

Assignment 1 - Volatility surface without interpolation

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May 2024

Implied volatility surface without interpolation

The provided data includes European call option prices for different strike prices and maturities, along with the current index price (S) and the risk-free interest rate (r).

Table 1: Market prices of European call options for different strikes and maturities

Strikes	T-0.25	T-0.5	T-1	T-1.5
60	40.2844	42.4249	50.8521	59.1664
70	30.5281	33.5355	42.6656	51.2181
80	21.0415	24.9642	34.4358	42.9436
90	12.2459	16.9652	26.4453	34.7890
100	5.2025	10.1717	19.4706	27.8938
110	1.3448	5.4318	14.4225	23.3305
120	1.2052	2.7647	11.2103	20.7206
130	0.0216	1.4204	9.1497	19.1828
140	0.0019	0.7542	7.7410	18.1858

Black-Scholes Call Price Formula

The Black-Scholes model is used to calculate the theoretical price of European call options. The formula is:

$$C(\sigma) = S_t \Phi(d_1) - K e^{-r(T-t)} \Phi(d_2)$$

$$d_1 = \frac{\log\left(\frac{S_t}{K}\right) + (r + 0.5\sigma^2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = d_1 - \sigma\sqrt{T-t}.$$

where:

S: Current stock price

K: Strike price

T: Time to maturity

r: Risk-free interest rate

σ : Volatility of the stock

Φ : Cumulative distribution function of the standard normal distribution

This model is used to determine the theoretical price of options based on inputs such as current stock price, strike price, time to maturity, risk-free rate, and volatility.

Newton-Raphson Method for Implied Volatility

The implied volatility is the value of σ that equates the Black-Scholes theoretical price to the market price. We use the Newton-Raphson method to solve for it.

Objective Function:

$$f(\sigma) = C_{BS}(\sigma) - C_{(market)}$$

Derivative of the Objective Function:

$$f'(\sigma) = \frac{\partial C_{BS}}{\partial \sigma}$$

Iteration Formula:

$$\sigma_{\text{new}} = \sigma_{\text{old}} - \frac{f'(\sigma_{\text{old}})}{f(\sigma_{\text{old}})}$$

This iterative method is used to find the implied volatility by solving the equation where the Black-Scholes model price equals the market price. The method involves iteratively refining an initial guess until convergence.

Volatility Surface Without Interpolation

The implied volatility is calculated directly for each combination of strike prices and maturities without any need for interpolation. Each data point on the volatility surface corresponds to a specific observed market price of an option. We have therefore used observed data directly.