2}i -$$

полюс 1-го порядка

$$= 2\pi i \lim_{z\to\frac{\sqrt{3}}{2}i} \frac{1}{2\sqrt{3}z^2-7iz-2\sqrt{3}} \left(z-\frac{\sqrt{3}}{2}i\right) = 2\pi i\cdot i = -2\pi$$

# 6 Вычислить интеграл:

$$\int_0^{2\pi} \frac{dt}{(\sqrt{7}+\cos t)^2} =$$

$$\left| \begin{matrix} z=e^{it} \quad dt=\frac{dz}{zi} \quad \cos t = \frac{e^{it}+e^{-it}}{2} = \frac{z+\frac{1}{z}}{2} \quad 0\le t\le\pi \mapsto |z|=1 \\ dz = ie^{it}dt \end{matrix} \right| =$$

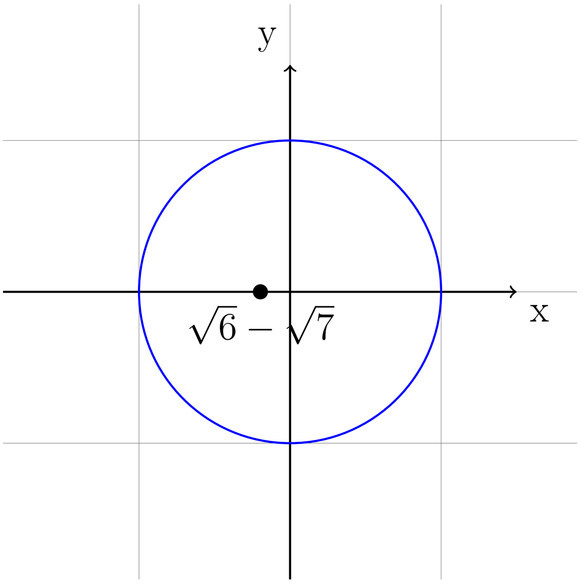
$$\oint\limits_{|z|=1} \frac{dz}{zi\left(\sqrt{7}+\frac{z+\frac{1}{z}}{2}\right)^2} = \oint\limits_{|z|=1} \frac{dz}{zi\left(7+\sqrt{7}\left(z+\frac{1}{z}\right)+\frac{z^2+2+\frac{1}{z^2}}{4}\right)} =$$

$$\oint\limits_{|z|=1} \frac{dz}{7iz+\sqrt{7}iz^2+\sqrt{7}i+\frac{z^3}{4}i+\frac{z}{2}i+\frac{1}{4z}i} =$$

$$\oint\limits_{|z|=1} \frac{dz}{\frac{28iz^2+4\sqrt{7}iz^3+4\sqrt{7}iz+iz^4+2iz^2+i}{4z}} =$$

$$\oint\limits_{|z|=1} \frac{4zdz}{(30z^2+4\sqrt{7}z^3+4\sqrt{7}z+z^4+1)i} =$$

$$-4i\oint\limits_{|z|=1} \frac{zdz}{z^4+4\sqrt{7}z^3+30z^2+4\sqrt{7}z+1} =$$




$z^4+4\sqrt{7}z^3+30z^2+4\sqrt{7}z+1=0| : z^2$

$z^2+4\sqrt{7}z+30+\dfrac{4\sqrt{7}}{z}+\dfrac{1}{z^2}=0$

$z^2+\dfrac{1}{z^2}+4\sqrt{7}\left(z+\dfrac{1}{z}\right)+30=0$

$\left(z+\dfrac{1}{z}\right)^2+4\sqrt{7}\left(z+\dfrac{1}{z}\right)+28=0|\ t=z+\dfrac{1}{z}$

$t^2+4\sqrt{7}t+28=0$

$D=16\cdot 7-4\cdot 28=0$

$t=\dfrac{-4\sqrt{7}}{2}=-2\sqrt{7}$

$z+\dfrac{1}{z}=-2\sqrt{7}$

$z^2+2\sqrt{7}z+1=0$

$D=4\cdot 7-4=24$

$z=\dfrac{-2\sqrt{7}-\sqrt{24}}{2}=\dfrac{-2\sqrt{7}-2\sqrt{6}}{2}=-\sqrt{7}-\sqrt{6}\notin\{z:|z|=1\}$

$z=\dfrac{-2\sqrt{7}+\sqrt{24}}{2}=\dfrac{-2\sqrt{7}+2\sqrt{6}}{2}=\sqrt{6}-\sqrt{7}\notin\{z:|z|=1\}$ – и.о.т.о.х.

$= 8\pi\operatorname{Res}_{z=\sqrt{6}-\sqrt{7}}\dfrac{z}{z^4+4\sqrt{7}z^3+30z^2+4\sqrt{7}z+1} =$

$\lim\limits_{z\to\sqrt{6}-\sqrt{7}}\dfrac{z}{z^4+4\sqrt{7}z^3+30z^2+4\sqrt{7}z+1}=\infty\Rightarrow z=\sqrt{6}-\sqrt{7}$ – полюс

$\lim\limits_{z\to\sqrt{6}-\sqrt{7}}\dfrac{z}{z^4+4\sqrt{7}z^3+30z^2+4\sqrt{7}z+1}(z-\sqrt{6}+\sqrt{7})^n=$

$\lim\limits_{z\to\sqrt{6}-\sqrt{7}}\dfrac{z}{(z+\sqrt{7}+\sqrt{6})^2(z-\sqrt{6}+\sqrt{7})^2}(z-\sqrt{6}+\sqrt{7})^n=$

$$\lim_{z\to\sqrt{6}-\sqrt{7}} \frac{z}{(z+\sqrt{7}+\sqrt{6})^2}(z-\sqrt{6}+\sqrt{7})^{n-2} = \begin{cases} -\dfrac{\sqrt{6}-\sqrt{7}}{24}, n=2 \\ 0, n>2 \\ \infty, n=1 \end{cases} \Rightarrow$$

$z = \sqrt{6}-\sqrt{7}$ – полюс 2-го порядка

$$= 8\pi \lim_{z\to\sqrt{6}\sqrt{7}} \left(\frac{z}{z^4+4\sqrt{7}z^3+30z^2+4\sqrt{7}z+1}(z-\sqrt{6}+\sqrt{7})^2\right)' =$$

$$8\pi \lim_{z\to\sqrt{6}\sqrt{7}} \left(\frac{z}{(z+\sqrt{7}+\sqrt{6})^2}\right)' =$$

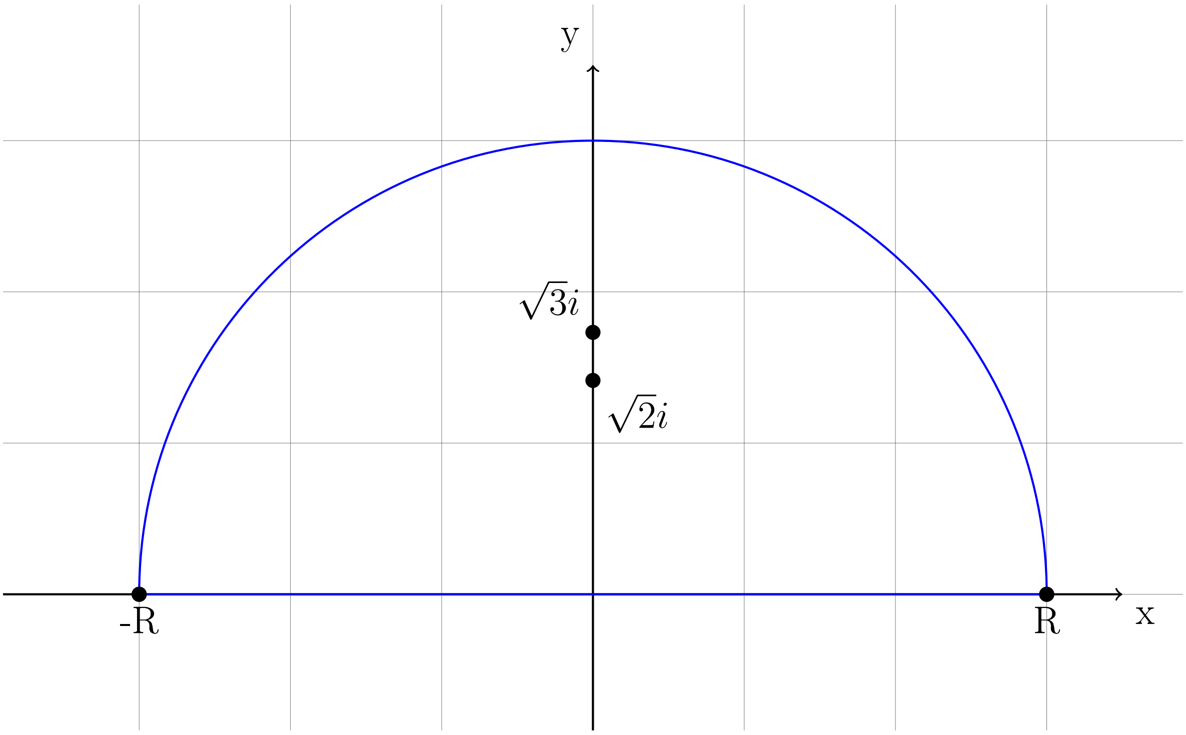
$$8\pi \lim_{z\to\sqrt{6}\sqrt{7}} \frac{(z+\sqrt{7}+\sqrt{6})^2-2z(z+\sqrt{7}+\sqrt{6})}{(z+\sqrt{7}+\sqrt{6})^4} =$$

$$8\pi \lim_{z\to\sqrt{6}\sqrt{7}} \frac{z+\sqrt{7}+\sqrt{6}-2z}{(z+\sqrt{7}+\sqrt{6})^3} = 8\pi \lim_{z\to\sqrt{6}\sqrt{7}} \frac{-z+\sqrt{7}+\sqrt{6}}{(z+\sqrt{7}+\sqrt{6})^3} =$$

$$8\pi\frac{-\sqrt{6}+\sqrt{7}+\sqrt{7}+\sqrt{6}}{(\sqrt{6}-\sqrt{7}+\sqrt{7}+\sqrt{6})^3} = 8\pi\frac{2\sqrt{7}}{(2\sqrt{6})^3} = 8\pi\frac{2\sqrt{7}}{8\cdot 6\sqrt{6}} = \frac{\pi\sqrt{7}}{3\sqrt{6}}$$

## 7 Вычислить интеграл:

$$\int_{-\infty}^{+\infty} \frac{x^2+5}{x^4+5x+6}\,dx =$$



$x^4 + 5x + 6 = 0|\ t = x^2$

$t^2 + 5t + 6 = 0$

$D = 25 - 24 = 1$

$t_1 = \dfrac{-5-1}{2} = -3$

$t_2 = \dfrac{-5+1}{2} = -2$

$z_1 = \sqrt{3}i \in \{z : \operatorname{Im} z > 0\}$ – и.о.т.о.х.

$z_2 = -\sqrt{3}i \notin \{z : \operatorname{Im} z > 0\}$

$z_3 = \sqrt{2}i \in \{z : \operatorname{Im} z > 0\}$ – и.о.т.о.х.

$z_4 = -\sqrt{2}i \notin \{z : \operatorname{Im} z > 0\}$

$$= 2\pi i \left( \operatorname{Res}_{z=\sqrt{2}i} \frac{z^2+5}{z^4+5z+6} + \operatorname{Res}_{z=\sqrt{3}i} \frac{z^2+5}{z^4+5z+6} \right) =$$

$\lim\limits_{z\to\sqrt{2}i} \dfrac{z^2+5}{z^4+5z+6} = \infty \Rightarrow z = \sqrt{2}i$ – полюс

$$\lim_{z\to\sqrt{2}i} \frac{z^2+5}{z^4+5z+6}(z-\sqrt{2}i)^n =$$

$$\lim_{z\to\sqrt{2}i} \frac{z^2+5}{(z-\sqrt{3}i)(z+\sqrt{3}i)(z-\sqrt{2}i)(z+\sqrt{2}i)}(z-\sqrt{2}i)^n =$$

$$\lim_{z\to\sqrt{2}i} \frac{z^2+5}{(z-\sqrt{3}i)(z+\sqrt{3}i)(z+\sqrt{2}i)}(z-\sqrt{2}i)^{n-1} = \begin{cases} \dfrac{3}{2\sqrt{2}i}, n=1 \\ 0, n>1 \end{cases} \Rightarrow$$

$z = \sqrt{2}i$ – полюс 1-го порядка

$\lim\limits_{z\to\sqrt{3}i} \dfrac{z^2+5}{z^4+5z+6} = \infty \Rightarrow z = \sqrt{3}i$ – полюс

$$\lim_{z\to\sqrt{3}i} \frac{z^2+5}{z^4+5z+6}(z-\sqrt{3}i)^n =$$

$$\lim_{z\to\sqrt{3}i} \frac{z^2+5}{(z-\sqrt{3}i)(z+\sqrt{3}i)(z-\sqrt{2}i)(z+\sqrt{2}i)}(z-\sqrt{3}i)^n =$$

$$\lim_{z\to\sqrt{3}i} \frac{z^2+5}{(z+\sqrt{3}i)(z-\sqrt{2}i)(z+\sqrt{2}i)}(z-\sqrt{3}i)^{n-1} = \begin{cases} -\dfrac{1}{\sqrt{3}i}, n=1 \\ 0, n>1 \end{cases} \Rightarrow$$

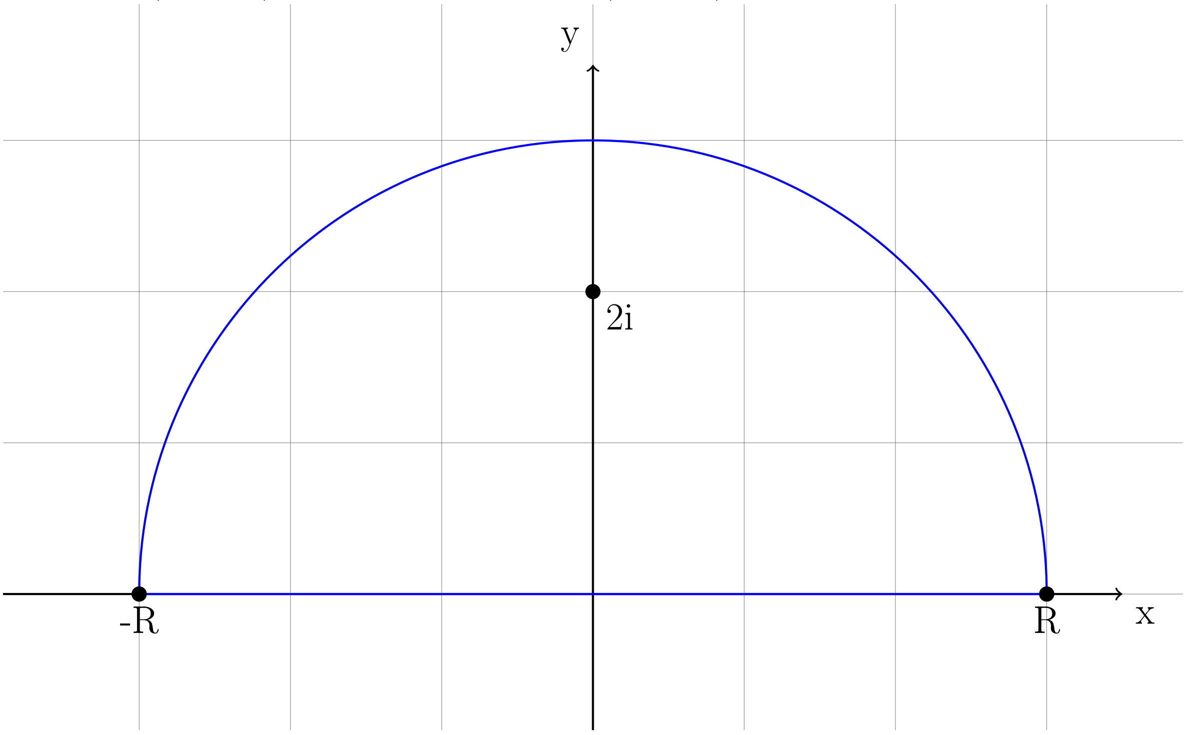
$z = \sqrt{3}i$ – полюс 1-го порядка

$$= 2\pi i \left( \lim_{z\to\sqrt{2}i} \frac{z^2+5}{z^4+5z+6}(z-\sqrt{2}i) \lim_{z\to\sqrt{3}i} \frac{z^2+5}{z^4+5z+6}(z-\sqrt{3}i) \right) =$$

$$\frac{6\pi i}{2\sqrt{2}i} - \frac{2\pi i}{\sqrt{3}i} = \frac{3\pi}{\sqrt{2}} - \frac{2\pi}{\sqrt{3}} = \frac{\pi(3\sqrt{3}-2\sqrt{2})}{\sqrt{6}}$$

# 8 Вычислить интеграл:

$$\int_{-\infty}^{+\infty} \frac{x\sin 2x - \sin x}{(x^2+4)^2}\,dx = \operatorname{Im}\int_{-\infty}^{+\infty} \frac{xe^{2ix} - e^{ix}}{(x^2+4)^2}\,dx \ominus$$




$x^2+4=0$

$x=-2i \notin \{z : \operatorname{Im} z > 0\}$

$x=2i \in \{z : \operatorname{Im} z > 0\}$

– и.о.т.о.х.

$$\ominus \operatorname{Im}\left(2\pi i \operatorname{Res}_{z=2i} \frac{ze^{2iz} - e^{iz}}{(z^2+4)^2}\right) \ominus$$

$$\lim_{z\to 2i} \frac{ze^{2iz} - e^{iz}}{(z^2+4)^2} = \infty \Rightarrow z = 2i \text{ – полюс}$$

$$\lim_{z\to 2i} \frac{ze^{2iz} - e^{iz}}{(z^2+4)^2}(z-2i)^n = \lim_{z\to 2i} \frac{ze^{2iz} - e^{iz}}{(z-2i)^2(z+2i)^2}(z-2i)^n =$$

$$\lim_{z\to 2i} \frac{ze^{2iz} - e^{iz}}{(z+2i)^2}(z-2i)^{n-2} = \begin{cases} -\dfrac{2ie^{-4} - e^{-2}}{-16}, n = 2 \\ 0, n > 2 \\ \infty, n = 1 \end{cases} \Rightarrow z = 2i\text{–полюс 2-го}$$

порядка

$$\ominus \operatorname{Im}\left(2\pi i \lim_{z\to 2i}\left(\frac{ze^{2iz} - e^{iz}}{(z^2+4)^2}(z-2i)^2\right)'\right) = \operatorname{Im}\left(2\pi i \lim_{z\to 2i}\left(\frac{ze^{2iz} - e^{iz}}{(z+2i)^2}\right)'\right) =$$

$$\operatorname{Im}\left(2\pi i \lim_{z\to 2i}\frac{(z2ie^{2iz}+e^{2iz}-ie^{iz})(z+2i)^2-(ze^{2iz}-e^{iz})2(z+2i)}{(z+2i)^4}\right)=$$
$$\operatorname{Im}\left(2\pi i \lim_{z\to 2i}\frac{(z2ie^{2iz}+e^{2iz}-ie^{iz})(z+2i)-2ze^{2iz}+2e^{iz}}{(z+2i)^3}\right)=$$
$$\operatorname{Im}\left(2\pi i\frac{((-4+1)e^{-4}-ie^{-2})4i-4ie^{-4}+2e^{-2}}{-64i}\right)=$$
$$\operatorname{Im}\left(\pi\frac{(-3e^{-4}-ie^{-2})2i-2ie^{-4}+e^{-2}}{-16}\right)=\operatorname{Im}\left(\pi\frac{-6ie^{-4}+2e^{-2}-2ie^{-4}+e^{-2}}{-16}\right)=$$
$$\operatorname{Im}\left(\pi\frac{-8ie^{-4}+3e^{-2}}{-16}\right)=\frac{-8\pi e^{-4}}{-16}=\frac{\pi}{2e^4}$$

# 9 Найти оригинал по заданному изображению:

$$\frac{5p}{(p+2)(p^2-2p+2)}=\frac{A}{p+2}+\frac{Bp+C}{p^2+2p+2}=$$
$$\frac{Ap^2-2Ap+2A+Bp^2+2Bp+Cp+2C}{(p+2)(p^2-2p+2)} =$$

$p^2: 0=A+B$

$p^1: 5=-2A+2B+C$

$p^0: 0=2A+2C$

$A=-1\ B=1\ C=1$

$$= \frac{-1}{p+2}+\frac{p+1}{p^2-2p+2}=\frac{-1}{p+2}+\frac{p+1}{(p-1)^2+1}=$$
$$\frac{-1}{p+2}+\frac{p-1}{(p-1)^2+1}+2\frac{1}{(p-1)^2+1}\fallingdotseq -e^{-2t}+e^t\cos t+2e^t\sin t$$

# 10 Найти решения дифференциального уравнения, удовлетворяющее условиям:

$$y''+y'=\frac{1}{1+e^t}$$
$$y'=z;\quad z=z(t)$$
$$z'+z=\frac{1}{1+e^t}$$
$$z=u(t)v(t)$$
$$u'v+uv'+uv=\frac{1}{1+e^t}$$

$$u'v + u(v' - v) = \frac{1}{1+e^t}$$
$$v' = -v$$
$$\frac{dv}{v} = -dt$$
$$\ln|v| = -t$$
$$v = e^-t$$
$$u'e^t = \frac{1}{1+e^t}$$
$$u' = \frac{1}{e^t(1+e^t)}$$
$$u = \int \frac{dt}{e^t(1+e^t)} = \left| \begin{array}{cc} \zeta = e^t & dt = \dfrac{d\zeta}{\zeta} \\ d\zeta = e^t dt & \end{array} \right| = \int \frac{d\zeta}{\zeta^2(1+\zeta)} \ominus$$
$$\frac{1}{\zeta^2(1+\zeta)} = \frac{A}{\zeta^2} + \frac{B}{\zeta} + \frac{C}{1+\zeta} = \frac{A + A\zeta + B\zeta + B\zeta^2 + C\zeta^2}{\zeta^2(1+\zeta)}$$
$$\zeta^2 : 0 = B + C$$
$$\zeta^1 : 0 = A + B$$
$$\zeta^0 : 1 = A$$
$$A = 1 \ B = -1 \ C = 1$$
$$\ominus \int \left( \frac{1}{\zeta^2} - \frac{1}{\zeta} + \frac{1}{1+\zeta} \right) d\zeta = -\frac{1}{\zeta} - \ln|\zeta| + \ln|1+\zeta| + c_1 =$$
$$-\frac{1}{e^t} + \ln\left|\frac{1+e^t}{e^t}\right| + c_1$$
$$z = -\frac{1}{e^{2t}} + \frac{1}{e^t}\ln\left|\frac{1+e^t}{e^t}\right| + \frac{c_1}{e^t}$$
$$y' = -\frac{1}{e^{2t}} + \frac{1}{e^t}\ln\left|\frac{1+e^t}{e^t}\right| + \frac{c_1}{e^t}$$
$$y = \int \left( -\frac{1}{e^{2t}} + \frac{1}{e^t}\ln\left|\frac{1+e^t}{e^t}\right| + \frac{c_1}{e^t} \right) dt = \frac{2}{e^{2t}} - \frac{c_1}{e^t} + \int \frac{1}{e^t}\ln\left|\frac{1+e^t}{e^t}\right| dt =$$
$$\left| \begin{array}{cc} u = \ln\left|\dfrac{1+e^t}{e^t}\right| & du = \dfrac{e^t}{1+e^t} \cdot \dfrac{e^{2t} - e^t - e^{2t}}{e^{2t}} dt = -\dfrac{dt}{1+e^t} \\ dv = \dfrac{dt}{e^t} & v = -\dfrac{1}{e^t} \end{array} \right| =$$
$$\frac{2}{e^{2t}} - \frac{c_1}{e^t} - \frac{1}{e^t}\ln\left|\frac{1+e^t}{e^t}\right| - \int \frac{dt}{e^t(1+e^t)} = \left| \begin{array}{cc} z = e^t & dt = \dfrac{dz}{z} \\ dz = e^t dt & \end{array} \right| =$$
$$\frac{2}{e^{2t}} - \frac{c_1}{e^t} - \frac{1}{e^t}\ln\left|\frac{1+e^t}{e^t}\right| - \int \frac{dz}{z^2(1+z)} =$$
$$\frac{2}{e^{2t}} - \frac{c_1}{e^t} - \frac{1}{e^t}\ln\left|\frac{1+e^t}{e^t}\right| - \int \left( \frac{1}{z^2} - \frac{1}{z} + \frac{1}{1+z} \right) dz =$$

$$\frac{2}{e^{2t}} - \frac{c_1}{e^t} - \frac{1}{e^t}\ln\left|\frac{1+e^t}{e^t}\right| + \frac{1}{2e^t} + \ln|e^t| - \ln|1+e^t| + c_2 =$$
$$\frac{2}{e^{2t}} + \frac{1-2c_1}{2e^t} - \frac{1}{e^t}\ln\left|\frac{1+e^t}{e^t}\right| + \ln\left|\frac{e^t}{1+e^t}\right| + c_2$$

# 11 Операционным методом решить задачу Коши:

$$\begin{cases} y'' + 4y' + 4y = t^3e^{2t} \\ y(0) = 1 \\ y'(0) = 2 \end{cases}$$

$$p^2\overline{y}(p) - py(0) - y'(0) + 4(p\overline{y}(p) - y(0)) + 4\overline{y}(p) = \frac{6}{(p-2)^4}$$
$$p^2\overline{y}(p) - p - 2 + 4p\overline{y}(p) - 4 + 4\overline{y}(p) = \frac{6}{(p-2)^4}$$
$$\overline{y}(p)(p^2+4p+4) = \frac{6}{(p-2)^4} + p + 2 + 4$$
$$\overline{y}(p)(p+2)^2 = \frac{6}{(p-2)^4} + p + 2 + 4$$
$$\overline{y}(p) = \frac{6}{(p-2)^4(p+2)^2} + \frac{1}{p+2} + \frac{4}{(p+2)^2}$$
$$\frac{6}{(p-2)^4(p+2)^2} \risingdotseq (t^3e^2t * te^{-2t}) = \int_0^t \tau^3e^{2\tau}(t-\tau)e^{-2(t-\tau)}\,d\tau =$$
$$\int_0^t \tau^3(t-\tau)e^{4\tau}e^{-2t}\,d\tau = e^{-2t}\int_0^t \tau^3(t-\tau)e^{4\tau}\,d\tau = e^{-2t}\int_0^t (\tau^3te^{4\tau} - \tau^4e^{4\tau})\,d\tau =$$

$$\int_0^t \tau^4e^{4\tau}\,d\tau = \left|\begin{matrix} u=\tau^4 & du = 4\tau^3d\tau \\ dv = e^{4\tau} & v = \frac{1}{4}e^{4\tau} \end{matrix}\right| = \frac{t^4e^{4t}}{4} - \int_0^t \tau^3e^{4\tau}\,d\tau =$$
$$\left|\begin{matrix} u=\tau^3 & du = 3\tau^2d\tau \\ dv = e^{4\tau} & v = \frac{1}{4}e^{4\tau} \end{matrix}\right| = \frac{t^4e^{4t}}{4} - \frac{t^3e^{4t}}{4} + \frac{3}{4}\int_0^t \tau^2e^{4\tau}\,d\tau = \left|\begin{matrix} u=\tau^2 & du = 2\tau d\tau \\ dv = e^{4\tau} & v = \frac{1}{4}e^{4\tau} \end{matrix}\right| =$$
$$\frac{t^4e^{4t}}{4} - \frac{t^3e^{4t}}{4} + \frac{3t^2e^{4t}}{16} - \frac{3}{8}\int_0^t \tau e^{4\tau}\,d\tau = \left|\begin{matrix} u=\tau & du = d\tau \\ dv = e^{4\tau} & v = \frac{1}{4}e^{4\tau} \end{matrix}\right| =$$
$$\frac{t^4e^{4t}}{4} - \frac{t^3e^{4t}}{4} + \frac{3t^2e^{4t}}{16} - \frac{3te^{4t}}{32} + \frac{3}{32}\int_0^t e^{4\tau}\,d\tau =$$
$$\frac{t^4e^{4t}}{4} - \frac{t^3e^{4t}}{4} + \frac{3t^2e^{4t}}{16} - \frac{3te^{4t}}{32} + \frac{3e^{4t}}{128} - \frac{3}{128}$$

$$\int_0^t \tau^3 e^{4\tau}\,d\tau = \frac{t^3e^{4t}}{4} - \frac{3t^2e^{4t}}{16} + \frac{3te^{4t}}{32} - \frac{3e^{4t}}{128} + \frac{3}{128}$$

$$= te^{-2t}\left(\frac{t^3e^{4t}}{4} - \frac{3t^2e^{4t}}{16} + \frac{3te^{4t}}{32} - \frac{3e^{4t}}{128} + \frac{3}{128}\right)$$

$$- e^{-2t}\left(\frac{t^4e^{4t}}{4} - \frac{t^3e^{4t}}{4} + \frac{3t^2e^{4t}}{16} - \frac{3te^{4t}}{32} + \frac{3e^{4t}}{128} - \frac{3}{128}\right) =$$

$$\frac{t^4e^{2t}}{4} - \frac{3t^3e^{2t}}{16} + \frac{3t^2e^{2t}}{32} - \frac{3te^{2t}}{128} + \frac{3t}{128e^{2t}} - \frac{t^4e^{2t}}{4} + \frac{t^3e^{2t}}{4} - \frac{3t^2e^{2t}}{16} + \frac{3te^{2t}}{32} -$$

$$\frac{3e^{2t}}{128} + \frac{3}{128e^{2t}} =$$

$$\frac{t^3e^{2t}}{16} - \frac{3t^2e^{2t}}{32} + \frac{9te^{2t}}{128} - \frac{3e^{2t}}{128} + \frac{3}{128e^{2t}}(t+1)$$

$$y = \frac{t^3e^{2t}}{16} - \frac{3t^2e^{2t}}{32} + \frac{9te^{2t}}{128} - \frac{3e^{2t}}{128} + \frac{3}{128e^{2t}}(t+1) + e^{-2t} + 4te^{-2t}$$

## 12 Найти решение системы дифференциальных уравнений, удовлетворяющее заданному начальному условию:

$$\begin{cases} x' = 3x + 3y \\ y' = \dfrac{5}{2}x - y + 2 \\ x(0) = 0 \\ y(0) = 1 \end{cases}$$

$$\begin{cases} p\overline{x}(p) = 3\overline{x}(p) + 2\overline{y}(p) \\ p\overline{y}(p) - 1 = \dfrac{5}{2}\overline{x}(p) - \overline{y}(p) + \dfrac{2}{p} \end{cases} \Leftrightarrow \begin{cases} \overline{x}(p)(p-3) = 2\overline{y}(p) \\ \overline{y}(p)(p+1) = \dfrac{5}{2}\overline{x}(p) + \dfrac{2}{p} + 1 \end{cases} \Leftrightarrow$$

$$\begin{cases} \overline{y}(p) = \dfrac{\overline{x}(p)(p-3)}{2} \\ \overline{x}(p)\dfrac{(p-3)(p+1)-5}{2} = \dfrac{2+p}{p} \end{cases} \Leftrightarrow \begin{cases} \overline{y}(p) = \dfrac{\overline{x}(p)(p-3)}{2} \\ \overline{x}(p)(p^2+p-3p-3-5) = \dfrac{4+2p}{p} \end{cases} \Leftrightarrow$$

$$\begin{cases} \overline{y}(p) = \dfrac{(2+p)(p-3)}{p(p^2-2p-8)} \\ \overline{x}(p) = \dfrac{2(2+p)}{p(p^2-2p-8)} \end{cases} \Leftrightarrow$$

$$p^2 - 2p - 8 = 0$$

$$D = 4 + 32 = 36$$

$$p_1 = \frac{2-6}{2} = -2$$

$$p_2 = \frac{2+6}{2} = 4$$

$$\Leftrightarrow \begin{cases} \overline{x}(p) = \dfrac{2(2+p)}{p(p+2)(p-4)} \\ \overline{y}(p) = \dfrac{(2+p)(p-3)}{p(p+2)(p-4)} \end{cases} \Leftrightarrow \begin{cases} \overline{x}(p) = \dfrac{2}{p(p-4)} \\ \overline{y}(p) = \dfrac{p-3}{p(p-4)} \end{cases} \Leftrightarrow$$

$$\frac{2}{p(p-4)} = \frac{A}{p} + \frac{B}{p-4} = \frac{Ap-4A+Bp}{p(p-4)}$$

$$p^1 : 0 = A + B$$

$$p^0 : 2 = -4A$$

$$A = -\frac{1}{2} \;\; B = \frac{1}{2}$$

$$\frac{p-3}{p(p-4)} = \frac{A}{p} + \frac{B}{p-4} = \frac{Ap-4A+Bp}{p(p-4)}$$

$$p^1 : 1 = A + B$$

$$p^0 : -3 = -4A$$

$$A = \frac{3}{4} \;\; B = \frac{1}{4}$$

$$\Leftrightarrow \begin{cases} \overline{x}(p) = -\dfrac{1}{2} \cdot \dfrac{1}{p} + \dfrac{1}{2} \cdot \dfrac{1}{p-4} \\ \overline{y}(p) = \dfrac{3}{4} \cdot \dfrac{1}{p} + \dfrac{1}{4} \cdot \dfrac{1}{p-4} \end{cases}$$

$$\begin{cases} x(t) = -\dfrac{1}{2} + \dfrac{1}{2} e^{4t} \\ y(t) = \dfrac{3}{4} + \dfrac{1}{4} e^{4t} \end{cases}$$