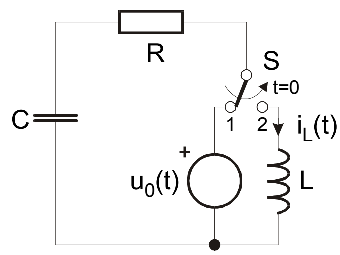
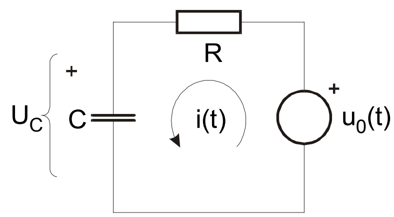
ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA

1. U trenutku t=0 sklopka S se prebaci iz položaja 1 u položaj 2. Izračunati odziv $i_L(t)$. Zadana je pobuda $u_0(t)=2\cos(2t)$ ($-\infty<t<+\infty$ stacionarna sinusna pobuda) i normirane vrijednosti elemenata: R=2, L=1, C=1/2.



Rješenje:

a) Za $t\leq 0$ stacionarna sinusna pobuda → fazori



$$U_0(j\omega)=I(j\omega)\cdot\left(R+\frac{1}{j\omega C}\right),\ U_C(j\omega)=I(j\omega)\cdot\frac{1}{j\omega C}$$

$$U_C(j\omega)=U_0(j\omega)\cdot\frac{1}{1+j\omega RC};\quad U_0(j\omega)=2\angle 0^\circ$$

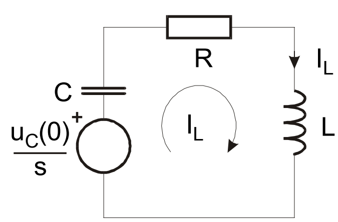
$$U_C(j\omega)=\frac{2}{1+j2\cdot 2\cdot\frac{1}{2}}=\frac{2}{1+j2}\cdot\frac{1-j2}{1-j2}=\frac{2}{5}(1-j2)=\frac{2}{\sqrt{5}}e^{-j\arctan(2)}$$

$$u_C(t)=\frac{2}{\sqrt{5}}\cos(2t-\arctan 2)\quad\Rightarrow\quad u_C(0)=\frac{2}{\sqrt{5}}\cos(\arctan 2)$$

$$\tan x=2,\quad \cos x=?\ \tan x=\frac{\sin x}{\cos x}=\frac{\sqrt{1-\cos^2 x}}{\cos x}\Rightarrow \tan^2 x=\frac{1-\cos^2 x}{\cos^2 x}\Rightarrow \cos^2 x(1+\tan^2 x)=1$$

$$\cos x=\frac{1}{\sqrt{1+\tan^2 x}},\ \cos x=\frac{1}{\sqrt{1+4}}=\frac{1}{\sqrt{5}},\Rightarrow\ u_C(0)=\frac{2}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}}=\frac{2}{5}=0.4$$

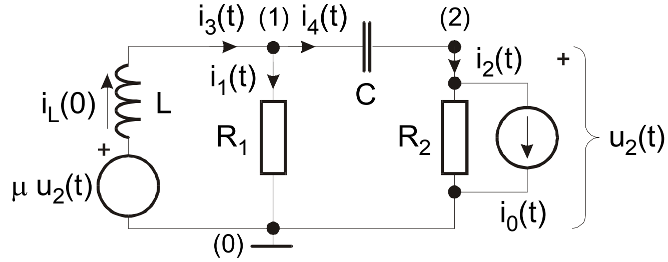
b) Za $t>0$ Laplaceova transformacija



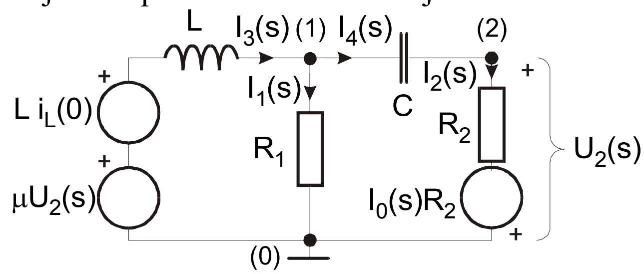
$$I_L(s)=\frac{\frac{u_C(0)}{s}}{R+sL+\frac{1}{sC}}=\frac{Cu_C(0)}{s^2LC+sRC+1}\quad\Rightarrow\quad I_L(s)=\frac{\frac{1}{2}\cdot 0.4}{\frac{s^2}{2}+s+1}=\frac{0.4}{s^2+2s+2}$$

$$I_L(s)=0.4\cdot\frac{1}{(s+1)^2+1}\quad\Rightarrow\quad \underline{i_L(t)=0.4\cdot e^{-t}\sin t\cdot S(t)}$$

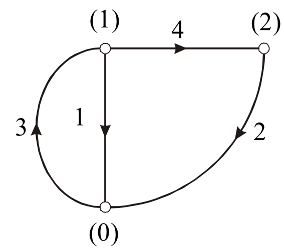
2. Zadan je električni krug prema slici. Poštujući oznake grana i čvorišta nacrtati pripadni orijentirani graf te ispisati reduciranu matricu incidencija **A**. Napisati naponsko-strujne jednadžbe grana u matričnom obliku te ispisati matricu admitancija grana $\mathbf{Y}_b$ i vektor strujnih izvora grana $\mathbf{I}_{0b}$. Topološkom analizom napisati sustav jednadžbi čvorova, odnosno odrediti matrice admitancija čvorova $\mathbf{Y}_v$ i vektor izvora čvorova $\mathbf{I}_v$.



Rješenje: Uz primjenu Laplaceove transformacije :



Električni krug



orijentirani graf

Matrica incidencija: $\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$

Naponsko strujne relacije grana (prvi oblik: naponi grana su izraženi pomoću struja grana):

$$\begin{aligned} U_1 &= I_1 R_1 \\ U_2 &= I_2 R_2 - I_0(s) R_2 \\ U_3 &= I_3 sL - \mu U_2 - L i_L(0) \\ U_4 &= I_4 \frac{1}{sC} \end{aligned} \quad \Rightarrow \quad \begin{aligned} U_1 &= I_1 R_1 \\ U_2 &= I_2 R_2 - I_0(s) R_2 \\ U_3 &= -\mu I_2 R_2 + I_3 sL + \mu I_0(s) R_2 - L i_L(0) \\ U_4 &= I_4 \frac{1}{sC} \end{aligned}$$

Naponsko-strujne relacije grana u matričnom obliku:

$$\underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b} = \underbrace{\begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & -\mu R_2 & sL & 0 \\ 0 & 0 & 0 & \frac{1}{sC} \end{bmatrix}}_{\mathbf{Z}_b} \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b} + \underbrace{\begin{bmatrix} 0 \\ -I_0 R_2 \\ \mu I_0(s) R_2 - L i_L(0) \\ 0 \end{bmatrix}}_{\mathbf{U}_{0b}}$$

Naponsko strujne relacije grana (drugi oblik: struje grana su izražene pomoću napona grana):

$$I_1 = \frac{1}{R_1} U_1$$

$$I_2 = \frac{1}{R_2} U_2 + I_0(s)$$

$$I_3 = \frac{1}{sL} U_3 + \frac{\mu}{sL} U_2 + \frac{i_L(0)}{s}$$

$$I_4 = sCU_4$$

Naponsko-strujne relacije grana u matričnom obliku:

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b} = \underbrace{\begin{bmatrix} \frac{1}{R_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ 0 & \frac{\mu}{sL} & \frac{1}{sL} & 0 \\ 0 & 0 & 0 & sC \end{bmatrix}}_{\mathbf{Y}_b} \underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b} + \underbrace{\begin{bmatrix} 0 \\ I_0 \\ \frac{i_L(0)}{s} \\ 0 \end{bmatrix}}_{\mathbf{I}_{0b}}$$

Sustav jednadžbi čvorova: $\mathbf{Y}_v \cdot \mathbf{U}_v = \mathbf{I}_v$, gdje su:

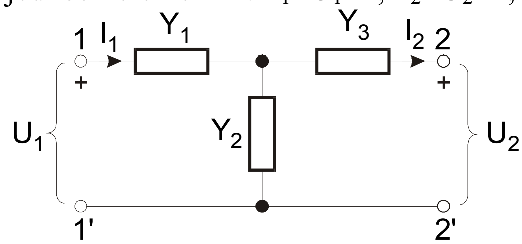
$$\mathbf{Y}_v = \mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{A}^T = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{R_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ 0 & \frac{\mu}{sL} & \frac{1}{sL} & 0 \\ 0 & 0 & 0 & sC \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 1 & -1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{1}{R_1} & -\frac{\mu}{sL} & -\frac{1}{sL} & sC \\ 0 & \frac{1}{R_2} & 0 & -sC \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{sL} + sC & -\frac{\mu}{sL} - sC \\ -sC & \frac{1}{R_2} + sC \end{bmatrix}$$

$$\mathbf{I}_v = \mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{U}_{0b} = \begin{bmatrix} \frac{1}{R_1} & -\frac{\mu}{sL} & -\frac{1}{sL} & sC \\ 0 & \frac{1}{R_2} & 0 & -sC \end{bmatrix} \begin{bmatrix} 0 \\ -I_0 R_2 \\ \mu I_0(s) R_2 - L i_L(0) \\ 0 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{\mu}{sL} I_0 R_2 - \frac{\mu}{sL} I_0 R_2 + \frac{i_L(0)}{s} \\ -I_0(s) \end{bmatrix} = \begin{bmatrix} \frac{i_L(0)}{s} \\ -I_0(s) \end{bmatrix}$$

3. Za T-četveropol prikazan slikom izračunati y-parametre i napisati matricu y-parametara. (Izraziti y-parametre pomoću admitancija elemenata Y_1, Y_2 i Y_3.) Ako je izlazni prilaz (2–2') zaključen admitancijom Y_L pomoću poznatih y-parametara izračunati: a) naponsku prijenosnu funkciju četveropola $T(s)=U_2(s)/U_1(s)$; b) ulaznu admitanciju u četveropol $Y_{ul1}(s)=I_1(s)/U_1(s)$. Najprije izraziti $T(s)$ i $Y_{ul1}(s)$ pomoću poznatih y-parametara izraženih admitancijama elemenata Y_1, Y_2 i Y_3, a zatim uvrstiti slijedeće vrijednosti elemenata: $Y_1=G_1=1$, $Y_2=G_2=2$, $Y_3=G_3=1$ i $Y_L=G_L=1$.

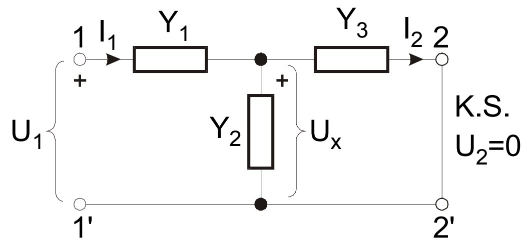


Rješenje:

[y]-parametri:

$$I_1 = y_{11}U_1 - y_{12}U_2$$
$$I_2 = y_{21}U_1 - y_{22}U_2$$

a) $U_2=0$



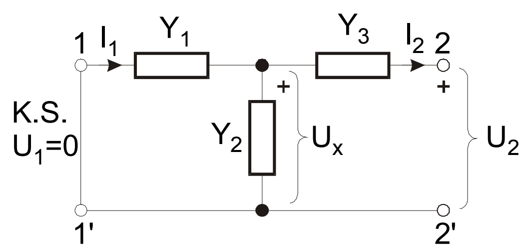
$$U_1 = I_1\left(\frac{1}{Y_1}+\frac{1}{Y_2+Y_3}\right) = I_1\frac{Y_1+Y_2+Y_3}{Y_1(Y_2+Y_3)}$$

$$U_x = I_2\frac{1}{Y_3} = I_1\frac{1}{Y_2+Y_3} \quad\Rightarrow\quad I_2 = I_1\frac{Y_3}{Y_2+Y_3} \Rightarrow I_1 = I_2\frac{Y_2+Y_3}{Y_3}$$

$$\Rightarrow U_1 = I_1\frac{Y_1+Y_2+Y_3}{Y_1(Y_2+Y_3)} = I_2\frac{Y_2+Y_3}{Y_3}\frac{Y_1+Y_2+Y_3}{Y_1(Y_2+Y_3)} = I_2\frac{Y_1+Y_2+Y_3}{Y_1Y_3}$$

$$y_{11} = \left.\frac{I_1}{U_1}\right|_{U_2=0} = \frac{Y_1(Y_2+Y_3)}{Y_1+Y_2+Y_3}; \quad y_{21} = \left.\frac{I_2}{U_1}\right|_{U_2=0} = \frac{Y_1Y_3}{Y_1+Y_2+Y_3}$$

b) $U_1=0$



$$U_2 = -I_2\left(\frac{1}{Y_3}+\frac{1}{Y_1+Y_2}\right) = -I_2\frac{Y_1+Y_2+Y_3}{Y_3(Y_1+Y_2)}$$

$$U_x = -I_2\frac{1}{Y_1+Y_2} = -I_1\frac{1}{Y_1} \quad\Rightarrow\quad I_1 = I_2\frac{Y_1}{Y_1+Y_2} \Rightarrow I_2 = I_1\frac{Y_1+Y_2}{Y_1}$$

$$\Rightarrow U_2 = -I_2 \frac{Y_1+Y_2+Y_3}{Y_3(Y_1+Y_2)} = -I_1 \frac{Y_1+Y_2}{Y_1}\frac{Y_1+Y_2+Y_3}{Y_3(Y_1+Y_2)} = -I_1 \frac{Y_1+Y_2+Y_3}{Y_1Y_3}$$

$$y_{12} = -\frac{I_1}{U_2}\bigg|_{U_1=0} = \frac{Y_1Y_3}{Y_1+Y_2+Y_3}\ ; \quad y_{22} = -\frac{I_2}{U_2}\bigg|_{U_1=0} = \frac{Y_3(Y_1+Y_2)}{Y_1+Y_2+Y_3}$$

$$[y] = \begin{bmatrix} \dfrac{Y_1(Y_2+Y_3)}{Y_1+Y_2+Y_3} & -\dfrac{Y_1Y_3}{Y_1+Y_2+Y_3} \\ \dfrac{Y_1Y_3}{Y_1+Y_2+Y_3} & -\dfrac{Y_3(Y_1+Y_2)}{Y_1+Y_2+Y_3} \end{bmatrix},\ [y] = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix}$$

Prijenosna funkcija napona:

$$T(s) = \frac{U_2}{U_1} = \frac{y_{21}}{Y_L + y_{22}} = \frac{\dfrac{Y_1Y_3}{Y_1+Y_2+Y_3}}{Y_L + \dfrac{Y_3(Y_1+Y_2)}{Y_1+Y_2+Y_3}} = \frac{Y_1Y_3}{Y_L(Y_1+Y_2+Y_3)+Y_3(Y_1+Y_2)}$$

uz uvrštene vrijednosti:

$$T(s) = \frac{U_2}{U_1} = \frac{G_1G_3}{G_L(G_1+G_2+G_3)+G_3(G_1+G_2)} = \frac{1}{1(1+2+1)+1(1+2)} = \frac{1}{7}$$

Ulazna admitancija u četveropol:

$$Y_{ul1}(s) = \frac{I_1}{U_1} = y_{11} - \frac{y_{12}y_{21}}{Y_L+y_{22}} = \frac{Y_1(Y_2+Y_3)}{Y_1+Y_2+Y_3} - \frac{\left(\dfrac{Y_1Y_3}{Y_1+Y_2+Y_3}\right)^2}{Y_L + \dfrac{Y_3(Y_1+Y_2)}{Y_1+Y_2+Y_3}} =$$

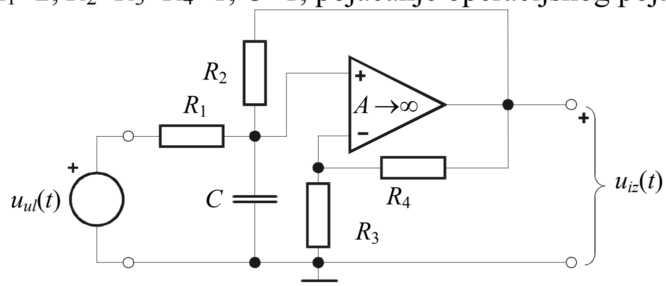
$$= \frac{1}{(Y_1+Y_2+Y_3)}\left[Y_1(Y_2+Y_3) - \frac{(Y_1Y_3)^2}{Y_L(Y_1+Y_2+Y_3)+Y_3(Y_1+Y_2)}\right]$$

uz uvrštene vrijednosti:

$$Y_{ul1} = \frac{1}{(G_1+G_2+G_3)}\left[G_1(G_2+G_3) - \frac{(G_1G_3)^2}{G_L(G_1+G_2+G_3)+G_3(G_1+G_2)}\right] =$$

$$= \frac{1}{(1+2+1)}\left[1(2+1) - \frac{(1)^2}{1(1+2+1)+1(1+2)}\right] = \frac{1}{4}\left[3-\frac{1}{7}\right] = \frac{1}{4}\cdot\frac{20}{7} = \frac{5}{7}$$

4. Za električni krug prikazan slikom: *a*) Odrediti prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$;
b) Izračunati polove i nule prijenosne funkcije i prikazati njihov raspored u *s*-ravnini;
c) Odrediti odziv mreže $u_{iz}(t)$ na pobudu $u_{ul}(t)=S(t)$. Skicirati odziv. Da li je električni krug stabilan? Zadano je: $R_1=2$, $R_2=R_3=R_4=1$, $C=1$, pojačanje operacijskog pojačala $A\rightarrow\infty$.



Rješenje: *a*) Naponska prijenosna funkcija: čvor (1) je na (+) ulazu u operacijsko pojačalo, a čvor (2) na (–) ulazu u operacijsko pojačalo. Slijede jednadžbe čvorova:

$$(1)\qquad \left(\frac{1}{R_1}+\frac{1}{R_2}+sC\right)U_1(s)-\frac{1}{R_1}U_{ul}(s)-\frac{1}{R_2}U_{iz}(s)=0$$

$$(2)\qquad \left(\frac{1}{R_3}+\frac{1}{R_4}\right)U_2(s)-\frac{1}{R_4}U_{iz}(s)=0$$

$$(3)\qquad A\left[U_1(s)-U_2(s)\right]=U_{iz}(s)\Rightarrow U_1(s)-U_2(s)=\frac{U_{iz}(s)}{A}$$

$$A\rightarrow\infty\Rightarrow U_1(s)=U_2(s)$$

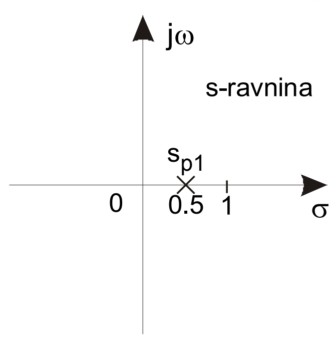
$$(2)\Rightarrow\quad \left(\frac{R_4}{R_3}+1\right)U_2(s)=U_{iz}(s),(3)\Rightarrow U_2(s)=U_1(s)\rightarrow(1)$$

$$(1)\qquad \left(\frac{1}{R_1}+\frac{1}{R_2}+sC\right)\frac{U_{iz}(s)}{\left(\frac{R_4}{R_3}+1\right)}-\frac{1}{R_1}U_{ul}(s)-\frac{1}{R_2}U_{iz}(s)=0$$

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+sC-\frac{1}{R_2}\frac{R_4}{R_3}-\frac{1}{R_2}\right)U_{iz}(s)=\frac{1}{R_1}\left(\frac{R_4}{R_3}+1\right)U_{ul}(s)$$

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{\frac{1}{R_1}\left(\frac{R_4}{R_3}+1\right)}{sC+\frac{1}{R_1}-\frac{1}{R_2}\frac{R_4}{R_3}}=\frac{0.5(1+1)}{s+0.5-1}=\frac{1}{s-0.5}$$

b) Polovi i nule:
polovi: $s-0.5=0\Rightarrow s_{p1}=0.5$ (pol je realan i nalazi se u desnoj poluravnini); nule: $s_{o1}\rightarrow\infty$



c) Odziv na Step $u_{ul}(t) = S(t) \Rightarrow U_{ul}(s) = \frac{1}{s}$

$$U_{iz}(s) = T(s)U_{ul}(s) = \frac{1}{s-0.5}\frac{1}{s} = \frac{A}{s-0.5} + \frac{B}{s} = \frac{As + Bs - 0.5B}{(s-0.5)s}$$

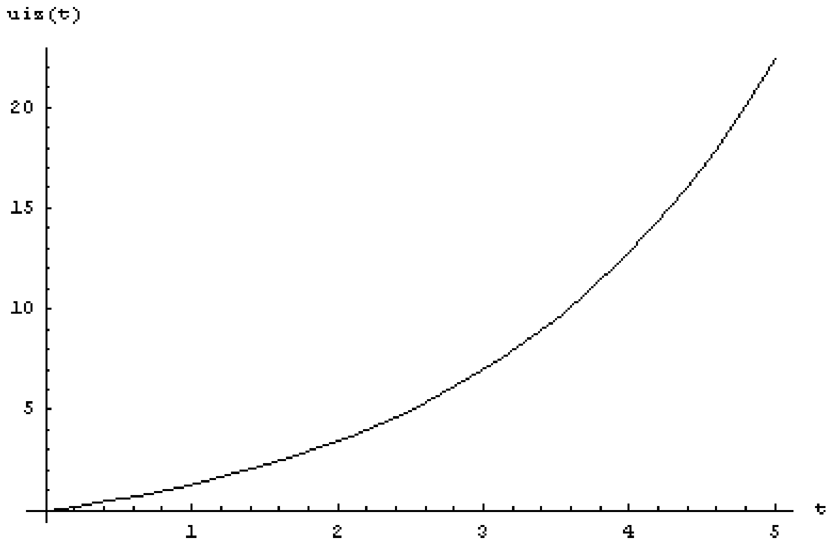
$$A + B = 0 \Rightarrow A = -B = 2$$

$$-0.5B = 1 \Rightarrow B = -2$$

$$U_{iz}(s) = \frac{2}{s-0.5} - \frac{2}{s}$$

$$\underline{u_{iz}(t) = 2e^{0.5t}S(t) - 2S(t) = 2(e^{0.5t} - 1)S(t)}$$

Skica odziva:



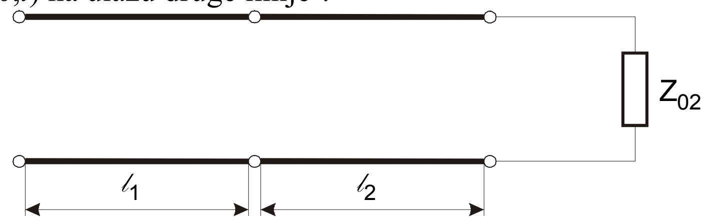
Odgovor: Odziv je raspirujući stoga je električni krug nestabilan.
To je zato jer se pol nalazi u desnoj poluravnini s-ravnine.

5. Na liniju bez gubitaka duljine $l_1=\lambda_1/2$, s primarnim parametrima L_1=2 mH/km i C_1=6 nF/km, priključena je linija bez gubitaka zadana sa L_2=0,6 mH/km i C_2=40 nF/km. Druga linija je zaključena svojom karakterističnom impedancijom.

a) Koliki je faktor refleksije prve linije na spojnom mjestu ?

b) Kolika je amplituda polaznog, a kolika reflektiranog vala na spojnom mjestu, ako je napon na ulazu prve linije $u_I(0,t)=2\cos 10^4 t$?

c) Koliki je napon $u_{II}(0,t)$ na ulazu druge linije ?



Rješenje:

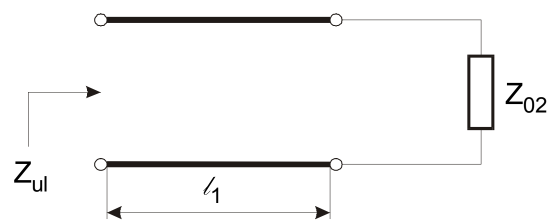
Linija bez gubitaka $\rightarrow$ $R=0$, $G=0$ $\qquad\Rightarrow\qquad$ $Z_0=\sqrt{L/C}$, $\gamma=s\sqrt{LC}$

$$Z_{01}=\sqrt{L_1/C_1}=1/\sqrt{3}\cdot 10^3\,\Omega$$

$$Z_{02}=\sqrt{L_2/C_2}=\sqrt{3/2}\cdot 10^2\,\Omega$$

a)
$$\Gamma=\frac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}}=\frac{\sqrt{3/2}\cdot 10^2-1/\sqrt{3}\cdot 10^3}{\sqrt{3/2}\cdot 10^2+1/\sqrt{3}\cdot 10^3}=\frac{3-10\sqrt{2}}{3+10\sqrt{2}}=-\frac{11.1}{17.1}=-0.65$$

Za $l=\lambda/2 \Rightarrow Z_{ul}=Z_2$



$Z_{ul}=Z_{02}$

$$U_p\left(\frac{\lambda_1}{2}\right)=\frac{U(0)+Z_{01}I(0)}{2}e^{-j\beta_1\frac{\lambda_1}{2}}=\frac{U(0)+Z_{01}I(0)}{2}e^{-j\pi}=$$

b)
$$=\frac{U(0)+Z_{01}\dfrac{U(0)}{Z_{ul}}}{2}e^{-j\pi}=\frac{U(0)}{2}\left(1+\frac{1}{\sqrt{3}}10^3\sqrt{\frac{2}{3}}10^{-2}\right)e^{-j\pi}=\frac{2}{2}\left(1+\frac{10\sqrt{2}}{3}\right)e^{-j\pi}=\underline{5.71405e^{-j\pi}}$$

$u_p(l_1,t)$= 5.71405 ($\cos 10^4 t$– 180°)= – 5.71405 ($\cos 10^4 t$)

$U_r=U_p\cdot(-0.65)$=5.71405·(–0.65)$\underline{=-3.71413}$

$u_r(l_1,t)$= 3.71413 ($\cos 10^4 t$)

c) $u_{II}(0,t)=u_p(l_1,t)+u_r(l_1,t)=U_p(1+\Gamma)$ ($\cos 10^4 t$– 180°)= – 5.71405·0.35 ($\cos 10^4 t$)=

$\underline{u_{II}(0,t)=-2.0\ (\cos 10^4 t)}$