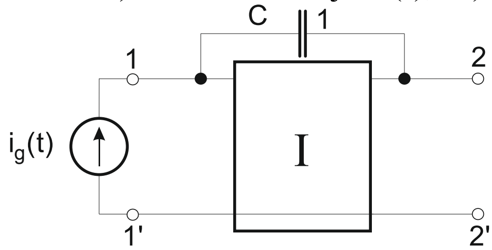
ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA 2015-2016 – Rješenja

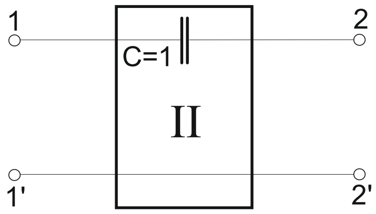
1. Mreža na slici predstavlja paralelni spoj dvaju četveropola. Ako je jedan od njih četveropol I: *a*) nacrtati drugi četveropol. *b*) odrediti *y*-parametre drugog četveropola; *c*) odrediti *y*-parametre paralelne kombinacije dva četveropola. Obzirom na 2–2' odrediti: *d*) Nortonovu struju $I_N(s)$; i e) Nortonovu admitanciju $Y_N(s)$.

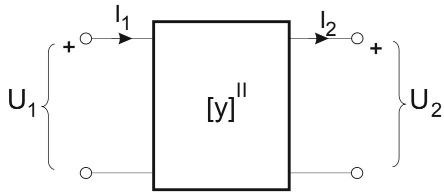
$$[z]=\begin{bmatrix} 2s+1 & -2s \\ 2s & -2s \end{bmatrix}$$



Rješenje:

a) drugi četveropol II:





(1 bod)

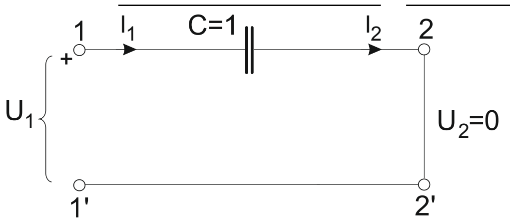
b) *y*-parametri drugog četveropola. Naponske jednadžbe za četveropol uz referentne oznake struja i napona:

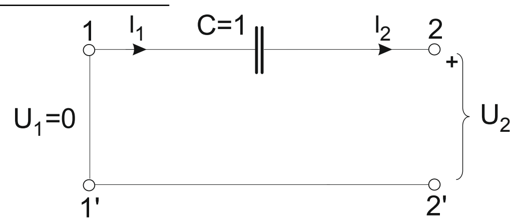
$$I_1 = y_{11}\cdot U_1 - y_{12}\cdot U_2$$
$$I_2 = y_{21}\cdot U_1 - y_{22}\cdot U_2$$

$$U_2 = 0, I_2 = I_1 \Rightarrow \quad y_{11} = I_1/U_1 = sC = s \quad y_{21} = I_2/U_1 = sC = s$$

$$U_1 = 0, I_2 = I_1 \Rightarrow \quad y_{12} = -I_1/U_2 = sC = s \quad y_{22} = -I_2/U_2 = sC = s$$

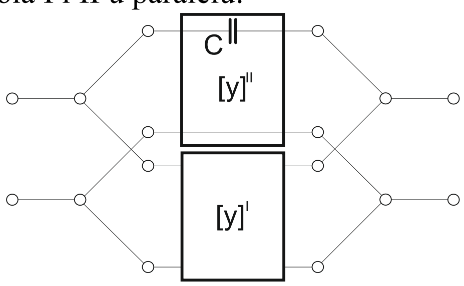
$$\Rightarrow [y]^{II} = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix} = \begin{bmatrix} s & -s \\ s & -s \end{bmatrix}$$





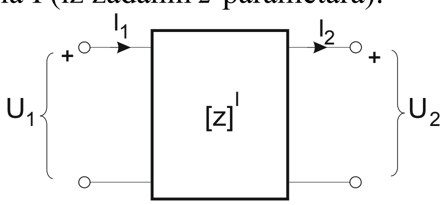
(1 bod)

c) Možemo promatrati dva četveropola I i II u paralelu:



Najprije nađimo *y*-parametre četveropola I (iz zadanih *z*-parametara):

$$U_1 = z_{11}\cdot I_1 - z_{12}\cdot I_2$$
$$U_2 = z_{21}\cdot I_1 - z_{22}\cdot I_2$$

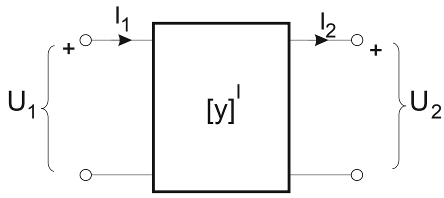


$$U_1 = (2s+1)\cdot I_1 - 2s\cdot I_2$$
$$U_2 = 2s\cdot I_1 - 2s\cdot I_2 \Rightarrow I_2 = I_1 - \frac{1}{2s}U_2$$

$$U_1 = (2s+1)\cdot I_1 - 2s\cdot\left(I_1 - \frac{1}{2s}\cdot U_2\right)$$
$$U_1 = (2s+1)\cdot I_1 - 2s\cdot I_1 + U_2$$

$$U_1 = 1\Omega\cdot I_1 + U_2$$
$$I_1 = \frac{1}{1\Omega}U_1 - \frac{1}{1\Omega}U_2$$

$$I_2=\frac{1}{1\Omega}U_1-\frac{1}{1\Omega}U_2-\frac{1}{2s}\cdot U_2$$
$$\underline{I_2=\frac{1}{1\Omega}U_1-U_2\left(1+\frac{1}{2s}\right)}$$



$$[y]^I=\begin{bmatrix}1 & -1\\ 1 & -\left(1+\dfrac{1}{2s}\right)\end{bmatrix}$$

Ili na drugi način: $[y]=[z]^{-1}$

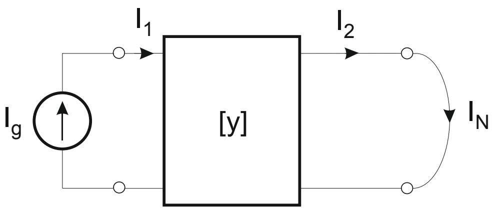
$$y_{11}=\frac{z_{22}}{|\Delta z|}=\frac{2s}{2s}=1;\quad y_{12}=\frac{z_{12}}{|\Delta z|}=\frac{2s}{2s}=1;\quad y_{21}=\frac{z_{21}}{|\Delta z|}=\frac{2s}{2s}=1;\quad y_{22}=\frac{z_{11}}{|\Delta z|}=\frac{2s+1}{2s}=1+\frac{1}{2s}$$

$|\Delta z|=(2s+1)\cdot 2s-4s^2=2s$ (Dobije se isti rezultat.)

Sada slijede y-parametri dva četveropola u paralelu:

$$[y]=[y]^I+[y]^{II}=\begin{bmatrix}1 & -1\\ 1 & -\left(1+\dfrac{1}{2s}\right)\end{bmatrix}+\begin{bmatrix}s & -s\\ s & -s\end{bmatrix}=\begin{bmatrix}1+s & -(1+s)\\ 1+s & -\left(1+s+\dfrac{1}{2s}\right)\end{bmatrix}$$ (1 bod)

d) Nortonova struja $I_N(s)$ iz y-parametara:



$$I_1=y_{11}\cdot U_1-y_{12}\cdot U_2$$
$$\underline{I_2=y_{21}\cdot U_1-y_{22}\cdot U_2}$$

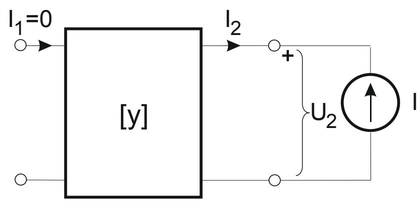
$i_g(t)=\delta(t)\ \Rightarrow\ I_g(s)=1,\ I_g=I_1;\ I_N=I_2$

$U_2=0$ kratki spoj na priključnicama 2–2' kroz koje teče Nortonova struja

$$\begin{aligned}I_1&=y_{11}\cdot U_1\\ \underline{I_2}&\underline{=y_{21}\cdot U_1}\end{aligned}\quad \frac{I_2}{I_1}=\frac{y_{21}}{y_{11}}\ \Rightarrow\ I_2=\frac{y_{21}}{y_{11}}\cdot I_1=\frac{1+s}{1+s}\cdot 1=1\ \Rightarrow\ I_N(s)=1$$ (1 bod)

Drugi način je upotrijebiti izraz : $H_i(s)=\dfrac{I_2}{I_1}=\dfrac{Y_L y_{21}}{\Delta y+y_{11}Y_L}\bigg|_{Y_L=\infty}=\dfrac{y_{21}}{y_{11}}$ što će dati isti rezultat.

e) Nortonova admitancija $Y_N(s)$ iz y-parametara :



$I_1=0$ prazni hod spoj na priključnicama 1–1' jer je isključen strujni izvor

$$I_1=-I_2;\quad Y_N=\frac{I}{U_2}=\frac{-I_2}{U_2};\ \Rightarrow\ \begin{aligned}0&=y_{11}\cdot U_1-y_{12}\cdot U_2\\ \underline{-I}&\underline{=y_{21}\cdot U_1-y_{22}\cdot U_2}\end{aligned}\quad \Rightarrow y_{11}\cdot U_1=y_{12}\cdot U_2\Rightarrow U_1=\left(y_{12}/y_{11}\right)\cdot U_2$$

$$\Rightarrow -I=y_{21}\cdot\frac{y_{12}}{y_{11}}U_2-y_{22}\cdot U_2=\frac{y_{12}y_{21}-y_{11}y_{22}}{y_{11}}\cdot U_2\Rightarrow Y_N=\frac{I}{U_2}=\frac{y_{11}y_{22}-y_{12}y_{21}}{y_{11}}=\frac{\Delta y}{y_{11}};\Delta y=y_{11}y_{22}-y_{12}y_{21}$$

$$\Delta y=(1+s)\left(1+s+\frac{1}{2s}\right)-(1+s)^2=(1+s)^2+\frac{1+s}{2s}-(1+s)^2=\frac{1+s}{2s};\quad Y_N(s)=\frac{\Delta y}{y_{11}}=\frac{(1+s)/(2s)}{1+s}=\frac{1}{2s}$$ (1 bod)

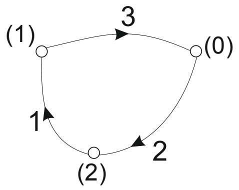
Drugi način je upotrijebiti izraz : $Y_{ul2}(s)=y_{22}-\dfrac{y_{12}y_{21}}{y_{11}+Y_1}\bigg|_{Y_1=0}=\dfrac{y_{11}y_{22}-y_{12}y_{21}}{y_{11}}$ što će dati isti rezultat.

2. Električni krug opisan je matricom incidencija **A**, matricom admitancija grana $\mathbf{Y}_b$, i vektorom strujnih izvora grana $\mathbf{I}_{0b}$. Nacrtati: a) pripadni orijentirani graf i b) električnu mrežu. c) Napisati sustav jednadžbi napona čvorova u matričnom obliku za dobivenu mrežu, d) odrediti matrice admitancija čvorova $\mathbf{Y}_v$ i e) vektor strujnih izvora u čvorovima $\mathbf{I}_{0v}$.

$$\mathbf{Y}_b = \begin{bmatrix} sC_1 & 0 & 0 \\ 0 & sC_2 & 0 \\ rGsC_1 & 0 & G \end{bmatrix},\quad \mathbf{I}_{0b} = \begin{bmatrix} U_g sC_1 + C_1 u_{C1}(0) \\ 0 \\ rGU_g sC_1 + rGC_1 u_{C1}(0) \end{bmatrix},\quad \mathbf{A} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

<u>Rješenje:</u>

a) Orijentirani graf slijedi iz matrice incidencija (reducirane) $\mathbf{A} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$



(1 bod)

b) Električna mreža: treba napisati strujno-naponske jednadžbe grana

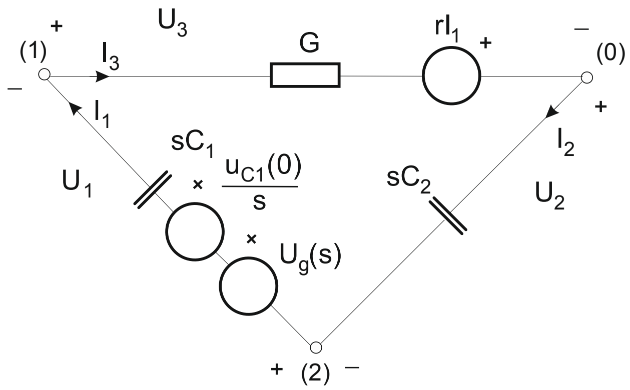
$$\mathbf{I}_b = \mathbf{Y}_b \cdot \mathbf{U}_b + \mathbf{I}_{0b}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} sC_1 & 0 & 0 \\ 0 & sC_2 & 0 \\ rGsC_1 & 0 & G \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix} + \begin{bmatrix} U_g sC_1 + C_1 u_{C1}(0) \\ 0 \\ rGU_g sC_1 + rGC_1 u_{C1}(0) \end{bmatrix}$$

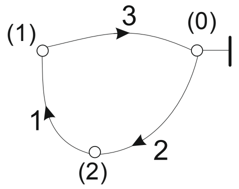
$$I_1 = U_1 \cdot sC_1 + U_g \cdot sC_1 + C_1 \cdot u_{C1}(0)$$
$$I_2 = U_2 \cdot sC_2$$
$$I_3 = U_1 rG \cdot sC_1 + U_3 G + rGU_g \cdot sC_1 + rGC_1 \cdot u_{C1}(0)$$

$$I_1 = \left[U_1 + U_g + \frac{u_{C1}(0)}{s} \right] \cdot sC_1$$
$$I_2 = U_2 \cdot sC_2$$
$$I_3 = U_3 G + rG \cdot sC_1 \left[U_1 + U_g + \frac{u_{C1}(0)}{s} \right] = (U_3 + rI_1) \cdot G$$

$G = \dfrac{1}{R}$ je vodljivost, odnosno otpor veličine $R = \dfrac{1}{G}$ je u mreži:



Orijentirani graf (čvor 0 je referentan):



c) Napisati sustav jednadžbi napona čvorova u matričnom obliku za dobivenu mrežu:
Sustav jednadžbi čvorova glasi: $\mathbf{Y}_v \cdot \mathbf{U}_v = \mathbf{I}_v$ rješenje vektor napona čvorišta $\mathbf{U}_v$.
b) + c) (2 boda)

d) matrica admitancija čvorova $\mathbf{Y}_v$:

$$\mathbf{Y}_v = \mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{A}^T = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} sC_1 & 0 & 0 \\ 0 & sC_2 & 0 \\ rGsC_1 & 0 & G \end{bmatrix} \cdot \begin{bmatrix} -1 & 1 \\ 0 & -1 \\ 1 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} -sC_1 + rGsC_1 & 0 & G \\ sC_1 & -sC_2 & 0 \end{bmatrix} \cdot \begin{bmatrix} -1 & 1 \\ 0 & -1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} sC_1 - rGsC_1 + G & -sC_1 + rGsC_1 \\ -sC_1 & sC_1 + sC_2 \end{bmatrix}$$

(1 bod)

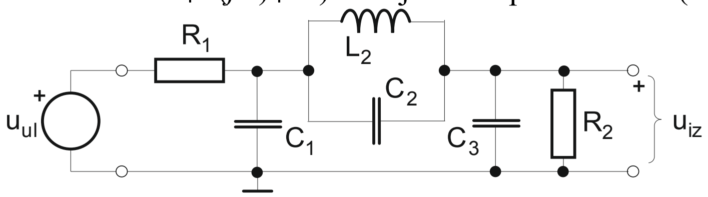
e) vektor strujnih izvora u čvorovima $\mathbf{I}_{0v}$:

$$\mathbf{I}_{0v} = -\mathbf{A} \cdot \mathbf{I}_{0b} = -\begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} \cdot \begin{bmatrix} U_g sC_1 + C_1 u_{C1}(0) \\ 0 \\ rGU_g sC_1 + rGC_1 u_{C1}(0) \end{bmatrix} =$$

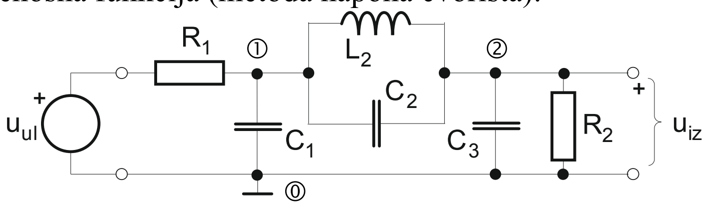
$$= \begin{bmatrix} U_g sC_1 + C_1 u_{C1}(0) - rGU_g sC_1 - rGC_1 u_{C1}(0) \\ -U_g sC_1 - C_1 u_{C1}(0) \end{bmatrix} = \begin{bmatrix} (1 - rG)(U_g sC_1 + C_1 u_{C1}(0)) \\ -U_g sC_1 - C_1 u_{C1}(0) \end{bmatrix}$$

(1 bod)

3. Za električni krug prikazan slikom odrediti: a) naponsku prijenosnu funkciju $H(s)=U_{iz}(s)/U_{ul}(s)$ kao funkciju varijable „s" i elemenata (R_i, L_i i C_i). b) Uvrstiti normalizirane vrijednosti elemenata: $R_1=1$, $R_2=1$, $C_1=1$, $C_2=1$, $C_3=1$, $L_2=1$. c) Prikazati raspored polova i nula u kompleksnoj ravnini. d) Izračunati i skicirati amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$. e) O kojem se tipu filtra radi (NP, VP, PP ili PB)?



Rješenje: a) Naponska prijenosna funkcija (metoda napona čvorišta):



$$(1)\ U_1\left(\frac{1}{R_1}+sC_1+sC_2+\frac{1}{sL_2}\right)-U_2\left(sC_2+\frac{1}{sL_2}\right)=U_{ul}\frac{1}{R_1}$$

$$(2)\ -U_1\left(sC_2+\frac{1}{sL_2}\right)+U_2\left(sC_2+\frac{1}{sL_2}+sC_3+\frac{1}{R_2}\right)=0$$

$$U_2=U_{iz}$$

$$(2)\Rightarrow U_1=U_2\frac{sC_2+\frac{1}{sL_2}+sC_3+\frac{1}{R_2}}{sC_2+\frac{1}{sL_2}}\rightarrow(1)\Rightarrow$$

$$U_2\frac{sC_2+\frac{1}{sL_2}+sC_3+\frac{1}{R_2}}{sC_2+\frac{1}{sL_2}}\left(\frac{1}{R_1}+sC_1+sC_2+\frac{1}{sL_2}\right)-U_2\left(sC_2+\frac{1}{sL_2}\right)=U_{ul}\frac{1}{R_1}\Big/\cdot\left(sC_2+\frac{1}{sL_2}\right)$$

$$U_2\left[s(C_2+C_3)+\frac{1}{sL_2}+\frac{1}{R_2}\right]\left[s(C_1+C_2)+\frac{1}{sL_2}+\frac{1}{R_1}\right]-U_2\left(sC_2+\frac{1}{sL_2}\right)^2=U_{ul}\frac{1}{R_1}\left(sC_2+\frac{1}{sL_2}\right)$$

$$U_{iz}\left[s^2(C_1+C_2)(C_2+C_3)+\frac{C_1+C_2}{L_2}+s\frac{C_1+C_2}{R_2}+\frac{C_2+C_3}{L_2}+\frac{1}{s^2L_2^2}+\frac{1}{sL_2R_2}+s\frac{C_2+C_3}{R_1}+\right.$$

$$\left.+\frac{1}{sL_2R_1}+\frac{1}{R_1R_2}-s^2C_2^2-2\frac{C_2}{L_2}-\frac{1}{s^2L_2^2}\right]=U_{ul}\frac{1}{R_1}\left(sC_2+\frac{1}{sL_2}\right)$$

$$U_{iz}\left[s^2(C_1C_2+C_1C_3+C_2C_3)+\frac{C_1+C_3}{L_2}+s\frac{C_1+C_2}{R_2}+\frac{1}{sL_2R_2}+s\frac{C_2+C_3}{R_1}+\frac{1}{sL_2R_1}+\frac{1}{R_1R_2}\right]=$$

$$=U_{ul}\frac{1}{R_1}\left(sC_2+\frac{1}{sL_2}\right)\Big/\cdot sL_2$$

$$U_{iz}\left[s^3L_2(C_1C_2+C_1C_3+C_2C_3)+s(C_1+C_3)+s^2L_2\frac{C_1+C_2}{R_2}+\frac{1}{R_2}+s^2L_2\frac{C_2+C_3}{R_1}+\frac{1}{R_1}+\frac{sL_2}{R_1R_2}\right]=$$

$$=U_{ul}\frac{1}{R_1}\left(s^2L_2C_2+1\right)$$

$$T(s)=\frac{\frac{1}{R_1}\left(s^2L_2C_2+1\right)}{s^3L_2(C_1C_2+C_1C_3+C_2C_3)+s^2L_2\left(\frac{C_1+C_2}{R_2}+\frac{C_2+C_3}{R_1}\right)+s\left(\frac{L_2}{R_1R_2}+C_1+C_3\right)+\frac{1}{R_1}+\frac{1}{R_2}}$$

Dobiveni izraz se može još urediti i prikazati na razne načine:

$$T(s)=\frac{R_2\left(s^2L_2C_2+1\right)}{s^3L_2R_1R_2(C_1C_2+C_1C_3+C_2C_3)+s^2L_2\left[R_1(C_1+C_2)+R_2(C_2+C_3)\right]+s\left[L_2+R_1R_2(C_1+C_3)\right]+R_1+R_2}\quad \text{ili}$$

$$T(s)=\frac{\frac{1}{L_2R_1(C_1C_2+C_1C_3+C_2C_3)}\left(s^2L_2C_2+1\right)}{s^3+s^2\frac{R_1(C_1+C_2)+R_2(C_2+C_3)}{R_1R_2(C_1C_2+C_1C_3+C_2C_3)}+s\frac{L_2+R_1R_2(C_1+C_3)}{L_2R_1R_2(C_1C_2+C_1C_3+C_2C_3)}+\frac{R_1+R_2}{L_2R_1R_2(C_1C_2+C_1C_3+C_2C_3)}},\quad \text{itd.}$$

b) Uvrštene normalizirane vrijednosti elemenata: R_1=1, R_2=1, C_1=1, C_2=1, C_3=1, L_2=1:

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{s^2+1}{3s^3+4s^2+3s+2}$$ a) + b) (3 boda)

c) Raspored polova i nula u kompleksnoj ravnini:

-polovi: $3s^3+4s^2+3s+2=3\left(s^3+\frac{4}{3}s^2+s+\frac{2}{3}\right)=0$

Pretpostavimo vrijednost pola: $s_{p1}=-1$

$$s_{p1}=-1\Rightarrow 3(-1)^3+4(-1)^2+3(-1)+2=0\Rightarrow -3+4-3+2=0\ \text{DA}$$

Podijelimo:

$$\left(3s^3+4s^2+3s+2\right):(s+1)=3s^2+\frac{s^2+3s+2}{s+1}$$
$$\underline{-3s^3-3s^2}$$
$$=s^2+3s+2$$

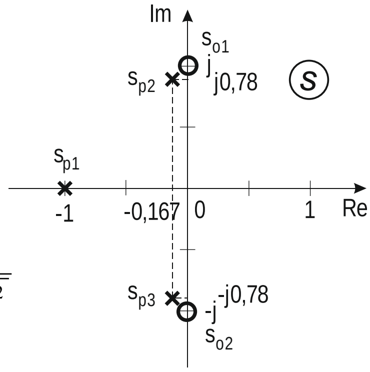
$$\left(s^2+3s+2\right):(s+1)=s+\frac{2s+2}{s+1}$$
$$\underline{-s^2-s}$$
$$=2s+2$$

$$\left(2s+2\right):(s+1)=2$$
$$\underline{-2s-2}$$
$$=0$$

Polovi: $3s^3+4s^2+3s+2=(s+1)(3s^2+s+2)=0$

$$3s^2+s+2=0\ \Rightarrow\ s_{p2,3}=\frac{-1\pm\sqrt{1-24}}{2\cdot 3}=\frac{-1\pm j\sqrt{23}}{6}=-0{,}16667\pm j0{,}799305$$

-nule: $s^2+1=0\ \Rightarrow s_{o1,2}=\pm j$ (1 bod)



d) amplitudno-frekvencijska karakteristika:

$$T(j\omega)=\frac{-\omega^2+1}{-3j\omega^3-4\omega^2+3j\omega+2}\ \Rightarrow\qquad \left|T(j\omega)\right|=\frac{\left|1-\omega^2\right|}{\sqrt{\left(2-4\omega^2\right)^2+\left(3\omega-3\omega^3\right)^2}}$$

Ako uvrstimo tri karakteristične točke:

s=0 u $T(s)\Rightarrow\ T(0)=\left|T(j\omega)\right|_{\omega=0}=\frac{1}{2}$; s=1 $\Rightarrow\ T(1)=\left|T(j\omega)\right|_{\omega=1}=0$; s=→∞ u $T(s)\Rightarrow\ T(\infty)=\left|T(j\omega)\right|_{\omega\to\infty}=0$

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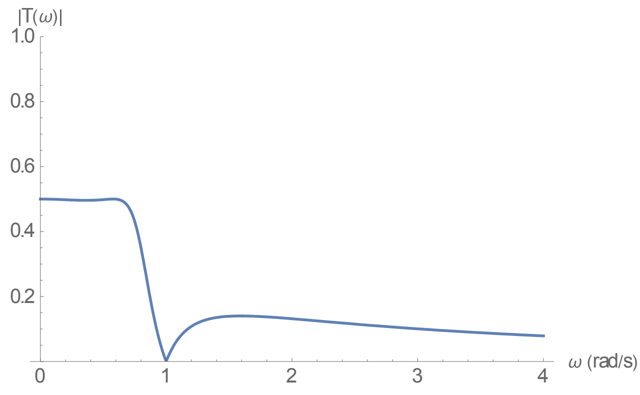
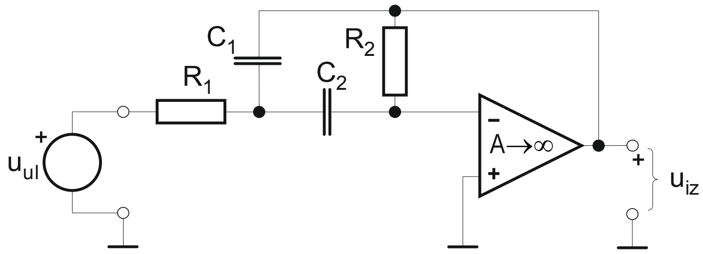


e) O kojem se tipu filtra radi (NP, VP, PP ili PB)?

Točna su dva odgovora: NP- niski propust (NP Notch) ili PB-pojasna brana d) + e) (1 bod)

4. Za pojasno-propusni električni filtar prikazan slikom zadana je naponska prijenosna funkcija $T(s)=U_{iz}(s)/U_{ul}(s)$. a) Usporedbom s općim oblikom PP prijenosne funkcije filtra 2. stupnja odrediti parametre k, ω_p, q_p kao funkcije elemenata filtra. b) Ako su zadane normalizirane vrijednosti parametara $\omega_p=1$ i $q_p=0{,}7071068$ te ako je $C_1=C_2=1$, izračunati normalizirane vrijednosti otpora R_1 i R_2 i pojačanje k. c) Prikazati raspored polova i nula u kompleksnoj ravnini. d) Izračunati i skicirati amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$. e) Izračunati denormirane elemente filtra za frekvenciju normalizacije $\omega_0=10^3$ rad/s i otpor R_0=1kΩ.



$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=-\frac{sR_2C_2}{s^2R_1C_1R_2C_2+s\left(R_1C_1+R_1C_2\right)+1}$$

<u>Rješenje:</u>

a) Parametri

Napišemo prijenosnu funkciju tako da je koeficijent uz najveću potenciju od s (s^2) jediničan.

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=-\frac{s\dfrac{1}{R_1C_1}}{s^2+s\dfrac{R_1C_1+R_1C_2}{R_1C_1R_2C_2}+\dfrac{1}{R_1C_1R_2C_2}}$$

Usporedba s općim oblikom:

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=k\cdot\frac{\dfrac{\omega_p}{q_p}s}{s^2+\dfrac{\omega_p}{q_p}s+\omega_p^2}$$

$$\omega_p^2=\frac{1}{R_1R_2C_1C_2}\;\Rightarrow\;\omega_p=\frac{1}{\sqrt{R_1R_2C_1C_2}}$$

$$\frac{\omega_p}{q_p}=\frac{R_1(C_1+C_2)}{R_1R_2C_1C_2}\;\Rightarrow\;q_p=\omega_p\frac{R_1R_2C_1C_2}{R_1(C_1+C_2)}=\frac{1}{\sqrt{R_1R_2C_1C_2}}\cdot\frac{R_1R_2C_1C_2}{R_1(C_1+C_2)}=\frac{\sqrt{R_1R_2C_1C_2}}{R_1(C_1+C_2)}$$

$$k\cdot\frac{\omega_p}{q_p}=\frac{1}{R_1C_1}\;\Rightarrow\;k=\frac{1}{R_1C_1}\cdot\frac{q_p}{\omega_p}=\frac{1}{R_1C_1}\cdot\frac{R_1C_1R_2C_2}{R_1(C_1+C_2)}=\frac{R_2C_2}{R_1(C_1+C_2)}$$

(1 bod)

b) proračun: u proračunu smo pretpostavili $C_1=C_2=C$ pa će izrazi za ω_p, q_p i k iz točke *a)* poprimiti jednostavniji oblik:

$$\omega_p=\frac{1}{C\sqrt{R_1R_2}},\quad q_p=\frac{1}{2}\sqrt{\frac{R_2}{R_1}},\quad k=\frac{R_2}{2R_1},$$

Izračun jednadžbi iz uvjeta za ω_p i q_p:

Iz $q_p=\dfrac{1}{\sqrt2}$ slijedi $\dfrac{1}{\sqrt2}=\dfrac{1}{2}\sqrt{\dfrac{R_2}{R_1}}\Rightarrow\dfrac{2}{\sqrt2}=\sqrt{\dfrac{R_2}{R_1}}\Rightarrow 2=\dfrac{R_2}{R_1}\Rightarrow R_2=2R_1$

Iz $\omega_p=1$ slijedi $1=\dfrac{1}{C\sqrt{R_1R_2}}\Rightarrow C\sqrt{R_1R_2}=1\Rightarrow C\sqrt{2R_1^2}=1\Rightarrow R_1=\dfrac{1}{C\sqrt2}$

Proračun elemenata: <u>Uz odabir C=1 je C_1=1, C_2=1 i računamo</u>:

$R_1=\dfrac{1}{\sqrt2}$, $R_2=\dfrac{2}{\sqrt2}=\sqrt2$.

Pojačanje u području propuštanja iznosi: $k=\dfrac{R_2}{2R_1}=\dfrac{\sqrt{2}}{2\dfrac{1}{\sqrt{2}}}=\dfrac{2}{2}=1$

(1 bod)

c) raspored polova i nula u kompleksnoj ravnini:

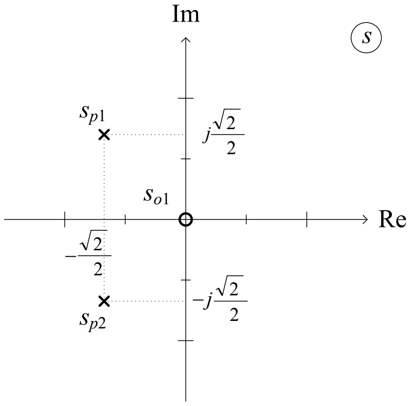
$$H_{PP}(s)=k\frac{(\omega_p/q_p)s}{s^2+(\omega_p/q_p)s+\omega_p^2}=\frac{\sqrt{2}s}{s^2+\sqrt{2}s+1}$$

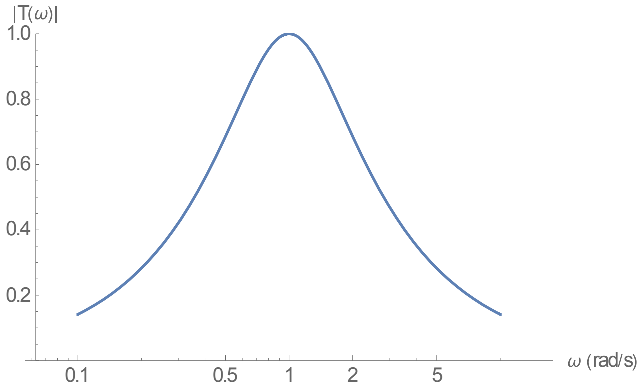
nule $\quad s_{o1}=0,\ s_{o2}=\infty$

polovi $\ s^2+\sqrt{2}\cdot s+1=0 \quad\Rightarrow\quad s_{p1,2}=-\dfrac{\sqrt{2}}{2}\pm\sqrt{\dfrac{2}{4}-1}=-\dfrac{\sqrt{2}}{2}\pm j\dfrac{\sqrt{2}}{2}$ (1 bod)

d) amplitudno-frekvencijska karakteristika:

$$T(j\omega)=\frac{-\omega\cdot\sqrt{2}}{-\omega^2+j\omega\cdot\sqrt{2}+1}\quad\Rightarrow\quad |T(j\omega)|=\frac{\left|\sqrt{2}\cdot\omega\right|}{\sqrt{(1-\omega^2)^2+\left(\omega\cdot\sqrt{2}\right)^2}}=\frac{\sqrt{2}\cdot\omega}{\sqrt{1+\omega^4}}$$ (1 bod)





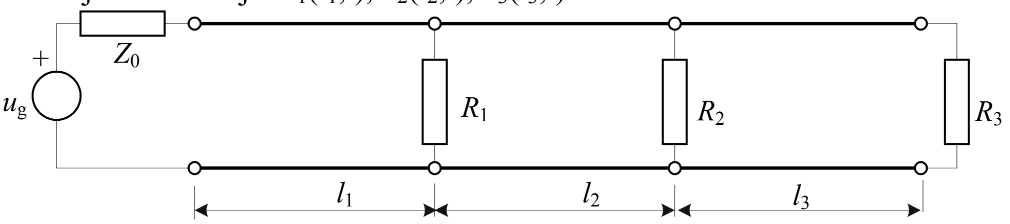
e) Denormalizacija elemenata po $\omega_0=10^3$ rad/s i R_0=1kΩ:

normalizirani elementi	izrazi za denormalizaciju	denormalizirani elementi
$R_1=1/\sqrt{2}$	$R=R_0\cdot R_n;$	$R_1=R_0\cdot 0{,}7071=707{,}1\Omega$
$R_2=\sqrt{2}$	$C=\dfrac{C_n}{\omega_0\cdot R_0};$	$R_2=R_0\cdot 1{,}4142=1414{,}2\Omega$
$C_1=1$	$L=\dfrac{L_n\cdot R_0}{\omega_0}$	$C_1=\dfrac{1}{\omega_0 R_0}\cdot 1=C_0\cdot 1=1\mu\text{F}$
$C_2=1$		$C_2=C_0\cdot 1=\mu\text{F}$

(1 bod)

5. Tri linije bez gubitaka spojene su u kaskadu prema slici. Zadano je: L=0,25mH/km, C=100nF/km, u_g=10 cos(2.5π 10^5 t) V, R_2 =150Ω, R_3=25Ω, l_1=3λ/4, l_2=λ/2 i l_3=λ/4. Odrediti:

a) valnu impedanciju i koeficijent prijenosa linija;
b) brzinu širenja vala na linijama i duljinu druge i treće linije;
c) otpor R_1 da bi prva linija bila prilagođena na izlazu;
d) faktore refleksije na krajevima druge i treće linije: Γ_{i2} i Γ_{i3};
e) napone na kraju svake linije: $u_1(l_1,t)$, $u_2(l_2,t)$, $u_3(l_3,t)$.



Rješenje:

a) $Z_0=\sqrt{\dfrac{L}{C}}=\sqrt{\dfrac{2,5\cdot 10^{-4}}{10^{-7}}}=50\Omega$ $\gamma=j\beta=j\omega_0\sqrt{LC}=j2,5\cdot\pi\cdot 10^5\sqrt{0,25\cdot 10^{-10}}=\dfrac{j5}{4}\pi$ [rad/km]

(1 bod)

b) $v=\dfrac{\omega}{\beta}=\dfrac{1}{\sqrt{LC}}=\dfrac{1}{\sqrt{0,25\cdot 10^{-10}}}=2\cdot 10^5$ [km/s] $\qquad \lambda=\dfrac{2\cdot\pi}{\beta}=\dfrac{2\cdot\pi}{1,25\cdot\pi}=\dfrac{8}{5}=1,6\text{ km}$

$l_3=\dfrac{\lambda}{4}=400$ [m]; $\qquad l_2=\dfrac{\lambda}{2}=800$ [m]; (1 bod)

c) $\gamma\cdot l_1=j\cdot\beta\cdot l_1=j\dfrac{3\pi}{2}$ $\qquad \gamma\cdot l_2=j\cdot\beta\cdot l_2=j\pi$; $\qquad \gamma\cdot l_3=j\cdot\beta\cdot l_3=j\dfrac{\pi}{2}$

$$Z_{ul3}=\frac{R_3 ch(\gamma\cdot l_3)+Z_0 sh(\gamma\cdot l_3)}{\dfrac{R_3}{Z_0}sh(\gamma\cdot l_3)+ch(\gamma\cdot l_3)}=\frac{R_3\cos(\beta\cdot l_3)+jZ_0\sin(\beta\cdot l_3)}{j\dfrac{R_3}{Z_0}\sin(\beta\cdot l_3)+\cos(\beta\cdot l_3)}=\frac{Z_0^2}{R_3}=\frac{2500}{25}=100\,\Omega$$

$$R_{eq2}=\frac{R_2\cdot Z_{ul3}}{R_2+Z_{ul3}}=\frac{150\cdot 100}{250}=60\,\Omega \qquad Z_{ul2}=\frac{R_{eq2}\cos(\beta\cdot l_2)+jZ_0\sin(\beta\cdot l_2)}{j\dfrac{R_{eq2}}{Z_0}\sin(\beta\cdot l_2)+\cos(\beta\cdot l_2)}=\frac{-R_{eq2}}{-1}=R_{eq2}=60\,\Omega$$

$$R_{eq1}=\frac{R_1\cdot Z_{ul2}}{R_1+Z_{ul2}}=\frac{R_1\cdot R_{eq2}}{R_1+R_{eq2}}=Z_0=50\,\Omega \qquad\Rightarrow\qquad R_1=\frac{Z_0\cdot R_{eq2}}{R_{eq2}-Z_0}=\frac{50\cdot 60}{10}=300\,\Omega$$

(1 bod)

d) $\Gamma_{i2}=\dfrac{R_{eq2}-Z_0}{R_{eq2}+Z_0}=\dfrac{10}{110}=\dfrac{1}{11}$ $\qquad \Gamma_{i3}=\dfrac{R_3-Z_0}{R_3+Z_0}=\dfrac{-25}{75}=-\dfrac{1}{3}$ (1 bod)

e) $\gamma\cdot l_1=j\beta\dfrac{3\lambda}{4}=j\dfrac{3\pi}{2}$ $\qquad U_1(l_1)=U(0)\cdot e^{-j\beta l_1}=5\cdot e^{-j3\pi/2}=5j$ $\qquad u_1(l_1,t)=5\cos\left(\omega t+\dfrac{\pi}{2}\right)$

$$U_2(l_2)=U_1(l_1)\cdot\cos(\beta\cdot l_2)-jU_1(l_1)\cdot\frac{Z_0}{Z_{ul2}}\sin(\beta\cdot l_2)=-U_1(l_1)=-5j \quad u_2(l_2,t)=-5\cos\left(\omega t+\frac{\pi}{2}\right)$$

$$U_3(l_3)=U_2(l_2)\cdot\cos(\beta\cdot l_3)-jU_2(l_2)\cdot\frac{Z_0}{Z_{ul3}}\sin(\beta\cdot l_3)=-jU_2(l_2)\cdot\frac{Z_0}{Z_{ul3}}\sin(\beta\cdot l_3)=-j(-5j)\frac{1}{2}=-\frac{5}{2}$$

$u_1(l_3,t)=-2,5\cdot\cos(\omega t)$ (1 bod)