~~Illustrated Study~~

Mathematics (dt: 23/02/2021)

Probability

Introduction:

Random Experiment (R.E):

An exp (E) is called a R.E if

(i) All possible outcomes are known in advance

(ii) It is impossible to predict the outcomes

(iii) E can be repeated.

Events: outcomes of a random experiment

Mutually Exclusive: E_i & E_j are ME events iff $E_i \cap E_j = \emptyset$

(S) → set of event space

Classical defn of probability.

(E) → random exp.

such that the event space S

contains 'n' no. of events which are

equally likely.

Now, if an event (A) has 'm' no. of event points, then $P(A) = \frac{m}{n}$

Drawbacks:

(i) If the events are not equally likely

(ii) $n \to \infty$

Identities

(i) $P(\emptyset_n) = 0$

(ii) $P(A^c) = 1 - P(A)$

(iii) $P(A+B) = P(A) + P(B) - P(A \cap B)$

Conditional Probability

(i) $P(A/B) = \frac{P(A \cap B)}{P(B)}$ provided $P(B) \neq 0$.

(ii) A & B are independent events iff (occ. of A doesn't depend on B & vice-versa)

$P(AB) = P(A \cap B) = P(A)\,P(B)$.

Baye's Th.

Let $A_1, A_2, A_3, \ldots A_n$ be 'n' pairwise mutually exclusive & exhaustive set of events, X be an arbitrary event connected with the sample space (S),

Exhaustive means, union of all the events will result in the event space S.

i.e $\bigcup_i E_i = S$

$\Rightarrow E_1 \cup E_2 \cup E_3 \ldots \cup E_n = S.$

$P(X/A_1), P(X/A_2) \ldots, P(X/A_n)$

$$P(A_i/X) = \frac{P(A_i) \cdot P(X/A_i)}{\sum_{i=1}^{n} P(A_i) \cdot P(X/A_i)} \qquad i = 1, 2, 3, \ldots, n$$

Expanding Sequence

A seqn $\{A_n\}$

monotonically decreasing $A_{n+1} \leq A_n$ for all n.

increasing if $A_n \leq A_{n+1}$ for all n.

$A_1 = \{1, 2\}, \quad A_2 = \{1, 2, 3\}, \quad A_3 = \{1, 2, 3, 4, 5\}$

$A_1 + A_2 = \{1, 2, 3\}$

$(A_1 + A_2) = \{1, 2, 3\} = A_2$

$(A_1 + A_2) + A_3 = \{1, 2, 3, 4, 5\} = A_3$

Generalizing:

$(A_1 + A_2 + \ldots + A_n) = A_n \Rightarrow \lim_{n \to \infty} A_n = \sum_{n=1}^{\infty} A_n$

Th. If $\{A_n\}$ is a monotonic sequence (increasing or decreasing) then,

$$P(\lim_{n\to\infty} A_n) = \lim_{n\to\infty} P(A_n)$$

$\{A_n\} \rightarrow$ monotonic increasing sequence

$B_1 = A_1$

$B_2 = A_2 - A_1$

$B_3 = A_3 - A_2$

$\vdots$

$B_n = A_n - A_{n-1}$ $(n \geq 2)$

$\therefore \sum_{n=1}^{\infty} A_n = \sum_{n=1}^{\infty} B_n$ & $A_n = \sum_{i=1}^{n} B_i$

$P(\lim_{n\to\infty} A_n)$

$= P(\sum_{n=1}^{\infty} A_n)$

$= P(\sum_{n=1}^{\infty} B_n) = \sum_{n=1}^{\infty} P(B_n)$

Taking summation out

$P(B_1 + B_2 + B_3 + \dots B_n) = P(B_1) + P(B_2) + \dots + P(B_n)$ ($\because B_1, B_2, B_3 \dots B_n$ are Mutually Exclusive)

OR

So $P(\lim_{n\to\infty} A_n) = \lim_{n\to\infty} \sum_{i=1}^{n} P(B_i)$

$= \lim_{n\to\infty} P(\sum_{i=1}^{n} B_i)$

$= \lim_{n\to\infty} P(A_n)$ (Hence Proved)

$A_1 = \{1,2,3\}$ Decreasing

$A_2 = \{1,2,3\}$

$A_3 = \{1,2\}$

$A_1 A_2 = (A_1 \cap A_2) = \{1,2,3\} = A_2$

$(A_1 A_2) A_3 = \{1,2\} = A_3$.

$\therefore \lim_{n\to\infty} A_1 A_2 A_3 \dots A_n = \lim_{n\to\infty} A_n$

$$\boxed{\lim_{n\to\infty} A_n = \prod_{n=1}^{\infty} A_n}$$

Th. Prove that if $\{A_n\}$ is either expanding or contracting seq^n, then we have

$$\boxed{\overline{\lim_{n\to\infty} A_n} = \lim_{n\to\infty} \overline{A_n}}$$

($\overline{A}$ means A^c (complement))

Let $\{A_n\}$ is expanding sequence.

$$\lim_{n\to\infty} A_n = \sum_{n=1}^{\infty} A_n$$

$$\Rightarrow \overline{\lim_{n\to\infty} A_n} = \overline{A_1 + A_2 + \dots + A_n}$$

$$= \overline{A_1}\,\overline{A_2}\,\overline{A_3} \dots \overline{A_n}$$

$$= \prod_{n=1}^{n} \overline{A_n} = \lim_{n\to\infty} \overline{A_n}$$ (Proved).