Übung 3.2

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Hilber-1

$\vdash \varphi \to \psi \to \varphi$
$\quad \varphi \vdash \psi \to \varphi$
$\quad \psi, \varphi \vdash \varphi$

Hilbert-2

$\vdash (\varphi \to \psi) \to \neg\psi \to \neg\varphi$
$\quad \varphi \to \psi \vdash \neg\psi \to \neg\varphi$
$\quad \neg\psi, \varphi \to \psi \vdash \neg\varphi$
$\quad \neg\psi \vdash \varphi, \neg\varphi$
$\quad \varphi, \neg\psi \vdash \varphi$

$\quad \neg\psi, \psi \vdash \neg\varphi$
$\quad \varphi, \neg\psi, \psi \vdash$
$\quad \varphi, \psi \vdash \psi$

Hilbert-3

$\vdash ((\varphi \to \psi) \to \varphi \to \chi) \to \varphi \to \psi \to \chi$
$\quad (\varphi \to \psi) \to \varphi \to \chi \vdash \varphi \to \psi \to \chi$
$\quad \varphi, (\varphi \to \psi) \to \varphi \to \chi \vdash \psi \to \chi$
$\quad \psi, \varphi, (\varphi \to \psi) \to \varphi \to \chi \vdash \chi$
$\quad \psi, \varphi \vdash \varphi \to \psi, \chi$
$\quad \varphi, \psi, \varphi \vdash \psi, \chi$

$\quad \psi, \varphi, \varphi \to \chi \vdash \chi$
$\quad \psi, \varphi \vdash \varphi, \chi$
$\quad \psi, \varphi, \chi \vdash \chi$

01-contradiction

$\varphi \wedge \neg\varphi \vdash$
$\quad \varphi, \neg\varphi \vdash$
$\quad \varphi \vdash \varphi$

02-de-morgan-or

$\vdash \neg(\varphi \lor \psi) \leftrightarrow \neg\varphi \land \neg\psi$
    $\neg(\varphi \lor \psi) \vdash \neg\varphi \land \neg\psi$
    $\neg(\varphi \lor \psi) \vdash \neg\varphi$
    $\varphi, \neg(\varphi \lor \psi) \vdash$
    $\varphi \vdash \varphi \lor \psi$
    $\varphi \vdash \varphi, \psi$

    $\neg(\varphi \lor \psi) \vdash \neg\psi$
    $\psi, \neg(\varphi \lor \psi) \vdash$
    $\psi \vdash \varphi \lor \psi$
    $\psi \vdash \varphi, \psi$

    $\neg\varphi \land \neg\psi \vdash \neg(\varphi \lor \psi)$
    $\varphi \lor \psi, \neg\varphi \land \neg\psi \vdash$
    $\varphi, \neg\varphi \land \neg\psi \vdash$
    $\varphi, \neg\varphi, \neg\psi \vdash$
    $\varphi, \neg\psi \vdash \varphi$

    $\psi, \neg\varphi \land \neg\psi \vdash$
    $\psi, \neg\varphi, \neg\psi \vdash$
    $\psi, \neg\varphi \vdash \psi$

03-composition

$\varphi \to \psi, \varphi \to \chi \vdash \varphi \to \psi \land \chi$
    $\varphi, \varphi \to \psi, \varphi \to \chi \vdash \psi \land \chi$
    $\varphi, \varphi \to \psi, \varphi \to \chi \vdash \psi$
    $\varphi, \varphi \to \chi \vdash \varphi, \psi$

    $\varphi, \psi, \varphi \to \chi \vdash \psi$

    $\varphi, \varphi \to \psi, \varphi \to \chi \vdash \chi$
    $\varphi, \varphi \to \chi \vdash \varphi, \chi$

    $\varphi, \psi, \varphi \to \chi \vdash \chi$
    $\varphi, \psi \vdash \varphi, \chi$
    $\varphi, \psi, \chi \vdash \chi$

04-distributivity-and

$\vdash \varphi \land (\psi \lor \chi) \leftrightarrow \varphi \land \psi \lor \varphi \land \chi$
    $\varphi \land (\psi \lor \chi) \vdash \varphi \land \psi \lor \varphi \land \chi$
    $\varphi \land (\psi \lor \chi) \vdash \varphi \land \psi, \varphi \land \chi$
    $\varphi \land (\psi \lor \chi) \vdash \varphi, \varphi \land \chi$
    $\varphi \land (\psi \lor \chi) \vdash \varphi, \varphi$

$$\varphi, \psi \lor \chi \vdash \varphi, \varphi$$

$$\varphi \land (\psi \lor \chi) \vdash \varphi, \chi$$

$$\varphi, \psi \lor \chi \vdash \varphi, \chi$$
$$\varphi \land (\psi \lor \chi) \vdash \psi, \varphi \land \chi$$

$$\varphi \land (\psi \lor \chi) \vdash \psi, \varphi$$
$$\varphi, \psi \lor \chi \vdash \psi, \varphi$$

$$\varphi \land (\psi \lor \chi) \vdash \psi, \chi$$

$$\varphi, \psi \lor \chi \vdash \psi, \chi$$
$$\varphi, \psi \vdash \psi, \chi$$

$$\varphi, \chi \vdash \psi, \chi$$

$$\varphi \land \psi \lor \varphi \land \chi \vdash \varphi \land (\psi \lor \chi)$$
$$\varphi \land \psi \lor \varphi \land \chi \vdash \varphi$$
$$\varphi \land \psi \vdash \varphi$$
$$\varphi, \psi \vdash \varphi$$

$$\varphi \land \chi \vdash \varphi$$
$$\varphi, \chi \vdash \varphi$$

$$\varphi \land \psi \lor \varphi \land \chi \vdash \psi \lor \chi$$
$$\varphi \land \psi \lor \varphi \land \chi \vdash \psi, \chi$$
$$\varphi \land \psi \vdash \psi, \chi$$
$$\varphi, \psi \vdash \psi, \chi$$

$$\varphi \land \chi \vdash \psi, \chi$$
$$\varphi, \chi \vdash \psi, \chi$$