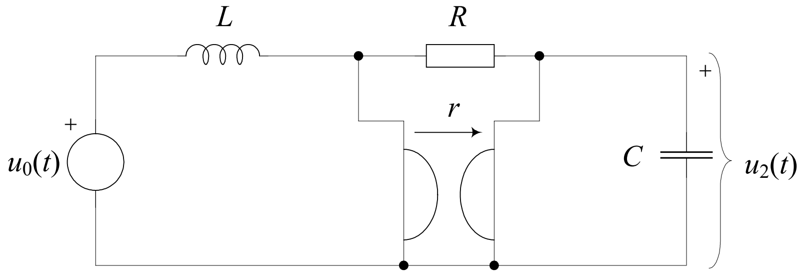
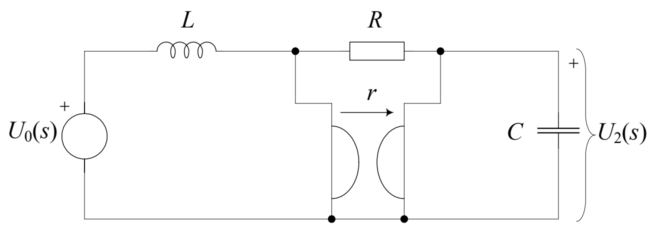
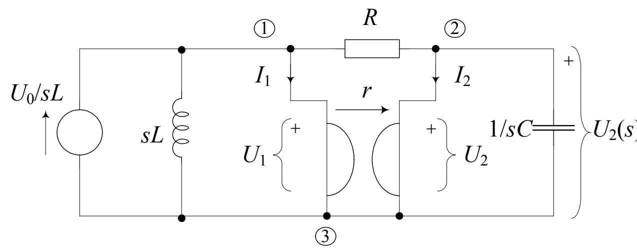
DRUGI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA

1. Za krug prikazan slikom izračunati odziv $u_2(t)$ ako je zadana pobuda $u_0(t)=2\delta(t)$, konstanta giratora $r=2$, a normirane vrijednosti elemenata su: $R=1$, $L=2$ i $C=1/2$. Početni uvjeti su jednaki nuli.



Rješenje: Jednadžbe čvorišta





(1) $$\frac{U_0(s)}{sL}-I_1(s)=\left(\frac{1}{sL}+\frac{1}{R}\right)U_1(s)-\frac{1}{R}U_2(s)$$

(2) $$-I_2(s)=-\frac{1}{R}U_1(s)+\left(sC+\frac{1}{R}\right)U_2(s)$$

$$U_1(s)=rI_2(s)\quad\Rightarrow\quad I_2(s)=\frac{1}{r}U_1(s)$$

$$U_2(s)=-rI_1(s)\quad\Rightarrow\quad I_1(s)=-\frac{1}{r}U_2(s)$$

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(1) $$\frac{U_0(s)}{sL}+\frac{U_2(s)}{r}=\left(\frac{1}{sL}+\frac{1}{R}\right)U_1(s)-\frac{1}{R}U_2(s)$$

(2) $$-\frac{U_1(s)}{r}=-\frac{1}{R}U_1(s)+\left(sC+\frac{1}{R}\right)U_2(s)$$

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(1) $$\frac{U_0(s)}{sL}=\left(\frac{1}{sL}+\frac{1}{R}\right)U_1(s)-\left(\frac{1}{R}+\frac{1}{r}\right)U_2(s)$$

(2) $$0=-\left(\frac{1}{R}-\frac{1}{r}\right)U_1(s)+\left(sC+\frac{1}{R}\right)U_2(s)$$

$$\Delta=\begin{vmatrix}\frac{1}{sL}+\frac{1}{R} & -\left(\frac{1}{R}+\frac{1}{r}\right)\\ -\left(\frac{1}{R}-\frac{1}{r}\right) & sC+\frac{1}{R}\end{vmatrix}=\left(\frac{1}{sL}+\frac{1}{R}\right)\left(sC+\frac{1}{R}\right)-\left(\frac{1}{R}+\frac{1}{r}\right)\left(\frac{1}{R}-\frac{1}{r}\right)=\frac{sC}{sL}+\frac{sC}{R}+\frac{1}{RsL}+\frac{1}{R^2}-\frac{1}{R^2}+\frac{1}{r^2}$$

$$=\frac{sC}{sL}+\frac{sC}{R}+\frac{1}{RsL}+\frac{1}{r^2}$$

$$\Delta_2 = \begin{vmatrix} \dfrac{1}{sL}+\dfrac{1}{R} & \dfrac{U_0}{sL} \\ -\left(\dfrac{1}{R}-\dfrac{1}{r}\right) & 0 \end{vmatrix} = \frac{U_0}{sL}\left(\frac{1}{R}-\frac{1}{r}\right)$$

Uvrstimo vrijednosti: r=2, R=1, L=2 i C=1/2, $U_0(s)$=2.

$$\Delta = \frac{1}{4}+\frac{1}{2}s+\frac{1}{2}\cdot\frac{1}{s}+\frac{1}{4} = \frac{1}{2}+\frac{1}{2}s+\frac{1}{2s}$$

$$\Delta_2 = \frac{2}{2s}\left(1-\frac{1}{2}\right) = \frac{1}{2s}$$

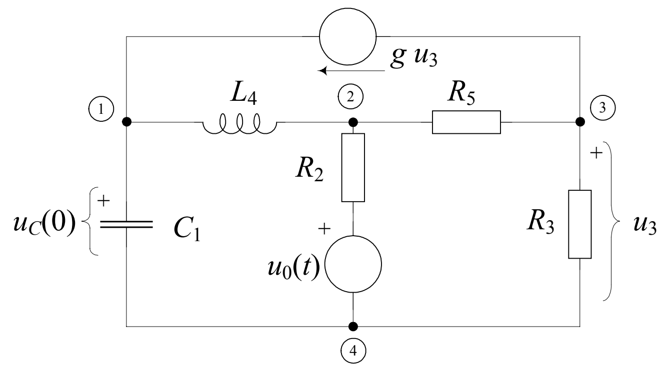
$$U_2(s) = \frac{\Delta_2}{\Delta} = \frac{\frac{1}{2s}}{\frac{1}{2}+\frac{1}{2}s+\frac{1}{2s}} = \frac{1}{s^2+s+1}$$

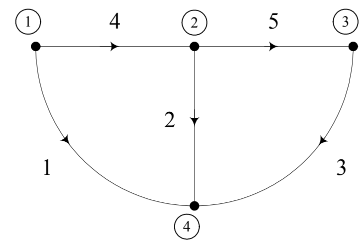
$$s^2+s+1=0 \Rightarrow s_{1,2} = \frac{-1\pm\sqrt{1-4}}{2} = -\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$$

$$U_2(s) = \frac{1}{s^2+s+\frac{1}{4}+\frac{3}{4}} = \frac{2}{\sqrt{3}}\cdot\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

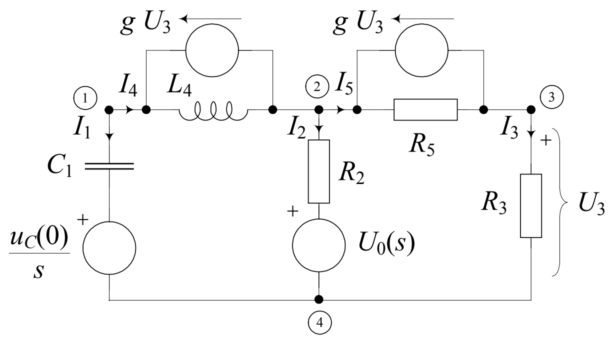
$$u_2(t) = \mathscr{L}^{-1}[U_2(s)] = \frac{2}{\sqrt{3}}\cdot e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right)\cdot S(t)$$

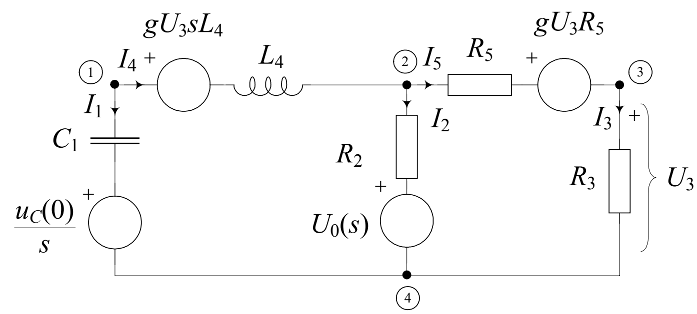
2. Za krug prikazan slikom i pridruženi orijentirani graf napisati temeljni sustav jednadžbi petlji u matričnom obliku (odrediti matrice $\mathbf{Z}_p$ i $\mathbf{U}_{0p}$ preko matrica impedancija grana $\mathbf{Z}_b$ i nezavisnih izvora grana $\mathbf{U}_{0b}$). Matrica $\mathbf{Z}_b$ mora biti regularna. (Uputa: grane stabla: 1, 2 i 3.)



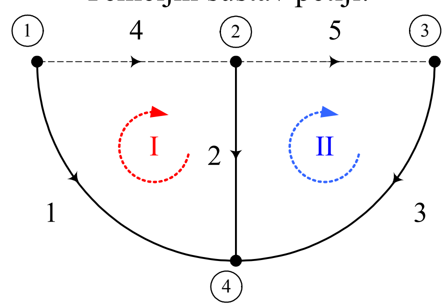


Rješenje: Primjena Laplaceove transformacije i posmicanje strujnog izvora





Temeljni sustav petlji:



Temeljna spojna matrica: $\mathbf{S} = \begin{bmatrix} -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 \end{bmatrix},$

Naponsko – strujne relacije grana:

$$\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$$

$$U_1 = I_1 \cdot \frac{1}{sC_1} + \frac{u_C(0)}{s}$$

$$U_2 = I_2 \cdot R_2 + U_0(s)$$

$$U_3 = I_3 \cdot R_3$$

$$U_4 = I_4 \cdot sL_4 + g \cdot U_3 \cdot sL_4 = gR_3 sL_4 \cdot I_3 + I_4 \cdot sL_4$$

$$U_5 = gU_3 \cdot R_5 + I_5 \cdot R_5 = gI_3 \cdot R_3 \cdot R_5 + I_5 \cdot R_5$$

$$\mathbf{Z}_b = \begin{bmatrix} \frac{1}{sC_1} & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 \\ 0 & 0 & gR_3sL_4 & sL_4 & 0 \\ 0 & 0 & gR_3R_5 & 0 & R_5 \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} \frac{u_C(0)}{s} \\ U_0(s) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Matrica $\mathbf{Z}_b$ je regularna jer nema niti jedan redak niti stupac jednak nuli.

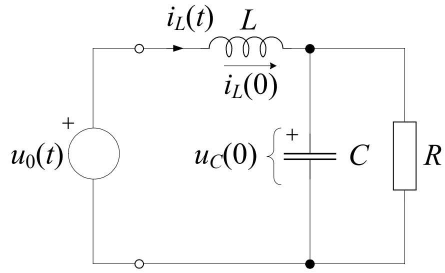
Temeljni sustav jednadžbi petlji: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$

$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \begin{bmatrix} -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{sC_1} & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 \\ 0 & 0 & gR_3sL_4 & sL_4 & 0 \\ 0 & 0 & gR_3R_5 & 0 & R_5 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{1}{sC_1} & R_2 & gR_3sL_4 & sL_4 & 0 \\ 0 & -R_2 & R_3 & 0 & R_5 \end{bmatrix} \cdot \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{sC_1} + R_2 + sL_4 & -R_2 + gR_3sL_4 \\ -R_2 & R_2 + R_3 + R_5 \end{bmatrix}$$

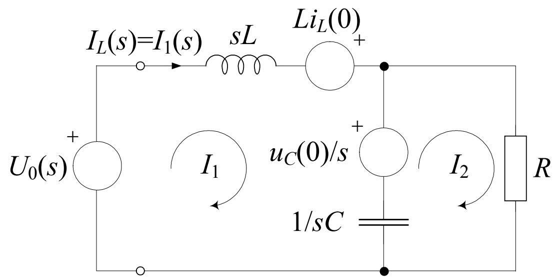
$$\mathbf{U}_{0p} = -\mathbf{S} \cdot \mathbf{U}_{0b} = -\begin{bmatrix} -1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{u_C(0)}{s} \\ U_0(s) \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{u_C(0)}{s} - U_0(s) \\ U_0(s) \end{bmatrix}$$

3. Odrediti odziv $i_L(t)$ mreže prema slici za $t \geq 0$, ako su zadane normirane vrijednosti elemenata: $R=1$, $L=2$ i $C=1$, početni uvjeti u mreži $u_C(0)=-1/5$ i $i_L(0)=1/5$, a pobuda je $u_0(t)=S(t)$.



Rješenje:

Za $t \geq 0$: Primjena Laplaceove transformacije → Jednadžbe petlji



(1) $$U_0(s)+Li_L(0)-\frac{u_C(0)}{s}=\left(sL+\frac{1}{sC}\right)I_1(s)-\frac{1}{sC}I_2(s)$$

(2) $$\frac{u_C(0)}{s}=-\frac{1}{sC}I_1(s)+\left(\frac{1}{sC}+R\right)I_2(s)$$

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(2) $$\Rightarrow \quad I_2(s)\frac{1+sRC}{sC}=\frac{1}{sC}I_1(s)+\frac{u_C(0)}{s} \quad \Rightarrow \quad I_2(s)=\frac{1}{1+sRC}I_1(s)+\frac{Cu_C(0)}{1+sRC} \quad \rightarrow \quad (1) \quad \Rightarrow$$

$$U_0(s)+Li_L(0)-\frac{u_C(0)}{s}=\frac{s^2LC+1}{sC}I_1(s)-\frac{1}{sC}\cdot\frac{I_1(s)+Cu_C(0)}{1+sRC}\Big/\cdot sC(1+sRC)$$

$$Cu_C(0)+(1+sRC)\left(sCU_0(s)+sCLi_L(0)-Cu_C(0)\right)=I_1(s)\left[(s^2LC+1)(1+sRC)-1\right]$$

$$I_1(s)=\frac{(1+sRC)\left[U_0(s)+Li_L(0)\right]-RCu_C(0)}{s^2RLC+sL+R}=\frac{U_0(s)\left(\frac{1}{R}+sC\right)+Li_L(0)\left(\frac{1}{R}+sC\right)-Cu_C(0)}{s^2LC+s\frac{L}{R}+1}$$

$$I_1(s)=\frac{\frac{1}{s}(1+s)+\frac{2}{5}(1+s)+\frac{1}{5}}{2s^2+2s+1}=\frac{1}{5s}\cdot\frac{5(1+s)+2s(1+s)+s}{2s^2+2s+1}$$

$$I_1(s)=\frac{1}{5s}\cdot\frac{2s^2+8s+5}{2s^2+2s+1}=\frac{1}{5}\left(\frac{5}{s}-4\frac{s+0{,}5}{(s+0{,}5)^2+0{,}25}+2\frac{0{,}5}{(s+0{,}5)^2+0{,}25}\right)$$

$$i_L(t)=i_1(t)=\frac{1}{5}\left(5-4e^{-0{,}5t}\cos(0{,}5t)+2e^{-0{,}5t}\sin(0{,}5t)\right)\cdot S(t)$$

4. Odziv neke mreže na pobudu $x(t)$=S(t) glasi: $y(t)$=e^{-3t}ch($2t$)·S(t). Odrediti funkciju mreže i fazor odziva na pobudu $x(t)$=2 cos($3t$+45˚).

Rješenje:

$$X(s)=\frac{1}{s}$$

$$Y(s)=\frac{s+3}{(s+3)^2-4}=\frac{s+3}{s^2+6s+5}$$

$$H(s)=\frac{s(s+3)}{(s+3)^2-4}=\frac{s(s+3)}{s^2+6s+5}$$

Fazori:

$$H(\mathrm{j}\omega)=\frac{\mathrm{j}\omega(\mathrm{j}\omega+3)}{(\mathrm{j}\omega)^2+6\mathrm{j}\omega+5}$$

$$X_1(\mathrm{j}\omega)=2e^{\mathrm{j}\pi/4}$$

$$Y_1(\mathrm{j}\omega)=H(\mathrm{j}\omega)X_1(\mathrm{j}\omega)=\frac{\mathrm{j}\omega(\mathrm{j}\omega+3)}{(\mathrm{j}\omega)^2+6\mathrm{j}\omega+5}X_1(\mathrm{j}\omega)=\frac{\mathrm{j}\omega(\mathrm{j}\omega+3)}{(\mathrm{j}\omega)^2+6\mathrm{j}\omega+5}2e^{\mathrm{j}\pi/4}$$

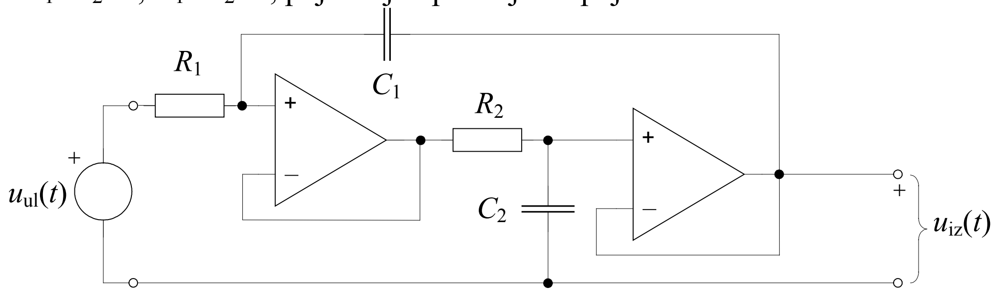
$$\omega=3$$

$$Y_1(\mathrm{j}3)=\frac{\mathrm{j}3\cdot(\mathrm{j}3+3)}{(\mathrm{j}3)^2+18\mathrm{j}+5}\cdot 2\cdot\left(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}\right)$$

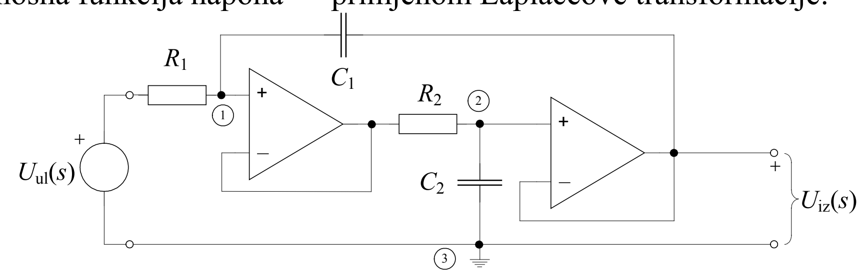
$$Y_1(\mathrm{j}3)=\frac{9\sqrt{2}}{85}\cdot(2+j9)$$

$$y_1(t)=\frac{9\sqrt{2}}{\sqrt{85}}\cos(3t+77.47°)$$

5. Za električni krug prikazan slikom: *a*) Odrediti prijenosnu funkciju napona $H(s)=U_{iz}(s)/U_{ul}(s)$;
b) Izračunati polove i nule prijenosne funkcije i prikazati njihov raspored u *s*-ravnini;
c) Izračunati i skicirati amplitudno-frekvencijsku karakteristiku $|H(j\omega)|$;
d) Izračunati i skicirati logaritamsku mjeru pojačanja $\alpha(\omega)$.
Zadano je: $R_1=R_2=1$, $C_1=C_2=1$, pojačanje operacijskih pojačala $A\to\infty$.



Rješenje: *a*) Prijenosna funkcija napona — primjenom Laplaceove transformacije:



Metoda napona čvorova (čvorište 3 je referentno) :

(1) $$\frac{U_1-U_0}{R_1}+(U_1-U_2)sC_1=0 \quad\Rightarrow\quad U_0=U_1(1+sC_1R_1)-U_2sC_1R_1$$

(2) $$\frac{U_2-U_1}{R_1}+U_2sC_2=0 \quad\Rightarrow\quad U_1=U_2(1+sC_2R_2)$$

$$\Rightarrow \qquad U_0=\left[(1+sC_1R_1)(1+sC_2R_2)-sC_1R_1\right]U_2$$

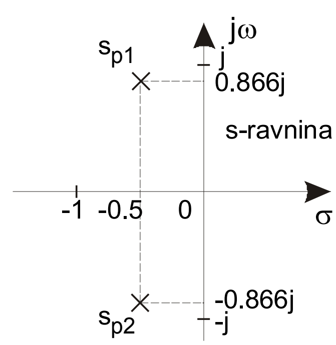
$$\Rightarrow \qquad U_2=\frac{U_0}{s^2C_1R_1C_2R_2+sC_2R_2+1}=\frac{\dfrac{U_0}{C_1R_1C_2R_2}}{s^2+s\dfrac{1}{C_1R_1}+\dfrac{1}{C_1R_1C_2R_2}}$$

$$H(s)=\frac{U_{iz}}{U_0}=\frac{\dfrac{1}{C_1R_1C_2R_2}}{s^2+s\dfrac{1}{C_1R_1}+\dfrac{1}{C_1R_1C_2R_2}}=\frac{1}{s^2+s+1}$$

b) Polovi i nule:

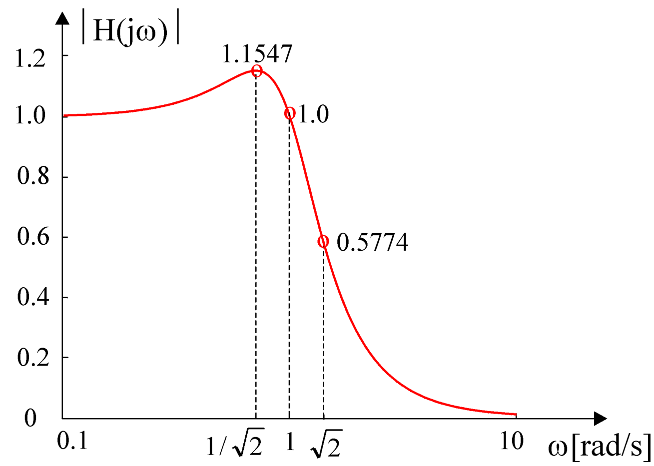
polovi: $s^2+s+1=0 \Rightarrow s_{p1,2}=-\frac{1}{2}\pm\sqrt{\frac{1}{4}-1}=-\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$

nule: $s_{o1,2}\to\infty$



c) A-f karakteristika $|H(j\omega)|$:

$$s = j\omega \Rightarrow \; H(j\omega) = \frac{1}{1-\omega^2 + j\omega} \Rightarrow |H(j\omega)| = \frac{1}{\sqrt{(1-\omega^2)^2 + \omega^2}} = \frac{1}{\sqrt{1-\omega^2+\omega^4}}$$



d) Logaritamska mjera pojačanja $\alpha(\omega)$:

$$\alpha(\omega) = 20\log|H(j\omega)| = -10\log(\omega^4 - \omega^2 + 1)$$