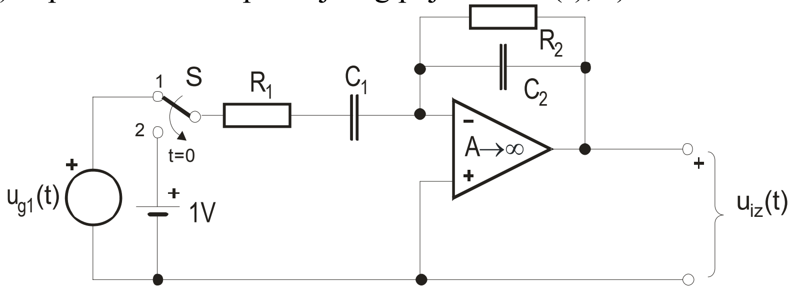
ZAVRŠNI ISPIT IZ PREDMETA ELEKTRIČNI KRUGOVI 2010/11

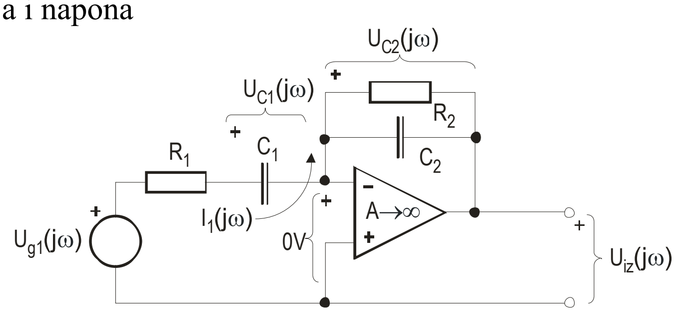
Rješenja i bodovi (svaki zadatak je bodovan od 0 do 5 bodova):

1. Za električni krug prikazan slikom se u trenutku t=0 prebaci sklopka S iz položaja 1 u 2 uzrokujući prijelaznu pojavu. Zadane su normalizirane vrijednosti elemenata: R_1=1, R_2=1, C_1=1/2, C_2=1, $u_{g1}(t)$=10sin(2t); $-\infty<t<\infty$ (sinusoidalno stacionarno stanje) i napon baterije $u_{g2}(t)$=1V (istosmjerni izvor). Odrediti za t<0: a) fazore napona na kapacitetima C_1 i C_2; b) valne oblike napona na kapacitetima $u_{C1}(t)$ i $u_{C2}(t)$; c) početne napone $u_{C1}(0)$ i $u_{C2}(0)$. Odrediti za $t\geq 0$: d) napon na izlazu operacijskog pojačala $U_{iz}(s)$; e) valni oblika napona $u_{iz}(t)$.



Rješenje:

a) za t<0 fazori struja i napona



$$I_1(j\omega)=\frac{U_{g1}(j\omega)}{R_1+\dfrac{1}{j\omega C_1}};\quad U_{C1}(j\omega)=I_1(j\omega)\cdot\frac{1}{j\omega C_1}=\frac{U_{g1}(j\omega)}{1+j\omega R_1C_1}$$

$$U_{C2}(j\omega)=-U_{iz}(j\omega)=\frac{I_1(j\omega)}{j\omega C_2+\dfrac{1}{R_2}}=\frac{U_{g1}(j\omega)}{\left(R_1+\dfrac{1}{j\omega C_1}\right)\left(j\omega C_2+\dfrac{1}{R_2}\right)}$$

Uz uvrštene vrijednosti elemenata:

$$U_{C1}(j\omega)=\frac{U_{g1}(j\omega)}{1+j\omega R_1C_1}=\frac{10\angle 0^\circ}{1+j2\cdot 1\cdot 1/2}=\frac{10}{1+j}=\frac{10}{1+j}\cdot\frac{1-j}{1-j}=\frac{10}{2}(1-j)=\underline{5(1-j)=5\sqrt{2}e^{-j45^\circ}}$$

$$U_{C2}(j\omega)=\frac{U_{g1}(j\omega)}{\left(R_1+\dfrac{1}{j\omega C_1}\right)\left(j\omega C_2+\dfrac{1}{R_2}\right)}=\frac{10\angle 0^\circ}{\left(1+\dfrac{1}{j2\cdot 1/2}\right)(j2\cdot 1+1)}=\frac{10}{(1-j)(1+2j)}$$

$$U_{C2}(j\omega)=\frac{10}{2\cdot 5}(1+j)(1-2j)=\underline{(1+j)(1-2j)=3-j}=\sqrt{2}e^{j45^\circ}\sqrt{5}e^{-j\arctan(2)}=$$

(1 bod)

$$=\sqrt{10}e^{j(45^\circ-63.435^\circ)}=\underline{\sqrt{10}e^{-j18.435^\circ}}\sqrt{2}e^{j45^\circ}\sqrt{5}e^{-j\arctan(2)}=\sqrt{10}e^{j(45^\circ-63.435^\circ)}=\underline{\sqrt{10}e^{-j18.435^\circ}}$$

b) za t<0 valni oblici napona na kapacitetima $u_{C1}(t)$ i $u_{C2}(t)$:

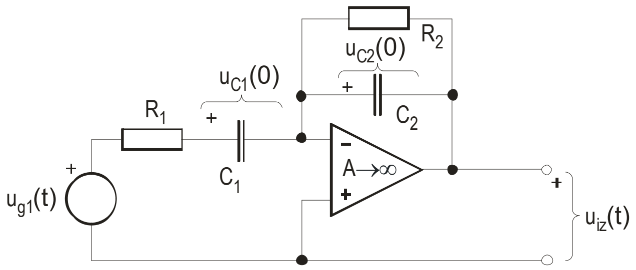
$$\underline{u_{C1}(t)=5\sqrt{2}\sin(2t-45^\circ)=7.071068\sin(2t-45^\circ)}$$

$$\underline{u_{C2}(t)=\sqrt{10}\sin(2t-18.435^\circ)=3.162278\sin(2t-18.435^\circ)}\quad \textbf{(1 bod)}$$

c) za t=0 početni naponi $u_{C1}(0)$ i $u_{C2}(0)$:

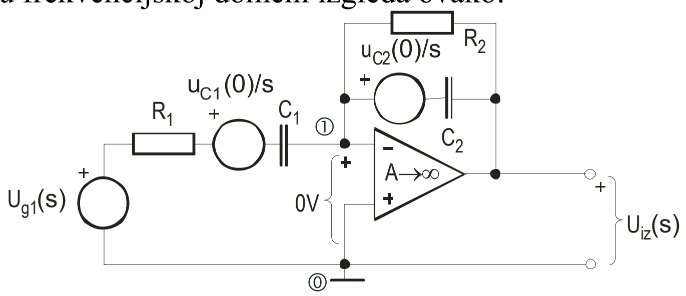
$$u_{C1}(0)=5\sqrt{2}\sin(-45°)=-5\sqrt{2}\frac{\sqrt{2}}{2}=\underline{-5[\mathrm{V}]}$$

$$u_{C2}(0)=\sqrt{10}\sin(-18.435°)=\underline{-1[\mathrm{V}]}$$ **(1 bod)**



d) za t≥0 Laplaceova transformacija

Uz poznate početne uvjete $u_{C1}(0)=-5$ i $u_{C2}(0)=-1$, te pobudu $u_{g1}(t)=S(t)$ (baterija), (za t≥0) električni krug u frekvencijskoj domeni izgleda ovako:



Jednadžba za čvor (1) glasi:

$$(1)\quad U_1(s)\left(\frac{1}{R_1+\frac{1}{sC_1}}+sC_2+\frac{1}{R_2}\right)=\frac{U_{g1}(s)-\frac{u_{C1}(0)}{s}}{R_1+\frac{1}{sC_1}}+\frac{\frac{u_{C2}(0)}{s}}{\frac{1}{sC_2}}+U_{iz}(s)\left(sC_2+\frac{1}{R_2}\right)$$

Zbog virtualnog kratkog spoja je $U_1(s)$=0 pa vrijedi:

$$U_{iz}(s)\left(sC_2+\frac{1}{R_2}\right)=-\frac{U_{g1}(s)-\frac{u_{C1}(0)}{s}}{R_1+\frac{1}{sC_1}}-C_2u_{C2}(0)\,;\; U_{iz}(s)=-\frac{U_{g1}(s)-\frac{u_{C1}(0)}{s}}{\left(R_1+\frac{1}{sC_1}\right)\left(sC_2+\frac{1}{R_2}\right)}-\frac{C_2u_{C2}(0)}{\left(sC_2+\frac{1}{R_2}\right)}$$

Uz uvrštene vrijednosti elemenata:

$$U_{iz}(s)=-\frac{\frac{1}{s}-\frac{-5}{s}}{(1+2/s)(s+1)}-\frac{1\cdot(-1)}{(s+1)}=-\frac{6}{(s+2)(s+1)}+\frac{1}{s+1}\Rightarrow \underline{U_{iz}(s)=\frac{s-4}{(s+2)(s+1)}}$$ **(1 bod)**

e) valni oblika napona $u_{iz}(t)$: Rastav na parcijalne razlomke:

$$\frac{-6}{(s+2)(s+1)}=\frac{A}{s+2}+\frac{B}{s+1}=\frac{As+A+Bs+2B}{(s+2)(s+1)}=\frac{(A+B)s+(A+2B)}{(s+2)(s+1)}=\frac{6}{s+2}+\frac{-6}{s+1}$$

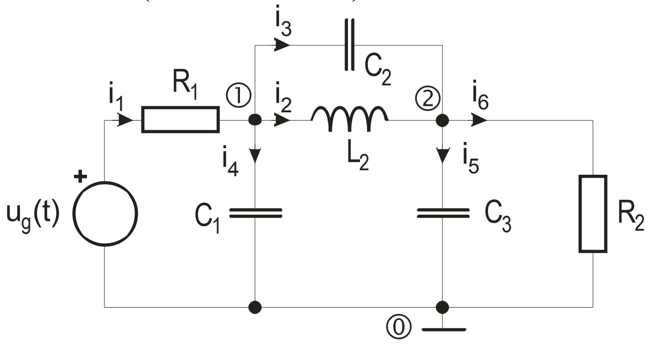
$$(1)\quad A+B=0\Rightarrow A=-B=6$$

$$\underline{(2)\quad A+2B=-6\Rightarrow -B+2B=B=-6}$$

$$U_{iz}(s)=-\frac{6}{(s+2)(s+1)}+\frac{1}{s+1}=\frac{6}{s+2}-\frac{6}{s+1}+\frac{1}{s+1}=\frac{6}{s+2}-\frac{5}{s+1}$$

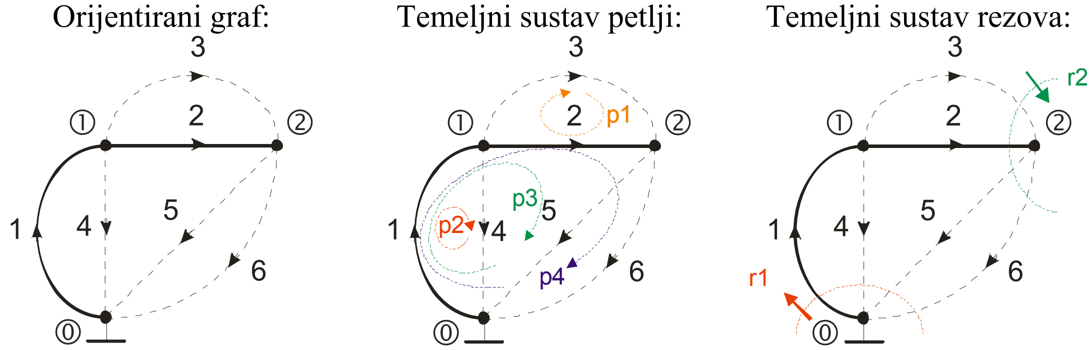
$$\Rightarrow \underline{u_{iz}(t)=(6e^{-2t}-5e^{-t})S(t)}$$ **(1 bod)**

2. Za električni krug prikazan slikom i pridružene orijentacije grana i čvorove nacrtati: a) orijentirani graf, temeljni sustav petlji i temeljni sustav rezova; b) napisati matricu incidencija $\mathbf{A}_a$, temeljnu spojnu matricu **S**, temeljnu rastavnu matricu **Q**; c) matricu impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$; d) matricu admitancija grana $\mathbf{Y}_b$ i vektor početnih uvjeta i nezavisnih strujnih izvora grana $\mathbf{I}_{0b}$; e) pomoću navedenih matrica odrediti sustav jednadžbi čvorova (matrice $\mathbf{Y}_v$ i $\mathbf{I}_{0v}$).



Rješenje:

a) orijentirani graf, temeljni sustav petlji i temeljni sustav rezova: **(1 bod)**



b) matrica incidencija $\mathbf{A}_a$, temeljna spojna matrica **S**, temeljna rastavna matrica **Q**: **(1 bod)**

Matrica incidencija:

$$\mathbf{A}_a = \begin{bmatrix} -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 1 \\ 1 & 0 & 0 & -1 & -1 & -1 \end{bmatrix}$$

Temeljna spojna matrica:

$$\mathbf{S} = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Temeljna rastavna matrica:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & -1 & -1 & -1 \\ 0 & 1 & 1 & 0 & -1 & -1 \end{bmatrix}$$

c) matrica impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$: **(1 bod)**

Naponsko – strujne relacije grana:

$$\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}, \text{ odn. } \mathbf{I}_b = \mathbf{Y}_b \cdot \mathbf{U}_b + \mathbf{I}_{0b}$$

$$U_1 = I_1 \cdot R_1 - U_g(s) \quad \Rightarrow \quad I_1 = \frac{1}{R_1} \cdot U_1 + \frac{U_g(s)}{R_1}$$

$$U_2 = I_2 \cdot sL_2 \quad \Rightarrow \quad I_2 = \frac{1}{sL_2} \cdot U_2, \text{ itd.}$$

$$U_3 = I_3 \cdot \frac{1}{sC_2},\ U_4 = I_4 \cdot \frac{1}{sC_1},\ U_5 = I_5 \cdot \frac{1}{sC_3},\ U_6 = I_6 \cdot R_2$$

Iz gornjeg sustava se mogu pročitati:

$$\mathbf{Z}_b = \begin{bmatrix} R_1 & & & & & \\ & sL_2 & & & & \\ & & \frac{1}{sC_2} & & 0 & \\ & & & \frac{1}{sC_1} & & \\ & 0 & & & \frac{1}{sC_3} & \\ & & & & & R_2 \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} -U_g(s) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

d) matrica admitancija grana $\mathbf{Y}_b$ i vektor početnih uvjeta i nezavisnih strujnih izvora grana $\mathbf{I}_{0b}$: **(1 bod)**

Jedan način je da se gornji sustav napiše tako da su s lijeve strane struje grana, a drugi način je da se invertira matrica $\mathbf{Y}_b = \mathbf{Z}_b^{-1}$. U slučaju dijagonalne matrice to je lako jer elementi na dijagonali inverzne matrice imaju recipročnu vrijednost elemenata originalne matrice.

$$\mathbf{Y}_b = \begin{bmatrix} \frac{1}{R_1} & & & & & \\ & \frac{1}{sL_2} & & & & \\ & & sC_2 & & 0 & \\ & & & sC_1 & & \\ & 0 & & & sC_3 & \\ & & & & & \frac{1}{R_2} \end{bmatrix}, \mathbf{I}_{0b} = \begin{bmatrix} U_g(s)/R_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

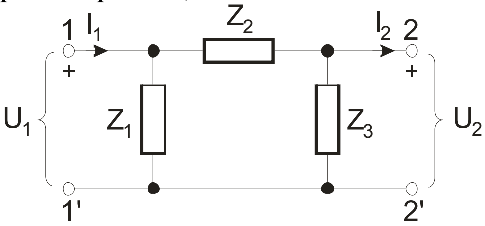
e) sustav jednadžbi čvorova (matrice $\mathbf{Y}_v$ i $\mathbf{I}_{0v}$): **(1 bod)**

$$\mathbf{Y}_v = \mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{A}^T = \begin{bmatrix} -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{R_1} & & & & & \\ & \frac{1}{sL_2} & & & & \\ & & sC_2 & & 0 & \\ & & & sC_1 & & \\ & 0 & & & sC_3 & \\ & & & & & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} -\frac{1}{R_1} & \frac{1}{sL_2} & sC_2 & sC_1 & 0 & 0 \\ 0 & -\frac{1}{sL_2} & -sC_2 & 0 & sC_3 & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & -1 \\ 1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{sL_2} + sC_1 + sC_2 & -\frac{1}{sL_2} - sC_2 \\ -\frac{1}{sL_2} - sC_2 & \frac{1}{sL_2} + sC_2 + sC_3 + \frac{1}{R_2} \end{bmatrix}$$

$$\mathbf{I}_{0v} = -\mathbf{A} \cdot \mathbf{I}_{0b} = -\begin{bmatrix} -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} U_g(s)/R_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} U_g(s)/R_1 \\ 0 \end{bmatrix}$$

3. Za Π-četveropol prikazan slikom izračunati z-parametre. Napisati: a) parametre z_{11}, z_{21}, z_{12} i z_{22} (izraziti z-parametre pomoću Z_1, Z_2 i Z_3). Ako je izlazni prilaz (2–2') zaključen impedancijom Z_L pomoću z-parametara izračunati: b) strujnu prijenosnu funkciju četveropola $H_i(s)=I_2(s)/I_1(s)$; c) ulaznu impedanciju u četveropol $Z_{ul1}(s)=U_1(s)/I_1(s)$ (izraziti $H_i(s)$ i $Z_{ul1}(s)$ pomoću gore izračunatih z-parametara izraženih općim impedancijama elemenata Z_1, Z_2 i Z_3). d) Izračunati $H_i(s)$ i $Z_{ul1}(s)$ uz slijedeće vrijednosti elemenata: $Z_1=R_1=1$, $Z_2=sL_2=s$, $Z_3=R_3=1$ i $Z_L=R_L=1$. e) Da li je četveropol recipročan, simetričan?



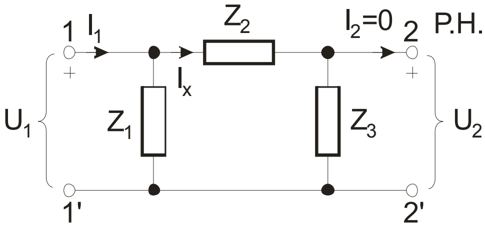
Rješenje:

a) [z]-parametri: **(1 bod)**

$$U_1 = z_{11}I_1 - z_{12}I_2$$

$$U_2 = z_{21}I_1 - z_{22}I_2$$

$I_2=0$ parametri z_{11} i z_{21}:



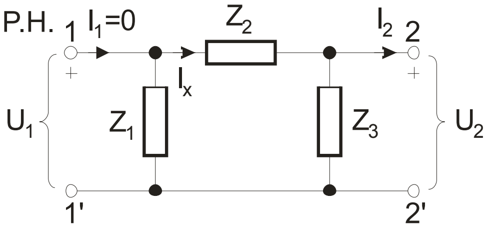
$$I_1 = U_1\left(\frac{1}{Z_1}+\frac{1}{Z_2+Z_3}\right) = U_1\frac{Z_1+Z_2+Z_3}{Z_1(Z_2+Z_3)}$$

$$I_x = U_2\frac{1}{Z_3} = U_1\frac{1}{Z_2+Z_3} \;\Rightarrow\; U_2 = U_1\frac{Z_3}{Z_2+Z_3} \Rightarrow U_1 = U_2\frac{Z_2+Z_3}{Z_3}$$

$$\Rightarrow\; I_1 = U_1\frac{Z_1+Z_2+Z_3}{Z_1(Z_2+Z_3)} = U_2\frac{Z_2+Z_3}{Z_3}\frac{Z_1+Z_2+Z_3}{Z_1(Z_2+Z_3)} = U_2\frac{Z_1+Z_2+Z_3}{Z_1Z_3}$$

$$z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0} = \frac{Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3}\;;\quad z_{21} = \left.\frac{U_2}{I_1}\right|_{I_2=0} = \frac{Z_1Z_3}{Z_1+Z_2+Z_3}$$

$I_1=0$ parametri z_{12} i z_{22}:



$$I_2 = -U_2\left(\frac{1}{Z_3}+\frac{1}{Z_1+Z_2}\right) = -U_2\frac{Z_1+Z_2+Z_3}{Z_3(Z_1+Z_2)}$$

$$I_x = -U_2\frac{1}{Z_1+Z_2} = -U_1\frac{1}{Z_1} \;\Rightarrow\; U_1 = U_2\frac{Z_1}{Z_1+Z_2} \Rightarrow U_2 = U_1\frac{Z_1+Z_2}{Z_1}$$

$$\Rightarrow\; I_2 = -U_2\frac{Z_1+Z_2+Z_3}{Z_3(Z_1+Z_2)} = -U_1\frac{Z_1+Z_2}{Z_1}\frac{Z_1+Z_2+Z_3}{Z_3(Z_1+Z_2)} = -U_1\frac{Z_1+Z_2+Z_3}{Z_1Z_3}$$

$$z_{12}=-\frac{U_1}{I_2}\bigg|_{I_1=0}=\frac{Z_1Z_3}{Z_1+Z_2+Z_3};\quad z_{22}=-\frac{U_2}{I_2}\bigg|_{I_1=0}=\frac{Z_3(Z_1+Z_2)}{Z_1+Z_2+Z_3}$$

$$[z]=\begin{bmatrix}\frac{Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3} & -\frac{Z_1Z_3}{Z_1+Z_2+Z_3}\\ \frac{Z_1Z_3}{Z_1+Z_2+Z_3} & -\frac{Z_3(Z_1+Z_2)}{Z_1+Z_2+Z_3}\end{bmatrix},\quad [z]=\begin{bmatrix}z_{11} & -z_{12}\\ z_{21} & -z_{22}\end{bmatrix}$$

b) Prijenosna funkcija struje: **(1 bod)**

$$H_i(s)=\frac{I_2}{I_1}=\frac{z_{21}}{Z_L+z_{22}}=\frac{\frac{Z_1Z_3}{Z_1+Z_2+Z_3}}{Z_L+\frac{Z_3(Z_1+Z_2)}{Z_1+Z_2+Z_3}}=\frac{Z_1Z_3}{Z_L(Z_1+Z_2+Z_3)+Z_3(Z_1+Z_2)}$$

c) Ulazna impedancija u četveropol: **(1 bod)**

$$Z_{ul1}(s)=\frac{U_1}{I_1}=z_{11}-\frac{z_{12}z_{21}}{Z_L+z_{22}}=\frac{Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3}-\frac{\left(\frac{Z_1Z_3}{Z_1+Z_2+Z_3}\right)^2}{Z_L+\frac{Z_3(Z_1+Z_2)}{Z_1+Z_2+Z_3}}=$$

$$=\frac{1}{(Z_1+Z_2+Z_3)}\left[Z_1(Z_2+Z_3)-\frac{(Z_1Z_3)^2}{Z_L(Z_1+Z_2+Z_3)+Z_3(Z_1+Z_2)}\right]$$

d) Uz uvrštene vrijednosti: **(1 bod)**

$$H_i(s)=\frac{I_2}{I_1}=\frac{R_1R_3}{R_L(R_1+R_2+R_3)+R_3(R_1+R_2)}=\frac{1}{1(1+s+1)+1(1+s)}=\frac{1}{3+2s}$$

$$Z_{ul1}=\frac{1}{(R_1+R_2+R_3)}\left[R_1(R_2+R_3)-\frac{(R_1R_3)^2}{R_L(R_1+R_2+R_3)+R_3(R_1+R_2)}\right]=$$

$$=\frac{1}{(1+s+1)}\left[1(s+1)-\frac{(1)^2}{1(1+s+1)+1(1+s)}\right]=\frac{1}{2+s}\left[s+1-\frac{1}{3+2s}\right]=$$

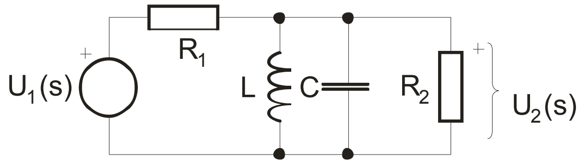
$$=\frac{1}{2+s}\left[\frac{3s+2s^2+3+2s-1}{3+2s}\right]=\frac{2s^2+5s+2}{(2+s)(3+2s)}=\frac{2s^2+5s+2}{2s^2+7s+6}$$

e) Recipročnost i simetričnost četveropola: **(1 bod)**

$$[z]=\begin{bmatrix}\frac{Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3} & -\frac{Z_1Z_3}{Z_1+Z_2+Z_3}\\ \frac{Z_1Z_3}{Z_1+Z_2+Z_3} & -\frac{Z_3(Z_1+Z_2)}{Z_1+Z_2+Z_3}\end{bmatrix}=\begin{bmatrix}\frac{s+1}{s+2} & -\frac{1}{s+2}\\ \frac{1}{s+2} & -\frac{s+1}{s+2}\end{bmatrix}$$

Četveropol je recipročan jer vrijedi $z_{21}=z_{12}$ i simetričan jer je $z_{11}=z_{22}$.

4. Za pojasno propusni električni filtar prikazan slikom zadane su vrijednosti elemenata L=1mH, R_1=50Ω, C=100nF i R_2=50Ω. Izračunati: a) naponsku prijenosnu funkciju $T(s)=U_2(s)/U_1(s)$; b) Q-faktor polova q_p, centralnu frekvenciju ω_0 te pojačanje u području propuštanja k; c) Kolika je širina pojasa propuštanja B, te gornja i donja granična frekvencija ω_g i ω_d? Izračunati i skicirati: d) amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$ u dB; e) fazno-frekvencijsku karakteristiku.



Rješenje:

a) Naponska prijenosna funkcija: **(1bod)**

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{Y_1}{Y_1+Y_2}$$

$$Y_1(s)=\frac{1}{R_1},\quad Y_2(s)=\frac{1}{sL}+\frac{1}{R_2}+sC$$

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{sL}+\frac{1}{R_2}+sC}\frac{/\cdot R_1R_2sL}{/\cdot R_1R_2sL}=\frac{sLR_2}{s^2R_1R_2LC+sL(R_1+R_2)+R_1R_2}$$

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{s\frac{LR_2}{R_1R_2LC}}{s^2+s\frac{L(R_1+R_2)}{R_1R_2LC}+\frac{R_1R_2}{R_1R_2LC}}=\frac{s\frac{1}{R_1C}}{s^2+s\frac{1}{R_pC}+\frac{1}{LC}}=\frac{R_p}{R_1}\cdot\frac{s\frac{1}{R_pC}}{s^2+s\frac{1}{R_pC}+\frac{1}{LC}}$$

Uz zadane vrijednosti:

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{2\cdot 10^5\cdot s}{s^2+4\cdot 10^5\cdot s+10^{10}}$$

b) Parametri prijenosne funkcije: q_p, ω_p i k: **(1bod)**

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{k\cdot\frac{\omega_p}{q_p}\cdot s}{s^2+\frac{\omega_p}{q_p}s+\omega_p^2}\quad\Rightarrow$$

$$\omega_p=\frac{1}{\sqrt{LC}},\qquad k=\frac{R_p}{R_1}=\frac{R_2}{R_1+R_2},\qquad R_p=\frac{R_1R_2}{R_1+R_2},$$

$$\frac{\omega_p}{q_p}=\frac{1}{R_pC}\quad\Rightarrow\quad q_p=\omega_p\cdot R_pC=\frac{1}{\sqrt{LC}}\cdot R_pC=R_p\cdot\sqrt{\frac{C}{L}}$$

Uz zadane vrijednosti:

$$\omega_p=\sqrt{10^{10}}=10^5\,\text{rad/s},\qquad \frac{\omega_p}{q_p}=4\cdot 10^5\quad\Rightarrow\quad q_p=\frac{10^5}{4\cdot 10^5}=\frac{1}{4},\quad k=\frac{1}{2}$$

c) Širina pojasa propuštanja B, te gornja i donja granična frekvencija ω_g i ω_d: **(1bod)**

$$\text{Širina pojasa propuštanja } B=\frac{\omega_p}{q_p}=4\cdot 10^5\ \text{[rad/s]}$$

Gornja i donja granična frekvencija su:

$$\omega_{g,d} = \omega_p \sqrt{1+\frac{1}{4q_p^2}} \pm \frac{\omega_p}{2q_p} = 10^5 \sqrt{1+\frac{1}{4\cdot 0.0625}} \pm \frac{4\cdot 10^5}{2} = 10^5(\sqrt{5}\pm 2)\ \text{[rad/s]}$$

ω_g=423606,8 [rad/s], ω_d=23606,8 [rad/s]

B=ω_g–ω_d=423606,8–23606,8=400 000= $4\cdot 10^5$ [rad/s]

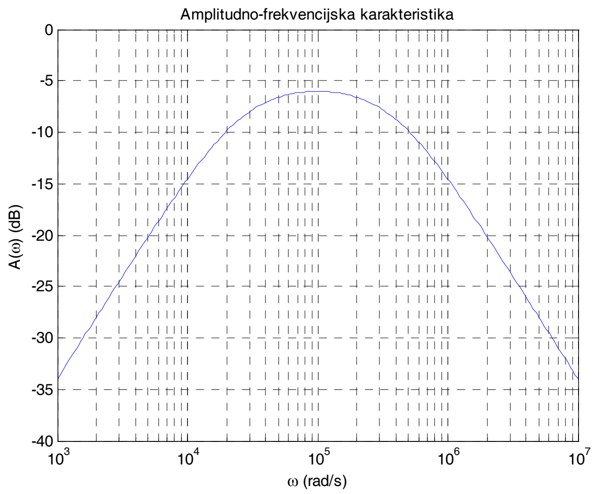
d) Amplitudno-frekvencijska karakteristika: **(1bod)**

$$\Rightarrow\ T(j\omega) = \frac{N(j\omega)}{D(j\omega)} = \frac{2\cdot 10^5\cdot j\omega}{-\omega^2 + 4\cdot 10^5\cdot j\omega + 10^{10}}\ \Rightarrow$$

$$|T(j\omega)| = \frac{2\cdot 10^5\cdot\omega}{\sqrt{(10^{10}-\omega^2)^2 + (4\cdot 10^5\cdot\omega)^2}}$$

$$\Rightarrow\ A(\omega)[\text{dB}] = 20\log|T(j\omega)| = 20\log\frac{2\cdot 10^5\cdot\omega}{\sqrt{(10^{10}-\omega^2)^2 + (4\cdot 10^5\cdot\omega)^2}}$$

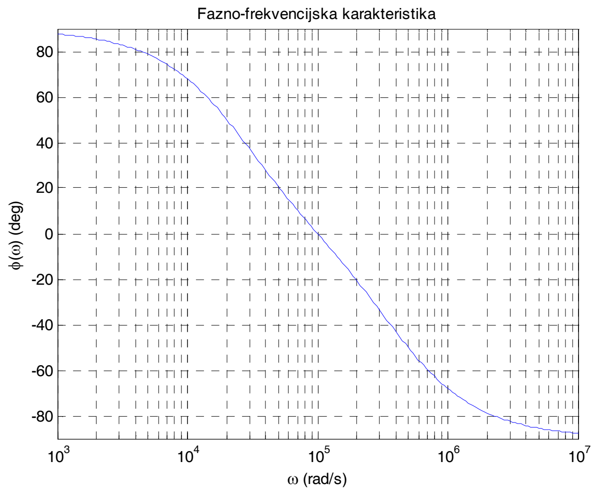
Amplitudno-frekvencijska karakteristika u Matlabu:



e) Fazno-frekvencijska karakteristika: **(1bod)**

$$\varphi(\omega) = \arctan\frac{\text{Im}[T(j\omega)]}{\text{Re}[T(j\omega)]} = \arctan\frac{\text{Im}[N(j\omega)]}{\text{Re}[N(j\omega)]} - \arctan\frac{\text{Im}[D(j\omega)]}{\text{Re}[D(j\omega)]} = \frac{\pi}{2} - \arctan\frac{4\cdot 10^5\cdot\omega}{10^{10}-\omega^2}$$

Fazno-frekvencijska karakteristika u Matlabu:



5. Zadana je linija bez gubitaka s L=400 μH/km i C=40 nF/km. Na ulaz linije priključen je naponski izvor $u_g(t) = 2\sin(\omega_0 t)$, u seriju s otporom R=50Ω, a na izlazu je karakteristična impedancija Z_0. Duljina linije l jednaka je njenoj valnoj duljini na frekvenciji $\omega_0=2\pi 10^5$ rad/s. Odrediti:

a) karakterističnu impedanciju Z_0;
b) faktor prijenosa γ;
c) duljinu linije l u km i brzinu širenja vala po liniji v;
d) ulaznu impedanciju Z_{ul} i ulazni napon linije u(0,t);
e) napon i struju na polovini linije.

Rješenje:

a) $Z_0=\sqrt{L/C}=\sqrt{4\cdot10^{-4}/4\cdot10^{-8}}=100\Omega$; **(1 bod)**

b) $\gamma=j\omega_0\sqrt{LC}=j2\pi\cdot10^5\cdot\sqrt{4\cdot10^{-4}\cdot4\cdot10^{-8}}=j2\pi\cdot0{,}4$ **(1 bod)**

c) $l=\lambda_0=\dfrac{2\pi}{\omega_0\sqrt{LC}}=\dfrac{2\pi}{2\pi\cdot10^5\cdot4\cdot10^{-6}}=2{,}5\text{km}$;

$$v=\frac{\omega}{\beta}=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{4\cdot10^{-4}\cdot4\cdot10^{-8}}}=\frac{10^6}{4}=250\,000\,\text{km/s}$$ **(1 bod)**

d) $Z_{ul}=\dfrac{U(0)}{I(0)}=Z_0$; $\quad U(0)=U_g\dfrac{Z_{ul}}{R+Z_{ul}}=2\dfrac{100}{50+100}=1{,}3333$ **(1 bod)**

e) $U(x)=U(0)\cdot ch\,\gamma x-I(0)Z_0 sh\,\gamma x=U(0)\cdot e^{-\gamma x}=U(0)\cdot e^{-j\pi}=-U(0)$

$u(x,t)=-1{,}333\sin(\omega_0 t)$

$i(x,t)=\dfrac{u(x,t)}{Z_0}=-\dfrac{1{,}3333}{Z_0}\sin(\omega_0 t)=-0{,}013333\sin(\omega_0 t)$ **(1 bod)**