have completed these the risk can then be 'closed' as no longer relevant. In any case, as noted earlier, the probability and impact of a risk are likely to change during the course of the project.

7.9 Evaluating risks to the schedule

In Section 7.6 we showed a probability chart – Figure 7.3. This illustrated the point that a forecast of the time needed to do a job is most realistically presented as a graph of likelihood of a range of figures, with the most likely duration as the peak and the chances of the job taking longer or shorter shown as curves sloping down on either side of the peak. Thus we can show that a job might take five days but that there is a small chance it might need four or six days, and a smaller chance of three or seven days and so on. If a task in a project takes longer than planned, we might hope that some other task might take less and thus compensate for this delay. In the following sections we will examine PERT, a technique which takes account of the uncertainties in the durations of activities within a project. We will also touch upon Monte Carlo simulation, which is a more powerful and flexible tool that tackles the same problem.

A drawback to the application of methods like PERT is that in practice there is a tendency for developers to work to the schedule even if a task could be completed more quickly. Even if tasks are completed earlier than planned, project managers are not always quick to exploit the opportunities to start subsequent activities earlier than scheduled. Critical chain management will be explored as a way of tackling this problem.

7.10 Applying the PERT technique

Using PERT to evaluate the effects of uncertainty

PERT (Program Evaluation and Review Technique) was published in the same year as CPM. Developed for the Fleet Ballistic Missiles Program, it is said to have saved considerable time in development of the Polaris missile.

PERT was developed to take account of the uncertainty surrounding estimates of task durations. It was developed in an environment of expensive, high-risk and state-of-the-art projects – not that dissimilar to many of today's large software projects.

The method is very similar to the CPM technique (indeed many practitioners use the terms PERT and CPM interchangeably) but, instead of using a single estimate for the duration of each task, PERT requires three estimates.

- *Most likely time*: the time we would expect the task to take under normal circumstances. We shall identify this by the letter m.
- *Optimistic time*: the shortest time in which we could expect to complete the activity, barring outright miracles. We shall use the letter a for this.
- *Pessimistic time*: the worst possible time, allowing for all reasonable eventualities but excluding 'acts of God and warfare' (as they say in most insurance exclusion clauses). We shall call this b.

PERT then combines these three estimates to form a single expected duration, t_e, using the formula

$$t_e = \frac{a + 4m + b}{6}$$

EXERCISE 7.5

Table 7.6 provides additional activity duration estimates for the network shown in Figure 6.29. There are new estimates for a and b and the original activity duration estimates have been used as the most likely times, m. Calculate the expected duration, t_e, for each activity.

Activity	Optimistic (a)	Activity durations (weeks) Most likely (m)	Pessimistic (b)
A	5	6	8
B	3	4	5
C	2	3	3
D	3.5	4	5
E	1	3	4
F	8	10	15
G	2	3	4
H	2	2	2.5

TABLE 7.6 PERT activity time estimates

Using expected durations

The expected durations are used to carry out a forward pass through a network, using the same method as the CPM technique. In this case, however, the calculated event dates are not the earliest possible dates but the dates by which we expect to achieve those events.

EXERCISE 7.6

Before reading further, use your calculated expected activity durations to carry out a forward pass through the network (Figure 6.29) and verify that the project duration is 13.5 weeks. What does an expected duration of 13.5 weeks mean in terms of the completion date for the project?

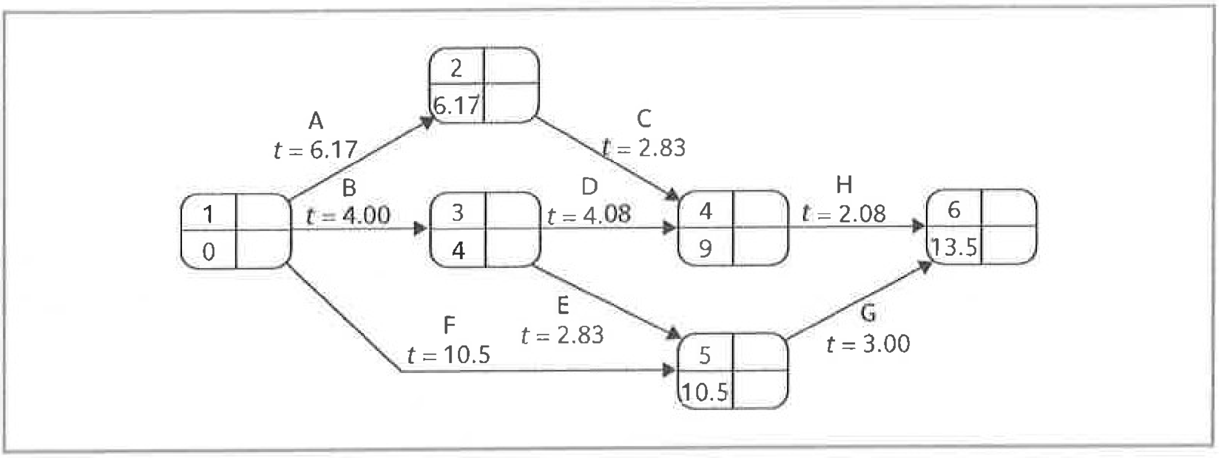

FIGURE 7.6 The PERT network after the forward pass

The PERT network illustrated in Figure 7.6 indicates that we expect the project to take 13.5 weeks. In Figure 7.6 we have used an activity-on-arrow network as this form of presentation makes it easier to separate visually the estimated activity data (expected durations and, later, their standard deviations) from the calculated data (expected completion dates and target completion dates). The method can, of course, be equally well supported by activity-on-node diagrams.

The PERT event labelling convention adopted here indicates event number and its target date along with the calculated values for expected time and standard deviation.

Unlike the CPM approach, the PERT method does not indicate the earliest date by which we could complete the project but the expected (or most likely) date. An advantage of this approach is that it places an emphasis on the uncertainty of the real world. Rather than being tempted to say '*the completion date for the project is . . .*' we are led to say '*we expect to complete the project by . . .*'.

It also focuses attention on the uncertainty of the estimation of activity durations. Requesting three estimates for each activity emphasizes the fact that we are not certain what will happen – we are forced to take into account the fact that estimates are approximate.

Activity standard deviations

A quantitative measure of the degree of uncertainty of an activity duration estimate may be obtained by calculating the standard deviation s of an activity time, using the formula

This standard deviation formula is based on the rationale that there are approximately six standard deviations between the extreme tails of many statistical distributions.

$$s = \frac{b - a}{6}$$

The activity standard deviation is proportional to the difference between the optimistic and pessimistic estimates, and can be used as a ranking measure of the degree of uncertainty or risk for each activity. The activity expected durations and standard deviations for our sample project are shown in Table 7.7.

Activity	Activity durations (weeks)				
	Optimistic (*a*)	Most likely (*m*)	Pessimistic (*b*)	Expected (t_e)	Standard deviation (*s*)
A	5	6	8	6.17	0.50
B	3	4	5	4.00	0.33
C	2	3	3	2.83	0.17
D	3.5	4	5	4.08	0.25
E	1	3	4	2.83	0.50
F	8	10	15	10.50	1.17
G	2	3	4	3.00	0.33
H	2	2	2.5	2.08	0.08

TABLE 7.7 Expected times and standard deviations

The likelihood of meeting targets

The main advantage of the PERT technique is that it provides a method for estimating the probability of meeting or missing target dates. There might be only a single target date – the project completion – but we might wish to set additional intermediate targets.

Suppose that we must complete the project within 15 weeks at the outside. We expect it will take 13.5 weeks but it could take more or, perhaps, less. In addition, suppose that activity C must be completed by week 10, as it is to be carried out by a member of staff who is scheduled to be working on another project, and that event 5 represents the delivery of intermediate products to the customer, which must take place by week 10. These three target dates are shown on the PERT network in Figure 7.7.

The PERT technique uses the following three-step method for calculating the probability of meeting or missing a target date:

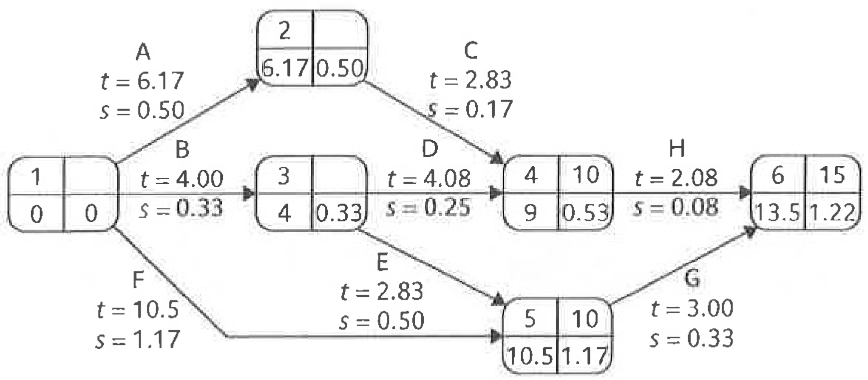

FIGURE 7.7 The PERT network with three target dates and calculated event standard deviations

- calculate the standard deviation of each project event;
- calculate the z value for each event that has a target date;
- convert z values to a probabilities.

Calculating the standard deviation of each project event

The square of the standard deviation is known as the variance. Standard deviations may not be added together but variances may.

Standard deviations for the project events can be calculated by carrying out a forward pass using the activity standard deviations in a manner similar to that used with expected durations. There is, however, one small difference – to add two standard deviations we must add their squares and then find the square root of the sum. Exercise 7.7 illustrates the technique. One practical outcome of this is that the contingency time to be allocated to a sequence of activities as a whole would be less than the sum of the contingency allowances for each of the component activities. This has implications that can be exploited in critical chain project management, which are discussed in the next section.

EXERCISE 7.7

The standard deviation for event 3 depends solely on that of activity B. The standard deviation for event 3 is therefore 0.33.

For event 5 there are two possible paths, B + E or F. The total standard deviation for path B + E is $\sqrt{(0.33^2 + 0.50^2)} = 0.6$ and that for path F is 1.17; the standard deviation for event 5 is therefore the greater of the two, 1.17.

Verify that the standard deviations for each of the other events in the project are as shown in Figure 7.7.

Calculating the z values

The z value is calculated for each node that has a target date. It is equivalent to the number of standard deviations between the node's expected and target dates. It is calculated using the formula

$$z = \frac{T - t_e}{s}$$

where t_e is the expected date and T the target date.

EXERCISE 7.8

The z value for event 4 is $(10 - 9.00)/0.53 = 1.8867$.

Calculate the z values for the other events with target dates in the network shown in Figure 7.7.

Converting *z* values to probabilities

A *z* value may be converted to the probability of not meeting the target date by using the graph in Figure 7.8.

This graph is the equivalent of tables of *z* values, also known as standard normal deviates, which may be found in most statistics textbooks.

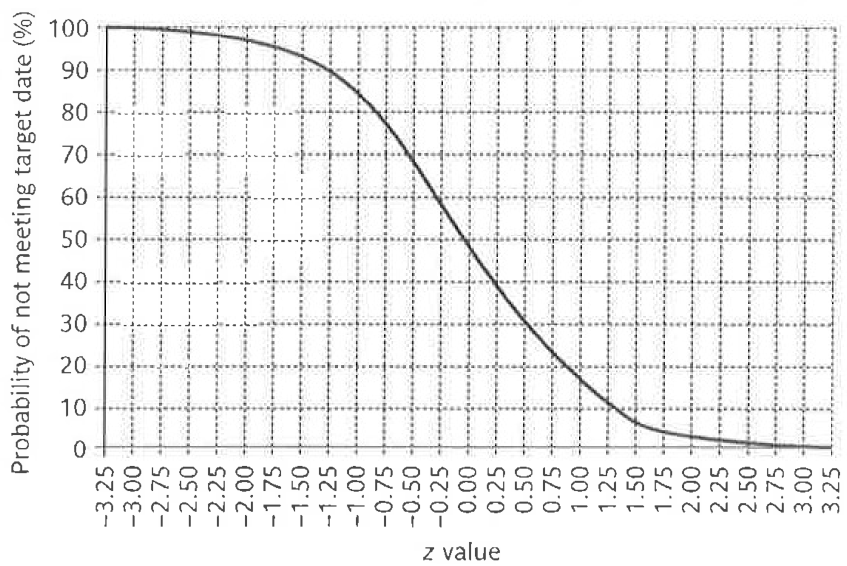

FIGURE 7.8 The probability of obtaining a value within *z* standard deviations of the mean for a normal distribution

EXERCISE 7.9

The z value for the project completion (event 6) is 1.23. Using Figure 7.8 we can see that this equates to a probability of approximately 11%, that is, there is an 11% risk of not meeting the target date of the end of week 15.

Find the probabilities of not achieving events 4 or 5 by their target dates of the end of week 10.

What is the likelihood of completing the project by week 14?

The advantages of PERT

We have seen that by requesting multi-valued activity duration estimates and calculating expected dates, PERT focuses attention on the uncertainty of forecasting. We can use the technique to calculate the standard deviation for each task and use this to rank them according to their degree of risk. Using this ranking, we can see, for example, that activity F is the one regarding which we have greatest uncertainty, whereas activity C should, in principle, give us relatively little cause for concern.

If we use the expected times and standard deviations for forward passes through the network we can, for any event or activity completion, estimate the probability of meeting any set target. In particular, by setting target dates along the critical path, we can focus on those activities posing the greatest risk to the project's schedule.

7.11 Monte Carlo simulation

As an alternative to the PERT technique, and to provide a greater degree of flexibility in specifying likely activity durations, we can use Monte Carlo simulation techniques to evaluate the risks of not achieving deadlines. The basis of this technique involves calculating activity completion times for a project network a large number of times, each time selecting estimated activity times randomly from a set of estimates for each activity. The results can then be tabulated, summarized or displayed as a graph such as that shown in Figure 7.9.

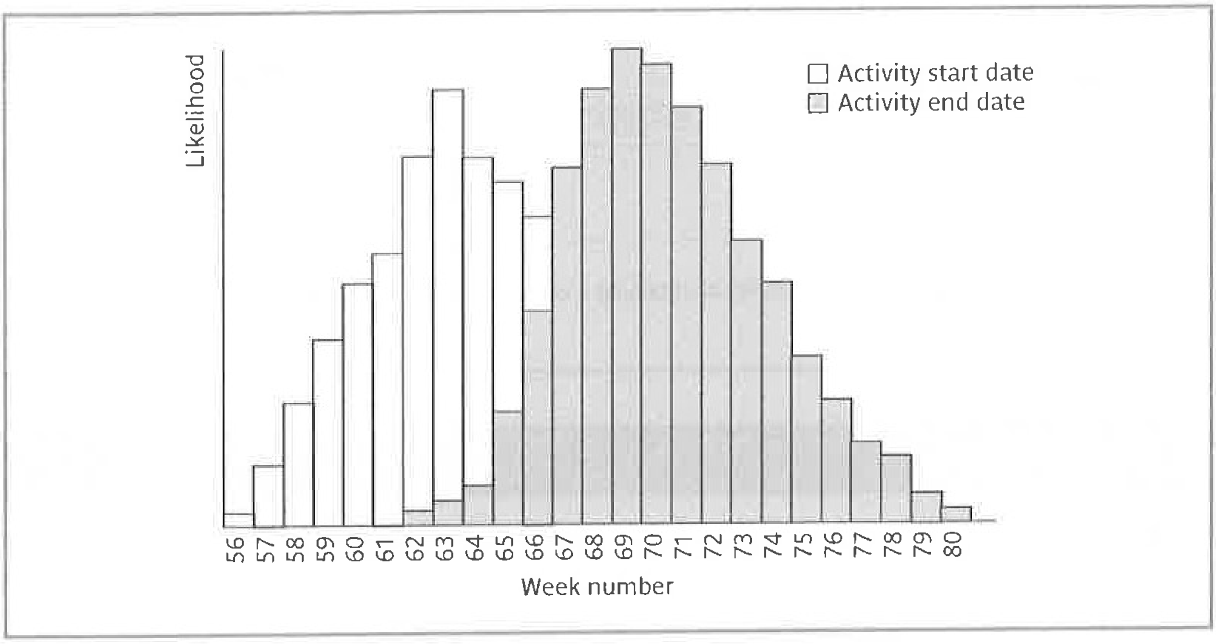

FIGURE 7.9 Risk profile for an activity generated using Monte Carlo simulation

Activity duration estimates can be specified in a variety of forms, depending upon the information available. If, for example, we have historic data available about the durations of similar activities, we might be able to specify durations as a probability distribution. With less information available we should, at least, be able to provide three time estimates as used by PERT.

The calculation required for this is clearly extensive as we may have to carry out the forward pass through the network many hundreds of times before obtaining a representative selection of possible completion times. Fortunately, there are a number of packages available for carrying out Monte Carlo simulation. Some will exchange data with project-scheduling applications and some interface to standard spreadsheet software.