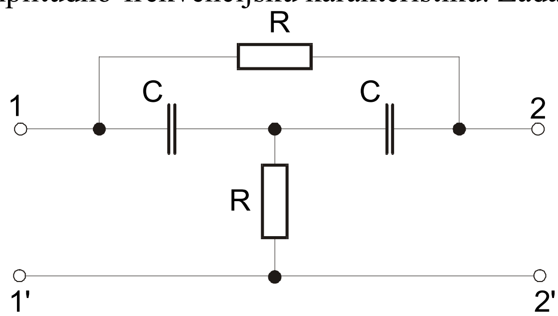
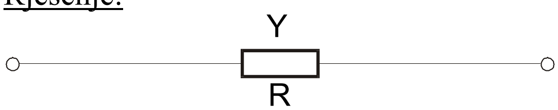
ZADACI ZA VJEŽBU IZ ELEKTRIČNIH KRUGOVA

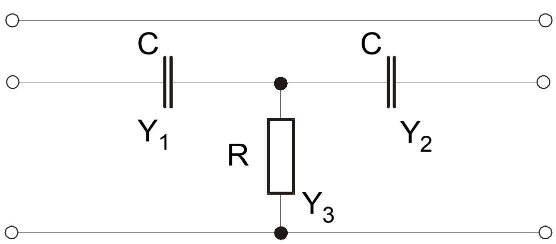
– četveropoli i linije –

1. Za četveropol prikazan slikom odrediti matricu [**y**]-parametara i prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$, za prazni hod na izlazu. Nacrtati raspored polova i nula u kompleksnoj s-ravnini i konstruirati amplitudno-frekvencijsku karakteristiku. Zadano je R=1, C=1.



Rješenje:





$$[\mathbf{y}]^{I}=\begin{bmatrix}\frac{1}{R} & -\frac{1}{R}\\ \frac{1}{R} & -\frac{1}{R}\end{bmatrix}$$

$$[\mathbf{y}]^{I}=\begin{bmatrix}Y & -Y\\ Y & -Y\end{bmatrix}$$

$$[\mathbf{y}]^{II}=\begin{bmatrix}\frac{Y_1(Y_2+Y_3)}{Y_1+Y_2+Y_3} & -\frac{Y_1Y_2}{Y_1+Y_2+Y_3}\\ \frac{Y_1Y_2}{Y_1+Y_2+Y_3} & -\frac{Y_2(Y_1+Y_3)}{Y_1+Y_2+Y_3}\end{bmatrix}$$

$$Y_1+Y_2+Y_3=2sC+\frac{1}{R}$$

$$Y_1(Y_2+Y_3)=sC\left(sC+\frac{1}{R}\right)=s^2C^2+\frac{sC}{R}$$

$$Y_1Y_2=s^2C^2$$

$$[\mathbf{y}]^{II}=\begin{bmatrix}\frac{sC(sRC+1)}{2sRC+1} & -\frac{s^2C^2R}{2sRC+1}\\ \frac{s^2C^2R}{2sRC+1} & -\frac{sC(sRC+1)}{2sRC+1}\end{bmatrix}$$

$$[\mathbf{y}] = [\mathbf{y}]^{I} + [\mathbf{y}]^{II} = \begin{bmatrix} \frac{sC(sRC+1)}{2sRC+1} + \frac{1}{R} & -\frac{s^2C^2R}{2sRC+1} - \frac{1}{R} \\ \frac{s^2C^2R}{2sRC+1} + \frac{1}{R} & -\left[\frac{sC(sRC+1)}{2sRC+1} + \frac{1}{R}\right] \end{bmatrix} =$$

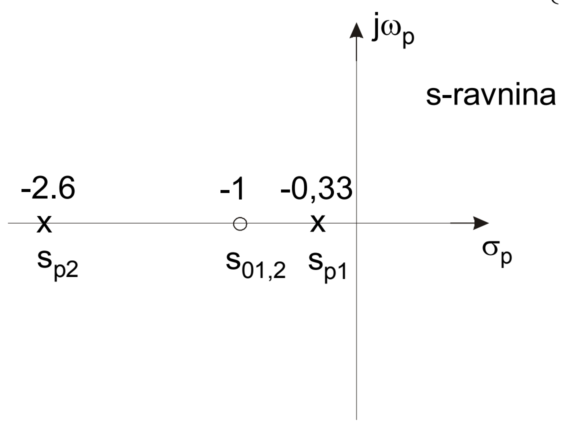
$$= \frac{1}{2s+1}\begin{bmatrix} s^2+3s+1 & -(s^2+2s)+1 \\ s^2+2s+1 & -(s^2+3s+1) \end{bmatrix}$$

$$T(s) = \frac{y_{21}}{y_{22}} = \frac{U_2}{U_1} = \frac{s^2+2s+1}{s^2+3s+1}$$

nule: $(s+1)^2 = 0 \quad S_{0_{1,2}} = -1$

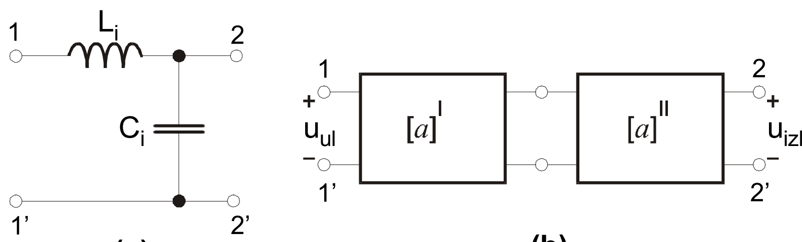
polovi: $s^2+3s+1=0$

$$S_{P_{12}} = \frac{-3 \pm \sqrt{9-4}}{2} = \frac{-3 \pm \sqrt{5}}{2} = \begin{cases} -2.618 \\ -0.382 \end{cases}$$



$$|T(j\omega)| = \frac{\sqrt{4\omega^2 - (1-\omega^2)^2}}{\sqrt{9\omega^2 + (1-\omega^2)^2}}$$

2. Za četveropol na slici **(a)** naći matricu prijenosnih parametara [a]. Za kaskadu dva takva četveropola, koja je prikazana na slici **(b),** naći ukupnu matricu prijenosnih parametara [a]. Pomoću matrice prijenosnih parametara izračunati naponsku prijenosnu funkciju kaskade $T(s)=U_{iz}(s)/U_{ul}(s)$, ako je na izlazu otpornik R=1. Zadano je $C_i=L_i=1$ (i=1, 2).



(a) **(b)**

Rješenje:

prijenosne jednadžbe četeveropola

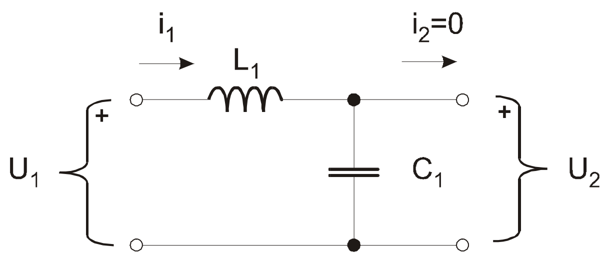
(1) $U_1 = A \cdot U_2 + B \cdot I_2$

(2) $I_1 = C \cdot U_2 + D \cdot I_2$

iz njih slijede prijenosni parametri

$$A = \frac{U_1}{U_2}\Big|_{I_2=0}; \quad B = \frac{U_1}{I_2}\Big|_{U_2=0}; \quad C = \frac{I_1}{U_2}\Big|_{I_2=0}; \quad D = \frac{I_1}{I_2}\Big|_{U_2=0}$$

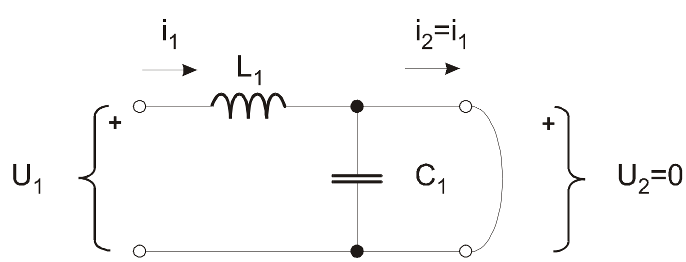
$I_2 = 0$



$$\frac{U_2}{U_1} = \frac{\frac{1}{sC_1}}{sL_1 + \frac{1}{sC_1}} = \frac{1}{s^2 L_1 C_1 + 1} \Rightarrow \qquad A = \frac{U_1}{U_2} = s^2 L_1 C_1 + 1$$

$$U_2 = I_1 \cdot \frac{1}{sC_1} \Rightarrow \qquad C = \frac{I_1}{U_2} = sC_1$$

$U_2 = 0$



$$U_1 = I_1 \cdot sL_1 = I_2 \cdot sL_1 \Rightarrow \qquad B = \frac{U_1}{I_2} = sL_1$$

$I_1 = I_2 \Rightarrow \qquad D = \dfrac{I_1}{I_2} = 1$

Konačno matrica prijenosnih parametara $[a]$ glasi

$[a]^I = \begin{bmatrix} 1+s^2L_1C_1 & sL_1 \\ sC_1 & 1 \end{bmatrix}$ analogno tome slijedi i $[a]^{II} = \begin{bmatrix} 1+s^2L_2C_2 & sL_2 \\ sC_2 & 1 \end{bmatrix}$

Spoj u kaskadu ili lanac:

$$[a] = [a]^I \cdot [a]^{II} = \begin{bmatrix} 1+s^2L_1C_1 & sL_1 \\ sC_1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1+s^2L_2C_2 & sL_2 \\ sC_2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (1+s^2L_1C_1)(1+s^2L_2C_2) + sL_1sC_2 & (1+s^2L_1C_1)sL_2 + sL_1 \\ sC_1(1+s^2L_2C_2) + sC_2 & sC_1sL_2 + 1 \end{bmatrix}$$

$$[a] = \begin{bmatrix} s^4L_1L_2C_1C_2 + s^2(L_1C_1 + L_2C_2 + L_1C_2) + 1 & s^3L_1C_1L_2 + s(L_1 + L_2) \\ s^3L_2C_2C_1 + s(C_1 + C_2) & s^2L_2C_1 + 1 \end{bmatrix}$$

uz uvrštene vrijednosti:

$$[a] = \begin{bmatrix} s^4 + 3s^2 + 1 & s^3 + 2s \\ s^3 + 2s & s^2 + 1 \end{bmatrix}$$

zaključenje $Z_2 = \dfrac{U_2}{I_2}$

(1) $\qquad U_1 = A \cdot U_2 + \dfrac{B}{Z_2} \cdot U_2 = \left(A + \dfrac{B}{Z_2} \right) \cdot U_2$

naponska prijenosna funkcija glasi:

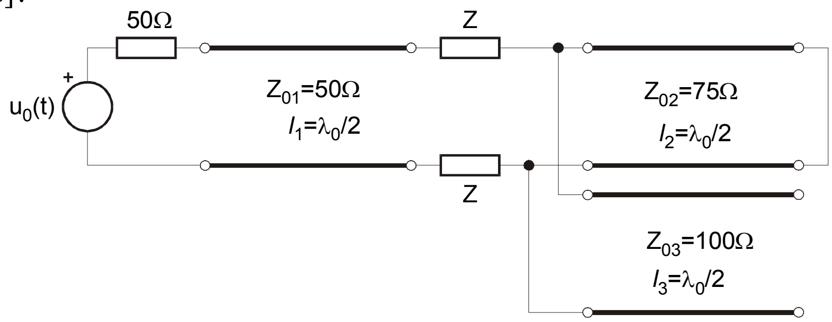
$T(s) = \dfrac{U_2}{U_1} = \dfrac{1}{A + \dfrac{B}{Z_2}}; \quad Z_2 = R$, odnosno:

$$T(s) = \frac{1}{s^4L_1L_2C_1C_2 + s^2(L_1C_1 + L_2C_2 + L_1C_2) + 1 + s^3\dfrac{L_1C_1L_2}{R} + s\dfrac{L_1 + L_2}{R}}$$

uz uvrštene vrijednosti:

$$T(s) = \frac{1}{s^4 + s^3 + 3s^2 + 2s + 1}$$

3. Zadan je sustav linija bez gubitaka prikazan slikom. Odrediti impedanciju Z da bi prva linija bila prilagođena po zrcalnim impedancijama. Odrediti napone na kraju svake linije ako je u_0=sin $(4\pi\ 10^5\ t)$[mV]. Koliko su duge linije ako je brzina širenja vala na linijama $v=4\cdot10^5$[km/s]?



Rješenje:
linija bez gubitaka uz sinusoidalnu pobudu

$$Z_0 = \sqrt{\frac{L}{C}} \qquad \gamma = \alpha + j\beta \Rightarrow \alpha = 0,\ \beta = \omega_0\sqrt{LC} \qquad \lambda = \frac{v}{f_0} = \frac{2\cdot\pi\cdot v}{\omega_0}$$

$f_0 = \dfrac{\omega_0}{2\pi}$ frekvencija signala

$v = \dfrac{\omega}{\beta}$ brzina širenja vala duž linije

$\lambda = \dfrac{2\pi}{\beta}$ valna dužina

$$\Rightarrow \lambda_0 = \frac{2\pi}{\omega_0\sqrt{LC}} \Rightarrow \lambda_0 = \frac{2\pi\cdot(4\cdot10^{-5})}{4\pi\cdot10^5} = 2km$$

$$l_1 = l_2 = l_3 = \frac{\lambda_0}{2} = 1km$$

prijenosne jednadžbe linije:

$$U(0) = U(x)\cdot ch(\gamma x) + I(x)Z_0 sh(\gamma x) = U(x)\cdot\cos(\beta x) + jI(x)Z_0\sin(\beta x)$$

$$I(0) = \frac{U(x)}{Z_0}sh(\gamma x) + I(x)ch(\gamma x) = j\frac{U(x)}{Z_0}\sin(\beta x) + I(x)\cos(\beta x)$$

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ako je $x = l \Rightarrow \gamma x = \gamma l = g$ i ako je linija zaključena impedancijom $Z_2 = \dfrac{U(l)}{I(l)}$ tada je ulazna impedancija $Z_{ul} = \dfrac{U(0)}{I(0)} = \dfrac{Z_2 ch\,g + Z_0 sh\,g}{\frac{Z_2}{Z_0}sh\,g + ch\,g}$

općenito vrijedi:
$$ch\,jx = \cos x$$
$$sh\,jx = j\sin x$$

ako je: $l = \dfrac{\lambda_0}{2} \Rightarrow g = \gamma\cdot l = j\beta\cdot\dfrac{1}{2}\cdot\dfrac{2\pi}{\beta} = j\dfrac{1}{2}\cdot2\pi = j\pi$

$$ch\,g = ch\,j\pi = \cos\pi = -1$$
$$sh\,g = sh\,j\pi = j\sin\pi = 0$$

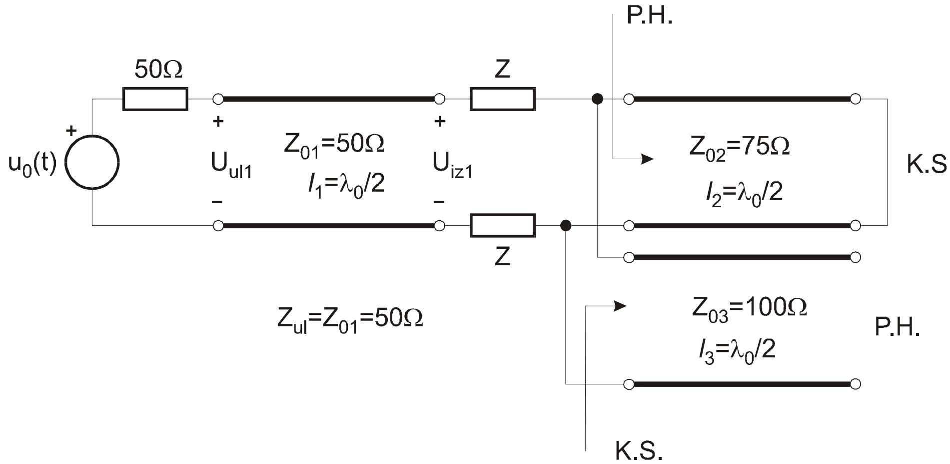
$$Z_{ul} = \frac{Z_2 \cdot (-1) + Z_0 \cdot 0}{\frac{Z_2}{Z_0} \cdot 0 + (-1)} = Z_2$$

imamo slijedeće slučajeve:

a) $Z_2 = \infty \Rightarrow \qquad Z_{ul} = \infty$

b) $Z_2 = 0 \Rightarrow \qquad Z_{ul} = 0$

c) $Z_2 = Z_0 \Rightarrow \qquad Z_{ul} = Z_0$



$$Z = \frac{Z_0}{2} = 25\Omega$$

prijenosna funkcija na prvoj liniji uz zaključenje $Z_2 = Z_0 = \dfrac{U_{iz1}}{I_{iz1}} = 50\Omega$

$$U(0) = U(l) ch\gamma l + \underbrace{I(l) \cdot Z_0}_{U(l)} \cdot sh\gamma l = U(l) \cdot (ch\gamma l + sh\gamma l) = U(l) \cdot e^g$$

$T(s) = \dfrac{U(l)}{U(0)} = e^{-g}; \qquad g = j\pi$, dolazi do zakreta faze za $-\pi$, a gušenja nema

$$sh\, x = \frac{e^{-x} - e^{-x}}{2}; \qquad ch\, x = \frac{e^{x} + e^{-x}}{2} \Rightarrow \qquad sh\, x + ch\, x = e^x$$

$$U_{ul1} = \frac{U_0}{2}$$

$$U_{iz1} = U_{ul1} \cdot e^{-j\pi} = \frac{U_0}{2} \cdot e^{-j\pi}$$

$$u_{iz1}(t) = \frac{1}{2} \sin(4\pi \cdot 10^5 t - \pi) \left[mV\right] \qquad u_{iz2}(t) = u_{iz3}(t) = 0 \left[V\right]$$

4. Zadana je linija bez gubitaka s primarnim parametrima L=1mH/km i C=400nF/km duljine l=314m prema slici. Na ulaz je spojen naponski izvor $u_g(t)$=10 sin $10^6 t$ unutrašnjeg otpora jednakog karakterističnoj impedanciji linije. Koliki mora biti omjer transformatora n da bi na izlazu bilo postignuto prilagođenje ako je R_2=800Ω. Koliki su napon i struja na izlazu linije $u_l(t)$ i $i_l(t)$?

Rješenje:

Linija bez gubitaka: $R=0;\quad G=0$

$$Z_0=\sqrt{\frac{L}{C}}=\sqrt{\frac{1\cdot 10^{-3}}{400\cdot 10^{-9}}}=50\Omega$$

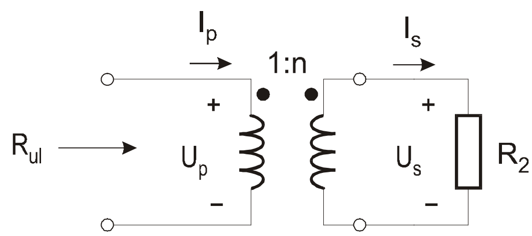
$$\gamma=s\cdot\sqrt{LC}=s\cdot\sqrt{10^{-3}\cdot 400\cdot 10^{-9}}=s\cdot 2\cdot 10^{-5}/km$$

$$\gamma=\alpha+j\beta=0+j\omega_0\sqrt{LC}$$

$$\nu=\frac{\omega_0}{\beta}$$

$$\lambda=\frac{2\pi}{\beta}=\frac{2\pi}{\omega_0\sqrt{LC}}=\frac{2\pi}{20}\cdot km=0.314\,km$$

$$\beta=\omega_0\cdot\sqrt{LC}=10^6\cdot 2\cdot 10^{-5}=20/km$$



$$U_s=n\cdot U_p$$

$$I_s=\frac{1}{n}\cdot I_p$$

$$U_s=I_s\cdot R_2$$

$$R_{ul}=\frac{U_p}{I_p}=\frac{\frac{U_s}{n}}{I_s\cdot n}=\frac{\frac{U_s}{I_s}}{n^2}=\frac{R_2}{n^2}$$

$$R_2=n^2\cdot R_{ul}$$

$$n^2=\frac{R_2}{R_{ul}}=\frac{800}{50}=16\Rightarrow\qquad n=4$$

$g = j\omega\sqrt{LC} \cdot l = j \cdot 10^6 \cdot 2 \cdot 10^{-5} \cdot 0.314 = j \cdot 2\pi$

prijenosne jednadžbe linije:

$$U(x) = U(0) \cdot ch\gamma x - I(0) \cdot Z(0) \cdot sh\gamma x$$
$$I(x) = -\frac{U(0)}{Z_0} \cdot sh\gamma x + I(0) \cdot ch\gamma x$$

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na mjestu $x = l$

$$U(l) = U(0) \cdot ch\, g - I(0) \cdot Z(0) \cdot sh\, g$$

na ulazu linije $I(0) = \dfrac{U(0)}{Z_0}$ jer je $Z_{ul} = Z_0 = \dfrac{R_2}{n^2} = 50\Omega$ (prilagođenje)

$$U(l) = U(0) \cdot (ch\, g - sh\, g) = U(0) \cdot e^{-g}$$
$$U(0) = \frac{U_g}{2} = 5\angle 0°$$
$$U(l) = 5 \cdot e^{-j2\pi} = 5 \cdot (\cos 2\pi - j \sin 2\pi) = -j5$$

$$I(l) = -\frac{U(0)}{Z_0} sh\, g + I(0) ch\, g$$
$$I(0) = \frac{U(0)}{Z_0}$$

$$I(l) = -\frac{U_0}{Z_0} sh\, g + \frac{U_0}{Z_0} ch\, g = \frac{U(0)}{Z_0}(ch\, g - sh\, g) = \frac{U(0)}{Z_0} \cdot e^{-g}$$
$$I(l) = \frac{U(l)}{Z_0} = \frac{-j5}{50} = -0.1j$$

$$u(l,t) = 5\sin(10^6 t - 90°)$$
$$i(l,t) = 0.1\sin(10^6 t - 90°)$$