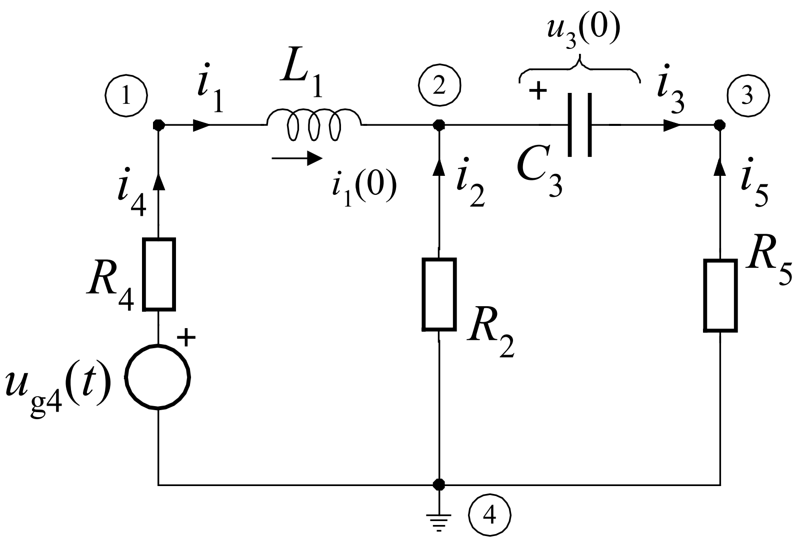
Električni krugovi

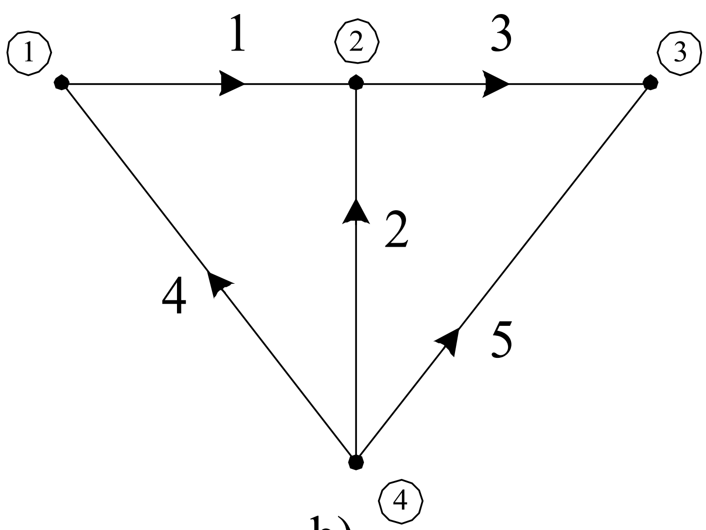
Grafovi i mreže
Primjeri

Graf i matrice grafa

- Primjer 1. Za krug na slici postaviti jednadžbe temeljnog sustava petlji, rezova i čvorišta.

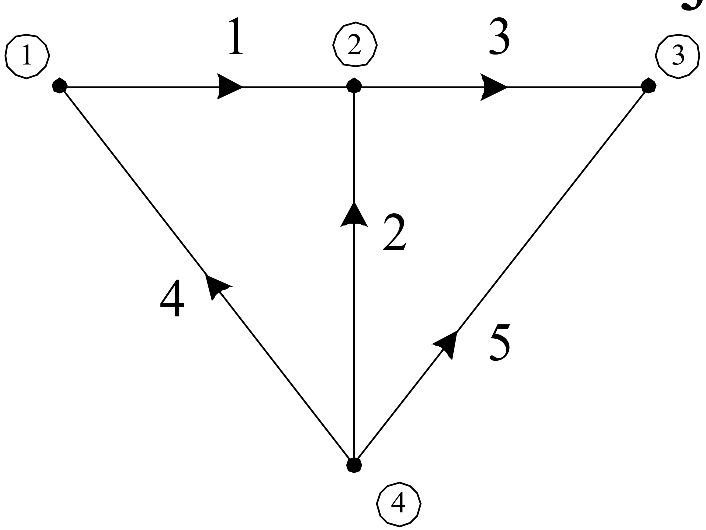


a)



b)

- Matrica incidencija



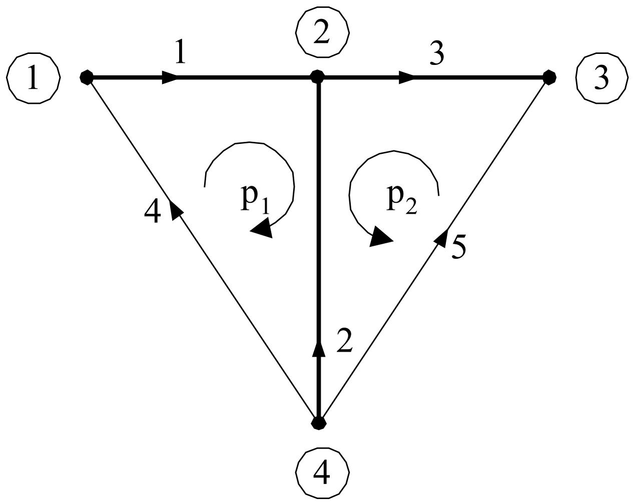
$$\mathbf{A}_a = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

- ***Reducirana matrica incidencija* A**

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix}$$

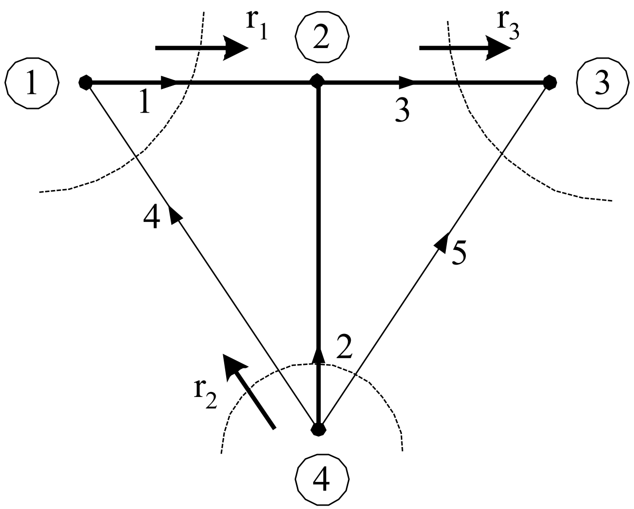
- Odbacivanjem 4. retka $\rightarrow$ čvorište 4 - referentno.

- ***Spojna matrica*** **S** → Stablo i temeljni sustav petlji



$$\mathbf{S}=\begin{bmatrix}1 & -1 & 0 & 1 & 0\\ 0 & -1 & -1 & 0 & 1\end{bmatrix}$$

- ***Rastavna matrica*** **Q** → Temeljni sustav rezova



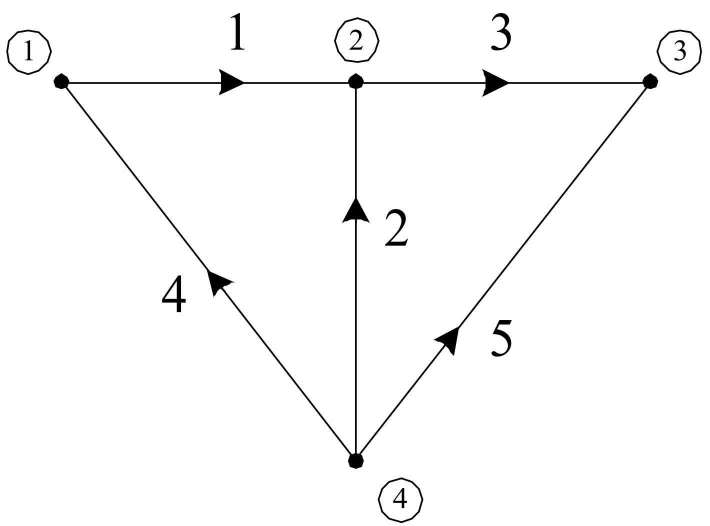
$$\mathbf{Q}=\begin{bmatrix}1 & 0 & 0 & -1 & 0\\ 0 & 1 & 0 & 1 & 1\\ 0 & 0 & 1 & 0 & 1\end{bmatrix}$$

- Vektori struja grana $\mathbf{i_b}$ i napona grana $\mathbf{u_b}$

$$\mathbf{i}_b = \begin{bmatrix} i_1(t) \\ i_2(t) \\ i_3(t) \\ i_4(t) \\ i_5(t) \end{bmatrix} \qquad \mathbf{u}_b = \begin{bmatrix} u_1(t) \\ u_2(t) \\ u_3(t) \\ u_4(t) \\ u_5(t) \end{bmatrix}$$

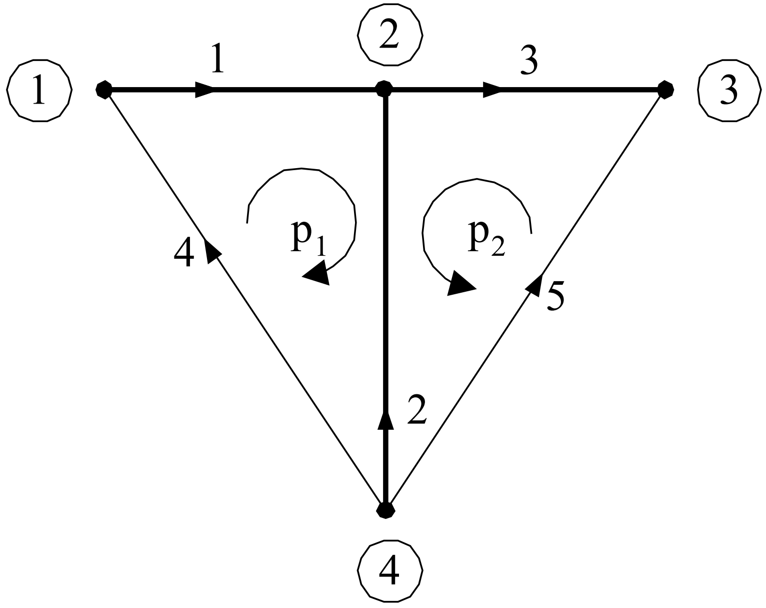
- Jednadžbe KZS za čvorišta 1, 2, i 3

$$\mathbf{A}\cdot\mathbf{i}_b=\begin{bmatrix}1 & 0 & 0 & -1 & 0\\ -1 & -1 & 1 & 0 & 0\\ 0 & 0 & -1 & 0 & -1\end{bmatrix}\cdot\begin{bmatrix}i_1(t)\\ i_2(t)\\ i_3(t)\\ i_4(t)\\ i_5(t)\end{bmatrix}=\begin{bmatrix}i_1(t)-i_4(t)\\ -i_1(t)-i_2(t)+i_3(t)\\ -i_3(t)-i_5(t)\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$



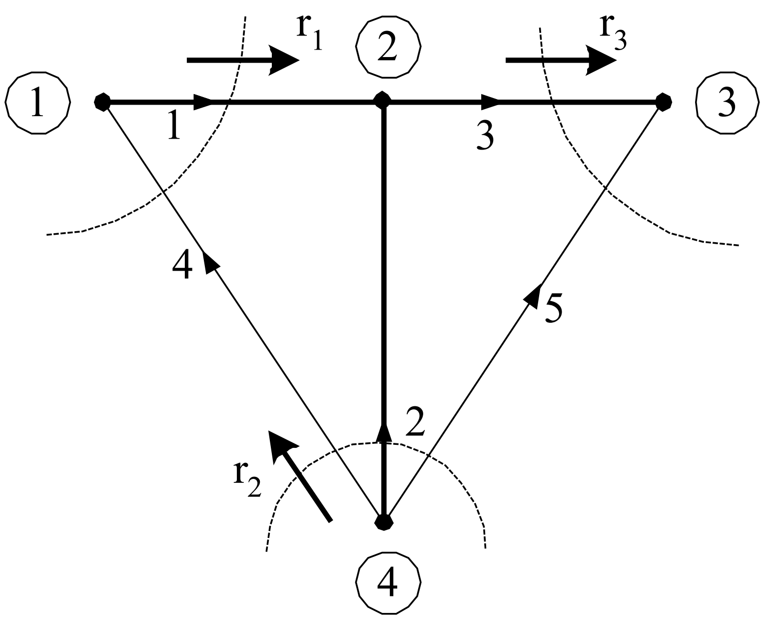
Jednadžbe KZN primjenjenoga na temeljne petlje

$$\mathbf{S}\cdot\mathbf{u}_b=\begin{bmatrix}1 & -1 & 0 & 1 & 0\\ 0 & -1 & -1 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}u_1(t)\\ u_2(t)\\ u_3(t)\\ u_4(t)\\ u_5(t)\end{bmatrix}=\begin{bmatrix}u_1(t)-u_2(t)+u_4(t)\\ -u_2(t)-u_3(t)+u_5(t)\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$



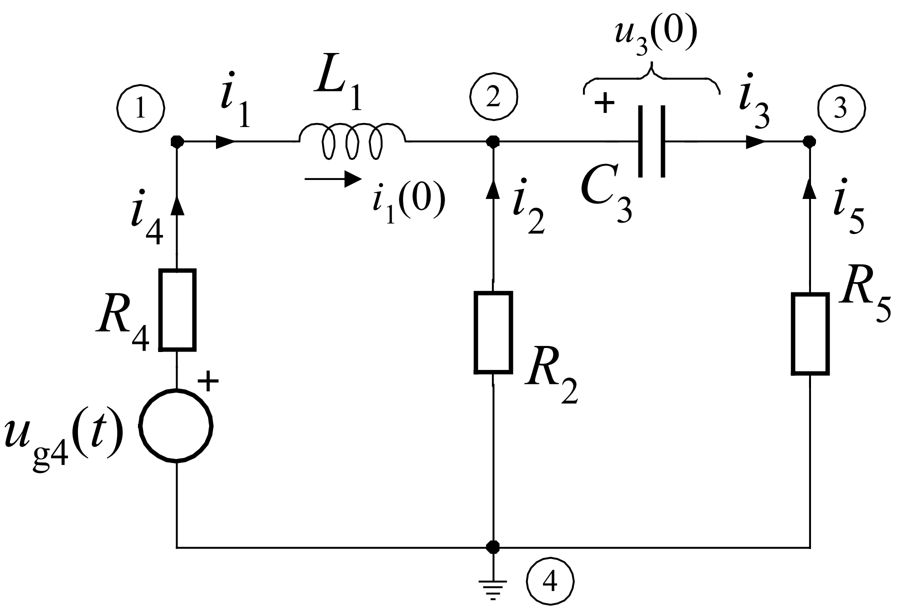
- Jednadžbe KZS za temeljne rezove glase

$$\mathbf{Q}\cdot\mathbf{i}_b=\begin{bmatrix}1 & 0 & 0 & -1 & 0\\ 0 & 1 & 0 & 1 & 1\\ 0 & 0 & 1 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}i_1(t)\\ i_2(t)\\ i_3(t)\\ i_4(t)\\ i_5(t)\end{bmatrix}=\begin{bmatrix}i_1(t)-i_4(t)\\ i_2(t)+i_4(t)+i_5(t)\\ i_3(t)+i_5(t)\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$



Jednadžbe temeljnih petlji

Strujno naponske jednadžbe grana



$$u_1(t) = L_1 \frac{\mathrm{d}i_1(t)}{\mathrm{d}t}$$

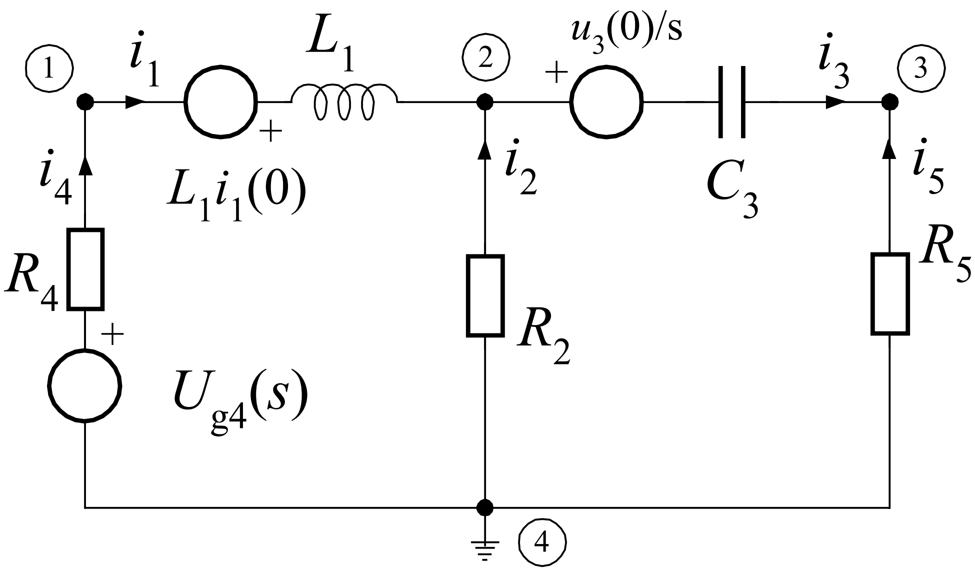
$$u_2(t) = R_2 i_2(t)$$

$$u_3(t) = \frac{1}{C_3}\int_0^t i_3(\tau)\,\mathrm{d}\tau + u_3(0)$$

$$u_4(t) = -u_{g4}(t) + R_4 i_4(t)$$

$$u_5(t) = R_5 i_5(t)$$

- Primjenom Laplaceove transformacije dobiva se



$$U_1(s) = sL_1 I_1(s) - L_1 i_1(0)$$

$$U_2(s) = R_2 I_2(s)$$

$$U_3(s) = \frac{1}{sC_3} I_3(s) + \frac{u_3(0)}{s}$$

$$U_4(s) = -U_{g4}(s) + R_4 I_4(s)$$

$$U_5(s) = R_5 I_5(s)$$

- Sustav jednadžbi u matričnoj formi ima oblik

$$\begin{bmatrix} U_1(s) \\ U_2(s) \\ U_3(s) \\ U_4(s) \\ U_5(s) \end{bmatrix} = \begin{bmatrix} sL_1 & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{1}{sC_3} & 0 & 0 \\ 0 & 0 & 0 & R_4 & 0 \\ 0 & 0 & 0 & 0 & R_5 \end{bmatrix} \cdot \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \\ I_4(s) \\ I_5(s) \end{bmatrix} + \begin{bmatrix} -L_1 \cdot i_1(0) \\ 0 \\ u_3(0)/s \\ -U_{g4}(s) \\ 0 \end{bmatrix}$$

$$\mathbf{U}_b = \mathbf{Z}_b \mathbf{I}_b + \mathbf{U}_{0b}$$

- $\mathbf{Z}_b$ je ***matrica impedancija grana***.
- Ona za ovaj slučaj ima oblik dijagonalne matrice

$$\mathbf{Z}_b = \begin{bmatrix} sL_1 & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{1}{sC_3} & 0 & 0 \\ 0 & 0 & 0 & R_4 & 0 \\ 0 & 0 & 0 & 0 & R_5 \end{bmatrix}$$

- Množenjem jednadžbe s lijeva sa spojnom matricom

$$\mathbf{S} \cdot \mathbf{U}_{\mathrm{b}} = \mathbf{0}$$

$$\mathbf{S} \cdot \mathbf{Z}_{b} \cdot \mathbf{I}_{b} + \mathbf{S} \cdot \mathbf{U}_{ob} = \mathbf{0}$$

- Struje grana izražene strujama petlji

$$\mathbf{I}_{\mathrm{b}} = \mathbf{S}^{\mathrm{t}} \cdot \mathbf{I}_{\mathrm{p}}$$

- $\mathbf{I}_{\mathrm{p}} \rightarrow$ vektor struja petlji

$$\mathbf{I}_{\mathrm{p}} = \begin{bmatrix} I_{p1}(s) \\ I_{p2}(s) \end{bmatrix}$$

$$\mathbf{S} \cdot \mathbf{Z}_{b} \cdot \mathbf{S}^{t} \cdot \mathbf{I}_{p} + \mathbf{S} \cdot \mathbf{U}_{ob} = \mathbf{0}$$

- Rezultat je

- $\mathbf{S}\mathbf{Z}_{b}\mathbf{S}^{\mathrm{t}} \rightarrow$ matrica impedancija temeljnoga sustava petlji $\mathbf{Z}_{\mathrm{p}}$

$$\mathbf{Z}_{p} = \mathbf{S} \cdot \mathbf{Z}_{b} \cdot \mathbf{S}^{t}$$

- U našem je slučaju

$$\mathbf{Z}_p = \begin{bmatrix} 1 & -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} sL_1 & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ 0 & 0 & \dfrac{1}{sC_3} & 0 & 0 \\ 0 & 0 & 0 & R_4 & 0 \\ 0 & 0 & 0 & 0 & R_5 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ -1 & -1 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{Z}_p = \begin{bmatrix} sL_1 + R_2 + R_4 & R_2 \\ R_2 & R_2 + \dfrac{1}{sC_3} + R_5 \end{bmatrix}$$

- Matrična jednadžba

$$\mathbf{Z}_p \mathbf{I}_p = -\mathbf{S} \cdot \mathbf{U}_{ob}$$

$$\mathbf{Z}_p \mathbf{I}_p = \mathbf{U}_{op}$$

$$\begin{bmatrix} sL_1 + R_2 + R_4 & R_2 \\ R_2 & R_2 + R_5 + \dfrac{1}{sC_3} \end{bmatrix} \cdot \begin{bmatrix} I_{p1} \\ I_{p2} \end{bmatrix} = \begin{bmatrix} U_{g4}(s) + L_1 \cdot i_1(0) \\ \dfrac{1}{s} u_3(0) \end{bmatrix}$$

- Množenjem s lijeva inverznom matricom $\mathbf{Z}_p$,

$\rightarrow$ rješenje sustava

$$\mathbf{I}_p = \mathbf{Z}_p^{-1} \cdot \mathbf{U}_{op}$$

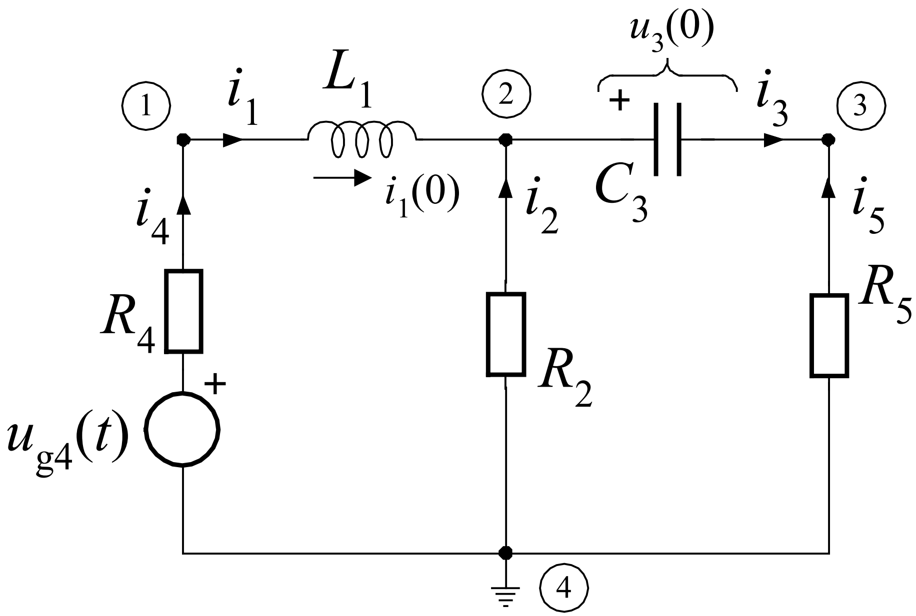
- odnosno

$$\begin{bmatrix} I_{p1} \\ I_{p2} \end{bmatrix} = \begin{bmatrix} sL_1 + R_2 + R_4 & R_2 \\ R_2 & R_2 + R_5 + \dfrac{1}{sC_3} \end{bmatrix}^{-1} \cdot \begin{bmatrix} U_{g4}(s) + L_1 \cdot i_1(0) \\ \dfrac{1}{s} u_3(0) \end{bmatrix}$$

▪ Jednadžbe čvorišta

▪ Za postavljanje jednadžbi čvorišta potrebno je struje grana izraziti naponima grana.

$$i_1(t) = \frac{1}{L_1}\int_0^t u_1(\tau)\mathrm{d}\tau + i_1(0)$$



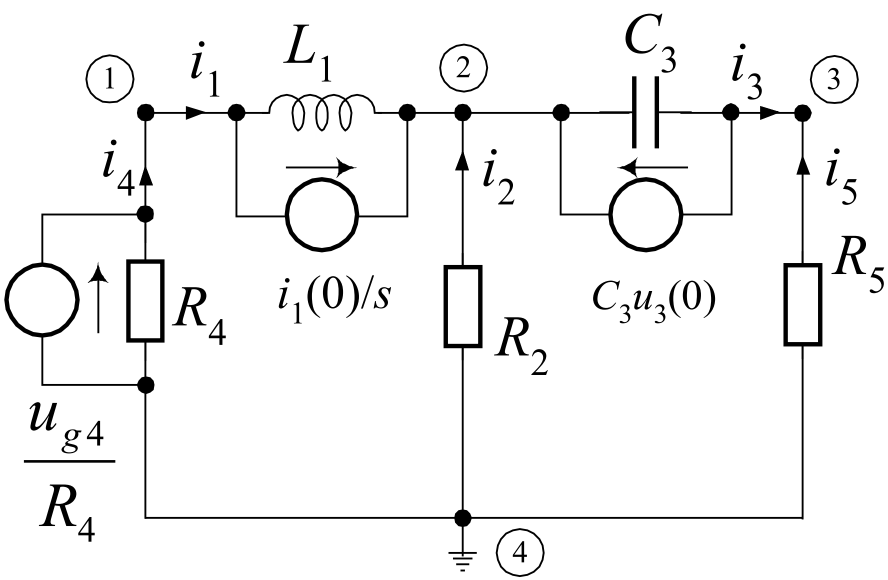
$$i_2(t) = \frac{u_2(t)}{R_2}$$

$$i_3(t) = C_3\frac{\mathrm{d}u_3(t)}{\mathrm{d}t}$$

$$i_4(t) = \frac{u_{g4}(t)}{R_4} + \frac{u_4(t)}{R_4}$$

$$i_5(t) = \frac{u_5(t)}{R_5}$$

- Primjenom Laplaceove transformacije dobiva se



$$I_1(s) = \frac{U_1(s)}{sL_1} + \frac{i_1(0)}{s}$$

$$I_2(s) = \frac{U_2(s)}{R_2}$$

$$I_3(s) = sC_3U_3(s) - C_3 i_3(0)$$

$$I_4(s) = \frac{U_{g4}(s)}{R_4} + \frac{U_4(s)}{R_4}$$

$$I_5(s) = \frac{U_5(s)}{R_5}$$

- Sustav jednadžbi u matričnoj formi

$$\mathbf{I}_b = \mathbf{Y}_b \cdot \mathbf{U}_b + \mathbf{I}_{ob}$$

$$\begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \\ I_4(s) \\ I_5(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{sL_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 & 0 \\ 0 & 0 & sC_3 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{R_5} \end{bmatrix} \cdot \begin{bmatrix} U_1(s) \\ U_2(s) \\ U_3(s) \\ U_4(s) \\ U_5(s) \end{bmatrix} + \begin{bmatrix} i_1(0)/s \\ 0 \\ -C_3 u_3(0) \\ U_{g4}(s)/R_4 \\ 0 \end{bmatrix}$$

- $\mathbf{Y}_b$ je ***matrica admitancija grana***, koja za ovaj slučaj ima oblik dijagonalne matrice.

$$\mathbf{Y}_b = \mathbf{Z}_b^{-1} = \begin{bmatrix} \dfrac{1}{sL_1} & 0 & 0 & 0 & 0 \\ 0 & \dfrac{1}{R_2} & 0 & 0 & 0 \\ 0 & 0 & sC_3 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1}{R_4} & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1}{R_5} \end{bmatrix}$$

- Množenjem s lijeva s matricom incidencija

$$\mathbf{A} \cdot \mathbf{I}_{\mathrm{b}} = \mathbf{0}$$

$$\mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{U}_b + \mathbf{A} \cdot \mathbf{I}_{ob} = \mathbf{0}$$

- Napone grana izraženi naponima čvorišta

$$\mathbf{U}_{\mathrm{b}} = \mathbf{A}^{\mathrm{t}} \cdot \mathbf{U}_{\mathrm{v}}$$

- $\mathbf{U}_{\mathrm{v}} \rightarrow$ vektor napona čvorišta

$$\mathbf{U}_{\mathrm{v}}(s) = \begin{bmatrix} U_{v1}(s) \\ U_{v2}(s) \\ U_{v3}(s) \end{bmatrix}$$

$$\mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{A}^t \cdot \mathbf{U}_v + \mathbf{A} \cdot \mathbf{I}_{ob} = \mathbf{0}$$

- $\mathbf{A}\mathbf{Y}_b\mathbf{A}^{\mathrm{t}}$ je matrica admitancija čvorišta $\mathbf{Y}_{\mathrm{v}}$.

$$\mathbf{Y}_v = \mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{A}^t$$

- Za zadani krug ona glasi

$$\mathbf{Y}_v=\begin{bmatrix}1 & 0 & 0 & -1 & 0\\ -1 & -1 & 1 & 0 & 0\\ 0 & 0 & -1 & 0 & -1\end{bmatrix}\cdot\begin{bmatrix}\frac{1}{sL_1} & 0 & 0 & 0 & 0\\ 0 & \frac{1}{R_2} & 0 & 0 & 0\\ 0 & 0 & sC_3 & 0 & 0\\ 0 & 0 & 0 & \frac{1}{R_4} & 0\\ 0 & 0 & 0 & 0 & \frac{1}{R_5}\end{bmatrix}\cdot\begin{bmatrix}1 & -1 & 0\\ 0 & -1 & 0\\ 0 & 1 & -1\\ -1 & 0 & 0\\ 0 & 0 & -1\end{bmatrix}$$

$$\mathbf{Y}_v=\begin{bmatrix}\frac{1}{sL_1}+\frac{1}{R_4} & -\frac{1}{sL_1} & 0\\ -\frac{1}{sL_1} & \frac{1}{sL_1}+\frac{1}{R_2}+sC_3 & -sC_3\\ 0 & -sC_3 & sC_3+\frac{1}{R_5}\end{bmatrix}$$

- Matrična se jednadžba može napisati u obliku

$$\mathbf{Y}_v \cdot \mathbf{U}_v = -\mathbf{A} \cdot \mathbf{I}_{ob}$$

$$\mathbf{Y}_v \cdot \mathbf{U}_v = \mathbf{I}_{ov}$$

$$\begin{bmatrix} \frac{1}{sL_1}+\frac{1}{R_4} & -\frac{1}{sL_1} & 0 \\ -\frac{1}{sL_1} & \frac{1}{sL_1}+\frac{1}{R_2}+sC_3 & -sC_3 \\ 0 & -sC_3 & sC_3+\frac{1}{R_5} \end{bmatrix} \cdot \begin{bmatrix} U_{v1} \\ U_{v2} \\ U_{v3} \end{bmatrix} = \begin{bmatrix} U_{g4}/R_4 + i_{L1}(0)/s \\ C_3 u_{C3}(0) - i_{L1}(0)/s \\ -C_3 u_{C3}(0) \end{bmatrix}$$

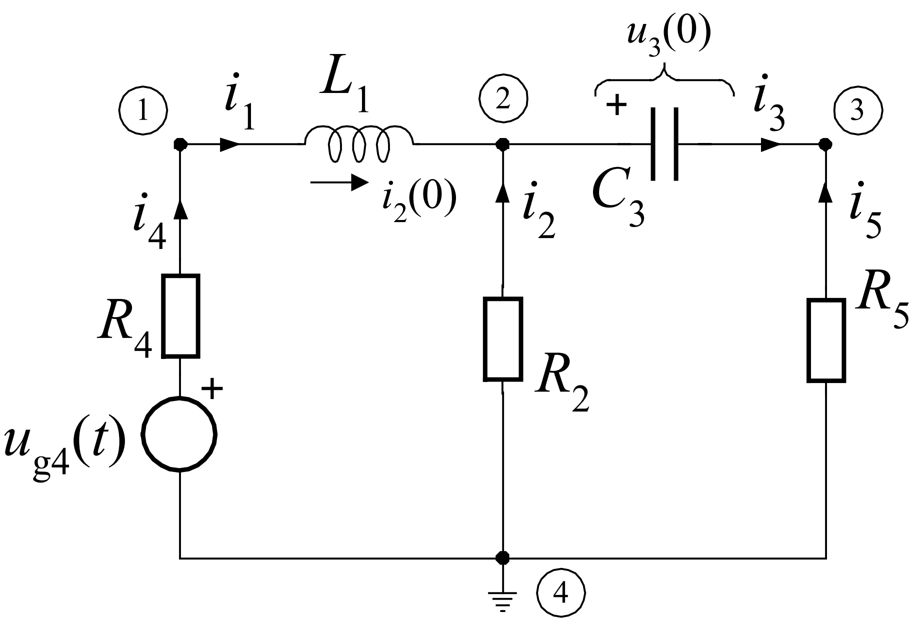
- Množenjem s lijeva s $\mathbf{Y}_v^{-1}$ → rješenje sustava

$$\mathbf{Y}_v^{-1} \cdot \mathbf{Y}_v \cdot \mathbf{U}_v = \mathbf{U}_v = \mathbf{Y}_v^{-1} \cdot \mathbf{I}_{ov}$$

- odnosno

$$\begin{bmatrix} U_{v1} \\ U_{v2} \\ U_{v3} \end{bmatrix} = \begin{bmatrix} \frac{1}{sL_1} + \frac{1}{R_4} & -\frac{1}{sL_1} & 0 \\ -\frac{1}{sL_1} & \frac{1}{sL_1} + \frac{1}{R_2} + sC_3 & -sC_3 \\ 0 & -sC_3 & sC_3 + \frac{1}{R_5} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \frac{U_{g4}}{R_4} - \frac{i_{L1}(0)}{s} \\ C_3 u_{C3}(0) + \frac{i_{L1}(0)}{s} \\ -C_3 u_{C3}(0) \end{bmatrix}$$

Jednadžbe rezova



$$i_1(t) = \frac{1}{L_1}\int_0^t u_1(\tau)\mathrm{d}\tau + i_1(0)$$

$$i_2(t) = \frac{u_2(t)}{R_2}$$

$$i_3(t) = C_3\frac{\mathrm{d}u_3(t)}{\mathrm{d}t}$$

$$i_4(t) = \frac{u_{g4}(t)}{R_4} + \frac{u_4(t)}{R_4}$$

$$i_5(t) = \frac{u_5(t)}{R_5}$$

- Primjenom Laplaceove transformacije dobiva se

$$I_1(s) = \frac{U_1(s)}{sL_1} + \frac{i_1(0)}{s}$$

$$I_2(s) = \frac{U_2(s)}{R_2}$$

$$I_3(s) = sC_3U_3\left(s\right) - C_3 i_3(0)$$

$$I_4(s) = \frac{U_{g4}(s)}{R_4} + \frac{U_4(s)}{R_4}$$

$$I_5(s) = \frac{U_5(s)}{R_5}$$

- Matrična jednadžba grana kruga glasi

$$\mathbf{I}_b = \mathbf{Y}_b \cdot \mathbf{U}_b + \mathbf{I}_{ob}$$

$$\begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \\ I_4(s) \\ I_5(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{sL_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 & 0 \\ 0 & 0 & sC_3 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{R_5} \end{bmatrix} \cdot \begin{bmatrix} U_1(s) \\ U_2(s) \\ U_3(s) \\ U_4(s) \\ U_5(s) \end{bmatrix} + \begin{bmatrix} i_1(0)/s \\ 0 \\ -C_3 u_3(0) \\ U_{g4}(s)/R_4 \\ 0 \end{bmatrix}$$

- Množenjen jednadžbe s lijeva s rastavnom matricom

$$\mathbf{Q} \cdot \mathbf{I}_{\mathrm{b}} = \mathbf{0}$$

$$\mathbf{Q} \cdot \mathbf{I}_{ob} + \mathbf{Q} \cdot \mathbf{Y}_{b} \cdot \mathbf{U}_{b} = \mathbf{0}$$

- Naponi grana izraženi naponima rezova

$$\mathbf{U}_{\mathrm{b}} = \mathbf{Q}^{\mathrm{t}} \cdot \mathbf{U}_{\mathrm{r}}$$

- $\mathbf{U}_{\mathrm{r}} \rightarrow$ vektor napona rezova

$$\mathbf{U}_{\mathrm{r}}(s) = \begin{bmatrix} U_{r1}(s) \\ U_{r2}(s) \\ U_{r3}(s) \end{bmatrix}$$

$$\mathbf{Q} \cdot \mathbf{Y}_{b} \cdot \mathbf{Q}^{t} \cdot \mathbf{U}_{r} + \mathbf{Q} \cdot \mathbf{I}_{ob} = \mathbf{0}$$

- $\mathbf{Q}\mathbf{Y}_{b}\mathbf{Q}^{\mathrm{t}} \rightarrow$ matrica admitancija rezova $\mathbf{Y}_{\mathrm{r}}$.

$$\mathbf{Y}_{r} = \mathbf{Q} \cdot \mathbf{Y}_{b} \cdot \mathbf{Q}^{t}$$

- Za zadani krug ona glasi

$$\mathbf{Y}_v = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{sL_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 & 0 \\ 0 & 0 & sC_3 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{R_5} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\mathbf{Y}_r = \begin{bmatrix} \frac{1}{sL_1} + \frac{1}{R_4} & -\frac{1}{R_4} & 0 \\ -\frac{1}{R_4} & \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5} & \frac{1}{R_5} \\ 0 & \frac{1}{R_5} & sC_3 + \frac{1}{R_5} \end{bmatrix}$$

- Matrična se jednadžba

$$\mathbf{Y}_r \cdot \mathbf{U}_r = -\mathbf{Q} \cdot \mathbf{I}_{ob}$$

$$\mathbf{Y}_r \cdot \mathbf{U}_r = \mathbf{I}_{or}$$

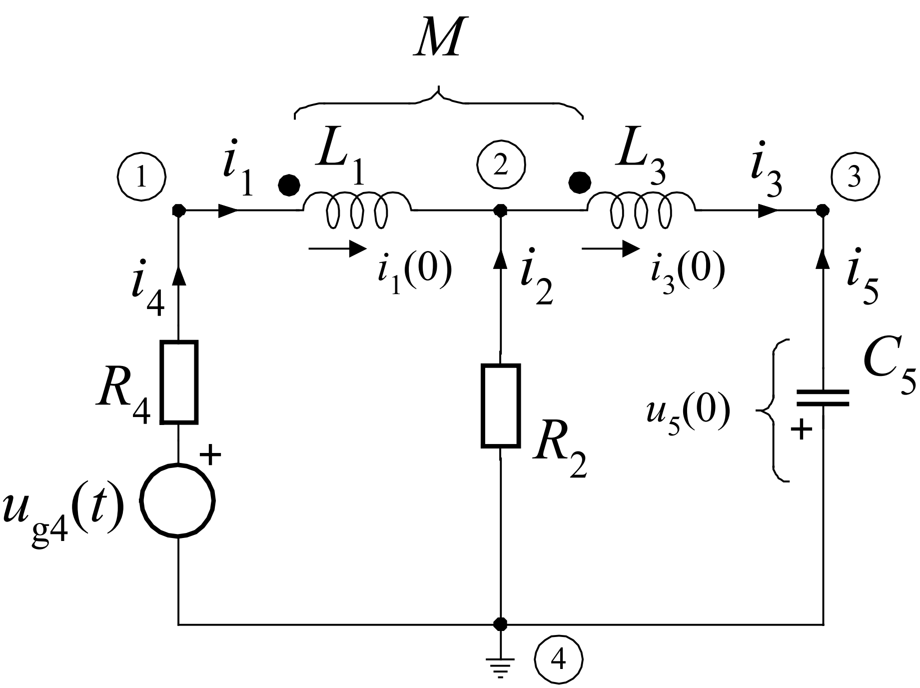
$$\begin{bmatrix} \frac{1}{sL_1} + \frac{1}{R_4} & -\frac{1}{R_4} & 0 \\ -\frac{1}{R_4} & \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5} & \frac{1}{R_5} \\ 0 & \frac{1}{R_5} & sC_3 + \frac{1}{R_5} \end{bmatrix} \cdot \begin{bmatrix} U_{r1} \\ U_{r2} \\ U_{r3} \end{bmatrix} = \begin{bmatrix} U_{g4}/R_4 + i_{L1}(0)/s \\ -U_{g4}/R_4 \\ C_3 u_{C3}(0) \end{bmatrix}$$

- Množenjem s lijeva s inverznom matricom $\mathbf{Y}_r(s)$, dobiva se rješenje sustava

$$\mathbf{Y}_r^{-1} \cdot \mathbf{Y}_r \cdot \mathbf{U}_r = \mathbf{U}_r = \mathbf{Y}_r^{-1} \cdot \mathbf{I}_{or}$$

$$\begin{bmatrix} U_{r1} \\ U_{r2} \\ U_{r3} \end{bmatrix} = \begin{bmatrix} \frac{1}{sL_1} + \frac{1}{R_4} & -\frac{1}{R_4} & 0 \\ -\frac{1}{R_4} & \frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5} & \frac{1}{R_5} \\ 0 & \frac{1}{R_5} & sC_3 + \frac{1}{R_5} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \frac{U_{g4}}{R_4} - \frac{i_{L1}(0)}{s} \\ -\frac{U_{g4}}{R_4} \\ C_3 u_{C3}(0) \end{bmatrix}$$

- Primjer 2. Za krug na slici postaviti jednadžbe temeljnog sustava petlji, rezova i čvorišta.



$$u_1(t) = L_1 \frac{\mathrm{d}i_1(t)}{\mathrm{d}t} + M \frac{\mathrm{d}i_3(t)}{\mathrm{d}t}$$

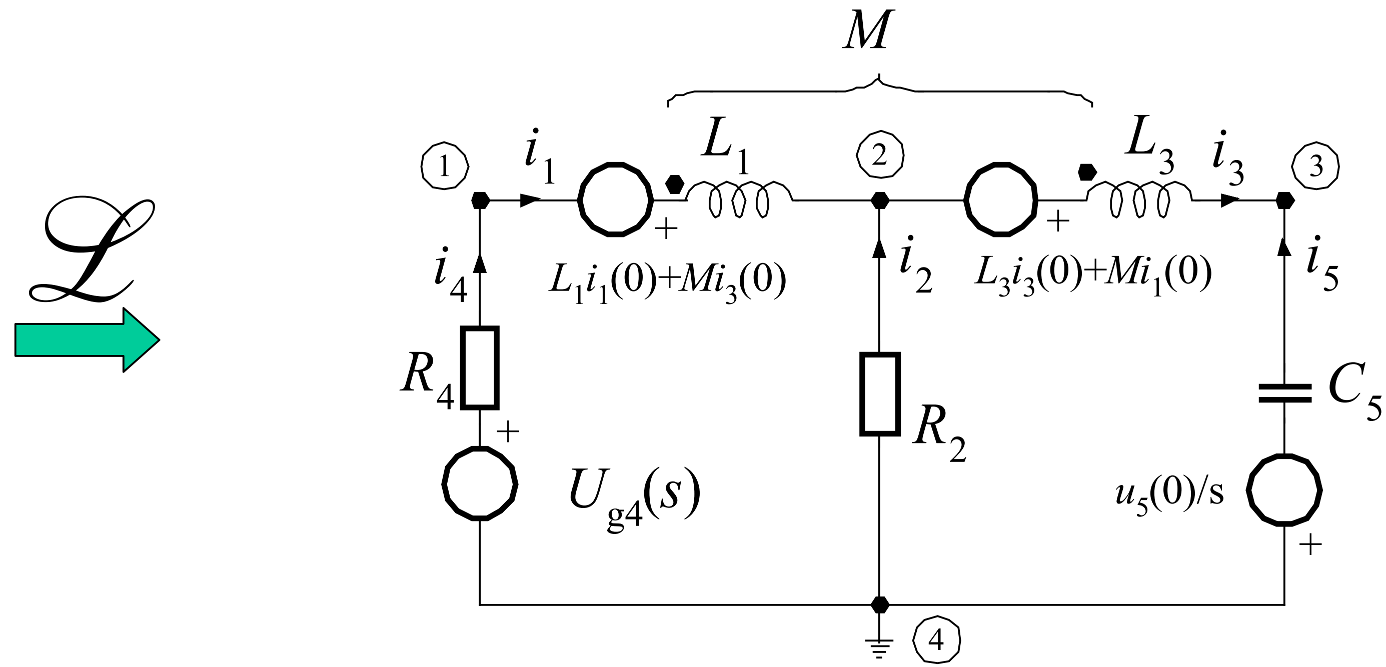
$$u_2(t) = R_2 i_2(t)$$

$$u_3(t) = M \frac{\mathrm{d}i_1(t)}{\mathrm{d}t} + L_3 \frac{\mathrm{d}i_3(t)}{\mathrm{d}t}$$

$$u_4(t) = -u_{g4}(t) + R_4 i_4(t)$$

$$u_5(t) = \frac{1}{C_5} \int_0^t i_5(\tau)\,\mathrm{d}\tau + u_5(0)$$

- Strujno naponske jednadžbe grana



$$U_1(s) = sL_1 I_1(s) - L_1 i_1(0) + sMI_3(s) - Mi_3(0) \qquad U_4(s) = -U_{g4}(s) + R_4 I_4(s)$$

$$U_2(s) = R_2 I_2(s) \qquad U_5(s) = \frac{1}{sC_5} I_5(s) + \frac{u_5(0)}{s}$$

$$U_3(s) = sMI_1(s) - Mi_1(0) + sL_3 I_3(s) - L_3 i_3(0)$$

- Jednadžbe grana u matričnome obliku

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix} = \begin{bmatrix} sL_1 & 0 & sM & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ sM & 0 & sL_3 & 0 & 0 \\ 0 & 0 & 0 & R_4 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1}{sC_5} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} + \begin{bmatrix} -L_1 i_1(0) - M i_3(0) \\ 0 \\ -M i_1(0) - L_3 i_3(0) \\ -U_{g4} \\ u_5(0)/s \end{bmatrix}$$

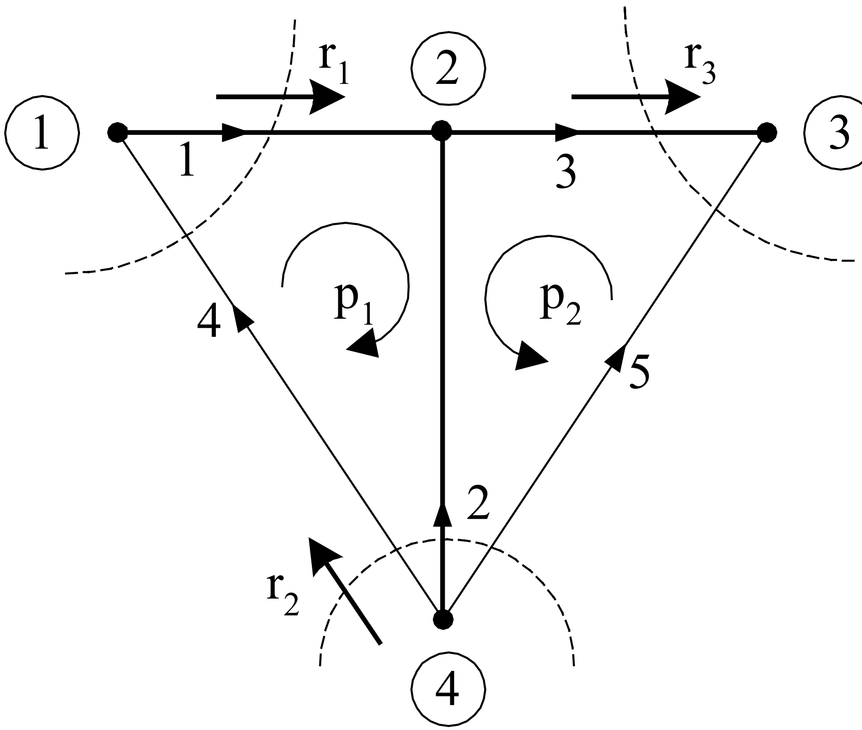
$$\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{ob}$$

- Matrica impedancija grana

$$\mathbf{Z}_b = \begin{bmatrix} sL_1 & 0 & sM & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ sM & 0 & sL_3 & 0 & 0 \\ 0 & 0 & 0 & R_4 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1}{sC_5} \end{bmatrix}$$

- Nije dijagonalna
- Simetrična oko glavne dijagonale $\mathbf{Z}_b^t = \mathbf{Z}_b$

- Matrice



$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix}$$

$$\mathbf{S} = \begin{bmatrix} 1 & -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \end{bmatrix}$$

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

- Jednadžbe temeljnih petlji

$$\mathbf{S}\cdot\mathbf{U}_b=\mathbf{0}$$

$$\mathbf{0}=\mathbf{S}\cdot\mathbf{U}_{ob}+\mathbf{S}\cdot\mathbf{Z}_b\cdot\mathbf{S}^t\cdot\mathbf{I}_p$$

$$\mathbf{0}=\mathbf{S}\cdot\mathbf{U}_{ob}+\mathbf{Z}_p\cdot\mathbf{I}_p$$

$$\mathbf{Z}_p=\mathbf{S}\cdot\mathbf{Z}_b\cdot\mathbf{S}^t=\begin{bmatrix} sL_1+R_2+R_4 & R_2-sM \\ R_2-sM & R_2+sL_3+\dfrac{1}{sC_5} \end{bmatrix}$$

$$\mathbf{Z}_p\cdot\mathbf{I}_p=-\mathbf{S}\cdot\mathbf{U}_{ob}$$

- Jednadžbe temeljnih petlji

$$\begin{bmatrix} sL_1 + R_2 + R_4 & R_2 - sM \\ R_2 - sM & R_2 + sL_3 + \dfrac{1}{sC_5} \end{bmatrix} \begin{bmatrix} I_{p1} \\ I_{p2} \end{bmatrix} = \begin{bmatrix} U_{g4} + L_1 i_1(0) + M i_3(0) \\ -M i_1(0) - L_3 i_3(0) + u_5(0)/s \end{bmatrix}$$

- Rješenje

$$\mathbf{I}_p = -\mathbf{Z}_p^{-1} \cdot \mathbf{S} \cdot \mathbf{U}_{ob}^{'}$$

$$\begin{bmatrix} I_{p1} \\ I_{p2} \end{bmatrix} = \begin{bmatrix} sL_1 + R_2 + R_4 & R_2 - sM \\ R_2 - sM & R_2 + sL_3 + \dfrac{1}{sC_5} \end{bmatrix}^{-1} \cdot \begin{bmatrix} U_{g4} + L_1 i_1(0) + M i_3(0) \\ -M i_1(0) - L_3 i_3(0) + u_5(0)/s \end{bmatrix}$$

- Za jednadžbe čvorišta i jednadžbe rezova treba definirati matricu admitancija grana.
- Jedan od načina – preuređenjem jednadžbi grana

$$U_1(s) = sL_1 I_1(s) - L_1 i_1(0) + sMI_3(s) - Mi_3(0)$$

$$U_2(s) = R_2 I_2(s)$$

$$U_3(s) = sMI_1(s) - Mi_1(0) + sL_3 I_3(s) - L_3 i_3(0)$$

$$U_4(s) = -U_{g4}(s) + R_4 I_4(s)$$

$$U_5(s) = \frac{1}{sC_5} I_5(s) + \frac{u_5(0)}{s}$$

- Struje grana izražene naponima

$$I_1(s)=\frac{sL_3}{\Delta M}U_1(s)-\frac{sM}{\Delta M}U_3(s)+i_1(0) \qquad \Delta M=s^2\left(L_1L_3-M^2\right)$$

$$I_2(s)=\frac{1}{R_2}U_2(s)$$

$$I_3(s)=-\frac{sM}{\Delta M}U_1(s)+\frac{sL_3}{\Delta M}U_3(s)+\frac{i_3(0)}{s}$$

$$I_4(s)=\frac{1}{R_4}U_4(s)+\frac{1}{R_4}U_{g4}(s)$$

$$I_5(s)=sC_5U_5(s)-C_5u_5(0)$$

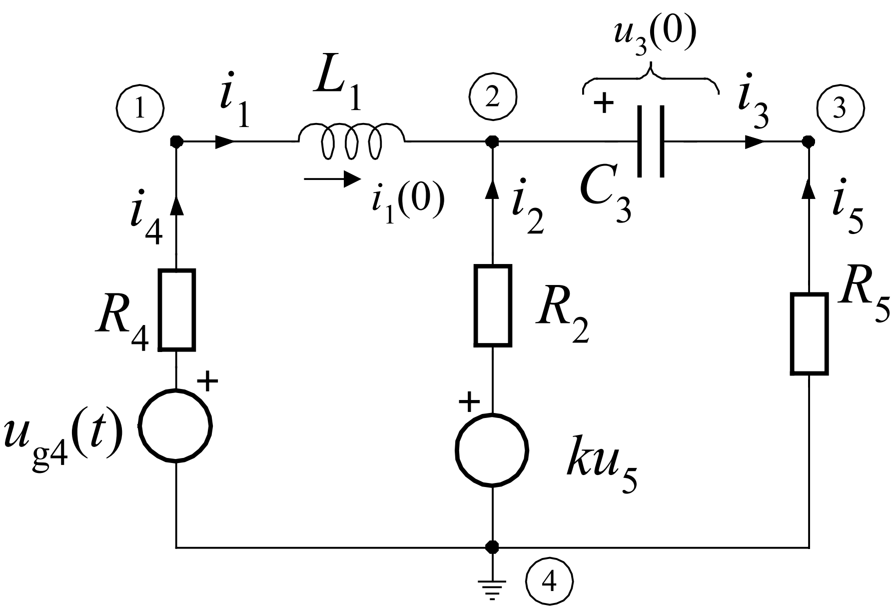
- Matrica admitancija grana

$$\mathbf{Y}_b = \begin{bmatrix} \frac{sL_3}{\Delta M} & 0 & -\frac{sM}{\Delta M} & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 & 0 \\ -\frac{sM}{\Delta M} & 0 & \frac{sL_1}{\Delta M} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_4} & 0 \\ 0 & 0 & 0 & 0 & sC_5 \end{bmatrix}$$

- Drugi način $\longrightarrow$ invertiranjem $\mathbf{Z}_\mathrm{b}$

$$\mathbf{Y}_b = \mathbf{Z}_b^{-1} = \begin{bmatrix} \frac{sL_3}{\Delta M} & 0 & -\frac{sM}{\Delta M} & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 & 0 \\ -\frac{sM}{\Delta M} & 0 & \frac{sL_1}{\Delta M} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_4} & 0 \\ 0 & 0 & 0 & 0 & sC_5 \end{bmatrix}$$

- Primjer 3. Za krug na slici postaviti jednadžbe temeljnog sustava petlji, rezova i čvorišta.



$$u_1(t) = L_1 \frac{\mathrm{d}i_1(t)}{\mathrm{d}t}$$

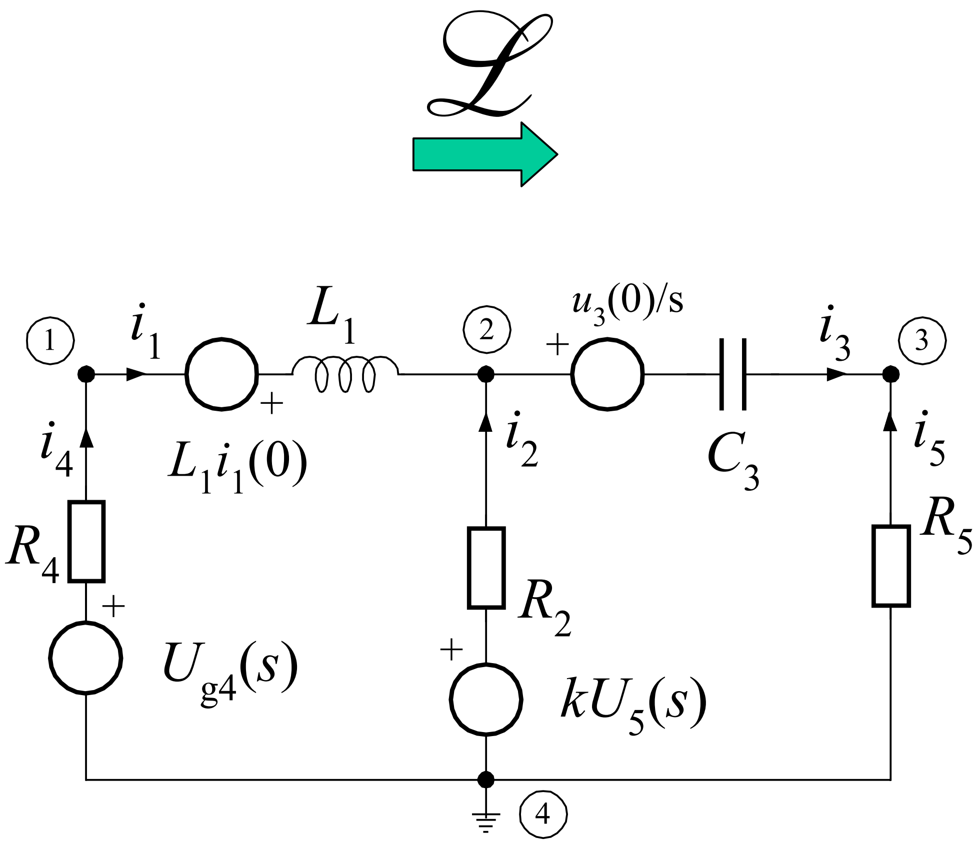
$$u_2(t) = R_2 i_2(t) - k \cdot u_5(t)$$

$$u_3(t) = \frac{1}{C_3} \int_0^t i_3(\tau)\mathrm{d}\tau + u_3(0)$$

$$u_4(t) = -u_{g4}(t) + R_4 i_4(t)$$

$$u_5(t) = R_5 i_5(t)$$

- Strujno naponske jednadžbe grana



$$U_1(s) = sL_1 I_1(s) - L_1 i_1(0)$$

$$U_2(s) = R_2 I_2(s) - k \cdot U_5(s)$$

$$U_3(s) = \frac{1}{sC_3} I_3(s) + \frac{u_3(0)}{s}$$

$$U_4(s) = -U_{g4}(s) + R_4 I_4(s)$$

$$U_5(s) = R_5 I_5(s)$$

- Jednadžbe grana u matričnome obliku

$$\begin{bmatrix} U_1(s) \\ U_2(s) \\ U_3(s) \\ U_4(s) \\ U_5(s) \end{bmatrix} = \begin{bmatrix} sL_1 & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -k\cdot R_5 \\ 0 & 0 & \dfrac{1}{sC_3} & 0 & 0 \\ 0 & 0 & 0 & R_4 & 0 \\ 0 & 0 & 0 & 0 & R_5 \end{bmatrix} \cdot \begin{bmatrix} I_1(s) \\ I_2(s) \\ I_3(s) \\ I_4(s) \\ I_5(s) \end{bmatrix} + \begin{bmatrix} -L_1\cdot i_1(0) \\ 0 \\ u_3(0)/s \\ -U_{g4}(s) \\ 0 \end{bmatrix}$$

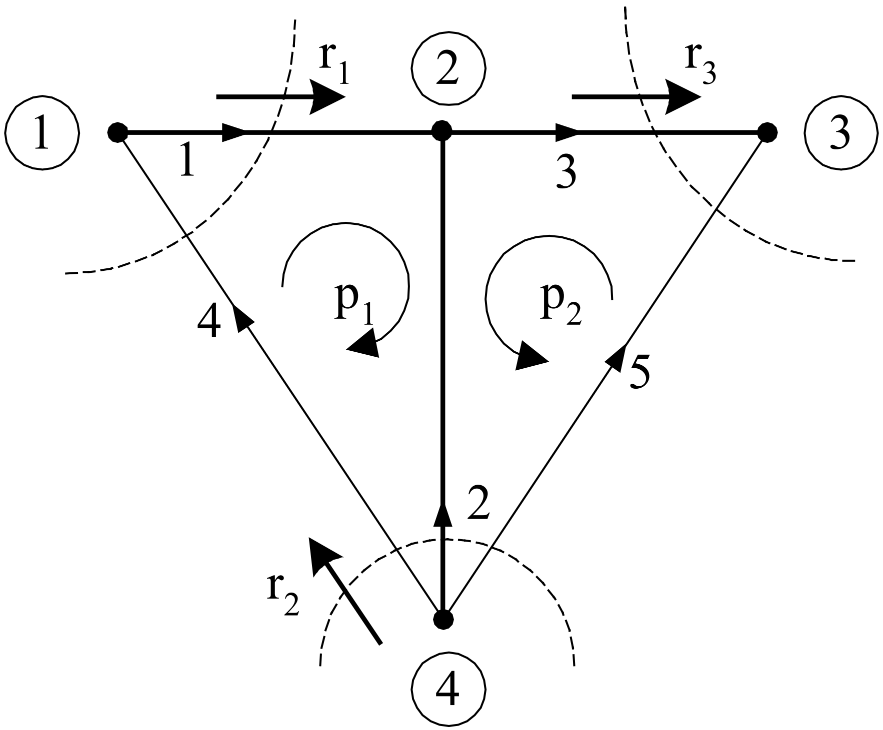
$$\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$$

- Matrica impedancija grana

$$\mathbf{Z}_b = \begin{bmatrix} sL_1 & 0 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & -k\cdot R_5 \\ 0 & 0 & \dfrac{1}{sC_3} & 0 & 0 \\ 0 & 0 & 0 & R_4 & 0 \\ 0 & 0 & 0 & 0 & R_5 \end{bmatrix}$$

- Nije dijagonalna
- Nije simetrična oko glavne dijagonale $\mathbf{Z}_b^t \neq \mathbf{Z}_b$
- Mreža je nerecipročna

- Matrice



$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ -1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix}$$

$$\mathbf{S} = \begin{bmatrix} 1 & -1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \end{bmatrix}$$

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

- Jednadžbe temeljnih petlji

$$\mathbf{S} \cdot \mathbf{U}_b = \mathbf{0}$$

$$\mathbf{0} = \mathbf{S} \cdot \mathbf{U}_{ob} + \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^t \cdot \mathbf{I}_p$$

$$\mathbf{0} = \mathbf{S} \cdot \mathbf{U}_{ob} + \mathbf{Z}_p \cdot \mathbf{I}_p$$

$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^t = \begin{bmatrix} sL + R_2 + R_4 & R_2 + kR_5 \\ R_2 & R_2 + \dfrac{1}{sC_3} + R_5 + kR_5 \end{bmatrix}$$

$$\mathbf{Z}_p \cdot \mathbf{I}_p = -\mathbf{S} \cdot \mathbf{U}_{ob}$$

- Jednadžbe temeljnih petlji

$$\begin{bmatrix} sL+R_2+R_4 & R_2+kR_5 \\ R_2 & R_2+\dfrac{1}{sC_3}+R_5+kR_5 \end{bmatrix} \cdot \begin{bmatrix} I_{p1} \\ I_{p2} \end{bmatrix} = \begin{bmatrix} U_{g4}(s)+L_1\cdot i_1(0) \\ \dfrac{1}{s}u_3(0) \end{bmatrix}$$

- Rješenje

$$\mathbf{I}_p = -\mathbf{Z}_p^{-1}\cdot\mathbf{S}\cdot\mathbf{U}_{ob}$$

$$\begin{bmatrix} I_{p1} \\ I_{p2} \end{bmatrix} = \begin{bmatrix} sL+R_2+R_4 & R_2+kR_5 \\ R_2 & R_2+\dfrac{1}{sC_3}+R_5+kR_5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} U_{g4}(s)+L_1\cdot i_1(0) \\ \dfrac{1}{s}u_3(0) \end{bmatrix}$$

- Matrica admitancija grana.
- Preuređenjem jednadžbi grana

$$I_1(s) = \frac{U_1(s)}{sL_1} + \frac{i_1(0)}{s}$$

$$I_2(s) = \frac{U_2(s)}{R_2} + k\frac{U_5(s)}{R_2}$$

$$I_3(s) = sC_3U_3\left(s\right) - sC_3i_3(0)$$

$$I_4(s) = \frac{U_{g4}(s)}{R_4} + \frac{U_4(s)}{R_4}$$

$$I_5(s) = \frac{U_5(s)}{R_5}$$

- Matrica admitancija grana

$$\mathbf{Y}_b = \begin{bmatrix} \frac{1}{sL_1} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 & \frac{k}{R_2} \\ 0 & 0 & sC_3 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R_4} & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{R_5} \end{bmatrix}$$

- Drugi način $\longrightarrow$ invertiranjem $\mathbf{Z}_b$

$$\mathbf{Y}_b = \mathbf{Z}_b^{-1} = \begin{bmatrix} \dfrac{1}{sL_1} & 0 & 0 & 0 & 0 \\ 0 & \dfrac{1}{R_2} & 0 & 0 & \dfrac{k}{R_2} \\ 0 & 0 & sC_3 & 0 & 0 \\ 0 & 0 & 0 & \dfrac{1}{R_4} & 0 \\ 0 & 0 & 0 & 0 & \dfrac{1}{R_5} \end{bmatrix}$$