

# Električni krugovi

Četveropoli

## Ekvivalentni četveropoli

Dva četveropola su ekvivalentna ako



njihovom međusobnom zamjenom

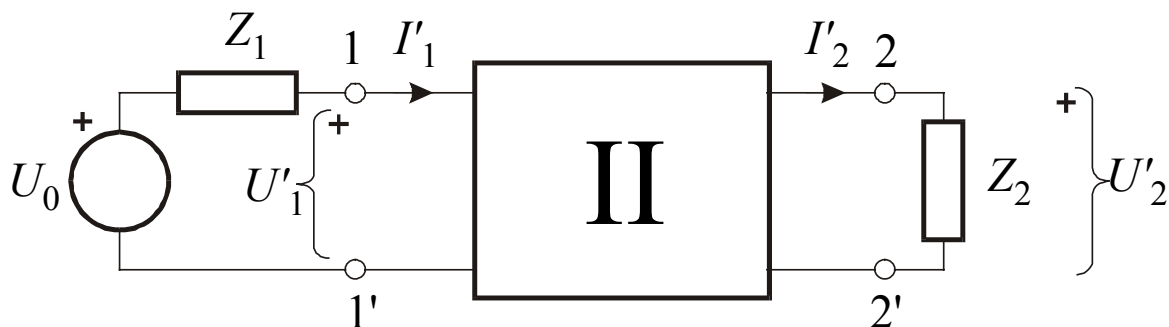
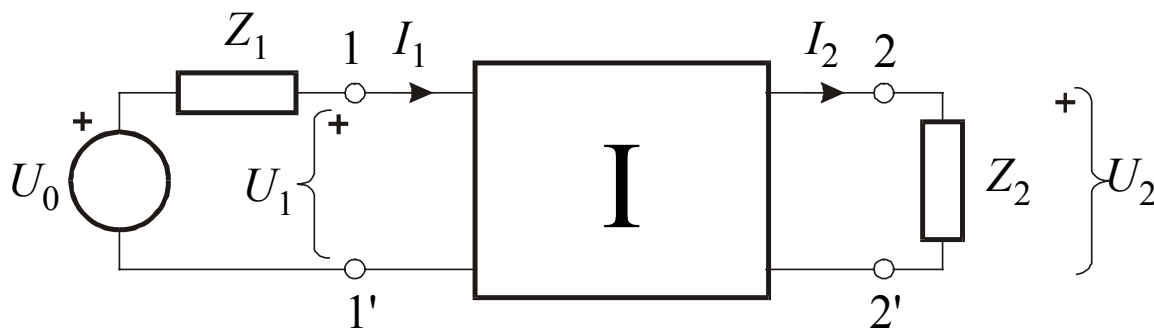


STRUJE I NAPONI NA PRIKLJUČNICAMA



OSTAJU NEPROMIJENJENI

# Prijenosne i ulazne funkcije četveropola



Ako je

$$U_1 = U'_1$$

$$U_2 = U'_2$$

$$I_1 = I'_1$$

$$I_2 = I'_2$$

***ekvivalentni*** četveropoli

## Nuždan i dovoljan uvjet

Parametri jednog = parametrima drugog

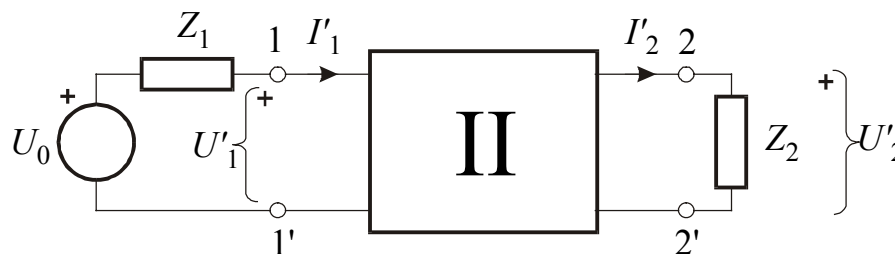
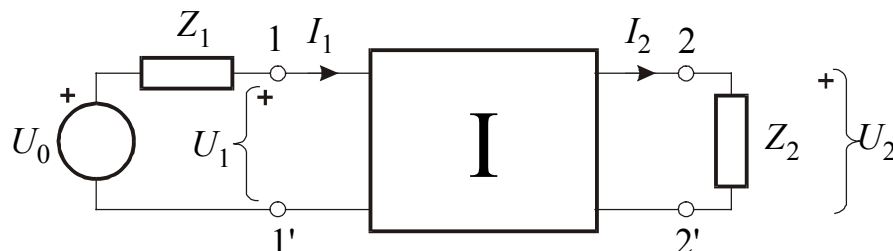
$$[z]^I = [z]^{II}$$

$$z_{11}^I = z_{11}^{II}$$

$$z_{12}^I = z_{12}^{II}$$

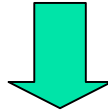
$$z_{21}^I = z_{21}^{II}$$

$$z_{22}^I = z_{22}^{II}$$

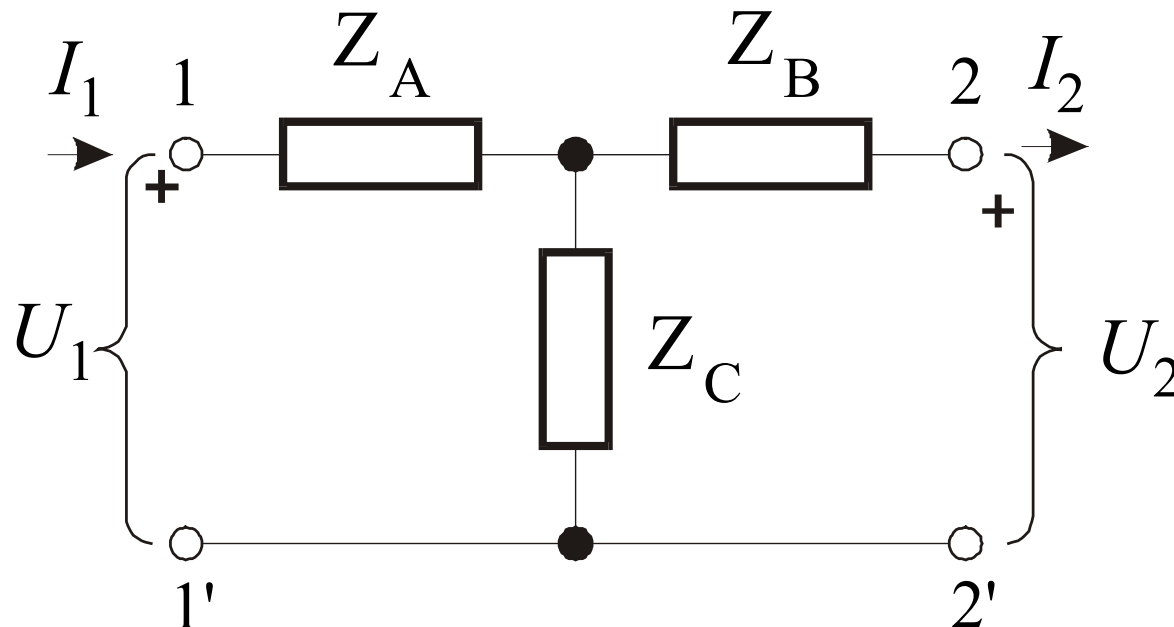


## Ekvivalentni četveropol u T-spoju

Svakom recipročnom četveropolu zadanom z-parametrima



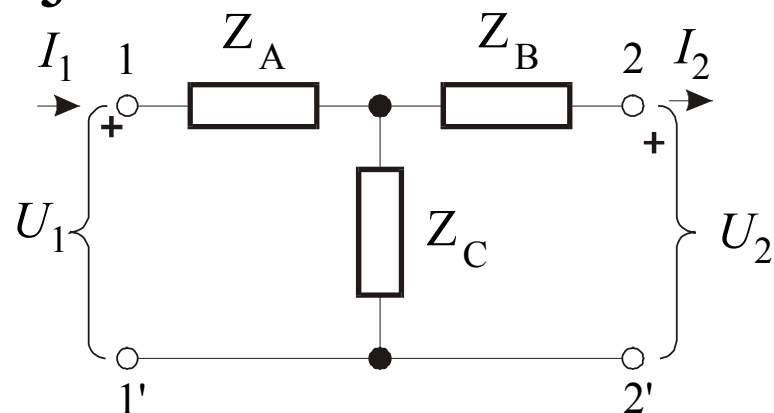
moguće je odrediti ekvivalentni četveropol u T-spoju



## z-parametri ekvivalentnoga T-spoja

$$U_1 = I_1 z_{11} - I_2 z_{12}$$

$$U_2 = I_1 z_{21} - I_2 z_{22}$$



## elementi ekvivalentnoga T-spoja

$$z_{11} = Z_A + Z_C$$

$$z_{12} = Z_C$$

$$z_{21} = Z_C$$

$$z_{22} = Z_B + Z_C$$

$$\left. \begin{array}{l} z_{11} = Z_A + Z_C \\ z_{12} = Z_C \\ z_{21} = Z_C \\ z_{22} = Z_B + Z_C \end{array} \right\} \Rightarrow \begin{array}{l} Z_A = z_{11} - z_{12} \\ Z_B = z_{22} - z_{12} \\ Z_C = z_{12} = z_{21} \end{array}$$

$Z_A$ ,  $Z_B$  i  $Z_C$  nisu uvijek realizabilne  $R$ ,  $L$  ili  $C$  elementima

## Nerecipročni četveropol $z_{12} \neq z_{21}$

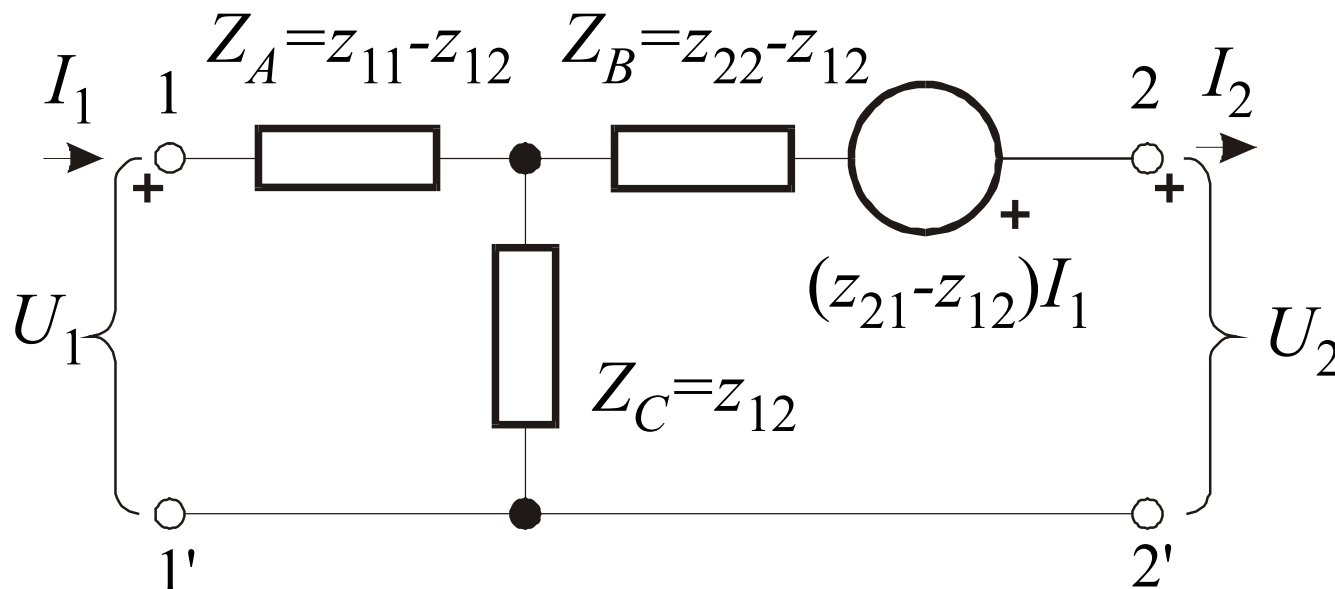
$$U_1 = I_1 z_{11} - I_2 z_{12}$$

$$U_1 = I_1 z_{11} - I_2 z_{12}$$

$$\underline{U_2 = I_1 z_{21} - I_2 z_{22}}$$

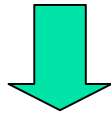
$$U_2 = I_1 z_{12} - I_2 z_{22} + \underbrace{I_1 (z_{21} - z_{12})}_{\text{naponski izvor}}$$

naponski izvor

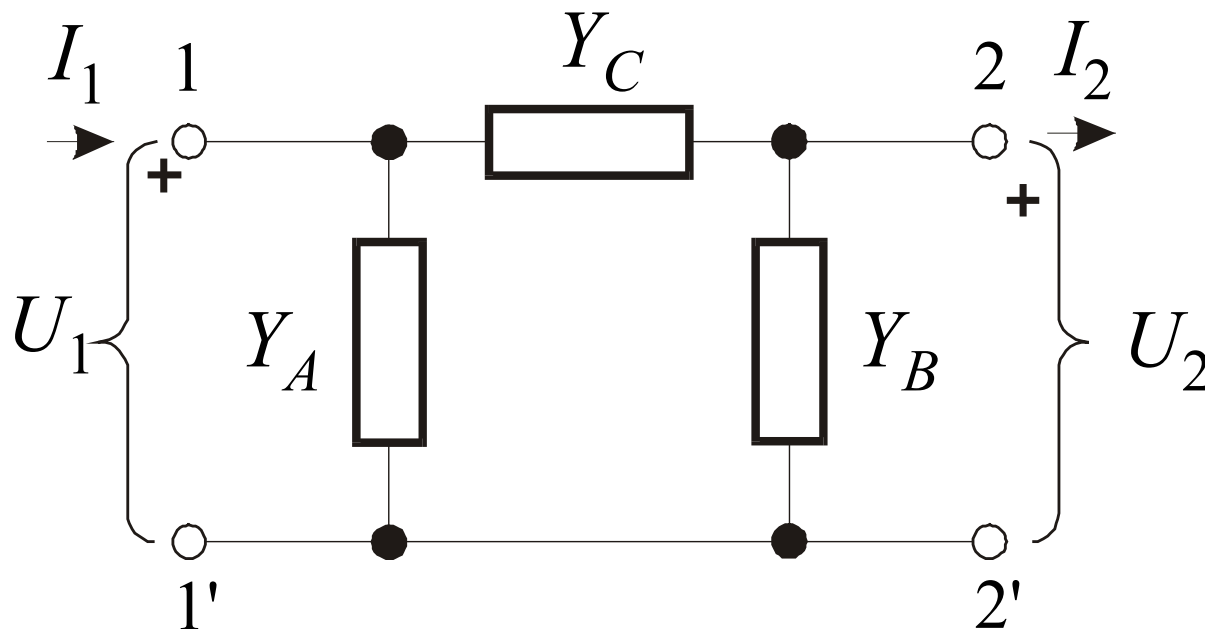


## Ekvivalentni četveropol u $\Pi$ -spoju

Svakom recipročnom četveropolu zadanom y-parametrima



moguće je odrediti ekvivalentni četveropol u  $\Pi$ -spoju

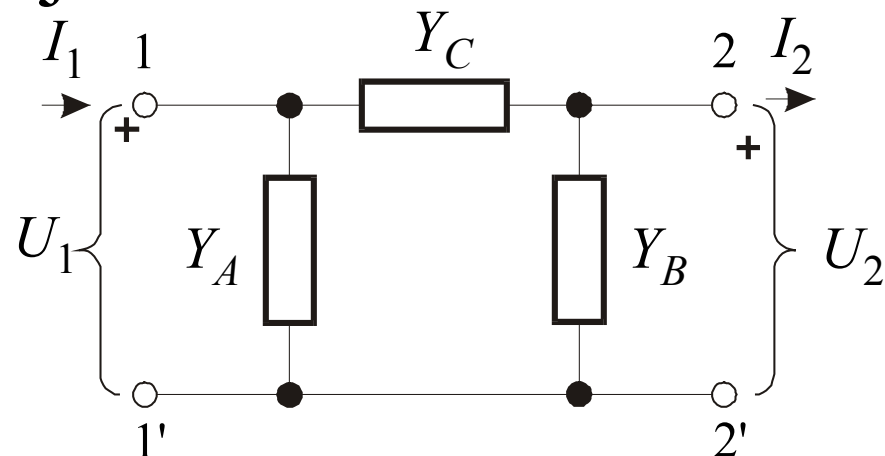




# y-parametri ekvivalentnoga $\Pi$ -spoja

$$I_1 = U_1 y_{11} - U_2 y_{12}$$

$$\underline{I_2 = U_1 y_{21} - U_2 y_{22}}$$



## elementi ekvivalentnoga $\Pi$ -spoja

$$\left. \begin{array}{ll} y_{11} = Y_A + Y_C & y_{12} = Y_C \\ y_{21} = Y_C & y_{22} = Y_B + Y_C \end{array} \right\} \Rightarrow \begin{array}{l} Y_A = y_{11} - y_{12} \\ Y_B = y_{22} - y_{12} \\ Y_C = y_{12} = y_{21} \end{array}$$

$Y_A$ ,  $Y_B$  i  $Y_C$  nisu uvijek realizabilne  $R$ ,  $L$  ili  $C$  elementima

# Nerecipročni četveropol $y_{12} \neq y_{21}$

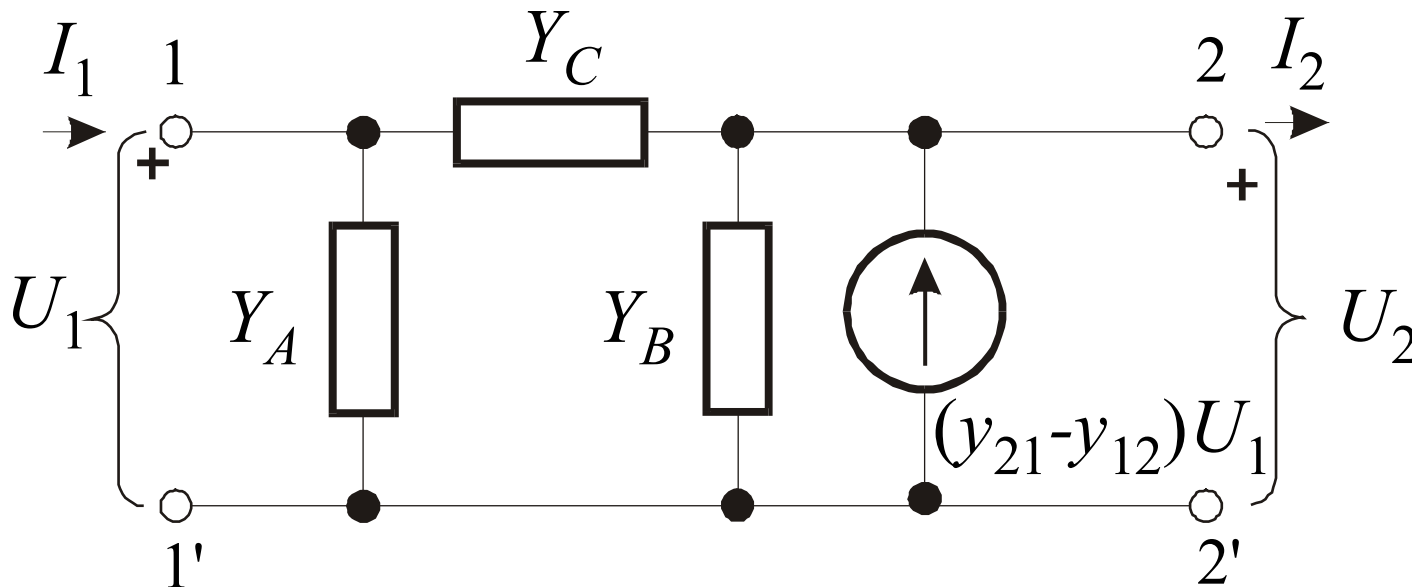
$$I_1 = y_{11}U_1 - y_{12}U_2$$

$$I_1 = y_{11}U_1 - y_{12}U_2$$

$$\underline{I_2 = y_{21}U_1 - y_{22}U_2}$$

$$I_2 = y_{12}U_1 - y_{22}U_2 + \underbrace{(y_{21} - y_{12})U_1}_{\text{strujni izvor}}$$

strujni izvor



# Zrcalni ili valni parametri četveropola

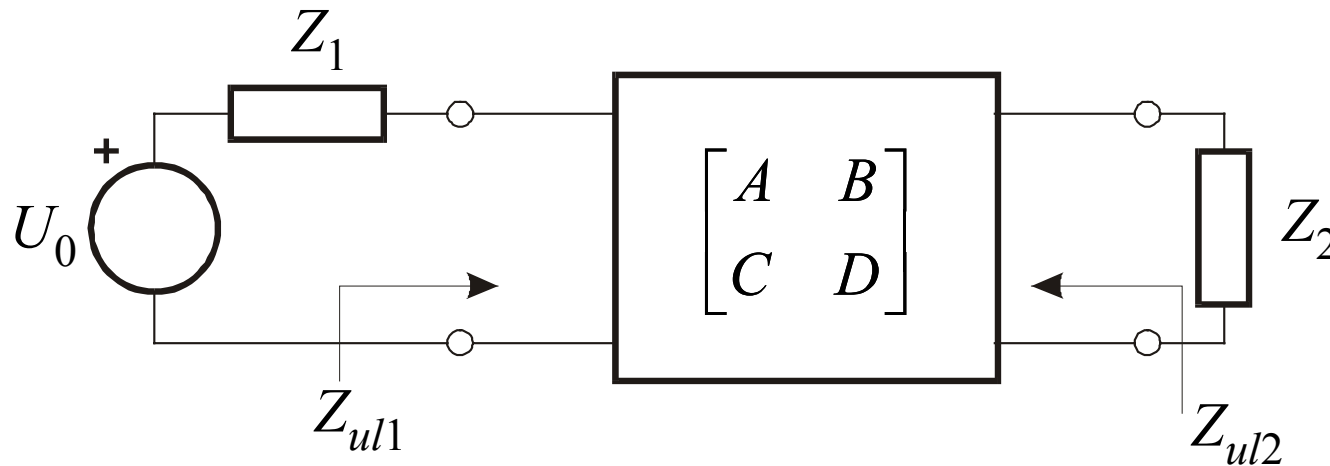
- Recipročne je četveropole moguće opisati s 3 parametra.
- Kod recipročnih četveropola često se primjenjuju

## *ZRCALNI ILI VALNI PARAMETRI*

- To su:

- *zrcalne impedancije  $Z_{C1}$  i  $Z_{C2}$*
- *zrcalna konstanta prijenosa  $g$*

- Za definiranje zrcalnih parametara → četveropol na slici



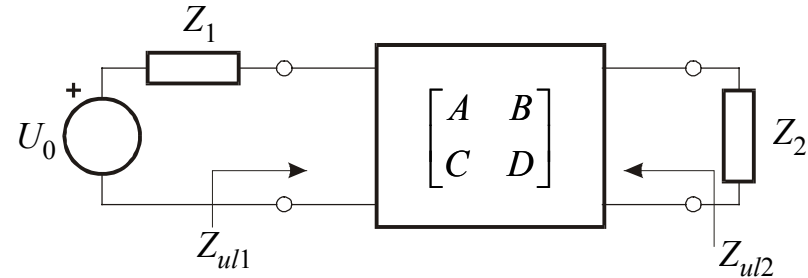
- Ulazne impedancije na 1-1' i 2-2' su

$$Z_{ul1} = \frac{AZ_2 + B}{CZ_2 + D}$$

$$Z_{ul2} = \frac{DZ_1 + B}{CZ_1 + A}$$

## Zrcalni parametri četveropola

- Uvjet:  $Z_{ul1} = Z_1$  i  $Z_{ul2} = Z_2$



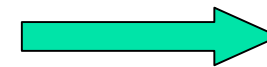
$$Z_{u1} = Z_1 \quad \longrightarrow \quad Z_1 = \frac{AZ_2 + B}{CZ_2 + D} \cdot (CZ_2 + D)$$

$$Z_{u2} = Z_2 \quad \longrightarrow \quad Z_2 = \frac{DZ_1 + B}{CZ_1 + A} \cdot (CZ_1 + A)$$

$$CZ_1Z_2 + DZ_1 - AZ_2 - B = 0$$

$$CZ_1Z_2 + AZ_2 - DZ_1 - B = 0 \quad / \quad +$$

$$CZ_1Z_2 - B = 0$$

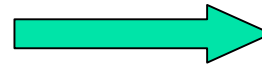


$$Z_1 \cdot Z_2 = \frac{B}{C}$$

$$\begin{array}{l} CZ_1Z_2 + DZ_1 - AZ_2 - B = 0 \\ CZ_1Z_2 + AZ_2 - DZ_1 - B = 0 \end{array} \quad \bigg| \quad -$$

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$$DZ_1 - AZ_2 = 0$$



$$\boxed{\frac{Z_1}{Z_2} = \frac{A}{D}}$$

$$\boxed{Z_1 = Z_{C1} = \sqrt{\frac{AB}{CD}}}$$

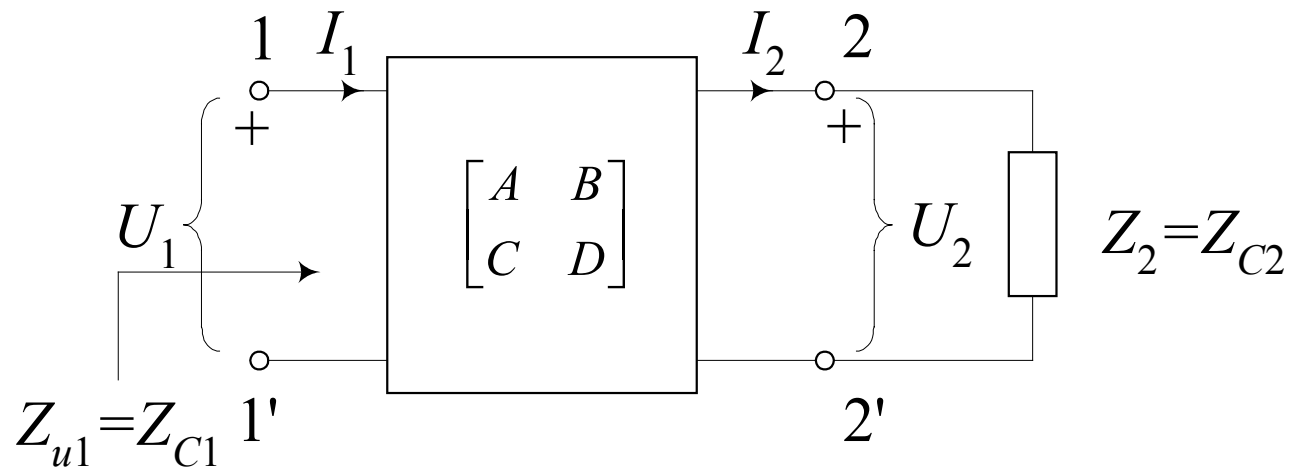
$$\boxed{Z_2 = Z_{C2} = \sqrt{\frac{BD}{AC}}}$$



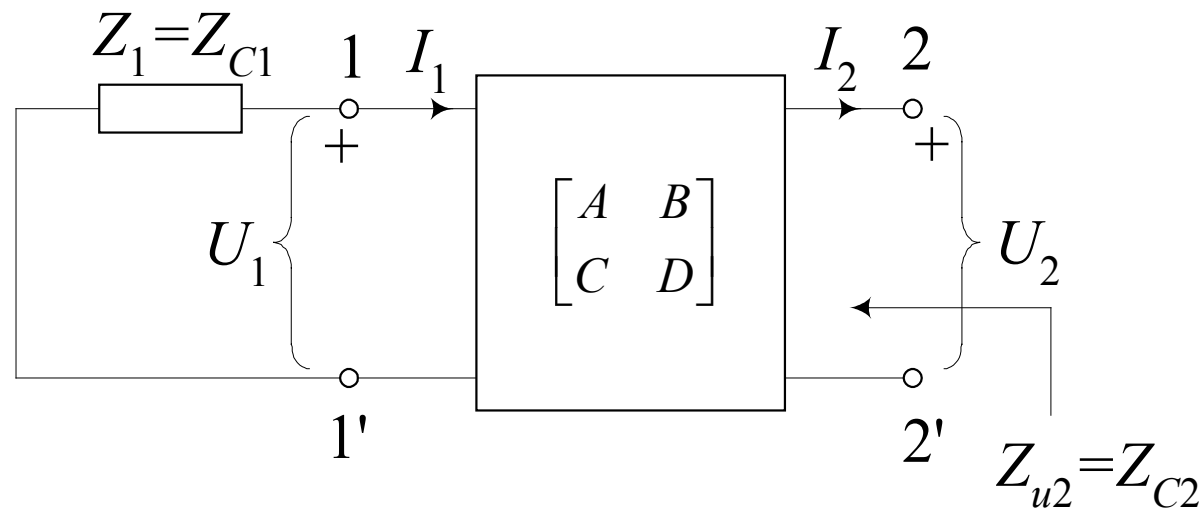
## ***ZRCALNE ILI VALNE IMPEDANCIJE***

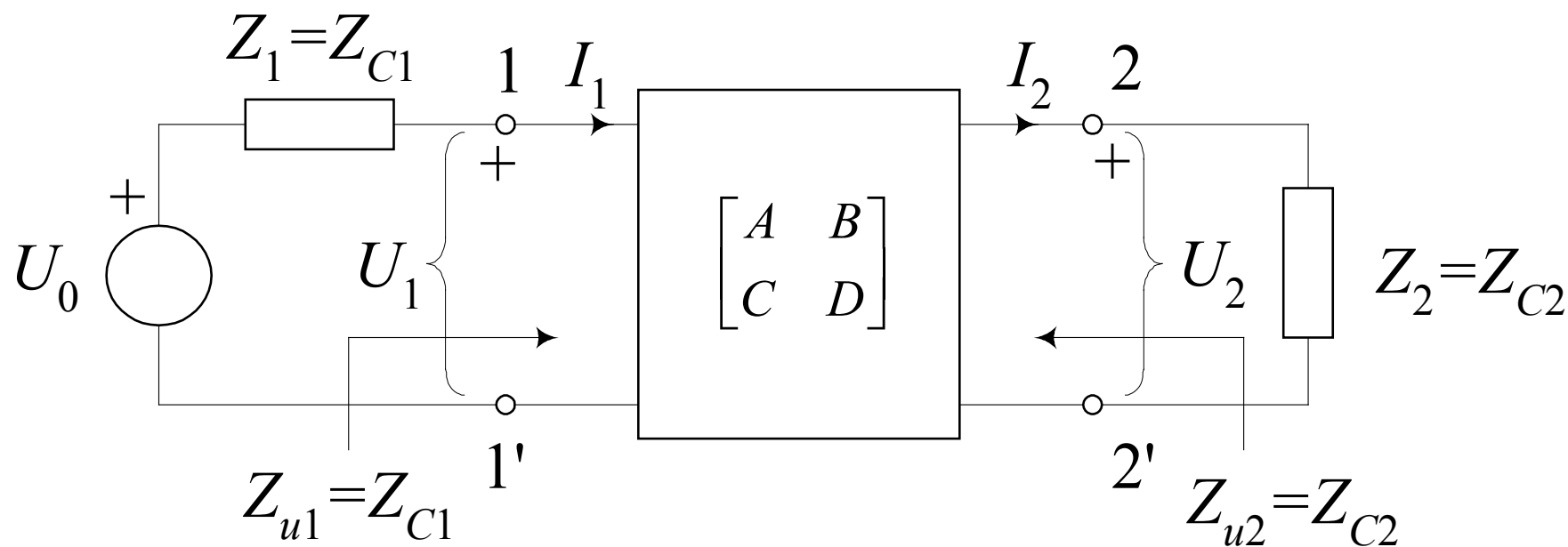
## Zrcalni parametri četveropola

■ Ako je  
 $Z_2 = Z_{C2}$   
 tada je  $Z_{ul1} = Z_{C1}$



■ Ako je  
 $Z_1 = Z_{C1}$   
 tada je  $Z_{ul2} = Z_{C2}$



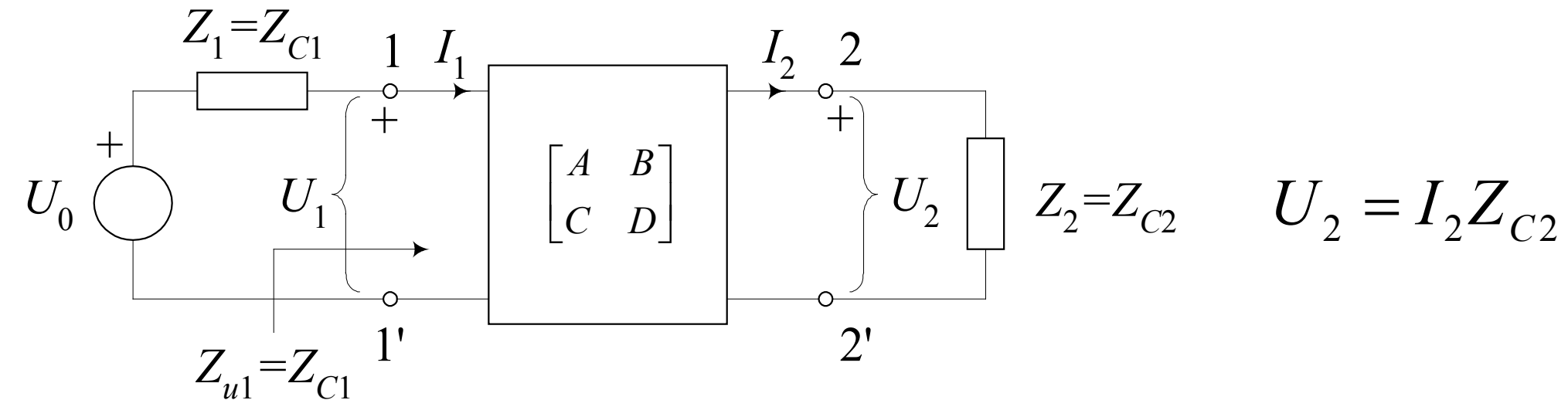


**Četveropol je prilagođen po zrcalnim impedancijama.**



# Prijenosne jednačbe i zrcalni parametri

Prijenos signala s 1-1' na 2-2'.



$$U_1 = AU_2 + BI_2 = AU_2 + \frac{B}{Z_{C2}}U_2 \quad \longrightarrow \quad \frac{U_1}{U_2} = A + \frac{B}{Z_{C2}}$$

$$I_1 = CU_2 + DI_2 = (CZ_{C2} + D)I_2 \quad \longrightarrow \quad \frac{I_1}{I_2} = CZ_{C2} + D$$

$$Z_{c2} = \sqrt{\frac{BD}{AC}}$$

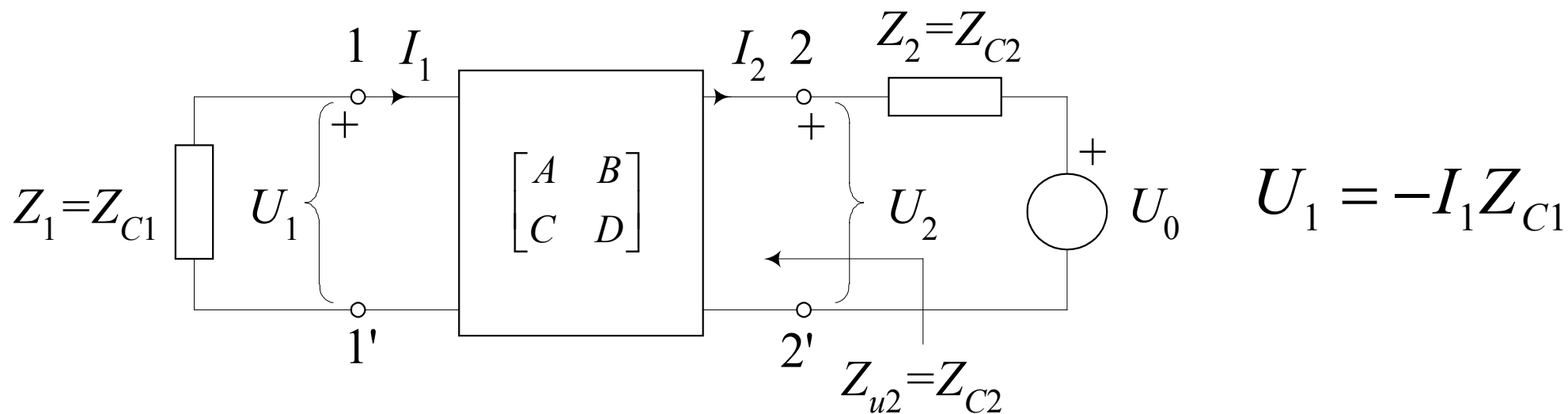
$$\frac{U_1}{U_2} = A + \frac{B}{\sqrt{\frac{BD}{AC}}} = \sqrt{\frac{A}{D}}(\sqrt{AD} + \sqrt{BC})$$

omjer prijenosa  
napona

$$\frac{I_1}{I_2} = C\sqrt{\frac{BD}{AC}} + D = \sqrt{\frac{D}{A}}(\sqrt{AD} + \sqrt{BC})$$

omjer prijenosa  
struje

Prijenos signala s 2-2' na 1-1'.



$$U_2 = DU_1 - BI_1 = DU_1 + \frac{B}{Z_{C1}}U_1 \quad \longrightarrow \quad \frac{U_2}{U_1} = D + \frac{B}{Z_{C1}}$$

$$-I_2 = CU_1 - AI_2 = -(CZ_{C1} + A)I_1 \quad \longrightarrow \quad \frac{I_2}{I_1} = CZ_{C1} + A$$

$$Z_{C1} = \sqrt{\frac{AB}{CD}}$$

$$\frac{U_2}{U_1} = D + \frac{B}{Z_{C1}} = \sqrt{\frac{D}{A}} (\sqrt{AD} + \sqrt{BC})$$

omjer prijenosa  
napona

$$\frac{I_2}{I_1} = CZ_{C1} + A = \sqrt{\frac{A}{D}} (\sqrt{AD} + \sqrt{BC})$$

omjer prijenosa  
struja

$$\frac{A}{D} = \frac{Z_{C1}}{Z_{C2}} \quad \sqrt{\frac{A}{D}} = \sqrt{\frac{Z_{C1}}{Z_{C2}}} = n$$

$$\frac{D}{A} = \frac{Z_{C2}}{Z_{C1}} \quad \sqrt{\frac{D}{A}} = \sqrt{\frac{Z_{C2}}{Z_{C1}}} = \frac{1}{n}$$

$n \rightarrow$  omjer transformacije

$$\sqrt{AD} + \sqrt{BC} = e^g \quad \longrightarrow \quad g = \ln(\sqrt{AD} + \sqrt{BC})$$

$g \rightarrow$  **ZRCALNI KOEFICIJENT PRIJENOSA**

- U uvjetima stacionarne sinusne pobude

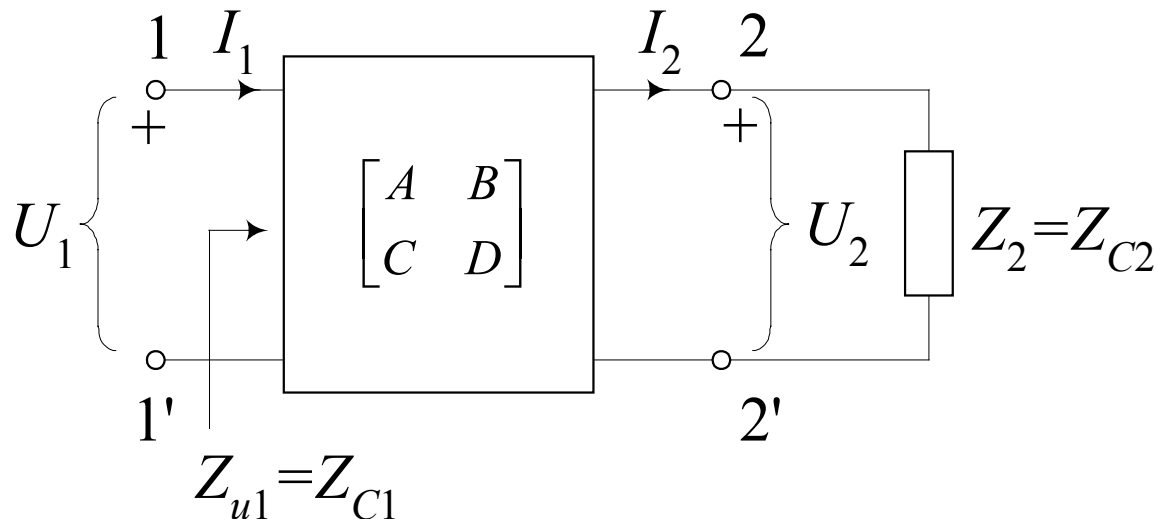
$$g = g(j\omega) = a(\omega) + jb(\omega)$$

- kompleksna veličina

$a \rightarrow$  zrcalni koeficijent gušenja

$b \rightarrow$  zrcalni koeficijent faze

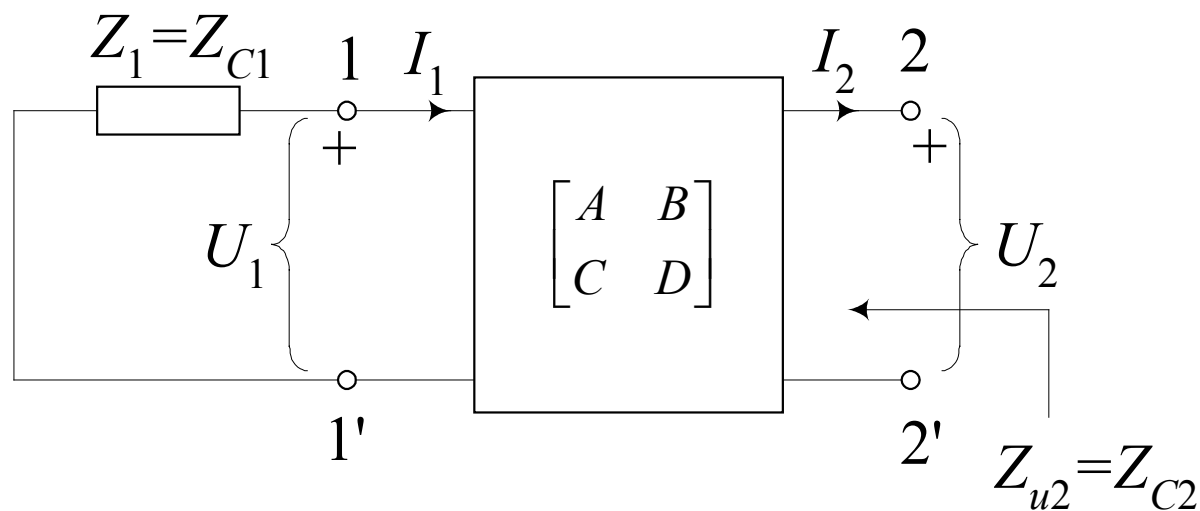
- Kod prilagođenja i prijenosa sa 1-1' na 2-2'



$$\frac{U_1}{U_2} = n \cdot e^g \quad \text{uz} \quad Z_2 = Z_{C2}$$

$$\frac{I_1}{I_2} = \frac{1}{n} \cdot e^g \quad \text{uz} \quad Z_2 = Z_{C2}$$

- Kod prilagođenja i prijenosa sa 2-2' na 1-1'



$$\frac{U_2}{U_1} = \frac{1}{n} \cdot e^g \quad \text{uz} \quad Z_1 = Z_{C1}$$

$$\frac{I_2}{I_1} = n \cdot e^g \quad \text{uz} \quad Z_1 = Z_{C1}$$

- Ako se izraz za zrcalni koeficijent prijenosa

$$e^g = \sqrt{AD} + \sqrt{BC}$$

pomnoži s izrazom  $(\sqrt{AD} - \sqrt{BC})$

dobiva se

$$e^g (\sqrt{AD} - \sqrt{BC}) = (\sqrt{AD} + \sqrt{BC})(\sqrt{AD} - \sqrt{BC}) = AD - BC$$

- Pošto se radi o recipročnome četveropolu, vrijedi

$$AD - BC = 1$$

pa je  $e^g (\sqrt{AD} - \sqrt{BC}) = 1 \quad \longrightarrow \quad (\sqrt{AD} - \sqrt{BC}) = e^{-g}$



- Ove izraze moguće je koristiti za određivanje parametara A,B,C i D iz poznatih zrcalnih  $Z_{C1}$ ,  $Z_{C2}$  i  $g$ .

$$\begin{array}{l} \sqrt{AD} + \sqrt{BC} = e^g \\ \sqrt{AD} - \sqrt{BC} = e^{-g} \end{array} \left| \begin{array}{l} + \\ - \end{array} \right| : 2 \quad \Rightarrow \quad \sqrt{AD} = \frac{e^g + e^{-g}}{2} = ch(g)$$

$$\begin{array}{l} \sqrt{AD} + \sqrt{BC} = e^g \\ \sqrt{AD} - \sqrt{BC} = e^{-g} \end{array} \left| \begin{array}{l} + \\ - \end{array} \right| : 2 \quad \Rightarrow \quad \sqrt{BC} = \frac{e^g - e^{-g}}{2} = sh(g)$$

## Zrcalni parametri četveropola

$$ch(g) = \sqrt{AD} \qquad \sqrt{\frac{Z_{C1}}{Z_{C2}}} = \sqrt{\frac{A}{D}}$$

$$sh(g) = \sqrt{BC} \qquad \sqrt{Z_{C1}Z_{C2}} = \sqrt{\frac{B}{C}}$$

$$A = \sqrt{\frac{Z_{C1}}{Z_{C2}}} \cdot chg$$

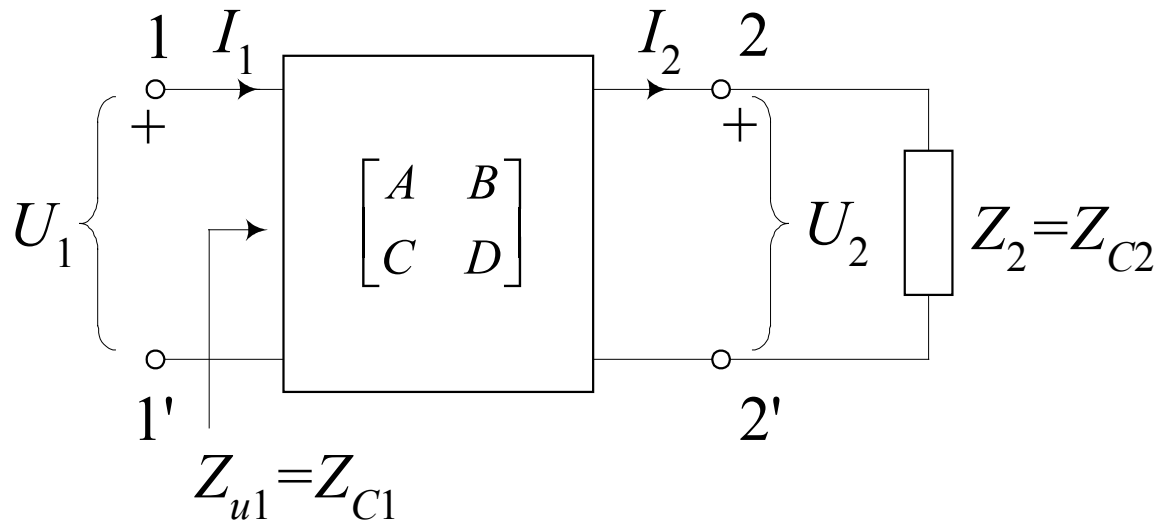
$$B = \sqrt{Z_{C1}Z_{C2}} \cdot shg = n \cdot Z_{C2} \cdot shg$$

$$C = \frac{1}{\sqrt{Z_{C1}Z_{C2}}} \cdot shg = \frac{1}{n \cdot Z_{C2}} \cdot shg$$

$$D = \sqrt{\frac{Z_{C2}}{Z_{C1}}} \cdot chg$$

# Prijenosne jednačbe

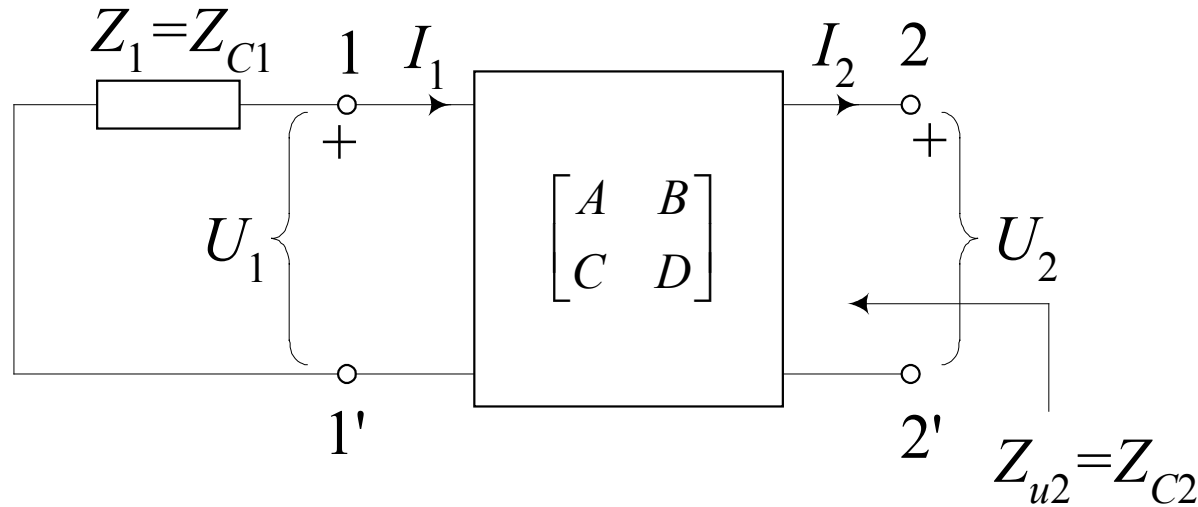
- Prilagođenje na 2-2' i prijenos s 1-1' na 2-2'



$$U_1 = n(U_2 \cdot chg + I_2 \cdot Z_{C2} \cdot shg)$$

$$I_1 = \frac{1}{n} \left( U_2 \cdot \frac{shg}{Z_{C2}} + I_2 \cdot chg \right)$$

- Prilagođenje na 1-1' i prijenos sa 2-2' na 1-1'



$$U_2 = \frac{1}{n} (U_1 \cdot chg - I_1 \cdot Z_{C1} \cdot shg)$$

$$-I_2 = n \left( \frac{U_1}{Z_{C1}} \cdot shg - I_1 \cdot chg \right)$$

# Impedancije na kratko i na prazno

$$U_1 = AU_2 + BI_2$$

$$I_1 = \underline{CU_2 + DI_2}$$

$$I_2 = 0 \quad \longrightarrow \quad Z_{p1} = z_{11} = \left. \frac{U_1}{I_1} \right|_{I_2=0} = \frac{A}{C}$$

$$U_2 = 0 \quad \longrightarrow \quad Z_{k1} = \frac{1}{y_{11}} = \left. \frac{U_1}{I_1} \right|_{U_2=0} = \frac{B}{D}$$


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$$Z_{c1} = \sqrt{\frac{AB}{CD}} = \sqrt{Z_{p1}Z_{k1}} = \sqrt{\frac{z_{11}}{y_{11}}}$$

$$U_1 = AU_2 + BI_2$$

$$I_1 = CU_2 + DI_2$$

$$I_1 = 0 \quad \longrightarrow \quad Z_{p2} = z_{22} = -\frac{U_2}{I_2} \Big|_{I_1=0} = \frac{D}{C}$$

$$U_1 = 0 \quad \longrightarrow \quad Z_{k2} = \frac{1}{y_{22}} = -\frac{U_2}{I_2} \Big|_{U_1=0} = \frac{B}{A}$$

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$$Z_{c2} = \sqrt{\frac{DB}{AC}} = \sqrt{Z_{p2}Z_{k2}} = \sqrt{\frac{z_{22}}{y_{22}}}$$

$$Z_{p1} = \frac{A}{C} \quad Z_{p2} = \frac{D}{C} \quad Z_{k1} = \frac{B}{D} \quad Z_{k2} = \frac{B}{A}$$

- Za omjere impedancija na prazno i na kratko vrijedi

$$\frac{Z_{p1}}{Z_{p2}} = \frac{A}{D} \quad \frac{Z_{k1}}{Z_{k2}} = \frac{A}{D}$$

- Odatle slijedi

$$\frac{Z_{p1}}{Z_{p2}} = \frac{Z_{k1}}{Z_{k2}} \quad \longrightarrow \quad \frac{Z_{p1}}{Z_{k1}} = \frac{Z_{p2}}{Z_{k2}}$$

$$th(g) = \frac{sh(g)}{ch(g)} = \sqrt{\frac{BC}{AD}} = \sqrt{\frac{Z_{k1}}{Z_{p1}}} = \sqrt{\frac{Z_{k2}}{Z_{p2}}}$$

- Zrcalni parametri izraženi s  $Z_{k1}$ ,  $Z_{k2}$ ,  $Z_{p1}$  i  $Z_{p2}$ , glase

$$Z_{c1} = \sqrt{Z_{p1} Z_{k1}}$$

$$Z_{c2} = \sqrt{Z_{p2} Z_{k2}}$$

$$th(g) = \sqrt{\frac{Z_{k1}}{Z_{p1}}} = \sqrt{\frac{Z_{k2}}{Z_{p2}}}$$

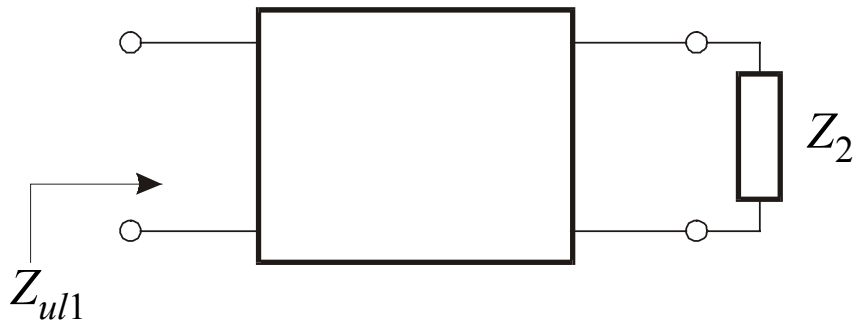


- $Z_{k1}$ ,  $Z_{k2}$ ,  $Z_{p1}$  i  $Z_{p2}$  je moguće izraziti zrcalnim parametrima

$$\left. \begin{aligned} Z_{c1} &= \sqrt{Z_{p1} Z_{k1}} \\ th(g) &= \sqrt{\frac{Z_{k1}}{Z_{p1}}} \end{aligned} \right\} \longrightarrow \left\{ \begin{aligned} Z_{k1} &= Z_{c1} th(g) \\ Z_{p1} &= Z_{c1} cth(g) \end{aligned} \right.$$

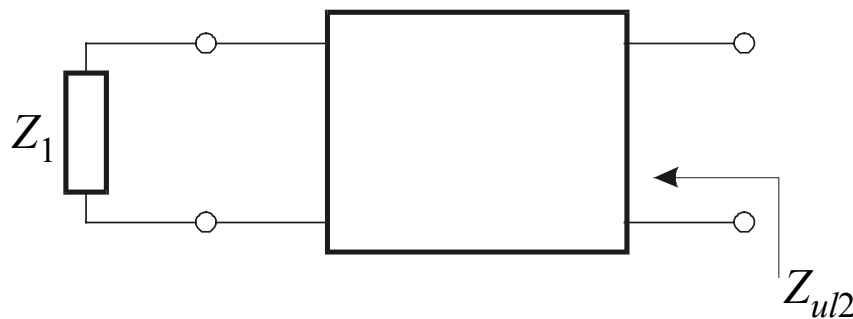
$$\left. \begin{aligned} Z_{c2} &= \sqrt{Z_{p2} Z_{k2}} \\ th(g) &= \sqrt{\frac{Z_{k2}}{Z_{p2}}} \end{aligned} \right\} \longrightarrow \left\{ \begin{aligned} Z_{k2} &= Z_{c2} th(g) \\ Z_{p2} &= Z_{c2} cth(g) \end{aligned} \right.$$

Ako četveropol nije prilagođen  $\rightarrow Z_2 \neq Z_{C2} \Rightarrow Z_{ul1} \neq Z_{c1}$



$$Z_{ul1} = \frac{U_1}{I_1} = \frac{shg + \frac{Z_2}{Z_{c2}} chg}{chg + \frac{Z_2}{Z_{c2}} shg} Z_{c1}$$

$$Z_1 \neq Z_{C1} \Rightarrow Z_{u2} \neq Z_{c2}$$



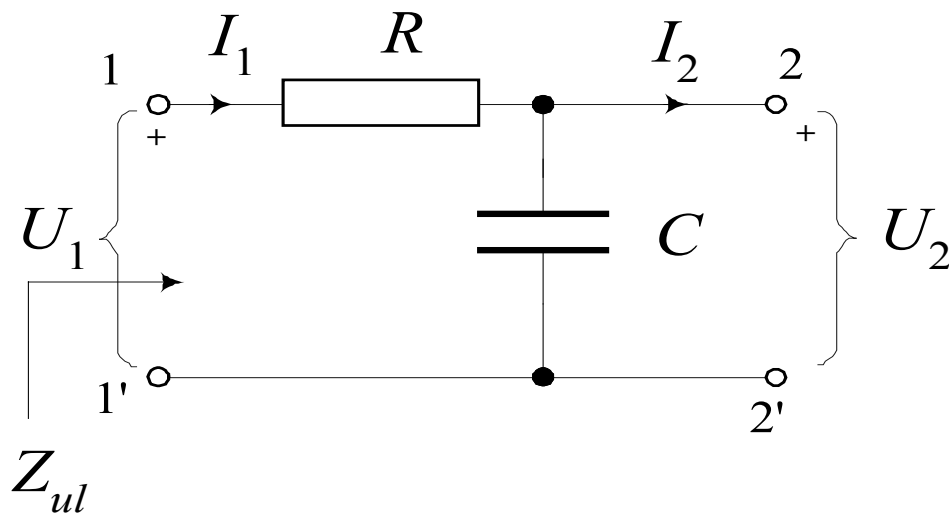
$$Z_{ul2} = -\frac{U_2}{I_2} = \frac{shg + \frac{Z_1}{Z_{c1}} chg}{chg + \frac{Z_1}{Z_{c1}} shg} Z_{c2}$$

## Mjera odstupanja od prilagođenja

$$p_1 = \frac{Z_1 - Z_{c1}}{Z_1 + Z_{c1}} \quad \rightarrow \text{pogreška prilagođenja ili koeficijent refleksije na ulazu}$$

$$p_2 = \frac{Z_2 - Z_{c2}}{Z_2 + Z_{c2}} \quad \rightarrow \text{pogreška prilagođenja ili koeficijent refleksije na izlazu}$$

Primjer: Izračunati zrcalne parametre četveropola na slici



$$Z_{k1} = R$$

$$Z_{k2} = \frac{R}{sRC + 1}$$

$$Z_{p1} = R + \frac{1}{sC}$$

$$Z_{p2} = \frac{1}{sC}$$

$$Z_{C1} = \sqrt{R \left( R + \frac{1}{sC} \right)} = \sqrt{\frac{R}{sC}} \sqrt{sRC + 1}$$

$$Z_{C2} = \sqrt{\frac{1}{sC} \cdot \frac{R}{sRC + 1}} = \sqrt{\frac{R}{sC}} \frac{1}{\sqrt{sRC + 1}}$$

$$th(g) = \sqrt{\frac{Z_{k1}}{Z_{p1}}} = \sqrt{\frac{R}{R + 1/sC}} = \sqrt{\frac{sRC}{sRC + 1}}$$

$$th(g) = \sqrt{\frac{Z_{k2}}{Z_{p2}}} = \sqrt{\frac{R}{(sRC + 1)/sC}} = \sqrt{\frac{sRC}{sRC + 1}}$$

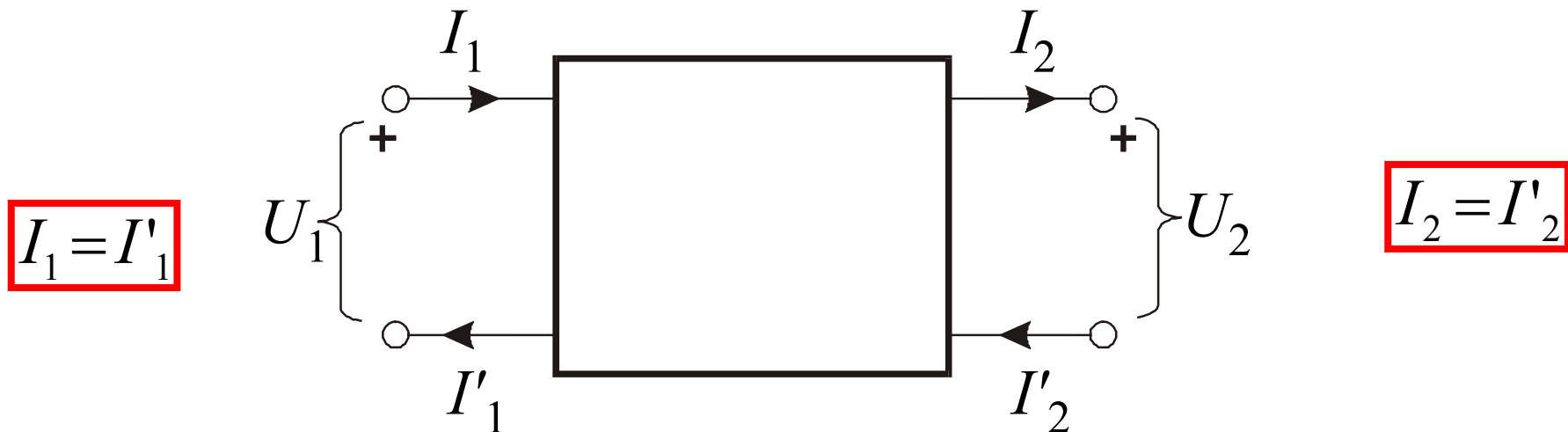
# Metode povezivanja četveropola

- Analiza složene mreže s dva pristupa može biti jednostavnija ako je prikazana  
→ kao kombinacija dva ili više jednostavnih četveropola
- Sinteza nekog složenog prijenosnog sistema može biti jednostavnija ako se sastavlja  
→ od više međusobno spojenih osnovnih četveropola.
- Složena mreža → također četveropol

Postoji 5 načina spajanja četveropola:

- serijski spoj četveropola
- paralelni spoj četveropola
- serijsko-paralelni spoj četveropola
- paralelno-serijski spoj četveropola
- spoj u lanac ili kaskadu četveropola

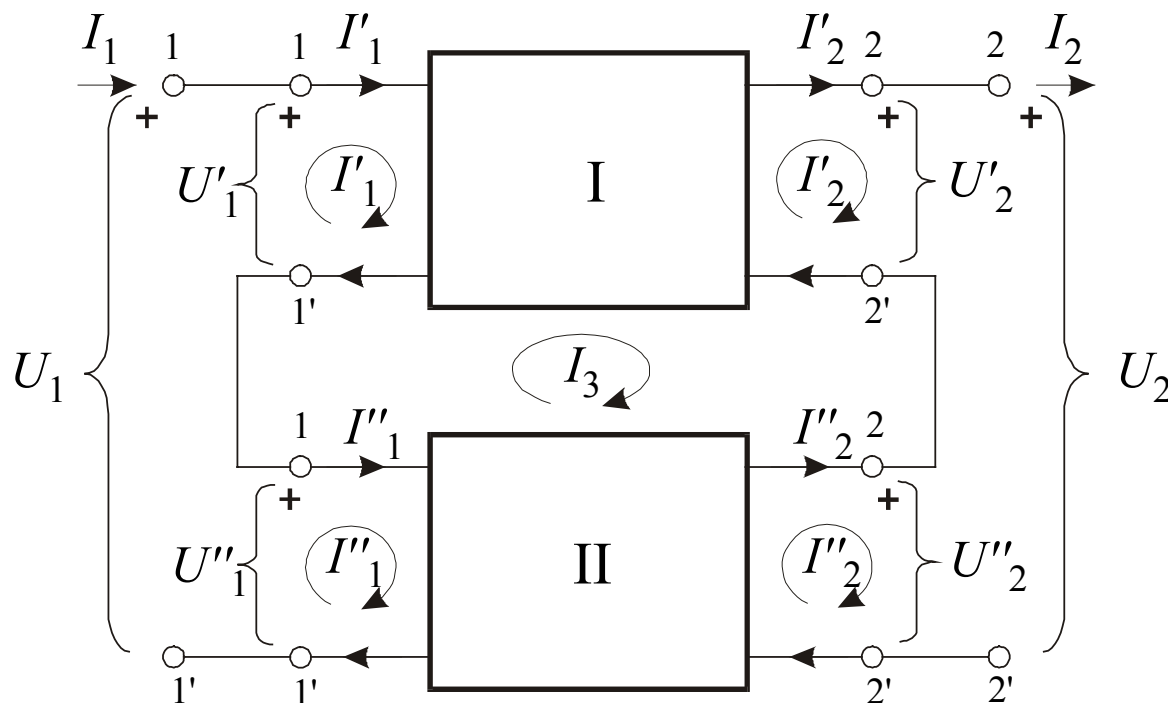
- Svrha:
- Prilikom povezivanja → parametre cijeloga sistema  
→ iz parametara pojedinih četveropola  
→ korištenjem matrica parametara
- Osnovni uvjet za korištenje operacija s matricama:
- Svaki četveropol u kombinaciji  
→ sačuvana svojstva na prilazima





## ■ Serijski spoj četveropola:

- Ulazne priključnice → spojene serijski
- Izlazne priključnice → spojene serijski



mora biti  
 $I_3=0$

Za ovaj spoj vrijedi:  $U_1 = U'_1 + U''_1$        $I_1 = I'_1 = I''_1$   
 $U_2 = U'_2 + U''_2$        $I_2 = I'_2 = I''_2$

$$\begin{bmatrix} U'_1 \\ U'_2 \end{bmatrix} = \begin{bmatrix} z'_{11} & -z'_{12} \\ z'_{21} & -z'_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \qquad \begin{bmatrix} U''_1 \\ U''_2 \end{bmatrix} = \begin{bmatrix} z''_{11} & -z''_{12} \\ z''_{21} & -z''_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \left( \begin{bmatrix} z'_{11} & -z'_{12} \\ z'_{21} & -z'_{22} \end{bmatrix} + \begin{bmatrix} z''_{11} & -z''_{12} \\ z''_{21} & -z''_{22} \end{bmatrix} \right) \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$[z] = [z'] + [z'']$$

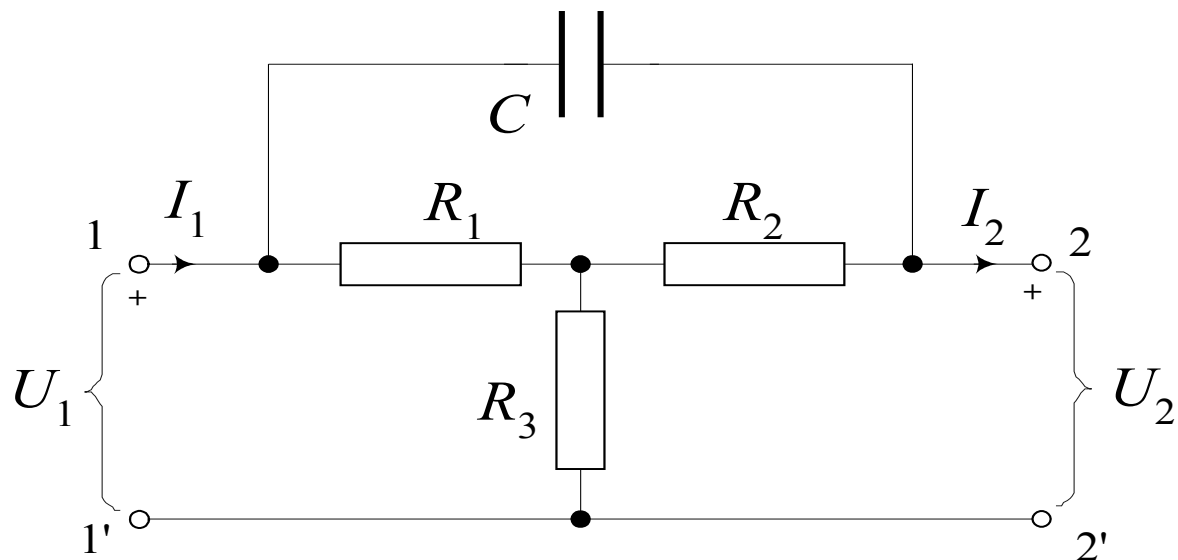
$$z_{11} = z'_{11} + z''_{11}$$

$$z_{12} = z'_{12} + z''_{12}$$

$$z_{21} = z'_{21} + z''_{21}$$

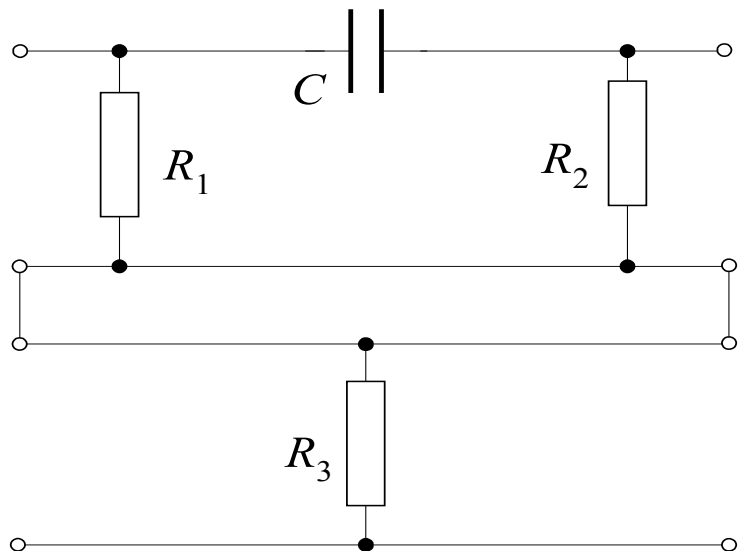
$$z_{22} = z'_{22} + z''_{22}$$

## ■ Primjer: Premošteni T-četveropol



moguće je prikazati kao serijski spoj dvaju četveropola

# Postupci povezivanja četveropola

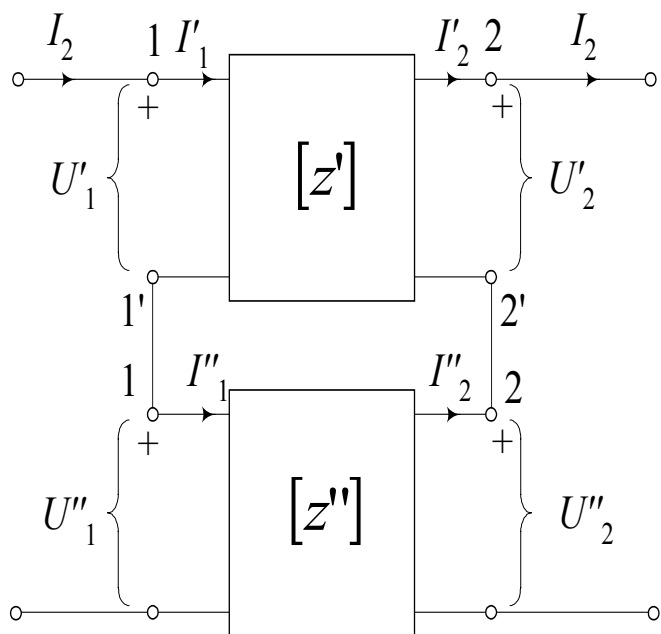


z-parametri tih četveropola su

$$z'_{11} = \frac{(sCR_2 + 1)R_1}{sC(R_1 + R_2) + 1}$$

$$z'_{12} = z'_{21} = \frac{R_1 R_2 sC}{sC(R_1 + R_2) + 1}$$

$$z'_{22} = \frac{(sCR_1 + 1)R_2}{sC(R_1 + R_2) + 1}$$



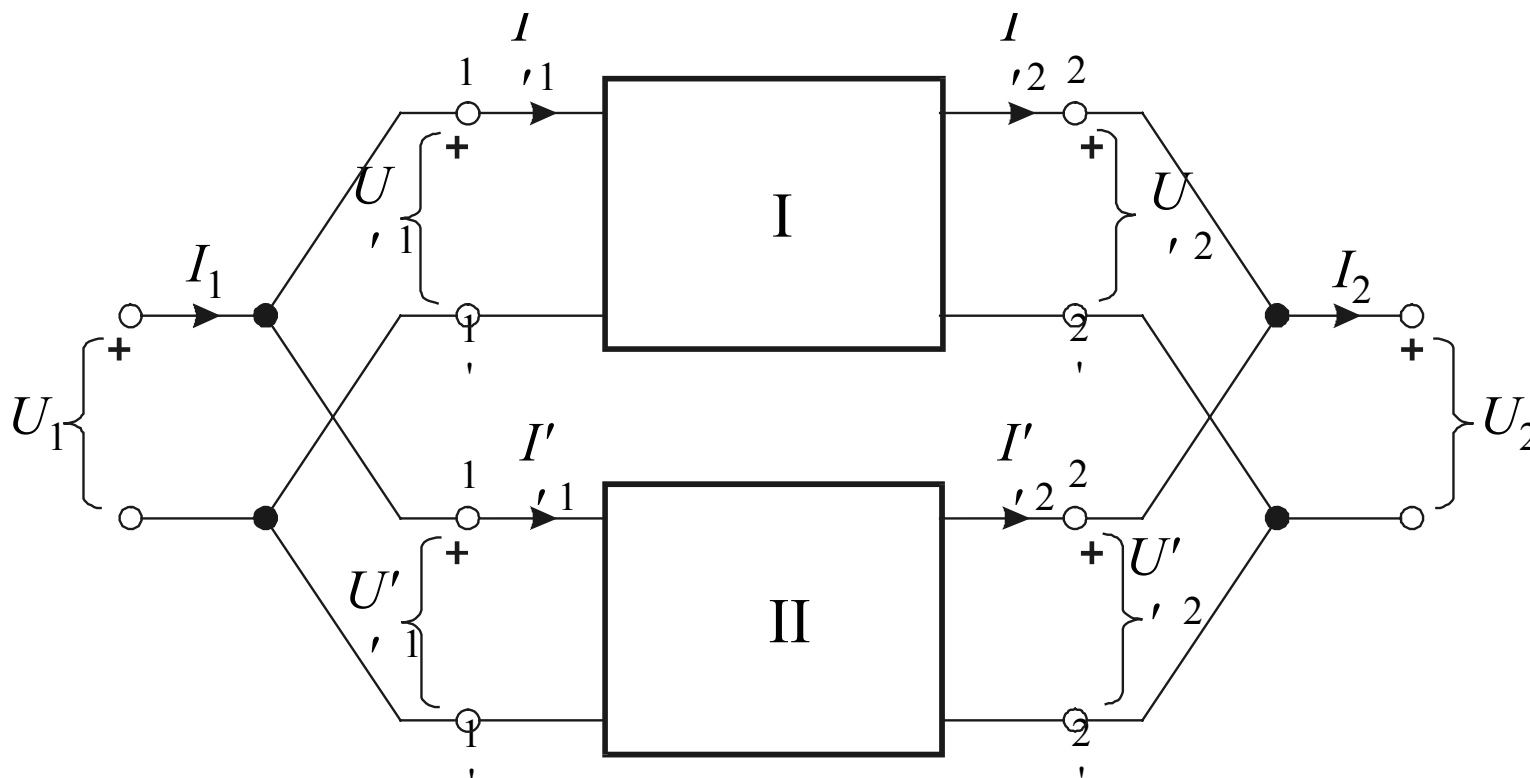
$$z''_{11} = z''_{12} = z''_{21} = z''_{22} = R_3$$

Matrica z-parametara kombinacije četveropola glasi

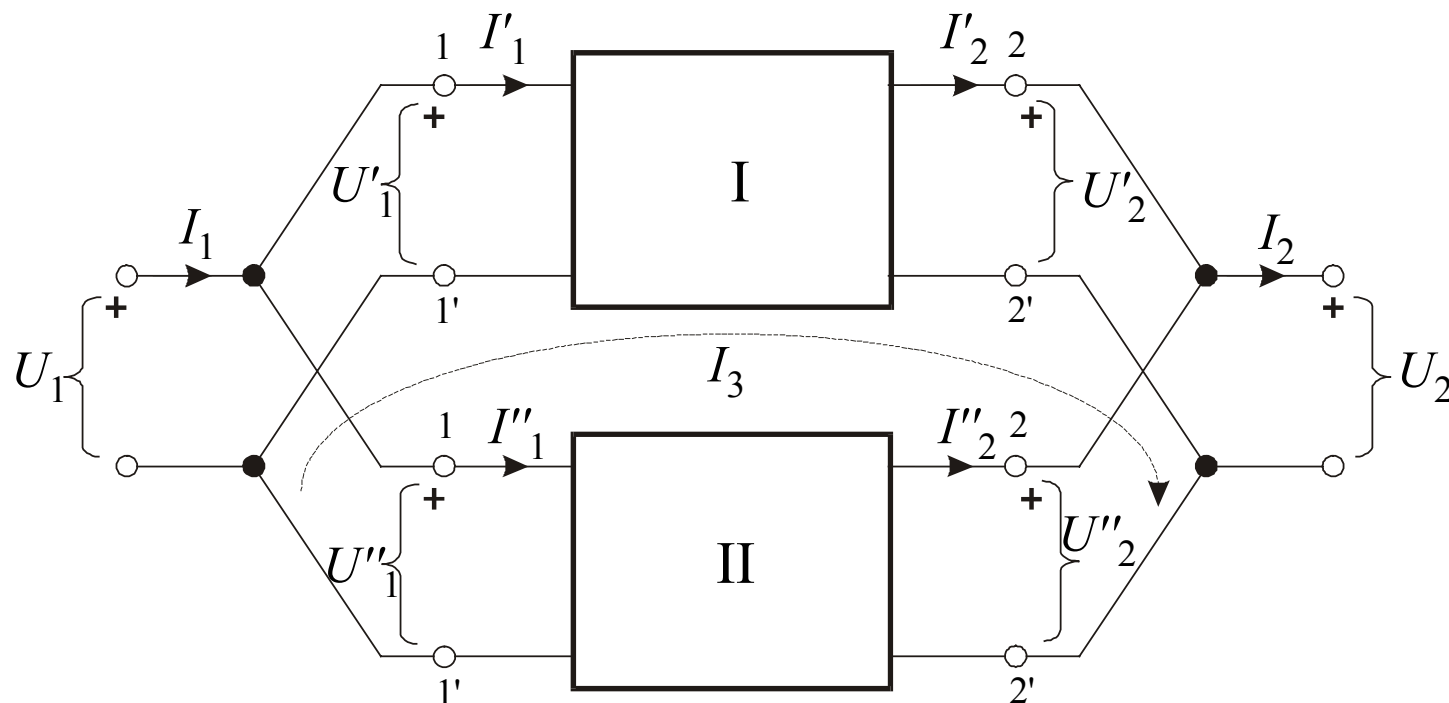
$$[z] = \begin{bmatrix} \frac{(sCR_2 + 1)R_1}{sC(R_1 + R_2) + 1} + R_3 & -\left( \frac{R_1R_2sC}{sC(R_1 + R_2) + 1} + R_3 \right) \\ \frac{R_1R_2sC}{sC(R_1 + R_2) + 1} + R_3 & -\left( \frac{(sCR_1 + 1)R_2}{sC(R_1 + R_2) + 1} + R_3 \right) \end{bmatrix}$$

## Paralelni spoj četveropola:

- Ulazne priključnice → spojene paralelno
- Izlazne priključnice → spojene paralelno



# Postupci povezivanja četveropola



Za ovaj spoj vrijedi:

$$U_1 = U'_1 = U''_1$$

$$U_2 = U'_2 = U''_2$$

$$I_1 = I'_1 + I''_1$$

$$I_2 = I'_2 + I''_2$$

mora biti

$$I_3 = 0$$

## Postupci povezivanja četveropola

$$\begin{bmatrix} I'_1 \\ I'_2 \end{bmatrix} = \begin{bmatrix} y'_{11} & -y'_{12} \\ y'_{21} & -y'_{22} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix} \quad \begin{bmatrix} I''_1 \\ I''_2 \end{bmatrix} = \begin{bmatrix} y''_{11} & -y''_{12} \\ y''_{21} & -y''_{22} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} I'_1 \\ I'_2 \end{bmatrix} + \begin{bmatrix} I''_1 \\ I''_2 \end{bmatrix} = \left( \begin{bmatrix} y'_{11} & -y'_{12} \\ y'_{21} & -y'_{22} \end{bmatrix} + \begin{bmatrix} y''_{11} & -y''_{12} \\ y''_{21} & -y''_{22} \end{bmatrix} \right) \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

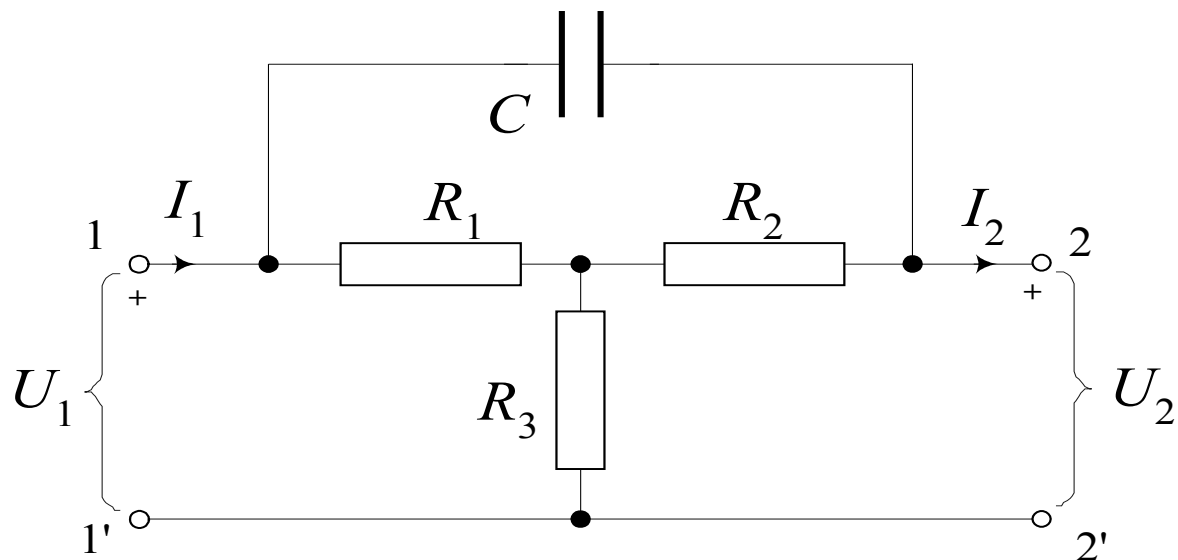
$$[y] = [y'] + [y'']$$

$$y_{11} = y'_{11} + y''_{11} \quad y_{12} = y'_{12} + y''_{12}$$

$$y_{21} = y'_{21} + y''_{21} \quad y_{22} = y'_{22} + y''_{22}$$

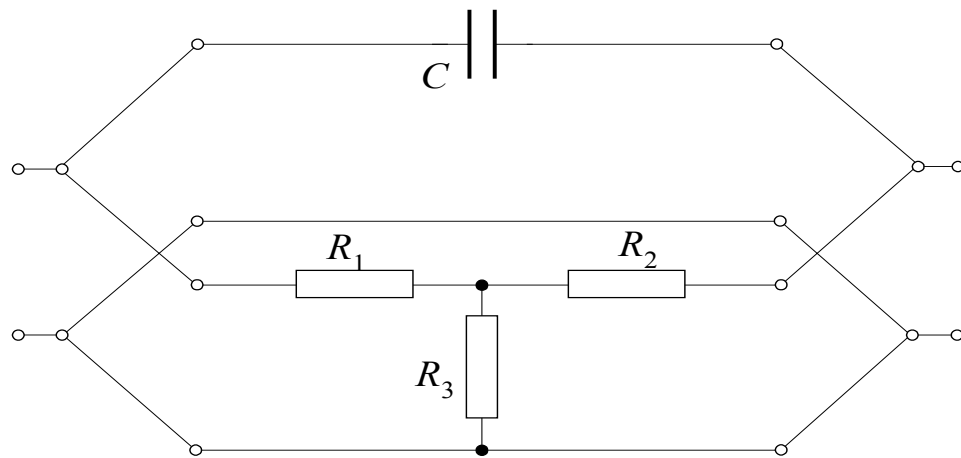


## ■ Primjer: Premošteni T-četveropol



moguće je prikazati kao paralelni spoj dvaju četveropola

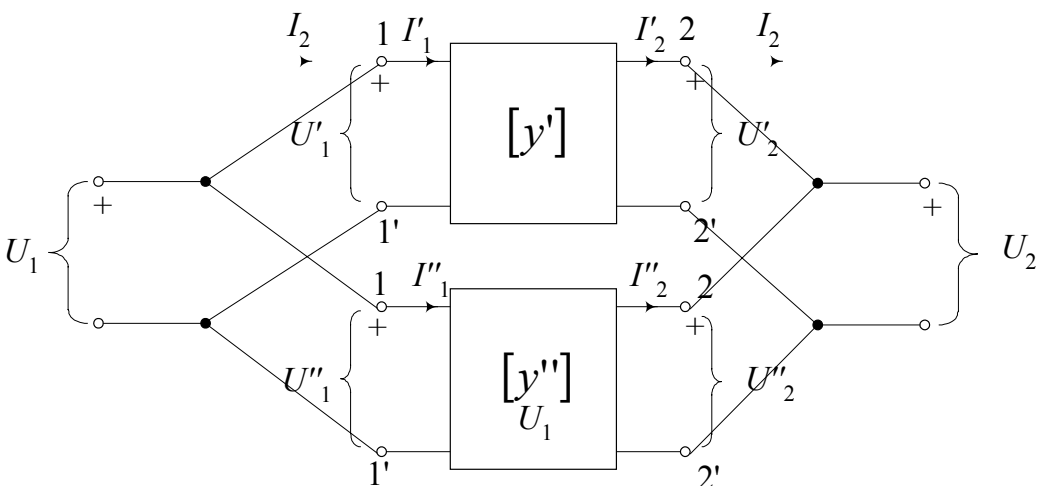
# Postupci povezivanja četveropola



$y$ -parametri tih četveropola su

$$y'_{11} = \frac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$y'_{21} = y'_{12} = \frac{R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$



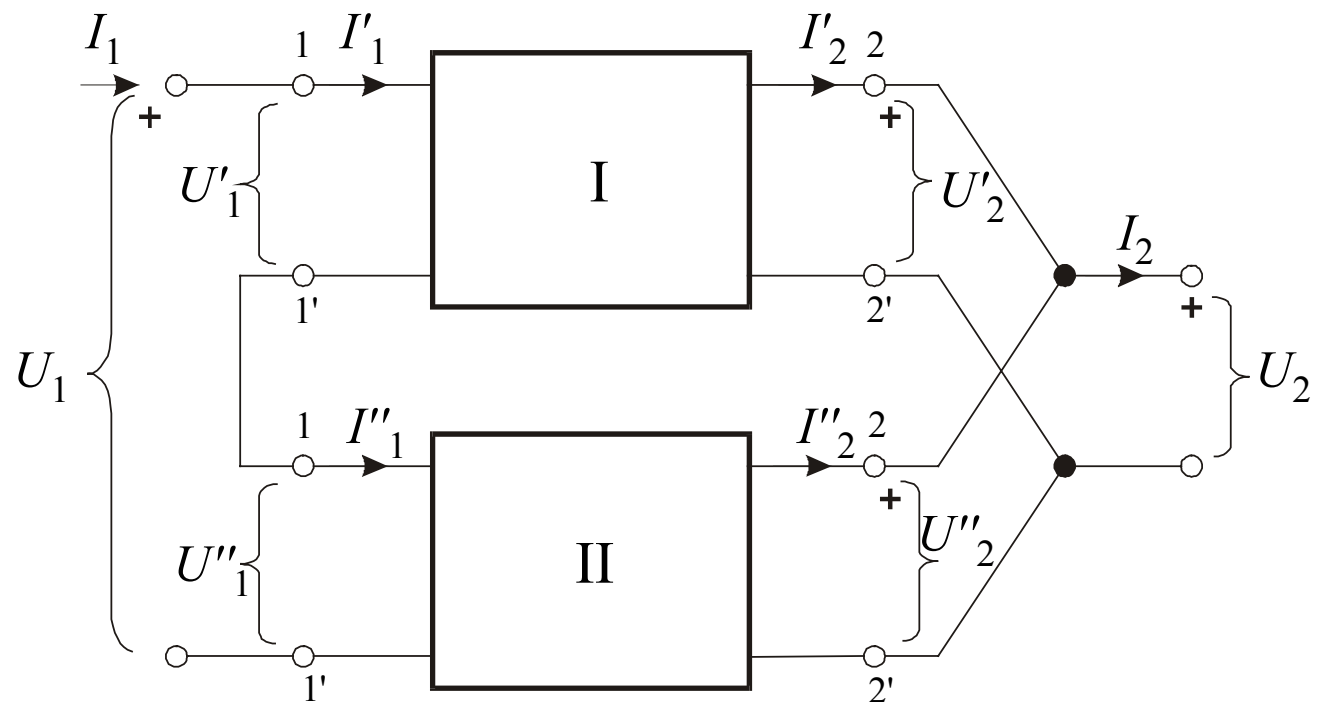
$$y'_{22} = \frac{R_1 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

$$y''_{11} = y''_{12} = y''_{21} = y''_{22} = sC$$

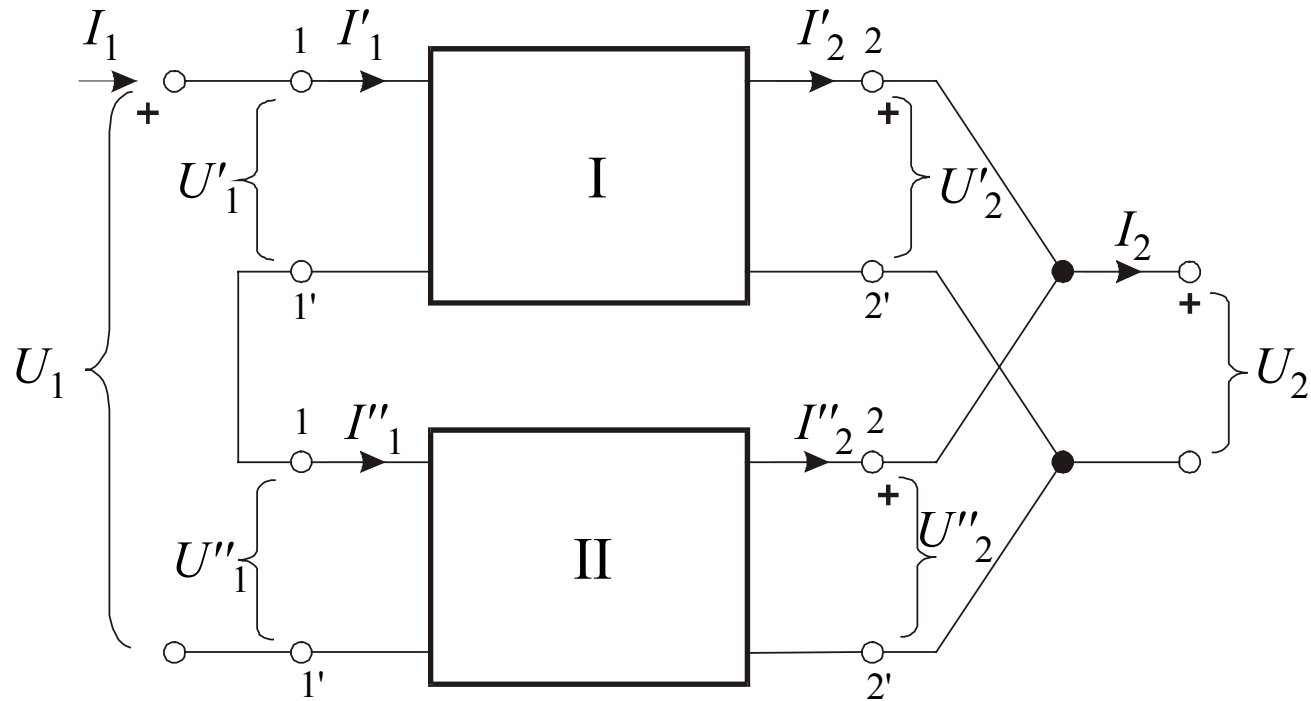
Matrica  $y$ -parametara kombinacije četveropola glasi

$$[y] = \begin{bmatrix} \frac{R_2 + R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} + sC & \frac{-R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} - sC \\ \frac{R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} + sC & \frac{-(R_1 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3} - sC \end{bmatrix}$$

- Serijsko-paralelni spoj četveropola:
- Ulazne priključnice → spojene serijski
- Izlazne priključnice → spojene paralelno



## Postupci povezivanja četveropola



Za ovaj spoj vrijedi:

$$U_1 = U'_1 + U''_1$$

$$I_1 = I'_1 = I''_1$$

$$U_2 = U'_2 + U''_2$$

$$I_2 = I'_2 + I''_2$$

## Postupci povezivanja četveropola

$$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} U'_1 \\ I'_2 \end{bmatrix} + \begin{bmatrix} U''_1 \\ I''_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} I'_1 \\ U'_2 \end{bmatrix} = \begin{bmatrix} I''_1 \\ U''_2 \end{bmatrix}$$

$$\begin{bmatrix} U'_1 \\ I'_2 \end{bmatrix} = \begin{bmatrix} h'_{11} & h'_{12} \\ h'_{21} & h'_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$$

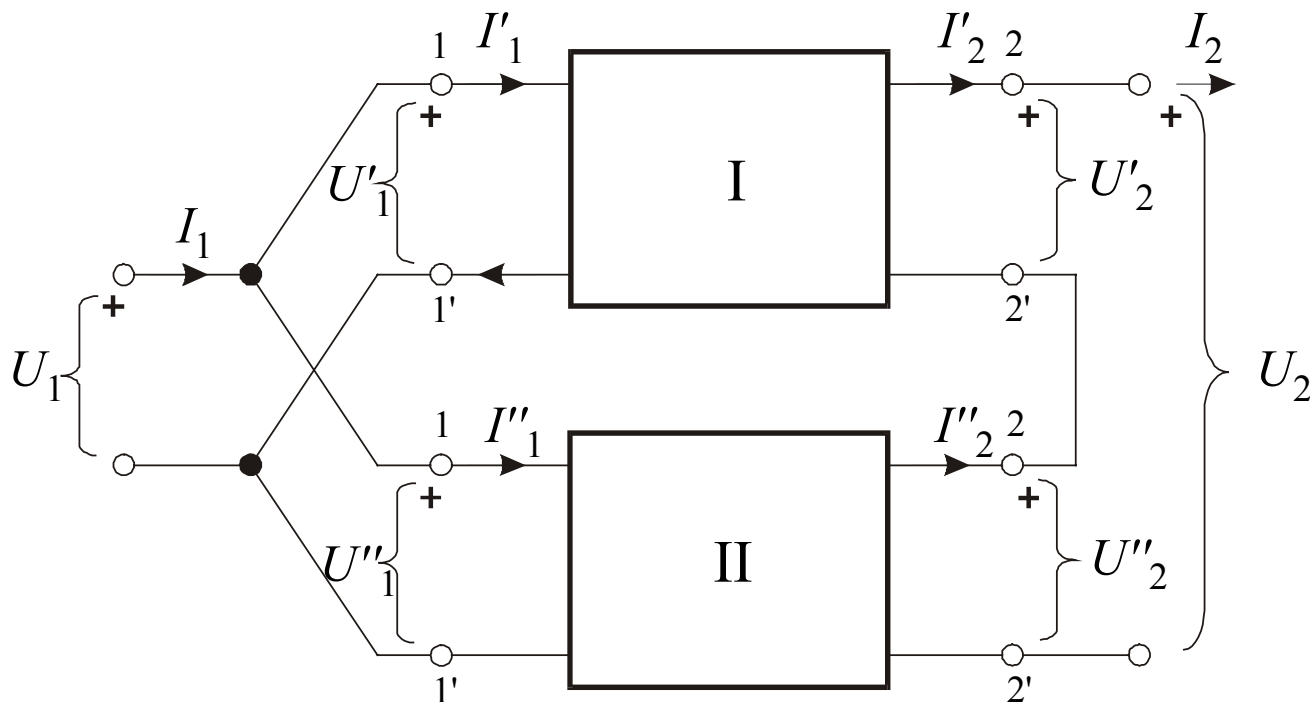
$$\begin{bmatrix} U''_1 \\ I''_2 \end{bmatrix} = \begin{bmatrix} h''_{11} & h''_{12} \\ h''_{21} & h''_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$$

$$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \left( \begin{bmatrix} h'_{11} & h'_{12} \\ h'_{21} & h'_{22} \end{bmatrix} + \begin{bmatrix} h''_{11} & h''_{12} \\ h''_{21} & h''_{22} \end{bmatrix} \right) \cdot \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$$

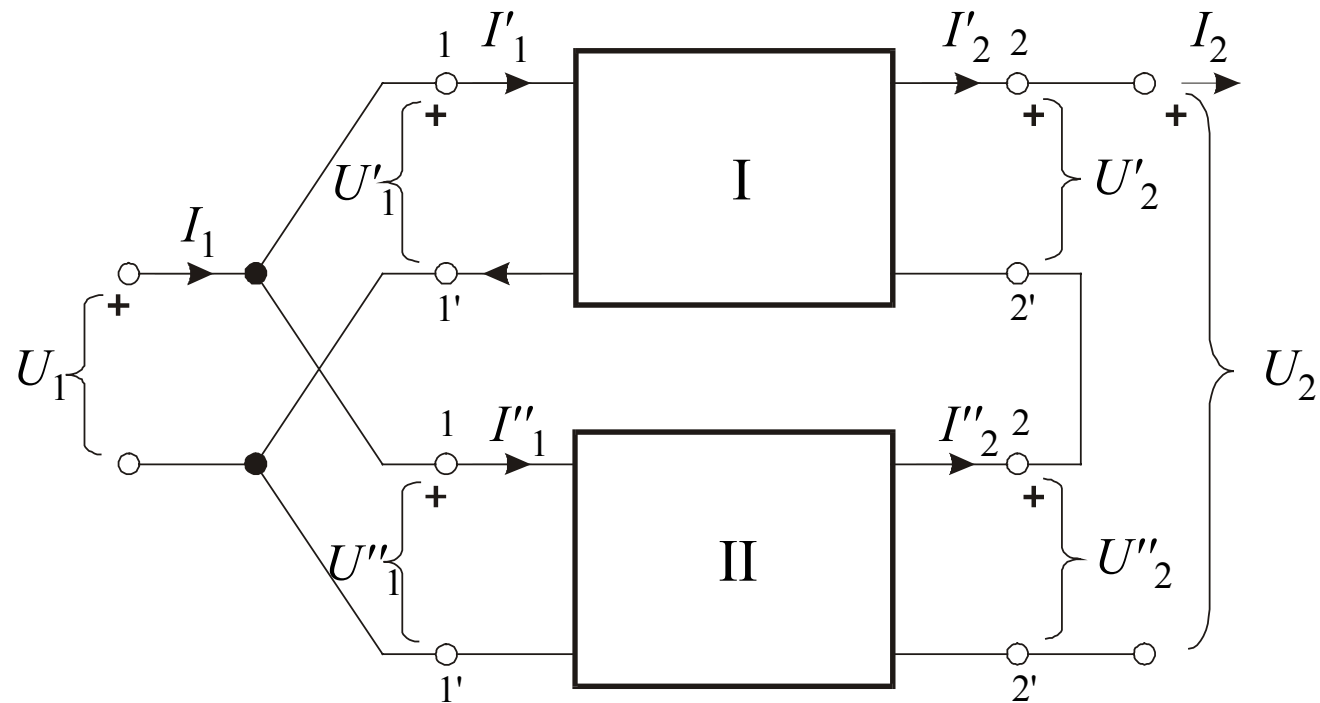
$$[h] = [h'] + [h'']$$

## Paralelno-serijski spoj četveropola:

- Ulazne priključnice → spojene paralelno
- Izlazne priključnice → spojene serijski



# Postupci povezivanja četveropola



Za ovaj spoj vrijedi:

$$U_1 = U'_1 = U''_1$$

$$I_1 = I'_1 + I''_1$$

$$U_2 = U'_2 + U''_2$$

$$I_2 = I'_2 = I''_2$$



## Postupci povezivanja četveropola

$$\begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} I'_1 \\ U'_2 \end{bmatrix} + \begin{bmatrix} I''_1 \\ U''_2 \end{bmatrix}$$

$$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} U'_1 \\ I'_2 \end{bmatrix} = \begin{bmatrix} U''_1 \\ I''_2 \end{bmatrix}$$

$$\begin{bmatrix} I'_1 \\ U'_2 \end{bmatrix} = \begin{bmatrix} g'_{11} & g'_{12} \\ g'_{21} & g'_{22} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I''_1 \\ U''_2 \end{bmatrix} = \begin{bmatrix} g''_{11} & g''_{12} \\ g''_{21} & g''_{22} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \left( \begin{bmatrix} g'_{11} & g'_{12} \\ g'_{21} & g'_{22} \end{bmatrix} + \begin{bmatrix} g''_{11} & g''_{12} \\ g''_{21} & g''_{22} \end{bmatrix} \right) \cdot \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$$

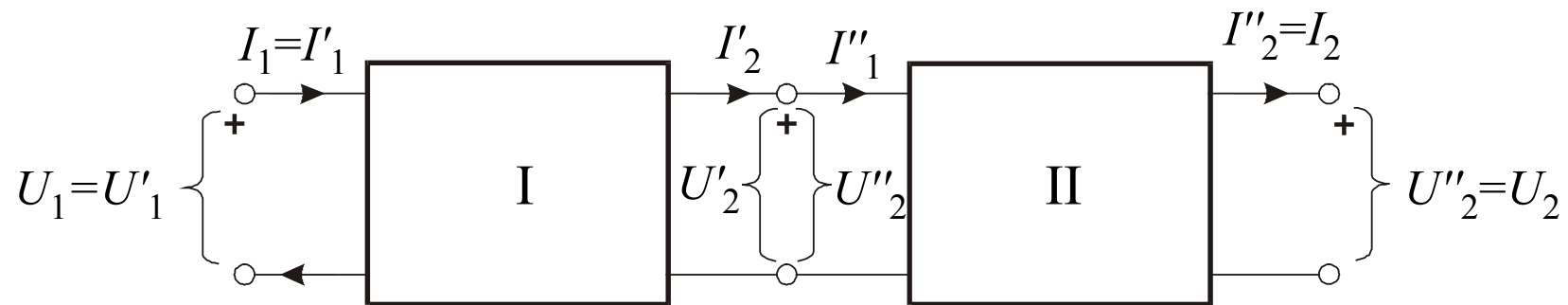
$$[g] = [g'] + [g'']$$

$$g_{11} = g'_{11} + g''_{11} \quad g_{12} = g'_{12} + g''_{12}$$

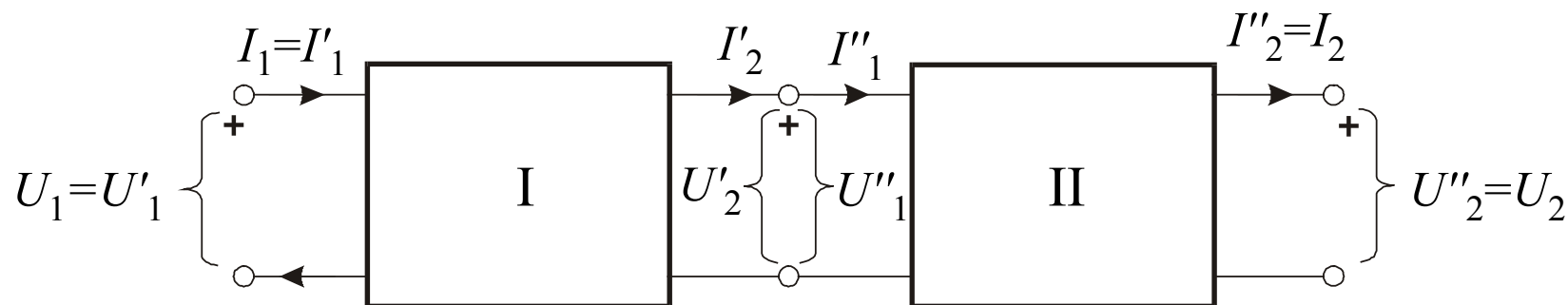
$$g_{21} = g'_{21} + g''_{21} \quad g_{22} = g'_{22} + g''_{22}$$

## Lanac ili kaskada četveropola

- Lanac dvaju četveropola:
- Izlazne priključnice prvog četveropola  
→ na ulazne priključnice drugog
- Ulaz prvoga → ulaz kombinacije
- Izlaz drugoga → izlaz kombinacije



## Postupci povezivanja četveropola



Za ovaj spoj vrijedi:

$$U_1 = U'_1$$

$$U'_2 = U''_1$$

$$U''_2 = U_2$$

$$I_1 = I'_1$$

$$I'_2 = I''_1$$

$$I''_2 = I_2$$

ili

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} U'_1 \\ I'_1 \end{bmatrix}$$

$$\begin{bmatrix} U'_2 \\ I'_2 \end{bmatrix} = \begin{bmatrix} U''_1 \\ I''_1 \end{bmatrix}$$

$$\begin{bmatrix} U''_2 \\ I''_2 \end{bmatrix} = \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$$

## Postupci povezivanja četveropola

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} U'_1 \\ I'_1 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \cdot \begin{bmatrix} U'_2 \\ I'_2 \end{bmatrix}$$

$$\begin{bmatrix} U'_2 \\ I'_2 \end{bmatrix} = \begin{bmatrix} U''_1 \\ I''_1 \end{bmatrix} = \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \cdot \begin{bmatrix} U''_2 \\ I''_2 \end{bmatrix} = \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \cdot \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$$

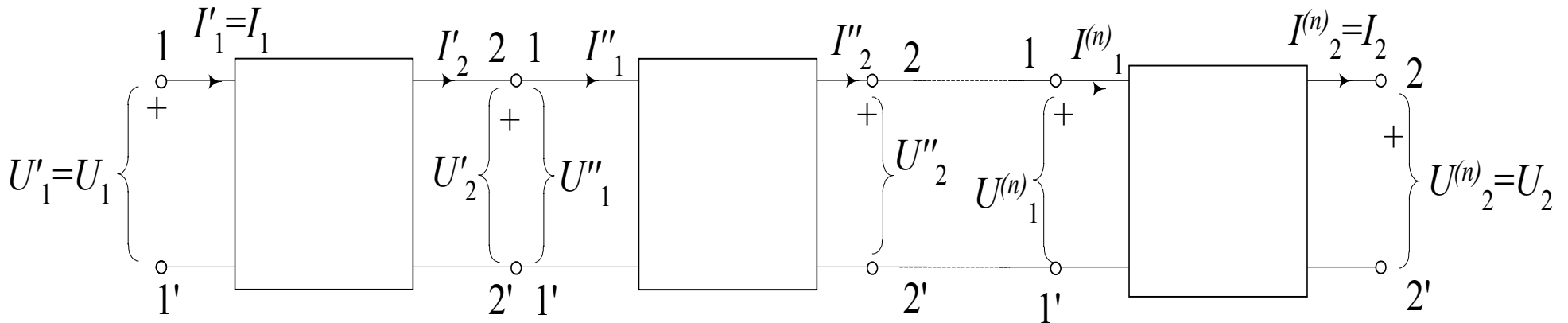
$$\begin{aligned} \begin{bmatrix} U_1 \\ I_1 \end{bmatrix} &= \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \cdot \begin{bmatrix} A'' & B'' \\ C'' & D'' \end{bmatrix} \cdot \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = \\ &= \begin{bmatrix} A'A''+B'C'' & A'B''+B'D'' \\ C'A''+D'C'' & C'B''+D'D'' \end{bmatrix} \cdot \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \end{aligned}$$

$$A = A' A'' + B' C'' \qquad B = A' B'' + B' D''$$

$$C = C' A'' + D' C'' \qquad D = C' B'' + D' D''$$

$$[a] = [a'] \cdot [a'']$$

Za N četveropola:

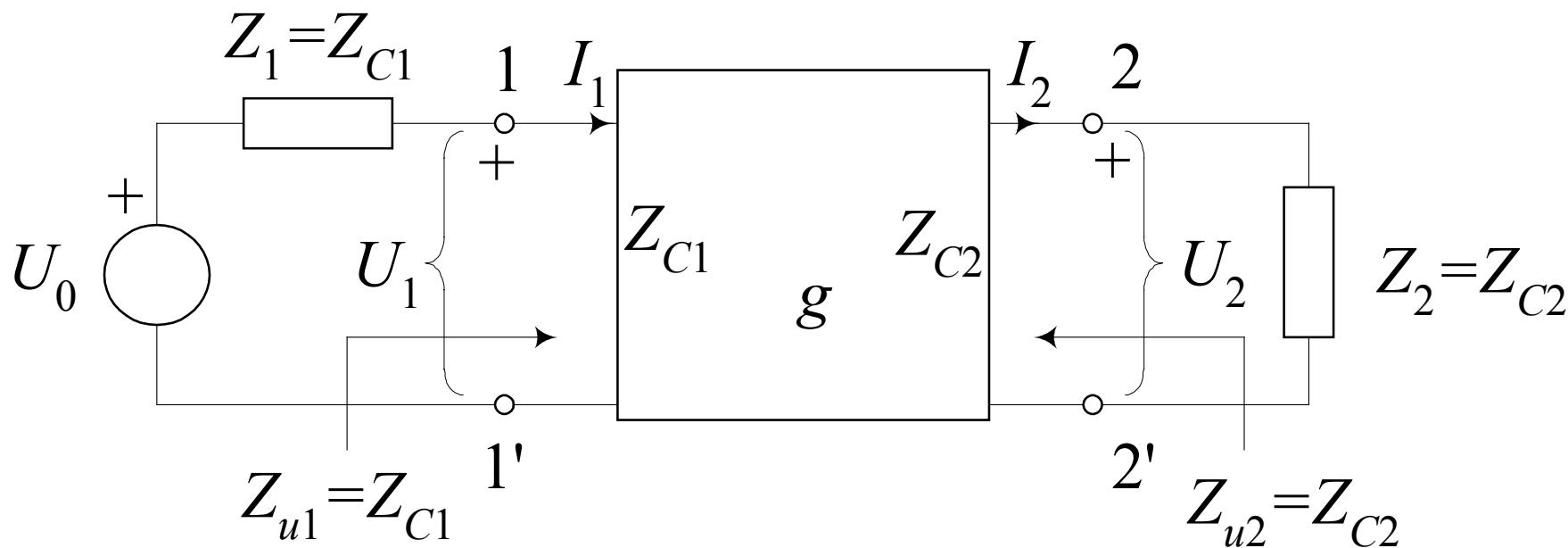


$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^I \cdot \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{II} \cdots \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{(n)}$$

# Lanac ili kaskada i zrcalni parametri

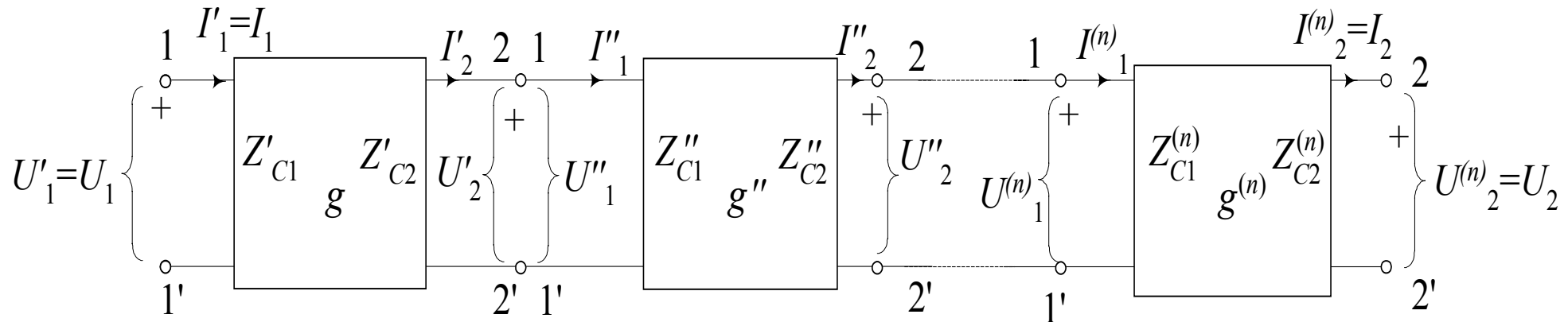
Prilagođenje po zrcalnim parametrima:

$$Z_{C_1}, Z_{C_2}, g$$



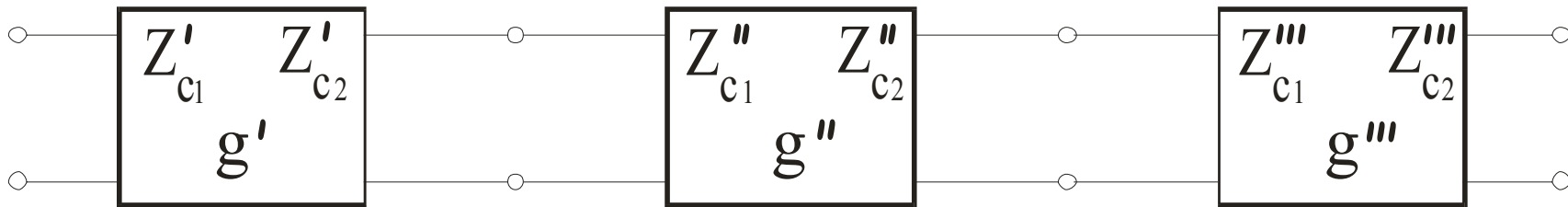
# Prilagođeni lanci

- Kod više četveropola spojenih u lanac  
 → često je potrebno ispuniti zahtjev prilagođenja





Za prilagođeni lanac mora biti



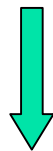
$$Z'_{c_2} = Z''_{c_1} \qquad Z''_{c_2} = Z'''_{c_1}$$

# Simetrični četveropoli

Ako je  $z_{11} = z_{22} \rightarrow$  simetričan četveropol

$$\begin{array}{c} \downarrow \\ Z_{p_1} = Z_{p_2} \end{array}$$

- Za četveropol su dovoljna samo 2 parametra
- Također slijedi iz:  $[y] = [z]^{-1}$



$$\boxed{y_{11} = y_{22}} \quad \text{tj.} \quad Z_{k_1} = Z_{k_2}$$

- Uvjet simetrije izražen ostalim parametrima

- Prijenosni parametri:  $A = D$

- Hibridni  $h$ -parametri: 
$$\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = -1$$

- Hibridni  $g$ -parametri: 
$$\begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = 1$$

■ Zrcalni parametri:

$$Z_{C_1} = Z_{C_2} = Z_C = \sqrt{\frac{B}{C}}$$

$$g = \ln(\sqrt{AD} + \sqrt{BC}) = \ln(A + \sqrt{BC})$$

pošto je četveropol recipročan  $\longrightarrow AD - BC = 1$

$$BC = AD - 1 \quad \longrightarrow \quad BC = A^2 - 1$$

pa je za simetrične i recipročne četveropole

$$g = \ln(A + \sqrt{A^2 - 1})$$

Prijenosne jednačbe glase:  $U_1 = n(U_2 ch(g) + I_2 Z_{C_2} sh(g))$

$$I_1 = \frac{1}{n} \left( \frac{U_2 sh(g)}{Z_{C_2}} + I_2 ch(g) \right)$$

Pošto je:  $n = \sqrt{\frac{Z_{C_1}}{Z_{C_2}}} = 1$   $Z_{C_2} = Z_C$

$$U_1 = U_2 ch g + I_2 Z_C sh g$$

$$I_1 = \frac{U_2}{Z_C} sh g + I_2 ch g$$