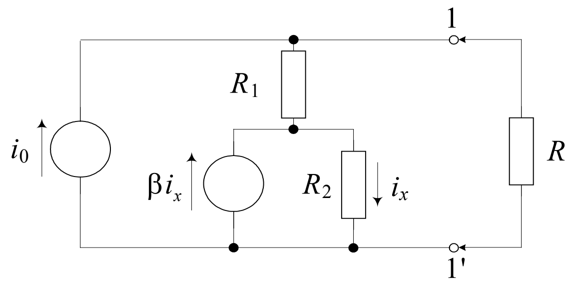
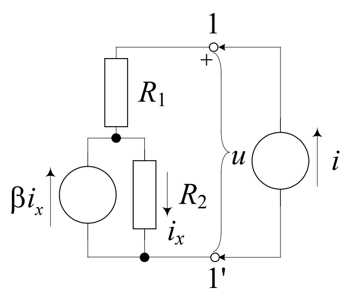
PRVI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA

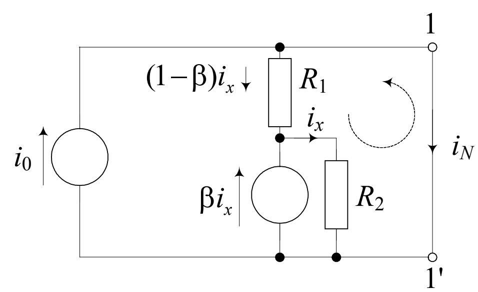
1. Za krug prikazan slikom:
   a) isključiti otpor R i odrediti nadomjesni spoj po Nortonu obzirom na priključnice 1–1';
   b) odrediti iznos konstante β za koji je $Y_N(s)=1/R$.
   c) uz uključen otpor R primjenom transformacija izvora i Kirchhoffovih zakona odrediti struju i_x;
   Zadana je pobuda $i_0=2$ A, i vrijednosti elemenata $R_1=R_2=4\ \Omega$, $R=16\ \Omega$.



Rješenje:

a) isključiti otpor R i odrediti nadomjesni spoj po Nortonu obzirom na priključnice 1–1';





Nortonova admitancija: $i_x=\beta i_x+i\Rightarrow i_x(1-\beta)=i\Rightarrow i_x=\dfrac{i}{1-\beta}$, $u=iR_1+\dfrac{i}{1-\beta}R_2\Rightarrow Z_T=\dfrac{u}{i}=R_1+\dfrac{R_2}{1-\beta}$,

$$Y_N=\frac{1}{Z_T}=\frac{1}{R_1+\dfrac{R_2}{1-\beta}}=\frac{1-\beta}{R_1(1-\beta)+R_2}$$

Nortonova struja: $i_N=(\beta-1)i_x+i_0$,

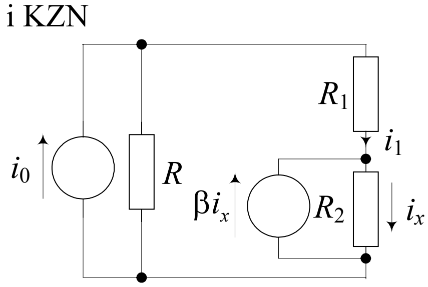
$$(1-\beta)i_xR_1+i_xR_2=0\ \Rightarrow\ (\beta-1)i_xR_1=i_xR_2\Rightarrow i_x=0\Rightarrow i_N=i_0=2A$$

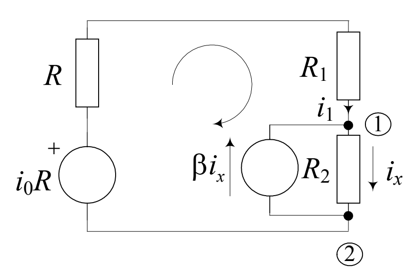
b) odrediti iznos konstante β za koji je $Y_N(s)=1/R$.

$$Z_T=R_1+\frac{R_2}{1-\beta}=R\Rightarrow 4+\frac{4}{1-\beta}=16\Rightarrow\frac{4}{1-\beta}=12\Rightarrow\frac{1}{1-\beta}=3\Rightarrow 1-\beta=\frac{1}{3}\Rightarrow\beta=1-\frac{1}{3}=\frac{2}{3},$$

konačno je: $Y_N=1/Z_T=1/16$.

c) Jednadžbe KZS i KZN





KZS za čvor (1): $0 = -i_1 - \beta i_x + i_x \quad \Rightarrow \quad i_1 = -(\beta - 1) i_x = (1-\beta) i_x$

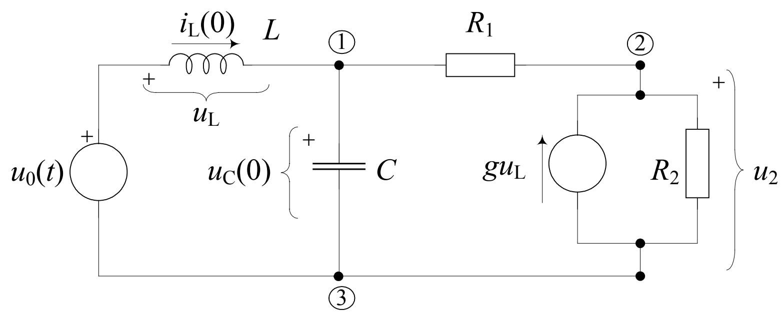
KZN za petlju: $i_1(R + R_1) + i_x R_2 = i_0 R$

$$\Rightarrow \quad (1-\beta) i_x (R + R_1) + i_x R_2 = i_0 R \quad \Rightarrow$$

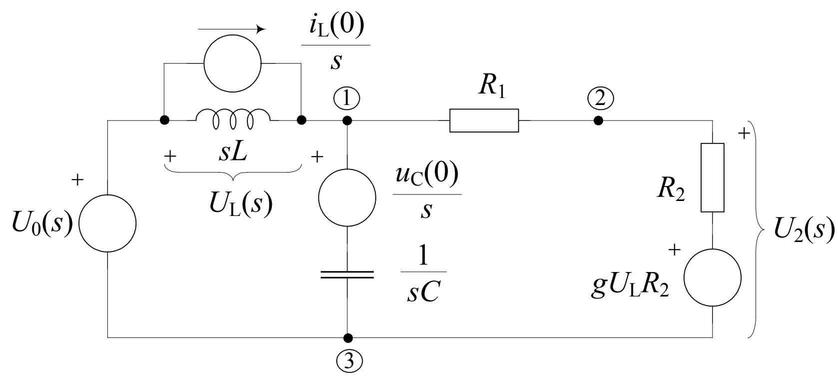
$$i_x = \frac{i_0 R}{(1-\beta)(R+R_1) + R_2}$$

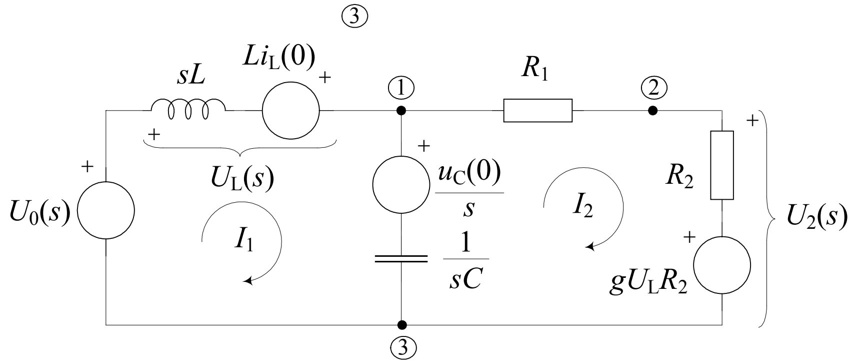
$$i_x = \frac{2 \cdot 16}{(1-2/3)(16+4)+4} = \frac{32}{(1/3)\cdot 20 + 4} = \frac{3 \cdot 32}{20+12} = \frac{96}{32} = 3A$$

2. Za krug prikazan slikom napisati jednadžbe petlji. Izračunati napon $U_2(s)$, ako je zadana pobuda $u_0(t)$= 2S(t), konstanta g=1, normirane vrijednosti elemenata su: R_1=R_2=1, L=1 i C=1, a početni uvjeti su $u_C(0)$=1, $i_L(0)$=1.



Rješenje: Primjena Laplaceove transformacije





Jednadžbe petlji:

(1) $\quad I_1(s) sL + [I_1(s) - I_2(s)] \frac{1}{sC} = U_0(s) + L i_L(0) - \frac{u_C(0)}{s}$

(2) $\quad -[I_1(s) - I_2(s)] \frac{1}{sC} + I_2(s) R_1 + I_2(s) R_2 = -g U_L(s) R_2 + \frac{u_C(0)}{s}$

$$U_L(s) = I_1(s)sL - Li_L(0)$$
$$U_2(s) = I_2(s)R_2 + gU_L(s)R_2 = I_2(s)R_2 + g\left[I_1(s)sL - Li_L(0)\right]R_2$$

$$(1) \qquad I_1(s)sL + \left[I_1(s) - I_2(s)\right]\frac{1}{sC} = U_0(s) + Li_L(0) - \frac{u_C(0)}{s}$$
$$(2) \qquad -\left[I_1(s) - I_2(s)\right]\frac{1}{sC} + I_2(s)R_1 + I_2(s)R_2 = -g\left[I_1(s)sL - Li_L(0)\right]R_2 + \frac{u_C(0)}{s}$$

$$(1) \qquad I_1(s)\left(sL + \frac{1}{sC}\right) - I_2(s)\frac{1}{sC} = U_0(s) + Li_L(0) - \frac{u_C(0)}{s}$$
$$(2) \qquad -I_1(s)\left(\frac{1}{sC} - gR_2 sL\right) + I_2(s)\left(\frac{1}{sC} + R_1 + R_2\right) = gR_2 Li_L(0) + \frac{u_C(0)}{s}$$

Uvrstimo vrijednosti elemenata:

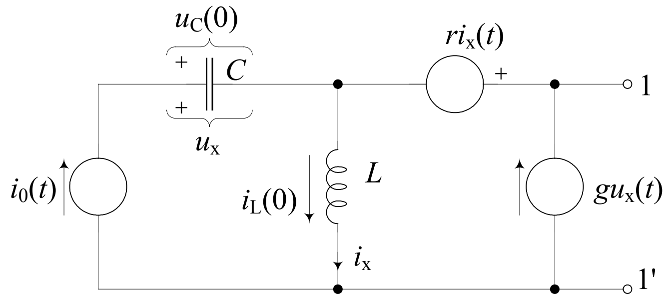
$$(1) \qquad I_1(s)\left(s + \frac{1}{s}\right) - I_2(s)\frac{1}{s} = \frac{2}{s} + 1 - \frac{1}{s} = 1 + \frac{1}{s} \Big/ \cdot s$$
$$(2) \qquad -I_1(s)\left(\frac{1}{s} - s\right) + I_2(s)\left(\frac{1}{s} + 2\right) = 1 + \frac{1}{s} \Big/ \cdot s$$

$$(1) \qquad I_1(s)(s^2 + 1) - I_2(s) = s + 1$$
$$(2) \qquad I_1(s)(s^2 - 1) + I_2(s)(2s + 1) = s + 1$$

$$\Delta = \begin{vmatrix} s^2 + 1 & -1 \\ s^2 - 1 & 2s + 1 \end{vmatrix} = (s^2 + 1)(2s + 1) + s^2 - 1 = 2s^3 + 2s + s^2 + 1 + s^2 - 1 = 2s^3 + 2s^2 + 2s$$
$$\Delta_1 = \begin{vmatrix} s + 1 & -1 \\ s + 1 & 2s + 1 \end{vmatrix} = (s + 1)(2s + 1) + s + 1 = 2s^2 + 2s + s + 1 + s + 1 = 2s^2 + 4s + 2$$
$$\Delta_2 = \begin{vmatrix} s^2 + 1 & s + 1 \\ s^2 - 1 & s + 1 \end{vmatrix} = (s^2 + 1)(s + 1) - (s^2 - 1)(s + 1) = (s + 1)(s^2 + 1 - s^2 + 1) = 2(s + 1)$$
$$I_1 = \frac{\Delta_1}{\Delta} = \frac{2s^2 + 4s + 2}{2s^3 + 2s^2 + 2s} = \frac{2(s^2 + 2s + 1)}{2(s^3 + s^2 + s)} = \frac{s^2 + 2s + 1}{s^3 + s^2 + s}$$
$$I_2 = \frac{\Delta_2}{\Delta} = \frac{2(s + 1)}{2(s^3 + s^2 + s)} = \frac{s + 1}{s^3 + s^2 + s}$$

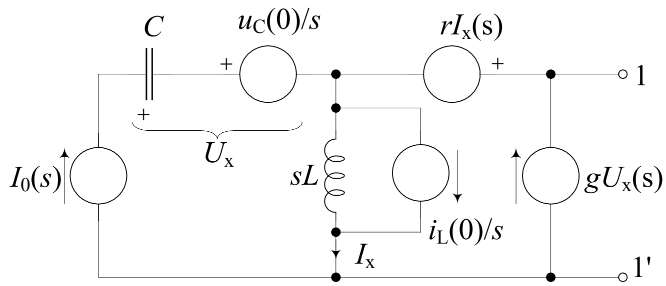
$$U_2(s) = I_2(s)R_2 + g\left[I_1(s)sL - Li_L(0)\right]R_2 = \frac{s + 1}{s^3 + s^2 + s} + \frac{s^2 + 2s + 1}{s^3 + s^2 + s}s - 1$$
$$U_2(s) = \frac{s + 1 + s^3 + 2s^2 + s - s^3 - s^2 - s}{s^3 + s^2 + s} = \frac{s^2 + s + 1}{s^3 + s^2 + s} = \frac{s^2 + s + 1}{s(s^2 + s + 1)} = \frac{1}{s}$$

3. Za krug prikazan slikom odrediti nadomjesnu shemu po Theveninu obzirom na priključnice 1-1', ako je pobuda $i_0(t)=S(t)$. Zadane su normirane vrijednosti elemenata: $R=1$, $L=1$, $C=1$, $r=0.5$, $g=0.5$, a početni uvjeti su $u_C(0)=1$ i $i_L(0)=1$.



Rješenje: Primjena Laplaceove transformacije

a) Theveninov napon $U_T(s)$



$$U_x(s)=I_0\frac{1}{sC}+\frac{u_C(0)}{s}$$

$$I_x(s)=I_0(s)+gU_x(s)=I_0(s)+g\left(I_0\frac{1}{sC}+\frac{u_C(0)}{s}\right)$$

$$U_T(s)=U_L(s)+rI_x(s)=sLI_x(s)-Li_L(0)+rI_x(s)=(sL+r)I_x(s)-Li_L(0)$$

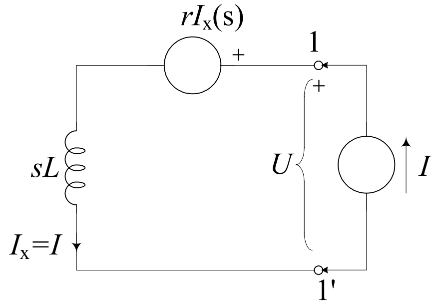
$$U_T(s)=(sL+r)\left[I_0(s)+g\left(I_0\frac{1}{sC}+\frac{u_C(0)}{s}\right)\right]-Li_L(0)$$

$$U_T(s)=(sL+r)\left(1+\frac{g}{sC}\right)I_0(s)+(sL+r)g\frac{u_C(0)}{s}-Li_L(0)$$

$$U_T(s)=\left(s+\frac{1}{2}\right)\left(1+\frac{1}{2s}\right)\frac{1}{s}+\left(s+\frac{1}{2}\right)\frac{1}{2s}-1=\frac{2s^2+5s+1}{4s^2}=\frac{1}{2}+\frac{5}{4s}+\frac{1}{4s^2}$$

$$u_T(t)=\frac{1}{2}\delta(t)+\frac{5}{4}S(t)+\frac{1}{4}tS(t)$$

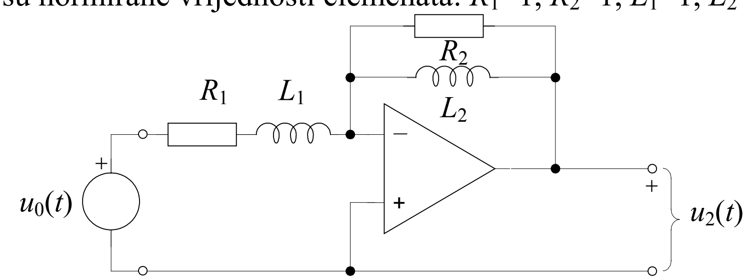
b) Theveninova impedancija $Z_T(s)$



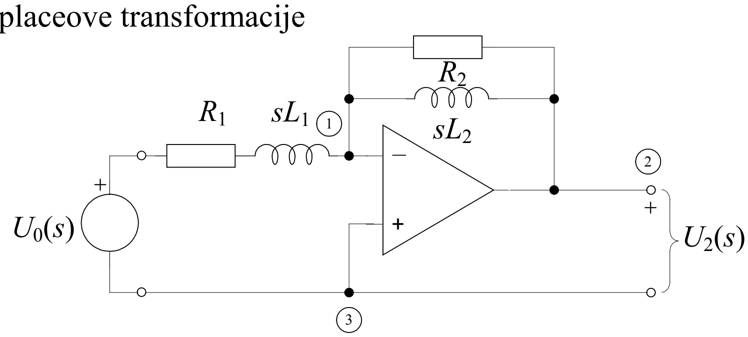
$$U = I_x sL + r \cdot I_x, I_x = I \Rightarrow U = I(sL + r)$$

$$Z_T(s) = \frac{U}{I} = sL + r = s + 1/2$$

4. Za krug prikazan slikom odrediti napon na izlazu operacijskog pojačala $u_2(t)$, ako je pobuda $u_0(t)=S(t)$. Zadane su normirane vrijednosti elemenata: $R_1=1$, $R_2=1$, $L_1=1$, $L_2=1/2$, $A\to\infty$.



Rješenje: Primjena Laplaceove transformacije



(1) $$\frac{U_0}{sL_1 + R_1} = U_1\left(\frac{1}{sL_1 + R_1} + \frac{1}{sL_2} + \frac{1}{R_2}\right) - U_2\left(\frac{1}{sL_2} + \frac{1}{R_2}\right)$$

$U_1 = 0$, jer $A \to \infty$

(1) $\Rightarrow$ $$U_2 = -\frac{U_0}{(sL_1 + R_1)\left(\frac{1}{sL_2} + \frac{1}{R_2}\right)} = -\frac{U_0}{\frac{L_1}{sR_2}\left(s + \frac{R_1}{L_1}\right)\left(s + \frac{R_2}{L_2}\right)}$$

$$U_2 = -\frac{\frac{R_2}{L_1}s}{\left(s + \frac{R_1}{L_1}\right)\left(s + \frac{R_2}{L_2}\right)}U_0 = -\frac{1 \cdot s \cdot \frac{1}{s}}{(s+1)(s+2)} = \frac{-1}{(s+1)(s+2)}$$

Rastav na parcijalne razlomke:

$$U_2(s)=\frac{-1}{(s+1)(s+2)}=\frac{A}{s+1}+\frac{B}{s+2}=\frac{A(s+2)+B(s+1)}{(s+1)(s+2)}=\frac{(A+B)s+(2A+B)}{(s+1)(s+2)}$$

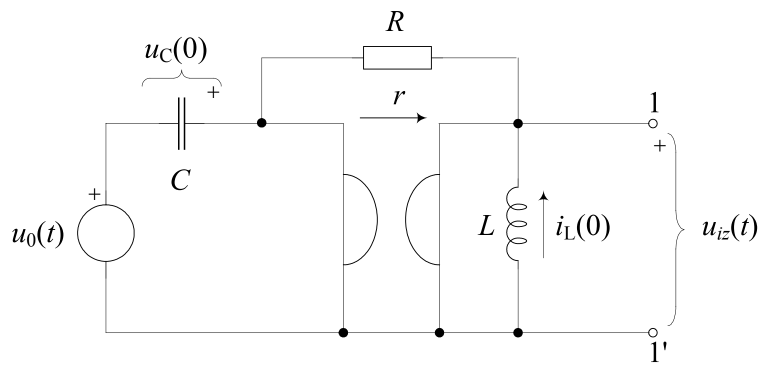
$$A+B=0 \;\Rightarrow\; B=-A$$

$$2A+B=-1 \;\Rightarrow\; 2A-A=-1 \;\Rightarrow\; A=-1 \Rightarrow\; B=1$$

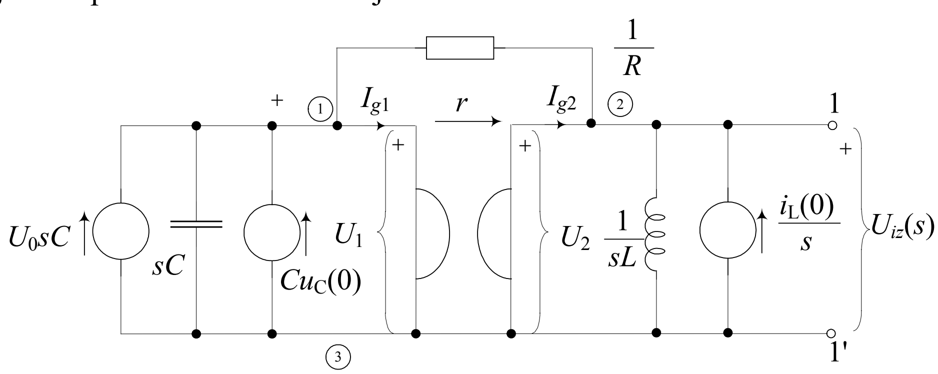
$$U_2(s)=\frac{-1}{s+1}+\frac{1}{s+2}$$

Konačno je: $u_2(t)=\left(-e^{-t}+e^{-2t}\right)\cdot S(t)$

5. Za krug prikazan slikom odrediti napon $u_{iz}(t)$ na priključnicama 1-1', koristeći postupak jednadžbi čvorišta, ako je pobuda $u_0(t)= S(t)$. Zadane su normirane vrijednosti elemenata: R=0.5, L=1, C=1, r=1 i početni uvjeti $u_C(0)$=2, $i_L(0)$=1.



Rješenje: Primjena Laplaceove transformacije



(1) $$U_1\left(sC+\frac{1}{R}\right)-U_2\frac{1}{R}=U_0(s)sC+Cu_C(0)-I_{g1} \qquad I_{g1}=-U_2\frac{1}{r}$$

(2) $$-U_1\frac{1}{R}+U_2\left(\frac{1}{R}+\frac{1}{sL}\right)=\frac{i_L(0)}{s}+I_{g2} \qquad I_{g2}=-U_1\frac{1}{r}$$

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(1) $$U_1\left(sC+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right)=U_0(s)sC+Cu_C(0)$$

(2) $$-U_1\left(\frac{1}{R}-\frac{1}{r}\right)+U_2\left(\frac{1}{R}+\frac{1}{sL}\right)=\frac{i_L(0)}{s}$$

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(2) $$\Rightarrow \qquad U_1=\frac{U_2\left(\frac{1}{R}+\frac{1}{sL}\right)-\frac{i_L(0)}{s}}{\frac{1}{R}-\frac{1}{r}}$$

(1) $$\frac{U_2\left(\frac{1}{R}+\frac{1}{sL}\right)-\frac{i_L(0)}{s}}{\frac{1}{R}-\frac{1}{r}}\left(sC+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right)=U_0sC+Cu_C(0) \qquad \Big/\cdot\left(\frac{1}{R}-\frac{1}{r}\right)$$

$$\left[U_2\left(\frac{1}{R}+\frac{1}{sL}\right)-\frac{i_L(0)}{s}\right]\left(sC+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right)\left(\frac{1}{R}-\frac{1}{r}\right)=\left[U_0sC+Cu_C(0)\right]\left(\frac{1}{R}-\frac{1}{r}\right)$$

$$U_2\left(\frac{1}{R}+\frac{1}{sL}\right)\left(sC+\frac{1}{R}\right)-U_2\left(\frac{1}{R^2}-\frac{1}{r^2}\right)=\left[U_0sC+Cu_C(0)\right]\left(\frac{1}{R}-\frac{1}{r}\right)+\frac{i_L(0)}{s}\left(sC+\frac{1}{R}\right)$$

$$U_2\left(\frac{sC}{R}+\frac{sC}{sL}+\frac{1}{R^2}+\frac{1}{RsL}-\frac{1}{R^2}+\frac{1}{r^2}\right)=\left[U_0sC+Cu_C(0)\right]\left(\frac{1}{R}-\frac{1}{r}\right)+\frac{i_L(0)}{s}\left(sC+\frac{1}{R}\right)$$

$$U_2(s)=\frac{\left[U_0sC+Cu_C(0)\right]\left(\frac{1}{R}-\frac{1}{r}\right)+\frac{i_L(0)}{s}\left(sC+\frac{1}{R}\right)}{\frac{sC}{R}+\frac{sC}{sL}+\frac{1}{RsL}+\frac{1}{r^2}}$$

$$U_2(s)=\frac{\left[U_0sC+Cu_C(0)\right]\left(\frac{1}{R}-\frac{1}{r}\right)+\frac{i_L(0)}{s}\left(sC+\frac{1}{R}\right)}{\frac{sC}{R}+\frac{sC}{sL}+\frac{1}{RsL}+\frac{1}{r^2}}$$

$$U_2(s)=\frac{\left[\frac{1}{s}s+1\cdot 2\right](2-1)+\frac{1}{s}(s+2)}{2s+1+\frac{2}{s}+1}=\frac{3+\frac{2}{s}+1}{2s+2+\frac{2}{s}}=\frac{4+\frac{2}{s}}{2s+2+\frac{2}{s}}=\frac{2+\frac{1}{s}}{s+1+\frac{1}{s}}=\frac{2s+1}{s^2+s+1}$$

$$U_2(s)=\frac{2s+1}{s^2+s+1}=2\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$u_2(t)=2e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{3}}{2}\right)\cdot S(t)$$