Električni krugovi

Električne prijenosne linije

Valni raspored napona i struja duž linije

Napon i struja na liniji su

$$U(x,s) = A_1 e^{-\gamma x} + A_2 e^{\gamma x}$$

$$I(x,s) = \frac{A_1}{Z_0} e^{-\gamma x} - \frac{A_2}{Z_0} e^{\gamma x}$$

Stacionarna sinusna pobuda

$$u(x,t) = |U| e^{j(\omega t + \phi)}$$

$$i(x,t) = |I| e^{j(\omega t + \psi)}$$

$$u(x,t) = A_1 e^{-\gamma x} + A_2 e^{\gamma x}$$

$$i(x,t) = B_1 e^{-\gamma x} + B_2 e^{\gamma x}$$

Valni raspored napona i struja duž linije

tada su i $A_1 = |A_1| e^{j(\omega t + \phi_1)}$ $A_2 = |A_2| e^{j(\omega t + \phi_2)}$

Z_0 i $\gamma \rightarrow$ funkcije od $j\omega$

$$\underbrace{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \qquad \gamma = \sqrt{(R + j\omega L)(G + j\omega C)}}_{}$$

KOMPLEKSNI

$\gamma = \alpha + j\beta$

$$\alpha = \sqrt{\frac{1}{2}\left(RG - \omega^2 LC + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (RC + LG)^2}\right)}$$

$$\beta = \sqrt{\frac{1}{2}\left(\omega^2 LC - RG + \sqrt{(RG - \omega^2 LC)^2 + \omega^2 (RC + LG)^2}\right)}$$

$\alpha \rightarrow$ Karakteristični ili zrcalni faktor **gušenja**

$\beta \rightarrow$ Karakteristični ili zrcalni faktor **faze**

Valni raspored napona i struja duž linije

$$u(x,t) = A_1 e^{-\gamma x} + A_2 e^{\gamma x} = A_1 e^{-(\alpha + j\beta)x} + A_2 e^{(\alpha + j\beta)x}$$

$$A_1 = |A_1| e^{j(\omega t + \phi_1)} \qquad A_2 = |A_2| e^{j(\omega t + \phi_2)}$$

$$u(x,t) = |A_1| e^{-(\alpha x + j\beta x)} \cdot e^{j(\omega t + \phi_1)} + |A_2| e^{\alpha x + j\beta x} \cdot e^{j(\omega t + \phi_2)}$$

$$i(x,t) = \frac{|A_1|}{Z_0} e^{-(\alpha x + j\beta x)} \cdot e^{j(\omega t + \phi_1)} - \frac{|A_2|}{Z_0} e^{\alpha x + j\beta x} \cdot e^{j(\omega t + \phi_2)}$$

$$u(x,t) = |A_1| e^{-\alpha x} \cdot e^{j(\omega t - \beta x + \phi_1)} + |A_2| e^{\alpha x} \cdot e^{j(\omega t + \beta x + \phi_2)}$$

$$f\big(\underbrace{\omega t - \beta x}_{y}\big) = f(y) = z$$

Valni raspored napona i struja duž linije

$$f(\omega t_1 - \beta x_1) = z_1 = f(y_1) \rightarrow$$ stanje na liniji na mjestu x_1 u času t_1

U času $t_2 = t_1 + \Delta t$ → isto stanje z_1 na mjestu $x_2 = x_1 + \Delta x$.

$$\omega t_2 - \beta x_2 = y_1$$

$$\cancel{\omega t_1} + \Delta t \cdot \omega - \beta(\cancel{x_1} + \Delta x) = \cancel{\omega t_1} - \cancel{\beta x_1}$$

$$\Delta t \cdot \omega - \beta \Delta x = 0$$

$$\frac{\Delta x}{\Delta t} = \frac{\omega}{\beta} = v$$

Valni raspored napona i struja duž linije

$$\frac{dx}{dt} = \frac{\omega}{\beta} \qquad \boxed{\frac{\omega}{\beta} = v}$$

To stanje putuje po liniji konačnom brzinom v

$v \rightarrow$ brzina širenja vala

$f(\omega t - \beta x) \rightarrow$ val koji se kreće od početka linije prema kraju brzinom $v = \omega / \beta$

Analogno tome izraz $A_2 \cdot e^{\alpha x} \cdot e^{j(\omega t + \beta x + \phi_2)}$

$\downarrow$

$g(\omega t + \beta x) \rightarrow$ **Val koji se kreće u negativnom smjeru od kraja linije prema početku brzinom** $v = \omega / \beta$

prema tome

$$u(x,t) = u_p(x,t) + u_r(x,t)$$

$$i(x,t) = i_p(x,t) - i_r(x,t)$$

polazni val reflektirani val

Periodične funkcije

Ako je $f(\omega t - \beta x)$ periodična tada je

$$f(y) = f(y + P)$$ P → period

$$f(y_2) = f(y_1) \text{ ako je } y_2 = y_1 + nP$$

Periodične funkcije

U trenutku t_1, je u (t_1, x_1) isto stanje kao u (t_1, x_2) ako je:

$$\omega t_1 - \beta x_2 = \omega t_1 - \beta x_1 + nP$$

$$\beta x_1 = \beta x_2 + nP \Rightarrow x_1 - x_2 = \frac{n \cdot P}{\beta}$$

Temeljni period → n=1

$$x_1 - x_2 = \frac{P}{\beta} = \lambda \rightarrow$$ VALNA DULJINA

Ako promatramo na istom mjestu x_1

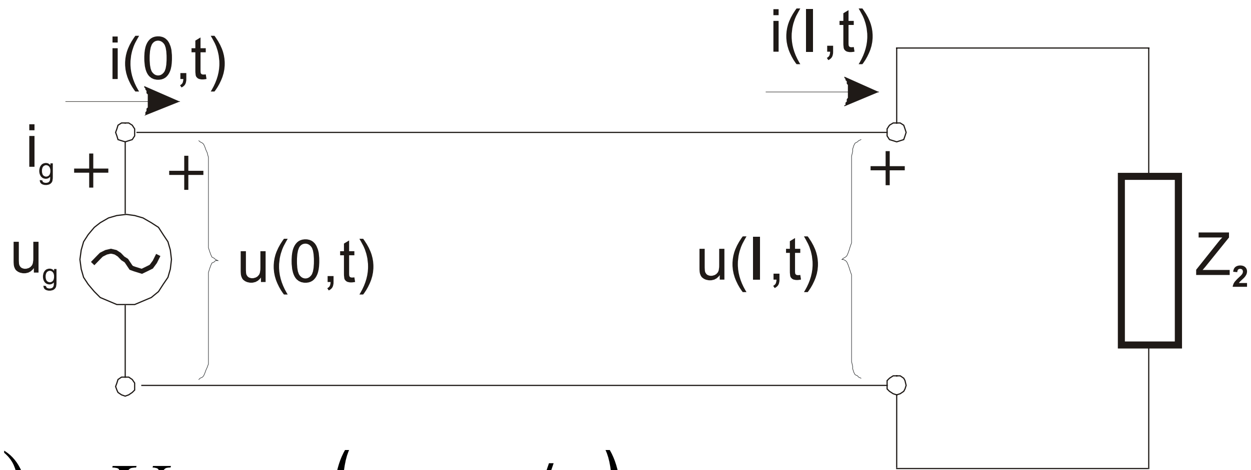
$$\omega t_2 - \cancel{\beta} x_1 = \omega t_1 - \cancel{\beta} x_1 + nP$$

$$\omega(t_2 - t_1) = n \cdot P$$

$$t_2 - t_1 = \frac{n \cdot P}{\omega} \Rightarrow \text{temeljni period n=1}$$

$$t_2 - t_1 = \frac{P}{\omega} = T \Rightarrow \text{trajanje jednog titraja}$$

Polazni i reflektirani val kod sinusnog signala



$$u_g(t) = U_g \cos(\omega t + \phi_g)$$

fazorski $\mathbf{U}_g = U_g \cdot e^{j\phi_g}$ pa je

$$u_g(t) = \mathrm{Re}\left[\mathbf{U}_g e^{j\omega t}\right]$$

$$i_g(t) = \mathrm{Re}\left[\frac{\mathbf{U}_g}{Z_{ul}} e^{j\omega t}\right] = I_g \cos(\omega t + \psi_g)$$

Polazni i reflektirani val kod sinusnog signala

$$u_g(t)=u(0,t) \quad i_g(t)=i(0,t) \quad u_2(t)=u(l,t) \quad i_2(t)=i(l,t)$$

<u>rubni uvjeti!!!</u>

$$A_1=\frac{\mathbf{U}_g+\mathbf{I}_g Z_0}{2}e^{j\omega t}=|A_1|e^{j(\omega t+\phi_1)}$$

$$A_2=\frac{\mathbf{U}_g-\mathbf{I}_g Z_0}{2}e^{j\omega t}=|A_2|e^{j(\omega t+\phi_2)}$$

$$Z_0=|Z_0|e^{j\zeta_0}$$

Polazni i reflektirani val kod sinusnog signala

$$u(x,t) = \mathrm{Re}\left\lfloor |A_1| e^{j\phi_1} e^{j\omega t} e^{-(\alpha x - j\beta x)} \right\rfloor + \mathrm{Re}\left\lfloor |A_2| e^{j\phi_2} e^{j\omega t} e^{(\alpha + j\beta)x} \right\rfloor$$

$$i(x,t) = \mathrm{Re}\left[\left| \frac{A_1}{Z_0} \right| e^{j(\phi_1 - \zeta_0)} e^{j\omega t} e^{-(\alpha x - j\beta x)} \right] -$$

$$- \mathrm{Re}\left[\left| \frac{A_2}{Z_0} \right| e^{j(\phi_2 - \zeta_0)} e^{-j\omega t} e^{-\alpha x + j\beta x} \right]$$

Polazni i reflektirani val kod sinusnog signala

$$u(x,t) = |A_1| e^{-\alpha x} \cos(\omega t - \beta x + \phi_1) + |A_2| e^{\alpha x} \cos(\omega t + \beta x + \phi_2)$$

$$i(x,t) = \left|\frac{A_1}{Z_0}\right| e^{-\alpha x} \cos(\omega t - \beta x + \phi_1 - \zeta_0) +$$

$$+ \left|\frac{A_2}{Z_0}\right| e^{\alpha x} \cos(\omega t + \beta x + \phi_2 - \zeta_0)$$

Polazni i reflektirani val kod sinusnog signala

$$u_p(x,t) = |A_1| e^{-\alpha x} \cos(\omega t - \beta x + \phi_1)$$

$$u_r(x,t) = |A_2| e^{\alpha x} \cos(\omega t + \beta x + \phi_2)$$

$$i_p(x,t) = \left|\frac{A_1}{Z_0}\right| e^{-\alpha x} \cos(\omega t - \beta x + \phi_1 - \zeta_0)$$

$$i_r(x,t) = \left|\frac{A_2}{Z_0}\right| e^{\alpha x} \cos(\omega t + \beta x + \phi_2 - \zeta_0)$$

Faktor refleksije

$$u_p(x,t) = |A_1| e^{-\alpha x} \cos(\omega t - \beta x + \phi_1)$$

$$A_1 = |A_1| e^{j(\omega t + \phi_1)} \qquad A_2 = \Gamma_2 \cdot A_1 \cdot e^{-2\gamma l} = \Gamma_2 \cdot A_1 \cdot e^{-2(\alpha + j\beta)l}$$

$$\Gamma_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0} = |\Gamma_2| e^{j\Theta_2} = \left|\frac{Z_2 - Z_0}{Z_2 + Z_0}\right| e^{j\Theta_2}$$

$$A_2 = |A_2| e^{j(\omega t + \phi_2)} \qquad = |A_1| \cdot |\Gamma_2| e^{-2\alpha l} e^{j(\omega t - 2\beta l + \phi_1 + \Theta_2)}$$

$$|A_2| = |A_1||\Gamma_2| e^{-2\alpha l} \qquad \phi_2 = \phi_1 + \Theta_2 - 2\beta l$$

Polazni i reflektirani val kod sinusnog signala

$$u_p(x,t) = |A_1| e^{-\alpha x} \cos(\omega t - \beta x + \phi_1)$$

$$u_r(x,t) = |A_2| e^{\alpha x} \cos(\omega t + \beta x + \phi_2)$$

$$= |A_1| |\Gamma_2| e^{-2\alpha l} e^{\alpha x} \cos(\omega t + \beta x + \phi_1 + \Theta_2 - 2\beta l)$$

Amplituda na mjestu $x=l$

ampl

$$\left.\begin{aligned} u_r(l,t) &= |A_1| |\Gamma_2| e^{\alpha l} \\ u_p(l,t) &= |A_1| e^{-\alpha l} \end{aligned}\right\} \quad \frac{ampl\ u_r(l,t)}{ampl\ u_p(l,t)} = |\Gamma_2|$$

$$faza\left[\frac{u_r(l,t)}{u_p(l,t)}\right] = -\beta l + \phi_1 - \beta l - \phi_1 + \Theta_2 + 2\beta l = -\Theta_2$$

Posebni slučajevi homogene linije

1. LINIJA BEZ GUBITAKA

$$R = 0 \quad G = 0$$

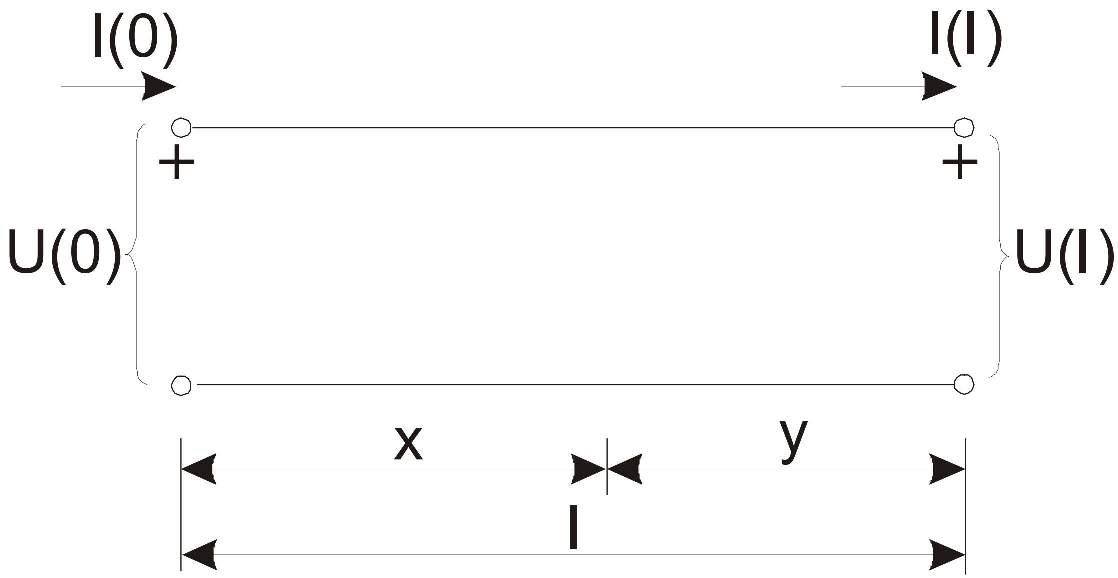
$$Z_0 = \sqrt{\frac{R + sL}{G + sC}} = \sqrt{\frac{L}{C}} = konst = R_0$$

$$\gamma = \sqrt{(R + sL)(G + sC)} = s\sqrt{LC}$$

Za $s = j\omega \rightarrow$ sinusna pobuda

$$\gamma = j\omega\sqrt{LC} = j\beta \qquad \alpha = 0$$

Posebni slučajevi homogene linije



$$y = l - x$$

$$U(y) = U(l)ch\gamma y + I(l)Z_0 sh\gamma y$$

$$I(y) = \frac{U(l)}{Z_0} sh\gamma y + I(l)ch\gamma y$$

Posebni slučajevi homogene linije

$$\gamma = j\omega\sqrt{LC} = j\beta$$

$$U(y) = U(l)\cos\beta y + j\,I(l)Z_0\sin\beta y$$

$$I(y) = j\frac{U(l)}{Z_0}\sin\beta y + I(l)\cos\beta y$$

$$U(l) = |U(l)|e^{j\phi}e^{j\omega t} \qquad I(l) = |I(l)|e^{j\psi}e^{j\omega t}$$

$$u(y,t) = \mathrm{Re}[U(y)] = \mathrm{Re}\left[|U(l)|e^{j(\omega t+\phi)}\cos\beta y + j|I(l)|Z_0e^{j(\omega t+\psi)}\sin\beta y\right]$$

$$= \left[|U(l)|\cos(\omega t+\phi)\cos\beta y - |I(l)|Z_0\sin\beta y\sin(\omega t+\psi)\right]$$

Posebni slučajevi homogene linije

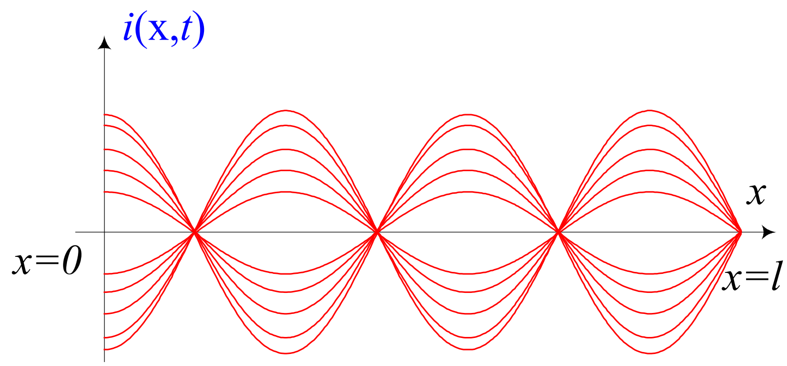
$$i(y,t)=\mathrm{Re}[I(y)]=\mathrm{Re}\left[j\frac{|U(l)|}{Z_0}e^{j(\omega t+\phi)}\sin\beta y+|I(l)|e^{j(\omega t+\psi)}\cos\beta y\right]=$$

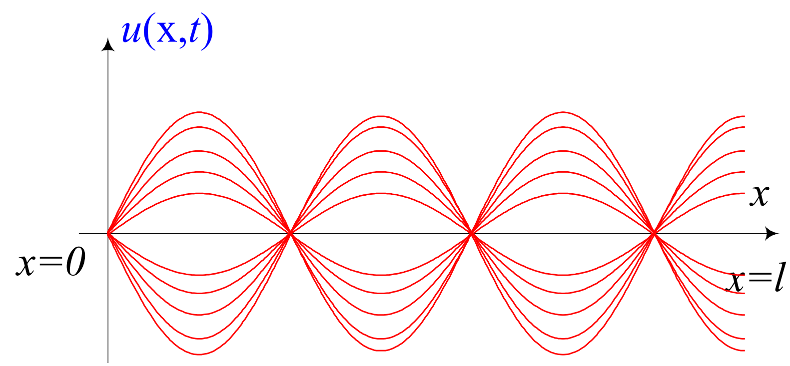
$$=-\frac{|U(l)|}{Z_0}\sin\beta y\,\sin(\omega t+\phi)+|I(l)|\cos\beta y\,\cos(\omega t+\psi)$$

<u>postoje 2 slučaja</u>:

1. $I_2 = 0 \rightarrow \Gamma_2 = 1 \qquad Z_2 = \infty$

$$\left.\begin{aligned} u(y,t) &= |U(l)| \cos\beta y \, \cos(\omega t + \phi) \\ i(y,t) &= -\frac{|U(l)|}{Z_0} \sin\beta y \, \sin(\omega t + \phi) \end{aligned}\right\} \text{stojni val}$$



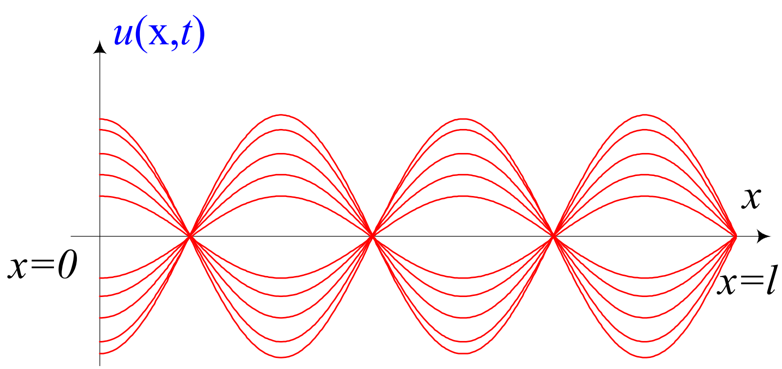


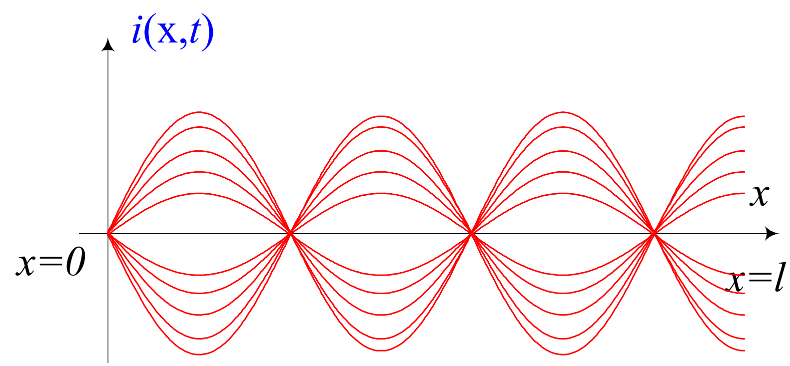
Posebni slučajevi homogene linije

2. $U_l = 0 \to Z_2 = 0 \qquad \Gamma_2 = -1$

$$u(y,t) = -|I(l)| Z_0 \sin\beta y \, \sin(\omega t + \phi)$$

$$i(y,t) = |I(l)| \cos\beta y \cos(\omega t + \psi)$$





2. LINIJA BEZ DISTORZIJE

$$\frac{R}{L}=\frac{G}{C}\Rightarrow\frac{R}{G}=\frac{L}{C}\Rightarrow RC=GL$$

$$Z_0=\sqrt{\frac{R+sL}{G+sC}}=\sqrt{\frac{\frac{R}{L}+s}{\frac{G}{C}+s}\cdot\frac{L}{C}}=\sqrt{\frac{L}{C}}$$

$$\gamma=\sqrt{(R+sL)(G+sC)}=\sqrt{LC\left(\frac{R}{L}+s\right)\left(\frac{G}{C}+s\right)}=\sqrt{LC}\left(\frac{R}{L}+s\right)=$$

$$=\sqrt{RG}+s\sqrt{LC}$$

Za slučaj sinusne pobude

$$s = j\omega \Rightarrow \gamma = \sqrt{RC} + j\omega\sqrt{LC}$$

$$\alpha = R\sqrt{\frac{C}{L}} = \sqrt{RG} \quad \beta = \omega\sqrt{LC}$$

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

3. RC-LINIJA

$$G = 0 \quad L = 0$$

$$Z_0 = \sqrt{\frac{R + sL}{G + sC}} = \sqrt{\frac{R}{sC}}$$

$$Z_0 = \sqrt{\frac{R}{j\omega C}} = \sqrt{\frac{R}{\omega C}} \cdot e^{-j45^\circ}$$

$$\gamma = \sqrt{R \cdot j\omega C} = \sqrt{\omega RC} e^{j45^\circ} = \sqrt{\frac{\omega RC}{2}} + j\sqrt{\frac{\omega RC}{2}}$$

$$\alpha = \beta = \sqrt{\frac{\omega RC}{2}}$$

4. LINIJA S MALIM GUBICIMA

$$\omega L >> R \quad \omega C >> G$$

$$Z_0 = \sqrt{\frac{L}{C}} \cdot e^{-j\left(\frac{R}{2\omega L} - \frac{G}{2\omega C}\right)}$$

$$\gamma = j\omega\sqrt{LC}\sqrt{\left(1 - j\frac{R}{\omega L}\right)\left(1 - j\frac{G}{\omega C}\right)} \cong$$

$$\cong j\omega\sqrt{LC}\left(1 - j\frac{R}{2\omega L}\right)\left(1 - j\frac{G}{2\omega C}\right)$$

$$\gamma \cong \left(\frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}\right) + j\omega\sqrt{LC}$$

Linija s malim gubicima

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}$$

$$\beta = \omega\sqrt{LC}$$

Ulazna impedancija linije

$$Z_{ul} = \frac{U(0)}{I(0)} \qquad Z_2 = \frac{U(l)}{I(l)}$$

$$Z_{ul} = \frac{U(l)\,\mathrm{ch}\,\gamma l + I(l) Z_0\,\mathrm{sh}\,\gamma l}{\frac{U(l)}{Z_0}\,\mathrm{sh}\,\gamma l + I(l)\,\mathrm{ch}\,\gamma l} = \frac{Z_2\,\mathrm{ch}\,\gamma l + Z_0\,\mathrm{sh}\,\gamma l}{\frac{Z_2}{Z_0}\,\mathrm{sh}\,\gamma l + \mathrm{ch}\,\gamma l}$$

$$Z_{ul} = Z_0 \frac{Z_2\,\mathrm{ch}\,\gamma l + Z_0\,\mathrm{sh}\,\gamma l}{Z_2\,\mathrm{sh}\,\gamma l + Z_0\,\mathrm{ch}\,\gamma l}$$

Ulazna impedancija linije

$$za\ \ Z_2 = 0 \Rightarrow Z_{ul} = Z_0 \operatorname{th} \gamma l = \frac{1}{y_{11}} = Z_k$$

$$za\ \ Z_2 = \infty \Rightarrow Z_{ul} = Z_0 \operatorname{cth} \gamma l = Z_p = z_{11}$$

$$Z_0 = \sqrt{Z_p \cdot Z_k} \qquad\qquad Z_{ul} = Z_0 \frac{e^{2\gamma l} + \Gamma_2}{e^{2\gamma l} - \Gamma_2}$$

$$za\ \ Z_2 = Z_0 \Rightarrow Z_{ul} = Z_0 \Rightarrow$$ prilagođenje

Beskonačno duga linija

$$Za\ \ l \to \infty\ \ nema\ \ povratnog\ vala$$

Samo polazni val

$$U(0) = A_1 \quad I(0) = \frac{A_1}{Z_0}$$

$$U(x) = U(0) \cdot e^{-\gamma x}$$

$$I(x) = \frac{U(0)}{Z_0} \cdot e^{-\gamma x}$$

$$Z_{ul} = \frac{U(0)}{I(0)} = Z_0$$

Linija zaključena sa Z_0 → kao beskonačno duga linija

$$Z_2 = Z_0 \Rightarrow \Gamma_2 = 0$$ nema refleksije

$$A_2 = \Gamma_2 \cdot A_1 \cdot e^{-2\gamma l} \Rightarrow 0$$

$$U(x) = A_1 e^{-\gamma x} = U(0) e^{-\gamma x}$$

$$I(x) = \frac{U(0)}{Z_0} e^{-\gamma x}$$

1. Linija bez gubitaka

Za liniju bez gubitaka vrijedi R=G=0

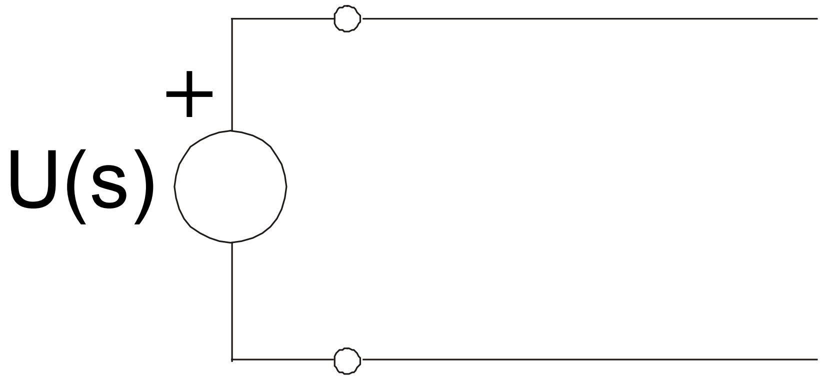
Tada je

$$\gamma = s \cdot \sqrt{LC} = \frac{s}{v}$$

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$U(x,s) = A_1 e^{-\frac{sx}{v}}$$

$$I(x,s) = \frac{A_1}{Z_0} e^{-\frac{sx}{v}}$$



Rubni uvjeti:

$$U(0,s) = U(s) = A_1$$

$$U(x,s) = U(s) \cdot e^{-\frac{sx}{v}}$$

$$\boxed{u(x,t) = u\left(t - \frac{x}{v}\right)}$$

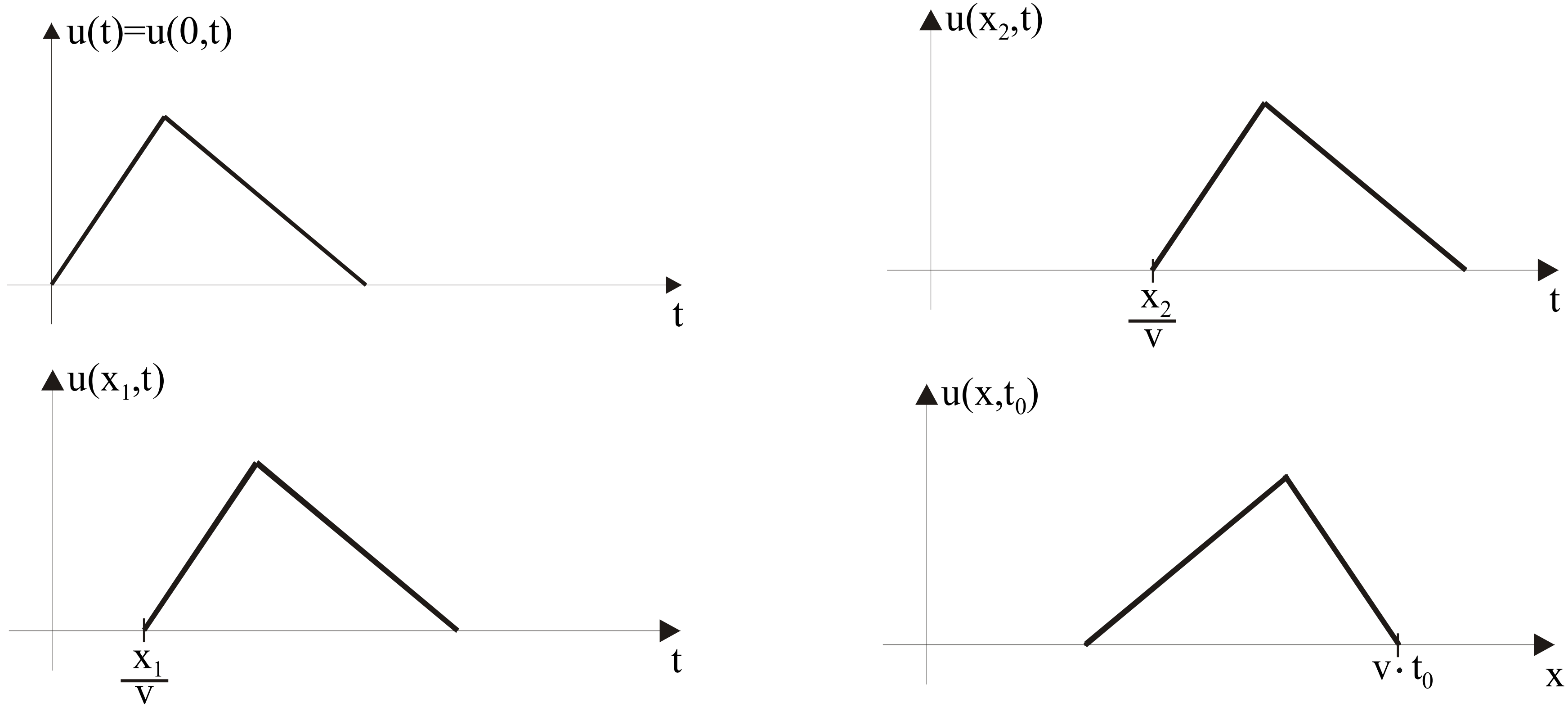
Signal se po obliku ne mijenja, ali kasni vremenski

$$\boxed{i(x,t) = \frac{u\left(t - \frac{x}{v}\right)}{Z_0}}$$

Jer je Z_0 konstanta , tj. $Z_0 = \sqrt{\dfrac{L}{C}}$

$$i(x,t)=\sqrt{\frac{C}{L}}\,u\left(t-\frac{x}{v}\right)$$

Primjer:



2. Linija bez distorzije

$$\gamma = \sqrt{RG} + s\sqrt{LC} = \alpha + \frac{s}{v}$$

$$Z_0 = \sqrt{\frac{R + sL}{G + sC}} = \sqrt{\frac{L}{C}}$$

$\alpha \neq 0$ $\longrightarrow$ postoji gušenje

Beskonačno duga linija

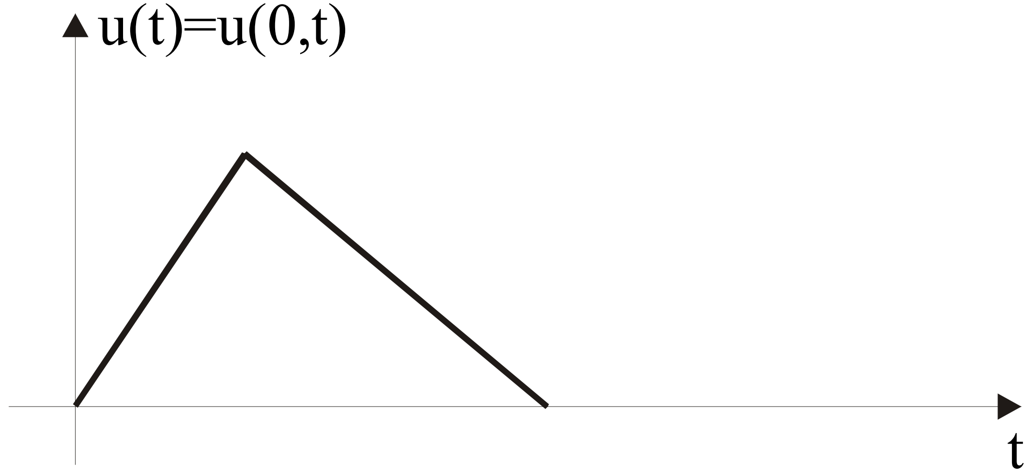
$$U(x,s)=U(s)\cdot e^{-\left(\alpha+\frac{s}{v}\right)x}=U(s)\cdot e^{-\alpha x}\cdot e^{-\frac{s\,x}{v}}$$

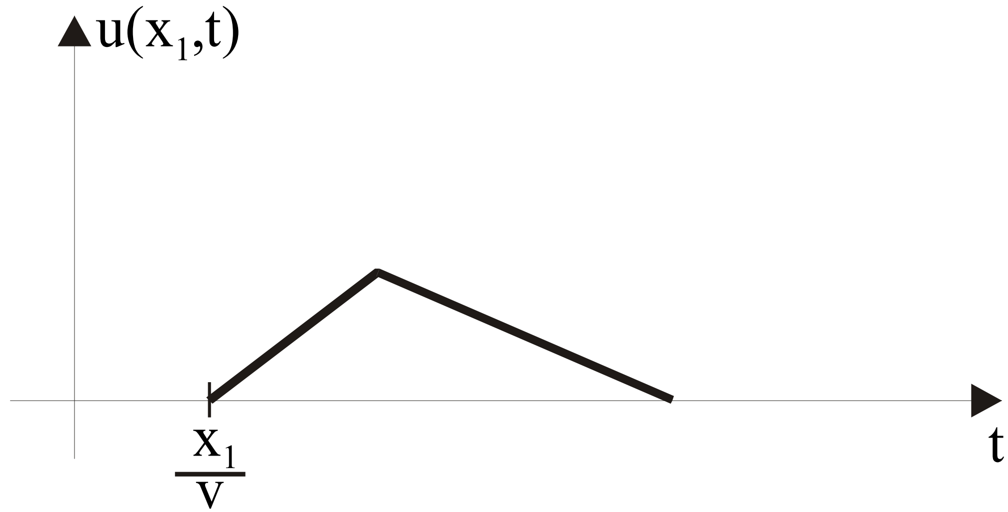
$$I(x,s)=\frac{U(s)}{Z_0}\cdot e^{-\alpha x}\cdot e^{-\frac{s\,x}{v}}$$

$$\left.\begin{aligned} u(x,t)&=e^{-\alpha x}\cdot u\left(t-\frac{x}{v}\right) \\ i(x,t)&=e^{-\alpha x}\cdot i\left(t-\frac{x}{v}\right) \end{aligned}\right\}$$

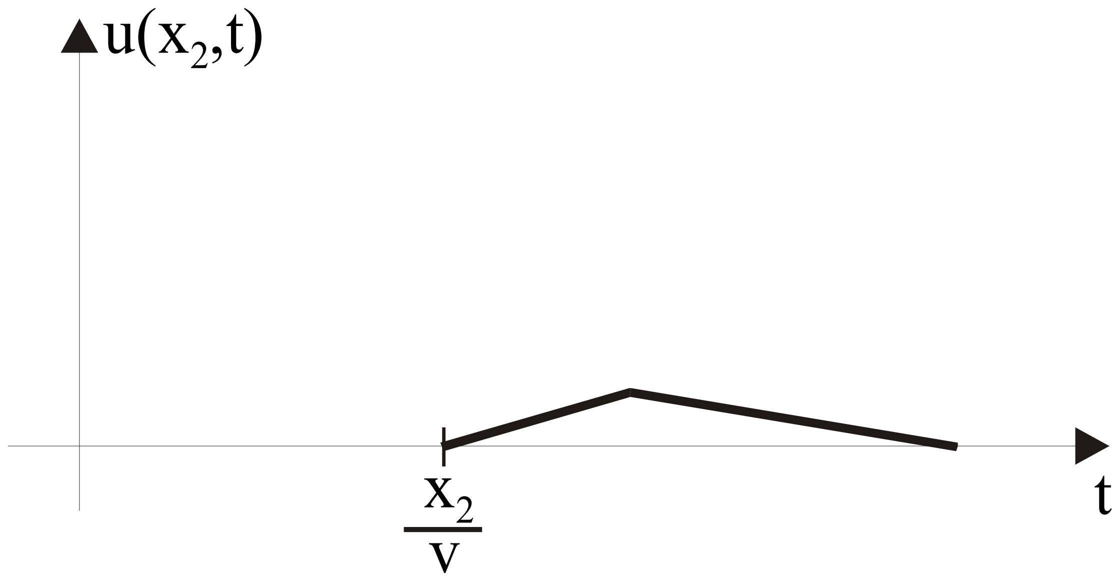
Signal je prigušen duž linije
i kasni vremenski

Beskonačno duga linija





Beskonačno duga linija



Linije konačne duljine

U praksi ∞ linija ne postoji.

Međutim analiza ∞ linije pomaže nam da bolje razumijemo prirodu prostiranja signala duž linije i na taj način da bolje shvatimo ponašanje realne linije tj. one konačne duljine.

Signal se linijom prostire brzinom v.

Ako je linija konačna i zaključena sa Z_L tada su napon i struja na svakom mjestu linije x dani izrazima:

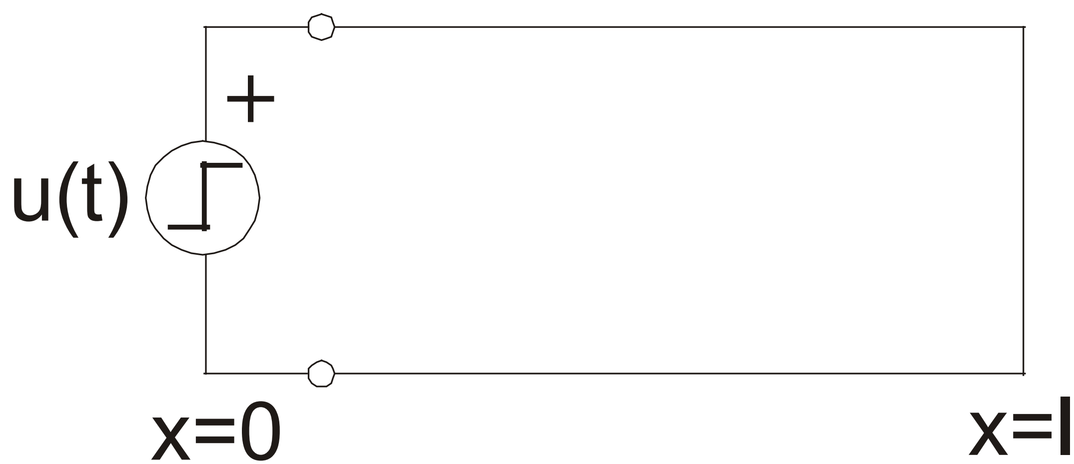
$$U(x,s)=U(s)\frac{e^{-\gamma(x-l)}+\Gamma_2 e^{\gamma(x-l)}}{e^{\gamma l}+\Gamma_2 e^{-\gamma l}}$$

$$I(x,s)=\frac{U(s)}{Z_0}\frac{e^{-\gamma(x-l)}-\Gamma_2 e^{\gamma(x-l)}}{e^{\gamma l}+\Gamma_2 e^{-\gamma l}}$$

pri čemu je

$$\Gamma_2(s)=\frac{Z_L(s)-Z_0(s)}{Z_L(s)+Z_0(s)}$$

Zamislimo radi ilustracije liniju bez gubitaka koja je kratko spojena na izlazu tj. Z_L=0.



Koeficijent refleksije je

$$\Gamma_2 = \frac{0 - Z_0}{0 + Z_0} = -1$$

Napon na mjestu x je:

$$U(x,s) = U(s)\frac{e^{-\gamma(x-l)} - e^{\gamma(x-l)}}{e^{\gamma l} - e^{-\gamma l}}$$

a struja:

$$I(x,s) = \frac{U(s)}{Z_0}\frac{e^{-\gamma(x-l)} + e^{\gamma(x-l)}}{e^{\gamma l} - e^{-\gamma l}}$$

Pretpostavimo da je u(t) step funkcija tj.:

$$u(t) = E \cdot S(t) \circ\!\!-\!\!\!-\!\!\!-\!\!\bullet\; U(s) = \frac{E}{s}$$

tada je

$$u(x,t) = \mathscr{L}^{-1}\left[\frac{E}{s}\frac{e^{-\gamma(x-l)} - e^{\gamma(x-l)}}{e^{\gamma l} - e^{-\gamma l}}\right] = \mathscr{L}^{-1}\left[\frac{E}{s}\frac{e^{-\gamma(x-l)}}{e^{\gamma l}} \cdot \frac{1 - e^{2\gamma(x-l)}}{1 - e^{-2\gamma l}}\right]$$

Pošto je:

$$\frac{1}{1 - e^{-2\gamma l}} = 1 + e^{-2\gamma l} + e^{-4\gamma l} + e^{-6\gamma l} + \cdots$$

nakon uvrštenja dobivamo

$$u(x,t)=\mathscr{L}^{-1}\left[\frac{E}{s}\left(e^{-\gamma x}-e^{-\gamma(2l-x)}\right)\left(1+e^{-2\gamma l}+e^{-4\gamma l}+e^{-6\gamma l}+\ldots\right)\right]=$$

$$=\mathscr{L}^{-1}\left[\frac{E}{s}\cdot e^{-\gamma x}-\frac{E}{s}\cdot e^{-\gamma(2l-x)}+\frac{E}{s}\cdot e^{-\gamma(2l+x)}-\frac{E}{s}\cdot e^{-\gamma(4l-x)}\pm\cdots\right]$$

Za liniju bez gubitaka je $\gamma=s/v$ pa je

$$u(x,t)=\mathscr{L}^{-1}\left[\frac{E}{s}e^{-\frac{x}{v}s}-\frac{E}{s}e^{-\frac{2l-x}{v}s}+\frac{E}{s}e^{-\frac{2l+x}{v}s}-\frac{E}{s}e^{-\frac{4l-x}{v}s}\pm\cdots\right]$$

odnosno

$$u(x,t) = E\left[S\left(t - \frac{x}{v}\right) - S\left(t - \frac{2l - x}{v}\right) + S\left(t - \frac{2l + x}{v}\right) - \right.$$

$$\left. - S\left(t - \frac{4l - x}{v}\right) \pm \cdots \right]$$

To je rješenje za napon na nekom mjestu x linije.

Signal sa ulaza dolazi na izlaz brzinom v.

Međutim na izlazu je kratki spoj, pa napon mora biti jednak nuli za svaki t.

Ali signal koji dolazi sa ulaza ima veličinu E.

Imamo dakle kontradikciju.

Ako se prisjetimo izraza

$$U(x,s)=U(s)\cdot\left(A_1 e^{-\gamma x}+A_2 e^{\gamma x}\right)$$

tada je za liniju bez gubitaka

$$U(x,s)=U(s)\cdot\left(A_1 e^{-\frac{x}{v}s}+A_2 e^{\frac{x}{v}s}\right)$$

odnosno

$$u(x,t)=A_1\cdot u\left(t-\frac{x}{v}\right)+A_2\cdot u\left(t+\frac{x}{v}\right)$$

Linije konačne duljine

Izraz $u(t-x/v)$ je signal u(t) koji putuje duž linije u pozitivnom smjeru (sa ulaza na izlaz) brzinom v.

Izraz $u(t+x/v)$ je signal u(t) koji putuje duž linije u negativnom smjeru (sa izlaza na ulaz) brzinom v.

Opće rješenje linije sadrži oba ova signala.

Jednadžbe linije su zadovoljene ako postoje signali koji propagiraju linijom u(+) i u (-) smjeru.

Dva kontradiktorna zahtjeva su zadovoljena jednim posebnim fenomenom kojeg zovemo

REFLEKSIJA.

U trenutku $t=l/v$ kad signal veličine E dođe na kraj linije, kratki spoj na izlazu uzrokuje napon veličine –E koji počinje putovati u negativnom smjeru tj. prema ulazu.

Totalni napon na izlazu je suma dolaznog i reflektiranog signala i jednaka je nuli.

Ovo rješenje premda atraktivno u početku stvara nam nevolje u daljnjem toku razmatranja signala.

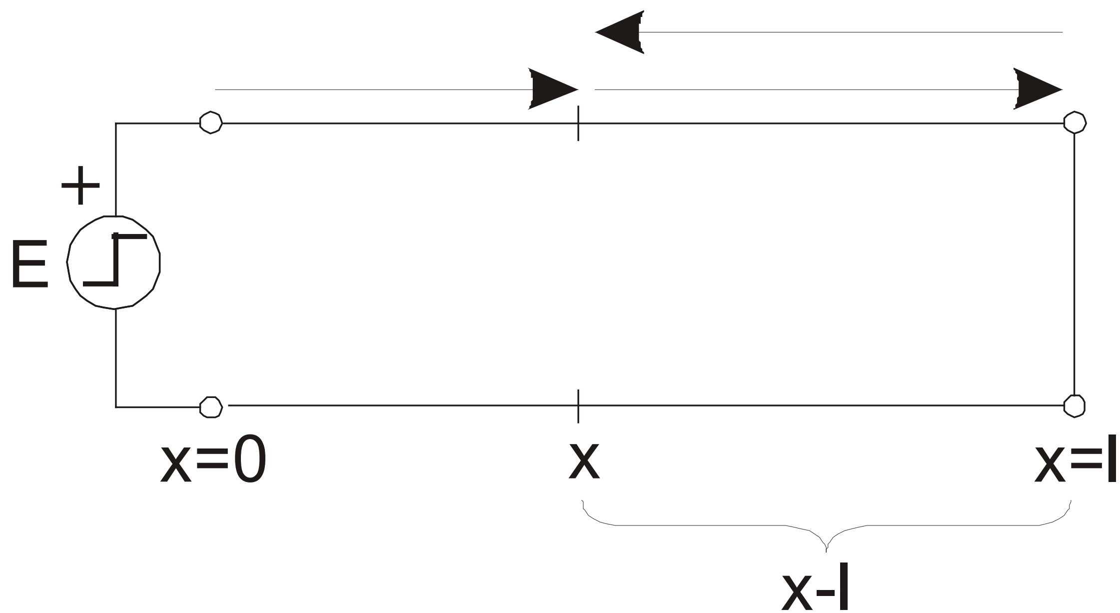
Pošto reflektirani signal ima amplitudu –E on putujući prema ulazu poništava dolazni signal +E i u trenutku $t=2l/v$ on dolazi na ulaz kao 0V.

Međutim na ulazu je izvor napona E pa je napon na njemu u svakom trenutku E.

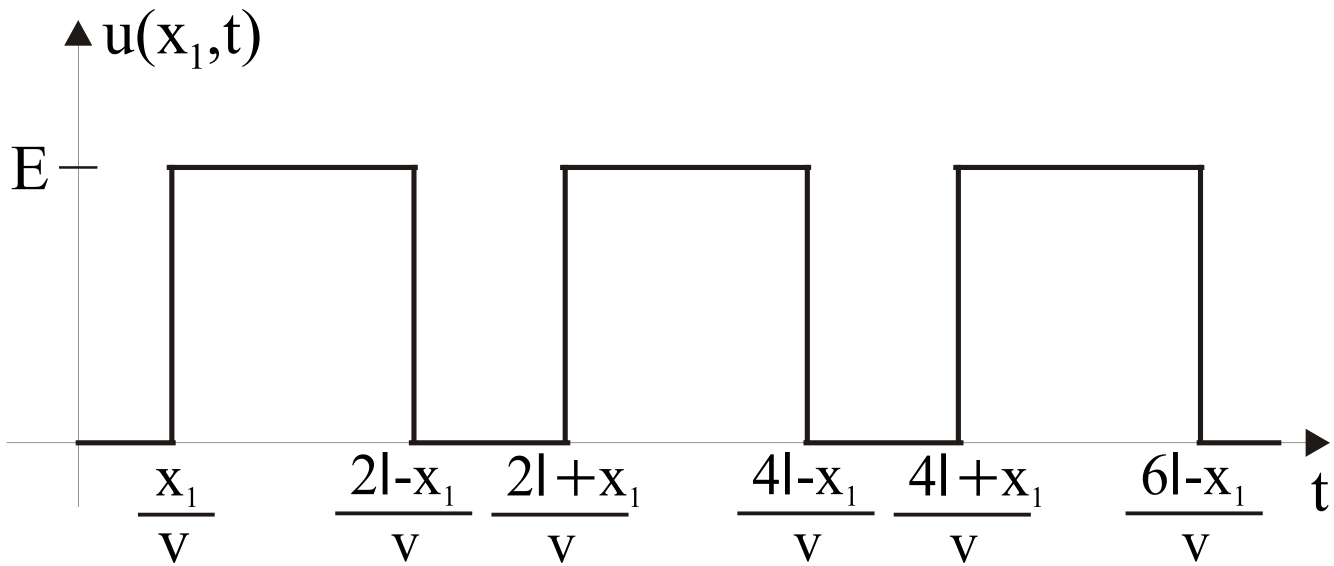
Da bi udovoljili rubne uvjete na ulazu, novi reflektirani val na ulazu veličine +E kreće prema izlazu.

Taj ciklus se ponavlja do u beskonačnost.

Linije konačne duljine



Uočimo li mjesto x_1 na liniji tada je napon na tom mjestu



Razmotrimo sada valni oblik struje na liniji

$$i(x,t)=\mathscr{L}^{-1}\left[\frac{U(s)}{Z_0}\frac{e^{-\gamma(x-l)}+e^{\gamma(x-l)}}{e^{\gamma l}-e^{-\gamma l}}\right]=$$

$$=\mathscr{L}^{-1}\left[\frac{E}{sZ_0}\frac{e^{-\gamma(x-l)}}{e^{\gamma l}}\cdot\frac{1+e^{2\gamma(x-l)}}{1-e^{-2\gamma l}}\right]$$

odnosno

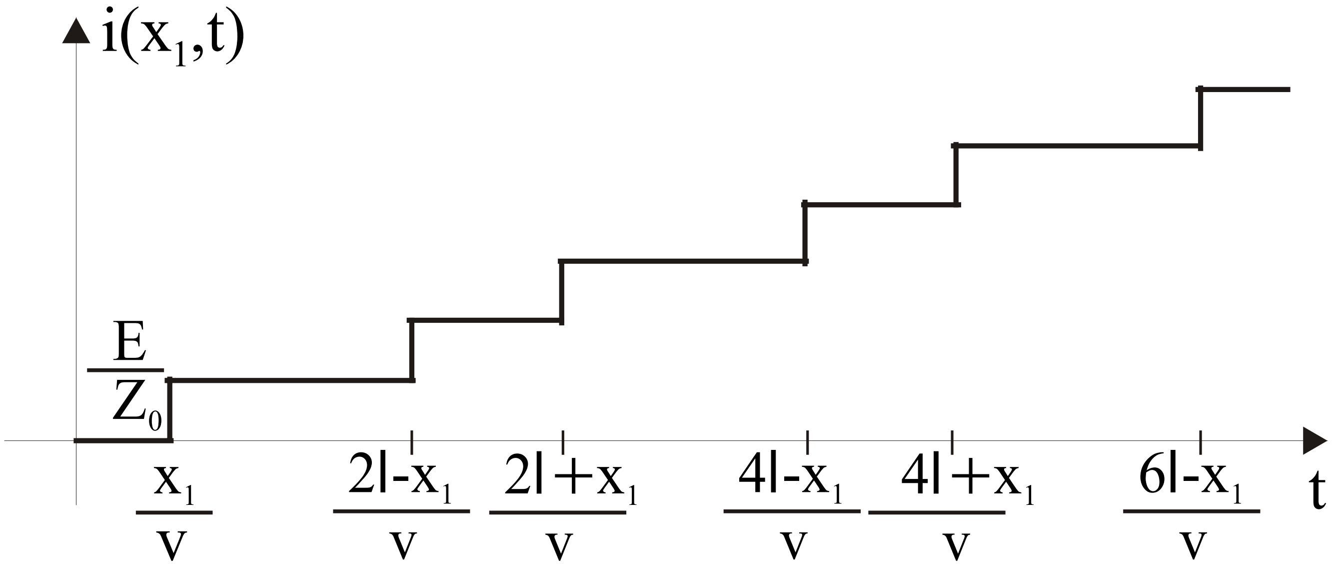
$$i(x,t) = \mathscr{L}^{-1}\left\{\frac{E}{sZ_0}\left[e^{-\gamma x} + e^{-\gamma(2l-x)}\right]\left[1 + e^{-2\gamma l} + e^{-4\gamma l} + \cdots\right]\right\}$$

Za liniju bez gubitaka je $\gamma = x/v$ i $Z_0 \neq Z_0(s)$

$$i(x,t) = \frac{1}{Z_0}\mathscr{L}^{-1}\left[\frac{E}{s}e^{-\frac{x}{v}s} + \frac{E}{s}e^{-\frac{2l-x}{v}s} + \frac{E}{s}e^{-\frac{2l+x}{v}s} + \frac{E}{s}e^{-\frac{4l-x}{v}s} + \cdots\right]$$

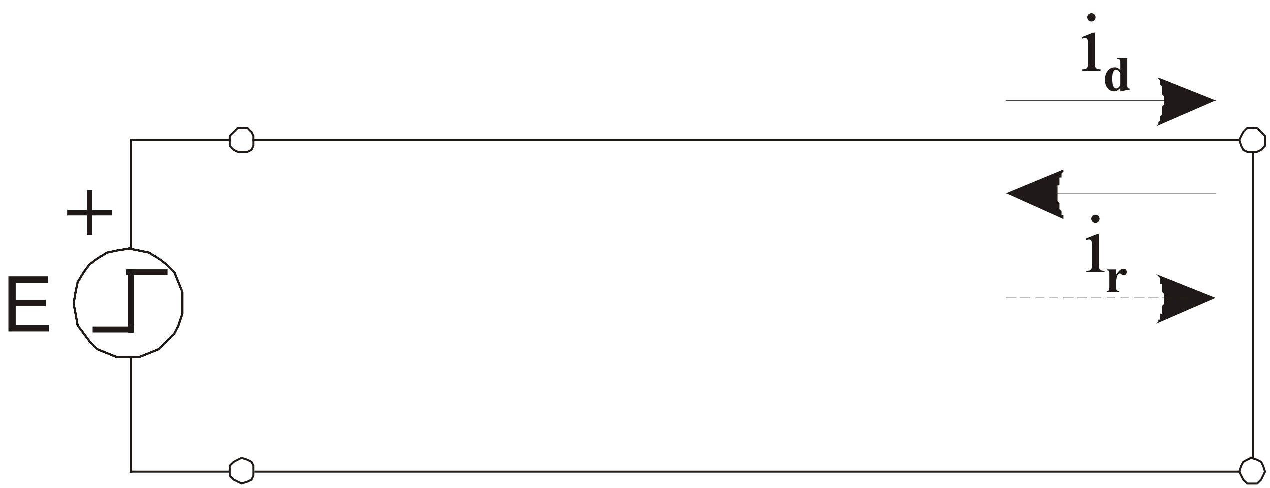
$$i(x,t)=\frac{E}{Z_0}\left[S\left(t-\frac{x}{v}\right)+S\left(t-\frac{2l-x}{v}\right)+S\left(t-\frac{2l+x}{v}\right)+\right.$$

$$\left.+S\left(t-\frac{4l-x}{v}\right)+\cdots\right]$$



Struja skokovito raste u koracima od E/Z_0 i teži u ∞.

Ovo se također moglo očekivati i po intuiciji zbog kratkog spoja na izlazu.



$$i_d = \frac{E}{Z_0}$$

$$i_r = -\frac{E}{Z_0}$$

Koeficijent refleksije

Kod slučaja $Z_L=0$ tj. kratkog spoja kratki spoj generira napon suprotnog polariteta i istog iznosa. To ne mora biti slučaj za neki opći iznos $Z_L \neq 0$.

Reflektirani val mora biti takav da zadovoljava granične uvjete na liniji.

Za slučaj da je $Z_L = Z_0 \rightarrow \Gamma_2 = 0$ nema refleksije.
Linija zaključena sa Z_0 ponaša se kao beskonačna linija.