Homework 7

Hengtao Guo

April 12, 2019

https://github.com/Tonight1121/Biology-Image-Analysis

1 Task 1

The landmarks coordinates in text files plotting onto the fixed image and moving image can be seen in Fig. 1. The orginial difference image can be seen in Fig. 2.

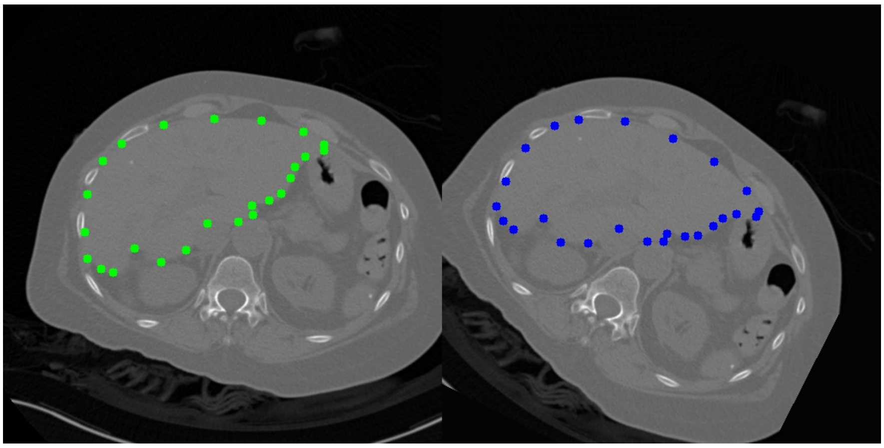

Figure 1: Green landmarks on left fixed image and blue landmarks on right moving image.

Figure 2: Difference image before any registration.

2 Task 2: SVD Registration

2.1 SVD

The original landmarks on the fixed image can be seen in Fig. 3. The SVD registration has the following procedure.

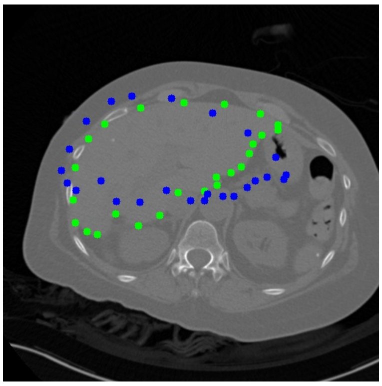

Figure 3: Original landmarks.

(1) Calculate the center points of fixed-image landmarks and moving-image landmarks.

(2) Recenter the two landmarks sets by subtracting the center points individually. Thus the center of two landmarks sets will both be (0,0), as shown in Fig. 4. Then we can apply singular value decomposition.

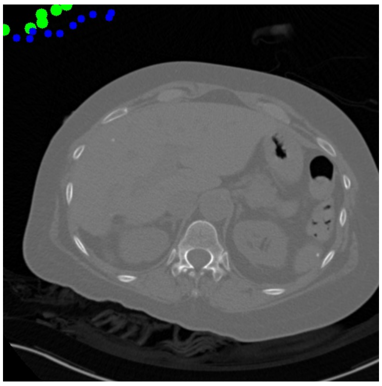

Figure 4: Move the center to the image origin (0,0).

(3) **Singular value decomposition.** Compute the correlation matrix $M = points\_fixed * transpose(points\_moving)$. Apply SVD to get u, s and vh using the numpy function np.linalg.svd. The rotation matrix $R = transpose(vh) * transpose(u)$. Finally we get the parameters of R in Fig. 5, where $cos\alpha = 0.891$ Applying the rotation matrix R to the moving image landmarks, we can basically match the same postion of landmarks in the fixed image. The superimposed points can be seen in 6.

```
R
[[ 0.89100652  0.4539905 ]
 [-0.4539905   0.89100652]]
```

Figure 5: Rotation matrix R from SVD.

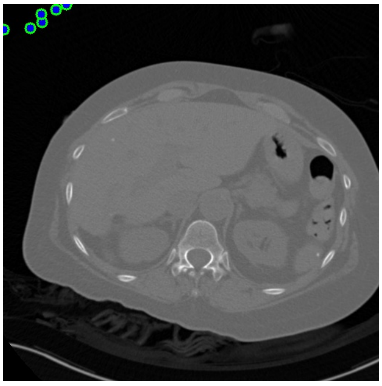

Figure 6: Matching landmarks around (0, 0.)

(4) Move back the registered points by adding the original center of the fixed image landmarks. Thus all landmarks are correctely located on the image which can be seen in Fig. **??**.

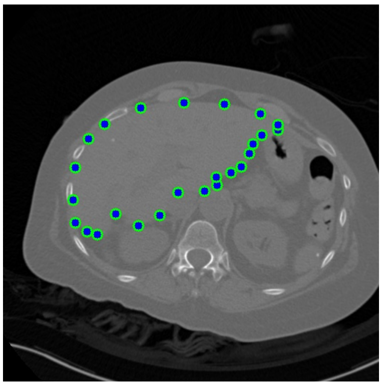

Figure 7: SVD registered landmarks.

2.2 Conclusion

The rotation degree is **27 degrees** calculated from the value of $cos\alpha = 0.891$. The translation in millimeters equals to **22.38mm in x** direction and **-13.78mm in y** direction, calculated from the difference of original centers of fixed landmarks and moving landmarks and the $elementspacing = 0.7128$ in .mhd header.

The singular value decomposition should be performed after we centering the landmarks to (0,0), otherwise it would cause error. This is a noticeable trick.

3 Task 3: Metric Registration

3.1 Comparison

Using the sitk library, I choose the Correlation, MeanSquares and JointHistogramMutualInformation as the comparison metric to perform registration. I only used the GradientDescent optimizer. The optimizer is set with learning rate 0.1 and iterations of 1000.

The registration evaluation can be seen in Fig. 8. Registration using the mean squared error as the metric performs the worst. The advantage of mutual information should be revealed when applying to images from different modalities, but it can still generate satisfactory result in our case. Moreover, the correlation metric is clearly the best among three, with the lowest landmarks $RMSE = 2.464 \times 10^{-12}$, which means significantly small error.

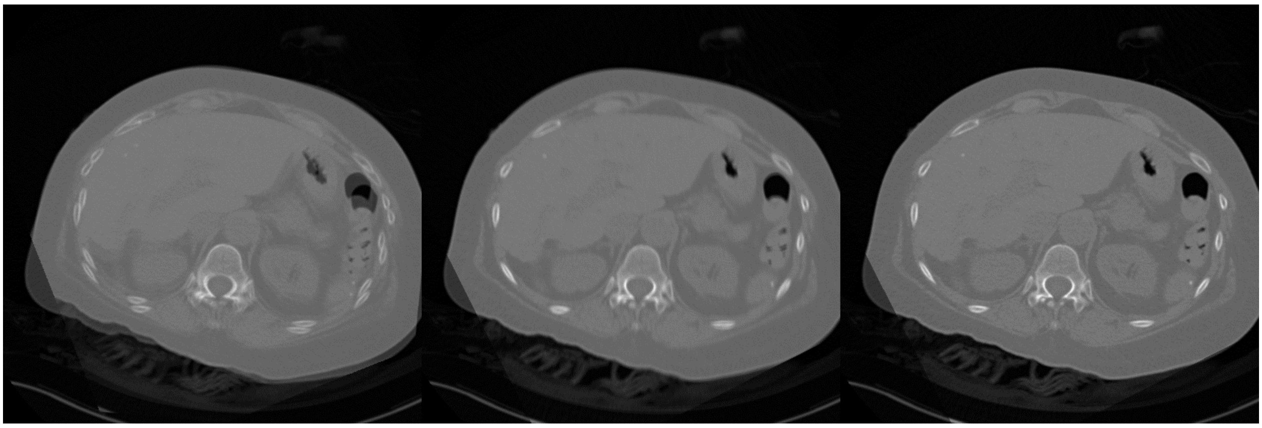

Figure 8: GD optimizer using different metric. **From left to right: mean squared error, joint histogram mutual information and correlation.** The transformed moving image with 50% transpareting superimposed onto the fixed image to see the registration quality.

The interpolations are compared between Gaussian, nearest neighbour and linear interpolation, which can be seen in Fig. 9. According to my observation, the Gaussian method is the most blurry, while nearest neighbour method and linear interpolation method appear no significant difference.

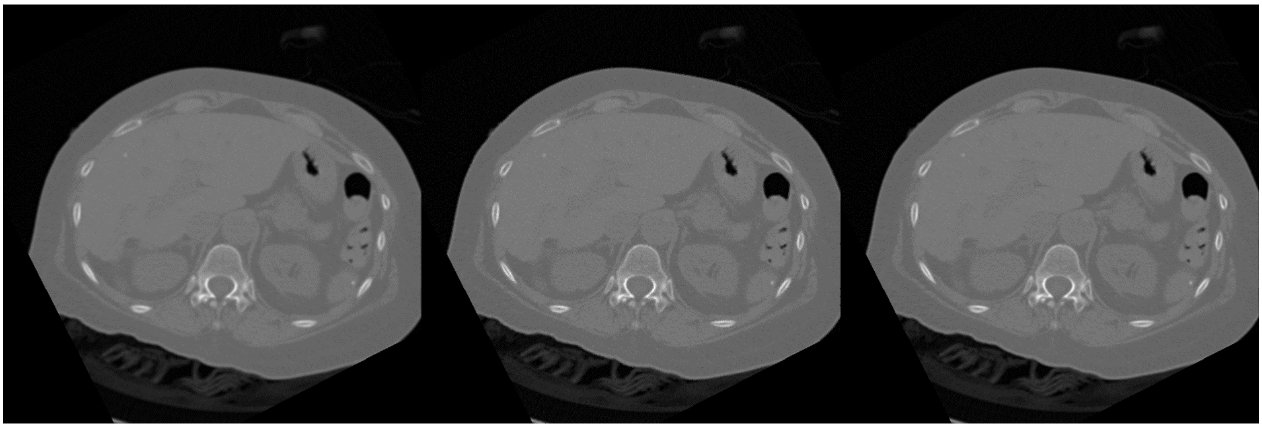

Figure 9: Different interpolation methods. **From left to right: Gaussian, nearest neighbour and linear interpolation.**

3.2 Conclusion

The difference image after registration can be seen in Fig. 10. The uniform pattern of the difference image is an indicator that the fixed image and moving image are aligned very well, with the lowest **landmarks** $RMSE = 2.464 \times 10^{-12}$.

The rotation degree is **27 degrees** counter-clock-wise. The translation in millimeters equals to **22.38mm in x** direction and **-13.78mm in y** direction.

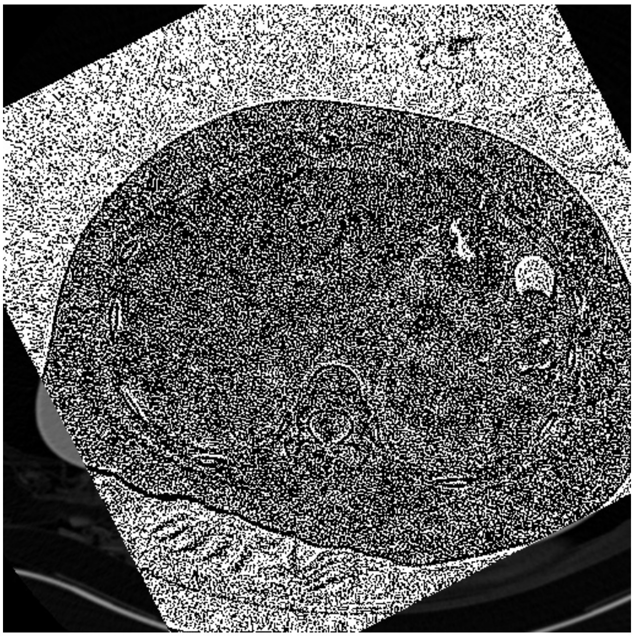

Figure 10: Difference image after registration, using correlation metric, gradient descent optimizer and linear interpolation.