

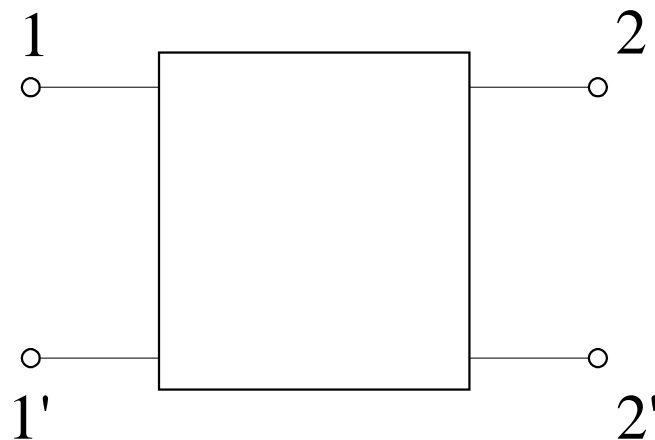
# Električni krugovi

## Četveropoli

Skripta: M. Plohl, Teorija četveropolnih sistema, 1987. , I-dio

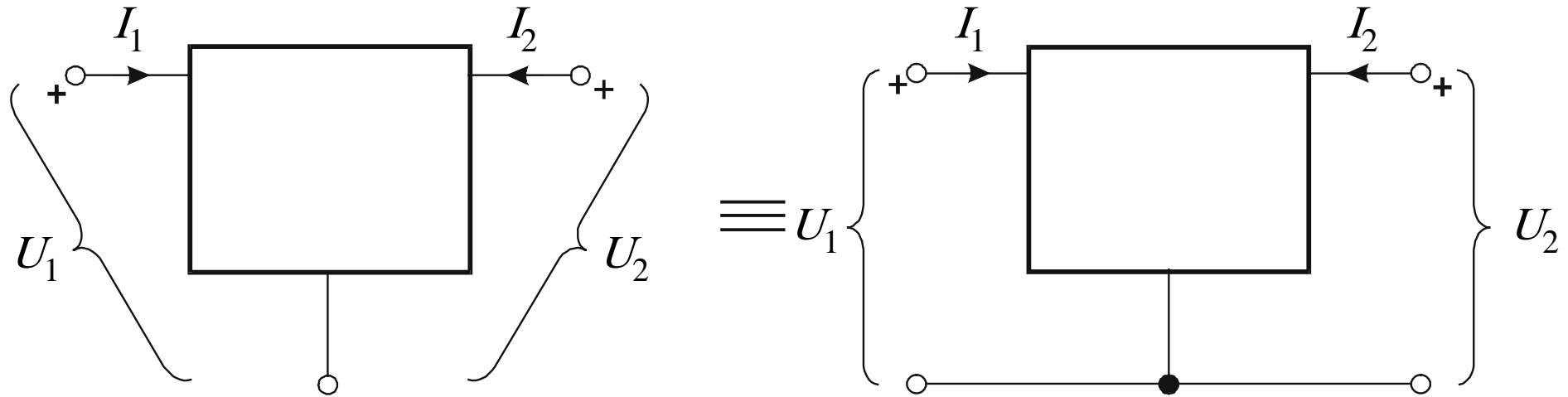
# Uvod u teoriju četveropola

- Četveropol je element s 4 priključnice ili pola.



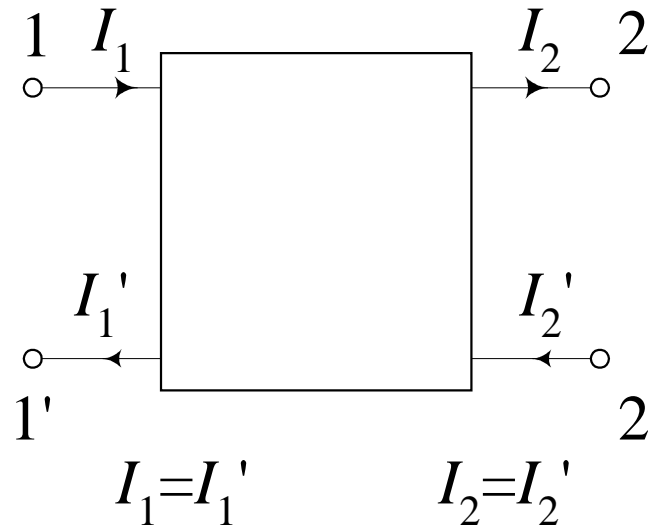
- Priključnice četveropola su uređene kao 2 para polova.
- Svaki par polova čini jedan prilaz.
- Zato se četveropoli nazivaju i *dvoprilaznim mrežama*.

- Primjer: Element s 3 priključnice →



- Dvoprilazni element → jedna priključnica zajednička.

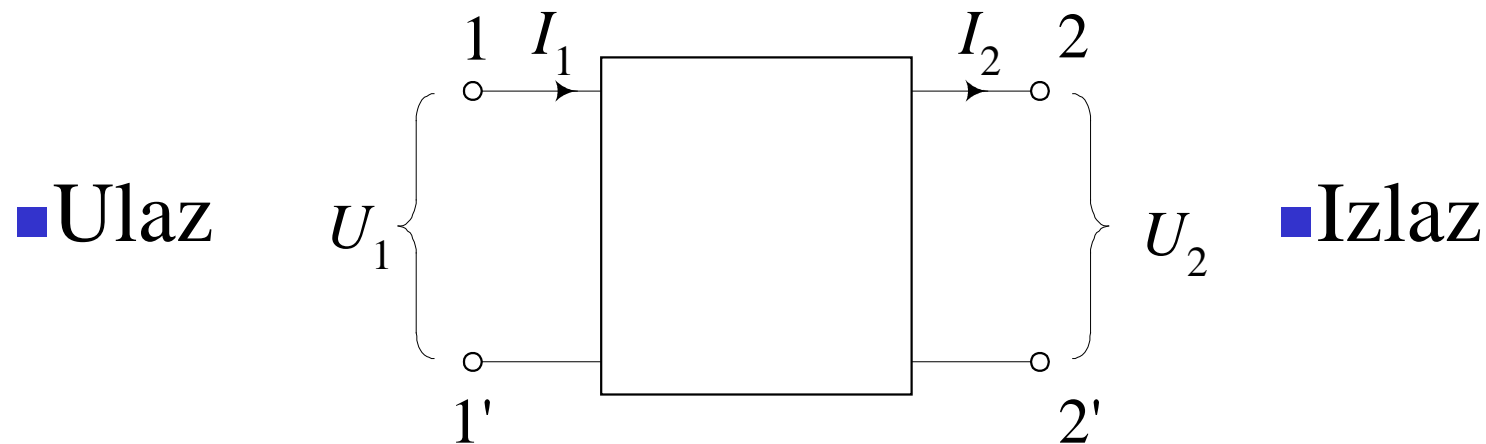
- Prilaz  $\rightarrow$  par polova sa svojstvom  $\rightarrow$  trenutni iznos struje koja ulazi na jedan = struji koja izlazi kroz drugi.



- U nastavku:
- Pojam *četveropol* označavat će *dvoprilaznu mrežu*.

- Četveropol može biti
  - element mreže
  - submreža.
- Analizu kruga moguće je pojednostavniti ako se pojedini dijelovi kruga promatraju kao *submreže*.
- Svaka submreža → složeni element s više priključnica.
- U pravilu → submreža gledana s vanjskih priključnica → jednostavnija za analizu.
- Posebna pogodnost → u slučajevima više istih submreža.

- Mreže s dva prilaza pojavljuju se u mnogim primjenama.
- Jedan od prilaza obično je označen kao ulaz.
- Drugi predstavlja izlaz signala



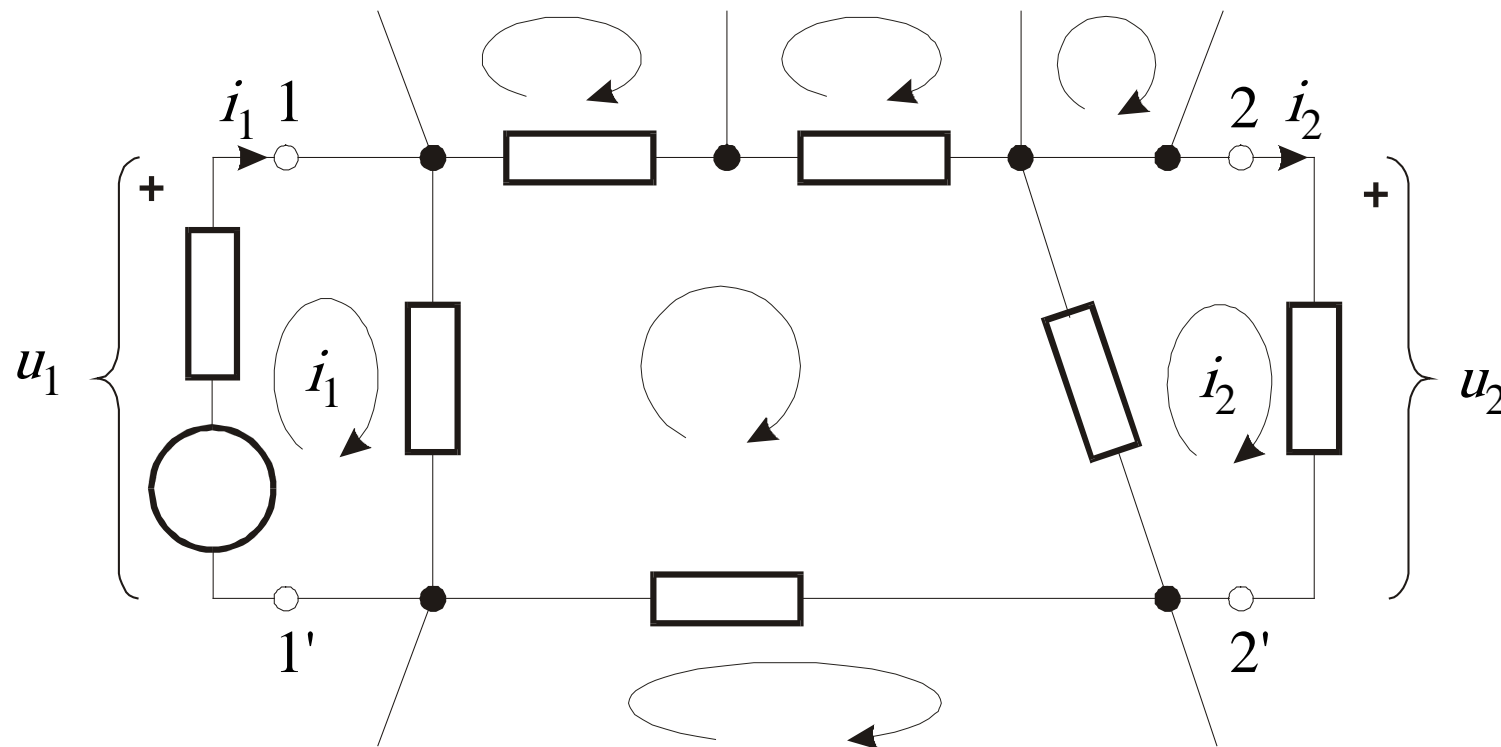
- Ulazne polove označavamo kao 1-1'.
- Izlazne polove označavamo kao 2-2'.

## ■ Jednadžbe četveropola

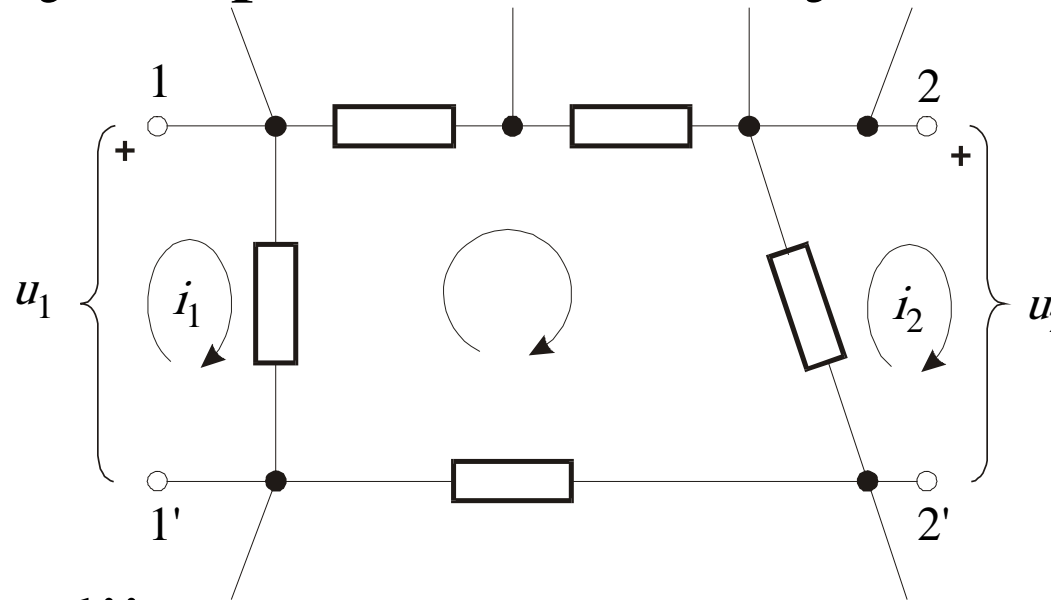
### ■ Primjer:

→ mreža s pobudom na 1-1'

→ odziv na 2-2'



- Napon i struja na prilazima  $\rightarrow$  varijable četveropola



- Jednadžbe petlji:
 
$$U_1 = I_1 Z_{11} - I_2 Z_{12} - I_3 Z_{13} - \dots - I_n Z_{1n}$$

$$U_2 = -I_1 Z_{21} + I_2 Z_{22} - I_3 Z_{23} - \dots - I_n Z_{2n}$$

$$0 = -I_1 Z_{31} - I_2 Z_{32} + I_3 Z_{33} - \dots - I_n Z_{3n}$$

$$\vdots$$

$$0 = -I_1 Z_{n1} - I_2 Z_{n2} - I_3 Z_{n3} - \dots + I_n Z_{nn}$$



Rješenja jednažbi petlji:  $\rightarrow$  struje  $I_1$  i  $I_2$

$$I_1 = \left( \frac{\Delta_{11}}{\Delta} \right)_z U_1 - \left( \frac{\Delta_{21}}{\Delta} \right)_z U_2$$

$$I_2 = \left( \frac{\Delta_{12}}{\Delta} \right)_z U_1 - \left( \frac{\Delta_{22}}{\Delta} \right)_z U_2$$


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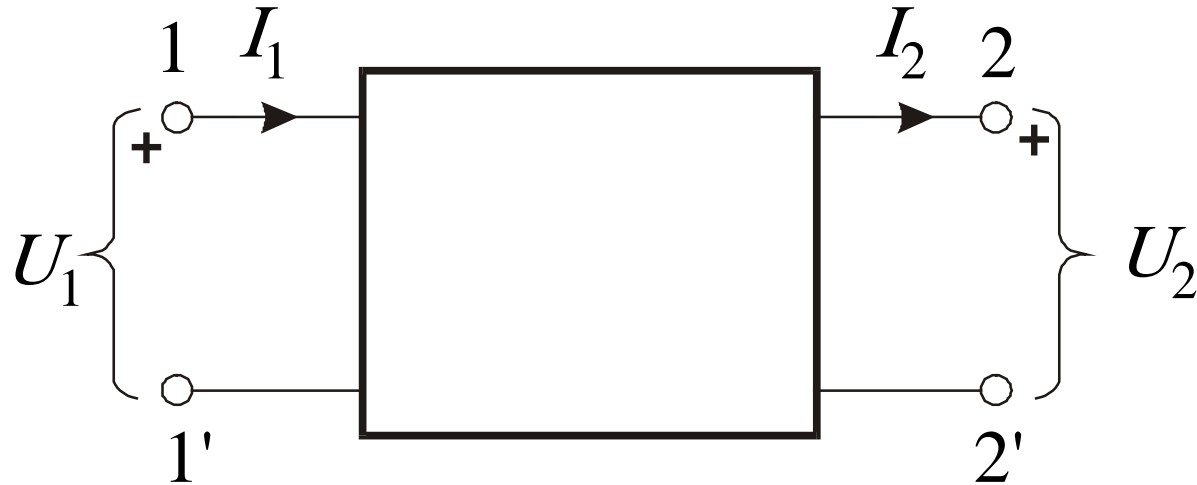
$\rightarrow$  elementi  
u determinantama  
su impedancije

$\Delta \rightarrow$  determinanta sustava

$\Delta_{ij} \rightarrow$  kofaktori det. sustava

$\frac{\Delta_{ij}}{\Delta} \rightarrow$  ima dimenziju  $\frac{1}{\Omega}$

- Cijelu je mrežu moguće promatrati kao

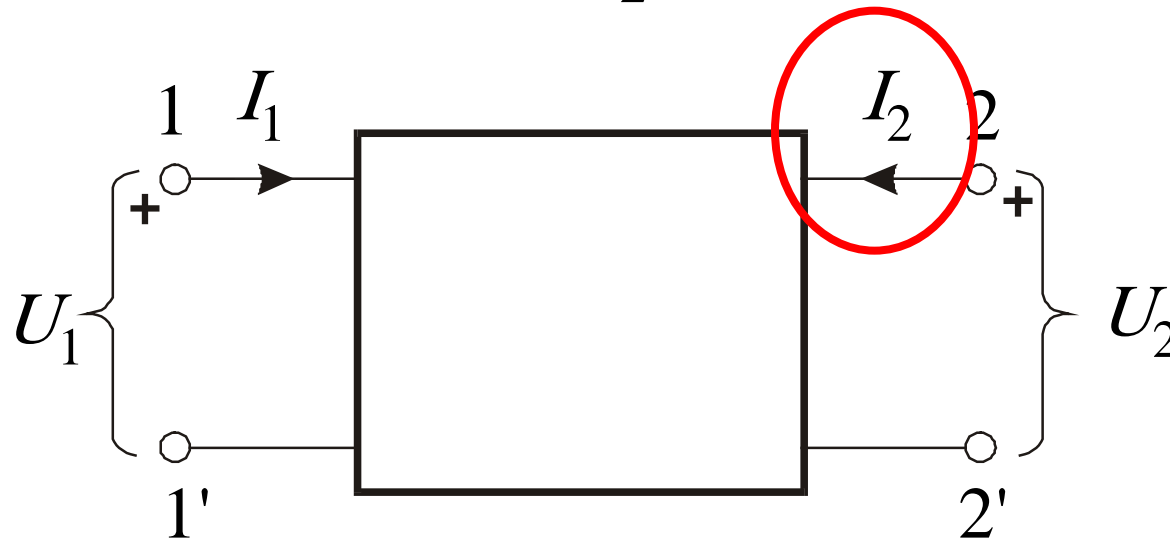


$$\begin{aligned} I_1 &= y_{11}U_1 - y_{12}U_2 \\ I_2 &= y_{21}U_1 - y_{22}U_2 \end{aligned}$$

*Strujne jednačbe četveropola*

$y_{11}, y_{12}, y_{21}, y_{22} \rightarrow y\text{-parametri četveropola}$

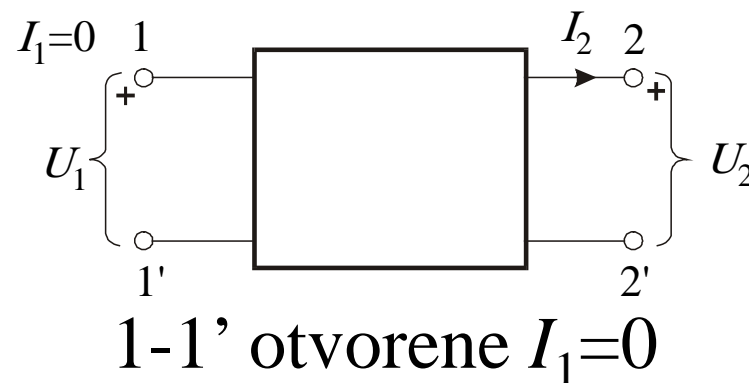
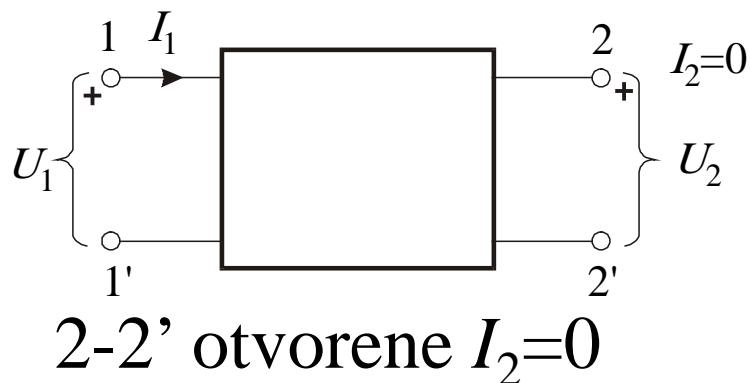
Ponekad je (u literaturi) struja  $I_2$  okrenuta suprotno:



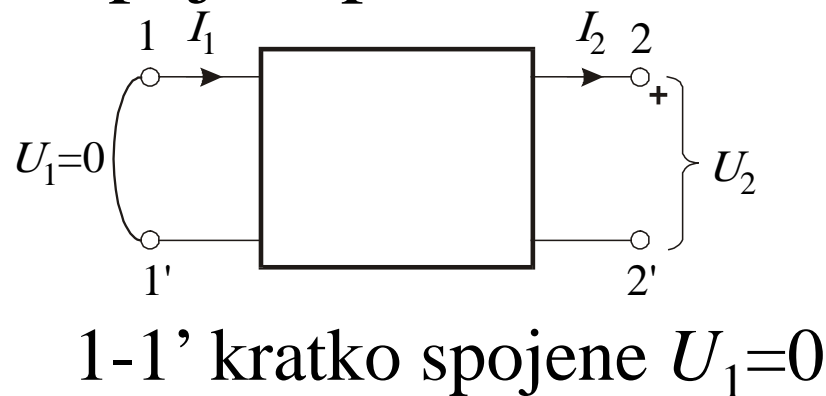
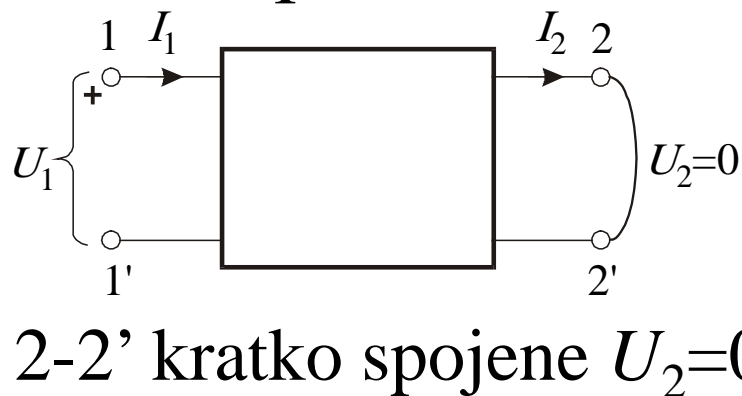
$$I_1 = y_{11}U_1 + y_{12}U_2$$

$$I_2 = y_{21}U_1 + y_{22}U_2$$

- Određivanje parametara četveropola:
- Parametri četveropola  $\rightarrow$  iz uvjeta na prazno i na kratko.
- Četveropol na prazno  $\rightarrow$  prazni hod na prilazu 1-1' ili 2-2'



- Četveropol na kratko  $\rightarrow$  kratki spoj na prilazu 1-1' ili 2-2'

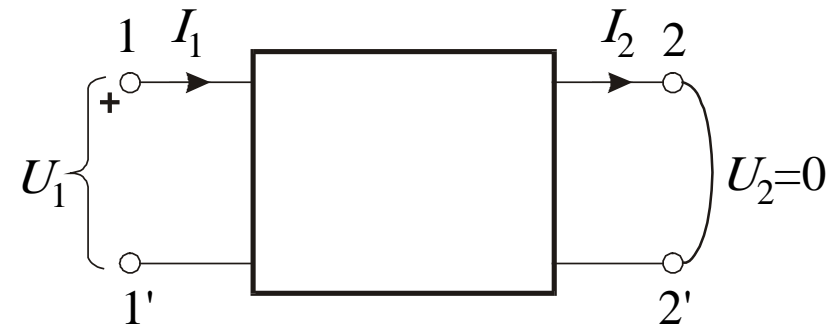


- Strujne jednačbe →  $y$ -parametri četveropola
- $y$ -parametri → iz četveropola na kratko

$$I_1 = U_1 y_{11} - U_2 y_{12}$$

$$\underline{I_2 = U_1 y_{21} - U_2 y_{22}}$$

- $\underline{U_2=0} \rightarrow$  kratki spoj na 2-2'



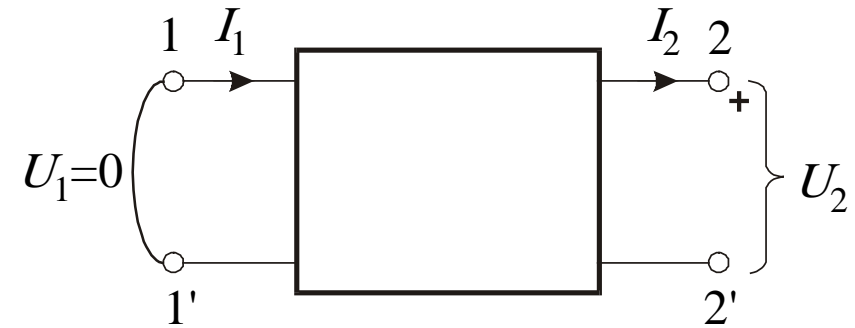
$$I_1 = U_1 y_{11} \quad \Rightarrow \quad y_{11} = \left. \frac{I_1}{U_1} \right|_{U_2=0}$$

ulazna admitancija  
na kratko  $U_2=0$

$$I_2 = U_1 y_{21} \quad \Rightarrow \quad y_{21} = \left. \frac{I_2}{U_1} \right|_{U_2=0}$$

prijenosna admitancija  
na kratko  $U_2=0$

uz  $U_1=0$   $\rightarrow$  kratki spoj na 1-1'



$$I_1 = -U_2 y_{12} \quad \Rightarrow \quad y_{12} = -\frac{I_1}{U_2} \bigg|_{U_1=0}$$

prijenosna admitancija  
na kratko  $U_1=0$

$$I_2 = -U_2 y_{22} \quad \Rightarrow \quad y_{22} = -\frac{I_2}{U_2} \bigg|_{U_1=0}$$

ulazna admitancija na  
kratko  $U_1=0$

- Ako je četveropol recipročan

$$\Delta_{12} = \Delta_{21} \text{ pa je}$$



$$y_{12} = y_{21}$$

uvjet  
recipročnosti  
četveropola

- Strujne jednačbe četveropola u matričnome obliku

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = [y] \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$[y] = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix}$$

***Matrica y-parametara***

## Naponske jednačbe $\rightarrow$ $z$ -parametri četveropola

- Strujne jednačbe  $\rightarrow$  struje  $I_1$  i  $I_2$  iz danih napona  $U_1$  i  $U_2$
- Problem: odrediti napone  $U_1$  i  $U_2$  za zadane struje  $I_1$  i  $I_2$ .

$$\begin{array}{l} I_1 = U_1 y_{11} - U_2 y_{12} \\ I_2 = U_1 y_{21} - U_2 y_{22} \end{array} \rightarrow \begin{array}{l} U_1 = \frac{y_{22}}{D_y} I_1 - \frac{y_{12}}{D_y} I_2 \\ U_2 = \frac{y_{21}}{D_y} I_1 - \frac{y_{11}}{D_y} I_2 \end{array}$$

$$D_y = y_{11} y_{22} - y_{12} y_{21} = \begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}$$

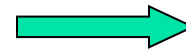


$$\begin{aligned} U_1 &= I_1 z_{11} - I_2 z_{12} \\ U_2 &= I_1 z_{21} - I_2 z_{22} \end{aligned}$$

■ *Naponske jednačbe četveropola*

$z_{11}, z_{12}, z_{21}, z_{22}, \rightarrow$  *z - parametri četveropola*

Za recipročni četveropol



$$z_{21} = z_{12}$$

uvjet  
recipročnosti  
četveropola

z-parametri  $\rightarrow$  iz četveropola na prazno

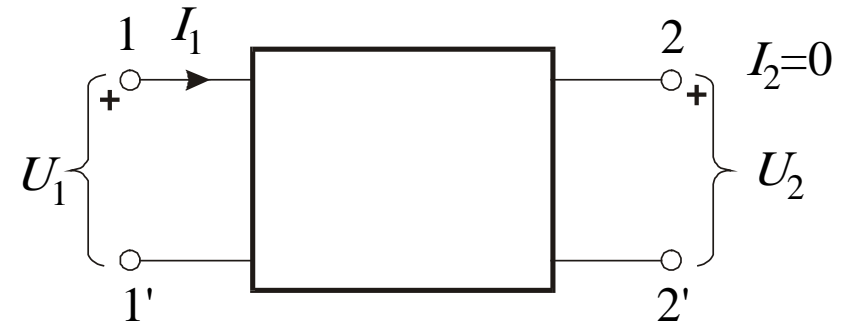
$$U_1 = I_1 z_{11} - I_2 z_{12}$$

$$U_2 = I_1 z_{21} - I_2 z_{22}$$

■  $I_2=0$  priključnice 2-2' otvorene

$$U_1 = I_1 z_{11} \quad \Rightarrow \quad z_{11} = \left. \frac{U_1}{I_1} \right|_{I_2=0}$$

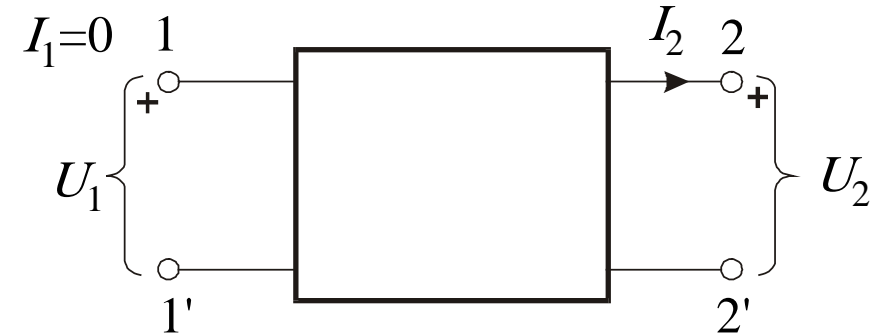
$$U_2 = I_1 z_{21} \quad \Rightarrow \quad z_{21} = \left. \frac{U_2}{I_1} \right|_{I_2=0}$$



ulazna impedancija  
na prazno  $I_2=0$

prijenosna impedancija  
na prazno  $I_2=0$

- $\underline{I_1} \equiv 0$  priključnice 1-1' otvorene



$$U_1 = -I_2 z_{12} \Rightarrow z_{12} = -\left. \frac{U_1}{I_2} \right|_{I_1=0}$$

prijenosna impedancija  
na prazno  $I_1=0$

$$U_2 = -I_1 z_{22} \Rightarrow z_{22} = -\left. \frac{U_2}{I_1} \right|_{I_1=0}$$

ulazna impedancija  
na prazno  $I_1=0$

- Naponske jednačbe četveropola u matričnome obliku

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = [z] \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[z] = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix}$$

***Matrica z-parametara***

$$[z] = [y]^{-1} \quad [z] = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix} = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix}^{-1}$$

## Prijenosne jednačbe → *a*-parametri četveropola

- Prijenos signala → s ulaza 1-1' na izlaz 2-2'
- Ovisnost napona  $U_1$  i struje  $I_1$  o  $U_2$  i  $I_2$  =?

$$\begin{aligned} I_1 &= U_1 y_{11} - U_2 y_{12} \\ I_2 &= U_1 y_{21} - U_2 y_{22} \end{aligned} \quad \Rightarrow \quad \begin{aligned} U_1 &= U_2 \frac{y_{22}}{y_{21}} + \frac{1}{y_{21}} I_2 \\ I_1 &= \frac{D_y}{y_{21}} U_2 + \frac{y_{11}}{y_{21}} I_2 \end{aligned}$$

$$\begin{aligned} U_1 &= AU_2 + BI_2 \\ \underline{I_1} &= \underline{CU_2 + DI_2} \end{aligned}$$

- *Prijenosne jednačbe četveropola*

$A, B, C, D,$  →

*prijenosni a-parametri četveropola*

- Određivanje prijenosnih parametara  
→ iz 2-2' na prazno i na kratko

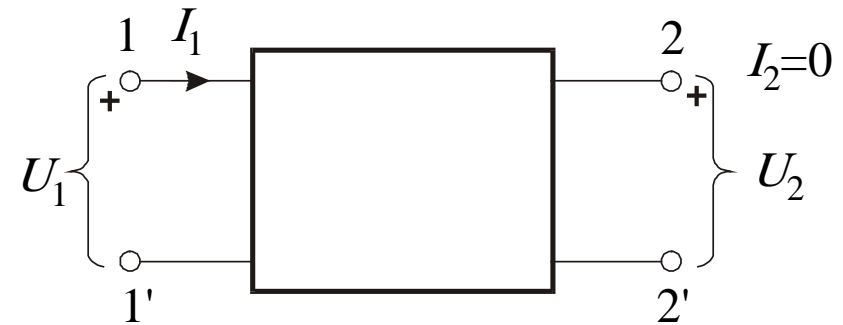
$$U_1 = AU_2 + BI_2$$

$$\underline{I_1 = CU_2 + DI_2}$$

- $I_2=0$  priključnice 2-2' otvorene

$$U_1 = AU_2 \Rightarrow A = \left. \frac{U_1}{U_2} \right|_{I_2=0}$$

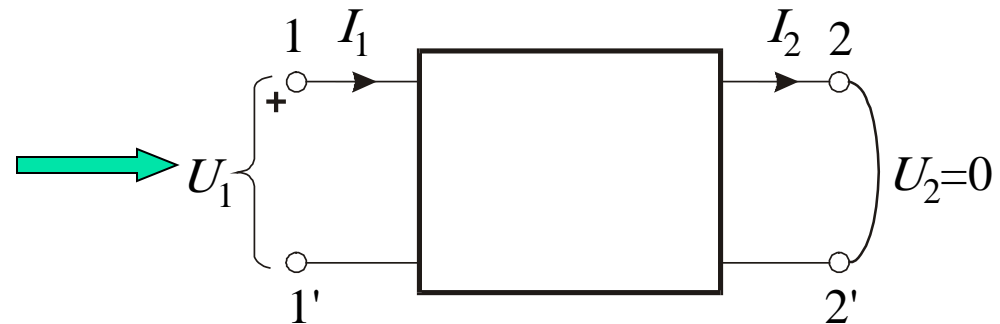
$$I_1 = CU_2 \Rightarrow C = \left. \frac{I_1}{U_2} \right|_{I_2=0}$$



omjer napona na prazno  $I_2=0$

recipročna vrijednost  
prijenosne impedancije na  
prazno  $I_2=0$

- $\underline{U_2=0} \rightarrow$  kratki spoj na 2-2'



$$U_1 = BI_2 \quad \Rightarrow \quad B = \left. \frac{U_1}{I_2} \right|_{U_2=0}$$

recipročna vrijednost  
prijenosne admitancije na  
kratko  $U_2=0$

$$I_1 = DI_2 \quad \Rightarrow \quad D = \left. \frac{I_1}{I_2} \right|_{U_2=0}$$

omjer prijenosa struja na  
kratko  $U_2=0$

- Prijenosne jednačbe četveropola u matričnome obliku

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$$

$$[a] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \quad \textit{Matrica prijenosnih parametara}$$



- Prijenos signala  $\rightarrow$  s prilaza 2-2' na prilaz 1-1'
- Ovisnost napona  $U_2$  i struje  $I_2$  o  $U_1$  i  $I_1$  =?

$$\begin{aligned} U_1 &= AU_2 + BI_2 \\ \underline{I_1} &= \underline{CU_2 + DI_2} \end{aligned} \quad \Rightarrow \quad \begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$$

- Množenjem s lijeva s inverznom prijenosnom matricom

$$\begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-1} \cdot \begin{bmatrix} U_1 \\ I_1 \end{bmatrix}$$

$$\begin{aligned} U_2 &= \frac{D}{\Delta_A} U_1 - \frac{B}{\Delta_A} I_1 \\ -I_2 &= \frac{C}{\Delta_A} U_1 - \frac{A}{\Delta_A} I_1 \end{aligned}$$

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$$\Delta_A = \begin{vmatrix} A & B \\ C & D \end{vmatrix}$$

- Za recipročne četveropole vrijedi:
- pa je

$$\Delta_A = \begin{vmatrix} A & B \\ C & D \end{vmatrix} = AD - BC = 1$$

$$\begin{aligned} U_2 &= DU_1 - BI_1 \\ -I_2 &= CU_1 - AI_1 \end{aligned}$$

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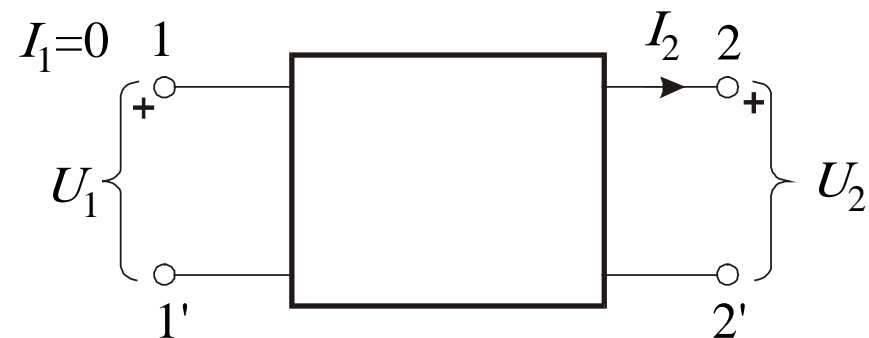
## ■ *Prijenosne jednačbe četveropola*

- Prijenosni parametri → iz 1-1' na prazno i na kratko

$$U_2 = DU_1 - BI_1$$

$$\underline{-I_2 = CU_1 - AI_1}$$

- $I_1=0$  priključnice 1-1' otvorene



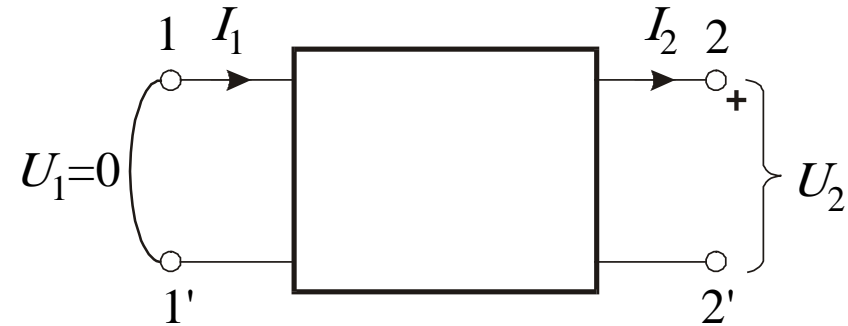
$$U_2 = DU_1 \Rightarrow D = \left. \frac{U_2}{U_1} \right|_{I_1=0}$$

omjer napona na prazno  $I_1=0$

$$-I_2 = CU_1 \Rightarrow C = - \left. \frac{I_2}{U_1} \right|_{I_1=0}$$

recipročna vrijednost  
prijenosne impedancije na  
prazno  $I_1=0$

■  $U_1=0$   $\rightarrow$  kratki spoj na 1-1'  $\rightarrow$



$$U_2 = -BI_1 \Rightarrow B = -\left. \frac{U_2}{I_1} \right|_{U_1=0}$$

recipročna vrijednost  
prijenosne admitancije na  
kratko  $U_1=0$

$$I_2 = AI_1 \Rightarrow A = \left. \frac{I_2}{I_1} \right|_{U_1=0}$$

omjer prijenosa struja na  
kratko  $U_1=0$

# Hibridne jednačbe → $h$ -parametri četveropola

Ako su zadane veličine  $I_1$  i  $U_2$  →  $U_1=?$  i  $I_2=?$

$$I_1 = U_1 y_{11} - U_2 y_{12}$$

⇒

$$I_2 = U_1 y_{21} - U_2 y_{22}$$

$$U_1 = \frac{1}{y_{11}} I_1 + U_2 \frac{y_{12}}{y_{11}}$$

$$I_2 = \frac{y_{21}}{y_{11}} I_1 + \frac{y_{12} y_{21} - y_{11} y_{22}}{y_{11}} U_2$$

$$\begin{aligned} U_1 &= h_{11} I_1 + h_{12} U_2 \\ I_2 &= h_{21} I_1 + h_{22} U_2 \end{aligned}$$

■ ***Hibridne jednačbe četveropola***

$h_{11}, h_{12}, h_{21}, h_{22}$  → ***hibridni  $h$ -parametri četveropola***

- Hibridne jednačbe četveropola u matričnome obliku

$$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$$

$$[h] = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

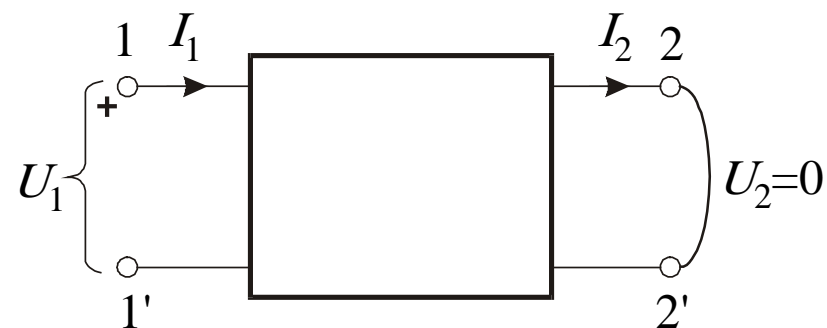
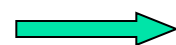
Matrica hibridnih  $h$ -parametara

- Hibridni  $h$ -parametri  $\rightarrow$  iz 1-1' na prazno i 2-2' na kratko

$$U_1 = h_{11}I_1 + h_{12}U_2$$

$$I_2 = h_{21}I_1 + h_{22}U_2$$

- $U_2=0$   $\rightarrow$  kratki spoj na 2-2'



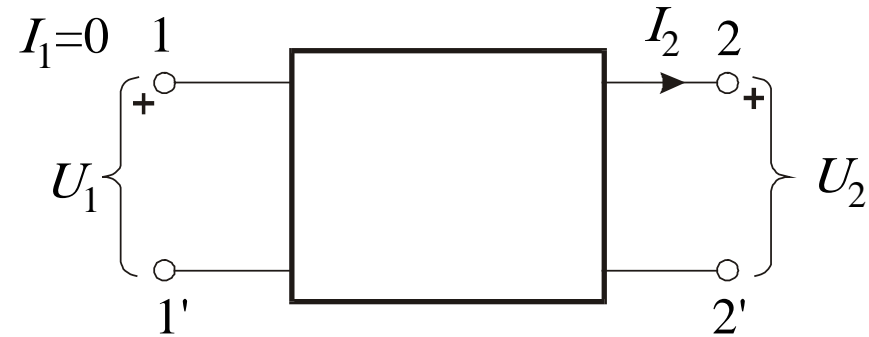
$$U_1 = h_{11}I_1 \quad \Rightarrow \quad h_{11} = \left. \frac{U_1}{I_1} \right|_{U_2=0}$$

Ulazna impedancija na  
kratko  $U_2=0$

$$I_2 = h_{21}I_1 \quad \Rightarrow \quad h_{21} = \left. \frac{I_2}{I_1} \right|_{U_2=0}$$

prijenosna funkcija struje  
na kratko  $U_2=0$

- $I_1=0$  priključnice 1-1' otvorene



$$U_1 = h_{12}U_2 \Rightarrow h_{12} = \left. \frac{U_1}{U_2} \right|_{I_1=0} \quad \text{omjer napona na prazno } I_1=0$$

$$I_2 = h_{22}U_2 \Rightarrow h_{22} = \left. \frac{I_2}{U_2} \right|_{I_1=0} \quad \text{ulazna admitancija na prazno } I_1=0$$



Hibridne jednačbe  $\rightarrow$  g-parametri četveropola

Neka su zadane veličine  $U_1$  i  $I_2 \rightarrow U_2=?$  i  $I_1=?$

- Matrična jednačba hibridnih  $h$ -parametara

$$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix} \rightarrow \begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}^{-1} \cdot \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$$

■ dobiva se

$$I_1 = \frac{h_{22}}{\Delta_h} U_1 - \frac{h_{21}}{\Delta_h} I_2$$
$$U_2 = -\frac{h_{12}}{\Delta_h} U_1 + \frac{h_{11}}{\Delta_h} I_2$$

$$\Delta_h = \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix}$$

$$\Delta_h = h_{11}h_{22} - h_{12}h_{21}$$

$$I_1 = g_{11}U_1 + g_{12}I_2$$
$$U_2 = g_{21}U_1 + g_{22}I_2$$

■ ***Hibridne jednačbe četveropola***

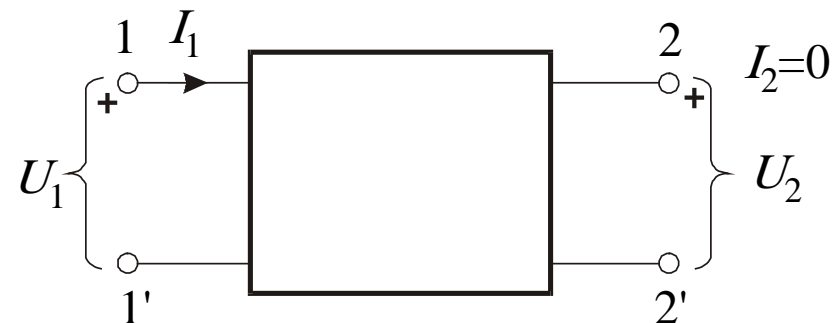
$g_{11}, g_{12}, g_{21}, g_{22} \rightarrow$  ***hibridni g-parametri četveropola***

- Hibridni  $g$ -parametri  $\rightarrow$  iz 2-2' na prazno i 1-1' na kratko

$$I_1 = g_{11}U_1 + g_{12}I_2$$

$$U_2 = g_{21}U_1 + g_{22}I_2$$

- $I_2=0$   $\rightarrow$  priključnice 2-2' otvorene



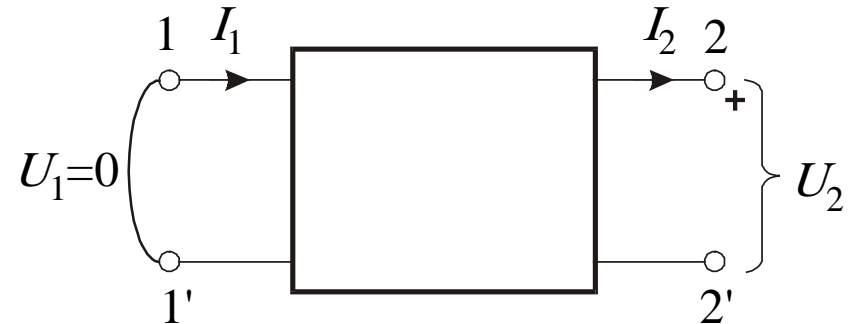
$$I_1 = g_{11}U_1 \Rightarrow g_{11} = \left. \frac{I_1}{U_1} \right|_{I_2=0}$$

Ulazna admitancija na prazno  $I_2=0$

$$U_2 = g_{21}U_1 \Rightarrow g_{21} = \left. \frac{U_2}{U_1} \right|_{I_2=0}$$

prijenosna funkcija napona na prazno  $I_2=0$

- $\underline{U_1=0}$  kratki spoj 1-1'



$$I_1 = g_{12} I_2 \Rightarrow g_{12} = \left. \frac{I_1}{I_2} \right|_{U_1=0}$$

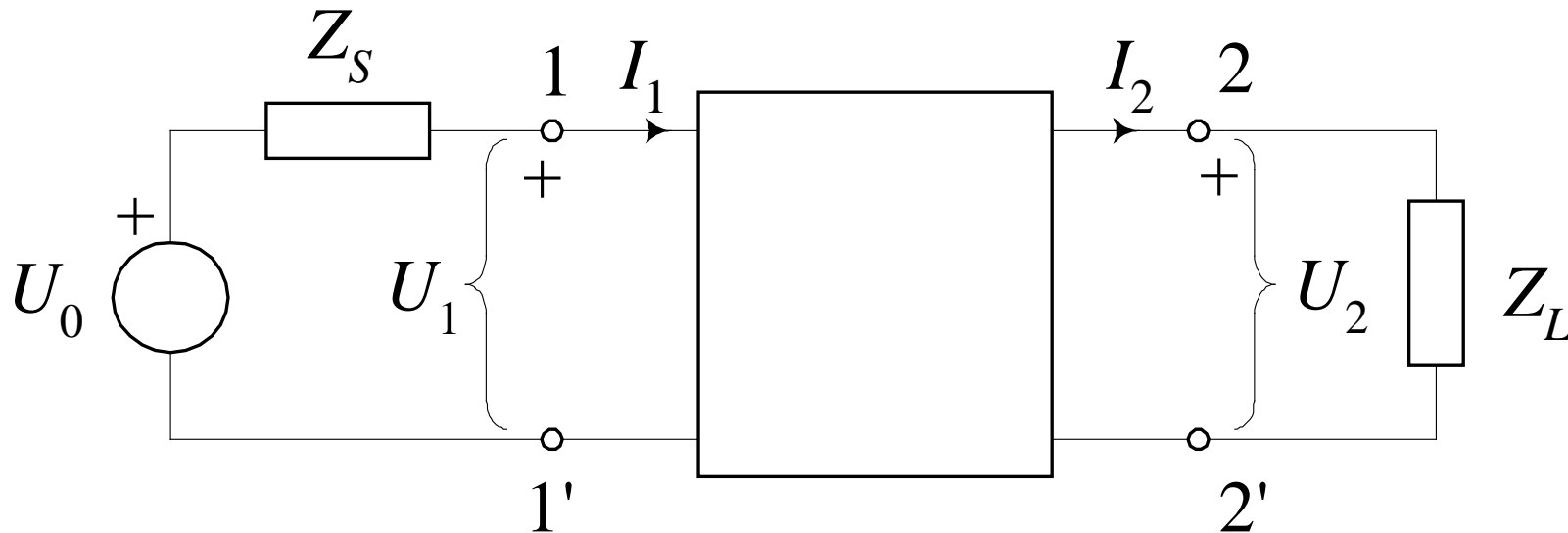
omjer struja na kratko  $U_1=0$

$$U_2 = g_{22} I_2 \Rightarrow g_{22} = \left. \frac{U_2}{I_2} \right|_{U_1=0}$$

ulazna impedancija  
na kratko  $U_1=0$

# Prijenosne i ulazne funkcije četveropola

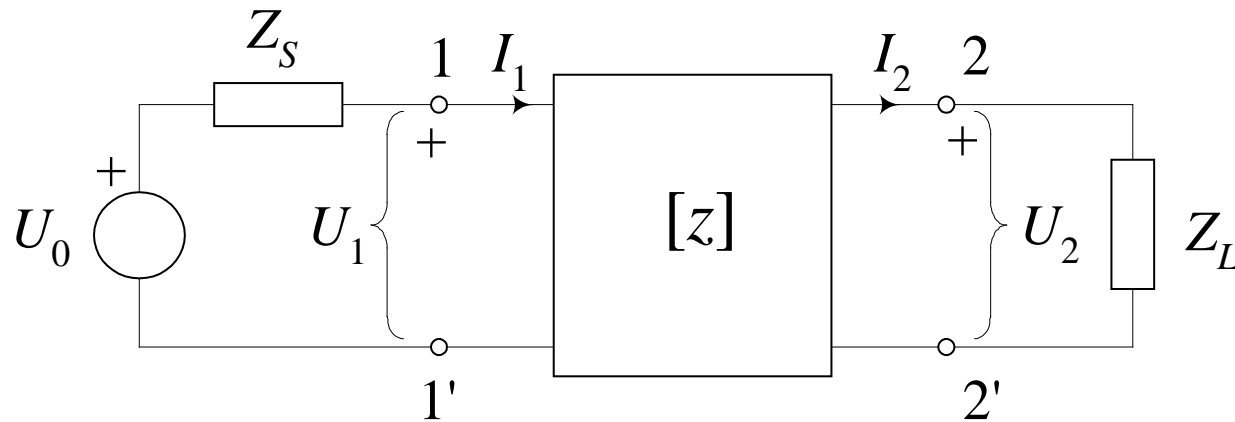
- Četveropol  $\rightarrow$  povezuje izvor signala s opterećenjem.
- Mrežu koja daje pobudni signal moguće je nadomjestiti po Theveninu ili Nortonu.
- Opterećenje  $\rightarrow$  impedancija  $Z_L$  priključena na 2-2'



- Cilj → analizirati mrežu koristeći parametre četveropola  
→ Dobiti rezultate koji vrijede za bilo koji četveropol
- Posebno odrediti funkcije vezane za prilaze četveropola:
  - Prijenosnu funkciju napona  $H_u(s)=U_2(s)/U_1(s)$
  - Prijenosnu funkciju struje  $H_i(s)=I_2(s)/I_1(s)$
  - Ekvivalentnu ulaznu impedanciju  $Z_u(s)=U_1(s)/I_1(s)$
  - Ekvivalentnu izlaznu impedanciju  $Z_i(s)=-U_2(s)/I_2(s)|_{U_0=0}$

# Prijenosne funkcije četveropola

## Prijenosne funkcije izražene z-parametrima



$$\begin{aligned} U_1 &= I_1 z_{11} - I_2 z_{12} \\ U_2 &= I_1 z_{21} - I_2 z_{22} \end{aligned}$$

- Četveropol je na 2-2' zaključen s  $Z_L$

$$U_2 = I_2 Z_L \longrightarrow U_2 = I_1 z_{21} - I_2 z_{22} \longrightarrow I_2 Z_L = I_1 z_{21} - I_2 z_{22}$$

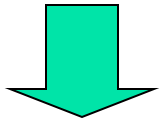
- Prijenosna funkcija struje

$$H_i(s) = \frac{I_2}{I_1} = \frac{z_{21}}{Z_L + z_{22}}$$

- Naponska prijenosna funkcija  $\rightarrow$  iz prve jednadžbe

$$U_1 = I_1 z_{11} - I_2 z_{12} = I_1 z_{11} - I_1 H_i(s) z_{12}$$

$$U_2 = I_1 z_{21} - I_2 z_{22} = I_1 z_{21} - I_1 H_i(s) z_{22}$$

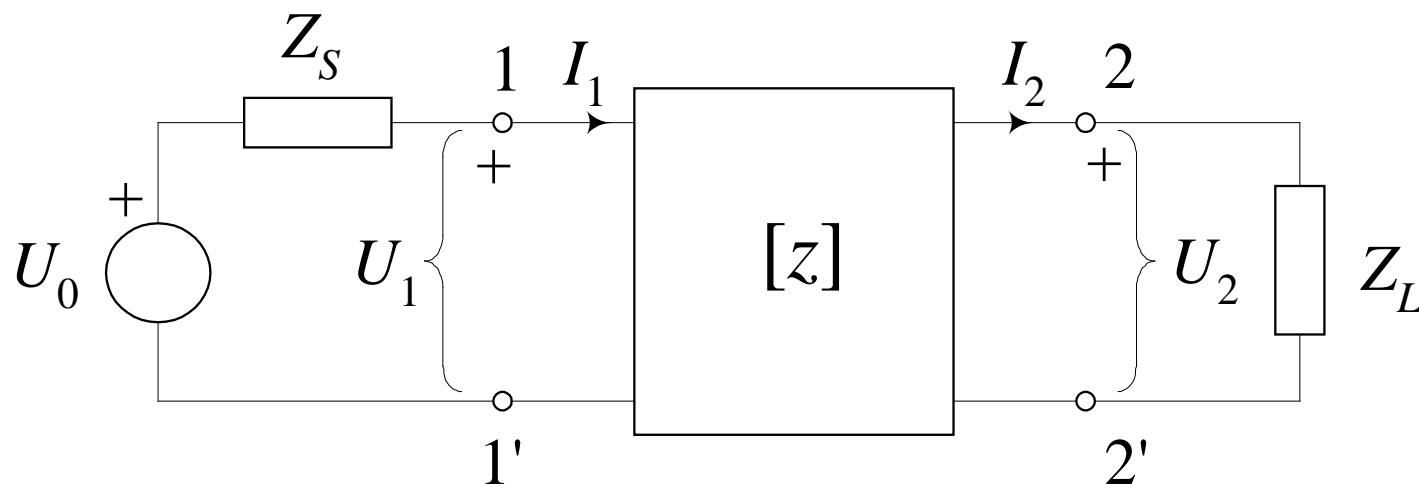


$$H_u(s) = \frac{U_2}{U_1} = \frac{Z_L z_{21}}{z_{11}(z_{22} + Z_L) - z_{12} z_{21}} = \frac{Z_L z_{21}}{\Delta_z + z_{11} Z_L}$$

$$\Delta_z = z_{11} z_{22} - z_{12} z_{21}$$



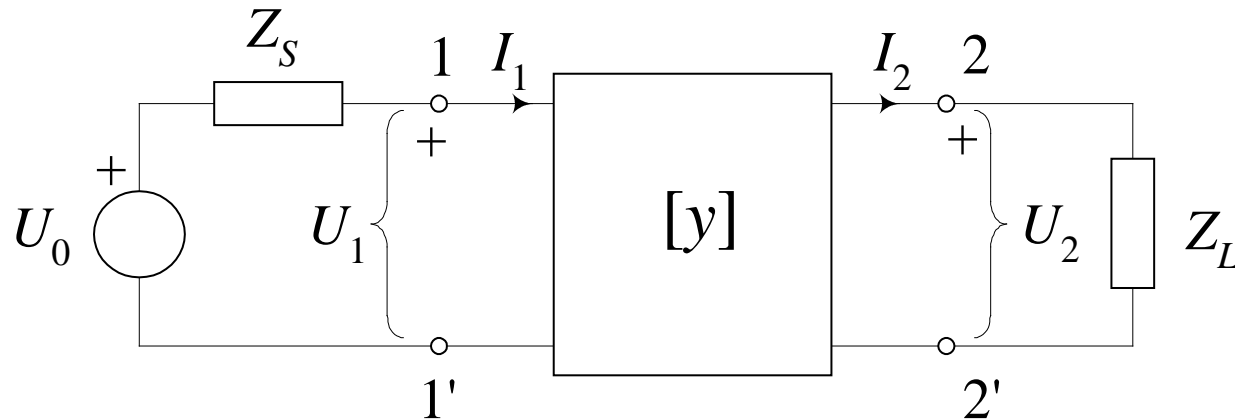
## ■ Prijenosna funkcija $U_2/U_0$



$$U_0(s) = U_1(s) + I_1(s)Z_s$$

$$H_u(s) = \frac{U_2}{U_0} = \frac{Z_L z_{21}}{(z_{11} + Z_s)(z_{22} + Z_L) - z_{12}z_{21}}$$

## Prijenosne funkcije izražene y-parametrima



$$\begin{aligned} I_1 &= y_{11}U_1 - y_{12}U_2 \\ I_2 &= y_{21}U_1 - y_{22}U_2 \end{aligned}$$

- Za napon  $U_2$  i struju  $I_2$  vrijedi

$$U_2 = I_2 Z_L \longrightarrow I_2 = y_{21}U_1 - y_{22}U_2 \longrightarrow U_2 Y_L = y_{21}U_1 - y_{22}U_2$$

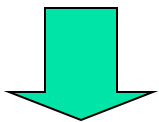
- Prijenosna funkcija napona

$$H_u(s) = \frac{U_2}{U_1} = \frac{y_{21}}{Y_L + y_{22}}$$

## ■ Strujna prijenosna funkcija

$$I_1 = U_1 y_{11} - U_2 y_{12} = U_1 y_{11} - U_1 H_u(s) y_{12}$$

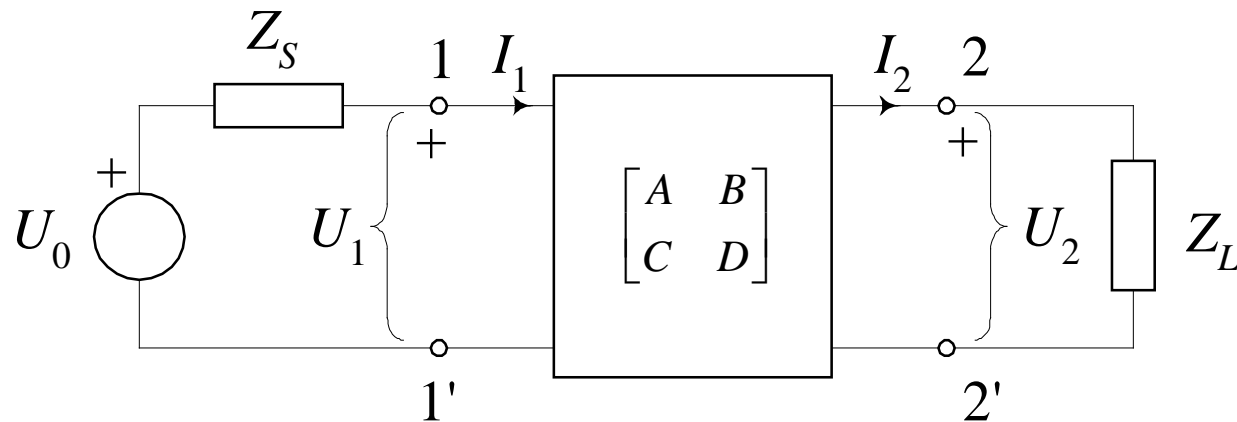
$$I_2 = U_1 y_{21} - U_2 y_{22} = U_1 y_{21} - U_1 H_u(s) y_{22}$$



$$H_i(s) = \frac{I_2}{I_1} = \frac{Y_L y_{21}}{y_{11}(y_{22} + Y_L) - y_{12} y_{21}} = \frac{Y_L y_{21}}{\Delta_y + y_{11} Y_L}$$

$$\Delta_y = y_{11} y_{22} - y_{12} y_{21}$$

# Prijenosne funkcije izražene prijenosnim parametrima



$$\begin{aligned} U_1 &= AU_2 + BI_2 \\ I_1 &= CU_2 + DI_2 \end{aligned}$$

- Za napon  $U_2$  i struju  $I_2$  vrijedi

$$U_2 = I_2 Z_L \longrightarrow U_1 = AU_2 + BI_2 \longrightarrow U_1 = AU_2 + B \frac{U_2}{Z_L}$$

- Prijenosna funkcija napona je

$$H_u(s) = \frac{U_2}{U_1} = \frac{Z_L}{AZ_L + B}$$

- Strujna prijenosna funkcija  $\rightarrow$  iz druge jednačbe

$$I_1 = CU_2 + DI_2 = CI_2Z_L + DI_2$$

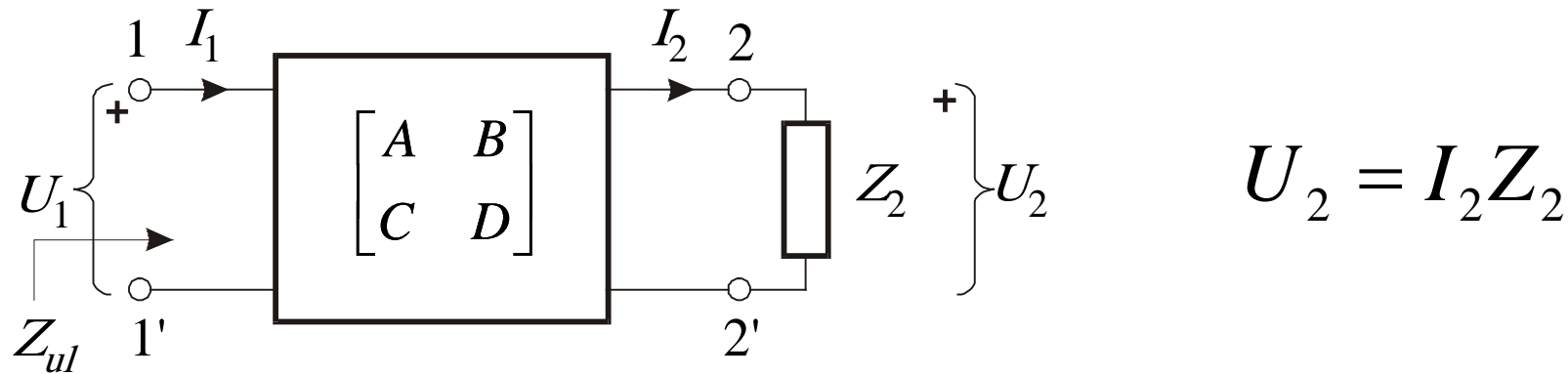
$$H_i(s) = \frac{I_2}{I_1} = \frac{1}{CZ_L + D}$$

- Naponska prijenosna funkcija  $U_2/U_0$

$$U_0 = U_1 + I_1Z_s = AU_2 + B\frac{U_2}{Z_L} + CU_2Z_s + D\frac{U_2}{Z_L}Z_s$$

$$H(s) = \frac{U_2}{U_0} = \frac{Z_L}{AZ_L + B + Z_s(CZ_L + D)}$$

# Ulazne funkcije četveropola → funkcije impedancije



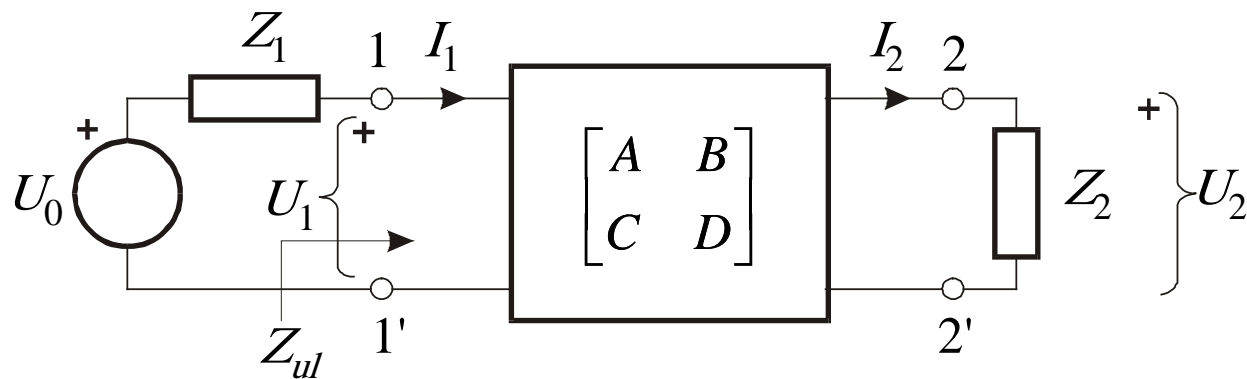
$$U_1 = AU_2 + BI_2 = (AZ_2 + B)I_2$$

$$I_1 = CU_2 + DI_2 = (CZ_2 + D)I_2$$

$$Z_{ul} = \frac{U_1}{I_1} = \frac{AZ_2 + B}{CZ_2 + D}$$

$$Y_{ul} = \frac{1}{Z_{ul}}$$

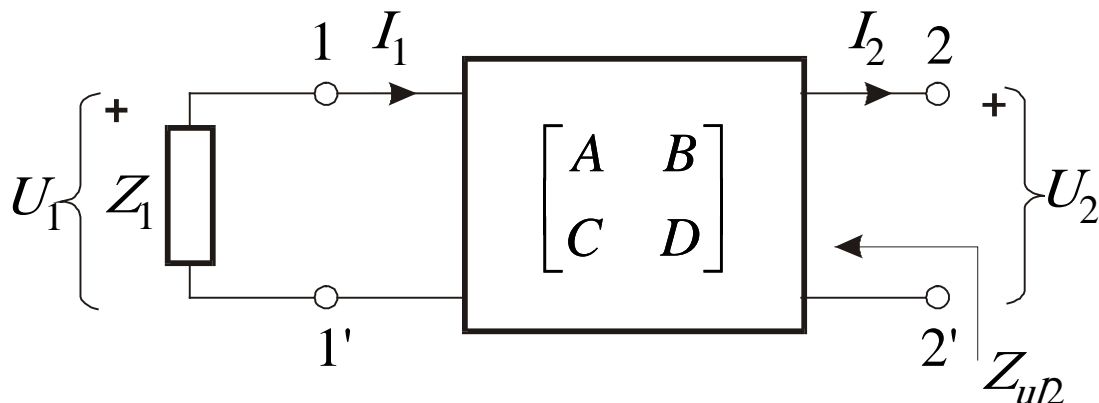
## Prijenosne i ulazne funkcije četveropola



$$I_1 = \frac{U_0}{Z_1 + Z_{ul}}$$

$$U_1 = I_1 Z_{ul} = U_0 \frac{Z_{ul}}{Z_1 + Z_{ul}}$$

Ako je izvor na 2-2'  $\rightarrow Z_{ul2} = Z_{iz}$



$$U_1 = -Z_1 I_1$$

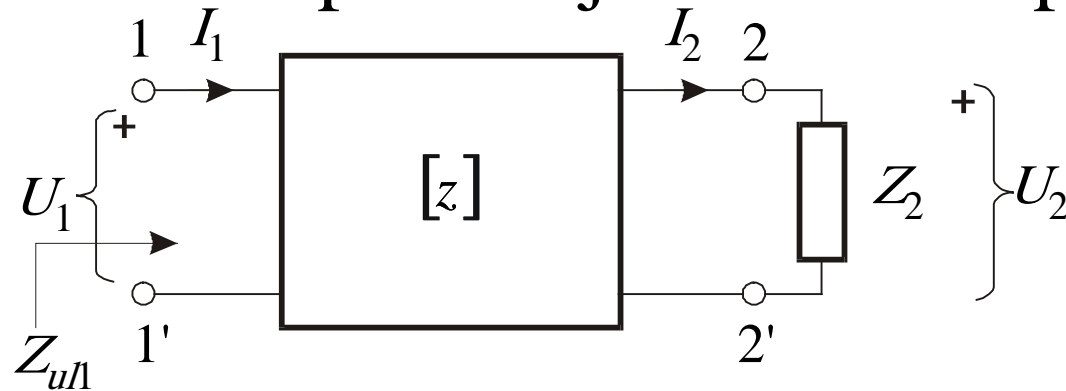
$$U_2 = DU_1 - BI_1 = -(DZ_1 + B)I_1$$

$$-I_2 = CU_1 - AI_1 = -(CZ_1 + A)I_1$$

$$Z_{ul2} = -\frac{U_2}{I_2} = \frac{DZ_1 + B}{CZ_1 + A}$$

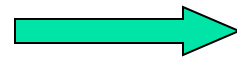


# Ulazna impedancija izražena z-parametrima



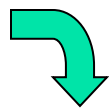
$$U_2 = I_2 Z_2$$

$$U_1 = I_1 z_{11} - I_2 z_{12}$$



$$\frac{U_1}{I_1} = z_{11} - z_{12} \frac{I_2}{I_1}$$

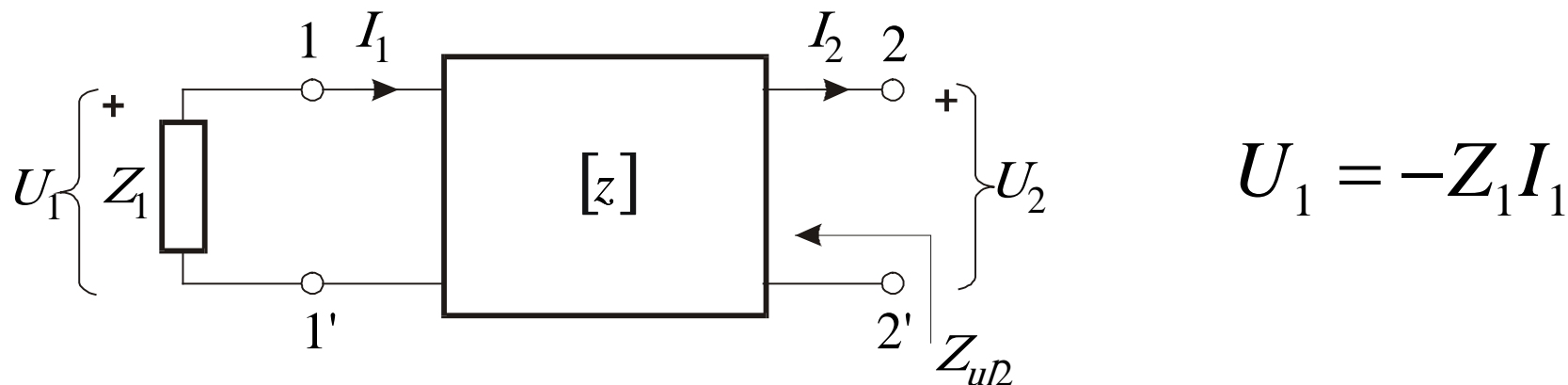
$$\underline{U_2 = I_1 z_{21} - I_2 z_{22}}$$



$$Z_2 I_2 = z_{21} I_1 - z_{22} I_2 \Rightarrow \frac{I_2}{I_1} = \frac{z_{21}}{Z_2 + z_{22}}$$

$$\boxed{Z_{ul1} = \frac{U_1}{I_1} = z_{11} - z_{12} \cdot \frac{z_{21}}{Z_2 + z_{22}}}$$

# Prijenosne i ulazne funkcije četveropola



$$U_1 = -Z_1 I_1$$

$$-Z_1 I_1 = I_1 z_{11} - I_2 z_{12} \Rightarrow \frac{I_1}{I_2} = \frac{z_{12}}{z_{11} + Z_1}$$

$$-\frac{U_2}{I_2} = -z_{21} \frac{I_1}{I_2} + z_{22}$$

$$Z_{ul2} = -\frac{U_2}{I_2} = z_{22} - \frac{z_{12} z_{21}}{z_{11} + Z_1}$$

# Ulazna admitancija izražena y-parametrima

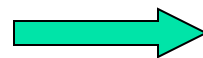
$$I_1 = U_1 y_{11} - U_2 y_{12}$$

$$I_2 = U_1 y_{21} - U_2 y_{22}$$

$$I_2 = U_2 Y_2$$

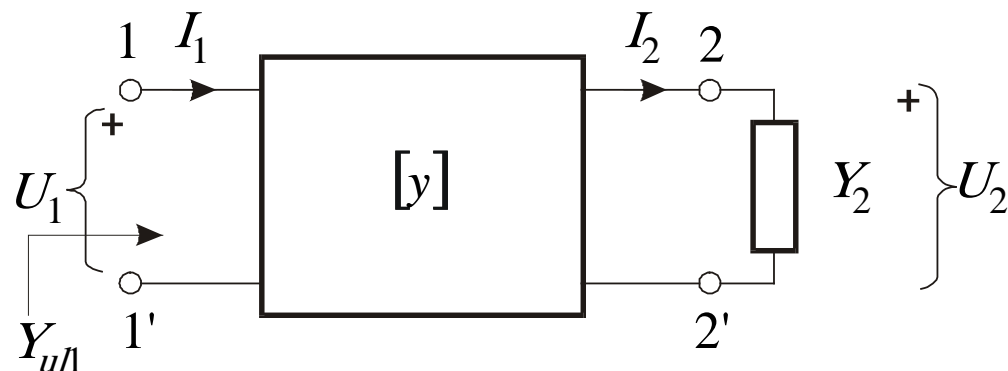
$$Y_{ul1} = \frac{I_1}{U_1} = y_{11} - \frac{U_2}{U_1} y_{12}$$

$$U_2 Y_2 = U_1 y_{21} - U_2 y_{22}$$



$$\frac{U_2}{U_1} = \frac{y_{21}}{Y_2 + y_{22}}$$

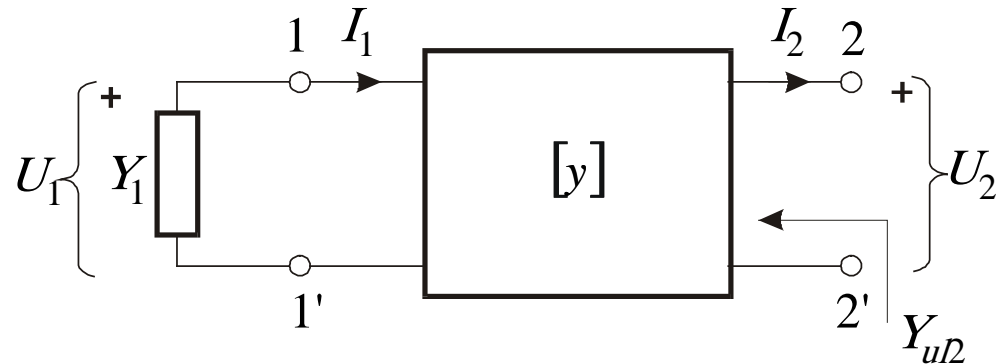
$$Y_{ul1} = y_{11} - \frac{y_{12} y_{21}}{Y_2 + y_{22}}$$



# Ulazna admitancija izražena y-parametrima

$$I_1 = U_1 y_{11} - U_2 y_{12}$$

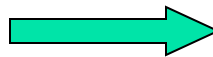
$$I_2 = U_1 y_{21} - U_2 y_{22}$$



$$Y_{ul2} = -\frac{I_2}{U_2} = y_{22} - \frac{U_1}{U_2} y_{21}$$

$$I_1 = -U_1 Y_1$$

$$-U_1 Y_1 = U_1 y_{11} - U_2 y_{12}$$



$$\frac{U_1}{U_2} = \frac{y_{21}}{Y_1 + y_{11}}$$

$$Y_{ul2} = y_{22} - \frac{y_{12} y_{21}}{Y_1 + y_{11}}$$