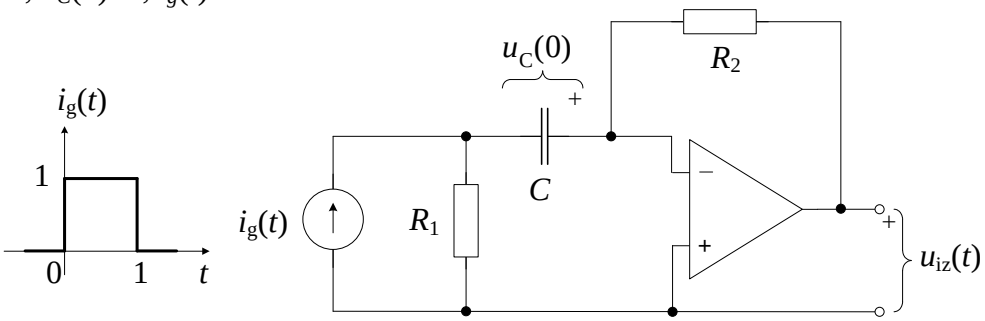
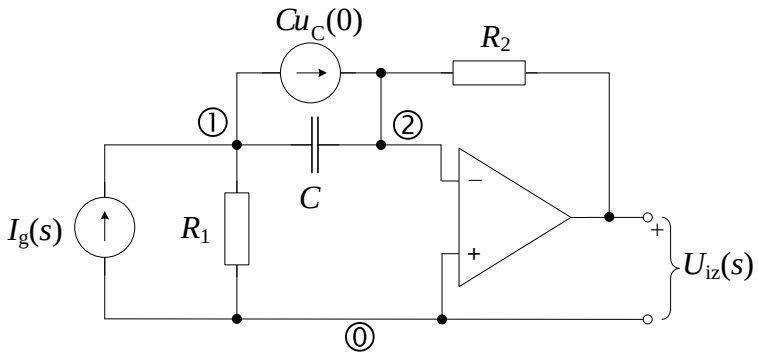
MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA 2012-2013 – Rješenja

1. Za električni krug prikazan slikom izračunati: a) napon na izlazu operacijskog pojačala $U_{iz}(s)$; b) valni oblik napona $u_{iz}(t)$. Zadane su normalizirane vrijednosti elemenata: $R_1=1$, $R_2=1/2$, $C=2$, $u_C(0)=1$, $i_g(t)$ zadan slikom.



Rješenje:

a) Jednadžbe čvorišta:



$$1)\ U_1\left(sC+\frac{1}{R_1}\right)-U_2sC=I_g(s)-Cu_C(0)$$

$$2)\ -U_1sC+U_2\left(sC+\frac{1}{R_2}\right)=\frac{U_{iz}(s)}{R_2}+Cu_C(0)$$

$$U_2=0$$

$$1)\ U_1\left(sC+\frac{1}{R_1}\right)=I_g(s)-Cu_C(0)$$

$$2)\ -U_1sC=\frac{U_{iz}(s)}{R_2}+Cu_C(0)\Rightarrow U_1=-\frac{U_{iz}(s)}{sCR_2}-\frac{u_C(0)}{s}$$ (1 bod)

$$\Rightarrow\ -\left(\frac{U_{iz}(s)}{sCR_2}+\frac{u_C(0)}{s}\right)\left(sC+\frac{1}{R_1}\right)=I_g(s)-Cu_C(0)$$

$$\Rightarrow\ U_{iz}(s)=-\frac{I_g(s)-Cu_C(0)}{sC+\frac{1}{R_1}}sCR_2-Cu_C(0)R_2$$ (1 bod)

$$i_g(t)=S(t)-S(t-1)\Rightarrow I_g(s)=\frac{1}{s}-\frac{1}{s}e^{-s}=\frac{1}{s}(1-e^{-s})$$ (1 bod)

$$U_{iz}(s) = -\frac{\frac{1}{s}(1-e^{-s})-2}{2s+1}s - 1 = \frac{2s-1}{2s+1} + \frac{e^{-s}}{2s+1} - 1 =$$

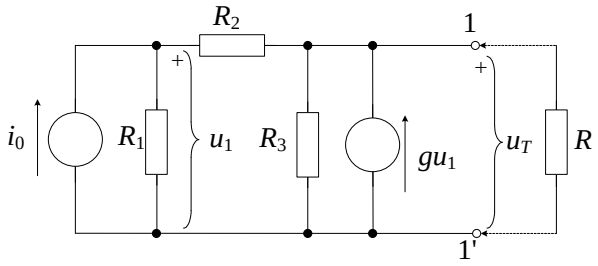
$$= \frac{2s+1-2}{2s+1} + \frac{e^{-s}}{2s+1} - 1 = 1 + \frac{-2}{2s+1} + \frac{e^{-s}}{2s+1} - 1 = \frac{-2}{2s+1} + \frac{e^{-s}}{2s+1} = -\frac{1}{s+\frac{1}{2}} + \frac{\frac{1}{2}}{s+\frac{1}{2}} \cdot e^{-s}$$

(1 bod)

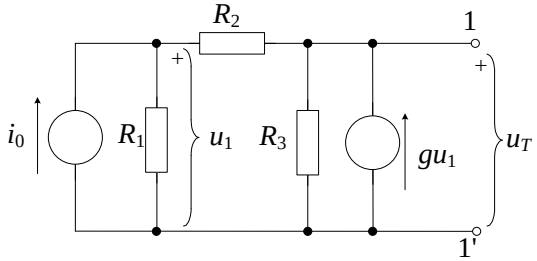
$$\Rightarrow \quad u_{iz}(t) = -e^{-t/2}S(t) + \frac{1}{2}e^{-(t-1)/2}S(t-1) \qquad \text{(1 bod)}$$

2. Za krug na slici obzirom na priključnice 1–1' i isključen otpor R odrediti:
a) Theveninov napon u_T; b) Theveninov otpor R_T; c) iznos konstante g za koji je $R_T=R$;
d) napon u_1 uz uključen otpor R [g iz zadatka c)]; e) iznos konstante g za koji je $R_T=\infty$.

Zadano je: i_0=2 A i $R_1=1\Omega$, $R_1=\frac{1}{2}\Omega$ i $R_3=R=\frac{1}{3}\Omega$.



Rješenje: a) Theveninov napon u_T:

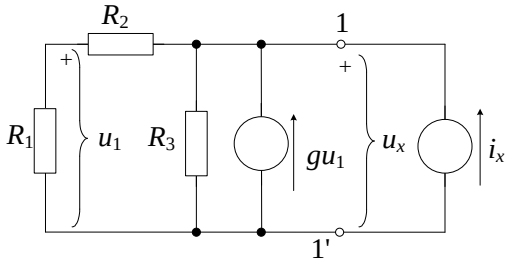


$$u_1(G_1+G_2)-u_2G_2=i_0$$

$$-u_1G_2+u_2(G_2+G_3)=gu_1 \quad \Rightarrow \quad u_1=u_2\frac{G_2+G_3}{g+G_2}$$

$$u_T=u_2=\frac{g+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2}i_0=\frac{2g+4}{11-2g}$$ (1 bod)

b) Theveninov otpor R_T:



$$u_1=u_x\frac{G_2}{G_1+G_2}$$

$$i_x+gu_1=u_x\left(G_3+\frac{G_1G_2}{G_1+G_2}\right) \quad \Rightarrow \quad i_x=u_x\left(-g\frac{G_2}{G_1+G_2}+G_3+\frac{G_1G_2}{G_1+G_2}\right)$$

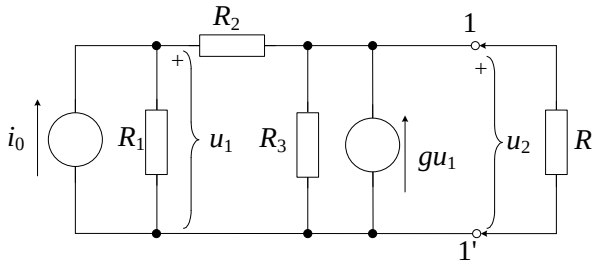
$$R_T=\frac{u_x}{i_x}=\frac{G_1+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2}=\frac{3}{11-2g}$$ (1 bod)

c) odrediti iznos konstante g za koji je $R_T=R$.

$$R_T=R=\frac{1}{3}=\frac{G_1+G_2}{G_3(G_1+G_2)+G_1G_2-gG_2} \quad \Rightarrow \quad \frac{1}{3}=\frac{1+2}{3(1+2)+2-2g} \quad \Rightarrow \quad g=1\ \Omega^{-1}$$

(1 bod)

d) napon u_1 uz uključen otpor R



$$-u_1G_2+u_2\left(G_2+G_3+G\right)=gu_1$$

$$u_1=u_2\frac{G_2+G_3+G}{g+G_2}=\frac{u_T}{2}\cdot\frac{G_2+G_3+G}{g+G_2}=\frac{1}{2}\cdot\frac{G_2+G_3+G}{g+G_2}\cdot\frac{g+G_2}{G_3\left(G_1+G_2\right)+G_1G_2-gG_2}i_0=\frac{8}{9}\,\mathrm{V}$$

(1 bod)

e) iznos konstante g za koji je $R_T=\infty$?

$$R_T=\frac{u_x}{i_x}=\frac{G_1+G_2}{G_3\left(G_1+G_2\right)+G_1G_2-gG_2}\,\Omega$$

$$G_3\left(G_1+G_2\right)+G_1G_2-gG_2=0\Rightarrow\qquad g=\frac{G_3\left(G_1+G_2\right)}{G_2}+G_1=\frac{11}{2}\ \Omega^{-1}$$ (1 bod)

3. Za neki električni krug poznate su slijedeće matrice:

$$\mathbf{Z}_b = \begin{bmatrix} R & 0 & 0 \\ \mu R & \dfrac{1}{sC} & 0 \\ 0 & 0 & sL \end{bmatrix},\quad \mathbf{U}_{0b} = \begin{bmatrix} 0 \\ \dfrac{u_C(0)}{s} \\ -I_0 sL + Li_L(0) \end{bmatrix},\quad \mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}.$$

Odrediti temeljni sustav jednadžbi petlji primjenom grafova i pritom: a) Nacrtati zadanu električnu shemu kruga. b) Nacrtati orijentirani graf i napisati spojnu matricu **S**. Napisati: c) matricu impedancija petlji $\mathbf{Z}_p$ i d) vektor početnih uvjeta i nezavisnih izvora petlji $\mathbf{U}_{0p}$.

Rješenje:

Naponsko – strujne relacije grana: $\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$

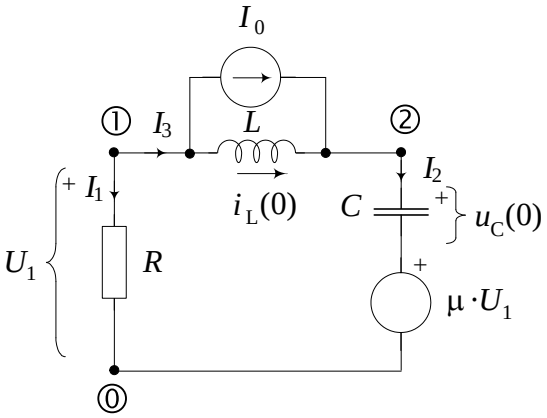
$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} = \begin{bmatrix} R & 0 & 0 \\ \mu R & \dfrac{1}{sC} & 0 \\ 0 & 0 & sL \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{u_C(0)}{s} \\ -I_0 sL + Li_L(0) \end{bmatrix}$$

(1) $U_1 = R \cdot I_1$

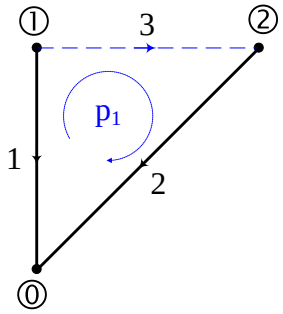
(2) $U_2 = \mu R \cdot I_1 + \dfrac{1}{sC} \cdot I_2 + \dfrac{u_C(0)}{s} = \mu \cdot U_1 + \dfrac{1}{sC} \cdot I_2 + \dfrac{u_C(0)}{s}$

(3) $U_3 = sL \cdot I_3 - I_0 sL + Li_L(0) = (I_3 - I_0)sL + Li_L(0)$

(1 bod)



(1 bod)



(1 bod)

Spojna matrica: $\mathbf{S} = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix}$

Temeljni sustav jednadžbi petlji u matričnom obliku: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$

$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} R & 0 & 0 \\ \mu R & \dfrac{1}{sC} & 0 \\ 0 & 0 & sL \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} =$$

(1 bod)

$$= \begin{bmatrix} -R + \mu R & \dfrac{1}{sC} & sL \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} R(1-\mu) + \dfrac{1}{sC} + sL \end{bmatrix}$$

$$\mathbf{U}_{0p} = -\mathbf{S} \cdot \mathbf{U}_{0b} = -\begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ \dfrac{u_C(0)}{s} \\ -I_0 sL + Li_L(0) \end{bmatrix} = \begin{bmatrix} -\dfrac{u_C(0)}{s} + I_0 sL - Li_L(0) \end{bmatrix}$$ (1 bod)

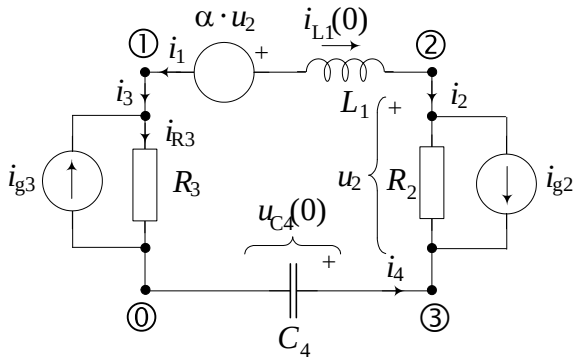
4. Za električni krug na slici i pridruženim orijentacijama grana zadane su normalizirane vrijednosti elemenata L_1=1, R_2=1, R_3=2, C_4=1/2, te α=2, $u_{C4}(0)$=2, $i_{L1}(0)$=1, $i_{g2}(t)$=$i_{g3}(t)$=$S(t)$. Koristeći KZS i KZN te oznake grana i čvorova prema slici, napisati:

a) Jednadžbe KZS i KZN;

b) Naponsko-strujne jednadžbe za grane;

c) Napon na otporu R_2: $U_2(s)$ i $u_2(t)$;

d) Struju kroz otpor R_3: $I_{R3}(s)$ i $i_{R3}(t)$;



Rješenje:

a) N_b=4 (broj grana)

N_v=4 (broj čvorova)

Broj jednadžbi KZS $= N_v - 1 = 4 - 1 = 3$

Broj jednadžbi KZN $= N_b - N_v + 1 = 4 - 4 + 1 = 1$

Slijede jednadžbe Kirchhoffovih zakona (4 jednadžbe):

1) $-I_1 + I_3 = 0$ KZS

2) $I_1 + I_2 = 0$ KZS

3) $-I_2 - I_4 = 0$ KZS

4) $U_1 - U_2 + U_3 + U_4 = 0$ KZN (1 bod)

b) Naponsko–strujne jednadžbe grana (4 jednadžbe):

1) $U_1 = sL_1 \cdot I_1 + L_1 i_{L1}(0) + \alpha U_2 = sL_1 \cdot I_1 + L_1 i_{L1}(0) + \alpha(I_2 - I_{g2})R_2$

2) $U_2 = (I_2 - I_{g2}) \cdot R_2$

3) $U_3 = (I_3 + I_{g3}) \cdot R_3$

4) $U_4 = \dfrac{1}{sC_4} \cdot I_4 - \dfrac{u_{C4}(0)}{s}$ (1 bod)

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c) Sustav ima ukupno $2N_b$=8 jednadžbi i 8 nepoznanica (sve struje i svi naponi grana)

Naponsko – strujne jednadžbe grana uvrstimo u jednadžbe Kirchhoffovih zakona (1)–(4) te dobivamo:

1) , 2), 3) $I_1 = -I_2 = I_3 = I_4$

4) $sL_1 I_1 + L_1 i_{L1}(0) + \alpha(I_2 - I_{g2})R_2 - (I_2 - I_{g2})R_2 + (I_3 + I_{g3})R_3 + \dfrac{1}{sC_4} I_4 - \dfrac{u_{C4}(0)}{s} = 0$

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1)→4) $\Rightarrow sL_1 I_1 + L_1 i_{L1}(0) - \alpha(I_1 + I_{g2})R_2 + (I_1 + I_{g2})R_2 + (I_1 + I_{g3})R_3 + \dfrac{1}{sC_4} I_1 - \dfrac{u_{C4}(0)}{s} = 0$

$$\Rightarrow I_1\left(sL_1 + (1-\alpha)R_2 + R_3 + \frac{1}{sC_4}\right) + L_1 i_{L1}(0) + (1-\alpha)I_{g2}R_2 + I_{g3}R_3 - \frac{u_{C4}(0)}{s} = 0$$

$$\Rightarrow I_1(s)=\frac{-L_1 i_{L1}(0)-(1-\alpha)I_{g2}R_2-I_{g3}R_3+\frac{u_{C4}(0)}{s}}{sL_1+(1-\alpha)R_2+R_3+\frac{1}{sC_4}}$$

$$I_1(s)=\frac{-1+\frac{1}{s}-\frac{2}{s}+\frac{2}{s}}{s-1+2+\frac{2}{s}}=\frac{-1+\frac{1}{s}}{s+1+\frac{2}{s}}=\frac{-s+1}{s^2+s+2}$$ (1 bod)

$$\Rightarrow I_2(s)=-I_1(s)=\frac{s-1}{s^2+s+2}\Rightarrow$$

$$U_2(s)=\left[I_2(s)-I_{g2}(s)\right]R_2=\frac{s-1}{s^2+s+2}-\frac{1}{s}=\frac{s-1}{s^2+s+\frac{1}{4}+\frac{7}{4}}-\frac{1}{s}=\frac{s+\frac{1}{2}-\frac{3}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{7}{4}}-\frac{1}{s}$$

$$U_2(s)=\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}-\frac{3}{\sqrt{7}}\frac{\frac{\sqrt{7}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}-\frac{1}{s}$$

$$u_2(t)=e^{-\frac{t}{2}}\left(\cos\frac{\sqrt{7}}{2}t-\frac{3}{\sqrt{7}}\sin\frac{\sqrt{7}}{2}t\right)S(t)-S(t)$$ (1 bod)

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d) Struja kroz R_3

$$\Rightarrow I_3(s)=I_1(s)=\frac{-s+1}{s^2+s+2}\Rightarrow$$

$$I_{R3}(s)=I_3(s)+I_{g3}(s)=-\frac{s-1}{s^2+s+2}+\frac{1}{s}=-\left(\frac{s-1}{s^2+s+2}-\frac{1}{s}\right)$$

[= minus izraz za $U_2(s)$ gore]

$$i_{R3}(t)=-e^{-\frac{t}{2}}\left(\cos\frac{\sqrt{7}}{2}t-\frac{3}{\sqrt{7}}\sin\frac{\sqrt{7}}{2}t\right)S(t)+S(t)$$ (1 bod)

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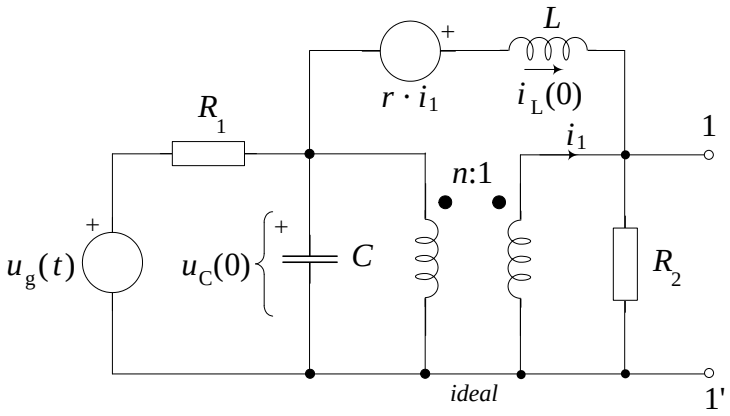
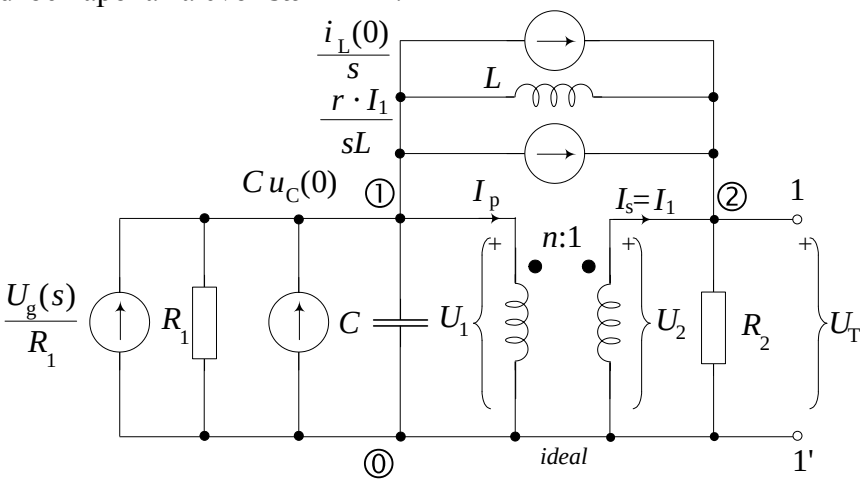
5. Za električni krug na slici zadane su normalizirane vrijednosti elemenata C=1/2, L=2, R_1=R_2=1 te n=2, r=4, $u_C(0)$=1, $i_L(0)$=1, $u_g(t)$=$S(t)$. Odrediti nadomjesne parametre mreže po Theveninu s obzirom na polove 1–1'. Koristiti metodu napona čvorišta. U zadatku je potrebno: a) Nacrtati sklop za izračunavanje Theveninovog napona i postaviti jednadžbe napona za čvorišta ① i ②; b) Odrediti Theveninov napon $U_T(s)$; c) Nacrtati sklop za izračunavanje Theveninove impedancije i postaviti jednadžbe napona za čvorišta ① i ②; d) Odrediti Theveninovu impedanciju $Z_T(s)$. e) Da li je električni krug recipročan? Zašto?



Rješenje:

a) Jednadžbe napona za čvorišta ① i ②:



1) $U_1\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\frac{U_g(s)}{R_1}+Cu_C(0)-\frac{rI_s(s)}{sL}-\frac{i_L(0)}{s}-I_p(s)$

2) $-U_1\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rI_s(s)}{sL}+\frac{i_L(0)}{s}+I_s(s)$

3) $U_2=\frac{1}{n}U_1$

4) $I_s=nI_p$

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1) $nU_2\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\frac{U_g(s)}{R_1}+Cu_C(0)-\frac{rnI_p(s)}{sL}-\frac{i_L(0)}{s}-I_p(s)$

2) $-nU_2\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rnI_p(s)}{sL}+\frac{i_L(0)}{s}+nI_p(s)$ (1 bod)

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b) Theveninov napon $U_T(s)=U_2(s)$:

Uvrstimo vrijednosti: $C=1/2$, $L=2$, $R_1=R_2=1$ te $n=2$, $r=4$, $u_C(0)=1$, $i_L(0)=1$, $u_g(t)=S(t)$.

$$1)\ 2U_2\left(\frac{s}{2}+1+\frac{1}{2s}\right)-U_2\frac{1}{2s}=\frac{1}{s}+\frac{1}{2}-\frac{4I_p(s)}{s}-\frac{1}{s}-I_p(s)\ \Rightarrow$$

$$U_2\left(s+2+\frac{1}{2s}\right)=\frac{1}{2}-I_p(s)\left(\frac{4}{s}+1\right)\ \Rightarrow\ I_p(s)=\frac{\frac{1}{2}-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}$$

$$2)\ -U_2\frac{1}{s}+U_2\left(1+\frac{1}{2s}\right)=\frac{4I_p(s)}{s}+\frac{1}{s}+2I_p(s)$$

$$U_2\left(1-\frac{1}{2s}\right)=\frac{1}{s}+2I_p(s)\left(1+\frac{2}{s}\right)$$

1), 2) $\Rightarrow$

$$U_2\left(1-\frac{1}{2s}\right)=\frac{1}{s}+2\frac{\frac{1}{2}-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}\left(1+\frac{2}{s}\right)=\frac{1}{s}+\frac{1-U_2 2\left(s+2+\frac{1}{2s}\right)}{(s+4)}(s+2)$$

$$\Rightarrow U_2(2s-1)=2+2s\frac{1-U_2 2(s+2+1/(2s))}{(s+4)}(s+2)$$

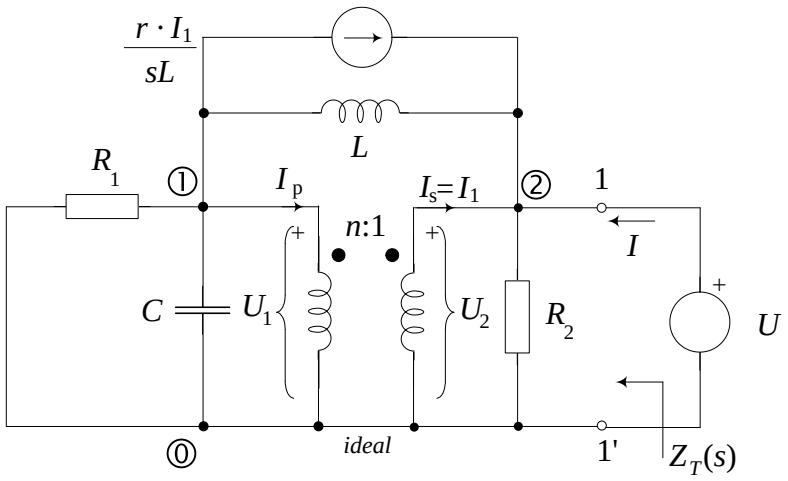
$$\Rightarrow U_2(2s-1)(s+4)=2(s+4)+2s(s+2)-U_2 4s\left[(s+2)^2+\frac{1}{2s}(s+2)\right]$$

$$\Rightarrow U_2\left[(2s-1)(s+4)+4s\left(s^2+4s+4+\frac{1}{2}+\frac{1}{s}\right)\right]=2(s+4)+2s(s+2)$$

$$\Rightarrow U_2\left[2s^2-s+8s-4+4s^3+16s^2+16s+2s+4\right]=2s+8+2s^2+4s$$

$$U_T(s)=U_2(s)=\frac{2s^2+6s+8}{4s^3+18s^2+25s}=\frac{2(s^2+3s+4)}{s(4s^2+18s+25)}$$ (1 bod)

c) Izračunavanje Theveninove impedancije pomoću jednadžbi napona čvorišta ① i ②



$$1)\ U_1\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=-\frac{rI_s(s)}{sL}-I_p(s)$$

$$2)\ -U_1\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rI_s(s)}{sL}+I_s(s)+I(s)$$

$$3)\ U_2=\frac{1}{n}U_1$$

$$4)\ I_s=nI_p$$

$$1)\ nU_2\left(sC+\frac{1}{R_1}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=-\frac{rnI_p(s)}{sL}-I_p(s)$$

$$2)\ -nU_2\frac{1}{sL}+U_2\left(\frac{1}{R_2}+\frac{1}{sL}\right)=\frac{rnI_p(s)}{sL}+nI_p(s)+I(s)$$ (1 bod)

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d) Theveninova impedancija $Z_T(s)=U_2(s)/I(s)$:

Uvrstimo vrijednosti: $C=1/2$, $L=2$, $R_1=R_2=1$ te $n=2$, $r=4$, $u_C(0)=1$, $i_L(0)=1$, $u_g(t)=S(t)$.

$$1)\ 2U_2\left(\frac{s}{2}+1+\frac{1}{2s}\right)-U_2\frac{1}{2s}=-\frac{4I_p(s)}{s}-I_p(s)\ \Rightarrow$$

$$U_2\left(s+2+\frac{1}{2s}\right)=-I_p(s)\left(\frac{4}{s}+1\right)\ \Rightarrow\ I_p(s)=\frac{-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}$$

$$2)\ -U_2\frac{1}{s}+U_2\left(1+\frac{1}{2s}\right)=\frac{4I_p(s)}{s}+2I_p(s)+I(s)$$

$$U_2\left(1-\frac{1}{2s}\right)=2I_p(s)\left(1+\frac{2}{s}\right)+I(s)$$

$$1),2)\Rightarrow U_2\left(1-\frac{1}{2s}\right)=2\frac{-U_2\left(s+2+\frac{1}{2s}\right)}{\left(\frac{4}{s}+1\right)}\left(1+\frac{2}{s}\right)+I(s)=\frac{-U_2 2\left[s+2+1/(2s)\right]}{(s+4)}(s+2)+I(s)$$

$$\Rightarrow U_2(2s-1)=2s\frac{-U_2 2\left(s+2+1/(2s)\right)}{(s+4)}(s+2)+2sI(s)$$

$$\Rightarrow U_2(2s-1)(s+4)=-U_2 4s\left[(s+2)^2+(s+2)/(2s)\right]+2s(s+4)I(s)$$

$$\Rightarrow U_2\left[(2s-1)(s+4)+4s(s+2)^2+4s(1/2+1/s)\right]=2s(s+4)I(s)$$

$$\Rightarrow U_2\left[2s^2-s+8s-4+4s^3+16s^2+16s+2s+4\right]=\left[2s^2+8s\right]I(s)$$

$$Z_T(s)=\frac{U_2(s)}{I(s)}=\frac{2s^2+8s}{4s^3+18s^2+25s}=\frac{2(s+4)}{4s^2+18s+25}$$ (1 bod)

e) Da li je električni krug recipročan? Zašto?

NE jer ima strujno ovisni naponski izvor. (1 bod)