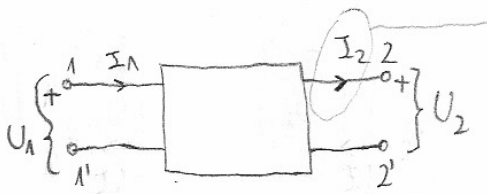
FER 2
Električni krugovi
Četveropoli

Napisao:
Sharp_Shooter

PDF pripremio i uredio:
Sharp_Shooter

ČETVEROPOLI



- ako je struja suprotnog smjera: Y_{12}, Y_{22}, z_{12} i z_{22} imaju '+' predznak

- strujne jednadžbe:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & -Y_{12} \\ Y_{21} & -Y_{22} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

- naponske jednadžbe

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

- Y-parametri: $\begin{bmatrix} Y_{11} & -Y_{12} \\ Y_{21} & -Y_{22} \end{bmatrix} = [Y]$ $\Rightarrow$ iz četveropola na KRATKO ($U_1=0$, pa $U_2=0$)

- z-parametri: $\begin{bmatrix} z_{11} & -z_{12} \\ Y_{21} & -Y_{22} \end{bmatrix} = [z]$ $\Rightarrow$ iz četveropola na PRAZNO ($I_1=0$, pa $I_2=0$)

- za recipročni četveropol: $Y_{12} = Y_{21}$

$z_{12} = z_{21}$

- veza između z i Y parametara:

$$[z] = [Y]^{-1}$$

- prijenosne jednadžbe:

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \qquad \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \cdot \begin{bmatrix} U_1 \\ I_1 \end{bmatrix}$$

- a-parametri: $\begin{bmatrix} A & B \\ C & D \end{bmatrix} = [a]$ $\Rightarrow [a]^{-1} = \begin{bmatrix} \frac{D}{\Delta_A} & \frac{-B}{\Delta_A} \\ \frac{-C}{\Delta_A} & \frac{A}{\Delta_A} \end{bmatrix}$

- iz 2-2' na PRAZNO I KRATKO

$$\Delta_A = \begin{vmatrix} A & B \\ C & D \end{vmatrix} = A \cdot D - C \cdot B$$

- za recipročne četveropole:

$$\Delta_A = \begin{vmatrix} A & B \\ C & D \end{vmatrix} = 1$$

- $[a]^{-1}$ $\Rightarrow$ iz 1-1' na PRAZNO I KRATKO

-hibridne jednadžbe

$$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_2 \\ U_2 \end{bmatrix} \qquad \begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} \cdot \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$$

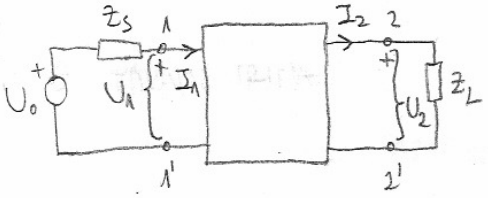
– h – parametri: $\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = [h]$ $\Rightarrow$ iz: 1-1' na prazno
2-2' na kratko

– g – parametri: $\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} = \begin{bmatrix} \frac{h_{12}}{\Delta_h} & \frac{-h_{21}}{\Delta_h} \\ \frac{-h_{12}}{\Delta_h} & \frac{h_{11}}{\Delta_h} \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = [g]$

$$\Delta_h = \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = h_{11}h_{22} - h_{12}h_{21}$$

$[g] \Rightarrow$ iz: 1-1' na KRATKO
2-2' na PRAZNO

PRIJENOSNE FUNKCIJE ČETVEROPOLA



- izražene:

① Z - PARAMETRIMA

- prijenosna funkcija struje:

$$H_i(s) = \frac{I_2}{I_1} = \frac{z_{21}}{z_L + z_{22}}$$

- napona:

$$H_u(s) = \frac{U_2}{U_1} = \frac{z_L z_{21}}{\Delta_z + z_{11} \cdot z_L}$$

$$H_u(s) = \frac{U_2}{U_0} = \frac{z_L \cdot z_{21}}{(z_{11} + z_S)(z_{22} + z_L) - z_{12} z_{21}}$$

② Y - PARAMETRIMA

$$H_i(s) = \frac{I_2}{I_1} = \frac{Y_L Y_{21}}{\Delta_Y + Y_{11} Y_L}$$

$$H_u(s) = \frac{U_2}{U_1} = \frac{Y_{21}}{Y_L + Y_{22}}$$

③ PRIJENOSNIM PARAMETRIMA

$$H_i(s) = \frac{I_2}{I_1} = \frac{1}{C \cdot Z_L + D}$$

$$H_u(s) = \frac{U_2}{I_2} = \frac{Z_L}{A \cdot Z_L + B}$$

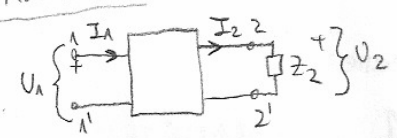
$$H_u(s) = \frac{U_2}{U_0} = \frac{Z_L}{A \cdot Z_L + B + Z_S \cdot (C \cdot Z_L + D)}$$

ULAZNE FUNKCIJE ČETVEROPOLA

– izražene preko:

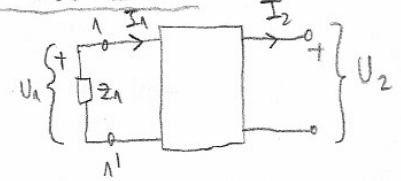
(1.) Z – PARAMETARA

– ulaz na 1-1':



$$Z_{ul_1} = \frac{U_1}{I_1} = z_{11} - z_{12} \cdot \frac{z_{21}}{z_2 + z_{22}}$$

– ulaz na 2-2':



$$Z_{ul_2} = -\frac{U_2}{I_2} = z_{22} - \frac{z_{12} z_{21}}{z_{11} + Z_1}$$

(2.) Y – PARAMETRIMA

– ulaz na 1-1':

$$Y_{ul_1} = Y_{11} - \frac{Y_{12} \cdot Y_{21}}{Y_2 + Y_{22}}$$

– ulaz na 2-2':

$$Y_{ul_2} = Y_{22} - \frac{Y_{12} \cdot Y_{21}}{Y_1 + Y_{11}}$$

(3.) PRIJENOSNIM PARAMETRIMA

– ulaz na 1-1':

$$Z_{ul_1} = \frac{U_1}{I_1} = \frac{A \cdot Z_2 + B}{C \cdot Z_2 + D}$$

– ulaz na 2-2':

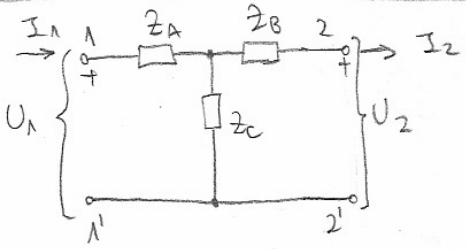
$$Z_{ul_2} = -\frac{U_2}{I_2} = \frac{D \cdot Z_1 + B}{C \cdot Z_1 + A}$$

EKVIVALENTNI ČETVEROPOLI

- nužan i dovoljan uvjet:

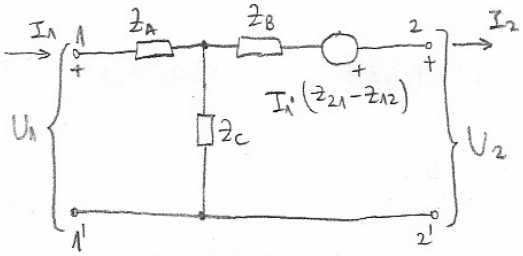
$$[Z]' = [Z]''$$

- ekvivalentni četveropoli u T spoju:



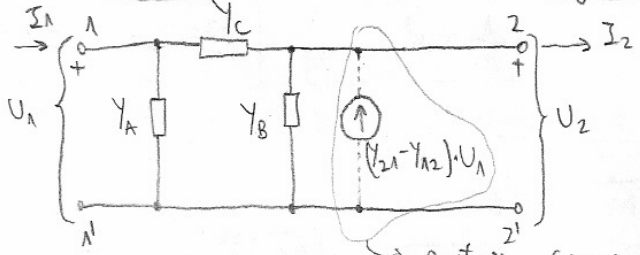
$$Z_A = Z_{11} - Z_{12}$$
$$Z_B = Z_{22} - Z_{12}$$
$$Z_C = Z_{12} = Z_{21}$$

- ako četveropol nije recipročan ($Z_{12} \neq Z_{21}$):



$$U_1 = I_1 \cdot Z_{11} - I_2 \cdot Z_{12}$$
$$U_2 = I_1 \cdot Z_{12} - I_2 \cdot Z_{22} + \underbrace{I_1 (Z_{21} - Z_{12})}_{\text{NAPONSKI IZVOR}}$$

- ekvivalentni četveropol u Π spoju:



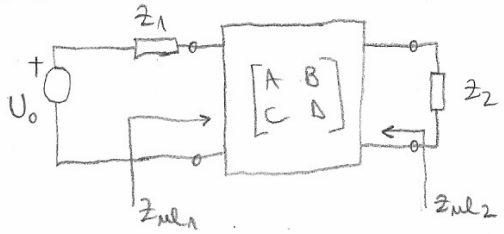
$$Y_A = Y_{11} - Y_{12}$$
$$Y_B = Y_{22} - Y_{12}$$
$$Y_C = Y_{12} = Y_{21}$$

→ postoji samo kada četveropol nije recipročan.

- ako je $Y_{12} \neq Y_{21}$

$$I_1 = Y_{11} \cdot U_1 - Y_{12} \cdot U_2$$
$$I_2 = Y_{12} \cdot U_1 - Y_{22} \cdot U_2 + \underbrace{(Y_{21} - Y_{12}) \cdot U_1}_{\text{STRUJNI IZVOR}}$$

ZRCALNI PARAMETRI



$$Z_{c1} = \sqrt{\frac{A \cdot B}{C \cdot D}}$$

$$Z_{c2} = \sqrt{\frac{B \cdot D}{A \cdot C}}$$

- ako je: $Z_2 = Z_{c2} \Rightarrow Z_{ul1} = Z_{c1}$

$Z_1 = Z_{c1} \Rightarrow Z_{ul2} = Z_{c2}$

- ako je $Z_1 = Z_{c1}$ i $Z_2 = Z_{c2}$, četveropol je prilagođen po zrcalnim impedancijama

$$\frac{Z_{c1}}{Z_{c2}} = \frac{A}{D}$$

$$m = \sqrt{\frac{Z_{c1}}{Z_{c2}}} \Rightarrow \text{OMJER TRANSFORMACIJE}$$

PRIJENOSNE JEDNADŽBE I ZRCALNI PARAMETRI

- prijenos sa 1-1' na 2-2':

$$\frac{U_1}{U_2} = A + \frac{B}{Z_{c2}}$$

$$\frac{I_1}{I_2} = C Z_{c2} + D$$

uz $Z_2 = Z_{c2}$ $\Leftarrow$ $\frac{U_1}{U_2} = m \cdot e^{g}$ $\Rightarrow$ uz $Z_2 = Z_{c2}$ $\frac{I_1}{I_2} = \frac{1}{m} \cdot e^{g}$

- prijenos signala sa 2-2' na 1-1':

$$\frac{U_2}{U_1} = D + \frac{B}{Z_{c1}}$$

$$\frac{I_2}{I_1} = C \cdot Z_{c1} + A$$

uz $Z_1 = Z_{c1}$ $\Leftarrow$ $\frac{U_2}{U_1} = \frac{1}{m} \cdot e^{g}$ $\Rightarrow$ uz $Z_1 = Z_{c1}$ $\frac{I_2}{I_1} = m \cdot e^{g}$

- ZRCALNI KOEFICIJENT PRIJENOSA (g):

$$\sqrt{AD} + \sqrt{BC} = e^{g}$$

$$g = \ln\left(\sqrt{AD} + \sqrt{BC}\right)$$

$$e^{-g} = \sqrt{AD} - \sqrt{BC}$$

$$\sqrt{AD} = \frac{e^{g} + e^{-g}}{2} = \operatorname{ch} g$$

$$\sqrt{BC} = \frac{e^{g} - e^{-g}}{2} = \operatorname{sh} g$$

$$A = m \cdot \operatorname{ch} g$$

$$B = m \cdot Z_{c2} \cdot \operatorname{sh} g$$

$$C = \frac{1}{m \cdot Z_{c2}} \cdot \operatorname{sh} g$$

$$D = \frac{1}{m} \cdot \operatorname{ch} g$$

PRIJENOSNE JEDNADŽBE SA g:

- prilagođenje na 2-2' i prijenos sa 1-1' na 2-2':

$Z_2 = Z_{c2}$

$Z_{ul_1} = Z_{c1}$

$$U_1 = m \cdot (U_2 \cdot ch g + I_2 \cdot Z_{c2} \cdot sh g)$$

$$I_1 = \frac{1}{m} \cdot \left(U_2 \cdot \frac{sh g}{Z_{c2}} + I_2 \cdot ch g\right)$$

- prilagođenje na 1-1' i prijenos sa 2-2' na 1-1':

$Z_1 = Z_{c1}$

$Z_{ul_2} = Z_{c2}$

$$U_2 = \frac{1}{m} \cdot (U_1 \cdot ch g - I_1 \cdot Z_{c1} \cdot sh g)$$

$$-I_2 = m \cdot \left(\frac{U_1}{Z_{c1}} \cdot sh g - I_1 \cdot ch g\right)$$

IMPEDANCIJE NA KRATKO I PRAZNO

$$Z_{p1} = \frac{A}{C} \qquad Z_{k1} = \frac{B}{D}$$

$$Z_{p2} = \frac{D}{C} \qquad Z_{k2} = \frac{B}{A}$$

$$\frac{Z_{p1}}{Z_{p2}} = \frac{Z_{k1}}{Z_{k2}} \Rightarrow \frac{Z_{p1}}{Z_{k1}} = \frac{Z_{p2}}{Z_{k2}}$$

$$Z_{c1} = \sqrt{Z_{p1} \cdot Z_k}$$

$$Z_{c2} = \sqrt{Z_{p2} \cdot Z_{k2}}$$

$$th(g) = \sqrt{\frac{Z_{k2}}{Z_p}}$$

$$Z_{k1} = Z_{c1} \cdot th(g)$$

$$Z_{p1} = Z_{c1} \, cth(g)$$

$$Z_{k2} = Z_{c2} \cdot th(g)$$

$$Z_{p2} = Z_{c2} \cdot cth(g)$$

NEPRILAGOĐENI ČETVEROPOL

- za: $Z_2 \neq Z_{c2} \Rightarrow Z_{ul1} \neq Z_{c1}$

$$Z_{ul1} = \frac{U_1}{I_1} = \frac{sh\gamma + \frac{Z_2}{Z_{c2}} \cdot ch\gamma}{ch\gamma + \frac{Z_2}{Z_{c2}} sh\gamma} \cdot Z_{c1}$$

- za $Z_1 \neq Z_{c1} \Rightarrow Z_{ul2} \neq Z_{c2}$

$$Z_{ul2} = \frac{-U_2}{I_2} = \frac{sh\gamma + \frac{Z_1}{Z_{c1}} \cdot ch\gamma}{ch\gamma + \frac{Z_1}{Z_{c1}} sh\gamma} \cdot Z_{c2}$$

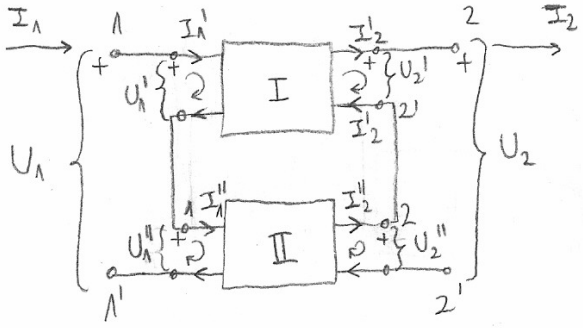
- POGREŠKA PRILAGOĐENJA:

$$\rho_1 = \frac{Z_1 - Z_{c1}}{Z_1 + Z_{c1}} \Rightarrow \text{NA ULAZU}$$

$$\rho_2 = \frac{Z_2 - Z_{c2}}{Z_2 + Z_{c2}} \Rightarrow \text{NA IZLAZU}$$

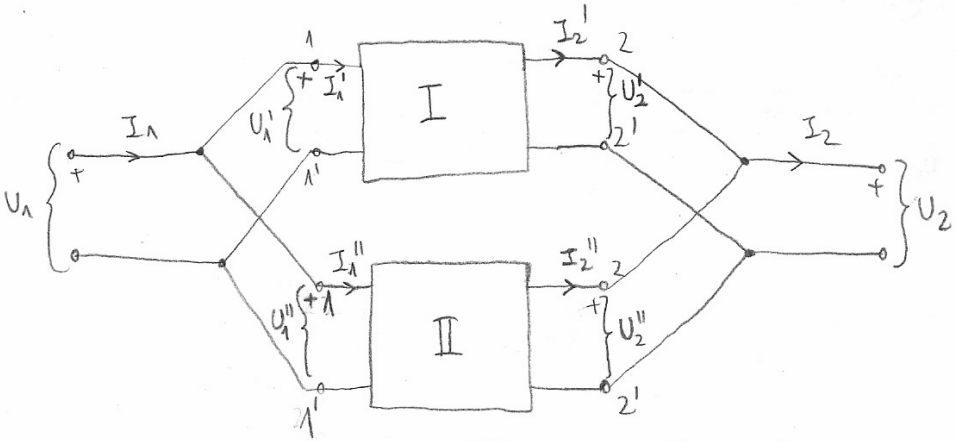
METODE POVEZIVANJA ČETVEROPOLA

1. SERIJSKI SPOJ ČETVEROPOLA



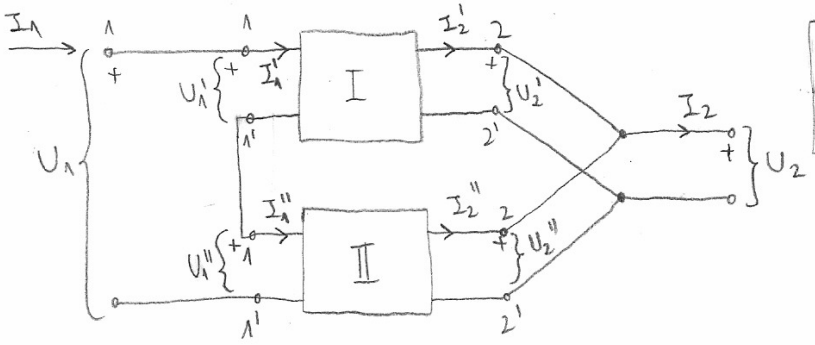
$$[z] = [z'] + [z'']$$

2. PARALELNI SPOJ ČETVEROPOLA



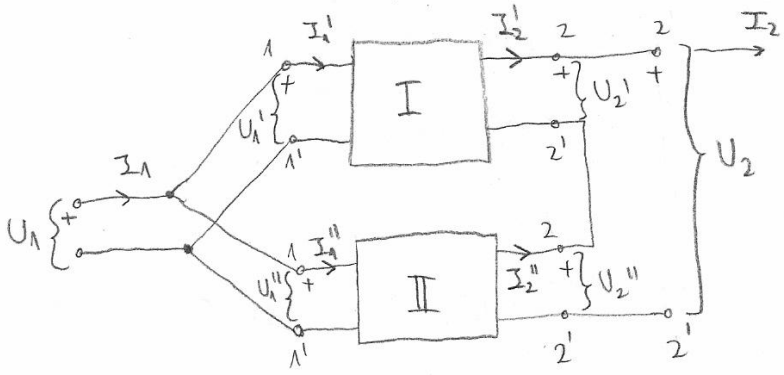
$$[Y] = [Y'] + [Y'']$$

3. SERIJSKO - PARALELNI SPOJ ČETVEROPOLA



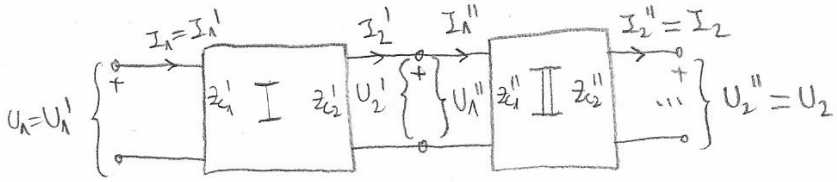
$$[h] = [h'] + [h'']$$

(4.) PARALELNO SERIJSKI SPOJ ČETVEROPOLA



$$[g] = [g'] + [g'']$$

(5.) KASKADA ČETVEROPOLA



$$[a] = [a'] \cdot [a''] \cdot \ldots \cdot [a^{(m)}]$$

– za prilagođeni lanac mora biti:

$$z'_{c2} = z''_{c1}, \quad z''_{c2} = z'''_{c1}, \ldots$$

SIMETRIČNI ČETVEROPOLI

$$Z_{11} = Z_{22} \Rightarrow Z_{P1} = Z_{P2}$$

$$Y_{11} = Y_{22} \Rightarrow Z_{K1} = Z_{K2}$$

$$[Y] = [Z^{-1}]$$

- kod ostalih parametara:

$$A = D, \quad AD - BC = 1 \Rightarrow BC = A^2 - 1$$

$$\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = -1$$

$$\begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = 1$$

- kod zrcalnih parametara:

$$Z_{c1} = Z_{c2} = Z_c = \sqrt{\frac{B}{C}}$$

$$g = \ln\left(\sqrt{AD} + \sqrt{BC}\right) = \ln\left(A + \sqrt{BC}\right)$$

$$g = \ln\left(A + \sqrt{A^2 - 1}\right)$$

- prijenosne jednadžbe:

$$Z_{c1} = Z_{c2} = Z_c$$

$$m = \sqrt{\frac{Z_{c1}}{Z_{c2}}} = 1$$

$$U_1 = U_2 \cdot \operatorname{ch} g + I_2 \cdot Z_c \cdot \operatorname{sh} g$$

$$I_1 = \frac{U_2}{Z_c} \operatorname{sh} g + I_2 \cdot \operatorname{ch} g$$

ČETVEROPOL	UVJET	BR. PARAMETARA
OBIČAN	/	4
RECIPROČAN	$z_{12} = z_{21}$	3
SIMETRIČAN	$z_{11} = z_{22}$	3
RECIPROČAN I SIMETRIČAN	$z_{12} = z_{21}$ $z_{11} = z_{22}$	2