Ex 1 The chairs of an auditorium are to be labeled with an uppercase english letter (A - Z) followed by a +ve integer not exceeding 100. What is the largest no. of chairs that can be labeled differently

Ans

Sol ①

$\underline{\ \ }_{\alpha}$ $\underline{\ \ \ \ \ \ }_{\text{Number}}$

↳ 26 possibilities

→ $\underline{0}$ $\underline{\ \ }$ $\underline{\ \ }$ (0-9, 0-9) → $1 \times 10 \times 10$ − 1 ↳ Due to condition of 000

→ $\underline{1}$ $\underline{0}$ $\underline{0}$ → 1

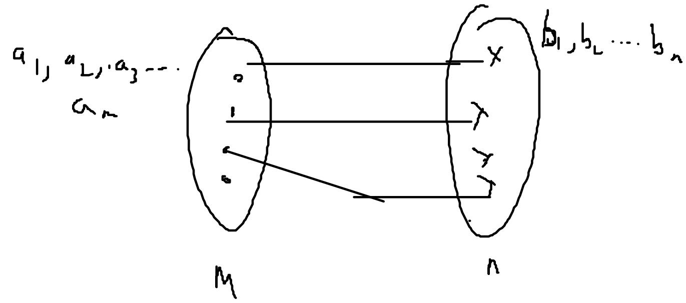
Total $26(\underbrace{1 \times 10 \times 10 - 1 + 1}_{\text{disjoint sets}}) = \underline{2600}$

Sol ②

$\underset{(26)}{\underline{\ \ }_{\alpha}}$ $\underline{\ \ \ \ }_{(1-100)}$ $\longrightarrow 26 \times 100 = \underline{2600}$.

Ex 2 How many one-to-one functions are there from of m elements to one with n-elements.

Ans



Possible if $\underline{m \leq n}$.

a_1 (n possibilities of b_i) a_2 (n-1 possibilities of b_i) ..... a_m (n-m+1 possibilities of b_i)

$= n \times (n-1) \times (n-2) \dots (n-m+1)$

Ex3 A student can choose a project from one of 3 lists. The 3 lists contain 23, 15 & 19 possible projects respectively. No project is on more than one list. How many possible ways to choose.

Ans Sol i) Since all projects are distinct, then all the lists can be combined & one can be chosen.

$\therefore$ combined lists contain

$23+15+19$.

Sol ii): Students can take project either from first list or second list or third list.

from first list $\rightarrow$ can take in 23 ways

2nd list $\rightarrow$ " " " 15 ways.

3rd list $\rightarrow$ " " " 19 ways.

$\therefore$ totals ways $= \underbrace{23+15+19}_{\text{either/or}}$

Question: If there are a projects common in list 1 & list 2
b . . . . . . list 1 & list 3
c . . . . . . list 2 & list 3

Sol: $|list\ 1| + |list\ 2| + |list\ 3| - a - b - c$.

Quest: If d projects are in L1, L2, List 3
& also the above.

Sol. $|L1| + |L2| + |L3| - a - b - c + d$.

Principal of inclusion-exclusion (Rosen).

(a) Subtraction rule :- If a task can be done in either n_1 ways or in n_2 ways, then the number of ways to do the task is $n_1 + n_2$ minus the number of ways to do the task that are common to the two different ways.

$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$

(b) The subtraction rule is known as the principal of inclusion-exclusion especially when it is used to count the number of elements in the union of two sets.

eg C → strongly typed language.

weakly typed (php, basic) language →
var. a & A → both are same
var. integer, Integer } both mean same

strongly typed language:
A & a → both mean same.

eg1 Say In a weakly typed language, the name of a variable is a string of one or two (numbers or characters) alphanumeric characters, where uppercase & lowercase letters are not distinguished. Moreover, a variable name must begin with a letter & must be different from the 5 strings of two characters that are reserved for programming use. How many different variable names

Ans Let us say that the total number of variables be V.

Let the single character variable be $V_1 = 26$

2 " " " " " " " $V_2 = 26.36$.

$\therefore V = V_1 + V_2$

$= 26 + 26.36$

$\therefore$ There are 5 strings common.

$\therefore$ total no. of variables is

$V = V_1 + V_2 - 5$.

Ex A company has 350 applications out of which

$A_1 \to$ 220 applicants from CS

$A_2 \to 147$ " " " MBA

51 applicants from CS+MBA

Question: How many applicants did neither in CS nor in MBA.

Ans. The total no. of candidates who did neither CS. nor MBA

$A = |A_1 \cup A_2| \longrightarrow |A_1| + |A_2|$ (All person who did CS & MBA.)

$\downarrow$

This contain who did both CS & MBA.

$\therefore$ Total no. of candidates who did

$$A = |A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$$

$$= 220 + 147 - 51$$

$$= 316 \quad \text{(Either CS / MBA \& CS + MBA)}$$

$\therefore$ Total no of applicants who did neither CS or MBA.

$$350 - 316 = 34.$$

(6) Division Rule:

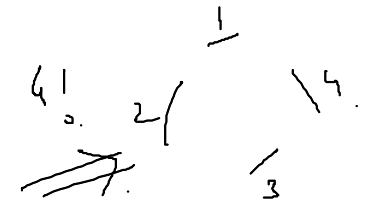
There are (n/d) ways to do a task if it can be done using a procedure that can be carried out in n ways, and for every way w, exactly d of the n ways correspond to way w.

$\Rightarrow$ If the finite set A is the union of n-pairwise disjoint subsets each with d elements, then $n = |A|/d$.

eg. How many different ways are there to seat 4 people around a circular table where two seatings are considered the same when each person has the same left neighbour & the same right neighbour

Soln. There can be a total of 4! arrangements for the 4 people in the seats.

$$\begin{array}{cccc} ① & ③ & ② & ① \\ \downarrow & \downarrow & \downarrow & \downarrow \\ \_ & \_ & \_ & \_ \\ S1 & S2 & S3 & S4 \end{array} \rightarrow 4!$$



Now, for seat 1, there are 4 choices.

but → 2 ways where the assignment of $\underbrace{\text{left \& right} \equiv \text{right \& left}}_{\text{2 arrangements}}$.

for seat 2 → there are 3 choices

but 2 ways where the assignment of $\underbrace{\text{left \& right} = \text{right \& left}}_{\text{2 arrangements}}$.

Total no. of ways $= \dfrac{24}{2 \times 2} = 6$ different seating arrangements of 4 people round the circular table.

Linear Diophantine Equation

(2 variable) Let a, b & c be integers with a & b be both not zero. Then the eq. $ax+by=c$ has an integral solution iff d divides c, where $d = \gcd(a,b)$. Furthermore, if (x_0, y_0) is a particular integral solution of this eq, then all the integral solutions for this eq are given by.

$$x = x_0 + \frac{b}{d} n, \quad y = y_0 - \frac{a}{d} n.$$

where n is any integer.

for multiple variable —

$$a_1 + a_2 + a_3 + \dots a_n = r. \quad — (1)$$

$a_i \in (0,1)$, find the combination for which eq. (1) satisfies.

$n > r$.

eg $\quad a_1 + a_2 + a_3 + a_4 = 3. \quad a_i \in (0,1)$. ??