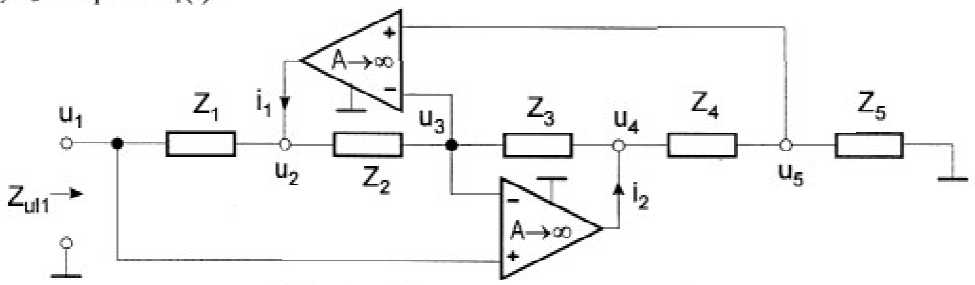
Električni krugovi - Lab

Lab 6. Priprema
Električni filtri 2. reda

Ime i Prezime: _____
Asistent: _____
Grupa: _____

Napomena: Ukoliko nema dovoljno prostora neka student doda list papira na kojemu će postupak koji je doveo do rješenja. **Lab Pripremu** treba odštampati dvostrano i popuniti je te pričvrstiti dodatnu stranicu papira pomoću spajalice. Popunjena **Lab Priprema** se predaje asistentu na početku laboratorijskih vježbi.

1. Za električni krug prikazan slikom izračunati napone i struje operacijskih pojačala. Izvesti izraze za ovisnost napona $u_2(t)$, $u_4(t)$ i struja $i_1(t)$, $i_2(t)$ o vrijednostima impedancija Z_1, Z_2, …, Z_5 i napona $u_1(t)$.



Slika 1 Opći konvertor impedancija (GIC).

$$U_2(s) = \left[1 - \frac{Z_2 \cdot Z_4}{Z_3 \cdot Z_5}\right] \cdot U_1(s) \qquad I_1(s) = \frac{(Z_1 + Z_2) \cdot Z_4}{Z_1 \cdot Z_3 \cdot Z_5} U_1(s)$$

$$U_4(s) = \left[1 + \frac{Z_4}{Z_5}\right] \cdot U_1(s) \qquad I_2(s) = -\frac{Z_3 + Z_4}{Z_3 \cdot Z_5} \cdot U_1(s)$$

IZVODI SU NA DODATNOJ STRANICI 1

2. Za električni krug prikazan slikom 1 izračunati napone $u_2(t)$, $u_4(t)$ i struje $i_1(t)$, $i_2(t)$ ako su zadane impedancije: $Z_1(s)=R_1$, i $Z_2(s)=R_2$, $Z_3(s)=R_3$, i $Z_4(s)=1/(sC_4)$, $Z_5(s)=R_5$ i vrijednosti elemenata R_1=1kΩ, R_2=100kΩ, R_3=1kΩ, C_4=100nF, R_5=1kΩ. Koliki je ekvivalentni induktivitet L_{ekv}? Ulazni napon je $u_1(t)$= sin $10^5 t$; $-\infty<t<\infty$.

$u_2(t) = 10.05 \cdot \sin(10^5 t + 84.2°)$ [V] $= 10.05 \cdot \sin(10^5 t + 1.471\,rad)$ [V]

$u_4(t) = 1.005 \sin(10^5 t - 5.71°)$ [V]

$i_1(t) = 10.1 \sin(10^5 t + 180°)$ [mA]

$i_2(t) = 1.05 \cdot \sin(10^5 t + 174.3°)$ [mA]

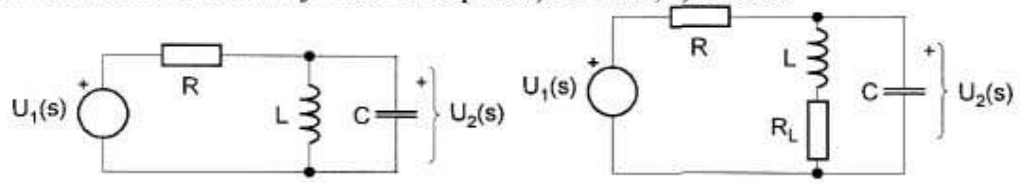
$L_{ekv} = 1\,mH$

IZVODI SU NA DODATNOJ STRANICI

Da bi električni krug na slici 1 mogao ispravno raditi, operacijska pojačala moraju biti u svom tzv. linearnom području rada. To znači da naponi na izlazima operacijskih pojačala $u_2(t)$ i $u_4(t)$ ne bi smjeli premašiti iznos od ±10V (ako je izvor napajanja jednak ±12V), a izlazne struje $i_1(t)$ i $i_2(t)$ ne bi smjele biti veće od ±20mA. Da li električni krug na slici 1 sa uvrštenim elementima i zadanom pobudom kao u ovom primjeru, ispunjava uvjete ispravnog rada ?

DA! - Napon $u_2(t)$ ima $U_{2max} = 10.1\,V \approx 10\,V$
- Napon $U_{4max} = 1.005\,V < 10\,V$
- $I_{1max} = 10.1\,mA < 20\,mA$
- $I_{2max} = 1.05\,mA < 20\,mA$

3. Za električne krugove prikazane slikom 2(a) i (b) izračunati naponske prijenosne funkcije $T(s)=U_2(s)/U_1(s)$, ako su zadane vrijednosti elemenata L=1mH, R_L=10Ω, C=100nF i a) R=1kΩ. Zatim uvrstiti vrijednosti za otpore b) R=200Ω, c) R=20Ω.



Slika 2 Pasivni RLC pojasno-propusni filtar 2. reda.

IZVODI I RJEŠENJA NA DODATNIM LISTOVIMA 3 i 4

Rješenje: a) b) c)

PRIJENOSNE FUNKCIJE

SLIKA (a)

$$T(s) = \frac{\frac{1}{RC}s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

SLIKA (b)

$$T(s) = \frac{\frac{1}{C}\left(s + \frac{R_L}{L}\right)}{s^2 + s\,\frac{R \cdot R_L \cdot C + L}{RLC} + \frac{R + R_L}{RLC}}$$

4. Za električne krugove prikazane slikom 2(a) i (b) uz zadane vrijednosti elemenata L=1mH, R_L=10Ω, C=100nF i a) R=1kΩ, b) R=200Ω, c) R=20Ω izračunati i nacrtati odziv na Step $u_1(t)=S(t)$.

Rješenje: a) b) c)

IZVODI, RJEŠENJA I SKICE ODZIVA SU NA DODATNIM LISTOVIMA BR 5 do 11

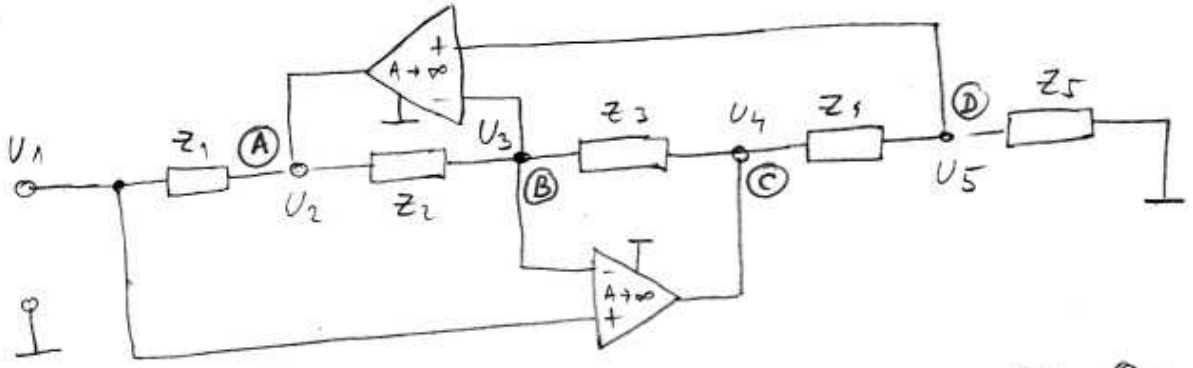
5. Za električne krugove prikazane slikom 2(a) i (b) uz zadane vrijednosti elemenata L=1mH, R_L=10Ω, C=100nF i a) R=1kΩ, b) R=200Ω, izračunati i nacrtati amplitudno-frekvencijsku karakteristiku. Za električni krug na slici 2(a) uz iste elemente: kolika je širina pojasa propuštanja B, koliki su Q-faktor i centralna frekvencija ω_0?

Rješenje: a) b)

IZVODI, RJEŠENJA I SKICE AMPLITUDNO-FREKVENCIJSKIH KARAKTERISTIKA SU NA DODATNIM STRANICAMA BR. 12 do 17

LabG. PRIPREMA – DODATNI LIST 1

1.



OP ima: 1) $R_{ul} \to \infty \to$ STRUJE NA ULAZIMA SU 0

2) $A_V \to \infty \to u_+ = u_-$ tj $\boxed{U_1 = U_3 = U_5}$

Napišemo KZS za čvorove (A), (B), (C) i (D)

(A) $I_1 = \frac{U_2 - U_1}{Z_1} + \frac{U_2 - U_1}{Z_2}$

(B) $\frac{U_2 - U_1}{Z_2} = \frac{U_1 - U_4}{Z_3}$

(C) $I_2 = \frac{U_4 - U_1}{Z_3} + \frac{U_4 - U_1}{Z_4}$

(D) $\frac{U_4 - U_1}{Z_4} = \frac{U_1}{Z_5}$

$Z_1 = Z_1(s)$
$Z_2 = Z_2(s)$
$Z_3 = Z_3(s)$
$Z_4 = Z_4(s)$
$Z_5 = Z_5(s)$

Iz (D) $\to$ $\boxed{U_4(s) = \left(1 + \frac{Z_4}{Z_5}\right) \cdot U_1(s)}$

Iz (B) $\to$ $(U_2 - U_1) Z_3 = \left(\cancel{U_1} - U_1 \frac{Z_4}{Z_5} - \cancel{U_1}\right) \cdot Z_2$

$U_2 Z_3 = U_1 \left(Z_3 - \frac{Z_2 \cdot Z_4}{Z_5}\right) \mid : Z_3$

$\boxed{U_2(s) = \left(1 - \frac{Z_2 \cdot Z_4}{Z_3 \cdot Z_5}\right) \cdot U_1(s)}$

Iz (A) $\to$ $I_1 = (U_1 - U_2)\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)$

$I_1 = \left(\cancel{U_1} - \cancel{U_1} + \frac{Z_2 \cdot Z_4}{Z_3 \cdot Z_5} \cdot U_1\right) \cdot \frac{Z_1 + Z_2}{Z_1 \cdot \cancel{Z_2}}$

$\boxed{I_1(s) = \frac{(Z_1 + Z_2) \cdot Z_4}{Z_1 \cdot Z_3 \cdot Z_5} \cdot U_1(s)}$

Iz (C) $\to$ $I_2 = (U_4 - U_1) \cdot \left(\frac{1}{Z_3} + \frac{1}{Z_4}\right) = \left(\cancel{U_1} - U_1 \frac{Z_4}{Z_5} - \cancel{U_1}\right) \cdot \frac{Z_3 + Z_4}{Z_3 \cdot \cancel{Z_4}}$

$\boxed{I_2(s) = -\frac{Z_3 + Z_4}{Z_3 Z_5} \cdot U_1(s)}$

(2)

$Z_1(s) = R_1 = 10^3\ \Omega$

$Z_2(s) = R_2 = 10^5\ \Omega$

$Z_3(s) = R_3 = 10^3\ \Omega$

$Z_4(s) = 1/(sC_4) \longrightarrow Z_4 = \frac{1}{j\omega C_4} = -j\frac{1}{10^5 \cdot 10^{-7}} = -j100 = 100\angle -90°$

$Z_5(s) = R_5 = 10^3\ \Omega$

$u_1(t) = \sin 10^5 t \longrightarrow \omega = 10^5$ rad/s, $U = 1\angle 0°$

$u_2(t), u_4(t), i_1(t), i_2(t), L_{ekv} = ?$

$$U_2 = \left(1 - \frac{Z_2 \cdot Z_4}{Z_1 \cdot Z_5}\right) \cdot U_1 = \left(1 - \frac{10^5 \cdot (-j100)}{10^3 \cdot 10^3}\right) \cdot 1\angle 0° = (1 + j10) =$$

$$= 10.05\angle 84.2° \text{ V}$$

$$= 10.05\angle 1.47 \text{ rad (V)}$$

$u_2(t) = 10.05 \sin(10^5 t + 84.2°)$

$\boxed{u_2(t) = 10.05 \sin(10^5 t + 1.471 \text{ rad})}$

$$U_4 = \left(1 + \frac{Z_4}{Z_5}\right) U_1 = \left(1 + \frac{-j100}{10^3}\right) \cdot 1\angle 0° = 1 - j0.1 = 1.005\angle -5.71°$$

$\boxed{u_4(t) = 1.005 \sin(10^5 \cdot t - 5.71°)}$

$$I_1 = \frac{(Z_1 + Z_2) \cdot Z_4}{Z_1 \cdot Z_3 \cdot Z_5} \cdot U_1 = \frac{(10^3 + 10^5) \cdot (-j100)}{10^3 \cdot 10^3 \cdot 10^3} \cdot 1\angle 0° =$$

$I_1 = 0 - j0.0101\ A = -j10.1\text{ mA} = 10.1\angle -90°$

$\boxed{i_1(t) = 10.1 \sin(10^5 t + 180°)}$

$$I_2 = -\frac{Z_1 + Z_4}{Z_3 \cdot Z_5} \cdot U_1 = -\frac{10^3 - j100}{10^3 \cdot 10^3} \cdot 1\angle 0° = -0.001 + j0.0001\ A$$

$I_2 = -1 + j0.1\text{ mA} = 1.05\angle 174.3°\text{ mA}$

$\boxed{i_2(t) = 1.05 \cdot \sin(10^5 t + 174.3°)}$

$$\boxed{L_{ekv} = \frac{R_1 \cdot R_3 \cdot R_5 \cdot C_4}{R_2} = \frac{10^3 \cdot 10^3 \cdot 10^3 \cdot 10^{-7}}{10^5} = 10^{-3}\text{ H} = 1\text{ mH}}$$

Lab 6 – PRIPREMA – DODATNI ZADATAK 3.

(3) slika (a)

$$Z_1 = R, \quad Z_2 = sL \,||\, (1/sC) = \frac{sL \cdot \frac{1}{sC}}{sL + \frac{1}{sC}} = \frac{\frac{L}{C}}{\frac{s^2LC+1}{sC}} = \frac{sL}{s^2LC+1} =$$

$$T(s) = \frac{U_2}{U_1} = \frac{Z_2}{Z_1+Z_2} = \frac{\frac{sL}{s^2LC+1}}{R + \frac{sL}{s^2LC+1}} = \frac{sL}{s^2RLC + R + sL}$$

$$\boxed{T(s) = \frac{s\frac{1}{RC}}{s^2 + s\frac{1}{RC} + \frac{1}{LC}}}$$

slika (b)

$$Z_1 = R, \quad Z_2 = (R_L + sL) \,||\, (1/sC) = \frac{(R_L+sL)\cdot\left(\frac{1}{sC}\right)}{R_L + sL + \frac{1}{sC}}$$

$$= \frac{R_L + sL}{R_L \cdot C \cdot s + s^2LC + 1}$$

$$T(s) = \frac{U_2}{U_1} = \frac{Z_2}{Z_1+Z_2} = \frac{\frac{R_L+sL}{s^2LC + sR_LC + 1}}{R + \frac{R_L+sL}{s^2LC + sR_LC + 1}} =$$

$$= \frac{R_L + sL}{s^2RLC + sR\cdot R_LC + R + R_L + sL} =$$

$$= \frac{R_L + sL}{RLC\left(s^2 + s\frac{R\cdot R_LC + L}{RLC} + \frac{R+R_L}{RLC}\right)} =$$

$$\boxed{T(s) = \frac{\frac{1}{RC}\cdot s + \frac{R_L}{RLC}}{s^2 + s\frac{R\cdot R_LC + L}{RLC} + \frac{R+R_L}{RLC}}}$$

Lab 6 – PRIPREMA – DODATNI LIST 4

(3) – NASTAVAK

UVRŠTENJE VRIJEDNOSTI

$L = 1mH = 10^{-3}$

$R_L = 10\Omega$

$C = 100uF = 10^{-7}$

SLIKA (a)

a) $R = 10^3\Omega$

$$T(s) = \frac{s \cdot \frac{1}{10^3 \cdot 10^{-7}}}{s^2 + s\frac{1}{10^3 \cdot 10^{-7}} + \frac{1}{10^{-3} \cdot 10^{-7}}} = \frac{10^4 \cdot s}{s^2 + 10^4 s + 10^{10}}$$

b) $R = 200\Omega$

$$T(s) = \frac{s \cdot \frac{1}{200 \cdot 10^{-7}}}{s^2 + s\frac{1}{200 \cdot 10^{-7}} + \frac{1}{10^{-3} \cdot 10^{-7}}} = \frac{5 \cdot 10^4 s}{s^2 + 5 \cdot 10^4 s + 10^{10}}$$

c) $R = 20\Omega$

$$T(s) = \frac{50 \cdot 10^4 s}{s^2 + 50 \cdot 10^4 s + 10^{10}}$$

SLIKA (b)

a) $R = 1k\Omega = 10^3\Omega$

$$T(s) = \frac{\frac{1}{10^3 \cdot 10^{-7}} s + \frac{10}{10^3 \cdot 10^{-3} \cdot 10^{-7}}}{s^2 + s\frac{10^3 \cdot 10 \cdot 10^{-7} + 10^{-3}}{10^3 \cdot 10^{-3} \cdot 10^{-7}} + \frac{10^3 + 10}{10^3 \cdot 10^{-3} \cdot 10^{-7}}}$$

$$T(s) = \frac{10^4 s + 10^6}{s^2 + 2 \cdot 10^4 s + 1.01 \cdot 10^{10}}$$

b) $R = 200\Omega$

$$T(s) = \frac{5 \cdot 10^4 \cdot s + 5 \cdot 10^6}{s^2 + 6 \cdot 10^4 s + 1.05 \cdot 10^{10}}$$

c) $R = 20\Omega$

$$T(s) = \frac{50 \cdot 10^4 s + 50 \cdot 10^6}{s^2 + 5.1 \cdot 10^5 s + 1.5 \cdot 10^{10}}$$

Lab 6 – PRIPREMA – DODATNI ZADATAK 5

④ SLIKA (a)

a) $T(s) = \dfrac{10^4 s}{s^2 + 10^4 s + 10^{10}}$, $U_1(s) = \dfrac{1}{s}$

$$\boxed{U_2(s) = \frac{10^4}{s^2 + 10^4 s + 10^{10}}}$$

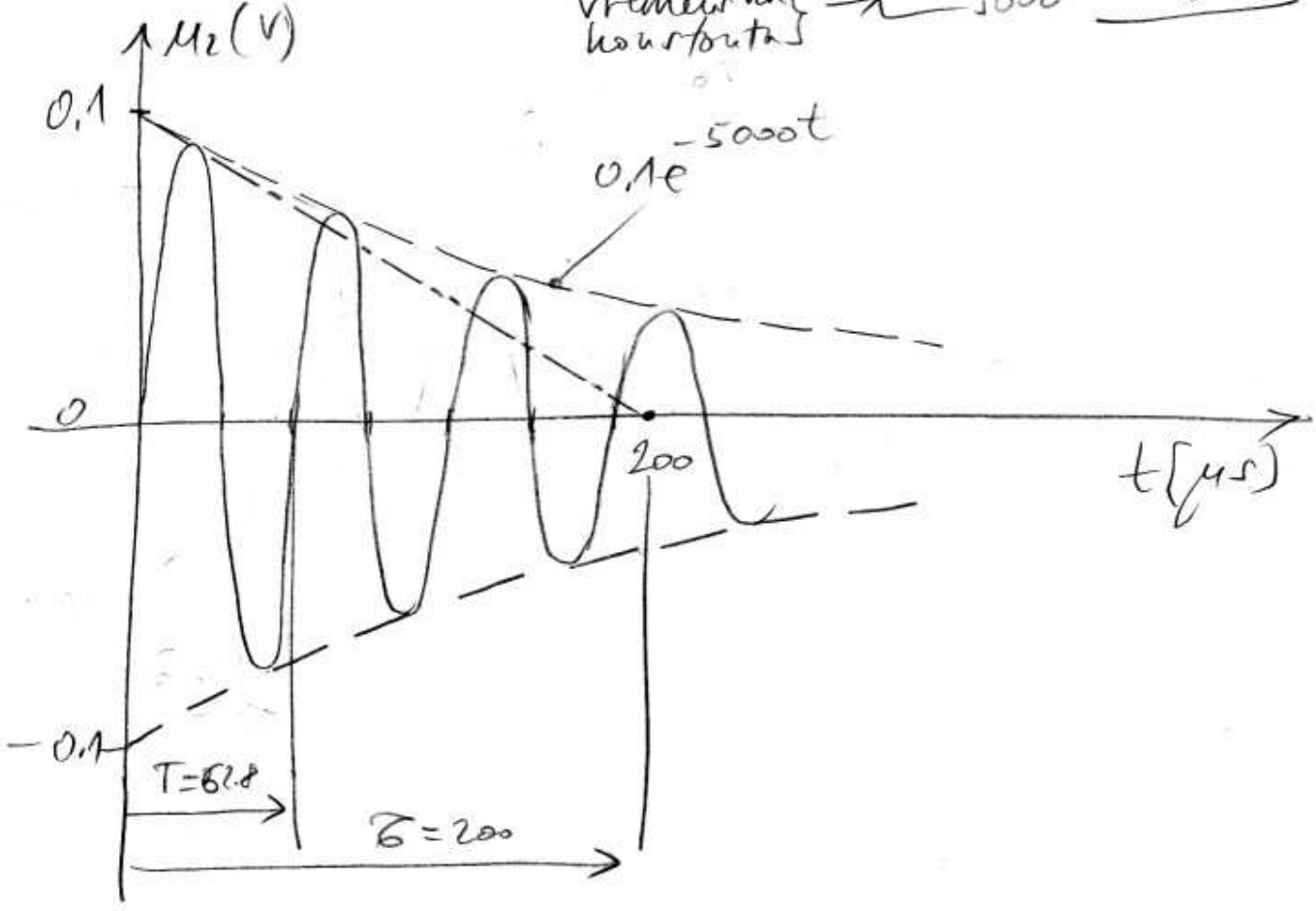
Polovi: $sp_{1,2} = \dfrac{10^4}{2} \pm \sqrt{\dfrac{10^8}{4} - \dfrac{4 \cdot 10}{4}}$

$$\boxed{sp_{1,2} = 5000 \pm j\,99875}$$

$$U_2(s) = \frac{10^4}{(s+5000)^2 + 99875^2} = \frac{10000}{99875} \cdot \frac{99875}{(s+1000)^2 + 99875^2}$$

$u_2(t) = 0.1 \cdot e^{-5000t} \cdot \sin 99875\,t$ → period $\boxed{T = 62.8\,\mu s}$

vremenska konstanta → $\boxed{\tau = \dfrac{1}{5000} = 200\,\mu s}$



Lab 6 – PRIPREMA — dodatni list 6

④ sliko (a)

b) $R = 200\ \Omega$ $\quad T(s) = \dfrac{5\cdot 10^4 s}{s^2 + 5\cdot 10^4 s + 10^{10}}$, $U_1(s) = \dfrac{1}{s}$

$$U_2(s) = \frac{5\cdot 10^4}{s^2 + 5\,10^4 s + 10^{10}}$$

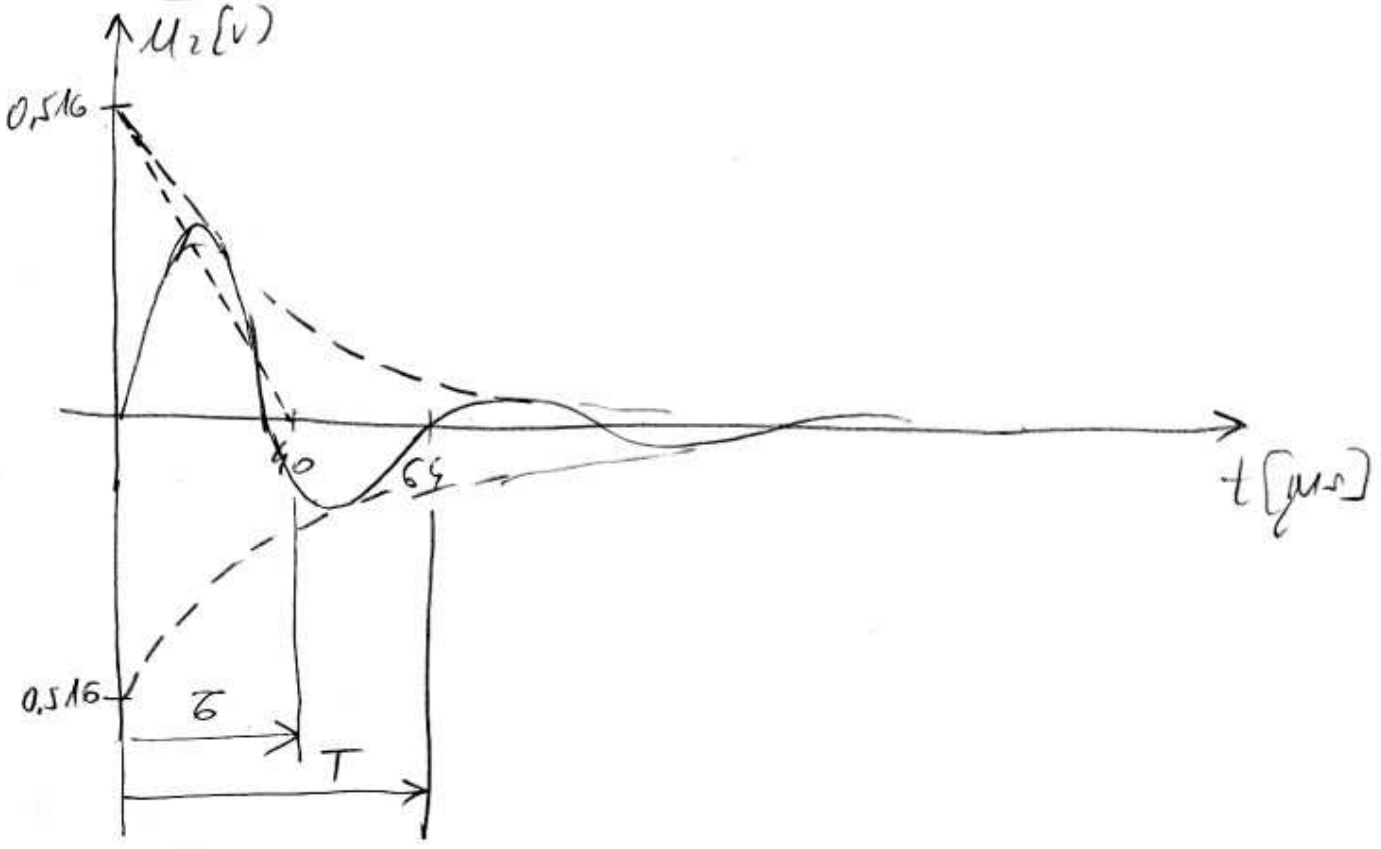
POLOVI: $s_{1,2} = -25000 \pm j\,96825 = \sigma_p \pm j\omega_p$

$$U_2(s) = \frac{50000}{(s+25000)^2 + 96825^2}$$

$$U_2(s) = \frac{50000}{96825} \cdot \frac{96825}{(s+25000)^2 + 96825^2}$$

$$u_2(t) = 0.516 \cdot e^{-25000t} \cdot \sin(96825\,t)$$

$\tau = 40\ \mu s$ $\quad \omega = 96825 \rightarrow T = \dfrac{96825}{2\pi} = 64\ \mu s$



Lab 6 – PRIPREMA – dodatni list 7

④ slika (a)

SLUČAJ C) $R = 20\,\Omega$, $T(s) = \dfrac{50 \cdot 10^4 s}{s^2 + 50 \cdot 10^4 s + 10^{10}}$, $U_1(s) = \dfrac{1}{s}$

$sp_1 = -20871$ } POLOVI REALNI I NEGATIVNI
$sp_2 = -479129$ } (STABILAN OPTIV)

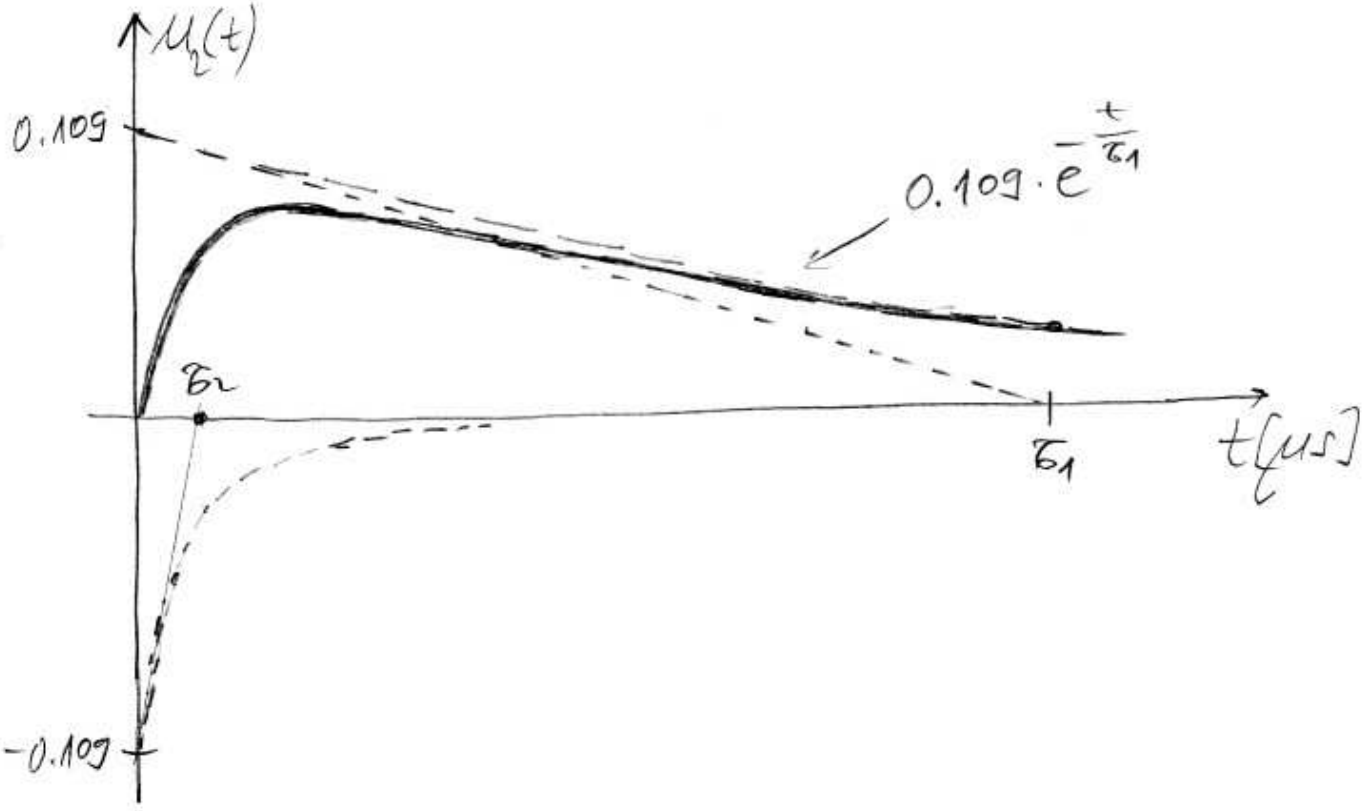
$$U_2(s) = \frac{50000}{(s+20871)(s+479129)} = \frac{k}{(s+a)(s+b)} =$$

$$= \frac{k}{b-a} \cdot \frac{b-a}{(s+a)(s+b)} \multimap \frac{k}{b-a}\left(e^{-at} - e^{-bt}\right)$$

$$U_2(s) = \frac{50\,000}{479129 - 20871} \cdot \frac{479129 - 20871}{(s+20871)(s+479129)}$$

$$\boxed{u_2(t) = 0.109 \cdot \left(e^{-20871\,t} - e^{-479129\,t}\right)}$$

$\tau_1 = 30.1\,\mu s$ $\tau_2 = 13.1\,\mu s$



Lab 6 – priprema — DODATNI LIST

④ SLIKA(b)

a) $R = 1\,k\Omega$, $T(s) = \dfrac{10^4 s + 10^6}{s^2 + 2\cdot 10^4 s + 1.01\cdot 10^{10}}$, $U_1(s) = \dfrac{1}{s}$

$$U_2(s) = \frac{10^4}{s^2 + 2\cdot 10^4 s + 1.01\cdot 10^{10}} + \frac{10^6}{s(s^2 + 2\cdot 10^4 s + 1.01\cdot 10^{10})}$$

$\boxed{sp_{1,2} = -10^4 \pm j10^5}$ LIJEVA POLUVRAVNINA

Dobije se oblik:

$$U_2(s) = \frac{K1}{s^2 + 2\cdot ab\, s + b^2} + \frac{K2}{s(s^2 + 2ab\cdot s + b^2)} =$$

$$= \frac{K1}{b^2}\cdot\frac{b^2}{s^2 + 2ab\, s + b^2} + \frac{K2}{b^2}\cdot\frac{b^2}{s(s^2 + 2ab\, s + b^2)}$$

i odatle

$$u_2(t) = \frac{K_1}{b^2}\cdot\frac{b}{\sqrt{1-a^2}}\cdot e^{-abt}\cdot \sin\left(b\sqrt{1-a^2}\,t\right) +$$

$$+ \frac{K2}{b^2}\left[1 - \frac{1}{\sqrt{1-a^2}}\cdot e^{-abt}\cdot \sin\left(b\sqrt{1-a^2}\,t + \cos^{-1} a\right)\right]$$

$K_1 = 10^4$, $K2 = 10^6$

$2ab = 2\cdot 10^4$

$b^2 = 1.01\cdot 10^{10}$ $\Big\}$ $a = 0.0995$, $b = 1.005\cdot 10^5 = 100\,500$

$$u_2(t) \approx 0.1\cdot e^{-10^4 t}\cdot \sin(10^5 t) + \underbrace{10^{-4}\left[1 - 1.005\cdot e^{-10^4 t}\cdot \sin(10^5 t + 84.3^\circ)\right]}_{\text{(1000 PUTA MANJE OD PRVOG ČLANA) } t_1 \approx 0}$$

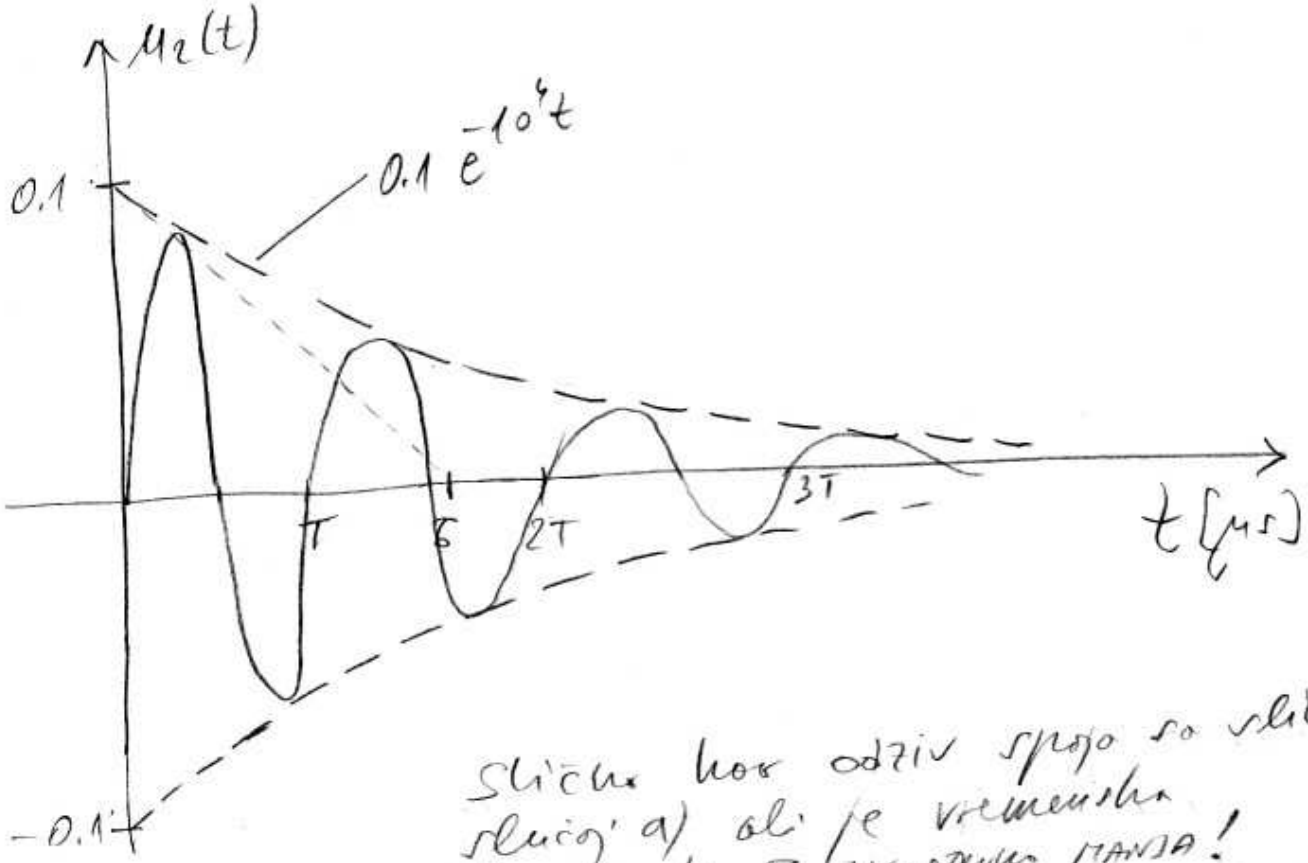
$\boxed{u_2(t) \approx 0.1\cdot e^{-10^4 t}\cdot \sin(10^5 t)}$

Lab 6 – priprema — DODATNI LIST 9

④ SLIKA (b)

a) = 1 kΩ skica odziva

$u_2(t) \approx 0.1 e^{-10^4 t} \sin(10^5 t) \rightarrow \tau = 100\,\mu s,\ T = 62.8\,\mu s$



Slična kor odziv sproja so slike (a), slici a) ali je vremenska konstanta τ dvakrat manša!

Lab 6 – priprema — DODATNI LIST 10

(4) slika (b)

b) $R = 200\,\Omega$, $T(s) = \dfrac{5\cdot10^4 s + 5\cdot10^6}{s^2 + 6\cdot10^4 s + 1.05\cdot10^{10}}$, $U_1(s) = \dfrac{1}{s}$

$$U_2(s) = \frac{5\cdot10^4}{s^2+6\cdot10^4 s+1.05\cdot10^{10}} + \frac{5\cdot10^6}{s(s^2+6\cdot10^4 s+1.05\cdot10^{10})}$$

(polovi: $s_{p1,2} = -3\cdot10^4 \pm j9.79\cdot10^4$)

$$U_2(s) = \frac{K_1}{b^2}\cdot\frac{b^2}{s^2+2ab s+b^2} + \frac{K_2}{b^2}\cdot\frac{b^2}{s(s^2+2ab s+b^2)}$$

$K_1 = 5\cdot10^4$, $K_2 = 5\cdot10^6$, $\left.\begin{matrix} 2ab = 6\cdot10^4 \\ b^2 = 1.05\cdot10^{10}\end{matrix}\right\}$ $a = 0.293$, $b = 1.025\cdot10^5$

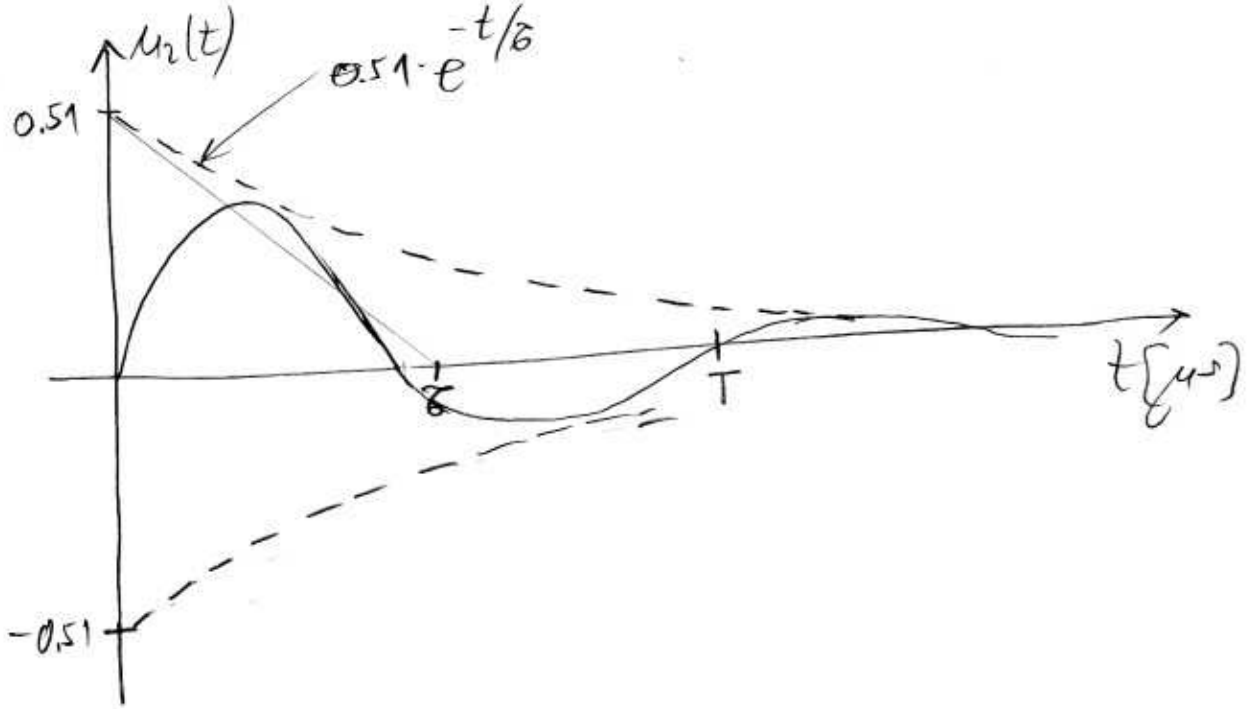
KAO NA PRETHODNOJ STRANICI DOBIJEMO

$$u_2(t) = \frac{K_1}{b}\cdot\frac{e^{-abt}}{\sqrt{1-a^2}}\cdot\sin\left(b\sqrt{1-a^2}\,t\right) + \frac{K_2}{b^2}\left[1 - \frac{e^{-abt}}{\sqrt{1-a^2}}\cdot\sin\left(b\sqrt{1-a^2}\,t + \cos^{-1}a\right)\right]$$

$$u_2(t) = 0.51\cdot e^{-3\cdot10^4 t}\cdot\sin\left(9.8\cdot10^4 t\right) + 4.759\cdot10^{-4}\left[1 - 1.046\cdot e^{-3\cdot10^4 t}\cdot\sin\left(9.8\cdot10^4 t + 73°\right)\right]$$

zanemarivo

$$\boxed{u_2(t) \approx 0.51\cdot e^{-3.18t}\cdot\sin\left(9.8\cdot10^4 t\right)} \qquad \tau = 33.3\,\mu s,\ T = 64.1\,\mu s$$



Lab 6 – priprema – DODATNI LIST 11

④ slika (b)

c) $R = 20\,\Omega$, $T(s) = \dfrac{50\cdot10^3 s + 50\cdot10^2}{s^2 + 5.1\cdot10^5 s + 1.5\cdot10^{10}}$, $U_1(s) = \dfrac{1}{s}$

$$U_2(s) = \frac{50\cdot10^4}{s^2+5.1\cdot10^5\cdot s+1.5\cdot10^{10}} + \frac{50\cdot10^6}{s(s^2+5.1\cdot10^5\cdot s+1.5\cdot10^{10})}$$

Pocovi: $s_{p1} = 0$, $s_{p2} = -3.13\cdot10^4$, $s_{p3} = -4.79\cdot10^5$

$$U_2(s) = \frac{K_1}{(s+a)(s+b)} + \frac{K_2}{s(s+a)(s+b)}$$

$$U_2(s) = \frac{K_1}{b-a}\cdot\frac{b-a}{(s+a)(s+b)} + \frac{K_2}{b-a}\cdot\frac{b-a}{s(s+a)(s+b)}$$

$$u_2(t) = \frac{K_1}{b-a}\cdot\left(e^{-at} - e^{-bt}\right) + \frac{K_2}{b-a}\cdot\left(\frac{1}{ab} - \frac{e^{-at}}{a(b-a)} - \frac{e^{-bt}}{b(a-b)}\right)$$

$$u_2(t) = \frac{K_1}{b-a}\left(e^{-at} - e^{-bt}\right) + \frac{K_2}{(b-a)\cdot ab}\cdot\left(1 - \frac{b}{b-a}\cdot e^{-at} + \frac{a}{b-a}e^{-bt}\right)$$

$K_1 = 5\cdot10^5$, $K_2 = 5\cdot10^7$, $a = 3.13\cdot10^4$, $b = 4.79\cdot10^5$

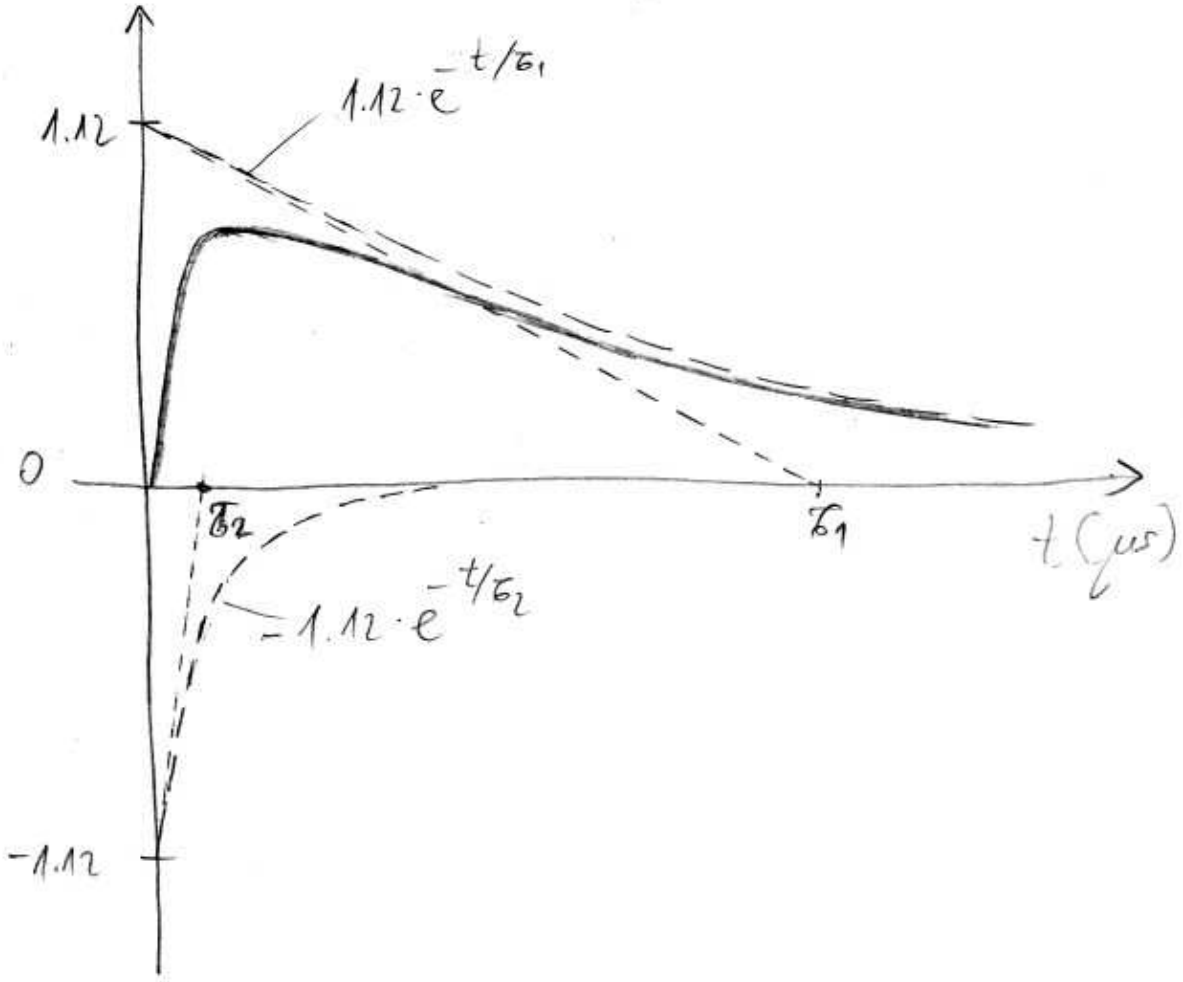
$$u_2(t) = 1.12\left(e^{-3.14\cdot10^4 t} - e^{-4.79\cdot10^5 t}\right) + \underbrace{7.45\cdot10^{-9}\left(1 - 1.07\cdot10^{-3.13\cdot10^4 t} + 0.07\cdot e^{-4.79\cdot10^5 t}\right)}_{\text{ZANEMARIVO } (\approx 0)}$$

$$\boxed{u_2(t) = 1.12\cdot\left(e^{-3.14\cdot10^4 t} - e^{-4.79\cdot10^5 t}\right)}$$

(4) (b), c), NASTAVAK Lab6 – priprema DODATNI LIST 12

$$u_2(t) = 1.12 \cdot \left(e^{-3.14\cdot10^4 t} - e^{-4.79\cdot10^5 t}\right) = 1.12\left(e^{-t/\tau_1} - e^{-t/\tau_2}\right)$$

$\tau_1 = 31.85\,\mu s$, $\tau_2 = 2.088\,\mu s$



⑤ slika 2(a) a) $R = 1\,k\Omega$

$$T(s) = \frac{10^4 s}{s^2 + 10^4 s + 10^{10}}$$

Opći oblik prijenosne funkcije PP filtra 2. reda je:

$$H_{pp}(s) = K \cdot \frac{s\frac{\omega_p}{Q_p}}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

$\omega_p^2 = 10^{10} \rightarrow \boxed{\omega_p = 10^5} \Rightarrow$ maksimum kod $\omega = \omega_p = 10^5 \left[\frac{rad}{s}\right]$

$\frac{\omega_p}{Q_p} = 10^4 \rightarrow \boxed{Q_p = \frac{\omega_p}{10^4} = 10}$ Q-faktor govori o širini pojasa B

$K \cdot \frac{\omega_p}{Q_p} = 10^4 \rightarrow \boxed{K = 1}$

– Amplitudno frekvencijska karakteristika

$$\boxed{a_{pp}(\omega) = \frac{K}{\sqrt{1 + Q_p^2\left(\frac{\omega}{\omega_p} - \frac{\omega_p}{\omega}\right)^2}} = \frac{1}{\sqrt{1 + 100\left(\frac{\omega}{10^5} - \frac{10^5}{\omega}\right)^2}}}$$

– Tok $a_{pp}(\omega)$: $\omega \rightarrow 0 \Rightarrow a_{pp}(\omega) \rightarrow 0$

$\omega < \omega_p \Rightarrow a_{pp}(\omega) \rightarrow$ RASTE

$\omega = \omega_p \Rightarrow a_{pp}(\omega) = 1$ (MAKSIMUM)

$\omega > \omega_p \Rightarrow a_{pp}(\omega) \rightarrow$ PADA

$\omega \rightarrow \infty \Rightarrow a_{pp}(\omega) \rightarrow 0$

(5) NASTAVAK

- Granične frekvencije: uvjet $a_{pp}(\omega) = \frac{K}{\sqrt{2}} = 0.707$

$$\omega_{g,d} = \omega_p \sqrt{1 + \underbrace{\frac{1}{4Q_p^2}}_{\approx 0}} \pm \frac{\omega_p}{2Q_p} = 10^5 \pm 5 \cdot 10^3$$

$$\omega_d = 10^5 - 5 \cdot 10^3 = 95 \cdot 10^3 = 95000 \ [rad/s]$$

$$\omega_g = 10^5 + 5 \cdot 10^3 = 105 \cdot 10^3 = 105000 \ [rad/s]$$

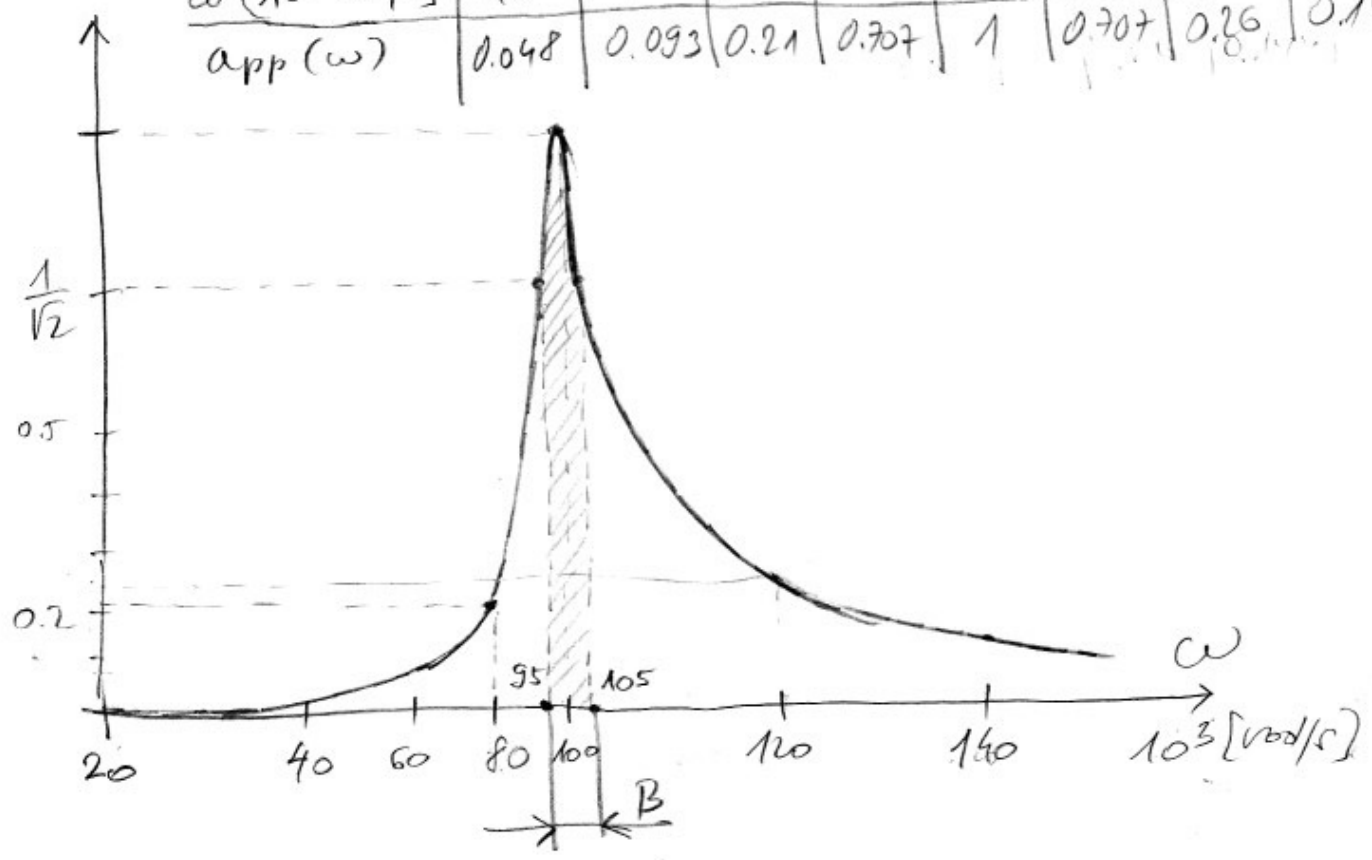
- ŠIRINA POJASA PROPUŠTANJA

$$B = \omega_g - \omega_d = 105 \cdot 10^3 - 95 \cdot 10^3 = 5 \cdot 10^3 = 5000 \ [rad/s]$$

(ili $B = \frac{\omega_p}{Q_p} = 5000 \ [rad/s]$)

- centralna frekvencija $\omega_0 = \omega_p = 10^5 \ [rad/s]$

$\omega \ (10^3 \ rad/s)$	40	60	80	95	100	105	120	140
$a_{pp}(\omega)$	0.048	0.093	0.21	0.707	1	0.707	0.26	0.144



(5) NASTAVAK

Slika 2(a), sklop b) $R = 200\,\Omega$

$$T(s) = \frac{5\cdot 10^5 s}{s^2 + 5\cdot 10^5 s + 10^{10}} \Rightarrow \boxed{\omega_p = 10^5,\ Q_p = 2} \quad \boxed{K = 1}$$

$Q_p = 2 \rightarrow$ ŠIROKOPOJASNI FILTER

- amplitudno-frekvencijska karakteristika

$$\boxed{a_{pp} = \frac{1}{\sqrt{1 + 4\left(\frac{\omega}{10^5} - \frac{10^5}{\omega}\right)^2}}}$$

- granične frekvencije:

$$\omega_{g,d} = 10^5 \sqrt{1 + \frac{1}{4\cdot 2^2}} \pm \frac{10^5}{2\cdot 2} = 103\cdot 10^3 \pm 25\cdot 10^3 \ [\text{rad/s}]$$

$$\boxed{\omega_d = 78\cdot 10^3\ [\text{rad/s}] \quad \omega_g = 128\cdot 10^3\ [\text{rad/s}]}$$

- širina propusnog područja

$$B = \omega_g - \omega_d = \frac{\omega_p}{Q_p} = 5\cdot 10^4 = 50\cdot 10^3\ [\text{rad/s}]$$

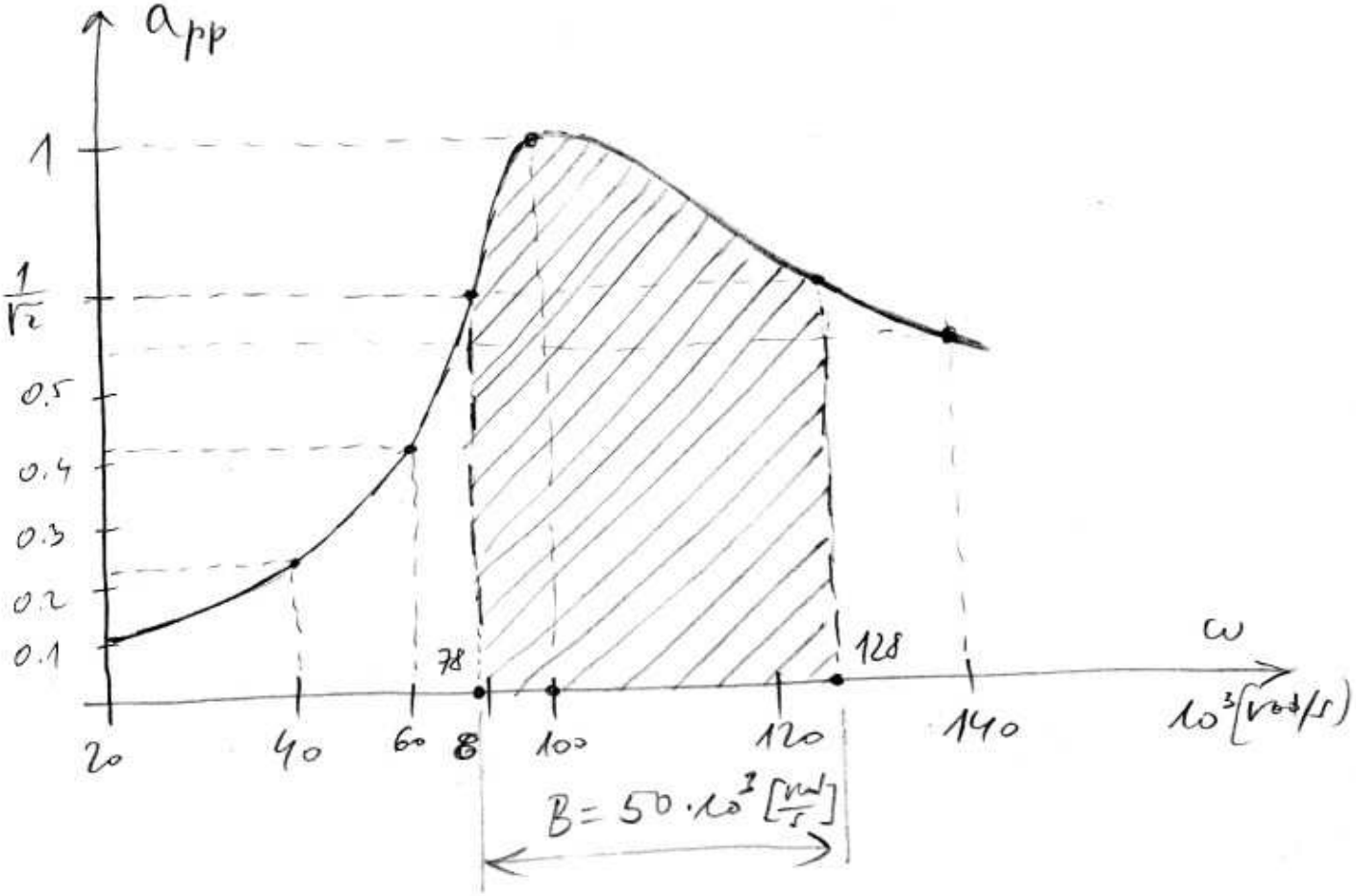
- centralna frekvencija $\omega_0 = \omega_p = 10^5$ rad/s

- Tablica za crtanje a-f karakteristike

ω ($10^3 \frac{\text{rad}}{\text{s}}$)	40	60	78	80	100	120	128	140
$a_{pp}(\omega)$	0.23	0.42	0.707	0.74	1	0.806	0.707	0.589

⑤ slika 2(a), sheme b) NASTAVAK

- graf amplitudno frekvencijske karakteristike



(5) slika 2(b) a) $R = 1k\Omega$

$$T(s) = \frac{10^4 s + 10^6}{s^2 + 2\cdot 10^4 s + 1.01\cdot 10^{10}}$$

$$T(j\omega) = \frac{10^4 j\omega + 10^6}{-\omega^2 + 2\cdot 10^4 j\omega + 1.01\cdot 10^{10}}$$

$$\boxed{|T(j\omega)| = \frac{\sqrt{10^{12} + 10^8\omega^2}}{\sqrt{(1.01\cdot 10^{10} - \omega^2)^2 + 4\cdot 10^8\omega^2}} = a_{pp}(\omega)}$$

$$\boxed{\omega_0 = \sqrt{1.01\cdot 10^{10}} = 100.5\cdot 10^3 \text{ [rad/s]}}$$

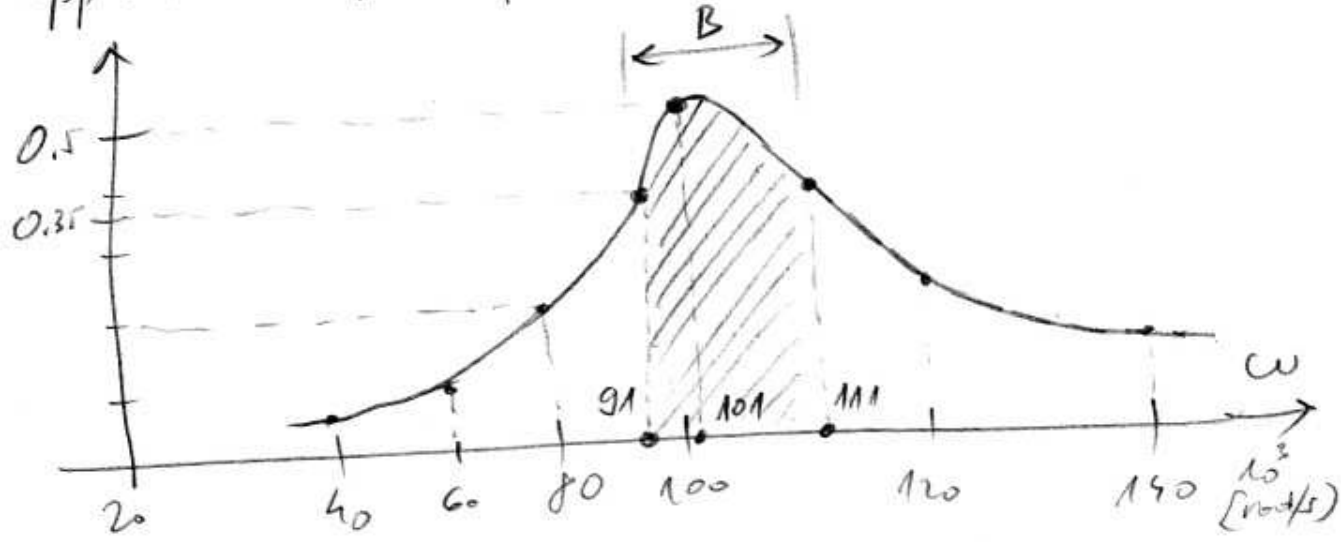
$$\boxed{Q = \frac{\omega_0}{2\cdot 10^4} = \frac{100.5\cdot 10^3}{2\cdot 10^4} = 5.02}$$

$$\omega_{g,d} = \omega_0 \cdot \sqrt{1 + \frac{1}{4Q^2}} \pm \frac{\omega_0}{2Q} = 101\cdot 10^3 \pm 10\cdot 10^3$$

$$\boxed{\omega_d = 91\cdot 10^3 \text{ (rad/s)}}, \quad \boxed{\omega_g = 111\cdot 10^3 \text{ [rad/s]}}$$

$$B = \omega_g - \omega_d = 20\cdot 10^3 \text{ [rad/s]}$$

				ω_d		ω_0	ω_g		
$\frac{\omega}{10^3}$ [rad/s]	40	60	80	91	100	101	111	120	140
$a_{pp}(\omega)$	0.05	0.09	0.20	0.35	0.50	0.50	0.35	0.24	0.14



(5) slika 2(b) b) $R = 200\ \Omega$

$$T(s) = \frac{5\cdot10^4 s + 5\cdot10^6}{s^2 + 6\cdot10^4 s + 1.05\cdot10^{10}}$$

$$T(j\omega) = \frac{5\cdot10^4 j\omega + 5\cdot10^6}{-\omega^2 + 6\cdot10^4 j\omega + 1.05\cdot10^{10}}$$

$$a_{pp}(\omega) = |T(j\omega)| = \frac{\sqrt{25\cdot10^8\omega^2 + 25\cdot10^{12}}}{\sqrt{(1.05\cdot10^{10} - \omega^2)^2 + 36\cdot10^8\omega^2}}$$

$$\omega_0 = \sqrt{1.05\cdot10^{10}} = 102.5\cdot10^3\ [rad/s]$$

$$Q = \frac{\omega_0}{6\cdot10^4} = 1.71$$

$$\omega_{g,d} = \omega_0\sqrt{1 + \frac{1}{4Q^2}} \pm \frac{\omega_0}{2Q} = 106.8\cdot10^3 \pm 30\cdot10^3$$

$$\omega_d = 76.8\cdot10^3\ [rad/s] \qquad \omega_g = 136.8\cdot10^3\ [rad/s]$$

ω 10^3 [rad/s]	40	60	76.8 (ω_d)	80	100	106.8 (ω_0)	120	136.8 (ω_g)	140
$a_{pp}(\omega)$	0.22	0.39	0.59	0.63	0.83	0.83	0.73	0.59	0.57

$B = \omega_g - \omega_d = 60\cdot10^3\ [rad/s]$