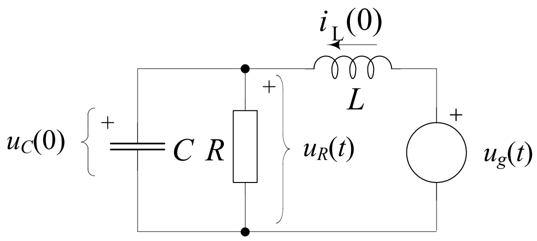
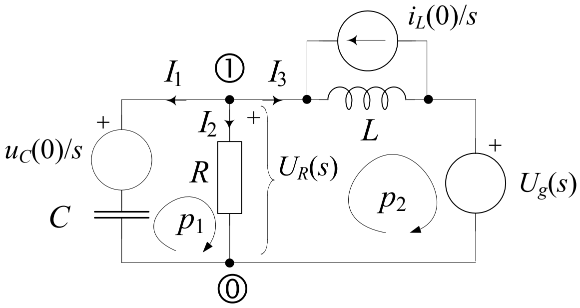
MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA 2013-2014 – Rješenja

1. Za električni krug prikazan slikom primjenom Kirchhoffovih zakona izračunati valni oblik napona $u_R(t)$ kao odziv, ako je zadana pobuda $u_g(t)=\delta(t)$. Zadane su normalizirane vrijednosti elemenata C=1, R=1, L=1/2, te početni uvjeti $u_C(0)$=8 i $i_L(0)$=2.



<u>Rješenje:</u> Primjena Laplaceove transformacije



(1 bod)

Mreža ima N_b=3 grane i N_v=2 čvora

Jednadžbe KZN

$$\text{(p1)}\ \ -U_1(s)+U_2(s)=0$$

$$\text{(p2)}\ \ -U_2(s)+U_3(s)=0$$

Jednadžbe KZS

$$\text{(č1)}\ \ I_1(s)+I_2(s)+I_3(s)=0$$ (1 bod)

Naponsko-strujne relacije grana

$$\text{(g1)}\ \ \left. U_1(s)=\frac{1}{sC}I_1(s)+\frac{u_C(0)}{s}\right/\cdot sC$$

$$\text{(g2)}\ \ U_2(s)=R\cdot I_2(s)$$

$$\text{(g3)}\ \ U_3(s)=sL\cdot\left[I_3(s)+\frac{i_L(0)}{s}\right]+U_g(s)$$ (1 bod)

$$\text{(g1)}\Rightarrow\ I_1(s)=sC\cdot U_1(s)-Cu_C(0)$$

$$\text{(g2)}\Rightarrow\ I_2(s)=U_2(s)\frac{1}{R}$$

$$\text{(g3)}\Rightarrow\ U_3(s)=sL\cdot I_3(s)+Li_L(0)+U_g(s)\ \Rightarrow\ I_3(s)=\frac{1}{sL}U_3(s)-\frac{i_L(0)}{s}-\frac{1}{sL}U_g(s)$$

$$\text{(č1)}\Rightarrow\ sC\cdot U_1(s)-Cu_C(0)+U_2(s)\frac{1}{R}+\frac{1}{sL}U_3(s)-\frac{i_L(0)}{s}-\frac{1}{sL}U_g(s)=0$$

$$\text{(p1), (p2)}\Rightarrow\ U_1(s)=U_2(s)=U_3(s),\ U_R(s)=U_2(s)$$

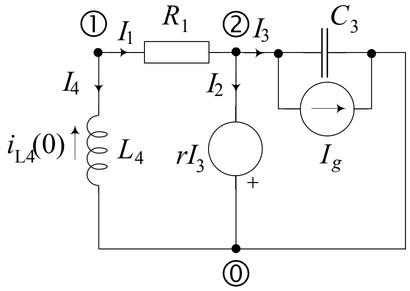
$$\Rightarrow \left(sC+\frac{1}{R}+\frac{1}{sL}\right)\cdot U_R(s)=Cu_C(0)+\frac{i_L(0)}{s}+\frac{1}{sL}U_g(s)$$

$$\Rightarrow U_R(s)=\frac{Cu_C(0)+\frac{i_L(0)}{s}+\frac{1}{sL}U_g(s)}{sC+\frac{1}{R}+\frac{1}{sL}}=\frac{8+\frac{2}{s}+\frac{2}{s}}{s+1+\frac{2}{s}}=\frac{8+\frac{4}{s}}{s+1+\frac{2}{s}}=\frac{8s+4}{s^2+s+2}$$ (1 bod)

$$U_R(s)=8\cdot\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{7}{4}}=8\cdot\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{7}}{2}\right)^2}$$

$$\Rightarrow \underline{u_R(t)=8\cdot e^{-\frac{t}{2}}\cos\left(\frac{\sqrt{7}}{2}t\right)\cdot S(t)}$$ (1 bod)

2. Za električni krug na slici zadane su normalizirane vrijednosti elemenata R_1=1, C_3=1/2, L_4=1, r=2, $u_{C3}(0)$=0, $i_{L4}(0)$=1, te pobuda $i_g(t)=S(t)$. Koristeći metodu petlji te oznake grana i čvorova prema slici kao odziv izračunati napon grane 1 $u_1(t)$.



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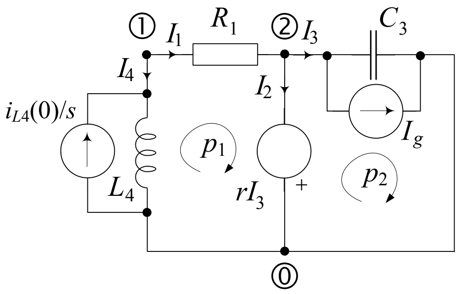
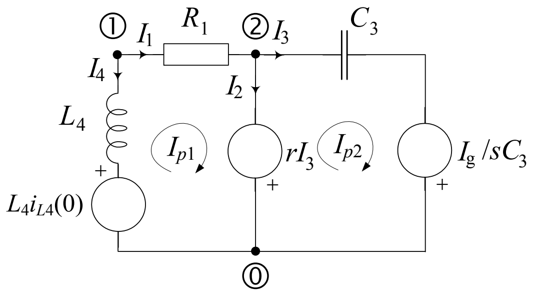




(1 bod)

Vidljivo je:

$$I_1(s) = I_{p1}(s); \quad I_2(s) = I_{p1}(s) - I_{p2}(s); \quad I_3(s) = I_{p2}(s)$$

Jednadžbe petlji

$$1)\ I_{p1}(s)\left(R_1 + sL_4\right) = L_4 i_{L4}(0) + r \cdot I_{p2}(s)$$

$$2)\ I_{p2}(s)\frac{1}{sC_3} = -r \cdot I_{p2}(s) + \frac{1}{sC_3} \cdot I_g(s)$$ (1 bod)

$$2) \Rightarrow I_{p2}(s)\left(\frac{1}{sC_3} + r\right) = \frac{1}{sC_3} \cdot I_g(s) \Rightarrow I_{p2}(s) = \frac{\frac{1}{sC_3}}{\frac{1}{sC_3} + r} \cdot I_g(s) \Rightarrow I_{p2}(s) = \frac{1}{1 + rsC_3} \cdot I_g(s)$$

$$1) \Rightarrow I_1(s) = I_{p1}(s) = \frac{L_4 i_{L4}(0) + r \cdot I_{p2}(s)}{R_1 + sL_4} = \frac{L_4 i_{L4}(0) + r \cdot \frac{1}{1 + rsC_3} \cdot I_g(s)}{R_1 + sL_4}$$

Uz uvrštene vrijednosti elemenata:

$$I_1(s) = \frac{1 + 2 \cdot \frac{1}{1+s} \cdot \frac{1}{s}}{1+s} = \frac{1}{1+s} + \frac{2}{s} \cdot \frac{1}{(1+s)^2};$$

$$\Rightarrow U_1(s) = R_1 \cdot I_1(s) = \frac{1}{1+s} + \frac{2}{s} \cdot \frac{1}{(1+s)^2}$$ (1 bod)

Rastav na parcijalne razlomke:

$$\frac{2}{s} \cdot \frac{1}{(s+1)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

$$2 = A(s+1)^2 + Bs(s+1) + Cs$$

$$2 = As^2 + 2As + A + Bs^2 + Bs + Cs$$
$$2 = (A+B)s^2 + (2A+B+C)s + A$$

$$A + B = 0 \Rightarrow B = -A = -2$$
$$2A + B + C = 0 \Rightarrow C = -2A - B = -4 + 2 = -2$$
$$\underline{A = 2}$$

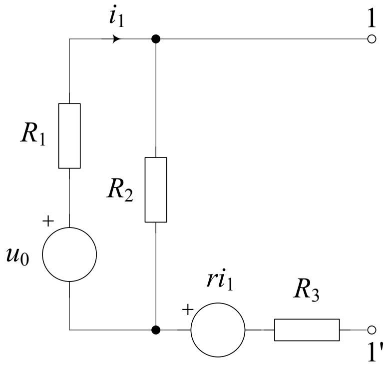
$$U_1(s) = \frac{1}{1+s} + \frac{2}{s} \cdot \frac{1}{(1+s)^2} = \frac{1}{1+s} + \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$
$$U_1(s) = \frac{1}{1+s} + \frac{2}{s} \cdot \frac{1}{(1+s)^2} = \frac{1}{1+s} + \frac{2}{s} - \frac{2}{s+1} - \frac{2}{(s+1)^2}$$
$$U_1(s) = \frac{2}{s} - \frac{1}{s+1} - \frac{2}{(s+1)^2}$$ (1 bod)

$$\Rightarrow \underline{u_1(t) = \left(2 - e^{-t} - 2 \cdot t \cdot e^{-t}\right) \cdot S(t)}$$ (1 bod)

3. Za električni krug na slici obzirom na priključnice 1–1' odrediti:

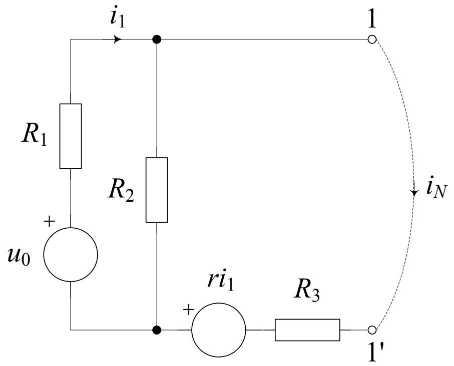
a) izraz za Nortonovu struju i_N;
b) izraz za Nortonovu admitanciju G_N;
c) iznos konstante r ako je G_N=1S;
d) struju i_1 kad se na priključnice 1–1' spoji otpor R=2Ω;
e) iznos konstante r za koji je G_N=1/2.

Zadano je: u_0=2V, R_1=3Ω, R_2=2Ω, R_3=1Ω.



Rješenje:

a) Nortonova struja i_N:



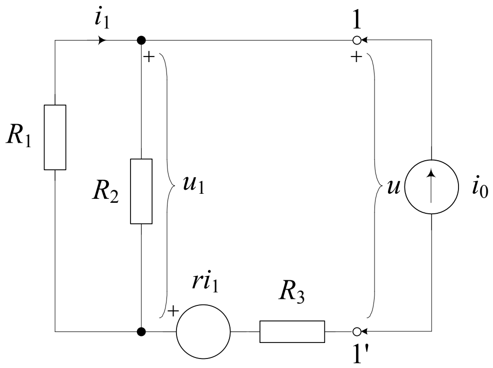
$$u_0 = i_1 \cdot (R_1 + R_2) - i_N \cdot R_2$$

$$0 = -i_1 \cdot R_2 - r \cdot i_1 + i_N \cdot (R_2 + R_3) \;\Rightarrow\; i_1 = i_N \cdot \frac{R_2 + R_3}{r + R_2}$$

$$u_0 = i_N \cdot \frac{(R_1 + R_2)(R_2 + R_3)}{r + R_2} - i_N \cdot R_2$$

$$\boxed{i_N = u_0 \cdot \frac{r + R_2}{R_1 R_2 + (R_1 + R_2) R_3 - r R_2}}$$ (1 bod)

b) Nortonova admitancija G_N:



$$i_1 = -\frac{u_1}{R_1} = -i_0 \frac{R_1 R_2}{R_1 + R_2} \cdot \frac{1}{R_1} = -i_0 \frac{R_2}{R_1 + R_2}$$

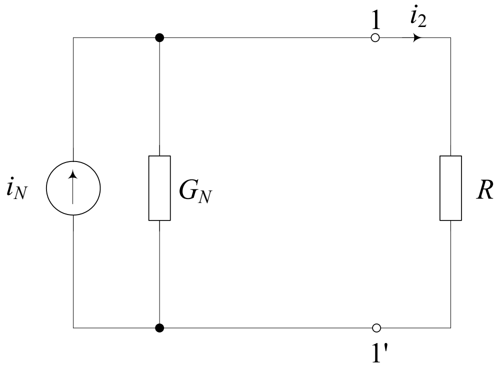
$$u = u_1 + r i_1 + i_0 R_3 = i_0 \frac{R_1 R_2}{R_1 + R_2} - r i_0 \frac{R_2}{R_1 + R_2} + i_0 R_3$$

$$\frac{u}{i_0} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3 - r R_2}{R_1 + R_2} \quad \Rightarrow \boxed{G_N = \frac{R_1 + R_2}{R_1 R_2 + (R_1 + R_2) R_3 - r R_2}} \text{ (1 bod)}$$

c) iznos konstante r ako je G_N=1S.

$$G_N = \frac{5}{11 - 2r} = 1 \quad \Rightarrow \boxed{r = 3} \text{ (1 bod)}$$

d) struju i_1 kad se na priključnice 1–1' spoji otpor R=2Ω



$$i_N = u_0 \frac{r + R_2}{R_1 R_2 + (R_1 + R_2) R_3 - r R_2} \quad \Rightarrow \quad i_N = 2 \cdot \frac{3 + 2}{3 \cdot 2 + (3 + 2) \cdot 1 - 3 \cdot 2} = 2 \cdot \frac{5}{5} = 2$$

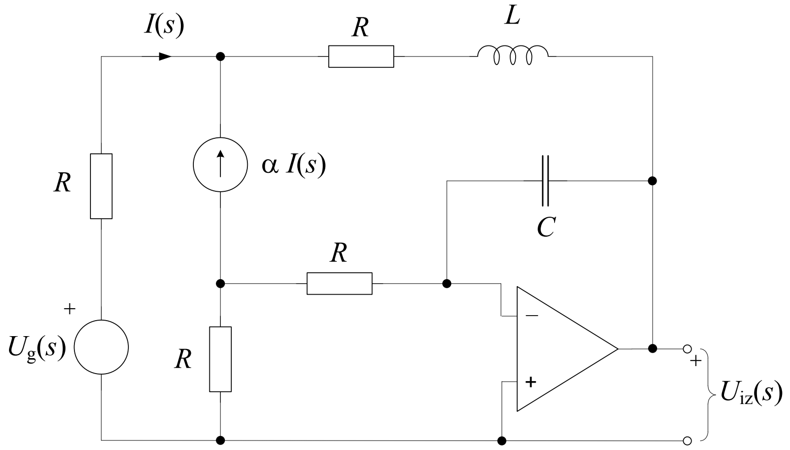
$$i_2 \cdot R = \frac{i_N - i_2}{G_N} \Rightarrow i_2 = i_N \frac{1}{R \cdot G_N + 1} = \frac{i_N}{3} \quad \Rightarrow \quad i_2 = \frac{2}{3}$$

$$i_1 = i_2 \cdot \frac{R_2 + R_3 + R}{r + R_2} = \frac{2}{3} \cdot \frac{5}{5} = \frac{2}{3} \text{ (1 bod)}$$

e) iznos konstante r za koji je G_N=1/2.

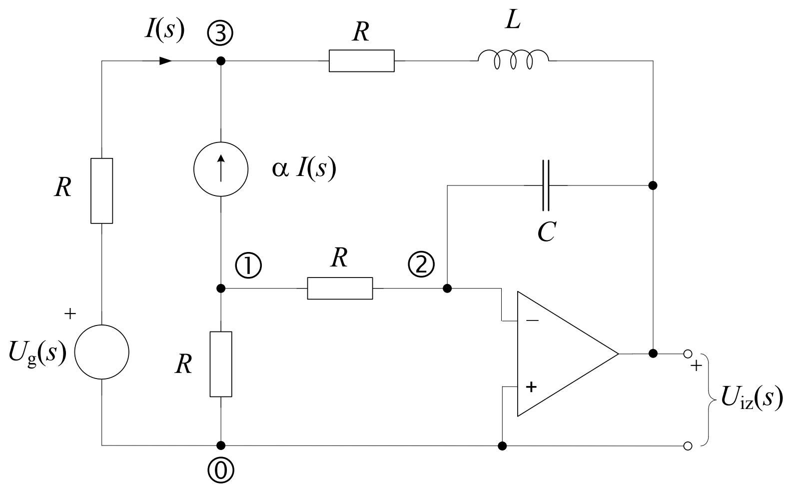
$$G_N = \frac{5}{11 - 2r} = \frac{1}{2} \Rightarrow r = \frac{1}{2} \text{ (1 bod)}$$

4. Za električni krug prikazan slikom izračunati odziv u frekvencijskoj domeni $U_{iz}(s)$ na pobudu $U_g(s)=E/s$. Zadane su normalizirane vrijednosti elemenata R=4, C=0.1 i L=1.25; te konstante E=6 i α=3. Operacijsko pojačalo je idealno. Početni uvjeti: $u_C(0)=0$, $i_L(0)=0$. Traženi odziv treba odrediti primjenom jednadžbi čvorišta.



Rješenje:

Primjenom Laplaceove transformacije dobivamo slijedeći električni krug u *frekvencijskoj* domeni. Postavimo jednadžbe čvorišta:



1) $U_1\left(\frac{1}{R}+\frac{1}{R}\right)-U_2\frac{1}{R}=-\alpha I(s);\ \Rightarrow\ I(s)=\frac{U_g(s)-U_3(s)}{R};$

2) $-U_1\frac{1}{R}+U_2\left(sC+\frac{1}{R}\right)=U_{iz}(s)sC;$

3) $U_3\left(\frac{1}{R+sL}\right)=U_{iz}\frac{1}{R+sL}+\alpha\cdot I(s)+I(s);$ (1 bod)

$U_2=0\ \Rightarrow$

1) $U_1\frac{2}{R}=-\alpha\frac{U_g(s)-U_3(s)}{R};$

2) $-U_1\frac{1}{R}=U_{iz}(s)sC;$

3) $U_3(s)\frac{1}{R+sL}=(1+\alpha)\cdot\frac{U_g(s)-U_3(s)}{R}+U_{iz}(s)\frac{1}{R+sL};$ (1 bod)

Nakon malo sređivanja:

$$1)\ 2U_1 - \alpha U_3 = -\alpha U_g(s) \Rightarrow U_1 = \frac{\alpha}{2}U_3(s) - \frac{\alpha}{2}U_g(s);$$

$$2)\ U_{iz}(s) = -\frac{1}{sRC}U_1;$$

$$3)\ U_3(s)\left[\frac{1}{R+sL} + (1+\alpha)\cdot\frac{1}{R}\right] = U_g(s)\cdot\frac{1+\alpha}{R} + U_{iz}(s)\frac{1}{R+sL}$$

$$3) \Rightarrow U_3(s)\left[R+(1+\alpha)(R+sL)\right] = U_g(s)\cdot(1+\alpha)(R+sL) + U_{iz}(s)R;$$

$$\Rightarrow U_3(s) = U_g(s)\cdot\frac{(1+\alpha)(R+sL)}{R+(1+\alpha)(R+sL)} + U_{iz}(s)\frac{R}{R+(1+\alpha)(R+sL)};$$

$$2) \Rightarrow U_{iz}(s) = -\frac{1}{sRC}U_1 = -\frac{1}{sRC}\frac{\alpha}{2}\left[U_3(s) - U_g(s)\right]$$

$$U_{iz}(s) = -\frac{1}{sRC}\frac{\alpha}{2}\left[\frac{(1+\alpha)(R+sL)}{R+(1+\alpha)(R+sL)}\cdot U_g(s) - U_g(s) + \frac{R}{R+(1+\alpha)(R+sL)}\cdot U_{iz}(s)\right]$$

$$-U_{iz}(s)\frac{2sRC}{\alpha} = \frac{-R}{R+(1+\alpha)(R+sL)}\cdot U_g(s) + \frac{R}{R+(1+\alpha)(R+sL)}\cdot U_{iz}(s)$$

$$U_{iz}(s)\left[\frac{2sRC}{\alpha} + \frac{R}{R+(1+\alpha)(R+sL)}\right] = \frac{R}{R+(1+\alpha)(R+sL)}\cdot U_g(s)$$

$$U_{iz}(s) = \frac{\dfrac{R}{R+(1+\alpha)(R+sL)}}{\dfrac{2sRC}{\alpha} + \dfrac{R}{R+(1+\alpha)(R+sL)}}\cdot U_g(s) = \frac{R\alpha}{2sRC\left[R+(1+\alpha)(R+sL)\right] + R\alpha}\cdot U_g(s)$$

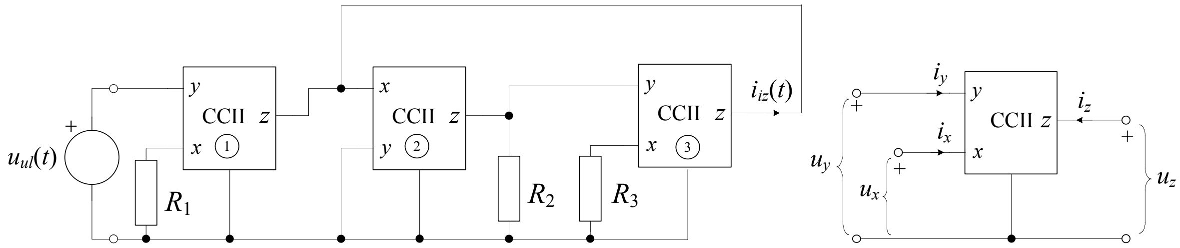
(2 boda)

Uz uvrštene vrijednosti elemenata:

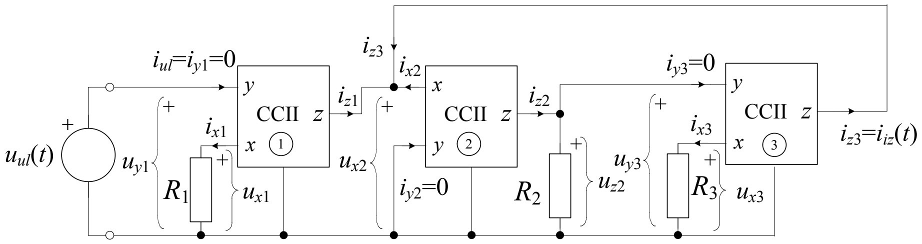
$$U_{iz}(s) = \frac{12}{2s0.4\left[4+(1+3)(4+s1.25)\right]+3\cdot 4}\cdot\frac{6}{s} = \frac{12}{0.8s\left[5s+20\right]+12}\cdot\frac{6}{s} = \frac{12}{4s^2+16s+12}\cdot\frac{6}{s}$$

$$\underline{U_{iz}(s) = \frac{3}{s^2+4s+3}\cdot\frac{6}{s} = \frac{18}{s\left(s^2+4s+3\right)}}$$ (1 bod)

5. Za električni krug prikazan slikom izračunati valni oblik struje $i_{iz}(t)$ za $t>0$ kao odziv, ako je zadana pobuda $u_{ul}(t)=E\cdot S(t)$. Zadane su normalizirane vrijednosti elemenata $R_1=2$, $R_2=4$, $R_3=1$, te konstanta $E=10$. Za strujni prijenosnik druge generacije (CCII) vrijede slijedeće definicijske jednadžbe: $u_x=u_y$, $i_y=0$, $i_z=i_x$ uz referentna usmjerenja struja i napona prilaza prikazana na slici.



Rješenje:



Za prvi CCII vrijedi:

$$u_{x1}=u_{y1}=u_{ul},\ i_{y1}=0,\ i_{x1}=\frac{u_{x1}}{R_1}=\frac{u_{ul}}{R_1},\ i_{z1}=i_{x1}=\frac{u_{ul}}{R_1}\ \text{(1 bod)}$$

Za drugi CCII vrijedi:

$$u_{x2}=u_{y2}=0,\ i_{x2}=-i_{z1}-i_{z3},\ i_{z2}=i_{x2}=-(i_{z1}+i_{z3})\ \text{(1 bod)}$$

$$u_{z2}=R_2 i_{z2}=-R_2(i_{z1}+i_{z3})$$

Za treći CCII vrijedi:

$$u_{y3}=u_{z2},\ i_{y3}=0,\ u_{x3}=u_{y3}=R_3 i_{x3}\ \text{(1 bod)}$$

$$i_{z3}=i_{x3}=\frac{u_{y3}}{R_3}=\frac{-(i_{z1}+i_{z3})R_2}{R_3}$$

$$i_{z3}=-i_{z1}\frac{R_2}{R_3}-i_{z3}\frac{R_2}{R_3}\Rightarrow i_{z3}+i_{z3}\frac{R_2}{R_3}=-i_{z1}\frac{R_2}{R_3}\Rightarrow i_{z3}\left(1+\frac{R_2}{R_3}\right)=-i_{z1}\frac{R_2}{R_3}$$

$$\Rightarrow i_{iz}=i_{z3}=-i_{z1}\cdot\frac{\frac{R_2}{R_3}}{1+\frac{R_2}{R_3}}=-\frac{u_{ul}}{R_1}\cdot\frac{\frac{R_2}{R_3}}{1+\frac{R_2}{R_3}}$$

Uz uvrštene vrijednosti elemenata:

$$i_{iz}=-\frac{10}{2}\cdot\frac{\frac{4}{1}}{1+\frac{4}{1}}=-\frac{10}{2}\cdot\frac{4}{5}=-4\ \text{(1 bod)}$$

$$\Rightarrow \underline{i_{iz}(t)=-4\cdot S(t)[A]}\ \text{(1 bod)}$$