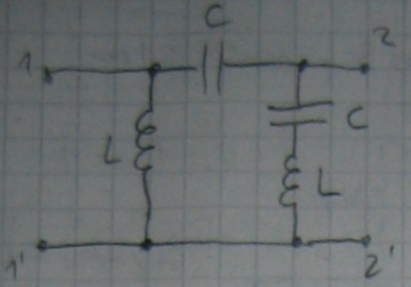
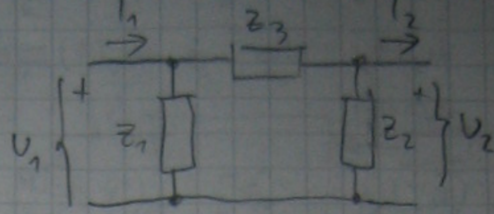
1. Prikazanom četveropolu odrediti ekvivalentni T-spoj



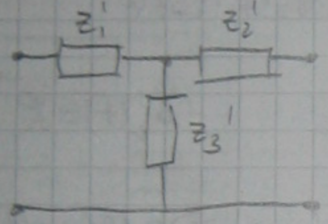


$Z_1 = sL$

$Z_2 = sL + \frac{1}{sC}$

$Z_3 = \frac{1}{sC}$

$$[Z] = \begin{bmatrix} \frac{Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3} & -\frac{Z_1 Z_2}{Z_1+Z_2+Z_3} \\ \frac{Z_1 Z_2}{Z_1+Z_2+Z_3} & -\frac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3} \end{bmatrix} = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix}$$



$$[Z]' = \begin{bmatrix} Z_1' + Z_3' & -Z_3' \\ Z_3' & -(Z_2' + Z_3') \end{bmatrix}$$

izjednačavanje $[Z] = [Z]' \Rightarrow Z_1', Z_2', Z_3'$

$$Z_1' = z_{11} - z_{12} = \frac{Z_1(Z_2+Z_3) - Z_1 Z_2}{Z_1+Z_2+Z_3} = \frac{Z_1 Z_3}{Z_1+Z_2+Z_3}$$

$$Z_2' = z_{22} - z_{12} = \frac{Z_2 Z_3}{Z_1+Z_2+Z_3}$$

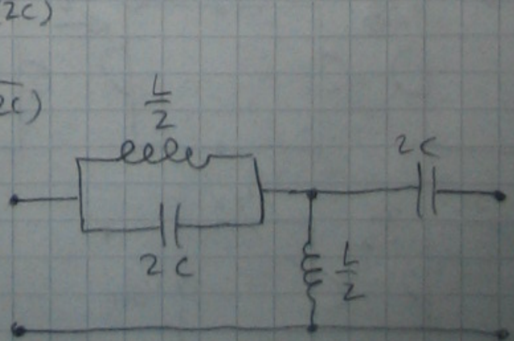
$$Z_3' = z_{12} = \frac{Z_1 Z_2}{Z_1+Z_2+Z_3}$$

$\Pi \Rightarrow T$

$$Z_1' = \frac{sL \cdot \frac{1}{sC}}{2\left(sL + \frac{1}{sC}\right)} = \frac{s\frac{L}{2}\,\frac{1}{s(2C)}}{s\frac{L}{2} + \frac{1}{s(2C)}} = \left(\frac{L}{2}\right) \| (2C)$$

$$Z_2' = \frac{\left(sL + \frac{1}{sC}\right)}{2\left(sL + \frac{1}{sC}\right)} \cdot \frac{1}{sC} = \frac{1}{s(2C)}$$

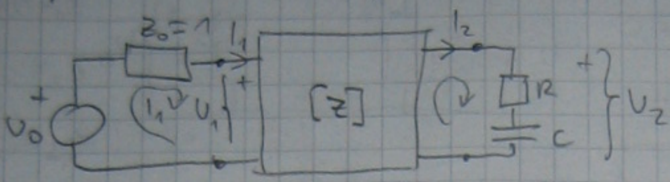
$$Z_3' = s\frac{L}{2}$$



2. Zadan je prijenosni sistem prema slici. Odrediti:

a) Napon $U_2(s)$ ako je $U_0(s)=\frac{1}{s}$

b) Napon $u_2(t)$ ako je $u_0(t)=\cos t$, $-\infty<t<+\infty$



$R_1=1,\ C=1$

$$[Z]=\begin{bmatrix} 3+2s & -(2+s) \\ 2+s & -(4+3s) \end{bmatrix}$$

$$Z_2=R+\frac{1}{sC}=1+\frac{1}{s}$$

$$\left.\begin{aligned} U_0&=I_1Z_0+U_1\\ U_2&=I_2Z_2\\ U_1&=I_1Z_{11}-I_2Z_{12}\\ U_2&=I_1Z_{21}-I_2Z_{22} \end{aligned}\right\}$$

$$T(s)=\frac{U_2}{U_0}=\frac{Z_2\cdot Z_{21}}{(Z_2+Z_{22})(Z_{11}+Z_0)-Z_{12}Z_{21}}=\frac{s^2+3s+2}{5s^3+18s^2+18s+4}$$

a) $U_2(s)=T(s)\,U_0(s)=\frac{s^2+3s+2}{5s^3+18s^2+18s+4}\cdot\frac{1}{s}$ $\qquad u_2(t)=\mathcal{L}^{-1}\left[U_2(s)\right]$

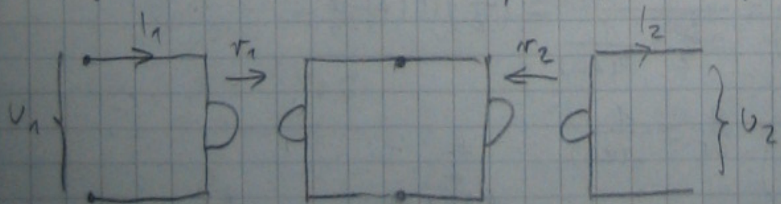
b) $s=j\omega$ $\quad T(j\omega)$ $\quad U_2(j\omega)=T(j\omega)\cdot\underbrace{U_0(j\omega)}_{1}\Rightarrow 0{,}166\cos(t-65{,}5^\circ)$

$$T(j\omega)=\frac{-\omega^2+3j\omega+2}{-5j\omega^3-18\omega^2+18j\omega+4}$$

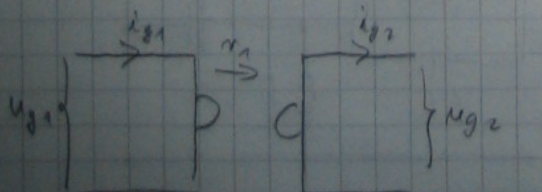
$$|T(j\omega)|=\left.\frac{\sqrt{(2-\omega^2)^2+(3\omega)^2}}{\sqrt{(4-18\omega^2)^2+(18\omega-5\omega^3)^2}}\right|_{\omega=1}$$

$$\angle T(j\omega)=\operatorname{arctg}\frac{3\omega}{2-\omega^2}-\operatorname{arctg}\frac{-5\omega^3+18\omega}{4-18\omega^2}$$

3. Da li prikazani lanac četveropola predstavlja recipročan četveropol?



$[a]=[a]^{I}\cdot[a]^{II}$



$u_{g1}=-i_{g2}\cdot r_1$

$u_{g2}=-i_{g1}\cdot r_1$

$U_1 = AU_2 + BI_2$

$I_1 = CU_2 + DI_2$

$[a]^{I} = \begin{bmatrix} 0 & -r_1 \\ -\frac{1}{r_1} & 0 \end{bmatrix}$

$[a]^{II} = \begin{bmatrix} 0 & +r_2 \\ +\frac{1}{r_2} & 0 \end{bmatrix}$

$$[a] = [a]^{I} \cdot [a]^{II} = \begin{bmatrix} 0 & -r_1 \\ -\frac{1}{r_1} & 0 \end{bmatrix} \begin{bmatrix} 0 & r_2 \\ \frac{1}{r_2} & 0 \end{bmatrix} = \begin{bmatrix} -\frac{r_1}{r_2} & 0 \\ 0 & -\frac{r_2}{r_1} \end{bmatrix}$$

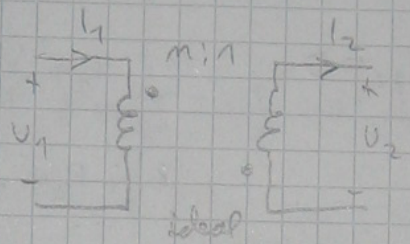
$\det([a]) = AD - BC = \left(-\frac{r_1}{r_2}\right)\left(-\frac{r_2}{r_1}\right) = 1$ ✓ DA

$U_1 = -\frac{r_1}{r_2} U_2$ $\qquad n = \frac{r_1}{r_2}$

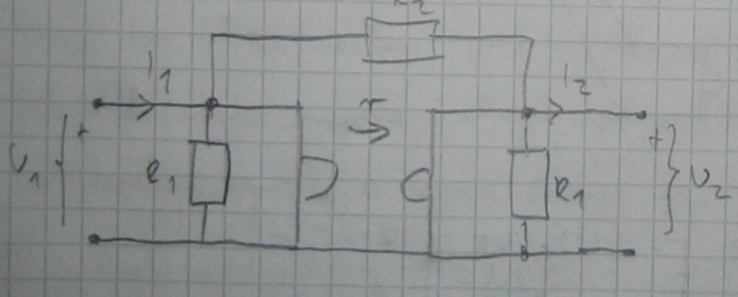
$I_1 = -\frac{r_2}{r_1} I_2$

$U_1 = -nU_2$

$I_1 = -\frac{1}{n} I_2$



4. Odrediti y-parametre i ulaznu admitanciju ako je zaključen kapacitetom C

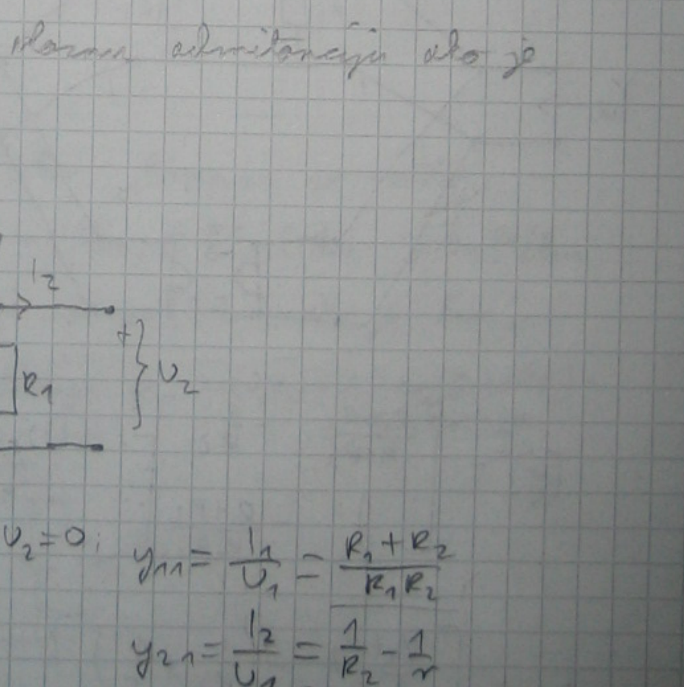


$I_1 = U_1 y_{11} - U_2 y_{12}$

$I_2 = U_1 y_{21} - U_2 y_{22}$

$U_2 = 0$: $\quad y_{11} = \frac{I_1}{U_1} = \frac{R_1 + R_2}{R_1 R_2}$

$y_{21} = \frac{I_2}{U_1} = \frac{1}{R_2} - \frac{1}{r}$



$\Rightarrow i_{g_1}=0 \qquad U_1\left(\frac{1}{R_1}+\frac{1}{R_2}\right) - \cancel{U_2\frac{1}{R_2}} = I_1 - \cancel{I_{g_1}}$

$$I_2 = \frac{U_1}{R_2} - \frac{U_1}{r}$$

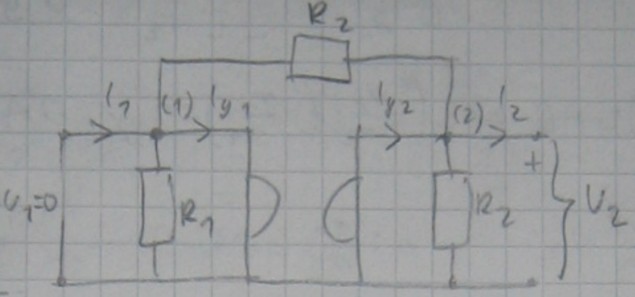
$$u_{g_1} = -r\cdot i_{g_2}$$

$$u_{g_2} = -r\cdot i_{g_1}$$

$U_1=0$:

$$y_{12} = -\frac{I_1}{U_2} = \frac{1}{R_2}+\frac{1}{r}$$

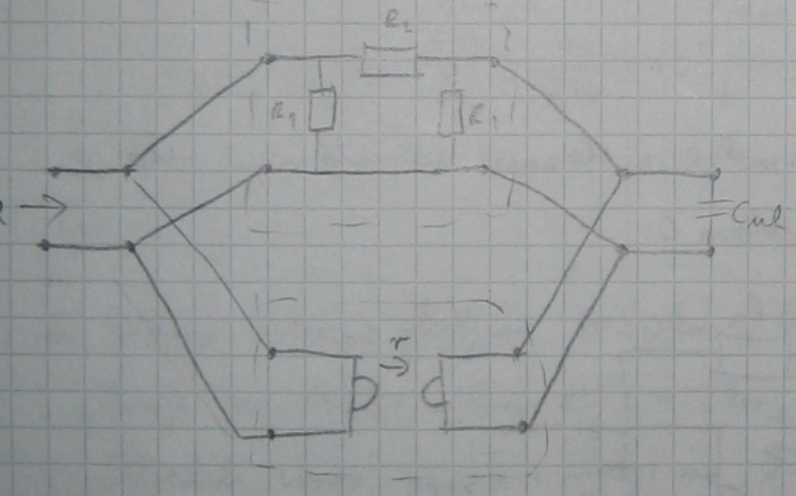
$$y_{22} = -\frac{I_2}{U_2} = \frac{R_1+R_2}{R_1R_2}$$



$\Rightarrow i_{g_2}=0 \qquad (1)\quad I_1 + \frac{U_2}{R_2} = -\frac{1}{r}U_2$

$(2)\quad I_2 + \frac{U_2}{R_1} + \frac{U_2}{R_2} = 0$

ili



$$[y]^{I} = \begin{bmatrix} \frac{R_1+R_2}{R_1R_2} & -\frac{1}{R_2} \\ \frac{1}{R_2} & -\frac{R_1+R_2}{R_1R_2} \end{bmatrix} \qquad [y_2]^{II} = \begin{bmatrix} 0 & -\frac{1}{r} \\ -\frac{1}{r} & 0 \end{bmatrix}$$

$$[y] = [y]^{I} + [y]^{II} = \begin{bmatrix} \frac{R_1+R_2}{R_1R_2} & -\left(\frac{1}{R_2}+\frac{1}{r}\right) \\ \frac{1}{R_2} - \frac{1}{r} & -\frac{R_1+R_2}{R_1R_2} \end{bmatrix}$$

$$Y_{ul} = y_{11} - \frac{y_{12} \cdot y_{21}}{y_{22}+Y_2} = \frac{R_1+R_2}{R_1 R_2} - \frac{\left(\frac{1}{R_2}+\frac{1}{r}\right)\left(\frac{1}{R_2}-\frac{1}{r}\right)}{\frac{R_1+R_2}{R_1 R_2}+sC}$$

5. Zadana je naponska prijenosna funkcija nekog PP filtra. Kolika je širina pojasa propuštanja B, Q-faktor i centralna frekvencija, te gornja i donja granična frekvencija?

$$T(s) = \frac{10^4 s}{s^2+10^4 s+10^{10}} \qquad T(s) = \frac{K \frac{\omega_p}{Q_p} s}{s^2 + \frac{\omega_p}{Q_p} s + \omega_p^2}$$

$\omega_p^2 = 10^{10} \Rightarrow \omega_p = 10^5 \text{ rad/s} = \omega_c$

$\frac{\omega_p}{Q_p} = 10^4 \Rightarrow Q_p = \frac{10^5}{10^4} = 10$

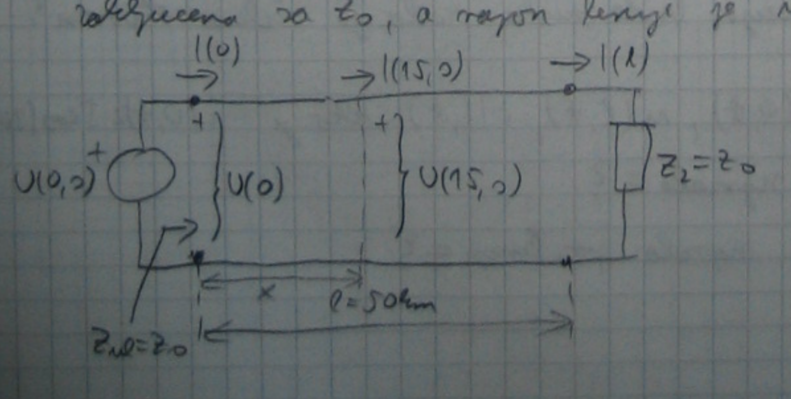
širina pojasa propuštanja $B = \frac{\omega_p}{Q_p} = \frac{10^5}{10} = 10^4 \text{ rad/s}$

2. način $B = \omega_g - \omega_d$

$$\omega_{g,d} = \omega_p \sqrt{1+\frac{1}{4Q_p^2}} \pm \frac{\omega_p}{2Q_p} = 10^5 \sqrt{1+\frac{1}{400}} \pm \frac{10^4}{2} = \begin{cases} \omega_g = 105125 \\ \omega_d = 95125 \end{cases}$$

Linije

6. Zadana je linija duljine $l = 50$ km. Primarni parametri linije su $R = 5{,}4\ \Omega/\text{km}$, $L = 2\ \text{mH/km}$, $G = 1\ \mu\text{S/km}$, $C = 6\ \text{nF/km}$. Odrediti iznos napona i struje na $x = 15$ km ako je linija zaključena sa Z_0, a napon linije je $u(0,t) = 10\cos(5 \cdot 10^3 t)$, $-\infty < t < +\infty$



$$Z_0=\sqrt{\frac{R+j\omega L}{G+j\omega C}}=\sqrt{\frac{5{,}4+j10}{10^{-6}(1+j30)}} \qquad s=j\omega$$

$$\gamma=\sqrt{(R+j\omega L)(G+j\omega C)}=10^{-3}\sqrt{(5{,}4+j10)(1+j30)}=0{,}0048+j0{,}0178$$

$$Z_0=\sqrt{10^5(3{,}39-j1{,}69)}=10^2(5{,}99-j1{,}41)$$

$$U(0)=U(x)\,ch\,\gamma x+I(x)Z_0\,sh\,\gamma x$$
$$I(0)=\frac{U(x)}{Z_0}\,sh\,\gamma x+I(x)\,ch\,\gamma x$$

$$U(x)=U(0)\,ch\,\gamma x-I(0)Z_0\,sh\,\gamma x$$
$$I(x)=-\frac{U(0)}{Z_0}\,sh\,\gamma x+I_0(0)\,ch\,\gamma x$$

$$U(0)=Z_0\,I(0)$$
$$U(x)=U(0)(ch\,\gamma x-sh\,\gamma x)=U(0)e^{-\gamma x}$$
$$I(x)=\frac{U(0)}{Z_0}(-sh\,\gamma x+ch\,\gamma x)=\frac{U(0)}{Z_0}e^{-\gamma x}$$
$$U(15)=10e^{-\gamma 15}=8{,}071-j2{,}458=9{,}3e^{-j15^\circ}\,[V]$$
$$I(15)=\frac{10}{Z}e^{-\gamma 15}=0{,}015-j0{,}00055=15{,}1e^{-j2^\circ}\,[mA]$$

$$u(15,t)=9{,}3\cos(\omega t-15^\circ)\,[V]$$
$$i(15,t)=15{,}1\cos(\omega t-2^\circ)\,[mA]$$

7. Zadana je linija bez gubitaka, $L=4\,mH/km$ i $C=8\,nF/km$

a) Koliko najmanje mora biti duga ova linija da kod $\omega=10^6\,rad/s$ ulazna impedancija bude nula kada je izlaz otvoren?

b) Koliki su $u(0,t)$, $u(l,t)$, $i(l,t)$ ako je $i(0,t)=5\cos(10^6 t)$?

d) Valna duljina signala = ?

c) Brzina širenja signala po liniji = ?

$Z_0 = \sqrt{\frac{L}{C}} \quad \gamma = s\sqrt{LC}$

$s = j\omega \rightarrow \gamma = j\omega\sqrt{LC} = j\beta$

a) $I(l) = 0$

$U(0) = U(l)\,\mathrm{ch}\,\gamma l + I(l) Z_0\,\mathrm{sh}\,\gamma l = U(l)\,\mathrm{ch}\,\gamma l$

$I(0) = \frac{U(l)}{Z_0}\mathrm{sh}\,\gamma l + I(l)\,\mathrm{ch}\,\gamma l = \frac{U(l)}{Z_0}\mathrm{sh}\,\gamma l$

$Z_{ul} = \frac{U(0)}{I(0)} = Z_0 \frac{\mathrm{ch}\,\gamma l}{\mathrm{sh}\,\gamma l} = 0 \Rightarrow \mathrm{ch}(\gamma l) = 0$

$\mathrm{ch}(\gamma l) = \mathrm{ch}(j\beta \cdot l) = \cos(\beta l) = \cos(\omega\sqrt{LC} \cdot l) = 0$

$\omega\sqrt{LC} \cdot l = \frac{\pi}{2} \Rightarrow l = \frac{\pi}{2\omega\sqrt{LC}} = \frac{\pi}{8\sqrt{2}}$ km

b) $u(0,t) = 0$

$i(0,t) = 5\cos(10^6 t)$

$i(l,t) = 0$

$U(l) = U(0)\,\mathrm{ch}\,\gamma l - I(0) Z_0\,\mathrm{sh}\,\gamma l = -I(0) Z_0\,\mathrm{sh}(j\beta l) = -I(0) Z_0 j \sin\left(\frac{\pi}{2}\right)$

$Z_0 = \sqrt{\frac{L}{C}} = \frac{10^3}{\sqrt{2}} \quad U(l) = -j I(0) Z_0 = -j\frac{5 \cdot 10^3}{\sqrt{2}} = \frac{5 \cdot 10^3}{\sqrt{2}} e^{-j\frac{\pi}{2}}$

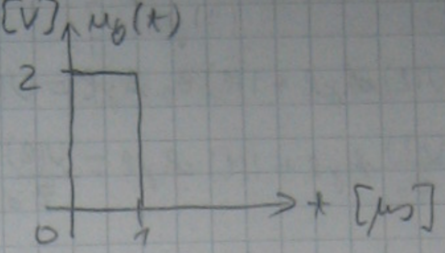
$u(l,t) = \frac{5 \cdot 10^3}{\sqrt{2}} \cos\left(10^6 t - \frac{\pi}{2}\right)$

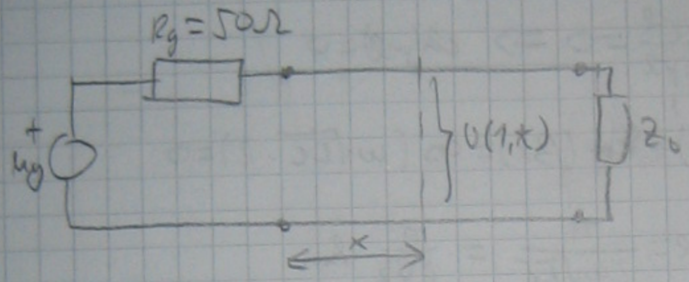
c) $v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \cdot 10^{-3} \cdot 8 \cdot 10^{-9}}} = \frac{10^6}{\sqrt{32}} = 176\,776{,}69$ km/s

d) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{2\pi}{10^6\sqrt{4 \cdot 10^{-3} \cdot 8 \cdot 10^{-9}}} = \frac{2\pi}{\sqrt{32}} = 1{,}1107$ km

8. Zadana je linija bez distorzije s $L = 250\,\mu H/km$, $C = 100\,nF/km$, $R = 50\,\Omega/km$, $G = 0{,}02\,S/km$, $l = 3\,km$

Odrediti napon na $x = \frac{1}{3} l = 1\,km$ ako je napon izvora zadan slikom





$$Z_{ul} = \frac{U(0)}{I(0)} = Z_0$$

$$U(0) = U_g(s)\,\frac{Z_{ul}}{R_g + Z_{ul}} = \frac{U_g}{2}$$

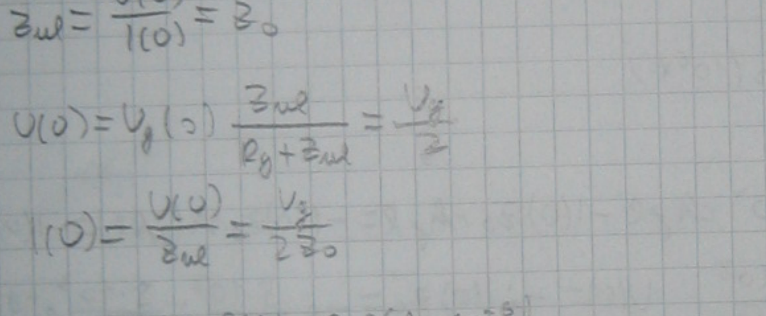
$$I(0) = \frac{U(0)}{Z_{ul}} = \frac{U_g}{2Z_0}$$

$$u_g(t) = 2\cdot S(t) - 2\,S(t - 10^{-6})$$

$$U_g(s) = \frac{2}{s} - \frac{2}{s}\,e^{-s\cdot 10^{-6}}$$

$$U(x) = U(0)\,e^{-\gamma x} \quad (\text{jer je } Z_2 = Z_0)$$

$$U(x) = \frac{U_g}{2}\,e^{-(1 + s\cdot 5\cdot 10^{-6})} = \frac{U_g}{2}\,e^{-1} e^{-s\cdot 5\cdot 10^{-6}}$$



$$U(x) = \left(\frac{1}{s} - \frac{1}{s}\,e^{-s\cdot 10^{-6}}\right) e^{-(1 + s\cdot 5\cdot 10^{-6})} = \frac{1}{s}\cdot e^{-1}\cdot e^{-s\cdot 5\cdot 10^{-6}} - \frac{1}{s}\,e^{-1}\,e^{-s(10^{-6} + 5\cdot 10^{-6})}$$

$$\circ\!\!-\!\!\bullet \quad u(x,t) = \frac{1}{e}\,S(t - 5\cdot 10^{-6}) - \frac{1}{e}\,S(t - 6\cdot 10^{-6}) \quad , \quad \frac{1}{e} = 0{,}368$$