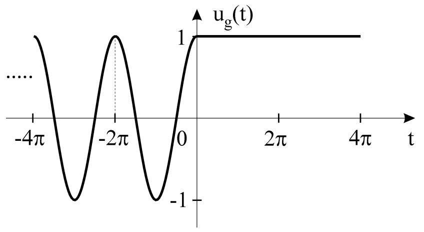
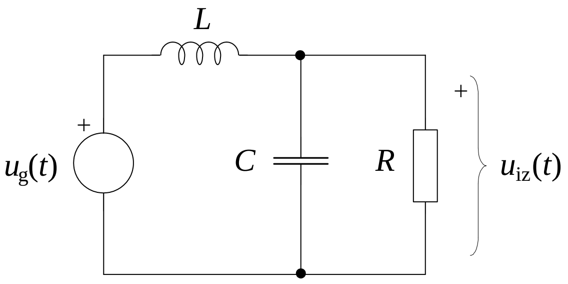
PISMENI ISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2012-2013

1. Za električni krug na slici zadane su normalizirane vrijednosti elemenata $R=1/\sqrt{2}$, $C=1$, $L=1$, te valni oblik pobude $u_g(t)$ prikazan slikom. Odrediti: a) valni oblik napona na kapacitetu $u_C(t)$ i struje kroz induktivitet $i_L(t)$ za $t<0$; b) početni napon $u_C(0)$ i struju $i_L(0)$; c) napon $U_{iz}(s)$; d) valni oblika napona $u_{iz}(t)$ za $t \geq 0$.





<u>Rješenje:</u>

a) <u>za $t<0$</u> napon generatora je svevremenska sinusoida koja se može opisati izrazom: $u_g(t)=\cos(\omega_0 t)$; $-\infty<t<\infty$, gdje se vidi iz slike: $T=2\pi$, $f_0=1/T=1/(2\pi)$ pa je $\omega_0=2\pi f_0=1$ [rad/s]. Stoga se mogu izračunati napon $u_C(t)$ i struja $i_L(t)$ korištenjem fazorskog računa.

$$I_L(j\omega)=\frac{U_g(j\omega)}{j\omega L+\dfrac{1}{j\omega C+1/R}};\quad U_C(j\omega)=U_{iz}(j\omega)=I_L(j\omega)\cdot\frac{1}{j\omega C+\dfrac{1}{R}};$$

$$U_g(j\omega)=1\angle 0^\circ \quad (\omega_0=1\text{rad/s})$$

$$U_C(j\omega)=\frac{U_g(j\omega)}{j\omega L+\dfrac{1}{j\omega C+1/R}}\cdot\frac{1}{j\omega C+\dfrac{1}{R}}=\frac{U_g(j\omega)}{j\omega L\left(j\omega C+\dfrac{1}{R}\right)+1}=\frac{U_g(j\omega)}{1-\omega^2 LC+j\omega\dfrac{L}{R}}$$

Uz uvrštene vrijednosti elemenata na frekvenciji signala ω_0:

$$I_L(j\omega)=\frac{U_g(j\omega)(j\omega C+1/R)}{j\omega L(j\omega C+1/R)+1}=\frac{U_g(j\omega)(j\omega C+1/R)}{1-\omega^2 LC+j\omega L/R}$$

$$I_L(j\omega)=\frac{1\angle 0^\circ\cdot\left(j+\sqrt{2}\right)}{1-1+j\sqrt{2}}=\frac{j+\sqrt{2}}{j\sqrt{2}}=\frac{1}{\sqrt{2}}-j$$

$$U_C(j\omega)=\frac{1}{j\sqrt{2}}=-j\frac{\sqrt{2}}{2}$$ **(1 bod)**

Za $t<0$ valni oblici struje kroz induktivitet $i_L(t)$ i napona na kapacitetu $u_C(t)$:

$$|I_L(j\omega)|=\sqrt{\left(\frac{1}{\sqrt{2}}\right)^2+1^2}=\sqrt{\frac{1}{2}+1}=\sqrt{\frac{3}{2}},$$

$$\angle I_L(j\omega)=\arctan\left(-\sqrt{2}\right)=-0.955317\text{rad}=-54.7356^\circ$$

$$\Rightarrow |U_C(j\omega)|=\frac{\sqrt{2}}{2},\quad \angle U_C(j\omega)=-\frac{\pi}{2}\text{rad}=-90^\circ$$

$$\underline{i_L(t)=\sqrt{3/2}\cos(\omega_0 t-54.7356^\circ)[A]}$$

$$\underline{u_C(t)=\sqrt{2}/2\cos(\omega_0 t-90^\circ)[V]}$$ **(1 bod)**

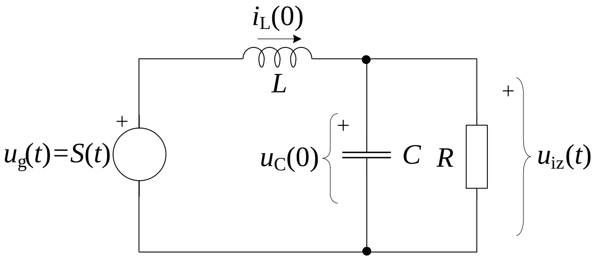
b) početni napon $u_C(0)$ i struja $i_L(0)$: u prethodne izraze uvrstimo $t=0$.

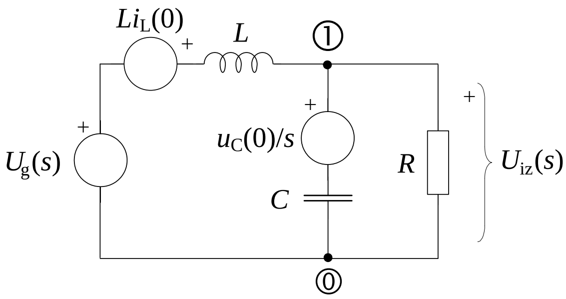
$$\underline{i_L(0)=\sqrt{3/2}\cos(-54.7356^\circ)=1/\sqrt{2}=\sqrt{2}/2=0.707[A]}$$

$$\underline{u_C(0)=\sqrt{2}/2\cos(-90^\circ)=0[V]}$$ **(1 bod)**

c) <u>za $t \geq 0$</u> Laplaceova transformacija.

Uz poznate početne uvjete $u_C(0)$ i $i_L(0)$, te pobudu $u_g(t)=S(t)$, za $t \geq 0$ električni krug u frekvencijskoj domeni izgleda ovako:





Napon čvorišta (1): $U_{iz}(s)\left(\frac{1}{sL}+sC+\frac{1}{R}\right)=\frac{U_g(s)+Li_L(0)}{sL}+Cu_C(0)$

$$\Rightarrow U_{iz}(s)=\frac{\frac{U_g(s)+Li_L(0)}{sL}+Cu_C(0)}{\left(\frac{1}{sL}+sC+\frac{1}{R}\right)}=\frac{U_g(s)+Li_L(0)+sLCu_C(0)}{s^2LC+s\frac{L}{R}+1}=\frac{U_g(s)\frac{1}{LC}+\frac{i_L(0)}{C}+su_C(0)}{s^2+s\frac{1}{RC}+\frac{1}{LC}}$$

$$U_{iz}(s)=\frac{U_g(s)\frac{1}{LC}+\frac{i_L(0)}{C}}{s^2+s\frac{1}{RC}+\frac{1}{LC}}=\frac{\frac{1}{s}+\frac{\sqrt{2}}{2}}{s^2+s\sqrt{2}+1}=\frac{s\frac{\sqrt{2}}{2}+1}{s\left(s^2+s\sqrt{2}+1\right)}$$ **(1 bod)**

d) Povratak u vremensku domenu (rastav na parcijalne razlomke):

$$U_{iz}(s)=\frac{s\cdot\sqrt{2}/2+1}{s\cdot\left(s^2+s\sqrt{2}+1\right)}=\frac{A}{s}+\frac{Bs+C}{s^2+s\sqrt{2}+1}=\frac{As^2+As\sqrt{2}+A+Bs^2+Cs}{s\left(s^2+s\sqrt{2}+1\right)}$$

$$(A+B)s^2+(A\sqrt{2}+C)s+A=s\cdot\sqrt{2}/2+1\Rightarrow$$

$$A=1,$$
$$A+B=0\Rightarrow B=-A=-1,$$
$$C+A\sqrt{2}=\sqrt{2}/2\Rightarrow C=-\sqrt{2}+\sqrt{2}/2=-\sqrt{2}/2$$

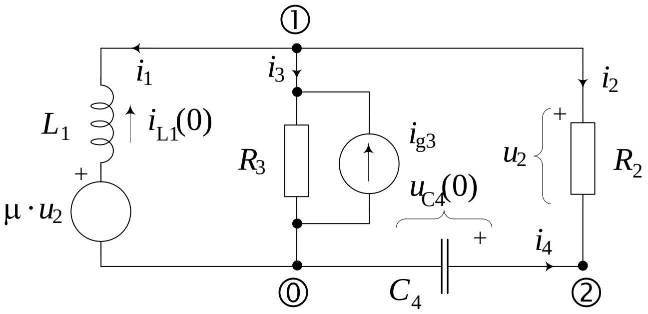
Polovi:

$$s^2+s\sqrt{2}+1=0\Rightarrow s_{p1,2}=-\frac{\sqrt{2}}{2}\pm\sqrt{\frac{1}{2}-1}=-\frac{\sqrt{2}}{2}\pm j\sqrt{\frac{1}{2}}=-\frac{\sqrt{2}}{2}\pm j\frac{\sqrt{2}}{2}$$

$$s=0\Rightarrow s_{p3}=0$$

$$U_{iz}(s)=\frac{1}{s}-\frac{\left(s+\frac{\sqrt{2}}{2}\right)}{\left(s+\frac{\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2}\qquad\Rightarrow\qquad u_{iz}(t)=\left[1-e^{-\frac{\sqrt{2}}{2}t}\cos\left(\frac{\sqrt{2}}{2}t\right)\right]\cdot S(t)$$ **(1 bod)**

2. Za električni krug na slici i pridruženim orijentacijama grana zadane su normalizirane vrijednosti elemenata L_1=1, R_2=1, R_3=1, C_4=1, te μ=2, $u_{C4}(0)$=1, $i_{L1}(0)$=1, $i_{g3}(t)=S(t)$. Koristeći KZS i KZN te oznake grana i čvorova prema slici, napisati: a) Jednadžbe KZS i KZN (odabrati referentne smjerove petlji u smjeru kazaljke na satu); b) Naponsko-strujne jednadžbe za grane; c) Napon na otporu R_2 $U_2(s)$; d) Napon na otporu R_2 $u_2(t)$; e) Da li je električni krug stabilan? Zašto?

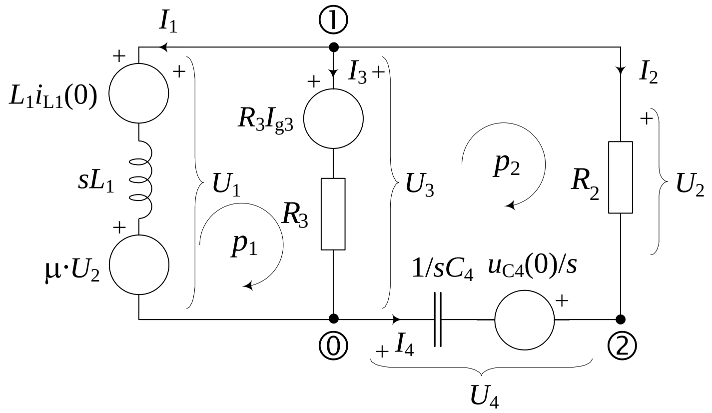


<u>Rješenje:</u> Laplaceova transformacija

a)

N_b=4 (broj grana)

N_v=3 (broj čvorova)



Broj jednadžbi

$\text{KZS} = N_v - 1 = 3 - 1 = 2$

Broj jednadžbi

$\text{KZN} = N_b - N_v + 1 = 4 - 3 + 1 = 2$

Slijede jednadžbe Kirchhoffovih zakona (4 jednadžbe):

1) $I_1 + I_2 + I_3 = 0$ KZS čvorište (1)

2) $-I_2 - I_4 = 0$ KZS čvorište (2)

3) $-U_1 + U_3 = 0$ KZN petlja p_1

4) $U_2 - U_3 - U_4 = 0$ KZN petlja p_2 **(1 bod)**

b) Naponsko–strujne jednadžbe grana (4 jednadžbe):

1) $U_1 = sL_1 \cdot I_1 + L_1 i_{L1}(0) + \mu U_2 = sL_1 \cdot I_1 + \mu R_2 \cdot I_2 + L_1 i_{L1}(0)$

2) $U_2 = R_2 \cdot I_2$

3) $U_3 = R_3 \cdot I_3 + I_{g3} R_3$

4) $U_4 = \frac{1}{sC_4} \cdot I_4 - \frac{u_{C4}(0)}{s}$ **(1 bod)**

c) Sustav ima ukupno $2N_b$=8 jednadžbi i 8 nepoznanica (sve struje i svi naponi grana) Naponsko – strujne jednadžbe grana uvrstimo u jednadžbe Kirchhoffovih zakona (1)–(4) te dobivamo:

1) $I_1 = -I_2 - I_3$

2) $I_2 = -I_4$

3) $-sL_1 \cdot I_1 - \mu R_2 \cdot I_2 - L_1 i_{L1}(0) + R_3 \cdot I_3 + I_{g3} R_3 = 0$

4) $R_2 \cdot I_2 - R_3 \cdot I_3 - I_{g3} R_3 - \frac{1}{sC_4} \cdot I_4 + \frac{u_{C4}(0)}{s} = 0$

$$1)\rightarrow 3)\Rightarrow\ sL_1\cdot(I_2+I_3)-\mu R_2\cdot I_2-L_1 i_{L1}(0)+R_3\cdot I_3+I_{g3}R_3=0$$

$$2)\rightarrow 4)\Rightarrow\ R_2\cdot I_2-R_3\cdot I_3-I_{g3}R_3+\frac{1}{sC_4}\cdot I_2+\frac{u_{C4}(0)}{s}=0\ \Rightarrow$$

$$(1')\ \ I_2(sL_1-\mu R_2)+I_3(sL_1+R_3)=L_1 i_{L1}(0)-I_{g3}R_3$$

$$(2')\ \left(R_2+\frac{1}{sC_4}\right)\cdot I_2-R_3\cdot I_3=I_{g3}R_3-\frac{u_{C4}(0)}{s}$$

$\Rightarrow I_2(s),\ I_3(s)$ koristimo metodu determinanti:

$$\begin{bmatrix} sL_1-\mu R_2 & sL_1+R_3 \\ R_2+\dfrac{1}{sC_4} & -R_3 \end{bmatrix}\cdot\begin{bmatrix} I_2(s) \\ I_3(s)\end{bmatrix}=\begin{bmatrix} L_1 i_{L1}(0)-I_{g3}R_3 \\ I_{g3}R_3-\dfrac{u_{C4}(0)}{s}\end{bmatrix}$$

$$\Delta=\begin{vmatrix} sL_1-\mu R_2 & sL_1+R_3 \\ R_2+\dfrac{1}{sC_4} & -R_3 \end{vmatrix}=-R_3(sL_1-\mu R_2)-(sL_1+R_3)\left(R_2+\frac{1}{sC_4}\right)$$

$$\Delta=-R_3 sL_1+\mu R_2R_3-sL_1R_2-R_3R_2-\frac{L_1}{C_4}-R_3\frac{1}{sC_4}=-s+2-s-1-1-\frac{1}{s}=-2s-\frac{1}{s}$$

$$\Delta_2=\begin{vmatrix} L_1 i_{L1}(0)-I_{g3}R_3 & sL_1+R_3 \\ I_{g3}R_3-\dfrac{u_{C4}(0)}{s} & -R_3 \end{vmatrix}=-R_3\left(L_1 i_{L1}(0)-I_{g3}R_3\right)-(sL_1+R_3)\left(I_{g3}R_3-\frac{u_{C4}(0)}{s}\right)$$

$$\Delta_2=-R_3L_1 i_{L1}(0)+R_3^2 I_{g3}-sL_1I_{g3}R_3-R_3^2 I_{g3}+sL_1\frac{u_{C4}(0)}{s}+R_3\frac{u_{C4}(0)}{s}=$$

$$=-R_3L_1 i_{L1}(0)-sL_1I_{g3}R_3+L_1u_{C4}(0)+R_3\frac{u_{C4}(0)}{s}=-1-s\cdot\frac{1}{s}+1+\frac{1}{s}=-1+\frac{1}{s}$$

$$I_2(s)=\frac{\Delta_2}{\Delta}=\frac{-1+\dfrac{1}{s}}{-2s-\dfrac{1}{s}}=\frac{1-\dfrac{1}{s}}{2s+\dfrac{1}{s}}=\frac{s-1}{2s^2+1};\quad R_2=1$$

$$U_2(s)=I_2(s)R_2=\frac{s-1}{2s^2+1}$$ **(1 bod)**

d) Napon na otporu R_2 $u_2(t)$:

$$U_2(s)=\frac{1}{2}\cdot\frac{s-1}{s^2+\frac{1}{2}}=\frac{1}{2}\cdot\frac{s}{s^2+\frac{1}{2}}-\frac{1}{2}\cdot\frac{1}{s^2+\frac{1}{2}}=\frac{1}{2}\cdot\frac{s}{s^2+\left(\frac{1}{\sqrt{2}}\right)^2}-\frac{1}{\sqrt{2}}\cdot\frac{\frac{1}{\sqrt{2}}}{s^2+\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$u_2(t)=\left[\frac{1}{2}\cdot\cos\left(\frac{t}{\sqrt{2}}\right)-\frac{1}{\sqrt{2}}\cdot\sin\left(\frac{t}{\sqrt{2}}\right)\right]S(t)$$ **(1 bod)**

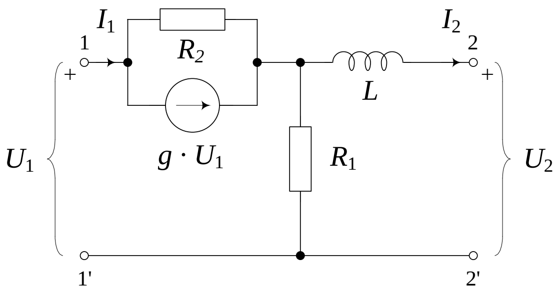
e) Stabilnost:

Električni krug je marginalno stabilan (na rubu stabilnosti).

Polovi $s^2+\frac{1}{2}=0\Rightarrow s_{p1,2}=\pm j\frac{\sqrt{2}}{2}$ su jednostruki i nalaze se na imaginarnoj osi.

(1 bod)

3. Za četveropol na slici izračunati: a) [a]-parametre. Zadano je R_1=1/2, R_2=1/2, L=1/2, g=2. b) Da li je četveropol: recipročan, simetričan ? Obrazložiti odgovore. Ako je izlazni prilaz (2–2') zaključen otporom R_L=1 pomoću [a]-parametara izračunati: c) ulaznu impedanciju $Z_{ul1}(s)=U_1(s)/I_1(s)$; d) ako je uz to na ulaz priključen generator ulaznog otpora R_g=1 izračunati prijenosnu funkciju napona $H(s)=U_2(s)/U_g(s)$.

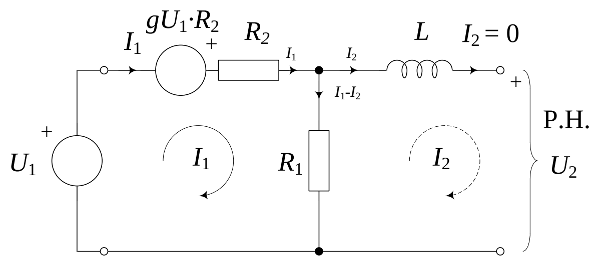


Rješenje:

a) [a]-parametri:

$$U_1 = A \cdot U_2 + B \cdot I_2$$
$$I_1 = C \cdot U_2 + D \cdot I_2$$

$$I_2 = 0 \qquad A = \left.\frac{U_1}{U_2}\right|_{I_2=0} \; ; \quad C = \left.\frac{I_1}{U_2}\right|_{I_2=0}$$



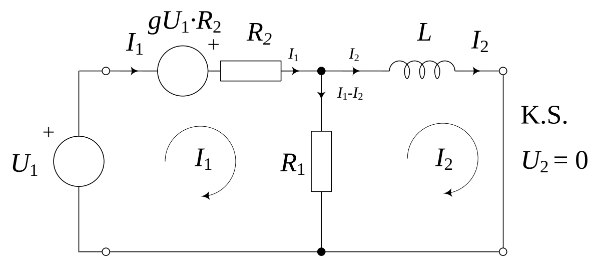
$$I_1 \cdot (R_1 + R_2) - g \cdot U_1 \cdot R_2 = U_1 \Rightarrow I_1 = \frac{g \cdot U_1 \cdot R_2 + U_1}{R_1 + R_2}$$

$$U_2 = I_1 \cdot R_1$$

$$\Rightarrow U_2 = U_1 \cdot (1 + gR_2) \cdot \frac{R_1}{R_1 + R_2} \qquad A = \left.\frac{U_1}{U_2}\right|_{I_2=0} = \frac{\frac{R_1 + R_2}{1 + gR_2}}{R_1} = \frac{\frac{1/2 + 1/2}{1+1}}{1/2} = \frac{1/2}{1/2} = 1$$

$$\Rightarrow U_2 = I_1 \cdot R_1, \qquad C = \left.\frac{I_1}{U_2}\right|_{I_2=0} = \frac{1}{R_1} = \frac{1}{1/2} = 2$$

$$U_2 = 0 \qquad B = \left.\frac{U_1}{I_2}\right|_{U_2=0} \; ; \quad D = \left.\frac{I_1}{I_2}\right|_{U_2=0}$$



$$I_1 \cdot (R_1 + R_2) - I_2 \cdot R_1 = U_1 + g \cdot U_1 \cdot R_2$$
$$- I_1 \cdot R_1 + I_2 \cdot (R_1 + sL) = 0$$

$$(2) \Rightarrow \; I_1 = I_2 \cdot \frac{R_1 + sL}{R_1}$$

$$(2) \rightarrow (1) \Rightarrow \; I_2 \cdot \frac{R_1 + sL}{R_1} \cdot (R_1 + R_2) - I_2 \cdot R_1 = U_1 \cdot (1 + gR_2)$$

$$I_2 \cdot \left[\left(1 + \frac{sL}{R_1} \right) \cdot (R_1 + R_2) - R_1 \right] = U_1 \cdot (1 + gR_2)$$

$$I_2 \cdot \left[R_1 + R_2 + sL + R_2 \frac{sL}{R_1} - R_1 \right] = U_1 \cdot (1 + gR_2)$$

$$B = \left. \frac{U_1}{I_2} \right|_{U_2=0} = \frac{R_2 + sL + R_2 \dfrac{sL}{R_1}}{1 + gR_2} = \frac{\dfrac{sLR_1 + R_2(R_1 + sL)}{1 + gR_2}}{R_1}$$

$$= \frac{\dfrac{s \cdot 1/4 + (1/2) \cdot (1/2 + s \cdot 1/2)}{1+1}}{1/2} = \frac{s \cdot 1/4 + (1/4) \cdot (1+s)}{1/4} = s + (1+s) = 2s + 1$$

$$I_1 = I_2 \cdot \frac{R_1 + sL}{R_1} \qquad D = \left. \frac{I_1}{I_2} \right|_{U_2=0} = \frac{R_1 + sL}{R_1} = \frac{1/2 + s \cdot 1/2}{1/2} = s + 1$$

Uvrstimo vrijednosti elemenata R_1=1/2, R_2=1/2, L=1/2, g=2.:

$$[a] = \begin{bmatrix} 1 & 2s+1 \\ 2 & s+1 \end{bmatrix}$$ **(2 boda)**

b) Da li je četveropol recipročan, simetričan? **(1bod)**

Za recipročnost vrijedi: $\Delta = AD - BC = 1$

$$\Delta = s + 1 - 4s - 2 = -3s - 1 \quad \Rightarrow$$ Čeveropol nije recipročan.

Za simetričnost vrijedi: A=D $\Rightarrow 1 \neq s + 1 \quad \Rightarrow$ Četveropol nije simetričan

Konačno iz jednadžbi $\begin{array}{l} U_1 = A \cdot U_2 + B \cdot I_2 \\ I_1 = C \cdot U_2 + D \cdot I_2 \end{array}$, $R_L = \dfrac{U_2}{I_2}$, $U_g = I_1 R_g + U_1$ slijede:

c) Ulazna impedancija u četveropol:

$$Z_{ul1}(s) = \frac{U_1}{I_1} = \frac{AU_2 + BI_2}{CU_2 + DI_2} = \frac{A\dfrac{U_2}{I_2} + B}{C\dfrac{U_2}{I_2} + D} = \frac{AR_L + B}{CR_L + D}$$

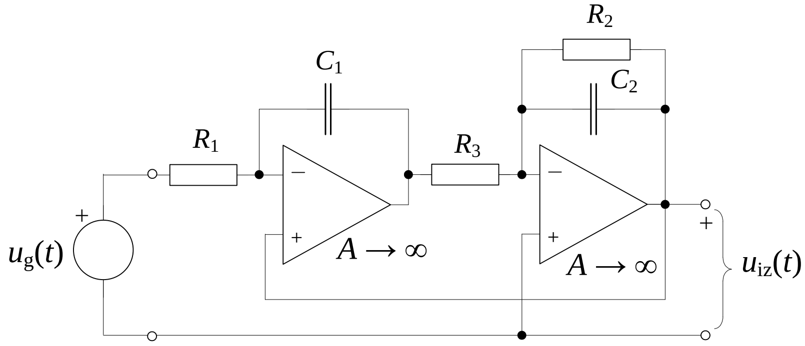
$$\Rightarrow Z_{ul1}(s) = \frac{1 \cdot 1 + 2s + 1}{2 \cdot 1 + s + 1} = \frac{2s + 2}{s + 3} = 2 \cdot \frac{s+1}{s+3}$$ **(1 bod)**

d) Prijenosna funkcija napona:

$$U_g = I_1 R_g + U_1 = \left(CU_2 + D\frac{U_2}{R_L} \right) R_g + AU_2 + B\frac{U_2}{R_L} \Rightarrow H(s) = \frac{U_2}{U_g} = \frac{R_L}{AR_L + B + R_g(CR_L + D)} \Rightarrow$$

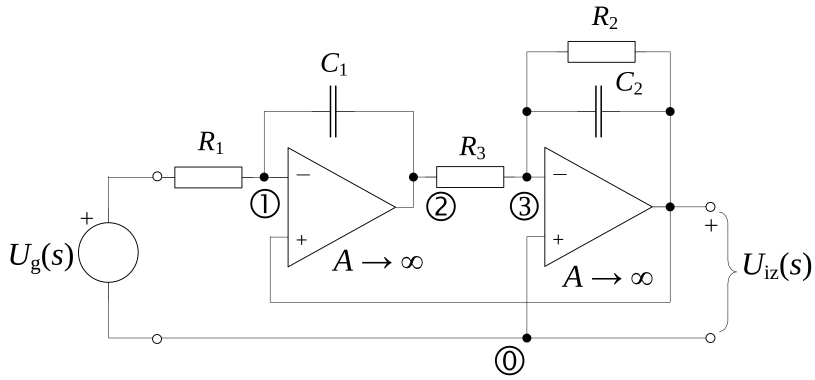
$$H(s) = \frac{1}{1 \cdot 1 + 2s + 1 + 1 \cdot [2 + s + 1]} = \frac{1}{2s + 2 + s + 3} = \frac{1}{3s + 5}$$ **(1 bod)**

4. Zadan je aktivni-RC električni filtar prikazan slikom. a) Izračunati njegovu naponsku prijenosnu funkciju $T(s)=U_{iz}(s)/U_g(s)$. b) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja odrediti parametre k, ω_0, Q. O kojem se tipu filtra radi (NP, VP, PP ili PB)? c) Ako su zadane normalizirane vrijednosti elemenata $C_1=1/2$, $C_2=2$, te $R_1=1$, $R_2=R_3=4$, izračunati parametre ω_0, Q i pojačanje k. d) Prikazati raspored polova i nula u kompleksnoj ravnini. e) Nacrtati amplitudno-frekvencijsku karakteristiku.



Rješenje:

a) Metoda čvorišta:



$$(1)\quad U_1\left(sC_1+\frac{1}{R_1}\right)-U_2sC_1=U_g\frac{1}{R_1}$$

$$(2)\quad -U_2\frac{1}{R_3}+U_3\left(sC_2+\frac{1}{R_2}+\frac{1}{R_3}\right)=U_{iz}\left(sC_2+\frac{1}{R_2}\right)$$

$$(3)\quad A(U_{iz}-U_1)=U_2,\ A\to\infty\Rightarrow U_1=U_{iz}$$

$$(4)\quad A(-U_3)=U_{iz},\ A\to\infty\Rightarrow U_3=0$$

$$(1)\quad U_{iz}\left(sC_1+\frac{1}{R_1}\right)-U_2sC_1=U_g\frac{1}{R_1}$$

$$(2)\quad -U_2\frac{1}{R_3}=U_{iz}\left(sC_2+\frac{1}{R_2}\right)$$

$$(2)\Rightarrow U_2=-U_{iz}\left(sC_2R_3+\frac{R_3}{R_2}\right)\to(1)\Rightarrow$$

$$U_{iz}\left(sC_1+\frac{1}{R_1}\right)+U_{iz}\left(sC_2R_3+\frac{R_3}{R_2}\right)sC_1=U_g\frac{1}{R_1}\Big/\cdot R_1$$

$$U_{iz}\left(sR_1C_1+1\right)+U_{iz}\left(sC_2R_3+\frac{R_3}{R_2}\right)sR_1C_1=U_g$$

$$U_{iz}\left(s^2R_1C_1C_2R_3+sR_1C_1\frac{R_3}{R_2}+sR_1C_1+1\right)=U_g\,;T(s)=\frac{U_{iz}}{U_g}=\frac{1}{s^2R_1C_1C_2R_3+sR_1C_1\frac{R_3}{R_2}+sR_1C_1+1}$$

$$T(s)=\frac{U_{iz}(s)}{U_g(s)}=\frac{\frac{1}{R_1C_1R_3C_2}}{s^2+s\frac{R_3/R_2+1}{C_2R_3}+\frac{1}{R_1C_1R_3C_2}}\quad\Leftrightarrow\quad T(s)=\frac{k\cdot\omega_0^{\,2}}{s^2+\frac{\omega_0}{Q}\cdot s+\omega_0^{\,2}}\ \text{(Opći oblik NP)}$$

(1 bod)

b) -o kojem se tipu filtra radi (NP, VP, PP ili PB)? $\Rightarrow$ NP (niski propust)

-parametri k, ω_0, Q:

$$\underline{\omega_0=\frac{1}{\sqrt{R_1R_3C_1C_2}}};\ \underline{k=1};\qquad \frac{\omega_0}{Q}=\frac{R_3/R_2+1}{R_3C_2}\Rightarrow$$

$$Q=\frac{\omega_0}{\frac{R_3/R_2+1}{R_3C_2}}=\frac{R_3C_2}{R_3/R_2+1}\cdot\frac{1}{\sqrt{R_1R_3C_1C_2}}=\underline{\frac{1}{R_3/R_2+1}\cdot\sqrt{\frac{R_3C_2}{R_1C_1}}}$$ **(1 bod)**

c) Ako su zadane normalizirane vrijednosti elemenata C_1=0.5, C_2=2, te R_1=1, R_2=R_3=4, izračunati parametre ω_0, Q i pojačanje k. **(1 bod)**

$$\omega_0=\frac{1}{\sqrt{4\cdot0.5\cdot2}}=\frac{1}{2},\ Q=\frac{1}{1+1}\cdot\sqrt{\frac{4\cdot2}{1\cdot0.5}}=\frac{\sqrt{16}}{2}=\frac{4}{2}=2,\ k=1$$

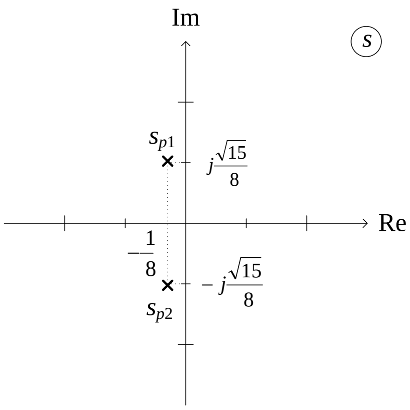
d) raspored polova i nula u kompleksnoj ravnini: **(1 bod)**

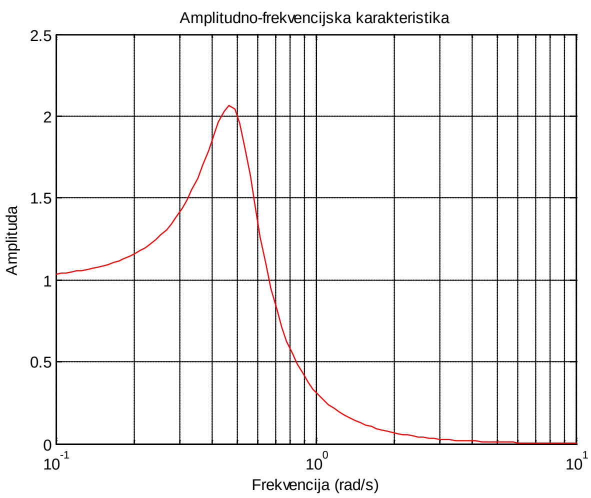
$$T(s)=\frac{\frac{1}{4}}{s^2+\frac{1/2}{2}s+\frac{1}{4}}=\frac{\frac{1}{4}}{s^2+\frac{1}{4}s+\frac{1}{4}}\qquad \text{nule}\quad s_{o1}=\infty,s_{o2}=\infty$$

$$\text{polovi}\quad s^2+\frac{1}{4}s+\frac{1}{4}=0\Rightarrow\qquad s_{p1,2}=-\frac{1}{8}\pm\sqrt{\frac{1}{64}-\frac{1}{4}}=-\frac{1}{8}\pm\sqrt{\frac{1-16}{64}}=-\frac{1}{8}\pm j\frac{\sqrt{15}}{8}$$

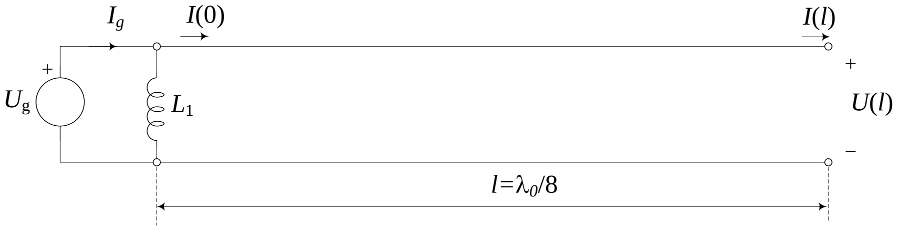
e) amplitudno-frekvencijska karakteristika: **(1 bod)**

$$s{=}j\omega\ \Rightarrow\ T(j\omega)=\frac{\frac{1}{4}}{-\omega^2+j\frac{1}{4}\omega+\frac{1}{4}}\qquad\Rightarrow\qquad \left|T(j\omega)\right|=\frac{\frac{1}{4}}{\sqrt{\left(\frac{1}{4}-\omega^2\right)^2+\left(\frac{1}{4}\omega\right)^2}}$$





5. Zadana je linija bez gubitaka s L=0,2 mH/km i C=80 nF/km. Na ulaz linije je priključen naponski izvor $u_0(t)$ = 20 cos(ω_0t) paralelno s induktivitetom L_1=0,5 mH. Duljina linije je $l=\lambda_0/8$, gdje je λ_0 valna duljina signala pri frekvenciji ω_0. Izlaz linije je otvoren. Odrediti: a) karakterističnu impedanciju Z_0; b) ulaznu impedanciju Z_{ul} linije i frekvenciju ω_0 na kojoj je struja I_g jednaka nuli; c) koeficijent prijenosa γ linije, duljinu linije l u km, valnu duljinu λ_0 signala frekvencije ω_0 i brzinu širenja vala na liniji v; d) struju $I(0)$ na ulazu u liniju; e) napon $U(l)$ i struju $I(l)$ na kraju linije.



Rješenje:

a) Linija bez gubitaka $\rightarrow$ $R=0,\ G=0 \quad \Rightarrow \quad Z_0=\sqrt{L/C},\ \gamma=s\sqrt{LC}$

Stac. sinusna pobuda $\rightarrow$ $s=j\omega \quad \Rightarrow \quad \gamma=j\omega\sqrt{LC}=j\beta$

$$Z_0=\sqrt{L/C}=\sqrt{2\cdot10^{-4}/8\cdot10^{-8}}=\sqrt{0{,}25\cdot10^{4}}=50\Omega$$ **(1 bod)**

b) $I(l)=0$

$$U(0)=U(l)\cdot ch(\gamma l)+I(l)Z_0 sh(\gamma l)=U(l)\cdot\cos(\beta l)+jI(l)Z_0\sin(\beta l)=U(l)\cdot\cos(\beta l)$$

$$I(0)=\frac{U(l)}{Z_0}sh(\gamma l)+I(l)ch(\gamma l)=\frac{U(l)}{Z_0}j\sin(\beta l)+I(l)\cos(\beta l)=\frac{U(l)}{Z_0}j\sin(\beta l)$$

$$Z_{ul}=\frac{U(0)}{I(0)}=-jZ_0\mathrm{ctg}(\beta l) \quad \Rightarrow \quad Y_{ul}=j\frac{\mathrm{tg}(\beta l)}{Z_0}=j\frac{\mathrm{tg}(\pi/4)}{Z_0}=j\frac{1}{Z_0}$$

$$Y_{ul}+\frac{1}{j\omega L_1}=j\frac{1}{Z_0}-j\frac{1}{\omega L_1}=0 \Rightarrow \quad \omega=\omega_0=\frac{Z_0}{L_1}=\frac{50}{0{,}5\cdot10^{-3}}=10^5[\mathrm{rad/s}]$$ **(1 bod)**

c) $$\gamma=j\omega_0\sqrt{LC}=j10^5\sqrt{2\cdot10^{-4}\cdot8\cdot10^{-8}}=j4\cdot10^{-1}=j0{,}4/\mathrm{km}$$

$$l=\lambda_0/8=\frac{2\pi}{8\beta}=\frac{\pi}{4\omega_0\sqrt{LC}}=\pi\cdot0{,}625\,\mathrm{km}=1{,}9635\mathrm{km}$$

$$\lambda_0=8l=\pi\cdot5\,\mathrm{km}=15{,}708\mathrm{km}$$

$$v=\frac{\omega_0}{\beta}=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{2\cdot10^{-4}\cdot8\cdot10^{-8}}}=2{,}5\cdot10^5\mathrm{km/s}$$ **(1 bod)**

d) $U_g=20\angle0^\circ$

$$I(0)=\frac{U(0)}{Z_{ul}}=\frac{U_g}{-jZ_0}=j\frac{U_g}{Z_0}=j\frac{20}{50}=0{,}4\angle90^\circ\ \mathrm{A}$$ **(1 bod)**

e) $$U(0)=U(l)\cdot\cos(\beta\cdot l)=U(l)\cdot\frac{\sqrt{2}}{2}$$

$$U(l)=U(0)\cdot\sqrt{2}=U_g\cdot\sqrt{2}=20\cdot\sqrt{2}\angle0^\circ\ \mathrm{V}$$ **(1 bod)**