Električni krugovi

Četveropoli

Ekvivalentni četveropoli

Dva četveropola su ekvivalentna ako

↓

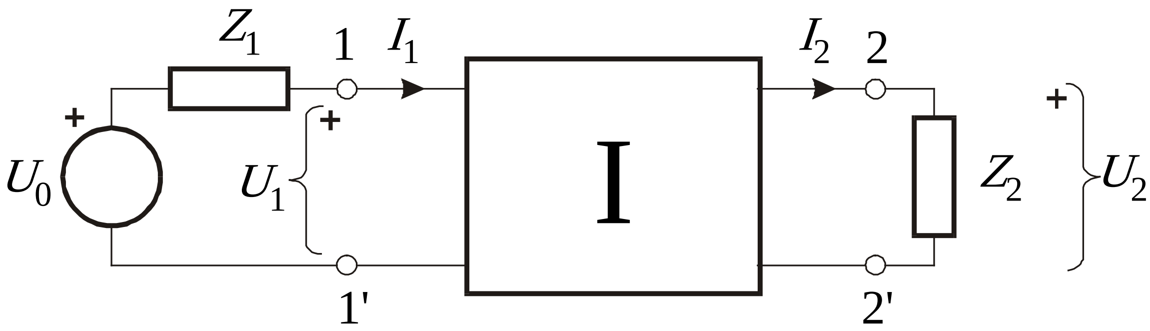
njihovom međusobnom zamjenom

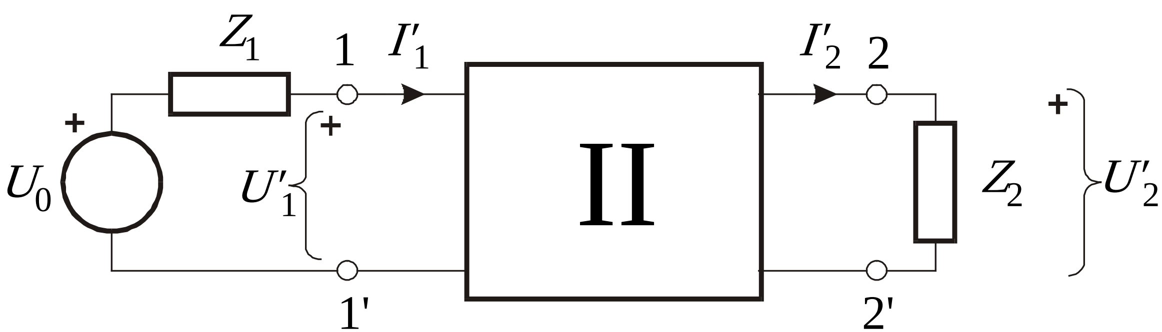
↓

STRUJE I NAPONI NA PRIKLJUČNICAMA

↓

OSTAJU NEPROMIJENJENI





Ako je $U_1 = U'_1$ $\qquad$ $U_2 = U'_2$

$I_1 = I'_1$ $\qquad$ $I_2 = I'_2$

ekvivalentni četveropoli

Nuždan i dovoljan uvjet

Parametri jednog = parametrima drugog

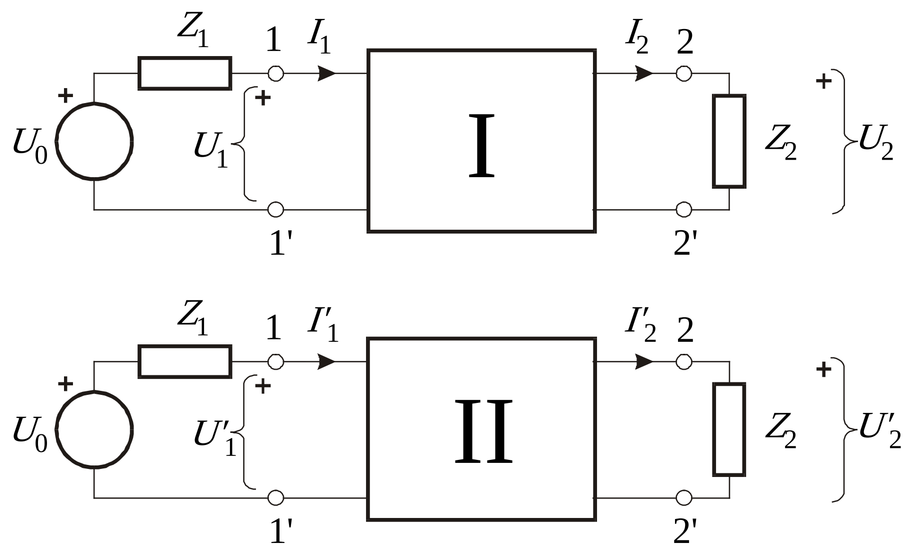
$$[z]^{I} = [z]^{II}$$

$$z_{11}^{I} = z_{11}^{II}$$

$$z_{12}^{I} = z_{12}^{II}$$

$$z_{21}^{I} = z_{21}^{II}$$

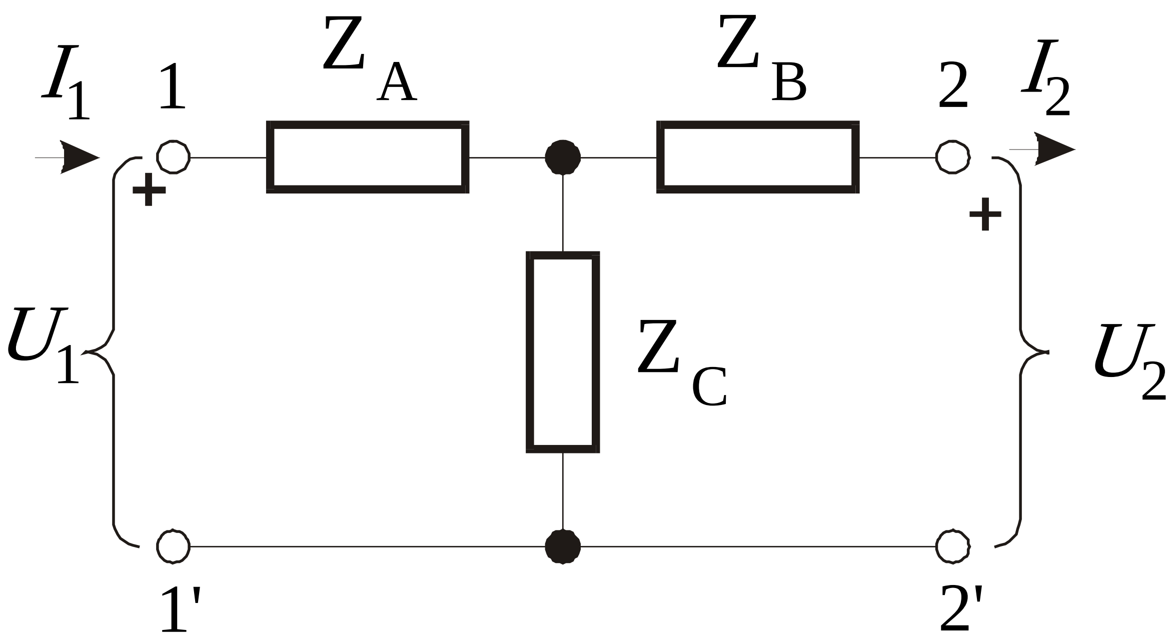
$$z_{22}^{I} = z_{22}^{II}$$



Ekvivalentni četveropol u T-spoju

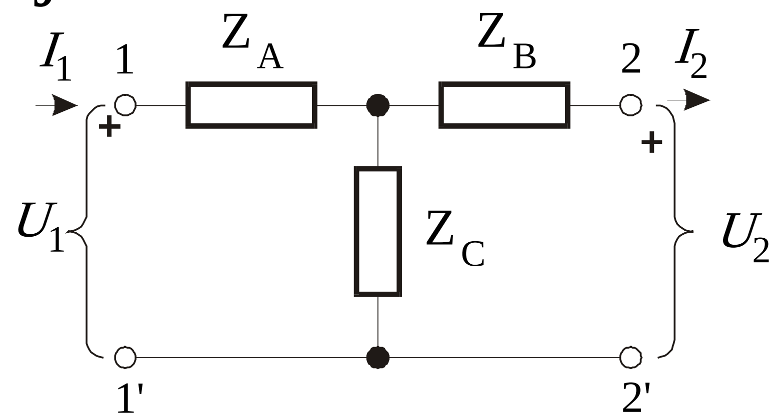
Svakom recipročnom četveropolu zadanom z-parametrima

moguće je odrediti ekvivalentni četveropol u T-spoju



z-parametri ekvivalentnoga T-spoja

$$U_1 = I_1 z_{11} - I_2 z_{12}$$
$$U_2 = I_1 z_{21} - I_2 z_{22}$$



elementi ekvivalentnoga T-spoja

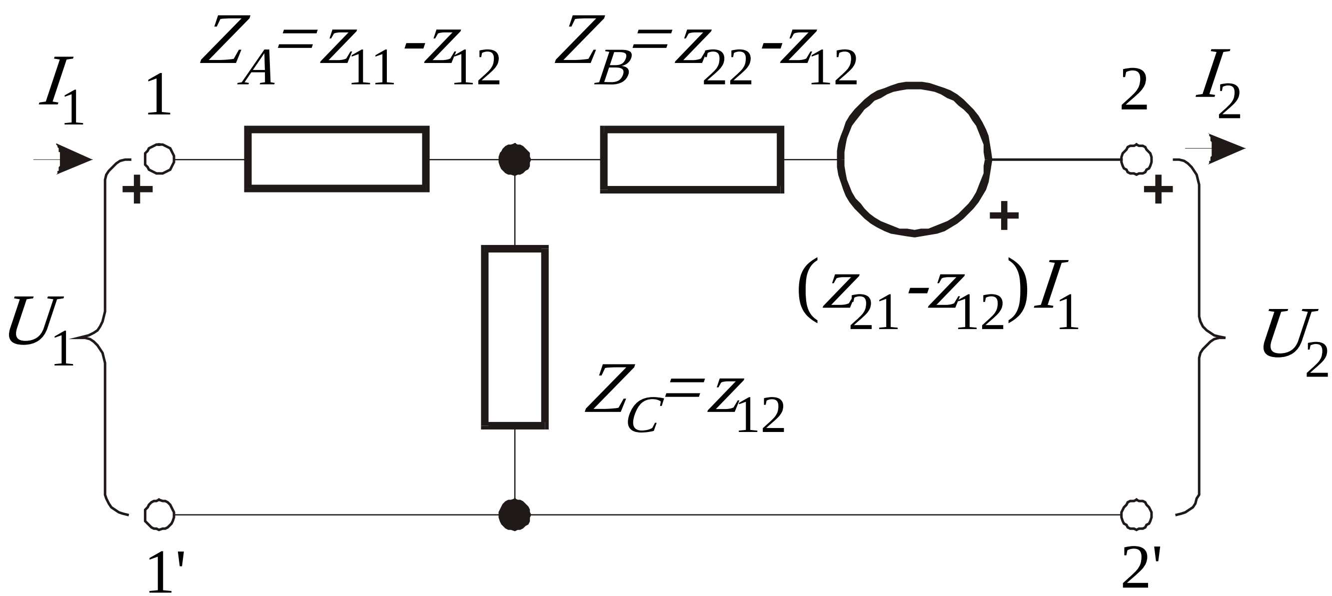
$$\left.\begin{array}{ll} z_{11} = Z_A + Z_C & z_{12} = Z_C \\ z_{21} = Z_C & z_{22} = Z_B + Z_C \end{array}\right\} \Rightarrow \begin{array}{l} Z_A = z_{11} - z_{12} \\ Z_B = z_{22} - z_{12} \\ Z_C = z_{12} = z_{21} \end{array}$$

Z_A, Z_B i Z_C nisu uvijek realizabilne R, L ili C elementima

Nerecipročni četveropol $z_{12} \neq z_{21}$

$$U_1 = I_1 z_{11} - I_2 z_{12}$$
$$U_2 = I_1 z_{21} - I_2 z_{22}$$

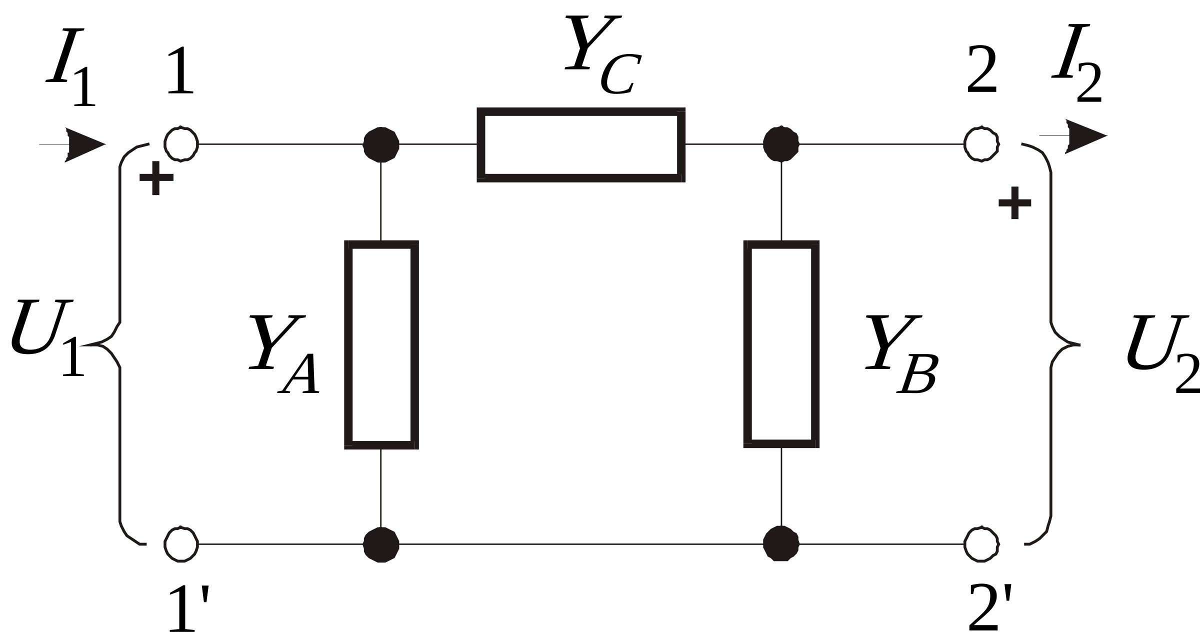
$$U_1 = I_1 z_{11} - I_2 z_{12}$$
$$U_2 = I_1 z_{12} - I_2 z_{22} + \underbrace{I_1(z_{21} - z_{12})}_{\text{naponski izvor}}$$



Ekvivalentni četveropol u Π-spoju

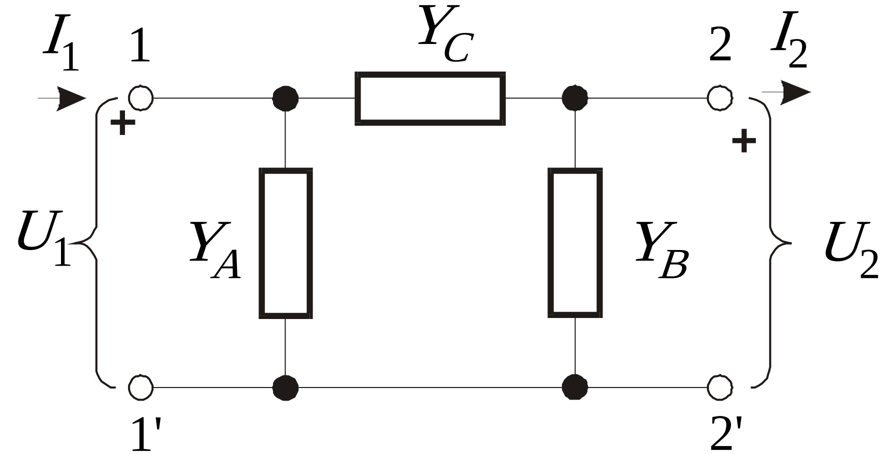
Svakom recipročnom četveropolu zadanom y-parametrima

moguće je odrediti ekvivalentni četveropol u Π -spoju



y-parametri ekvivalentnoga Π-spoja

$$I_1 = U_1 y_{11} - U_2 y_{12}$$
$$I_2 = U_1 y_{21} - U_2 y_{22}$$



elementi ekvivalentnoga Π-spoja

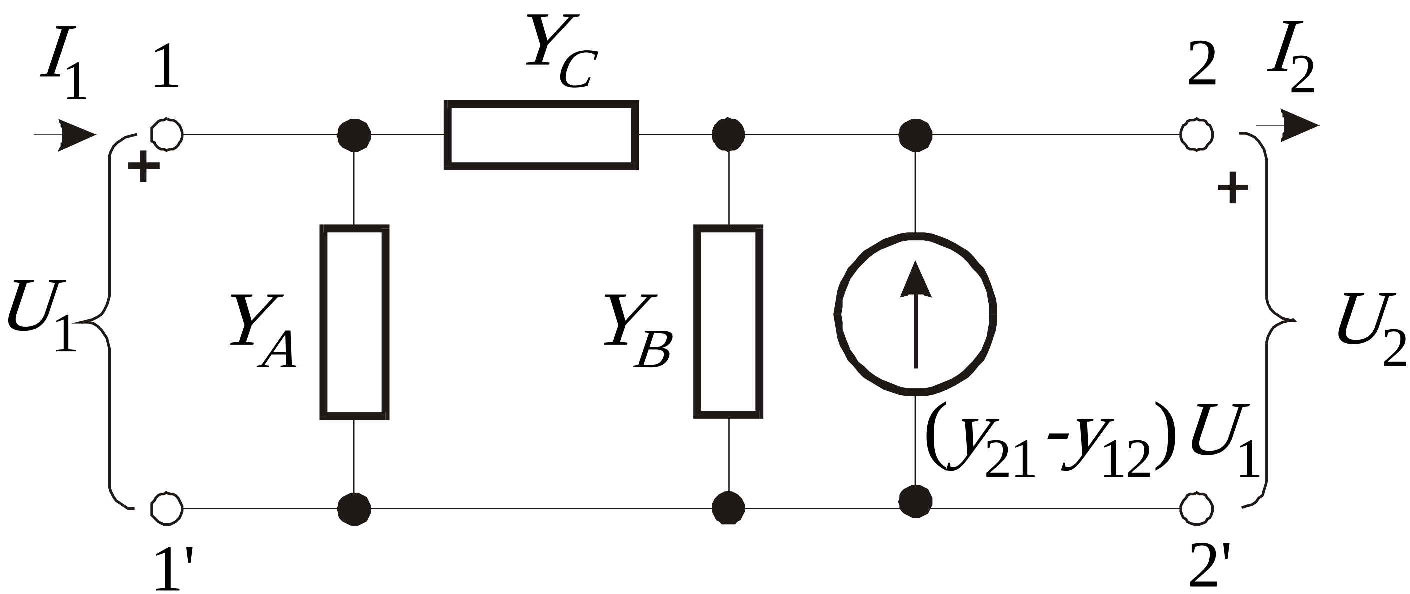
$$\left.\begin{aligned} y_{11} &= Y_A + Y_C & y_{12} &= Y_C \\ y_{21} &= Y_C & y_{22} &= Y_B + Y_C \end{aligned}\right\} \Rightarrow \begin{aligned} Y_A &= y_{11} - y_{12} \\ Y_B &= y_{22} - y_{12} \\ Y_C &= y_{12} = y_{21} \end{aligned}$$

Y_A, Y_B i Y_C nisu uvijek realizabilne R, L ili C elementima

Nerecipročni četveropol $y_{12} \neq y_{21}$

$$I_1 = y_{11}U_1 - y_{12}U_2 \qquad\qquad I_1 = y_{11}U_1 - y_{12}U_2$$

$$\underline{I_2 = y_{21}U_1 - y_{22}U_2} \qquad\qquad I_2 = y_{12}U_1 - y_{22}U_2 + \underbrace{(y_{21} - y_{12})U_1}_{\text{strujni izvor}}$$



Zrcalni ili valni parametri četveropola

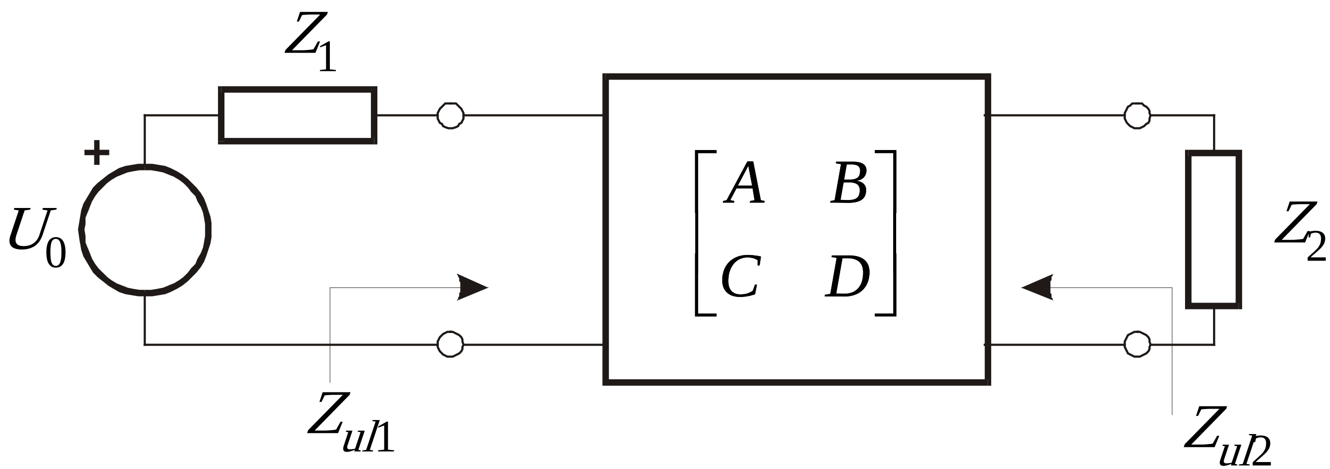
- Recipročne je četveropole moguće opisati s 3 parametra.
- Kod recipročnih četveropola često se primjenjuju

ZRCALNI ILI VALNI PARAMETRI

- To su:

- zrcalne impedancije Z_{C1} i Z_{C2}
- zrcalna konstanta prijenosa g

- Za definiranje zrcalnih parametara → četveropol na slici

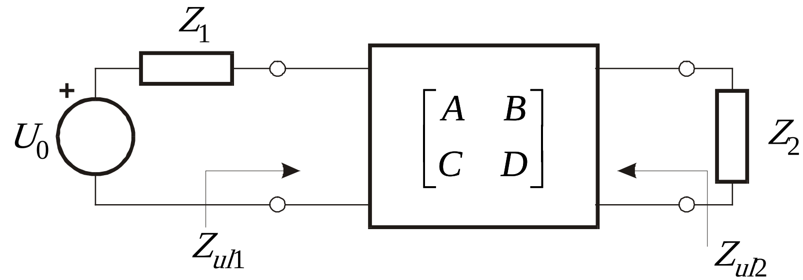


- Ulazne impedancije na 1-1' i 2-2' su

$$Z_{ul1} = \frac{AZ_2 + B}{CZ_2 + D} \qquad Z_{ul2} = \frac{DZ_1 + B}{CZ_1 + A}$$

Zrcalni parametri četveropola

- Uvjet: $Z_{ul1} = Z_1$ i $Z_{ul2} = Z_2$



$$Z_{u1} = Z_1 \quad \Longrightarrow \quad Z_1 = \frac{AZ_2 + B}{CZ_2 + D} \Bigg/ \cdot (CZ_2 + D)$$

$$Z_{u2} = Z_2 \quad \Longrightarrow \quad Z_2 = \frac{DZ_1 + B}{CZ_1 + A} \Bigg/ \cdot (CZ_1 + A)$$

$$\begin{aligned} CZ_1Z_2 + DZ_1 - AZ_2 - B &= 0 \\ CZ_1Z_2 + AZ_2 - DZ_1 - B &= 0 \quad \Big/ + \end{aligned}$$

$$CZ_1Z_2 - B = 0 \quad \Longrightarrow \quad \boxed{Z_1 \cdot Z_2 = \frac{B}{C}}$$

$$\begin{array}{l} CZ_1Z_2 + DZ_1 - AZ_2 - B = 0 \\ CZ_1Z_2 + AZ_2 - DZ_1 - B = 0 \end{array} \Bigg/ \; -$$

$$DZ_1 - AZ_2 = 0 \quad \Rightarrow \quad \boxed{\frac{Z_1}{Z_2} = \frac{A}{D}}$$

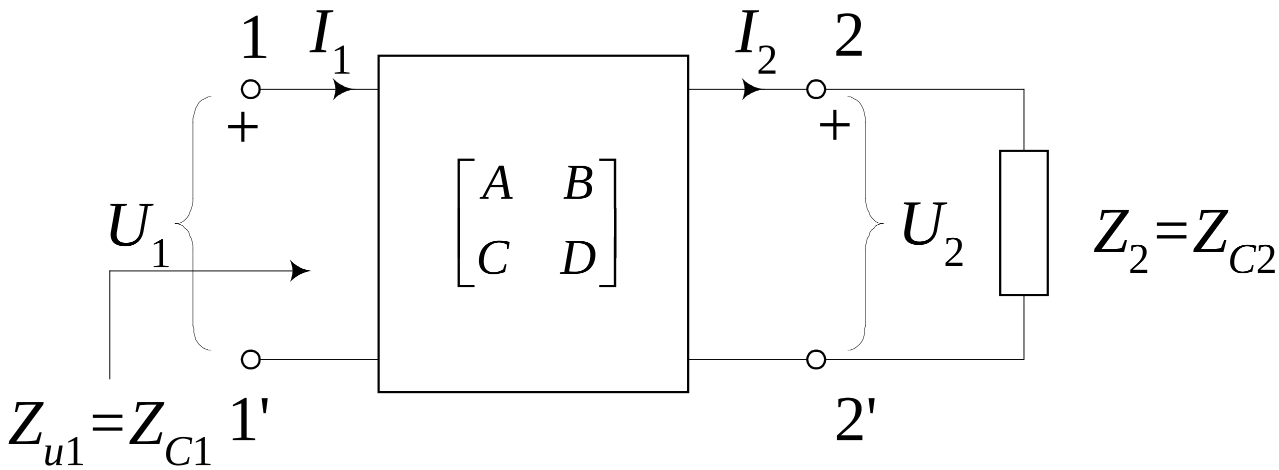
$$\underbrace{\boxed{Z_1 = Z_{C1} = \sqrt{\frac{AB}{CD}}} \qquad \boxed{Z_2 = Z_{C2} = \sqrt{\frac{BD}{AC}}}}$$

ZRCALNE ILI VALNE IMPEDANCIJE

- Ako je

$Z_2 = Z_{C2}$

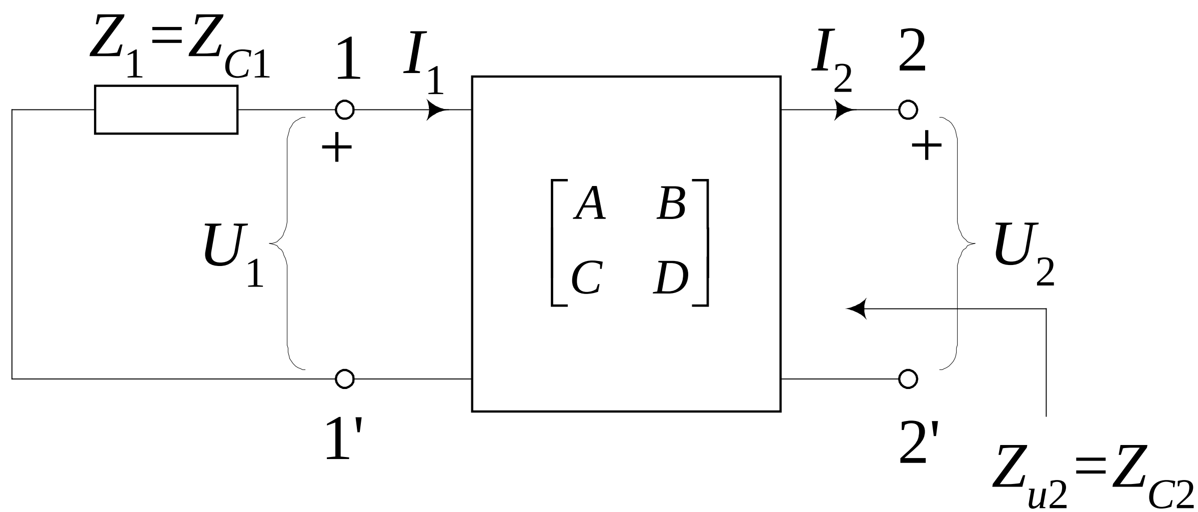
tada je $Z_{ul1} = Z_{C1}$

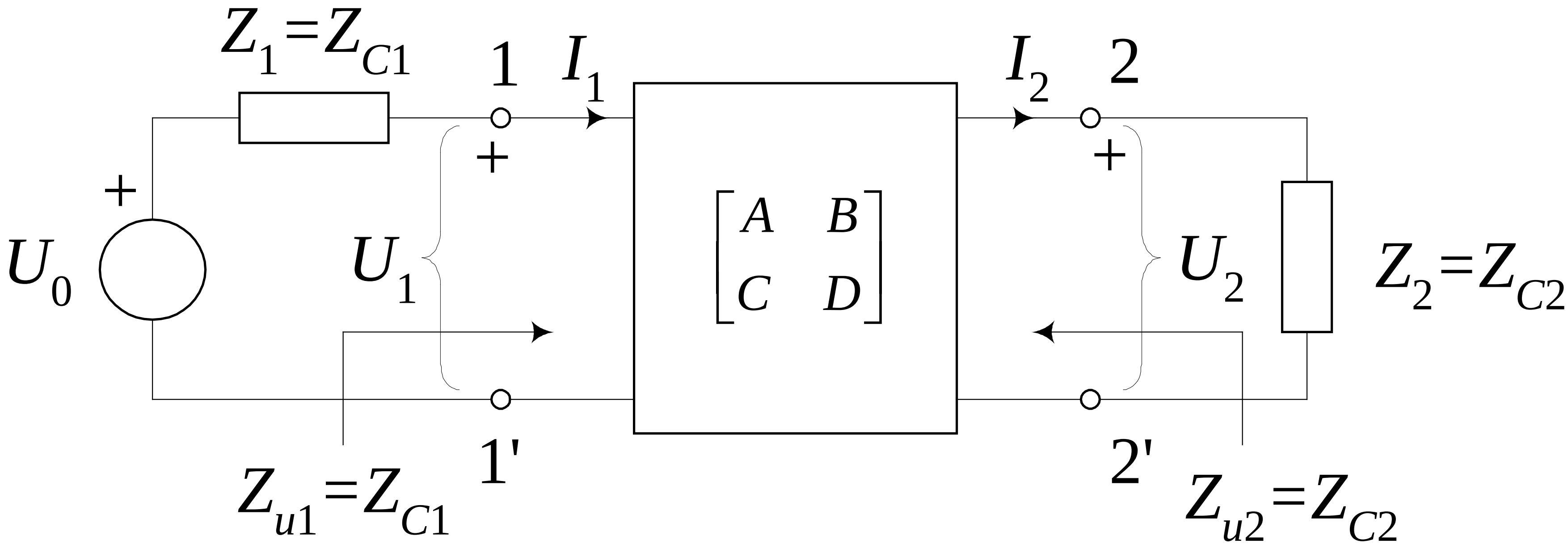


- Ako je

$Z_1 = Z_{C1}$

tada je $Z_{ul2} = Z_{C2}$

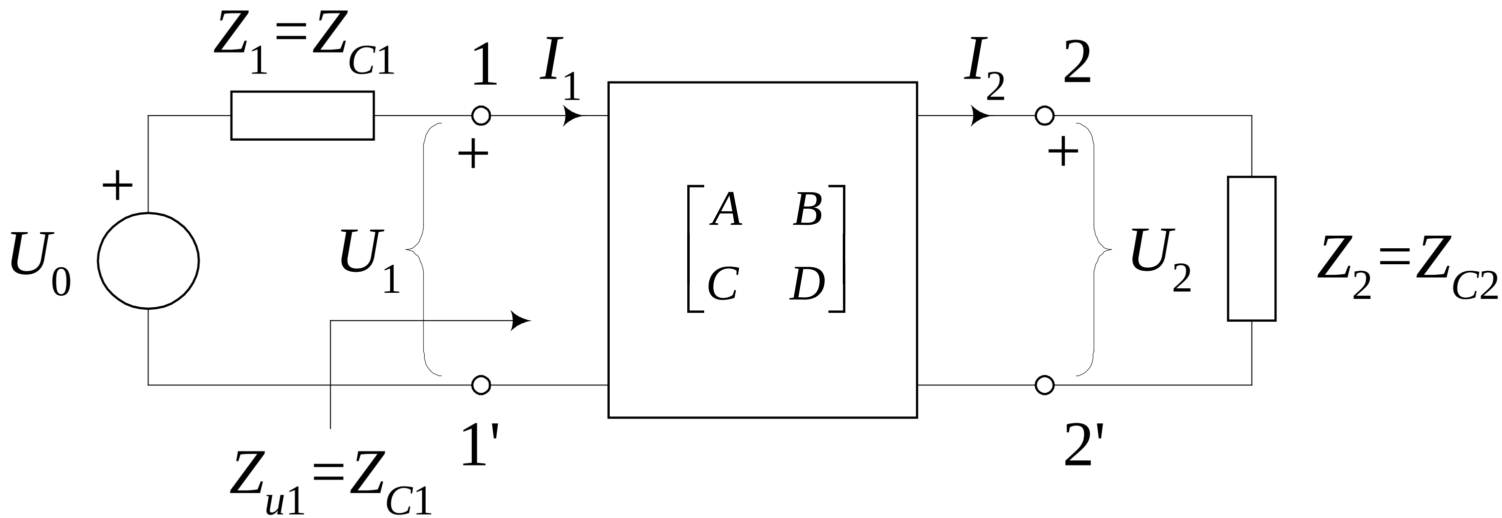




Četveropol je prilagođen po zrcalnim impedancijama.

Prijenosne jednadžbe i zrcalni parametri

Prijenos signala s 1-1’ na 2-2’.



$$U_2 = I_2 Z_{C2}$$

$$U_1 = AU_2 + BI_2 = AU_2 + \frac{B}{Z_{C2}} U_2 \quad \Rightarrow \quad \frac{U_1}{U_2} = A + \frac{B}{Z_{C2}}$$

$$I_1 = CU_2 + DI_2 = (CZ_{C2} + D) I_2 \quad \Rightarrow \quad \frac{I_1}{I_2} = CZ_{C2} + D$$

$$Z_{C2} = \sqrt{\frac{BD}{AC}}$$

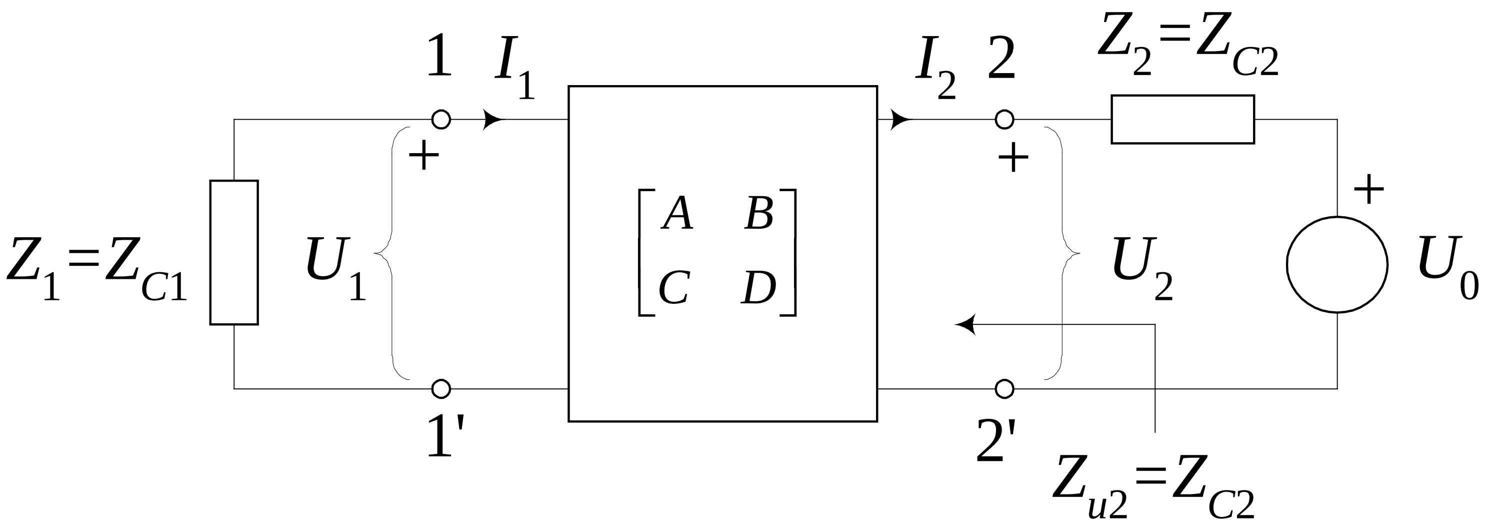
$$\frac{U_1}{U_2} = A + \frac{B}{\sqrt{\frac{BD}{AC}}} = \sqrt{\frac{A}{D}}\left(\sqrt{AD} + \sqrt{BC}\right)$$

omjer prijenosa napona

$$\frac{I_1}{I_2} = C\sqrt{\frac{BD}{AC}} + D = \sqrt{\frac{D}{A}}\left(\sqrt{AD} + \sqrt{BC}\right)$$

omjer prijenosa struje

Prijenos signala s 2-2’ na 1-1’.



$$U_1 = -I_1 Z_{C1}$$

$$U_2 = DU_1 - BI_1 = DU_1 + \frac{B}{Z_{C1}} U_1 \quad \Longrightarrow \quad \frac{U_2}{U_1} = D + \frac{B}{Z_{C1}}$$

$$-I_2 = CU_1 - AI_1 = -(CZ_{C1} + A) I_1 \quad \Longrightarrow \quad \frac{I_2}{I_1} = CZ_{C1} + A$$

Zrcalni parametri četveropola

$$Z_{C1} = \sqrt{\frac{AB}{CD}}$$

$$\frac{U_2}{U_1} = D + \frac{B}{Z_{C1}} = \sqrt{\frac{D}{A}}\left(\sqrt{AD} + \sqrt{BC}\right)$$

omjer prijenosa napona

$$\frac{I_2}{I_1} = CZ_{C1} + A = \sqrt{\frac{A}{D}}\left(\sqrt{AD} + \sqrt{BC}\right)$$

omjer prijenosa struja

Zrcalni parametri četveropola

$$\frac{A}{D} = \frac{Z_{C1}}{Z_{C2}} \qquad \sqrt{\frac{A}{D}} = \sqrt{\frac{Z_{C1}}{Z_{C2}}} = n$$

$$\frac{D}{A} = \frac{Z_{C2}}{Z_{C1}} \qquad \sqrt{\frac{D}{A}} = \sqrt{\frac{Z_{C2}}{Z_{C1}}} = \frac{1}{n}$$

$n \rightarrow$ omjer transformacije

$$\sqrt{AD} + \sqrt{BC} = e^{g} \quad \Longrightarrow \quad g = \ln(\sqrt{AD} + \sqrt{BC})$$

$g \rightarrow$ ***ZRCALNI KOEFICIJENT PRIJENOSA***

- U uvjetima stacionarne sinusne pobude

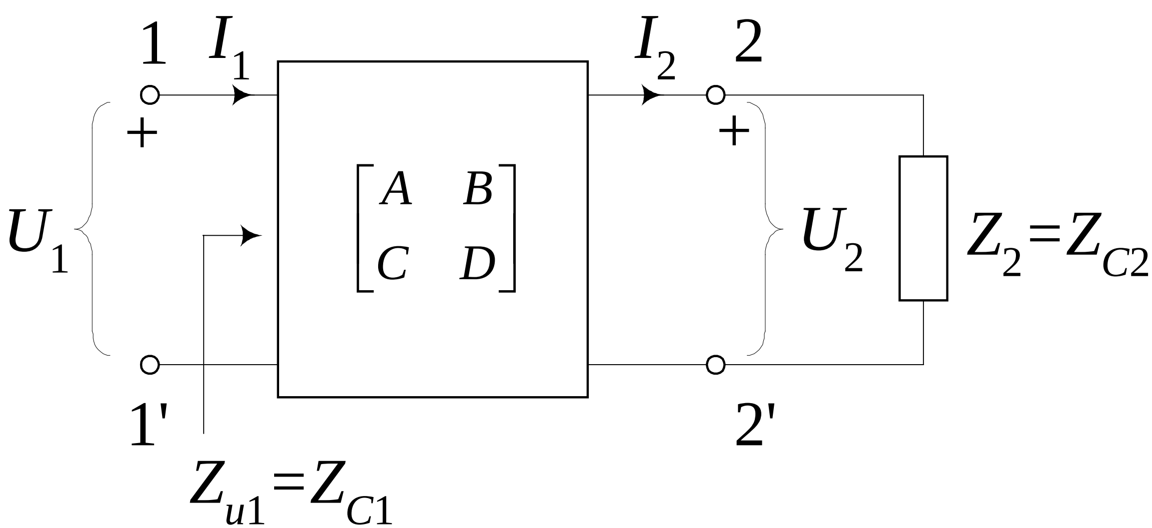
$$g = g(j\omega) = a(\omega) + jb(\omega)$$

- kompleksna veličina

$a \rightarrow$ zrcalni koeficijent gušenja

$b \rightarrow$ zrcalni koeficijent faze

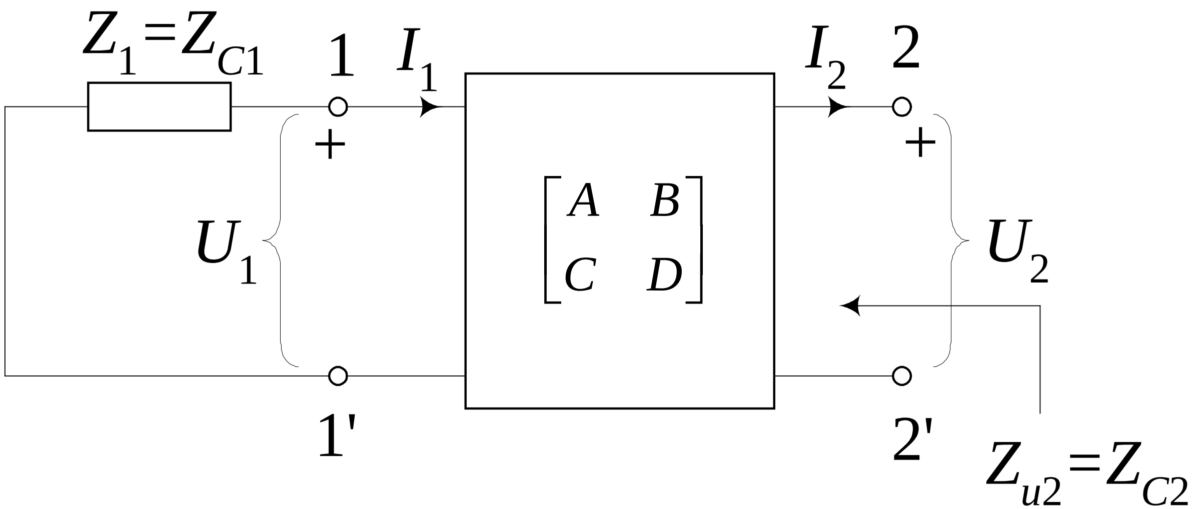
- Kod prilagođenja i prijenosa sa 1-1' na 2-2'



$$\frac{U_1}{U_2} = n \cdot e^g \quad \text{uz} \quad Z_2 = Z_{C2}$$

$$\frac{I_1}{I_2} = \frac{1}{n} \cdot e^g \quad \text{uz} \quad Z_2 = Z_{C2}$$

- Kod prilagođenja i prijenosa sa 2-2’ na 1-1’



$$\frac{U_2}{U_1} = \frac{1}{n} \cdot e^g \quad \text{uz} \quad Z_1 = Z_{C1}$$

$$\frac{I_2}{I_1} = n \cdot e^g \quad \text{uz} \quad Z_1 = Z_{C1}$$

- Ako se izraz za zrcalni koeficijent prijenosa

$$e^{g}=\sqrt{AD}+\sqrt{BC}$$

pomnoži s izrazom $\left(\sqrt{AD}-\sqrt{BC}\right)$

dobiva se

$$e^{g}\left(\sqrt{AD}-\sqrt{BC}\right)=\left(\sqrt{AD}+\sqrt{BC}\right)\left(\sqrt{AD}-\sqrt{BC}\right)=AD-BC$$

- Pošto se radi o recipročnome četveropolu, vrijedi

$$AD-BC=1$$

pa je $e^{g}(\sqrt{AD}-\sqrt{BC})=1 \quad \Longrightarrow \quad (\sqrt{AD}-\sqrt{BC})=e^{-g}$

- Ove izraze moguće je koristiti za određivanje parametara A,B,C i D iz poznatih zrcalnih Z_{C1}, Z_{C2} i g.

$$\left.\begin{aligned}\sqrt{AD}+\sqrt{BC}&=e^{g}\\ \sqrt{AD}-\sqrt{BC}&=e^{-g}\end{aligned}\right/ + \quad \Big/ :2 \quad \Rightarrow \quad \sqrt{AD}=\frac{e^{g}+e^{-g}}{2}=ch(g)$$

$$\left.\begin{aligned}\sqrt{AD}+\sqrt{BC}&=e^{g}\\ \sqrt{AD}-\sqrt{BC}&=e^{-g}\end{aligned}\right/ - \quad \Big/ :2 \quad \Rightarrow \quad \sqrt{BC}=\frac{e^{g}-e^{-g}}{2}=sh(g)$$

Zrcalni parametri četveropola

$$ch(g) = \sqrt{AD} \qquad \sqrt{\frac{Z_{C1}}{Z_{C2}}} = \sqrt{\frac{A}{D}}$$

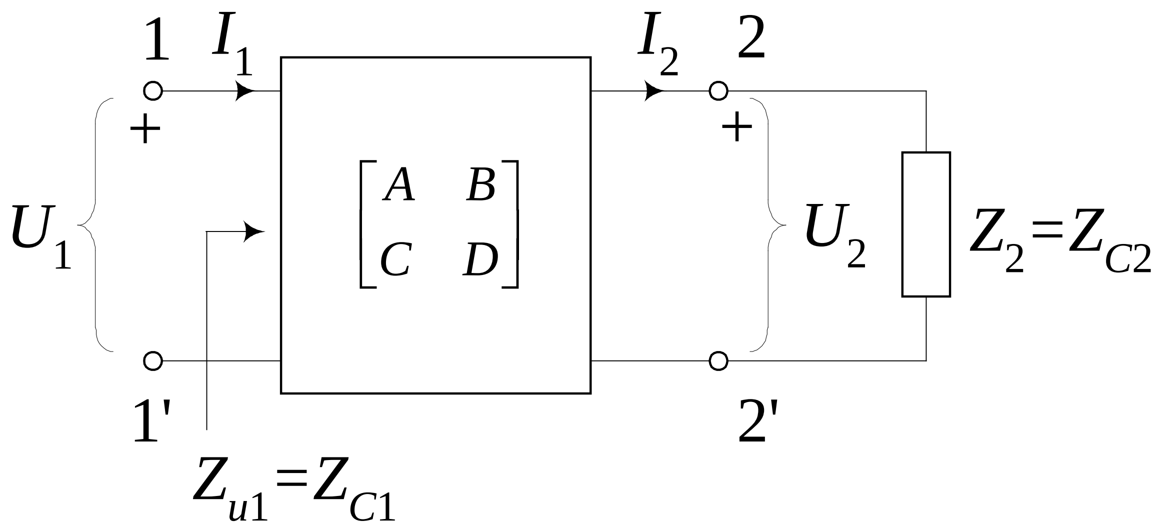
$$sh(g) = \sqrt{BC} \qquad \sqrt{Z_{C1}Z_{C2}} = \sqrt{\frac{B}{C}}$$

$$\boxed{A = \sqrt{\frac{Z_{C1}}{Z_{C2}}} \cdot chg} \qquad \boxed{B = \sqrt{Z_{C1}Z_{C2}} \cdot shg = n \cdot Z_{C2} \cdot shg}$$

$$\boxed{C = \frac{1}{\sqrt{Z_{C1}Z_{C2}}} \cdot shg = \frac{1}{n \cdot Z_{C2}} \cdot shg} \qquad \boxed{\mathrm{D} = \sqrt{\frac{Z_{C2}}{Z_{C1}}} \cdot chg}$$

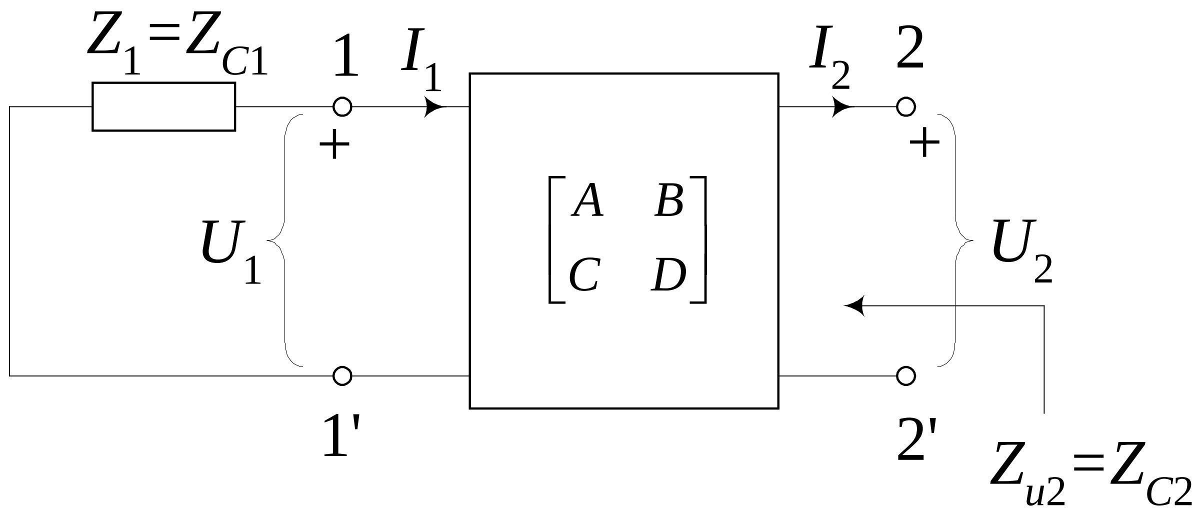
Prijenosne jednadžbe

- Prilagođenje na 2-2' i prijenos s 1-1' na 2-2'



$$\boxed{\begin{aligned} U_1 &= n(U_2 \cdot chg + I_2 \cdot Z_{C2} \cdot shg) \\ I_1 &= \frac{1}{n}\left(U_2 \cdot \frac{shg}{Z_{C2}} + I_2 \cdot chg\right) \end{aligned}}$$

- Prilagođenje na 1-1' i prijenos sa 2-2' na 1-1'



$$U_2 = \frac{1}{n}(U_1 \cdot chg - I_1 \cdot Z_{C1} \cdot shg)$$

$$-I_2 = n(\frac{U_1}{Z_{C1}} \cdot shg - I_1 \cdot chg)$$

Impedancije na kratko i na prazno

$$U_1 = AU_2 + BI_2$$

$$I_1 = CU_2 + DI_2$$

$$I_2 = 0 \quad \Rightarrow \quad Z_{p1} = z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0} = \frac{A}{C}$$

$$U_2 = 0 \quad \Rightarrow \quad Z_{k1} = \frac{1}{y_{11}} = \left.\frac{U_1}{I_1}\right|_{U_2=0} = \frac{B}{D}$$

$$Z_{c1} = \sqrt{\frac{AB}{CD}} = \sqrt{Z_{p1} Z_{k1}} = \sqrt{\frac{z_{11}}{y_{11}}}$$

Zrcalni parametri četveropola

$$U_1 = AU_2 + BI_2$$

$$I_1 = CU_2 + DI_2$$

$$I_1 = 0 \quad \Rightarrow \quad Z_{p2} = z_{22} = -\frac{U_2}{I_2}\bigg|_{I_1=0} = \frac{D}{C}$$

$$U_1 = 0 \quad \Rightarrow \quad Z_{k2} = \frac{1}{y_{22}} = -\frac{U_2}{I_2}\bigg|_{U_1=0} = \frac{B}{A}$$

$$Z_{c2} = \sqrt{\frac{DB}{AC}} = \sqrt{Z_{p2} Z_{k2}} = \sqrt{\frac{z_{22}}{y_{22}}}$$

$$Z_{p1}=\frac{A}{C}\qquad Z_{p2}=\frac{D}{C}\qquad Z_{k1}=\frac{B}{D}\qquad Z_{k2}=\frac{B}{A}$$

- Za omjere impedancija na prazno i na kratko vrijedi

$$\frac{Z_{p1}}{Z_{p2}}=\frac{A}{D}\qquad \frac{Z_{k1}}{Z_{k2}}=\frac{A}{D}$$

- Odatle slijedi

$$\frac{Z_{p1}}{Z_{p2}}=\frac{Z_{k1}}{Z_{k2}}\quad\Longrightarrow\quad\frac{Z_{p1}}{Z_{k1}}=\frac{Z_{p2}}{Z_{k2}}$$

$$th(g)=\frac{sh(g)}{ch(g)}=\sqrt{\frac{BC}{AD}}=\sqrt{\frac{Z_{k1}}{Z_{p1}}}=\sqrt{\frac{Z_{k2}}{Z_{p2}}}$$

- Zrcalni parametri izraženi s Z_{k1}, Z_{k2}, Z_{p1} i Z_{p2}, glase

$$\boxed{Z_{c1} = \sqrt{Z_{p1} Z_{k1}}}$$

$$\boxed{Z_{c2} = \sqrt{Z_{p2} Z_{k2}}}$$

$$\boxed{th(g) = \sqrt{\frac{Z_{k1}}{Z_{p1}}} = \sqrt{\frac{Z_{k2}}{Z_{p2}}}}$$

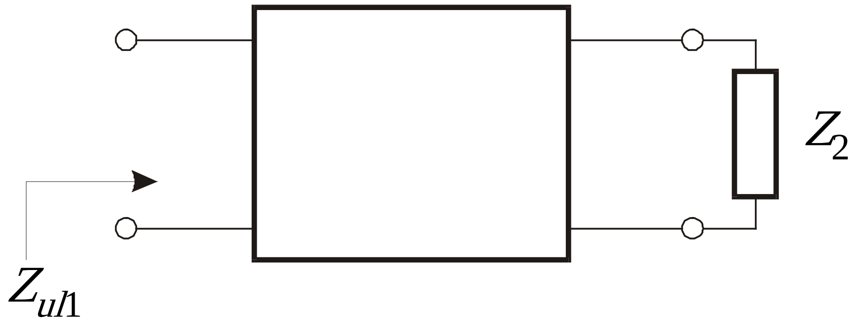
- Z_{k1}, Z_{k2}, Z_{p1} i Z_{p2} je moguće izraziti zrcalnim parametrima

$$\left.\begin{aligned} Z_{c1} &= \sqrt{Z_{p1} Z_{k1}} \\ th(g) &= \sqrt{\frac{Z_{k1}}{Z_{p1}}} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} Z_{k1} &= Z_{c1}\, th(g) \\ Z_{p1} &= Z_{c1}\, cth(g) \end{aligned}\right.$$

$$\left.\begin{aligned} Z_{c2} &= \sqrt{Z_{p2} Z_{k2}} \\ th(g) &= \sqrt{\frac{Z_{k2}}{Z_{p2}}} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} Z_{k2} &= Z_{c2}\, th(g) \\ Z_{p2} &= Z_{c2}\, cth(g) \end{aligned}\right.$$

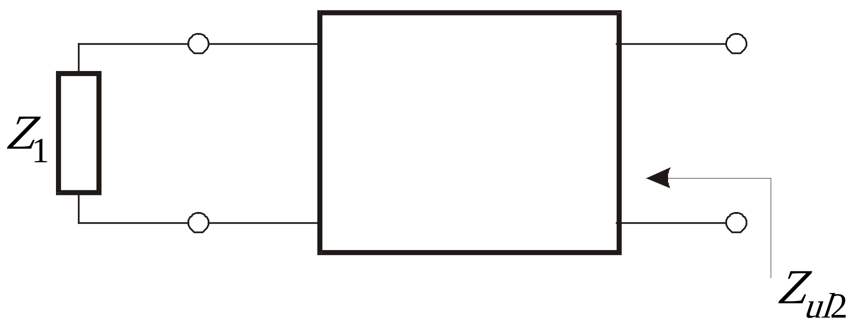
Zrcalni parametri četveropola

Ako četveropol nije prilagođen → $Z_2 \neq Z_{C2} \Rightarrow Z_{u1} \neq Z_{c1}$



$$Z_{ul1} = \frac{U_1}{I_1} = \frac{shg + \dfrac{Z_2}{Z_{c2}} chg}{chg + \dfrac{Z_2}{Z_{c2}} shg} Z_{c1}$$

$Z_1 \neq Z_{C1} \Rightarrow Z_{u2} \neq Z_{c2}$



$$Z_{ul2} = -\frac{U_2}{I_2} = \frac{shg + \dfrac{Z_1}{Z_{c1}} chg}{chg + \dfrac{Z_1}{Z_{c1}} shg} Z_{c2}$$

Mjera odstupanja od prilagođenja

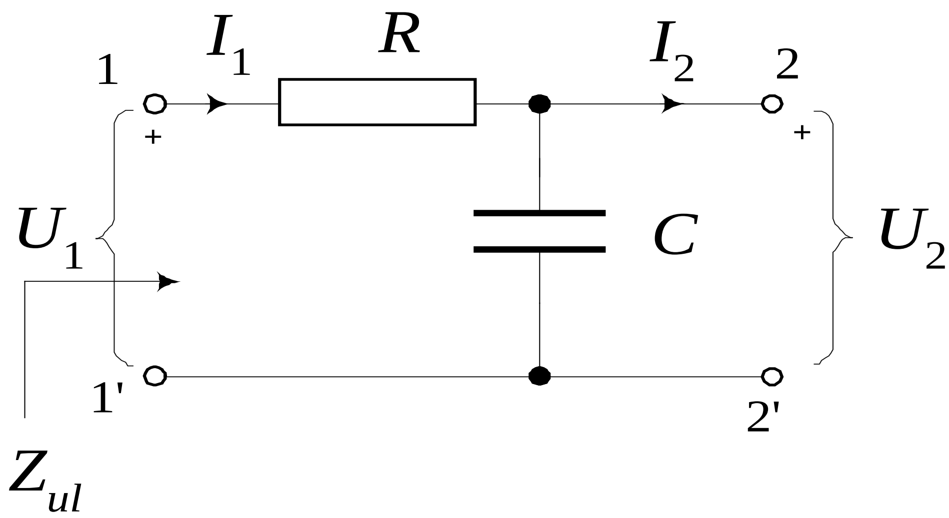
$$p_1 = \frac{Z_1 - Z_{c1}}{Z_1 + Z_{c1}}$$

$\rightarrow$ pogreška prilagođenja ili koeficijent refleksije na ulazu

$$p_2 = \frac{Z_2 - Z_{c2}}{Z_2 + Z_{c2}}$$

$\rightarrow$ pogreška prilagođenja ili koeficijent refleksije na izlazu

Primjer: Izračunati zrcalne parametre četveropola na slici



$$Z_{k1} = R \qquad\qquad Z_{k2} = \frac{R}{sRC+1}$$

$$Z_{p1} = R + \frac{1}{sC} \qquad\qquad Z_{p2} = \frac{1}{sC}$$

Zrcalni parametri četveropola

$$Z_{C1}=\sqrt{R\left(R+\frac{1}{sC}\right)}=\sqrt{\frac{R}{sC}}\sqrt{sRC+1}$$

$$Z_{C2}=\sqrt{\frac{1}{sC}\cdot\frac{R}{sRC+1}}=\sqrt{\frac{R}{sC}}\frac{1}{\sqrt{sRC+1}}$$

$$th(g)=\sqrt{\frac{Z_{k1}}{Z_{p1}}}=\sqrt{\frac{R}{R+1/sC}}=\sqrt{\frac{sRC}{sRC+1}}$$

$$th(g)=\sqrt{\frac{Z_{k2}}{Z_{p2}}}=\sqrt{\frac{R}{(sRC+1)/sC}}=\sqrt{\frac{sRC}{sRC+1}}$$

Metode povezivanja četveropola

- Analiza složene mreže s dva pristupa može biti jednostavnija ako je prikazana

→ kao kombinacija dva ili više jednostavnih četveropola

- Sinteza nekog složenog prijenosnog sistema može biti jednostavnija ako se sastoji

→ od više međusobno spojenih osnovnih četveropola.

- Složena mreža → također četveropol

Postoji 5 načina spajanja četveropola:

- serijski spoj četveropola
- paralelni spoj četveropola
- serijsko-paralelni spoj četveropola
- paralelno-serijski spoj četveropola
- spoj u lanac ili kaskadu četveropola

- Svrha:
- Prilikom povezivanja → parametre cijeloga sistema

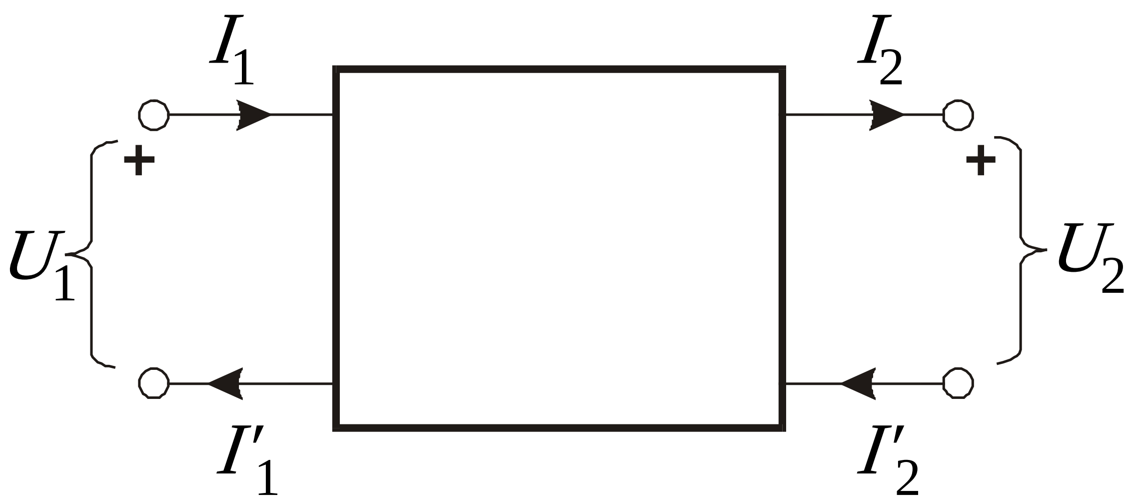
→iz parametara pojedinih četveropola

→korištenjem matrica parametara

- Osnovni uvjet za korištenje operacija s matricama:
- Svaki četveropol u kombinaciji

→ sačuvana svojstva na prilazima

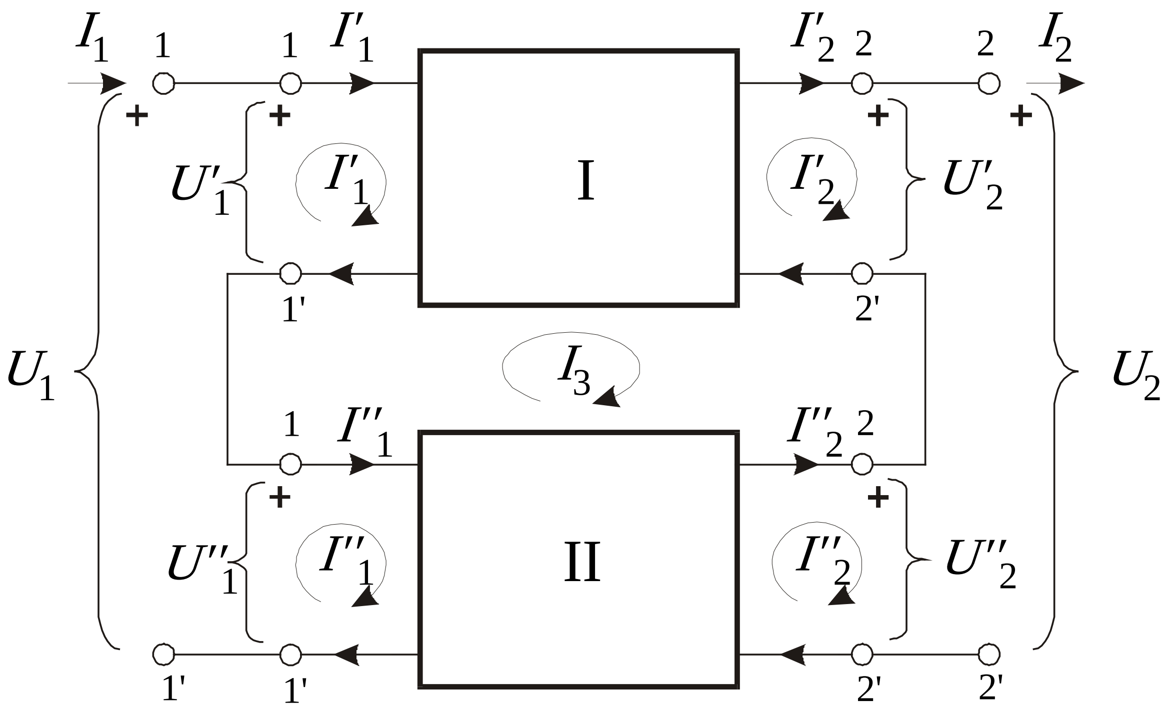
$$\boxed{I_1 = I'_1}$$



$$\boxed{I_2 = I'_2}$$

- <u>Serijski spoj četveropola:</u>

- Ulazne priključnice →spojene serijski
- Izlazne priključnice →spojene serijski



mora biti

I_3=0

Za ovaj spoj vrijedi:
$$U_1 = U'_1 + U''_1 \qquad I_1 = I'_1 = I''_1$$
$$U_2 = U'_2 + U''_2 \qquad I_2 = I'_2 = I''_2$$

$$\begin{bmatrix} U'_1 \\ U'_2 \end{bmatrix} = \begin{bmatrix} z'_{11} & -z'_{12} \\ z'_{21} & -z'_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \begin{bmatrix} U''_1 \\ U''_2 \end{bmatrix} = \begin{bmatrix} z''_{11} & -z''_{12} \\ z''_{21} & -z''_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

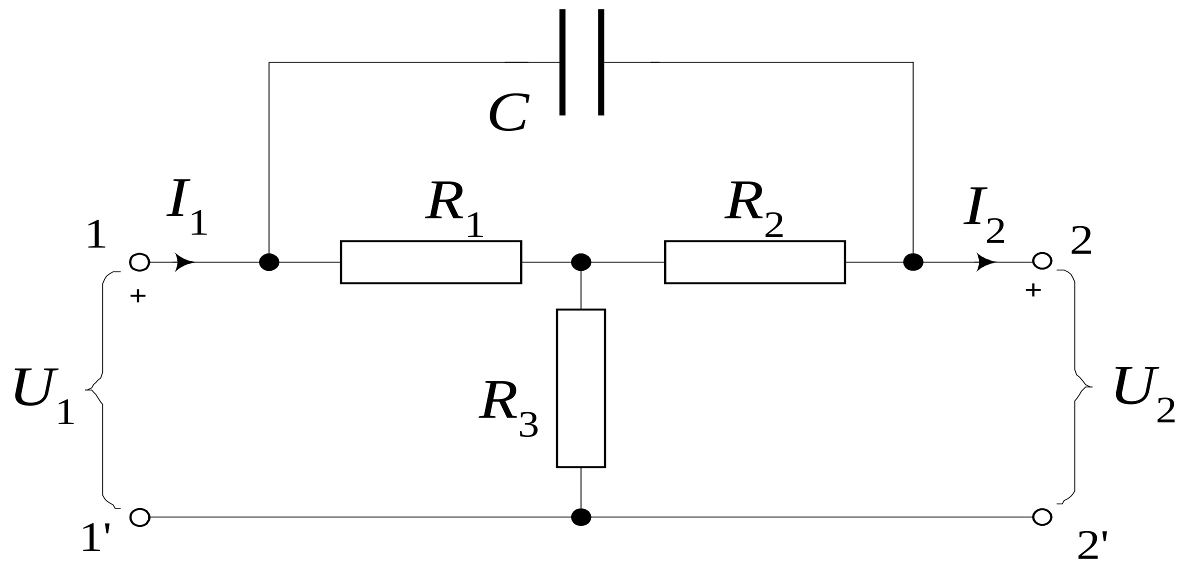
$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \left(\begin{bmatrix} z'_{11} & -z'_{12} \\ z'_{21} & -z'_{22} \end{bmatrix} + \begin{bmatrix} z''_{11} & -z''_{12} \\ z''_{21} & -z''_{22} \end{bmatrix} \right) \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\boxed{[z] = [z'] + [z'']}$$

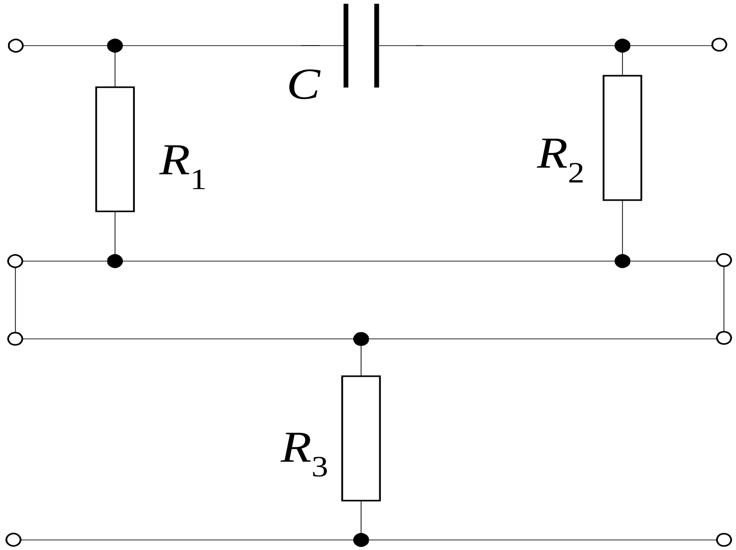
$$\boxed{z_{11} = z'_{11} + z''_{11}} \qquad \boxed{z_{12} = z'_{12} + z''_{12}}$$

$$\boxed{z_{21} = z'_{21} + z''_{21}} \qquad \boxed{z_{22} = z'_{22} + z''_{22}}$$

- Primjer: Premošteni T-četveropol



moguće je prikazati kao serijski spoj dvaju četveropola

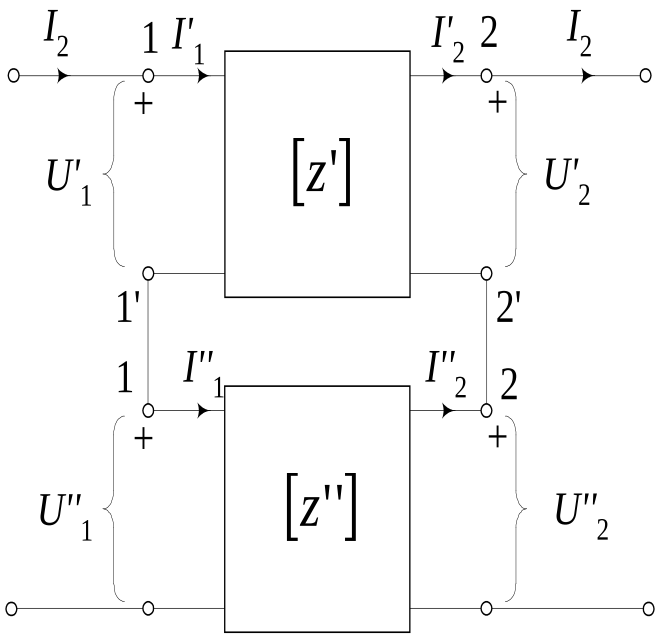


z-parametri tih četveropola su

$$z'_{11} = \frac{(sCR_2+1)R_1}{sC(R_1+R_2)+1}$$

$$z'_{12} = z'_{21} = \frac{R_1R_2sC}{sC(R_1+R_2)+1}$$

$$z'_{22} = \frac{(sCR_1+1)R_2}{sC(R_1+R_2)+1}$$



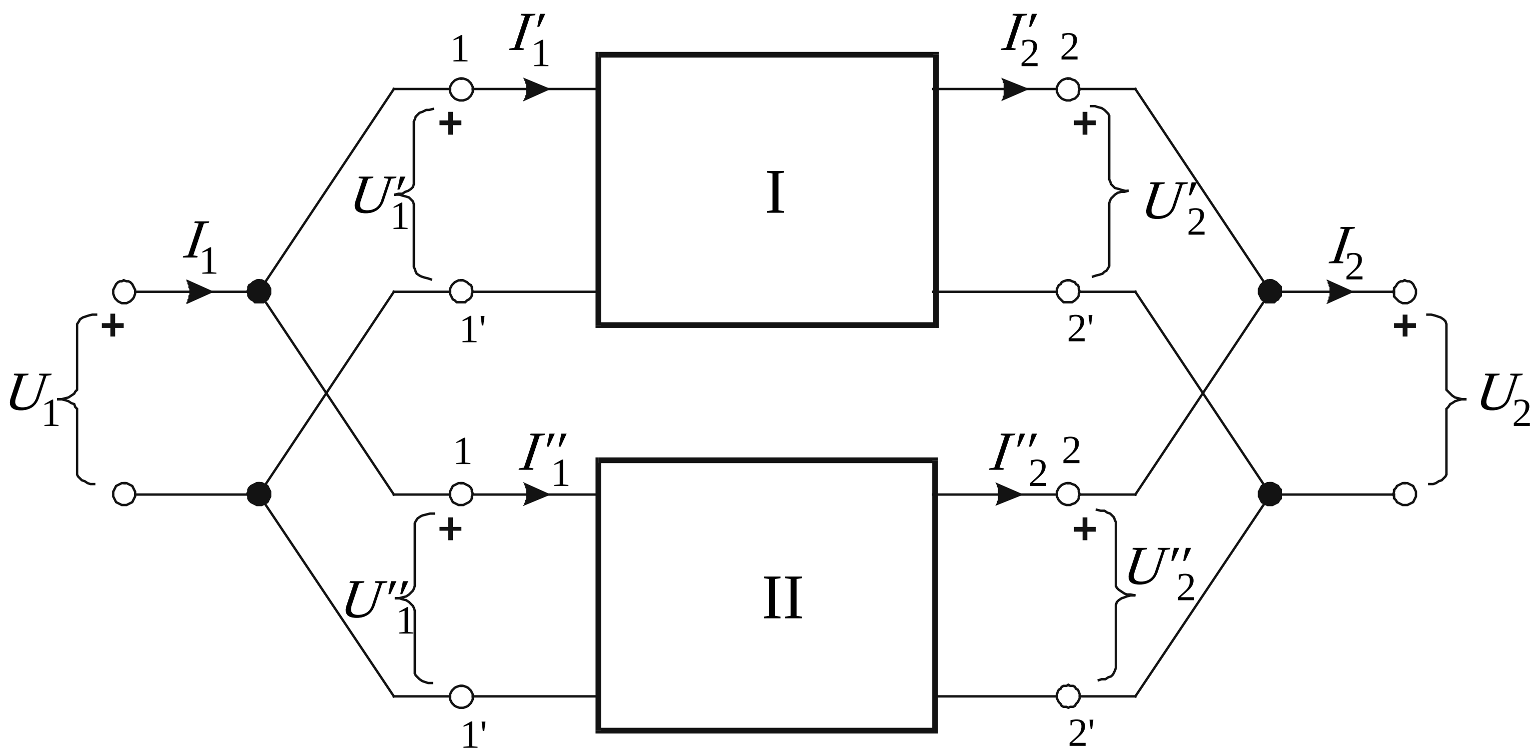
$$z''_{11} = z''_{12} = z''_{21} = z''_{22} = R_3$$

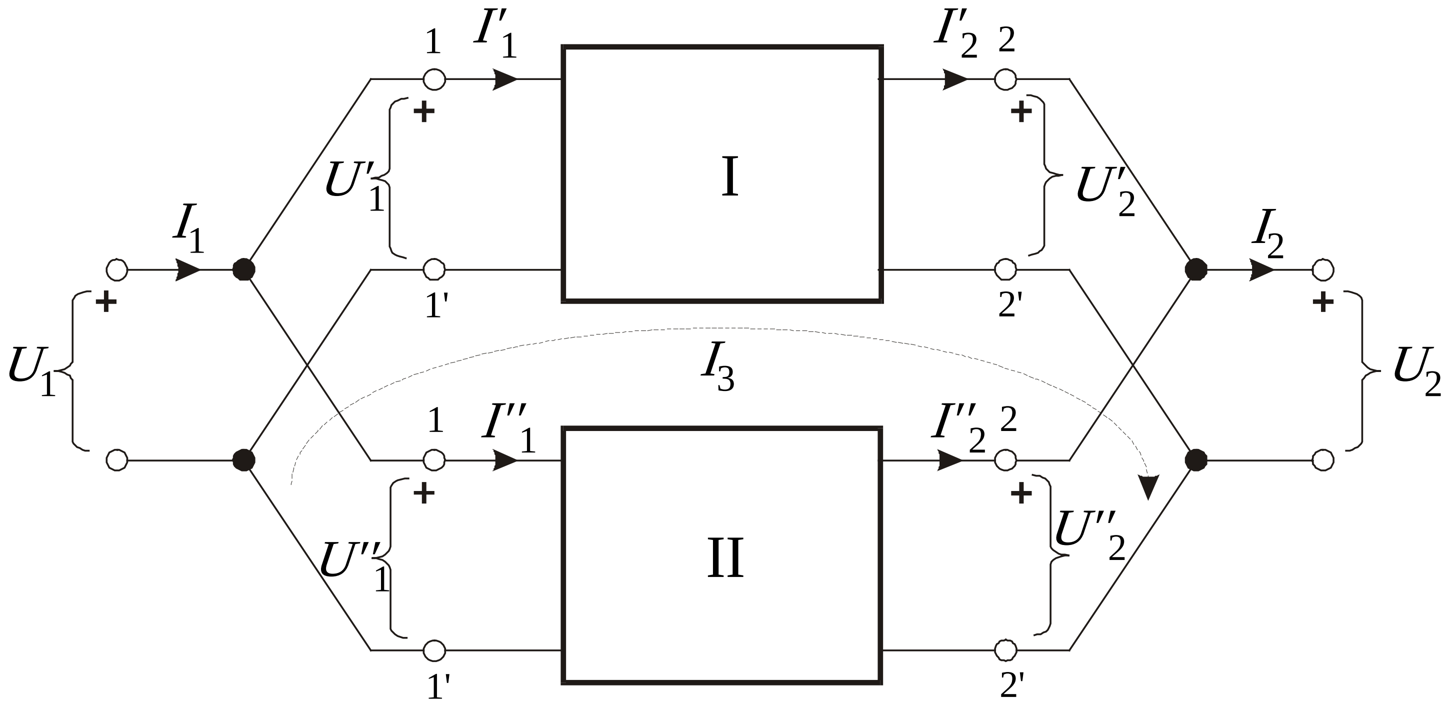
Matrica *z*-parametara kombinacije četveropola glasi

$$[z]=\begin{bmatrix} \dfrac{(sCR_2+1)R_1}{sC(R_1+R_2)+1}+R_3 & -\left(\dfrac{R_1R_2sC}{sC(R_1+R_2)+1}+R_3\right) \\ \dfrac{R_1R_2sC}{sC(R_1+R_2)+1}+R_3 & -\left(\dfrac{(sCR_1+1)R_2}{sC(R_1+R_2)+1}+R_3\right) \end{bmatrix}$$

Paralelni spoj četveropola:

- Ulazne priključnice →spojene paralelno
- Izlazne priključnice →spojene paralelno





Za ovaj spoj vrijedi:

$$U_1 = U'_1 = U''_1 \qquad I_1 = I'_1 + I''_1$$

$$U_2 = U'_2 = U''_2 \qquad I_2 = I'_2 + I''_2$$

mora biti

$I_3 = 0$

Postupci povezivanja četveropola

$$\begin{bmatrix} I'_1 \\ I'_2 \end{bmatrix} = \begin{bmatrix} y'_{11} & -y'_{12} \\ y'_{21} & -y'_{22} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \qquad \begin{bmatrix} I''_1 \\ I''_2 \end{bmatrix} = \begin{bmatrix} y''_{11} & -y''_{12} \\ y''_{21} & -y''_{22} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

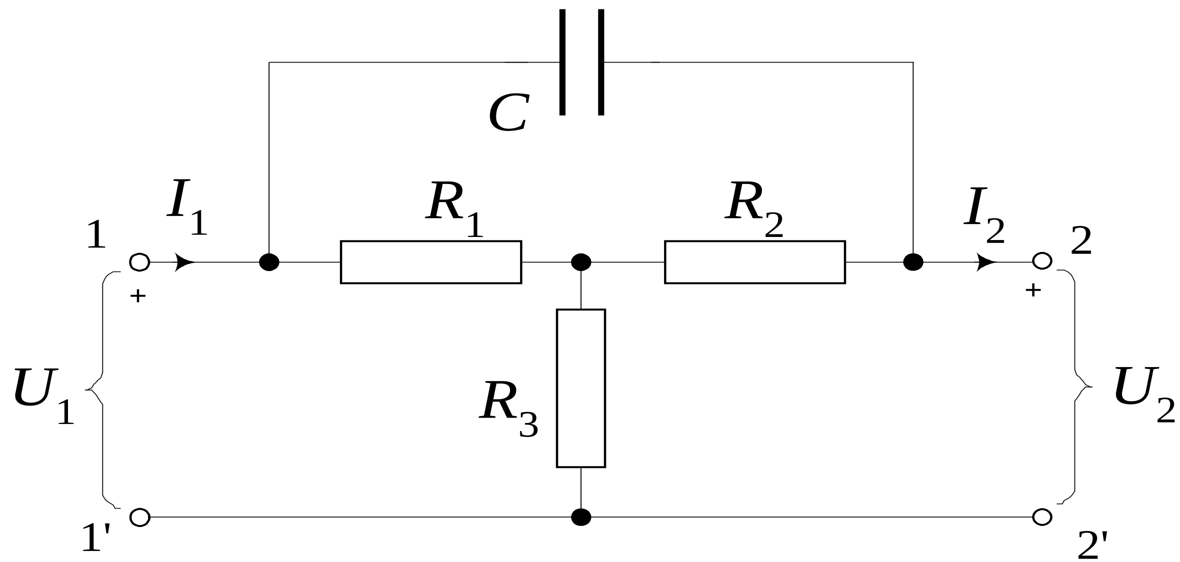
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I'_1 \\ I'_2 \end{bmatrix} + \begin{bmatrix} I''_1 \\ I''_2 \end{bmatrix} = \left(\begin{bmatrix} y'_{11} & -y'_{12} \\ y'_{21} & -y'_{22} \end{bmatrix} + \begin{bmatrix} y''_{11} & -y''_{12} \\ y''_{21} & -y''_{22} \end{bmatrix} \right) \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$[y] = [y'] + [y'']$$

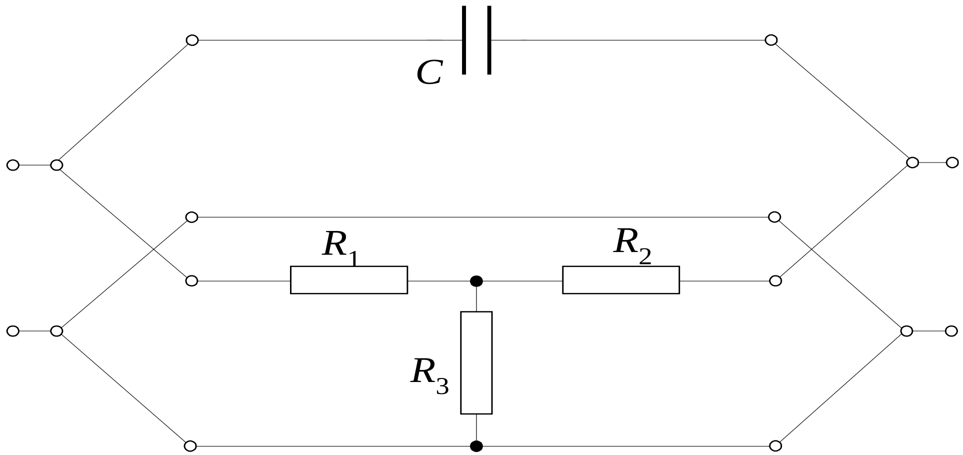
$$y_{11} = y'_{11} + y''_{11} \qquad y_{12} = y'_{12} + y''_{12}$$

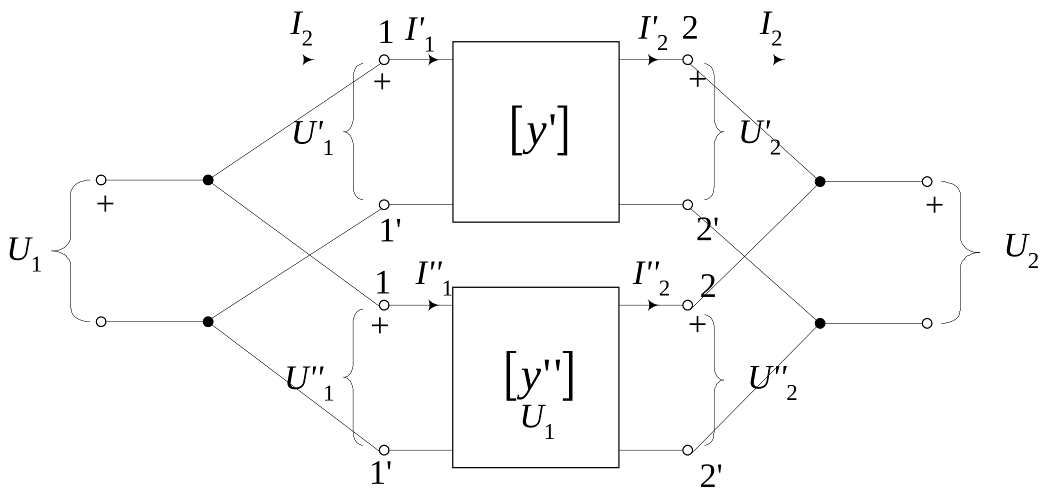
$$y_{21} = y'_{21} + y''_{21} \qquad y_{22} = y'_{22} + y''_{22}$$

- Primjer: Premošteni T-četveropol



moguće je prikazati kao paralelni spoj dvaju četveropola





y-parametri tih četveropola su

$$y'_{11} = \frac{R_2 + R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$y'_{21} = y'_{12} = \frac{R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

$$y'_{22} = \frac{R_1 + R_3}{R_1R_2 + R_1R_3 + R_2R_3}$$

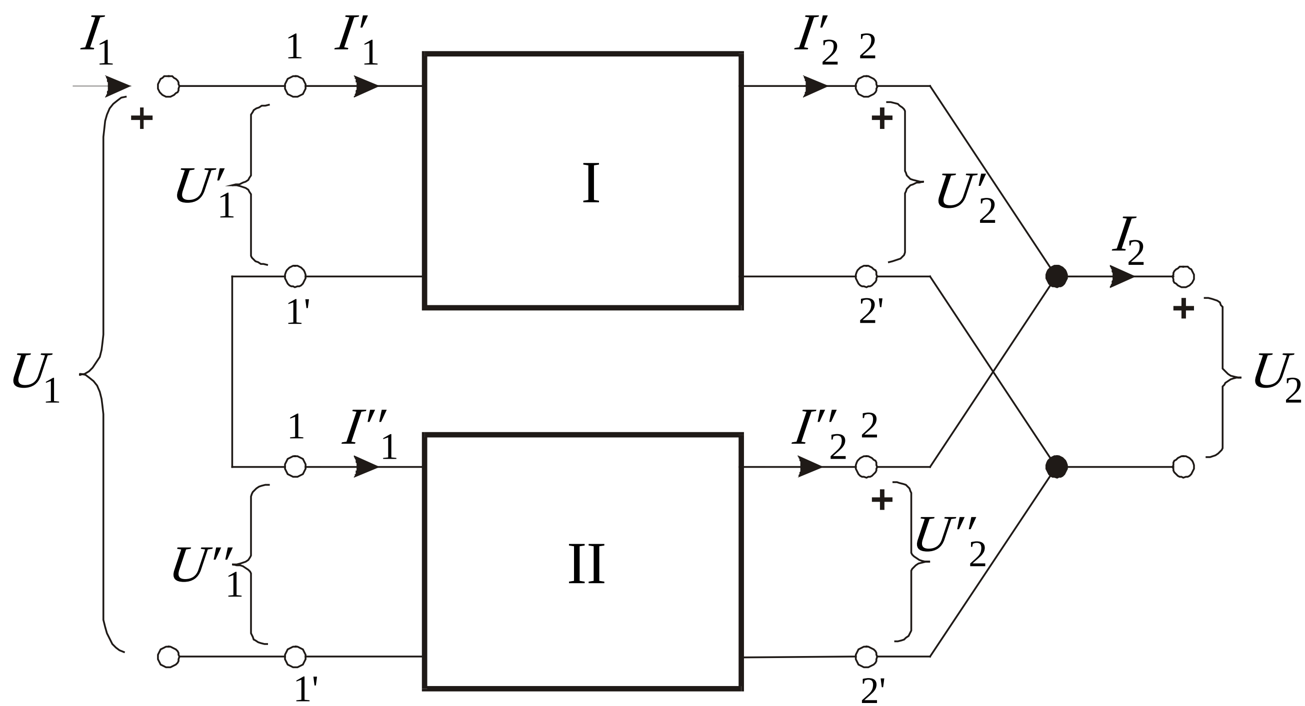
$$y''_{11} = y''_{12} = y''_{21} = y''_{22} = sC$$

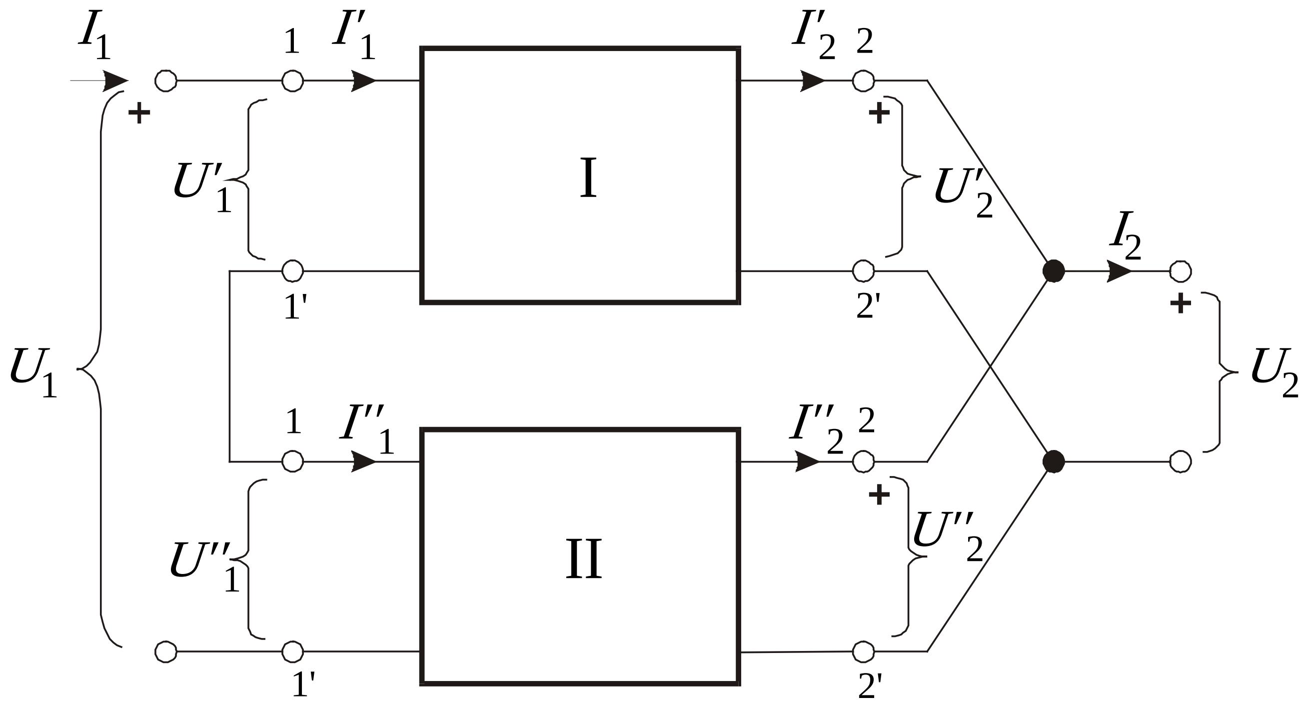
Matrica *y*-parametara kombinacije četveropola glasi

$$[y]=\begin{bmatrix}\dfrac{R_2+R_3}{R_1R_2+R_1R_3+R_2R_3}+sC & \dfrac{-R_3}{R_1R_2+R_1R_3+R_2R_3}-sC\\ \dfrac{R_3}{R_1R_2+R_1R_3+R_2R_3}+sC & \dfrac{-\left(R_1+R_3\right)}{R_1R_2+R_1R_3+R_2R_3}-sC\end{bmatrix}$$

- Serijsko-paralelni spoj četveropola:

- Ulazne priključnice → spojene serijski
- Izlazne priključnice → spojene paralelno





Za ovaj spoj vrijedi:

$$U_1 = U'_1 + U''_1 \qquad I_1 = I'_1 = I''_1$$

$$U_2 = U'_2 = U''_2 \qquad I_2 = I'_2 + I''_2$$

Postupci povezivanja četveropola

$$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} U'_1 \\ I'_2 \end{bmatrix} + \begin{bmatrix} U''_1 \\ I''_2 \end{bmatrix} \qquad \begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} I'_1 \\ U'_2 \end{bmatrix} = \begin{bmatrix} I''_1 \\ U''_2 \end{bmatrix}$$

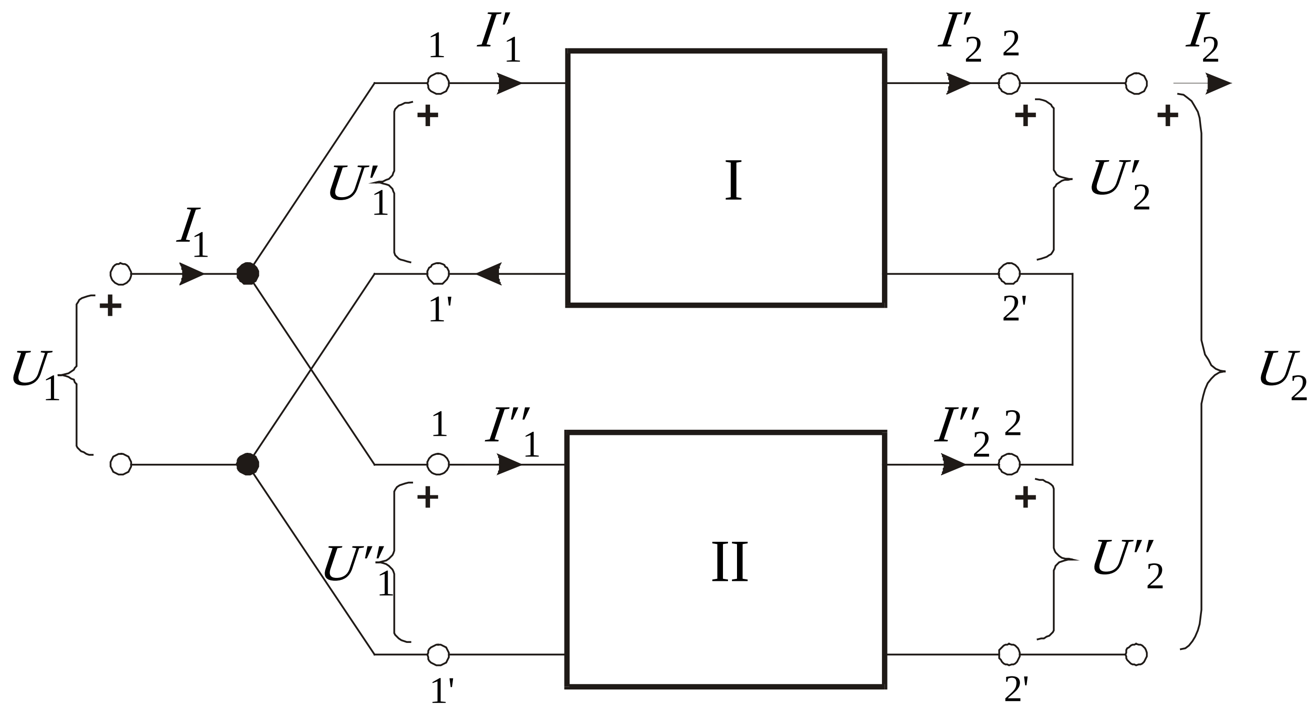
$$\begin{bmatrix} U'_1 \\ I'_2 \end{bmatrix} = \begin{bmatrix} h'_{11} & h'_{12} \\ h'_{21} & h'_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ U_2 \end{bmatrix} \qquad \begin{bmatrix} U''_1 \\ I''_2 \end{bmatrix} = \begin{bmatrix} h''_{11} & h''_{12} \\ h''_{21} & h''_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$$

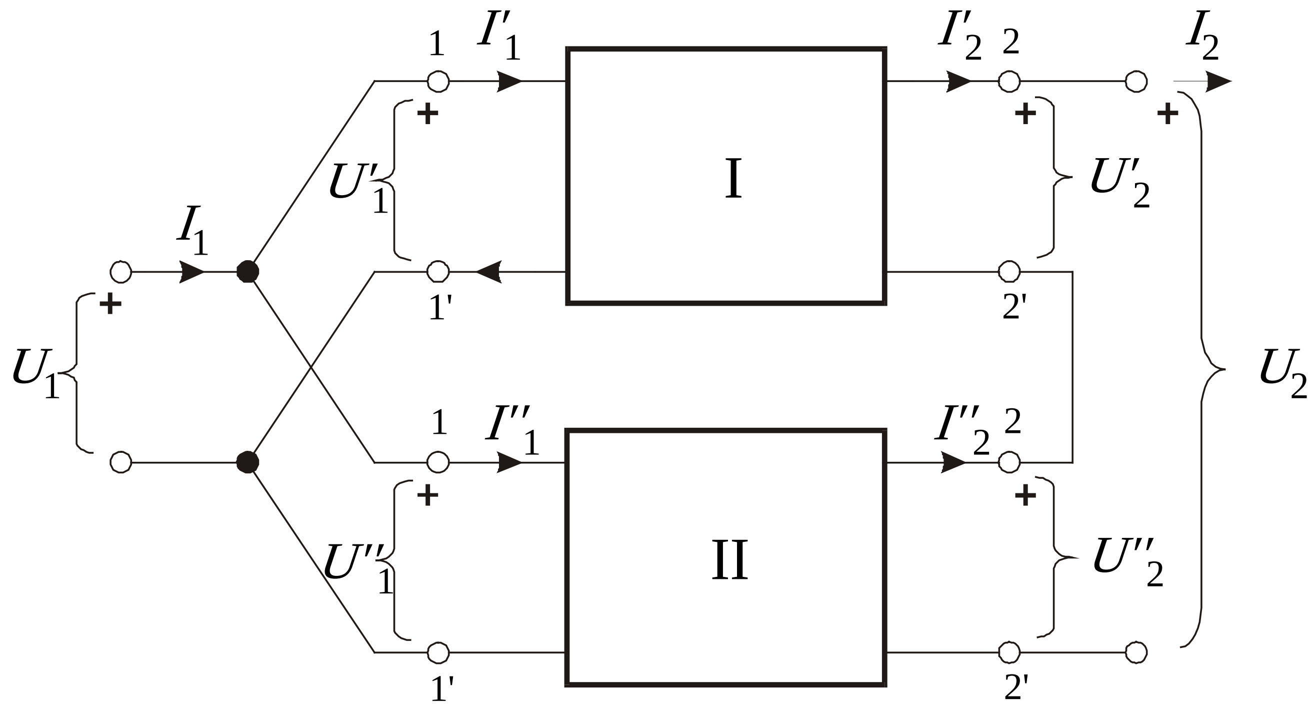
$$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \left(\begin{bmatrix} h'_{11} & h'_{12} \\ h'_{21} & h'_{22} \end{bmatrix} + \begin{bmatrix} h''_{11} & h''_{12} \\ h''_{21} & h''_{22} \end{bmatrix} \right) \cdot \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$$

$$[h] = [h'] + [h'']$$

Paralelno-serijski spoj četveropola:

- Ulazne priključnice →spojene paralelno
- Izlazne priključnice →spojene serijski





Za ovaj spoj vrijedi:

$$U_1 = U'_1 = U''_1 \qquad I_1 = I'_1 + I''_1$$

$$U_2 = U'_2 + U''_2 \qquad I_2 = I'_2 = I''_2$$

Postupci povezivanja četveropola

$$\begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} I'_1 \\ U'_2 \end{bmatrix} + \begin{bmatrix} I''_1 \\ U''_2 \end{bmatrix} \qquad \begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} U'_1 \\ I'_2 \end{bmatrix} = \begin{bmatrix} U''_1 \\ I''_2 \end{bmatrix}$$

$$\begin{bmatrix} I'_1 \\ U'_2 \end{bmatrix} = \begin{bmatrix} g'_{11} & g'_{12} \\ g'_{21} & g'_{22} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ I_2 \end{bmatrix} \qquad \begin{bmatrix} I''_1 \\ U''_2 \end{bmatrix} = \begin{bmatrix} g''_{11} & g''_{12} \\ g''_{21} & g''_{22} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \left(\begin{bmatrix} g'_{11} & g'_{12} \\ g'_{21} & g'_{22} \end{bmatrix} + \begin{bmatrix} g''_{11} & g''_{12} \\ g''_{21} & g''_{22} \end{bmatrix} \right) \cdot \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$$

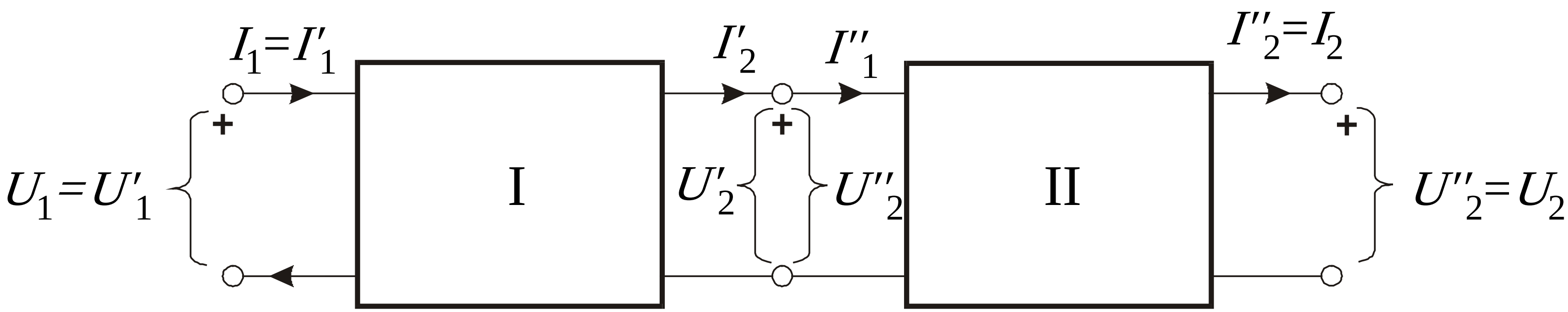
$$g_{11} = g'_{11} + g''_{11} \qquad g_{12} = g'_{12} + g''_{12}$$

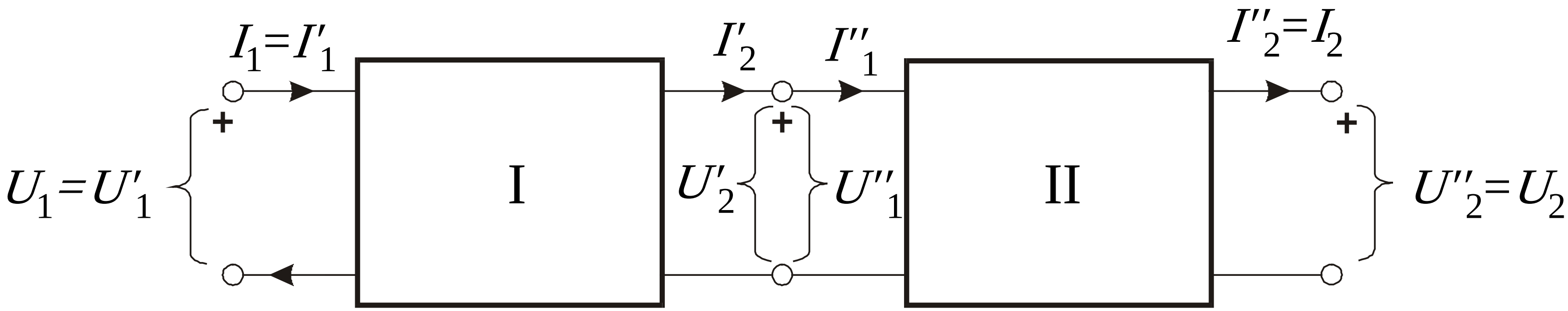
$$[g] = [g'] + [g'']$$

$$g_{21} = g'_{21} + g''_{21} \qquad g_{22} = g'_{22} + g''_{22}$$

Lanac ili kaskada četveropola

- Lanac dvaju četveropola:
- Izlazne priključnice prvog četveropola
  → na ulazne priključnice drugog
- Ulaz prvoga → ulaz kombinacije
- Izlaz drugoga → izlaz kombinacije





Za ovaj spoj vrijedi:

$$U_1 = U'_1 \qquad U'_2 = U''_1 \qquad U''_2 = U_2$$

$$I_1 = I'_1 \qquad I'_2 = I''_1 \qquad I''_2 = I_2$$

ili

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} U'_1 \\ I'_1 \end{bmatrix} \qquad \begin{bmatrix} U'_2 \\ I'_2 \end{bmatrix} = \begin{bmatrix} U''_1 \\ I''_1 \end{bmatrix} \qquad \begin{bmatrix} U''_2 \\ I''_2 \end{bmatrix} = \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$$

Postupci povezivanja četveropola

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} U'_1 \\ I'_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \cdot \begin{bmatrix} U'_2 \\ I'_2 \end{bmatrix}$$

$$\begin{bmatrix} U'_2 \\ I'_2 \end{bmatrix} = \begin{bmatrix} U''_1 \\ I''_1 \end{bmatrix} = \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \cdot \begin{bmatrix} U''_2 \\ I''_2 \end{bmatrix} = \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \cdot \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \cdot \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \cdot \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} =$$

$$= \begin{bmatrix} A'A''+B'C'' & A'B''+B'D'' \\ C'A''+D'C'' & C'B''+D'D'' \end{bmatrix} \cdot \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$$

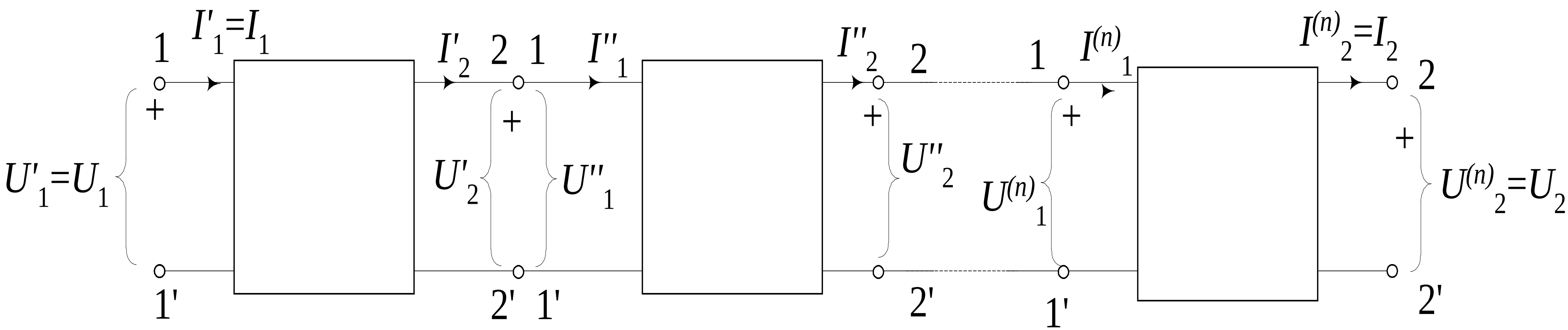
Postupci povezivanja četveropola

$$A = A'A'' + B'C'' \qquad B = A'B'' + B'D''$$

$$C = C'A'' + D'C'' \qquad D = C'B'' + D'D''$$

$$[a] = [a'] \cdot [a'']$$

Za N četveropola:

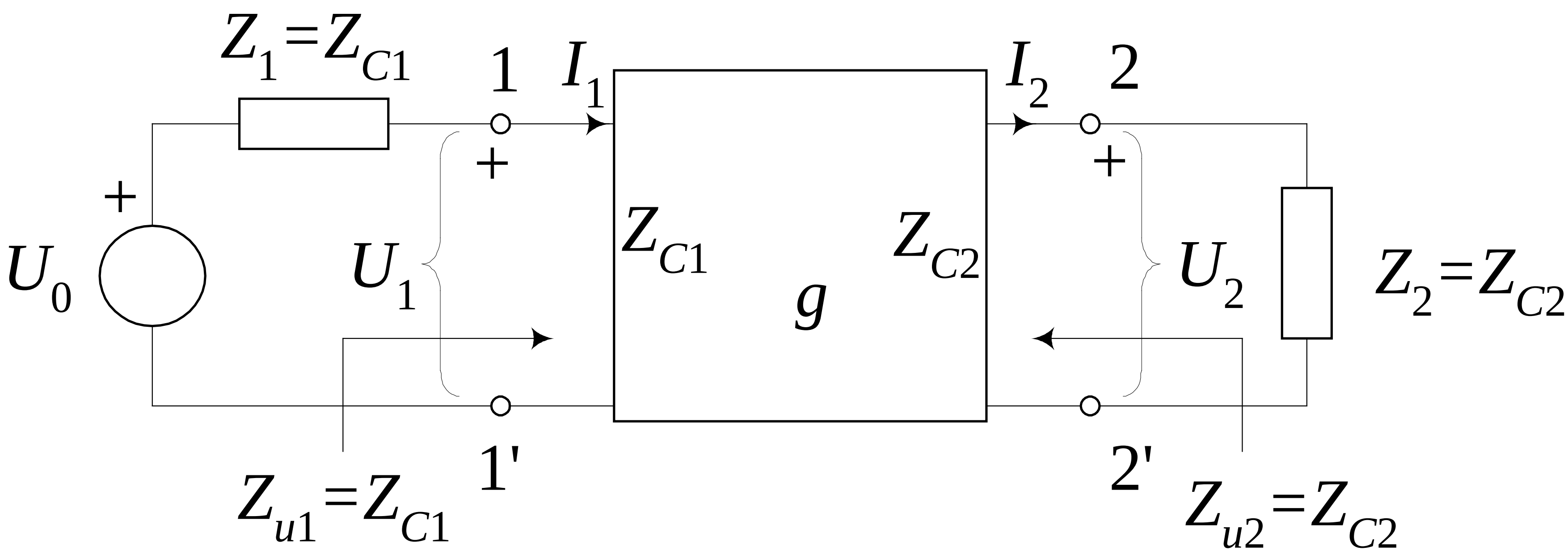


$$\boxed{\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{I} \cdot \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{II} \cdot \ldots \cdot \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{(n)}}$$

Lanac ili kaskada i zrcalni parametri

Prilagođenje po zrcalnim parametrima:

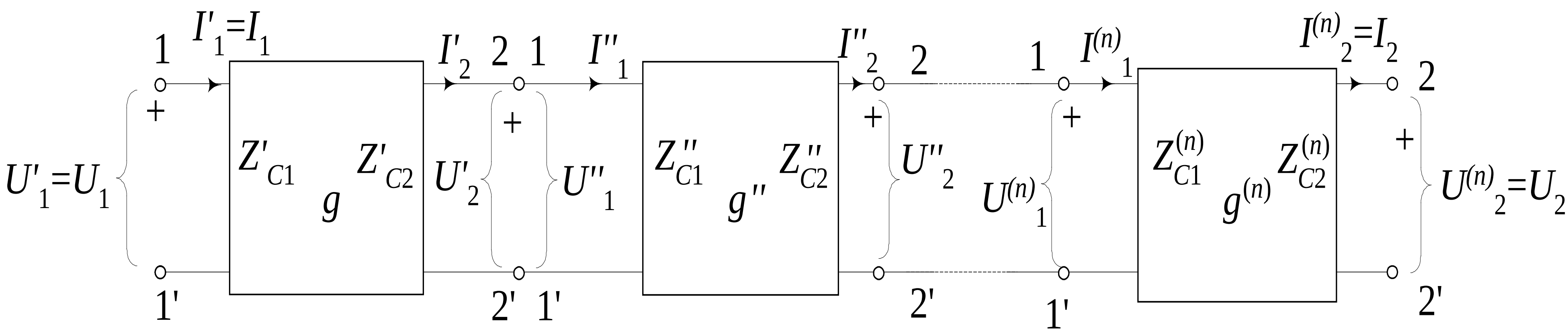
$$Z_{C_1},\ Z_{C_2},\ g$$



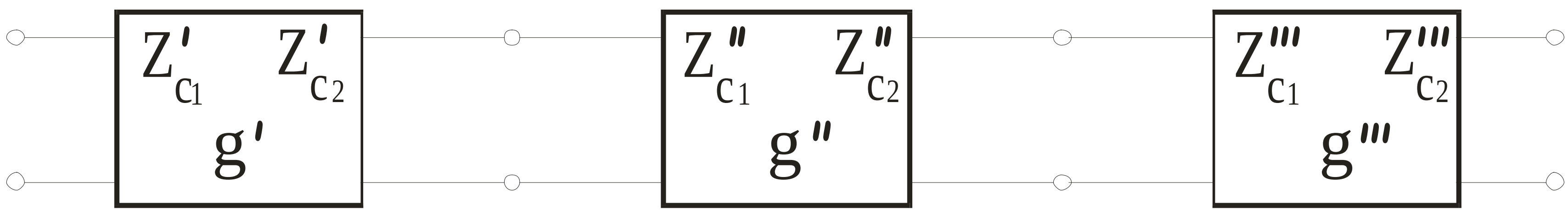
Prilagođeni lanci

- Kod više četveropola spojenih u lanac

→ često je potrebno ispuniti zahtjev prilagođenja



Za prilagođeni lanac mora biti



$$Z_{C_2}^{'} = Z_{C_1}^{''} \qquad Z_{C_2}^{''} = Z_{C_1}^{'''}$$

Simetrični četveropoli

Ako je $z_{11} = z_{22}$ → simetričan četveropol

$$\downarrow$$

$$Z_{p_1} = Z_{p_2}$$

- Za četveropol su dovoljna samo 2 parametra
- Također slijedi iz: $[y] = [z]^{-1}$

$$\Downarrow$$

$$\boxed{y_{11} = y_{22}} \quad \text{tj.} \quad Z_{k_1} = Z_{k_2}$$

- Uvjet simetrije izražen ostalim parametrima

- Prijenosni parametri:

$$A = D$$

- Hibridni *h*-parametri:

$$\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = -1$$

- Hibridni *g*-parametri:

$$\begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = 1$$

- Zrcalni parametri:

$$Z_{C_1} = Z_{C_2} = Z_C = \sqrt{\frac{B}{C}}$$

$$g = \ln\left(\sqrt{AD} + \sqrt{BC}\right) = \ln\left(A + \sqrt{BC}\right)$$

pošto je četveropol recipročan $\rightarrow$ $AD - BC = 1$

$BC = AD - 1$ $\rightarrow$ $BC = A^2 - 1$

pa je za simetrične i recipročne četveropole

$$\boxed{g = \ln\left(A + \sqrt{A^2 - 1}\right)}$$

Prijenosne jednadžbe glase:

$$U_1 = n\left(U_2 ch(g) + I_2 Z_{C_2} sh(g)\right)$$

$$I_1 = \frac{1}{n}\left(\frac{U_2 sh(g)}{Z_{C_2}} + I_2 ch(g)\right)$$

Pošto je:

$$n = \sqrt{\frac{Z_{C_1}}{Z_{C_2}}} = 1 \qquad Z_{C_2} = Z_C$$

$$U_1 = U_2\, ch\, g + I_2\, Z_C\, sh\, g$$

$$I_1 = \frac{U_2}{Z_C}\, sh\, g + I_2\, ch\, g$$