FER 2

Električni krugovi

Teorija Linija

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Sharp_Shooter

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TEORIJA LINIJA

$$\frac{\partial^2 i(x,t)}{\partial x^2} = LC \cdot \frac{\partial^2 i(x,t)}{\partial t^2} + (LG + RC) \cdot \frac{\partial i(x,t)}{\partial t} + R \cdot G \cdot i(x,t)$$

$$\frac{\partial^2 u(x,t)}{\partial x^2} = L \cdot C \cdot \frac{\partial^2 u(x,t)}{\partial t^2} + \frac{\partial u(x,t)}{\partial t} + RG \cdot u(x,t)$$

LAPLACE:

$$\frac{d^2 U(x,s)}{dx^2} = (R + sL)(G + sC) \cdot U(x,s)$$

$$\frac{d^2 I(x,s)}{dx^2} = (R + sL)(G + sC) \cdot I(x,s)$$

$$\boxed{\gamma^2 = (R + sL) \cdot (G + sC)}$$ $\Longrightarrow$ FAKTOR PRIJENOSA (PROPAGACIJE) HOMOGENE LINIJE

RJEŠENJE DIF. JEDNADŽBE:

$$U(x,s) = A \cdot e^{\lambda x}$$

$$\lambda^2 - \gamma^2 = 0$$

$$\lambda_{12} = \pm \gamma$$

$$\boxed{\begin{aligned} U(x,s) &= A_1 \cdot e^{-\gamma x} + A_2 \cdot e^{\gamma x} \\ I(x,s) &= B_1 \cdot e^{-\gamma x} + B_2 \cdot e^{\gamma x} \end{aligned}}$$

A_1, A_2, B_1, B_2 – određuju se iz rubnih uvjeta $U(0,s)$, $U(l,s)$, $I(0,s)$, $I(l,s)$

$$\boxed{Z_0 = \frac{R + sL}{\gamma} = \sqrt{\frac{R + sL}{G + sC}}}$$ $\Longrightarrow$ VALNA (KARAKTERISTIČNA) IMPEDANCIJA HOMOGENE LINIJE

$$B_1 = \frac{A_1}{Z_0}$$

$$B_2 = -\frac{A_2}{Z_0}$$

$$U(x,s) = A_1 \cdot e^{-\gamma x} + A_2 e^{\gamma x}$$

$$I(x,s) = \frac{A_1}{Z_0} e^{-\gamma x} - \frac{A_2}{Z_0} e^{\gamma x}$$

RUBNI UVJETI:

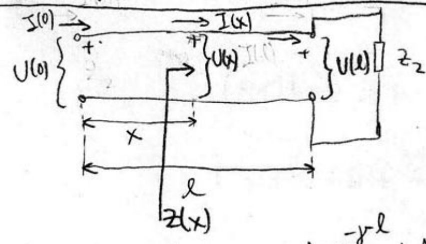
$$U(0) = A_1 + A_2$$

$$I(0) = \frac{A_1}{Z_0} - \frac{A_2}{Z_0}$$

$$A_1 = \frac{U(0) + I(0) \cdot Z_0}{2}$$

$$A_2 = \frac{U(0) - I(0) \cdot Z_0}{2}$$

HOMOGENA LINIJA ZAKLJUČENA IMPEDANCIJOM Z_2



$$U(l) = A_1 \cdot e^{-\gamma l} + A_2 e^{\gamma \cdot l}$$

$$Z_0 \cdot I(l) = A_1 \cdot e^{-\gamma l} - A_2 e^{\gamma l}$$

$$A_1 = \frac{I(l)}{2} \cdot (Z_2 + Z_0) \cdot e^{\gamma l} \qquad A_2 = \frac{I(l)}{2} \cdot (Z_2 - Z_0) e^{-\gamma l}$$

$$Z_2 = \frac{U(l)}{I(l)} = Z_0 \cdot \frac{A_1 e^{-\gamma l} + A_2 e^{\gamma l}}{A_1 e^{-\gamma l} - A_2 e^{\gamma l}}$$

$$\boxed{\Gamma_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0}} \Rightarrow \text{KOEFICIJENT REFLEKSIJE NA IZLAZU}$$

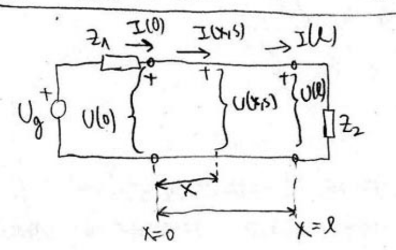
$$\frac{A_2}{A_1} = \Gamma_2 \cdot e^{-2\gamma l}$$

$$U(x,s) = U(0,s) \cdot \frac{e^{-\gamma \cdot (x-l)} + \Gamma_2 e^{\gamma(x-l)}}{e^{\gamma l} + \Gamma_2 e^{-\gamma l}}$$

$$I(x,s) = \frac{U(0,s)}{Z_0} \cdot \frac{e^{-\gamma(x-l)} - \Gamma_2 e^{\gamma(x-l)}}{e^{\gamma l} + \Gamma_2 \cdot e^{-\gamma l}}$$

$$Z(x,s) = \frac{U(x,s)}{I(x,s)} = Z_0 \cdot \frac{e^{-\gamma(x-l)} + \Gamma_2 e^{\gamma(x-l)}}{e^{-\gamma(x-l)} - \Gamma_2 e^{\gamma(x-l)}}$$

LINIJA ZAKLJUČENA NA OBA KRAJA



$$U(0) = U_g - Z_1 \cdot I(0) \qquad U(0) = A_1 + A_2$$

$$U(l) = I(l) \cdot Z_2 \qquad I(0) = \frac{1}{Z_0} \cdot (A_1 - A_2)$$

$$\boxed{\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}} \Rightarrow \text{KOEFICIJENT REFLEKSIJE NA ULAZU}$$

$$A_1 - \Gamma_1 A_2 = U_g \cdot \frac{Z_0}{Z_0 + Z_1}$$

$$A_2 = A_1 \cdot \Gamma_2 \cdot e^{-2\gamma l}$$

$$A_1 = U_g \cdot \frac{Z_0}{Z_0 + Z_1} \cdot \frac{1}{1 - \Gamma_1 \Gamma_2 e^{-2\gamma l}}$$

$$A_2 = U_g \cdot \frac{Z_0}{Z_0 + Z_1} \cdot \frac{\Gamma_2 e^{-2\gamma l}}{1 - \Gamma_1 \Gamma_2 e^{-2\gamma l}}$$

PRIJENOSNE JEDNADŽBE LINIJA

$$U(0) = U(x) \cdot ch(\gamma x) + Z_0 \cdot I(x) \cdot sh(\gamma x)$$

$$I(0) = \frac{U(x)}{Z_0} \cdot sh(\gamma x) + I(x)\, ch(\gamma x)$$

ZA $x = l$

$$U(0) = A \cdot U(l) + B \cdot I(l)$$

$$I(0) = C \cdot U(l) + D \cdot I(l)$$

$$A = ch(\gamma l) = D$$

$$B = Z_0 \cdot sh(\gamma l)$$

$$C = \frac{sh(\gamma l)}{Z_0}$$

-ako je $A = D$, četveropol je simetričan

$$U(x) = U(0)\, ch\gamma x - Z_0 \cdot I(0) \cdot sh\gamma x$$

$$I(x) = \frac{-U(0)}{Z_0} sh\gamma x + I(0)\, ch\gamma x$$

VALNI RASPORED NAPONA I STRUJA DUŽ LINIJE

$$u(x,t) = A_1 e^{-\gamma x} + A_2 e^{\gamma x}$$
$$i(x,t) = B_1 e^{-\gamma x} + B_2 e^{\gamma x}$$

$$u(x,t) = |U| \cdot e^{j(\omega t + \phi)}$$
$$i(x,t) = |I| \cdot e^{j(\omega t + \psi)}$$
$$A_1 = |A_1| \cdot e^{j(\omega t + \phi_1)} \qquad A_2 = |A_2| e^{j(\omega t + \phi_2)}$$

Z_0 i γ $\Rightarrow$ FUNKCIJE OD $j\omega$

$$\gamma = \alpha + j\beta$$

$\alpha \Rightarrow$ KARAKTERISTIČNI ILI ZRCALNI FAKTOR GUŠENJA

$\beta \Rightarrow$ KARAKTERISTIČNI ILI ZRCALNI FAKTOR FAZE

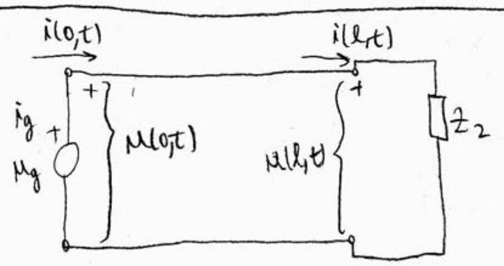
$$\boxed{\begin{aligned} \alpha &= \sqrt{\frac{1}{2}\left(R \cdot G - \omega^2 LC + \sqrt{(RG + \omega^2 LC)^2 + \omega^2 (RC + LG)^2}\right)} \\ \beta &= \sqrt{\frac{1}{2}\left(\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (RC + LG)^2}\right)} \end{aligned}}$$

$$f(\underbrace{\omega t - \beta x}) = f(Y) = z$$
$$\boxed{Y = \omega t - \beta x}$$

$\boxed{v = \dfrac{\omega}{\beta}} \Rightarrow$ BRZINA ŠIRENJA VALA

$$\boxed{\begin{aligned} &\text{POLAZNI VAL: } f(\omega t - \beta x) \\ &\text{REFLEKTIRANI VAL: } f(\omega t + \beta x) \end{aligned}}$$

POLAZNI I REFLEKTIRANI VAL KOD SINUSNOG SIGNALA



$$A_1 = \frac{U_g + I_g \cdot Z_0}{2} \cdot e^{j\omega t}$$

$$A_2 = \frac{U_g - I_g \cdot Z_0}{2} \cdot e^{j\omega t}$$

$$Z_0 = |Z_0| \cdot e^{j\xi_0}$$

POLAZNI VAL:

$$u_p(x,t) = |A_1| e^{-\alpha x} \cdot \cos(\omega t - \beta x + \phi_1)$$

$$i_p(x,t) = \left|\frac{A_1}{Z_0}\right| e^{-\alpha x} \cdot \cos(\omega t - \beta x + \phi_1 - \xi_0)$$

REFLEKTIRANI VAL

$$u_R(x,t) = |A_2| \cdot e^{\alpha x} \cdot \cos(\omega t + \beta x + \phi_2)$$

$$i_R(x,t) = \left|\frac{A_2}{Z_0}\right| \cdot e^{\alpha x} \cdot \cos(\omega t + \beta x + \phi_2 - \xi_0)$$

$$\text{faza}\left[\frac{u_R(l,t)}{u_P(l,t)}\right] = -\Theta_2$$

FAKTOR REFLEKSIJE

$$|A_2| = |A_1| \cdot |\Gamma_2| \cdot e^{-2\alpha \cdot l}$$

$$\phi_2 = \phi_1 - 2\beta l + \Theta_2$$

POSEBNI SLUČAJEVI HOMOGENE LINIJE

1. LINIJA BEZ GUBITAKA

$$R = 0$$
$$G = 0$$

$$Z_0 = \sqrt{\frac{L}{C}} = \text{konst.} = R_0$$

$$\gamma = s\sqrt{LC}$$

- za $s = j\omega$ (sinusna pobuda)

$$\gamma = j\omega\sqrt{LC} = j\beta$$
$$\beta = \omega\sqrt{LC}$$
$$\alpha = 0$$

- postoje 2 slučaja:

$\boxed{1.}$ $I_2 = 0 \rightarrow \Gamma_2 = 1, \; Z_2 = \infty$

$$\left.\begin{aligned} u(y,t) &= |U(l)| \cos(\beta y) \cos(\omega t + \phi) \\ i(y,t) &= -\frac{|U(l)|}{Z_0} \sin(\beta y) \sin(\omega t + \phi) \end{aligned}\right\} \text{STOJNI VAL}$$

$\boxed{2.}$ $U_1 = 0 \rightarrow \Gamma_2 = -1, \; Z_2 = 0$

$$u(y,t) = -|I(l)|\, Z_0 \sin(\beta y) \sin(\omega t + \phi)$$
$$i(y,t) = |I(l)| \cos(\beta y) \cos(\omega t + \psi)$$

2. LINIJA BEZ DISTORZIJE

$$\frac{R}{L} = \frac{G}{C}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\gamma = \sqrt{RG} + s\sqrt{LC}$$

- za $s = j\omega$

$$\gamma = \sqrt{RG} + j\omega\sqrt{LC} \Rightarrow \alpha = \sqrt{RG}$$
$$\beta = \omega\sqrt{LC}$$

3. RC - LINIJA

$$G = 0$$
$$L = 0$$

$$Z_0 = \sqrt{\frac{R}{sC}}$$

- za $s = j\omega$

$$Z_0 = \sqrt{\frac{R}{j\omega C}} = \sqrt{\frac{R}{\omega C}} \cdot e^{-j45^\circ}$$

$$\gamma = \sqrt{R \cdot j\omega C} = \sqrt{\frac{\omega RC}{2}} + j\sqrt{\frac{\omega RC}{2}}$$

$$\alpha = \sqrt{\frac{\omega RC}{2}}$$

$$\beta = \sqrt{\frac{\omega RC}{2}}$$

4. LINIJA S MALIM GUBITCIMA

$$\omega L \gg R$$
$$\omega C \gg G$$

$$Z_0 = \sqrt{\frac{L}{C}} \cdot e^{-j\left(\frac{R}{2\omega L} - \frac{G}{2\omega C}\right)}$$

$$\gamma = j\omega\sqrt{LC} \cdot \sqrt{\left(1 - j\frac{R}{\omega L}\right)\left(1 - j\frac{G}{\omega C}\right)}$$

$$\gamma \approx \left(\frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}\right) + j\omega\sqrt{LC}$$

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}$$

$$\beta = \omega\sqrt{LC}$$

ULAZNA IMPEDANCIJA LINIJE

$$Z_{ul} = \frac{U(0)}{I(0)}$$

$$= \frac{U(l)\,ch\gamma l + I(l) Z_0\, sh\gamma l}{\frac{U(l)}{Z_0} sh\gamma l + I(l) ch\gamma l}$$

$$Z_{ul} = Z_0 \cdot \frac{Z_2\, ch\gamma l + Z_0\, sh\gamma l}{Z_2\, sh\gamma l + Z_0 ch\gamma l} = Z_0 \cdot \frac{e^{2\gamma l} + T_2'}{e^{2\gamma l} - T_2'}$$

- za: $Z_2 = 0 \Rightarrow Z_{ul} = Z_0\, th\gamma l = Z_k$

$Z_2 = \infty \Rightarrow Z_{ul} = Z_0\, cth\gamma l = Z_p$

$Z_2 = Z_0 \Rightarrow Z_{ul} = Z_0 \Rightarrow$ PRILAGOĐENJE

$$Z_0 = \sqrt{Z_p \cdot Z_k}$$

BESKONAČNO DUGA LINIJA

① LINIJA BEZ GUBITAKA

$$U(x,s) = U(s) \cdot e^{-\frac{sx}{v}}$$

$$u(x,t) = u\left(t - \frac{x}{v}\right)$$

$$i(x,t) = \frac{u(t - \frac{x}{v})}{Z_0}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

- signal se po obliku ne mijenja, ali kasni vremenski

② LINIJA BEZ DISTORZIJE

$$U(x,s) = U(s) \cdot e^{-(\alpha + \frac{s}{v})x}$$

$$I(x,s) = \frac{U(s)}{Z_0} \cdot e^{-\alpha x} \cdot e^{-\frac{sx}{v}}$$

$$u(x,t) = e^{-\alpha x} \cdot u\left(t - \frac{x}{v}\right)$$

$$i(x,t) = e^{-\alpha x} \cdot i\left(t - \frac{x}{v}\right)$$

- signal je prigušen duž linije, i kasni vremenski

LINIJE KONAČNE DULJINE

- ako je linija zaključena sa $Z_2 = Z_0$, ona se ponaša kao ∞ - linija