



Bauman Moscow State University
Th. Computer Science Dept.

Pushdown Machines: Visible and Not



Antonina Nepeivoda
a_nevod@mail.ru

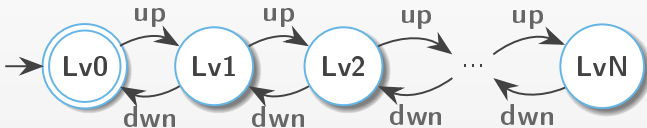
Lecture Outline



Finite Automata is Enough?

Real-world machines are finite. Do finite models suffice?

Recall “elevator automaton” with a unique final state on the “ground floor”, breaking if asked to reach an non-existing floor:



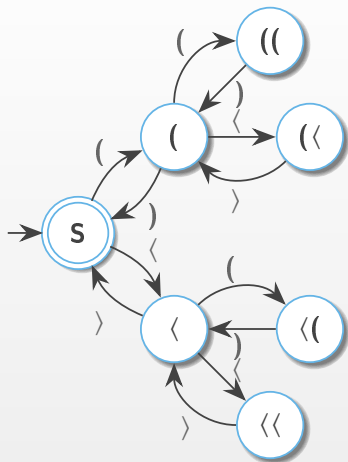
“up” and “down” instructions can be interpreted wrt a parentheses structure. That is, parsing string “((” we move to **Lv2**, and “(()))” returns us to **Lv1**.

Real-world nesting depth is limited (even in Lisp-like languages), and linear blow-up in state size seems satisfactory.

Until we decide to use several sorts of brackets...



Myhill–Nerode Congruence for Many-Sorted Brackets



Congruence Table

	ε)	>))))
ε	+	-	-	-	-	-	-
(-	+	-	-	-	-	-
<	-	-	+	-	-	-	-
((-	-	-	+	-	-	-
(<	-	-	-	-	+	-	-
(<(<	-	-	-	-	-	+	-
(<(<(<	-	-	-	-	-	-	+

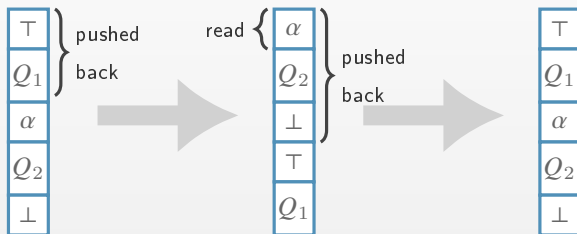
- N -depth balanced sequences of 2 sorts of brackets $\Rightarrow 2^{N+1} - 1$ states in a min NFA.
- N -depth balanced sequences of K sorts of brackets $\Rightarrow \frac{K^{N+1}-1}{K-1}$ states in a min NFA.

Finite automata cannot track nested structures efficiently.



Memoising Counters via Additional Memory

- Queue as a memory — can be considered as an additional tape with the write access, since it can be “re-rolled” to any wanted position with no memory loss.



- Stack as a memory — information given in Q_1 cannot be stored except in states when α is read. More restrictive, natural for tracking nested structures.



Natural Idea: Call–Return Counters

Input alphabet Σ is split into disjoint union $\Sigma_I \cup \Sigma_C \cup \Sigma_R$, where:
 Σ_I is internal alphabet (symbols not affecting the stack),
 Σ_C is call alphabet (symbols that push on the stack),
 Σ_R is return alphabet (symbols that pop from stack).

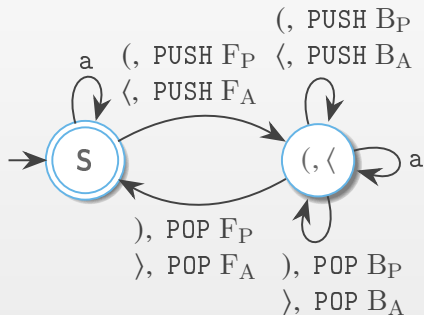
Balanced parentheses language:

- Σ_I — all non-brackets;
- Σ_C — all opening brackets;
- Σ_R — all closing brackets.

With several sorts of brackets:

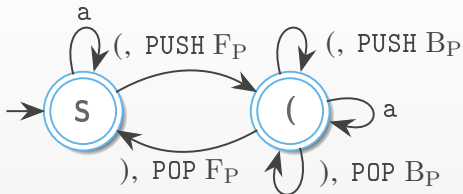
- state space does not increase;
- transitions set grows linearly.

For simplicity, henceforth we usually use an unique sort of brackets.



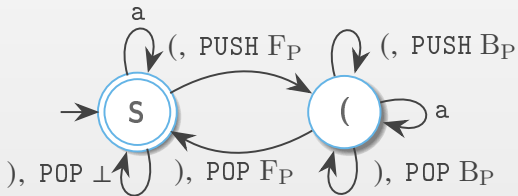
Handling Imbalance

Language of valid prefixes of balanced parentheses?



The final configuration can admit non-empty stack.

Language of valid suffixes of balanced parentheses?



Return symbols can be safely popped from the empty stack (\perp).

