Električni krugovi - Lab

Lab 4. Priprema
RLC Električni Krugovi 2. reda

Ime i Prezime: _____
Asistent: _____
Grupa: _____

Napomena: Ukoliko nema dovoljno prostora neka student doda list papira na kojemu će postupak koji je doveo do rješenja. **Lab Pripremu** treba odštampati dvostrano i popuniti je te pričvrstiti dodatnu stranicu papira pomoću spajalice. Popunjena **Lab Priprema** se predaje asistentu na početku laboratorijskih vježbi.

Zadatak: Zadana su četiri slučaja vrijednosti električnih elemenata:

a) R=1kΩ, L=1mH, C=100nF;
b) R=200Ω, L=1mH, C=100nF;
c) R=20Ω, L=1mH, C=100nF;
d) R=0Ω, L=1mH, C=100nF.

1. Za četiri slučaja elemenata koji su zadani u zadatku izvršiti normalizaciju po frekvenciji $\omega_0=10^5$rad/s i impedanciji R_0=100Ω, odn. izračunati normalizirane vrijednosti elemenata.

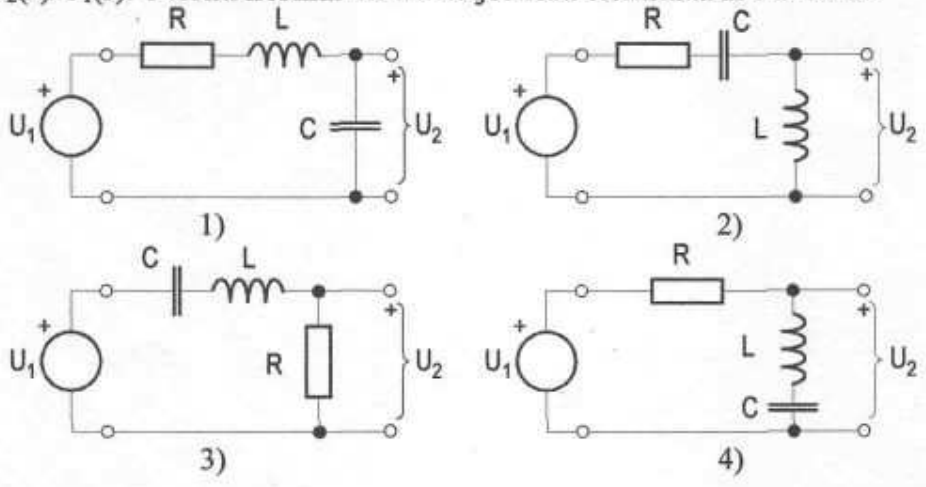
a) R= 10 , L= 1 , C= 1 ;

b) R= 2 , L= 1 , C= 1 ;

c) R= 0.2 , L= 1 , C= 1 ;

d) R= 0 , L= 1 , C= 1 .

2. Za električne krugove prikazane slikom izračunati naponske prijenosne funkcije $T(s)=U_2(s)/U_1(s)$. Uvrstiti **normalizirane** vrijednosti elemenata iz 1 zadatka.



Naponske prijenosne funkcije :

1) $T(s)=\dfrac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$

a) $T(s)=\dfrac{1}{s^2+10s+1}$

b) $T(s)=\dfrac{1}{s^2+2s+1}$

c) $T(s)=\dfrac{1}{s^2+0.2s+1}$

d) $T(s)=\dfrac{1}{s^2+1}$

2) $T(s)=\dfrac{s^2}{s^2+\frac{R}{L}s+\frac{1}{LC}}$

a) $T(s)=\dfrac{s^2}{s^2+10s+1}$

b) $T(s)=\dfrac{s^2}{s^2+2s+1}$

c) $T(s)=\dfrac{s^2}{s^2+0.2s+1}$

d) $T(s)=\dfrac{s^2}{s^2+1}$

3) $T(s)=\dfrac{\frac{R}{L}s}{s^2+\frac{R}{L}s+\frac{1}{LC}}$

a) $T(s)=\dfrac{10s}{s^2+10s+1}$

b) $T(s)=\dfrac{2s}{s^2+2s+1}$

c) $T(s)=\dfrac{0.2s}{s^2+0.2s+1}$

d) $T(s)=0$

4) $T(s)=\dfrac{s^2+\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$

a) $T(s)=\dfrac{s^2+1}{s^2+10s+1}$

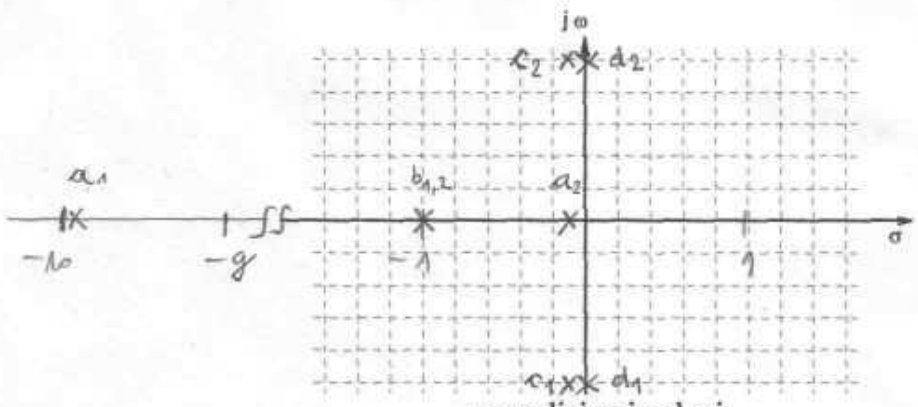
b) $T(s)=\dfrac{s^2+1}{s^2+2s+1}$

c) $T(s)=\dfrac{s^2+1}{s^2+0.2s+1}$

d) $T(s)=1$

3. Za električne krugove prikazane slikom u prethodnom zadatku izračunati normalizirane polove naponske prijenosne funkcije i prikazati ih u kompleksnoj s-ravnini. Uvrstiti normalizirane vrijednosti elemenata. Izračunati **normalizirane** vrijednosti parametara ω_p i q_p te realni i imaginarni dio polova (vidi uputu).

- slučaj *a*:	$q_p=0.1$	$\omega_p=1$	$Re=-5\pm\sqrt{24}$	$Im=0$	$s_1\approx-9.9$ $s_2\approx-0.1$
- slučaj *b*:	$q_p=0.5$	$\omega_p=1$	$Re=-1$	$Im=0$	$s_1=-1$ $s_2=-1$
- slučaj *c*:	$q_p=5$	$\omega_p=1$	$Re=-0.1$	$Im=\pm0.995$	$s_1\approx-0.1-j1$ $s_2\approx-0.1j1$
- slučaj *d*:	$q_p=\infty$	$\omega_p=1$	$Re=0$	$Im=\pm1$	$s_1=-j1$ $s_2=j1$



normalizirani polovi

4. Za 4 električna kruga prikazana slikom u zadatku 2 izračunati izraze za amplitudno-frekvencijsku i fazno-frekvencijsku karakteristiku. Uvrstiti **normalizirane** vrijednosti elemenata za slučaj b).

1)

$$|T(j\omega)| = \frac{1}{\omega^2+1}$$

$\omega<1 \rightarrow \varphi(\omega) = \operatorname{arctg}\frac{2\omega}{\omega^2-1}$

$\omega>1 \rightarrow \varphi(\omega) = \operatorname{arctg}\frac{2\omega}{\omega^2-1} - 180°$

$\omega=1 \rightarrow \varphi = -90°$

2)

$$|T(j\omega)| = \frac{\omega^2}{\omega^2+1}$$

$\omega<1 \rightarrow \varphi(\omega) = 180° + \operatorname{arctg}\frac{2\omega}{\omega^2-1}$

$\omega>1 \rightarrow \varphi(\omega) = \operatorname{arctg}\frac{2\omega}{\omega^2-1}$

$\omega=1 \rightarrow \varphi = +90°$

3)

$$|T(j\omega)| = \frac{2\omega}{\omega^2+1}$$

$\omega\neq 1 \rightarrow \varphi(\omega) = 90° + \operatorname{arctg}\frac{2\omega}{\omega^2-1}$

$\omega=1 \rightarrow \varphi = 0°$

4)

$$\omega<1 \begin{cases} |T(j\omega)| = \frac{1-\omega^2}{\omega^2+1} \\ \varphi(\omega) = \operatorname{arctg}\frac{2\omega}{\omega^2-1} \end{cases}$$

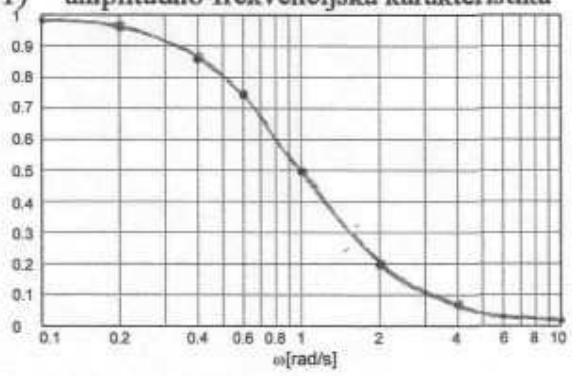
$\omega = 1^- \rightarrow \varphi = -90°$

$\omega = 1^+ \rightarrow \varphi = +90°$ $\quad |T(j\omega)| = 0$

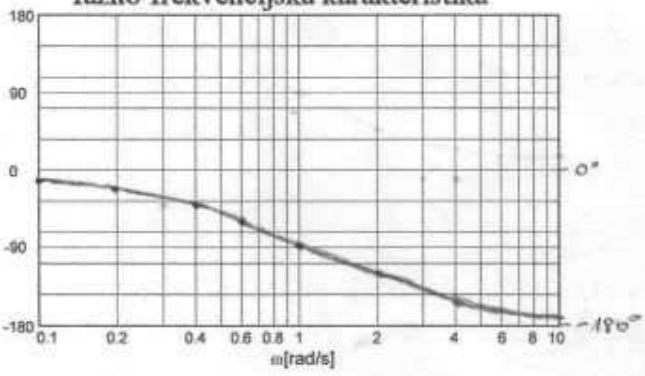
$$\omega>1 \begin{cases} |T(j\omega)| = \frac{\omega^2-1}{\omega^2+1} \\ \varphi(\omega) = \operatorname{arctg}\frac{2\omega}{\omega^2-1} \end{cases}$$

5. Za 4 električna kruga prikazana slikom u zadatku 2 i uz uvrštene normalizirane vrijednosti elemenata za slučaj *b*) nacrtati funkcije amplitudno-frekvencijske i fazno-frekvencijske karakteristike. Prikaz karakteristika neka bude u području frekvencija ω=0.1 rad/s do 10 rad/s (dvije dekade).

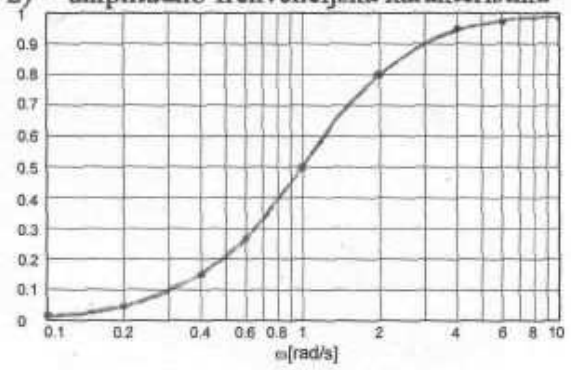
1) amplitudno-frekvencijska karakteristika



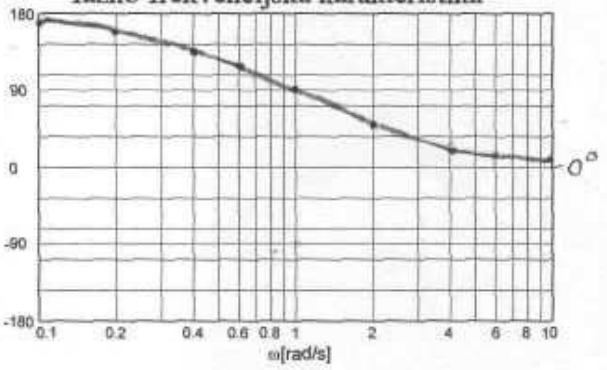
fazno-frekvencijska karakteristika



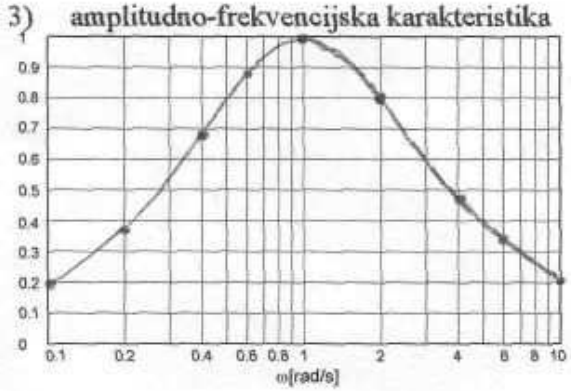
2) amplitudno-frekvencijska karakteristika



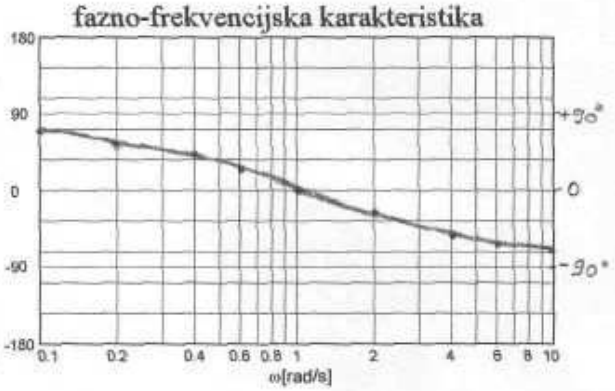
fazno-frekvencijska karakteristika



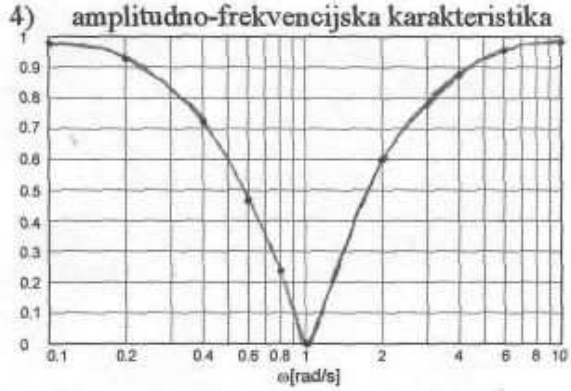
3) amplitudno-frekvencijska karakteristika



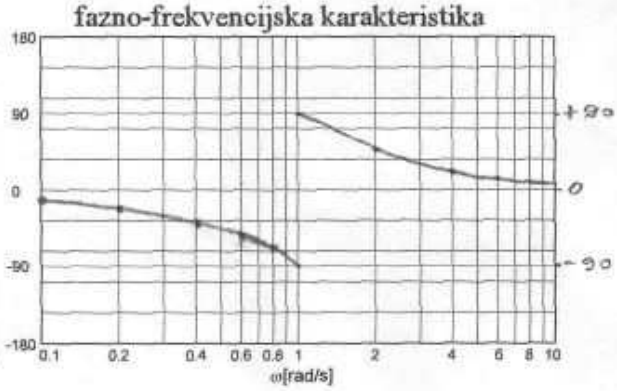
fazno-frekvencijska karakteristika



4) amplitudno-frekvencijska karakteristika



fazno-frekvencijska karakteristika



(1.) NORMALIZACIJA UZ $\omega_0 = 10^5$ rad/s, $R_0 = 100\Omega$

FORMULE: $\boxed{R_m = \frac{R}{R_0}}$ $\boxed{L_m = \frac{\omega_0 L}{R_0}}$ $\boxed{C_m = \omega_0 C R_0}$

a) $R_m = \frac{1000}{100} = 10$ $L_m = \frac{10^5 \cdot 10^{-3}}{100} = 1$ $C_m = 10^5 \cdot 10^{-9} \cdot 10^2 = 1$

b) $R_m = \frac{200}{100} = 2$ $L_m = 1$ $C_m = 1$

c) $R_m = \frac{20}{100} = 0.2$ $L_m = 1$ $C_m = 1$

d) $R_m = \frac{0}{100} = 0$ $L_m = 1$ $C_m = 1$

(2.) SHEMA 1) $T(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$

a) $R=10, L=1, C=1 \rightarrow T(s) = \frac{1}{s^2+10s+1}$

b) $R=2, L=1, C=1 \rightarrow T(s) = \frac{1}{s^2+2s+1}$

c) $R=0.2, L=1, C=1 \rightarrow T(s) = \frac{1}{s^2+0.2s+1}$

d) $R=0, L=1, C=1 \rightarrow T(s) = \frac{1}{s^2+1}$

SHEMA 2) $T(s) = \frac{s^2}{s^2 + \frac{R}{L}s + \frac{1}{C}}$

a) $R=10, L=1, C=1 \rightarrow T(s) = \frac{s^2}{s^2+10s+1}$

b) $R=2, L=1, C=1 \rightarrow T(s) = \frac{s^2}{s^2+2s+1} = \frac{s^2}{(s+1)^2}$

c) $R=0.2, L=1, C=1 \rightarrow T(s) = \frac{s^2}{s^2+0.2s+1}$

d) $R=0, L=1, C=1 \rightarrow T(s) = \frac{s^2}{s^2+1}$

SHEMA 3) $T(s)=\dfrac{\frac{R}{L}s}{s^2+\frac{R}{L}s+\frac{1}{LC}}$

a) $R=10,\ L=1,\ C=1 \rightarrow T(s)=\dfrac{10s}{s^2+10s+1}$

b) $R=2,\ L=1,\ C=1 \rightarrow T(s)=\dfrac{2s}{s^2+2s+1}=\dfrac{2s}{(s+1)^2}$

c) $R=0.2,\ L=1,\ C=1 \rightarrow T(s)=\dfrac{0.2s}{s^2+0.2s+1}$

d) $R=0,\ L=1,\ C=1 \rightarrow T(s)=0$

SHEMA 4) $T(s)=\dfrac{s^2+\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$

a) $R=10,\ L=1,\ C=1 \rightarrow T(s)=\dfrac{s^2+1}{s^2+10s+1}$

b) $R=2,\ L=1,\ C=1 \rightarrow T(s)=\dfrac{s^2+1}{s^2+2s+1}=\dfrac{s^2+1}{(s+1)^2}$

c) $R=0.2,\ L=1,\ C=1 \rightarrow T(s)=\dfrac{s^2+1}{s^2+0.2s+1}$

d) $R=0,\ L=1,\ C=1 \rightarrow T(s)=\dfrac{s^2+1}{s^2+1}=1$

(3.) NORMALIZIRANI POLOVI NAPONSKE PRIJENOSNE FUNKCIJE $T(s)$
→ RJEŠENJA KVADRATNE JEDNADŽBE U NAZIVNIKU $T(s)$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

a) $s^2 + 10s + 1 = 0 \rightarrow s_{p1,2} = -\frac{10}{2} \pm \frac{\sqrt{10^2-4}}{2} = -5 \pm \sqrt{24}$

$s_{p1} = -9.899 \quad s_{p2} = -0.101$

b) $s^2 + 2s + 1 = 0 \rightarrow s_{p1,2} = -\frac{2}{2} \pm \frac{\sqrt{2^2-4}}{2} = -1 + 0 = -1$

$s_{p1} = -1 \quad s_{p2} = -1$

c) $s^2 + 0.2s + 1 = 0 \rightarrow s_{p1,2} = -\frac{0.2}{2} \pm \frac{\sqrt{0.2^2-4}}{2} = -0.1 \pm j0.955$

$s_{p1} = -0.1 - j0.955 \quad s_{p2} = -0.1 + j0.955$

d) $s^2 + 1 = 0 \rightarrow s_{p1,2} = \pm j$

$s_{p1} = -j, \quad s_{p2} = +j$

NORMALIZIRANE VRIJEDNOSTI PARAMETARA W_p I Q_p

$$W_p = \frac{1}{\sqrt{LC}} \qquad Q_p = \frac{1}{R}\sqrt{\frac{L}{C}}$$

a) $R = 10, \; L = 1, \; C = 1 \rightarrow W_p = 1, \; Q_p = 0.1$

b) $R = 2, \; L = 1, \; C = 1 \rightarrow W_p = 1, \; Q_p = 0.5$

c) $R = 0.2, \; L = 1, \; C = 1 \rightarrow W_p = 1, \; Q_p = 5$

d) $R = 0, \; L = 1, \; C = 1 \rightarrow W_p = 1, \; Q_p = \infty$

(4.) SLUČAJ b) $R=2$, $L=1$, $C=1$

SHEMA 1) $T(s)=\frac{1}{s^2+2s+1}$, uz $s=jw$ → $\boxed{T(jw)=\frac{1}{-w^2+j2w+1}}$

$$|T(jw)|=\frac{|\text{brojnik}|}{|\text{nazivnik}|}=\frac{1}{\sqrt{(1-w^2)^2+(2w)^2}}=\frac{1}{\sqrt{1-2w^2+w^4+4w^2}}=$$

$$=\frac{1}{\sqrt{w^4+2w^2+1}}=\frac{1}{\sqrt{(w^2+1)^2}} \rightarrow \boxed{T(jw)=\frac{1}{w^2+1}}$$

$$\varphi(w)=\varphi[\text{brojnik od } T(jw)]-\varphi[\text{nazivnik od } T(jw)]$$

$$\boxed{\varphi(w)=0-\operatorname{arctg}\frac{\operatorname{Im}[\text{nazivnik}]}{\operatorname{Re}[\text{nazivnik}]}=-\operatorname{arctg}\frac{2w}{1-w^2}=\operatorname{arctg}\frac{2w}{w^2-1}}$$

FUNKCIJA TANGENS JE PERIODIČNA SA 180° PA ZATO TREBA DODATNO UREDITI IZRAZ ZA RAZLIČITE OPSEGE w.

$w<1 \rightarrow \boxed{\varphi(w)=\operatorname{arctg}\frac{2w}{w^2-1}}$ uz $\varphi(w)\in\langle -90°, +90°\rangle$

$w=1 \rightarrow \boxed{\varphi=-90°}$ JER JE $\lim\limits_{w\to 1}\frac{2w}{w^2-1}=\pm\infty$

$\varphi=\operatorname{arctg}(-\infty)=-90°$

$\varphi=\operatorname{arctg}(\infty)-180°=90°-180°=-90°$

$w>1 \rightarrow \boxed{\varphi(w)=\operatorname{arctg}\frac{2w}{w^2-1}-180°}$

SHEMA 2) $T(s)=\frac{s^2}{s^2+2s+1} \rightarrow \boxed{T(jw)=\frac{-w^2}{-w^2+2jw+1}}$

$$\boxed{|T(jw)|=\frac{|-w^2|}{|-w^2+j2w+1|}=\frac{w^2}{w^2+1}}$$

$$\varphi(w)=\varphi[-w^2]-\varphi[-w^2+j2w+1]=\pm 180+\operatorname{arctg}\frac{2w}{w^2-1}$$

$$\boxed{\varphi(w)=\begin{cases} w<1,\ \varphi(w)=180+\operatorname{arctg}\frac{2w}{w^2-1} \\ w=1,\ \varphi(1)=90° \\ w>1,\ \varphi(w)=\operatorname{arctg}\frac{2w}{w^2-1} \end{cases}}$$

SHEMA 3) $T(s)=\frac{2s}{s^2+2s+1} \rightarrow \boxed{T(j\omega)=\frac{j2\omega}{-\omega^2+j2\omega+1}}$

$$\boxed{|T(j\omega)|=\frac{|j2\omega|}{|-\omega^2+j2\omega+1|}=\frac{2\omega}{\omega^2+1}}$$

$$\varphi(\omega)=\varphi[j2\omega]-\varphi[-\omega+j2\omega+1]=90^\circ+\operatorname{arctg}\frac{2\omega}{\omega^2-1}$$

$$\boxed{\varphi(\omega)=\begin{cases}\omega\neq 1,\ \varphi(\omega)=90^\circ+\operatorname{arctg}\frac{2\omega}{\omega^2-1}\\ \omega=1,\ \varphi(1)=0^\circ\end{cases}}$$

SHEMA 4) $T(s)=\frac{s^2+1}{s^2+2s+1} \rightarrow \boxed{T(j\omega)=\frac{-\omega^2+1}{-\omega^2+j2\omega+1}}$

$$|T(j\omega)|=\frac{|1-\omega^2|}{|1-\omega^2+j2\omega|}=\frac{|1-\omega^2|}{\omega^2+1}$$

$$\omega\leq 1 \rightarrow |T(j\omega)|=\frac{1-\omega^2}{\omega^2+1}$$

$$\omega>1 \rightarrow |T(j\omega)|=\frac{\omega^2-1}{\omega^2+1}$$

$$\varphi(\omega)=\varphi[-\omega^2+1]-\varphi[-\omega^2+j2\omega+1]$$

$$\boxed{\varphi(\omega)=\begin{cases}\omega\neq 1 \rightarrow \varphi(\omega)=\operatorname{arctg}\frac{2\omega}{\omega^2-1}\\ \omega=1^- \rightarrow \varphi=-90^\circ\\ \omega=1^+ \rightarrow \varphi=+90^\circ\end{cases}}$$

(5.) IZRAČUNAVANJE TOČAKA ZA GRAFOVE $|T(j\omega)|$ I $\varphi(\omega)$

b) $R=2$, $L=1$, $C=1$

FORMULE IZ ZADATKA 4.

SHEMA 1) TABLICA

ω[rad/s]	0.1	0.2	0.4	0.6	0.8	1	2	4	6	10
$\lvert T(j\omega)\rvert$	0.99	0.96	0.86	0.74	0.61	0.5	0.2	0.05	0.03	0.01
$\varphi(\omega)$[°]	-11	-23	-44	-62	-77	-90	-127	-152	-161	-169

SHEMA 2) TABLICA

ω[rad/s]	0.1	0.2	0.4	0.6	0.8	1	2	4	6	10
$\lvert T(j\omega)\rvert$	0.01	0.04	0.14	0.26	0.39	0.5	0.80	0.94	0.97	0.99
$\varphi(\omega)$[°]	169	157	136	118	103	90	53	28	19	11

SHEMA 3) TABLICA

ω[rad/s]	0.1	0.2	0.4	0.6	0.8	1	2	4	6	10
$\lvert T(j\omega)\rvert$	0.20	0.38	0.69	0.28	0.98	1	0.8	0.47	0.32	0.2
$\varphi(\omega)$[°]	79	67	46	28	13	0	-37	-62	-72	-79

SHEMA 4) TABLICA

ω[rad/s]	0.1	0.2	0.4	0.6	0.8	1	2	4	6	10
$\lvert T(j\omega)\rvert$	0.98	0.92	0.72	0.47	0.22	0	0.6	0.88	0.95	0.98
$\varphi(\omega)$[°]	-11	-23	-44	-62	-77	-90° / +90°	53	28	19	11

U TOČKI $\omega=1$ FUNKCIJA $\varphi(\omega)$ IMA DVOSTRUKU VRIJEDNOST $\varphi(\omega=1^{-})=-90°$, $\varphi(\omega=1^{+})=+90°$ ŠTO SE VIDI NA GRAFU $\varphi(\omega)$. TO JE TRENUTAK REZONANCIJE KADA JE LIJEVO OD $\omega=1$ VEĆI UTJECAJ X_C, A DESNO VEĆI UTJECAJ X_L.