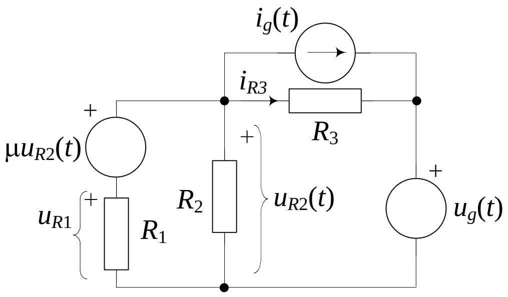
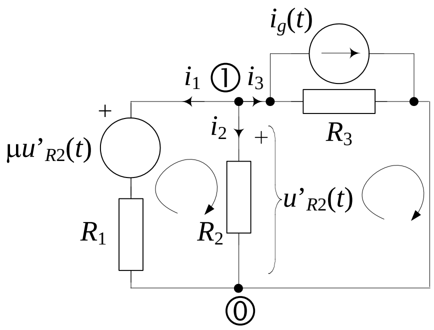
MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA 2014-2015 – Rješenja

1. Za električni krug prikazan slikom primjenom metode superpozicije izračunati napon $u_{R2}(t)$. Zadane su normalizirane vrijednosti elemenata $R_1=2$, $R_2=1$, $R_3=1/2$ i $\mu=2$ i pobude $u_g(t)=S(t)$ i $i_g(t)=S(t)$. Izračunati također napon $u_{R1}(t)$ na otporu R_1 i struju $i_{R3}(t)$ kroz R_3. U proračunu primijeniti Kirchhoffove zakone.



Rješenje: Primjena metode superpozicije.

a) Isključen naponski izvor $u_g=0$. Ovisni izvor (NONI) $\mu\, u_{R2}$ ostaje uključen.



Mreža ima $N_b=3$ grane i $N_v=2$ čvora

Jednadžbe KZN

$$\text{(p1)}\ \ -u_1(t)+u_2(t)=0$$

$$\text{(p2)}\ \ -u_2(t)+u_3(t)=0 \Rightarrow u_1(t)=u_2(t)=u_3(t),\ u'_{R2}(t)=u_2(t)$$

Naponsko-strujne relacije grana

$$\text{(g1)}\ \ u_1(t)=\mu u'_{R2}(t)+R_1 i_1(t) \Rightarrow i_1(t)=\frac{1}{R_1}\cdot\left[u_1(t)-\mu u_2(t)\right]$$

$$\text{(g2)}\ \ u'_{R2}(t)=u_2(t)=R\cdot i_2(t) \Rightarrow i_2(t)=u_2(t)\frac{1}{R_2}$$

$$\text{(g3)}\ \ u_3(s)=R_3\cdot\left[i_3(t)-i_g(t)\right] \Rightarrow i_3(t)=\frac{1}{R_3}u_3(t)+i_g(t)$$

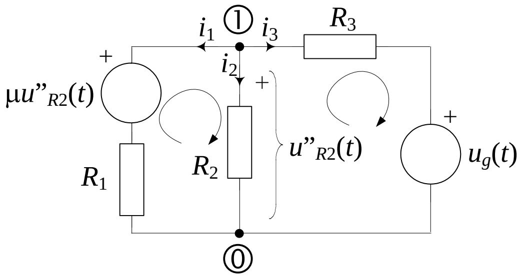
Jednadžbe KZS

$$\text{(č1)}\ \ i_1(t)+i_2(t)+i_3(t)=0 \Rightarrow u_1(t)\cdot\frac{1}{R_1}-\mu u_2(t)\frac{1}{R_1}+u_2(t)\frac{1}{R_2}+\frac{1}{R_3}u_3(t)+i_g(t)=0$$

$$\Rightarrow u'_{R2}(t)\cdot\left[\frac{1-\mu}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]=-i_g(t)$$

$$\Rightarrow u'_{R2}(t)=\frac{-i_g(t)}{\frac{1-\mu}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}=\frac{-S(t)}{\frac{1-2}{2}+1+2}=\frac{-S(t)}{\frac{1}{2}+\frac{4}{2}}=\frac{-2}{5}S(t)\ \ \text{(1 bod)}$$

b) Isključen strujni izvor $i_g(t)$=0. Ovisni izvor (NONI) $\mu\, u_{R2}$ ostaje uključen.



Jednadžbe KZN (iste kao i u slučaju a)

$$\text{(p1)} \quad -u_1(t)+u_2(t)=0$$

$$\text{(p2)} \quad -u_2(t)+u_3(t)=0 \Rightarrow u_1(t)=u_2(t)=u_3(t)\,,\; u''_{R2}(t)=u_2(t)$$

Naponsko-strujne relacije grana (g1 i g2 iste kao i u slučaju a)

$$\text{(g1)} \quad u_1(t)=\mu u'_{R2}(t)+R_1 i_1(t) \Rightarrow i_1(t)=\frac{1}{R_1}\cdot\left[u_1(t)-\mu u_2(t)\right]$$

$$\text{(g2)} \quad u''_{R2}(t)=u_2(t)=R\cdot i_2(t) \Rightarrow i_2(t)=u_2(t)\frac{1}{R_2}$$

$$\text{(g3)} \quad u_3(s)=R_3\cdot i_3(t)+u_g(t) \Rightarrow i_3(t)=\frac{1}{R_3}\left[u_3(t)-u_g(t)\right]$$

Jednadžbe KZS

$$\text{(č1)} \quad i_1(t)+i_2(t)+i_3(t)=0 \Rightarrow u_1(t)\cdot\frac{1}{R_1}-\mu u_2(t)\frac{1}{R_1}+u_2(t)\frac{1}{R_2}+\frac{1}{R_3}\left[u_3(t)-u_g(t)\right]=0$$

$$\Rightarrow u''_{R2}(t)\cdot\left[\frac{1-\mu}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]=\frac{1}{R_3}u_g(t)$$

$$\Rightarrow u''_{R2}(t)=\frac{\frac{1}{R_3}u_g(t)}{\frac{1-\mu}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}=\frac{2S(t)}{\frac{1-2}{2}+1+2}=\frac{2S(t)}{\frac{1}{2}+\frac{4}{2}}=\frac{4}{5}S(t) \quad \text{(1 bod)}$$

c) Superpozicija:

$$u_{R2}(t)=u'_{R2}(t)+u''_{R2}(t)=\frac{-i_g(t)+\frac{1}{R_3}u_g(t)}{\frac{1-\mu}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}=\frac{-S(t)+2S(t)}{\frac{1-2}{2}+1+2}=\frac{S(t)}{\frac{1}{2}+\frac{4}{2}}=\frac{2}{5}S(t) \quad \text{(1 bod)}$$

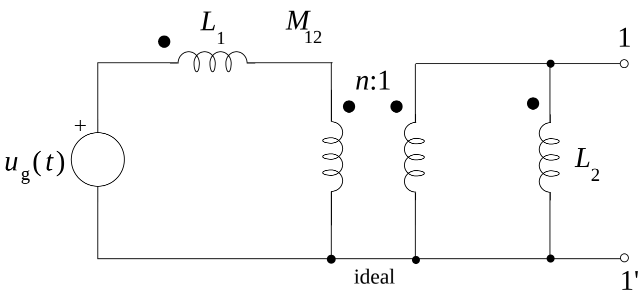
d) Napon u_{R1}

$$u_1=u_{R2}-\mu\cdot u_{R2}=(1-\mu)u_{R2}=-u_{R2}=-\frac{2}{5}S(t) \quad \text{(1 bod)}$$

e) Struja i_{R3}

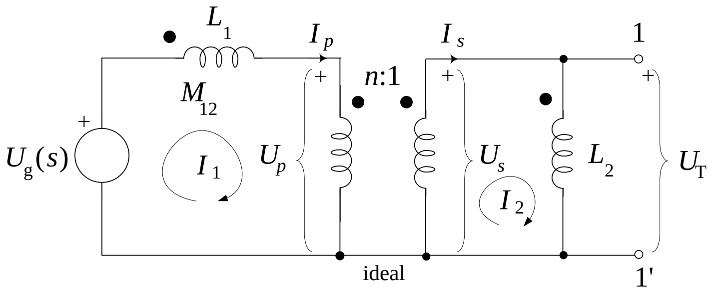
$$i_{R3}=\frac{u_{R2}-u_g}{R_3}=2\left(\frac{2}{5}-1\right)S(t)=-\frac{6}{5}S(t) \quad \text{(1 bod)}$$

2. Za električni krug na slici zadane su normalizirane vrijednosti elemenata L_1=2, L_2=1, M_{12}=1 te n=2, $u_g(t)$=$S(t)$. Odrediti nadomjesne parametre mreže po Theveninu s obzirom na polove 1–1'. Koristiti metodu struja petlji. U zadatku je potrebno: a) Nacrtati sklop za izračunavanje Theveninovog napona i postaviti jednadžbe petlji; b) Odrediti Theveninov napon $U_T(s)$; c) Nacrtati sklop za izračunavanje Theveninove impedancije i postaviti jednadžbe petlji; d) Odrediti Theveninovu impedanciju $Z_T(s)$. e) Da li je električni krug recipročan? Zašto?



Rješenje:

a) Jednadžbe petlji:



Iz sheme je vidljivo da vrijedi: $I_p = I_1,\;\; I_s = I_2,\;\; U_s = U_T$

1) $I_1 sL_1 + I_2 sM_{12} + U_p(s) = U_g(s)$

2) $I_1 sM_{12} + I_2 sL_2 = U_s(s)$

3) $U_s = \dfrac{1}{n} U_p \;\Rightarrow\; U_p = nU_s$

4) $I_s = nI_p \;\Rightarrow\; I_2 = nI_1$

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1) $I_1 sL_1 + I_2 sM_{12} + nU_s(s) = U_g(s)$

2) $I_1 sM_{12} + I_2 sL_2 = U_s(s)$ (1 bod)

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b) Theveninov napon $U_T(s)$=$U_s(s)$:

1), 2) $\Rightarrow$ $I_1 sL_1 + I_2 sM_{12} + n\left(I_1 sM_{12} + I_2 sL_2\right) = U_g(s)$

$I_1 sL_1 + nI_1 sM_{12} + n\left(I_1 sM_{12} + nI_1 sL_2\right) = U_g(s)$

$I_1\left(sL_1 + 2nsM_{12} + n^2 sL_2\right) = U_g(s)$

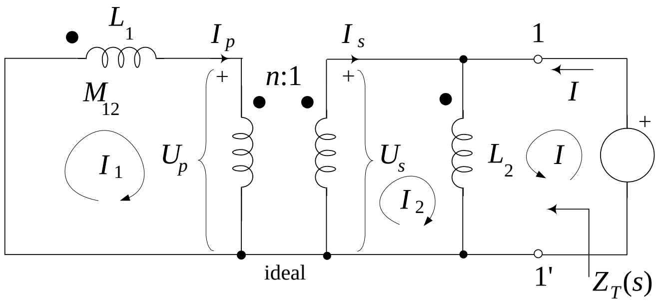
Uvrstimo vrijednosti: L_1=2, L_2=1, M_{12}=1, n=2, $u_g(t)$=$S(t)$.

$$I_1(s) = \frac{U_g(s)}{s\left(L_1 + 2nM_{12} + n^2 L_2\right)} = \frac{1}{s}\cdot\frac{1}{s(2+4+4)} = \frac{1}{s}\cdot\frac{1}{s(2+4+4)} = \frac{1}{10s^2}$$

$$I_2(s) = n\cdot I_1(s) = 2\cdot\frac{1}{10s^2} = \frac{1}{5s^2}$$

$$U_T(s)=U_s(s)=sM_{12}I_1(s)+sL_2I_2(s)=s\frac{1}{10s^2}+s\frac{2}{10s^2}=\frac{3}{10s}$$ (1 bod)

c) Izračunavanje Theveninove impedancije pomoću jednadžbi petlji



Iz sheme je vidljivo da vrijedi: $I_p=I_1,\quad I_s=I_2,\ U(s)=U_s(s),\ Z_T(s)=\dfrac{U(s)}{I(s)}$

1) $I_1sL_1+(I_2+I)sM_{12}+U_p(s)=0$

2) $I_1sM_{12}+(I_2+I)sL_2=U_s(s)$

3) $U_s(s)=U(s)$

4) $U_s=\dfrac{1}{n}U_p\ \Rightarrow\ U_p=nU_s$

5) $I_s=nI_p\ \Rightarrow\ I_2=nI_1$

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1) $I_1sL_1+(nI_1+I)sM_{12}+nU_s(s)=0$

2) $I_1sM_{12}+(nI_1+I)sL_2=U_s(s)$

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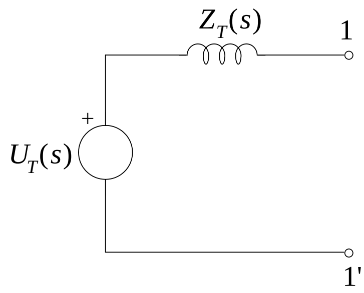
1) $I_1(sL_1+nsM_{12})+IsM_{12}+nU(s)=0$

2) $I_1(sM_{12}+nsL_2)+IsL_2=U(s)$ (1 bod)



d) Theveninova impedancija $Z_T(s)=U(s)/I(s)$:

$$2)\Rightarrow\ I_1=\frac{U(s)-IsL_2}{sM_{12}+nsL_2}\ \rightarrow 1)\ \frac{U(s)-IsL_2}{sM_{12}+nsL_2}(sL_1+nsM_{12})+IsM_{12}+nU(s)=0$$

$$U(s)\frac{L_1+nM_{12}}{M_{12}+nL_2}+nU(s)=I(s)sL_2\frac{L_1+nM_{12}}{M_{12}+nL_2}-I(s)sM_{12}$$

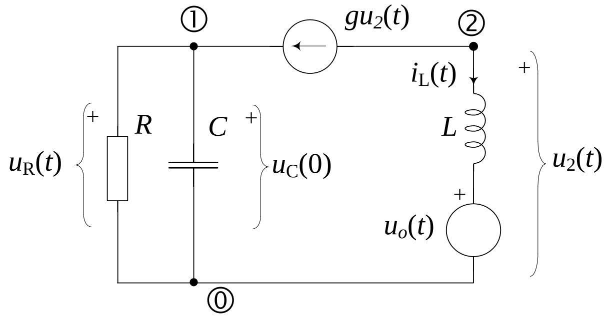
$$Z_T(s)=\frac{U(s)}{I(s)}=\frac{sL_2\dfrac{L_1+nM_{12}}{M_{12}+nL_2}-sM_{12}}{\dfrac{L_1+nM_{12}}{M_{12}+nL_2}+n}=\frac{s\dfrac{2+2}{1+2}-s}{\dfrac{2+2}{1+2}+2}=\frac{s\dfrac{4}{3}-s}{\dfrac{4}{3}+\dfrac{6}{3}}=\frac{4s-3s}{10}=\frac{s}{10}=sL_T$$

(1 bod)

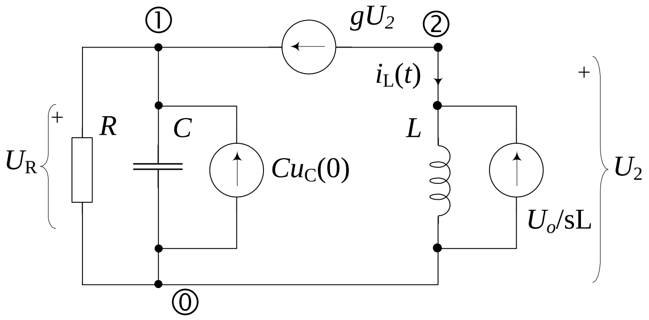
e) Da li je električni krug recipročan? Zašto?

DA! Jer ima idealni transformator i vezane induktivitete (nema ovisne izvore ili girator).
(1 bod)

3. Za električni krug na slici zadane su normalizirane vrijednosti elemenata R=1, C=1, L=2, g=2, početni napon na kapacitetu $u_C(0)$=1, te pobuda $u_o(t)$=$S(t)$. Primjenom Laplaceove transformacije i koristeći metodu čvorišta izračunati napon $u_R(t)$ na otporu R i struju $i_L(t)$.



Rješenje: Primjena Laplaceove transformacije



(1 bod)

$$U_1(s)\left(\frac{1}{R}+sC\right)=Cu_C(0)+g\cdot U_2(s) \;\Rightarrow\; U_1\left(\frac{1}{R}+sC\right)-g\cdot U_2=Cu_C(0)$$

$$U_2(s)\frac{1}{sL}=-g\cdot U_2(s)+\frac{1}{sL}\cdot U_o(s) \;\Rightarrow\; U_2\left(\frac{1}{sL}+g\right)=\frac{U_o}{sL}\Rightarrow\; U_2=\frac{U_o}{1+sgL}\text{(1 bod)}$$

$$U_1\frac{1}{R}(1+sCR)=\frac{gU_o}{1+sgL}+Cu_C(0) \;\Rightarrow\; U_1=\frac{R\left(gU_o+Cu_C(0)(1+sgL)\right)}{(1+sCR)(1+sgL)}\text{(1 bod)}$$

$$U_1=\frac{\frac{2}{s}+1+4s}{(1+s)(1+4s)}=\frac{s^2+\frac{s}{4}+\frac{1}{2}}{s(s+1)\left(s+\frac{1}{4}\right)}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{s+\frac{1}{4}}$$

$$s^2+\frac{s}{4}+\frac{1}{2}=A(s+1)\left(s+\frac{1}{4}\right)+Bs\left(s+\frac{1}{4}\right)+Cs(s+1)$$

$$A+B+C=1$$

$$\frac{5A}{4}+\frac{B}{4}+C=\frac{1}{4}$$

$$\frac{A}{4}=\frac{1}{2}\;\Rightarrow\quad A=2\;\Rightarrow\quad B+C=1-A=-1$$

$$\frac{5A}{4}+\frac{B}{4}+C=\frac{1}{4}\;\Rightarrow\;\frac{3}{4}C=\frac{1}{4}-\frac{5A}{4}-\frac{B+C}{4}=\frac{1}{4}-\frac{10}{4}-\frac{(-1)}{4}\;\Rightarrow\; C=-\frac{8}{3}$$

$$B=-1-C=\frac{5}{3}$$

$$U_1=\frac{2}{s}+\frac{5}{3}\cdot\frac{1}{s+1}-\frac{8}{3}\cdot\frac{1}{s+\frac{1}{4}} \qquad \Rightarrow \qquad u_R=u_1=\left(2+\frac{5}{3}\cdot e^{-t}-\frac{8}{3}\cdot e^{-t/4}\right)S(t)$$ (1 bod)

$$I_L=\frac{U_2-U_o}{sL}=\frac{U_o}{sL}\left(\frac{1}{1+sgL}-1\right)=\frac{-gU_o}{1+sgL}=\frac{-\frac{2}{s}}{1+4s}=-\frac{1}{2}\cdot\frac{1}{s(s+1/4)}$$

$$I_L=-\frac{1}{2}\cdot\frac{1}{s(s+1/4)}=\frac{A}{s}+\frac{B}{s+1/4}$$

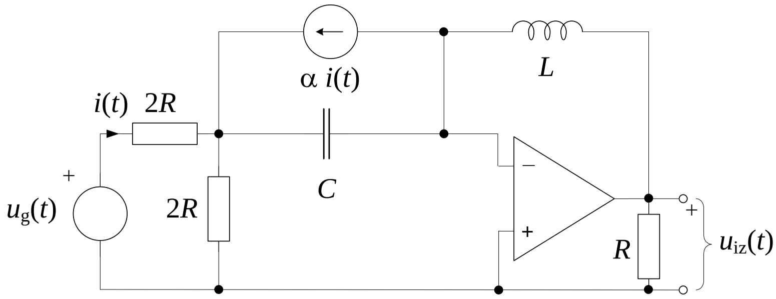
$$-\frac{1}{2}=A(s+1/4)+Bs$$

$$A+B=0$$

$$\frac{A}{4}=-\frac{1}{2} \qquad \Rightarrow \qquad A=-2 \qquad \Rightarrow \qquad B=2$$

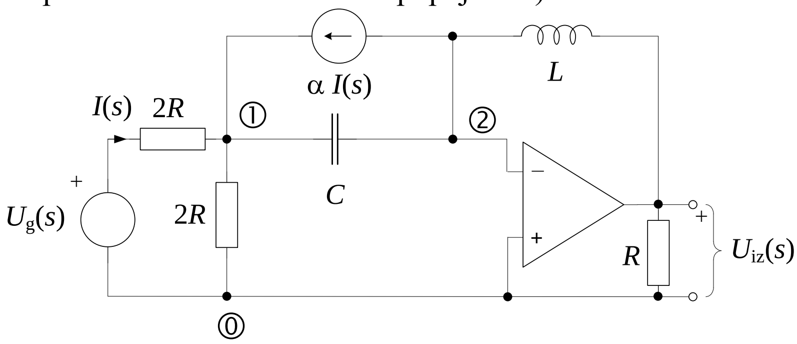
$$I_L=-\frac{2}{s}+\frac{2}{s+1/4} \qquad \Rightarrow \qquad i_L=-2\left(1-e^{-t/4}\right)S(t)$$ (1 bod)

4. Za električni krug prikazan slikom izračunati odziv $u_{iz}(t)$ na pobudu $u_g(t)=S(t)$. Zadane su normalizirane vrijednosti elemenata $R=1$, $C=1$ i $L=2$; te konstanta ovisnog izvora $\alpha=1$. Operacijsko pojačalo je idealno. Početni uvjeti su jednaki nuli. Traženi odziv treba odrediti primjenom jednadžbi čvorišta.



Rješenje:

a) Primjenom Laplaceove transformacije dobivamo slijedeći električni krug u *frekvencijskoj* domeni. Postavimo jednadžbe čvorišta (otpor R na izlazu op. pojačala se zanemaruje, jer je paralelno spojen naponskom izvoru na izlazu op. pojačala):



$$1)\ U_1\left(\frac{1}{2R}+\frac{1}{2R}+sC\right)-U_2sC=\alpha I(s)+\frac{U_g(s)}{2R};\ \Rightarrow\ I(s)=\frac{U_g(s)-U_1(s)}{2R};$$

$$2)\ -U_1sC+U_2\left(sC+\frac{1}{sL}\right)=U_{iz}(s)\frac{1}{sL}-\alpha I(s);$$

Virtualni kratki spoj $\Rightarrow U_2=0 \Rightarrow$

$$1)\ U_1\left(\frac{2}{2R}+sC\right)=\alpha\frac{U_g(s)-U_1(s)}{2R}+\frac{U_g(s)}{2R};$$

$$2)\ -U_1sC=U_{iz}(s)\frac{1}{sL}-\alpha\frac{U_g(s)-U_1(s)}{2R};$$

Nakon malo sređivanja:

$$1)\ U_1\left(\frac{2+\alpha}{2R}+sC\right)=\frac{U_g(s)}{2R}(1+\alpha)$$

$$\Rightarrow U_1(s)=\frac{\frac{1}{2R}(1+\alpha)}{\frac{2+\alpha}{2R}+sC}U_g(s)=\frac{1+\alpha}{2+\alpha+s(2RC)}U_g(s);$$

$$2)\ U_1\left(\frac{\alpha}{2R}+sC\right)=-U_{iz}(s)\frac{1}{sL}+U_g(s)\frac{\alpha}{2R};$$

$$1),\,2)\;\Rightarrow\;\frac{1+\alpha}{2+\alpha+s(2RC)}\left(\frac{\alpha}{2R}+sC\right)U_g(s)=-U_{iz}(s)\frac{1}{sL}+U_g(s)\frac{\alpha}{2R};$$

$$U_{iz}(s)\frac{1}{sL}=-\frac{1+\alpha}{2+\alpha+s(2RC)}\left(\frac{\alpha}{2R}+sC\right)U_g(s)+\frac{\alpha}{2R}U_g(s)$$

$$U_{iz}(s)=-sL\left[\frac{1+\alpha}{2+\alpha+s(2RC)}\left(\frac{\alpha}{2R}+sC\right)-\frac{\alpha}{2R}\right]U_g(s)$$ (3 boda)

b) Uz uvrštene vrijednosti elemenata: R=1, C=1, L=2; $U_g(s)=1/s$ i α=1

$$U_{iz}(s)=-2s\left[\frac{1+1}{2+1+2s}\left(\frac{1}{2}+s\right)-\frac{1}{2}\right]\cdot\frac{1}{s}=-2\left[\frac{2}{3+2s}\left(\frac{1+2s}{2}\right)-\frac{1}{2}\right]$$

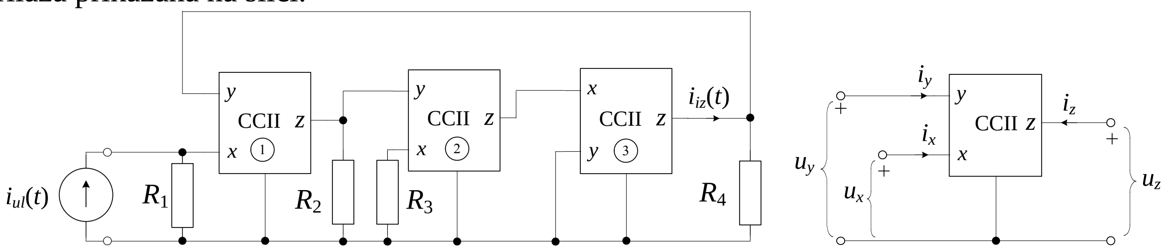
$$U_{iz}(s)=-2\left[\frac{1+2s}{3+2s}-\frac{1}{2}\right]$$

$$U_{iz}(s)=-2\left[\frac{3+2s-2}{3+2s}-\frac{1}{2}\right]=-2\left[1-\frac{2}{3+2s}-\frac{1}{2}\right]=-2\left[\frac{1}{2}-\frac{1}{s+\frac{3}{2}}\right]=-1+\frac{2}{s+\frac{3}{2}}$$ (1 bod)

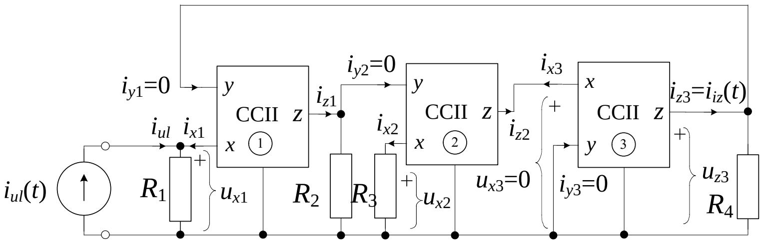
c) Inverzna Laplaceova transformacija izlaznog napona:

$$\underline{u_{iz}(t)=L^{-1}\left[U_{iz}(s)\right]=-\delta(t)+2e^{-3/2t}\cdot S(t)}$$ (1 bod)

5. Za električni krug prikazan slikom izračunati valni oblik struje $i_{iz}(t)$ za $t>0$ kao odziv, ako je zadana pobuda $i_{ul}(t)=E\cdot S(t)$[A]. Zadane su normalizirane vrijednosti elemenata $R_1=1$, $R_2=2$, $R_3=3$, $R_4=4$, te konstanta $E=5{,}5$. Za strujni prijenosnik druge generacije (CCII) vrijede slijedeće definicijske jednadžbe: $u_x=u_y$, $i_y=0$, $i_z=i_x$ uz referentna usmjerenja struja i napona prilaza prikazana na slici.



Rješenje:



a) Za prvi CCII vrijedi: (1 bod)

$$u_{x1}=u_{y1}=u_{z3},\ i_{y1}=0,\ i_{x1}+i_{ul}=\frac{u_{x1}}{R_1},\ i_{z1}=i_{x1}$$

b) Za drugi CCII vrijedi: (1 bod)

$$u_{x2}=u_{y2}=i_{z1}R_2,\ i_{x2}=\frac{u_{x2}}{R_3},\ i_{z2}=i_{x2}$$

$$i_{z2}=i_{x2}=\frac{u_{x2}}{R_3}=\frac{u_{y2}}{R_3}=i_{z1}\frac{R_2}{R_3}\Rightarrow\boxed{i_{z2}=i_{z1}\frac{R_2}{R_3}}$$

c) Za treći CCII vrijedi: (1 bod)

$$u_{x3}=u_{y3}=0,\ i_{z3}=i_{x3}=-i_{z2}\Rightarrow\boxed{i_{z3}=-i_{z2}}$$

$$u_{z3}=i_{z3}R_4$$

d) Nakon sređivanja do sada napisanih izraza: (1 bod)

$$\boxed{i_{iz}=i_{z3}},\ \boxed{i_{z3}=-i_{z1}\frac{R_2}{R_3}},\ i_{z1}=i_{x1}=\frac{u_{x1}}{R_1}-i_{ul}=\frac{u_{z3}}{R_1}-i_{ul}=i_{z3}\frac{R_4}{R_1}-i_{ul}$$

$$\Rightarrow i_{iz}=-\left(i_{iz}\frac{R_4}{R_1}-i_{ul}\right)\frac{R_2}{R_3}\Rightarrow i_{iz}\left(1+\frac{R_2R_4}{R_1R_3}\right)=i_{ul}\frac{R_2}{R_3}\Rightarrow\boxed{i_{iz}=\frac{\frac{R_2}{R_3}}{1+\frac{R_2R_4}{R_1R_3}}i_{ul}}$$

e) Uz uvrštene vrijednosti elemenata: (1 bod)

$$i_{iz}(t)=\frac{\frac{2}{3}}{1+\frac{2\cdot4}{1\cdot3}}i_{ul}(t)=\frac{2}{3+8}i_{ul}(t)=\frac{2}{11}i_{ul}(t)=\frac{2\cdot5{,}5}{11}S(t)=1\cdot S(t)[A]$$

$$\Rightarrow \underline{i_{iz}(t)=1\cdot S(t)[A]}$$