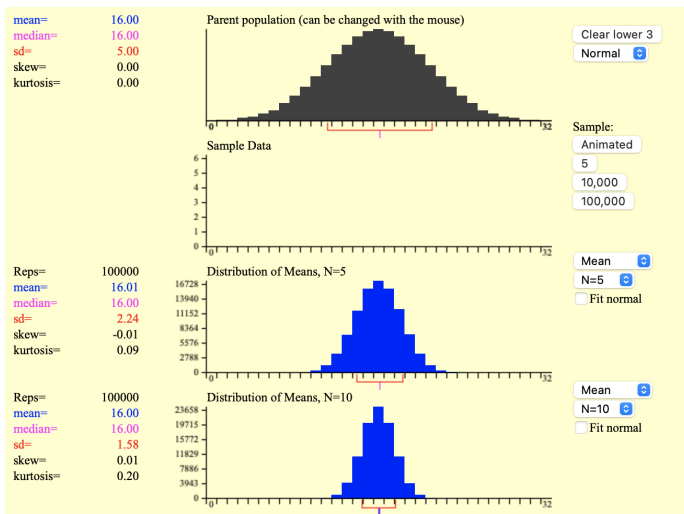


# Lab 3: Central Limit Theorem

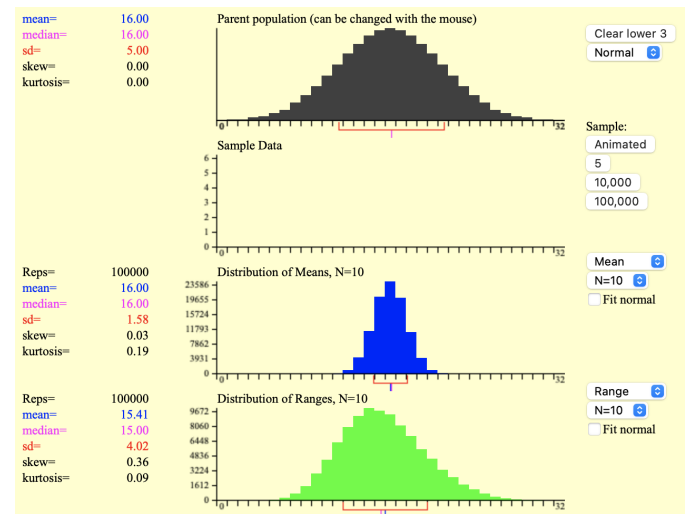
Siraphop Mukdaphetcharat 64011614

## Standard Error

1. The standard error is the standard deviation of the sampling distribution. Approximate the sampling distribution of the mean for  $N=5$ . The standard deviation of the distribution is the standard error of the mean. Find the standard error of the mean and the standard error of the range for  $N=10$  using the normal distribution.



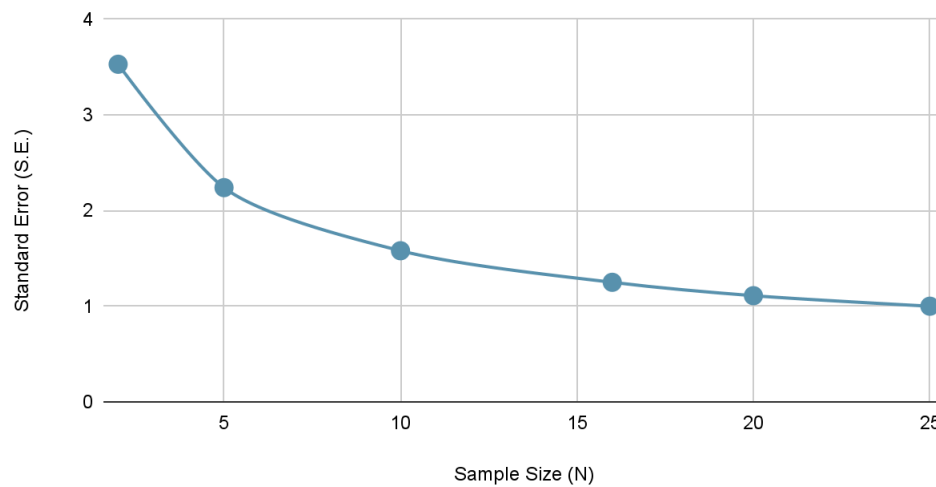
*Difference between  $N = 5$  and  $N = 10$*



*Difference between distro of range*

2. Determine how the standard error is affected by sample size. Plot the standard error of the mean as a function of sample size for different standard deviations? Can you discover a formula relating the standard error of the mean to the sample size and the standard deviation? If so, see if it holds for distributions other than the normal distribution.

Standard Error and Sample Size



Formula for standard error:

$$S.E. = \frac{S}{\sqrt{N}}$$

Where:

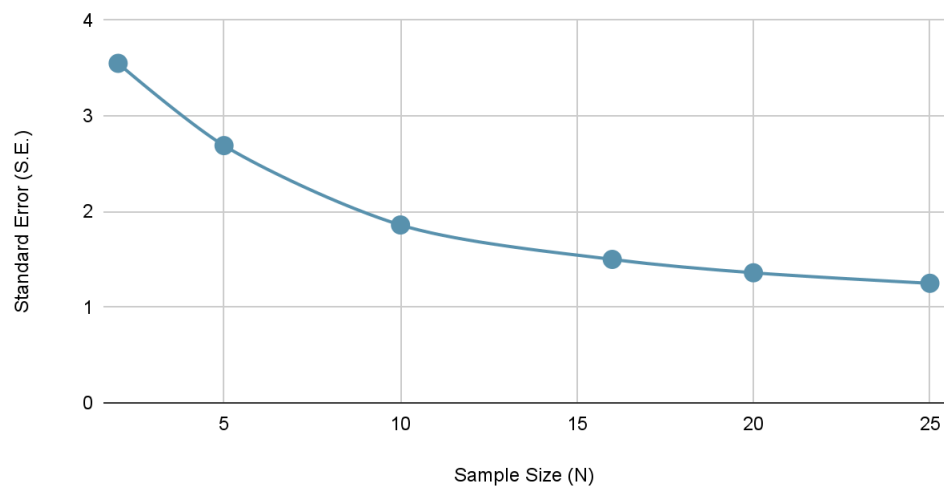
S.E. is the standard error

S is the sample standard deviation

N is the sample size

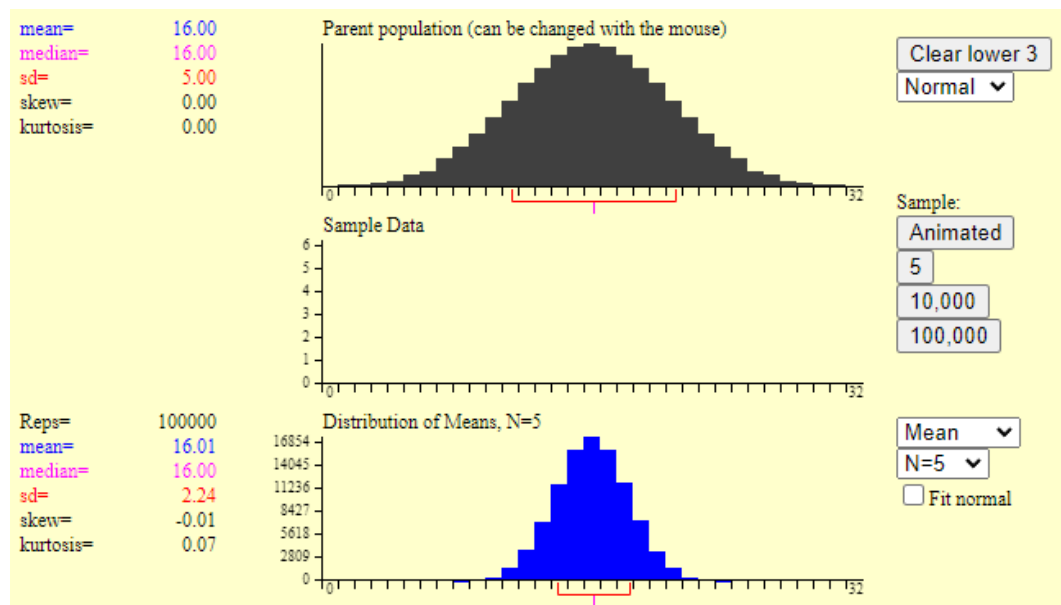
- Redo #2 above for the median.

Standard Error and Sample Size

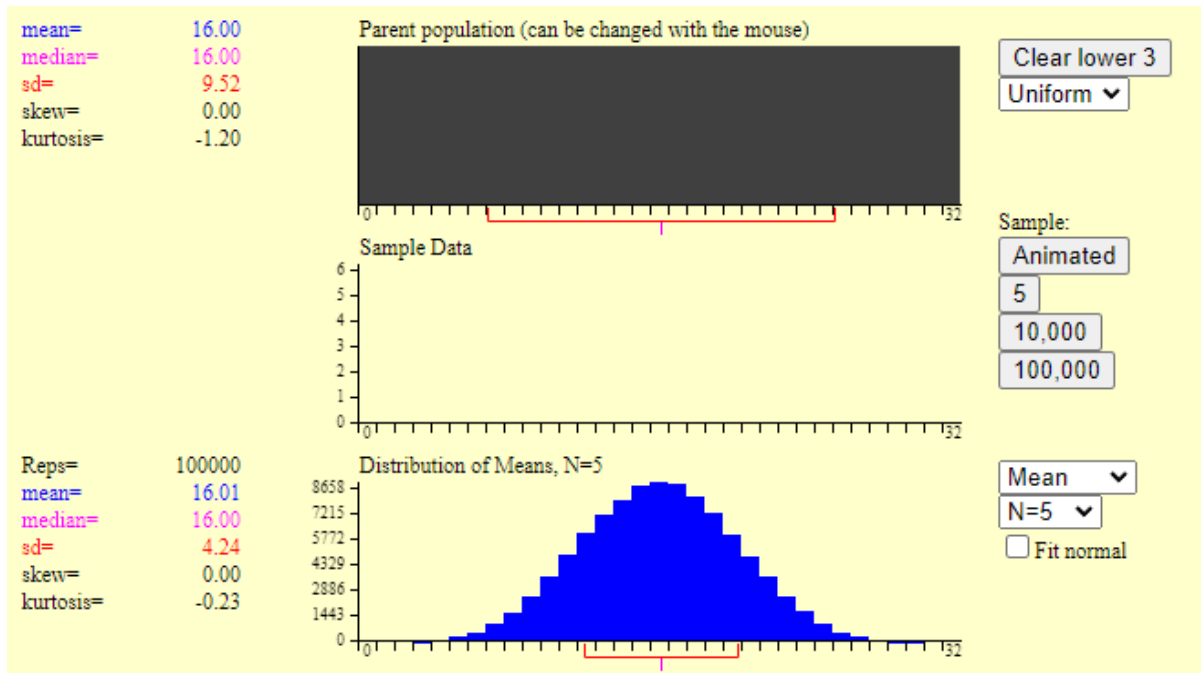


## Bias

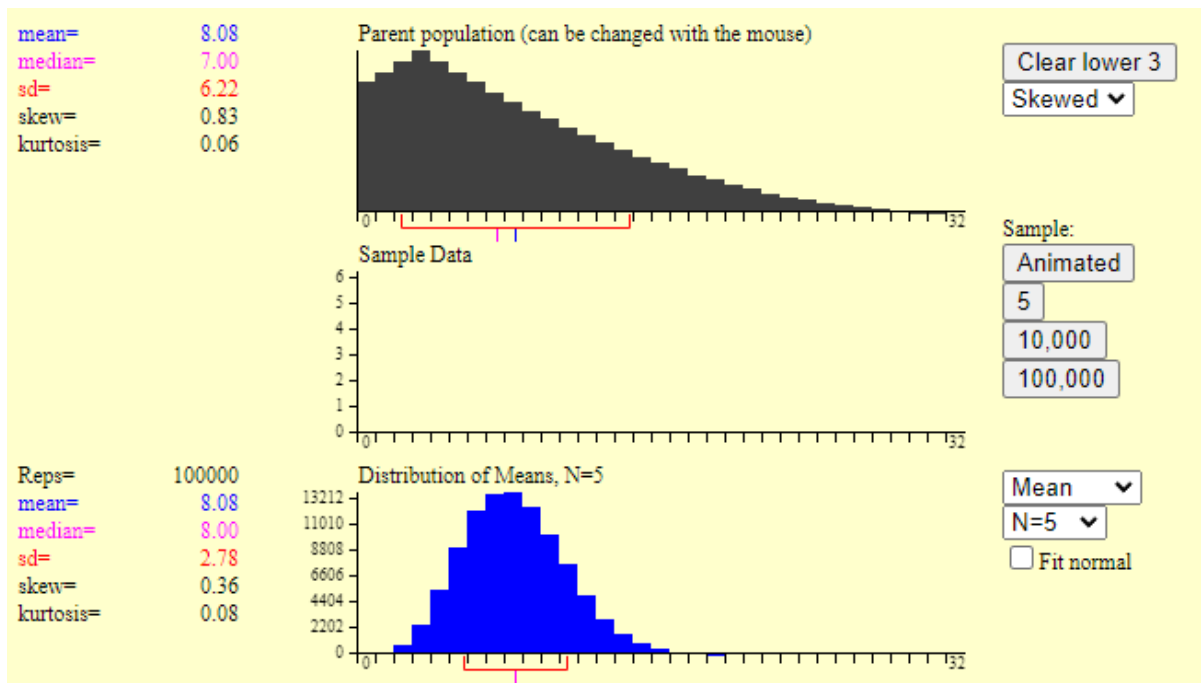
- A statistic is unbiased if the mean of the sampling distribution of the statistic is the parameter. Test to see if the sample mean is an unbiased estimate of the population mean. Try out different sample sizes and distributions.



Normal distribution

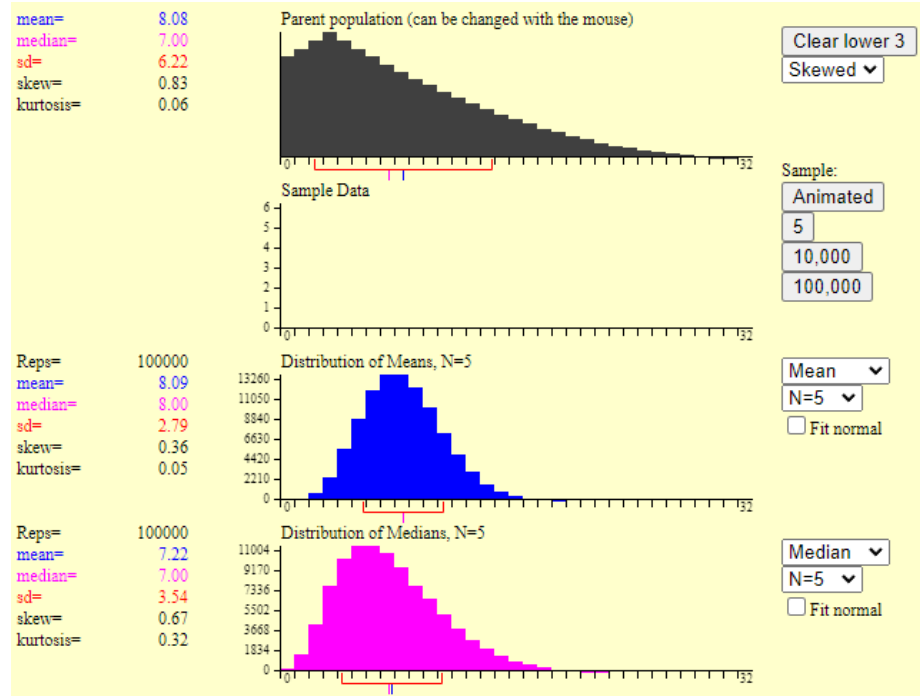


Uniform Distribution

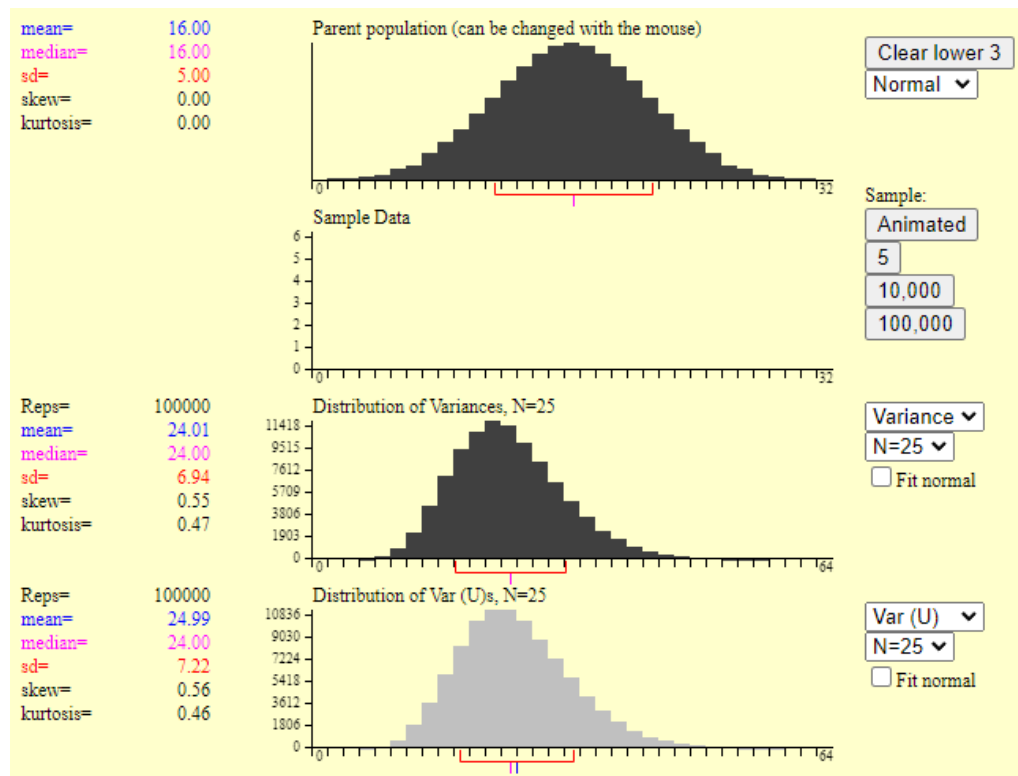


Skewed Distribution

- Find a distribution/sample size combination for which the sample median is a biased estimate of the population median.



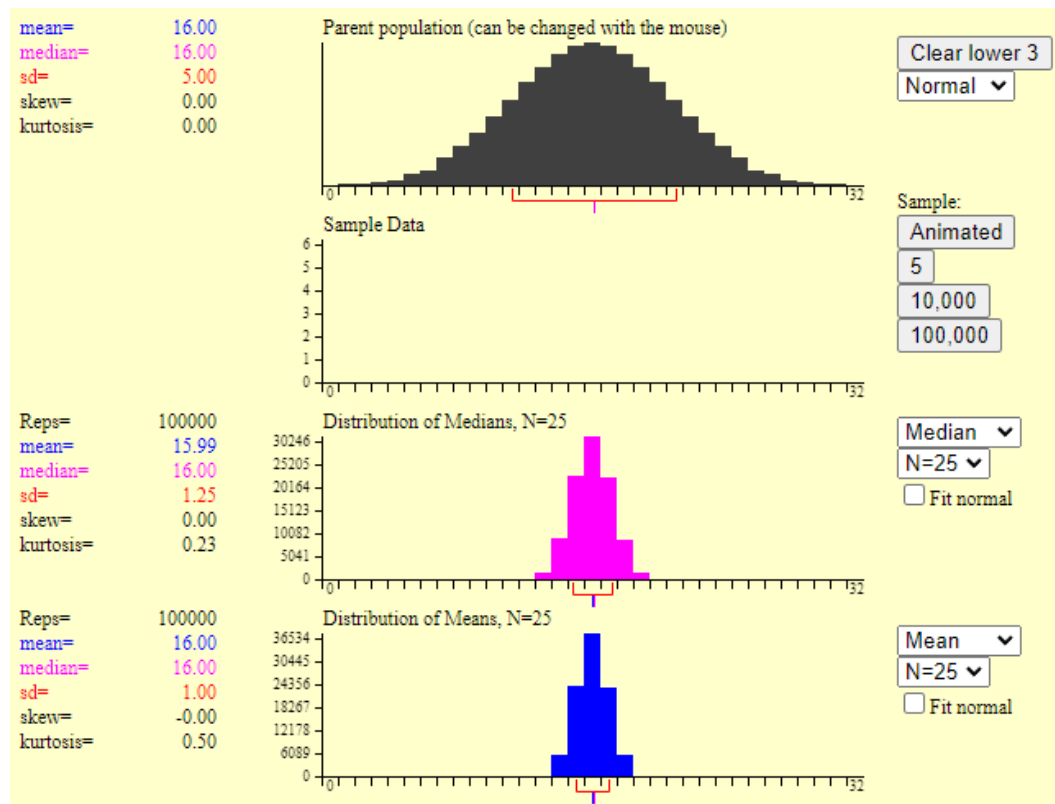
3. Is the sample variance an unbiased estimate of the population variance? If not, see if you can find a correction based on sample size. Does the correction hold for distributions other than the normal distribution?



- For what statistic is the mean of the sampling distribution dependent on sample size?  
Range

## Efficiency

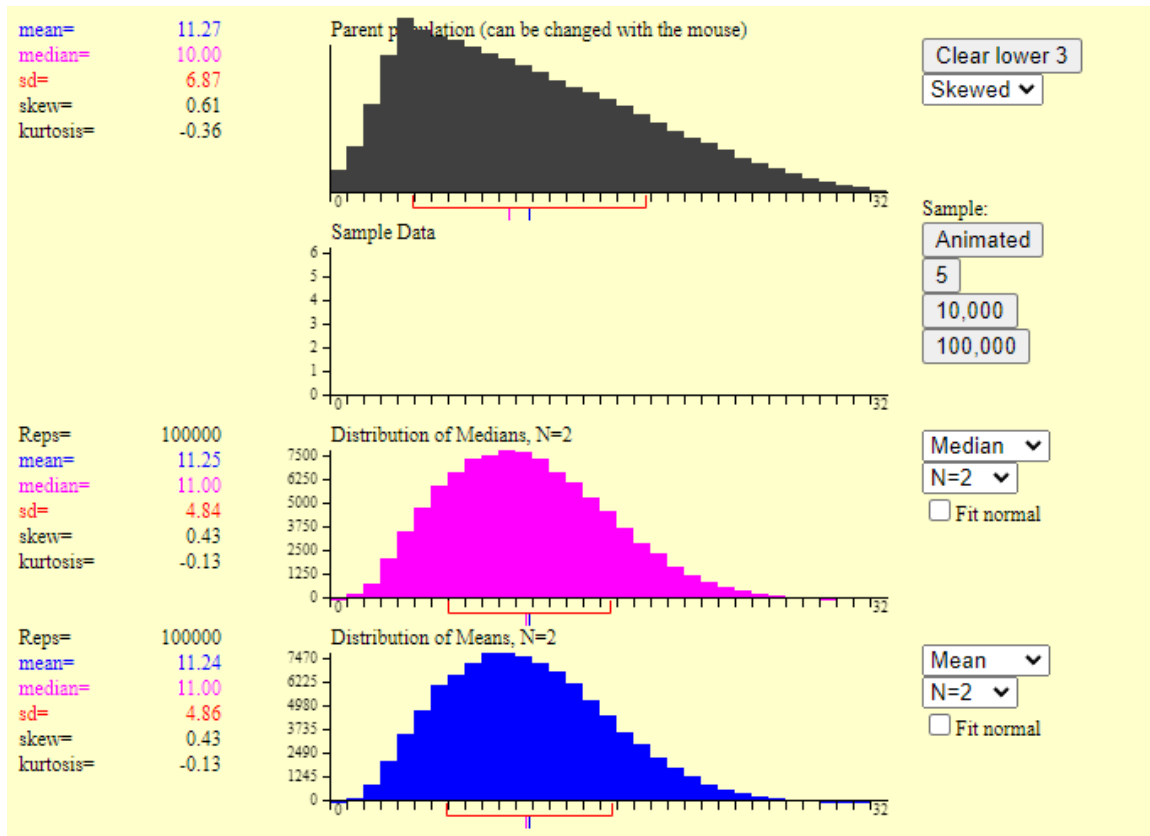
- For a normal distribution, compare the size of the standard error of the median and the standard error of the mean. Find a relationship that holds (approximately) across sample sizes?



- Does this relationship hold for a uniform distribution?

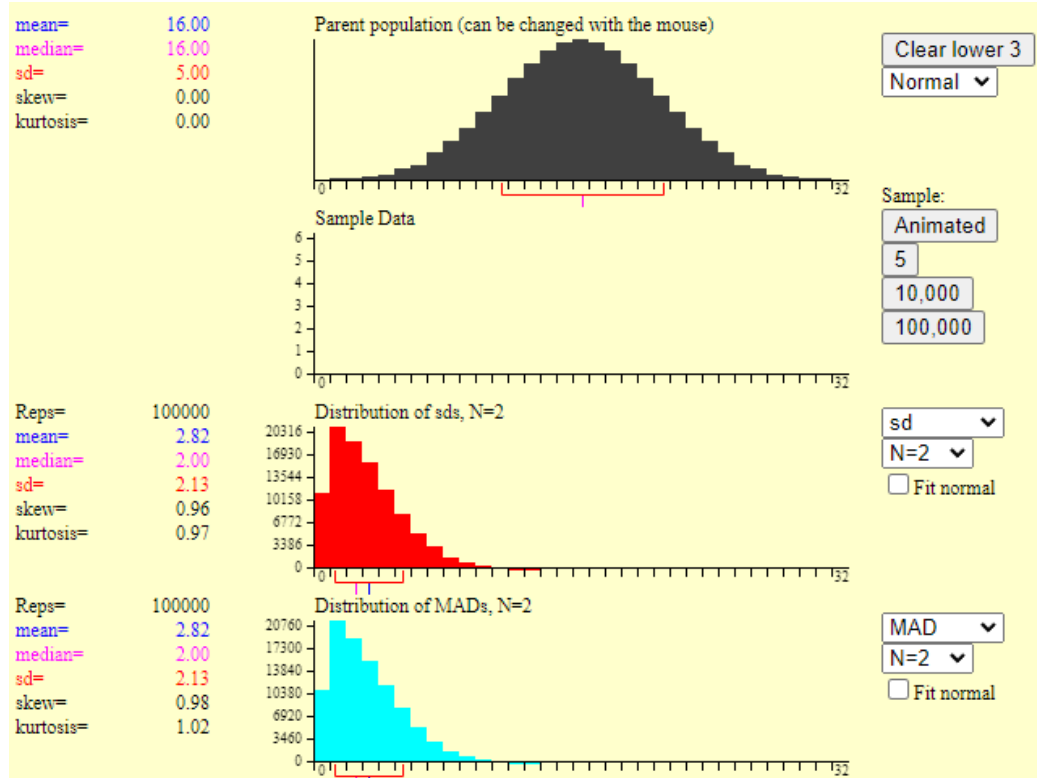
No, since the middle is more likely to occur and it is not normally distributed

- Find a distribution for which the standard error of the median is smaller than the standard error of the mean. (You may find this difficult, but don't give up.)

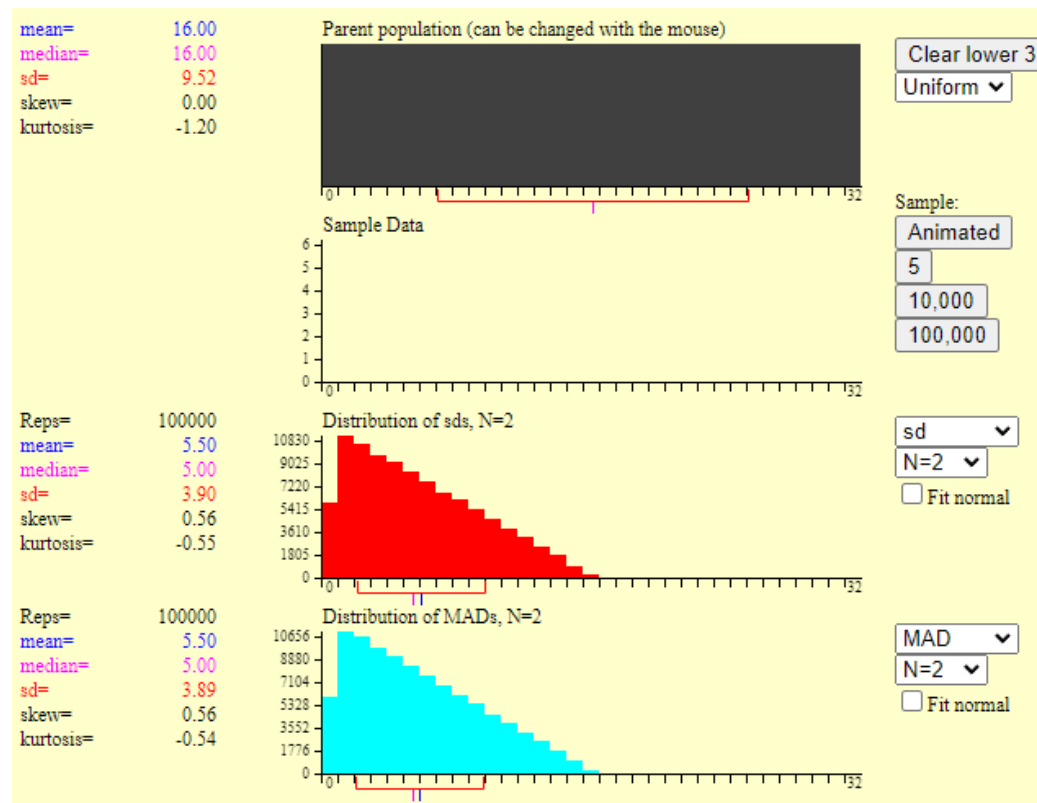


Extremely Skewed distribution with small sample size

4. Compare the standard error of the standard deviation and the standard error of the mean absolute deviation from the mean (MAD). Does the relationship depend on the distribution?

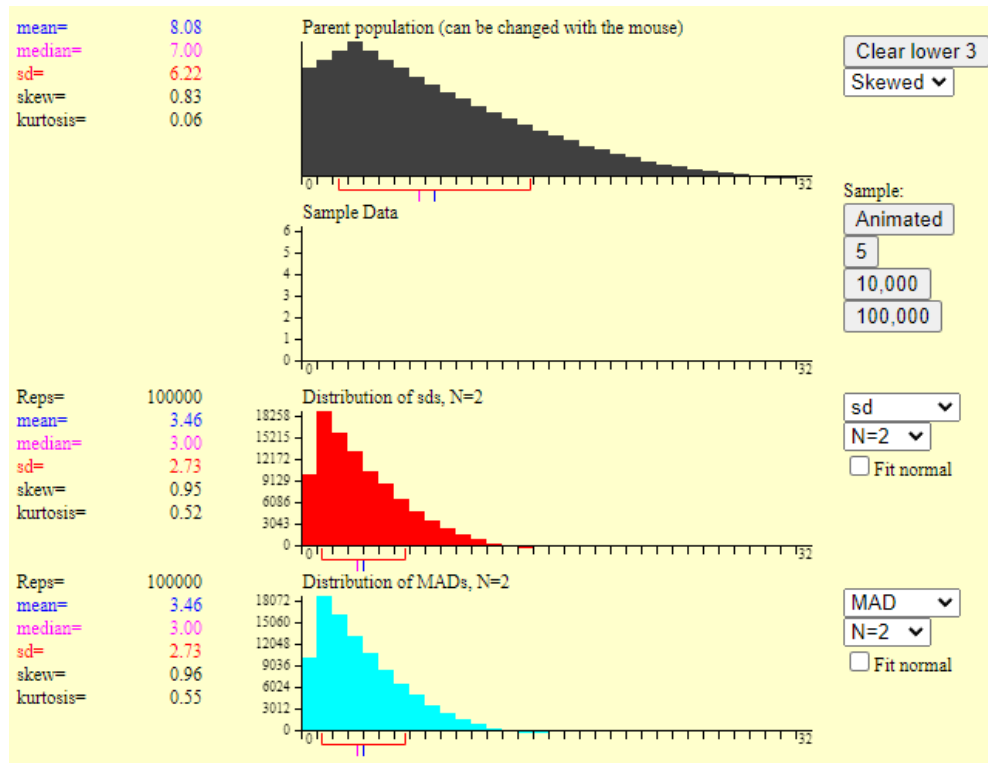


Normal distribution



Uniform Distribution

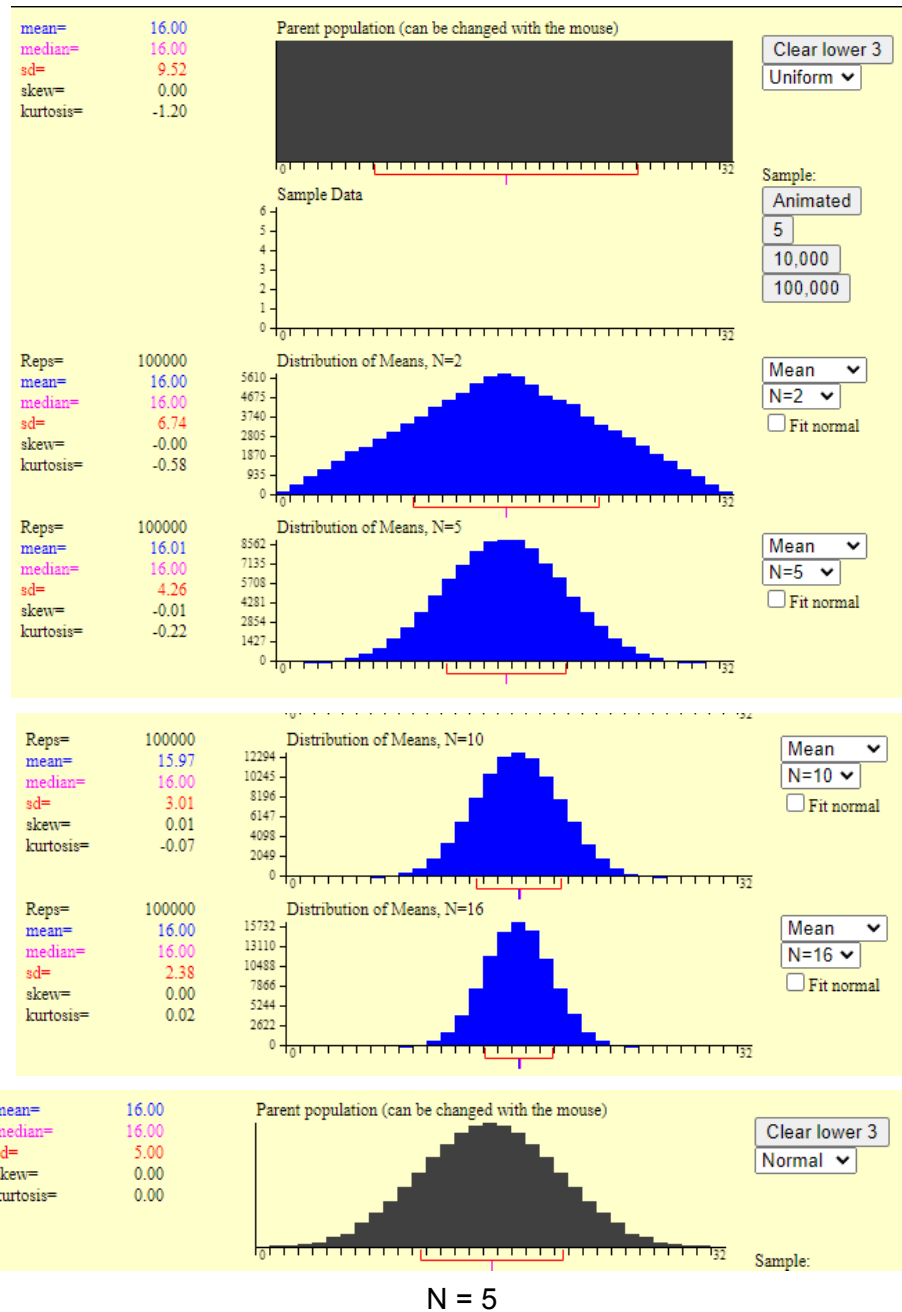




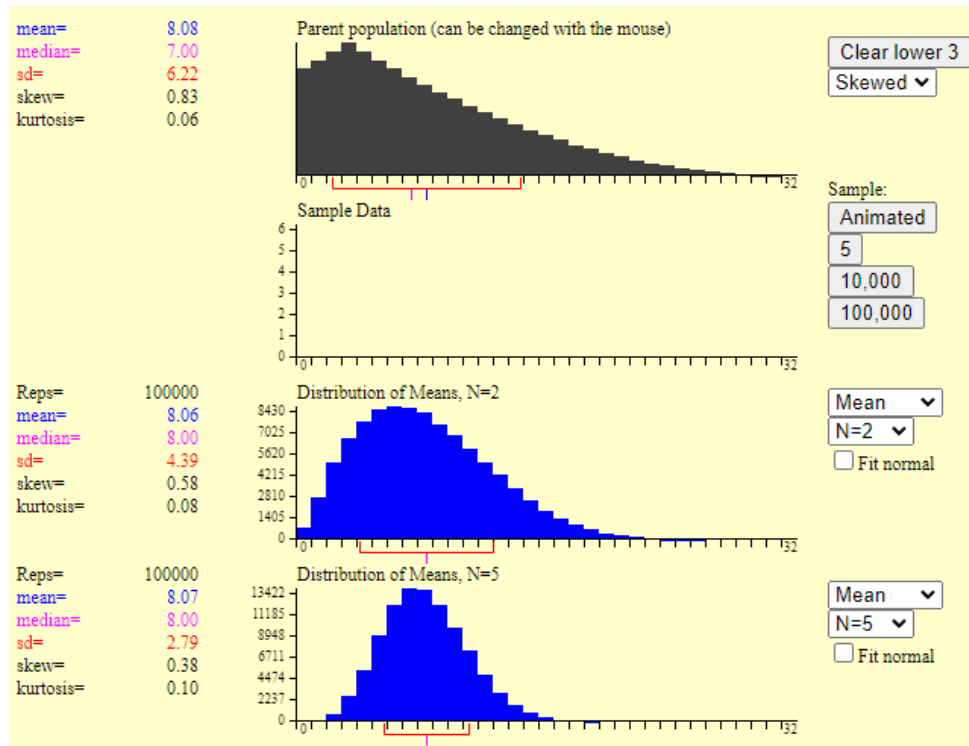
Skewed Distribution

## Central Limit Theorem

1. The central limit theorem states that the sampling distribution of the mean approaches a normal distribution as the sample size increases. Sample from the uniform distribution and determine how large a sample size is needed for the distribution to be a very close approximation of the normal distribution.



2. Do the same thing sampling from the skewed distribution.



- Determine whether the sampling distribution of the median approaches a normal distribution as sample size increases.

