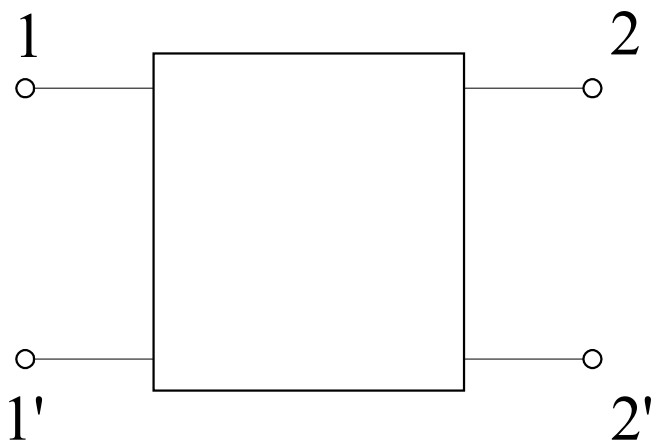
Električni krugovi

Četveropoli

Skripta: M. Plohl, Teorija četveropolnih sistema, 1987. , I-dio

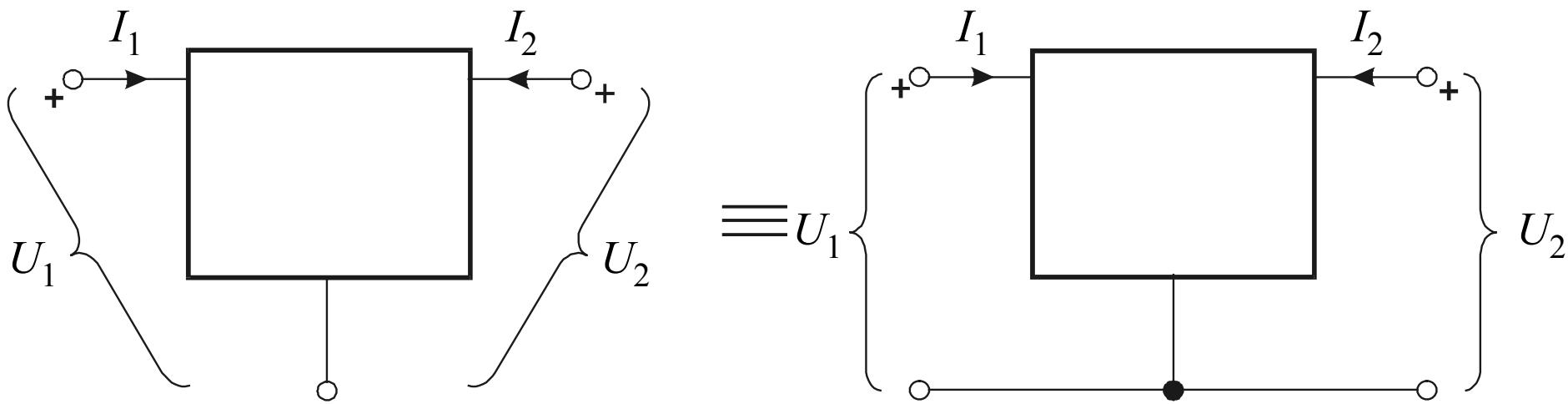
Uvod u teoriju četveropola

- Četveropol je element s 4 priključnice ili pola.



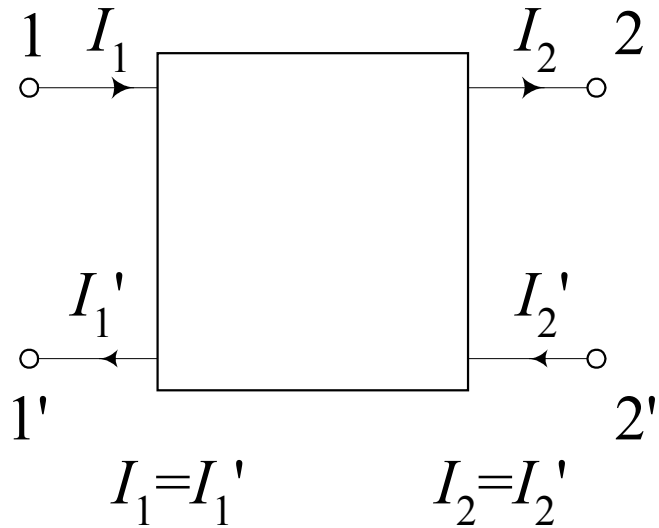
- Priključnice četveropola su uređene kao 2 para polova.
- Svaki par polova čini jedan prilaz.
- Zato se četveropoli nazivaju i ***dvoprilaznim mrežama.***

- Primjer: Element s 3 priključnice →



- Dvoprilazni element → jedna priključnica zajednička.

- Prilaz → par polova sa svojstvom → trenutni iznos struje koja ulazi na jedan = struji koja izlazi kroz drugi.

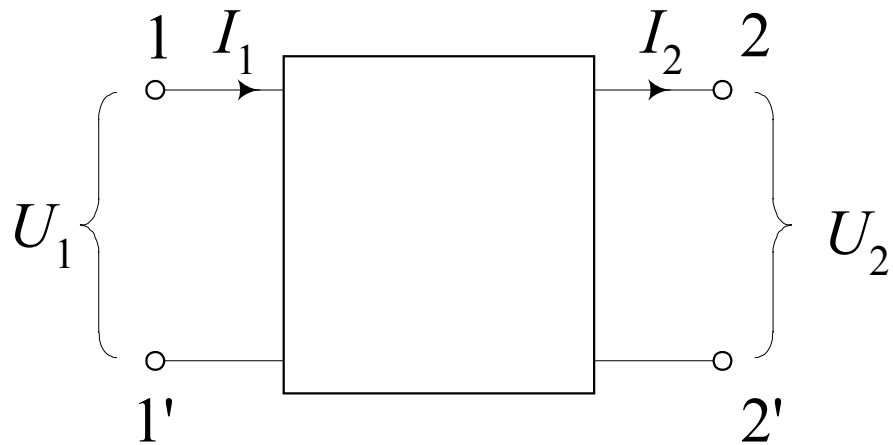


- U nastavku:
- Pojam ***četveropol*** označavat će ***dvoprilaznu mrežu***.

- Četveropol može biti
  → element mreže
  → submreža.
- Analizu kruga moguće je pojednostavniti ako se pojedini dijelovi kruga promatraju kao ***submreže.***
- Svaka submreža → složeni element s više priključnica.
- U pravilu → submreža gledana s vanjskih priključnica
  → jednostavnija za analizu.
- Posebna pogodnost → u slučajevima više istih submreža.

- Mreže s dva prilaza pojavljuju se u mnogim primjenama.
- Jedan od prilaza obično je označen kao ulaz.
- Drugi predstavlja izlaz signala

- Ulaz



- Izlaz

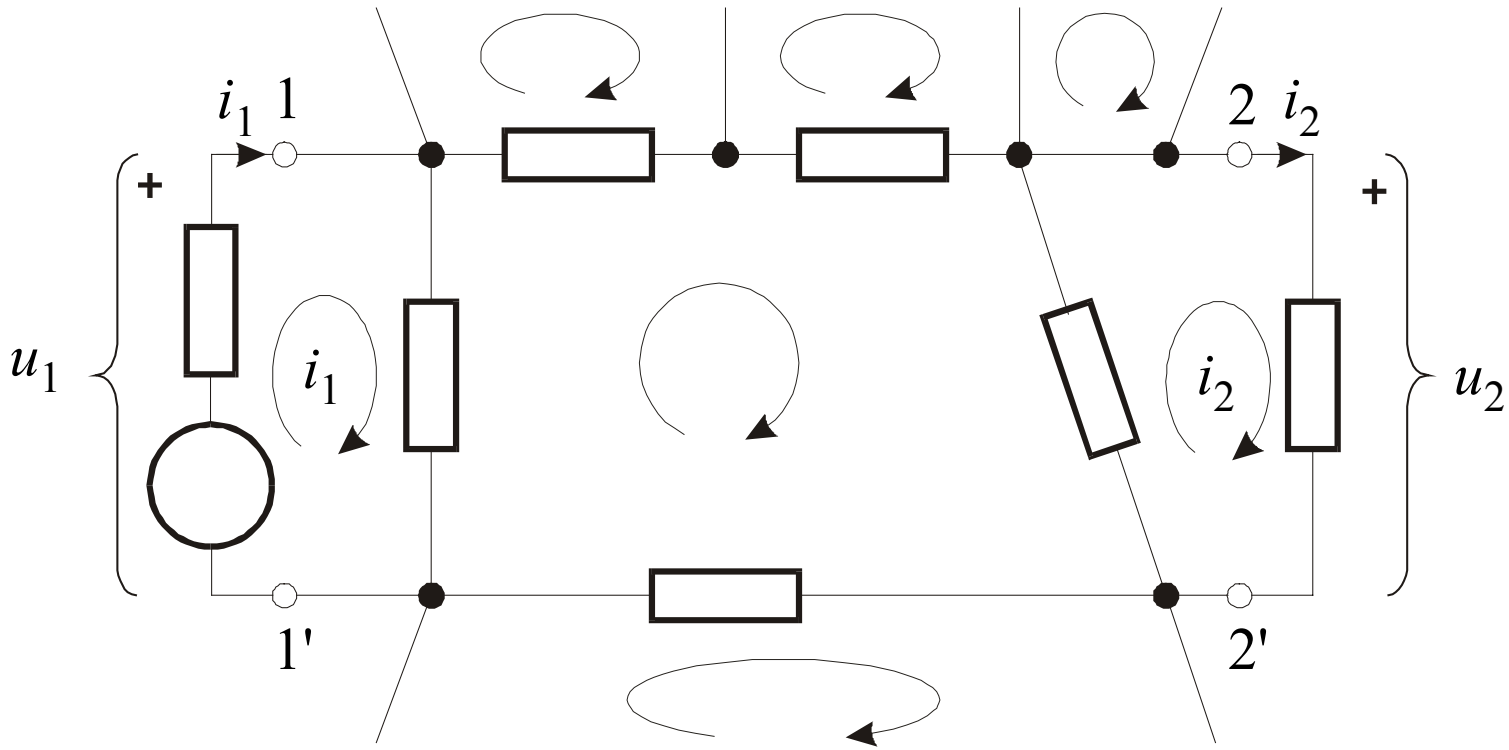
- Ulazne polove označavamo kao 1-1’.
- Izlazne polove označavamo kao 2-2’.

- Jednadžbe četveropola

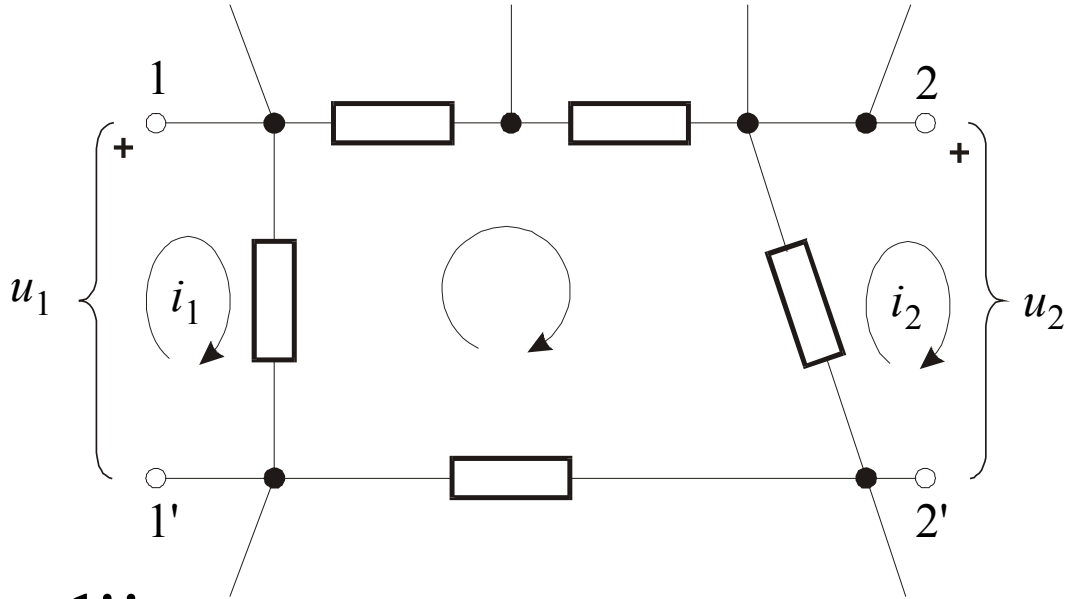
- Primjer:

  → mreža s pobudom na 1-1'

  → odziv na 2-2'



- Napon i struja na prilazima → varijable četveropola



- Jednadžbe petlji:

$$U_1 = I_1 Z_{11} - I_2 Z_{12} - I_3 Z_{13} - \ldots - I_n Z_{1n}$$

$$U_2 = -I_1 Z_{21} + I_2 Z_{22} - I_3 Z_{23} - \ldots - I_n Z_{2n}$$

$$0 = -I_1 Z_{31} - I_2 Z_{32} + I_3 Z_{33} - \ldots - I_n Z_{3n}$$

$$\vdots$$

$$0 = -I_1 Z_{n1} - I_2 Z_{n2} - I_3 Z_{n3} - \ldots + I_n Z_{nn}$$

Rješenja jednadžbi petlji: → struje I_1 i I_2

$$I_1 = \left(\frac{\Delta_{11}}{\Delta}\right)_z U_1 - \left(\frac{\Delta_{21}}{\Delta}\right)_z U_2$$

$$I_2 = \left(\frac{\Delta_{12}}{\Delta}\right)_z U_1 - \left(\frac{\Delta_{22}}{\Delta}\right)_z U_2$$

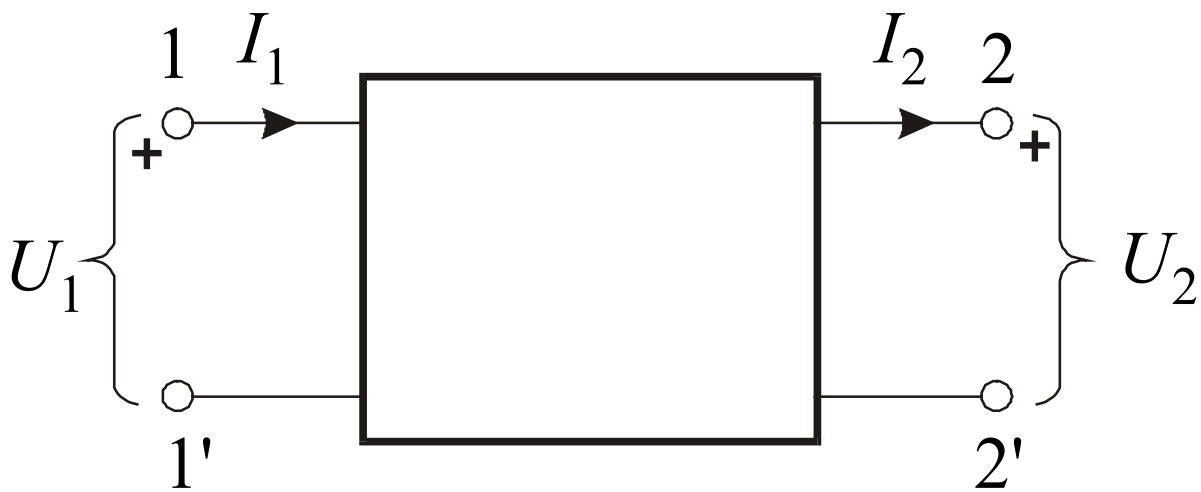
→ elementi u determinantama su impedancije

$\Delta \rightarrow$ determinanta sustava

$\Delta_{ij} \rightarrow$ kofaktori det. sustava

$\frac{\Delta_{ij}}{\Delta} \rightarrow$ ima dimenziju $\frac{1}{\Omega}$

- Cijelu je mrežu moguće promatrati kao

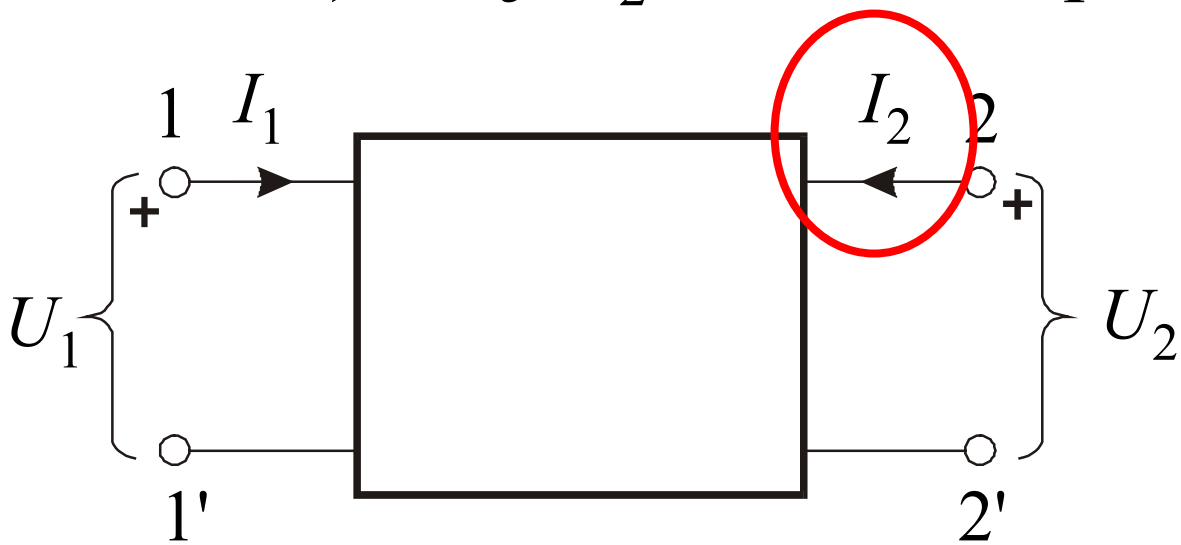


$$I_1 = y_{11}U_1 - y_{12}U_2$$
$$I_2 = y_{21}U_1 - y_{22}U_2$$

Strujne jednadžbe četveropola

$y_{11}, y_{12}, y_{21}, y_{22}$ → ***y-parametri četveropola***

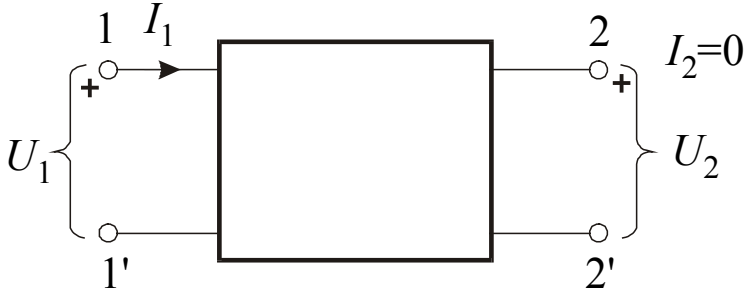
Ponekad je (u literaturi) struja I_2 okrenuta suprotno:



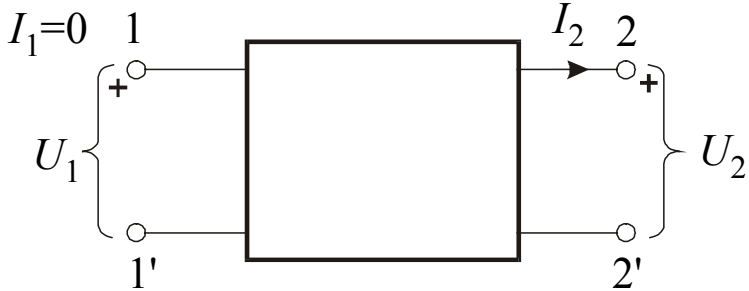
$$I_1 = y_{11}U_1 + y_{12}U_2$$

$$I_2 = y_{21}U_1 + y_{22}U_2$$

- <u>Određivanje parametara četveropola:</u>
- Parametri četveropola → iz uvjeta na prazno i na kratko.
- Četveropol na prazno →prazni hod na prilazu 1-1’ ili 2-2’

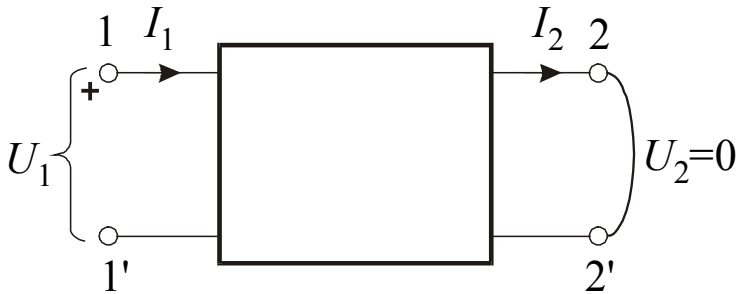


2-2’ otvorene I_2=0

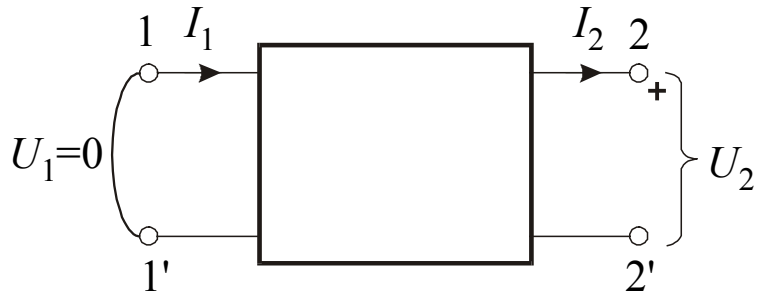


1-1’ otvorene I_1=0

- Četveropol na kratko → kratki spoj na prilazu 1-1’ ili 2-2’



2-2’ kratko spojene U_2=0



1-1’ kratko spojene U_1=0

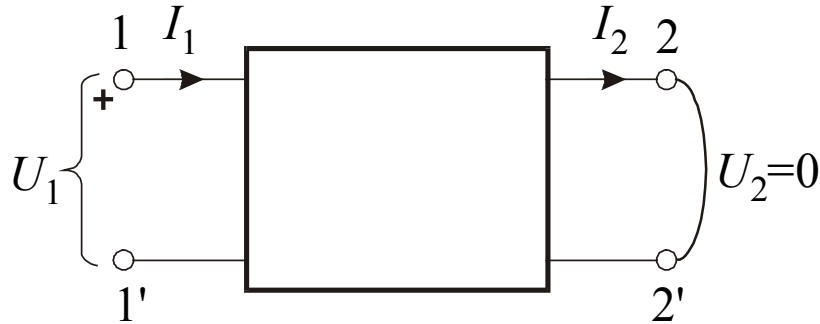
Strujne jednadžbe → *y*-parametri četveropola

- *y*-parametri → iz četveropola na kratko

$$I_1 = U_1 y_{11} - U_2 y_{12}$$

$$I_2 = U_1 y_{21} - U_2 y_{22}$$

- $U_2=0$ → kratki spoj na 2-2'



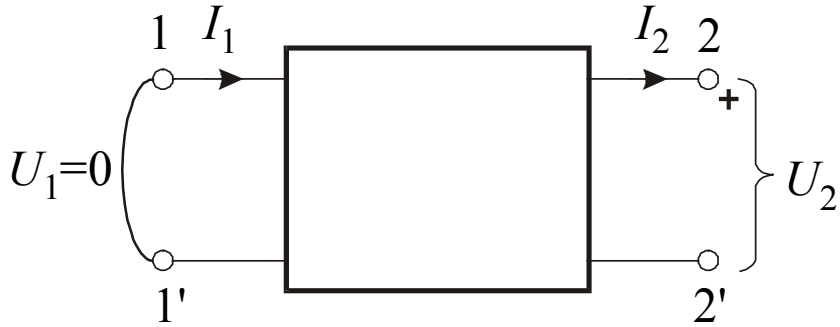
$$I_1 = U_1 y_{11} \quad \Rightarrow \quad y_{11} = \left.\frac{I_1}{U_1}\right|_{U_2=0}$$

ulazna admitancija na kratko $U_2=0$

$$I_2 = U_1 y_{21} \quad \Rightarrow \quad y_{21} = \left.\frac{I_2}{U_1}\right|_{U_2=0}$$

prijenosna admitancija na kratko $U_2=0$

uz $U_1=0$ → kratki spoj na 1-1'



$$I_1 = -U_2 y_{12} \quad \Rightarrow \quad y_{12} = -\left.\frac{I_1}{U_2}\right|_{U_1=0}$$

prijenosna admitancija na kratko $U_1=0$

$$I_2 = -U_2 y_{22} \quad \Rightarrow \quad y_{22} = -\left.\frac{I_2}{U_2}\right|_{U_1=0}$$

ulazna admitancija na kratko $U_1=0$

- Ako je četveropol recipročan

$$\Delta_{12} = \Delta_{21} \text{ pa je } \rightarrow \boxed{y_{12} = y_{21}}$$

uvjet recipročnosti četveropola

- Strujne jednadžbe četveropola u matričnome obliku

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = [y] \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$[y] = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix}$$

Matrica y-parametara

Naponske jednadžbe → *z*-parametri četveropola

- Strujne jednadžbe → struje I_1 i I_2 iz danih napona U_1 i U_2
- Problem: odrediti napone U_1 i U_2 za zadane struje I_1 i I_2.

$$\begin{aligned} I_1 &= U_1 y_{11} - U_2 y_{12} \\ I_2 &= U_1 y_{21} - U_2 y_{22} \end{aligned} \quad \rightarrow$$

$$U_1 = \frac{y_{22}}{D_y} I_1 - \frac{y_{12}}{D_y} I_2$$

$$U_2 = \frac{y_{21}}{D_y} I_1 - \frac{y_{11}}{D_y} I_2$$

$$D_y = y_{11}y_{22} - y_{12}y_{21} = \begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}$$

$$U_1 = I_1 z_{11} - I_2 z_{12}$$
$$U_2 = I_1 z_{21} - I_2 z_{22}$$

- ***Naponske jednadžbe četveropola***

$z_{11}, z_{12}, z_{21}, z_{22}, \rightarrow$ ***z - parametri četveropola***

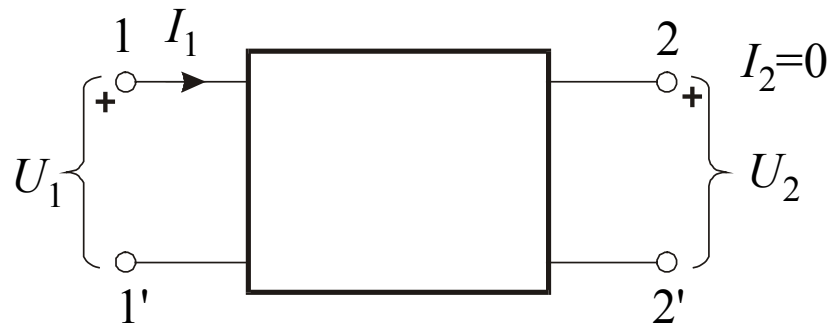
Za recipročni četveropol → $z_{21} = z_{12}$

uvjet recipročnosti četveropola

z-parametri → iz četveropola na prazno

$$U_1 = I_1 z_{11} - I_2 z_{12}$$

$$U_2 = I_1 z_{21} - I_2 z_{22}$$



- I_2=0 priključnice 2-2' otvorene

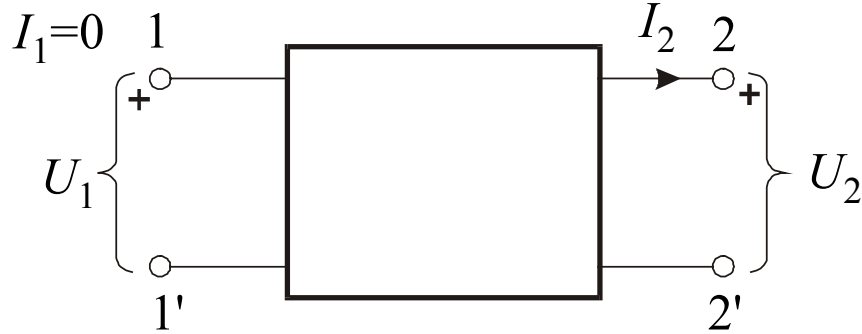
$$U_1 = I_1 z_{11} \quad \Rightarrow \quad z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0}$$

ulazna impedancija na prazno I_2=0

$$U_2 = I_1 z_{21} \quad \Rightarrow \quad z_{21} = \left.\frac{U_2}{I_1}\right|_{I_2=0}$$

prijenosna impedancija na prazno I_2=0

- $\underline{I_1=0}$ priključnice 1-1' otvorene



$$U_1 = -I_2 z_{12} \quad \Rightarrow \quad z_{12} = -\left.\frac{U_1}{I_2}\right|_{I_1=0}$$

prijenosna impedancija na prazno $I_1=0$

$$U_2 = -I_1 z_{22} \quad \Rightarrow \quad z_{22} = -\left.\frac{U_2}{I_2}\right|_{I_1=0}$$

ulazna impedancija na prazno $I_1=0$

- Naponske jednadžbe četveropola u matričnome obliku

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[z] = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix}$$

Matrica z-parametara

$$[z] = [y]^{-1} \qquad [z] = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix} = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix}^{-1}$$

Prijenosne jednadžbe → *a*-parametri četveropola

- Prijenos signala → s ulaza 1-1' na izlaz 2-2'
- Ovisnost napona U_1 i struje I_1 o U_2 i I_2 =?

$$I_1 = U_1 y_{11} - U_2 y_{12}$$
$$I_2 = U_1 y_{21} - U_2 y_{22}$$

$$\Rightarrow \quad U_1 = U_2 \frac{y_{22}}{y_{21}} + \frac{1}{y_{21}} I_2$$
$$I_1 = \frac{D_y}{y_{21}} U_2 + \frac{y_{11}}{y_{21}} I_2$$

$$U_1 = AU_2 + BI_2$$
$$I_1 = CU_2 + DI_2$$

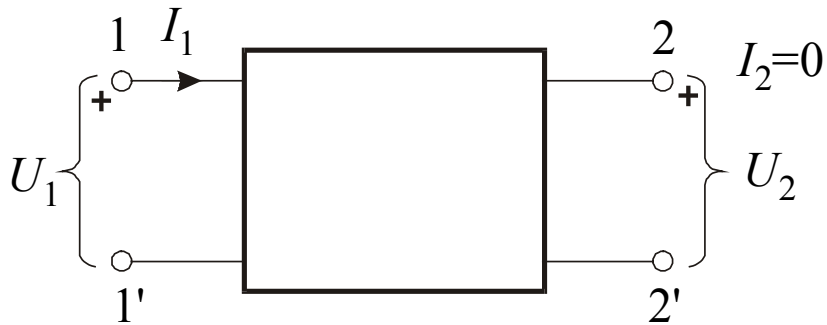
- ***Prijenosne jednadžbe četveropola***

$A, B, C, D, \rightarrow$ ***prijenosni a-parametri četveropola***

- Određivanje prijenosnih parametara
  → iz 2-2' na prazno i na kratko

$$U_1 = AU_2 + BI_2$$

$$I_1 = CU_2 + DI_2$$



- I_2=0 priključnice 2-2' otvorene

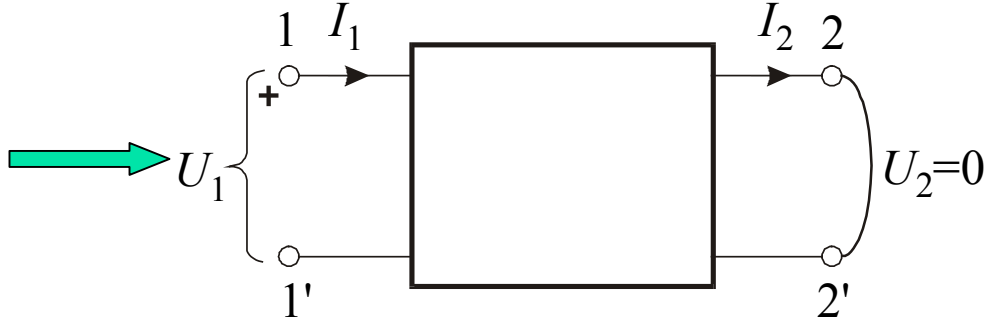
$$U_1 = AU_2 \Rightarrow A = \left.\frac{U_1}{U_2}\right|_{I_2=0}$$

omjer napona na prazno I_2=0

$$I_1 = CU_2 \Rightarrow C = \left.\frac{I_1}{U_2}\right|_{I_2=0}$$

recipročna vrijednost prijenosne impedancije na prazno I_2=0

- $\underline{U_2=0}$ → kratki spoj na 2-2'



$$U_1 = BI_2 \quad \Rightarrow \quad B = \left.\frac{U_1}{I_2}\right|_{U_2=0}$$

recipročna vrijednost prijenosne admitancije na kratko $U_2=0$

$$I_1 = DI_2 \quad \Rightarrow \quad D = \left.\frac{I_1}{I_2}\right|_{U_2=0}$$

omjer prijenosa struja na kratko $U_2=0$

- Prijenosne jednadžbe četveropola u matričnome obliku

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$$

$$[a] = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Matrica prijenosnih parametara

- Prijenos signala → s prilaza 2-2' na prilaz 1-1'
- Ovisnost napona U_2 i struje I_2 o U_1 i I_1 =?

$$\begin{aligned} U_1 &= AU_2 + BI_2 \\ I_1 &= CU_2 + DI_2 \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$$

- Množenjem s lijeva s inverznom prijenosnom matricom

$$\begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \cdot \begin{bmatrix} U_1 \\ I_1 \end{bmatrix}$$

$$U_2 = \frac{D}{\Delta_A} U_1 - \frac{B}{\Delta_A} I_1$$

$$-I_2 = \frac{C}{\Delta_A} U_1 - \frac{A}{\Delta_A} I_1$$

$$\Delta_A = \begin{vmatrix} A & B \\ C & D \end{vmatrix}$$

- Za recipročne četveropole vrijedi: $\Delta_A = \begin{vmatrix} A & B \\ C & D \end{vmatrix} = AD - BC = 1$
- pa je

$$U_2 = DU_1 - BI_1$$

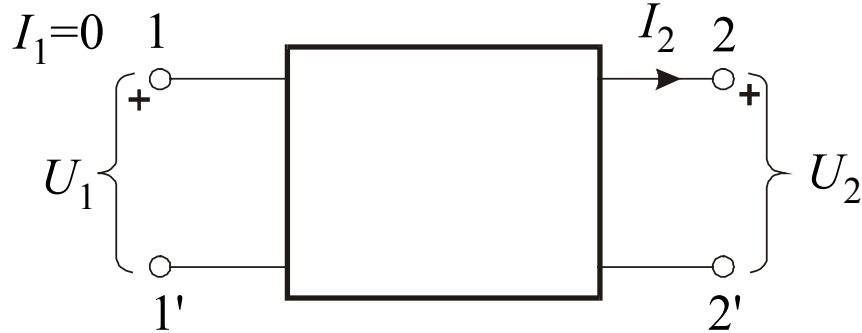
$$-I_2 = CU_1 - AI_1$$

- ***Prijenosne jednadžbe četveropola***

- Prijenosni parametri $\rightarrow$ iz 1-1' na prazno i na kratko

$$U_2 = DU_1 - BI_1$$

$$-I_2 = CU_1 - AI_1$$



- I_1=0 priključnice 1-1' otvorene

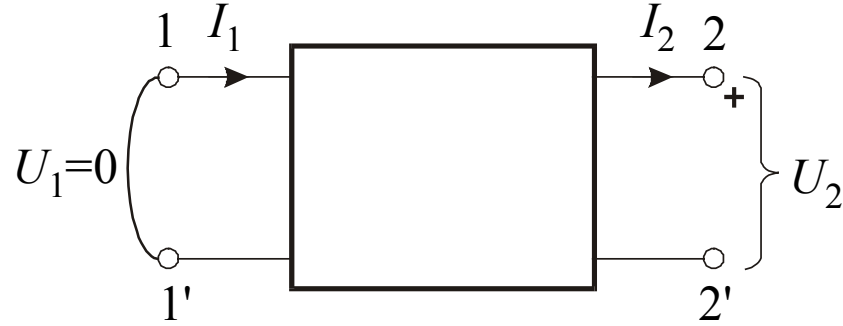
$$U_2 = DU_1 \Rightarrow D = \left.\frac{U_2}{U_1}\right|_{I_1=0}$$

omjer napona na prazno I_1=0

$$-I_2 = CU_1 \Rightarrow C = -\left.\frac{I_2}{U_1}\right|_{I_1=0}$$

recipročna vrijednost prijenosne impedancije na prazno I_1=0

- $\underline{U_1=0}$ → kratki spoj na 1-1'



$$U_2 = -BI_1 \quad \Rightarrow \quad B = -\left.\frac{U_2}{I_1}\right|_{U_1=0}$$

recipročna vrijednost prijenosne admitancije na kratko $U_1=0$

$$I_2 = AI_1 \quad \Rightarrow \quad A = \left.\frac{I_2}{I_1}\right|_{U_1=0}$$

omjer prijenosa struja na kratko $U_1=0$

Hibridne jednadžbe → *h*-parametri četveropola

Ako su zadane veličine I_1 i U_2 → U_1=? i I_2=?

$$I_1 = U_1 y_{11} - U_2 y_{12}$$
$$I_2 = U_1 y_{21} - U_2 y_{22}$$

$$\Rightarrow$$

$$U_1 = \frac{1}{y_{11}} I_1 + U_2 \frac{y_{12}}{y_{11}}$$

$$I_2 = \frac{y_{21}}{y_{11}} I_1 + \frac{y_{12}y_{21} - y_{11}y_{22}}{y_{11}} U_2$$

$$U_1 = h_{11} I_1 + h_{12} U_2$$
$$I_2 = h_{21} I_1 + h_{22} U_2$$

- ***Hibridne jednadžbe četveropola***

$h_{11}, h_{12}, h_{21}, h_{22}$ → ***hibridni h-parametri četveropola***

- Hibridne jednadžbe četveropola u matričnome obliku

$$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_2 \\ U_2 \end{bmatrix}$$

$$[h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

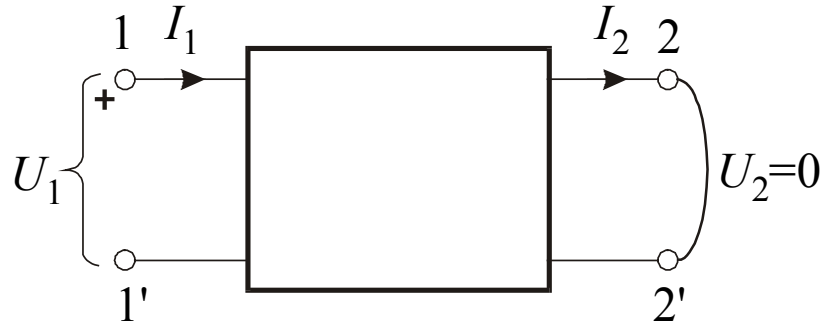
Matrica hibridnih *h*-parametara

- Hibridni *h*-parametri → iz 1-1' na prazno i 2-2' na kratko

$$U_1 = h_{11} I_1 + h_{12} U_2$$

$$I_2 = h_{21} I_1 + h_{22} U_2$$

- $\underline{U_2=0}$ → kratki spoj na 2-2'



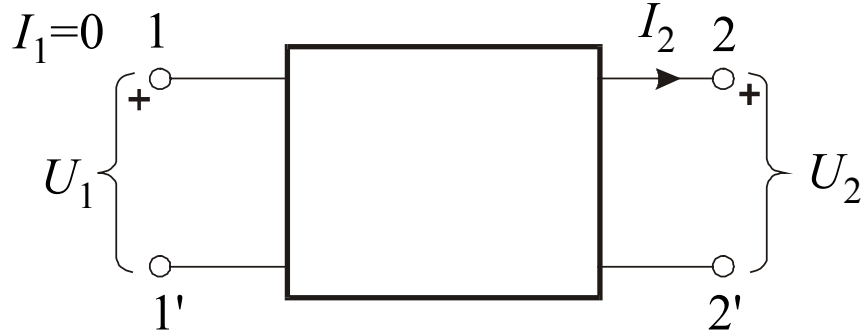
$$U_1 = h_{11} I_1 \quad \Rightarrow \quad h_{11} = \left.\frac{U_1}{I_1}\right|_{U_2=0}$$

Ulazna impedancija na kratko $U_2=0$

$$I_2 = h_{21} I_1 \quad \Rightarrow \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{U_2=0}$$

prijenosna funkcija struje na kratko $U_2=0$

- $\underline{I_1=0}$ priključnice 1-1' otvorene



$$U_1 = h_{12}U_2 \Rightarrow \quad h_{12} = \left.\frac{U_1}{U_2}\right|_{I_1=0}$$

omjer napona na prazno $I_1=0$

$$I_2 = h_{22}U_2 \Rightarrow \quad h_{22} = \left.\frac{I_2}{U_2}\right|_{I_1=0}$$

ulazna admitancija na prazno $I_1=0$

Hibridne jednadžbe → *g*-parametri četveropola

Neka su zadane veličine U_1 i $I_2 \rightarrow$ U_2=? i I_1=?

- Matrična jednadžba hibridnih *h*-parametara

$$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix} \rightarrow \begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} \cdot \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$$

- dobiva se

$$I_1 = \frac{h_{12}}{\Delta_h} U_1 - \frac{h_{21}}{\Delta_h} I_2$$

$$U_2 = -\frac{h_{12}}{\Delta_h} U_1 + \frac{h_{11}}{\Delta_h} I_2$$

$$\Delta_h = \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix}$$

$$\Delta_h = h_{11}h_{22} - h_{12}h_{21}$$

$$I_1 = g_{11}U_1 + g_{12}I_2$$

$$U_2 = g_{21}U_1 + g_{22}I_2$$

- ***Hibridne jednadžbe četveropola***

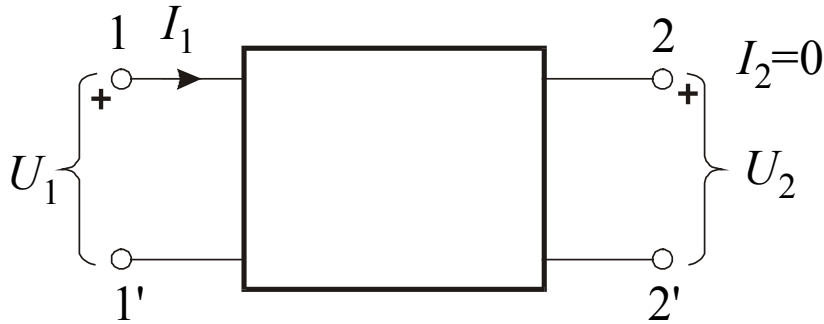
$g_{11}, g_{12}, g_{21}, g_{22}$ → ***hibridni g-parametri četveropola***

- Hibridni *g*-parametri → iz 2-2' na prazno i 1-1' na kratko

$$I_1 = g_{11}U_1 + g_{12}I_2$$

$$U_2 = g_{21}U_1 + g_{22}I_2$$

- $\underline{I_2=0}$ → priključnice 2-2' otvorene



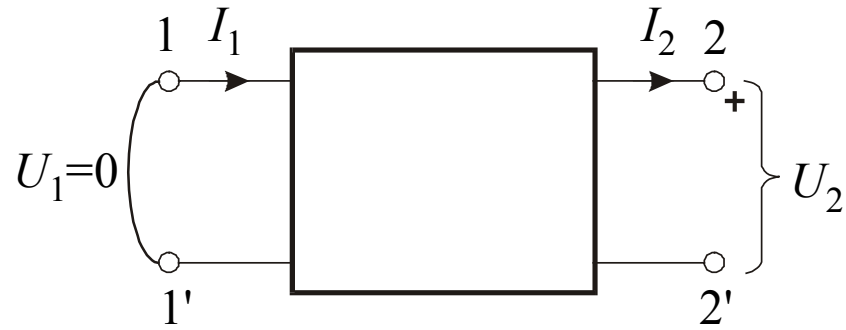
$$I_1 = g_{11}U_1 \quad \Rightarrow \quad g_{11} = \left.\frac{I_1}{U_1}\right|_{I_2=0}$$

Ulazna admitancija na prazno I_2=0

$$U_2 = g_{21}U_1 \quad \Rightarrow \quad g_{21} = \left.\frac{U_2}{U_1}\right|_{I_2=0}$$

prijenosna funkcija napona na prazno I_2=0

- $\underline{U_1=0}$ kratki spoj 1-1'



$$I_1 = g_{12} I_2 \quad \Rightarrow \quad g_{12} = \left.\frac{I_1}{I_2}\right|_{U_1=0}$$

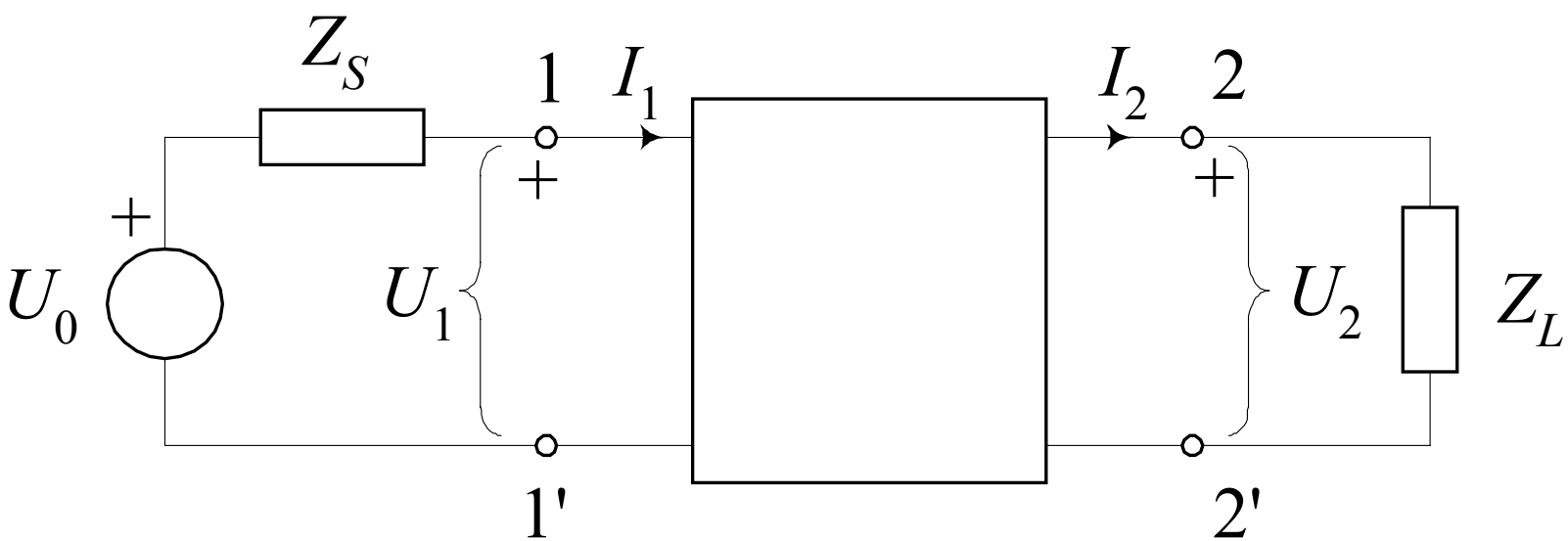
omjer struja na kratko $U_1=0$

$$U_2 = g_{22} I_2 \Rightarrow \quad g_{22} = \left.\frac{U_2}{I_2}\right|_{U_1=0}$$

ulazna impedancija na kratko $U_1=0$

Prijenosne i ulazne funkcije četveropola

- Četveropol → povezuje izvor signala s opterećenjem.
- Mrežu koja daje pobudni signal moguće je nadomjestiti po Theveninu ili Nortonu.
- Opterećenje → impedancija Z_L priključena na 2-2'

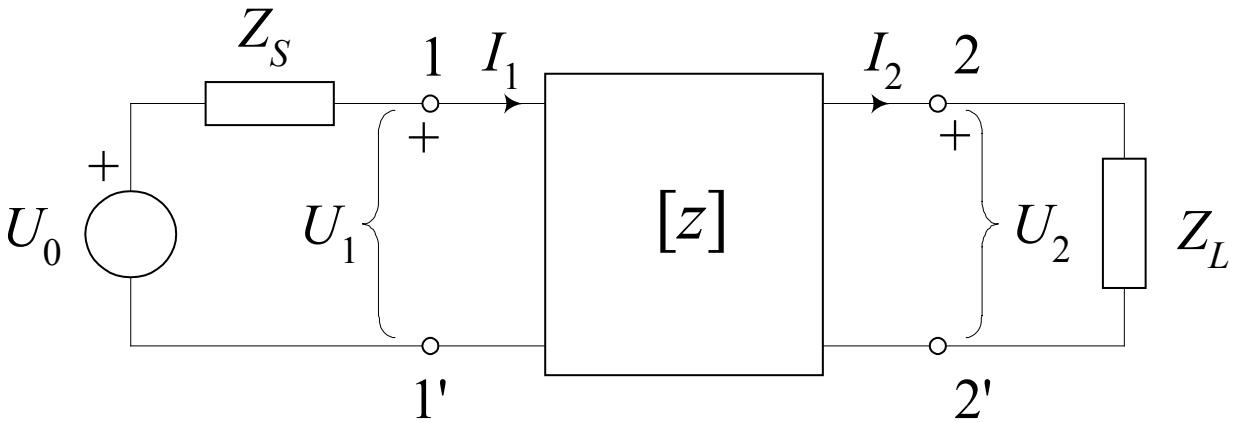


Prijenosne i ulazne funkcije četveropola

- Cilj → analizirati mrežu koristeći parametre četveropola
  →Dobiti rezultate koji vrijede za bilo koji četveropol
- Posebno odrediti funkcije vezane za prilaze četveropola:
  - Prijenosnu funkciju napona $H_u(s)=U_2(s)/U_1(s)$
  - Prijenosnu funkciju struje $H_i(s)=I_2(s)/I_1(s)$
  - Ekvivalentnu ulaznu impedanciju $Z_u(s)=U_1(s)/I_1(s)$
  - Ekvivalentnu izlaznu impedanciju $Z_i(s)=-U_2(s)/I_2(s)|_{U_0=0}$

Prijenosne funkcije četveropola

Prijenosne funkcije izražene z-parametrima



$$U_1 = I_1 z_{11} - I_2 z_{12}$$

$$U_2 = I_1 z_{21} - I_2 z_{22}$$

- Četveropol je na 2-2' zaključen s Z_L

$$U_2 = I_2 Z_L \Rightarrow U_2 = I_1 z_{21} - I_2 z_{22} \Rightarrow I_2 Z_L = I_1 z_{21} - I_2 z_{22}$$

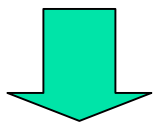
- Prijenosna funkcija struje

$$H_i(s) = \frac{I_2}{I_1} = \frac{z_{21}}{Z_L + z_{22}}$$

- Naponska prijenosna funkcija → iz prve jednadžbe

$$U_1 = I_1 z_{11} - I_2 z_{12} = I_1 z_{11} - I_1 H_i(s) z_{12}$$

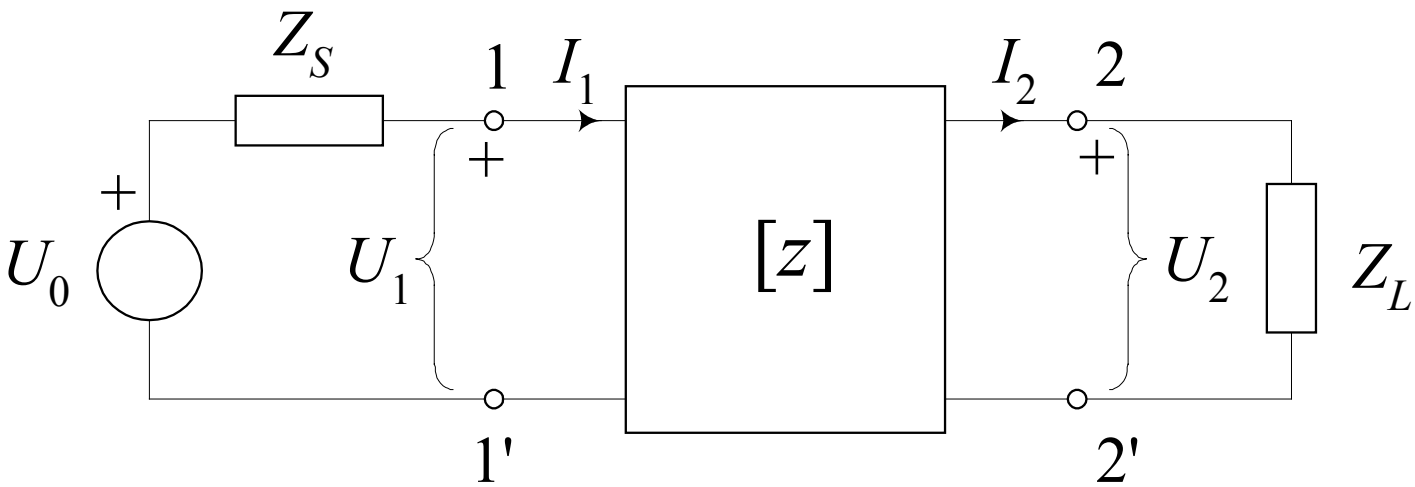
$$U_2 = I_1 z_{21} - I_2 z_{22} = I_1 z_{21} - I_1 H_i(s) z_{22}$$



$$H_u(s) = \frac{U_2}{U_1} = \frac{Z_L z_{21}}{z_{11}(z_{22} + Z_L) - z_{12} z_{21}} = \frac{Z_L z_{21}}{\Delta_z + z_{11} Z_L}$$

$$\Delta_z = z_{11} z_{22} - z_{12} z_{21}$$

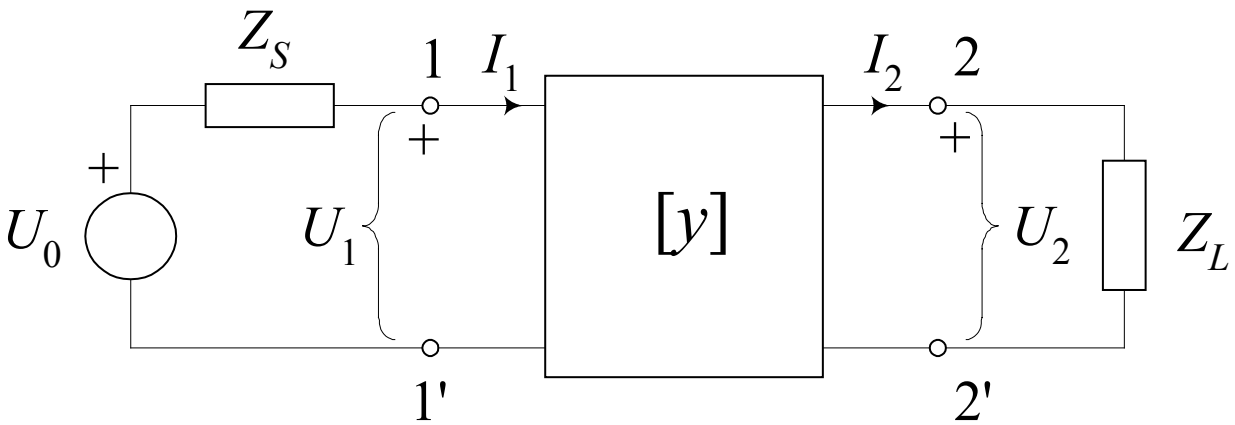
- Prijenosna funkcija U_2/U_0



$$U_0(s) = U_1(s) + I_1(s) Z_s$$

$$H_u(s) = \frac{U_2}{U_0} = \frac{Z_L z_{21}}{(z_{11} + Z_s)(z_{22} + Z_L) - z_{12} z_{21}}$$

Prijenosne funkcije izražene y-parametrima



$$I_1 = y_{11}U_1 - y_{12}U_2$$
$$I_2 = y_{21}U_1 - y_{22}U_2$$

- Za napon U_2 i struju I_2 vrijedi

$$U_2 = I_2 Z_L \Rightarrow I_2 = y_{21}U_1 - y_{22}U_2 \Rightarrow U_2 Y_L = y_{21}U_1 - y_{22}U_2$$

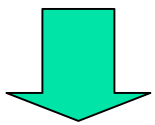
- Prijenosna funkcija napona

$$H_u(s) = \frac{U_2}{U_1} = \frac{y_{21}}{Y_L + y_{22}}$$

- Strujna prijenosna funkcija

$$I_1 = U_1 y_{11} - U_2 y_{12} = U_1 y_{11} - U_1 H_u(s) y_{12}$$

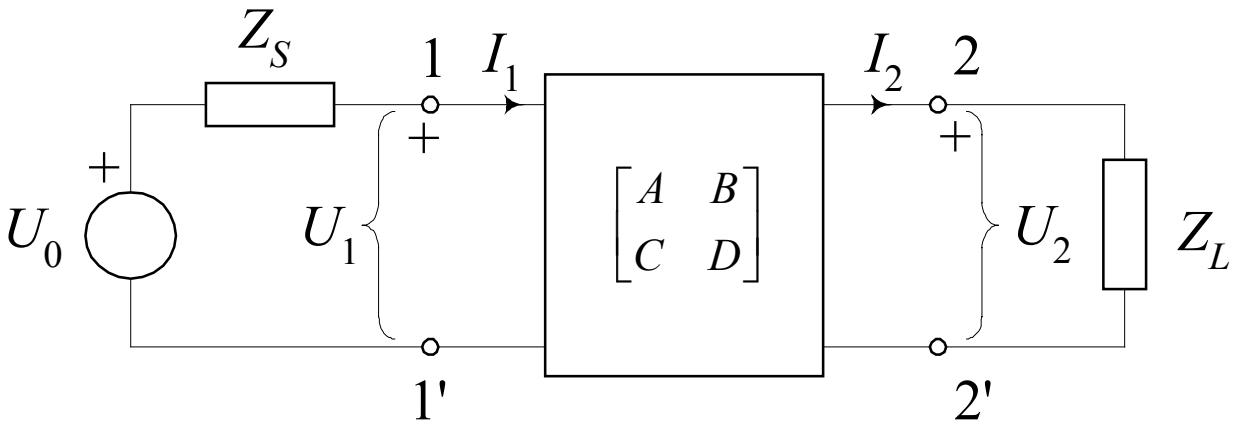
$$I_2 = U_1 y_{21} - U_2 y_{22} = U_1 y_{21} - U_1 H_u(s) y_{22}$$



$$H_i(s) = \frac{I_2}{I_1} = \frac{Y_L y_{21}}{y_{11}(y_{22} + Y_L) - y_{12} y_{21}} = \frac{Y_L y_{21}}{\Delta_y + y_{11} Y_L}$$

$$\Delta_y = y_{11} y_{22} - y_{12} y_{21}$$

Prijenosne funkcije izražene prijenosnim parametrima



$$U_1 = AU_2 + BI_2$$
$$I_1 = CU_2 + DI_2$$

- Za napon U_2 i struju I_2 vrijedi

$$U_2 = I_2 Z_L \Rightarrow U_1 = AU_2 + BI_2 \Rightarrow U_1 = AU_2 + B\frac{U_2}{Z_L}$$

- Prijenosna funkcija napona je

$$H_u(s) = \frac{U_2}{U_1} = \frac{Z_L}{AZ_L + B}$$

- Strujna prijenosna funkcija → iz druge jednadžbe

$$I_1 = CU_2 + DI_2 = CI_2 Z_L + DI_2$$

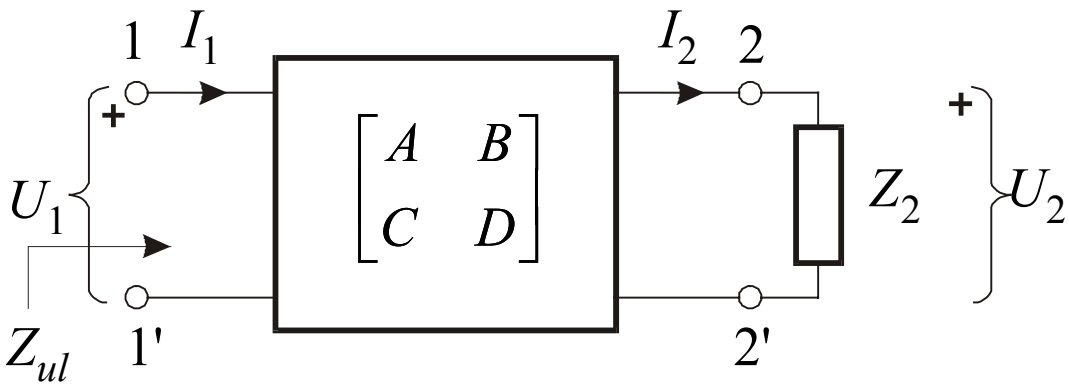
$$H_i(s) = \frac{I_2}{I_1} = \frac{1}{CZ_L + D}$$

- Naponska prijenosna funkcija U_2/U_0

$$U_0 = U_1 + I_1 Z_s = AU_2 + B\frac{U_2}{Z_L} + CU_2 Z_s + D\frac{U_2}{Z_L} Z_s$$

$$H(s) = \frac{U_2}{U_0} = \frac{Z_L}{AZ_L + B + Z_s(CZ_L + D)}$$

Ulazne funkcije četveropola → funkcije impedancije



$$U_2 = I_2 Z_2$$

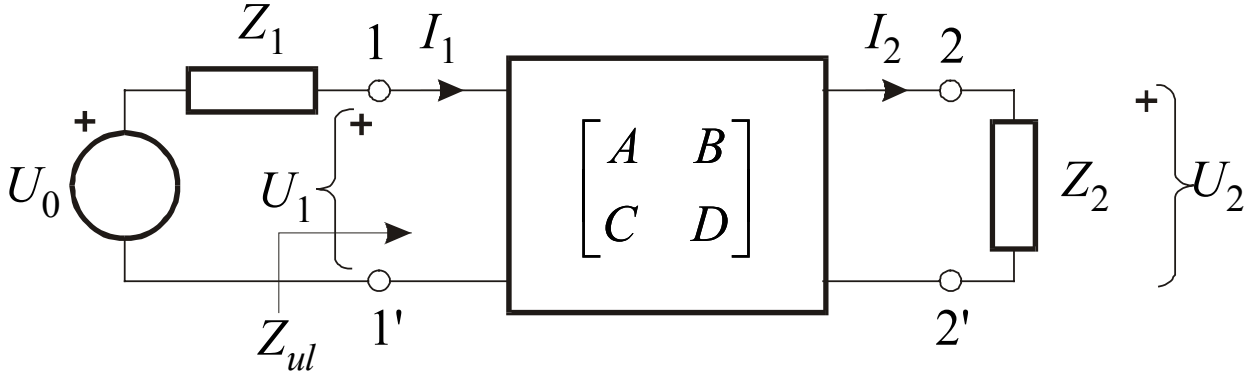
$$U_1 = AU_2 + BI_2 = (AZ_2 + B)I_2$$

$$I_1 = CU_2 + DI_2 = (CZ_2 + D)I_2$$

$$Z_{ul} = \frac{U_1}{I_1} = \frac{AZ_2 + B}{CZ_2 + D}$$

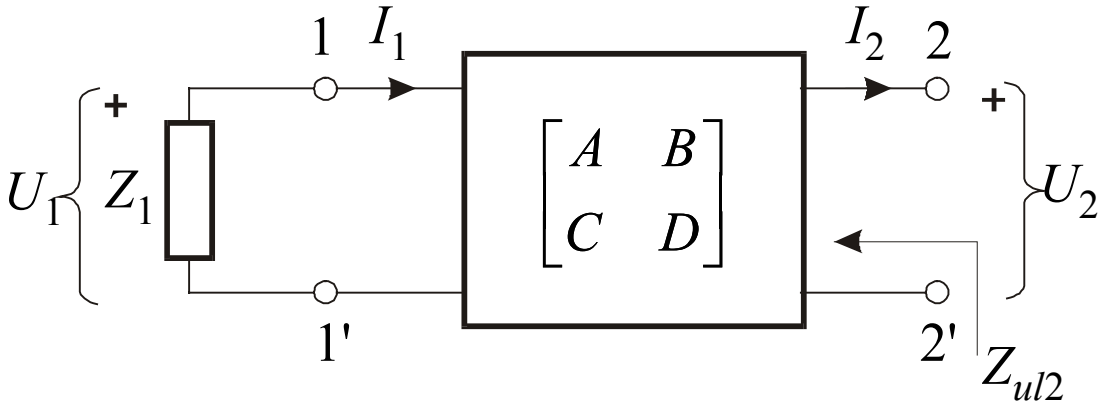
$$Y_{ul} = \frac{1}{Z_{ul}}$$

Prijenosne i ulazne funkcije četveropola



$$I_1 = \frac{U_0}{Z_1 + Z_{ul}} \qquad U_1 = I_1 Z_{ul} = U_0 \frac{Z_{ul}}{Z_1 + Z_{ul}}$$

Ako je izvor na 2-2' → $Z_{ul2} = Z_{iz}$



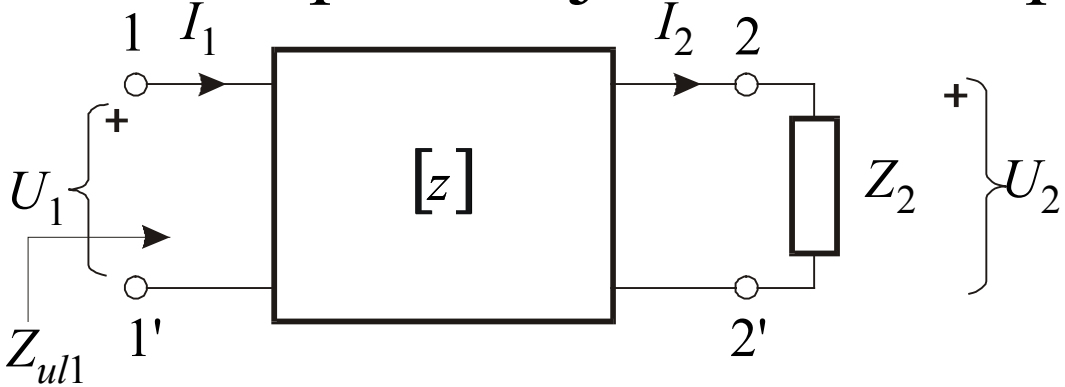
$$U_1 = -Z_1 I_1$$

$$U_2 = DU_1 - BI_1 = -(DZ_1 + B)I_1$$

$$-I_2 = CU_1 - AI_1 = -(CZ_1 + A)I_1$$

$$Z_{ul2} = -\frac{U_2}{I_2} = \frac{DZ_1 + B}{CZ_1 + A}$$

Ulazna impedancija izražena z-parametrima



$$U_2 = I_2 Z_2$$

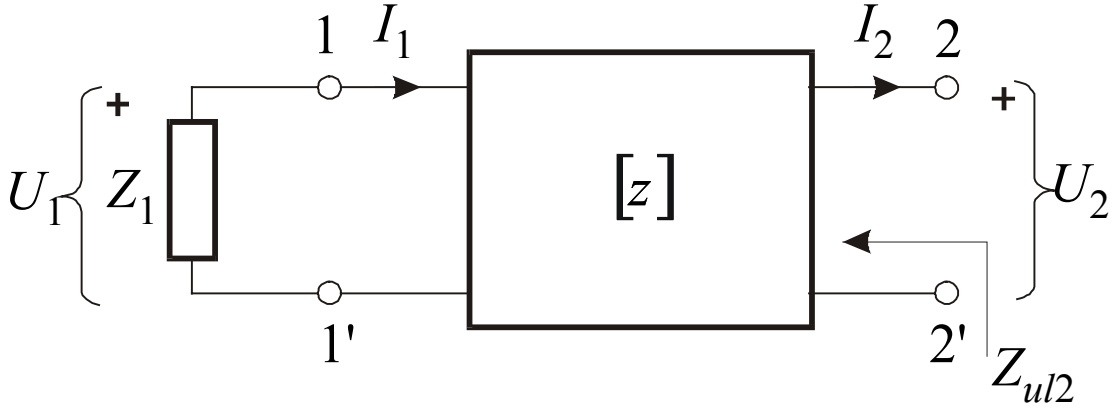
$$U_1 = I_1 z_{11} - I_2 z_{12} \quad \Rightarrow \quad \frac{U_1}{I_1} = z_{11} - z_{12}\frac{I_2}{I_1}$$

$$U_2 = I_1 z_{21} - I_2 z_{22}$$

$$Z_2 I_2 = z_{21} I_1 - z_{22} I_2 \Rightarrow \quad \frac{I_2}{I_1} = \frac{z_{21}}{Z_2 + z_{22}}$$

$$\boxed{Z_{ul1} = \frac{U_1}{I_1} = z_{11} - z_{12} \cdot \frac{z_{21}}{Z_2 + z_{22}}}$$

Prijenosne i ulazne funkcije četveropola



$$U_1 = -Z_1 I_1$$

$$-Z_1 I_1 = I_1 z_{11} - I_2 z_{12} \Rightarrow \qquad \frac{I_1}{I_2} = \frac{z_{12}}{z_{11} + Z_1}$$

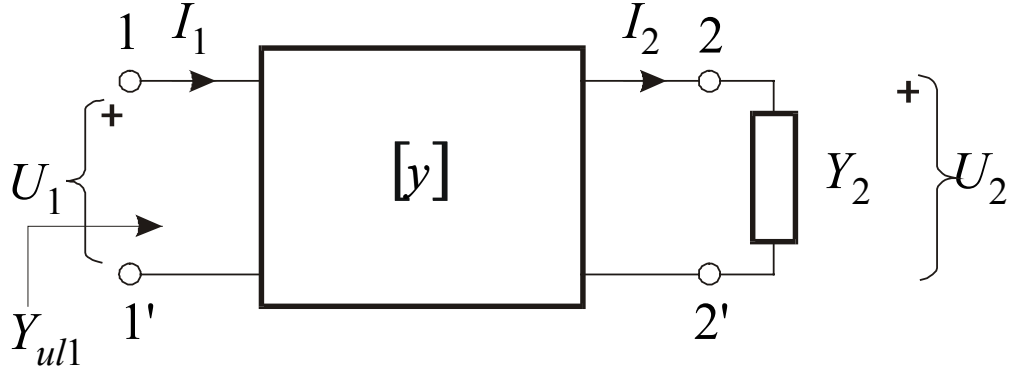
$$-\frac{U_2}{I_2} = -z_{21}\frac{I_1}{I_2} + z_{22}$$

$$\boxed{Z_{ul2} = -\frac{U_2}{I_2} = z_{22} - \frac{z_{12} z_{21}}{z_{11} + Z_1}}$$

Ulazna admitancija izražena *y*-parametrima

$$I_1 = U_1 y_{11} - U_2 y_{12}$$

$$I_2 = U_1 y_{21} - U_2 y_{22}$$



$$I_2 = U_2 Y_2$$

$$Y_{ul1} = \frac{I_1}{U_1} = y_{11} - \frac{U_2}{U_1} y_{12}$$

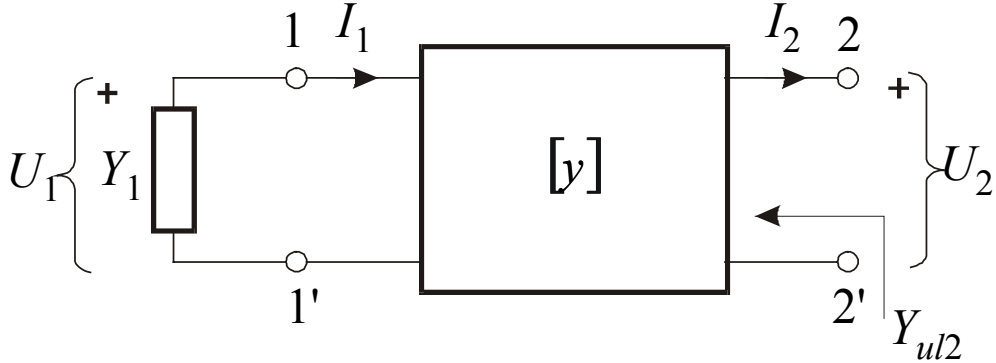
$$U_2 Y_2 = U_1 y_{21} - U_2 y_{22} \quad \Rightarrow \quad \frac{U_2}{U_1} = \frac{y_{21}}{Y_2 + y_{22}}$$

$$\boxed{Y_{ul1} = y_{11} - \frac{y_{12} y_{21}}{Y_2 + y_{22}}}$$

Ulazna admitancija izražena *y*-parametrima

$$I_1 = U_1 y_{11} - U_2 y_{12}$$

$$I_2 = U_1 y_{21} - U_2 y_{22}$$



$$Y_{ul2} = -\frac{I_2}{U_2} = y_{22} - \frac{U_1}{U_2} y_{21}$$

$$I_1 = -U_1 Y_1$$

$$-U_1 Y_1 = U_1 y_{11} - U_2 y_{12} \quad \Rightarrow \quad \frac{U_1}{U_2} = \frac{y_{21}}{Y_1 + y_{11}}$$

$$\boxed{Y_{ul2} = y_{22} - \frac{y_{12} y_{21}}{Y_1 + y_{11}}}$$