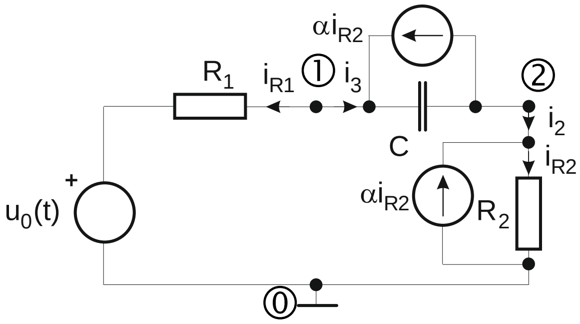
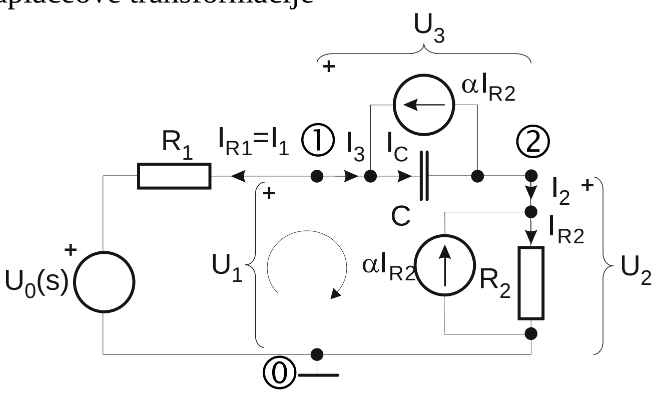
MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA 2016-2017 – Rješenja

1. Poštujući oznake čvorova i grana za električni krug na slici, napisati jednadžbe Kirchhoffovih zakona i izračunati odziv $i_{R1}(t)$ na poticaj $u_0(t)=S(t)$. Zadane su vrijednosti normaliziranih elemenata: $R_1=1$, $R_2=1$, $C=1$, te $\alpha=1/2$.



Rješenje: Primjena Laplaceove transformacije



a) Kirchhoffovi zakoni: (1 bod)

1) $I_1+I_3=0$; KZS

2) $-I_3+I_2=0$; KZS

$$\Rightarrow I_3=-I_1=I_2$$

3) $-U_1+U_2+U_3=0$; KZN

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b) Naponsko strujne relacije grana: (1 bod)

1) $U_1=I_1R_1+U_0$;

2) $U_2=I_2R_2+\alpha\cdot I_{R2}R_2$; $I_{R2}=\dfrac{U_2}{R_2}$;

3) $U_3=I_3\dfrac{1}{sC}+\alpha\cdot I_{R2}\dfrac{1}{sC}$;

$$U_2=I_2R_2+\alpha\cdot U_2 \Rightarrow U_2(1-\alpha)=I_2R_2 \Rightarrow U_2=\frac{R_2}{1-\alpha}\cdot I_2$$

$$U_3=I_3\frac{1}{sC}+\alpha\cdot\frac{1}{sR_2C}\cdot U_2 \Rightarrow U_3=I_3\frac{1}{sC}+\alpha\cdot\frac{1}{sR_2C}\cdot\frac{R_2}{1-\alpha}\cdot I_2$$

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1) $U_1 = I_1 R_1 + U_0$;

2) $U_2 = \dfrac{R_2}{1-\alpha} \cdot I_2$;

3) $U_3 = \dfrac{\alpha}{1-\alpha} \cdot \dfrac{1}{sC} \cdot I_2 + \dfrac{1}{sC} \cdot I_3$

(1 bod)
Uvrstimo jedno u drugo:

$$-U_1 + U_2 + U_3 = 0,\ I_3 = -I_1 = I_2 \Rightarrow$$

$$-I_1 R_1 - U_0 + \frac{R_2}{1-\alpha} \cdot I_2 + \frac{\alpha}{1-\alpha} \cdot \frac{1}{sC} \cdot I_2 + \frac{1}{sC} \cdot I_3 = 0$$

$$-I_1 R_1 - U_0 - \frac{R_2}{1-\alpha} \cdot I_1 - \frac{\alpha}{1-\alpha} \cdot \frac{1}{sC} \cdot I_1 - \frac{1}{sC} \cdot I_1 = 0$$

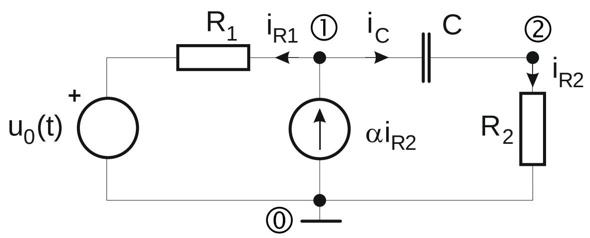
$$I_1 R_1 + \frac{R_2}{1-\alpha} \cdot I_1 + \left(\frac{\alpha}{1-\alpha} + 1 \right) \frac{1}{sC} \cdot I_1 = -U_0$$

$$\left(R_1 + \frac{R_2}{1-\alpha} + \frac{1}{1-\alpha} \frac{1}{sC} \right) \cdot I_1 = -U_0$$

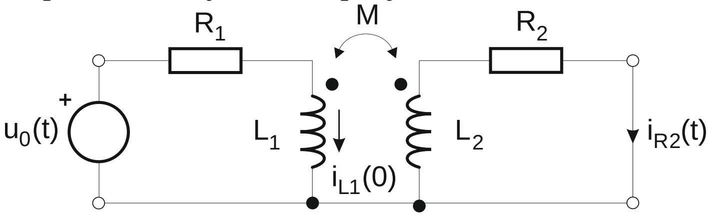
$$\Rightarrow I_1 = \frac{-U_0}{R_1 + \dfrac{1}{1-\alpha}\left(R_2 + \dfrac{1}{sC} \right)} = \frac{-\dfrac{1}{s}}{1 + \left(1 + \dfrac{1}{s} \right) 2} = \frac{-\dfrac{1}{s}}{3 + \dfrac{2}{s}} = \frac{-1}{3s+2} = -\frac{1}{3} \cdot \frac{1}{s + \dfrac{2}{3}}$$ (1 bod)

$\Rightarrow \underline{i_1(t) = -\frac{1}{3} \cdot e^{-\frac{2}{3}t} S(t)}$ (1 bod)

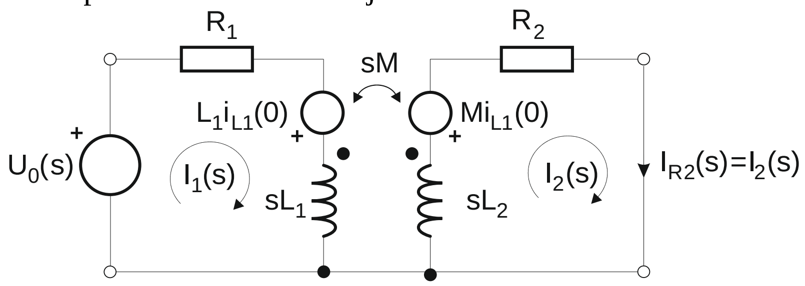
Napomena: Moguće je nacrtati ekvivalentni električni krug koji ima samo jedan strujno ovisni strujni izvor $\alpha\, i_{R2}$. (Izvršeno je posmicanje strujnog izvora.)



2. Za električni krug na slici zadane su normalizirane vrijednosti elemenata $R_1=R_2=1$, $L_1=L_2=2$, $M=1$, početne struje kroz induktivitete $i_{L1}(0)=1$, $i_{L2}(0)=0$, te pobuda $u_0(t)=S(t)$. Primjenom Laplaceove transformacije i koristeći metodu petlji izračunati struju $i_{R2}(t)$ kroz otpor R_2 kao odziv. Napisati sustav jednadžbi petlji u matričnom obliku.



Rješenje: Primjena Laplaceove transformacije



Metoda petlji:

1) $I_1(R_1+sL_1)-I_2sM-L_1i_{L1}(0)-U_0(s)=0$;

2) $-I_1sM+I_2(R_2+sL_2)+Mi_{L1}(0)=0$;

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1) $I_1(R_1+sL_1)-I_2sM=L_1i_{L1}(0)+U_0(s)$; (1 bod)

2) $-I_1sM+I_2(R_2+sL_2)=-Mi_{L1}(0)$; (1 bod)

$$\begin{bmatrix} R_1+sL_1 & -sM \\ -sM & R_2+sL_2 \end{bmatrix}\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix}=\begin{bmatrix} L_1i_{L1}(0)+U_0(s) \\ -Mi_{L1}(0) \end{bmatrix}$$

$$\Delta=\begin{vmatrix} R_1+sL_1 & -sM \\ -sM & R_2+sL_2 \end{vmatrix}=(R_1+sL_1)(R_2+sL_2)-(sM)^2$$

$$\Delta_2=\begin{vmatrix} R_1+sL_1 & L_1i_{L1}(0)+U_0(s) \\ -sM & -Mi_{L1}(0) \end{vmatrix}=-Mi_{L1}(0)(R_1+sL_1)+sM\left(L_1i_{L1}(0)+U_0(s)\right)$$

$$I_2(s)=\frac{\Delta_2}{\Delta}=\frac{-Mi_{L1}(0)(R_1+sL_1)+sM\left(L_1i_{L1}(0)+U_0(s)\right)}{(R_1+sL_1)(R_2+sL_2)-(sM)^2}\quad \text{(1 bod)}$$

a) Uz uvrštene vrijednosti elemenata i uz pobudu $u_0(t)=S(t)$:

$$I_2(s)=\frac{-(1+2s)+s(2+1/s)}{(1+2s)^2-s^2}=0\quad \text{(2 boda)}$$

b) Uz uvrštene vrijednosti elemenata i uz pobudu $u_0(t)=\delta(t)$:

$$I_2(s)=\frac{-(1+2s)+s(2+1)}{(1+2s)^2-s^2}=\frac{s-1}{3s^2+4s+1}=\frac{1}{3}\cdot\frac{s-1}{s^2+\frac{4}{3}s+\frac{1}{3}}=\frac{1}{3}\cdot\frac{s-1}{\left(s+\frac{2}{3}\right)^2-\frac{1}{9}}$$ (1 bod)

$$s^2+\frac{4}{3}s+\frac{1}{3}=0 \Rightarrow s_{1,2}=-\frac{2}{3}\pm\sqrt{\left(\frac{2}{3}\right)^2-\frac{1}{3}}=-\frac{2}{3}\pm\sqrt{\frac{1}{9}}=-\frac{2}{3}\pm\frac{1}{3} \Rightarrow s_1=-\frac{1}{3};s_2=-1.$$

Rastav na parcijalne razlomke:

$$\frac{s-1}{s^2+\frac{4}{3}s+\frac{1}{3}}=\frac{A}{s+\frac{1}{3}}+\frac{B}{s+1} \Rightarrow \frac{s-1}{s^2+\frac{4}{3}s+\frac{1}{3}}=\frac{A(s+1)+B\left(s+\frac{1}{3}\right)}{\left(s+\frac{1}{3}\right)(s+1)}=\frac{(A+B)s+\left(A+B\frac{1}{3}\right)}{s^2+\frac{4}{3}s+\frac{1}{3}}$$

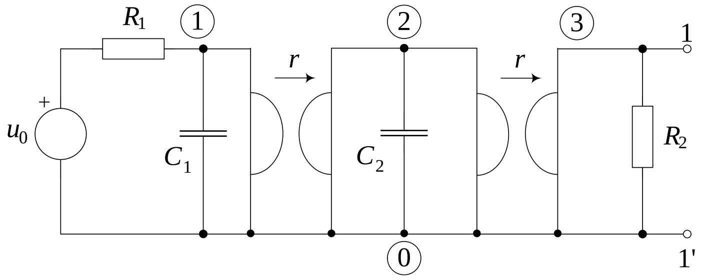
$$\begin{array}{lcl} A+B=1 & & A=1-B=-2 \\ A+B\frac{1}{3}=-1 & \Rightarrow & B\frac{2}{3}=2\Rightarrow B=3 \end{array}$$

$$I_2(s)=\frac{1}{3}\left(\frac{A}{s+\frac{1}{3}}+\frac{B}{s+1}\right)=\frac{1}{3}\left(\frac{-2}{s+\frac{1}{3}}+\frac{3}{s+1}\right)=-\frac{2}{3}\frac{1}{s+\frac{1}{3}}+\frac{1}{s+1}$$

$$\Rightarrow \underline{i_2(t)=\left(-\frac{2}{3}e^{-\frac{1}{3}t}+e^{-t}\right)S(t)}$$ (1 bod)

Napomena: Ovaj dio zadatka je bodovan i ako su studenti uvrstili bilo koju (npr. pogrešnu $u_0(t)=\delta(t)$) pobudu i/ili ako su zaboravili početni uvjet $Mi_{L1}(0)$, dobili su bodove na pretvaranje iz Laplaceove u vremensku domenu. Dobili su bodove na ispravan postupak.

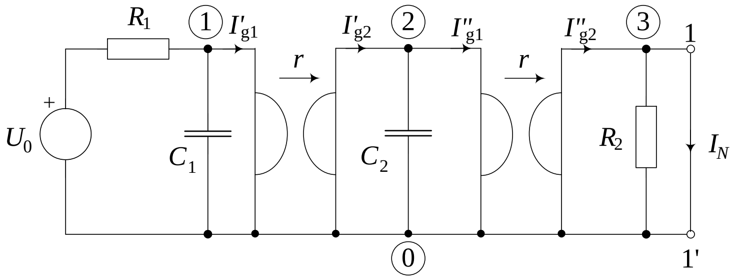
3. Za električni krug na slici zadane su normalizirane vrijednosti elemenata $C_1 = C_2 = \sqrt{2}$, $R_1 = R_2 = 1$, te $r = 1$. Odrediti: nadomjesne parametre mreže po Nortonu obzirom na polove 1–1': a) Nortonovu struju $I_N(s)$ i b) Nortonovu admitanciju $Y_N(s)$.



Rješenje:

a) Nortonova struja:

Jednadžbe napona za čvorišta i Laplaceova transfomacija.



$$\frac{U_0 - U_1}{R_1} = U_1 s C_1 + I'_{g1} \qquad I'_{g1} = -\frac{U_2}{r}, \quad I'_{g2} = -\frac{U_1}{r}$$

$$I'_{g2} - I''_{g1} = U_2 s C_2 \qquad I''_{g1} = -\frac{U_3}{r}, \quad I''_{g2} = -\frac{U_2}{r}$$

$$I_N = I''_{g2}$$

$$\frac{U_0}{R_1} = U_1\left(\frac{1}{R_1} + sC_1\right) - \frac{U_2}{r}$$

$$0 = \frac{U_1}{r} + U_2 s C_2 - \frac{U_3}{r}; \; U_3 = 0 \qquad \Rightarrow \qquad U_1 = -rU_2 s C_2$$

$$I_N = I''_{g2} = -\frac{U_2}{r}$$

$$\frac{U_0}{R_1} = -U_2 s r C_2\left(sC_1 + \frac{1}{R_1}\right) - \frac{U_2}{r} \Big/ \cdot rR_1$$

$$rU_0 = -U_2 s r^2 R_1 C_2\left(sC_1 + \frac{1}{R_1}\right) - U_2 R_1$$

$$rU_0 = -U_2\left[sr^2 C_2 (sR_1C_1 + 1) + R_1\right]$$

$$U_2 = -\frac{rU_0}{sr^2 C_2 (sR_1C_1 + 1) + R_1}$$

$$I_N(s) = -\frac{U_2}{r} = \frac{U_0}{sr^2C_2(sR_1C_1+1)+R_1} = \frac{U_0}{s^2r^2R_1C_1C_2+sr^2C_2+R_1}$$

$$I_N(s) = \frac{U_0}{2s^2+s\sqrt{2}+1} = \frac{1}{2}\cdot\frac{U_0}{s^2+s\sqrt{2}/2+1/2}$$ (3 boda)

b) Nortonova admitancija $Y_N(s)$:

$$Z_{iz1} = \frac{r^2}{R_1 \| \frac{1}{sC_1}} = \frac{r^2}{R_1\cdot\frac{1}{sC_1}}\left(R_1+\frac{1}{sC_1}\right) = r^2\left(sC_1+\frac{1}{R_1}\right)$$

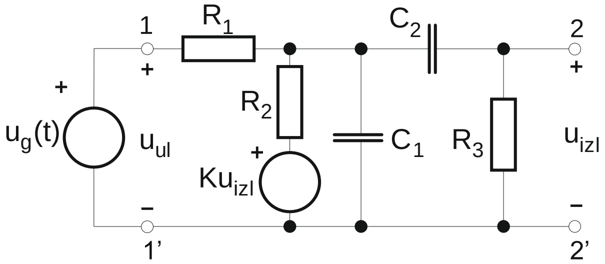
$$Z_{iz2} = \frac{r^2}{Z_{iz1} \| \frac{1}{sC_2}} = \frac{r^2}{Z_{iz1}\cdot\frac{1}{sC_2}}\left(Z_{iz1}+\frac{1}{sC_2}\right) = r^2\left(sC_2+\frac{1}{Z_{iz1}}\right) = r^2\left[\frac{1}{r^2(1/R_1+sC_1)}+sC_2\right]$$

$$Y_N(s) = \frac{1}{R_2}+\frac{1}{Z_{iz2}} = \frac{1}{R_2}+\frac{1}{\frac{1}{1/R_1+sC_1}+sr^2C_2} = \frac{1}{R_2}+\frac{1/R_1+sC_1}{1+sr^2C_2(1/R_1+sC_1)}$$

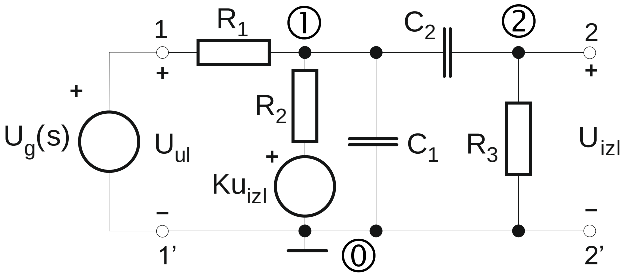
$$Y_N(s) = \frac{1+sr^2C_2(1/R_1+sC_1)+R_2(1/R_1+sC_1)}{R_2\left[1+sr^2C_2(1/R_1+sC_1)\right]} = \frac{1+s\sqrt{2}(1+s\sqrt{2})+(1+s\sqrt{2})}{1+s\sqrt{2}(1+s\sqrt{2})}$$

$$Y_N(s) = \frac{2s^2+s2\sqrt{2}+2}{2s^2+s\sqrt{2}+1} = \frac{s^2+s\sqrt{2}+1}{s^2+s\sqrt{2}/2+1/2}$$ (2 boda)

4. Odrediti funkciju napona na izlazu u frekvencijskoj $U_{izl}(s)$ i vremenskoj $u_{izl}(t)$ domeni za električni krug prikazan slikom. Koristiti metodu napona čvorišta. Zadano je: napon generatora na ulazu $u_g(t)=S(t)$, i normirane vrijednosti elemenata $R_1=R_2=2$, $R_3=1$, $C_1=C_2=1$ i $K=2$.



<u>Rješenje:</u> Metoda napona čvorišta i Laplaceova transformacija



1) $U_1\left(\frac{1}{R_1}+\frac{1}{R_2}+sC_1+sC_2\right)-U_2sC_2=\frac{U_g(s)}{R_1}+K\frac{U_{iz}(s)}{R_2}$; (1 bod)

2) $-U_1sC_2+U_2\left(sC_2+\frac{1}{R_3}\right)=0$; (1 bod)

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2) $\Rightarrow U_1=U_2\left(1+\frac{1}{sC_2R_3}\right)\rightarrow 1)$

$$U_2(s)\left[\left(1+\frac{1}{sC_2R_3}\right)\left(\frac{1}{R_1}+\frac{1}{R_2}+sC_1+sC_2\right)-sC_2-\frac{K}{R_2}\right]=\frac{U_g(s)}{R_1}$$

$$U_2\left[\frac{1}{R_1}+\frac{1}{R_2}+sC_1+sC_2+\frac{1}{R_1R_3sC_2}+\frac{1}{R_2R_3sC_2}+\frac{sC_1}{sC_2R_3}+\frac{sC_2}{sC_2R_3}-sC_2-\frac{K}{R_2}\right]=\frac{U_g}{R_1}\Big/\cdot R_1R_2R_3sC_2$$

$$U_2\left[s^2C_1R_1R_2R_3C_2+sR_2R_3C_2+sR_1R_2(C_1+C_2)+s(1-K)R_1R_3C_2+R_1+R_2\right]=U_gR_2R_3sC_2$$

(1 bod račun)

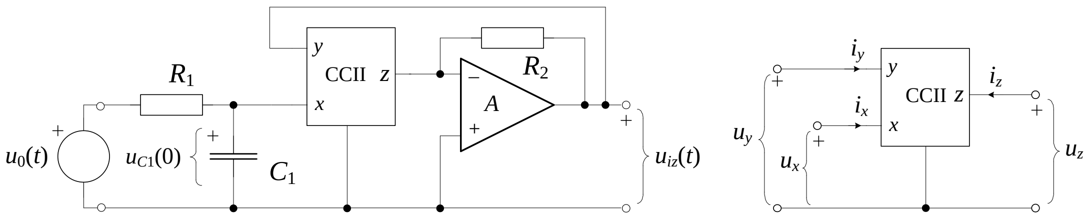
$$U_2(s)=U_g(s)\cdot\frac{R_2R_3sC_2}{s^2C_1R_1R_2R_3C_2+s\left[R_2R_3C_2+R_1R_2(C_1+C_2)+(1-K)R_1R_3C_2\right]+R_1+R_2}$$

$$U_2(s)=\frac{2s}{4s^2+s[2+8-2]+4}\cdot U_g(s)=\frac{1}{4}\cdot\frac{2s}{s^2+2s+1}\cdot U_g(s)$$

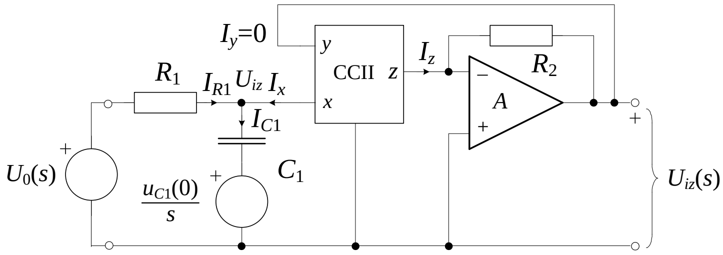
$$U_{iz}(s)=U_2(s)=\frac{1}{4}\cdot\frac{2s}{s^2+2s+1}\cdot\frac{1}{s}=\frac{1}{2}\cdot\frac{1}{(s+1)^2}$$ (1 bod)

$$u_{iz}(t)=\frac{1}{2}t\cdot e^{-t}\cdot S(t)$$ (1 bod)

5. Za električni krug prikazan slikom izračunati valni oblik napona na izlazu $u_{iz}(t)$ za $t>0$ kao odziv, ako je zadana pobuda $u_0(t)=S(t)$ te početni napon na kapacitetu $u_{C1}(0)=1$. Zadane su normalizirane vrijednosti elemenata $R_1=1$, $R_2=1$, $C_1=1$. Operacijsko pojačalo je idealno, a za strujni prijenosnik druge generacije (CCII) vrijede slijedeće definicijske jednadžbe: $u_x=u_y$, $i_y=0$, $i_z=i_x$ uz referentna usmjerenja struja i napona prilaza prikazana na slici.



Rješenje: Laplaceova transformacija



Za CCII vrijedi:

$$I_{C1}=I_{R1}+I_x \;\Rightarrow\; I_x=I_{C1}-I_{R1}=I_z \text{ (1 bod)}$$

$$U_x=U_y=U_{iz}$$

$$I_{C1}=\left[U_{iz}-\frac{u_C(0)}{s}\right]\cdot sC_1,\; I_y=0$$

$$I_{R1}=\frac{U_0-U_{iz}}{R_1}$$

$$U_{iz}=-I_zR_2$$

$$U_{iz}=-(I_{C1}-I_{R1})R_2=-\left(U_{iz}sC_1-\frac{U_0-U_{iz}}{R_1}\right)R_2+C_1u_{C1}(0)R_2$$

$$U_{iz}=-U_{iz}sC_1R_2+\frac{U_0}{R_1}R_2-\frac{U_{iz}}{R_1}R_2+C_1u_{C1}(0)R_2$$

$$U_{iz}+U_{iz}sC_1R_2+U_{iz}\frac{R_2}{R_1}=U_0\frac{R_2}{R_1}+R_2C_1u_{C1}(0)$$

$$U_{iz}\left(1+sC_1R_2+\frac{R_2}{R_1}\right)=U_0\frac{R_2}{R_1}+R_2C_1u_{C1}(0)$$

$$U_{iz}(s)=\frac{\frac{R_2}{R_1}}{\left(1+sC_1R_2+\frac{R_2}{R_1}\right)}\cdot U_0+\frac{R_2C_1}{\left(1+sC_1R_2+\frac{R_2}{R_1}\right)}\cdot u_{C1}(0) \text{ (1 bod)}$$

Uz uvrštene vrijednosti elemenata

$$U_{iz}(s)=\frac{1}{(s+2)}\cdot\frac{1}{s}+\frac{1}{(s+2)} \text{ (1 bod)}$$

Rastav na parcijalne razlomke

$$\frac{1}{(s+2)}\cdot\frac{1}{s}=\frac{A}{s+2}+\frac{B}{s}=\frac{As+B(s+2)}{(s+2)\cdot s}=\frac{(A+B)s+2B}{(s+2)\cdot s}=\frac{1}{(s+2)\cdot s}$$

$$\begin{array}{l} A+B=0 \\ \underline{2B=1} \end{array} \Rightarrow \begin{array}{l} A=-B=-1/2 \\ \underline{2B=1\Rightarrow B=1/2} \end{array}$$

$$U_{iz}(s)=\frac{1}{2}\left(\frac{-1}{s+2}+\frac{1}{s}\right)+\frac{1}{s+2}=\frac{1}{2}\left(\frac{1}{s+2}+\frac{1}{s}\right)$$ (1 bod)

$$\Rightarrow \underline{u_{iz}(t)=\frac{1}{2}\left(e^{-2t}+1\right)\cdot S(t)}$$ (1 bod)

Ili imamo slijedeću mogućnost:

$$U_{iz}(s)=\frac{1}{(s+2)}\cdot\frac{1}{s}+\frac{1}{(s+2)}=\frac{1+s}{s(s+2)}=\frac{1+s}{s^2+2s}=\frac{1+s}{s^2+2s+1-1}=\frac{s+1}{(s+1)^2-1}$$

$$\underline{u_{iz}(t)=e^{-t}\cdot ch(t)\cdot S(t)=e^{-t}\cdot\frac{e^{-t}+e^{t}}{2}\cdot S(t)=\frac{1}{2}\left(e^{-2t}+1\right)\cdot S(t)}$$