Skenirana rješenja

Prijenosne funkcije i a.-f. karakteristika: Zadaci sa rješenjima za vježbu

by:

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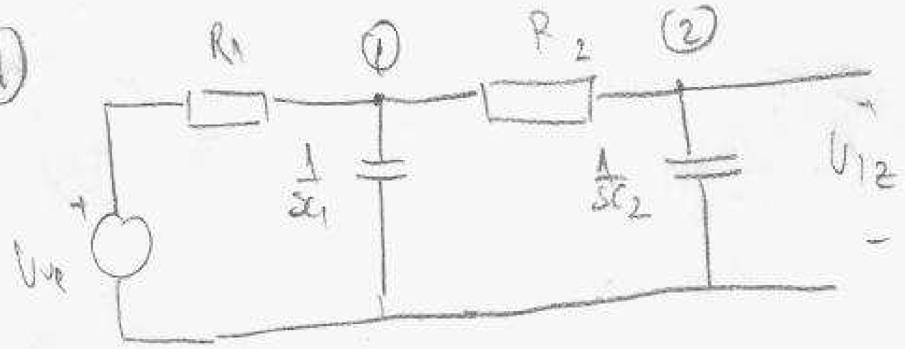
(1)

$R_1 = R_2 = C_1 = C_2 = 1$



$$(1)\quad U_1\left(\frac{1}{R_1}+sC_1+\frac{1}{R_2}\right)-U_2\left(\frac{1}{R_2}\right)=\frac{U_{ul}}{R_1}$$

$$(2)\quad U_2\left(\frac{1}{R_2}+sC_2\right)-U_1\left(\frac{1}{R_2}\right)=0$$

$$(1)\quad U_1(1+s+1)-U_2=U_{ul}$$

$$(2)\quad U_2(1+s)-U_1=0 \rightarrow \boxed{U_1=(s+1)U_2}$$

$$\Rightarrow U_2\left[(s+1)(s+2)\right]-U_2=U_{ul}$$

$$U_2\left[s^2+s+2s+2-1\right]=U_{ul}$$

$$\boxed{U_2 \cdot \left[s^2+3s+1\right]=U_{ul}}$$

$$T(s)=\frac{U_{iz}}{U_{ul}}=\frac{U_2}{U_{ul}}=\frac{\cancel{U_2}}{\cancel{U_2}\left[s^2+3s+1\right]}=\frac{1}{s^2+3s+1}$$

$$T(j\omega)=\frac{1}{-\omega^2+3j\omega+1}=\frac{1}{1-\omega^2+3j\omega}$$

$$|T(j\omega)|=\frac{1}{\sqrt{(1-\omega^2)^2+9\omega^2}}$$

$$|T(j\omega)| = \frac{1}{\sqrt{(1-\omega^2)^2 + 9\omega^2}}$$

$$|T(0)| = 1$$

$$|T(\infty)| = 0$$

$$|T(1)| = \frac{1}{\sqrt{9}} = \frac{1}{3}$$

(2) $R_1 = R_2 = 10\,k\Omega = 10 \cdot 10^3\,\Omega$

$C_1 = C_2 = 1\,nF = 1 \cdot 10^{-9}\,F$

$k = 2$

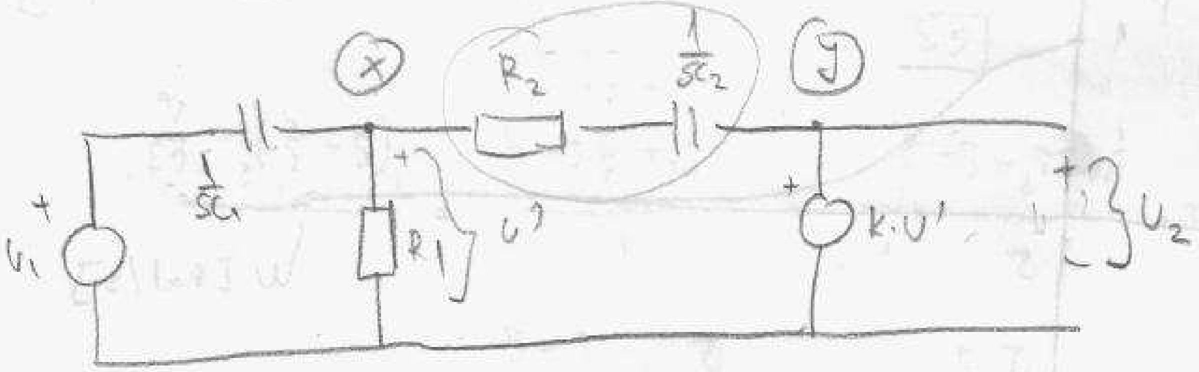
$\Rightarrow \omega_0 = 10^5\ Rad/s, \quad R_0 = 10^4\ k\Omega$

$$\Rightarrow R_m = \frac{R}{R_0} = \frac{10^4}{10^4} = 1$$

$$\Rightarrow C_m = \omega_0 R_0 C_0 = 10^5 \cdot 10^4 \cdot 10^{-9} = 1$$

$$\Rightarrow \boxed{R_1 = R_2 = 1, \quad C_1 = C_2 = 1, \quad k = 2}$$

Z_1



$$Z_1 = R_2 + \frac{1}{sC_2} = \frac{R_2 sC_2 + 1}{sC_2}$$

$$(1)\ U_x\left(sC_1 + R_1 + \frac{sC_2}{R_2 sC_2 + 1}\right) - U_y\left(\frac{sC_2}{R_2 sC_2 + 1}\right) = U_1 sC_1$$

$$(2)\ U_y = k \cdot U' \Rightarrow U_y = k \cdot U_x \Rightarrow \boxed{U_y = 2 \cdot U_x} \Rightarrow \boxed{U_x = \frac{U_y}{2}}$$

$$U_x\left(s + 1 + \frac{s}{s+1}\right) - U_y\left(\frac{s}{s+1}\right) = U_1 s$$

$$U_x\left(\frac{s^2 + s + s + 1 + s}{s+1}\right) - U_y\left(\frac{s}{s+1}\right) = U_1 s$$

$$U_x\left(\frac{s^2+3s+1}{s+1}\right) - U_y\left(\frac{s}{s+1}\right) = U_1 s$$

$$U_y\left(\frac{s^2+3s+1}{2(s+1)}\right) - U_y\left(\frac{s}{s+1}\right) = U_1 s$$

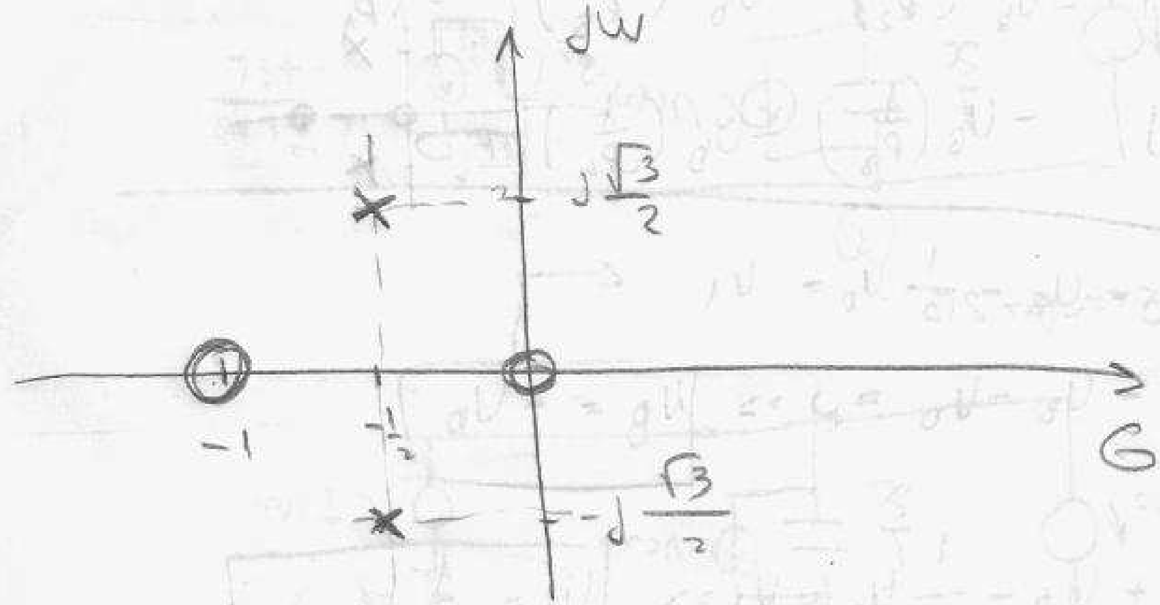
$$U_y\left(\frac{s^2+3s+1}{2(s+1)} - \frac{s}{s+1}\right) = U_1 s$$

$$U_y\left(\frac{s^2+s+1}{2(s+1)}\right) = U_1 s \Rightarrow \boxed{U_1 = \frac{s^2+s+1}{2s(s+1)}\, U_y}$$

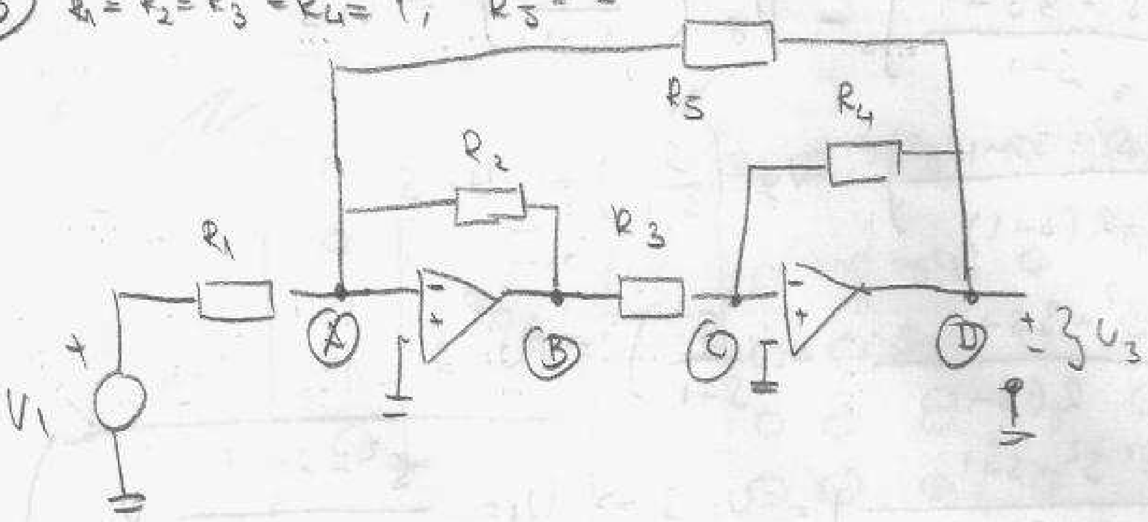
$$T(s) = \frac{U_2}{U_1} = \frac{U_y}{U_1} = \frac{2s(s+1)}{s^2+s+1}$$

POLOVI: $s^2+s+1=0 \Rightarrow s_{p12} = \frac{-1\pm\sqrt{1-4}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

NULE: $2s(s+1)=0 \Rightarrow s_{01}=0,\ s_{02}=-1$



(3) $R_1 = R_2 = R_3 = R_4 = 1, \quad R_5 = 2$



$$\Rightarrow (A) \quad U_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5}\right) - U_B\left(\frac{1}{R_2}\right) - U_D\left(\frac{1}{R_5}\right) = \frac{V_1}{R_1}$$

$$(C) \quad U_C\left(\frac{1}{R_3} + \frac{1}{R_4}\right) - U_B\left(\frac{1}{R_3}\right) - U_D\left(\frac{1}{R_4}\right) = 0$$

$$\Rightarrow \boxed{U_A = U_C = 0}$$

$$\Rightarrow (A) \quad -U_B\left(\frac{1}{R_2}\right) - U_D\left(\frac{1}{R_5}\right) = \frac{U_1}{R_1}$$

$$(C) \quad -U_B\left(\frac{1}{R_3}\right) - U_D\left(\frac{1}{R_4}\right) = 0$$

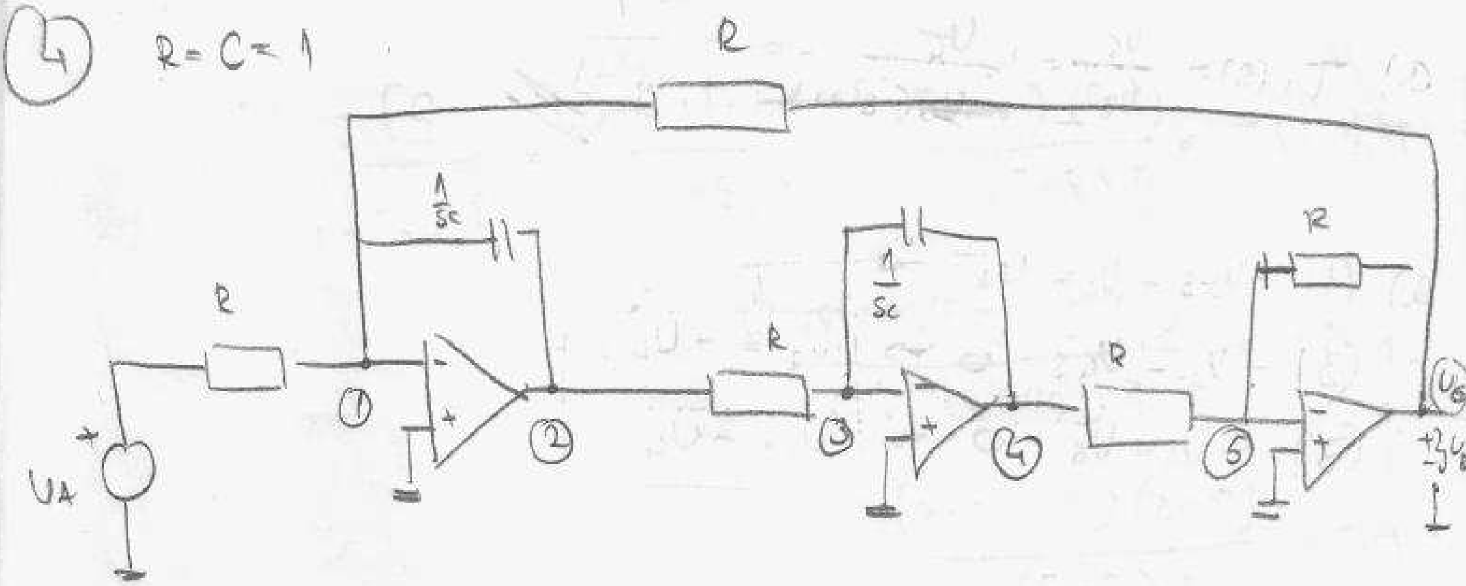
$$-U_B - \frac{1}{2} U_D = U_1$$

$$-U_B - U_D = 0 \Rightarrow \boxed{U_B = -U_D}$$

$$+U_D - \frac{1}{2} U_D = V_1 \Rightarrow \boxed{V_1 = \frac{1}{2} U_D}$$

$$T(s) = \frac{U_3}{V_1} = \frac{U_D}{U_1} = \frac{\cancel{U_D}}{\frac{1}{2}\cancel{U_D}} = \frac{1}{\frac{1}{2}} = 2$$

④ $R = C = 1$



(1) $U_1\left(\frac{1}{R} + sC + \frac{1}{R}\right) - U_2(sC) - U_6\left(\frac{1}{R}\right) = \frac{U_A}{R}$

(3) $U_3\left(\frac{1}{R} + sC\right) - U_2\left(\frac{1}{R}\right) - U_4(sC) = 0$

(5) $U_5\left(\frac{1}{R} + \frac{1}{R}\right) - U_4\left(\frac{1}{R}\right) - U_6\left(\frac{1}{R}\right) = 0$

$\Rightarrow \boxed{U_1 = U_3 = U_5 = 0}$

(1) $-U_2(sC) - U_6\left(\frac{1}{R}\right) = \frac{U_A}{R}$

(3) $-U_2\left(\frac{1}{R}\right) - U_4 sC = 0$

(5) $-U_4\left(\frac{1}{R}\right) - U_6\left(\frac{1}{R}\right) = 0$

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(1) $-U_2 s - U_6 = U_A$

(3) $-U_2 - U_4 s = 0$

(5) $-U_4 - U_6 = 0 \Rightarrow U_4 = -U_6$

(3) $\Rightarrow -U_2 + U_6 s = 0 \Rightarrow \boxed{U_2 = U_6 s}$

$-U_6 s^2 - U_6 = U_A \Rightarrow \boxed{U_A = -U_6(s^2+1)}$

a) $T_1(s) = \frac{U_6}{U_A} = \frac{U_6}{-\cancel{U_6}(s^2+1)} = -\frac{1}{s^2+1}$

a) (1) $-V_2 s - U_6 = U_A$

(3) $-V_2 - U_4 s = 0 \Rightarrow \boxed{U_2 = -U_4 s}$

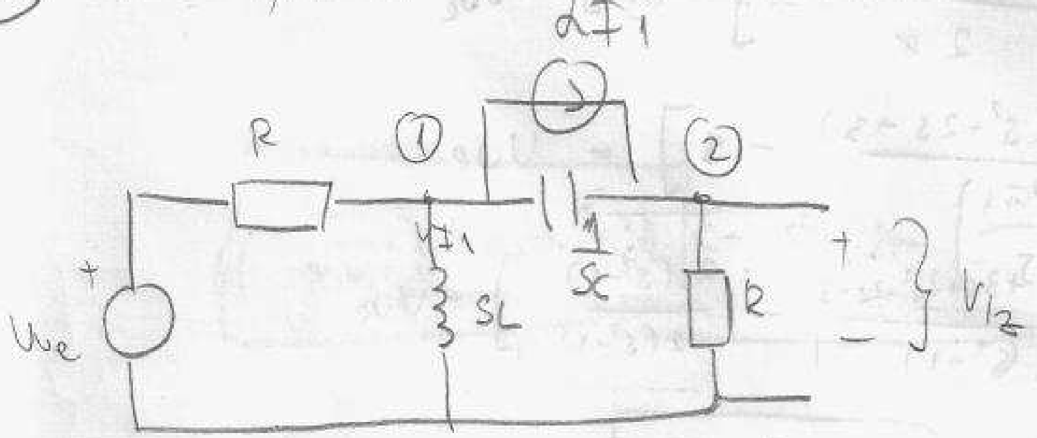
(5) $-U_4 - U_6 = 0 \Rightarrow \boxed{U_6 = -U_4}$

(1) $U_4 s^2 + U_4 = U_A$

$U_4(s^2+1) = U_A$

$T(s) = \frac{U_4}{U_A} = \frac{\cancel{U_4}}{\cancel{U_4}(s^2+1)} = \frac{1}{s^2+1}$

(5) $R = C = 1,\ L = \alpha = 2$



(1) $V_1\left(\frac{1}{R} + \frac{1}{sL} + sC\right) - V_2(sC) = \frac{V_{ue}}{R} - \alpha I_1$

(2) $V_2\left(sC + \frac{1}{R}\right) - V_1(sC) = \alpha I_1$

(3) $I_1 = \frac{V_1}{sL}$

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(1) $V_1\left(1 + \frac{1}{2s} + s\right) - V_2 s = V_{ue} - \cancel{2}\left(\frac{V_1}{\cancel{2}s}\right)$

(2) $V_2(s+1) - V_1 s = \cancel{2}\left(\frac{V_1}{\cancel{2}s}\right)$

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(1) $V_1\left(\frac{2s + 1 + 2s^2}{2s}\right) - V_2 s = V_{ue} - \frac{V_1}{s}$

(2) $V_2(s+1) - V_1 s = \frac{V_1}{s} \Rightarrow V_2(s+1) = V_1\left(\frac{1}{s} + s\right)$

$\Rightarrow V_2(s+1) = V_1\left(\frac{s^2+1}{s}\right) \Rightarrow \boxed{V_1 = V_2\left[\frac{s(s+1)}{s^2+1}\right]}$

(1) $V_1\left(\frac{2s^2 + 2s + 1}{2s} + \frac{1}{s}\right) - V_2 s = V_{ue}$

$V_1\left(\frac{2s^2 + 2s + 3}{2s}\right) - V_2 s = V_{ue}$

$$U_2\left[\frac{\cancel{s}(s+1)}{s^2+1}\;\frac{2s^2+2s+3}{2\cancel{s}}\right]-U_2\,s=U_{ue}$$

$$U_2\left[\frac{(s+1)(2s^2+2s+3)}{2(s^2+1)}-s\right]=U_{ue}$$

$$U_2\left[\frac{2s^3+2s^2+3s+2s^2+2s+3}{2(s^2+1)}-\frac{2s(s^2+1)}{2(s^2+1)}\right]=U_{ue}$$

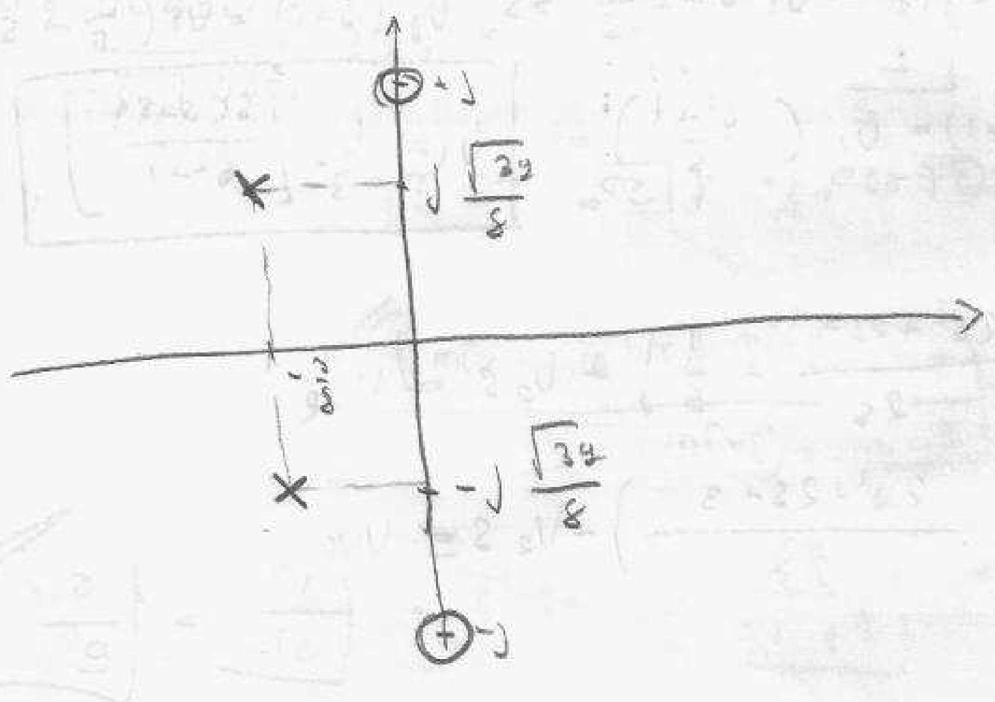
$$U_2\left[\frac{\cancel{2s^3}+4s^2+5s+3-\cancel{2s^3}-2s}{2(s^2+1)}\right]=U_{ue}$$

$$U_2 * \left(\frac{4s^2+3s+3}{2(s^2+1)}\right)=U_{ue}$$

$$T(s)=\frac{U_{iz}}{U_{ue}}=\frac{U_2}{U_{ue}}=\frac{2(s^2+1)}{4s^2+3s+3}$$

Polovi: $4s^2+3s+3=0 \Rightarrow s_{p12}=\frac{-3\pm\sqrt{9-48}}{8}=-\frac{3}{8}\pm j\frac{\sqrt{39}}{8}$

NULE: $2(s^2+1)=0 \Rightarrow s^2+1=0 \Rightarrow \boxed{s_{o1,2}=\pm j}$



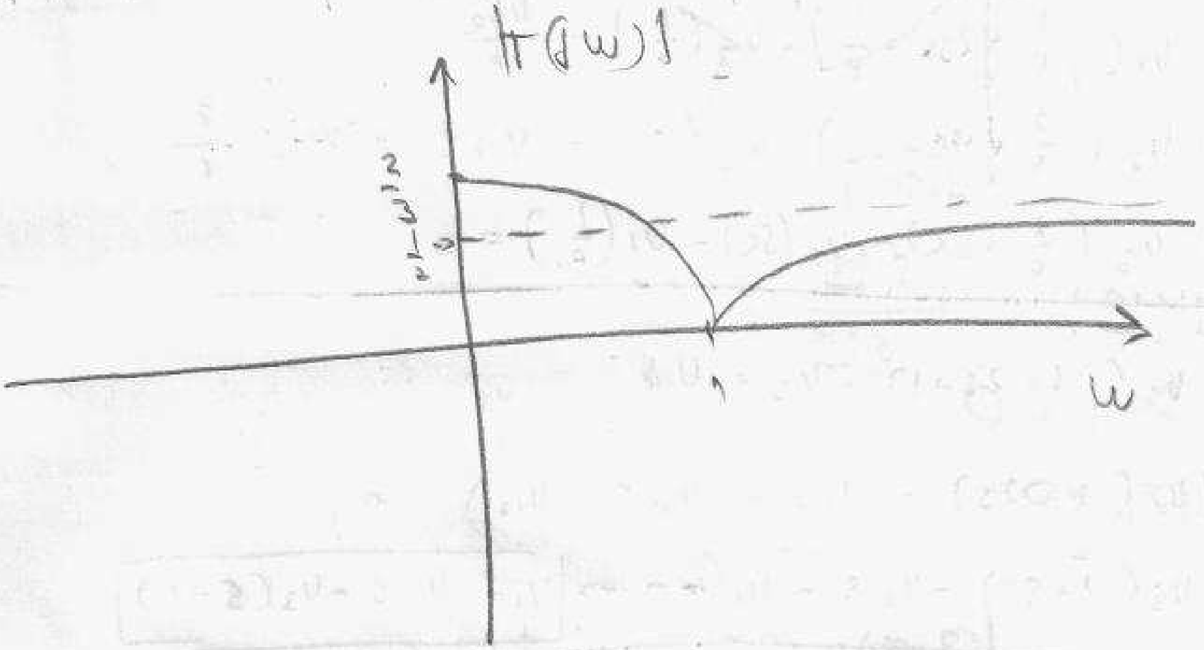
$$T(j\omega) = \frac{2\,(1-\omega^2)}{-4\omega^2 + 3j\omega + 3} = 2\,\frac{(1-\omega^2)}{(3-4\omega^2) + 3j\omega}$$

$$|T(j\omega)| = 2\,\frac{\sqrt{(1-\omega^2)^2}}{\sqrt{(3-4\omega^2)^2 + 9\omega^2}}$$

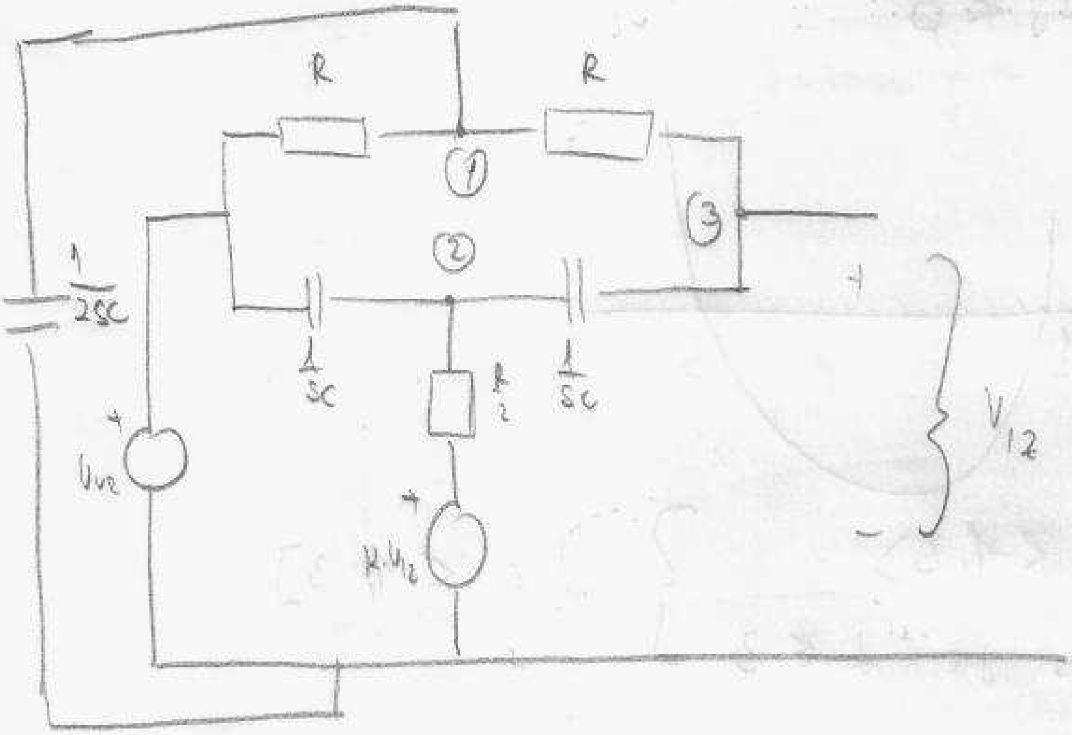
$$|T(0)| = 2\,\frac{1}{\sqrt{3^2}} = \frac{2}{3}$$

$$|T(\infty)| = 2\,\frac{1}{4} = \frac{1}{2}$$

$$|T(1)| = 0$$



(6) $R = C = 1, \ k = \frac{3}{2}$



(1) $U_1\left(\frac{1}{R} + 2sC + \frac{1}{R}\right) - U_3\left(\frac{1}{R}\right) = \frac{U_{ve}}{R}$

(2) $U_2\left(sC + \frac{2}{R} + sC\right) - U_3(sC) = k U_3 \cdot \frac{2}{R} + U_{ve} sC$

(3) $U_3\left(sC + \frac{1}{R}\right) - U_2(sC) - U_1\left(\frac{1}{R}\right) = 0$

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(1) $U_1(2 + 2s) - U_3 = U_{ve}$

(2) $U_2(2 + 2s) - U_3 s = 3U_3 + U_{ve} s$

(3) $U_3(s+1) - U_2 s - U_1 = 0 \Rightarrow \boxed{U_1 = U_3(s+1) - U_2 s}$

$U_2(2+2s) = 3U_3 + sU_3 + U_{ve} s$

$U_2(2+2s) = U_3(s+3) + U_{ve} s \Rightarrow \boxed{U_2 = U_3 \frac{s+3}{2(s+1)} + U_{ve} \frac{s}{2(s+1)}}$

$$\left[V_3(s+1) - V_2 s\right] 2(s+1) - V_3 = U_{ve}$$

$$V_3\, 2(s+1)^2 - V_2\, 2s(s+1) - V_3 = U_{ve}$$

$$V_3\left[2(s^2+2s+1) - 1\right] - V_2\left[2s(s+1)\right] = U_{ve}$$

$$V_3\left[2s^2+4s+1\right] - \left[V_3 \frac{s+3}{2(s+1)} + U_{ve}\frac{s}{2(s+1)}\right] \cdot 2s(s+1) = U_{ve}$$

$$V_3\left[2s^2+4s+1\right] - V_3\, s(s+3) - U_{ve}\, s^2 = U_{ve}$$

$$V_3\left[\frac{2s^3+4s^2+s}{s} - \frac{s^2(s+3)}{s}\right] = U_{ve}(s^2+1)$$

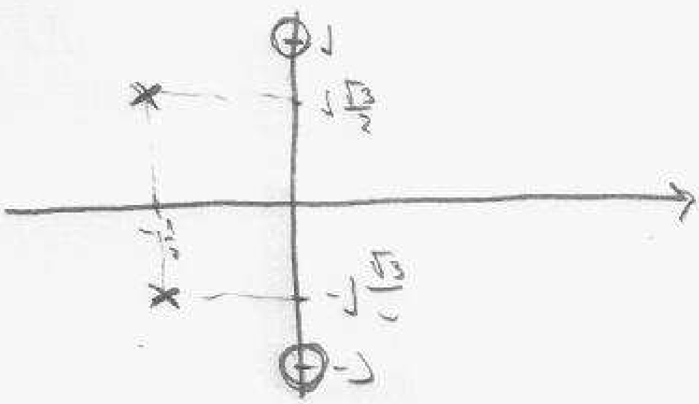
$$V_3\left(\frac{2s^3+4s^2+s-s^3-3s^2}{s}\right) \cdot \frac{1}{s^2+1} = U_{ve}$$

$$U_{ve} = V_3\left(\frac{s^3+s^2+s}{s(s^2+1)}\right) = V_3\,\frac{s^2+s+1}{s^2+1}$$

$$T(s) = \frac{V_3}{U_{ve}} = \frac{s^2+1}{s^2+s+1}$$

POLOVI: $s^2+s+1=0 \Rightarrow s_{p1,2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

NULE: $s^2+1=0 \Rightarrow s_{01,2} = \pm j$



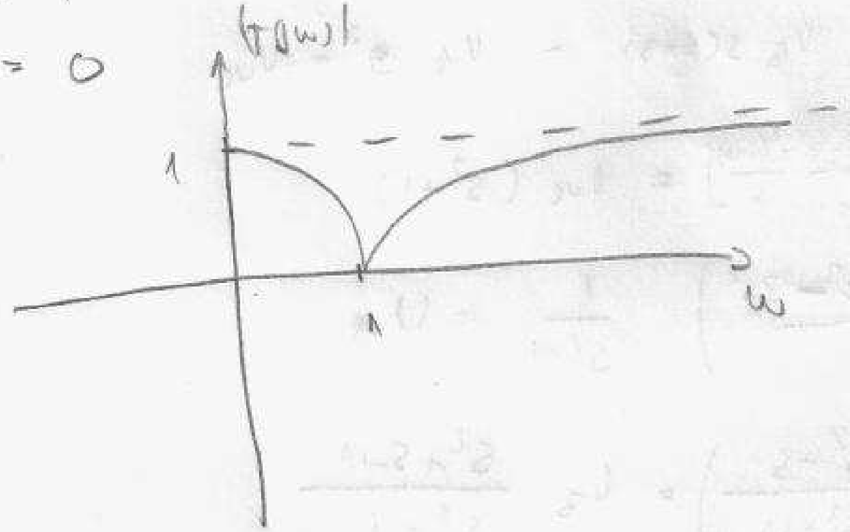
$$T(s) = \frac{s^2+1}{s^2+s+1}$$

$$T(jw) = \frac{1-w^2}{1-w^2+jw} \Rightarrow |T(jw)| = \frac{|1-w^2|}{\sqrt{(1-w^2)^2+w^2}}$$

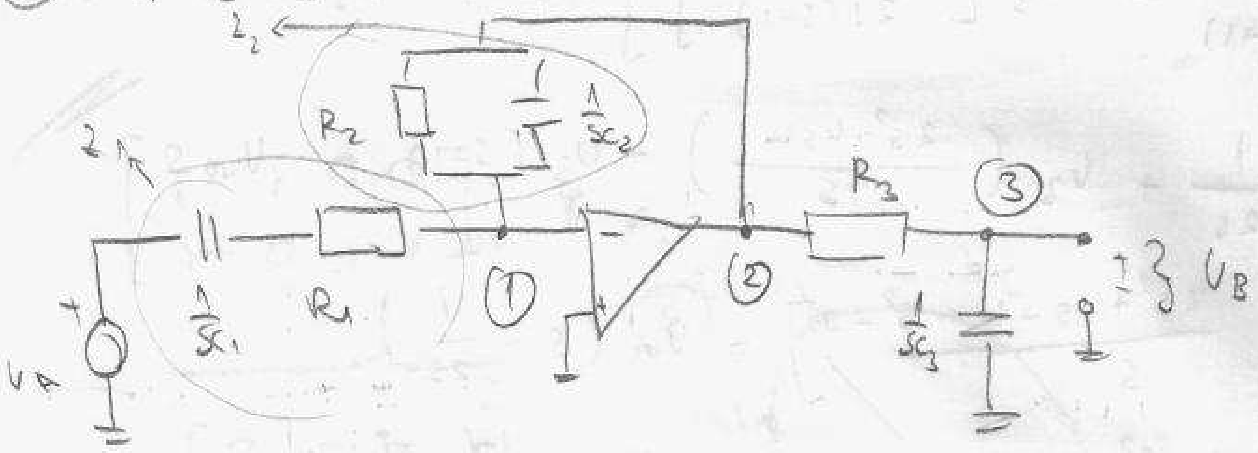
$T(0) = 1$

$|T(\infty)| = 1$

$|T(1)| = 0$



⑦ $R_1 = R_2 = R_3 = 1, \quad C_1 = 1, \quad C_2 = \frac{1}{2}, \quad C_3 = 2$



$$Z_1 = \frac{1}{sC_1} + R_1 = \frac{1 + R_1 sC_1}{sC_1}$$

$$Z_2 = \left(\frac{1}{R_2} + sC_2\right)^{-1} = \left(\frac{1 + R_2 sC_2}{R_2}\right)^{-1} \Rightarrow Z_2 = \frac{R_2}{1 + R_2 sC_2}$$

$$(1) \quad V_1\left(\frac{sC_1}{1+R_1 sC_1} + \frac{1+R_2 sC_2}{R_2}\right) - V_2\left(\frac{1+R_2 sC_2}{R_2}\right) = V_A \frac{sC_1}{1+R_1 sC_1}$$

$$(3) \quad V_3\left(\frac{1}{R_3} + sC_3\right) - V_2\left(\frac{1}{R_3}\right) = 0$$

$$\Rightarrow V_1 = 0$$

$$(1) \; -V_2\left(\frac{1+\frac{1}{2}s}{1}\right) = V_A \frac{s}{1+s}$$

$$(3) \quad V_3(1+2s) - V_2 = 0 \Rightarrow \boxed{V_3(2s+1) = V_2}$$

$$(1) \; -V_3(2s+1)\left(\frac{\frac{2+s}{2}}{1}\right) = V_A \frac{s}{s+1}$$

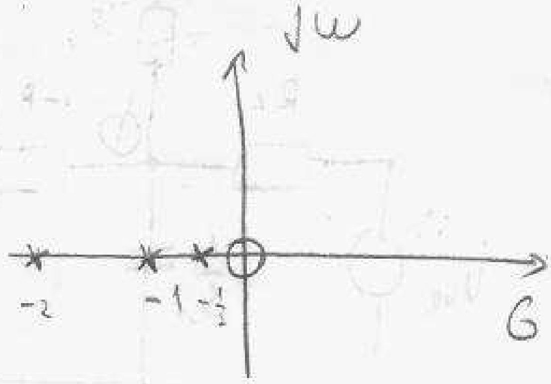
$$-V_3 \frac{(2s+1)(s+2)}{2} = V_A \frac{s}{s+1} \Rightarrow \boxed{V_A = -\frac{(s+2)(2s+1)(s+1)}{2s} V_3}$$

$$T(s) = \frac{U_3}{U_1} = -\frac{2s}{(s+2)(s+1)(2s+1)}$$

$\Rightarrow$ Polovi $\Rightarrow$ $s+2=0 \rightarrow s_{p1} = -2$;

$s+1=0 \rightarrow s_{p2} = -1$

$2s+1=0 \rightarrow s_{p3} = -\frac{1}{2}$



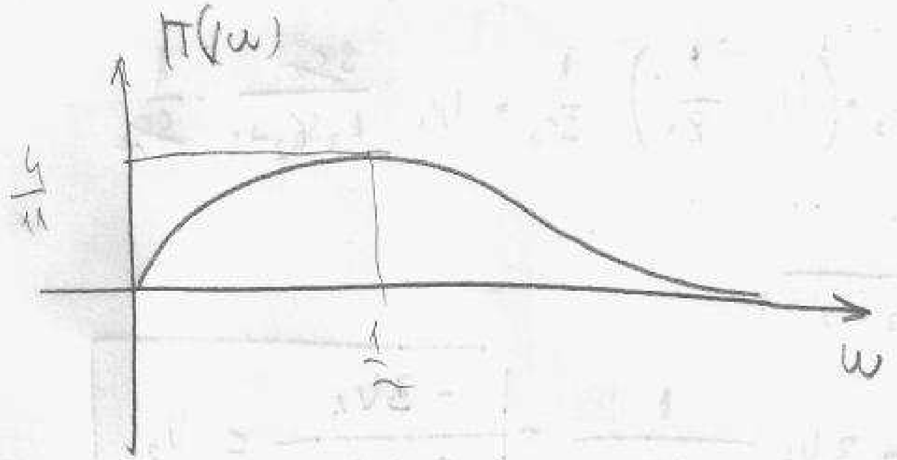
NULE: $s_{0,1} = 0$

$s_{0,2} = \infty$, $s_{0,3} = \infty$

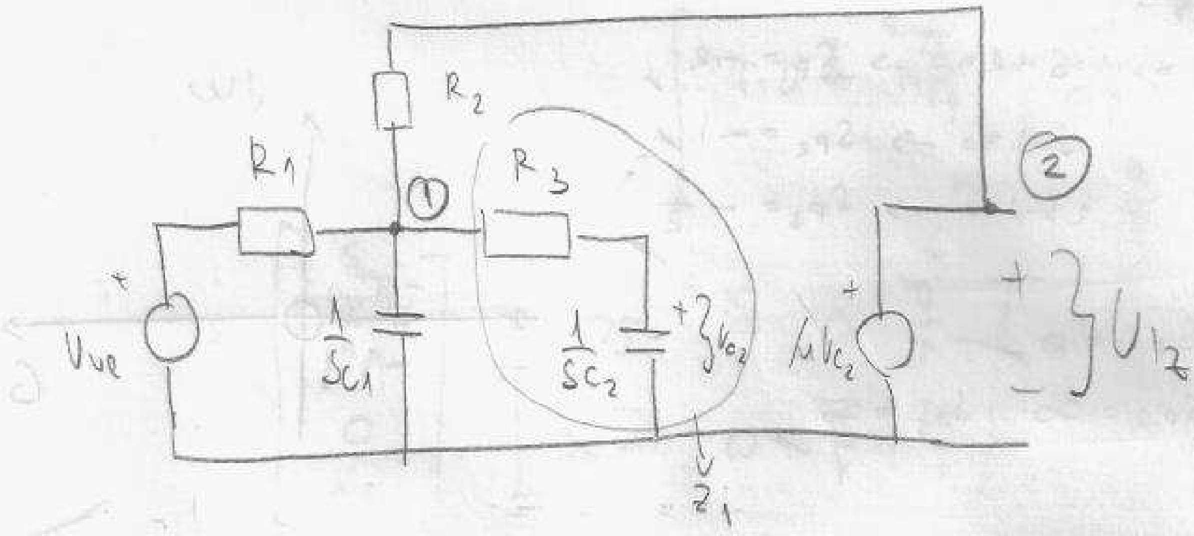
$$T(j\omega) = \frac{-2j\omega}{(2+j\omega)(1+j\omega)(1+2j\omega)} \Rightarrow |T(j\omega)| = \frac{|-2j\omega|}{\sqrt{[(\ )(\ )(\ )]^2}}$$

$|T(0)| = 0$

$|T(\infty)| = 0$



8 $R_1 = R_2 = \frac{1}{2}, \quad R_3 = C_1 = C_2 = 1, \quad \mu = -3$



$$Z_1 = R_3 + \frac{1}{sC_2} = \frac{R_3 sC_2 + 1}{sC_2}$$

(1) $U_1\left(\frac{1}{R_1} + \frac{1}{R_2} + sC_1 + \frac{sC_2}{R_3 sC_2 + 1}\right) - U_2\left(\frac{1}{R_2}\right) = \frac{U_{ul}}{R_1}$

(2) $V_2 = \mu V_{C_2}$

(3) $V_{C_2} = I_x \cdot \frac{1}{sC_2} = \left(U_1 \cdot \frac{1}{Z_1}\right)\frac{1}{sC_2} = U_1 \frac{\cancel{sC_2}}{R_3 sC_2 + 1} \cdot \frac{1}{\cancel{sC_2}}$

$$\Rightarrow V_{C_2} = V_1 \frac{1}{R_3 sC_2 + 1}$$

$\Rightarrow$ (2) $V_2 = \mu V_{C_2} = -3 V_1 \frac{1}{R_3 sC_2 + 1} = \boxed{\frac{-3V_1}{s+1} = V_2}$

$$\Downarrow$$

$$\boxed{U_1 = -U_2 \frac{s+1}{3}}$$

① $U_1\left(2+2+s+\frac{s}{s+1}\right)-2U_2-2U_{ve}$

$U_1\left[\frac{4(s+1)+s(s+1)+s}{s+1}\right]-2U_2=2U_{ve}$

$-U_2\left(\frac{\cancel{s+1}}{3}\cdot\frac{4s+4+s^2+s+s}{\cancel{(s+1)}}\right]-2U_2=2U_{ve}$

$-U_2\left[\frac{s^2+6s+4}{3}+2\right]=2U_{ve}$

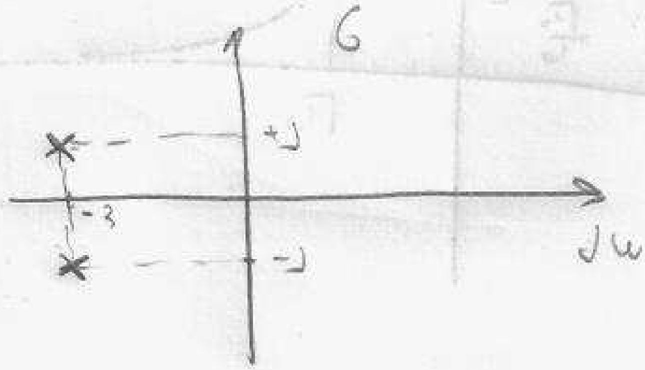
$-U_2\left[\frac{s^2+6s+4+6}{3}\right]=2U_{ve} \Rightarrow \boxed{-U_2\left[\frac{s^2+6s+10}{6}\right]=U_{ve}}$

$$T(s)=\frac{U_2}{U_{ve}}=\frac{-6}{s^2+6s+10}$$

$\Rightarrow$ polovi: $s^2+6s+10=0 \Rightarrow s_{p1,2}=\frac{-6\pm\sqrt{36-40}}{2}=\frac{-6\pm j2}{2}$

$\Rightarrow s_{p1,2}=-3\pm j$

$\boxed{s_{o1,2}=\infty}$



$$T(s) = \frac{-6}{s^2+6s+10}$$

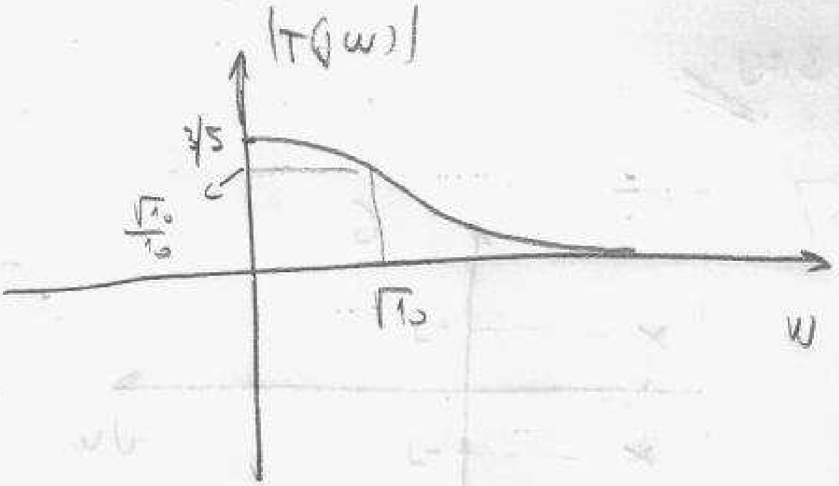
$$T(jw) = \frac{-6}{-w^2+6jw+10} = \frac{-6}{10-w^2+6jw}$$

$$|T(jw)| = \frac{|-6|}{\sqrt{(10-w^2)^2+36w^2}} = \frac{6}{\sqrt{(10-w^2)^2+36w^2}}$$

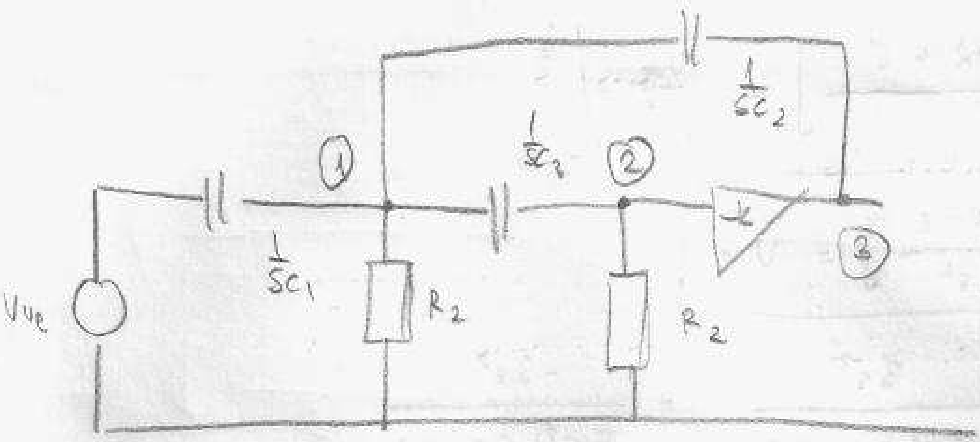
$$|T(0)| = \frac{6}{10} = \frac{3}{5}$$

$$|T(\infty)| = 0$$

$$|T(\sqrt{10})| = \frac{6}{\sqrt{360}} = \frac{6}{6\sqrt{10}} = \frac{\sqrt{10}}{10}$$



(9) $R_1 = R_2 = C_3 = 1$, $C_1 = C_2 = \frac{1}{2}$, $|k| = 3$



(1) $U_1(sC_1 + \frac{1}{R_2} + sC_2 + sC_3) - V_2(sC_3) - V_3(sC_2) = U_{ve}\, sC_1$

(2) $V_2(sC_3 + \frac{1}{R_2}) - U_1(sC_3) = 0$

(3) $U_3 = -k \cdot U_2 \Rightarrow U_3 = -3U_2 \Rightarrow \boxed{U_2 = -\frac{1}{3}U_3}$

---

(1) $U_1(\frac{s}{2} + 1 + \frac{s}{2} + s) - U_2 s - U_3\frac{s}{2} = \frac{U_{ve}}{2}\cdot s$

(2) $U_2(s+1) - U_1 s = 0$

---

(1) $U_1(2s+1) - U_2 s - U_3\frac{s}{2} = \frac{U_{ve}}{2}\cdot s$

(2) $U_1 s = U_2(s+1) \Rightarrow U_1 = U_2\frac{s+1}{s} = \boxed{-U_3\frac{s+1}{3s} = U_1}$

---

(1) $-U_3\frac{s+1}{3s}(2s+1) + U_3\frac{s}{3} - U_3\frac{s}{2} = \frac{U_{ve}}{2}\cdot s$

$$U_3\left[\frac{-(s+1)(2s+1)}{3s} + \frac{s}{3} - \frac{s}{2}\right] = \frac{U_{ve}}{2}\cdot s$$

$$U_3\left[\frac{-(2s^2+s+2s+1)}{3s} + \frac{2s-3s}{6}\right] = \frac{U_{ve}}{2}\cdot s$$

$$-U_3\left[\frac{2s^2+3s+1}{3s}+\frac{s}{6}\right]=\frac{U_{ul}}{2}\cdot s$$

$$-U_3\left[\frac{4s^2+6s+2+s^2}{6s}\right]=\frac{U_{ul}}{2}\cdot s \Big/ \frac{2}{s}$$

$$\boxed{-U_3\left[\frac{5s^2+6s+2}{3s^2}\right]=U_{ul}}$$

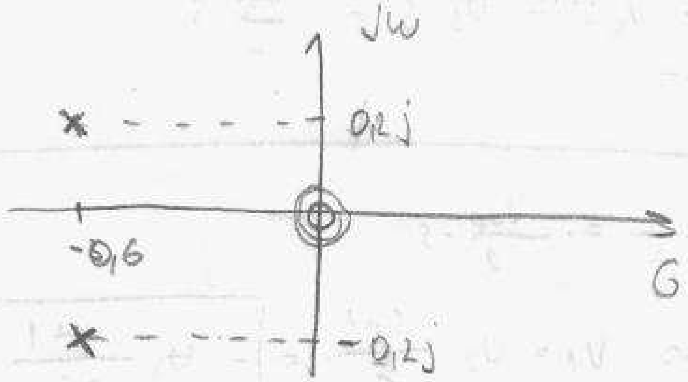
$$T(s)=\frac{U_3}{U_{ul}}=-\frac{3s^2}{5s^2+6s+2}=\frac{-3s^2}{5s^2+6s+2}$$

Polovi:

$$5s^2+6s+2=0 \Rightarrow s_{p1,2}=\frac{-6\pm\sqrt{36-40}}{2}=\frac{-6\pm 2j}{10}=-0,6\pm 0,2j$$

NULE:

$$-3s^2=0 \Rightarrow s_{01,2}=0$$



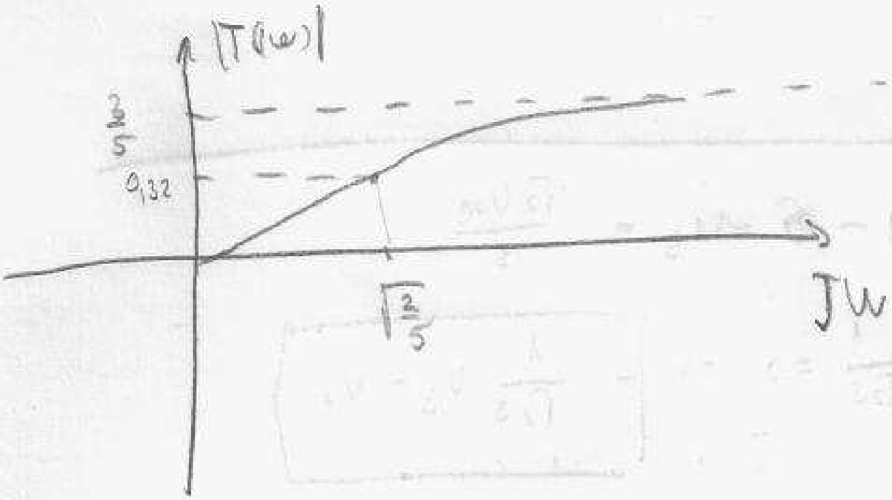
$$T(j\omega) = \frac{3\omega^2}{-5\omega^2 + 6j\omega + 2} = \frac{3\omega^2}{(2-5\omega^2) + 6j\omega}$$

$$|T(j\omega)| = \frac{3\omega^2}{\sqrt{(2-5\omega^2)^2 + 36\omega^2}}$$

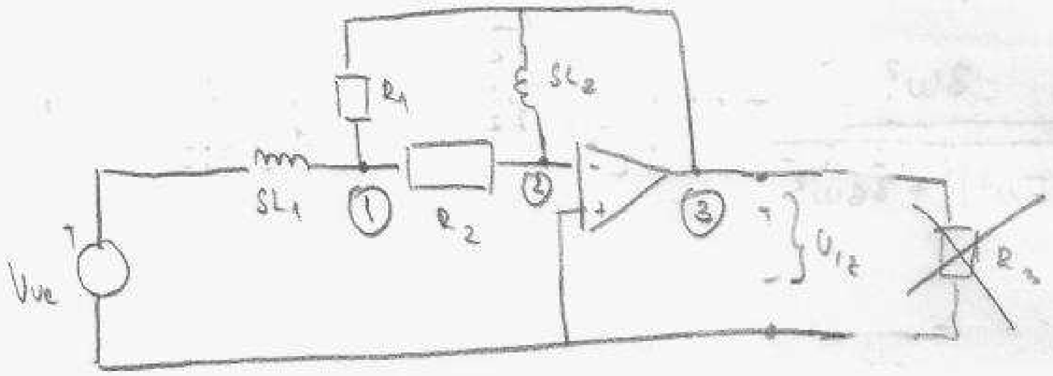
$$|T(0)| = 0$$

$$|T(\infty)| = \frac{3}{5}$$

$$\left|T\left(\sqrt{\tfrac{2}{5}}\right)\right| = \frac{\frac{6}{5}}{\sqrt{36\cdot\frac{2}{5}}} = \frac{\frac{6}{5}}{6\sqrt{\frac{2}{5}}} = \frac{1}{5}\cdot\frac{\sqrt{5}}{\sqrt{2}} = \frac{\sqrt{5}}{5\sqrt{2}} = \frac{\sqrt{10}}{10} = 0{,}32$$



(10) $R_1 = R_2 = R_3 = 1$, $L_1 = \frac{1}{\sqrt{2}}$, $L_2 = \sqrt{2}$



$$(1)\quad U_1\left(\frac{1}{sL_1} + \frac{1}{R_2} + \frac{1}{R_1}\right) - U_2\frac{1}{R_2} - U_3\frac{1}{R_1} = \frac{U_{ul}}{sL_1}$$

$$(2)\quad U_2\left(\frac{1}{sL_2} + \frac{1}{R_2}\right) - U_1\left(\frac{1}{R_2}\right) - U_3\left(\frac{1}{sL_2}\right) = 0$$

$$\Rightarrow U_2 = 0$$

$$(1)\quad U_1\left(\frac{\sqrt{2}}{s} + 1 + 1\right) - 0 - U_3 = \frac{\sqrt{2}\,U_{ul}}{s}$$

$$(2)\quad 0 - U_1 - U_3\frac{1}{\sqrt{2}s} = 0 \Rightarrow \boxed{-\frac{1}{\sqrt{2}s}U_3 = U_1}$$

$$(1)\quad U_1\left(\frac{\sqrt{2}+2s}{s}\right) - U_3 = \frac{\sqrt{2}}{s}U_{ul}$$

$$-\frac{1}{\sqrt{2}s}\left(\frac{\sqrt{2}+2s}{s^2}\right)U_3 - U_3 = \frac{\sqrt{2}}{s}U_{ul}$$

$$-U_3\left[\frac{\sqrt{2}+2s}{\sqrt{2}s^2} + 1\right] = \frac{\sqrt{2}}{s}U_{ul}$$

$$-U_3\left[\frac{\sqrt{2}+2s+\sqrt{2}s^2}{\sqrt{2}s^2}\right] = \frac{\sqrt{2}}{s}U_{ul} \quad / \cdot\frac{s}{\sqrt{2}}$$

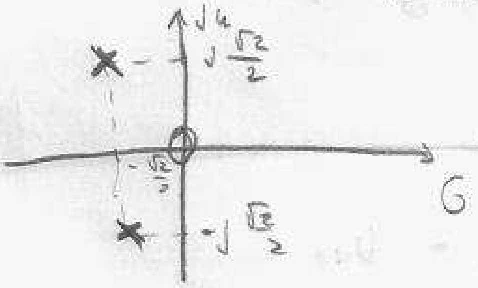
$$\boxed{U_{ul} = -U_3\left[\frac{\sqrt{2}s^2+2s+\sqrt{2}}{2s}\right]}$$

$$T(s) = \frac{U_3}{U_{ul}} = -\frac{2s}{\sqrt{2}s^2+2s+\sqrt{2}} \Big/ \frac{\sqrt{2}}{\sqrt{2}} = -\frac{2\sqrt{2}s}{2s^2+2\sqrt{2}s+2} =$$

$$T(s) = \frac{-\sqrt{2}s}{s^2+\sqrt{2}s+1}$$

Polovi: $s_{12} = s^2+\sqrt{2}s+1=0 \rightarrow s_{12} = \frac{-\sqrt{2}\pm\sqrt{2-4}}{2} = -\frac{\sqrt{2}}{2} \pm j\frac{\sqrt{2}}{2}$

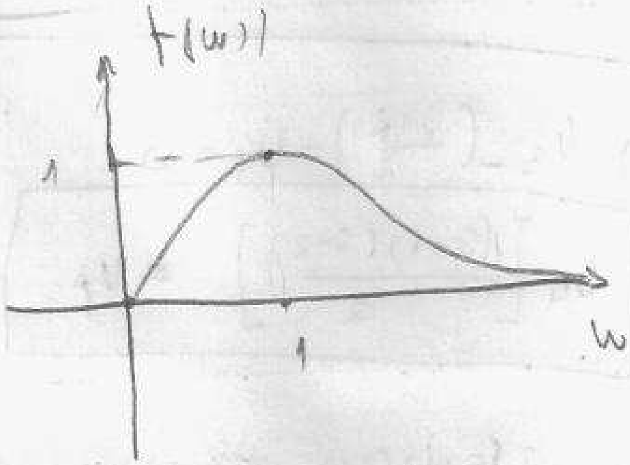
NULE: $-\sqrt{2}s=0 \rightarrow \boxed{s_{01}=0}$, $\boxed{s_{02}=\infty}$



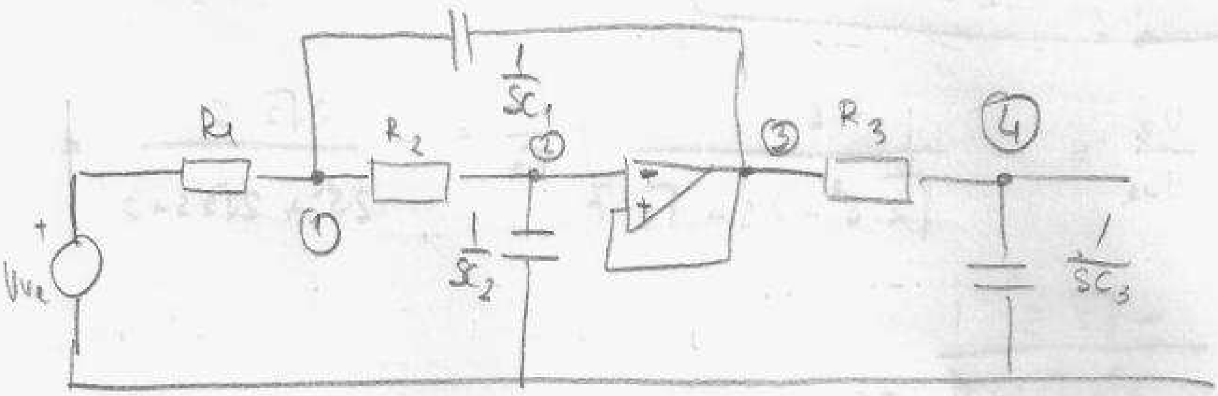
$$T(j\omega) = \frac{-\sqrt{2}j\omega}{-\omega^2+\sqrt{2}j\omega+1} = \frac{-j\sqrt{2}\omega}{(1-\omega^2)+\sqrt{2}j\omega}$$

$$|T(j\omega)| = \frac{\sqrt{2}\omega}{\sqrt{(1-\omega^2)^2+2\omega^2}}$$

$$\left.\begin{aligned} |T(0)| &= 0 \\ |T(\infty)| &= 0 \\ |T(1)| &= \frac{\sqrt{2}\omega}{\sqrt{2}\omega} = 1 \end{aligned}\right\}$$



(11) $R_1 = R_2 = R_3 = C_3 = 1,\quad C_1 = 2,\quad C_2 = \frac{1}{2}$



(1) $U_1\left(\frac{1}{R_2} + \frac{1}{R_1} + sC_1\right) - U_2\left(\frac{1}{R_2}\right) - U_3(sC_1) = \frac{U_{ue}}{R_1}$

(2) $U_2\left(sC_2 + \frac{1}{R_2}\right) - U_1\left(\frac{1}{R_2}\right) = 0$

(4) $U_4\left(\frac{1}{R_3} + sC_3\right) - U_3\left(\frac{1}{R_3}\right) = 0$

$\to U_2 = U_3$

---

(1) $U_1(1 + 1 + 2s) - U_3 - U_3 2s = U_{ue}$

(2) $U_3\left(\frac{s}{2} + 1\right) - U_1 = 0$

(4) $U_4(s+1) - U_3 = 0 \Rightarrow \boxed{U_3 = U_4(s+1)}$

---

(2) $U_3\left(\frac{s+2}{2}\right) - U_1 = 0$

$$\boxed{U_4\left[\frac{(s+1)(s+2)}{2}\right] = U_1}$$

(1) $U_4\left[\frac{(s+1)(s+2)\cancel{2}(s+1)}{\cancel{2}}\right] - U_4(s+1)(1+2s) = U_{ue}$

$U_4\left[(s+1)^2(s+2)\right] - U_4(s+1)(2s+1) = U_{ue}$

$$u_4 \left[(s+1) \left[(s+1)(s+2) - 2s - 1 \right] \right] = u_{ve}$$

$$v_4 \left[(s+1)\left(s^2 + s + \cancel{2s} + 2 - \cancel{2s} - 1 \right) \right] = v_{v1}$$

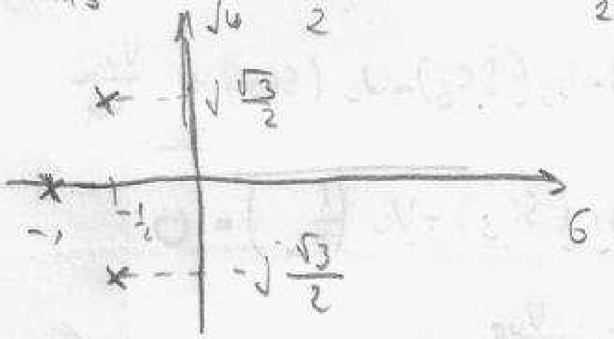
$$U_4 \left[(s+1)\left(s^2 + s + 1 \right) \right] = U_{ve}$$

$$T(s) = \frac{V_4}{V_{ve}} = \frac{1}{(s+1)(s^2+s+1)} = \frac{1}{s^3+s^2+s+s^2+s+1} = \frac{1}{s^3+2s^2+2s+1}$$

Polovi: $(s+1) = 0 \Rightarrow \boxed{s_1 = -1}$

$$s^2+s+1 = 0 \Rightarrow s_{2,3} = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

NULE: $s_{01,2,3} = \infty$

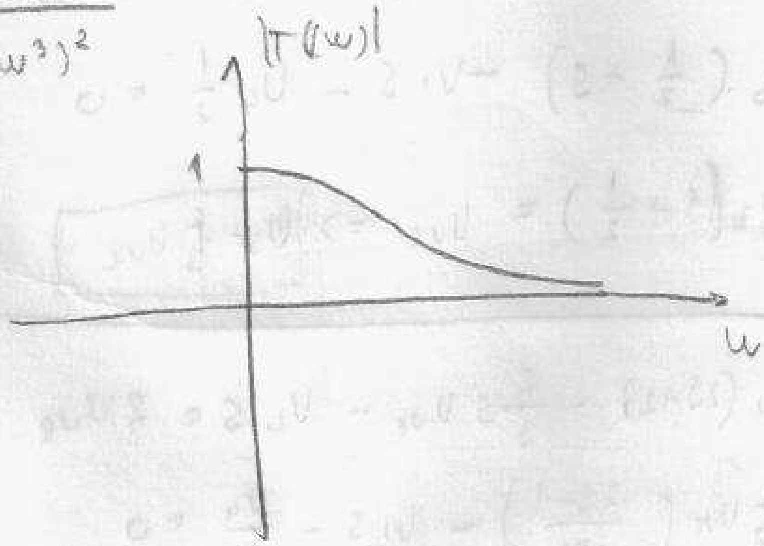


$$T(j\omega) = \frac{1}{-j\omega^3 - 2\omega^2 + 2j\omega + 1} = \frac{1}{1-2\omega^2 + j(2\omega - \omega^3)}$$

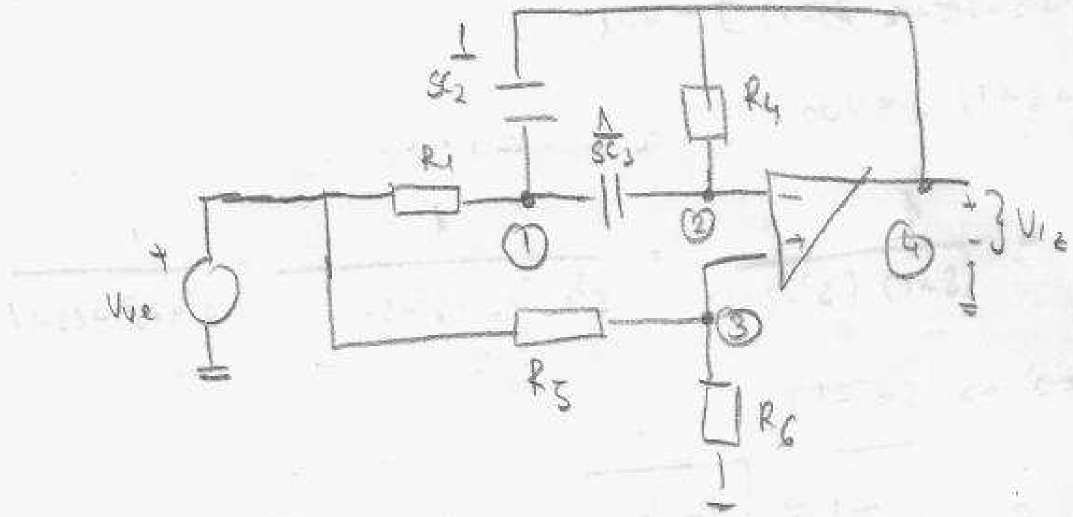
$$|T(j\omega)| = \frac{1}{\sqrt{(1-2\omega^2)^2 + (2\omega-\omega^3)^2}}$$

$$|T(0)| = 1$$

$$|T(\infty)| = 0$$



(12) $R_1 = \frac{1}{2}$, $R_4 = R_6 = 2$, $R_5 = C_2 = C_3 = 1$



$$(1)\quad U_1\left(\frac{1}{R_1} + sC_2 + sC_3\right) - U_2(sC_3) - U_4(sC_2) = \frac{U_{ve}}{R_1}$$

$$(2)\quad U_2\left(\frac{1}{R_4} + sC_3\right) - U_1(sC_3) - U_4\left(\frac{1}{R_4}\right) = 0$$

$$(3)\quad U_3\left(\frac{1}{R_6} + \frac{1}{R_5}\right) = \frac{U_{ve}}{R_5}$$

$\Rightarrow U_2 = U_3$

$$(1)\quad U_1(2 + s + s) - U_3 s - U_4 s = 2U_{ve}$$

$$(2)\quad U_3\left(\frac{1}{2} + s\right) - U_1 s - U_4 \frac{1}{2} = 0$$

$$(3)\quad U_3\left(1 + \frac{1}{2}\right) = U_{ve} \Rightarrow \boxed{U_3 = \frac{2}{3}U_{ve}}$$

$$(1)\quad U_1(2s+2) - \frac{2}{3}s\,U_{ve} - U_4 s = 2U_{ve}$$

$$(2)\quad \frac{2}{3}U_{ve}\left(\frac{2s+1}{2}\right) - U_1 s - \frac{U_4}{2} = 0$$

$$U_1 s = \frac{2s+1}{3}U_{ve} - \frac{U_4}{2} \Rightarrow \boxed{U_1 = \frac{2s+1}{3s}U_{ve} - \frac{1}{2s}U_4}$$

$$\left[\frac{(2s+1)}{3s}U_{ul} - \frac{1}{2s}U_u\right]2(s+1) - U_u s = \frac{(6+2s)}{3}U_{ul}$$

$$\frac{2(s+1)(2s+1)}{3s}U_{ul} - \frac{s+1}{s}U_u - U_u s = \frac{(6+2s)}{3}U_{ul}$$

$$-U_u\left(\frac{s+1}{s}+s\right) = \left[\frac{6+2s}{3} - \frac{2(2s^2+s+2s+1)}{3s}\right]U_{ul}$$

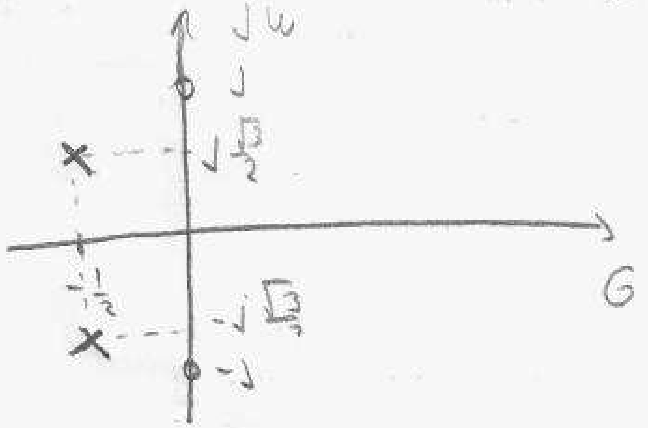
$$-U_u\left(\frac{s^2+s+1}{s}\right) = \left[\frac{\cancel{6s}+2s^2-4s^2-\cancel{6s}-2}{3s}\right]U_{ul}$$

$$\cancel{-}U_u\left(\frac{s^2+s+1}{s}\right) = \cancel{-}\frac{2(s^2+1)}{3s}U_{ul}$$

$$U_{ul} = \frac{3(s^2+s+1)}{2(s^2+1)}U_u \Rightarrow T(s) = \frac{U_u}{U_{ul}} = \frac{2}{3}\frac{s^2+1}{s^2+s+1}$$

$\Rightarrow$ NULE: $s^2+1=0 \Rightarrow s_{01} = \pm j$

$\Rightarrow$ POLOVI: $s^2+s+1=0 \Rightarrow s_{P12} = \frac{-1\pm\sqrt{1-4}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$



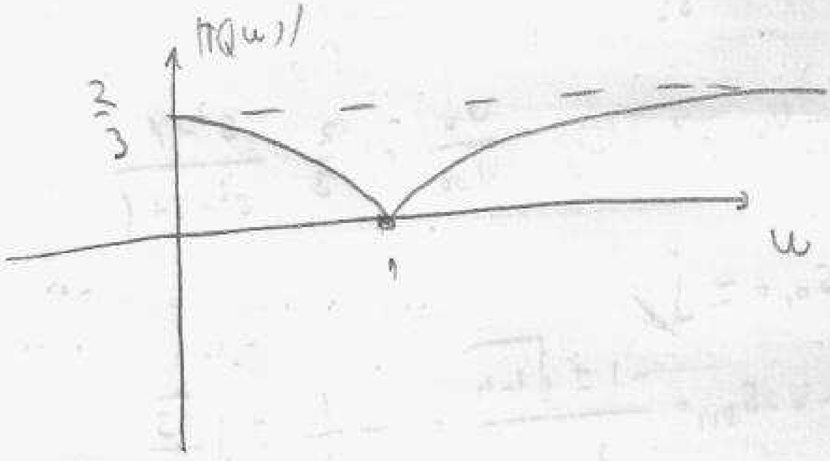
$$T(j\omega) = \frac{2}{3} \frac{1-\omega^2}{(1-\omega^2)+j\omega}$$

$$|T(j\omega)| = \frac{2}{3} \frac{|1-\omega^2|}{\sqrt{(1-\omega^2)^2+\omega^2}}$$

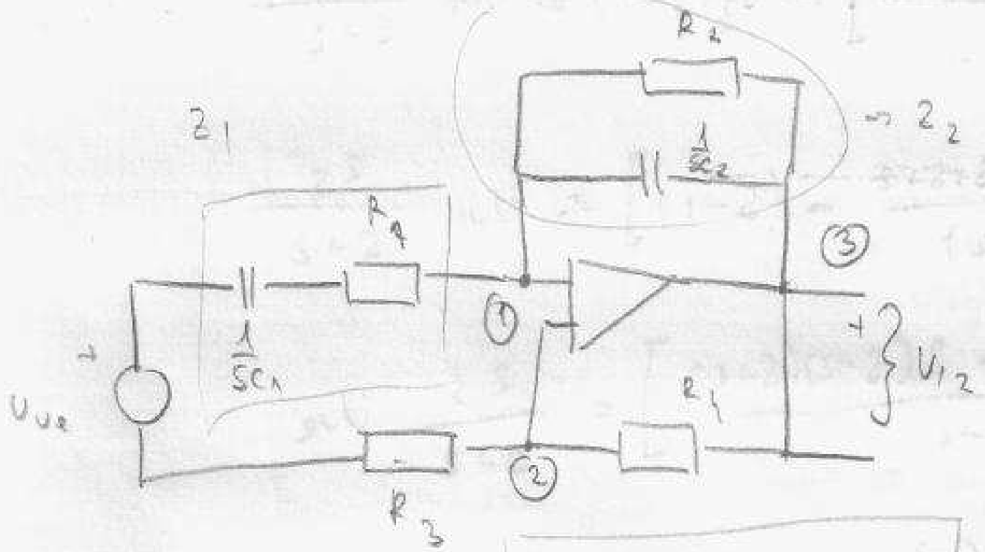
$$|T(0)| = \frac{2}{3}$$

$$|T(\infty)| = \frac{2}{3}$$

$$|T(1)| = 0$$



(13) $R_1 = \frac{1}{2}, \quad R_2 = R_3 = R_4 = C_1 = C_2 = 1$



$$Z_1 = \left(\frac{1}{sC_1} + R_1\right) = \boxed{\frac{1+R_1 sC_1}{sC_1} = Z_1}$$

$$Z_2 = \left(sC_2 + \frac{1}{R_2}\right)^{-1} = \left(\frac{R_2 sC_2 + 1}{R_2}\right)^{-1} = \boxed{\frac{R_2}{R_2 sC_2 + 1} = Z_2}$$

---

(1) $U_1\left(\frac{sC_1}{1+R_1 sC_1} + \frac{R_2 sC_2 + 1}{R_2}\right) - U_3\left(\frac{R_2 sC_2 + 1}{R_2}\right) = U_{ul}\frac{sC_1}{1+R_1 sC_1}$

(2) $U_2\left(\frac{1}{R_3} + \frac{1}{R_4}\right) - U_3\left(\frac{1}{R_4}\right) = 0$

(3) $U_2 = U_1$

---

(1) $U_2\left(\frac{s}{1+\frac{s}{2}} + \frac{s+1}{1}\right) - U_3\left(\frac{s+1}{1}\right) = U_{ul}\frac{s}{1+\frac{1}{2}s}$

(2) $U_2(1+1) - U_3 = 0 \Rightarrow 2U_2 = U_3 \Rightarrow \boxed{U_2 = \frac{U_3}{2}}$

---

(1) $U_2\left[\frac{2s}{s+2} + (s+1)\right] - U_3(s+1) = U_{ul}\frac{2s}{s+2}$

$$\frac{U_3}{2}\left[\frac{2s+(s+1)(s+2)}{s+2}\right]-U_3(s+1)=U_{ve}\,\frac{2s}{s+2}$$

$$U_3\left(\frac{2s+s^2+2s+s+2}{2(s+2)}-(s+1)\right]=U_{ve}\,\frac{2s}{s+2}$$

$$U_3\left(\frac{s^2+5s+2-2(s+2)(s+1)}{2(s+2)}\right]=\frac{2s}{s+2}U_{ve}$$

$$U_3\left(\frac{s^2+5s+2-2(s^2+2s+s+2)}{2(s+2)}\right)=\frac{2s}{s+2}U_{ve}$$

$$U_3\left(\frac{s^2+5s+2-2s^2-6s-4}{2(s+2)}\right)=\frac{2s}{s+2}\,U_{ve}$$

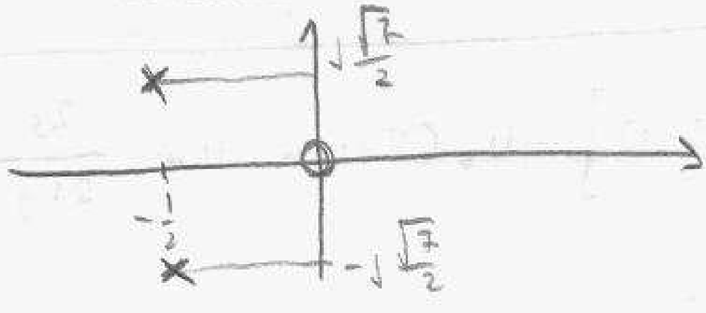
$$U_3\,\frac{-s^2-s-2}{2(\cancel{s+2})}\;\frac{\cancel{s+2}}{2s}=U_{ve}$$

$$U_3\,\frac{-(s^2+s+2)}{4s}=U_{ve}$$

$$T(s)=\frac{U_3}{U_{ve}}=\frac{-4s}{s^2+s+2}$$

Polovi: $s^2+s+2=0 \Rightarrow s_{p1,2}=\dfrac{-1\pm\sqrt{1-8}}{2}=-\dfrac{1}{2}\pm j\dfrac{\sqrt{7}}{2}$

NULE: $\boxed{s_1=0}$, $\boxed{s_2=\infty}$



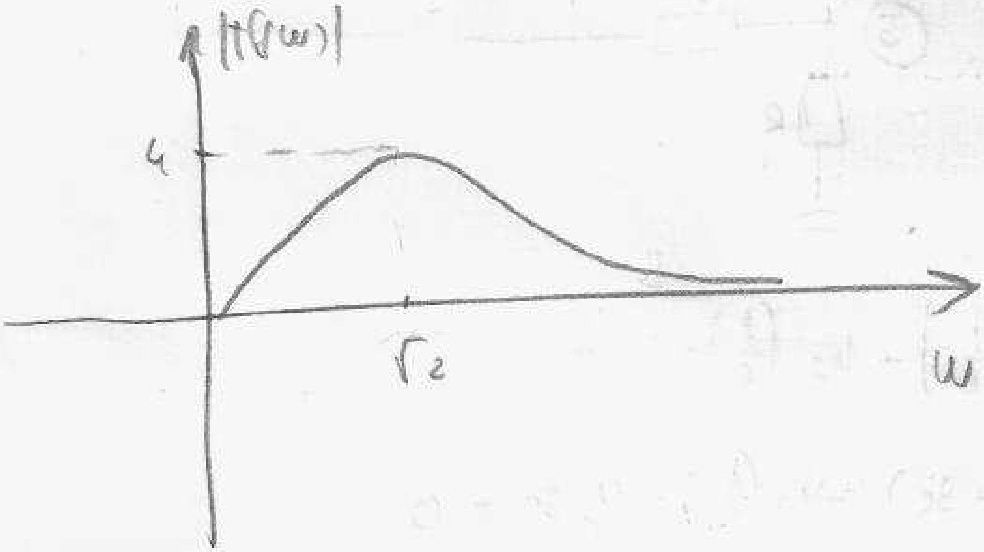
$$T(j\omega) = \frac{-4j\omega}{(2-\omega^2)+j\omega}$$

$$|T(j\omega)| = \frac{4\omega}{\sqrt{(2-\omega^2)^2+\omega^2}}$$

$$|T(0)| = 0$$

$$|T(\infty)| = 0$$

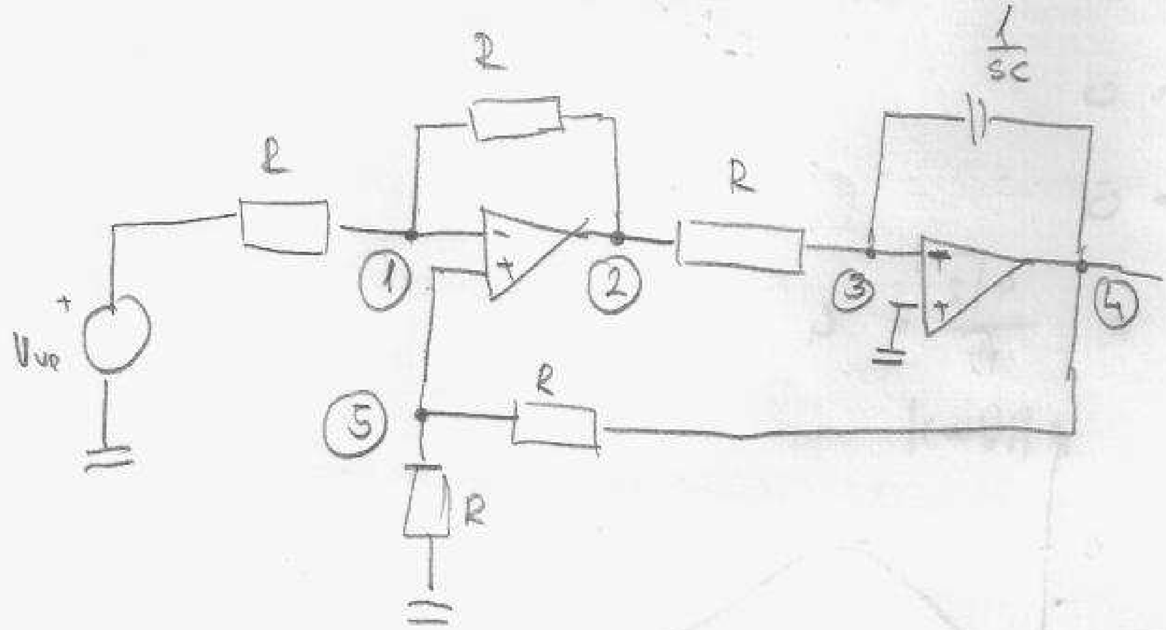
$$|T(\sqrt{2})| = \frac{4\sqrt{2}}{\sqrt{2}} = 4$$



(14) NEMAM BAŠ VOLJE ZA OVO, PA RECIMO SLJEDEĆE:

$R = C = 1$

$\Rightarrow T(s) = \dfrac{1}{s+1}$ (KAD SE UVRSTI $R=C=1$ U $R|_o$)



(1) $U_1\left(\frac{1}{R}+\frac{1}{R}\right) - U_2\left(\frac{1}{R}\right) = \frac{U_{ul}}{R}$

(3) $U_3\left(\frac{1}{R}+sC\right) - U_2\left(\frac{1}{R}\right) - U_4\, sC = 0$

(5) $U_5\left(\frac{1}{R}+\frac{1}{R}\right) - U_4\left(\frac{1}{R}\right) = 0$

$\Rightarrow U_1 = U_5, \quad U_3 = 0$

---

(1) $2U_1 - U_2 = U_{ul}$

(3) $-U_2 - U_4 s = 0 \Rightarrow$ $\boxed{U_2 = -sU_4}$

(5) $2U_1 - U_4 = 0 \Rightarrow$ $\boxed{U_1 = \frac{1}{2}U_4}$

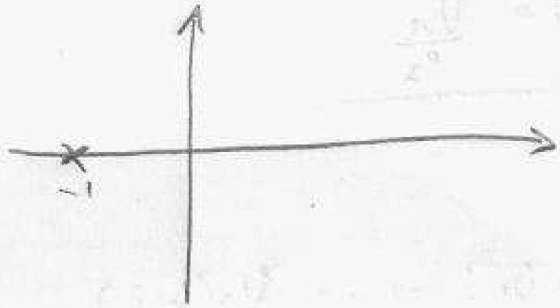
$U_4 + sU_4 = V_{ue}$

$V_{ue} = U_4(s+1)$

$$T(s) = \frac{U_4}{U_{ue}} = \frac{1}{s+1}$$

Polovi: $s+1=0 \Rightarrow \boxed{s_{p1} = -1}$

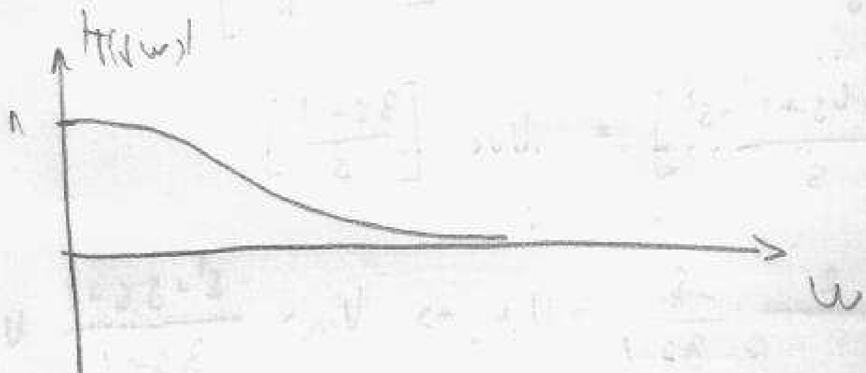
Nule: $\boxed{s_0 = \infty}$



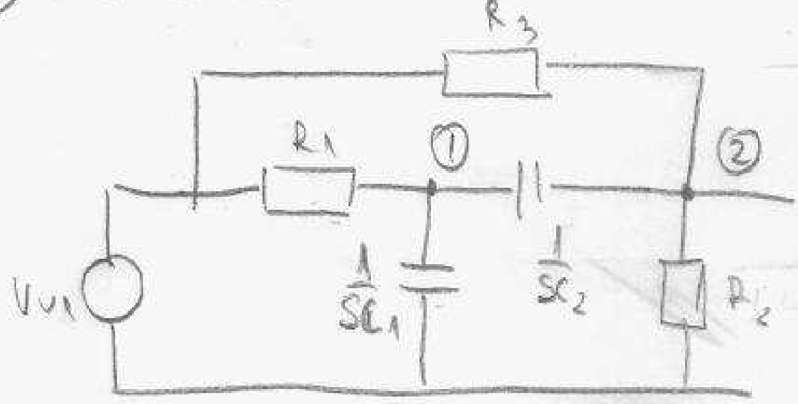
$$T(j\omega) = \frac{1}{1+j\omega} \Rightarrow |T(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

$|T(0)| = 1$

$|T(\infty)| = 0$



(15) $R_1 = R_2 = R_3 = C_1 = C_2 = 1$



(1) $U_1\left(\frac{1}{R_1} + sC_1 + sC_2\right) - U_2 sC_2 = \frac{U_{ve}}{R_1}$

(2) $U_2\left(sC_2 + \frac{1}{R_2} + \frac{1}{R_3}\right) - U_1 (sC_2) = \frac{U_{ve}}{R_3}$

---

(1) $U_1(1 + s + s) - U_2 s = U_{ve}$

(2) $U_2(s+2) - U_1 s = U_{ve} \Rightarrow \underbrace{U_1 s = -U_{ve} + U_2(s+2)}$

$U_1 = -\frac{U_{ve}}{s} + U_2 \frac{s+2}{s}$

(1) $\left[\frac{U_{ve}}{s} - U_2 \frac{s+2}{s}\right](2s+1) - U_2 s = U_{ve}$

$U_2 \frac{(s+2)(2s+1)}{s} - U_2 s = U_{ve}\left[1 + \frac{(2s+1)}{s}\right]$

$U_2\left[\frac{2s^2 + s + 4s + 2 - s^2}{s}\right] = U_{ve}\left[\frac{3s+1}{s}\right]$

$U_2 \frac{s^2 + 5s + 2}{\cancel{s}} \frac{\cancel{s}}{3s+1} = U_{ve} \Rightarrow U_{ve} = \frac{s^2+5s+2}{3s+1} U_2$

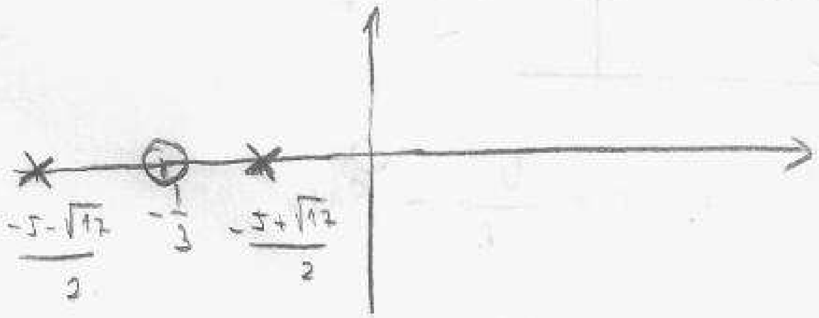
$T(s) = \frac{U_2}{U_{ve}} = \frac{3s+1}{s^2+5s+1}$

Polovi:

$$s^2+5s+2=0 \Rightarrow s_{p_{12}} = \frac{-5 \pm \sqrt{25-8}}{2} = \frac{-5 \pm \sqrt{17}}{2}$$

NULE:

$$3s+1=0 \Rightarrow s_{01} = -\frac{1}{3}, \quad s_{02} = \infty$$

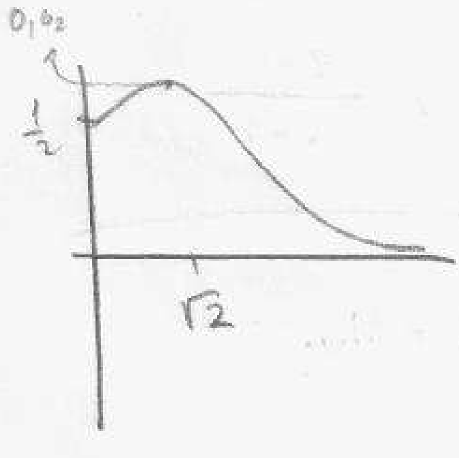


$$T(j\omega) = \frac{1+3j\omega}{(2-\omega^2)+5j\omega} \Rightarrow |T(j\omega)| = \frac{\sqrt{1+9\omega^2}}{\sqrt{(2-\omega^2)^2+25\omega^2}}$$

$|T(0)| = \frac{1}{2}$

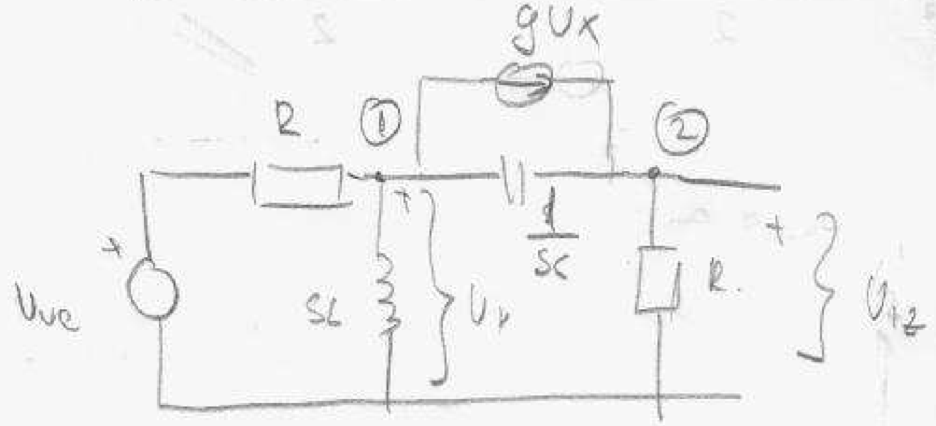
$|T(\infty)| = 0$

$|T(\sqrt{2})| = \frac{\sqrt{19}}{5\sqrt{2}} = 0{,}62$



(16) $R = C = 1, \quad L = g = 2$

$u_1(t) = 10\cos t$



(1) $U_1\left(\frac{1}{R} + \frac{1}{sL} + sC\right) - U_2\, sC = \frac{U_{ve}}{R} - gU_x$

(2) $U_2\left(sC + \frac{1}{R}\right) - U_1(sC) - gU_x$

(3) $U_x = U_1$

(1) $U_1\left(1 + \frac{1}{2s} + s\right) - U_2 s = U_{ve} - 2U_1$

(2) $U_2(s+1) - U_1 s = 2U_1$

(1) $U_1\left(\frac{2s^2+2s+1}{2s} + 2\right) - U_2 s = U_{ve}$

(2) $\Rightarrow U_2(s+1) = U_1(s+2) \Rightarrow U_1 = U_2\,\frac{s+1}{s+2}$

$U_2\left(\frac{s+1}{s+2}\cdot\frac{2s^2+2s+1+4s}{2s}\right) - U_2 s = U_{ve}$

$U_2\left(\frac{(s+1)(2s^2+6s+1)}{2s(s+2)} - \frac{2s^2(s+2)}{2s(s+2)}\right] = U_{ve}$

$$U_{ve} = U_2 \left[\frac{\cancel{2s^3}+6s^2+s+2s^2+6s+1-\cancel{2s^3}-4s^2}{2s(s+2)} \right]$$

$$U_{ve} = U_2 \left[\frac{4s^2+7s+1}{2s(s+2)} \right] \Rightarrow T(s) = \frac{U_2}{U_{ve}} = \frac{2s(s+2)}{4s^2+7s+1}$$

$$T(s) = \frac{2s^2+4s}{4s^2+7s+1} \rightarrow T(j\omega) = \frac{-2\omega^2+4j\omega}{-4\omega^2+7j\omega+1} = \frac{4j\omega-2\omega^2}{(1-4\omega^2)+7j\omega}$$

$$\Rightarrow U_{ve} = 10\cos t = 10\angle 0° \Rightarrow \omega = 1$$

$$T(j\cdot 1) = \frac{j4-2}{-3+7j} = \frac{4{,}47\angle 116{,}56°}{7{,}61\angle 113{,}19°} = 0{,}587\angle 3{,}37°$$

$$T(j\cdot 1) = 0{,}587\angle 3{,}37°$$

$$T(j\cdot 1) = \frac{U_{i2}}{U_{ve}} \Rightarrow U_{i2} = T(j\cdot 1)\cdot U_{ve} = 0{,}587\angle 3{,}37° \cdot 10\angle 0°$$

$$U_{i2} = 5{,}87\angle 3{,}37 = 5{,}87\cos(t+3{,}37°)$$

17 $x(t) = \delta(t) \Rightarrow X(s) = \frac{1}{s}$

$y(t) = e^{-3t} ch(2t) \cdot \delta(t) \Rightarrow Y(s) = \frac{(s+3)}{(s+3)^2 - 4}$

$$Y(s) = \frac{s+3}{s^2+6s+9-4} = \frac{s+3}{s^2+6s+5}$$

$$T(s) = \frac{Y(s)}{X(s)} = \frac{s+3}{s^2+6s+5} \cdot \frac{s}{1} = \frac{s^2+3s}{s^2+6s+5}$$

$$\boxed{T(j\omega) = \frac{-\omega^2 + 3j\omega}{(5-\omega^2) + 6j\omega}}$$

$x_1(t) = 2\cos(3t + 45°) = 2 \angle 45° \Rightarrow \omega = 3$

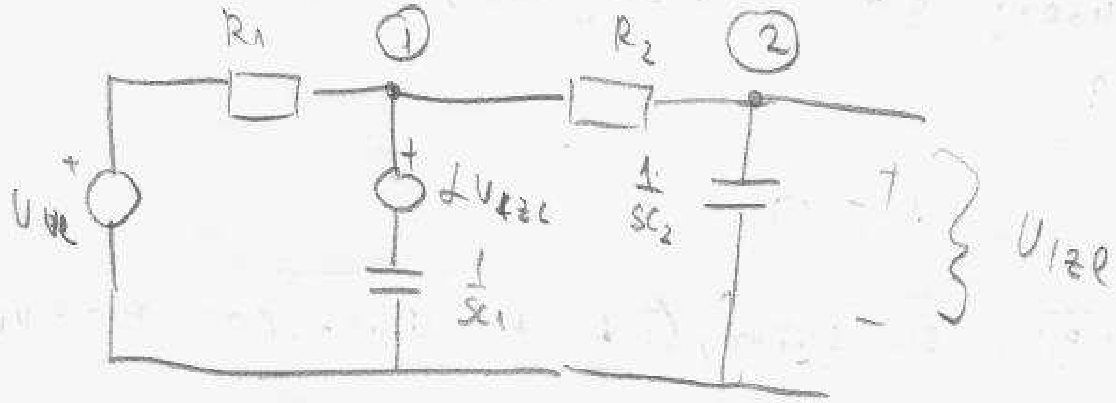
$$T(j \cdot 3) = \frac{-9+9j}{-4+18j} = \frac{12{,}73 \angle 135°}{18{,}44 \angle 102{,}53°} = 0{,}69 \angle 32{,}47$$

$$T(j \cdot 3) = \frac{y(t)}{x(t)} \Rightarrow \boxed{y(t) = T(j3) \cdot x(t)}$$

$y(t) = 0{,}69 \angle 32{,}47 \cdot 2 \angle 45 = 1{,}38 \angle 77{,}47°$

$y(t) = 1{,}38 \cos(3t + 77{,}47°)$

18 i 19 $\quad R_1 = R_2 = C_1 = C_2 = 1$



① $U_1\left(\frac{1}{R_1} + sC_1 + \frac{1}{R_2}\right) - U_2\left(\frac{1}{R_2}\right) = \frac{U_{ve}}{R_1} + \alpha U_{iz} \cdot sC$

② $U_2\left(\frac{1}{R_2} + sC_2\right) - U_1\left(\frac{1}{R_2}\right) = 0$

---

① $U_1(1 + s + 1) - U_2 = U_{ve} + s\alpha U_2$

② $U_2(1 + s) - U_1 = 0 \Rightarrow \boxed{U_1 = U_2(s+1)}$

---

① $U_2\left[(s+1)(s+2)\right] - U_2 - s\alpha U_2 = U_{ve}$

$U_2\left[s^2 + 2s + s + 2 - 1 - \alpha\right] = U_{ve}$

$U_{ve} = U_2\left(s^2 + 3s + 1 - \alpha s\right) = U_2\left[s^2 + (3-\alpha)s + 1\right]$

$$T(s) = \frac{U_2}{U_{ve}} = \frac{1}{s^2 + (3-\alpha)s + 1}$$

$s^2+(3-\alpha)s+1=0$

=> POLOVI MORAJU BITI U LIJEVOJ POLURAVNINI

TJ.

$s^2+(3-\alpha)s+1 \le 0$

=> DA BI POLOVI BILI LIJEVO, $(3-\alpha)$ MORA UVIJEK BITI POZITIVNO

$3-\alpha \ge 0 \Rightarrow -\alpha \ge -3 \Rightarrow \boxed{\alpha \le 3}$

=> $s^2+(3-\alpha)s+1=0$

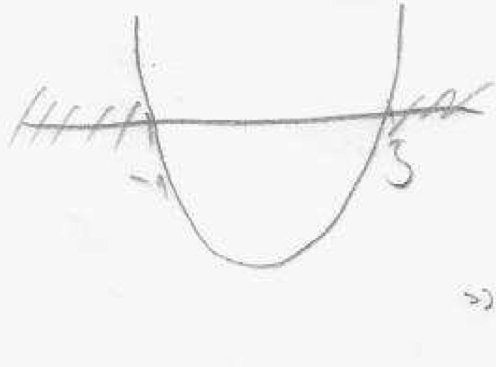
$$s_{12}=\frac{-(3-\alpha)\pm\sqrt{(3-\alpha)^2-4}}{2}$$

=> DA BI RJ. BILA REALNA $(3-\alpha)^2-4\ge 0$

$9-6\alpha+\alpha^2-4\ge 0 \Rightarrow \alpha^2-6\alpha+5\ge 0$

$\alpha^2-6\alpha+5=0 \Rightarrow \alpha_{12}=\frac{6\pm\sqrt{36-20}}{2}=\frac{6\pm 4}{2}$

$\alpha_1=5,\ \alpha_2=1$



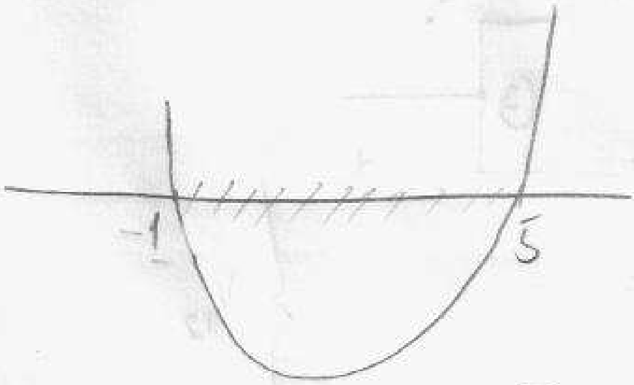
$\alpha \in [-\infty,1] \cup [5,\infty]$

=> ALI, KAKO VRIJEDI $\alpha \le 3$

$\boxed{\alpha \le 1}$

→ DA BI POLOVI BILI KOMPLEKSNI :

$$\alpha^2 - 6\alpha + 5 < 0$$



$$\left.\begin{array}{l} \alpha \in \langle 1, 5 \rangle \\ \text{ALI KAKO JE } \alpha \leq 3 \end{array}\right\} \quad \alpha \in \langle 1, 3]$$