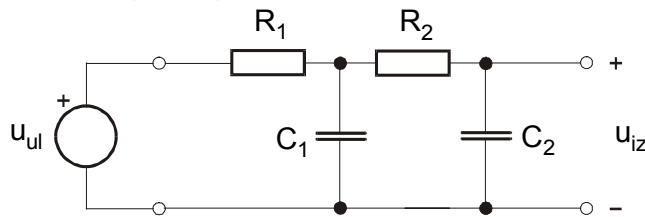
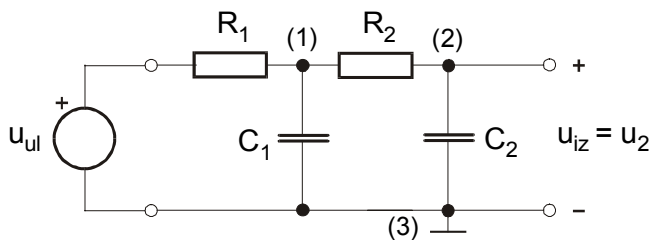


Prijenosne funkcije i a.-f. karakteristika: Zadaci sa rješenjima za vježbu

1. Za električni krug na slici odrediti prijenosnu funkciju napona $T(s) = \frac{U_{iz}(s)}{U_{ul}(s)}$. Prikazati frekvencijsku karakteristiku $|T(j\omega)|$. Zadano je: $R_1 = 1$, $R_2 = 1$, $C_1 = 1$, $C_2 = 1$.

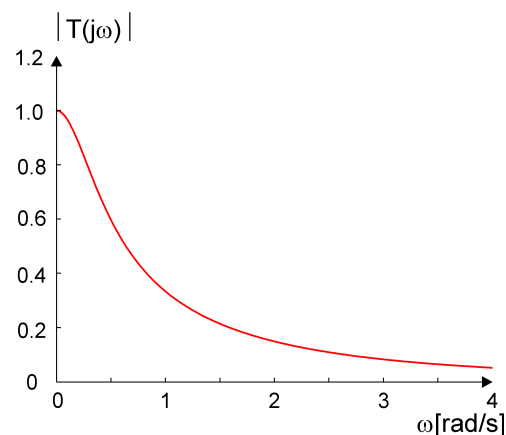


Rješenje:



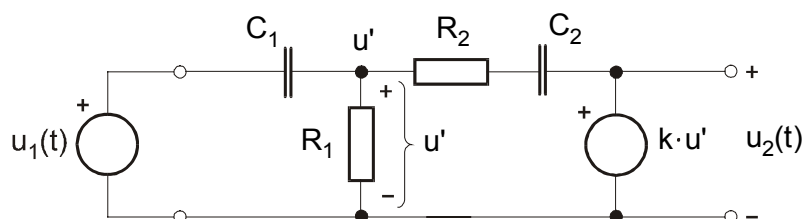
$$T(s) = \frac{U_{iz}}{U_{ul}} = \frac{1}{s^2 R_1 R_2 C_1 C_2 + s(R_1 C_1 + R_2 C_2 + R_1 C_2) + 1} = \frac{1}{s^2 + 3s + 1}$$

$$T(j\omega) = \frac{1}{-\omega^2 + 3j\omega + 1} = \frac{1}{(1 - \omega^2) + 3j\omega} \Rightarrow |T(j\omega)| = \frac{1}{\sqrt{(1 - \omega^2)^2 + 9\omega^2}}$$



2. Odrediti prijenosnu funkciju $T(s) = \frac{U_2(s)}{U_1(s)}$ za električni krug na slici. Zadane su vrijednosti

elemenata: $R_1 = R_2 = 10\text{ K}\Omega$, $C_1 = C_2 = 1\text{ nF}$, i parametar $k=2$. Izvršiti normalizaciju elemenata po frekvenciji $\omega_0 = 10^5\text{ rad/s}$ i impedanciji $R_0 = 10^4\Omega$. Odrediti normaliziranu prijenosnu funkciju. Prikazati raspored nula i polova u s ravnini za normaliziranu prijenosnu funkciju.



Rješenje:

$$T(s) = \frac{U_2(s)}{U_1(s)} = \frac{k \cdot s \cdot \left(s + \frac{1}{R_2 C_2} \right)}{s^2 + s \frac{R_1 C_1 + R_2 C_2 + R_1 C_2 (1-k)}{R_1 R_2 C_1 C_2} + \frac{1}{R_1 R_2 C_1 C_2}}$$

uz uvrštene vrijednosti elemenata:

$$T(s) = \frac{U_2(s)}{U_1(s)} = \frac{2 \cdot s \cdot (s + 10^5)}{s^2 + 10^5 s + 10^{10}}$$

normalizacija na: $R_0 = 10^4 \Omega$, $\omega_0 = 10^5 \text{ rad/s}$:

$$R_n = \frac{R}{R_0} = \frac{10^4}{10^4} = 1 \Rightarrow R_1 = R_2 = 1$$

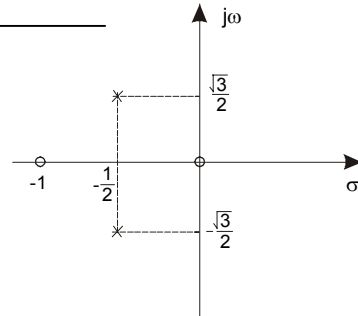
$$C_n = \omega_0 \cdot R_0 \cdot C = 10^5 \cdot 10^4 \cdot 10^{-9} \Rightarrow C_1 = C_2 = 1$$

uz uvrštene normalizirane vrijednosti elemenata:

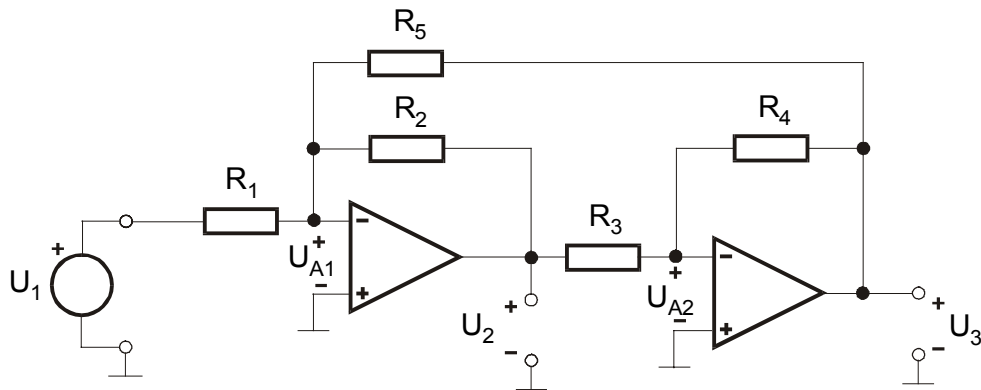
$$T(s) = \frac{U_2}{U_1} = \frac{2s(s+1)}{s^2 + s + 1}$$

nule: $s_{01} = 0$, $s_{02} = -1$

$$\text{polovi: } s_{p1,2} = \frac{-1 \pm \sqrt{1-4}}{2} = -\frac{1}{2} \pm j \frac{\sqrt{3}}{2}$$



3. Za električni krug na slici odrediti naponsku prijenosnu funkciju $T(s) = \frac{U_3(s)}{U_1(s)}$. Zadane su normalizirane vrijednosti elemenata: $R_1 = R_2 = R_3 = R_4 = 1$, $R_5 = 2$.



Rješenje:

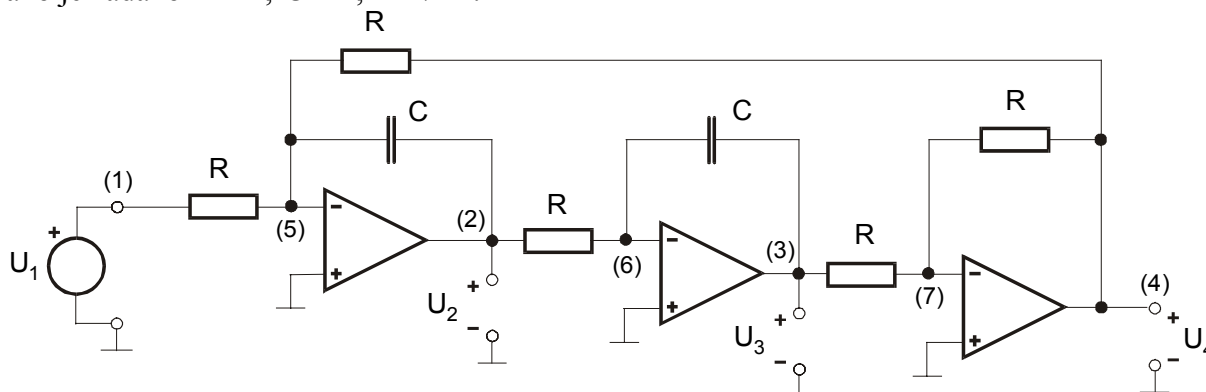
$$T(s) = \frac{U_3}{U_1} = \frac{R_2 R_4 R_5}{R_1 (R_3 R_5 - R_2 R_4)} = 2$$

4. Za električni krug prikazan slikom naći prijenosnu funkciju napona:

A) $T(s) = \frac{U_4(s)}{U_1(s)}$

B) $T(s) = \frac{U_3(s)}{U_1(s)}$

ako je zadano $R=1$, $C=1$, $A \rightarrow \infty$.

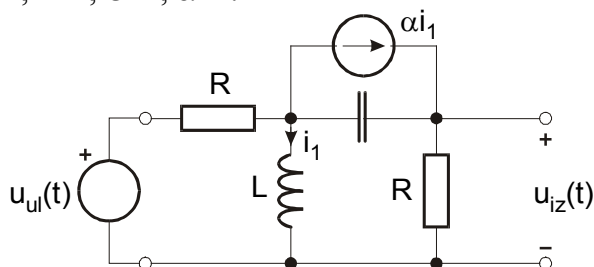


Rješenje:

A) $T(s) = \frac{U_4}{U_1} = -\frac{1}{1+s^2}$

B) $T(s) = \frac{U_3}{U_1} = \frac{1}{1+s^2}$

5. Odrediti prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$ za električni krug prikazan slikom. Nacrtati raspored polova i nula te funkcije u kompleksnoj s -ravnini. Odrediti i nacrtati funkciju $|T(j\omega)|$. Zadano je $R=1$, $L=2$, $C=1$, $\alpha=2$.

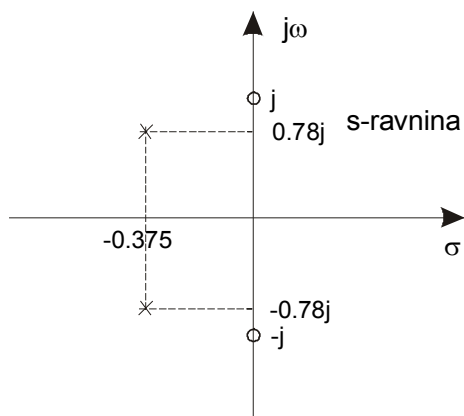


Rješenje:

$$T(s) = \frac{U_{iz}}{U_{ul}} = \frac{2(s^2+1)}{(s+1)(2s^2+2s+3)-2s(s^2+1)} = \frac{2(s^2+1)}{4s^2+3s+3} = \frac{1}{2} \cdot \frac{s^2+1}{s^2+\frac{3}{4}s+\frac{3}{4}}$$

nule: $s^2+1=0 \Rightarrow s^2=-1 \Rightarrow s_{o1,2}=\pm j$

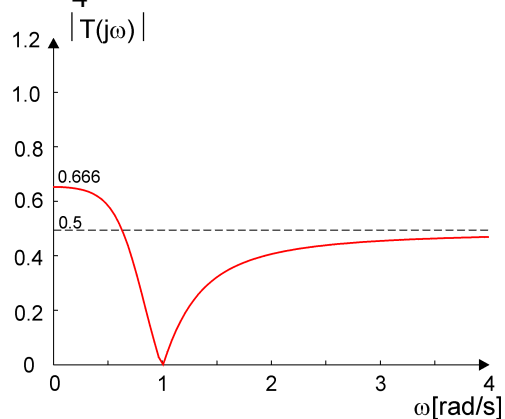
polovi: $s^2+\frac{3}{4}s+\frac{3}{4}=0 \Rightarrow s_{p1,2}=\frac{-3}{8} \pm \sqrt{\frac{9}{64}-\frac{3}{4}} = \frac{-3}{8} \pm \sqrt{\frac{9-48}{64}} = \frac{-3}{8} \pm j\frac{\sqrt{39}}{8}$
 $\Rightarrow s_{p1,2}=-0.375 \pm j0.78065$



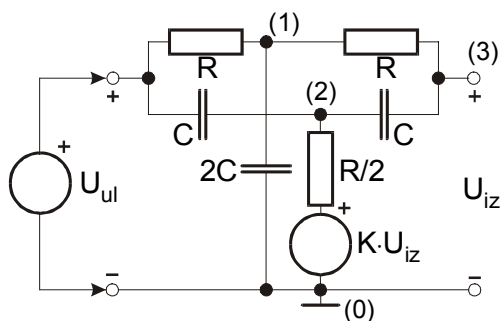
$|T(j\omega)|$:

$$T(j\omega) = \frac{1}{2} \cdot \frac{-\omega^2 + 1}{-\omega^2 + \frac{3}{4}j\omega + \frac{3}{4}} \Rightarrow |T(j\omega)| = \frac{1}{2} \cdot \frac{|1 - \omega^2|}{\sqrt{\left(\frac{3}{4} - \omega^2\right)^2 + \left(\frac{3}{4}\omega\right)^2}}$$

karakteristične točke: $T(0) = \frac{1}{2} \cdot \frac{1}{\frac{3}{4}} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3} = 0.666$, $T(\infty) = \lim_{s \rightarrow \infty} \frac{1}{2} \cdot \frac{1 + \frac{1}{s^2}}{1 + \frac{3}{4} \frac{1}{s} + \frac{3}{4} \frac{1}{s^2}} = \frac{1}{2} = 0.5$



6. Odrediti prijenosnu funkciju napona $T(s) = U_{iz}(s)/U_{ul}(s)$ za električni krug prikazan slikom, ako je zadano: $R=1$, $C=1$, $k=1.5$. Nacrtati raspored polova i nula u kompleksnoj s -ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$.



Rješenje:

$$T(s) = \frac{U_{iz}}{U_{ul}} = \frac{s^2 + 1}{s^2 + s + 1}$$

Polovi:

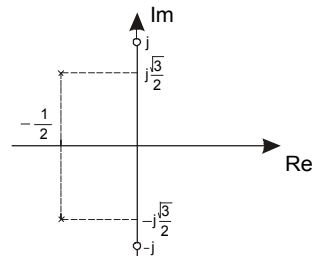
$$s^2 + s + 1 = 0$$

$$s_{p1,2} = -\frac{1}{2} \pm j \frac{\sqrt{3}}{2}$$

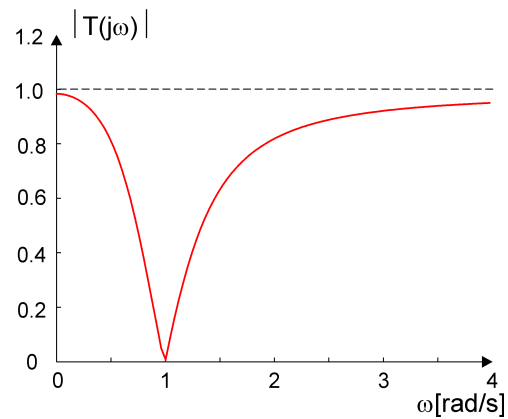
Nule:

$$s^2 + 1 = 0$$

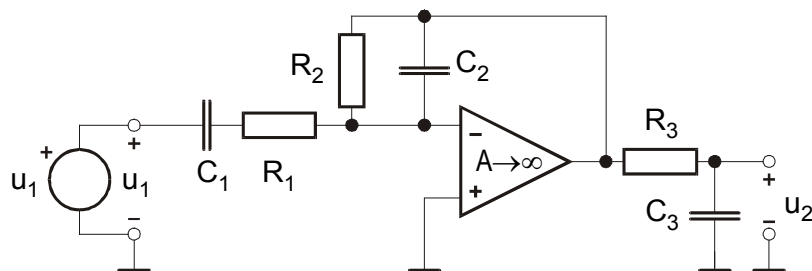
$$s_{o1,2} = \pm j$$



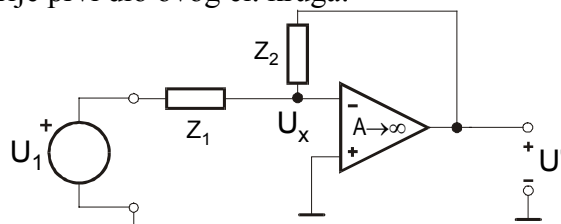
a-f karakteristika:



7. Naći naponsku prijenosnu funkciju $T(s)=U_2(s)/U_1(s)$ električnog kruga na slici. Nacrtati raspored nula i polova u s -ravnini i konstruirati amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$, ako su zadane normalizirane vrijednosti elemenata $R_1=R_2=R_3=1$, $C_1=1$, $C_2=1/2$, $C_3=2$.



Rješenje: Proučimo najprije prvi dio ovog el. kruga:



$$\frac{U'}{U_1} = -\frac{\frac{1}{Z_1}}{\frac{1}{A}\left(\frac{1}{Z_1} + \frac{1}{Z_2}\right) + \frac{1}{Z_2}}$$

$$A \rightarrow \infty \Rightarrow \frac{U'}{U_1} = -\frac{\frac{1}{Z_1}}{\frac{1}{Z_2}} = -\frac{Z_2}{Z_1}, \quad Z_1(s) = R_1 + \frac{1}{sC_1} = \frac{1 + sR_1C_1}{sC_1}, \quad Z_2(s) = \frac{R_2 \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{1 + sR_2C_2}$$

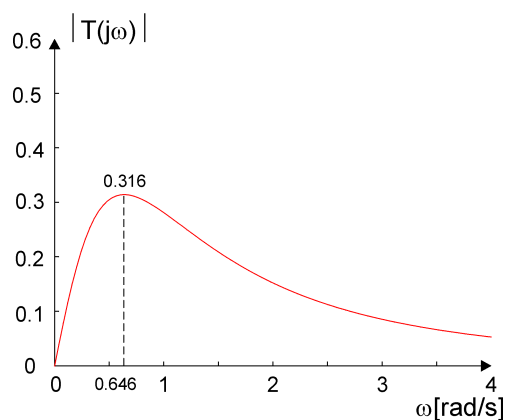
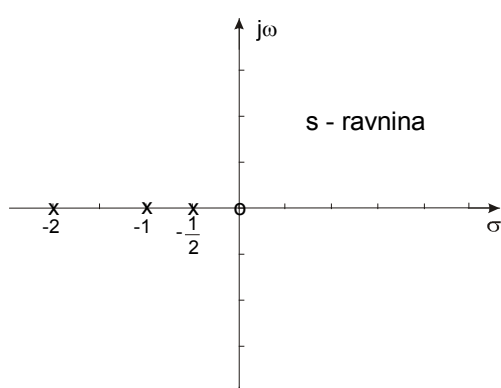
$$\frac{U'}{U_1} = -\frac{sC_1R_2}{(1 + sR_1C_1)(1 + sR_2C_2)} = \frac{-s \frac{1}{R_1C_2}}{\left(s + \frac{1}{R_1C_1}\right)\left(s + \frac{1}{R_2C_2}\right)}$$

$$\frac{U_2}{U'} = \frac{\frac{1}{sC_3}}{R_3 + \frac{1}{sC_3}} = \frac{1}{sR_3C_3 + 1} = \frac{\frac{1}{R_3C_3}}{s + \frac{1}{R_3C_3}}$$

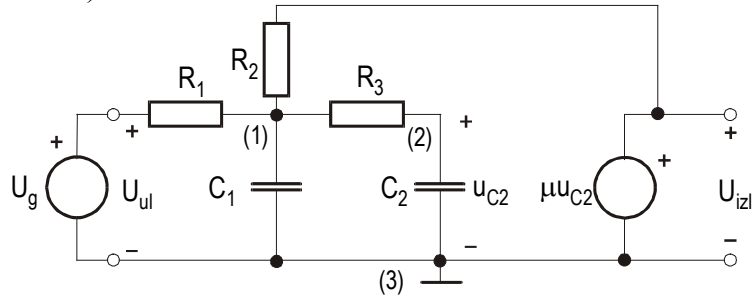
$$T(s) = \frac{U_2}{U_1} = \frac{U'}{U_1} \cdot \frac{U_2}{U'} = \frac{-s \frac{1}{R_1C_2}}{\left(s + \frac{1}{R_1C_1}\right)\left(s + \frac{1}{R_2C_2}\right)} \cdot \frac{\frac{1}{R_3C_3}}{\left(s + \frac{1}{R_3C_3}\right)}$$

$$R_1 = R_2 = R_3 = 1; \quad C_1 = 1; \quad C_2 = \frac{1}{2}; \quad C_3 = 2$$

$$T(s) = \frac{U_2}{U_1} = \frac{-s}{(s+1)\left(s + \frac{1}{2}\right)(s+2)}$$



8. Za električni krug prikazan slikom naći prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$. Nacrtati raspored nula i polova u kompleksnoj s -ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$. Zadane su vrijednosti elemenata: $R_1=R_2=0.5$, $R_3=1$, $C_1=C_2=1$, $\mu=-3$, (pažnja: naponsko pojačanje μ naponski upravljanog naponskog izvora u mreži ima negativnu vrijednost).



Rješenje:

$$T(s) = \frac{U_2}{U_1} = \frac{-\mu \cdot R_2}{s^2(R_1 R_2 R_3 C_1 C_2) + s(R_1 R_2 C_1 + R_2 R_3 C_2 + R_1 R_3 C_2 + R_1 R_2 C_2) + R_1 \cdot (1 + \mu) + R_2}$$

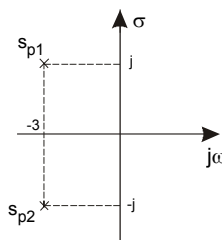
$$T(s) = \frac{-3 \cdot 0.5}{0.25s^2 + s(0.25 + 0.5 + 0.5 + 0.25) + 0.5 + 4 \cdot 0.5} = \frac{-1.5}{0.25s^2 + 1.5s + 2.5} \cdot 4 =$$

$$= \frac{-6}{s^2 + 6s + 10} = \frac{-\frac{3}{5} \cdot 10}{s^2 + 6s + 10}$$

Na frekvenciji $\omega=0$ pojačanje prijenosne funkcije je $T(0)=-3/5$.

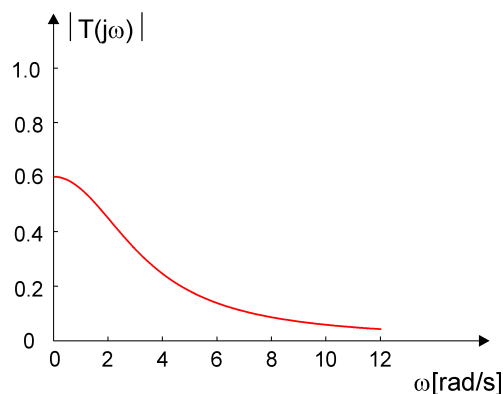
polovi: $s^2 + 6s + 10 = 0$; $\Delta = b^2 - 4ac = 36 - 4 \cdot 10 = -4$

$$s_{p1,2} = \frac{-6 \pm j2}{2} = -3 \pm j$$

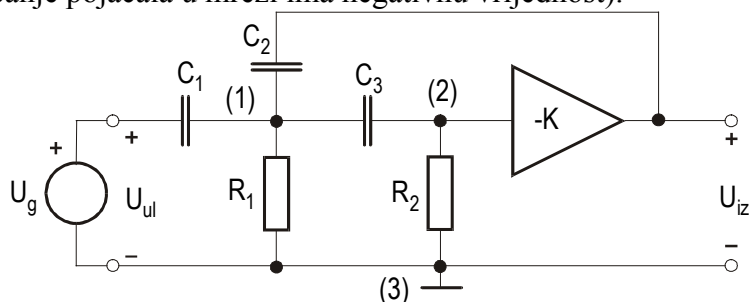


nule: dvije nule u beskonačnosti

$$|T(j\omega)| = \left| \frac{-6}{-\omega^2 + 6j\omega + 10} \right| = \frac{6}{\sqrt{(10 - \omega^2)^2 + 36\omega^2}}$$



9. Za električni krug prikazanu slikom naći prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$. Nacrtati raspored nula i polova u kompleksnoj s -ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$. Zadane su vrijednosti elemenata: $R_1=R_2=1$, $C_1=C_2=0.5$, $C_3=1$, $|K|=3$, (pažnja: pojačanje pojačala u mreži ima negativnu vrijednost).



Rješenje:

$$T(s) = \frac{U_{iz}}{U_{ul}} = \frac{-Ks^2 R_1 R_2 C_1 C_3}{s^2 R_1 R_2 C_3 (C_1 + (1+K)C_2) + s[R_2 C_3 + R_1 (C_1 + C_2) + R_1 C_3] + 1}$$

$$T(s) = -\frac{1.5s^2}{2.5s^2 + s \cdot [2+1] + 1} = -\frac{0.6s^2}{s^2 + 1.2s + 0.4}$$

Na frekvenciji $\omega=\infty$ pojačanje prijenosne funkcije je $T(0)=-0.6$.

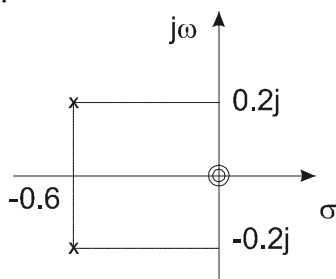
polovi: $s^2 + 1.2s + 0.4 = 0$

$$s_{p1,2} = \frac{-1.2 \pm \sqrt{1.2^2 - 4 \cdot 0.4}}{2} = -0.6 \pm 0.2j$$

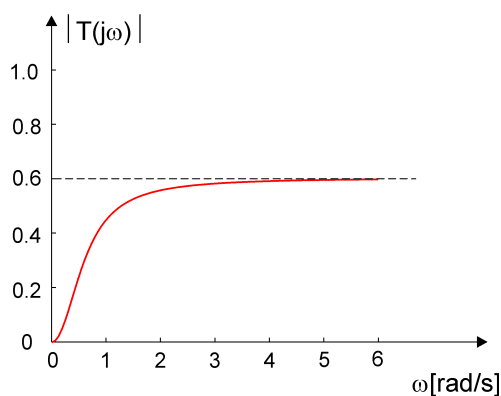
nule: $s^2 = 0$

$s_{01,2} = 0$ dvostruka nula u ishodištu

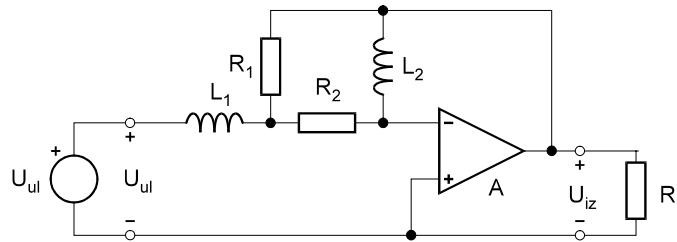
raspored nula i polova u s -ravnini:



$$|T(j\omega)| = \left| \frac{-0.6\omega^2}{-\omega^2 + 1.2j\omega + 0.4} \right| = \frac{0.6\omega^2}{\sqrt{(0.4 - \omega^2)^2 + (1.2\omega)^2}}$$



10. Za električni krug prikazan slikom treba naći prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$, položaj polova i nula u kompleksnoj s -ravnini i konstruirati amplitudno-frekvencijsku karakteristiku. Zadane su normalizirane vrijednosti elemenata: $R_1=1, R_2=1, R_3=1, L_1=1/\sqrt{2}, L_2=\sqrt{2}$ te $A \rightarrow \infty$.



Rješenje:

$$T(s) = \frac{U_{izl}}{U_{ul}} = \frac{-s \cdot \frac{R_1}{L_1}}{s^2 + s \frac{R_1 + R_2}{L_2} + \frac{R_1 R_2}{L_1 L_2}}$$

uz uvrštene vrijednosti dobivamo:

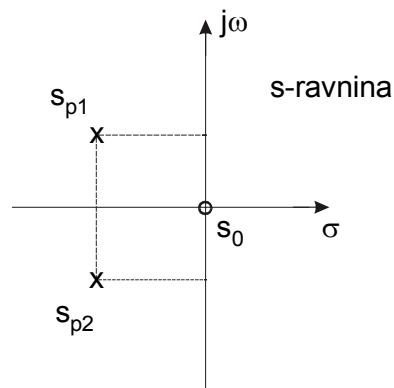
$$T(s) = \frac{U_{iz}}{U_{ul}} = -\frac{s \cdot \sqrt{2}}{s^2 + \frac{2}{\sqrt{2}}s + \frac{1}{\frac{1}{\sqrt{2}} \cdot \sqrt{2}}} = -\frac{s \cdot \sqrt{2}}{s^2 + \sqrt{2} \cdot s + 1}$$

polovi: $s^2 + \sqrt{2} \cdot s + 1 = 0$

$$\Rightarrow s_{p1,2} = \frac{-\sqrt{2} \pm \sqrt{2-4}}{2} = \frac{-\sqrt{2} \pm j\sqrt{2}}{2}$$

$$s_{p1,2} = -\frac{\sqrt{2}}{2} \pm j \frac{\sqrt{2}}{2}$$

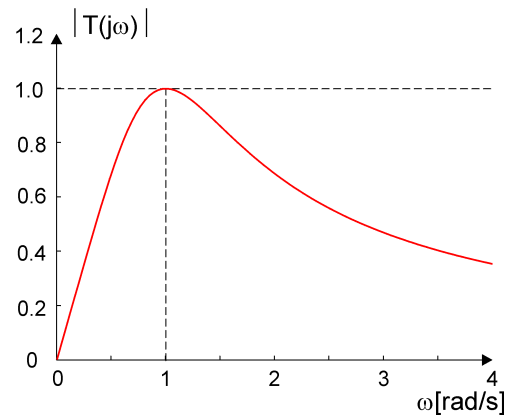
nule: $s_{01} = 0$ jedna nula u ishodištu, a druga u beskonačnosti



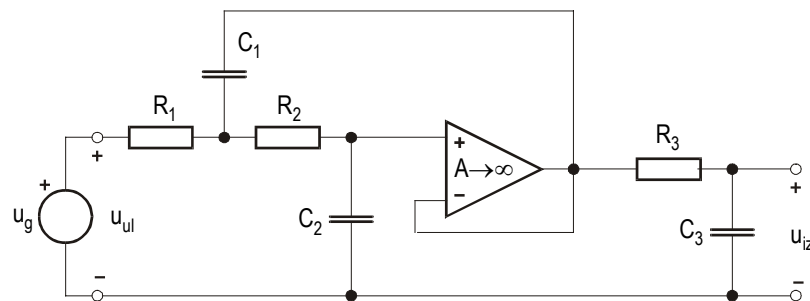
a-f karakteristika (uvrstimo $s=j\omega$ u $T(s)$):

$$T(j\omega) = \frac{U_{iz}(j\omega)}{U_{ul}(j\omega)} = -\frac{-j\omega \cdot \sqrt{2}}{-\omega^2 + \sqrt{2} \cdot j\omega + 1}$$

$$|T(j\omega)| = \frac{\omega\sqrt{2}}{\sqrt{(1-\omega^2)^2 + 2\omega^2}} = \frac{\omega\sqrt{2}}{\sqrt{1-2\omega^2+\omega^4+2\omega^2}} = \sqrt{2} \cdot \frac{\omega}{\sqrt{1+\omega^4}}$$



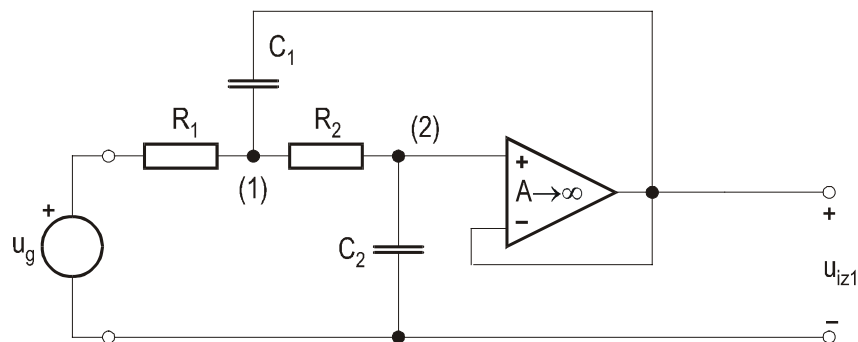
11. Za električni krug prikazan slikom odrediti prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$. Nacrtati položaj nula i polova u kompleksnoj s -ravnini te nacrtati amplitudno-frekvencijsku karakteristiku prijenosne funkcije i označiti karakteristične točke. Zadano je $R_1=1$, $R_2=1$, $R_3=1$, $C_1=2$, $C_2=1/2$, $C_3=1$.



Rješenje:

Ukupna prijenosna funkcija sastoji se od dva dijela i ima slijedeći oblik: $T(s) = T_1(s) \cdot T_2(s)$

a) prvi dio:

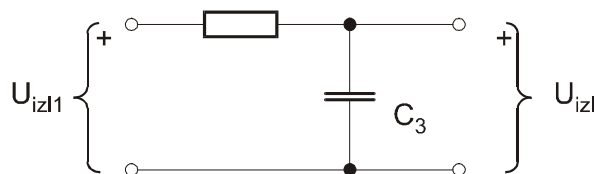


$$T_1(s) = \frac{U_{iz1}}{U_g} = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + sC_1 + s\frac{R_2C_2}{R_1} + sC_2 + s^2R_2C_1C_2 - \frac{1}{R_2} - sC_1}$$

$$= \frac{1}{s^2R_1R_2C_1C_2 + s(R_1C_2 + R_2C_2) + 1} = \frac{1}{s^2 + s + 1}$$

b) drugi dio: $T_2(s) = \frac{U_{iz}}{U_{iz1}}$

$$T_2(s) = \frac{\frac{1}{sC_3}}{R_3 + \frac{1}{sC_3}} = \frac{\frac{1}{R_3C_3}}{s + \frac{1}{R_3C_3}} = \frac{1}{s+1}$$



Konačno je ukupna prijenosna funkcija: $T(s) = \frac{1}{(s+1)(s^2+s+1)}$

polovi: $s^2 + s + 1 = 0$

$$s_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm j\sqrt{3}}{2}$$

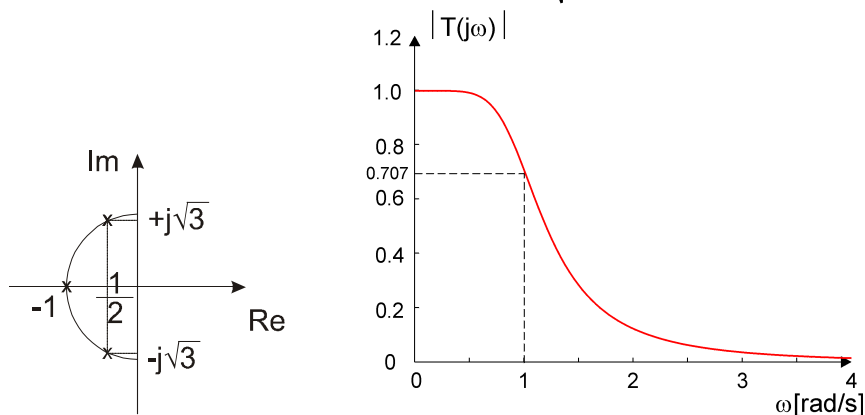
$$s+1=0 \Rightarrow s_3 = -1$$

$$s_1 = s_2^*$$

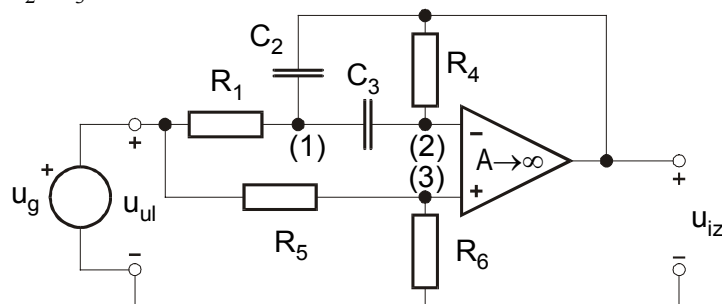
nule: postoje tri nule u beskonačnosti

a-f karakteristika:

$$T(j\omega) = \frac{1}{(j\omega+1)(-\omega^2+j\omega+1)} \Rightarrow |T(j\omega)| = \frac{1}{\sqrt{\omega^2+1}\sqrt{(1-\omega^2)^2+\omega^2}}$$



12. Za električni krug prikazan slikom naći prijenosnu funkciju napona $T(s) = U_{iz}(s)/U_{ul}(s)$. Nacrtati položaj nula i polova u kompleksnoj s -ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$. Zadane su normalizirane vrijednosti elemenata: $R_1=1/2$, $R_4=R_6=2$, $R_5=1$, $C_2=C_3=1$.



Rješenje:

$$T(s) = \frac{U_{iz}}{U_{ul}} = \alpha \cdot \frac{s^2 C_2 C_3 R_4 R_1 + s \left[\left(1 - \frac{1}{\alpha}\right) C_3 R_4 + C_2 R_1 + C_3 R_1 \right] + 1}{s^2 C_2 C_3 R_4 R_1 + s(C_2 R_1 + C_3 R_1) + 1}, \quad \alpha = \frac{R_6}{R_5 + R_6} \quad (0 < \alpha < 1);$$

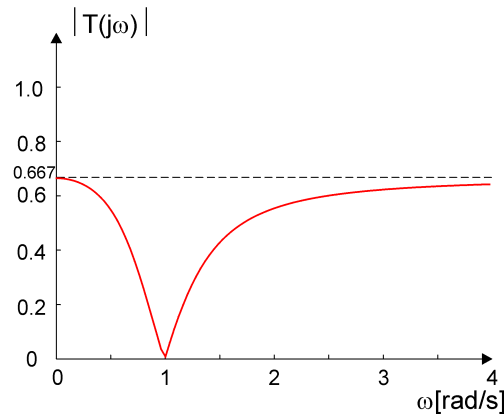
Uvrstimo vrijednosti, konačno je prijenosna funkcija:

$$T(s) = \frac{2}{3} \cdot \frac{s^2 + 1}{s^2 + s + 1}$$

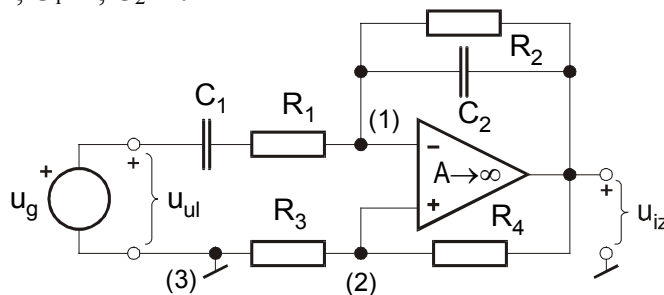
nule: $s_{01,2} = \pm j$

polovi: $s_{p1,2} = -\frac{1}{2} \pm j \frac{\sqrt{3}}{2}$

$$T(j\omega) = \frac{2}{3} \frac{|1 - \omega^2|}{3 - \omega^2 + j\omega + 1} \Rightarrow |T(j\omega)| = \frac{2}{3} \frac{|1 - \omega^2|}{\sqrt{(1 - \omega^2)^2 + \omega^2}}$$



13. Za električni krug prikazan slikom naći prijenosnu funkciju napona $T(s) = U_{iz}(s)/U_{ul}(s)$. Nacrtati položaj nula i polova u kompleksnoj s -ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$. Zadane su normalizirane vrijednosti elemenata: $R_1=1/2$, $R_2=1$, $R_3=1$, $R_4=1$, $C_1=1$, $C_2=1$.



Rješenje:

$$T(s) = \frac{U_{iz}(s)}{U_{ul}(s)} = \frac{-\left(1 + \frac{R_3}{R_4}\right)s \frac{1}{R_1 C_2}}{s^2 + s \frac{R_1 C_1 + R_2 C_2 - R_2 C_1 R_3 / R_4}{R_1 C_1 R_2 C_2} + \frac{1}{R_1 C_1 R_2 C_2}}$$

uz uvrštene vrijednosti elemenata je prijenosna funkcija:

$$T(s) = \frac{-4 \cdot s}{s^2 + s + 2}$$

Polovi: $s^2 + s + 2 = 0$

$$s_{p1,2} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} - 2} = -\frac{1}{2} \pm j\frac{\sqrt{7}}{2}$$

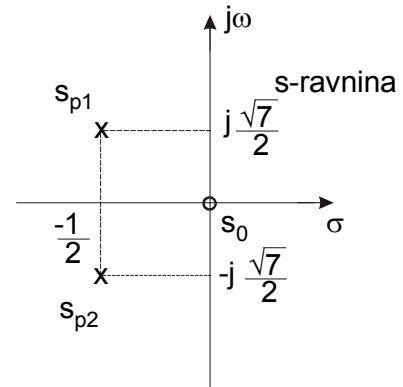
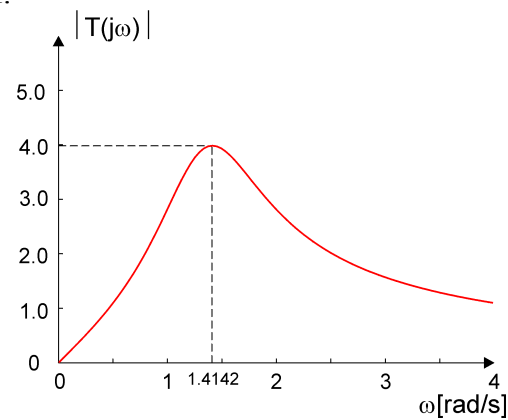
Nule: $s=0$

$$s_{o1} = 0, \quad s_{o2} = \infty$$

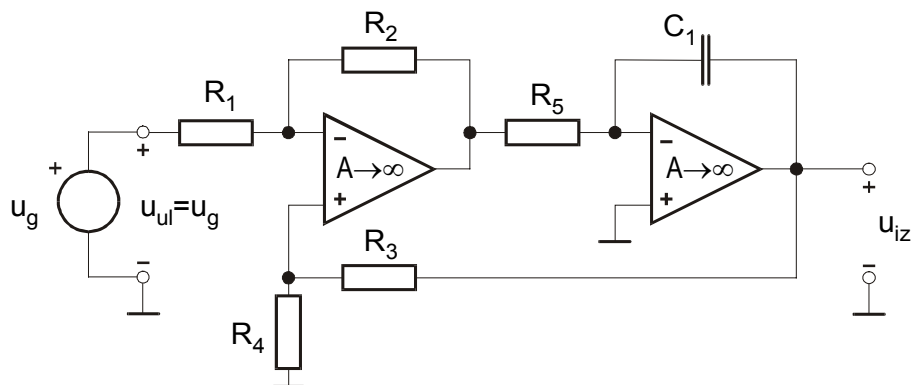
a-f karakteristika : $T(j\omega) = \frac{-4 \cdot j\omega}{-\omega^2 + j\omega + 2} \Rightarrow$

$$|T(j\omega)| = \frac{4 \cdot \omega}{\sqrt{(2 - \omega^2)^2 + \omega^2}}$$

Graf:



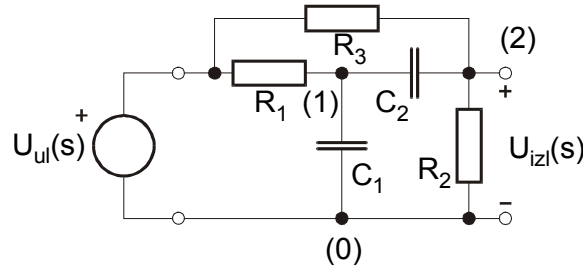
14. Odrediti prijenosnu funkciju napona $T(s) = U_{iz}(s)/U_{ul}(s)$ za električni krug prikazan slikom.



Rješenje:

$$\begin{aligned} T(s) = \frac{U_6}{U_1} &= \frac{1}{s \frac{R_1}{R_2} R_5 C_1 + \frac{R_4}{R_3 + R_4} \cdot \frac{R_1 + R_2}{R_2}} = \frac{\frac{R_2}{R_1} \cdot \frac{1}{R_5 C_1}}{s + \frac{R_4}{R_3 + R_4} \cdot \frac{R_1 + R_2}{R_1} \cdot \frac{1}{R_5 \cdot C_1}} = \\ &= \frac{R_2(R_3 + R_4)}{s C_1 R_1 R_5 (R_3 + R_4) + R_4(R_1 + R_2)} \end{aligned}$$

15. Naći naponsku prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$ električnog kruga na slici. Nacrtati položaj nula i polova u s -ravlini i konstruirati amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$, ako su zadane normalizirane vrijednosti elemenata $R_1=R_2=R_3=1$, $C_1=C_2=1$.



Rješenje:

$$U_1 \left(\frac{1}{R_1} + sC_1 + sC_2 \right) - U_2 \cdot sC_2 = U_{ul} \cdot \frac{1}{R_1}$$

$$-U_1 \cdot sC_2 + U_2 \left(\frac{1}{R_2} + \frac{1}{R_3} + sC_2 \right) = U_{ul} \cdot \frac{1}{R_3}$$

$$U_2 = \frac{\Delta_2}{\Delta}; \quad T(s) = \frac{U_{iz}}{U_{ul}}; \quad U_{iz} = U_2$$

$$\Delta = \begin{vmatrix} \frac{1}{R_1} + sC_1 + sC_2 & -sC_2 \\ -sC_2 & \frac{1}{R_2} + \frac{1}{R_3} + sC_2 \end{vmatrix} = \left(\frac{1}{R_1} + sC_1 + sC_2 \right) \cdot \left(\frac{1}{R_2} + \frac{1}{R_3} + sC_2 \right) - (sC_2)^2$$

$$\Delta_2 = \begin{vmatrix} \frac{1}{R_1} + sC_1 + sC_2 & U_{ul} \cdot \frac{1}{R_1} \\ -sC_2 & U_{ul} \cdot \frac{1}{R_3} \end{vmatrix} = U_{ul} \cdot \left[\frac{1}{R_3} \left(\frac{1}{R_1} + sC_1 + sC_2 \right) + \frac{sC_2}{R_1} \right]$$

$$T(s) = \frac{U_{iz}}{U_{ul}} = \frac{\Delta_2 / \Delta}{U_{ul}} = \frac{\frac{1}{C_1 C_2 R_1 R_2} + s \left[\left(\frac{1}{C_1} + \frac{1}{C_2} \right) \frac{1}{R_3} + \frac{1}{R_1 C_1} \right]}{\frac{1}{R_1 C_1 C_2} \left(\frac{1}{R_2} + \frac{1}{R_3} \right) + s \left[\left(\frac{1}{C_1} + \frac{1}{C_2} \right) \cdot \left(\frac{1}{R_2} + \frac{1}{R_3} \right) + \frac{1}{R_1 C_1} \right] + s^2}$$

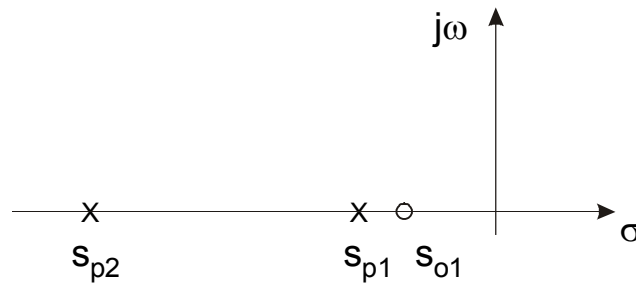
Uz uvrštene vrijednosti elemenata $R_1 = R_2 = R_3 = 1$, $C_1 = C_2 = 1$, prijenosna funkcija glasi:

$$T(s) = \frac{1 + 3s}{2 + 5s + s^2}$$

$$\text{nule: } 1 + 3s = 0 \Rightarrow s_{o1} = -\frac{1}{3}, \quad s_{o2} = \infty$$

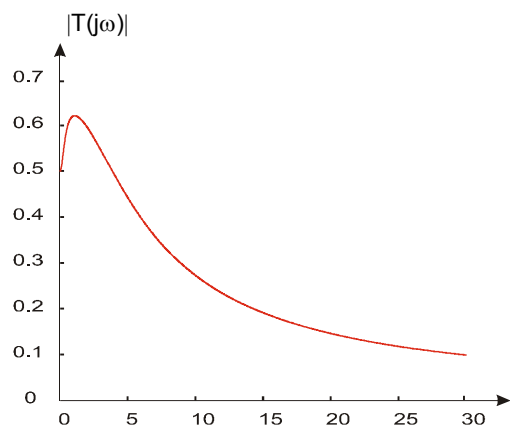
$$\text{polovi: } s^2 + 5s + 2 = 0 \Rightarrow s_{p1,2} = \frac{-s \pm \sqrt{25 - 8}}{2} = \frac{-5 \pm \sqrt{17}}{2}$$

$$s_{p1} = -0.4385, \quad s_{p2} = -4.561$$

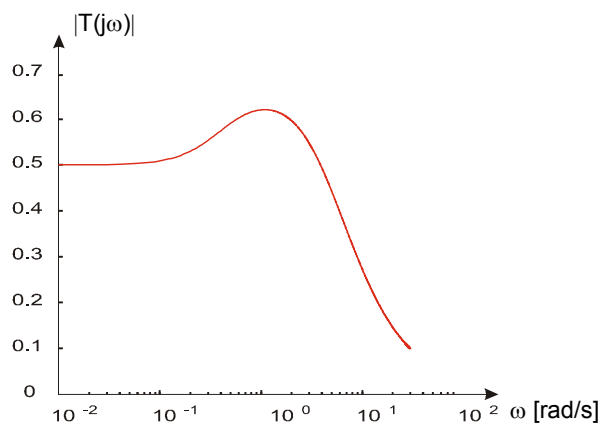


a-f karakteristika:

$$T(j\omega) = \frac{1 + 3j\omega}{2 + 5j\omega - \omega^2} \Rightarrow |T(j\omega)| = \frac{\sqrt{1 + (3\omega)^2}}{\sqrt{(2 - \omega^2)^2 + (5\omega)^2}}$$

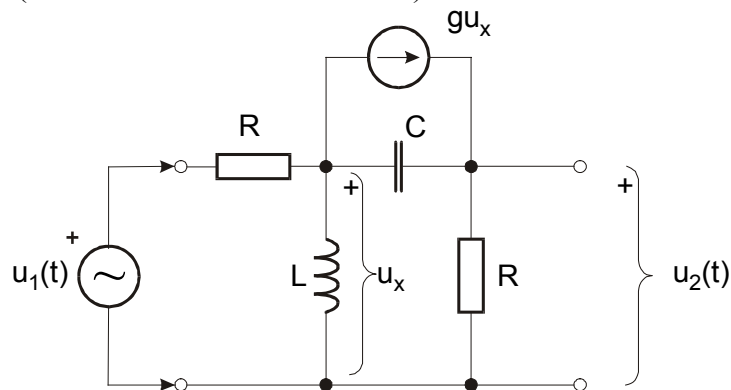


frekvencijska os je u linearnom mjerilu



frekvencijska os je u logaritamskom mjerilu

16. Za električni krug prikazan slikom naći prijenosnu funkciju napona $T(s) = \frac{U_2(s)}{U_1(s)}$ ako su zadane normalizirane vrijednosti elemenata: $R=1$, $C=1$, $L=2$ i parametar $g=2$. Za pobudu $u_1(t) = 10 \cos t$ (stacionarni sinusni valni oblik) naći valni oblik izlaznog napona $u_2(t)$.



Rješenje:

$$\text{Prijenosna funkcija napona glasi: } T(s) = \frac{U_2(s)}{U_1(s)} = \frac{2s^2 + 4s}{4s^2 + 7s + 1}$$

Pošto je pobuda sinusoidalnog stacionarnog valnog oblika u prienosnu funkciju $T(s)$ supstituiramo $s = j\omega$ i dobivamo:

$$T(j\omega) = \frac{-2\omega^2 + 4j\omega}{(1 - 4\omega^2) + 7j\omega} \quad (*)$$

Posljednji izraz se može napisati tako da se može razdvojiti realni i imaginarni dio :

$$T(j\omega) = \frac{-2\omega^2 + 4j\omega}{(1 - 4\omega^2) + 7j\omega} \cdot \frac{(1 - 4\omega^2) - 7j\omega}{(1 - 4\omega^2) - 7j\omega} = \frac{2\omega^2(13 + 4\omega^2) + 2j\omega(2 - \omega^2)}{(1 - 4\omega^2)^2 + (7\omega)^2} \quad (**)$$

Frekvencija pobudnog signala je jednaka 1 stoga uvrštavamo $\omega = 1$ u gornje izraze za $T(j\omega)$.

$$T(1j) = \frac{-2 + 4j}{-3 + 7j} \quad (*); \text{ ili } T(1j) = 2 \frac{17 + j}{9 + 49} = \frac{17 + j}{29} \quad (**)$$

Fazor pobudnog signala je : $U_1(j\omega) = 10 \angle 0^\circ$

Fazor odzivnog signala se izračuna iz : $U_2(j\omega) = T(j\omega) U_1(j\omega)$, odnosno:

Amplituda izlaznog signala je: $|U_2(j\omega)| = |T(j\omega)|_{\omega=1} \cdot |U_1(j\omega)|$

Fazni kut izlaznog signala je: $\arg U_2(j\omega) = \arg T(j\omega)_{\omega=1} + \arg U_1(j\omega)$

Amplituda izlaznog signala:

(*) \Rightarrow

$$|U_2(j\omega)| = 10 \cdot \frac{\sqrt{4\omega^4 + 16\omega^2}}{\sqrt{(1 - 4\omega^2)^2 + (7\omega)^2}} \Big|_{\omega=1} = 10 \frac{\sqrt{2^2 + 4^2}}{\sqrt{3^2 + 7^2}} = 10 \frac{\sqrt{20}}{\sqrt{58}} = 5.8722 \text{ (jednostavniji način)}$$

(**) \Rightarrow

$$|U_2(j\omega)| = 10 \cdot \frac{\sqrt{4\omega^4(13 + 4\omega^2)^2 + 4\omega^2(2 - \omega^2)^2}}{(1 - 4\omega^2)^2 + (7\omega)^2} \Big|_{\omega=1} = 10 \frac{\sqrt{17^2 + 1^2}}{29} = 5.8722$$

Fazni kut izlaznog signala:

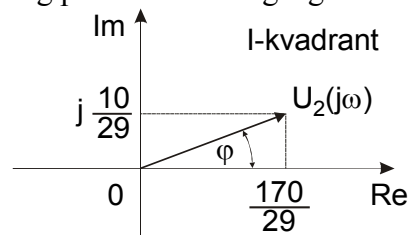
(*) \Rightarrow

$$\begin{aligned} \varphi &= \varphi_{\text{Brojnika}} - \varphi_{\text{Nazivnika}} = \arctg \frac{4\omega}{-2\omega^2} \Big|_{\omega=1} - \arctg \frac{7\omega}{1 - 4\omega^2} \Big|_{\omega=1} = \\ &= \arctg \frac{4}{-2} - \arctg \frac{7}{-3} = -63.43 - (-66.8) = 3.37^\circ \text{ (jednostavniji način)} \end{aligned}$$

(**) \Rightarrow

$$\varphi = \arctg \frac{2\omega(2 - \omega^2)}{2\omega^2(13 + 4\omega^2)} \Big|_{\omega=1} = \arctg \frac{1}{17} = 3.37^\circ$$

Uputno je nacrtati fazor izlaznog signala $U_2(j\omega)$ kako bi se vidjelo u kojem kvadrantu se fazor nalazi i odredio točan iznos faznog pomaka izlaznog signala.



Konačno rješenje tj. valni oblik odziva je: $u_2(t) = 5.87 \cos(t + 3.37^\circ)$.

17. Odziv nekog električnog kruga na pobudu $x(t)=S(t)$ glasi: $y(t)=e^{-3t}\text{ch}(2t)S(t)$. Odrediti funkciju mreže i fazor odziva na pobudu $x_1(t)=2 \cos(3t+45^\circ)$ (pobuda je sinusoidalni stacionarni signal).

Rješenje:

$$X(s) = \frac{1}{s}$$

$$Y(s) = \frac{s+3}{(s+3)^2 - 4} = \frac{s+3}{s^2 + 6s + 5}$$

$$H(s) = \frac{s(s+3)}{(s+3)^2 - 4} = \frac{s(s+3)}{s^2 + 6s + 5}$$

Fazori:

$$H(j\omega) = \frac{j\omega(j\omega + 3)}{(j\omega)^2 + 6j\omega + 5}$$

$$X_1(j\omega) = 2e^{j\pi/4}$$

$$Y_1(j\omega) = H(j\omega)X_1(j\omega) = \frac{j\omega(j\omega + 3)}{(j\omega)^2 + 6j\omega + 5} X_1(j\omega) = \frac{j\omega(j\omega + 3)}{(j\omega)^2 + 6j\omega + 5} 2e^{j\pi/4}$$

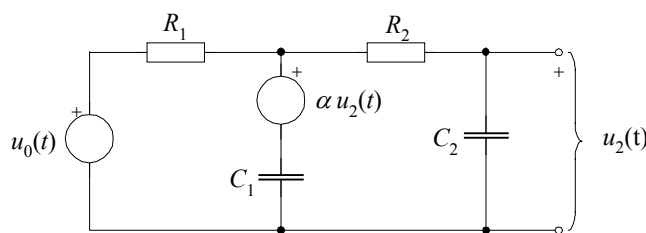
$$\omega = 3$$

$$Y_1(j3) = \frac{j3 \cdot (j3 + 3)}{(j3)^2 + 18j + 5} \cdot 2 \cdot \left(\frac{\sqrt{2}}{2} + j \frac{\sqrt{2}}{2} \right)$$

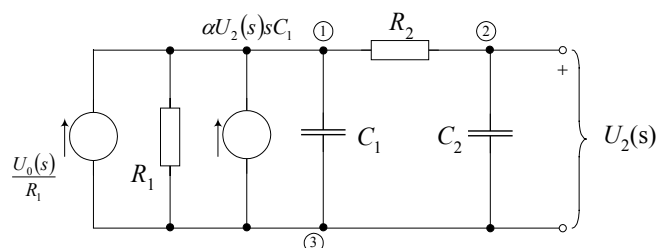
$$Y_1(j3) = \frac{9\sqrt{2}}{85} \cdot (2 + j9) \quad (\text{prvi kvadrant: } 0 < \varphi < 90^\circ)$$

$$y_1(t) = \frac{9\sqrt{2}\sqrt{85}}{85} \cos(3t + 77.47^\circ)$$

18. Odrediti prijenosnu funkciju $H(s)=U_2(s)/U_0(s)$ električnog kruga prema slici. U kojim se granicama mora kretati iznos konstante α da bi polovi funkcije $H(s)$ bili realni? Zadane su normirane vrijednosti elemenata: $R_1=1, R_2=1, C_1=1, C_2=1$.



Rješenje:



$$H(s) = \frac{U_2}{U_0} = \frac{1}{s^2 C_1 C_2 R_1 R_2 + s(C_2 R_1 + C_2 R_2 + (1-\alpha)C_1 R_1) + 1}$$

$$H(s) = \frac{U_2}{U_0} = \frac{1}{s^2 + s(3-\alpha) + 1} \Rightarrow s_{p1,2} = -\frac{3-\alpha}{2} \pm \sqrt{\left(\frac{3-\alpha}{2}\right)^2 - 1}$$

$$1. \text{ uvjet } \left(\frac{3-\alpha}{2}\right)^2 - 1 \geq 0 \Rightarrow \left(\frac{3-\alpha}{2}\right)^2 \geq 1 \Rightarrow 9 - 6\alpha + \alpha^2 \geq 4 \Rightarrow \alpha^2 - 6\alpha + 5 \geq 0$$

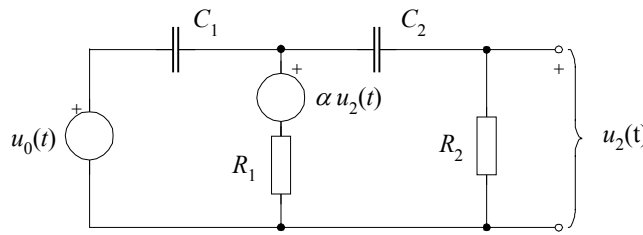
$$\Rightarrow \alpha_{1,2} = 3 \pm \sqrt{9-5} = 3 \pm 2 \Rightarrow \alpha \leq 1 \text{ \& } \alpha \geq 5$$

2. uvjet stabilnost polovi moraju biti u lijevoj poluravnini:

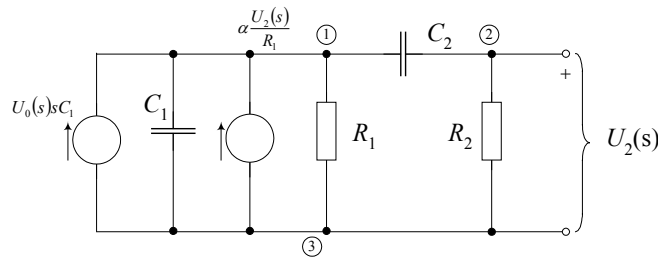
$$p_{1,2} = -\frac{3-\alpha}{2} \pm \sqrt{\left(\frac{3-\alpha}{2}\right)^2 - 1} \leq 0 \Rightarrow \alpha \leq 1$$

konačno α mora biti $\alpha \leq 1$.

19. Odrediti prijenosnu funkciju $H(s) = U_2(s)/U_0(s)$ električnog kruga prema slici. U kojim se granicama mora kretati iznos konstante α da bi polovi funkcije $H(s)$ bili kompleksni? Zadane su normirane vrijednosti elemenata: $R_1=1, R_2=1, C_1=1, C_2=1$.



Rješenje:



$$H(s) = \frac{U_2}{U_0} = \frac{s^2 C_1 C_2 R_2 R_1}{s^2 C_1 C_2 R_2 R_1 + (1-\alpha)s R_2 C_2 + s C_1 R_1 + s C_2 R_1 + 1}$$

$$H(s) = \frac{U_2}{U_0} = \frac{s^2}{s^2 + s(3-\alpha) + 1} \Rightarrow s_{p1,2} = -\frac{3-\alpha}{2} \pm \sqrt{\left(\frac{3-\alpha}{2}\right)^2 - 1}$$

$$1. \text{ uvjet } \left(\frac{3-\alpha}{2}\right)^2 - 1 < 0 \Rightarrow \left(\frac{3-\alpha}{2}\right)^2 < 1 \Rightarrow 9 - 6\alpha + \alpha^2 < 4 \Rightarrow \alpha^2 - 6\alpha + 5 < 0$$

$$\Rightarrow \alpha_{1,2} = 3 \pm \sqrt{9-5} = 3 \pm 2 \Rightarrow 1 < \alpha < 5$$

$$2. \text{ uvjet stabilnost : } \operatorname{Re}[p_{1,2}] = -\frac{3-\alpha}{2} \leq 0 \Rightarrow \alpha \leq 3$$

konačno α mora biti u granicama $1 < \alpha \leq 3$.