Bachelors of Computer Science and Engineering 2022

(2nd Year, 2nd Semester)

Mathematics IV

Full Marks: 100

USE SEPARATE ANSWER SCRIPTS FOR GROUP A AND GROUP B

Group A

FULL MARKS: 50

Answer question 1 any SIX from the rest:

1. Define a limit point of a nonempty subset of the set of real numbers, $\mathbb{R}$. Find the limit points of the open interval $(0,1)$ in $\mathbb{R}$. 2

2. Let A, B, C be three subsets of a set X. Prove or disprove the following: 8

   (i) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$. (ii) $A \times (B \Delta C) = (A \times B) \Delta (A \times C)$

3. Define an equivalence relation on a nonempty set. Let $S = \mathbb{Z} \times \mathbb{Z}$, where $\mathbb{Z}$ is the set of integers. Let ρ be a binary relation on S defined by

$$(a, b)\rho(c, d) \text{ if and only if } a + d = b + c.$$

   Determine whether ρ is an equivalence relation on S or not. 8

4. Define cardinal numbers of a set. Let α, β, γ be three cardinal numbers. Prove that $(\alpha^{\beta})^{\gamma} = \alpha^{\beta\gamma}$. 8

5. Define a countable set. Prove that the set of rational numbers is countable. 8

6. What is the least upper bound property. Show that the set of rational numbers does not have the least upper bound property. 8

[Turn over

7. What is a tautology? Prove that $p \vee \neg(p \wedge q)$ is tautology. 8

8. Define injective, surjective and bijective functions. Let $f : A \longrightarrow B$ and $g : B \longrightarrow C$ be two functions, where A, B, C be three nonempty sets. If $g \circ f$ is bijective, then prove that f is injective and g is surjective. 8

9. Use mathematical induction to prove the following

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \cdots + n \cdot (n+1) = \frac{n(n+1)(n+2)}{3},$$

for any positive integer n. 8

CSE/BS/B/Math/T/221

Bachelor Of Computer Science and Engineering

(2nd Year, 2nd Semester, 2022)

Mathematics IV

(Algebra, Probability and Stochastic Processes)

Full Marks: 100

USE SEPARATE ANSWER SCRIPTS FOR GROUP A AND GROUP B

Symbols/notations used have their usual meaning

Group B

Probability and Stochastic Processes

FULL MARKS: 50

Answer question numbers 1 and 2 any THREE from the rest:

1. (i) State Markov's inequality.

(ii) State Bayes' theorem on conditional probability.

(iii) If A and B are two events such that $P(A) = P(B) = 1$, then show that $P(A + B) = 1, P(AB) = 1$.

(iv) A bag contains tickets numbered from 1 to 20. Two tickets are drwn. Find the probability that

(a) Both tickets have prime number on them.

(b) On one them is a prime number and on the other is a multiple of 4.

2+1+2+5

2. (i) A point is chosen at random on a semi-circle having centre at the origin and radius unity and projected on the diameter. Prove that the distance of the point of projection from the centre has the probability density $\frac{1}{\pi\sqrt{1-x^2}}$ for $-1 < x < 1$ and zero elsewhere.

[Turn over

(ii) A point X is chosen at random on a line segment AB whose middle point is O. Find the probability that AX, BX and AO form the sides of a triangle.
5 + 5

3. (i) Assuming that the lifespan of transistor is normal, find the mean and standard deviation if 84% of the transistors have lifespan less than 65.2 months and 68% have lifespan lying between 65.2 and 62.8 months. (Given that $\Phi(0.9) = 0.84$ and $\Phi(-0.9) = 0.16$)

(ii) Derive the mean and standard deviation of a Binomial distribution with parameters n and p

4. (i) The joint density function of X and Y is given by

$$f(x,y) = \begin{cases} 2e^{-x}e^{-2y} & 0 < x < \infty, 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

compute:

(a) $P(X > 1, Y < 1)$

(b) $P(X < Y)$

(c) $P(X < a)$

(ii) The joint density of X and Y is given by

$$f(x,y) = \begin{cases} \frac{12}{5}x(2-x-y) & 0 < x < 1, 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Compute the conditional density of X given that $Y = y$, where $0 < y < 1$.

(iii) Suppose that the joint density of X and Y given by,

$$f(x,y) = \begin{cases} \frac{e^{\frac{x}{y}} \cdot e^{-y}}{y} & 0 < x < \infty, 0 < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

Find $P(X > 1 \mid Y = y)$ 4+4+2

5. (i) Let X_n, $n \geq 0$, be a Markov chain whose state space S is a subset of $\{0, 1, 2, \cdots\}$ and whose transition function P is such that

$$\sum_y yP(x,y) = Ax + B, x \in S,$$

for some constants A and B.

(a) Show that $EX_n + 1 = AEX_n + B$.

(b) Show that if $A \neq 1$, then

$$EX_n = \frac{B}{1-A} + A^n(EX_0 - \frac{B}{1-A}).$$

(ii) State and prove Chapman-Kolmogorov equation 5+5

6. The men's department of a large store employs one tailor for customer fittings. The number of customers requiring fittings appears to follow a Poisson distribution with mean arrival rate of 24 per hour. Customers are fitted in a first come first serve basis. The time it takes to fit a customer appears to be exponentially distributed with mean 2 minutes. (a) What is the average number of customers in the fitting room? (b)How much time should customer expect to spend in the fitting room? (c) What is the percentage of the time when tailor is idle? (d) What is the probability that a customer have to wait for more than 10 minutes for tailor's service? 10.