Bauman Moscow State University
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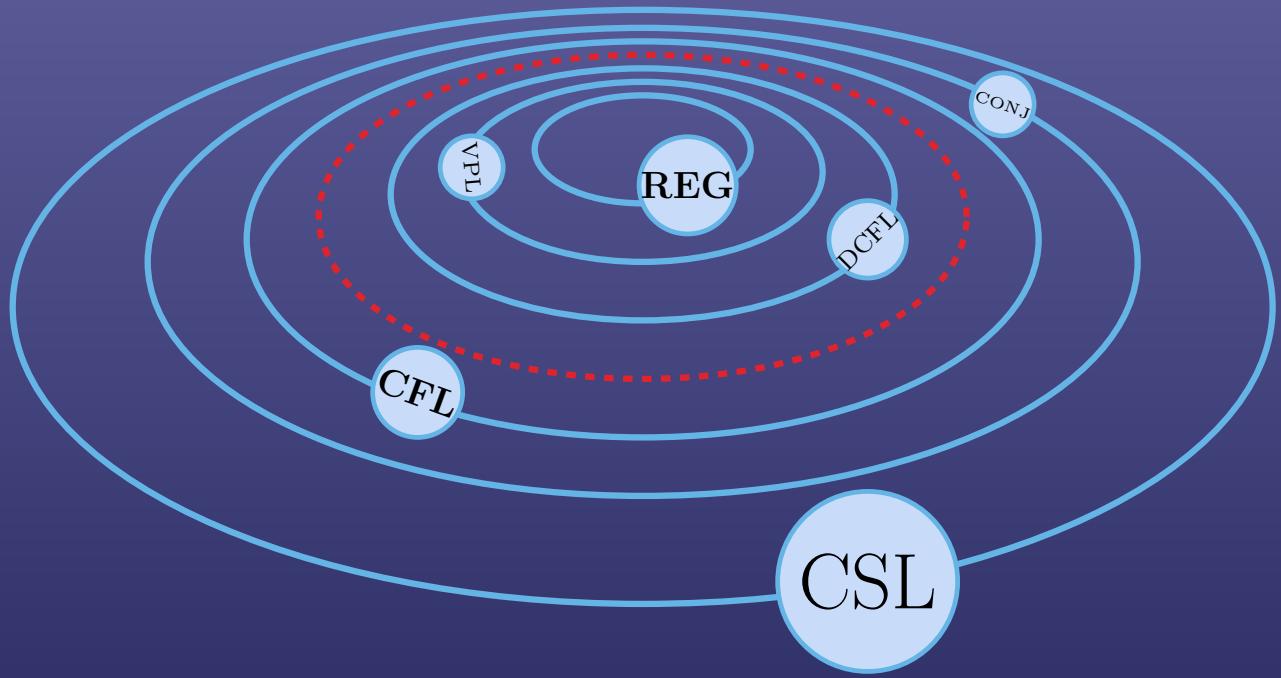
Pushdown Machines: Visible and Not



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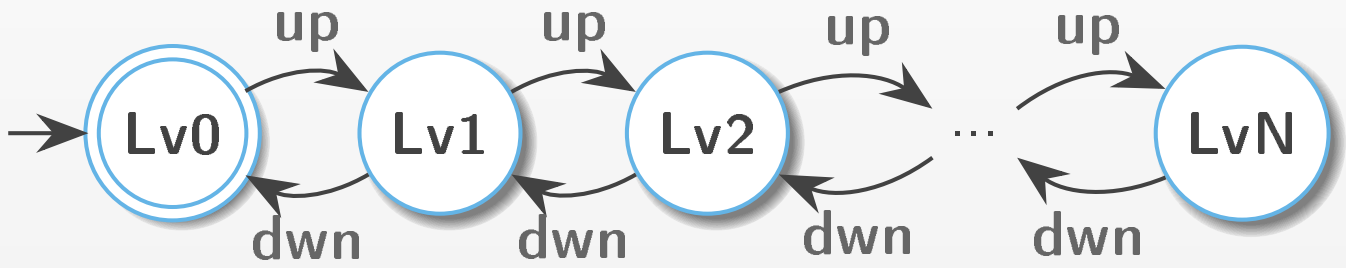
Lecture Outline

Finite Automata is Enough?

Real-world machines are finite. Do finite models suffice?

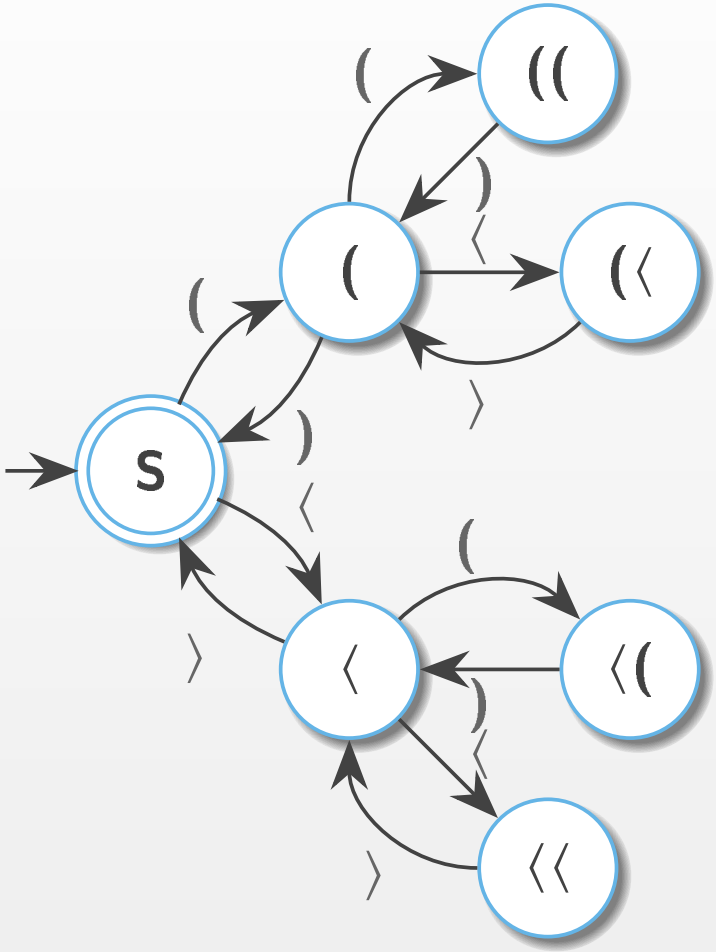
Recall "elevator automaton" with a unique final state on the "ground floor", breaking if asked to reach an non-existing floor:



"**up**" and "**dwn**" instructions can be interpreted wrt a parentheses structure. That is, parsing string "((" we move to **Lv2**, and "(()())" returns us to **Lv1**.
Real-world nesting depth is limited (even in Lisp-like languages), and linear blow-up in state size seems satisfactory.

Until we decide to use several sorts of brackets...

Myhill–Nerode Congruence for Many-Sorted Brackets



Congruence Table

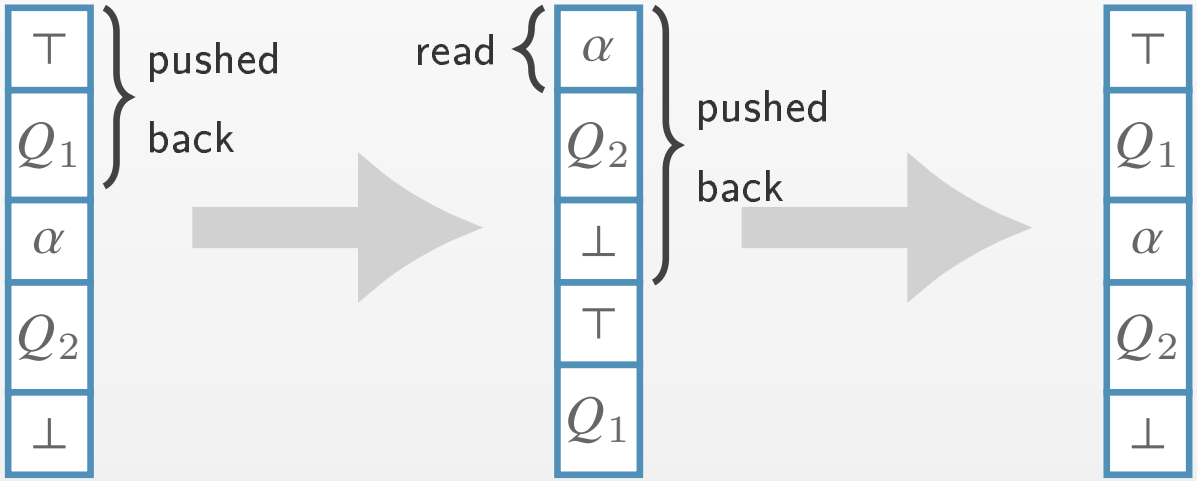
	ε	)	$\rangle$	))	$\rangle)$	$)\rangle$	$\rangle\rangle$
ε	+	−	−	−	−	−	−
(	−	+	−	−	−	−	−
$\langle$	−	−	+	−	−	−	−
((	−	−	−	+	−	−	−
(⟨	−	−	−	−	+	−	−
⟨(	−	−	−	−	−	+	−
⟨⟨	−	−	−	−	−	−	+

- N-depth balanced sequences of 2 sorts of brackets $\Rightarrow$ $2^{N+1} - 1$ states in a min NFA.
- N-depth balanced sequences of K sorts of brackets $\Rightarrow$ $\frac{K^{N+1}-1}{K-1}$ states in a min NFA.

Finite automata cannot track nested structures efficiently.

Memoising Counters via Additional Memory

- Queue as a memory — can be considered as an additional tape with the write access, since it can be "re-rolled" to any wanted position with no memory loss.



- Stack as a memory — information given in Q_1 cannot be stored except in states when α is read. More restrictive, natural for tracking nested structures.

Natural Idea: Call–Return Counters

Input alphabet Σ is split into disjoint union $\Sigma_I \cup \Sigma_C \cup \Sigma_R$, where:

Σ_I is internal alphabet (symbols not affecting the stack),

Σ_C is call alphabet (symbols that push on the stack),

Σ_R is return alphabet (symbols that pop from stack).

- We say a word is balanced if every its prefix contains not less symbols from Σ_C than from Σ_R, and the total counts of both coincide.
- Accepting balanced words $\Rightarrow$ accepting valid parentheses languages.
- Accepting words with the symbols left on the stack $\Rightarrow$ accepting valid prefixes of balanced words.
- Still allowing "return" symbols when the stack is empty $\Rightarrow$ accepting valid suffixes of balanced words.