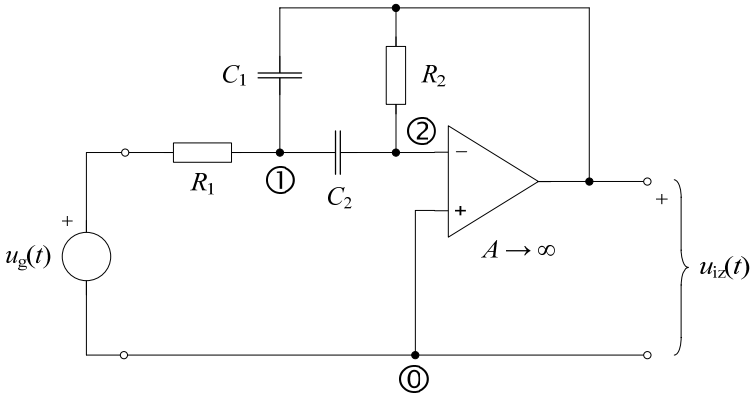
ZAVRŠNI ISPIT IZ PREDMETA ELEKTRIČNI KRUGOVI 2008/09

Rješenja i bodovi (svaki zadatak je bodovan od 0 do 5 bodova):

1. Za električni krug na slici zadana je pobuda $u_g(t)=e^{-t}\cdot S(t)$ i normalizirane vrijednosti elemenata: $R_1=R_2=1$, $C_1=1$, $C_2=2$, $A\to\infty$. Odrediti: *a*) jednadžbe čvorišta; *b*) prijenosnu funkciju: $T(s)=U_{iz}(s)/U_g(s)$; c) polove i nule $T(s)$ i odziv $U_{iz}(s)$; *d*) odziv $u_{iz}(t)$.



Rješenje:

a) jednadžbe čvorišta

$$1)\ U_1\left(sC_1+\frac{1}{R_1}+sC_2\right)-U_2(sC_2)=\frac{U_g(s)}{R_1}+\frac{U_{iz}(s)}{\frac{1}{sC_1}}$$

$$2)\ U_2\left(sC_2+\frac{1}{R_2}\right)-U_1(sC_2)=\frac{U_{iz}(s)}{R_2}\qquad U_2=0$$

(1 bod)

b) prijenosna funkcija: $T(s)=U_{iz}(s)/U_g(s)$

$$2)\quad\rightarrow\quad U_1=-\frac{U_{iz}(s)}{R_2sC_2}$$

$$-\frac{U_{iz}(s)}{R_2sC_2}\left(sC_1+\frac{1}{R_1}+sC_2\right)=\frac{U_g(s)}{R_1}+U_{iz}(s)\cdot sC_1$$

$$-U_{iz}(s)\left(\frac{sC_1}{R_2sC_2}+\frac{1}{R_1R_2sC_2}+\frac{1}{R_2}+sC_1\right)=\frac{U_g(s)}{R_1}\Big/\cdot R_1R_2sC_2$$

$$-U_{iz}(s)\left(sC_1R_1+1+R_1sC_2+s^2R_1R_2C_1C_2\right)=U_g(s)\cdot R_2sC_2$$

$$T(s)=\frac{U_{iz}(s)}{U_g(s)}=-\frac{sR_2C_2}{s^2R_1R_2C_1C_2+s(C_1R_1+R_1C_2)+1}=$$

$$=-\frac{2s}{2s^2+3s+1}=-\frac{s}{s^2+\frac{3}{2}s+\frac{1}{2}}=-\frac{s}{(s+1)\left(s+\frac{1}{2}\right)}$$

(1 bod)

c) polovi i nule $T(s)$ i odziv $U_{iz}(s)$

-nule: $s_{o,1}=0$, $s_{o,2}=\infty$

-polovi: $s^2+\frac{3}{2}s+\frac{1}{2}=0 \Rightarrow s_{p1,2}=-\frac{3}{4}\pm\sqrt{\frac{9}{16}-\frac{8}{16}}=-\frac{3}{4}\pm\frac{1}{4} \;\Rightarrow\; s_{p1}=-\frac{1}{2};\; s_{p2}=-1$

-odziv $U_{iz}(s)$:

$$U_{iz}(s)=T(s)\cdot U_g(s)$$

$$U_g(s)=\frac{1}{s+1}$$

$$U_{iz}(s)=\frac{-s}{(s+1)\left(s^2+\frac{3}{2}s+\frac{1}{2}\right)}=\frac{-s}{(s+1)(s+1)\left(s+\frac{1}{2}\right)}=\frac{-s}{(s+1)^2\left(s+\frac{1}{2}\right)}$$

(1 bod)

d) odziv $u_{iz}(t)$

Rastav na parcijalne razlomke:

$$U_{iz}(s)=\frac{-s}{(s+1)^2\left(s+\frac{1}{2}\right)}=\frac{A}{(s+1)^2}+\frac{B}{s+1}+\frac{C}{s+\frac{1}{2}}$$

$$A\left(s+\frac{1}{2}\right)+B(s+1)\left(s+\frac{1}{2}\right)+C(s+1)^2=-s$$

$$As+\frac{A}{2}+Bs^2+\frac{3}{2}Bs+\frac{B}{2}+Cs^2+2Cs+C=-s$$

$$(B+C)s^2+\left(A+\frac{3}{2}B+2C\right)s+\frac{A}{2}+\frac{B}{2}+C=-s$$

$$\left.\begin{array}{c} B+C=0 \\ A+\frac{3}{2}B+2C=-1 \\ \frac{A}{2}+\frac{B}{2}+C=0 \end{array}\right\} \Rightarrow \begin{array}{c} B=-C \\ \frac{1}{2}A=-1 \\ A=B \end{array}$$

$$A=-2 \qquad B=-2 \qquad C=2$$

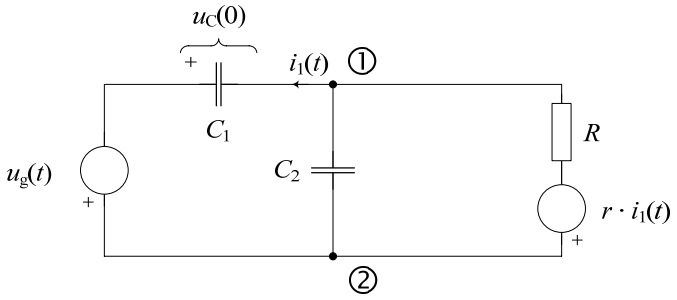
$$U_{iz}(s)=-2\left(\frac{1}{(s+1)^2}+\frac{1}{s+1}-\frac{1}{s+\frac{1}{2}}\right)$$

(1 bod)

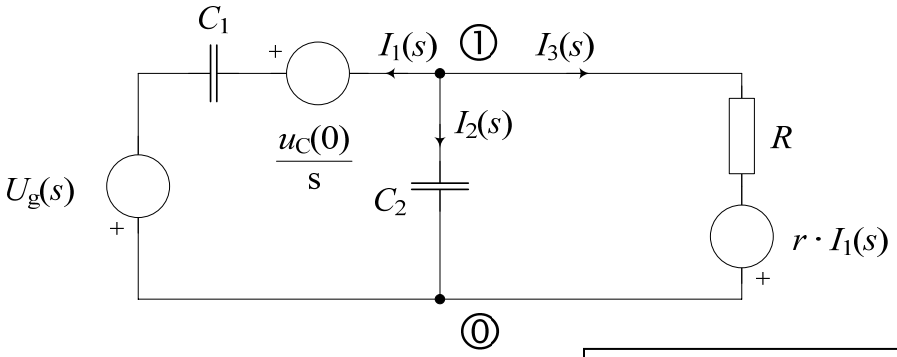
$$u_{iz}(t)=-2\cdot\left(t\cdot e^{-t}+e^{-t}-e^{-\frac{t}{2}}\right)\cdot S(t)$$

(1 bod)

2. Za električni krug prikazan slikom odrediti: *a*) orijentirani graf; *b*) matricu incidencija **A**; *c*) strujno-naponske jednadžbe grana; *d*) sustav jednadžbi čvorova u matričnom obliku (matrice $\mathbf{Y}_v$ i $\mathbf{I}_{0v}$ preko matrica $\mathbf{Y}_b$ i $\mathbf{I}_{0b}$). Matrica $\mathbf{Y}_b$ mora biti regularna.



Rješenje: Primjena $\mathcal{L}$-transformacije



c) Strujno naponske jednadžbe grana:

$$U_1 = I_1 \cdot \frac{1}{sC_1} - \frac{u_C(0)}{s} - U_g(s)$$

$$U_2 = I_2 \cdot \frac{1}{sC_2}$$

$$U_1 = I_3 \cdot R - r \cdot I_1(s)$$

Odnosno:

$$I_1 = sC_1 \cdot U_1 + C_1 \cdot u_C(0) + sC_1 \cdot U_g(s)$$

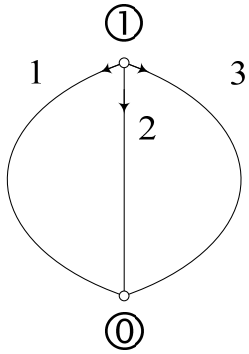
$$I_2 = sC_2 \cdot U_2$$

$$I_3 = \frac{1}{R} \cdot U_3 + \frac{r}{R} \cdot I_1(s)$$

$$I_3 = \frac{1}{R} \cdot U_3 + \frac{r}{R} \cdot sC_1 \cdot U_1 + \frac{r}{R} \cdot C_1 \cdot u_C(0) + \frac{r}{R} \cdot sC_1 \cdot U_g(s)$$

U matričnom obliku: $\mathbf{I_b} = \mathbf{Y_b} \cdot \mathbf{U_b} + \mathbf{I_{0b}}$

a) Orijentirani graf:



b) Matrica incidencija (reducirana):

$$\mathbf{A} = \begin{array}{c} \\ 1 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \end{array}$$

(1 bod)

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{\mathbf{I_b}} = \underbrace{\begin{bmatrix} sC_1 & 0 & 0 \\ 0 & sC_2 & 0 \\ \frac{r}{R}\cdot sC_1 & 0 & \frac{1}{R} \end{bmatrix}}_{\mathbf{Y_b}} \cdot \underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}}_{\mathbf{U_b}} + \underbrace{\begin{bmatrix} C_1 \cdot u_C(0) + sC_1 \cdot U_g(s) \\ 0 \\ \frac{r}{R}\cdot C_1 \cdot u_C(0) + \frac{r}{R}\cdot sC_1 \cdot U_g(s) \end{bmatrix}}_{\mathbf{I_{0b}}}$$

(2 boda)

d) Sustav jednadžbi napona čvorova u matričnom obliku $\mathbf{Y_v} \cdot \mathbf{U_v} = \mathbf{I_{0v}}$, gdje su:

$$\mathbf{Y_v} = \mathbf{A}\cdot\mathbf{Y_b}\cdot\mathbf{A^T} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} sC_1 & 0 & 0 \\ 0 & sC_2 & 0 \\ \frac{r}{R}\cdot sC_1 & 0 & \frac{1}{R} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} sC_1 + \frac{r}{R}\cdot sC_1 & sC_2 & \frac{1}{R} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} =$$

$$= \left[sC_1\left(1+\frac{r}{R}\right) + sC_2 + \frac{1}{R} \right]$$

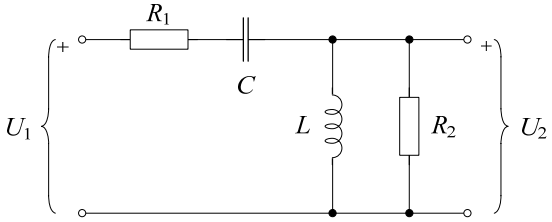
(1 bod)

$$\mathbf{I_{ov}} = -\mathbf{A}\cdot\mathbf{I_{0b}} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} C_1 \cdot u_C(0) + sC_1 \cdot U_g(s) \\ 0 \\ \frac{r}{R}\cdot C_1 \cdot u_C(0) + \frac{r}{R}\cdot sC_1 \cdot U_g(s) \end{bmatrix} =$$

$$= -\left[C_1 \cdot u_C(0) \cdot \left(1+\frac{r}{R}\right) + sC_1 \cdot U_g(s) \cdot \left(1+\frac{r}{R}\right) \right]$$

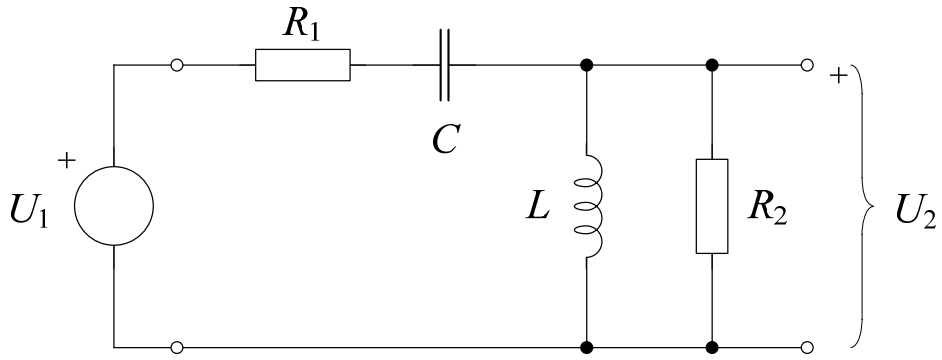
(1 bod)

3. Za četveropol na slici zadane su normalizirane vrijednosti elemenata R_1=R_2=1, C=2, L=1/2. *a*) Odrediti prijenosnu funkciju $T(s)=U_2(s)/U_1(s)$. *b*) Prikazati raspored polova i nula u kompleksnoj ravnini. *c*) Nacrtati amplitudno-frekvencijsku karakteristiku. *d*) O kojem se tipu filtra radi (NP, VP, PP ili PB)? *e*) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja odrediti parametre k, ω_0, Q.



Rješenje:

a) prijenosna funkciju $T(s)=U_2(s)/U_1(s)$:



$$T(s)=\frac{U_2}{U_1}=\frac{\dfrac{sL\cdot R_2}{sL+R_2}}{R_1+\dfrac{1}{sC}+\dfrac{sL\cdot R_2}{sL+R_2}}=\frac{sL\cdot R_2}{\left(R_1+\dfrac{1}{sC}\right)(sL+R_2)+sL\cdot R_2}=$$

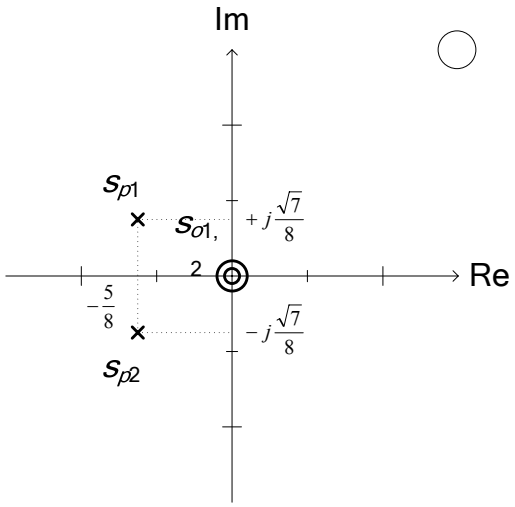
$$=\frac{sL\cdot R_2}{R_1sL+R_1R_2+\dfrac{sL}{sC}+\dfrac{R_2}{sC}+sL+R_2}\begin{matrix}/\cdot sC\\ /\cdot sC\end{matrix}=\frac{s^2LCR_2}{s^2R_1LC+sR_1R_2C+sL+R_2+s^2LR_2C}=$$

$$=\frac{s^2LCR_2}{s^2LC(R_1+R_2)+s(R_1R_2C+L)+R_2}=\frac{s^2}{2\cdot s^2+\dfrac{5}{2}\cdot s+1}=\frac{\dfrac{1}{2}\cdot s^2}{s^2+\dfrac{5}{4}\cdot s+\dfrac{1}{2}}$$ (1 bod)

b) raspored polova i nula u kompleksnoj ravnini:

nule $s_{o1,2}=0$

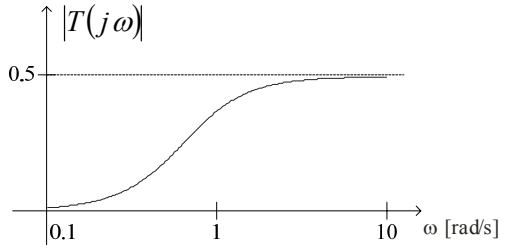
polovi $s^2+\dfrac{5}{4}\cdot s+\dfrac{1}{2}=0 \quad\Rightarrow\quad s_{p1,2}=-\dfrac{5}{8}\pm\sqrt{\dfrac{25}{64}-\dfrac{32}{64}}=-\dfrac{5}{8}\pm j\dfrac{\sqrt{7}}{8}$



(1 bod)

c) amplitudno-frekvencijska karakteristika

$$T(j\omega)=\frac{-\omega^2\cdot\frac{1}{2}}{-\omega^2+j\omega\cdot\frac{5}{4}+\frac{1}{2}} \quad \Rightarrow \quad |T(j\omega)|=\frac{\frac{1}{2}\cdot\omega^2}{\sqrt{\left(\frac{1}{2}-\omega^2\right)^2+\left(\omega\cdot\frac{5}{4}\right)^2}}$$



(1 bod)

d) o kojem se tipu filtra radi (NP, VP, PP ili PB)? $\Rightarrow$ VP (visoki propust)
(1 bod)

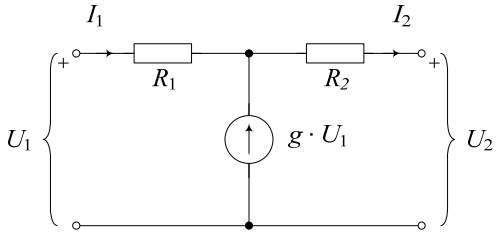
e) parametri k, ω_0, Q:

$$T(s)=\frac{\frac{1}{2}\cdot s^2}{s^2+\frac{5}{4}\cdot s+\frac{1}{2}}=\frac{k\cdot s^2}{s^2+\frac{\omega_0}{Q}\cdot s+{\omega_0}^2} \qquad \text{Opći oblik VP}$$

$$\omega_0=\frac{1}{\sqrt{2}} \qquad \frac{\omega_0}{Q}=\frac{5}{4}\Rightarrow Q=\frac{\omega_0}{\frac{5}{4}}=\frac{\frac{1}{\sqrt{2}}}{\frac{5}{4}}=\frac{4}{5\sqrt{2}}=\frac{2\sqrt{2}}{5} \qquad k=\frac{1}{2}$$

(1 bod)

4. Za četveropol na slici izračunati: *a*) [**y**] parametre. Da li je četveropol: *b*) recipročan; *c*) simetričan ? Obrazložiti odgovore.

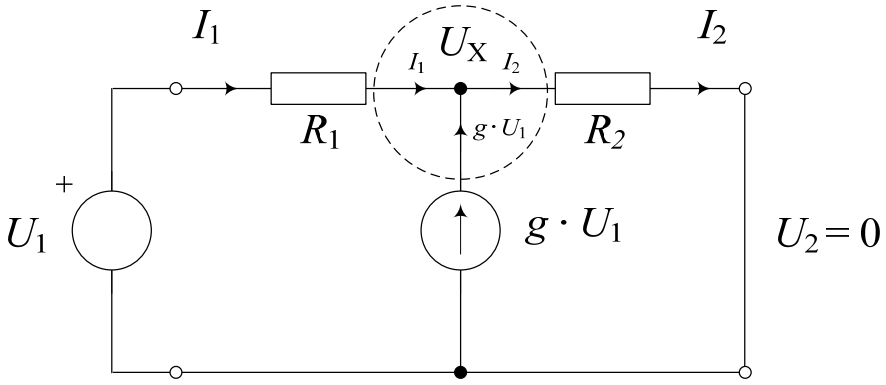


Rješenje:

a) izračun [**y**] parametara:

$$I_1 = y_{11} \cdot U_1 + y_{12} \cdot U_2$$
$$I_2 = y_{21} \cdot U_1 - y_{22} \cdot U_2$$

$U_2 = 0$



$$I_1 + g \cdot U_1 = I_2$$
$$U_X = I_2 \cdot R_2 = U_1 - I_1 \cdot R_1$$
$$\Rightarrow I_2 = \frac{U_1}{R_2} - I_1 \frac{R_1}{R_2}$$
$$\Rightarrow I_1 \cdot R_1 = U_1 - I_2 \cdot R_2$$
$$I_1 = \frac{1}{R_1} \cdot U_1 - I_2 \frac{R_2}{R_1}$$
$$\frac{1}{R_1} \cdot U_1 - I_2 \frac{R_2}{R_1} + g \cdot U_1 = I_2$$
$$U_1 \left(\frac{1}{R_1} + g \right) = I_2 \left(1 + \frac{R_2}{R_1} \right)$$

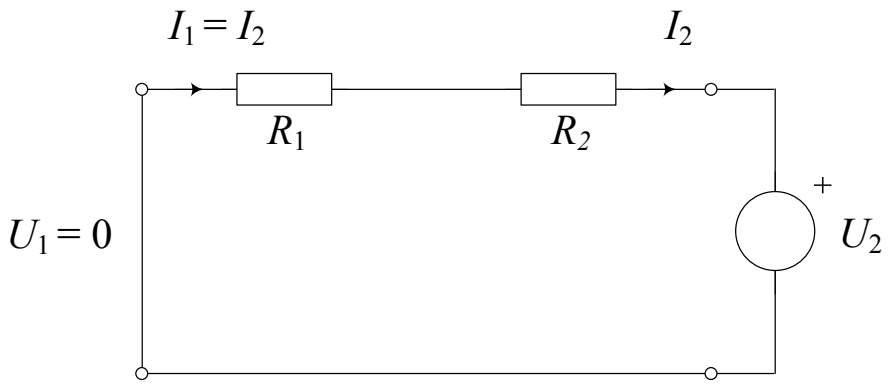
$$I_1 + g \cdot U_1 = \frac{U_1}{R_2} - I_1 \frac{R_1}{R_2}$$

$$I_1\left(1 + \frac{R_1}{R_2}\right) = U_1\left(\frac{1}{R_2} - g\right)$$

$$y_{11} = \frac{I_1}{U_1} = \frac{\frac{1}{R_2} - g}{1 + \frac{R_1}{R_2}} = \frac{1 - g \cdot R_2}{R_1 + R_2}$$

$$y_{21} = \frac{I_2}{U_1} = \frac{\frac{1}{R_1} + g}{1 + \frac{R_2}{R_1}} = \frac{\frac{1}{R_2} + \frac{R_1}{R_2} g}{1 + \frac{R_1}{R_2}} = \frac{1 + g \cdot R_1}{R_1 + R_2}$$

$U_1 = 0$



$$y_{12} = -\frac{I_1}{U_2} = \frac{1}{R_1 + R_2}$$

$$y_{22} = -\frac{I_2}{U_2} = \frac{1}{R_1 + R_2}$$

$$[\mathbf{y}] = \begin{bmatrix} \frac{1 - g \cdot R_2}{R_1 + R_2} & -\frac{1}{R_1 + R_2} \\ \frac{1 + g \cdot R_1}{R_1 + R_2} & -\frac{1}{R_1 + R_2} \end{bmatrix}$$

(do sada: maksimum 3 boda – ako su sva 4 parametra točna; 2 boda ako 1 fali; 1 bod ako 2 fale, 0 bodova ako 3 ili 4 parametra nisu točna)

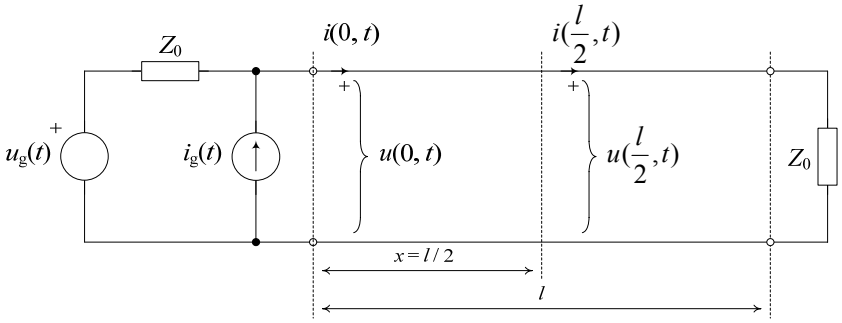
b) Da li je četveropol recipročan ?

Ne, jer za recipročnost mora vrijediti $y_{12}=y_{21}$. To očigledno ne vrijedi, a razlog tomu je zavisni izvor. (1 bod)

c) Da li je četveropol simetričan ?

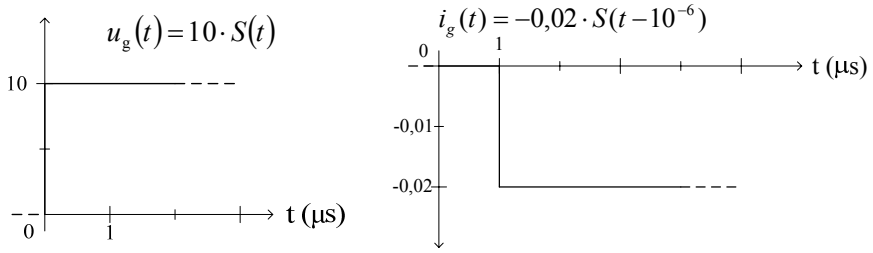
Ne, jer za simetričnost mora vrijediti $y_{11}=y_{22}$. (1 bod)

5. Linija duljine l=1 km sa primarnim parametrima: R= 1Ω/km, L= 3mH/km, G= 4μS/km i C= 12nF/km, zaključena je s obje strane svojom karakterističnom impedancijom Z_0. Na liniju su spojeni naponski izvor $u_g(t)=10S(t)$ i strujni izvor $i_g(t)=-0{,}02S(t-10^{-6})$ prema slici. *a*) Nacrtati valni oblik napona $u_g(t)$ i struje $i_g(t)$. Odrediti izraz za *b*) napon i *c*) struju na polovini linije. *d*) Nacrtati valne oblike traženih napona i struja.

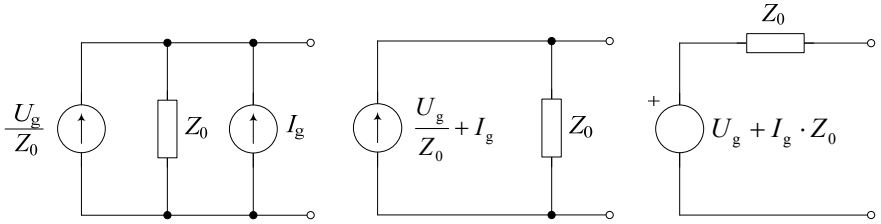


Rješenje:

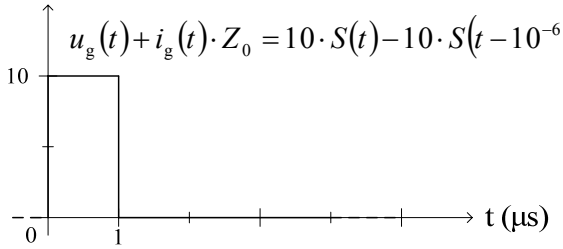
a) valni oblici napona $u_g(t)$ i struje $i_g(t)$: (1 bod)



- napon i struja na početku linije (transformacija izvora na ulazu u liniju) : (1 bod)



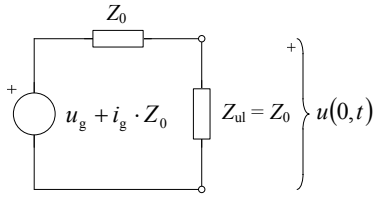
pa je ukupni valni oblik napona obaju generatora:

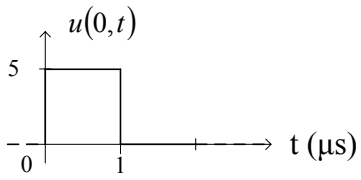


te napona i struje na ulazu linije:

$$Z_{ul} = Z_0 \Rightarrow \quad u(0,t) = (u_g + i_g \cdot Z_0)\frac{Z_0}{Z_0 + Z_0} = \frac{u_g + i_g \cdot Z_0}{2} \qquad U(0) = I(0) \cdot Z_0$$

$$i(0,t) = \frac{u(0,t)}{Z_0} \qquad I(0) = \frac{U(0)}{Z_0}$$





Prijenosne jednadžbe linije:

$$U(x) = U(0)\cdot \mathrm{ch}(\gamma x) - I(0)\cdot Z_0 \cdot \mathrm{sh}(\gamma x)$$
$$I(x) = -U(0)/Z_0 \cdot \mathrm{sh}(\gamma x) + I(0)\cdot \mathrm{ch}(\gamma x)$$

Sekundarni parametri linije:

$$\gamma = \sqrt{(R+sL)(G+sC)}$$
$$Z_0 = \sqrt{\frac{R+sL}{G+sC}}$$

Specijalni slučaj: $\frac{R}{L} = \frac{G}{C}$ $\qquad \frac{1}{3\cdot 10^{-3}} = \frac{4\cdot 10^{-6}}{12\cdot 10^{-9}}$

Linija bez distorzije:

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{3\cdot 10^{-3}}{12\cdot 10^{-9}}} = \sqrt{\frac{1}{4}\cdot 10^6} = \frac{1}{2}\cdot 10^3 \Omega$$

$$\gamma = \sqrt{RG} + s\sqrt{LC}$$
$$= \sqrt{1\cdot 4\cdot 10^{-6}} + s\sqrt{3\cdot 10^{-3}\cdot 12\cdot 10^{-9}}$$
$$= 2\cdot 10^{-3} + 6\cdot 10^{-6} s \ /\mathrm{km}$$

$x = l/2$ polovina linije (traži se napon i struja)
$x = 0$ početak linije (zadan izvor)

$$U(x) = U(0)\cdot \mathrm{ch}(\gamma x) - U(0)\cdot \mathrm{sh}(\gamma x) = U(0)(\mathrm{ch}(\gamma x) - \mathrm{sh}(\gamma x)) = U(0)\cdot e^{-\gamma x}$$
$$I(x) = I(0)(-\mathrm{sh}(\gamma x) + \mathrm{ch}(\gamma x)) = I(0)\cdot e^{-\gamma x}$$

$x = 1/2\mathrm{km}$ $\qquad \gamma x = \left(2\cdot 10^{-3} + 6\cdot 10^{-6} s\right)/\mathrm{km}\cdot 1/2\mathrm{km} = 1\cdot 10^{-3} + 3\cdot 10^{-6} s$

-izrazi za napon i struju na polovini linije:

b) napon: $U(1/2\mathrm{km}) = U(0)\cdot e^{-\left(10^{-3}+3\cdot 10^{-6}\cdot s\right)} = U(0)\cdot e^{-10^{-3}}\cdot e^{-3\cdot 10^{-6} s}$ (1 bod)

c) struja: $I(1/2\mathrm{km}) = I(0)\cdot e^{-\left(\gamma\frac{l}{2}\right)} = \frac{U(0)}{Z_0}\cdot e^{-\left(10^{-3}+3\cdot 10^{-6}\cdot s\right)} = U(0)\cdot 2\cdot 10^{-3}\cdot e^{-10^{-3}}\cdot e^{-3\cdot 10^{-6} s}$ (1 bod)

d) valni oblici napona i struje na polovini linije: (1 bod)