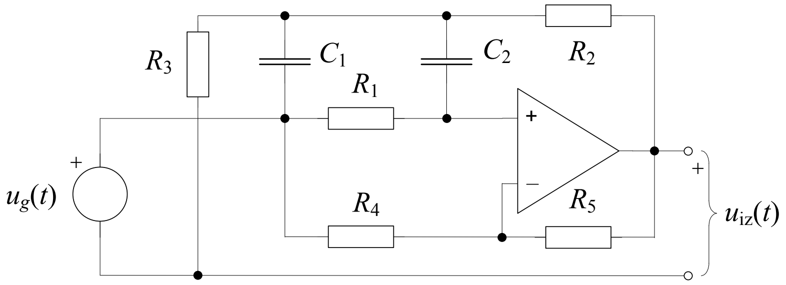
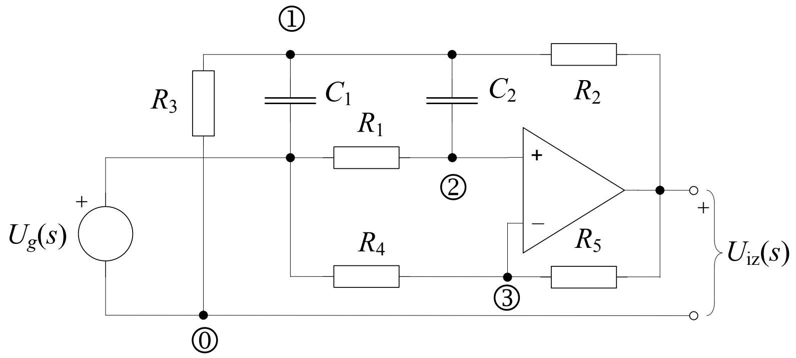
# ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA 2013-2014 – Rješenja

1. Za električni krug prikazan slikom izračunati naponsku prijenosnu funkciju $H(s)=U_{iz}(s)/U_g(s)$ metodom napona čvorova. Zadane su normalizirane vrijednosti elemenata $R_1=1$, $R_2=1$, $R_3=1$, $R_4=1$, $R_5=1$, $C_1=1$, $C_2=1$. Operacijsko pojačalo je idealno. Početni uvjeti: $u_{C1}(0)=u_{C2}(0)=0$. U jednadžbe čvorišta uvrstiti odmah vrijednosti elemenata. Izračunati valni oblik napona $u_{iz}(t)$ kao odziv, ako je zadana pobuda (stacionarni sinusni signal) $u_g(t)=\sin(\sqrt{2}t)$; $-\infty<t<+\infty$.




Rješenje:

Primjenom Laplaceove transformacije dobivamo slijedeći električni krug. Postavimo jednadžbe čvorišta:




$$1)\ U_1\left(sC_1+sC_2+\frac{1}{R_2}+\frac{1}{R_3}\right)-U_2sC_2=U_g(s)sC_1+U_{iz}(s)\frac{1}{R_2}$$

$$2)\ -U_1sC_2+U_2\left(sC_2+\frac{1}{R_1}\right)=\frac{U_g(s)}{R_1};$$

$$3)\ U_3\left(\frac{1}{R_4}+\frac{1}{R_5}\right)=\frac{U_g(s)}{R_4}+\frac{U_{iz}(s)}{R_5};$$

$$4)\ U_{iz}(s)=A\cdot[U_2(s)-U_3(s)]\ \Rightarrow\ U_2(s)-U_3(s)=\frac{U_{iz}(s)}{A};$$

$$A\to\infty\ \Rightarrow\ U_2(s)=U_3(s)$$

**(2 boda)**

$$3),\,4)\Rightarrow U_2=U_3=\frac{R_5}{R_4+R_5}U_g(s)+\frac{R_4}{R_4+R_5}U_{iz}(s)$$

Uvedimo oznaku $\alpha$, i uvrstimo vrijednosti elemenata:

$$\alpha=\frac{R_5}{R_4+R_5}=\frac{1}{2};\quad (1-\alpha)=\frac{R_4}{R_4+R_5}=\frac{1}{2}$$

$$\Rightarrow U_2=U_3=\alpha U_g(s)+(1-\alpha)U_{iz}(s)=\frac{U_g(s)}{2}+\frac{U_{iz}(s)}{2}$$

$$2) \Rightarrow U_1 = U_2\left(1+\frac{1}{sC_2R_1}\right) - U_g(s)\frac{1}{sC_2R_1} = U_2\left(1+\frac{1}{s}\right) - U_g(s)\frac{1}{s} \rightarrow 1) \Rightarrow$$

$$\left[U_2\left(1+\frac{1}{sC_2R_1}\right) - U_g(s)\frac{1}{sC_2R_1}\right]\left(sC_1 + sC_2 + \frac{1}{R_2} + \frac{1}{R_3}\right) - U_2 sC_2 = U_g(s)sC_1 + U_{iz}(s)\frac{1}{R_2}$$

$$\Rightarrow$$

$$\left[U_2\left(1+\frac{1}{s}\right) - U_g(s)\frac{1}{s}\right](2s+2) - U_2 s = U_g(s)s + U_{iz}(s)$$

$$2\left[U_2 + \frac{U_2}{s} - U_g(s)\frac{1}{s}\right](s+1) - U_2 s = U_g(s)s + U_{iz}(s)$$

$$2U_2 s + 2U_2 - 2U_g(s) + 2U_2 + 2\frac{U_2}{s} - 2\frac{U_g(s)}{s} - U_2 s = U_g(s)s + U_{iz}(s)$$

$$U_2 s + 4U_2 + \frac{2}{s}U_2 = 2U_g(s) + U_g(s)s + \frac{2}{s}U_g(s) + U_{iz}(s)$$

$$\left(\frac{U_g(s)}{2} + \frac{U_{iz}(s)}{2}\right)\left(s+4+\frac{2}{s}\right) = U_g(s)\left(2+s+\frac{2}{s}\right) + U_{iz}(s) \Big/ \cdot 2s$$

$$\left(U_g(s) + U_{iz}(s)\right)\left(s^2+4s+2\right) = U_g(s)\left(2s^2+4s+4\right) + U_{iz}(s)\cdot 2s$$

$$U_g(s)\left(s^2+4s+2\right) + U_{iz}(s)\left(s^2+4s+2\right) = U_g(s)\left(2s^2+4s+4\right) + U_{iz}(s)\cdot 2s$$

$$U_{iz}(s)\left(s^2+2s+2\right) = U_g(s)\left(s^2+2\right)$$

$$H(s) = \frac{U_{iz}(s)}{U_g(s)} = \frac{s^2+2}{s^2+2s+2}$$ **(2 boda)**

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Pobuda je svevremenska sinusoida: $u_g(t) = \sin(\sqrt{2}t)$; $-\infty<t<+\infty$.

Pridruženi fazor napona je $U_g(j\omega) = 1\angle 0°$.

Frekvencijska karakteristika naponske prijenosne funkcije je:

$$H(j\omega) = H(s)\big|_{s=j\omega} = \frac{U_{iz}(j\omega)}{U_g(j\omega)} = \frac{2-\omega^2}{2j\omega + (2-\omega^2)}$$

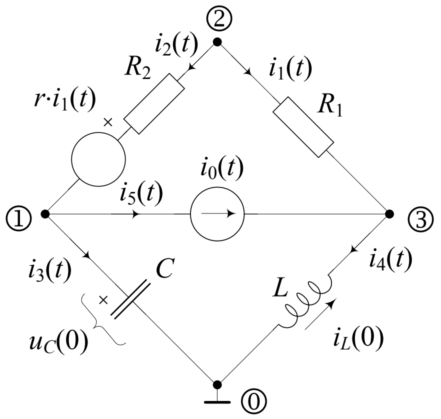
Na frekvenciji signala: $\omega_1 = \sqrt{2}$ je amplitudno frekvencijska karakteristika jednaka:

$$\left|H(j\omega)\right|\Big|_{\omega^2=\omega_1^2=2} = \left.\frac{\left|2-\omega^2\right|}{\sqrt{4\omega^2 + (2-\omega^2)^2}}\right|_{\omega^2=\omega_1^2=2} = \frac{|2-2|}{\sqrt{8+(2-2)^2}} = 0$$

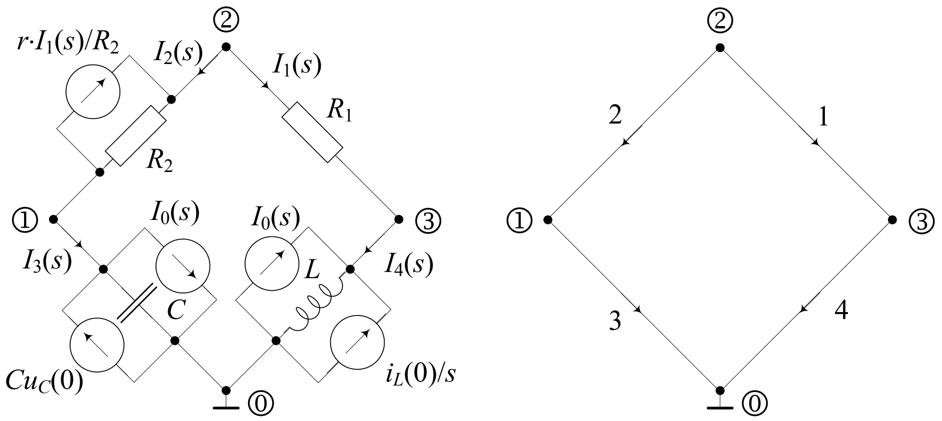
$$\left|U_{iz}(j\omega)\right| = \left|H(j\omega_1)\right|\cdot\left|U_g(j\omega)\right| = 0\cdot 1\angle 0° \;\Rightarrow\; u_{iz}(t) = 0$$

Zbog nule u prijenosnoj funkciji na frekvenciji ulaznog signala izlazni signal je jednak nuli. **(1 bod)**

2. Zadan je električni krug prema slici. Poštujući oznake grana i čvorišta nacrtati pripadni orijentirani graf. Napisati reduciranu matricu incidencija **A**. Napisati strujno-naponske jednadžbe grana u matričnom obliku te ispisati matricu admitancija grana $\mathbf{Y}_b$ i vektor strujnih izvora grana $\mathbf{I}_{0b}$. Matrica $\mathbf{Y}_b$ mora biti regularna. Napisati sustav jednadžbi napona čvorova, odnosno odrediti matrice admitancija čvorova $\mathbf{Y}_v$ i vektor strujnih izvora u čvorovima $\mathbf{I}_{0v}$.




<u>Rješenje:</u> Uz primjenu Laplaceove transformacije i posmicanje strujnog izvora:




Električni krug                orijentirani graf

**(1 bod)**

Matrica incidencija (reducirana):

$$\mathbf{A} = \begin{array}{c} \\ (1) \\ (2) \\ (3) \end{array}\!\!\begin{array}{c} \overbrace{\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array}}^{grane} \\ \begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

**(1 bod)**

$\uparrow$

*cvorovi*

Naponsko strujne relacije grana (struje izražene pomoću napona):

$$I_1 = U_1 \frac{1}{R_1}$$

$$I_2 = U_2 \frac{1}{R_2} - rI_1 \frac{1}{R_2} = -U_1 \frac{r}{R_1 R_2} + U_2 \frac{1}{R_2}$$

$$I_3 = U_3 sC + I_0 - Cu_C(0)$$

$$I_4 = U_4 \frac{1}{sL} - I_0 - \frac{i_L(0)}{s}$$

Naponsko-strujne relacije grana u matričnom obliku: $\mathbf{I}_b = \mathbf{Y}_b \cdot \mathbf{U}_b + \mathbf{I}_{0b}$

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b} = \underbrace{\begin{bmatrix} \frac{1}{R_1} & 0 & 0 & 0 \\ -\frac{r}{R_1R_2} & \frac{1}{R_2} & 0 & 0 \\ 0 & 0 & sC & 0 \\ 0 & 0 & 0 & \frac{1}{sL} \end{bmatrix}}_{\mathbf{Y}_b} \underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ I_0 - Cu_C(0) \\ -I_0 - \frac{i_L(0)}{s} \end{bmatrix}}_{\mathbf{I}_{0b}}$$ **(1 bod)**

Sustav jednadžbi napona čvorova u matričnom obliku $\mathbf{Y}_v \cdot \mathbf{U}_v = \mathbf{I}_{0v}$, gdje su (matrice $\mathbf{Y}_v$ i $\mathbf{I}_{0v}$):

$$\mathbf{Y}_v = \mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{A}^T = \begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{R_1} & 0 & 0 & 0 \\ -\frac{r}{R_1R_2} & \frac{1}{R_2} & 0 & 0 \\ 0 & 0 & sC & 0 \\ 0 & 0 & 0 & \frac{1}{sL} \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

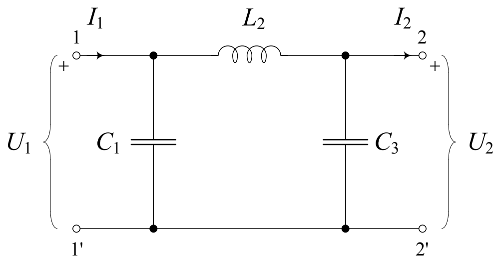
$$= \begin{bmatrix} \frac{r}{R_1R_2} & -\frac{1}{R_2} & sC & 0 \\ \frac{1}{R_1} - \frac{r}{R_1R_2} & \frac{1}{R_2} & 0 & 0 \\ -\frac{1}{R_1} & 0 & 0 & \frac{1}{sL} \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & -1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_2} + sC & \frac{r}{R_1R_2} - \frac{1}{R_2} & -\frac{r}{R_1R_2} \\ -\frac{1}{R_2} & \frac{1}{R_1} + \frac{1}{R_2} - \frac{r}{R_1R_2} & -\frac{1}{R_1} + \frac{r}{R_1R_2} \\ 0 & -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{sL} \end{bmatrix}$$

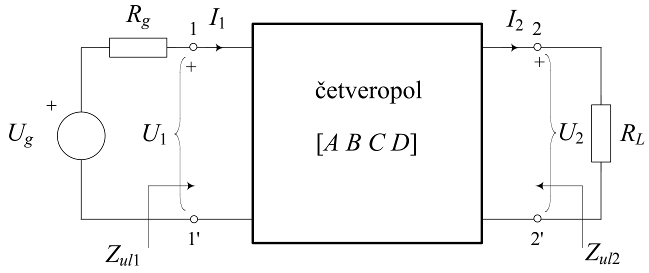
**(1 bod)**

$$\mathbf{I}_{0v} = -\mathbf{A} \cdot \mathbf{I}_{0b} = -\begin{bmatrix} 0 & -1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ I_0 - Cu_C(0) \\ -I_0 - \frac{i_L(0)}{s} \end{bmatrix} = \begin{bmatrix} -I_0 + Cu_C(0) \\ 0 \\ I_0 + \frac{i_L(0)}{s} \end{bmatrix}$$

**(1 bod)**

3. Za Π-četveropol prikazan lijevom slikom izračunati prijenosne *a*-parametre. a) Napisati parametre $A$, $B$, $C$ i $D$ pomoću $C_1$, $L_2$ i $C_3$ te uvrstiti normirane vrijednosti elemenata: $C_1=1$, $L_2=2$, $C_3=1$. Četveropol je spojen u električni krug na desnoj slici. b) Ako je izlazni prilaz (2–2') zaključen otporom $R_L=1$ pomoću *a*-parametara izračunati: ulaznu impedanciju $Z_{ul1}(s)=U_1(s)/I_1(s)$ gledano sa priključnica 1–1'; c) ako je $R_g=1$ izračunati izlaznu impedanciju $Z_{ul2}(s)=-U_2(s)/I_2(s)$ gledano sa priključnica 2–2'; i d) izračunati prijenosnu funkciju napona $H(s)=U_2(s)/U_g(s)$.



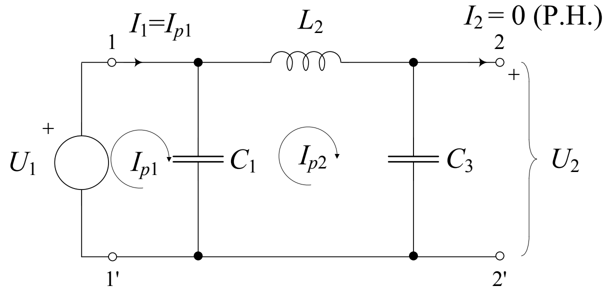




Rješenje:

a) [*a*]-parametri:

$$U_1 = A \cdot U_2 + B \cdot I_2$$
$$I_1 = C \cdot U_2 + D \cdot I_2$$

$$I_2 = 0 \qquad A = \left.\frac{U_1}{U_2}\right|_{I_2=0} \; ; \quad C = \left.\frac{I_1}{U_2}\right|_{I_2=0}$$




$$(1)\;\; U_1 = (I_{p1} - I_{p2})\frac{1}{sC_1} = I_{p2}\left(sL_2 + \frac{1}{sC_3}\right)$$

$$(2)\;\; U_2 = I_{p2}\frac{1}{sC_3} \Rightarrow I_{p2} = sC_3U_2$$

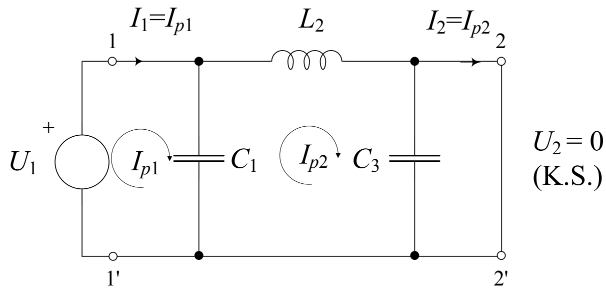
$$\Rightarrow \;\; A = \frac{U_1}{U_2} = \frac{I_{p2}\left(sL_2 + \dfrac{1}{sC_3}\right)}{I_{p2}\dfrac{1}{sC_3}} = s^2L_2C_3 + 1;$$

$$I_{p1} = I_{p2}\left(s^2L_2C_1 + \frac{C_1}{C_3} + 1\right) = sC_3U_2\left(s^2L_2C_1 + \frac{C_1}{C_3} + 1\right) = U_2\left(s^3L_2C_1C_3 + sC_1 + sC_3\right)$$

$$C = \frac{I_1}{U_2} = s^3L_2C_1C_3 + sC_1 + sC_3;$$

A i C **(1 bod)**

$$U_2 = 0 \qquad B = \left.\frac{U_1}{I_2}\right|_{U_2=0} \; ; \quad D = \left.\frac{I_1}{I_2}\right|_{U_2=0}$$



$$(1)\ U_1=(I_{p1}-I_{p2})\frac{1}{sC_1}$$

$$(2)\ U_1=I_{p2}sL_2 \Rightarrow I_{p2}=\frac{U_1}{sL_2}$$

$$U_1=\left(I_{p1}-\frac{U_1}{sL_2}\right)\frac{1}{sC_1}\Rightarrow U_1\left(1+\frac{1}{s^2C_1L_2}\right)=I_{p1}\frac{1}{sC_1}\Rightarrow I_{p1}=U_1\left(sC_1+\frac{1}{sL_2}\right)$$

$$\Rightarrow\ B=\frac{U_1}{I_2}=sL_2;\quad D=\frac{I_1}{I_2}=\frac{U_1\left(sC_1+\frac{1}{sL_2}\right)}{\frac{U_1}{sL_2}}=s^2C_1L_2+1;$$

B i D **(1 bod)**

Ovo su $[a]$-parametri četveropola:

$$[a]=\begin{bmatrix}A & B\\ C & D\end{bmatrix};\quad [a]=\begin{bmatrix}s^2L_2C_3+1 & s^3L_2C_1C_3+sC_1+sC_3\\ sL_2 & s^2C_1L_2+1\end{bmatrix}=\begin{bmatrix}2s^2+1 & 2s^3+2s\\ 2s & 2s^2+1\end{bmatrix}$$

b) Ulazna impedancija u četveropol:

Konačno iz jednadžbi

$$\begin{array}{l}U_1=A\cdot U_2+B\cdot I_2\\ I_1=C\cdot U_2+D\cdot I_2\end{array},\quad R_L=\frac{U_2}{I_2},\quad U_g=I_1R_g+U_1$$

slijede:

$$Z_{ul1}(s)=\frac{U_1}{I_1}=\frac{AU_2+BI_2}{CU_2+DI_2}=\frac{A\frac{U_2}{I_2}+B}{C\frac{U_2}{I_2}+D}=\frac{AR_L+B}{CR_L+D};\quad R_L=\frac{U_2}{I_2}$$

$$Z_{ul1}(s)=\frac{(2s^2+1)\cdot 1+(2s^3+2s)}{(2s)\cdot 1+(2s^2+1)}=\frac{2s^3+2s^2+2s+1}{2s^2+2s+1}\ \textbf{(1 bod)}$$

c) Izlazna impedancija iz četveropola: pošto je četveropol recipročan det($[a]$)=$AD-BC$=1 i simetričan ($A$=$D$) vrijedi:

$$Z_{ul2}(s)\big|_{U_g=0}=-\frac{U_2}{I_2}=\frac{DU_1-BI_1}{CU_1-AI_1}=\frac{D\frac{U_1}{I_1}-B}{C\frac{U_1}{I_1}-A}=\frac{DR_g+B}{CR_g+A};\quad R_g=-\frac{U_1}{I_1}$$

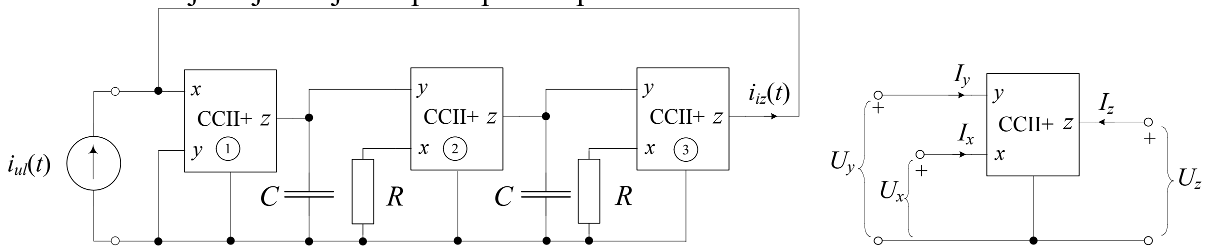
$$\Rightarrow Z_{ul2}(s)=Z_{ul1}(s)=\frac{2s^3+2s^2+2s+1}{2s^2+2s+1}\ \textbf{(1 bod)}$$

d) Prijenosna funkcija napona:

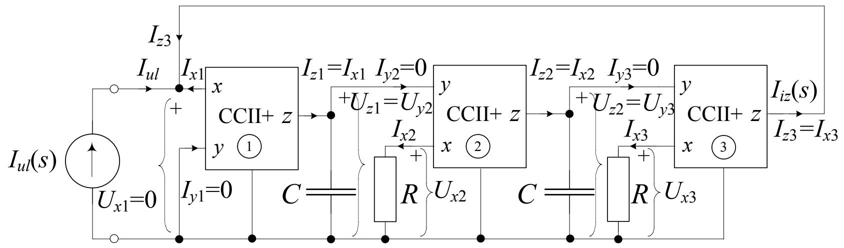
$$U_g=I_1R_g+U_1=\left(CU_2+D\frac{U_2}{R_L}\right)R_g+AU_2+B\frac{U_2}{R_L}\ \Rightarrow\ H(s)=\frac{U_2}{U_g}=\frac{R_L}{AR_L+B+R_g(CR_L+D)}$$

$$H(s)=\frac{1}{(2s^2+1)\cdot 1+2s^3+2s+1\cdot\left[(2s)\cdot 1+2s^2+1\right]}=\frac{\frac{1}{2}}{s^3+2s^2+2s+1}\ \textbf{(1 bod)}$$

4. Za električni krug prikazan slikom izračunati strujnu prijenosnu funkciju $H_i(s)=I_{iz}(s)/I_{ul}(s)$. Zadane su normalizirane vrijednosti elemenata $R$=1, $C$=1. Ako je zadana pobuda $i_{ul}(t)=S(t)$ izračunati i skicirati valni oblik struje $i_{iz}(t)$ za $t$>0 kao odziv. Za pozitivni strujni prijenosnik druge generacije (CCII+) vrijede slijedeće definicijske jednadžbe: $U_x=U_y$, $I_y=0$, $I_z=I_x$ uz referentna usmjerenja struja i napona prilaza prikazana na slici.




Rješenje:




Za prvi CCII+ vrijedi:

$$I_{x1}=-I_{ul}-I_{z3};\ U_{x1}=U_{y1}=U_{ul}=0,\ I_{y1}=0,\ I_{z1}=I_{x1}\ \textbf{(1 bod)}$$

Za drugi CCII+ vrijedi:

$$U_{z1}=U_{y2}=I_{z1}\frac{1}{sC},\ I_{x2}=\frac{U_{x2}}{R}=\frac{U_{y2}}{R};\ I_{y2}=0,\ I_{z2}=I_{x2}\ \textbf{(1 bod)}$$

Za treći CCII+ vrijedi:

$$U_{z2}=U_{y3}=I_{z2}\frac{1}{sC},\ I_{x3}=\frac{U_{x3}}{R}=\frac{U_{y3}}{R};\ I_{y3}=0,\ I_{iz}=I_{z3}=I_{x3}\ \textbf{(1 bod)}$$

Nakon malo sređivanja:

$$I_{iz}=I_{z3}$$

$$I_{z3}=I_{x3}=\frac{U_{x3}}{R}=\frac{U_{y3}}{R}=\frac{1}{sRC}I_{z2}$$

$$I_{z2}=I_{x2}=\frac{U_{x2}}{R}=\frac{U_{y2}}{R}=\frac{1}{sRC}I_{z1}$$

$$I_{iz}=I_{z3}=\frac{1}{sRC}I_{z2}=\left(\frac{1}{sRC}\right)^2 I_{z1}\quad\Rightarrow\quad I_{z1}=(sRC)^2 I_{iz}$$

$$I_{z1}=I_{x1}=-I_{ul}-I_{z3}=-I_{ul}-\left(\frac{1}{sRC}\right)^2 I_{z1}\quad\Rightarrow\quad I_{z1}\cdot\left(1+\frac{1}{(sRC)^2}\right)=-I_{ul}$$

$$I_{iz}\cdot(sRC)^2\cdot\left(1+\frac{1}{(sRC)^2}\right)=-I_{ul}\quad\Rightarrow\quad \underline{H_i(s)=\frac{I_{iz}(s)}{I_{ul}(s)}=-\frac{1}{1+(sRC)^2}}$$

Uz uvrštene vrijednosti elemenata:

$$\underline{H_i(s)=\frac{I_{iz}(s)}{I_{ul}(s)}=-\frac{1}{s^2+1}}\ \textbf{(1 bod)}$$

Za zadanu pobudu: $i_{ul}(t) = S(t) \Rightarrow I_{ul}(s) = \frac{1}{s}$

Uz uvrštene vrijednosti elemenata:

$$I_{iz}(s) = H_i(s) \cdot I_{ul}(s) = -\frac{1}{s} \cdot \frac{1}{s^2+1}$$

Rastav na parcijalne razlomke:

$$\frac{1}{s} \cdot \frac{1}{s^2+1} = \frac{A}{s} + \frac{Bs+C}{s^2+1} = \frac{A(s^2+1)+s(Bs+C)}{s(s^2+1)} = \frac{(A+B)s^2+Cs+A}{s(s^2+1)} = \frac{1}{s} - \frac{s}{s^2+1}$$

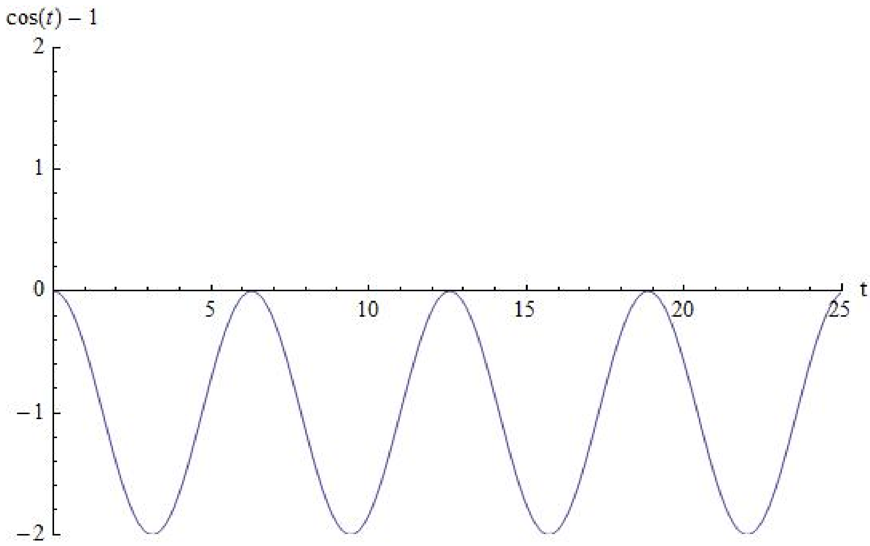
$$A + B = 0 \Rightarrow B = -A = -1$$

$$C = 0$$

$$\underline{A = 1}$$

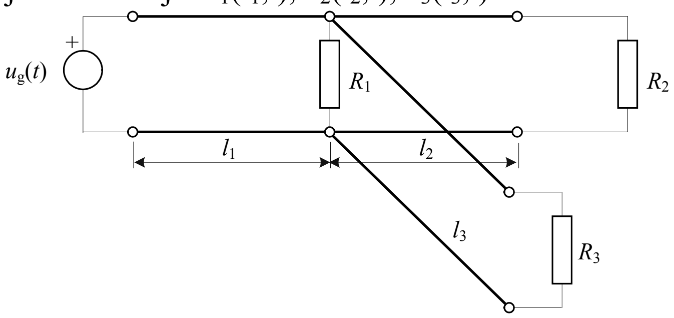
$$I_{iz}(s) = -\frac{1}{s} + \frac{s}{s^2+1} \quad \Rightarrow \quad \underline{i_{iz}(t) = [-1+\cos(t)] \cdot S(t)}$$ **(1 bod)**

Skica odziva $i_{iz}(t)$



5. Tri linije bez gubitaka spojene su prema slici. Zadano je: $L$=0,2mH/km, C=80nF/km, $u_g$= 10cos(2.5$\pi 10^5$ $t$) V, $R_2$ = 25 Ω, $R_3$= 100 Ω, $l_1$=3$\lambda$/4, $l_2$=$\lambda$/4 i $l_3$=$\lambda$/2. Odrediti:

a) valnu impedanciju i koeficijent prijenosa linija;
b) brzinu širenja vala na linijama i duljinu druge i treće linije;
c) otpor $R_1$ da bi prva linija bila prilagođena na izlazu;
d) faktore refleksije na krajevima druge i treće linije: $\Gamma_{i2}$ i $\Gamma_{i3}$;
e) napone na kraju svake linije: $u_1(l_1,t)$, $u_2(l_2,t)$, $u_3(l_3,t)$.




Rješenje:

a) $Z_0 = \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{2\cdot 10^{-4}}{8\cdot 10^{-8}}} = 50\Omega\,;\gamma = j\beta\,;$

$\beta = \omega_0\sqrt{LC} = 2.5\cdot\pi\cdot 10^5\sqrt{2\cdot 10^{-4}\cdot 8\cdot 10^{-8}} = \pi$ [rad/km] **(1 bod)**

b) $v = \dfrac{\omega}{\beta} = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{2\cdot 10^{-4}\cdot 8\cdot 10^{-8}}} = \dfrac{10^6}{4} = 25\cdot 10^4$ [km/s]

$\lambda = \dfrac{2\cdot\pi}{\beta} = \dfrac{2\cdot\pi}{\pi} = 2$ [km] $\qquad l_2 = \dfrac{\lambda}{4} = 500$ [m]; $\qquad l_2 = \dfrac{\lambda}{2} = 1$ [km]; **(1 bod)**

c) $\gamma\cdot l_2 = j\cdot\beta\cdot l_2 = j\dfrac{\pi}{2}\,; \qquad \gamma\cdot l_3 = j\cdot\beta\cdot l_3 = j\pi$

$$Z_{ul2} = \frac{R_2 ch(\gamma\cdot l_2) + Z_0 sh(\gamma\cdot l_2)}{\dfrac{R_2}{Z_0} sh(\gamma\cdot l_2) + ch(\gamma\cdot l_2)} = \frac{R_2\cos(\beta\cdot l_2) + jZ_0\sin(\beta\cdot l_2)}{j\dfrac{R_2}{Z_0}\sin(\beta\cdot l_2) + \cos(\beta\cdot l_2)} = \frac{Z_0^2}{R_2} = \frac{2500}{25} = 100\,\Omega$$

$$Z_{ul3} = \frac{R_3\cos(\beta\cdot l_3) + jZ_0\sin(\beta\cdot l_3)}{j\dfrac{R_3}{Z_0}\sin(\beta\cdot l_2) + \cos(\beta\cdot l_3)} = \frac{-R_3}{-1} = R_3 = 100\,\Omega \quad Z_{ul2}\| Z_{ul3} = 50\,\Omega \Rightarrow R_1 = \infty$$ **(1 bod)**

d) $\Gamma_{i2} = \dfrac{R_2 - Z_0}{R_2 + Z_0} = \dfrac{-25}{75} = -\dfrac{1}{3} \qquad \Gamma_{i3} = \dfrac{R_3 - Z_0}{R_3 + Z_0} = \dfrac{50}{150} = \dfrac{1}{3}$ **(1 bod)**

e) $\gamma\cdot l_1 = j\beta\dfrac{3\lambda}{4} = j\beta\dfrac{3\cdot 2\pi}{4\cdot\beta} = j\dfrac{3\pi}{2}\,;$

$U_1(l_1) = U(0)\cdot e^{-j\beta l_1} = 10\cdot e^{-j3\pi/2} = 10j\,;\;\; u_1(l_1,t) = 10\cos\left(\omega t + \dfrac{\pi}{2}\right)$

$U_2(l_2) = U_1(l_1)\cdot\cos(\beta\cdot l_2) - jU_1(l_1)\cdot\dfrac{Z_0}{Z_{ul2}}\sin(\beta\cdot l_2) = -jU_1(l_1)\cdot\dfrac{Z_0}{Z_{ul2}} = j5\cdot e^{-j\pi/2} = 5$

$u_1(l_2,t) = 5\cos(\omega t)$

$U_3(l_3) = U(l_1)\cdot\cos(\beta\cdot l_3) - jU(l_1)\cdot\dfrac{Z_0}{Z_{ul3}}\sin(\beta\cdot l_3) = -U(l_1) = -10\cdot e^{-j3\pi/2} = 10\cdot e^{-j\pi/2}$

$u_1(l_3,t) = 10\cos(\omega t - \pi/2)$ **(1 bod)**