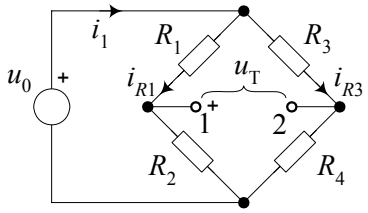
PRVI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA 2008 - Rješenja

1. Za dvopol s priključnicama 1 i 2 primjenom teorema superpozicije odrediti struju $i_1(t)$. Zadane su vrijednosti elemenata: R_1=1kΩ, R_2=750 Ω, R_3=1kΩ i u_0=4V. Odrediti:

a) struju i_1 ako je R_4=1,5 kΩ
b) elemente nadomjesnoga dvopola po Teveninu u_T i R_T
c) vrijednost otpora R_4 za koju je Teveninov napon jednak nuli
d) omjer struja i_{R1} i i_{R3}, kad je u_T=0.



Rješenje:

a) $$i_1 = i_{R1} + i_{R3} = \frac{u_0}{R_1 + R_2} + \frac{u_0}{R_3 + R_4} = \frac{4}{1{,}75 \cdot 10^3} + \frac{4}{2{,}5 \cdot 10^3} = 3{,}88mA$$

b) $$u_T = u_{R2} - u_{R4} = i_{R1}R_2 - i_{R3}R_4 = \frac{u_0 R_2}{R_1 + R_2} - \frac{u_0 R_4}{R_3 + R_4} = 4\left(\frac{3}{7} - \frac{R_4}{10^3 + R_4}\right)$$

$$R_4 = 1{,}5k\Omega \quad \rightarrow \quad u_T = 4\left(\frac{3}{7} - \frac{R_4}{10^3 + R_4}\right) = 4\left(\frac{3}{7} - \frac{3}{5}\right) = -\frac{24}{35} = -0{,}6857\text{V}$$

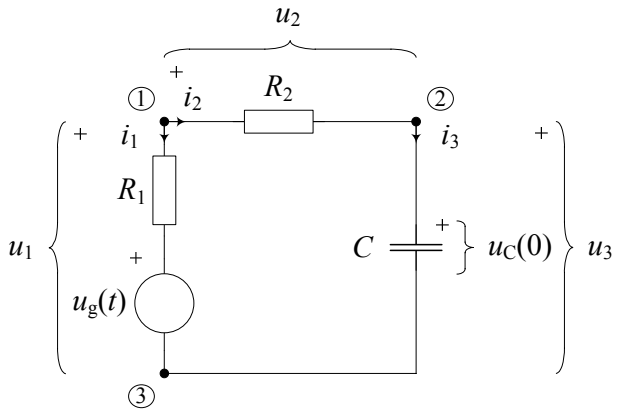
$$R_T = \frac{R_1 \cdot R_2}{R_1 + R_2} + \frac{R_3 \cdot R_4}{R_3 + R_4} = \left(\frac{3}{7} + \frac{3}{5}\right) \cdot 10^3 = 1{,}028k\Omega$$

c) $$u_T = u_0\left(\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4}\right) = 0 \quad \rightarrow \quad \frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} = 0$$

$$\frac{R_2}{R_1 + R_2} - \frac{R_4}{R_3 + R_4} \quad \rightarrow \quad R_4 = \frac{R_2 \cdot R_3}{R_1} = 750\Omega$$

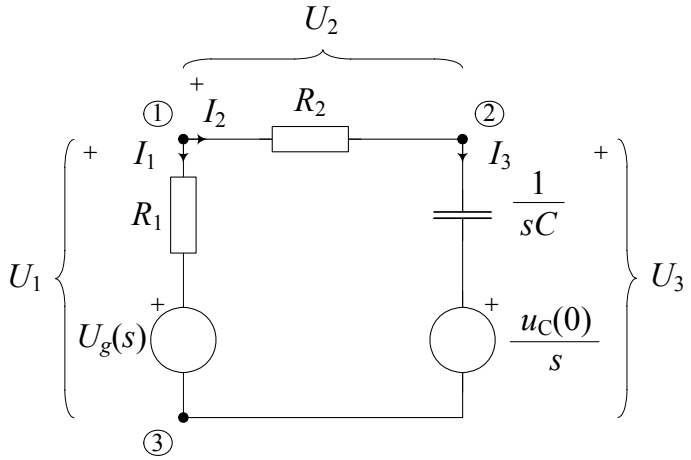
d) $$\frac{i_{R1}}{i_{R3}} = \frac{\dfrac{u_0}{R_1 + R_2}}{\dfrac{u_0}{R_3 + R_4}} = \frac{R_3 + R_4}{R_1 + R_2} = 1$$

2. Za električni krug na slici izračunati struju $i_2(t)$ ako su zadane normalizirane vrijednosti elemenata: $R_1 = R_2 = 1$, $C = 1$ te $u_C(0) = 2$, $u_g(t) = S(t)$. Koristiti metodu KZS i KZN, te oznake grana i čvorova prema slici.



Rješenje:

Primjena $\mathcal{L}$-transformacije na električni krug:



$N_b = 3$ (broj grana)

$N_v = 3$ (broj čvorova)

Broj jednadžbi:

KZS: $N_v - 1 = 3 - 1 = 2$

KZN: $N_b - N_v + 1 = 3 - 3 + 1 = 1$

Jednadžbe KZ (3 jednadžbe):

KZS: 1) $I_1 + I_2 = 0$

2) $I_2 - I_3 = 0$

KZN: 3) $-U_1 + U_2 + U_3 = 0$

Naponsko-strujne jednadžbe grana (3 jednadžbe):

4) $U_1 = I_1 R_1 + U_g$

5) $U_2 = I_2 R_2$

6) $U_3 = I_3 \dfrac{1}{sC} + \dfrac{u_C(0)}{s}$

Ukupno 6 jednadžbi treba riješiti da se izračuna električni krug.
Iz prethodnih 6 jednadžbi slijede jednadžbe:

$$1)\quad -I_1R_1 - U_g + I_2R_2 + I_3\frac{1}{sC} + \frac{u_C(0)}{s} = 0$$

$$2)\quad I_1 + I_2 = 0 \Rightarrow I_2 = -I_1$$

$$3)\quad I_2 - I_3 = 0 \Rightarrow I_2 = I_3$$

$$1)\quad -I_1R_1 + I_2R_2 + I_3\frac{1}{sC} = U_g - \frac{u_C(0)}{s}$$

2), 3) $\rightarrow$ 1)

$$I_2R_1 + I_2R_2 + I_2\frac{1}{sC} = U_g - \frac{u_C(0)}{s}$$

$$I_2\left(R_1 + R_2 + \frac{1}{sC}\right) = U_g - \frac{u_C(0)}{s}$$

$$u_g(t) = S(t) \Rightarrow U_g(s) = \frac{1}{s}$$

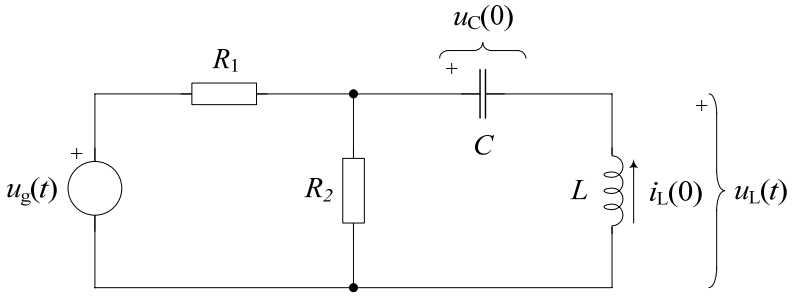
$$u_C(0) = 2$$

$$I_2 = \frac{U_g - \frac{u_C(0)}{s}}{R_1 + R_2 + \frac{1}{sC}} = \frac{\frac{1}{s} - \frac{2}{s}}{1 + 1 + \frac{1}{s}} = \frac{-\frac{1}{s}}{\frac{2s+1}{s}} = -\frac{1}{2s+1} = \frac{-\frac{1}{2}}{s + \frac{1}{2}}$$

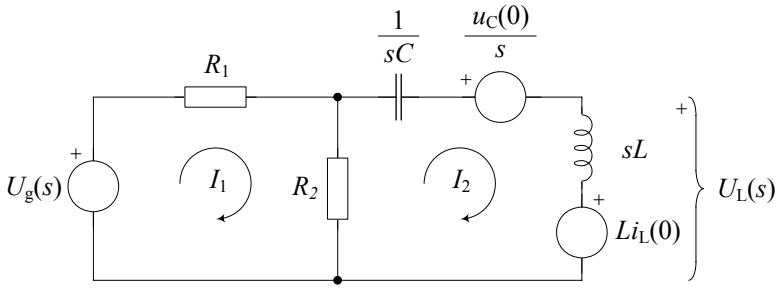
$$i_2(t) = i_3(t) = -i_1(t)$$

$$I_2(s) = \frac{-\frac{1}{2}}{s + \frac{1}{2}} \Rightarrow \mathcal{L}^{-1} \Rightarrow i_2(t) = -\frac{1}{2}e^{-\frac{t}{2}} \cdot S(t)$$

3. Za električni krug na slici izračunati napon na induktivitetu $u_L(t)$ ako su zadane normalizirane vrijednosti elemenata: $R_1 = R_2 = 1$, $L = 1$, $C = 1$ te $i_L(0) = 1$, $u_C(0) = 1$, $u_g(t) = S(t)$. Koristiti metodu konturnih struja.



Rješenje: Primjena $\mathscr{L}$-transformacije na električni krug:



Jednadžbe petlji:

$$1)\ I_1(R_1+R_2)-I_2R_2=U_g(s)$$

$$2)\ -I_1R_2+I_2\left(R_2+sL+\frac{1}{sC}\right)=-Li_L(0)-\frac{u_C(0)}{s}$$

$$U_L(s)=I_2\cdot sL+Li_L(0)$$

$$1)\ 2I_1-I_2=\frac{1}{s}$$

$$2)\ -I_1+I_2\left(1+s+\frac{1}{s}\right)=-1-\frac{1}{s}$$

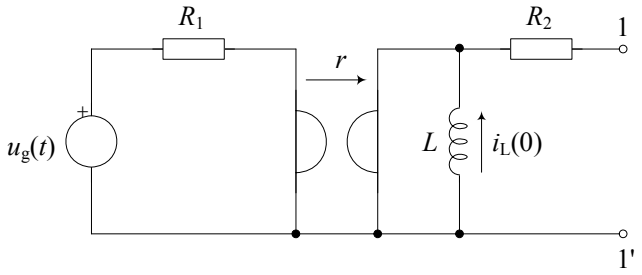
$$I_2(s)=\frac{\begin{vmatrix}2 & 1/s\\ -1 & -1-1/s\end{vmatrix}}{\begin{vmatrix}2 & -1\\ -1 & 1+s+1/s\end{vmatrix}}=\frac{2\left(-1-\frac{1}{s}\right)+\frac{1}{s}}{2+2s+\frac{2}{s}-1}=\frac{-2-\frac{2}{s}+\frac{1}{s}}{1+2s+\frac{2}{s}}=\frac{-2-\frac{1}{s}}{1+2s+\frac{2}{s}}=\frac{-\frac{2s+1}{s}}{\frac{2s^2+s+2}{s}}=-\frac{2s+1}{2s^2+s+2}$$

$$U_L(s)=-\frac{2s+1}{2s^2+s+2}\cdot s+1=-\frac{2s^2+s}{2s^2+s+2}+1=-\frac{2s^2+s+2-2}{2s^2+s+2}+1=-\left(1-\frac{2}{2s^2+s+2}\right)+1=\frac{2}{2s^2+s+2}$$

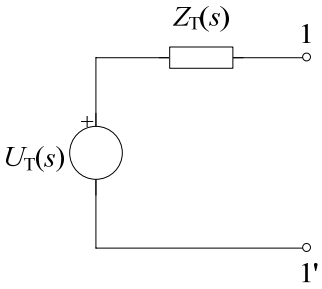
$$U_L(s)=\frac{1}{s^2+\frac{s}{2}+1}=\frac{1}{s^2+\frac{s}{2}+\frac{1}{16}+\frac{15}{16}}=\frac{1}{\left(s+\frac{1}{4}\right)^2+\frac{15}{16}}=\frac{1\cdot\frac{4}{\sqrt{15}}\cdot\frac{\sqrt{15}}{4}}{\left(s+\frac{1}{4}\right)^2+\left(\frac{\sqrt{15}}{4}\right)^2}=\frac{4}{\sqrt{15}}\frac{\frac{\sqrt{15}}{4}}{\left(s+\frac{1}{4}\right)^2+\left(\frac{\sqrt{15}}{4}\right)^2}$$

$$u_L(t)=\frac{4}{\sqrt{15}}e^{-\frac{t}{4}}\sin\left(\frac{\sqrt{15}}{4}t\right)\cdot S(t)$$

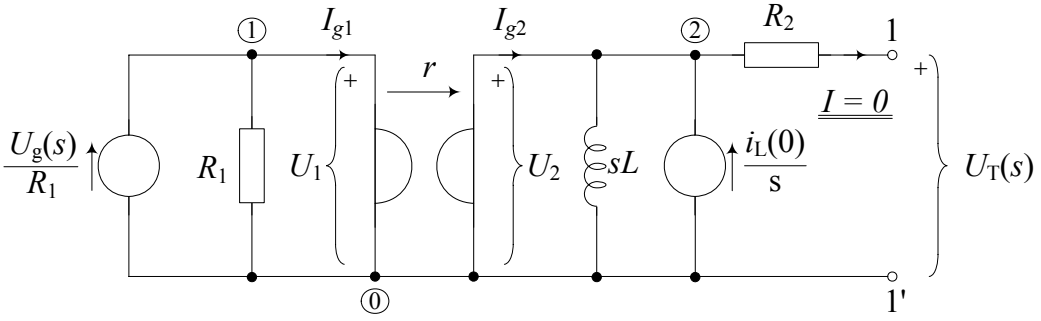
4. Za mrežu na slici izračunati nadomjesne parametre $U_T(s)$ i $Z_T(s)$ nadomjesnog kruga po Theveninu s obzirom na stezaljke 1 – 1'. Zadane su normalizirane vrijednosti elemenata: $R_1 = R_2 = 1$, $L = 1$, $i_L(0) = 1$, $r = 2$, $u_g(t) = S(t)$. Koristiti metodu napona čvorova.



Rješenje:



a) Theveninov napon $U_T(s)$ primjenom $\mathcal{L}$-transformacije na električni krug:



Jednadžbe giratora:

$$U_1 = -r \cdot I_{g2} \Rightarrow I_{g2} = -\frac{U_1}{r}$$

$$U_2 = -r \cdot I_{g1} \Rightarrow I_{g1} = -\frac{U_2}{r}$$

U_1 i U_2 su naponi čvorova ① i ② prema referentnom čvoru ⓪.

1) $U_1 \dfrac{1}{R_1} = \dfrac{U_g}{R_1} - I_{g1}$

2) $U_2 \dfrac{1}{sL} = I_{g2} + \dfrac{i_L(0)}{s}$

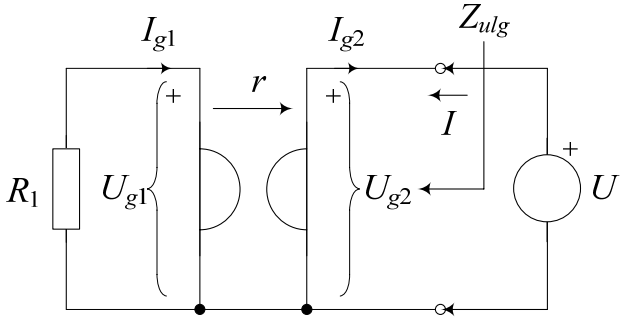
Sredimo jednadžbe:

$$\left.\begin{aligned} U_1 \frac{1}{R_1} &= \frac{U_g}{R_1} + \frac{U_2}{r} \\ U_2 \frac{1}{sL} &= -\frac{U_1}{r} + \frac{i_L(0)}{s} \end{aligned}\right\} \qquad \begin{aligned} U_1 \frac{1}{R_1} - U_2 \frac{1}{r} &= \frac{U_g}{R_1} \\ \frac{U_1}{r} + U_2 \frac{1}{sL} &= \frac{i_L(0)}{s} \end{aligned}$$

$$U_T(s) = U_2(s) = \frac{\begin{vmatrix} \frac{1}{R_1} & \frac{U_g}{R_1} \\ \frac{1}{r} & \frac{i_L(0)}{s} \end{vmatrix}}{\begin{vmatrix} \frac{1}{R_1} & -\frac{1}{r} \\ \frac{1}{r} & \frac{1}{sL} \end{vmatrix}} = \frac{\frac{1}{R_1}\frac{i_L(0)}{s} - \frac{1}{r}\frac{U_g}{R_1}}{\frac{1}{R_1}\frac{1}{sL} + \frac{1}{r^2}} = \frac{\frac{1}{s} - \frac{1}{2s}}{\frac{1}{s} + \frac{1}{4}} = \frac{\frac{1}{2s}}{\frac{1}{s} + \frac{1}{4}} \cdot \frac{4s}{4s} = \frac{2}{s+4}$$

b) Theveninova impedancija $Z_T(s)$:

1. način (pojednostavljen): Tako da se izračuna ulazna impedancija u giratoru zaključenim s R_1 na ulazu. Označimo je s Z_{ulg}. Na izlaz giratora priključimo pomoćni naponski (ili strujni) izvor kao poticaj te izračunamo struju (ili napon) kao odziv. Ulazna impedancija je tada omjer:



$$Z_{u\lg} = \frac{U}{I}$$

$$\Downarrow$$

$$Z_{u\lg} = \frac{r^2}{R_1}$$

$$I_{g1} = -\frac{U_{g1}}{R_1} \Rightarrow \frac{U_{g1}}{I_{g1}} = -R_1$$

$$U = U_{g2}$$

$$I = -I_{g2}$$

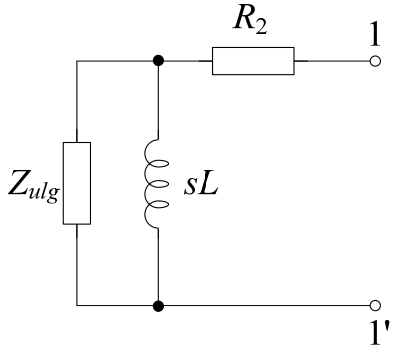
Jednadžbe giratora su:

$$U_{g1} = -r \cdot I_{g2} \Rightarrow I_{g2} = -\frac{U_{g1}}{r}$$

$$U_{g2} = -r \cdot I_{g1} \Rightarrow I_{g1} = -\frac{U_{g2}}{r}$$

$$Z_{u\lg} = \frac{U}{I} = -\frac{U_{g2}}{I_{g2}} = -\frac{-r \cdot I_{g1}}{-\frac{U_{g1}}{r}} = -\frac{r^2}{\frac{U_{g1}}{I_{g1}}} = -\frac{r^2}{-R_1} = \frac{r^2}{R_1}$$

Tada je Teveninova impedancija:



$$\leftarrow Z_T(s) = R_2 + sL \,||\, Z_{u\lg}$$

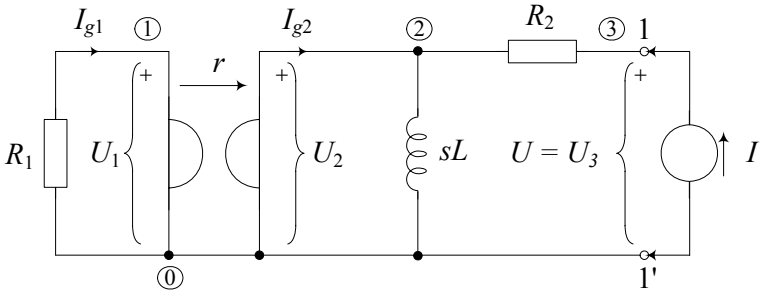
$$Z_T(s) = R_2 + \frac{\frac{r^2}{R_1} \cdot sL}{\frac{r^2}{R_1} + sL} = 1 + \frac{4s}{4+s} = \frac{5s+4}{s+4}$$

2. način: Pomoću metode čvorova.

1) $U_1 \frac{1}{R_1} = -I_{g1}$

2) $U_2 \frac{1}{sL} = I_{g2} + I$

3) $-\frac{U_2}{R_2} + \frac{U_3}{R_2} = I$



Gase se svi neovisni izvori i početni uvjeti. Ne gase se ovisni izvori (ukoliko ih ima). Dodaje se pomoćni strujni izvor *I* kao poticaj, a računa se napon na stezaljkama 1 – 1' kao odziv. Tada je: $Z_T(s) = \frac{U}{I}$

Vrijede jednadžbe giratora:

$$U_{g1} = -r \cdot I_{g2} \Rightarrow I_{g2} = -\frac{U_{g1}}{r} \Rightarrow I_{g2} = -\frac{U_2}{r}$$

$$U_{g2} = -r \cdot I_{g1} \Rightarrow I_{g1} = -\frac{U_{g2}}{r} \Rightarrow I_{g1} = -\frac{U_1}{r}$$

1) $U_1 \frac{1}{R_1} = \frac{U_2}{r}$

2) $U_2 \frac{1}{sL} = -\frac{U_1}{r} + I$

3) $U = U_2 + IR_2$

$$U_1 = U_2 \frac{R_1}{r}$$

$$U_2 = -U_1 \frac{sL}{r} + I \cdot sL = -U_2 \frac{R_1 \cdot sL}{r^2} + I \cdot sL$$

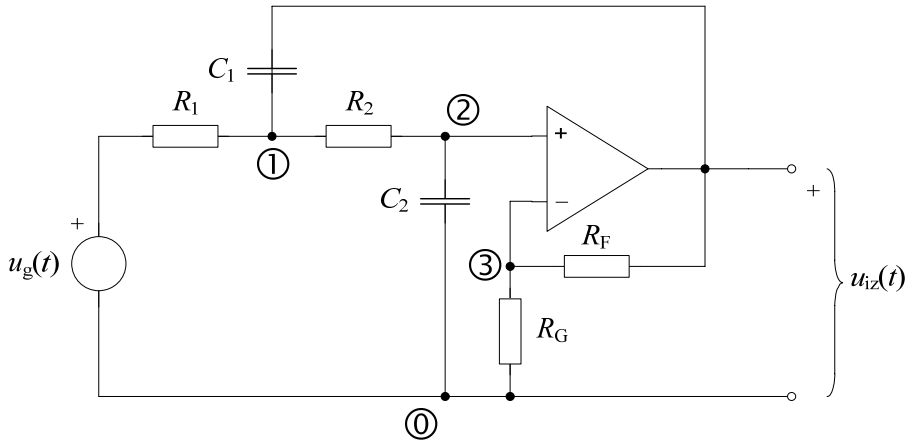
$$U_2 \left(1 + \frac{R_1 \cdot sL}{r^2}\right) = I \cdot sL$$

$$U_2 = I \cdot \frac{sL}{1 + \frac{R_1 \cdot sL}{r^2}} = I \cdot \frac{1}{\frac{1}{sL} + \frac{R_1}{r^2}}$$

$$U = I \cdot \frac{1}{\frac{1}{sL} + \frac{R_1}{r^2}} + IR_2$$

$$Z_T(s) = \frac{U}{I} = \frac{1}{\frac{1}{sL} + \frac{R_1}{r^2}} + R_2 = \frac{r^2 \cdot sL}{r^2 + R_1 \cdot sL} + R_2 = \frac{\frac{r^2}{R_1} \cdot sL}{\frac{r^2}{R_1} + sL} + R_2 = \frac{5s+4}{s+4}$$

5. Zadan je električni krug prema slici. Odrediti $U_{iz}(s)$ ako je zadano: $R_1=R_2=1$, $C_1=C_2=1$, $R_G=R_F=1$, $u_g(t)=S(t)$. Početni uvjeti su jednaki nula.



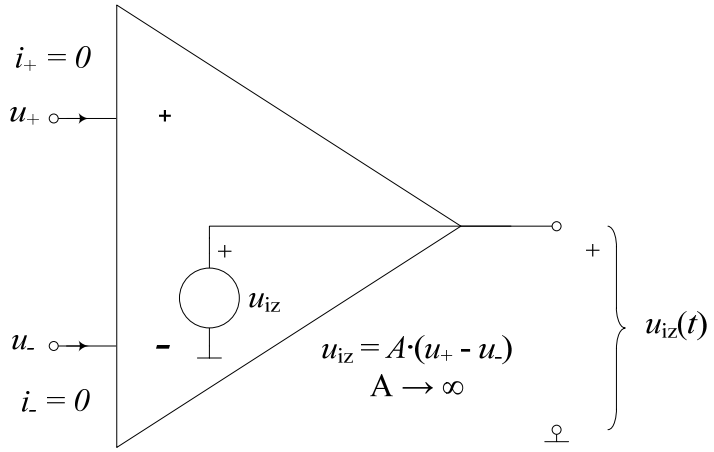
Rješenje: metoda napona čvorova: Postoje tri čvora ①, ② i ③ te referentni čvor ⓪.

$$1)\ U_1\left(\frac{1}{R_1}+\frac{1}{R_2}+sC_1\right)-U_2\frac{1}{R_2}=U_{iz}\cdot sC_1+\frac{U_g}{R_1}$$

$$2)\ -U_1\frac{1}{R_2}+U_2\left(\frac{1}{R_2}+sC_2\right)=0$$

$$3)\ U_3\left(\frac{1}{R_G}+\frac{1}{R_F}\right)=U_{iz}\frac{1}{R_F}$$

Operacijsko pojačalo:



$$U_{iz}=A(U_2-U_3)\ /:A$$

$$\frac{U_{iz}}{A}=U_2-U_3 \quad \Big/ A\to\infty$$

$$\Rightarrow U_2=U_3$$

Između čvorova ② i ③ vlada virtualni kratki spoj, što znači da je između njih napon nula i struja nula. U gore napisane jednadžbe čvorova se stoga uvrštava $U_2=U_3$ i time je utjecaj operacijskog pojačala u potpunosti uzet u proračun. Osim toga vidljivo je da je izlaz operacijskog pojačala, u stvari, (ovisni) naponski izvor pa se za taj čvor ne piše jednadžba.

3) $U_{iz} = U_3\left(1+\frac{R_G}{R_F}\right) = U_2\left(1+\frac{R_G}{R_F}\right)$

2) $U_1 = U_2\left(1+sR_2C_2\right)$

$\downarrow$

1) $U_2\left(1+sR_2C_2\right)\left(\frac{1}{R_1}+\frac{1}{R_2}+sC_1\right)-U_2\frac{1}{R_2} = U_2\left(1+\frac{R_F}{R_G}\right)\cdot sC_1+\frac{U_g}{R_1}$

$$\Rightarrow U_2\left(1+s\left(R_2C_2+R_1C_2-\frac{R_F}{R_G}R_1C_1\right)+s^2R_1R_2C_1C_2\right) = U_g$$

$$\Rightarrow U_{iz} = U_2\left(1+\frac{R_F}{R_G}\right) = \frac{U_g\left(1+\frac{R_F}{R_G}\right)}{1+s\left(R_2C_2+R_1C_2-\frac{R_F}{R_G}R_1C_1\right)+s^2R_1R_2C_1C_2}$$

Uvrstimo vrijednosti:

$$\Rightarrow U_{iz}(s) = \frac{2}{1+s+s^2}\cdot\frac{1}{s}$$