

Übung 3.2

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Hilber-1

$$\begin{array}{l} \vdash \varphi \rightarrow \psi \rightarrow \varphi \\ \varphi \vdash \psi \rightarrow \varphi \\ \psi, \varphi \vdash \varphi \end{array}$$

Hilbert-2

$$\begin{array}{l} \vdash (\varphi \rightarrow \psi) \rightarrow \neg\psi \rightarrow \neg\varphi \\ \varphi \rightarrow \psi \vdash \neg\psi \rightarrow \neg\varphi \\ \neg\psi, \varphi \rightarrow \psi \vdash \neg\varphi \\ \neg\psi \vdash \varphi, \neg\varphi \\ \varphi, \neg\psi \vdash \varphi \\ \\ \neg\psi, \psi \vdash \neg\varphi \\ \varphi, \neg\psi, \psi \vdash \\ \varphi, \psi \vdash \psi \end{array}$$

Hilbert-3

$$\begin{array}{l} \vdash ((\varphi \rightarrow \psi) \rightarrow \varphi \rightarrow \chi) \rightarrow \varphi \rightarrow \psi \rightarrow \chi \\ (\varphi \rightarrow \psi) \rightarrow \varphi \rightarrow \chi \vdash \varphi \rightarrow \psi \rightarrow \chi \\ \varphi, (\varphi \rightarrow \psi) \rightarrow \varphi \rightarrow \chi \vdash \psi \rightarrow \chi \\ \psi, \varphi, (\varphi \rightarrow \psi) \rightarrow \varphi \rightarrow \chi \vdash \chi \\ \psi, \varphi \vdash \varphi \rightarrow \psi, \chi \\ \varphi, \psi, \varphi \vdash \psi, \chi \\ \\ \psi, \varphi, \varphi \rightarrow \chi \vdash \chi \\ \psi, \varphi \vdash \varphi, \chi \\ \psi, \varphi, \chi \vdash \chi \end{array}$$

01-contradiction

$$\begin{array}{l} \varphi \wedge \neg\varphi \vdash \\ \varphi, \neg\varphi \vdash \\ \varphi \vdash \varphi \end{array}$$

02-de-morgan-or

$\vdash \neg(\varphi \vee \psi) \leftrightarrow \neg\varphi \wedge \neg\psi$
 $\neg(\varphi \vee \psi) \vdash \neg\varphi \wedge \neg\psi$
 $\neg(\varphi \vee \psi) \vdash \neg\varphi$
 $\varphi, \neg(\varphi \vee \psi) \vdash$
 $\varphi \vdash \varphi \vee \psi$
 $\varphi \vdash \varphi, \psi$

$\neg(\varphi \vee \psi) \vdash \neg\psi$
 $\psi, \neg(\varphi \vee \psi) \vdash$
 $\psi \vdash \varphi \vee \psi$
 $\psi \vdash \varphi, \psi$

$\neg\varphi \wedge \neg\psi \vdash \neg(\varphi \vee \psi)$
 $\varphi \vee \psi, \neg\varphi \wedge \neg\psi \vdash$
 $\varphi, \neg\varphi \wedge \neg\psi \vdash$
 $\varphi, \neg\varphi, \neg\psi \vdash$
 $\varphi, \neg\psi \vdash \varphi$

$\psi, \neg\varphi \wedge \neg\psi \vdash$
 $\psi, \neg\varphi, \neg\psi \vdash$
 $\psi, \neg\varphi \vdash \psi$

03-composition

$\varphi \rightarrow \psi, \varphi \rightarrow \chi \vdash \varphi \rightarrow \psi \wedge \chi$
 $\varphi, \varphi \rightarrow \psi, \varphi \rightarrow \chi \vdash \psi \wedge \chi$
 $\varphi, \varphi \rightarrow \psi, \varphi \rightarrow \chi \vdash \psi$
 $\varphi, \varphi \rightarrow \chi \vdash \varphi, \psi$

$\varphi, \psi, \varphi \rightarrow \chi \vdash \psi$

$\varphi, \varphi \rightarrow \psi, \varphi \rightarrow \chi \vdash \chi$
 $\varphi, \varphi \rightarrow \chi \vdash \varphi, \chi$

$\varphi, \psi, \varphi \rightarrow \chi \vdash \chi$
 $\varphi, \psi \vdash \varphi, \chi$
 $\varphi, \psi, \chi \vdash \chi$

04-distributivity-and

$\vdash \varphi \wedge (\psi \vee \chi) \leftrightarrow \varphi \wedge \psi \vee \varphi \wedge \chi$
 $\varphi \wedge (\psi \vee \chi) \vdash \varphi \wedge \psi \vee \varphi \wedge \chi$
 $\varphi \wedge (\psi \vee \chi) \vdash \varphi \wedge \psi, \varphi \wedge \chi$
 $\varphi \wedge (\psi \vee \chi) \vdash \varphi, \varphi \wedge \chi$
 $\varphi \wedge (\psi \vee \chi) \vdash \varphi, \varphi$

$$\varphi, \psi \vee \chi \vdash \varphi, \varphi$$

$$\varphi \wedge (\psi \vee \chi) \vdash \varphi, \chi$$

$$\varphi, \psi \vee \chi \vdash \varphi, \chi$$

$$\varphi \wedge (\psi \vee \chi) \vdash \psi, \varphi \wedge \chi$$

$$\varphi \wedge (\psi \vee \chi) \vdash \psi, \varphi$$

$$\varphi, \psi \vee \chi \vdash \psi, \varphi$$

$$\varphi \wedge (\psi \vee \chi) \vdash \psi, \chi$$

$$\varphi, \psi \vee \chi \vdash \psi, \chi$$

$$\varphi, \psi \vdash \psi, \chi$$

$$\varphi, \chi \vdash \psi, \chi$$

$$\varphi \wedge \psi \vee \varphi \wedge \chi \vdash \varphi \wedge (\psi \vee \chi)$$

$$\varphi \wedge \psi \vee \varphi \wedge \chi \vdash \varphi$$

$$\varphi \wedge \psi \vdash \varphi$$

$$\varphi, \psi \vdash \varphi$$

$$\varphi \wedge \chi \vdash \varphi$$

$$\varphi, \chi \vdash \varphi$$

$$\varphi \wedge \psi \vee \varphi \wedge \chi \vdash \psi \vee \chi$$

$$\varphi \wedge \psi \vee \varphi \wedge \chi \vdash \psi, \chi$$

$$\varphi \wedge \psi \vdash \psi, \chi$$

$$\varphi, \psi \vdash \psi, \chi$$

$$\varphi \wedge \chi \vdash \psi, \chi$$

$$\varphi, \chi \vdash \psi, \chi$$