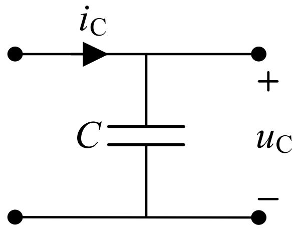
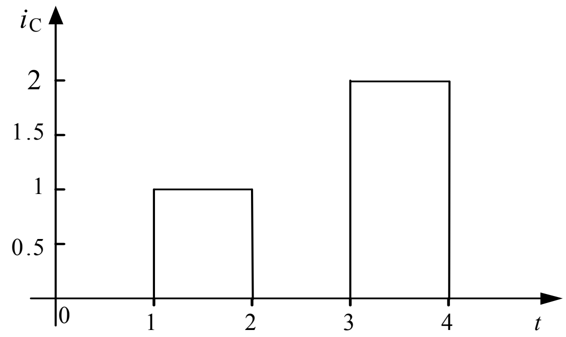
1. MREŽE

ZADATAK 1. Odrediti napon na kapacitetu C=2 ako je zadana struja $i_C(t)$.

a)





Po definiciji je napon na kapacitetu jednak

$$u_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(t)dt = \frac{1}{C}\int_{0}^{t} i_C(t)dt + u_C(0)$$

To znači da će napon na kapacitetu biti jednak integralu protekle struje kroz kapacitet pomnožen s recipročnom vrijednosti kapaciteta plus vrijednost napona na početku integracije. Primjenimo li to, slijedi:

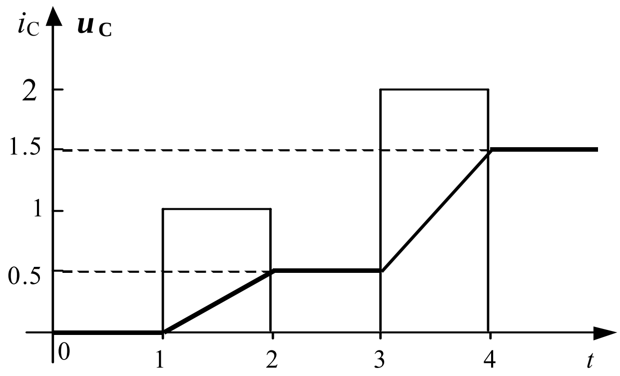
$$u_C(t) = \frac{1}{C}\int_{t1}^{t2} a\, dt + u_C(t1) = \frac{a}{C}t\Big|_{t1}^{t2} + u_C(t1)$$

Uz zadane vrijednosti dobije se:

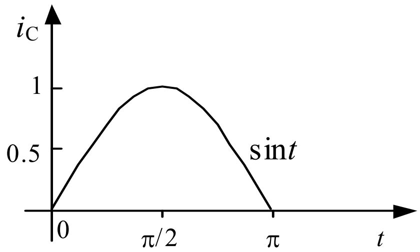
$$u_{C1}(t) = \frac{1}{2}\int_{1}^{2} 1\,dt + 0 = \frac{1}{2}t\Big|_{1}^{2} = \frac{1}{2}(2-1) = \frac{1}{2}$$

$$u_{C2}(t) = \frac{1}{2}\int_{3}^{4} 2\,dt + u_C(3) = \frac{1}{2}2t\Big|_{3}^{4} + \frac{1}{2} = 4 - 3 + \frac{1}{2} = \frac{3}{2}$$

Grafički prikazano rješenje (bold linija):



b)



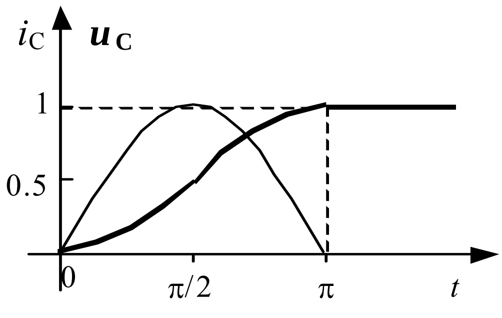
Integracijom funkcije struje dobije se:

$$u_C(t) = \frac{1}{2}\int_0^t sin(t)\,dt = \frac{1}{2}\left[-\cos(t)\right]_0^t = \frac{1}{2}\left[-\cos(t)+1\right]$$

Uz zadane vrijednosti slijedi

$$u_C(\pi) = \frac{1}{2}\left[1+1\right] = 1$$

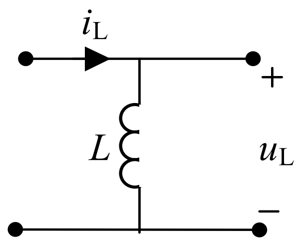
Grafički prikazano rješenje (bold linija):

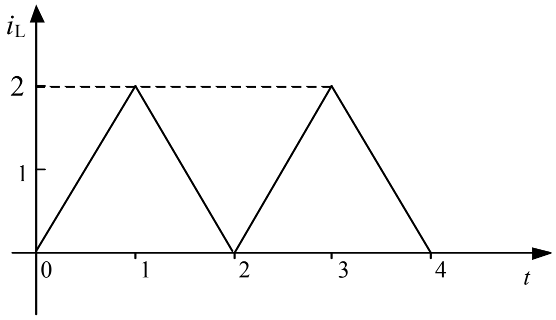


Primjetimo da nakon prestanka protjecanja struje kroz kapacitet na njemu i dalje ostaje dostignuta vrijednost napona.

ZADATAK 2. Odrediti napon na induktivitetu L=1/2 ako je zadana struja $i_L(t)$.

a)





Po definiciji je napon na induktivitetu jednak

$$u_L(t) = L\frac{d\,i_L(t)}{dt}$$

To znači da će napon na induktivitetu biti jednak derivaciji protekle struje kroz induktivitet pomnoženoj s induktivitetom. I ovaj ćemo zadatak riješavati po odsječcima vremena. Slijedi:

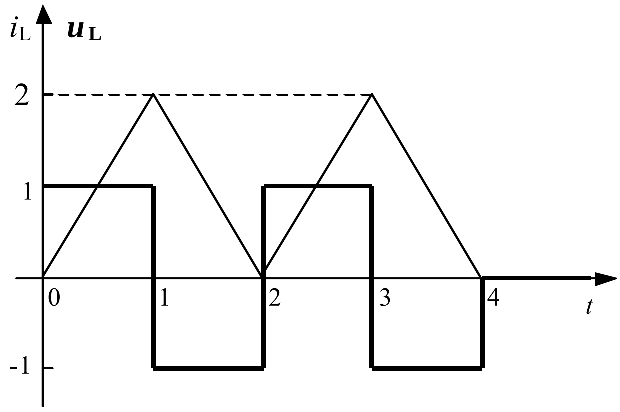
$0 \le t \le 1$ $$u_{L1}(t) = \frac{1}{2}\frac{d}{dt}(2t) = \frac{1}{2}\cdot 2 = 1$$

$1 \le t \le 2$ $$u_{L2}(t) = \frac{1}{2}\frac{d}{dt}(4-2t) = \frac{1}{2}\cdot(-2) = -1$$

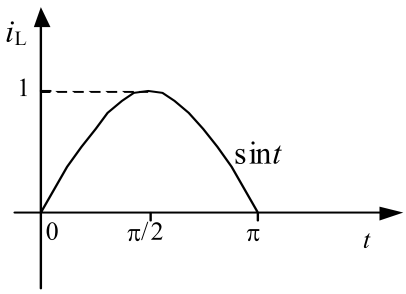
$2 \le t \le 3$ $$u_{L3}(t) = \frac{1}{2}\frac{d}{dt}(2t-4) = \frac{1}{2}\cdot 2 = 1$$

$3 \le t \le 4$ $$u_{L4}(t) = \frac{1}{2}\frac{d}{dt}(8-2t) = \frac{1}{2}\cdot(-2) = -1$$

Grafički prikazano rješenje (bold linija):



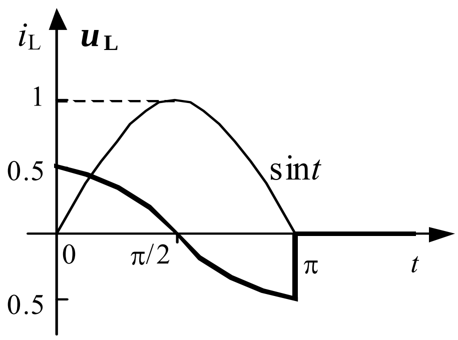
b)



Derivacijom funkcije struje dobije se:

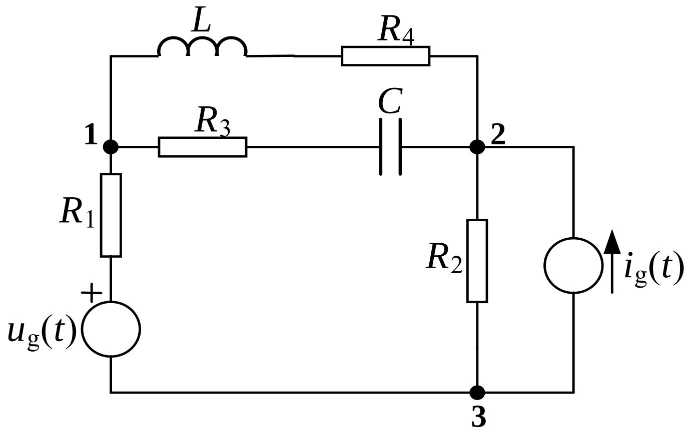
$$u_L(t) = \frac{1}{2}\frac{d}{dt}[\sin(t)] = \frac{1}{2}\cdot\cos(t)$$

Grafički prikazano rješenje (bold linija).

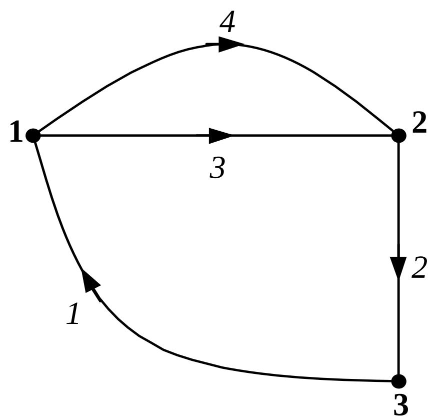


Primjetimo da nakon prestanka protjecanja struje kroz induktivitet nestaje napona na njemu.

ZADATAK 3. Napisati naponsko-strujne relacije grana.



pripadni graf: grane i čvorovi



proizvoljno: broj grane i smjer grane

Plus polaritet grane je pridružen čvoru iz kojeg grana izlazi, a minus čvoru u koji grana ulazi.

Strelica - smjer struje - polaritet napona

Naponsko-strujne relacije grana:

grana **3**

$$\underline{u_3(t) = i_3(t)\cdot R_3 + \frac{1}{C}\int_{-\infty}^{t} i_3(t)dt}$$

grana **4**

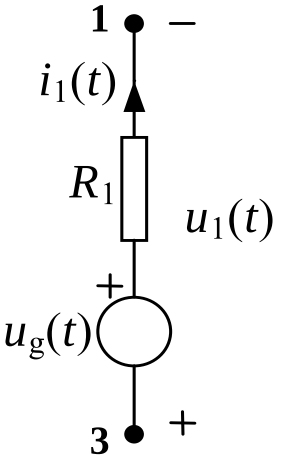
$$\underline{u_4(t) = L\frac{d\, i_4(t)}{dt} + i_4(t)\cdot R_4}$$

grana **1**

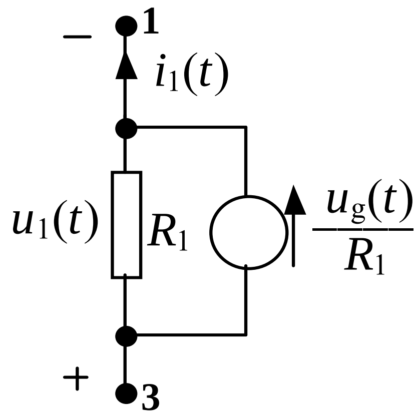
$$\underline{u_1(t) = i_1(t)\cdot R_1 - u_g(t)}\ .$$

struja grane:

$$i_1(t) = \frac{u_1(t)}{R_1} + \frac{u_g(t)}{R_1}\ .$$

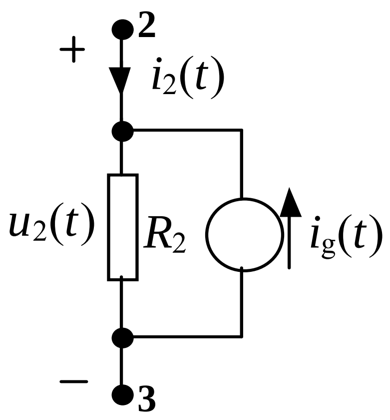


$\Rightarrow$

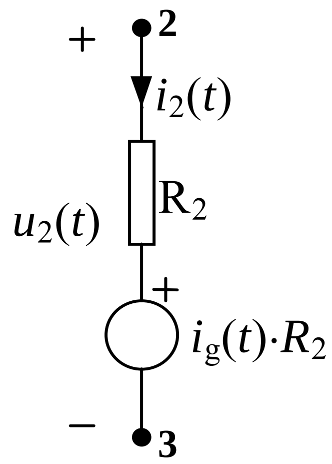


grana **2**

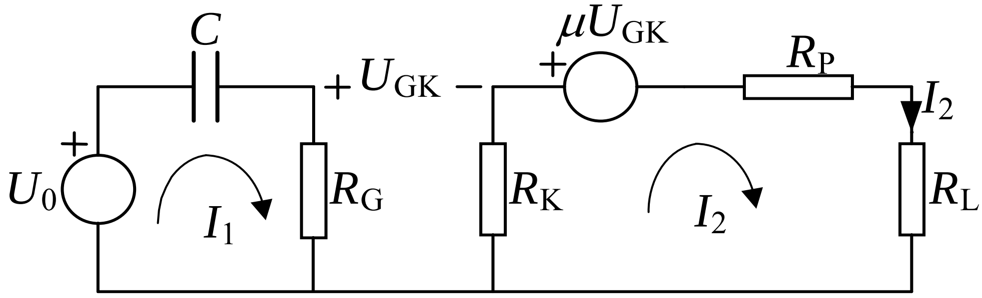
$$\underline{u_2(t) = i_2(t) \cdot R_2 + i_g(t) \cdot R_2}$$



$\Rightarrow$



ZADATAK 4. Odrediti fazor struje I_2. Zadano je: R_G=500kΩ, R_K=200Ω, R_P=11kΩ, R_L=5kΩ, C=0.5μF, μ=60 i $u_0(t)$=2sin(1600πt)V.



zavisni izvor, naponsko zavisni naponski izvor

iznos ovisi o naponu U_{GK} preko faktora zavisnosti μ

Jednadžbe petlji:

$$(1)\quad I_1\left(R_G+\frac{1}{j\omega C}\right)=U_0$$

$$(2)\quad I_2\left(R_K+R_P+R_L\right)=-\mu U_{GK}\quad .$$

Dvije jednadžbe (1) i (2) s tri nepoznanice I_1, I_2 i U_{GK}.

Treća jednadžba:

$$(3)\quad U_{GK}=I_1R_G+I_2R_K\quad .$$

Uvrstivši

$U_{\mathrm{GK}} \rightarrow$ (2) te

iz (1) $\rightarrow I_1 \rightarrow$ (2)

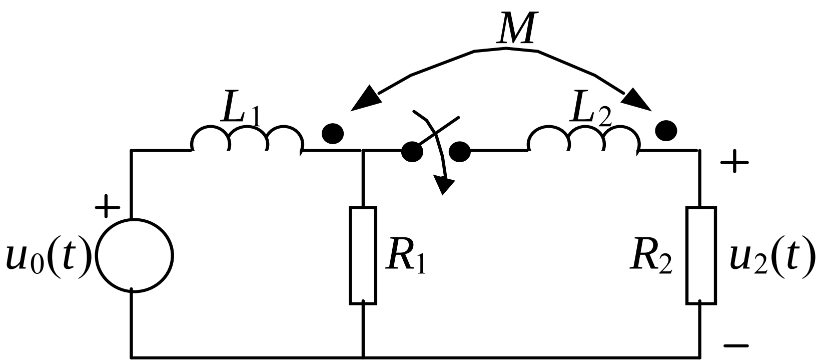
slijedi:

$$I_2 = -\frac{\mu \cdot R_G \cdot U_0}{\left(R_G + \frac{1}{j\omega C}\right)\left(R_P + R_L + (1+\mu)R_K\right)}$$

Fazor struje:

$$\underline{I_2 = -4.255\angle\left(45.38 \cdot 10^{-3}\right)^{\mathrm{o}} \mathrm{mA}}$$

ZADATAK 5. Izračunati napon $u_2(t)$ nakon što se u t=0 zatvori sklopka. Zadano je: L_1=L_2=1, R_1=R_2=1, M=1, $u_0(t)$=5.

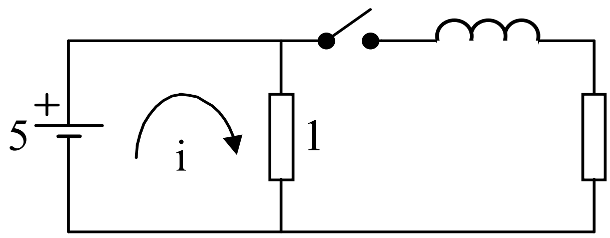


međuinduktivna veza M $$M = k\sqrt{L_1 L_2}$$

sklopka - dva vremenska odsječka - stanje mreže

***t*≤0:**

Istosmjerna pobuda:

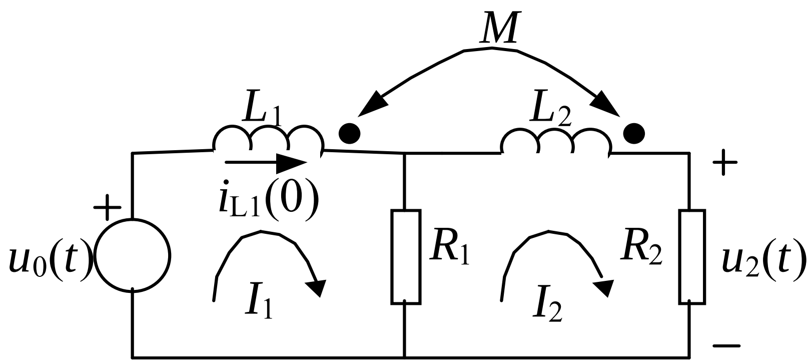


Struja je $$i(t) = \frac{5}{1} = 5$$

$$i(0) = i_{L1}(0) = 5$$

$$i_{L2}(0) = 0$$

***t*>0:**



Jednadžbe petlji:

$$(1)\quad I_1(sL_1+R_1)-I_2R_1+I_2sM=U_0+L_1i_{L1}(0)$$

$$(2)\quad I_2(R_1+R_2+sL_2)-I_1R_1+\left(I_1-\frac{i_{L1}(0)}{s}\right)sM=0$$

Iz (2) struja $\quad I_1=I_2\dfrac{2+s}{1-s}-\dfrac{5}{1-s}$

uvrstimo u (1): $\quad I_2=\dfrac{\frac{5}{s}+5}{5s+1}=\dfrac{\frac{1}{s}+1}{s+\frac{1}{5}}$

Napon: $$U_2(s) = I_2(s) R_2 = \frac{1}{s\left(s+\frac{1}{5}\right)} + \frac{1}{s+\frac{1}{5}}$$

Rastav na parcijalne razlomke:

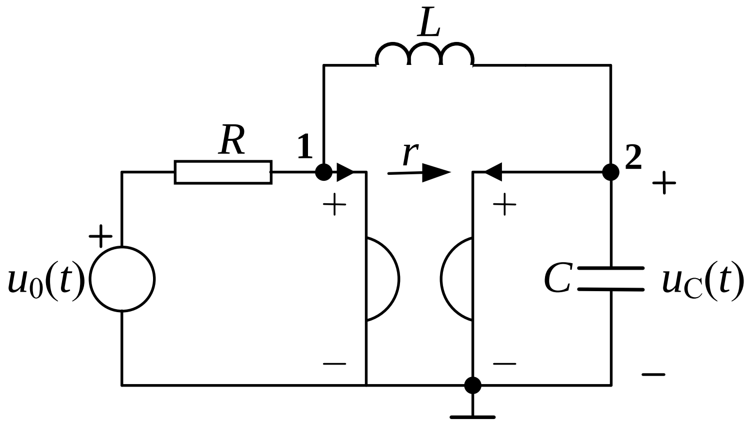
$$U_2(s) = \frac{5}{s} - \frac{5}{s+\frac{1}{5}} + \frac{1}{s+\frac{1}{5}} = \frac{5}{s} - \frac{4}{s+\frac{1}{5}}$$

U vremenskoj domeni

$$\underline{u_2(t) = \left(5 - 4e^{-\frac{1}{5}t}\right) S(t)}$$

Napomena: Ako se u mreži nalazi međuinduktivna veza onda je zadatak bitno lakše riješiti metodom jednadžbi petlji.

ZADATAK 6. Odrediti napon $u_C(t)$ ako je zadano: R=r=1, L=1, C=1, $u_0(t)$=$S(t)$.



Jednadžbe čvorova:

$$(1)\quad U_1\left(\frac{1}{sL}+\frac{1}{R}\right)-U_2\frac{1}{sL}-U_0\frac{1}{R}=-I_{g1}$$

$$(2)\quad U_2\left(\frac{1}{sL}+sC\right)-U_1\frac{1}{sL}=-I_{g2}$$

dvije jednadžbe s četiri nepoznanice

Dodatne jednažbe - jednadžbe giratora:

$$(3)\quad I_{g2}=\frac{U_1}{r} \qquad\qquad (4)\quad I_{g1}=-\frac{U_2}{r}$$

(3), (4) $\rightarrow$ (2), (1)   te   iz (2) $\rightarrow U_1 \rightarrow$ (1)

Dobije se:

$$U_2(s) = \frac{-s+1}{s\left(s^2+2s+1\right)}$$

$$U_2(s) = \frac{A}{s} + \frac{Bs+C}{s^2+2s+1}$$

$$U_2(s) = \frac{1}{s} - \frac{s+3}{s^2+2s+1} = \frac{1}{s} - \frac{s+1}{(s+1)^2} - 2\frac{1}{(s+1)^2}$$

$$U_2(s) = \frac{1}{s} - \frac{1}{s+1} - 2\frac{1}{(s+1)^2}$$

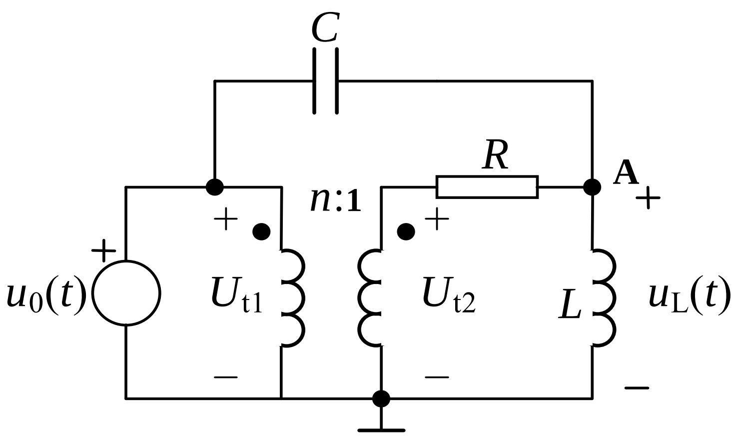
tablične transformacije

U vremenskoj domeni:

$$u_2(t) = \left(1 - e^{-t} - 2te^{-t}\right)S(t)$$

$$\underline{u_C(t) = u_2(t) = \left[1 - e^{-t}(1+2t)\right]\, S(t)}$$

ZADATAK 7. Odrediti napon $u_L(t)$ ako je zadano: R=1/2, L=1, C=1, n=2, $u_0(t)=\delta(t)$.



Jednadžba čvora **A**:

$$(\mathrm{A}) \quad U_A\left(\frac{1}{R}+\frac{1}{sL}+sC\right)-U_{t2}\frac{1}{R}-U_0 sC=0$$

Jednadžba transformatora:

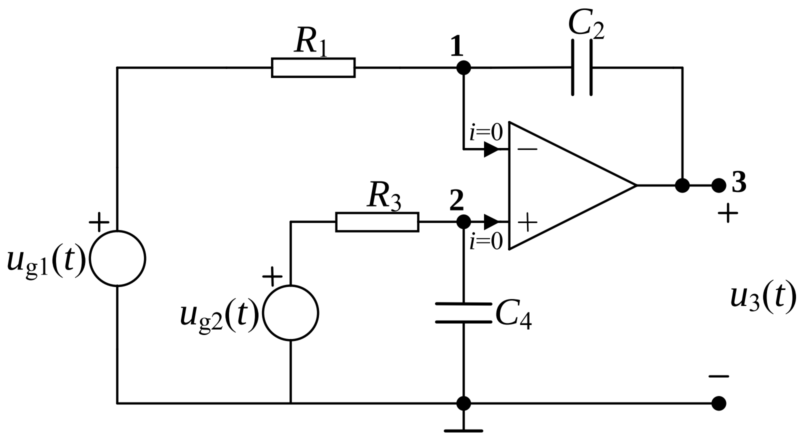
$$(2) \quad U_{t2}=\frac{1}{n}U_{t1}=\frac{1}{n}U_0$$

(2) u (A): $$U_A(s)=\frac{s^2+s}{s^2+2s+1}=\frac{s}{s+1}$$

$$U_A(s)=\frac{s+1-1}{s+1}=1-\frac{1}{s+1}$$

$$\underline{u_L(t)=u_A(t)=\delta(t)-e^{-t}S(t)}$$

ZADATAK 8. Odrediti napon $u_3(t)$ ako je zadano: $R_1=R_3=1$, $C_2=C_4=1$, $u_{g1}(t)=3S(t)$, $u_{g2}(t)=S(t)$.



Jednadžbe čvorova:

$$(1) \quad U_1\left(\frac{1}{R_1}+sC_2\right)-U_3 sC_2=U_{g1}\frac{1}{R_1}$$

$$(2) \quad U_2\left(\frac{1}{R_3}+sC_4\right)=U_{g2}\frac{1}{R_3}$$

Definicijska jednadžba operacijskog pojačala:

$$(3) \quad U_3=A(U_2-U_1)$$

iz (1): $$U_1=U_{g1}\frac{1}{1+sC_2R_1}+U_3\frac{sC_2R_1}{1+sC_2R_1}$$

iz (2): $$U_2=U_{g2}\frac{1}{1+sC_4R_3}$$

u (3):

$$U_3 = \frac{U_{g2}\dfrac{1}{1+sC_4R_3} - U_{g1}\dfrac{1}{1+sC_2R_1}}{\dfrac{1}{A} + \dfrac{sC_2R_1}{1+sC_2R_1}}$$

Uz zadane vrijednosti i $A \to \infty$:

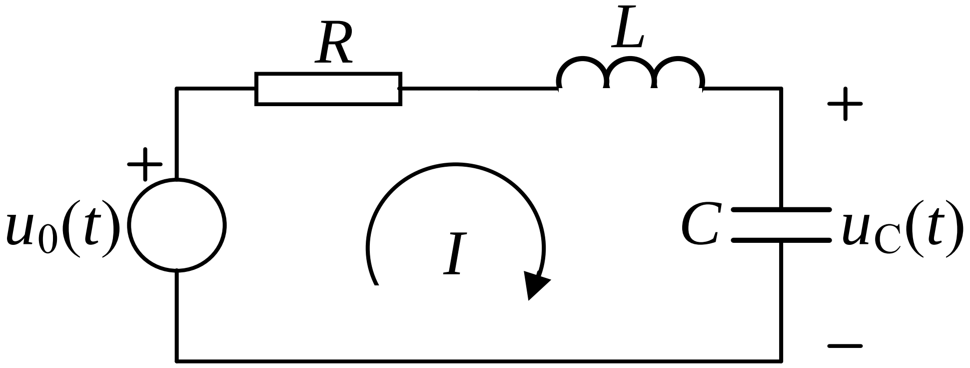
$$U_3(s) = -2\frac{1}{s^2}$$

$$\underline{u_3(t) = -2tS(t)}$$

VJEŽBA: Odrediti isti napon kad su kapaciteti zamijenjeni induktivitetima $L_2 = L_4 = 1$.

Rješenje: $u_3(t) = -2\delta(t)$.

ZADATAK 9. Odrediti napon $u_C(t)$ ako je zadano: R=2, L=2, C=1/4, $u_0(t)=\begin{cases} 2\sin 2t\,, & t\le 0 \\ 2S(t), & t>0 \end{cases}$.



<u>***t*≤0**</u>:

Jednadžba petlje:

$$I\left(R+j\omega L+\frac{1}{j\omega C}\right)=U_0$$

Struja na induktivitetu:

$$I=\frac{j}{-1+j}=\frac{1}{2}-j\frac{1}{2}$$

$$i(t)=\frac{\sqrt{2}}{2}\sin\left(2t-45^0\right)$$

u t=0:

$$\underline{i(0)=i_L(0)=-\frac{1}{2}}$$

Napon na kapacitetu:

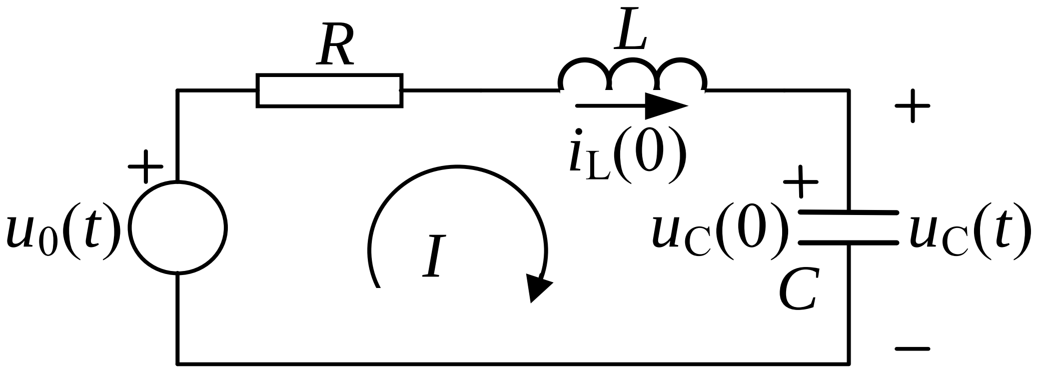
$$U_C = I_C \frac{1}{j\omega C} = -1 - j$$

$$u_C(t) = \sqrt{2}\sin\left(2t + 225^0\right)$$

u t=0:

$$\underline{u_C(0) = -1}$$

<u>***t*>0:**</u>



Jednadžba petlje:

$$I(s)\left(R + sL + \frac{1}{sC}\right) = U_0(s) + Li_L(0) - \frac{u_C(0)}{s}$$

Struja:

$$I(s) = \frac{3-s}{2\left(s^2+s+2\right)}$$

Napon:

$$U_C(s) = I(s)\frac{1}{sC} + \frac{u_C(0)}{s}$$

$$U_C(s) = \frac{2(3-s)}{s\left(s^2+s+2\right)} - \frac{1}{s}$$

$$U_C(s) = 2\frac{1}{s} - 3\frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2} - \sqrt{7}\frac{\frac{\sqrt{7}}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2}$$

$$\underline{u_C(t) = \left[2 - 3e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{7}}{2}t\right) - \sqrt{7}e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{7}}{2}t\right)\right]S(t)}$$

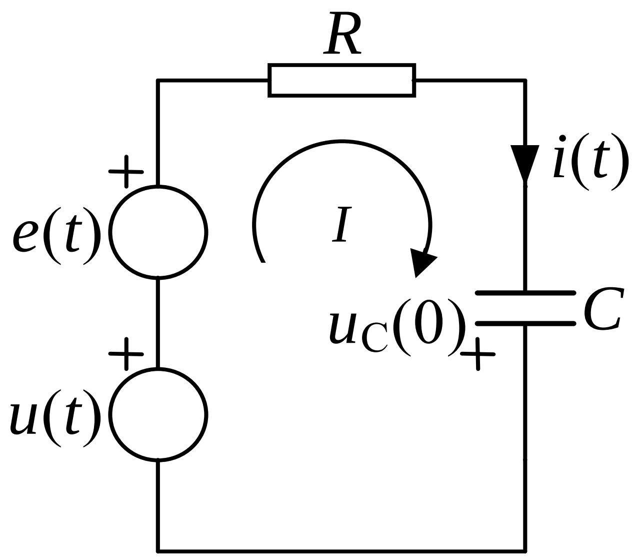
prijelazne pojave i stacionarno stanje (limes kad $t \to \infty$)

VJEŽBA: $u_0(t)=\begin{cases} 2\,, & t\le 0 \\ 2\sin 2t\,S(t), & t>0 \end{cases}$ i C=1/2, R=4, L=2.

Rješenje:

$$u_C(t)=\left[\frac{58}{25}e^{-t}+\frac{70}{25}te^{-t}-\frac{8}{25}\cos(2t)-\frac{6}{25}\sin(2t)\right]S(t)$$

ZADATAK 10. Odrediti slobodni i prisilni odziv mreže $i(t)$ ako je zadano: R=1, C=1, $e(t)$=S(t), $u_C(0)$=1, $u(t)$=sin$t$ $S(t)$.



diferencijalna jednadžba (složenije)

primjena Laplaceove transformacije

Jednadžba petlje:

$$I(s)\left(R+\frac{1}{sC}\right)=E(s)+U(s)+\frac{u_C(0)}{s}$$

Struja:

$$I(s)=\frac{E(s)}{R+\frac{1}{sC}}+\frac{U(s)}{R+\frac{1}{sC}}+\frac{\frac{u_C(0)}{s}}{R+\frac{1}{sC}}$$

Prisilni odziv $I_P(s)$:

$$I_P(s) = \frac{E(s)}{R + \frac{1}{sC}} + \frac{U(s)}{R + \frac{1}{sC}}$$

$$I_P(s) = \frac{1}{s+1} + \frac{s}{\left(s^2+1\right)\,(s+1)}$$

$$\frac{s}{\left(s^2+1\right)\,(s+1)} = \frac{As+B}{s^2+1} + \frac{C}{s+1}$$

$$As^2 + As + Bs + B + Cs^2 + C = s$$

3 jedn. - 3 nepozn. rješenja:

$$\left.\begin{aligned} \mathrm{A} + C &= 0 \\ A + B &= 1 \\ B + C &= 0 \end{aligned}\right\}$$

$$A = \frac{1}{2}$$

$$B = \frac{1}{2}$$

$$C = -\frac{1}{2}$$

$$I_P(s) = \frac{1}{s+1} - \frac{1}{2}\frac{1}{s+1} + \frac{1}{2}\frac{s}{s^2+1} + \frac{1}{2}\frac{1}{s^2+1}$$

$$\underline{i_P(t) = \left[\frac{1}{2}e^{-t} + \frac{1}{2}\cos t + \frac{1}{2}\sin t\right]S(t)}$$

Slobodni odziv $I_S(s)$:

$$I_S(s) = \frac{\frac{u_C(0)}{s}}{R + \frac{1}{sC}}$$

$$I_S(s) = \frac{1}{s\left(1 + \frac{1}{s}\right)} = \frac{1}{s+1}$$

$$\underline{i_S(t) = e^{-t}S(t)}$$

Ukupni odziv:

$$\underline{i_{uk}(t) = i_P(t) + i_S(t)}$$

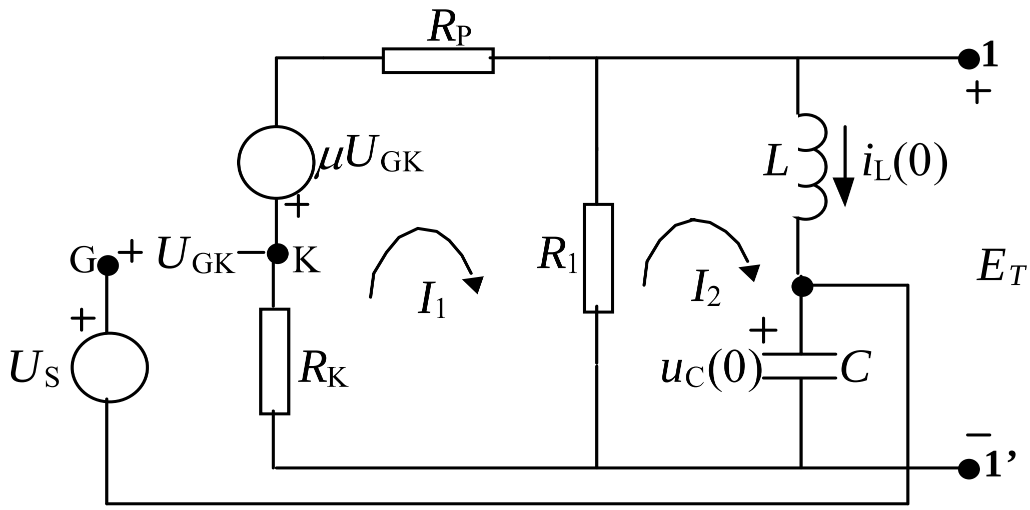
$$\underline{i_{uk}(t) = \frac{1}{2}\left[3e^{-t} + \cos t + \sin t\right]S(t)}$$

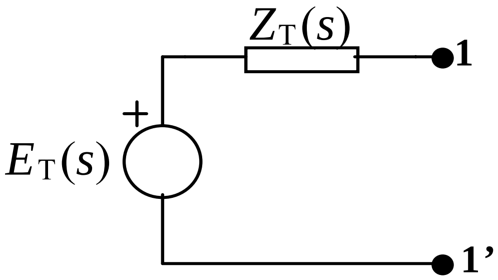
VJEŽBA: Odrediti istu struju ako se kapacitet C zamijeni induktivitetom L.

$$i_P(t)=\left[\frac{E}{R}-\frac{E}{R}e^{-\frac{R}{L}t}+\frac{\omega LU}{R^2+(\omega L)^2}e^{-\frac{R}{L}t}+\frac{RU}{R^2+(\omega L)^2}\sin\omega t-\frac{\omega LU}{R^2+(\omega L)^2}\cos\omega t\right]S(t)$$

$$i_S(t)=i_L(0)e^{-\frac{R}{L}t}\,S(t).$$

ZADATAK 11. Zadanu mrežu nadomjestiti po Theveninu na stezaljkama 1-1'.





Theveninov napon $E_T(s)$:

$$E_T(s) = I_2\left(sL + \frac{1}{sC}\right) - Li_L(0) + \frac{u_C(0)}{s}$$

Jednadžbe petlji:

$$(1) \quad I_1\left(R_P + R_1 + R_K\right) - I_2 R_1 = -\mu U_{GK}$$

$$(2) \quad I_2\left(R_1 + sL + \frac{1}{sC}\right) - I_1 R_1 = L i_L(0) - \frac{u_C(0)}{s}$$

$$U_{GK} = U_S + I_2 \frac{1}{sC} + \frac{u_C(0)}{s} + I_1 R_K$$

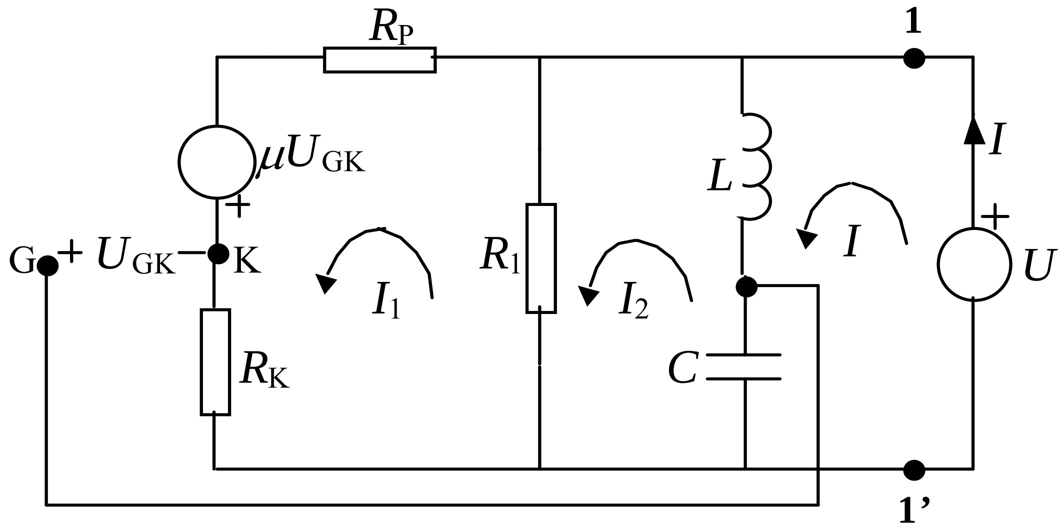
$U_{GK} \rightarrow (1)$

iz (2) $\rightarrow I_1 \rightarrow (1)$

iz (1) $\rightarrow I_2 \rightarrow E_T(s)$

Theveninova impedancija $Z_T(s)$:

- ugasiti nezavisne izvore i početna stanja
- na stezaljke spojiti naponski izvor napona U koji daje struju $I \rightarrow Z_T(s)=U/I$.



Jednadžbe petlji:

$$(1) \quad I_1(R_P + R_1 + R_K) - I_2 R_1 = \mu U_{GK}$$

$$(2) \quad I_2\left(R_1 + sL + \frac{1}{sC}\right) - I_1 R_1 - I\left(sL + \frac{1}{sC}\right) = 0$$

$$(3) \quad I\left(sL + \frac{1}{sC}\right) - I_2\left(sL + \frac{1}{sC}\right) = U$$

tri jednadžbe s pet nepoznanica

$$U_{GK} = (I - I_2)\frac{1}{sC} - I_1 R_K$$

$U_{\mathrm{GK}} \rightarrow (1);$

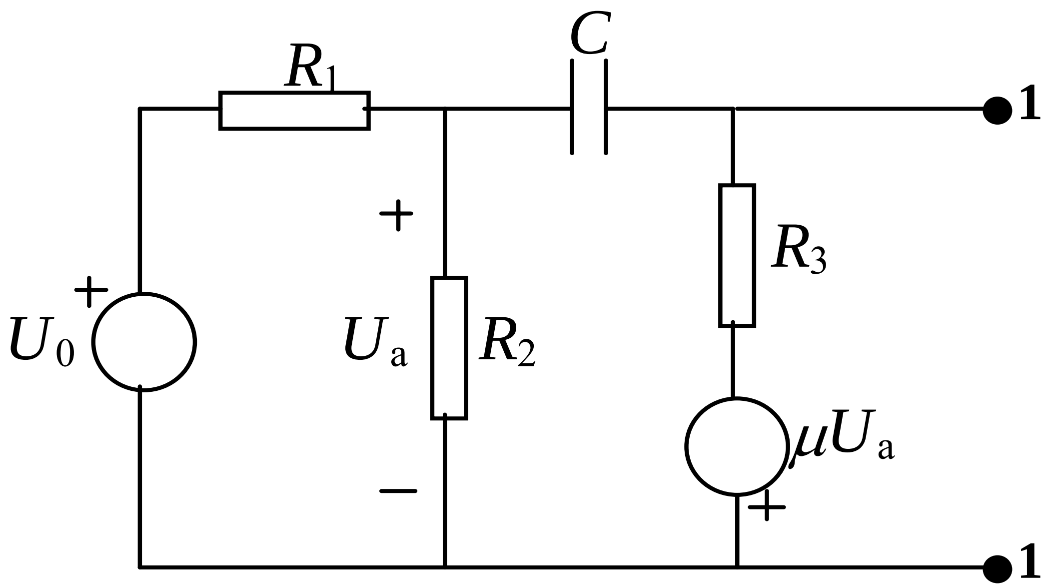
iz $(3) \rightarrow I_2 \rightarrow (1),(2);$

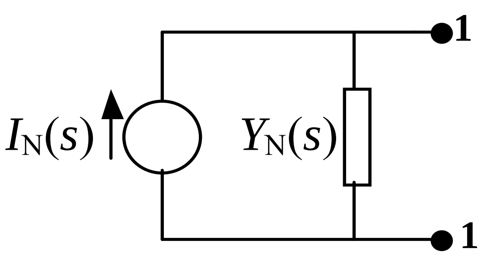
iz $(2) \rightarrow I_1 \rightarrow (1)$

$$\underline{Z_T(s) = \frac{U(s)}{I(s)}} \quad .$$

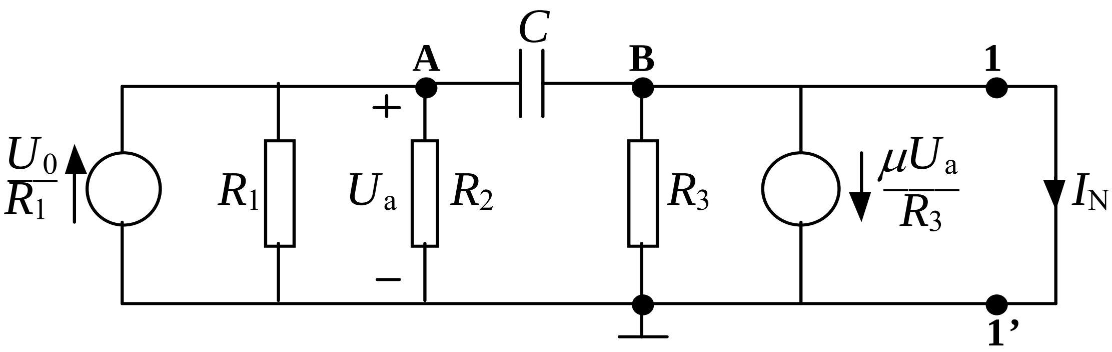
$E_{\mathrm{T}}(s)$ i $Z_{\mathrm{T}}(s)$ su funkcije frekvencije.

ZADATAK 12. Zadanu mrežu nadomjestiti po Nortonu na stezaljkama 1-1'. Zadano je: R_1=4, R_2=2, R_3=1, C=2, μ=60, $U_0(s)$=2/s.





Nortonova struja $I_N(s)$:



Vrijedi: $U_a = U_A$ i $U_B = 0$

Jednadžbe čvorova:

$$\text{(A)} \quad U_A\left(\frac{1}{R_1}+\frac{1}{R_2}+sC\right)=\frac{U_0}{R_1}$$

$$\text{(B)} \quad -U_A sC = -I_N - \frac{\mu U_A}{R_3}$$

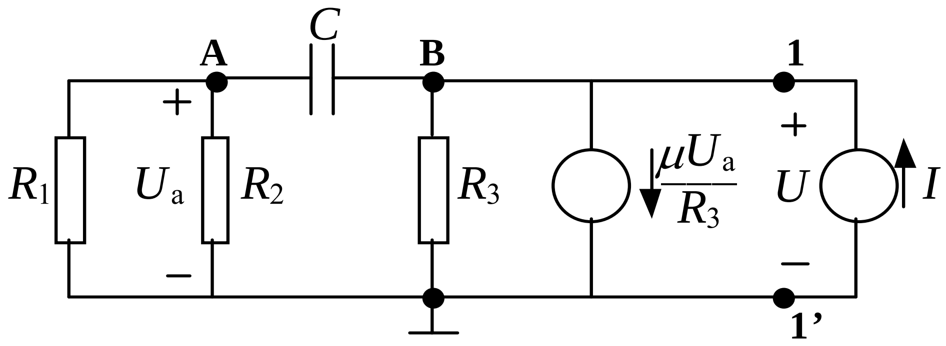
iz (A) $\rightarrow U_A \rightarrow$ (B)

$$I_N = U_0 \frac{sC - \frac{\mu}{R_3}}{1+\frac{R_1}{R_2}+sCR_1}$$

$$\underline{I_N = \frac{4s-120}{s(8s+3)}}$$

Nortonova admitancija $Y_N(s)$.

- ugasiti nezavisne izvore i početna stanja
- na stezaljke spojiti strujni izvor struje I na kojem je napon $U \rightarrow Y_N(s)=I/U$.



Vrijedi: $U_a = U_A$ i $U_B = U$

Jednadžbe čvorova:

$$(A) \quad U_A\left(\frac{1}{R_1}+\frac{1}{R_2}+sC\right)-U_B sC=0$$

$$(B) \quad -U_A sC+U_B\left(\frac{1}{R_3}+sC\right)=I-\frac{\mu U_A}{R_3} \quad .$$

iz (A) napon $U_A \rightarrow$ u (B)

$$I = U_B \left[\frac{1}{R_3} + sC + \frac{sC\left(\dfrac{\mu}{R_3} - sC\right)}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + sC} \right]$$

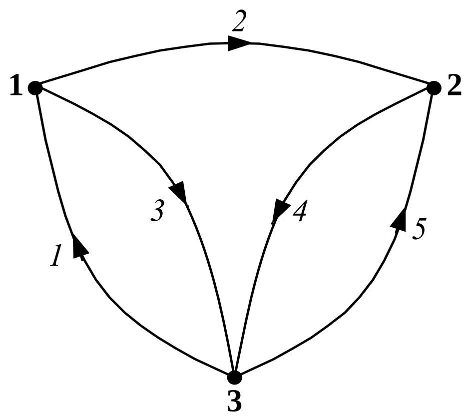
$$\underline{Y_N(s) = \frac{I(s)}{U_B(s)} = \frac{494s + 3}{8s + 3}}$$

VJEŽBA: Za isti zadatak odrediti Theveninove parametre. Rješenja provjeriti preko relacija pretvorbi izvora.

Napomena: Iako je u ovim zadacima nadomještavanje mreže po Theveninu bilo određeno metodom jednadžbi petlji, a nadomještavanje po Nortonu metodom jednadžbi čvorova to nije pravilo. Rješavanje zadane mreže vrši se na jednostavniji način tako da ima manje jednadžbi, bilo petlji bilo čvorova. Iznimka su mreže koje sadržavaju operacijsko pojačalo koje se rješavaju metodom jednadžbi čvorova i mreže koje sadržavaju međuinduktivnu vezu koje se rješavaju metodom jednadžbi petlji.

2. TOPOLOŠKA ANALIZA MREŽA

ZADATAK 13. Odrediti matricu incidencije te za jedno odabrano stablo temeljnu spojnu i temeljnu rastavnu matricu. Kojeg je ranga i nuliteta graf?



a) matrica incidencije V

građena je:

$$\begin{array}{cc} & \textit{G R A N E} \\ \mathbf{V} = & \begin{bmatrix} & & \\ & & \\ & & \end{bmatrix} \end{array} \begin{array}{c} \textit{ČVO} \\ \textit{RO} \\ \textit{VI} \end{array}$$

element u matrici je:

1 ... grana izlazi iz čvora

-1 ... grana ulazi u čvor

0 ... grana nije u čvoru

$$\mathbf{V} = \begin{array}{c} \begin{array}{ccccc} \mathit{1} & \mathit{2} & \mathit{3} & \mathit{4} & \mathit{5} \end{array} \\ \begin{bmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 \\ 1 & 0 & -1 & -1 & 1 \end{bmatrix} \end{array} \begin{array}{c} \mathbf{1} \\ \mathbf{2} \\ \mathbf{3} \end{array}$$

- $\mathbf{V_R}$ - nema jedan redak

b) temeljna spojna matrica S

građena je:

$$S = \begin{array}{cl} \textit{G R A N E} & \\ \left[\begin{array}{c} \\ \\ \\ \end{array}\right] & \begin{array}{l} \textit{TEME} - \\ \textit{LJNE} \\ \textit{PE} - \\ \textit{TLJE} \end{array} \end{array}$$

element u matrici je:

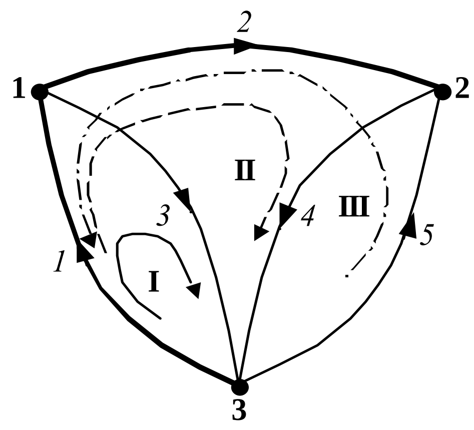
1 ... grana u smjeru petlje

-1 ... grana protivna smjeru

0 ... grana nije u petlji

stablo - 1, 2:

Sustav temeljnih petlji → temeljna spojna matrica:



$$\mathbf{S} = \begin{array}{cc} \begin{array}{ccccc} \mathit{1} & \mathit{2} & \mathit{3} & \mathit{4} & \mathit{5} \end{array} & \\ \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 \\ -1 & -1 & 0 & 0 & 1 \end{array}\right] & \begin{array}{c} \mathrm{I} \\ \mathrm{II} \\ \mathrm{III} \end{array} \end{array}$$

c) temeljna rastavna matrica Q (matrica rezova)

građena je: element u matrici je:

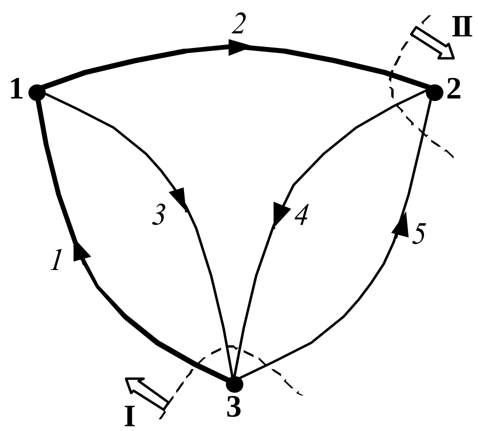
$$
Q = \begin{array}{c}
\text{G R A N E} \\
\left[\begin{array}{c} \quad \quad \quad \quad \end{array} \right]
\end{array}
\begin{array}{l} \textit{TEME –} \\ \textit{LJNI} \\ \textit{REZO –} \\ \textit{VI} \end{array}
$$

1 ... grana u smjeru reza

-1 ... grana protivna smjeru

0 ... grana nije u rezu

Sustav temeljnih rezova → temeljna matrica rezova:

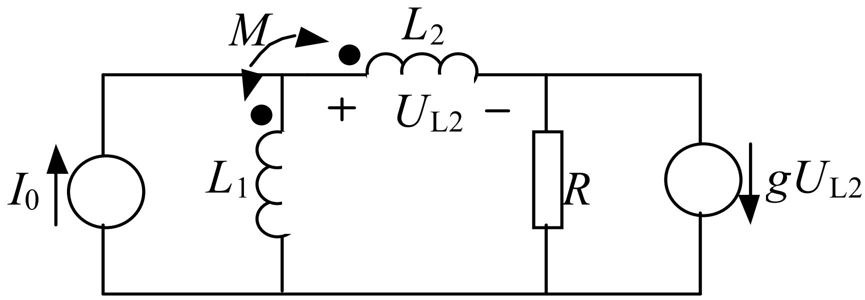


$$
\mathbf{Q} = \begin{array}{c} \begin{array}{ccccc} \mathit{1} & \mathit{2} & \mathit{3} & \mathit{4} & \mathit{5} \end{array} \\ \left[\begin{array}{cc|ccc} 1 & 0 & -1 & -1 & 1 \\ 0 & 1 & 0 & -1 & 1 \end{array} \right] \end{array} \begin{array}{l} \\ \text{I} \\ \text{II} \end{array}
$$

Rang grafa je broj stablenih grana: <u>R=2</u>.

Nulitet grafa je broj spojnih grana: <u>N=3</u>.

ZADATAK 14. Topološkom analizom odrediti temeljni sustav jednadžbi petlji.



Temeljni sustav jednadžbi petlji u matričnom obliku

$$\mathbf{Z_m}(s)\cdot\mathbf{I_m}(s)=\mathbf{E_m}(s)$$

$$\mathbf{Z_m}(s)=\mathbf{S}\cdot\mathbf{Z_b}(s)\cdot\mathbf{S^T}$$

$$\mathbf{E_m}(s)=-\mathbf{S}\cdot\mathbf{E_b}(s)$$

S temeljna spojna matrica

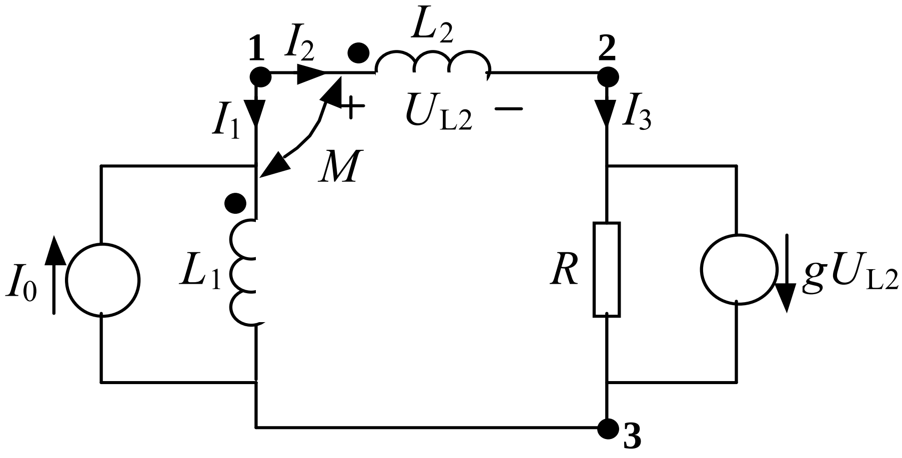
$\mathbf{S^T}$ transponirana matrica **S**

$\mathbf{Z_b}$(s) matrica impedancija grana

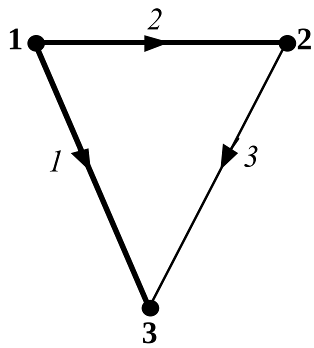
$\mathbf{E_b}(s)$ matrica nezavisnih izvora i početnih stanja

Matrice $\mathbf{Z_b}(s)$ i $\mathbf{E_b}(s)$ - iz naponsko-strujnih relacija grana.

Graf s tri grane (regularne matrice):



Pripadni graf:



stablene grane *1* i *2*   spona grana 3

Temeljna spojna matrica:

$$S = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix}$$

Naponsko-strujne relacije grana su:

$$U_1 = (I_1 + I_0)sL_1 + I_2sM = I_1sL_1 + I_2sM + I_0sL_1$$

$$U_2 = I_2sL_2 + (I_1 + I_0)sM = I_1sM + I_2sL_2 + I_0sM$$

$$U_3 = (I_3 - gU_{L2})R = (I_3 - gU_2)R = I_3R - gR[I_1sM + I_2sL_2 + I_0sM] =$$

$$= -I_1gRsM - I_2gRsL_2 + I_3R - I_0gRsM$$

elementi mreže, struje i naponi grana

Zavisne veličine obavezno je izraziti preko prije spomenutih veličina.

Oni članovi koji množe struje grana dolaze u matricu $\mathbf{Z_b}(s)$,

ostali (koji množe nezavisne izvore i početna stanja) u matricu $\mathbf{E_b}(s)$.

Dobivene matrice:

$$Z_{\mathbf{b}}(s) = \begin{bmatrix} sL_1 & sM & 0 \\ sM & sL_2 & 0 \\ -gRsM & -gRsL_2 & R \end{bmatrix} \qquad E_{\mathbf{b}}(s) = \begin{bmatrix} I_0sL_1 \\ I_0sM \\ -I_0gRsM \end{bmatrix}$$

Množenjem slijedi:

$$\mathbf{Z_m}(s) = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} sL_1 & sM & 0 \\ sM & sL_2 & 0 \\ -gRsM & -gRsL_2 & R \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$\underline{\mathbf{Z_m}(s) = \left[R + sL_1 + sL_2(1-gR) - sM(2-gR)\right]}$$

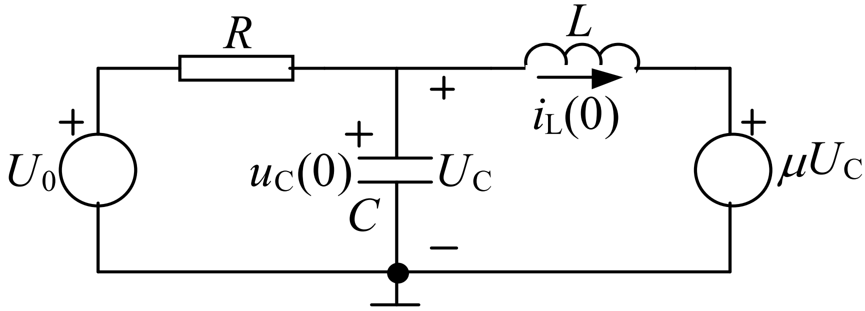
$$\mathbf{E_m}(s) = \begin{bmatrix} 1 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} I_0 sL_1 \\ I_0 sM \\ -I_0 gRsM \end{bmatrix}$$

$$\underline{\mathbf{E_m}(s) = \left[I_0\left(sL_1 - sM(1-gR)\right)\right]}$$

$$\underline{\mathbf{I_m}(s) = \left[I_1\right]}$$

VJEŽBA: Doći do istog rezultata direktno pišući jednadžbu petlje (bez topološke analize).

ZADATAK 15. Topološkom analizom odrediti temeljni sustav jednadžbi čvorova.



Temeljni sustav jednadžbi čvorova u matričnom obliku

$$\mathbf{Y_n}(s)\cdot\mathbf{U_n}(s)=\mathbf{I_n}(s)$$

$$\mathbf{Y_n}(s)=\mathbf{V_R}\cdot\mathbf{Z_b^{-1}}(s)\cdot\mathbf{V_r^T}$$

$$\mathbf{I_n}(s)=\mathbf{V_R}\cdot\mathbf{Z_b^{-1}}(s)\cdot\mathbf{E_b}(s)$$

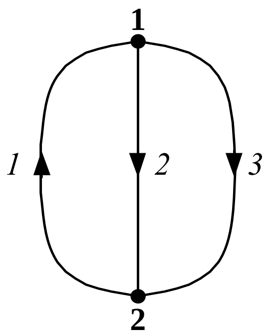
$\mathbf{V_R}$ reducirana matrica incidencije

$\mathbf{V_R}^{\mathbf{T}}$ transponirana matrica $\mathbf{V_R}$

$\mathbf{Z_b}^{-1}$(s) inverzna matrica impedancija grana

$\mathbf{E_b}(s)$ matrica nezavisnih izvora i početnih stanja.
Matrice $\mathbf{Z_b}(s)$ i $\mathbf{E_b}(s)$ - iz naponsko-strujnih relacija grana.

Graf:



Reducirana matrica incidencije

$$\mathbf{V_R} = \begin{bmatrix} -1 & 1 & 1 \end{bmatrix}$$

Naponsko-strujne relacije grana:

$$U_1 = I_1 R - U_0$$

$$U_2 = I_2 \frac{1}{sC} + \frac{u_C(0)}{s}$$

$$U_3 = I_3 sL - Li_L(0) + \mu U_C = I_3 sL - Li_L(0) + \mu U_2 = I_2 \frac{\mu}{sC} + I_3 sL - Li_L(0) + \frac{\mu u_C(0)}{s}$$

Oni članovi koji množe struje grana - u matricu $\mathbf{Z_b}(s)$,

ostali (koji množe nezavisne izvore i početna stanja) u matricu $\mathbf{E_b}(s)$.

Dobivene matrice:

$$Z_{\mathbf{b}}(s)=\begin{bmatrix} R & 0 & 0 \\ 0 & \dfrac{1}{sC} & 0 \\ 0 & \dfrac{\mu}{sC} & sL \end{bmatrix} \qquad E_{\mathbf{b}}(s)=\begin{bmatrix} -U_0 \\ \dfrac{u_C(0)}{s} \\ \dfrac{\mu \cdot u_C(0)}{s}-Li_L(0) \end{bmatrix}$$

Inverzija matrice $\mathbf{Z_b}(s)$ (3x3):

$$Z_{\mathrm{b}}^{*}(s)=\begin{bmatrix} \dfrac{1}{sC} & 0 \\ \dfrac{\mu}{sC} & sL \end{bmatrix} \qquad Z_{\mathbf{b}}(s)=\begin{bmatrix} R & 0 & 0 \\ 0 & & \\ 0 & & \end{bmatrix}$$

$$\left(Z_{\mathrm{b}}^{*}\right)^{-1}(s)=\begin{bmatrix} sC & 0 \\ -\dfrac{\mu}{sL} & \dfrac{1}{sL} \end{bmatrix}$$

$$Z_{\mathrm{b}}^{-1}(s)=\begin{bmatrix} \dfrac{1}{R} & 0 & 0 \\ 0 & sC & 0 \\ 0 & -\dfrac{\mu}{sL} & \dfrac{1}{sL} \end{bmatrix}$$

Množenje matrica:

$$\mathbf{Y_n}(s) = [-1 \quad 1 \quad 1] \cdot \begin{bmatrix} \frac{1}{R} & 0 & 0 \\ 0 & sC & 0 \\ 0 & -\frac{\mu}{sL} & \frac{1}{sL} \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$$\underline{\mathbf{Y_n}(s) = \left[\frac{1}{R} + sC + \frac{1}{sL}(1-\mu) \right]}$$

$$\mathbf{I_n}(s) = [-1 \quad 1 \quad 1] \cdot \begin{bmatrix} \frac{1}{R} & 0 & 0 \\ 0 & sC & 0 \\ 0 & -\frac{\mu}{sL} & \frac{1}{sL} \end{bmatrix} \cdot \begin{bmatrix} -U_0 \\ \frac{u_C(0)}{s} \\ \frac{\mu \cdot u_C(0)}{s} - Li_L(0) \end{bmatrix}$$

$$\mathbf{I_n}(s) = \left[\frac{U_0}{R} + Cu_C(0) - \frac{i_L(0)}{s} \right]$$

$$\underline{\mathbf{U_n}(s) = [U_1]}$$

VJEŽBA: Doći do istog rezultata direktno pišući jednadžbu čvora (bez topološke analize).

VJEŽBA: Za mrežu iz **ZAD.14** napisati temeljni sustav jednadžbi čvorova, a za mrežu iz **ZAD.15** napisati temeljni sustav jednadžbi petlji.

ZADATAK 16. Nacrtati graf ako je zadana temeljna spojna matrica.

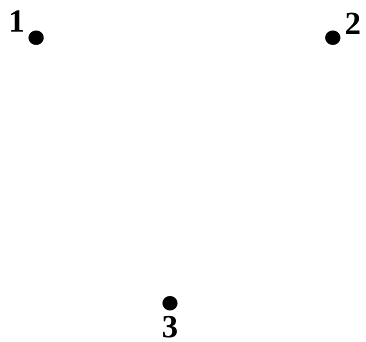
$$S = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix}$$

2 reda - graf ima 2 spone

ukupan broj grana - broj stupaca i iznosi 4

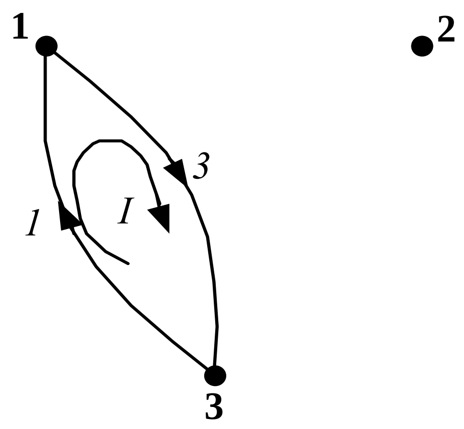
broj stablenih grana 4-2=2

mreža ima 2+1=3 čvora:

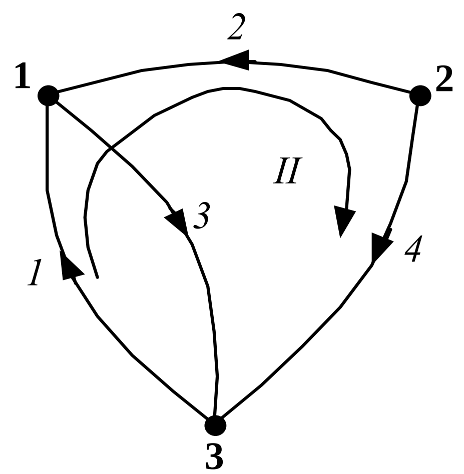


Iz matrice - stablene grane *1* i *2*, a spone *3* i *4*.

Prva petlja - spojna grana *3*, s njom je u petlji grana *1* i ima isti smjer:



Druga petlja - spojna grana *4*, s njom su u petlji stablene grane *1* uz isti smjer i *2* uz protivan smjer:



VJEŽBA: Nacrtati graf ako zadana matrica predstavlja matricu rezova Q. Rješenje provjerite obrnutim postupkom.

3. FUNKCIJE MREŽA

Funkcije imitancija

Potrebni i dovoljni uvjeti da bi funkcija $F(s)$ bila funkcija imitancije:

1) $F(s)$ nema polova u desnoj poluravnini kompleksne s-ravnine

2) Polovi na $j\omega$ osi (uključujući $s=0$ i $s=\infty$) su jednostruki s realnim pozitivnim reziduumima

3) $\mathrm{Re}[F(j\omega)]\geq 0$ za sve ω osim u polovima

Dodatni zahtjevi za lakšu provjeru:

a) polinom u brojniku i nazivniku funkcije $F(s)$ ima realne pozitivne koeficijente

b) stupanj polinoma u brojniku i nazivniku funkcije $F(s)$ može se razlikovati najviše za 1 (kako najviši tako i najniži)

ZADATAK 17. Ispitati da li je funkcija $W(s)$ funkcija imitancije (tj. PRF).

A) $$W(s)=\frac{s^3+s^2+2s}{s^3+s^2+s+1}=\frac{s\left(s^2+s+2\right)}{(s+1)\left(s^2+1\right)}$$

a) koeficijenti su realni i veći od 0

b) stupnjevi polinoma se ne razlikuju za više od 1 (ni najveći ni najmanji)

1) polovi funkcije (nultočke nazivnika):

$$s_{p1}=-1,\quad s_{p2}=-j,\quad s_{p3}=j$$

i ne nalaze se u desnoj poluravnini.

2) Polovi na jω osi, s_{p2} i s_{p3}, su jednostruki s reziduumima:

$$K_{p2}=K_{p3}=\left.\frac{s^2+1}{s}W(s)\right|_{s=\pm j}=\left.\frac{s^2+s+2}{(s+1)}\right|_{s=\pm j}=\frac{1\pm j}{1\pm j}=1>0$$

što daje realan pozitivan reziduum.

3) Realan dio funkcije $W(j\omega)$ je

$$\operatorname{Re}\left[W(s)\right]\Big|_{s=j\omega}=\frac{\omega^2}{\omega^2+1}\geq 0$$

$W(s)$ je funkcija imitancije.

B) $$W(s)=\frac{(s^2+1)(s^2+2)}{s(s^2+3)}=\frac{s^4+3s^2+2}{s^3+3s}$$

a) i b) su ispunjeni; koeficijenti su pozitivni i stupanj brojnika je za 1 veći i manji od nazivnika

1) polovi funkcije su:

$$s_{p1}=0, \quad s_{p2}=-j\sqrt{3}, \quad s_{p3}=j\sqrt{3}$$

i ne nalaze se u desnoj poluravnini.

2) Polovi su jednostruki s reziduumima:

$$K_{p1}=\lim_{s\to 0}[s\cdot W(s)]=\frac{2}{3}>0$$

$$K_{p2,3}=\frac{s^2+3}{s}W(s)\bigg|_{s^2=-3}=\frac{(s^2+1)(s^2+2)}{s^2}\bigg|_{s^2=-3}=-\frac{2}{3}<0$$

Realan negativan reziduum - $W(s)$ nije funkcija imitancije.

C) $$W(s)=\frac{s^3+7s^2+9}{s^4+6s^2+9}=\frac{s^3+7s^2+9}{\left(s^2+3\right)^2}$$

Ova funkcija nije PRF jer ima dvostruke polove na jω osi, $j\sqrt{3}$ i $-j\sqrt{3}$.

D) $$W(s)=\frac{s^2+2s+1}{s^2}=\frac{(s+1)^2}{s^2}\quad.$$

Ova funkcija nije PRF jer ima dvostruki pol u nuli i najniži stupanj nazivnika je za 2 veći od najnižeg stupnja brojnika

ZADATAK 18. Zadane su nule i polovi funkcije impedancije nekog dvopola: $s_{o1,2}=\pm j3$, $s_{p1,2}=-2\pm j5$. Za $s\rightarrow\infty$ vrijedi $lim[Z(s)]=5$. Odrediti struju $I(s)$ na priključnicama tog dvopola ako je napon $u(t)=2\cos 3t\ S(t)$.

Opći oblik funkcije impedancije:

$$Z(s)=k\frac{(s-s_{o1})(s-s_{o2})\cdot\ldots\cdot(s-s_{om})}{(s-s_{p1})(s-s_{p2})\cdot\ldots\cdot(s-s_{pn})}$$

Uvrste li se zadane nule i polovi:

$$Z(s)=k\frac{(s-j3)(s+j3)}{(s+2-j5)(s+2+j5)}=k\frac{s^2+9}{(s+2)^2+25}$$

Za određivanje konstante k primjenit će se poznata vrijednost funkcije u jednoj točki:

$$\lim_{s\rightarrow\infty}[Z(s)]=\lim_{s\rightarrow\infty}\left[k\frac{1+\frac{9}{s^2}}{\left(1+\frac{2}{s}\right)^2+\frac{25}{s^2}}\right]=k=5$$

Funkcija impedancije:

$$Z(s) = 5\frac{s^2+9}{(s+2)^2+25}$$

Uz napon

$$U(s) = 2\frac{s}{s^2+9}$$

slijedi struja:

$$I(s) = \frac{U(s)}{Z(s)} = \frac{2}{5}\frac{s\left[(s+2)^2+25\right]}{\left(s^2+9\right)^2}$$

ZADATAK 19. Zadana je funkcija poticaja dvopola $u(t)=S(t)$ i odziv $i(t)=(5/4)e^{-3t}\sin 4t\ S(t)$. Odrediti impedanciju dvopola i odziv struje na pobudu $v(t)=\delta(t)$.

pobuda: $$U(s)=\frac{1}{s}$$

odziv: $$I(s)=\frac{5}{(s+3)^2+16}$$

impedancija: $$Z(s)=\frac{U(s)}{I(s)}=\frac{(s+3)^2+16}{5s}$$

nova pobuda: $$V(s)=1$$

struja: $$I_2(s)=\frac{V(s)}{Z(s)}=5\frac{s}{(s+3)^2+16}$$

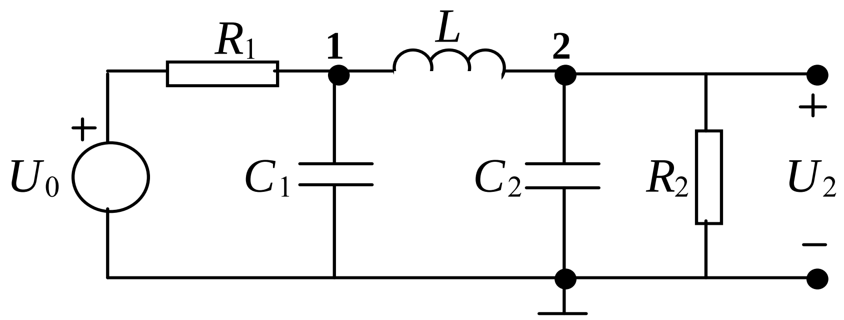
$$I_2(s)=5\frac{s+3-3}{(s+3)^2+16}=5\frac{s+3}{(s+3)^2+16}-\frac{15}{4}\frac{4}{(s+3)^2+16}$$

$$\underline{i_2(t)=\left[5e^{-3t}\cos 4t-\frac{15}{4}e^{-3t}\sin 4t\right]S(t)}$$

novi poticaj → novi odziv, mreža ostaje ista!

Prijenosne funkcije

ZADATAK 20. Odrediti prijenosnu funkciju $F(s)=U_2(s)/U_0(s)$, prikazati raspored polova i nula te nacrtati amplitudno-frekvencijsku karakteristiku. Zadano je: $R_1=R_2=1$, $C_1=C_2=1$, $L=2$.



Jednadžbe čvorova:

$$(1)\quad U_1\left(\frac{1}{R_1}+sC_1+\frac{1}{sL}\right)-U_2\frac{1}{sL}=\frac{U_0}{R_1}$$

$$(2)\quad -U_1\frac{1}{sL}+U_2\left(\frac{1}{R_2}+sC_2+\frac{1}{sL}\right)=0$$

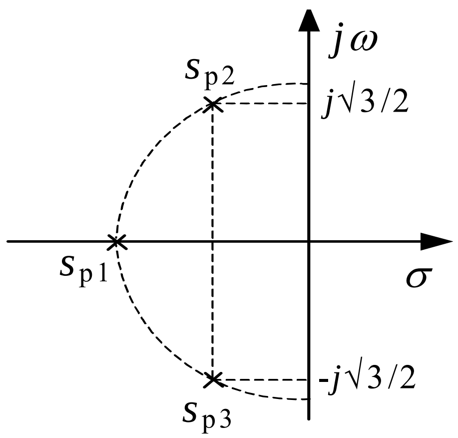
iz (2) $\rightarrow U_1 \rightarrow$ (1)

$$\underline{F(s)=\frac{U_2}{U_0}=\frac{1}{2\left(s^3+2s^2+2s+1\right)}=\frac{1}{2(s+1)\left(s^2+s+1\right)}}$$

polovi (nultočke nazivnika):

$$s_{p1} = -1 \quad , \quad s_{p2,3} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

nula: $\qquad s_{o1} = \infty$



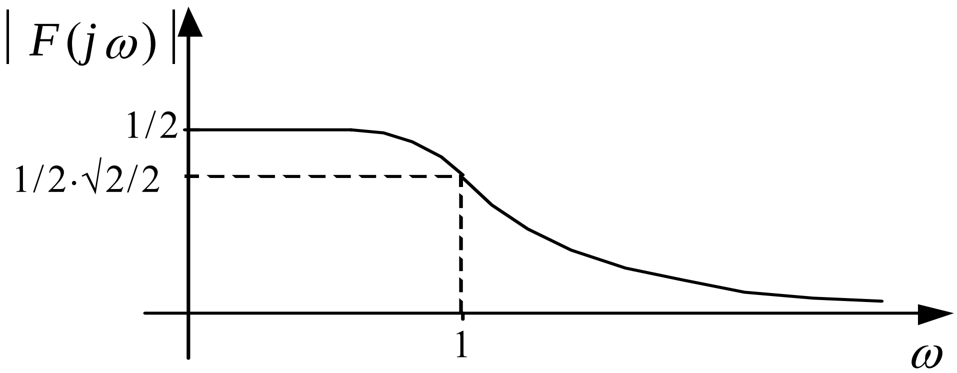
Amplitudno-frekvencijska karakteristika je modul prijenosne funkcije uz s=jω:

$$\left|F(j\omega)\right| = \frac{1}{2\left|\ (1+j\omega)\left(1-\omega^2+j\omega\right)\ \right|} = \frac{1}{2\left|1+j\omega\right|\left|1-\omega^2+j\omega\right|}$$

$$\underline{\left|F(j\omega)\right| = \frac{1}{2\sqrt{1+\omega^2}\sqrt{\left(1-\omega^2\right)^2+\omega^2}} = \frac{1}{2\sqrt{1+\omega^6}}}$$

Analiza toka funkcije $$\left|F(j\omega)\right| = \frac{1}{2\sqrt{1+\omega^6}}$$

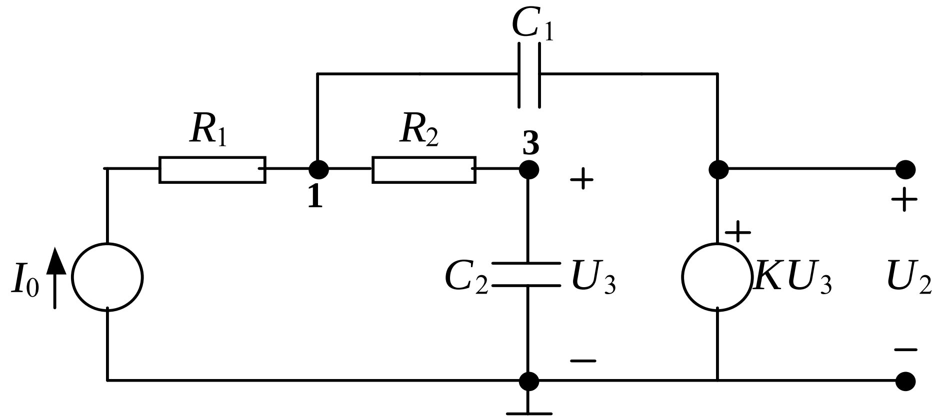
Funkcija nema polova, a nula je u beskonačnosti. Prva derivacija daje ekstreme, a druga derivacija točku infleksije. Vrijednost funkcije u 0 je 1/2, a karakteristična frekvencija je ω=1 gdje amplituda padne za 1/√2=0.707 što je ekvivalentno -3dB u decibelskom sustavu.



VJEŽBA: Odrediti izlaznu impedanciju $Z_{\mathrm{iz}}(s)$ zadane mreže.

Rješenje: $$Z_{iz}(s) = \frac{2s^2+2s+1}{2\left(s^3+2s^2+2s+1\right)}$$

ZADATAK 21. Odrediti prijenosnu funkciju $T(s)=U_2(s)/I_0(s)$, prikazati raspored polova i nula te nacrtati amplitudno-frekvencijsku karakteristiku za vrijednost faktora $K=1$. Zadano je: $R_1=R_2=1$, $C_1=C_2=1$.



Jednadžbe čvorova:

$$(1) \qquad I_0-\frac{U_1-U_3}{R_2}-(U_1-U_2)sC_1=0$$

$$(3) \qquad \frac{U_3-U_1}{R_2}+U_3sC_2=0$$

Iz slike: $\qquad U_2=KU_3$

$U_3 \rightarrow (3)$

iz (3) $\rightarrow U_1 \rightarrow$ (1)

Prijenosna funkcija:

$$T(s) = \frac{U_2}{I_0} = \frac{K}{s^2 C_1 C_2 R_2 + sC_2 - sC_1(K-1)}$$

$$T(s) = \frac{K}{s^2 + s(2-K)} = \underline{\frac{K}{s(s+2-K)}}$$

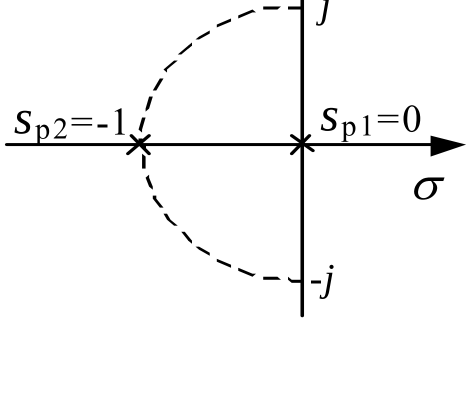
Polovi su rješenja jednadžbe:

$$s(s+2-K)s = 0$$

$$s_{p1} = 0$$

$$s_{p2} = K - 2$$

Raspored polova za *K*=1:



Amplitudno frekvencijska karakteristika:

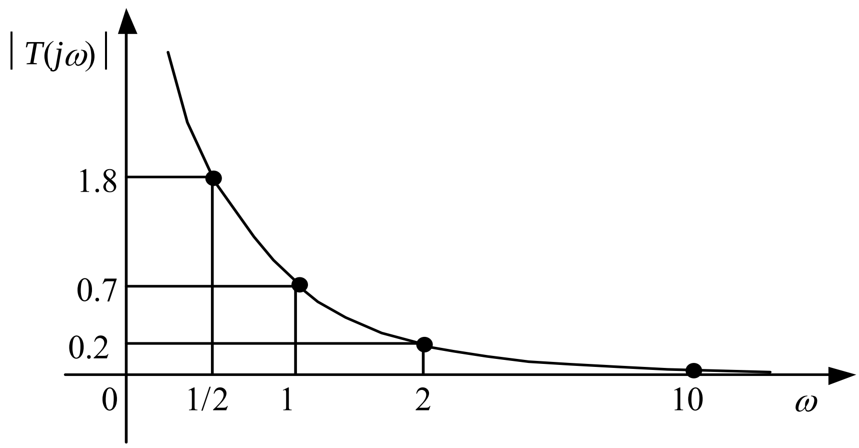
$$|T(j\omega)| = \left|T(s)\right|_{s=j\omega}\Big| = \frac{1}{\left|-\omega^2 + j\omega\right|} = \frac{1}{\sqrt{\left(-\omega^2\right)^2 + \omega^2}} = \frac{1}{\omega\sqrt{\omega^2 + 1}}$$

Do grafičkog rješenja dolazi se analizom toka funkcije $|T(j\omega)|$. Pol funkcije je na frekvenciji 0, a nula funkcije je u beskonačnosti. Vidljivo je da kako frekvencija ω raste tako se vrijednost modula smanjuje.

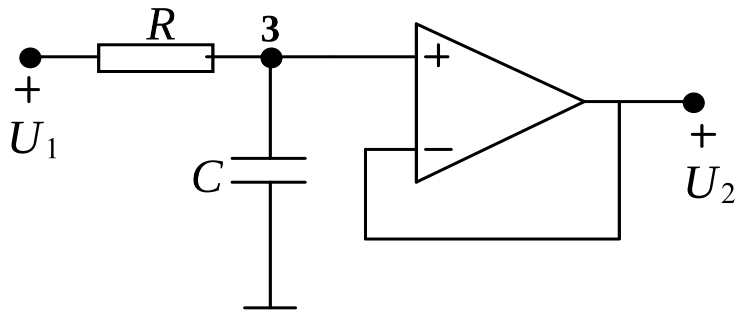
Tablica:

ω	0	1/2	1	2	10	∞
$\lvert T(j\omega)\rvert$	∞	1.79	0.71	0.22	0.01	0

Graf:



ZADATAK 22. Odrediti prijenosnu funkciju $T(s)=U_2(s)/U_1(s)$ i nacrtati amplitudno-frekvencijsku karakteristiku. Zadano je: R=1, C=1.



Jednadžba čvora **3**: $$(3) \quad U_3\left(\frac{1}{R}+sC\right)-\frac{U_1}{R_1}=0$$

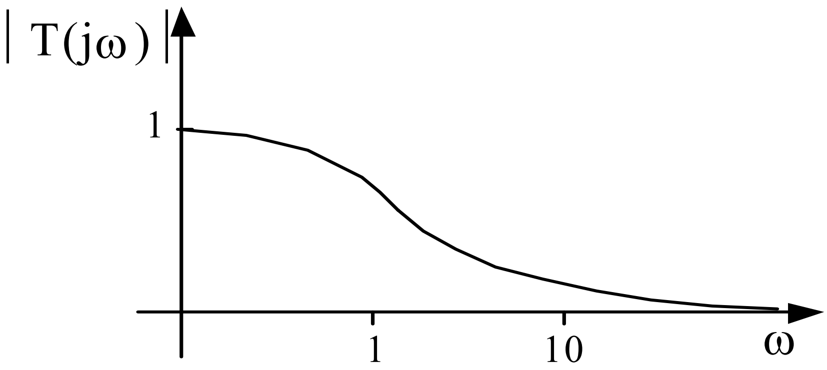
Operacijsko pojačalo: $$U_2 = A(U_3 - U_2)$$

iz (3) → U_3: $$U_2\left(\frac{1}{A}+1\right)=U_1\,\frac{1}{sCR+1}$$

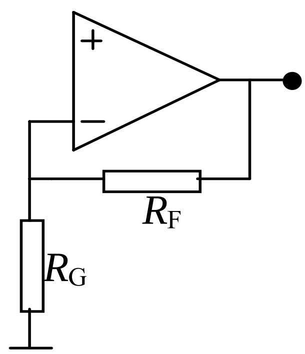
$A\rightarrow\infty$: $$T(s)=\frac{U_2}{U_1}=\frac{1}{sCR+1}=\underline{\frac{1}{s+1}}$$

Amplitudno-frekvencijska karakteristika:

$$\left|T(j\omega)\right| = \frac{1}{\left|j\omega+1\right|} = \underline{\frac{1}{\sqrt{\omega^2+1}}}$$



VJEŽBA: Isti zadatak uz povratnu vezu:

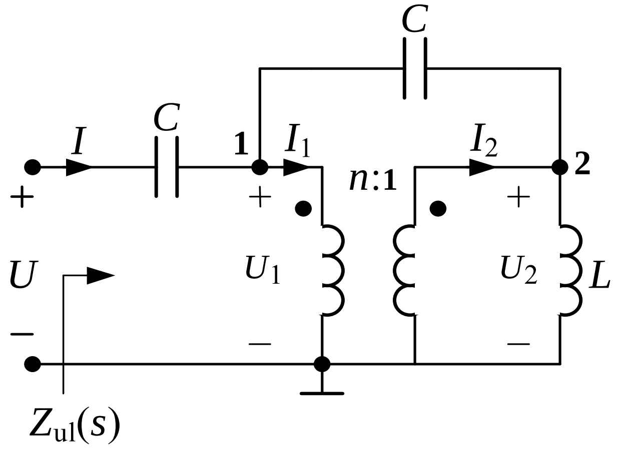


Rješenje: $$T(s) = \left(1+\frac{R_F}{R_G}\right)\frac{\frac{1}{RC}}{s+\frac{1}{RC}}$$

4. SINTEZA DVOPOLA

Fosterova realizacija

ZADATAK 23. Dvopol nadomjestiti *I* Fosterovim oblikom ako je zadano: L=1, C=1, n=2.



Jednadžbe čvorova:

$$(1) \quad I - I_1 = (U_1 - U_2)sC$$

$$(2) \quad I_2 = (U_2 - U_1)sC + U_2 \frac{1}{sL}$$

Dodatne jednadžbe:

$$(3) \quad U = I \frac{1}{sC} + U_1$$

$$(4) \quad U_1 = nU_2$$

$$(5) \quad I_1 = \frac{1}{n} I_2$$

(3), (4), (5) $\rightarrow$ (1), (2)

$$U = I\left(\frac{1}{sC} + \frac{n^2 sL}{s^2 LC\left(1 - 2n + n^2\right) \; + 1} \right)$$

$$Z_{ul}(s) = \frac{U}{I} = 5\,\frac{s^2 + \frac{1}{5}}{s\left(s^2 + 1\right)}$$

koeficijenti: $k_0 = \lim\limits_{s \to 0}\left[s \cdot Z(s)\right] = 1$

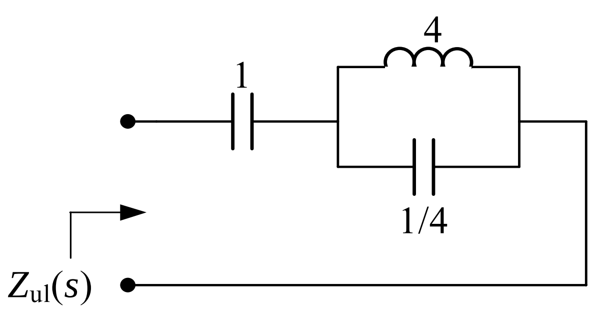
$$k_2 = \lim_{s^2 \to -1}\left[\frac{s^2 + 1}{s} Z(s)\right] = 4$$

$$k_\infty = \lim_{s \to \infty}\left[\frac{1}{s} Z(s)\right] = 0$$

elementi: $\underline{C_0 = \frac{1}{k_0} = 1}$ $\underline{C_2 = \frac{1}{k_2} = \frac{1}{4}}$

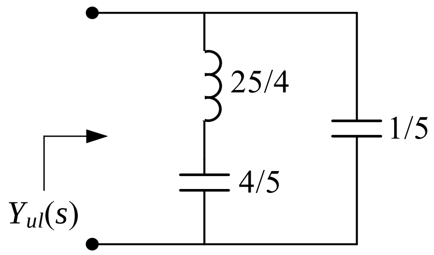
$\underline{L_\infty = k_\infty = 0}$ $\underline{L_2 = \frac{k_2}{\omega_2^2} = 4}$

realizacija:



VJEŽBA: Za isti dvopol odrediti *II* Fosterov kanonski oblik.

Rješenje:



ZADATAK 24. Ispitati koja od funkcija predstavlja funkciju admitancije *LC* dvopola i odrediti njen *II* Fosterov oblik.

$$F_1(s)=\frac{s^4+4s^2+3}{s^3+2s} \qquad F_2(s)=\frac{s^4+11s^2+30}{s^3+4s}$$

Faktorizirani oblici:

$$F_1(s)=\frac{(s^2+1)(s^2+3)}{s(s^2+2)} \qquad F_2(s)=\frac{(s^2+5)(s^2+6)}{s(s^2+4)}$$

Nule i polovi:

funkcija:	F_1	F_2
nule:	$\omega_{o1}=\sqrt{1}$, $\omega_{o2}=\sqrt{3}$	$\omega_{o1}=\sqrt{5}$, $\omega_{o2}=\sqrt{6}$
polovi:	$\omega_{p1}=0$, $\omega_{p2}=\sqrt{2}$	$\omega_{p1}=0$, $\omega_{p2}=\sqrt{4}$

Sve nule i polovi su jednostruki, leže na $j\omega$ osi, ali za funkciju $F_2(s)$ ne alterniraju te ona nije funkcija *LC* dvopola (reaktancije).

$$F_1(s)=Y(s)=\frac{(s^2+1)(s^2+3)}{s(s^2+2)}$$

koeficijenti: $$k_0 = \lim_{s \to 0} \left[s \cdot Y(s) \right] = \frac{3}{2}$$

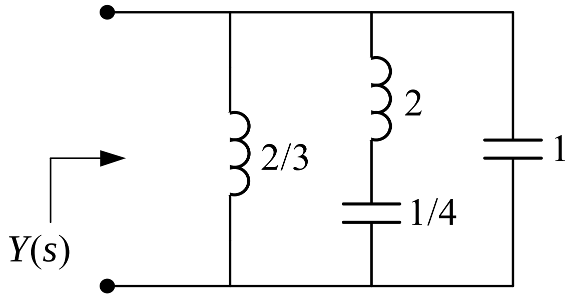
$$k_2 = \lim_{s^2 \to -2} \left[\frac{s^2 + 2}{s} Y(s) \right] = \frac{1}{2}$$

$$k_\infty = \lim_{s \to \infty} \left[\frac{1}{s} Y(s) \right] = 1$$

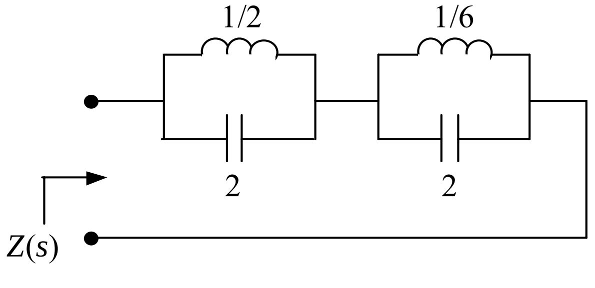
elementi: $$\underline{L_0 = \frac{1}{k_0} = \frac{2}{3}} \qquad \underline{L_2 = \frac{1}{k_2} = 2}$$

$$\underline{C_\infty = k_\infty = 1} \qquad \underline{C_2 = \frac{k_2}{\omega_2^2} = \frac{1}{4}}$$

realizacija:



VJEŽBA: Za isti dvopol odrediti *I* Fosterov kanonski oblik.



Cauerova realizacija

Ovom se metodom realiziraju ljestvičasti dvopoli.

ZADATAK 25. Realizirati *II* Cauerovom realizacijom dvopol ulazne admitancije

$$Y(s)=\frac{4s^2+1}{2s^3+s}$$

Dijeljenje:

$$\left(1+4s^2\right):\left(s+2s^3\right)=\frac{1}{s}=\frac{1}{sL_1} \quad \rightarrow \quad \underline{L_1=1}$$

$$\frac{-1 \pm 2s^2}{2s^2}$$

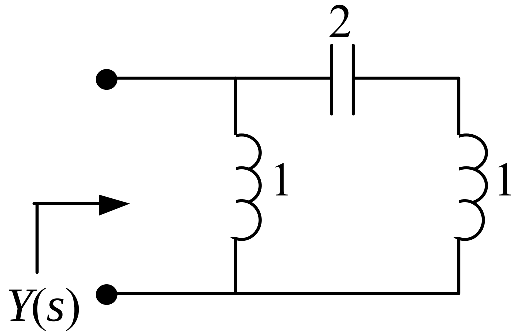
$$\left(s+2s^3\right):2s^2=\frac{1}{2s}=\frac{1}{sC_2} \quad \rightarrow \quad \underline{C_2=2}$$

$$\frac{-s}{2s^3}$$

$$2s^2:2s^3=\frac{1}{s}=\frac{1}{sL_3} \quad \rightarrow \quad \underline{L_3=1}$$

$$\frac{-2s^2}{0}$$

realizacija:



VJEŽBA: Odrediti *II* Fosterov kanonski oblik za istu admitanciju.

Mogućnost realizacije mreža koje sadrže otpore.

- nadomjesni ljestvičasti *RL* odnosno *RC* dvopol

Funkcija ne mora ispunjavati uvjete za funkcije reaktancije nego uvjete za funkcije imitancija.

ZADATAK 26. Realizirati *I* Cauerovom realizacijom dvopol ulazne impedancije

$$Z(s)=\frac{2s^2+14s+20}{s^2+4s+3}$$

$$\left(2s^2+14s+20\right):\left(s^2+4s+3\right)=2=R_1 \rightarrow \underline{R_1=2}$$

$$\frac{-2s^2 \pm 8s \pm 6}{6s+14}$$

$$\left(s^2+4s+3\right):\left(6s+14\right)=\frac{1}{6}s=sC_2 \rightarrow \underline{C_2=\frac{1}{6}}$$

$$\frac{-s^2 \pm \frac{7}{3}s}{\frac{5}{3}s+3}$$

$$(6s+14):\left(\frac{5}{3}s+3\right)=\frac{18}{5}=R_3 \rightarrow \underline{R_3=\frac{18}{5}}$$

$$\begin{array}{l} -6s \pm \frac{54}{5} \\ \hline \quad \frac{16}{5} \end{array}$$

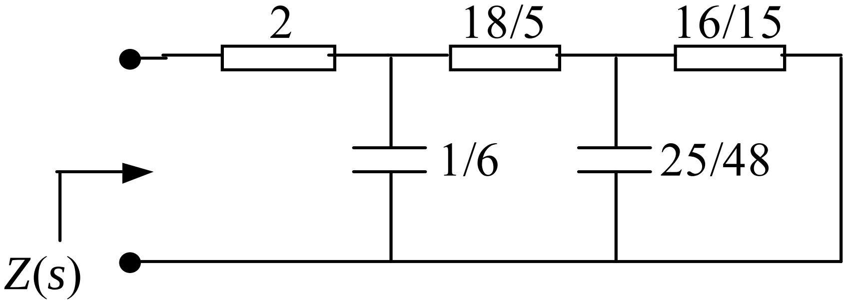
$$\left(\frac{5}{3}s+3\right):\frac{16}{5}=\frac{25}{48}s=sC_4 \rightarrow \underline{C_4=\frac{25}{48}}$$

$$\begin{array}{l} -\frac{5}{3}s \\ \hline \quad 3 \end{array}$$

$$\frac{16}{5}:3=\frac{16}{15}=R_5 \rightarrow \underline{R_5=\frac{16}{15}}$$

$$\begin{array}{l} -\frac{16}{5} \\ \hline \quad 0 \end{array}$$

realizacija:

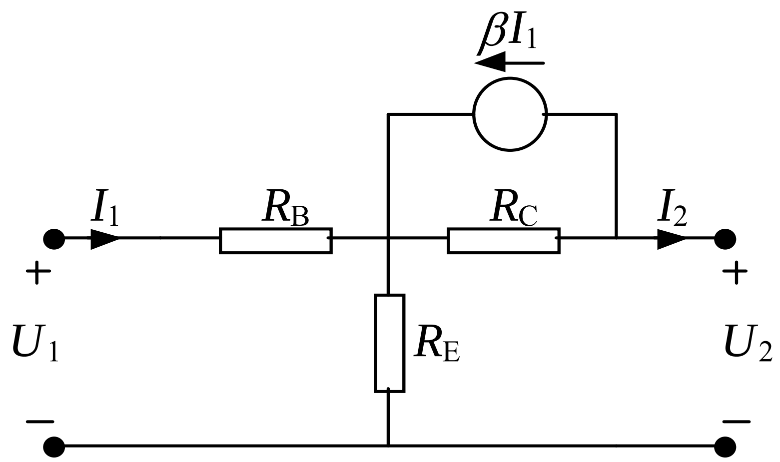


provjera: za $\omega=0 \rightarrow Z(0)=\frac{20}{3}$

suma otpora: $Z(0)=2+\frac{18}{5}+\frac{16}{15}=\frac{30+54+16}{15}=\frac{20}{3}$

5. ČETVEROPOLI

ZADATAK 27. Odrediti *z* i *A* parametre.



a) *z* parametri

Uvjet I_2=0:

$$U_1 = I_1(R_B + R_E)$$

$$U_2 = I_1 R_E - \beta I_1 R_C$$

Slijede parametri:

$$z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0} = R_B + R_E$$

$$z_{21} = \left.\frac{U_2}{I_1}\right|_{I_2=0} = R_E - \beta R_C$$

Uvjet I_1=0: (i βI_1=0)

$$U_1 = -I_2 R_E$$
$$U_2 = -I_2 (R_E + R_C)$$

Slijede parametri:

$$\underline{z_{12} = -\left.\frac{U_1}{I_2}\right|_{I_1=0} = R_E}$$

$$\underline{z_{22} = -\left.\frac{U_2}{I_2}\right|_{I_1=0} = R_E + R_C}$$

Matrica z parametara:

$$\underline{[z] = \begin{bmatrix} R_B + R_E & -R_E \\ R_E - \beta R_C & -(R_E + R_C) \end{bmatrix}}$$

b) *A* parametri

Uvjet I_2=0: (vrijede jednadžbe iz a) dijela)

$$A = \left.\frac{U_1}{U_2}\right|_{I_2=0} = \frac{R_B + R_E}{R_E - \beta R_C}$$

$$C = \left.\frac{I_1}{U_2}\right|_{I_2=0} = \frac{1}{R_E - \beta R_C}$$

Uvjet je U_2=0:

$$U_1 = I_1 R_B + (I_1 - I_2) R_E$$
$$0 = (I_2 - I_1) R_E + (I_2 + \beta I_1) R_C$$

$$U_1 = I_1 (R_B + R_E) - I_2 R_E$$
$$I_2 (R_E + R_C) = I_1 (R_E - \beta R_C)$$

Slijede parametri:

$$D = \left.\frac{I_1}{I_2}\right|_{U_2=0} = \frac{R_E + R_C}{R_E - \beta R_C}$$

$$B = \left.\frac{U_1}{I_2}\right|_{U_2=0} = \frac{R_B (R_E + R_C) + R_E R_C (1 + \beta)}{R_E - \beta R_C}$$

Matrica A parametara:

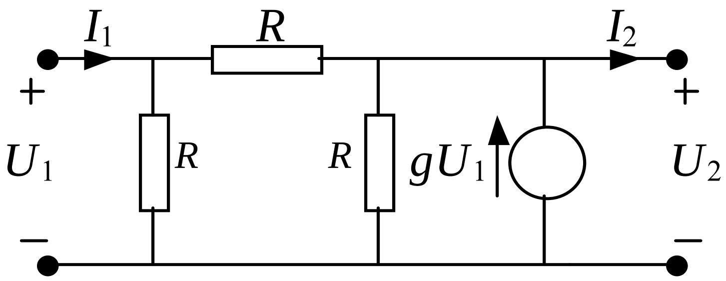
$$[A] = \begin{bmatrix} \dfrac{R_B + R_E}{R_E - \beta R_C} & \dfrac{R_B(R_E + R_C) + R_E R_C (1+\beta)}{R_E - \beta R_C} \\ \dfrac{1}{R_E - \beta R_C} & \dfrac{R_E + R_C}{R_E - \beta R_C} \end{bmatrix}$$

Prijenosni parametri iz z parametara:

$$A = \frac{z_{11}}{z_{21}} \qquad C = \frac{1}{z_{21}}$$

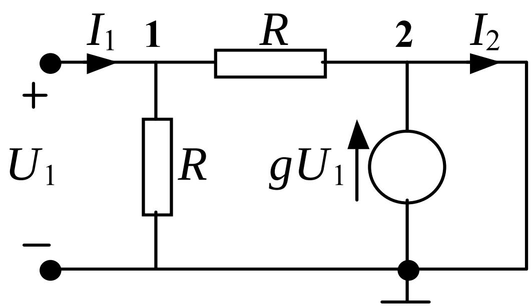
$$B = \frac{z_{11} z_{22}}{z_{21}} - z_{12} \qquad D = \frac{z_{22}}{z_{21}}$$

ZADATAK 28. Ispitati da li je četveropol simetričan i recipročan.



provjera preko *y* parametara

Uvjet U_2=0:

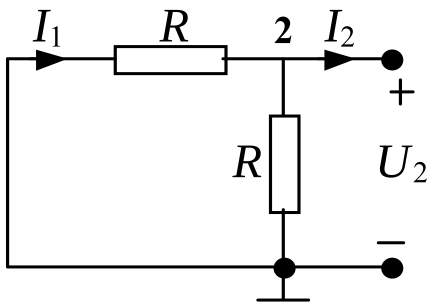


Jednadžbe čvorova: dva parametra:

(1) $U_1\left(\frac{1}{R}+\frac{1}{R}\right)=I_1$ $\qquad y_{11}=\left.\frac{I_1}{U_1}\right|_{U_2=0}=\frac{2}{R}$

(2) $-U_1\frac{1}{R}=gU_1-I_2$ $\qquad y_{21}=\left.\frac{I_2}{U_1}\right|_{U_2=0}=\frac{1}{R}+g$

Uvjet U_1=0:



Jednadžbe čvorova:

$$(2) \quad U_2\left(\frac{1}{R}+\frac{1}{R}\right)=-I_2$$

$$I_1=-U_2\frac{1}{R}$$

dva parametra:

$$y_{22}=-\frac{I_2}{U_2}\bigg|_{U_1=0}=\frac{2}{R}$$

$$y_{12}=-\frac{I_1}{U_2}\bigg|_{U_1=0}=\frac{1}{R}$$

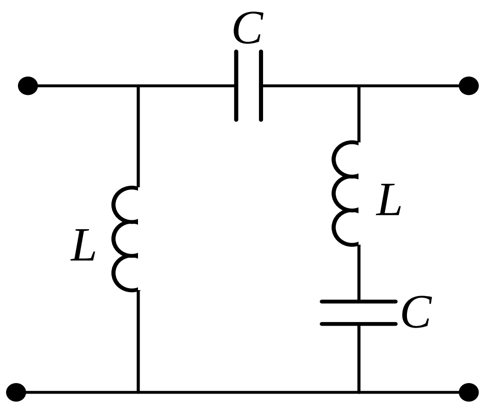
provjera uvjeta recipročnosti: $\underline{y_{12}\neq y_{21}}$

četveropol nije recipročan

provjera uvjeta simetričnosti: $\underline{y_{11}=y_{22}}$

četveropol je simetričan.

ZADATAK 29. Zadanom četveropolu odrediti ekvivalentni T-spoj.



Dva su četveropola ekvivalentna ako su im matrice bilo kojih parametara jednake. Zadani je četveropol u Π-spoju i ima slijedeću matricu *z* parametara:

$$[z]_{\Pi} = \begin{bmatrix} \dfrac{Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3} & -\dfrac{Z_1Z_2}{Z_1+Z_2+Z_3} \\ \dfrac{Z_1Z_2}{Z_1+Z_2+Z_3} & -\dfrac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3} \end{bmatrix}$$

pri čemu je:

$$Z_1 = sL \quad , \quad Z_2 = sL + \frac{1}{sC} \quad , Z_3 = \frac{1}{sC}$$

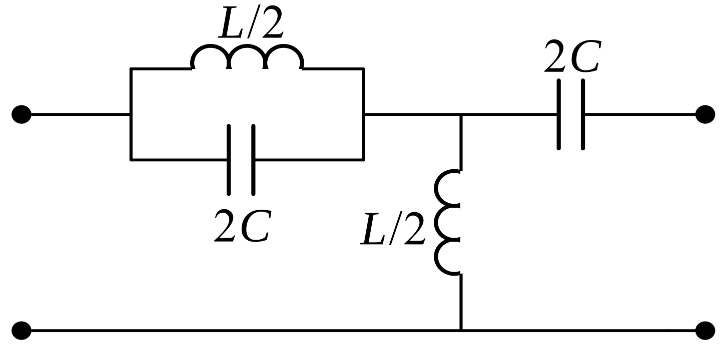
Elementi T-spoja:

$$Z_C = z_{21} = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3} = \frac{sL\left(sL + \frac{1}{sC}\right)}{2sL + \frac{2}{sC}} = s\frac{L}{2}$$

$$Z_A = z_{11} - z_{21} = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3} = \frac{\frac{L}{C}}{2sL + \frac{2}{sC}} = \frac{s\frac{L}{2} \cdot \frac{1}{s2C}}{s\frac{L}{2} + \frac{1}{s2C}}$$

$$Z_B = z_{22} - z_{21} = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3} = \frac{\frac{L}{C} + \frac{1}{s^2 C^2}}{2sL + \frac{2}{sC}} = \frac{1}{s2C}$$

T-spoj:



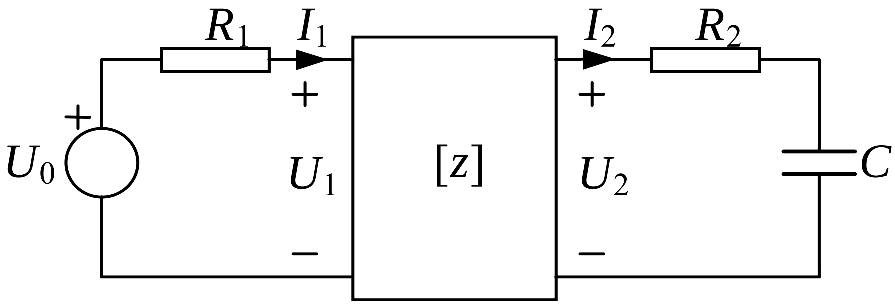
ZADATAK 30. Odrediti:

a) napon $U_0(s)$ ako je $U_2(s)=10/s$

b) napon $u_2(t)$ ako je $u_0(t)=\cos t$, uz $-\infty<t<\infty$.

Zadano je: $R_1=R_2=1$, $C=1$ i matrica z parametara

$$[z]=\begin{bmatrix} 3+2s & -(2+s) \\ 2+s & -(4+3s) \end{bmatrix}$$



a) za ovako označenu električnu mrežu vrijede slijedeće jednadžbe:

Dvije jednadžbe četveropola:

$$(1) \quad U_1 = z_{11}I_1 - z_{12}I_2$$

$$(2) \quad U_2 = z_{21}I_1 - z_{22}I_2$$

Dvije jednadžbe petlji:

$$(3) \quad U_0 = I_1R_1 + U_1$$

$$(4) \quad U_2 = I_2\left(R_2 + \frac{1}{sC}\right)$$

sustav 4 jednadžbe s 4 nepoznanice

iz (3) $\rightarrow$ U_1 $\rightarrow$ (1)

iz (4) $\rightarrow$ I_2 $\rightarrow$ (2)

te

iz (2) $\rightarrow$ I_1 $\rightarrow$ (1)

Slijedi:

$$U_0 = \frac{(Z_2 + z_{22})(R_1 + z_{11}) - z_{12}z_{21}}{z_{21}Z_2} U_2$$

$$Z_2 = R_2 + \frac{1}{sC}$$

Uz zadane vrijednosti:

$$\underline{U_0(s) = \frac{10}{s} \frac{5s^3 + 18s^2 + 18s + 4}{s^2 + 3s + 2}}$$

b) određivanje odziva mreže iz poznate prijenosne funkcije, pobuda je sinusoidalna $-\infty<t<\infty$.

Naponska prijenosna funkcija:

$$T(s)=\frac{U_2(s)}{U_0(s)}=\frac{s^2+3s+2}{5s^3+18s^2+18s+4}$$

Uvrstivši $s=j\omega$ slijedi:

$$T(j\omega)=\frac{\left(2-\omega^2\right)+3j\omega}{\left(4-18\omega^2\right)+j\left(18\omega-5\omega^3\right)}$$

za $\omega=1$:

$$T(j\omega)=\frac{1+3j}{-14+13j}\frac{-14-13j}{-14-13j}=\frac{25-55j}{365}$$

$$T(j\omega)=\frac{5}{73}-j\frac{11}{73}$$

$$U_0(j\omega)=1$$

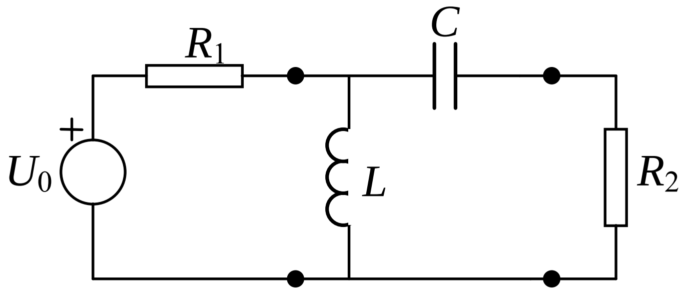
Odziv u frekvencijskoj domeni:

$$U_2(j\omega)=T(j\omega)\cdot U_0(j\omega)=0.166\angle-65.6^{\circ}$$

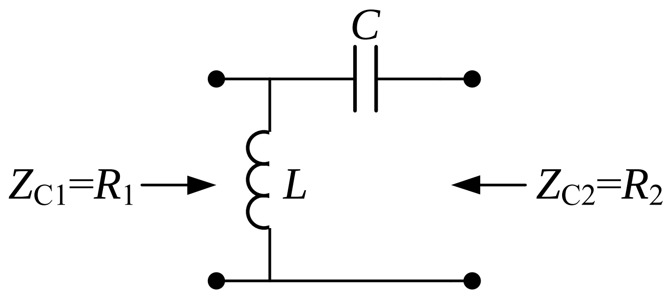
Odziv u vremenskoj domeni:

$$\underline{u_2(t)=0.166\cos\left(t-65.6^{\circ}\right)}$$

ZADATAK 31. Koliko iznose L i C izraženi s R_1 i R_2 da bi postojalo prilagođenje po zrcalnim impedancijama na frekvenciji ω_1.



Izdvojeni četveropol:



Ulazne impedancije na kratko i na prazno:

$$Z_{K1} = \left.\frac{U_1}{I_1}\right|_{U_2=0} = \frac{j\omega L \cdot \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1-\omega^2 LC}$$

$$Z_{P1} = \left.\frac{U_1}{I_1}\right|_{I_2=0} = j\omega L$$

$$Z_{K2} = -\left.\frac{U_2}{I_2}\right|_{U_1=0} = \frac{1}{j\omega C}$$

$$Z_{P2} = -\left.\frac{U_2}{I_2}\right|_{I_1=0} = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

Zrcalne impedancije i uvjet prilagođenja:

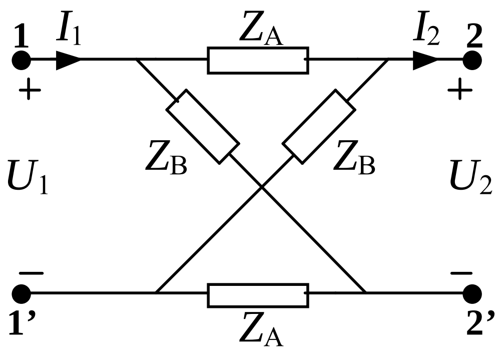
$$Z_{C1} = \sqrt{Z_{K1} \cdot Z_{P1}} = \sqrt{\frac{\omega_1^2 L^2}{\omega_1^2 LC - 1}} = R_1$$

$$Z_{C2} = \sqrt{Z_{K2} \cdot Z_{P2}} = \sqrt{\frac{\omega_1^2 LC - 1}{\omega_1^2 C^2}} = R_2$$

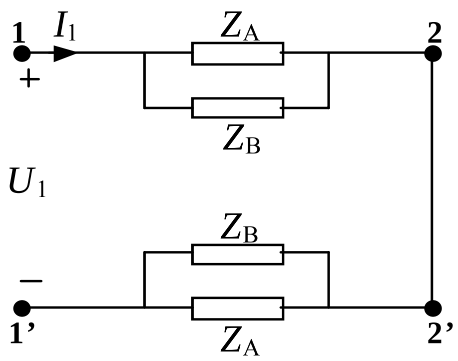
Rješenja su:

$$\underline{L = \frac{R_1 R_2}{\omega_1 \sqrt{R_1 R_2 - R_2^2}}} \qquad \underline{C = \frac{1}{\omega_1 \sqrt{R_1 R_2 - R_2^2}}}$$

ZADATAK 32. Odrediti zrcalne parametre **X**-spoja.

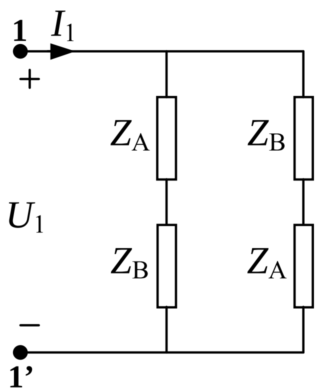


simetričan → impedancije samo na jednoj strani



$$Z_{K1} = \left.\frac{U_1}{I_1}\right|_{U_2=0}$$

$$Z_{K1} = 2\,\frac{Z_A Z_B}{Z_A + Z_B}$$



$$Z_{P1} = \left.\frac{U_1}{I_1}\right|_{I_2=0}$$

$$Z_{P1} = \frac{Z_A + Z_B}{2}$$

$$\underline{Z_{C1} = Z_{C2} = Z_C = \sqrt{Z_{K1}\cdot Z_{P1}} = \sqrt{Z_A Z_B}}$$

$$\underline{\operatorname{th} g = \sqrt{\frac{Z_{K1}}{Z_{P1}}} = \sqrt{\frac{Z_{K2}}{Z_{P2}}} = \frac{2\sqrt{Z_A Z_B}}{Z_A + Z_B}}$$

ZADATAK 33. Odrediti elemente X-spoja ako je $Z_C(\omega)$=100Ω i th(g/2)=$j\omega 10^{-3}$.

Za polučlan X-spoja poznato je:

$$Z_C' = \sqrt{Z_K' Z_P'} = \sqrt{Z_1 Z_2} = 100\Omega$$

i

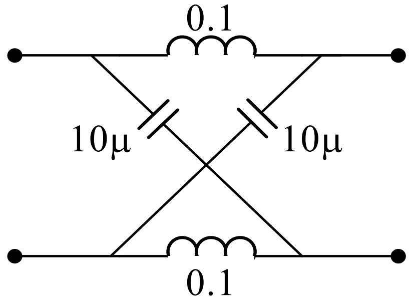
$$\text{th}\, g' = \text{th}\,\frac{g}{2} = \sqrt{\frac{Z_K'}{Z_P'}} = \sqrt{\frac{Z_1}{Z_2}} = j\omega \cdot 10^{-3}$$

Zrcalna impedancija polučlana jednaka je zrcalnoj impedanciji cijelog četveropola, a zrcalna konstanta prijenosa polučlana je dvostruko manja.

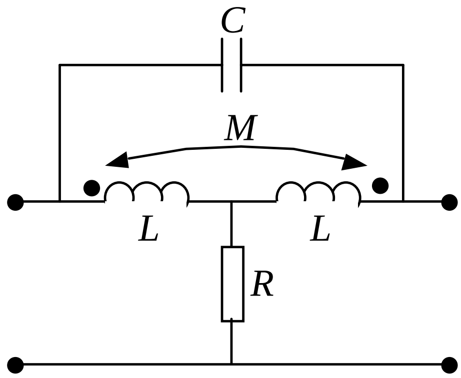
Sustav dvije jednadžbe s dvije nepoznanice Z_1 i Z_2.

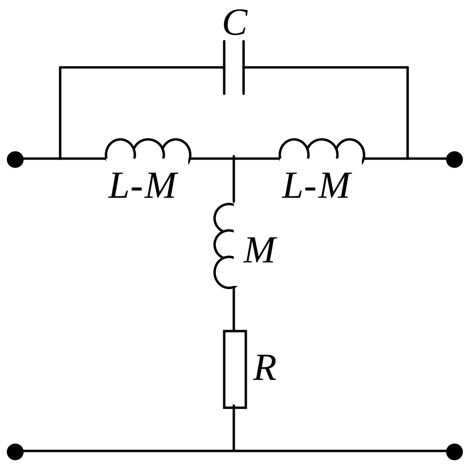
$$Z_1 = j\omega \cdot 10^{-1} = j\omega L \qquad Z_2 = \frac{1}{j\omega \cdot 10^{-5}} = \frac{1}{j\omega C}$$

Elementi: $\underline{L = 0.1\text{H}}$ $\underline{C = 10\mu\text{F}}$

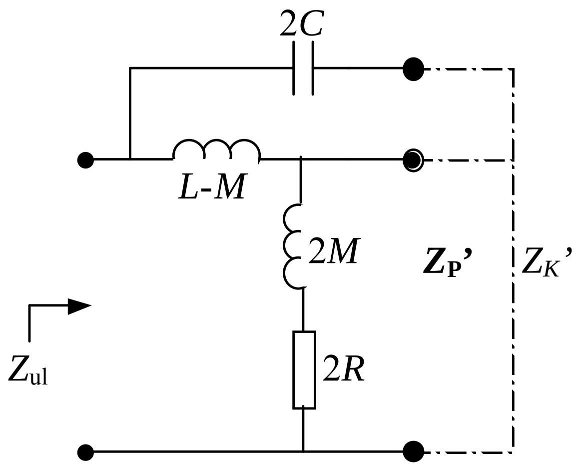


ZADATAK 34. Odrediti zadanom četveropolu ekvivalentan X-spoj.

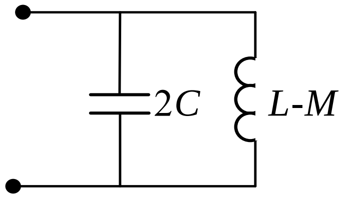




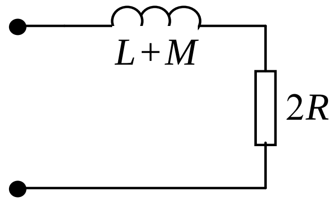
Polučlan:



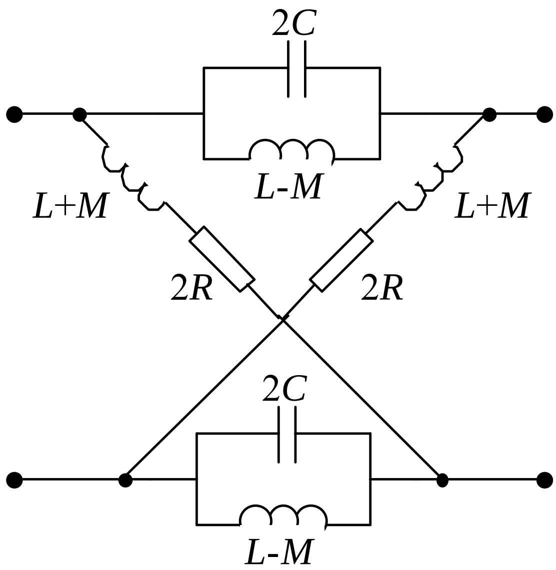
Z_K':



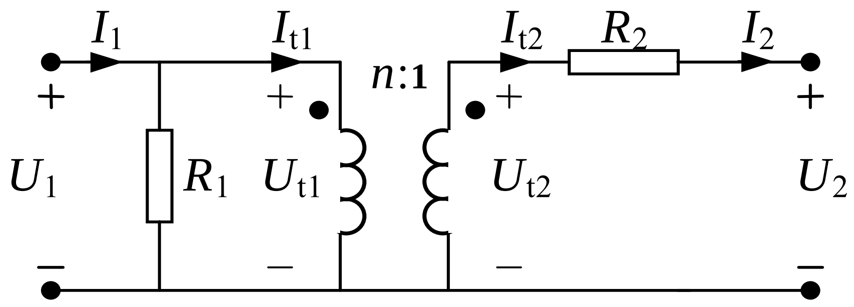
Z_P':



Bartlettov teorem - X-spoj:



ZADATAK 35. Odrediti *n* idealnog transformatora da bi četveropol bio simetričan. Odrediti ekvivalentni T- i X-spoj. Zadano je: R_1=4, R_2=3.



korištenje z parametara

Uvjet I_2=0:

$$U_1 = (I_1 - I_{t1})R_1 = \left(I_1 - \frac{1}{n} I_{t2}\right)R_1 = I_1 R_1$$

$$U_2 = \frac{1}{n} U_1 = \frac{1}{n} I_1 R_1$$

Parametri:

$$z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0} = R_1 = 4$$

$$z_{21} = \left.\frac{U_2}{I_1}\right|_{I_2=0} = \frac{1}{n} R_1 = \frac{4}{n}$$

Uvjet I_1=0:

$$U_2 = -I_2 R_2 + U_{t2} = -I_2 R_2 + \frac{1}{n} U_{t1} = -I_2 R_2 + \frac{1}{n}\left(-I_{t1} R_1\right) = -I_2 R_2 - \frac{1}{n} R_1 \frac{1}{n} I_2$$

Parametar:

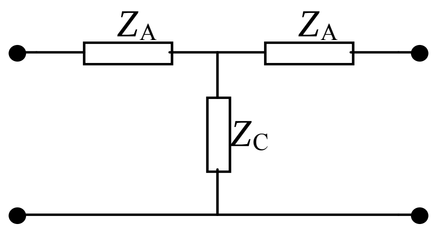
$$z_{22} = -\left.\frac{U_2}{I_2}\right|_{I_1=0} = R_2 + \frac{1}{n^2} R_1 = 3 + \frac{4}{n^2}$$

Uvjet simetričnosti: $z_{11} = z_{22}$

$$4 = 3 + \frac{4}{n^2}$$

$$\underline{n = 2}$$

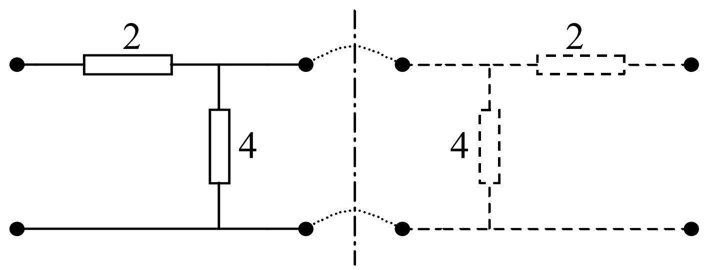
Elementi simetričnog T-spoja:



$$Z_A = z_{11} - z_{21} = 4 - 2 = 2$$

$$Z_C = z_{21} = 2$$

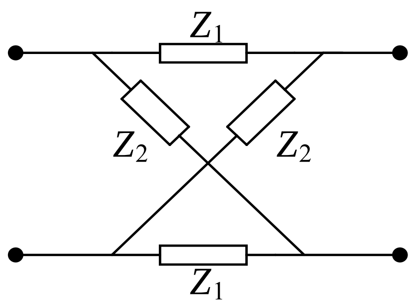
Polučlan je:



$$Z_K^{'} = 2$$

$$Z_P^{'} = 6$$

Traženi X-spoj:

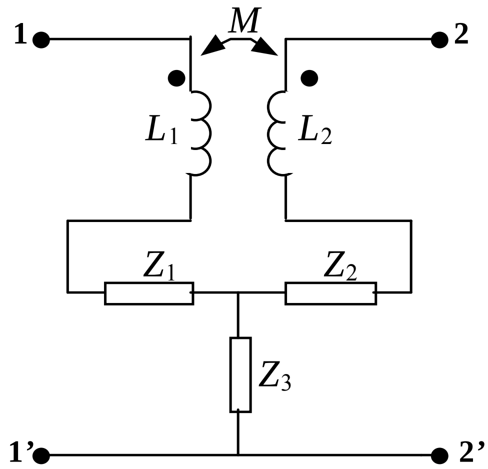


$$Z_1 = Z_K^{'} = 2$$

$$Z_2 = Z_P^{'} = 6$$

VJEŽBA: Odrediti elemente ekvivalentnog Π-spoja.
Rješenje: $Y_A = Y_B = Y_C = 1/6$

ZADATAK 36. Odrediti z parametre kombinacije četveropola.



serijski spoj dva četveropola:

$$[z] = [z]_1 + [z]_2$$

dva četveropola: vezani induktiviteti i T-spoj

a) naponske jednadžbe za vezane induktivitete:

$$U_1 = I_1 sL_1 - I_2 sM$$

$$U_2 = I_1 sM - I_2 sL_2$$

z parametri:

$$[z]_1 = \begin{bmatrix} sL_1 & -sM \\ sM & -sL_2 \end{bmatrix}$$

b) naponske jednadžbe za T-spoj:

$$U_1 = I_1 Z_1 + (I_1 - I_2) Z_3$$

$$U_2 = (I_1 - I_2) Z_3 - I_2 Z_3$$

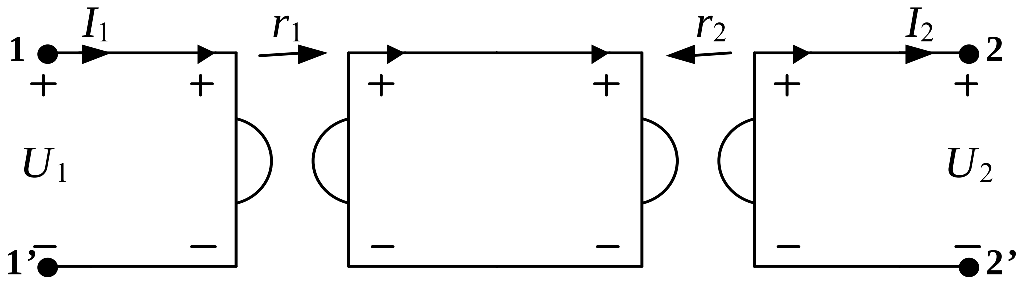
z parametri

$$[z]_2 = \begin{bmatrix} Z_1 + Z_3 & -Z_3 \\ Z_3 & -(Z_2 + Z_3) \end{bmatrix}$$

Matrica *z* parametara cijelog spoja:

$$\underline{[z] = [z]_1 + [z]_2 = \begin{bmatrix} Z_1 + Z_3 + sL_1 & -(Z_3 + sM) \\ Z_3 + sM & -(Z_2 + Z_3 + sL_2) \end{bmatrix}}$$

ZADATAK 37. Ispitati da li prikazani spoj četveropola predstavlja recipročan četveropol. Koji je osnovni četveropol ekvivalentan zadanom?



kaskadni (lančani) spoj:

$$[A] = [A]_1 \cdot [A]_2$$

dva giratora

prvi girator:

$$U_1 = r_1\left(-I_2^{'}\right)$$

$$I_1 = -\frac{1}{r_1}U_2^{'}$$

$$[A]_1 = \begin{bmatrix} 0 & -r_1 \\ -\dfrac{1}{r_1} & 0 \end{bmatrix}$$

drugi girator:

$$U_1^{'} = (-r_2)(-I_2)$$

$$I_1^{'} = -\left(-\frac{1}{r_2}\right)U_2$$

$$[A]_2 = \begin{bmatrix} 0 & r_2 \\ \dfrac{1}{r_2} & 0 \end{bmatrix}$$

Matrica A parametara cijelog spoja:

$$[A]=[A]_1\cdot[A]_2=\begin{bmatrix}-\dfrac{r_1}{r_2} & 0\\ 0 & -\dfrac{r_2}{r_1}\end{bmatrix}$$

Uvjet recipročnosti:

$$\underline{Det[A]=\left(-\frac{r_1}{r_2}\right)\cdot\left(-\frac{r_2}{r_1}\right)=1}$$

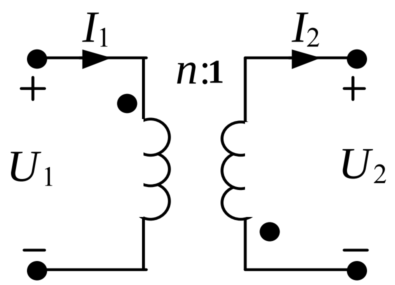
četveropol je recipročan.

Postavi li se da je $\dfrac{r_1}{r_2}=n$ slijede jednadžbe:

$$U_1=-nU_2$$

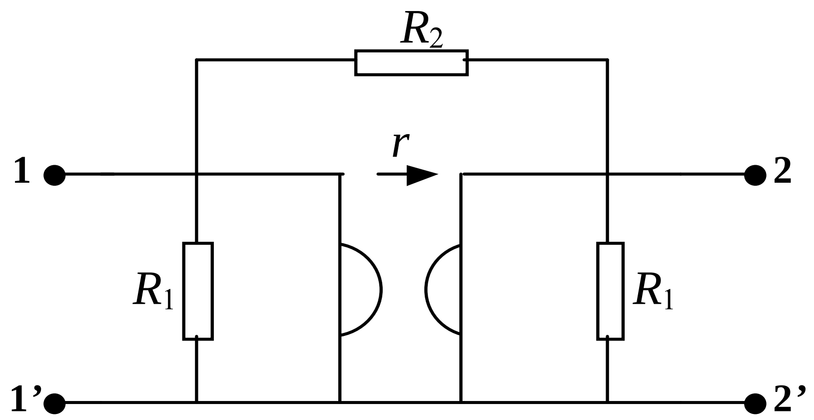
$$I_1=-\frac{1}{n}I_2$$

Jednadžbe idealnog transformatora:



VJEŽBA: Odrediti matrice z i y parametara giratora.

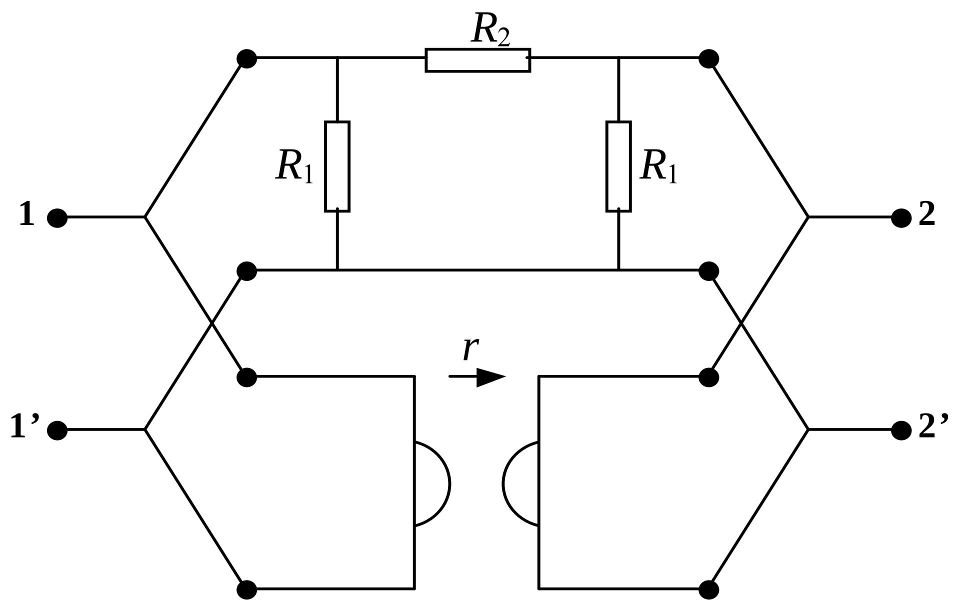
ZADATAK 38. Odrediti y parametre prikazanog četveropola i njegovu ulaznu admitanciju kad je zaključen kapacitetom C.



paralelni spoj dva četveropola:

$$[y]=[y]_1+[y]_2$$

dva četveropola: girator i simetričan Π-spoj



Matrice y parametara svakog pojedinog četveropola:

Π-spoj: $$[y]_1 = \begin{bmatrix} \dfrac{R_1 + R_2}{R_1 R_2} & -\dfrac{1}{R_2} \\ \dfrac{1}{R_2} & -\dfrac{R_1 + R_2}{R_1 R_2} \end{bmatrix}$$

girator: $$[y]_2 = \begin{bmatrix} 0 & -\dfrac{1}{r} \\ -\dfrac{1}{r} & 0 \end{bmatrix}$$

Matrica y parametara cijelog spoja:

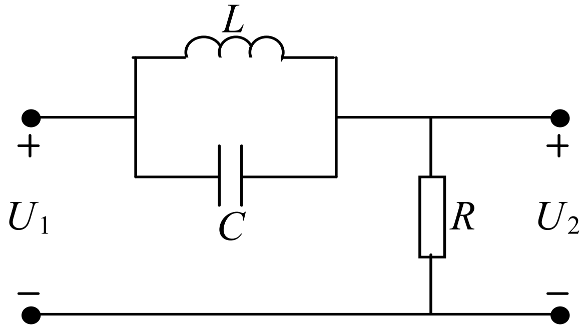
$$\underline{[y] = [y]_1 + [y]_2 = \begin{bmatrix} \dfrac{R_1 + R_2}{R_1 R_2} & -\left(\dfrac{1}{R_2} + \dfrac{1}{r}\right) \\ \dfrac{1}{R_2} - \dfrac{1}{r} & -\dfrac{R_1 + R_2}{R_1 R_2} \end{bmatrix}}$$

Ulazna admitancija:

$$\underline{Y_{ul} = y_{11} - \frac{y_{12} y_{21}}{y_{22} + Y_2}} \;, \qquad Y_2 = sC$$

6. FILTRI

ZADATAK 39. Odrediti parametre pasivnog filtra 2.reda ako je zadano: *R*=1, *C*=1, *L*=1. Nactrati raspored polova i nula prijenosne funkcije filtra te amplitudno-frekvencijsku i fazno-frekvencijsku karakteristiku. O kojem se tipu filtra radi?



Prijenosna funkcija:

$$T(s)=\frac{U_2}{U_1}=\frac{R}{R+\dfrac{sL\cdot\dfrac{1}{sC}}{sL+\dfrac{1}{sC}}}=\frac{s^2+\dfrac{1}{LC}}{s^2+\dfrac{1}{RC}s+\dfrac{1}{LC}}$$

Opća prijenosna funkcija:

$$T(s)=k\,\frac{s^2+\omega_0^2}{s^2+\dfrac{\omega_0}{Q}s+\omega_0^2}$$

Parametri filtra:

centralna frekvencija $\underline{\omega_0 = \dfrac{1}{\sqrt{LC}} = 1}$

Q-faktor $\underline{Q = RC\omega_0 = R\sqrt{\dfrac{C}{L}} = 1}$

pojačanje $\underline{k=1}$

Prijenosna funkcija: $T(s) = \dfrac{s^2+1}{s^2+s+1}$

nule:

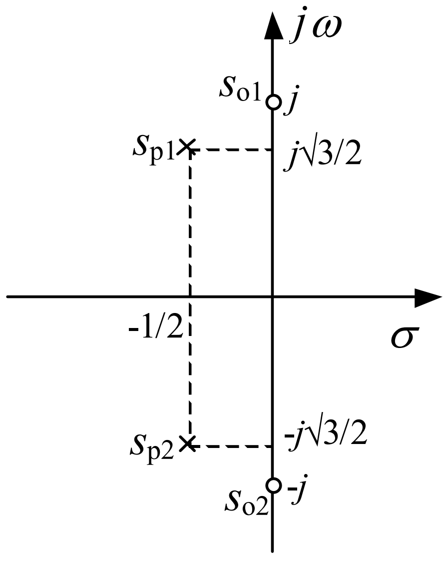
$s^2+1=0$

$(s_o)_{1,2} = \pm j$

polovi:

$s^2+s+1=0$

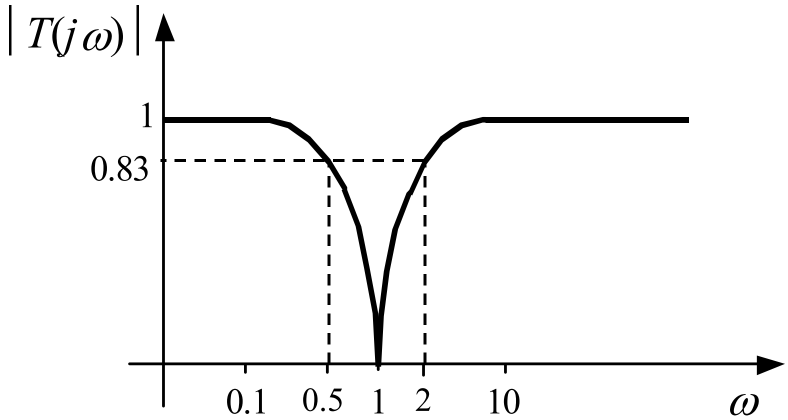
$(s_p)_{1,2} = -\dfrac{1}{2} \pm j\dfrac{\sqrt{3}}{2}$



A-f karakteristika:

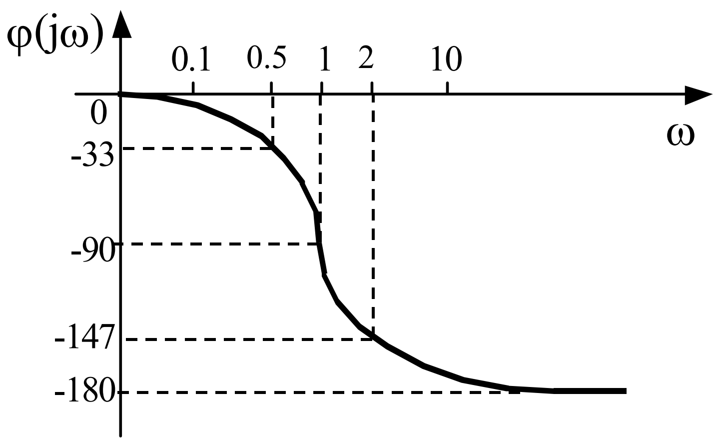
$$|T(j\omega)| = \left|\frac{1-\omega^2}{1-\omega^2+j\omega}\right| = \frac{|1-\omega^2|}{|1-\omega^2+j\omega|} = \frac{\sqrt{(1-\omega^2)^2}}{\sqrt{(1-\omega^2)^2+\omega^2}}$$

$$|T(j\omega)| = \frac{\sqrt{(1-\omega^2)^2}}{\sqrt{\omega^4-\omega^2+1}}$$



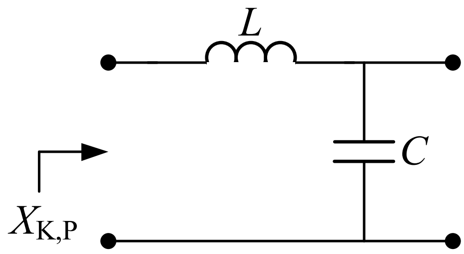
F-f karakteristika:

$$\varphi(\omega) = \arctan\left(-\frac{\omega}{1-\omega^2}\right)$$



Pojasna brana (PB)

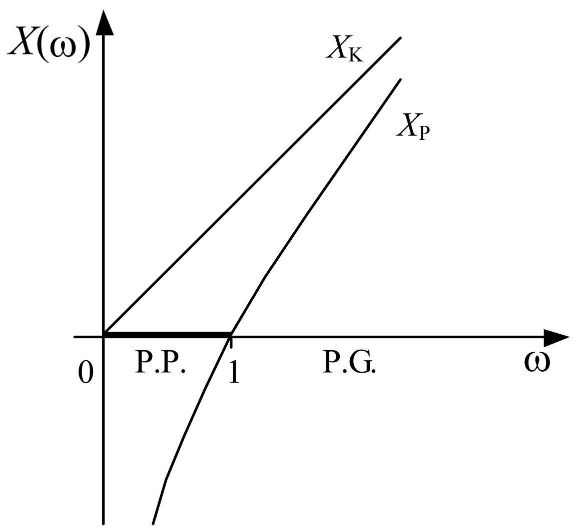
ZADATAK 40. Odrediti područje propuštanja i gušenja signala za zadani reaktantni četveropol. Zadano je L=2, C=1/2.



Ulazne reaktancije na kratko X_K i na prazno X_P

$$X_K = \omega L = 2\omega$$

$$X_P = \omega L - \frac{1}{\omega C} = 2\omega - \frac{2}{\omega} = \frac{2\left(\omega^2 - 1\right)}{\omega}$$



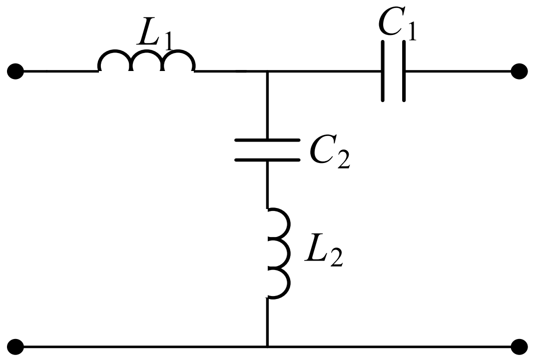
Područje propuštanja (PP): $\underline{\omega \in [0, 1]}$

Područje gušenja (PG): $\underline{\omega \in (1, \infty)}$

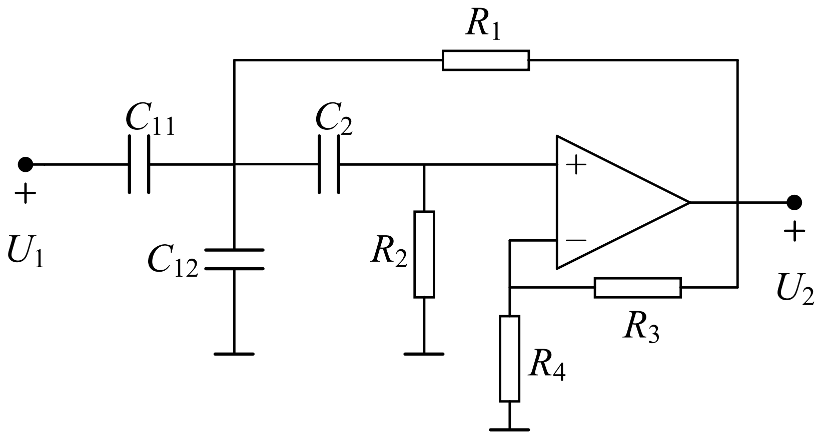
NP filtar

VJEŽBA: Isti tekst uz $L_1=L_2=C_1=C_2=1$.

Rješenje: PP za $\omega \in [0.618, 0.707] \cup [(1.414, 1.618)]$



ZADATAK 41. Odrediti elemente aktivnog VP filtra 2.reda ako su zadani parametri filtra: granična frekvencija f_g=4kHz, Q-faktor polova Q=10 i pojačanje k=1. Nactrati raspored polova i nula prijenosne funkcije filtra te amplitudno-frekvencijsku karakteristiku.



Prijenosna funkcija filtra (jednadžbe čvorova):

$$T(s)=\frac{U_2}{U_1}=\frac{\dfrac{C_{11}}{C_1}\left(1+\dfrac{R_3}{R_4}\right)s^2}{s^2+\left(\dfrac{1}{R_2C_1}+\dfrac{1}{R_2C_2}-\dfrac{R_3}{R_1R_4C_1}\right)s+\dfrac{1}{R_1R_2C_1C_2}}$$

$$C_1=C_{11}+C_{12}$$

$$T(s)=k\frac{s^2}{s^2+\dfrac{\omega_g}{Q}s+\omega_g^2}$$

Parametri filtra:

granična frekvencija $$\omega_g = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}$$

Q-faktor $$Q = \frac{\sqrt{\dfrac{C_1 R_2}{C_2 R_1}}}{1 + \dfrac{C_1}{C_2} - \dfrac{R_2 R_3}{R_1 R_4}}$$

pojačanje $$k = \frac{C_{11}}{C_1}\left(1 + \frac{R_3}{R_4}\right)$$

sustav tri jednadžbe sa 7 nepoznanica

Proizvoljno odabrano:

$$\underline{C_1 = C_2 = C = 10\text{nF}} \qquad \underline{R_4 = 1\text{k}\Omega}$$

$$R_1 = R_2 = R$$

Elementi filtra:

$$\omega_g = \frac{1}{RC} \quad \Rightarrow \quad R = \frac{1}{2\pi f_g C} = 3{,}98\text{k}\Omega$$

$$\underline{R_1 = R_2 = 3{,}98\text{k}\Omega}$$

$$Q=\frac{1}{2-\dfrac{R_3}{R_4}} \quad\Rightarrow\quad \underline{R_3=R_4\left(2-\frac{1}{Q}\right)=1{,}9\text{k}\Omega}$$

$$k=\frac{C_{11}}{C_1}\left(1+\frac{R_3}{R_4}\right) \quad\Rightarrow\quad \underline{C_{11}=\frac{kC_1}{1+\dfrac{R_3}{R_4}}=3{,}45\text{nF}}$$

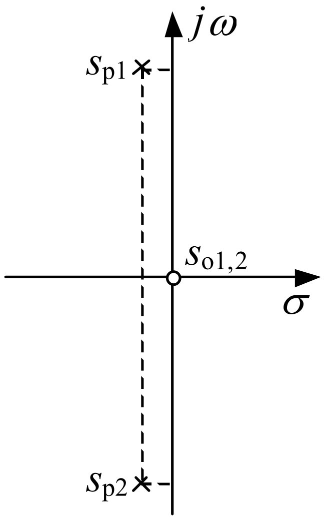
$$\underline{C_{12}=C_1-C_{11}=6{,}55\text{nF}}$$

Prijenosna funkcija:

$$T(s)=\frac{s^2}{s^2+2513{,}27s+6{,}316\cdot 10^8}$$

nule:

$$(s_o)_{1,2}=0$$



polovi:

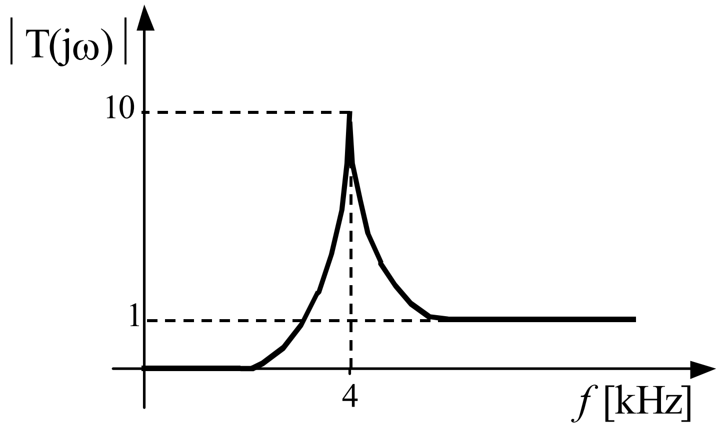
$$(s_p)_{1,2}=-1256{,}64\ \pm j\ 25101{,}31$$

A-f karakteristika:

$$\left|T(j\omega)\right| = \left|\frac{-\omega^2}{-\omega^2 + j2513{,}27\omega + 6{,}316\cdot 10^8}\right|$$

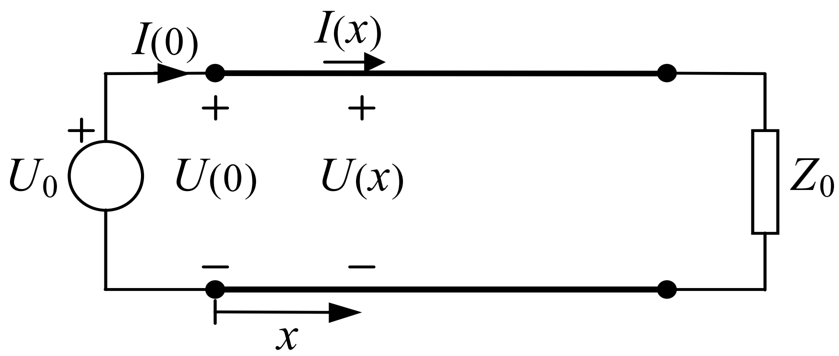
$$\left|T(j\omega)\right| = \frac{\omega^2}{\sqrt{\left(6{,}316\cdot 10^8 - \omega^2\right)^2 + \left(2513{,}27\omega\right)^2}}$$

$$\left|T(j\omega)\right| = \frac{\omega^2}{\sqrt{\omega^4 - 1{,}257\cdot 10^9 \cdot \omega^2 + 3{,}99\cdot 10^{17}}}$$



7. LINIJE

ZADATAK 42. Zadana je linija duljine l=50km s primarnim parametrima: R=5,4Ω/km, G=1S/km, L=2mH/km, C=6nF/km. Odrediti izraz za napon i struju na 15km od početka linije ako je zadano: $Z_2=Z_0$, $u(0,t)=10\cos(5\cdot10^3 t)$. Hiperbolne funkcije ne treba izračunavati.



Napon i struja na nekom mjestu na liniji:

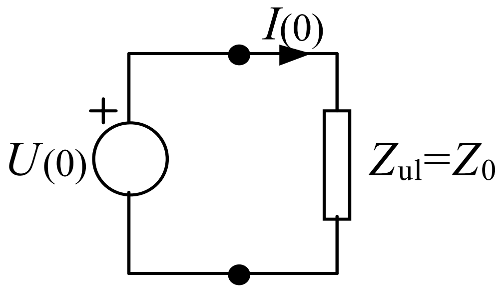
$$(1)\quad U(x) = U(0)\operatorname{ch}\gamma x - I(0)Z_0 \operatorname{sh}\gamma x$$

$$(2)\quad I(x) = -U(0)\frac{\operatorname{sh}\gamma x}{Z_0} + I(0)\operatorname{ch}\gamma x$$

Nadomještanje ulaznom impedancijom:

$$Z_{ul} = Z_0 \frac{Z_2 \operatorname{ch} g + Z_0 \operatorname{sh} g}{Z_2 \operatorname{sh} g + Z_0 \operatorname{ch} g}$$

$$Z_2=Z_0 \qquad \rightarrow \quad Z_{ul} = Z_0$$



$$I(0)=\frac{U(0)}{Z_0}$$

$$\underline{(1)\quad U(x)=U(0)[\text{ch}\,\gamma x-\text{sh}\,\gamma x]=U(0)\text{e}^{-\gamma x}}$$

$$\underline{(2)\quad I(x)=\frac{U(0)}{Z_0}[\text{ch}\,\gamma x-\text{sh}\,\gamma x]=\frac{U(0)}{Z_0}\text{e}^{-\gamma x}}$$

gdje je:

$\underline{x=15\text{km}}$ $\qquad$ $\underline{U(0)=10}$

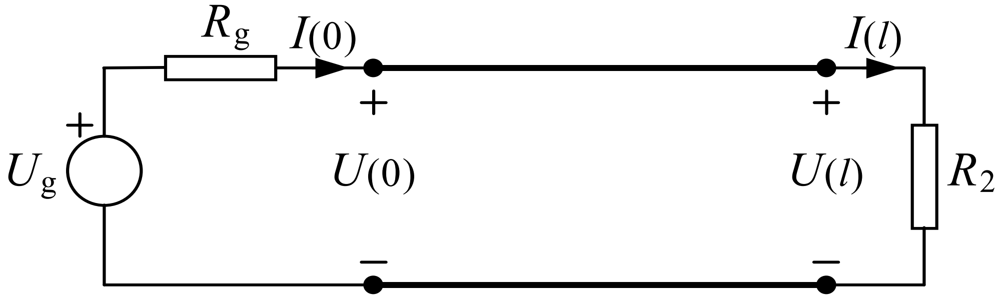
$$Z_0=\sqrt{\frac{R+sL}{G+sC}}=\sqrt{\frac{R+j\omega L}{G+j\omega C}}=\sqrt{\frac{5.4+j10}{1+j3\cdot10^{-5}}}$$

$$\underline{Z_0\cong\sqrt{5.4+j10}\;\Omega}$$

$$\gamma=\sqrt{(R+sL)(G+sC)}=\sqrt{(R+j\omega L)(G+j\omega C)}$$

$$\underline{\gamma\cong\sqrt{5.4+j10}\;/\text{km}}$$

ZADATAK 43. Zadana je linija bez gubitaka s L=0.8mH/km i C=80nF/km, duljine $l=\lambda_0/4$ (λ_0 je valna duljina pri kružnoj frekvenciji ω_0). Na frekvenciji $\omega_0=10^5$rad/s ulazna impedancija linije treba biti prilagođena na R_g. Koliko iznosi R_g? Koliko je duga linija? Odrediti $u_l(t)$ i $i_l(t)$ ako je zadano: R_2=1kΩ, $u_g(t)=4\cos\omega_0 t$.



linija bez gubitaka: $R = G = 0$

Sekundarni parametri:

$$Z_0 = \sqrt{\frac{L}{C}} = 100\Omega$$

$$g = \gamma \cdot l = \gamma \cdot k \cdot \lambda = j\omega\sqrt{LC} \cdot \frac{1}{4} \cdot \frac{2\pi}{\omega\sqrt{LC}} = j\frac{\pi}{2}$$

$$\operatorname{ch} g = \operatorname{ch} j\frac{\pi}{2} = \cos\frac{\pi}{2} = 0$$

$$\operatorname{sh} g = \operatorname{sh} j\frac{\pi}{2} = j\sin\frac{\pi}{2} = j$$

Ulazna impedancija linije:

$$Z_{ul} = Z_0 \frac{Z_2 \operatorname{ch} g + Z_0 \operatorname{sh} g}{Z_2 \operatorname{sh} g + Z_0 \operatorname{ch} g}$$

$$Z_{ul} = \frac{Z_0^2}{R_2} = R_g$$

(uslijed prilagođenja što se zahtijeva u zadatku).

Slijedi: $\underline{R_g = 10\Omega}$

Duljina linije je:

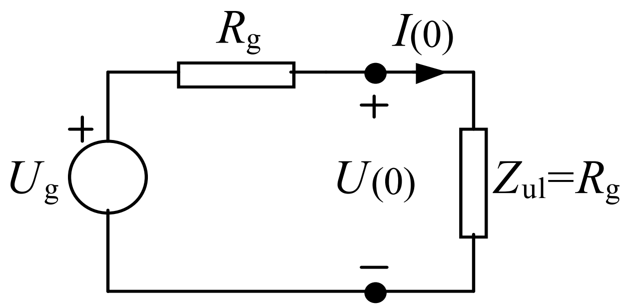
$$\underline{l = \frac{1}{4} \frac{2\pi}{\omega_0 \sqrt{LC}} = \frac{\pi}{1.6} \cong 2\text{km}}$$

Napon i struja na kraju linije:

$$U(l) = U(0) \operatorname{ch} g - I(0) Z_0 \operatorname{sh} g$$

$$I(l) = -U(0) \frac{\operatorname{sh} g}{Z_0} + I(0) \operatorname{ch} g$$

Nadomještanje linije ulaznom impedancijom:



$$U(0)=\frac{U_g}{2}=2 \qquad I(0)=\frac{U_g}{2R_g}=0.2$$

$$U(l)=-I(0)Z_0\,\mathrm{sh}\,g=-j20=20\angle-90^{\circ}$$

$$I(l)=-U(0)\frac{\mathrm{sh}\,g}{Z_0}=-j0.02=0.02\angle-90^{\circ}$$

U vremenskoj domeni:

$$\underline{u_l(t)=20\cos\left(10^5 t-90^{\circ}\right)}$$

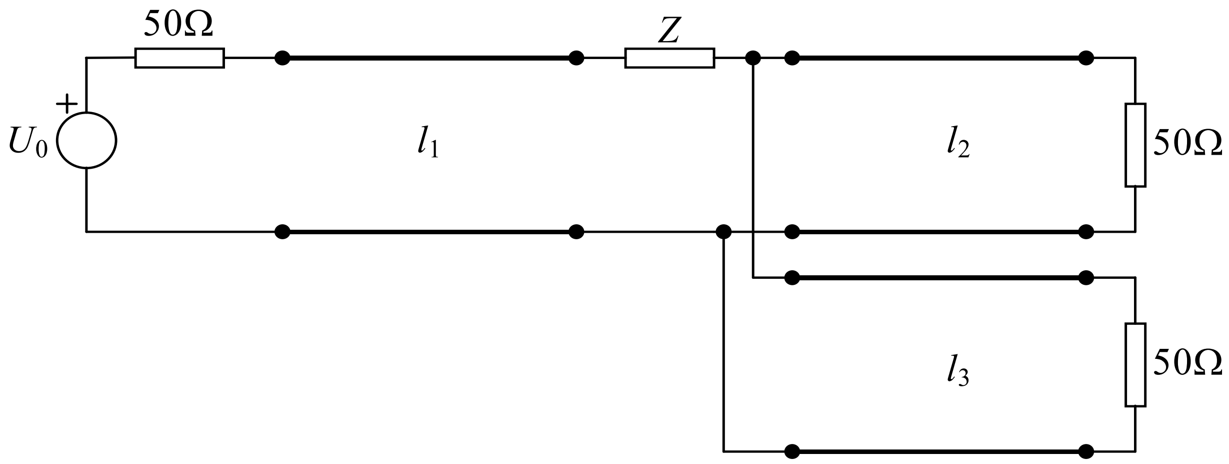
$$\underline{i_l(t)=0.02\cos\left(10^5 t-90^{\circ}\right)}$$

VJEŽBA: Isti zadatak uz $l=\lambda_0/2$.

Rješenje: $R_g=1\mathrm{k}\Omega$, $u_l(t)=-2\cos\left(10^5 t\right)$,

$i_l(t)=-0.002\cos\left(10^5 t\right)$

ZADATAK 44. Odrediti impedanciju Z da bi sustav linija bez gubitaka bio prilagođen po zrcalnim impedancijama. Primarni parametri svih linija su isti i iznose: L=0.8mH/km i C=0.32μF/km.



Prilagođenje po impedancijama: ulazna impedancija jednaka zrcalnoj.

Zrcalne impedancije zadanih linija:

$$Z_0 = \sqrt{\frac{L}{C}} = 50\Omega$$

Uslijed zaključenja linija *2* i *3* s 50Ω to su i ulazne impedancije tih linija jednake zrcalnoj:

$$\left(Z_{ul}\right)_2 = \left(Z_{ul}\right)_3 = 50\Omega$$

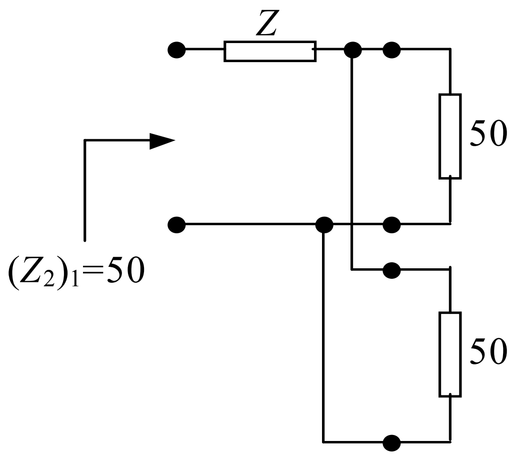
Paralela tih dviju impedancija u seriji s traženom impedancijom Z daje zaključenje linije *1*.

S druge strane, linija *1* će biti prilagođena ako njeno zaključenje iznosi također 50Ω.

Tada je

$$\left(Z_{ul}\right)_1 = Z_0 \frac{Z_0 \operatorname{ch} g + Z_0 \operatorname{sh} g}{Z_0 \operatorname{sh} g + Z_0 \operatorname{ch} g} = Z_0 = 50\Omega$$

Slikom prikazano:



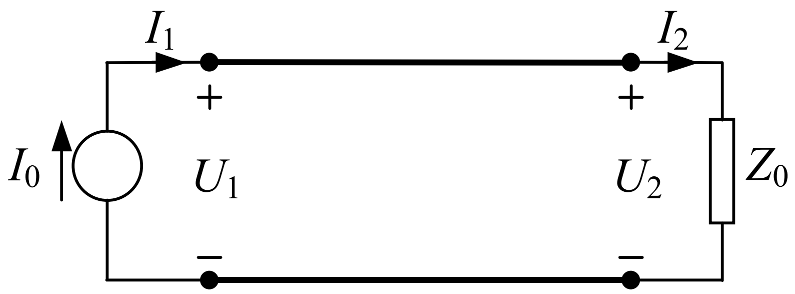
Slijedi: $\left(Z_2\right)_1 = Z + \left(Z_{ul}\right)_2 \| \left(Z_{ul}\right)_3$

$$50 = Z + 50 \| 50$$

$$\underline{Z = 25\Omega}$$

VJEŽBA: Na impedanciju *Z* je spojeno 10 istih linija. Odrediti *Z*. Rješenje: *Z*=45Ω

ZADATAK 45. Zadana je linija duljine l=10^3km s primarnim parametrima R=1Ω/km, G=3S/km, L=3nH/km, C=9nF/km. Odrediti napon na izlazu linije ako je $i_0(t)=S(t)$.



linija bez izobličenja: $$\frac{R}{L}=\frac{G}{C} \qquad RC = GL$$

Napon na kraju linije:

$$U_2 = U_1 \operatorname{ch} g - I_1 Z_0 \operatorname{sh} g$$

Kako je Z_2=Z_0 $\rightarrow$ Z_{ul}=Z_0

Ulazni napon u liniju:

$$U_1 = I_1 Z_0$$

$$U_2 = I_1 Z_0 (\operatorname{ch} g - \operatorname{sh} g) = I_1 Z_0\, e^{-g}$$

Sekundarni parametri:

$$Z_0 = \frac{\sqrt{3}}{3}\Omega$$

$$g = \gamma \cdot l = \left(\sqrt{3} + s \cdot 3\sqrt{3} \cdot 10^{-9}\right) \cdot 10^3 = \sqrt{3} \cdot 10^3 + s \cdot 3\sqrt{3} \cdot 10^{-6}$$

Izlazni napon:

$$U_2(s) = \frac{1}{s}\frac{\sqrt{3}}{3}\mathrm{e}^{-\left(\sqrt{3} \cdot 10^3 + s \cdot 3\sqrt{3} \cdot 10^{-6}\right)}$$

$$U_2(s) = \frac{1}{s}\frac{\sqrt{3}}{3}\mathrm{e}^{-\sqrt{3} \cdot 10^3}\,\mathrm{e}^{-s \cdot 3\sqrt{3} \cdot 10^{-6}}$$

U vremenskoj domeni:

$$\underline{u_2(t) = \frac{\sqrt{3}}{3}\mathrm{e}^{-\sqrt{3} \cdot 10^3}\; S\left(t - 3\sqrt{3} \cdot 10^{-6}\right)}$$

amplitudno prigušenje i vremenski pomak