Asymptotic Notations using Limits

$f = \Theta(g)$	f grows at the same rate as g	There exists an n_0 and constants $c_1, c_2 > 0$ such that for all $n > n_0$, $c_1 g(n) \le \lvert f(n) \rvert \le c_2 g(n)$.
$f = O(g)$	f grows no faster than g	There exists an n_0 and a constant $c > 0$ such that for all $n > n_0$, $\lvert f(n) \rvert \le c g(n)$.
$f = \Omega(g)$	f grows at least as fast as g	There exists an n_0 and a constant $c > 0$ such that for all $n > n_0$, $c g(n) \le \lvert f(n) \rvert$.

$$\lim_{n\to\infty} f(n)/g(n) \neq 0, \infty \quad \Rightarrow \quad f = \Theta(g)$$
$$\lim_{n\to\infty} f(n)/g(n) \neq \infty \quad \Rightarrow \quad f = O(g)$$
$$\lim_{n\to\infty} f(n)/g(n) \neq 0 \quad \Rightarrow \quad f = \Omega(g)$$

$$f(n) = n^2 \qquad \boxed{f(n) = O(n^3)}$$

$$g(n) = n^3$$

$$\lim_{n \to \infty} \frac{n^2}{n^3} = \lim_{n \to \infty} \frac{1}{n} = 0$$

$$f(n) = n^3$$

$$g(n) = n^2$$

$$\lim_{n \to \infty} \frac{n^3}{n^2} = \infty$$

$$f(n) = \Omega(g(n))$$

$$\frac{f(n)}{g(n)} = \frac{5n^2+6n+7}{n^2}$$

$$= 5 + 6/n + \frac{7}{n^2}$$

$$\lim_{n\to\infty}\left(5+\frac{6}{n}+\frac{7}{n^2}\right) = \boxed{5} \neq 0, \infty$$

$$\boxed{f(n) = \Theta(n^2)}$$