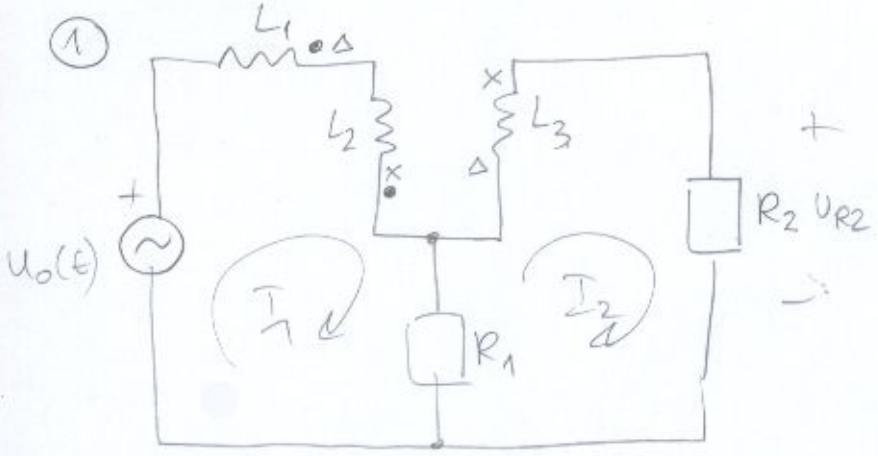
1. KZ 2006 TMK

① 

$R_1 = R_2 = L_1 = 1$

$L_2 = M_{13} = 2$

$L_3 = 4$

$M_{12} = 0{,}5$

$M_{23} = 3$

$U_0(t) = \sin t = 1\angle 0°$

$\omega = 1$

$U_{R2}(t) = ?. \Rightarrow U_{R2} = I_2 \cdot R_2 \Rightarrow I_2 = ?$

$$\begin{cases} U_0 = I_1(j\omega L_1 + j\omega L_2 + R_1 + 2j\omega M_{12}) - I_2(R_1 + j\omega M_{13} - j\omega M_{23}) \\ 0 = -I_1(R_1 + j\omega M_{13} - j\omega M_{23}) + I_2(R_1 + R_2 + j\omega L_3) \end{cases}$$

$$\begin{cases} 1 = I_1 \cdot (j + 2j + 2 \cdot 0{,}5j + 1) - I_2(1 + 2j - 3j) \\ 0 = -I_1(1 + 2j - 3j) + I_2(2 + 4j) \end{cases}$$

$$1 = (4j+1)I_1 - (1-j)I_2$$

$$0 = -(1-j)I_1 + (2+4j)I_2 \Rightarrow I_1 = I_2 \cdot \frac{2+4j}{1-j} = I_2 \cdot (-1+3j)$$

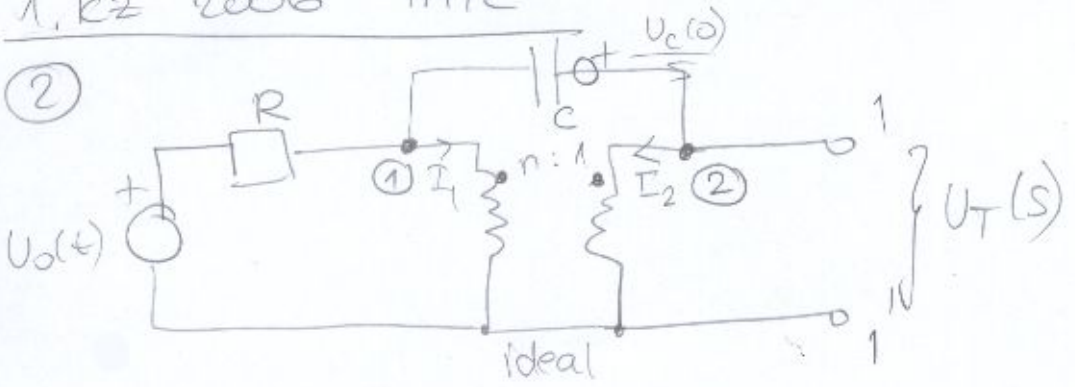
$$1 = (4j+1)(-1+3j)I_2 - (1-j)I_2$$

$$I_2 = -\frac{1}{14} = 71{,}43\,\text{mA}\ \angle 180°$$

$$\boxed{U_{R2} = I_2 \cdot R_2 = 71{,}43\,\text{mV}\ \angle 180°}$$

1. kz 2006 TMIL

②



$R = C = 1$

$n = 2$

$U_c(0) = 1$

$U_0(t) = s(t) \circ\!-\!\circ \frac{1}{s}$

$U_T(s)$, $Z_T(s) = ?$

① → tražim $U_T(s) = U_2(s)$

$U_1 = nU_2 = 2U_2$

$I_1 = -\frac{1}{n} I_2 = -\frac{1}{2} I_2$

① $\frac{U_1 - U_0}{R} + I_1 + \left(U_1 + \frac{U_c(0)}{s} - U_2\right) \cdot sC = 0$

② $\left(U_2 - U_1 - \frac{U_c(0)}{s}\right) \cdot sC + I_2 = 0$

$2U_2 - \frac{1}{s} - \frac{1}{2} I_2 + s \cdot \left[2U_2 + \frac{1}{s} - U_2\right] = 0$

$\left(U_2 - 2U_2 - \frac{1}{s}\right) \cdot s + I_2 = 0 \Rightarrow I_2 = 1 + sU_2$

$2U_2 - \frac{1}{s} - \frac{1}{2}(1 + sU_2) + U_2 \cdot s + 1 = 0$

$2U_2 - \frac{1}{s} + \frac{1}{2} + \frac{1}{2} sU_2 = 0$

$U_2 \left(2 + \frac{1}{2} s\right) = \frac{1}{s} + \frac{1}{2} = \frac{2+s}{2s}$

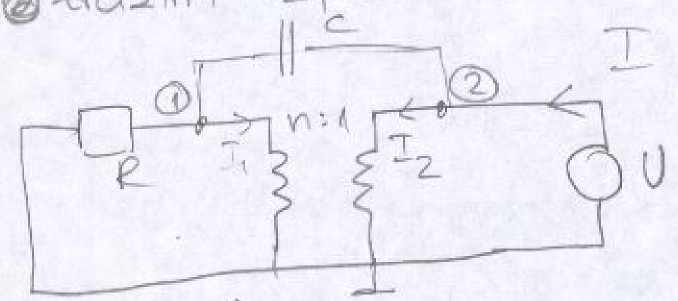
$$U_2 = \frac{\frac{2+s}{2s}}{\frac{4+s}{2}} = \frac{2+s}{s(4+s)} = U_T(s)$$

1, KZ 2006 TMIK

② tražim $Z_T(s)$

$C = R = 1$

$n = 2$



$Z_T(s) = \frac{U}{I}$

$U_1 = nU_2 = 2U_2$

$I_1 = -\frac{1}{n} I_2 = -\frac{1}{2} I_1$

$\boxed{U_2 = U}$

① $\frac{U_1}{R} + I_1 + (U_1 - U_2) sC = 0$

② $(U_2 - U_1) sC + I_2 - I = 0$

② $-U \cdot s + I_2 - I = 0$

① $2U - \frac{1}{2} I_2 + U \cdot s = 0 \Rightarrow I_2 = 2U(2+s)$

$Us - 2U(2+s) + I = 0$

$U(s - 4 - 2s) + I = 0$

$$\boxed{\frac{U}{I} = \frac{1}{4+s}} = Z_T(s)$$

$u_0(t) = \sin(t)\ ;\ -\infty < t < \infty$

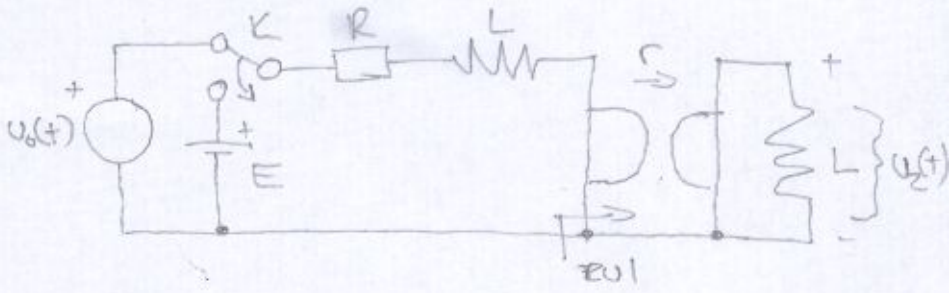
$E = 1$

$L = 1$

$R = 1$

$r = \sqrt{2}$

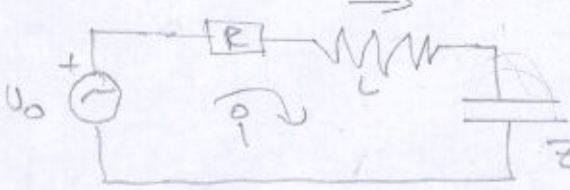
$u_2(t) = ?$



$z_{ul} = r^2 \cdot \frac{1}{z_2}$ → ulazni otpor giratora zaključenog impedancijom

① $t < 0$

$i_L(0) = -\frac{1}{2}$



$z_{ul} = 2 \cdot \frac{1}{jwL}$

$u_0 = i \cdot R + i \cdot jwL + i \cdot \frac{2}{jwL}$

$i = \frac{u_0}{R + jwL + \frac{2}{jwL}} = \frac{1}{1 + j + \frac{2}{j}} = \frac{1}{1 - j} = \frac{\sqrt{2}}{2} \angle 45^\circ$

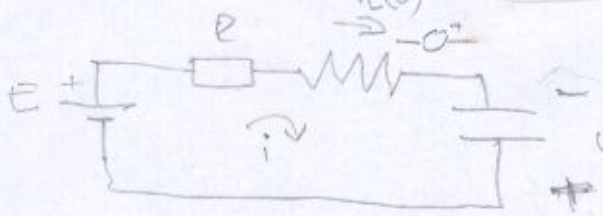
$= \frac{\sqrt{2}}{2} \sin\left(t + \frac{\pi}{4}\right)$

$t = 0 \quad i = \frac{\sqrt{2}}{2} \sin\left(\frac{\pi}{4}\right) = \frac{1}{2}$

$u_{zul} = i \cdot z_{ul} = \frac{1}{1-j} \cdot -2j = \sqrt{2} \angle -45^\circ = \sqrt{2} \sin\left(t + \frac{\pi}{4}\right)$

$= -1$

② $t > 0$



$z_{ul} = 2 \cdot \frac{1}{sL} = \frac{2}{s}$

$\frac{E}{s} = i \cdot R + i \cdot sL - L \cdot i_L(0) + \frac{1}{sC} \cdot i + \frac{u_C(0)}{s}$

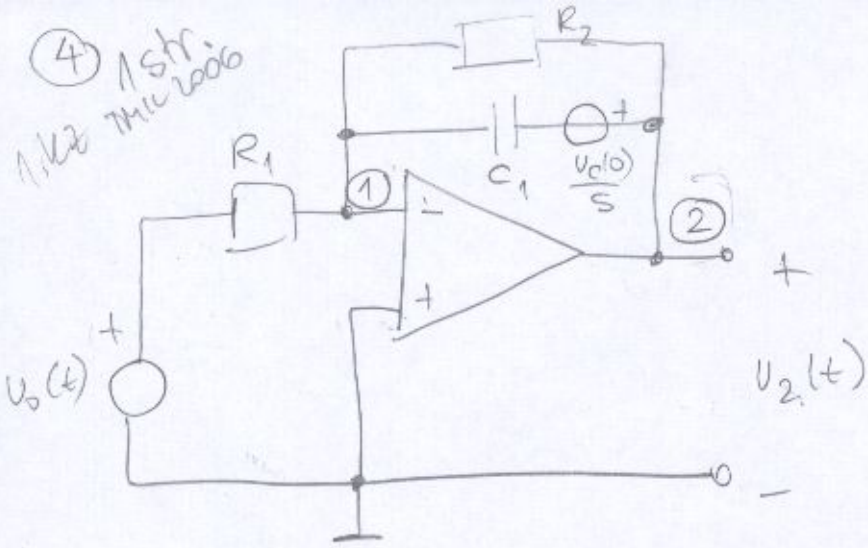
$i = \frac{\frac{E}{s} + L \cdot i_L(0) - \frac{u_C(0)}{s}}{R + sL + \frac{1}{sC}} = \frac{\frac{1}{s} + \frac{1}{2} + \frac{1}{s}}{1 + s + \frac{2}{s}}$ ✓ ovo je dobro

$= \frac{s+4}{2s^2 + 2s + 4}$

$u_2 = -r \cdot I_{g1}$

$= -\frac{\sqrt{2} \cdot (s+4)}{2s^2 + 2s + 4} = \circ\!\!-\!\!\circ = -\frac{\sqrt{2}}{2} \cdot e^{-\frac{t}{2}} \cdot \left(\cos\left(\frac{\sqrt{7} \cdot t}{2}\right) + \sqrt{7} \cdot \sin\left(\frac{\sqrt{7} \cdot t}{2}\right)\right)$

④ 1 str.



$u_2(t) = ?$

$u_0(t) = S(t) \circ\!-\!\circ \frac{1}{s}$

0 $U_1 = \emptyset$

$$\frac{\emptyset_1 - U_0}{R_1} + \left[\emptyset_1 + \frac{u_c(0)}{s} - U_2\right] \cdot sC_1 + \frac{\emptyset_1 - U_2}{R_2} = 0$$

$$-\frac{U_0}{R_1} + C_1 \cdot u_c(0) = U_2 \cdot sC_1 + \frac{U_2}{R_2}$$

slobodni: $C_1 u_c(0)$ — prisilni: $\frac{U_0}{R_1}$

$$U_2 = \frac{C_1 u_c(0) - \frac{U_0}{R_1}}{sC_1 + \frac{1}{R_2}}$$

$$U_{2slob}(s) = \frac{u_c(0)}{s + \frac{1}{C_1 R_2}} \circ\!-\!\circ u_c(0) \cdot e^{-\frac{t}{C_1 R_2}}$$

$$U_{2pris}(s) = -\frac{U_0}{sC_1R_1 + \frac{R_1}{R_2}} = -\frac{R_2}{s(sC_1R_1R_2 + R_1)}$$

$$= -\frac{1}{C_1R_1} \cdot \frac{1}{s\left(s + \frac{1}{C_1R_2}\right)}$$

$$\frac{1}{s\left(s+\frac{1}{C_1R_2}\right)} = \frac{A}{s} + \frac{B}{s+\frac{1}{C_1R_2}}$$

$1 = A\left(s + \frac{1}{C_1R_2}\right) + Bs$

$s = 0 \quad 1 = A\,\frac{1}{C_1R_2} \Rightarrow A = C_1R_2$

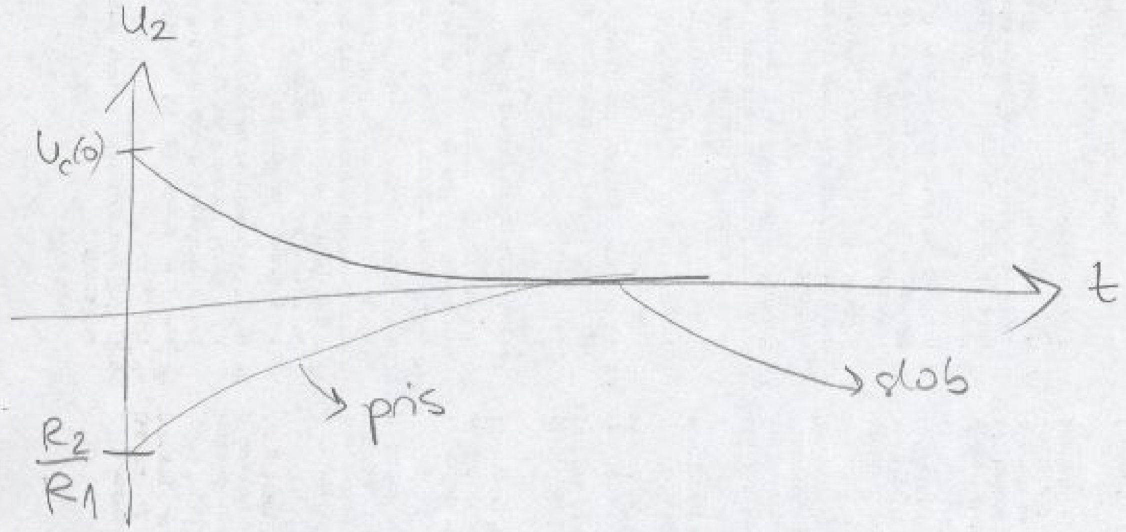
$s = -\frac{1}{C_1R_2} \quad 1 = -B \cdot \frac{1}{C_1R_2} \Rightarrow B = -C_1R_2$

④ 1. KZTMIL 2006

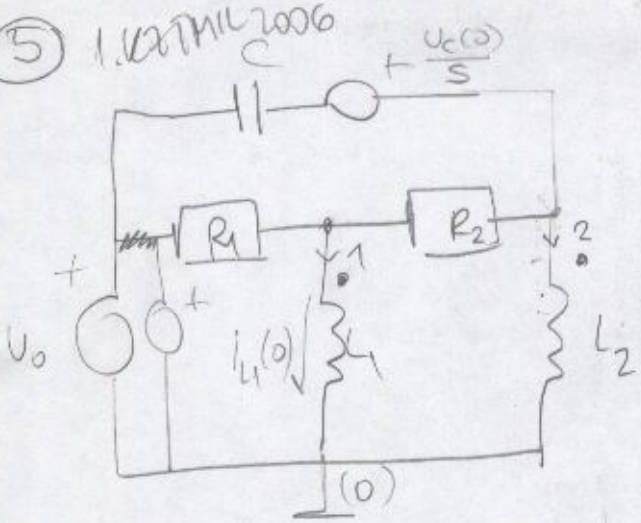
$$u_{2\,pris}(s) = -\frac{1}{C_1R_1}\cdot\left[\frac{C_1R_2}{s} - \frac{C_1R_2}{s+\frac{1}{C_1R_2}}\right]$$

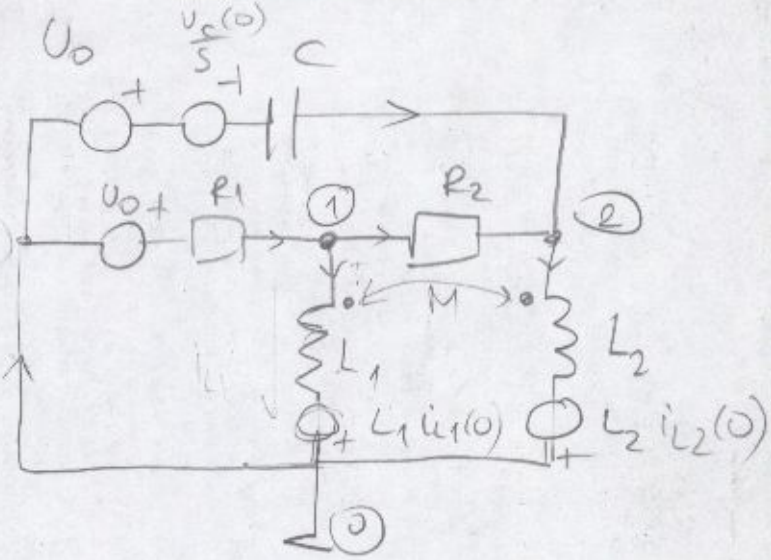
$$= +\frac{R_2}{R_1}\cdot\left(\frac{-1}{s} + \frac{1}{s+\frac{1}{C_1R_2}}\right)$$

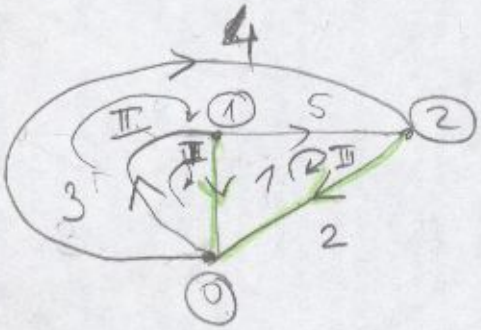
$$\circ\!\!-\!\!\bullet\ \frac{R_2}{R_1}\left(-1 + e^{-\frac{t}{C_1R_2}}\right)$$



(5) 1. KOLOKVIJ 2006







temeljni sustav pdžo petlji = ?

Z_m, E_m, Z_b, E_b

↳ regularna!

$$U_1 = U_{10} = -L_1 i_{L_1}(0) + I_1 \cdot sL_1 + I_2 sM$$

$$U_2 = U_{20} = I_1 sM + I_2 sL_2 - L_2 i_{L_2}(0)$$

$$U_3 = U_{01} = -U_0 + I_3 R_1$$

$$U_4 = U_{02} = -U_0 - \frac{u_c(0)}{s} + I_4 \cdot \frac{1}{sC}$$

$$U_5 = U_{12} = I_5 R_2$$

$$\boxed{U_b = Z_b I_b + E_b}$$

$$Z_b = \begin{bmatrix} sL_1 & sM & 0 & 0 & 0 \\ sM & sL_2 & 0 & 0 & 0 \\ 0 & 0 & R_1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{sC} & 0 \\ 0 & 0 & 0 & 0 & R_2 \end{bmatrix}$$

$$E_b = \begin{bmatrix} -L_1 i_{L_1}(0) \\ -L_2 i_{L_2}(0) \\ -U_0 \\ -U_0 - \frac{u_c(0)}{s} \\ 0 \end{bmatrix}$$

$$S = \begin{array}{c} \\ U_1 \\ U_2 \\ U_3 \\ E_3 \end{array} \begin{array}{c} \begin{array}{ccccc} \text{STA.GRANE} & & & \text{SPONE} & \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 1 \end{bmatrix} \\ \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \end{array} \quad \leftarrow E$$

(5) 2. Str. 1. Klausur 2006

$$Z_m = S Z_b S^T$$

$$SZ_b = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} sL_1 & sM & 0 & 0 & 0 \\ sM & sL_2 & 0 & 0 & 0 \\ 0 & 0 & R_1 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{sC} & 0 \\ 0 & 0 & 0 & 0 & R_2 \end{bmatrix}$$

$$= \begin{bmatrix} sL_1 & sM & R_1 & 0 & 0 \\ sM & sL_2 & 0 & \frac{1}{sC} & 0 \\ -sL_1+sM & -sM+sL_2 & 0 & 0 & R \end{bmatrix}$$

$$SZ_b S^T = \begin{bmatrix} sL_1 & sM & R_1 & 0 & 0 \\ sM & sL_2 & 0 & \frac{1}{sC} & 0 \\ -sL_1+sM & -sM+sL_2 & 0 & 0 & R \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} sL_1+R_1 & sM & -sL_1+sM \\ sM & sL_2+\frac{1}{sC} & -sM+sL_2 \\ -sL_1+sM & -sM+sL_2 & sL_1+sL_2-2sM+R \end{bmatrix}$$

$$E_m = -S \cdot E_b(s) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} L_1 i_{L1}(0) \\ L_2 i_{L2}(0) \\ U_0 \\ U_0 + \frac{U_c(0)}{s} \\ 0 \end{bmatrix} = \begin{bmatrix} L_1 i_{L1}(0) + U_0 \\ L_2 i_{L2}(0) + U_0 + \frac{U_c(0)}{s} \\ -L_1 i_{L1}(0) + L_2 i_{L2}(0) \end{bmatrix}$$