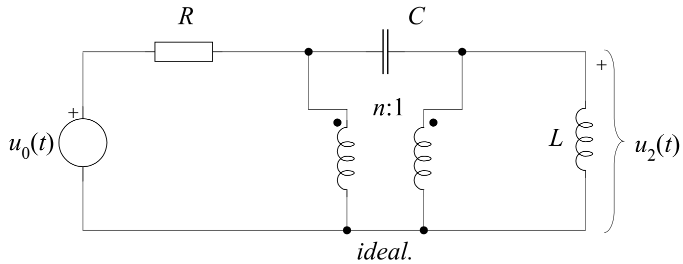
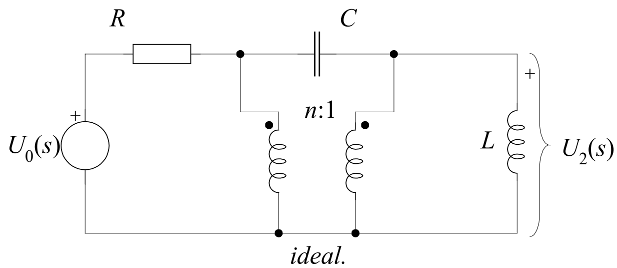
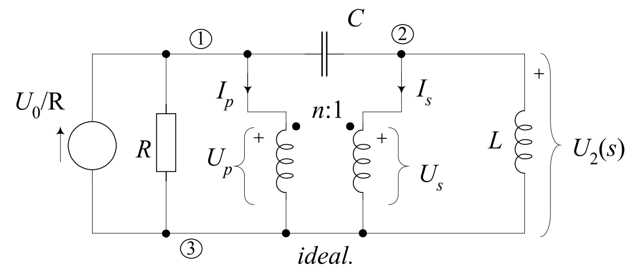
DRUGI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA

1. Za krug prikazan slikom izračunati odziv $u_2(t)$ ako je zadana pobuda $u_0(t)=S(t)$, prijenosni omjer n=1/2, a normirane vrijednosti elemenata su: R =2, L=1 i C=1. Početni uvjeti su jednaki nuli.



<u>Rješenje: Jednadžbe čvorišta</u>





$$(1)\qquad \frac{U_0(s)}{R} - I_p(s) = \left(\frac{1}{R} + sC\right)U_1(s) - sCU_2(s) \qquad\qquad U_p(s) = nU_s(s) \Rightarrow U_1(s) = nU_2(s)$$

$$(2)\qquad -I_s(s) = -sCU_1(s) + \left(sC + \frac{1}{sL}\right)U_2(s) \qquad\qquad I_p(s) = -\frac{1}{n}I_s(s)$$

$$(1)\qquad \frac{U_0}{R} + \frac{I_s}{n} = \left(\frac{1}{R} + sC\right)nU_2 - sCU_2$$

$$(2)\qquad -I_s(s) = -sCnU_2 + \left(sC + \frac{1}{sL}\right)U_2(s) \qquad \Rightarrow \qquad I_s(s) = \left(sCn - sC - \frac{1}{sL}\right)U_2(s)$$

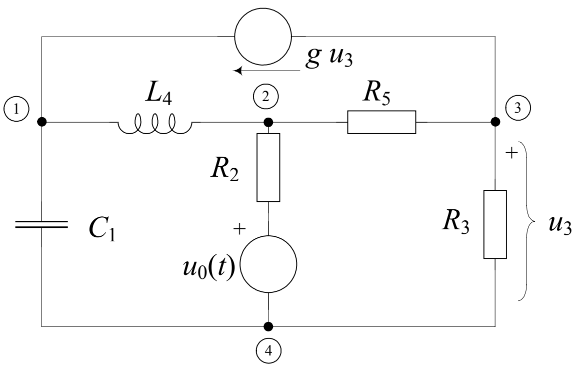
$$\frac{U_0}{R} = \left(-sC + \frac{1}{n}\left(sC + \frac{1}{sL}\right)\right)U_2 + \left(\frac{1}{R} + sC\right)nU_2 - sCU_2$$

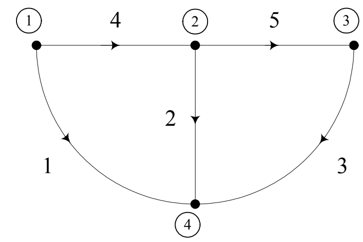
$$\frac{U_0}{R} = \left(\frac{1}{n}\left(sC + \frac{1}{sL}\right) + \left(\frac{1}{R} + sC\right)n - 2sC\right)U_2$$

$$U_2(s) = \frac{nU_0}{R\left(\left(sC + \frac{1}{sL}\right) + \left(\frac{1}{R} + sC\right)n^2 - 2nsC\right)} = \frac{snU_0}{RC\left(s^2\left(1 + n^2 - 2n\right) + s\frac{n^2}{RC} + \frac{1}{LC}\right)} = \frac{2}{2s^2 + s + 8}$$

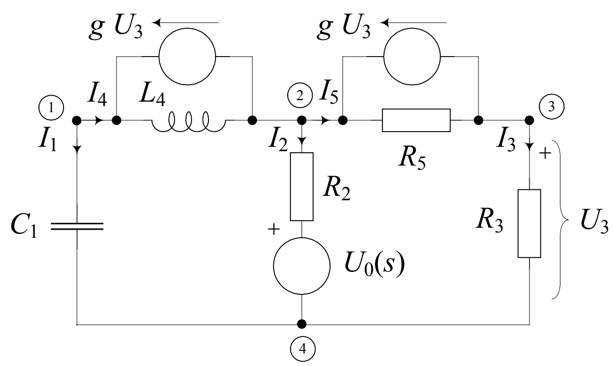
$$U_2(s) = \frac{1}{s^2 + \frac{s}{2} + 4} = \frac{1}{\left(s + \frac{1}{4}\right)^2 + \frac{63}{16}} = \frac{4}{\sqrt{63}} \cdot \frac{\frac{\sqrt{63}}{4}}{\left(s + \frac{1}{4}\right)^2 + \left(\frac{\sqrt{63}}{4}\right)^2} \qquad u_2(t) = \frac{4}{3\sqrt{7}} \cdot e^{-t/4} \sin\left(\frac{3\sqrt{7}}{4}\right)S(t)$$

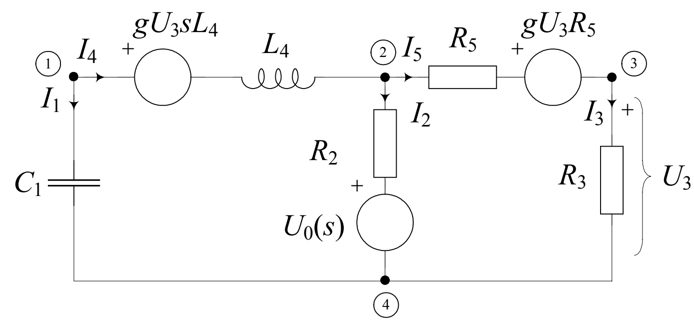
2. Za krug prikazan slikom i pridruženi orijentirani graf napisati matricu incidencija $\mathbf{A}_a$, temeljnu spojnu matricu $\mathbf{S}$, temeljnu rastavnu matricu $\mathbf{Q}$, matricu impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$. Matrica $\mathbf{Z}_b$ mora biti regularna. (Uputa: grane stabla: 1, 2 i 3.)





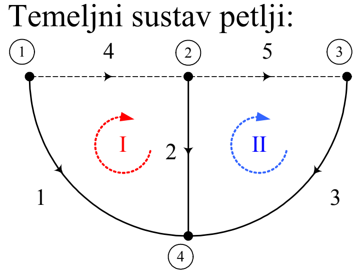
Rješenje: Primjena Laplaceove transformacije i posmicanje strujnog izvora



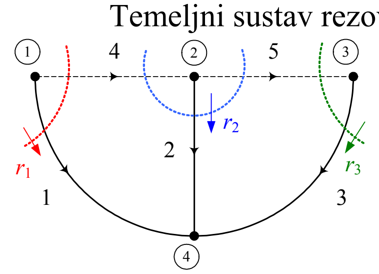


Matrica incidencija: $\mathbf{A}_a = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 \\ -1 & -1 & -1 & 0 & 0 \end{bmatrix}$

Temeljni sustav petlji:



Temeljni sustav rezova:



Temeljna spojna matrica: $\mathbf{S} = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 1 \end{bmatrix}$,

Temeljna rastavna matrica: $\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix}$

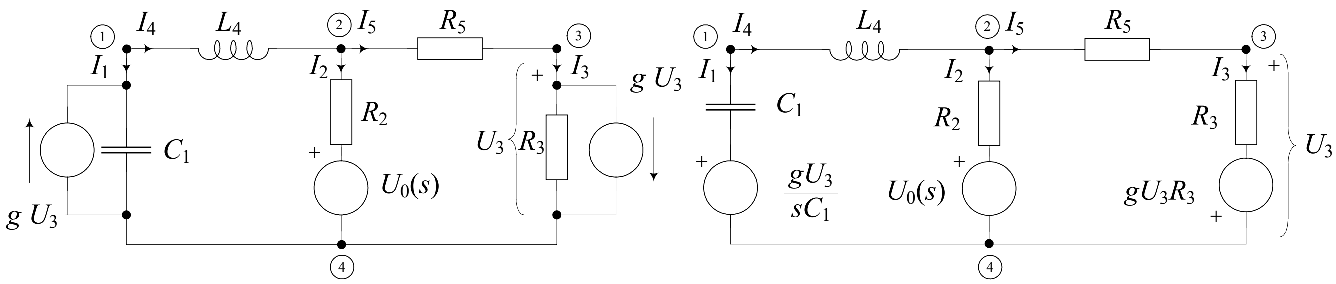
Naponsko – strujne relacije grana: $\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$

$$U_1 = I_1 \cdot \frac{1}{sC_1}$$
$$U_2 = I_2 \cdot R_2 + U_0(s)$$
$$U_3 = I_3 \cdot R_3$$
$$U_4 = I_4 \cdot sL_4 + g \cdot U_3 \cdot sL_4 = gR_3 sL_4 \cdot I_3 + I_4 \cdot sL_4$$
$$U_5 = gU_3 \cdot R_5 + I_5 \cdot R_5 = gI_3 \cdot R_3 \cdot R_5 + I_5 \cdot R_5$$

$$\mathbf{Z}_b = \begin{bmatrix} \frac{1}{sC_1} & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 \\ 0 & 0 & gR_3 sL_4 & sL_4 & 0 \\ 0 & 0 & gR_3 R_5 & 0 & R_5 \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} 0 \\ U_0(s) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Matrica $\mathbf{Z}_b$ je regularna jer nema niti jedan redak niti stupac jednak nuli.

<u>2. način:</u> posmicanje ovisnog strujnog izvora u čvor 4



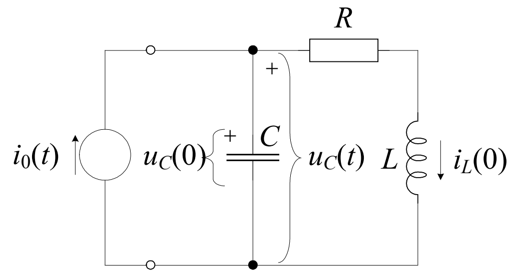
Naponsko – strujne relacije grana: $\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$

$$U_1 = I_1 \cdot \frac{1}{sC_1} + gU_3 \cdot \frac{1}{sC_1} \Rightarrow U_3 = I_3 \cdot \frac{R_3}{1+gR_3} \Rightarrow U_1 = I_1 \cdot \frac{1}{sC_1} + I_3 \cdot \frac{gR_3}{1+gR_3} \cdot \frac{1}{sC_1}$$
$$U_2 = I_2 \cdot R_2 + U_0(s)$$
$$U_3 = I_3 \cdot R_3 - gU_3 \cdot R_3 \Rightarrow U_3 \cdot (1+gR_3) = I_3 \cdot R_3 \Rightarrow U_3 = I_3 \cdot \frac{R_3}{1+gR_3}$$
$$U_4 = I_4 \cdot sL_4$$
$$U_5 = I_5 \cdot R_5$$

$$\mathbf{Z}_b = \begin{bmatrix} \frac{1}{sC_1} & 0 & \frac{gR_3}{(1+gR_3)sC_1} & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ 0 & 0 & \frac{R_3}{1+gR_3} & 0 & 0 \\ 0 & 0 & 0 & sL_4 & 0 \\ 0 & 0 & 0 & 0 & R_5 \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} 0 \\ U_0(s) \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

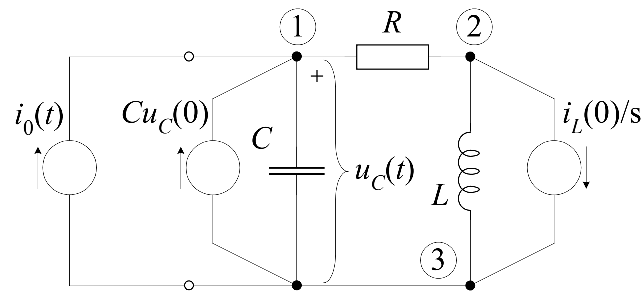
ostalo je sve isto

3. Odrediti odziv $u_C(t)$ mreže prema slici za $t \geq 0$, ako su zadane normirane vrijednosti elemenata: $R=1$, $C=2$ i $L=1$, početni uvjeti u mreži $u_C(0)=1/5$ i $i_L(0)=-1/5$, a pobuda je $i_0(t)=S(t)$.



<u>Rješenje:</u>

Za $t \geq 0$: Primjena Laplaceove transformacije → Jednadžbe čvorišta



$$I_0(s)+Cu_C(0)=\left(sC+\frac{1}{R}\right)U_1(s)-\frac{1}{R}U_2(s)$$

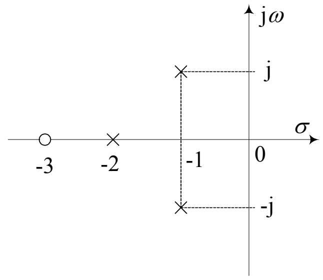
$$\frac{-i_L(0)}{s}=-\frac{1}{R}U_1(s)+\left(\frac{1}{sL}+\frac{1}{R}\right)U_2(s) \qquad \Rightarrow \qquad U_2(s)=\frac{sL}{(R+sL)}U_1(s)-i_L(0)\frac{RL}{(R+sL)}$$

$$U_1(s)=\frac{(R+sL)I_0(s)+C(R+sL)u_C(0)-Li_L(0)}{s^2LC+sRC+1}=\frac{(1+s)+2s(1+s)u_C(0)-si_L(0)}{s(2s^2+2s+1)}$$

$$U_1(s)=\frac{1}{5s}\cdot\frac{5(1+s)+2s(1+s)+s}{2s^2+2s+1}=\frac{1}{5s}\cdot\frac{2s^2+8s+5}{2s^2+2s+1}=\frac{1}{5}\left(\frac{5}{s}-4\frac{s+0{,}5}{(s+0{,}5)^2+0{,}25}+2\frac{0{,}5}{(s+0{,}5)^2+0{,}25}\right)$$

$$u_1(t)=\frac{1}{5}\left(5-4e^{-0{,}5t}\cos(0{,}5t)+2e^{-0{,}5t}\sin(0{,}5t)\right)S(t)$$

4. Zadan je raspored polova i nula prijenosne funkcije $H(s)=U_{iz}(s)/U_{ul}(s)$ nekoga električnog kruga prema slici. Odrediti prijenosnu funkciju ako se traži da bude $|H(j\omega)|=1$ za $\omega=1$. Odrediti fazor odziva ako je pobuda $u_{ul}(t)=\cos(3t+20^\circ)$.



Rješenje:

Prijenosna funkcija

$$H(s)=\frac{k(s+3)}{(s+1+\mathrm{j})(s+1-\mathrm{j})(s+2)}=\frac{k(s+3)}{(s+2)(s^2+2s+2)}$$

$$H(\mathrm{j}\omega)\Big|_{\omega=1}=\left|\frac{k(\mathrm{j}\omega+3)}{(\mathrm{j}\omega+2)(-\omega^2+2\mathrm{j}\omega+2)}\right|_{\omega=1}=\left.\frac{k\sqrt{\omega^2+9}}{\sqrt{\omega^2+4}\sqrt{(2-\omega^2)^2+4\omega^2}}\right|_{\omega=1}=k\sqrt{\frac{2}{5}}=1 \qquad \Rightarrow\ k=\sqrt{\frac{5}{2}}$$

$$H(s)=\sqrt{\frac{5}{2}}\cdot\frac{(s+3)}{(s+2)(s^2+2s+2)}$$

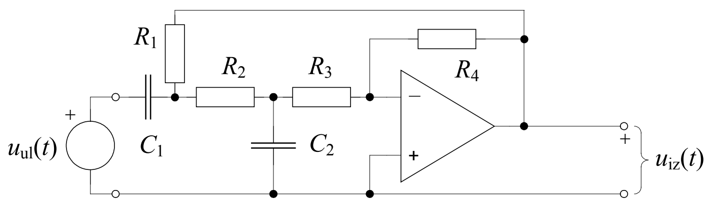
Fazor odziva

$$U_{iz}(\mathrm{j}\omega)=H(\mathrm{j}\omega)\cdot U_{ul}(\mathrm{j}\omega)=\sqrt{\frac{5}{2}}\cdot\frac{(\mathrm{j}\omega+3)\cdot U_{ul}(\mathrm{j}\omega)}{(\mathrm{j}\omega+2)(-\omega^2+2\mathrm{j}\omega+2)}$$

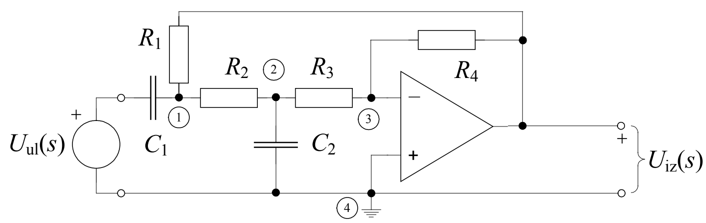
$$U_{ul}(\mathrm{j}\omega)\Big|_{\omega=3}=1\angle 20^\circ=e^{j20^\circ}=\cos(20^\circ)+j\sin(20^\circ)=0{,}9397+j0{,}342$$

$$U_{iz}(\mathrm{j}\omega)\Big|_{\omega=3}=\sqrt{\frac{5}{2}}\cdot\frac{(\mathrm{j}3+3)\cdot U_{ul}(\mathrm{j}\omega)}{(\mathrm{j}3+2)(-7+6\mathrm{j})}=\sqrt{\frac{5}{2}}\cdot\frac{3\sqrt{2}\cdot e^{j45^\circ}\cdot e^{j20^\circ}}{\sqrt{13}\cdot e^{j56{,}3^\circ}\sqrt{85}\cdot e^{j139{,}4^\circ}}=0{,}201\cdot e^{-j130{,}7^\circ}$$

3. Odrediti prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$ za električni krug prikazan slikom. Nacrtati raspored polova i nula te funkcije u kompleksnoj s-ravnini. Izračunati i nacrtati amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$. Zadano je $R_1=R_2=R_3=1$, $R_4=2$, $C_1=C_2=1$.



Rješenje: Primjena Laplaceove transformacije



Metoda napona čvorova (čvorište 4 je referentno) :

(1) $$U_{ul}(s)sC_1+U_{iz}(s)\frac{1}{R_1}=U_1\left(sC_1+\frac{1}{R_1}+\frac{1}{R_2}\right)-U_2\frac{1}{R_2}$$

(2) $$0=-U_1\frac{1}{R_2}+U_2\left(sC_2+\frac{1}{R_2}+\frac{1}{R_3}\right)$$

(3) $$U_{iz}\frac{1}{R_4}=-U_2\frac{1}{R_3}+U_3\left(\frac{1}{R_3}+\frac{1}{R_4}\right)$$

$$A\to\infty,\quad U_3=0\ \Rightarrow\ (3)\ U_{iz}(s)\frac{1}{R_4}=-U_2\frac{1}{R_3}\bigg/\cdot R_4$$

$$(3)\Rightarrow U_{iz}=-\frac{R_4}{R_3}U_2$$

$$(2)\Rightarrow U_1=\left(sR_2C_2+1+\frac{R_2}{R_3}\right)U_2\ \to(1)\Rightarrow U_{ul}sC_1+U_{iz}\frac{1}{R_1}=\left(\left(sR_2C_2+1+\frac{R_2}{R_3}\right)\left(sC_1+\frac{1}{R_1}+\frac{1}{R_2}\right)-\frac{1}{R_2}\right)U_2$$

$$U_{ul}(s)sC_1+U_{iz}(s)\frac{1}{R_1}=-\frac{R_3}{R_4}\left(\left(sR_2C_2+\frac{R_2}{R_3}\right)\left(sC_1+\frac{1}{R_1}+\frac{1}{R_2}\right)+sC_1+\frac{1}{R_1}\right)U_{iz}$$

$$\frac{U_{iz}}{U_{ul}}=-\frac{R_4}{R_3}\frac{sC_1}{s^2R_2C_1C_2+sC_1\frac{R_2}{R_3}+sR_2C_2\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+\frac{R_2}{R_3}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)+sC_1+\left(1+\frac{R_4}{R_3}\right)\frac{1}{R_1}}$$

$$T(s)=\frac{U_{iz}}{U_{ul}}=\frac{-\frac{R_4}{R_3}\cdot\frac{s}{R_2C_2}}{s^2+s\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{1}{R_2C_2}+\frac{1}{R_3C_2}\right)+\frac{R_1+R_2+R_3+R_4}{R_1R_2R_3C_1C_2}}$$

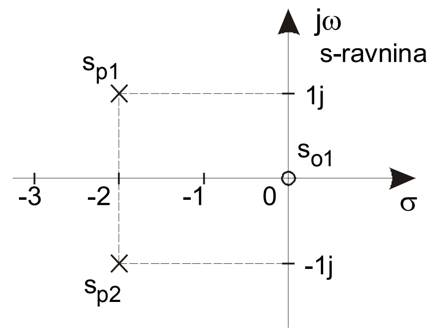
Uvrštenjem vrijednosti elemenata:

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{-\frac{2}{1}\frac{1}{1\cdot 1}\cdot s}{s^2+\left(1+1+1+1\right)\cdot s+\frac{1+1+1+2}{1\cdot 1\cdot 1\cdot 1\cdot 1}}=\frac{-2\cdot s}{s^2+4\cdot s+5}$$

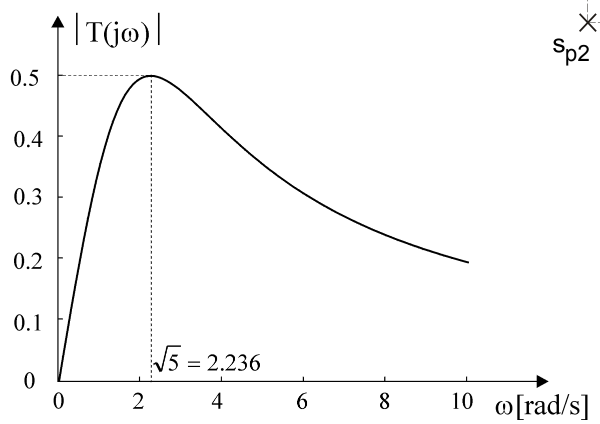
nule: $s_{o1}=0,\quad s_{o2}=\infty$

polovi: $s^2+4\cdot s+5=0 \Rightarrow s_{p1,2}=-2\pm\sqrt{4-5}=-2\pm j$

$$T(j\omega)=\frac{-2\cdot j\omega}{-\omega^2+4\cdot j\omega+5}\Rightarrow \left|T(j\omega)\right|=\frac{2\cdot|\omega|}{\sqrt{(5-\omega^2)^2+16\omega^2}}$$



a-f karakteristika:



2. jednostavniji način: odmah uvrstiti vrijednosti elemenata u jednadžbe čvorova :

(1) $\quad U_{ul}s+U_{iz}=U_1\left(s+2\right)-U_2$

(2) $\quad 0=-U_1+U_2\left(s+2\right)$

(3) $\quad \frac{U_{iz}}{2}=-U_2+U_3\frac{3}{2}$

$A\to\infty,\quad U_3=0\ \Rightarrow\ (3)\ U_2=-\frac{U_{iz}}{2}$

$(2)\Rightarrow\ U_1=U_2(s+2)=-U_{iz}\frac{(s+2)}{2}$

$U_{ul}s+U_{iz}=-U_{iz}\frac{(s+2)}{2}\left(s+2\right)+U_{iz}\frac{1}{2}$

$2U_{ul}s=-\left(s^2+4s+5\right)U_{iz}$

$\frac{U_{iz}}{U_{ul}}=-\frac{2s}{s^2+4s+5}$

$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{-2\cdot s}{s^2+4\cdot s+5}$,

ostali dio postupka je isti.