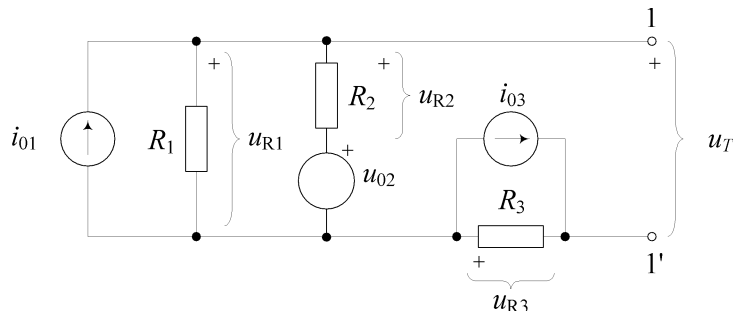
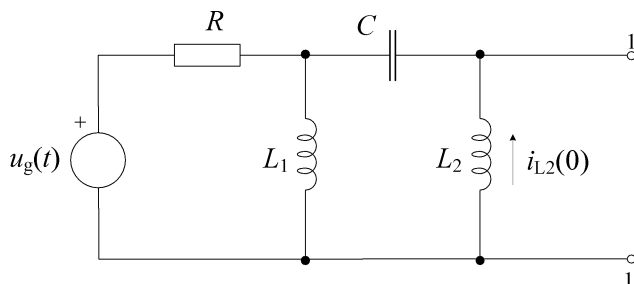


## PRVI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA 2009/10

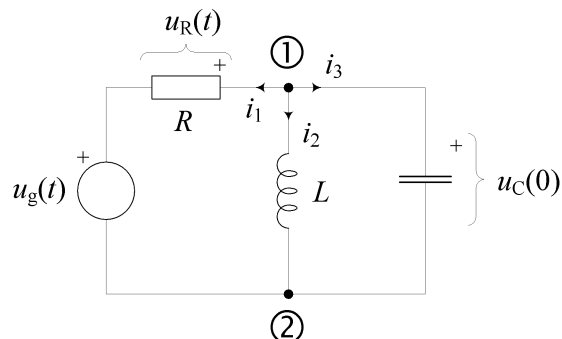
1. Za električni krug na slici odrediti:  
 a) Theveninov napon  $u_T(t)$ ; b) Theveninov otpor  $R_T$ ; c) napon na otporu  $R_3$ ; d) napon na otporu  $R_2$  i e) napon na otporu  $R_1$ . Zadano je:  $R_1 = 5\text{k}\Omega$ ,  $R_2 = 10\text{k}\Omega$ ,  $R_3 = 10\text{k}\Omega$ ,  $i_{01} = 3\text{mA}$ ,  $u_{02} = 3\text{V}$ ,  $i_{03} = 1\text{mA}$ .



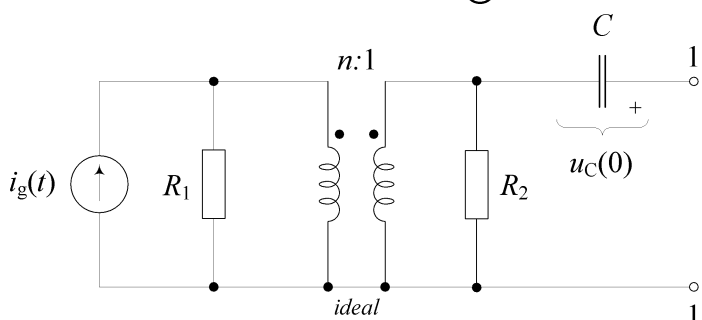
2. Za električni krug na slici pomoću teorema superpozicije odrediti parametre nadomjesnog kruga po Nortonu s obzirom na priključnice 1-1':  
 a)  $I_N(s)$ ; b)  $Y_N(s)$ . Zadane su normalizirane vrijednosti elemenata:  $R = 1$ ,  $L_1 = L_2 = 1$ ,  $C = 2$  te  $i_{L2}(0) = 1$ ,  $u_g(t) = S(t)$ . Koristiti jednačbe čvorova.



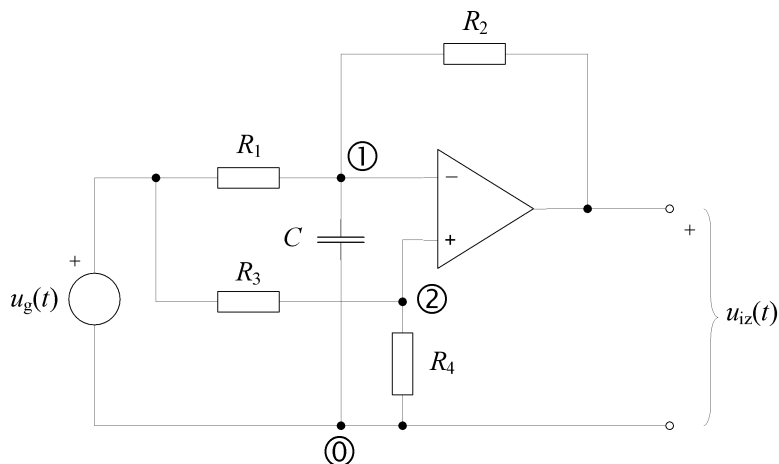
3. Za električni krug na slici izračunati napon  $U_R(s)$  ako su zadane normalizirane vrijednosti elemenata  $R=1$ ,  $L=1$ ,  $C=1$ ,  $u_C(0) = 2$ , te  $u_g(t) = S(t)$ . Koristiti KZS i KZN te oznake grana i čvorova prema slici.



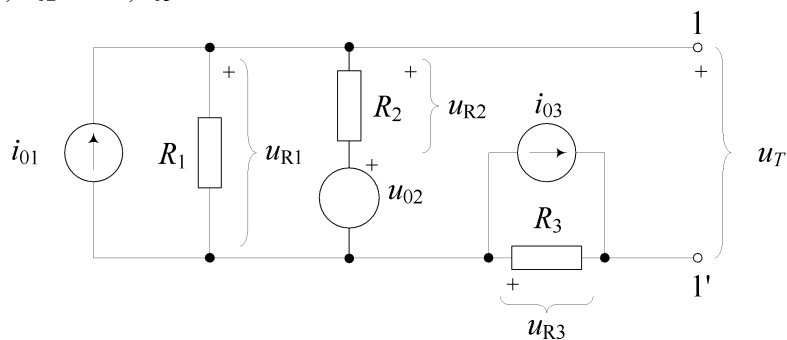
4. Za električni krug na slici izračunati parametre nadomjesnog kruga po Theveninu s obzirom na stezaljke 1-1':  
 a)  $U_T(s)$  i b)  $Z_T(s)$ . Zadane su normalizirane vrijednosti elemenata:  $R_1 = R_2 = 1$ ,  $C = 1$ ,  $u_C(0) = 1$ ,  $n = 2$ ,  $i_g(t) = S(t)$ . Koristiti jednačbe petlji.



5. Zadan je električni krug prema slici. Odrediti napon na izlazu  $u_{iz}(t)$  ako je zadano:  $R_1 = R_2 = R_3 = 1$ ,  $R_4 = 2$ ,  $C = 1$ ,  $u_g(t) = S(t)$ . Početni uvjeti su jednaki nuli. Koristiti metodu jednačbi čvorova.



1. Za električni krug na slici odrediti: a) Theveninov napon  $u_T(t)$ ; b) Theveninov otpor  $R_T$ ; c) napon na otporu  $R_3$ ; d) napon na otporu  $R_2$  i e) napon na otporu  $R_1$ . Zadano je:  $R_1 = 5\text{k}\Omega$ ,  $R_2 = 10\text{k}\Omega$ ,  $R_3 = 10\text{k}\Omega$ ,  $i_{01} = 3\text{mA}$ ,  $u_{02} = 3\text{V}$ ,  $i_{03} = 1\text{mA}$ .



Rješenje:

$$u_T = \frac{i_{01}R_2 + u_{02}}{R_1 + R_2} R_1 - i_{03}R_3 = i_{01} \cdot 3,333 \cdot 10^{-3} + u_{02} \cdot 0,333 - i_{03} \cdot 10^{-4} \quad [1 \text{ bod}]$$

$$u_T = 10\text{V} + 1\text{V} - 10\text{V} = 1\text{V}$$

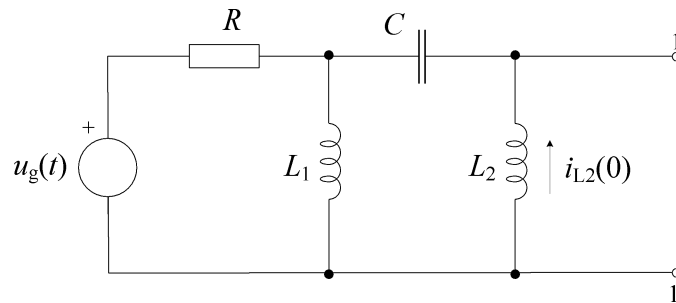
$$R_T = \frac{R_1 R_2}{R_1 + R_2} + R_3 = \frac{5 \cdot 10}{5 + 10} + 10 = 13,333\text{k}\Omega \quad [1 \text{ bod}]$$

$$u_{R1} = u_T - u_{R3} = 1 - (-10) = 11\text{V} \quad [1 \text{ bod}]$$

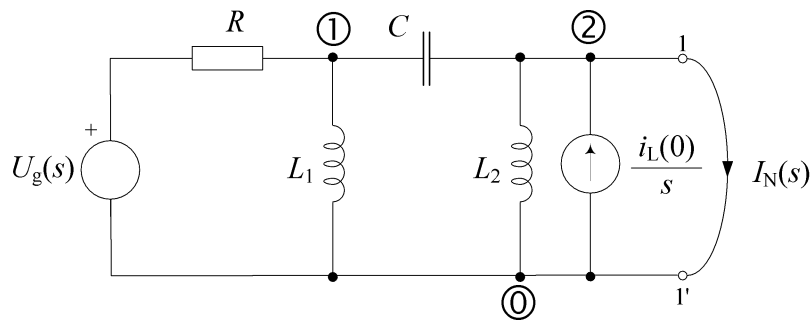
$$u_{R2} = u_T - u_{R3} - u_{02} = 11 - 3 = 8\text{V} \quad [1 \text{ bod}]$$

$$u_{R3} = -i_{03} \cdot R_3 = -10^{-3} \cdot 10 \cdot 10^3 = -10\text{V} \quad [1 \text{ bod}]$$

2. Za električni krug na slici pomoću teorema superpozicije odrediti parametre nadomjesnog kruga po Nortonu s obzirom na priključnice 1-1': a)  $I_N(s)$ ; b)  $Y_N(s)$ . Zadane su normalizirane vrijednosti elemenata:  $R = 1$ ,  $L_1 = L_2 = 1$ ,  $C = 2$  te  $i_{L2}(0) = 1$ ,  $u_g(t) = S(t)$ . Koristiti jednačbe čvorova.

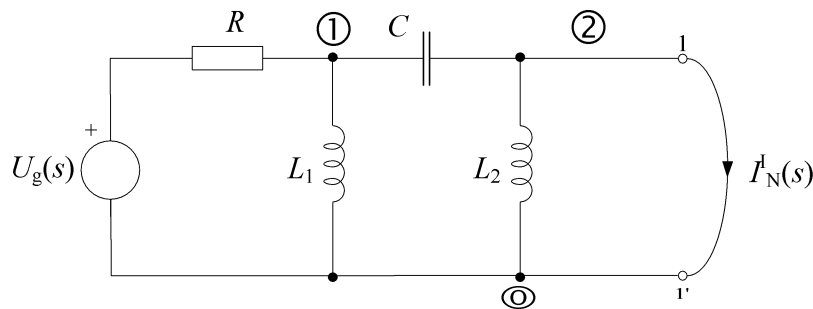


Rješenje: Primjena  $\mathcal{L}$ -transformacije na električni krug:



a) Nortonova struja  $I_N(s)$ :

a.1) početni uvjet je isključen i traži se komponenta Nortonove struje  $I_N^I(s)$ .



$$(1) \quad \frac{U_g(s)}{R} = U_1 \left( \frac{1}{R} + \frac{1}{sL_1} + sC \right) - U_2 sC$$

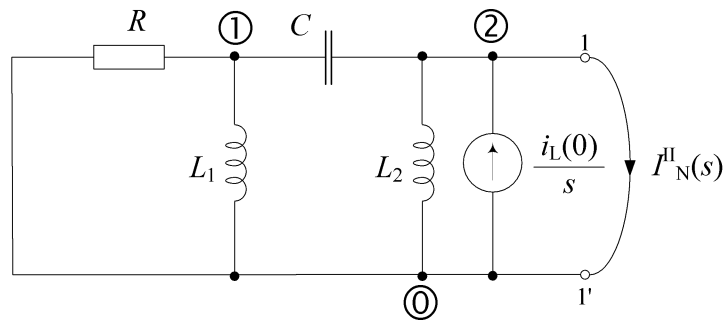
$$(2) \quad -I_N^I(s) = -U_1 sC + U_2 \left( \frac{1}{sL_2} + sC \right)$$

---


$$U_2 = 0$$

$$(1) \Rightarrow U_1 = \frac{U_g(s)}{R \left( \frac{1}{R} + \frac{1}{sL_1} + sC \right)} ; (2) \Rightarrow I_N^I(s) = \frac{U_g(s)}{R \left( \frac{1}{R} + \frac{1}{sL_1} + sC \right)} sC \quad [1 \text{ bod}]$$

a.2) naponski izvor je isključen i traži se komponenta Nortonove struje  $I_N^{\text{II}}(s)$ .



$$(1) \quad 0 = U_1 \left( \frac{1}{R} + \frac{1}{sL_1} + sC \right) - U_2 sC$$

$$(2) \quad \frac{i_{L2}(0)}{s} - I_N^{\text{II}}(s) = -U_1 sC + U_2 \left( \frac{1}{sL_2} + sC \right)$$

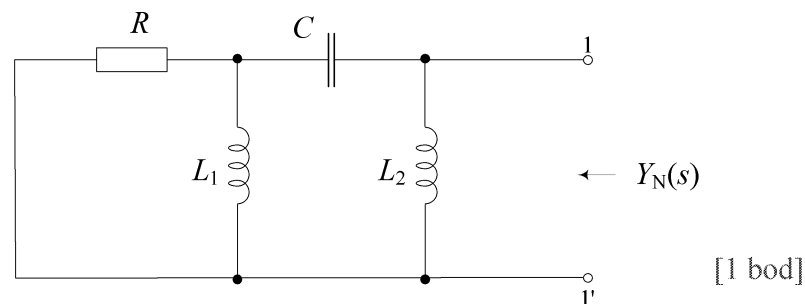
$$U_2 = 0 \Rightarrow U_1 = 0$$

$$(2) \Rightarrow I_N^{\text{II}}(s) = \frac{i_{L2}(0)}{s} \quad [1 \text{ bod}]$$

Ukupna Nortonova struja je:

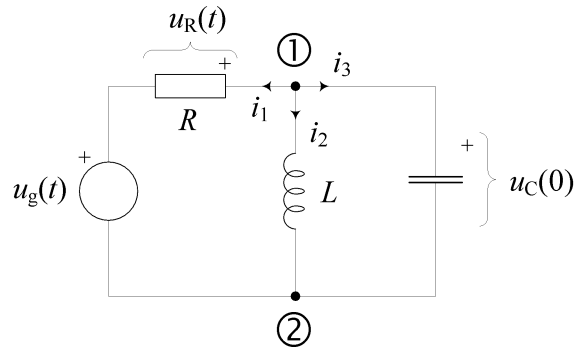
$$I_N(s) = I_N^{\text{I}}(s) + I_N^{\text{II}}(s) = \frac{U_g(s)}{R \left( \frac{1}{R} + \frac{1}{sL_1} + sC \right)} sC + \frac{i_{L2}(0)}{s} = \frac{\frac{1}{s} 2s}{1 \left( 1 + \frac{1}{s} + 2s \right)} + \frac{1}{s} = \frac{4s^2 + s + 1}{s(2s^2 + s + 1)} \quad [1 \text{ bod}]$$

b) Nortonova admitancija  $Y_N(s)$ :

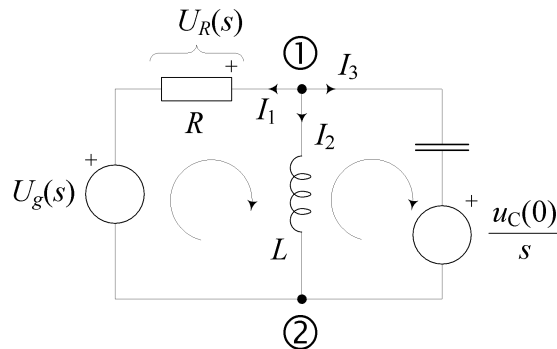


$$Y_N(s) = \frac{1}{sL_2} + \frac{1}{\frac{1}{sC} + \frac{RsL_1}{R + sL_1}} = \frac{2s^3 + 4s^2 + s + 1}{s(2s^2 + s + 1)} \quad [1 \text{ bod}]$$

3. Za električni krug na slici izračunati napon  $U_R(s)$  ako su zadane normalizirane vrijednosti elemenata  $R = 1, L = 1, C = 1, u_C(0) = 2$ , te  $u_g(t) = S(t)$ . Koristiti KZS i KZN te oznake grana i čvorova prema slici.



Rješenje: Primjena  $\mathcal{L}$ -transformacije



$$N_b = 3 \quad (\text{broj grana})$$

$$N_v = 2 \quad (\text{broj čvorova})$$

$$\text{Broj jednađžbi KZS} = N_v - 1 = 2 - 1 = 1$$

$$\text{Broj jednađžbi KZN} = N_b - N_v + 1 = 3 - 2 + 1 = 2$$

Jednađžbe Kirchhoffovih zakona (3 jednađžbe):

$$1) I_1 + I_2 + I_3 = 0 \quad \text{KZS}$$

$$2) -U_1 + U_2 = 0 \quad \text{KZN}$$

$$3) -U_2 + U_3 = 0 \quad \text{KZN} \quad [1 \text{ bod}]$$

Naponsko – strujne jednađžbe grana (3 jednađžbe):

$$4) U_1 = I_1 \cdot R + U_g$$

$$5) U_2 = I_2 \cdot sL$$

$$6) U_3 = I_3 \cdot \frac{1}{sC} + \frac{u_C(0)}{s} \quad [1 \text{ bod}]$$

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Sustav ima ukupno  $2N_b=6$  jednađžbi i 6 nepoznanica (sve struje i svi naponi grana)

Naponsko – strujne jednačbe grana uvrstimo u (1) jednačbu:

$$\left. \begin{array}{l} 4) I_1 = \frac{U_1}{R} - \frac{U_g}{R} \\ 5) I_2 = \frac{1}{sL} \cdot U_2 \\ 6) I_3 = sC \cdot U_3 - C \cdot u_c(0) \end{array} \right\} \rightarrow (1)$$

$$1) \frac{U_1}{R} - \frac{U_g}{R} + \frac{1}{sL} \cdot U_2 + sC \cdot U_3 - C \cdot u_c(0) = 0 \Rightarrow \frac{U_1}{R} + \frac{1}{sL} \cdot U_2 + sC \cdot U_3 = \frac{U_g}{R} + C \cdot u_c(0)$$

$$2) U_1 = U_2 \rightarrow (1)$$

$$3) U_2 = U_3 \rightarrow (1) \Rightarrow U_1 \left( \frac{1}{R} + \frac{1}{sL} + sC \right) = \frac{U_g}{R} + C \cdot u_c(0)$$


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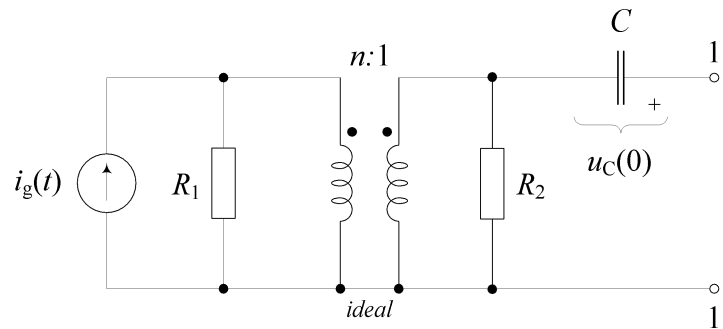
$$\Rightarrow U_1 = \frac{\frac{U_g}{R} + C \cdot u_c(0)}{\left( \frac{1}{R} + \frac{1}{sL} + sC \right)} \quad [1 \text{ bod}]$$

$$I_1 = \frac{U_1}{R} - \frac{U_g}{R} = \frac{\frac{U_g}{R} + C \cdot u_c(0)}{R \left( \frac{1}{R} + \frac{1}{sL} + sC \right)} - \frac{U_g}{R} \quad [1 \text{ bod}]$$

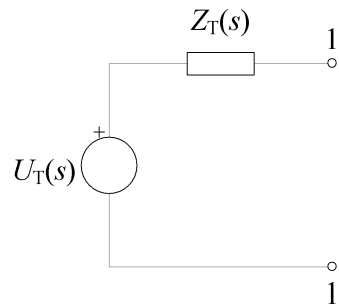
$$= \frac{\frac{1}{s} + 2}{1 + \frac{1}{s} + s} - \frac{1}{s} = \frac{1 + 2s}{1 + s + s^2} - \frac{1}{s} = \frac{s + 2s^2 - 1 - s - s^2}{s(1 + s + s^2)} = \frac{s^2 - 1}{s(1 + s + s^2)}$$

$$U_{R1}(s) = I_1 \cdot R = \frac{s^2 - 1}{s(s^2 + s + 1)} \quad [1 \text{ bod}]$$

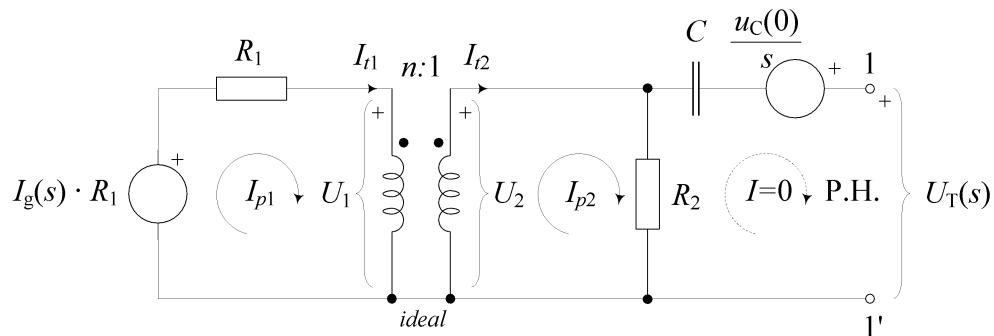
4. Za električni krug na slici izračunati nadomjesne parametre nadomjesnog kruga po Theveninu s obzirom na stezaljke 1 – 1': a)  $U_T(s)$  i b)  $Z_T(s)$ . Zadane su normalizirane vrijednosti elemenata:  $R_1 = R_2 = 1$ ,  $C = 1$ ,  $u_C(0) = 1$ ,  $n = 2$ ,  $i_g(t) = S(t)$ . Koristiti metodu petlji.



Rješenje:



- a) Theveninov napon  $U_T(s)$  primjenom  $\mathcal{L}$ -transformacije na električni krug:



Jednadžbe idealnog transformatora:

$$U_1 = n \cdot U_2 \Rightarrow U_2 = \frac{U_1}{n}$$

$$I_{t1} = \frac{1}{n} \cdot I_{t2} \Rightarrow I_{t2} = n \cdot I_{t1}$$

$$1) \ I_{p1} R_1 = -U_1 + I_g R_1 \quad I_{p1} = I_{t1}$$

$$2) \ I_{p2} R_2 = U_2 \quad I_{p2} = I_{t2} \quad [1 \text{ bod}]$$

Nakon sređivanja jednadžbe glase:

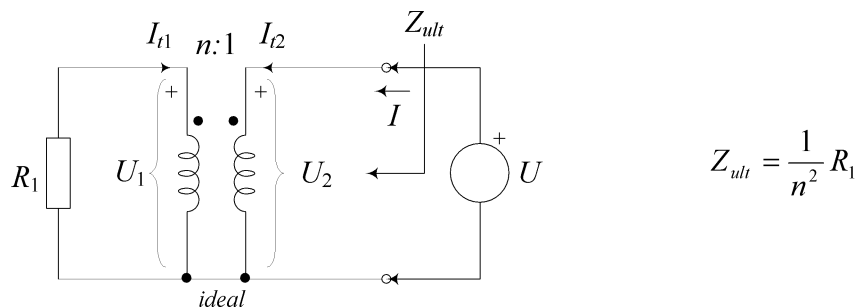
$$\left. \begin{array}{l} 1) \ I_{p1} R_1 + n U_2 = I_g R_1 \\ 2) \ -n I_{p1} R_2 + U_2 = 0 \end{array} \right\} \left. \begin{array}{l} I_{p1} R_1 + n U_2 = I_g R_1 \\ I_{p1} = \frac{U_2}{n R_2} \end{array} \right\} \frac{U_2}{n R_2} R_1 + n U_2 = I_g R_1 \left. \right\}$$

$$U_2 \left( \frac{1}{nR_2} R_1 + n \right) = I_g R_1 \quad \left. \vphantom{U_2} \right\} \quad U_2 = \frac{I_g R_1}{\frac{n^2 R_2 + R_1}{nR_2}} = I_g \cdot \frac{nR_1 R_2}{n^2 R_2 + R_1} \quad [1 \text{ bod}]$$

$$U_T(s) = U_2(s) + \frac{u_C(0)}{s} = I_g \cdot \frac{nR_1 R_2}{n^2 R_2 + R_1} + \frac{u_C(0)}{s} = \frac{1}{s} \cdot \frac{2}{5} + \frac{1}{s} = \frac{1}{s} \cdot \frac{7}{5} \quad [1 \text{ bod}]$$

b) Theveninova impedancija  $Z_T(s)$ :

1. način (pojednostavljen): Najprije izračunati ulaznu impedanciju u transformator zaključen s  $R_1$ . Označimo je s  $Z_{ult}$ .



$$I_{t1} = -\frac{U_1}{R_1} \Rightarrow \frac{U_1}{I_{t1}} = -R_1$$

$$U = U_2$$

$$I = I_{t2}$$

Jednadžbe od transformatora su (obratiti pažnju na referentno usmjerenje  $I_{t2}$ ):

$$U_1 = n \cdot U_2 \Rightarrow U_2 = \frac{U_1}{n}$$

$$I_{t1} = -\frac{1}{n} \cdot I_{t2} \Rightarrow I_{t2} = -n \cdot I_{t1}$$

$$Z_{ult} = \frac{U}{I} = \frac{U_2}{I_{t2}} = \frac{\frac{U_1}{n}}{-n \cdot I_{t1}} = -\frac{\frac{U_1}{I_{t1}}}{n^2} = -\frac{-R_1}{n^2} = \frac{R_1}{n^2}$$

Tada je Teveninova impedancija:

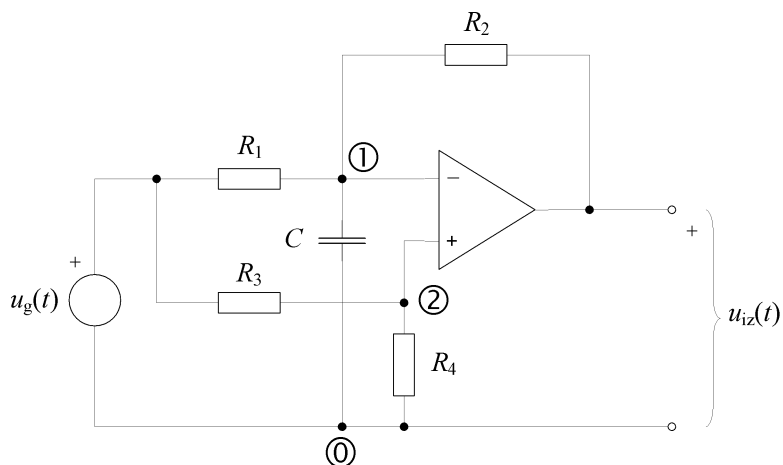
$$\leftarrow Z_T(s) = \frac{1}{sC} + R_2 \parallel Z_{ult} \quad [1 \text{ bod}]$$

$$Z_T(s) = \frac{1}{sC} + \frac{\frac{R_1}{n^2} \cdot R_2}{\frac{R_1}{n^2} + R_2} = \frac{1}{s} + \frac{\frac{1}{4}}{\frac{1}{4} + 1} = \frac{1}{s} + \frac{\frac{1}{4}}{\frac{5}{4}} = \frac{1}{s} + \frac{1}{5} \quad [1 \text{ bod}]$$

2. način: metodom petlji (nije ovdje prikazan, ali se priznaje u potpunosti).

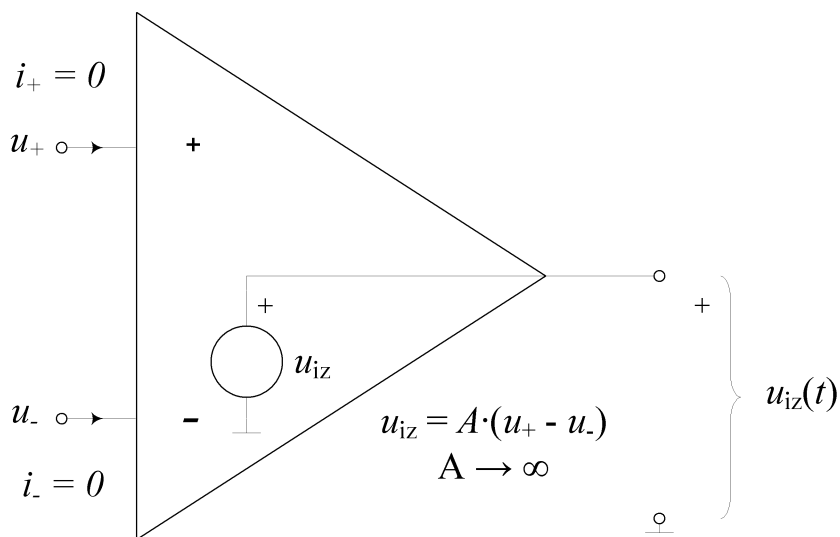


5. Zadan je električni krug prema slici. Odrediti napon na izlazu  $u_{iz}(t)$  ako je zadano:  $R_1 = R_2 = R_3 = 1$ ,  $R_4 = 2$ ,  $C = 1$ ,  $u_g(t) = S(t)$ . Početni uvjeti su jednaki nula. Koristiti metodu napona čvorova.



Rješenje:

Nadomjesni sklop operacijskog pojačala:



Jednadžbe čvorova:

$$1) U_1 \left( \frac{1}{R_1} + \frac{1}{R_2} + sC \right) - U_{iz} \frac{1}{R_2} - U_g \frac{1}{R_1} = 0 \quad [1 \text{ bod}]$$

$$2) U_2 \left( \frac{1}{R_3} + \frac{1}{R_4} \right) - U_g \frac{1}{R_3} = 0$$

---


$$U_{iz} = A(U_2 - U_1)$$

$$\frac{U_{iz}}{A} = U_2 - U_1 \quad A \rightarrow \infty$$

$$\Rightarrow U_1 = U_2 \quad [1 \text{ bod}]$$

$$1) U_1 \left( \frac{1}{R_1} + \frac{1}{R_2} + sC \right) - U_{iz} \frac{1}{R_2} - U_g \frac{1}{R_1} = 0 \Rightarrow U_{iz} = U_1 R_2 \left( \frac{1}{R_1} + \frac{1}{R_2} + sC \right) - U_g \frac{R_2}{R_1}$$

$$2) U_2 \left( \frac{1}{R_3} + \frac{1}{R_4} \right) - U_g \frac{1}{R_3} = 0 \Rightarrow U_2 = U_1 = U_g \frac{1}{R_3 \cdot \frac{R_3 + R_4}{R_3 R_4}} = U_g \frac{R_4}{R_3 + R_4}$$

↓

$$\begin{aligned} 1) U_{iz} &= U_g \frac{R_4}{R_3 + R_4} R_2 \left( \frac{R_1 + R_2 + sR_1 R_2 C}{R_1 R_2} \right) - U_g \frac{R_2}{R_1} = U_g \left( \frac{R_4 R_2}{R_3 + R_4} \left( \frac{R_1 + R_2 + sR_1 R_2 C}{R_1 R_2} \right) - \frac{R_2}{R_1} \right) \\ &\Rightarrow U_g \left( \frac{R_4 R_2}{R_3 + R_4} \frac{R_1 + R_2}{R_1 R_2} - \frac{R_2}{R_1} + \frac{R_4 R_2}{R_3 + R_4} \frac{sR_1 R_2 C}{R_1 R_2} \right) = U_g \left( \frac{R_4}{R_3 + R_4} \frac{R_1 + R_2}{R_1} - \frac{R_2}{R_1} + \frac{R_4 R_2}{R_3 + R_4} sC \right) = \\ U_{iz} &= U_g \left( \frac{R_1 R_4 + R_2 R_4}{R_1 (R_3 + R_4)} - \frac{R_2 R_3 + R_2 R_4}{R_1 (R_3 + R_4)} + \frac{R_4 R_2}{R_3 + R_4} sC \right) = U_g \frac{1}{R_3 + R_4} \left( \frac{R_1 R_4 - R_2 R_3}{R_1} + sR_4 R_2 C \right) \end{aligned}$$

Uvrstimo vrijednosti:

[2 boda]

$$\Rightarrow U_{iz}(s) = \frac{1}{3} \left( \frac{2-1}{1} + 2s \right) = \frac{1}{3} + \frac{2}{3} s$$

$$\Rightarrow u_{iz}(t) = \frac{1}{3} S(t) + \frac{2}{3} \delta(t) \quad [1 \text{ bod}]$$