(i) X → ...

Variance

$= E((X-\mu)^2)$

$\sigma^2 = E((X-\mu)^2) = \sum_{i=1}^{n} (x_i - \mu)^2 \, P(X = x_i)$

$S.D. = \sqrt{Var(X)} = \sigma$

(ii) X → continuous

$= \sqrt{E(X-\mu)^2}$

$$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\, dx$$

Binomial Distribution

$X \rightarrow$ discrete random var

$f(x) \rightarrow$ pmf of rv X

$f(x) = {}^nC_x \, p^x q^{n-x}$ $\qquad x = 1, 2, 3 \ldots n$ & this

$\qquad p + q = 1$

then $X \sim B(n, p)$ parameters.

$\boxed{\text{Mean}}$ → If $X \sim B(n,p)$ then $E(X) = np$, $var(X) = npq$

$$E(x) = \sum_{i=0}^{n} i f(i) = \sum_{i=0}^{n} i \binom{n}{i} p^i q^{n-i}$$

$$= np \sum_{i=1}^{n} \frac{(n-1)!}{(i-1)!(n-i)!} p^{i-1} q^{n-i}$$

$$= np\,(p+q)^{n-\cancel{i}+\cancel{i}-1} = np\,(1)^{n-1} = np.$$

$\boxed{\text{Var } X}$

$E(x(x-1))$

$$= \sum_{i=0}^{n} i(i-1)\binom{n}{i} p^i q^{n-i} = \sum_{i=2}^{n} \cdots = \sum i(i-1) \frac{n!}{i!(n-i)!} p^i q^{n-i}$$

$$= n(n-1)p^2 \sum_{i=2}^{n} \frac{(n-2)!}{(i-2)!(n-i)!} p^{i-2} q^{n-i}$$

$$= n(n-1)p^2$$

~~Var E(X)=~~ $\text{Var}(X) = E(X(X-1)) - E(X)(E(X)-1)$

$$= n(n-1)p^2 - np(np-1)$$

$$= np - np^2 = np(1-p) = npq.$$

Q) 10% of screws produced in a certain factory turn out to be defective. Find the prob that in a sample of 10 screws, 2

$\text{Var}(X) = E(X(X-1)) - E(X)(E(X)-1)$

$= n(n-1)p^2 - np(np-1)$

$= np - np^2 = np(1-p) = npq$

Q> 10% of screws produced in a certain factory turn out to be defective. Find the prob that in a sample of 10 screws, 2 are defective

$\cancel{p = \frac{9}{10} \quad q = \frac{1}{10}} \quad p = \frac{1}{10} \quad q = \frac{9}{10}$

$X \sim B(10, \frac{1}{10})$

$f(2) = {}^{10}C_2 \left(\frac{1}{10}\right)^2 \left(\frac{9}{10}\right)^8 = \boxed{\frac{1}{2} \times \left(\frac{9}{10}\right)^9}$

Q> The probability that a ~~number~~ man aged 60 will live to 70 is 0.65. What is the prob that out of 10 men, at least 7 → 70 years

$p = 0.65$
$q = 0.35$

$\binom{10}{7} p^7 q^3 + \binom{10}{8} p^8 q^2 + \binom{10}{9} p^9 q + \binom{10}{10} p^{10}$

$= 0.512$

Q> In a basket, there are 1 red, 2 white & 3 black balls. One ball is drawn 3 times in succession (with repetation)

(i) All white

(ii) 2 balls are white.

$p = \frac{1}{3}$
$q = \frac{2}{3}$

(i) $\binom{3}{3} p^3$

(ii) $\binom{3}{2} p^2 q$

Poisson Distribution

$$X \longrightarrow f(x) = \frac{e^{-\mu}\mu^{x}}{x!} \qquad x = 0, 1, 2, \ldots n$$

$\mu \longrightarrow$ parameter of distribution.

PROP: $X \sim P(\mu)$, then $E(X) = \mu$, $Var(X) = \mu$

$$\Rightarrow E(X) = \sum_{i=0}^{\infty} i f(i) = \sum i \frac{e^{-\mu}\mu^{i}}{i!}$$

$$= \sum \frac{i\, e^{-\mu}\mu^{i}}{i!} = \mu e^{-\mu} \sum \frac{\mu^{i-1}}{(i-1)!}$$

$$= \mu e^{-\mu} e^{\mu} = \boxed{\mu}$$

$$= \sum \frac{i e^{-\mu} \mu^i}{i!} = \mu e^{-\mu} \sum \frac{\mu^{i-1}}{(i-1)!}$$

$$= \mu e^{-\mu} e^{\mu} = \underline{\mu}$$

Q> A hospital switchboard receive an average 4 emergency calls in 5 min. Prob → (i) at most 2 emergency calls
(ii) exactly 3.

$\mu = 4 \qquad X \sim P(4)$

I> $P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$

$$= e^{-4} 4^0 + e^{-4} 4^1 + \frac{e^{-4} 4^2}{2}$$

II> $P(X=3) = \dfrac{e^{-4} 4^3}{3!}$

Q> A car hire firm has 2 cars which it hires out by a day. The number of demands of a car on each day is $P(x) \sim 1.5 : 4$.
calculate the proportion of day on which neither of car is used & - - - - - - - car demand can't be made due to lack of cars

$X \longrightarrow$ no of demands for a car on any day
$\mu = 1.5$

i) $P(x=0) = e^{-1.5}\mu^{0} = e^{-1.5}$

ii) $P(x \geq 3) = 1 - P(x=0) - P(x=1) - P(x=2)$