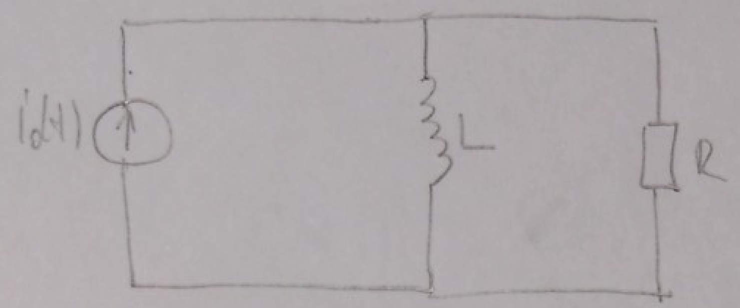
(1) $i_0(t) = 2\cos(2t)$

$R = 2$

$L = 2$

$C = 1$

(a) $t < 0$ → fazaski račun ; $w = 2$; $\underline{I_0 = 2\angle 0^\circ}$



Trazimo $i_L(0)$

Imamo strujno djelilo:

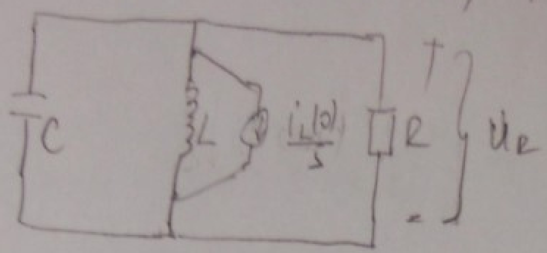
$$I_L = I_0 \cdot \frac{R}{R + jwL} = I_0 \cdot \frac{2}{2 + j \cdot 2 \cdot 2} = I_0 \cdot \frac{1}{1 + 2j} = 2\angle 0^\circ \cdot 0{,}447 \underline{\angle -63{,}435^\circ}$$

$$= 0{,}894 \underline{\angle -63{,}435^\circ}$$

$$i_L(t) = 0{,}894 \cdot \cos(2t - 63{,}435^\circ)$$

$$i_L(0) = 0{,}894 \cdot \cos(-63{,}435^\circ) = 0{,}4$$

b) $t \geq 0$; Laplaceovo područje ; $i_L(0) = 0{,}4$



$$U_R = \frac{i_L(0)}{s} \cdot Z_{ekv}$$

$$Z_{ekv} = R \| L \| C$$

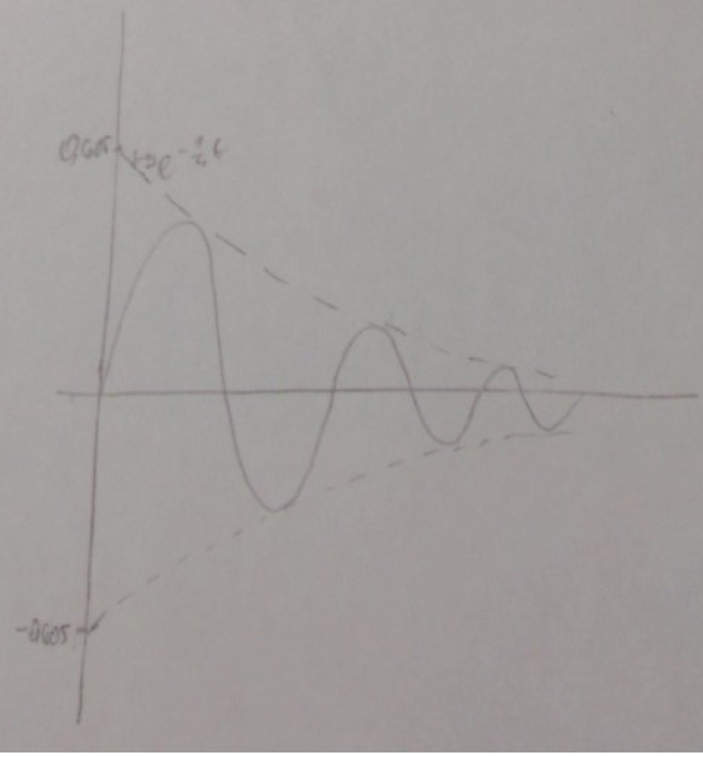
$$\frac{1}{Z_{ekv}} = \frac{1}{R} + \frac{1}{sL} + sC = \frac{1}{2} + \frac{1}{2s} + s = \frac{s+1+2s^2}{2s}$$

$$Z_{ekv} = \frac{2s}{2s^2+s+1} = \frac{s}{s^2+\frac{1}{2}s+\frac{1}{2}}$$

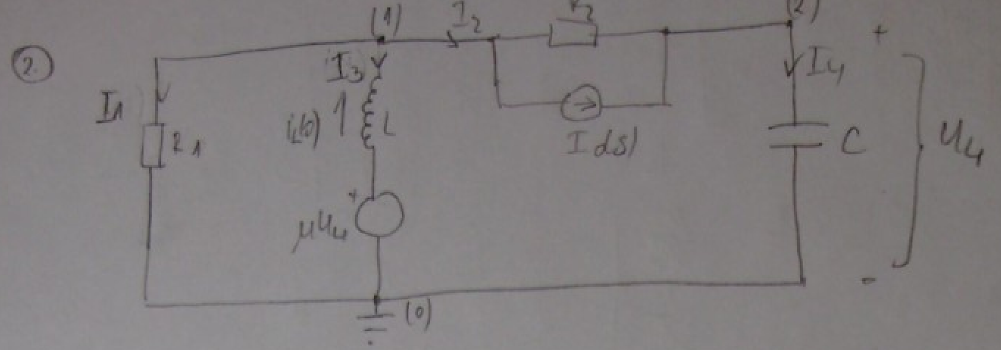
$$U_R = \frac{0.4}{\cancel{s}} \cdot \frac{\cancel{s}}{s^2+\frac{1}{2}s+\frac{1}{2}} = 0.4 \cdot \frac{1}{s^2+\frac{1}{2}s+\frac{1}{2}} = \frac{0.4}{\left(s+\frac{1}{4}\right)^2+\frac{7}{16}} = \frac{2}{5} \cdot \frac{4}{\sqrt{7}} \frac{\frac{\sqrt{7}}{4}}{\left(s+\frac{1}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2} =$$

$$= \frac{8}{5\sqrt{7}} \cdot \frac{\frac{\sqrt{7}}{4}}{\left(s+\frac{1}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2} = \frac{8\sqrt{7}}{35} \cdot \frac{\frac{\sqrt{7}}{4}}{\left(s+\frac{1}{4}\right)^2+\left(\frac{\sqrt{7}}{4}\right)^2}$$

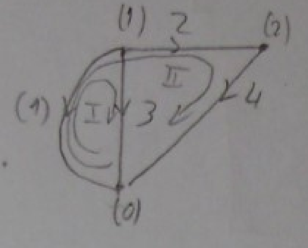
$$u_R(t) = \frac{8\sqrt{7}}{35} \sin\left(\frac{\sqrt{7}}{4}t\right) \cdot e^{-\frac{1}{4}t} \cdot S(t) = 0.605 \cdot \sin\left(\frac{\sqrt{7}}{4}t\right) \cdot e^{-\frac{1}{4}t} \cdot S(t)$$



2.



Orijentiram graf



1 i 2 grane stabla
3 i 4 spone

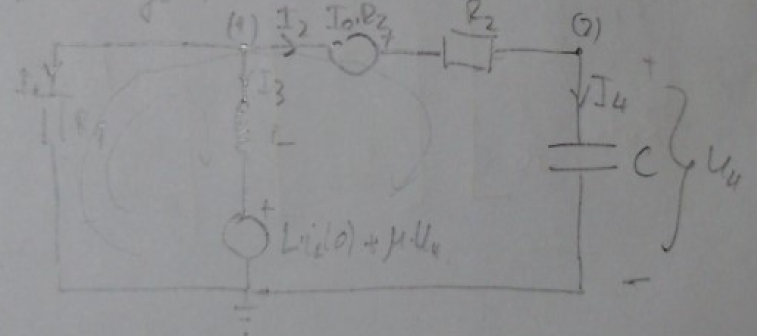
spojna matrica

$$S = \begin{array}{c} \\ \mathrm{I} \\ \mathrm{II} \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$\frac{i_L(0)}{s} \cdot sL$

$L \cdot i_L(0) +$

U-I relacije :



$U_1 = I_1 \cdot R_1$

$U_2 = I_2 R_2 - I_0 \cdot R_2$

$U_3 = I_3 \cdot sL + L \cdot i_L(0) + \mu U_4 = I_3 \cdot sL + \mu \cdot I_4 \cdot \frac{1}{sC} + L\, i_L(0)$

$U_4 = I_4 \cdot \frac{1}{sC}$

$$\underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{U_b} = \underbrace{\begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & 0 & sL & \frac{\mu}{sC} \\ 0 & 0 & 0 & \frac{1}{sC} \end{bmatrix}}_{Z_b} \cdot \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{I_b} + \underbrace{\begin{bmatrix} 0 \\ -I_0 R_2 \\ L \cdot i_L(0) \\ 0 \end{bmatrix}}_{U_{0b}}$$

1

$$Z_p = S \cdot Z_0 \cdot S^T = \begin{bmatrix} -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & 0 & sL & \frac{\mu}{sC} \\ 0 & 0 & 0 & \frac{1}{sC} \end{bmatrix} \cdot \begin{bmatrix} -1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} =$$

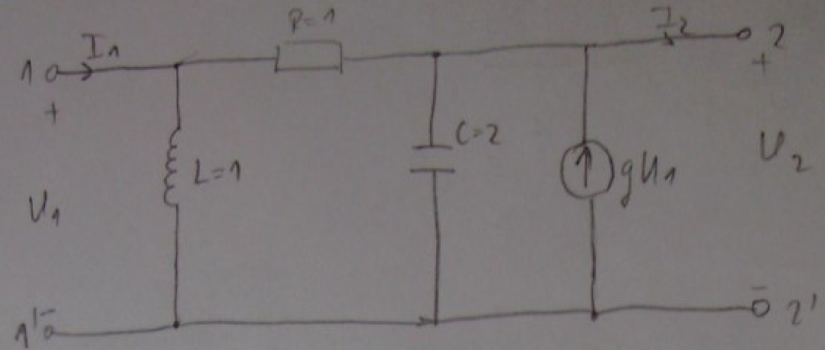
$$= \begin{bmatrix} -R_1 & 0 & sL & \frac{\mu}{sC} \\ -R_1 & R_2 & 0 & \frac{1}{sC} \end{bmatrix} \begin{bmatrix} -1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} R_1 + sL & R_1 + \frac{\mu}{sC} \\ R_1 & R_1 + R_2 + \frac{1}{sC} \end{bmatrix}$$

$$U_{0p} = -S \cdot U_{0b} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 1 & -1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -I_0 R_2 \\ L\, i_L(0) \\ 0 \end{bmatrix} = \begin{bmatrix} -L\, i_L(0) \\ I_0 \cdot R_2 \end{bmatrix}$$

$$\begin{bmatrix} R_1 + sL & R_1 + \frac{\mu}{sC} \\ R_1 & R_1 + R_2 + \frac{1}{sC} \end{bmatrix} \cdot \begin{bmatrix} I_{p1} \\ I_{p2} \end{bmatrix} = \begin{bmatrix} -L\, i_L(0) \\ I_0 \cdot R_2 \end{bmatrix}$$

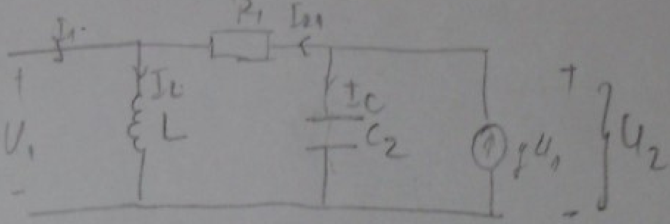
③



$$U_1 = I_1 \cdot z_{11} - I_2 \cdot z_{12}$$

$$U_2 = I_1 \cdot z_{21} - I_2 \cdot z_{22}$$

$$z_{11} = \frac{U_1}{I_1} \bigg|_{I_2=0}$$



$$I_1 + I_{R1} = I_L \quad (1)$$

$$I_C + I_{R1} = gU_1 \quad (2)$$

$$I_C \cdot \frac{1}{sC} - I_{R1} \cdot R_1 = U_1 \quad (3)$$

$$\frac{I_C}{2s} - I_{R1} = U_1 \Rightarrow I_C = 2s(U_1 + I_{R1}) \quad \text{uvrstimo u (2)}$$

$$2s \cdot U_1 + 2s \cdot I_{R1} + I_{R1} = gU_1 \Rightarrow I_{R1}(1+2s) = 2U_1 - 2sU_1 \quad (g=2)$$

$$I_{R1} = U_1 \cdot \left(\frac{2-2s}{1+2s}\right) = U_1 \cdot \frac{(1-s)}{\left(s+\frac{1}{2}\right)}$$

$$I_L = \frac{U_1}{sL} = \frac{U_1}{s}$$

$$I_1 = I_L - I_{R1} = \frac{U_1}{s} - \frac{U_1 (1-s)}{s+\frac{1}{2}} = \frac{U_1}{s} + \frac{U_1 \cdot (s-1)}{s+\frac{1}{2}}$$

$$= U_1 \frac{s + \frac{1}{2} + s^2 \cancel{- s}}{s \cdot \left(s+\frac{1}{2}\right)} = U_1 \cdot \frac{s^2 + \frac{1}{2}}{s\left(s+\frac{1}{2}\right)}$$

$$z_{11} = \frac{U_1}{I_1}\Big|_{I_2=0} = \frac{U_1}{U_1\,\frac{s^2+\frac{1}{2}}{s\cdot(s+\frac{1}{2})}} = \frac{s\cdot(s+\frac{1}{2})}{s^2+\frac{1}{2}}$$

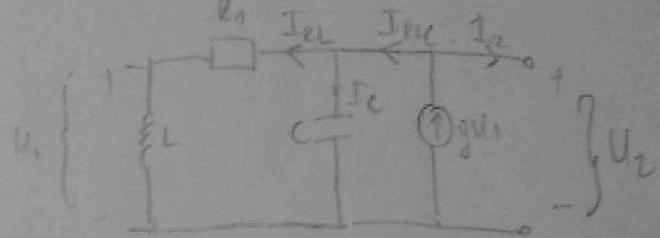
$$z_{21} = \frac{U_2}{I_1}\Big|_{I_2=0} = ?$$

$$U_2 = I_C\,\frac{1}{sC} = \frac{I_C}{2s} = \frac{2s(U_1+I_{R1})}{2s} = U_1 + I_{R1} = U_1 + U_1\cdot\frac{(1-s)}{s(s+\frac{1}{2})} = U_1\,\frac{s^2+\frac{1}{2}s+1-s}{s\cdot(s+\frac{1}{2})}$$

$$= U_1\cdot\frac{s^2-\frac{1}{2}s+1}{s\cdot(s+\frac{1}{2})}$$

$$z_{21} = \frac{U_1\cdot\frac{s^2-\frac{1}{2}s+1}{s(s+\frac{1}{2})}}{U_1\cdot\frac{s^2+\frac{1}{2}}{s\cdot(s+\frac{1}{2})}} = \frac{s^2-\frac{1}{2}s+1}{s^2+\frac{1}{2}}$$

$$z_{12} = -\frac{U_1}{I_2}\Big|_{I_1=0}$$



$$I_{RL} + I_C = I_{RLC}$$

$$I_2 = gU_1 - I_{RLC}$$

$$I_{RL} = I_{RLC}\cdot\frac{\frac{1}{sC}}{\frac{1}{sC}+R_1+sL} = I_{RLC}\cdot\frac{\frac{1}{2s}}{\frac{1}{2s}+1+s} = I_{RLC}\,\frac{\frac{1}{2s}}{\frac{2s^2+2s+1}{2s}}$$

$$I_{RL} = \frac{U_1}{sL} = \frac{U_1}{s}$$

$$\Rightarrow I_{RLC} = I_{RL}\,(2s^2+2s+1) = \frac{U_1}{s}\,(2s^2+2s+1)$$

$$I_2 = 2U_1 - U_1\,\frac{2s^2+2s+1}{s} = U_1\,\frac{-2s^2-1}{s}$$

$$z_{12} = \frac{-\cancel{U_1}}{\cancel{U_1}\cdot\frac{(-2s^2-1)}{s}} = \frac{s}{2s^2+1}$$

$$U_2 = U_1 + I_{RL}\cdot R_1 = U_1 + \frac{U_1}{s}\cdot 1 = U_1\,\frac{s+1}{s}$$

$$z_{22} = -\frac{U_2}{I_2}\Big|_{I_1=0} = \frac{-U_1\cdot\frac{s+1}{s}}{U_1\cdot\frac{-2s^2-1}{s}} = \frac{s+1}{2s^2+1}$$

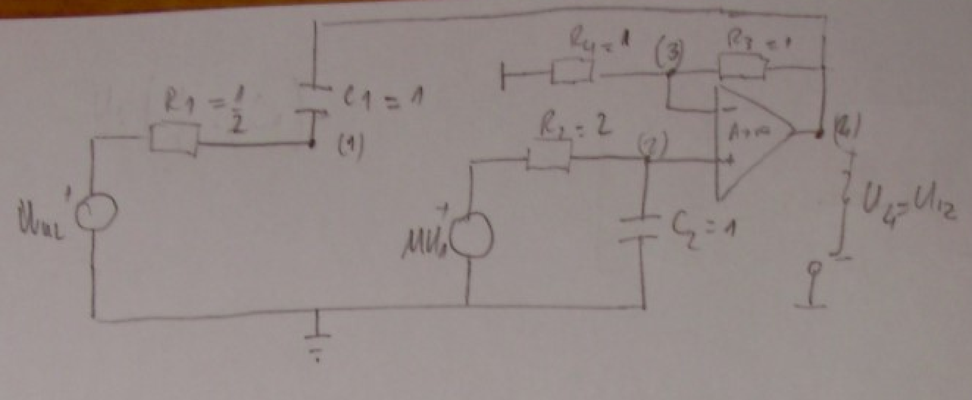
5 network

$$z = \begin{bmatrix} \frac{s(s+\frac{1}{2})}{s^2+\frac{1}{2}} & -\frac{s}{2s^2+1} \\ \frac{s^2+\frac{1}{2}s+1}{s^2+\frac{1}{2}} & -\frac{s+1}{2s^2+1} \end{bmatrix}$$

a) Budući da je $z_{12} \neq z_{21}$, četveropol nije recipročan zbog zavisnog strujnog izvora).

b) Zbog $z_{11} \neq z_{22}$ četveropol nije simetričan.

4.



(1) $U_1\left(\frac{1}{R_1} + sC_1\right) - U_4 \cdot sC_1 = \frac{U_{ul}}{R_1} \Rightarrow U_1 \cdot (2+s) - U_4 \cdot s = 2U_{ul}$

(2) $U_2 \cdot \left(\frac{1}{R_2} + sC_2\right) = \frac{\mu U_1}{R_2} \Rightarrow U_2\left(\frac{1}{2} + s\right) = \frac{U_1}{2}$

(3) $U_3 \cdot \left(\frac{1}{R_4} + \frac{1}{R_3}\right) - U_4 \frac{1}{R_3} = 0 \qquad 2U_3 = U_4$

(4) $U_2 = U_3$

---

iz (2) $U_1 = 2 \cdot U_2 \cdot \left(\frac{2s+1}{2}\right) = U_2(2s+1)$ (4)

iz (3) i (4) $U_2 = U_3 = \frac{U_4}{2}$; $U_1 = U_2(2s+1) = U_4\left(s+\frac{1}{2}\right)$ (5)

Uvrstimo (5) u (1)

$$U_4 \cdot \left(s+\frac{1}{2}\right)(s+2) - U_4 s = 2U_{ul}$$

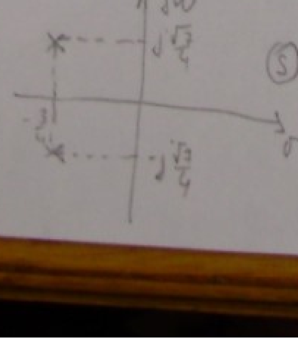
$$U_4\left(s^2 + \frac{5}{2}s + 1 - s\right) = 2U_{ul}$$

$$U_4 \cdot \left(s^2 + \frac{3}{2}s + 1\right) = 2U_{ul}$$

$$T(s) = \frac{U_{iz}}{U_{ul}} = \frac{U_4}{U_{ul}} = \frac{2}{s^2 + \frac{3}{2}s + 1}$$

nule : $s_{01} = s_{02} = \infty$

polovi $s^2 + \frac{3}{2}s + 1 = 0 \qquad s_{p1,2} = \frac{-\frac{3}{2} \pm \sqrt{\frac{9}{4} - 4}}{2} = \frac{-\frac{3}{2} \pm j\sqrt{\frac{7}{4}}}{2} = \frac{-3}{4} \pm j\frac{\sqrt{7}}{4}$

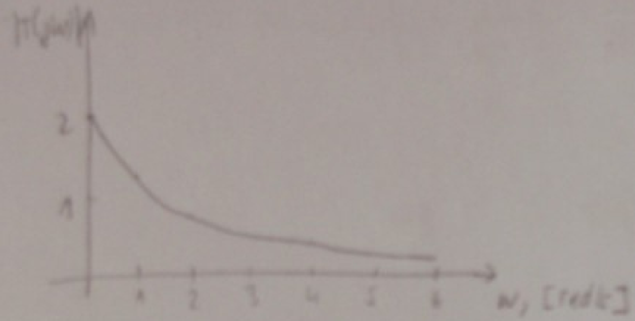


5

b) $T(j\omega) = \frac{2}{1-\omega^2 + \frac{3}{2}j\omega}$

Amplitudno frekvencijska karakteristika:

$$|T(j\omega)| = \frac{2}{\sqrt{(1-\omega^2)^2 + \left(\frac{3}{2}\omega\right)^2}}$$



⑤ $L = 4\,mH/km$
$C = 8\,nF/km$
$l = 40\,km$

Linija bez gubitaka $\gamma = s\cdot\sqrt{LC} = s\cdot\beta$

Za sinusnu pobudu $\gamma = \alpha + j\beta = j\beta = j\underbrace{\omega\sqrt{LC}}_{\beta}$

a) $l = \frac{\lambda}{4} \Rightarrow \lambda = 4l$

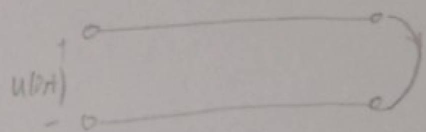
$\frac{2\pi}{\beta} = \lambda$; $\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega}$

$\frac{2\pi}{\omega_1\sqrt{LC}} = \lambda \Rightarrow \omega_1 = \frac{2\pi}{\lambda\sqrt{LC}} = \frac{2\pi}{4l\cdot\sqrt{LC}} = \frac{2\pi}{4l\sqrt{LC}} = 6942\ \frac{rad}{s}$

b) $Z_0 = \sqrt{\frac{L}{C}} = 707{,}1\ \Omega$

$v = \frac{\omega_1}{\omega_1\beta} = \frac{1}{\sqrt{LC}} = 176\,776{,}7\ \frac{km}{s}$

c) $u(0,t) = 2\sin(10^5 t + \frac{\pi}{6})$, fazor $U(0) = 2\angle\pi/6$



$\gamma = j\beta = j\omega\sqrt{LC}$

Prijenosne jednadžbe

$U(0) = U(l)ch(\gamma l) + Z_0 I(l) sh(\gamma l)$

$I(0) = \frac{U(l)}{Z_0} sh(\gamma l) + I(l) ch(\gamma l)$

Za kratki spoj na izlazu $U(l) = 0$

Ulazna impedancija $Z(0) = \frac{U(0)}{I(0)} = \frac{Z_0\cdot I(l)\,sh(\gamma l)}{I(l)\,ch(\gamma l)} = Z_0\cdot\frac{sh(j\beta l)}{ch(j\beta l)} =$

$= Z_0\cdot\frac{j\sin(\beta l)}{\cos\beta l} = j\cdot Z_0\cdot tg(\beta l) = j\cdot 522{,}38 = 522{,}38\angle\pi/2$

$Z(0) = 522{,}38\angle\pi/2$

(5) nastavak

$$I(0) = \frac{U(0)}{Z(0)} = \frac{2\angle \pi/6}{522,35\angle \pi/2} = 0,00383\angle -\frac{\pi}{3}$$

$$i(0,t) = 0,00383 \sin\left(10^5 t - \frac{\pi}{3}\right)$$

$$u(l,t) = 0 \quad \text{(zbog kratkog spoja)}$$

$$I(l) = -\frac{U(0)}{Z_0} \cdot sh(\gamma l) + I(0) \cdot ch(\gamma l) \quad \dots \text{prijenosna jednadžba}$$

$$I(l) = -\frac{U(0)}{Z_0} \cdot j \sin(\beta l) + I(0) \cdot \cos(\beta l)$$

$$= -\frac{2}{522,35\cancel{j}} \cancel{j} \sin(\omega \cdot \sqrt{LC} \cdot l) + (0,001915 - 0,003316j)(-0,804)$$

$$= \dots = 0,004676\angle -35,08^\circ$$

$$i(l,t) = 0,004676 \sin(10^5 t - 35,08^\circ)$$