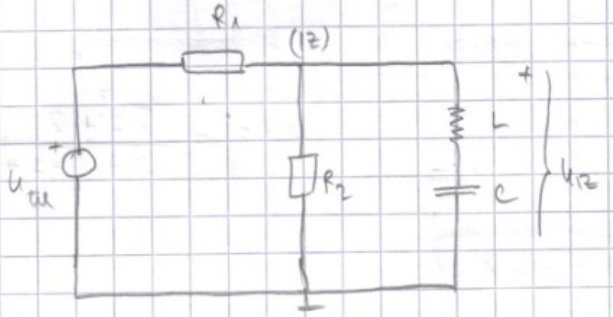
2. MASS (12.11.2011)

PRIJENOSNA FUNKCIJA



- ako su svi polovi u lijevoj poluravnini - sustav je stabilan
- sustav je GRANIČNO STABILAN ako je barem 1 pol na y-osi
- sustav je nestabilan ako se barem 1 pol nalazi u desnoj poluravnini, ili 1 dvostruki pol leži na y-osi

$$H(s) = \frac{U_{2Z}(s)}{U_{ul}(s)}$$

$$U_{2Z}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{sL + \frac{1}{sC}}\right) = \frac{U_{ul}}{R_1}$$

$$U_{2Z}\left(1 + \frac{1}{2} + \frac{1}{s + \frac{1}{s}}\right) = U_{ul}$$

$$U_{2Z}\left(\frac{3}{2} + \frac{s}{s^2+1}\right) = U_{ul}$$

$$U_{2Z}\left(\frac{3s^2 + 3 + 2s}{2(s^2+1)}\right) = U_{ul}$$

$$H(s) = \frac{2s^2+2}{3s^2+2s+3}$$

nazivnik = 0 } polovi

$3s^2 + 2s + 3 = 0$

$$s_{1,2} = \frac{-2 \pm \sqrt{4 - 36}}{6}$$

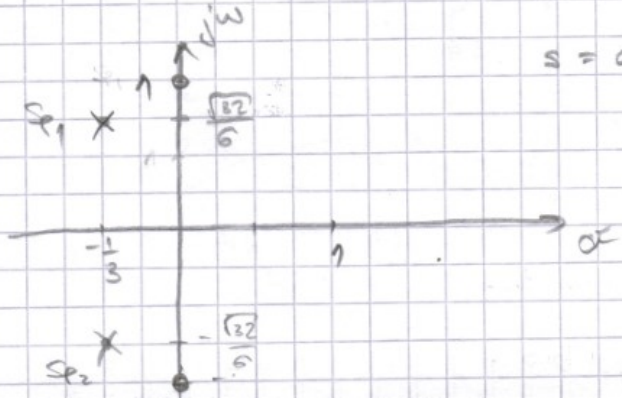
$$= -\frac{1}{3} \pm j\frac{\sqrt{32}}{6}$$

brojnik = 0 } nule

$2s^2 + 2 = 0$

$s_{0\,1,2} = \pm j$

- kada smo dobili polove i nule idemo u kompleksnu ravninu



$s = \sigma \pm j\omega$

nula - O
polovi - X

ODZIV U STACIONARNOM STANJU NA POBUDU $u_{ul}(t) = 4\cos(3t + 30°)$

$\omega_0 = 3$

$s = \sigma + j\omega$

$s = j\omega \Rightarrow$ u stacionarnom stanju

$$H(j\omega) = \frac{2 - 2\omega^2}{3 - 3\omega^2 + 2j\omega} \qquad H(s) = \frac{2s^2 + 2}{3s^2 + 2s + 3}$$

$$H(j\omega) = \frac{-16}{-24 + 6j}$$

$\boxed{j^2 = -1}$

$$|H(j3)| = \frac{16}{\sqrt{24^2 + 6^2}}$$

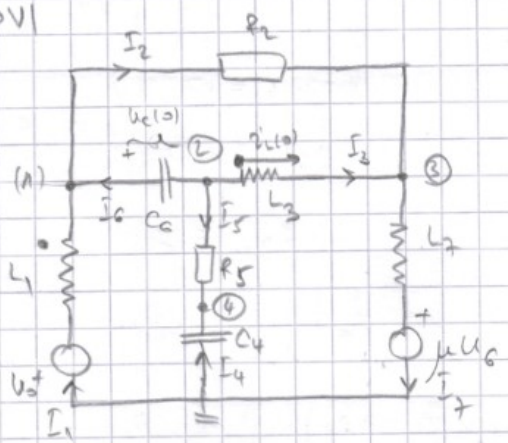
$$\sphericalangle H(j3) = -180° - \left(\operatorname{arctg}\frac{6}{-24} + 180°\right)$$

$$= \operatorname{arctg}\frac{6}{24}$$

$$u_{iz} = A_{ul} \cdot |H(j3)| \cdot \cos(3t + \alpha_{ul} + \sphericalangle H(j3))$$

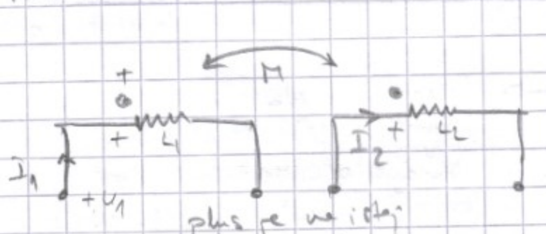
$$= 4 \cdot \frac{16}{\sqrt{24^2 + 6^2}} \cdot \cos\left(3t + 30° + \operatorname{arctg}\frac{6}{24}\right)$$

GRAFOVI



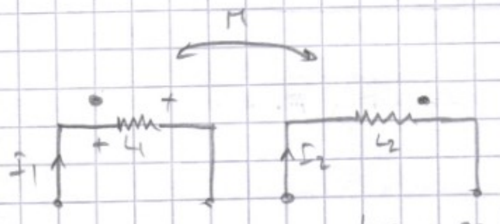
$M = M_{13}$

MEĐUINDUKTIVITET



plus je na istoj strani

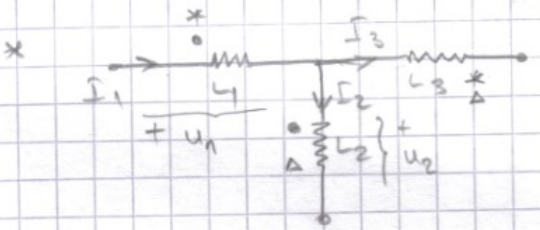
$U_1 = sL_1 I_1 + sM I_2$



plus je sa kontra strane

$U_1 = sL_1 I_1 - sM I_2$

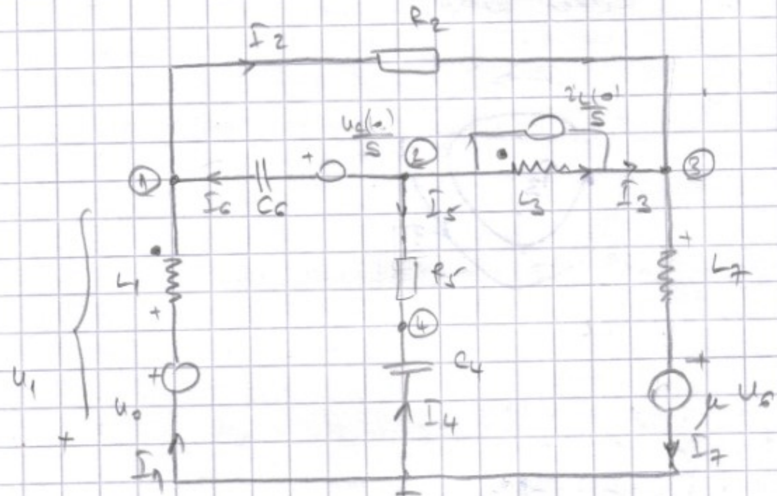
- ako struje ulaze na ista mjesta (jedna di je •, druga di je •) onda su predznaci isti
- ako struje ulaze na različita mjesta (jedna di je •, druga di nije •) predznaci su različiti



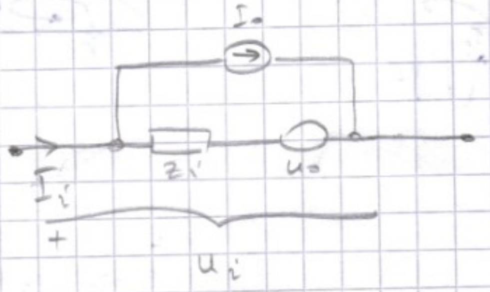
$$U_1 = sL_1 I_1 + sM_{\bullet} I_2 - sM_{*} I_3$$

$$U_2 = sL_2 I_2 + sM_{\bullet} I_1 + sM_{\Delta} I_3$$

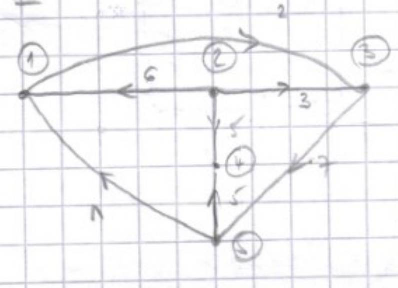
$$U_3 = sL_3 I_3 + sM_{\Delta} I_2 - sM_{*} I_1$$



GRANA



$$U_i = I_i \cdot Z_i - I_0 Z_0 + U_0$$



$N_b = 7$
$N_v = 5$

MATRICA INCIDENCIJE

$A_a \Rightarrow N_v \times N_b$

REDUCIRANA MATRICA INCIDENCIJE / matrica čvorova

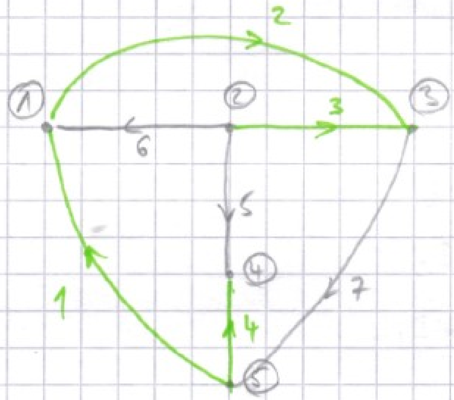
$A \Rightarrow (N_v - 1) \times N_b$

$(N_v - 1)$ → izbacuje se uzemljeni čvor

		g_1	g_2	g_3	g_4	g_5	g_6	g_7
$A =$	$\bar{c}_1$	-1	1	0	0	0	-1	0
	$\bar{c}_2$	0	0	1	0	1	1	0
	$\bar{c}_3$	0	-1	-1	0	0	0	1
	$\bar{c}_4$	0	0	0	-1	-1	0	0

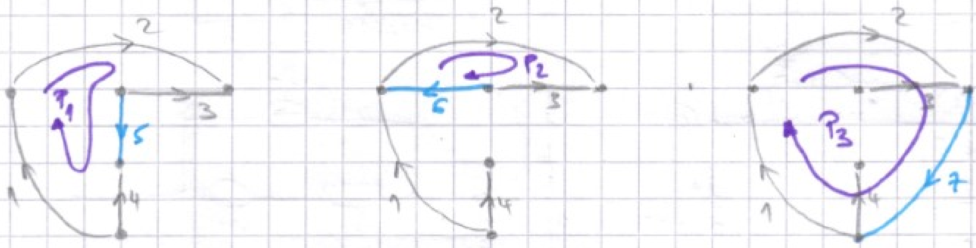
stablene grane – dio grafa da kroz njega ne možemo provući niti jednu petlju

$N_t = N_v - 1 = 4$ → prve 4 grane po numeraciji



SPOJNA MATRICA

$N_s = N_b - N_t$ / matrica petlji



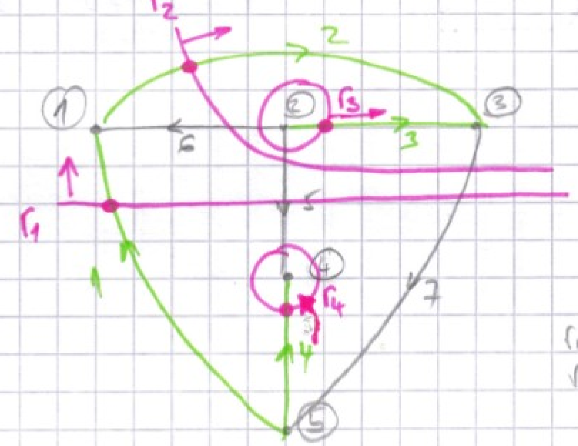
$S \Rightarrow N_s \times N_b$

		g_1	g_2	g_3	g_4	g_5	g_6	g_7
$S =$	p_1	1	1	-1	-1	1	0	0
	p_2	0	1	-1	0	0	1	0
	p_3	1	1	0	0	0	0	1

RASTAVNA MATRICA $Q = N_t \times N_b$ / matrica rezova

$$
Q = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 0 & -1 & -1 & -1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}
\begin{matrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{matrix}
$$

(columns: $g_1\ g_2\ g_3\ g_4\ g_5\ g_6\ g_7$)



smijemo prerezati samo 1 stablenu granu

r_4 - ulazi u kružnicu
r_3 - izlazi iz kružnice

MATRICE STRUJNO-NAPONSKIH ODNOSA

$$U_b = Z_b I_b + U_{0b}$$

$$I_b = Y_b U_b + I_{0b}$$

jednadžba petlji $Z_p I_r = U_{0p}$

jednadžba čvorova $Y_v U_v = I_{0v}$

jednadžba rezova $Y_r U_r = I_{0r}$

$$U_1 = sL_1 \cdot I_1 - U_0 - sM\left(I_3 - \frac{i_L(0)}{s}\right) = sL_1 I_1 - U_0 - sM I_3 + M i_L(0)$$

$$U_2 = R_2 \cdot I_2$$

$$U_3 = sL_3\left(I_3 - \frac{i_L(0)}{s}\right) - sM I_1 = sL_3 I_3 - L_3 i_L(0) - sM I_1$$

$$U_4 = I_4 \cdot \frac{1}{sC_4}$$

$$U_5 = R_5 \cdot I_5$$

$$U_6 = \frac{1}{sC_6} \cdot I_6 - \frac{u_C(0)}{s}$$

$$U_7 = sL_7 \cdot I_7 + \mu U_6 = sL_7 \cdot I_7 + \mu \frac{1}{sC_6} \cdot I_6 - \mu \frac{u_C(0)}{s}$$

$$U_b = Z_b \cdot I_b + U_{0b}$$

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \\ U_7 \end{bmatrix} = \begin{bmatrix} sL_1 & 0 & -sM & 0 & 0 & 0 & 0 \\ 0 & R & 0 & 0 & 0 & 0 & 0 \\ -sM & 0 & sL_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/sC_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & R & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/sC_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & \mu/sC_6 & sL_7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \end{bmatrix} + \begin{bmatrix} -U_0 + M i_{L_1}(0) \\ 0 \\ -L_3 i_{L_3}(0) \\ 0 \\ 0 \\ -U_C(0)/s \\ -\mu U_C(0)/s \end{bmatrix}$$

7×7 $\quad$ $N_b \times N_b$ $\qquad$ 7×1 $\quad$ $N_b \times 1$

$$Y_v U_v = I_{0v}$$

$$Y_v = A \cdot Y_b \cdot A^T$$

$$I_{0v} = -A \cdot I_{0b}$$

$$I_{0v} = A \cdot Y_b \cdot U_{0b}$$

$$Y_b = Z_b^{-1}$$

$$Y_b = \begin{bmatrix} \square & \square & \square & 0 & 0 & 0 & 0 \\ \square & \square & \square & 0 & 0 & 0 & 0 \\ \square & \square & \square & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & sC_4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/R & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & sC_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\mu/sL_7 & 1/sL_7 \end{bmatrix}$$

$$\begin{bmatrix} 1/sC_6 & 0 \\ \mu/sC_6 & sL_7 \end{bmatrix}^{-1} = \frac{1}{\frac{1}{sC_6} \cdot sL_7 - 0 \cdot \frac{\mu}{sC_6}} \begin{bmatrix} sL_7 & -\mu/sC_6 \\ -0 & 1/sC_6 \end{bmatrix}^T$$

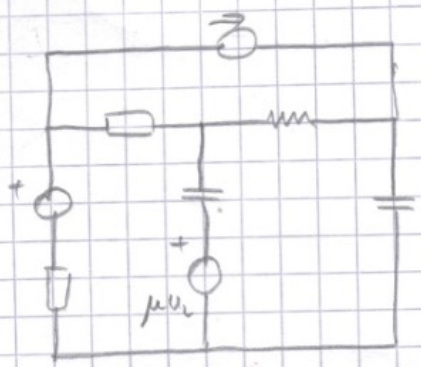
$$= \frac{C_6}{L_7} \begin{bmatrix} sL_7 & 0 \\ -\mu/sC_6 & 1/sC_6 \end{bmatrix} = \begin{bmatrix} sC_6 & 0 \\ -\mu/sL_7 & 1/sL_7 \end{bmatrix}$$

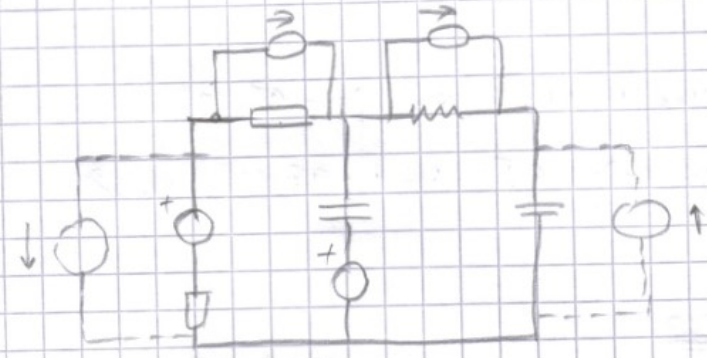
$$\left[\begin{array}{ccc|ccc} sL_1 & 0 & -sM & 1 & 0 & 0 \\ 0 & R & 0 & 0 & 1 & 0 \\ -sM & 0 & sL_3 & 0 & 0 & 1 \end{array}\right] \sim \ldots$$

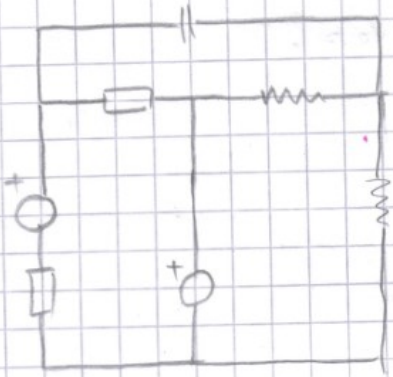
powetniji način je da rastavimo skupe:

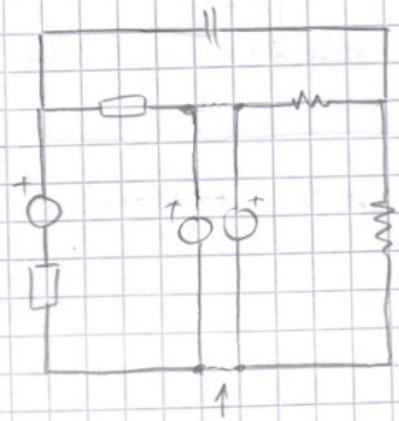
$$I_b = Y_b U_b + I_{0b}$$

POSTUPANJE IZVORA



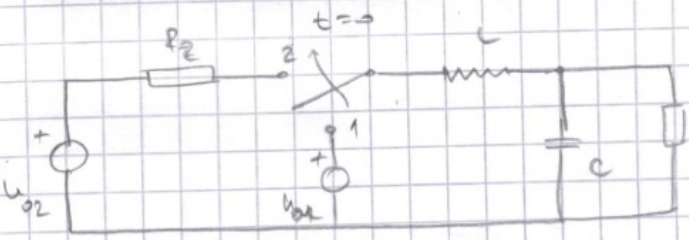






postupak radimo od ovih dijelova, bilo kojim

SKLOPKE



$u_{ul} = 10 \sin(2t)$

$R_1 = 6$

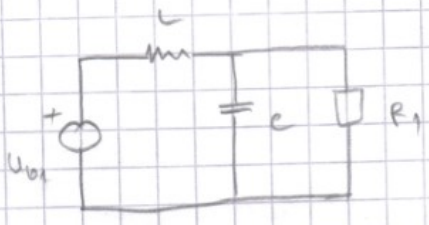
$R_2 = 5$

$u_{R1}(t)$ $C = 1/12$

$L = 3,5$

$u_{02} = s(t) \leftarrow \frac{1}{s}$

$t < 0$



$s = j\omega$

$X_L = j\omega L = j7$

$X_C = \frac{1}{j\omega C} = -j6$

$Z_{EC} = 3 - j3$

$Z_{ul} = j7 + (6 \parallel -j6) = j7 + \frac{6 \cdot (-j6)}{6 - j6} = 3 + j4$

$= 5 \angle 53,13$

$i_L(t) = 2 \sin(2t - 53,13°)$

$i_L(0) = 2 \sin(-53,13°)$

$= -1,6$

$i_L = i_{ul} = \frac{u_{ul}}{Z_{ul}} = \frac{10 \angle 0}{5 \angle 53,13°} = 2 \angle -53,13°$

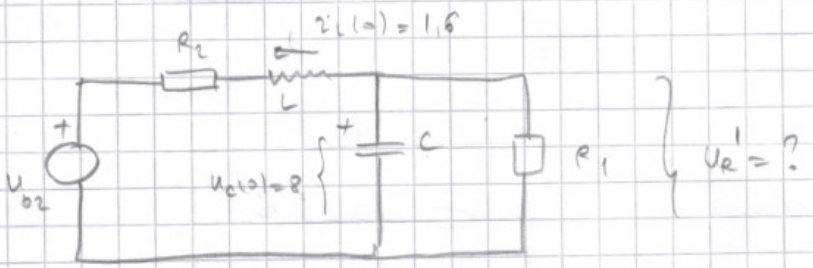
$$u_{RC} = \underline{I}_{uk} \cdot Z_{RC}$$

$$= 2\angle -53.13^\circ \cdot 3\sqrt{2}\angle -45^\circ = 6\sqrt{2}\angle -98.13^\circ$$

$$u_{RC}(t) = 6\sqrt{2}\sin(2t - 98.13^\circ)$$

$$u_{RC}(0) = 6\sqrt{2}\sin(-98.13^\circ) \approx 8$$

$t > 0$



preko Laplacea

$$u_R(t) = \begin{cases} 6\sqrt{2}\sin(2t - 98,13^\circ), & t < 0 \\ u_R'(t), & t \geq 0 \end{cases}$$