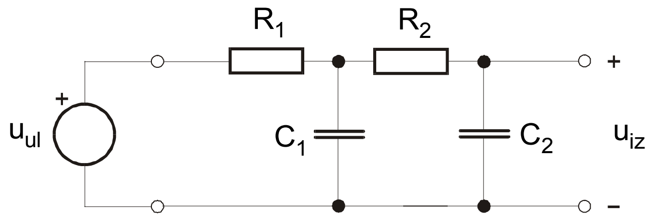
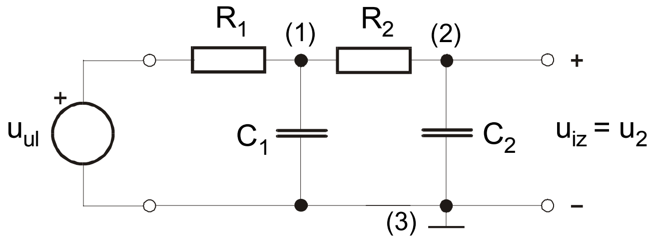
Prijenosne funkcije i a.-f. karakteristika: Zadaci sa rješenjima za vježbu

1. Za električni krug na slici odrediti prijenosnu funkciju napona $T(s)=\dfrac{U_{iz}(s)}{U_{ul}(s)}$. Prikazati frekvencijsku karakteristiku $|T(j\omega)|$. Zadano je: $R_1=1$, $R_2=1$, $C_1=1$, $C_2=1$.

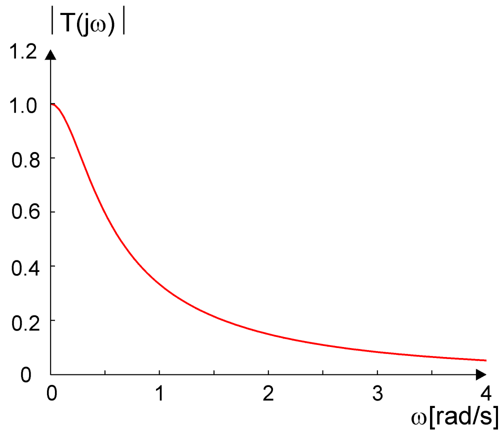


Rješenje:

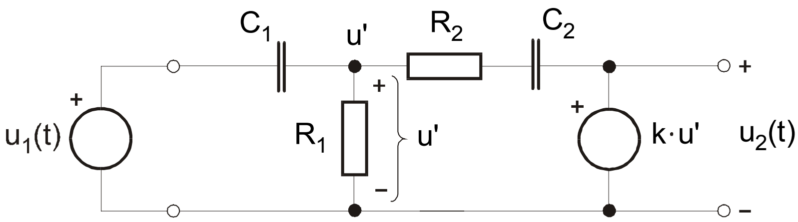


$$T(s)=\frac{U_{iz}}{U_{ul}}=\frac{1}{s^2R_1R_2C_1C_2+s(R_1C_1+R_2C_2+R_1C_2)+1}=\frac{1}{s^2+3s+1}$$

$$T(j\omega)=\frac{1}{-\omega^2+3j\omega+1}=\frac{1}{(1-\omega^2)+3j\omega}\qquad\Rightarrow\qquad |T(j\omega)|=\frac{1}{\sqrt{(1-\omega^2)^2+9\omega^2}}$$



2. Odrediti prijenosnu funkciju $T(s)=\dfrac{U_2(s)}{U_1(s)}$ za električni krug na slici. Zadane su vrijednosti elemenata: $R_1=R_2=10\,K\Omega$, $C_1=C_2=1nF$, i parametar k=2. Izvršiti normalizaciju elemenata po frekvenciji $\omega_0=10^5$rad/s i impedanciji $R_0=10^4\Omega$. Odrediti normaliziranu prijenosnu funkciju. Prikazati raspored nula i polova u s ravnini za normaliziranu prijenosnu funkciju.



Rješenje:

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{k\cdot s\cdot\left(s+\frac{1}{R_2C_2}\right)}{s^2+s\frac{R_1C_1+R_2C_2+R_1C_2(1-k)}{R_1R_2C_1C_2}+\frac{1}{R_1R_2C_1C_2}}$$

uz uvrštene vrijednosti elemenata:

$$T(s)=\frac{U_2(s)}{U_1(s)}=\frac{2\cdot s\cdot\left(s+10^5\right)}{s^2+10^5s+10^{10}}$$

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normalizacija na: $R_0=10^4\,\Omega$, $\omega_0=10^5\ rad/s$:

$$R_n=\frac{R}{R_0}=\frac{10^4}{10^4}=1 \qquad \Rightarrow \qquad R_1=R_2=1$$

$$C_n=\omega_0\cdot R_0\cdot C=10^5\cdot 10^4\cdot 10^{-9} \qquad \Rightarrow \qquad C_1=C_2=1$$

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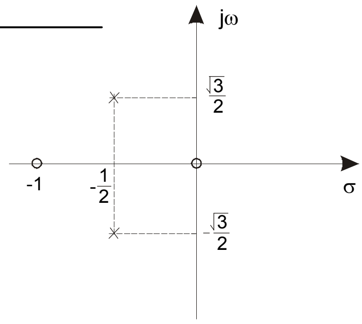
uz uvrštene normalizirane vrijednosti elemenata:

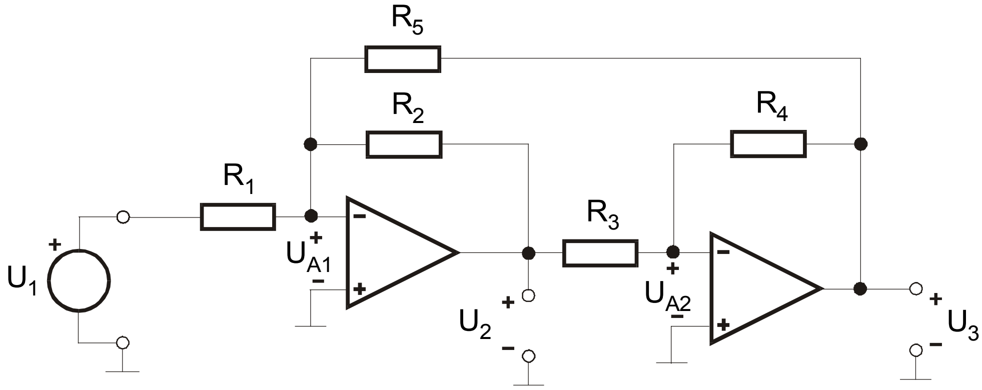
$$T(s)=\frac{U_2}{U_1}=\frac{2s(s+1)}{s^2+s+1}$$

nule: $s_{01}=0,\quad s_{02}=-1$

polovi: $s_{p1,2}=\dfrac{-1\pm\sqrt{1-4}}{2}=-\dfrac{1}{2}\pm j\dfrac{\sqrt{3}}{2}$



3. Za električni krug na slici odrediti naponsku prijenosnu funkciju $T(s)=\dfrac{U_3(s)}{U_1(s)}$. Zadane su normalizirane vrijednosti elemenata: $R_1=R_2=R_3=R_4=1$, $R_5=2$.



Rješenje:

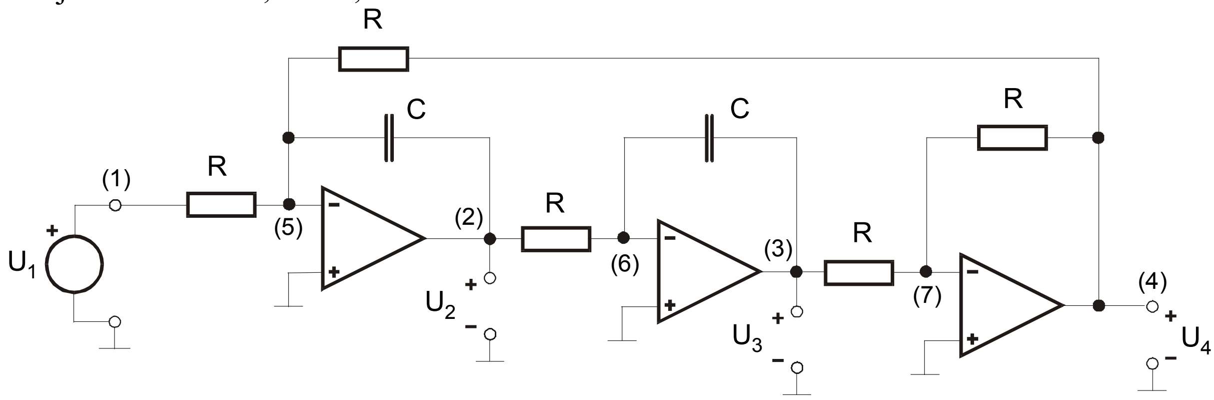
$$T(s)=\frac{U_3}{U_1}=\frac{R_2R_4R_5}{R_1(R_3R_5-R_2R_4)}=2$$

4. Za električni krug prikazan slikom naći prijenosnu funkciju napona:

A) $T(s) = \dfrac{U_4(s)}{U_1(s)}$

B) $T(s) = \dfrac{U_3(s)}{U_1(s)}$

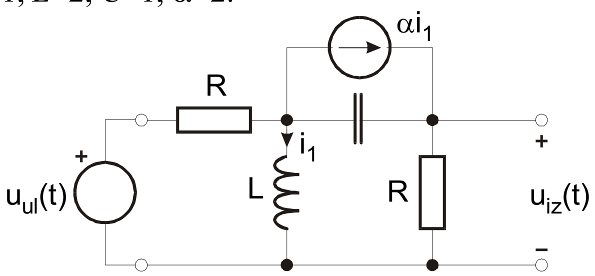
ako je zadano $R = 1$, $C = 1$, $A \to \infty$.



Rješenje:

A) $T(s) = \dfrac{U_4}{U_1} = -\dfrac{1}{1+s^2}$ B) $T(s) = \dfrac{U_3}{U_1} = \dfrac{1}{1+s^2}$

5. Odrediti prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$ za električni krug prikazan slikom. Nacrtati raspored polova i nula te funkcije u kompleksnoj s-ravnini. Odrediti i nacrtati funkciju $|T(j\omega)|$. Zadano je R=1, L=2, C=1, α=2.



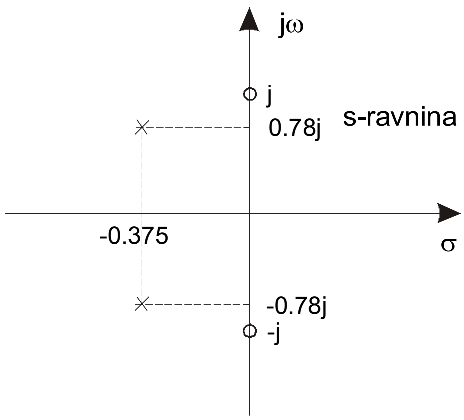
Rješenje:

$$T(s) = \frac{U_{iz}}{U_{ul}} = \frac{2(s^2+1)}{(s+1)(2s^2+2s+3)-2s(s^2+1)} = \frac{2(s^2+1)}{4s^2+3s+3} = \frac{1}{2}\cdot\frac{s^2+1}{s^2+\frac{3}{4}s+\frac{3}{4}}$$

nule: $s^2+1=0 \Rightarrow s^2=-1 \;/\sqrt{} \;\Rightarrow s_{o1,2}=\pm j$

polovi: $s^2+\frac{3}{4}s+\frac{3}{4}=0 \Rightarrow s_{p1,2}=\frac{-3}{8}\pm\sqrt{\frac{9}{64}-\frac{3}{4}}=\frac{-3}{8}\pm\sqrt{\frac{9-48}{64}}=\frac{-3}{8}\pm j\frac{\sqrt{39}}{8}$

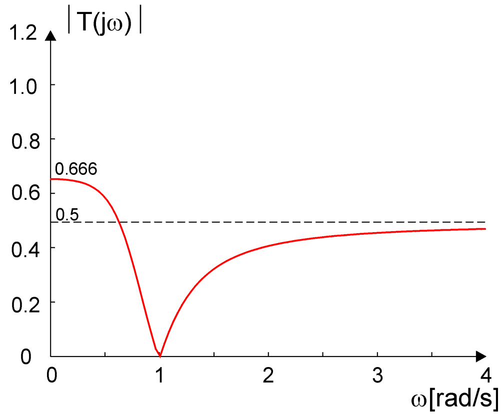
$\Rightarrow s_{p1,2}=-0.375\pm j0.78065$



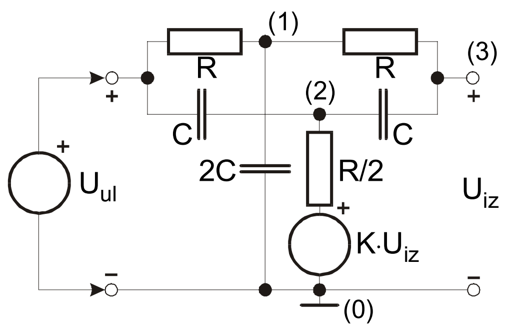
$|T(j\omega)|$:

$$T(j\omega)=\frac{1}{2}\cdot\frac{-\omega^2+1}{-\omega^2+\frac{3}{4}j\omega+\frac{3}{4}} \Rightarrow |T(j\omega)|=\frac{1}{2}\cdot\frac{|1-\omega^2|}{\sqrt{\left(\frac{3}{4}-\omega^2\right)^2+\left(\frac{3}{4}\omega\right)^2}}$$

karakteristične točke: $T(0)=\frac{1}{2}\cdot\frac{1}{\frac{3}{4}}=\frac{1}{2}\cdot\frac{4}{3}=\frac{2}{3}=0.666$, $T(\infty)=\lim_{s\to\infty}\frac{1}{2}\cdot\frac{1+\frac{1}{s^2}}{1+\frac{3}{4}\frac{1}{s}+\frac{3}{4}\frac{1}{s^2}}=\frac{1}{2}=0.5$



6. Odrediti prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$ za električni krug prikazan slikom, ako je zadano: $R=1$, $C=1$, $k=1.5$. Nacrtati raspored polova i nula u kompleksnoj s-ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$.



Rješenje:

$$T(s)=\frac{U_{iz}}{U_{ul}}=\frac{s^2+1}{s^2+s+1}$$

Polovi:

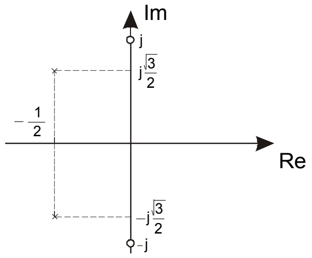
$$s^2 + s + 1 = 0$$

$$s_{P1,2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

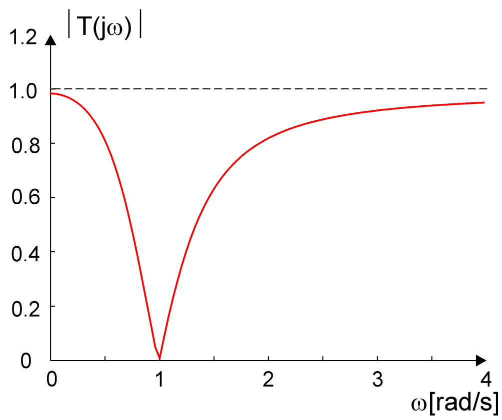
Nule:

$$s^2 + 1 = 0$$

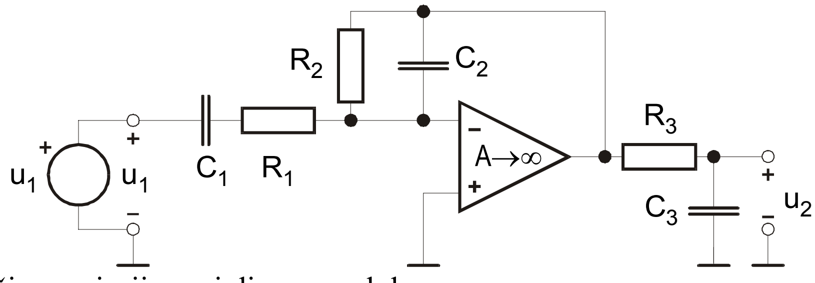
$$s_{o1,2} = \pm j$$



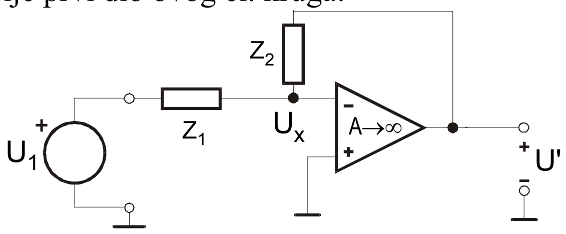
a-f karakteristika:



7. Naći naponsku prijenosnu funkciju $T(s)=U_2(s)/U_1(s)$ električnog kruga na slici. Nacrtati raspored nula i polova u s-ravnini i konstruirati amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$, ako su zadane normalizirane vrijednosti elemenata $R_1=R_2=R_3=1$, $C_1=1$, $C_2=1/2$, $C_3=2$.



Rješenje: Proučimo najprije prvi dio ovog el. kruga:



$$\boxed{\frac{U'}{U_1}=-\frac{\frac{1}{Z_1}}{\frac{1}{A}\left(\frac{1}{Z_1}+\frac{1}{Z_2}\right)+\frac{1}{Z_2}}}$$

$$A\to\infty\ \Rightarrow\ \frac{U'}{U_1}=-\frac{\frac{1}{Z_1}}{\frac{1}{Z_2}}=-\frac{Z_2}{Z_1},\quad Z_1(s)=R_1+\frac{1}{sC_1}=\frac{1+sR_1C_1}{sC_1},\ Z_2(s)=\frac{R_2\frac{1}{sC_2}}{R_2+\frac{1}{sC_2}}=\frac{R_2}{1+sR_2C_2}$$

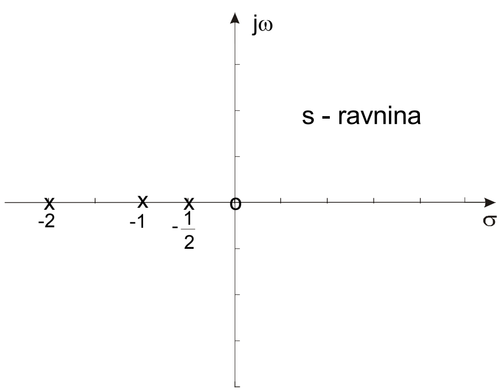
$$\frac{U'}{U_1}=-\frac{sC_1R_2}{(1+sR_1C_1)(1+sR_2C_2)}=\frac{-s\frac{1}{R_1C_2}}{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}$$

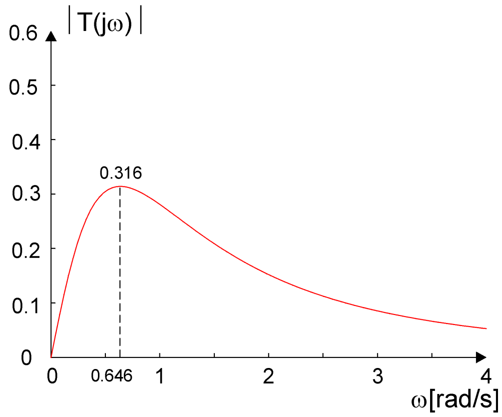
$$\frac{U_2}{U'}=\frac{\frac{1}{sC_3}}{R_3+\frac{1}{sC_3}}=\frac{1}{sR_3C_3+1}=\frac{\frac{1}{R_3C_3}}{s+\frac{1}{R_3C_3}}$$

$$T(s)=\frac{U_2}{U_1}=\frac{U'}{U_1}\cdot\frac{U_2}{U'}=\frac{-s\frac{1}{R_1C_2}}{\left(s+\frac{1}{R_1C_1}\right)\left(s+\frac{1}{R_2C_2}\right)}\cdot\frac{\frac{1}{R_3C_3}}{\left(s+\frac{1}{R_3C_3}\right)}$$

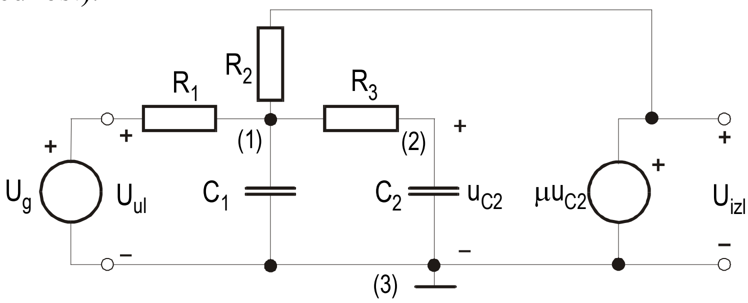
$$R_1=R_2=R_3=1;\quad C_1=1;\quad C_2=\frac{1}{2};\quad C_3=2$$

$$T(s)=\frac{U_2}{U_1}=\frac{-s}{(s+1)\left(s+\frac{1}{2}\right)(s+2)}$$





8. Za električni krug prikazan slikom naći prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$. Nacrtati raspored nula i polova u kompleksnoj s-ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$. Zadane su vrijednosti elemenata: $R_1=R_2=0.5$, $R_3=1$, $C_1=C_2=1$, $\mu=-3$, (pažnja: naponsko pojačanje μ naponski upravljanog naponskog izvora u mreži ima negativnu vrijednost).



Rješenje:

$$T(s)=\frac{U_2}{U_1}=\frac{-\mu\cdot R_2}{s^2(R_1R_2R_3C_1C_2)+s(R_1R_2C_1+R_2R_3C_2+R_1R_3C_2+R_1R_2C_2)+R_1\cdot(1+\mu)+R_2}$$

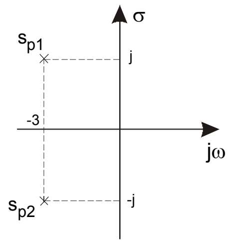
$$T(s)=\frac{-3\cdot 0.5}{0.25s^2+s(0.25+0.5+0.5+0.25)+0.5+4\cdot 0.5}=\frac{-1.5 \quad /\cdot 4}{0.25s^2+1.5s+2.5 \quad /\cdot 4}=$$

$$=\frac{-6}{s^2+6s+10}=\frac{-\frac{3}{5}\cdot 10}{s^2+6s+10}$$

Na frekvenciji ω=0 pojačanje prijenosne funkcije je T(0)=–3/5.

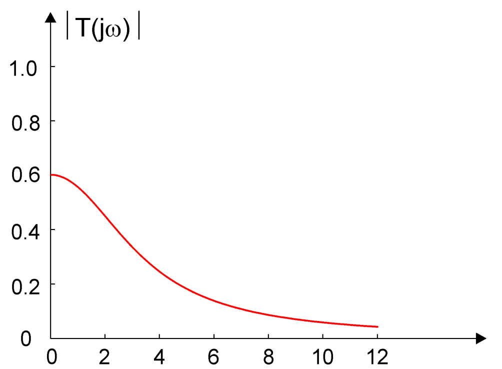
polovi: $s^2+6s+10=0;\qquad \Delta=b^2-4ac=36-4\cdot 10=-4$

$$s_{p1,2}=\frac{-6\pm j2}{2}=-3\pm j$$

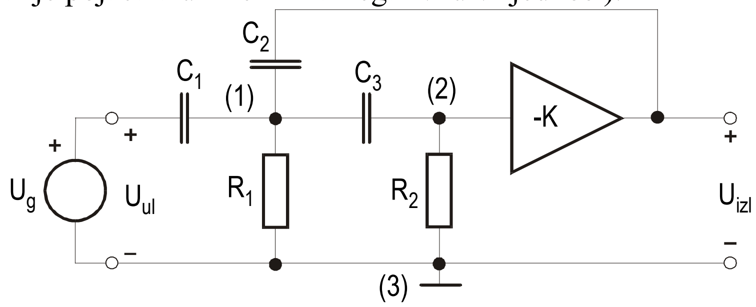


nule: dvije nule u beskonačnosti

$$|T(j\omega)|=\left|\frac{-6}{-\omega^2+6j\omega+10}\right|=\frac{6}{\sqrt{(10-\omega^2)^2+36\omega^2}}$$



9. Za električni krug prikazanu slikom naći prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$. Nacrtati raspored nula i polova u kompleksnoj s-ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$. Zadane su vrijednosti elemenata: $R_1=R_2=1$, $C_1=C_2=0.5$, $C_3=1$, $|K|=3$, (pažnja: pojačanje pojačala u mreži ima negativnu vrijednost).



Rješenje:

$$T(s)=\frac{U_{iz}}{U_{ul}}=\frac{-Ks^2R_1R_2C_1C_3}{s^2R_1R_2C_3\left(C_1+(1+K)C_2\right)+s\left[R_2C_3+R_1(C_1+C_2)+R_1C_3\right]+1}$$

$$T(s)=-\frac{1.5s^2}{2.5s^2+s\cdot[2+1]+1}=-\frac{0.6s^2}{s^2+1.2s+0.4}$$

Na frekvenciji ω=∞ pojačanje prijenosne funkcije je T(0)=–0.6.

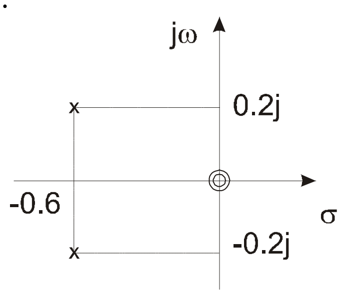
polovi: $s^2+1.2s+0.4=0$

$$s_{p1,2}=\frac{-1.2\pm\sqrt{1.2^2-4\cdot 0.4}}{2}=-0.6\pm 0.2j$$

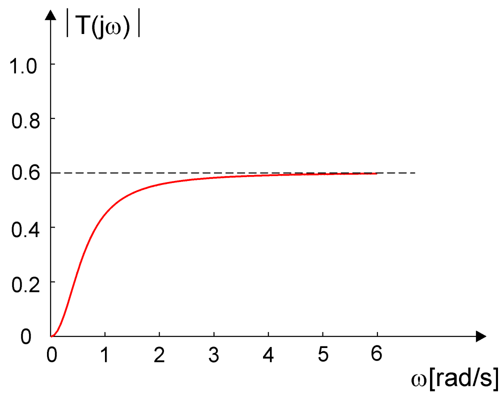
nule: $s^2=0$

$s_{01,2}=0$ dvostruka nula u ishodištu

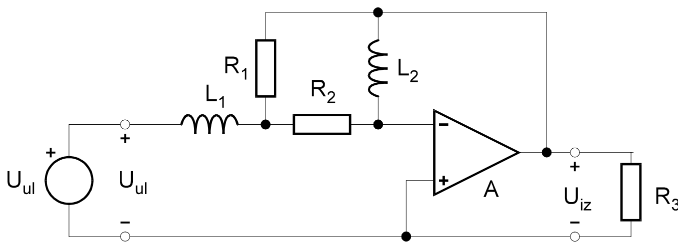
raspored nula i polova u s-ravnini:



$$|T(j\omega)|=\left|\frac{-0.6\omega^2}{-\omega^2+1.2j\omega+0.4}\right|=\frac{0.6\omega^2}{\sqrt{(0.4-\omega^2)^2+(1.2\omega)^2}}$$



10. Za električni krug prikazan slikom treba naći prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$, položaj polova i nula u kompleksnoj s-ravnini i konstruirati amplitudno-frekvencijsku karakteristiku. Zadane su normalizirane vrijednosti elemenata: $R_1=1$, $R_2=1$, $R_3=1$, $L_1=1/\sqrt{2}$, $L_2=\sqrt{2}$ te $A\to\infty$.



Rješenje:

$$T(s)=\frac{U_{izl}}{U_{ul}}=\frac{-s\cdot\dfrac{R_1}{L_1}}{s^2+s\dfrac{R_1+R_2}{L_2}+\dfrac{R_1R_2}{L_1L_2}}$$

uz uvrštene vrijednosti dobivamo:

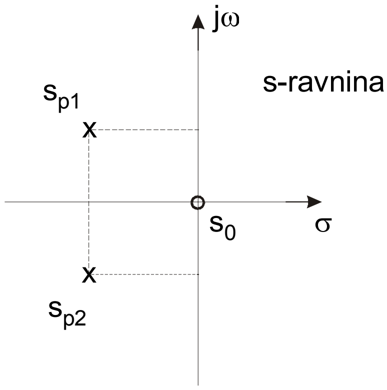
$$T(s)=\frac{U_{iz}}{U_{ul}}=-\frac{s\cdot\sqrt{2}}{s^2+\dfrac{2}{\sqrt{2}}s+\dfrac{1}{\dfrac{1}{\sqrt{2}}\cdot\sqrt{2}}}=-\frac{s\cdot\sqrt{2}}{s^2+\sqrt{2}\cdot s+1}$$

polovi: $s^2+\sqrt{2}\cdot s+1=0$

$$\Rightarrow\ s_{p1,2}=\frac{-\sqrt{2}\pm\sqrt{2-4}}{2}=\frac{-\sqrt{2}\pm j\sqrt{2}}{2}$$

$$s_{p1,2}=-\frac{\sqrt{2}}{2}\pm j\frac{\sqrt{2}}{2}$$

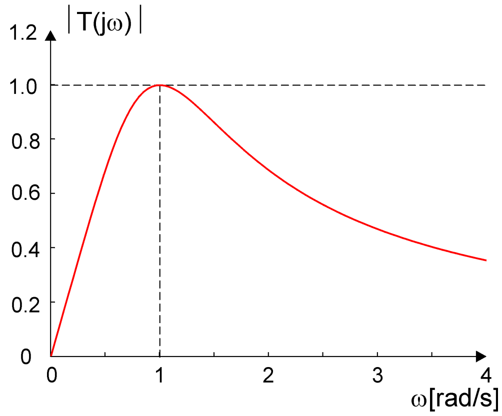
nule: $s_{01}=0$ jedna nula u ishodištu, a druga u beskonačnosti



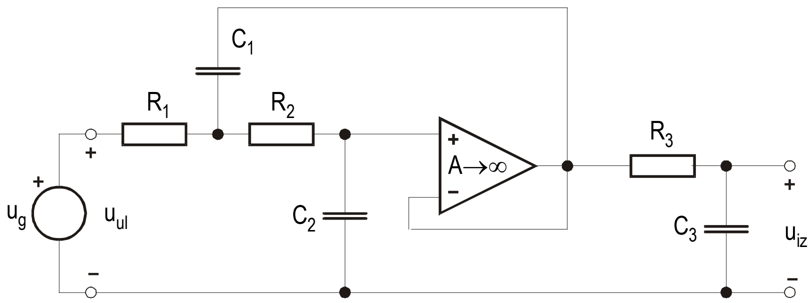
a-f karakteristika (uvrstimo $s=j\omega$ u $T(s)$):

$$T(j\omega)=\frac{U_{iz}(j\omega)}{U_{ul}(j\omega)}=-\frac{-j\omega\cdot\sqrt{2}}{-\omega^2+\sqrt{2}\cdot j\omega+1}$$

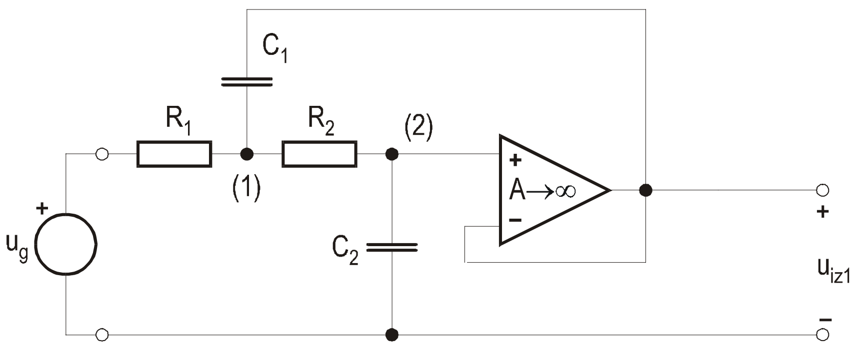
$$\left|T(j\omega)\right|=\frac{\omega\sqrt{2}}{\sqrt{\left(1-\omega^2\right)^2+2\omega^2}}=\frac{\omega\sqrt{2}}{\sqrt{1-2\omega^2+\omega^4+2\omega^2}}=\sqrt{2}\cdot\frac{\omega}{\sqrt{1+\omega^4}}$$



11. Za električni krug prikazan slikom odrediti prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$. Nacrtati položaj nula i polova u kompleksnoj s-ravnini te nacrtati amplitudno-frekvencijsku karakteristiku prijenosne funkcije i označiti karakteristične točke. Zadano je $R_1=1$, $R_2=1$, $R_3=1$, $C_1=2$, $C_2=1/2$, $C_3=1$.



Rješenje:
Ukupna prijenosna funkcija sastoji se od dva dijela i ima slijedeći oblik: $T(s)=T_1(s)\cdot T_2(s)$
a) prvi dio:

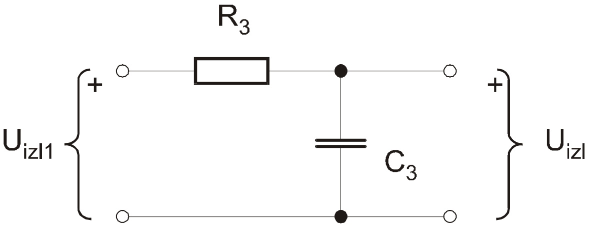


$$T_1(s)=\frac{U_{iz1}}{U_g}=\frac{\frac{1}{R_1}}{\frac{1}{R_1}+\frac{1}{R_2}+sC_1+s\frac{R_2C_2}{R_1}+sC_2+s^2R_2C_1C_2-\frac{1}{R_2}-sC_1}$$

$$=\frac{1}{s^2R_1R_2C_1C_2+s(R_1C_2+R_2C_2)+1}=\frac{1}{s^2+s+1}$$

b) drugi dio: $T_2(s) = \dfrac{U_{iz}}{U_{iz1}}$

$$T_2(s) = \frac{\dfrac{1}{sC_3}}{R_3 + \dfrac{1}{sC_3}} = \frac{\dfrac{1}{R_3C_3}}{s + \dfrac{1}{R_3C_3}} = \frac{1}{s+1}$$



Konačno je ukupna prijenosna funkcija: $T(s) = \dfrac{1}{(s+1)(s^2+s+1)}$

polovi: $s^2 + s + 1 = 0$

$$s_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm j\sqrt{3}}{2}$$

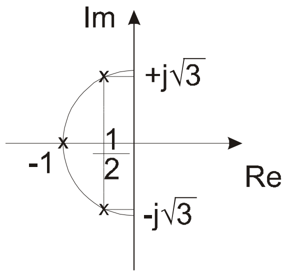
$s + 1 = 0 \Rightarrow \qquad s_3 = -1$

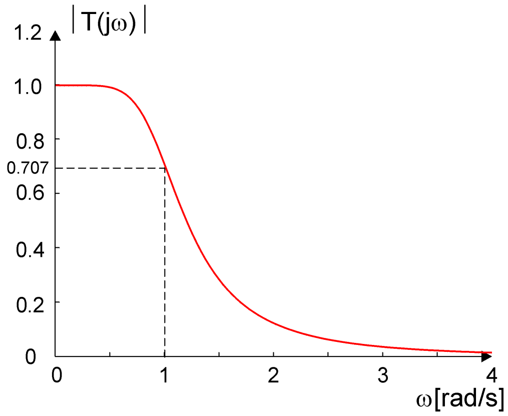
$s_1 = s_2^*$

nule: postoje tri nule u beskonačnosti

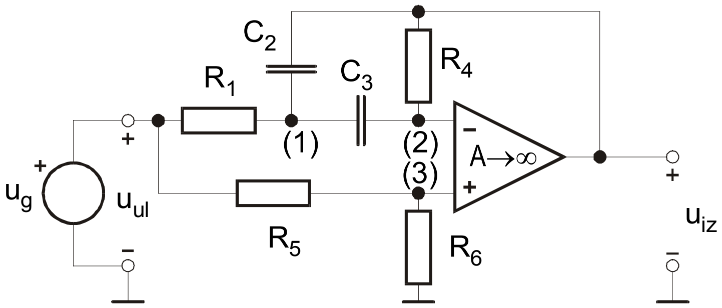
a-f karakteristika:

$$T(j\omega) = \frac{1}{(j\omega+1)(-\omega^2 + j\omega + 1)} \qquad \Rightarrow \qquad |T(j\omega)| = \frac{1}{\sqrt{\omega^2+1}\sqrt{(1-\omega^2)^2+\omega^2}}$$





12. Za električni krug prikazan slikom naći prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$. Nacrtati položaj nula i polova u kompleksnoj s-ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$. Zadane su normalizirane vrijednosti elemenata: $R_1=1/2$, $R_4=R_6=2$, $R_5=1$, $C_2=C_3=1$.



Rješenje:

$$T(s)=\frac{U_{iz}}{U_{ul}}=\alpha\cdot\frac{s^2C_2C_3R_4R_1+s\left[\left(1-\frac{1}{\alpha}\right)C_3R_4+C_2R_1+C_3R_1\right]+1}{s^2C_2C_3R_4R_1+s(C_2R_1+C_3R_1)+1},\ \alpha=\frac{R_6}{R_5+R_6}\ (0<\alpha<1);$$

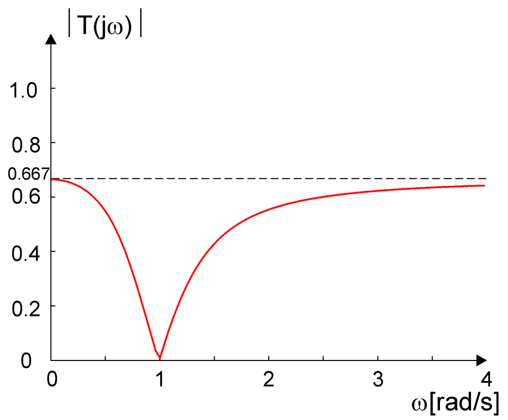
Uvrstimo vrijednosti, konačno je prijenosna funkcija:

$$\boxed{T(s)=\frac{2}{3}\cdot\frac{s^2+1}{s^2+s+1}}$$

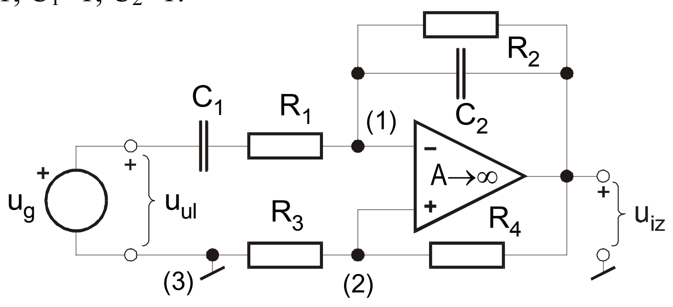
$$\text{nule}: s_{o1,2}=\pm j$$

$$\text{polovi}: s_{p1,2}=-\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$$

$$T(j\omega)=\frac{2}{3}\frac{\left|1-\omega^2\right|}{-\omega^2+j\omega+1}\Rightarrow\left|T(j\omega)\right|=\frac{2}{3}\frac{\left|1-\omega^2\right|}{\sqrt{(1-\omega^2)^2+\omega^2}}$$



13. Za električni krug prikazan slikom naći prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$. Nacrtati položaj nula i polova u kompleksnoj s-ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$. Zadane su normalizirane vrijednosti elemenata: $R_1=1/2$, $R_2=1$, $R_3=1$, $R_4=1$, $C_1=1$, $C_2=1$.



Rješenje:

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{-\left(1+\frac{R_3}{R_4}\right)s\frac{1}{R_1C_2}}{s^2+s\frac{R_1C_1+R_2C_2-R_2C_1R_3/R_4}{R_1C_1R_2C_2}+\frac{1}{R_1C_1R_2C_2}}$$

uz uvrštene vrijednosti elemenata je prijenosna funkcija:

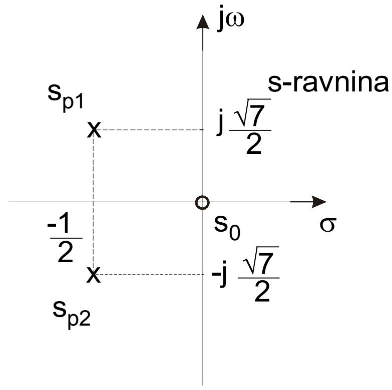
$$T(s)=\frac{-4\cdot s}{s^2+s+2}$$

Polovi: $s^2+s+2=0$

$$s_{p1,2}=-\frac{1}{2}\pm\sqrt{\frac{1}{4}-2}=-\frac{1}{2}\pm j\frac{\sqrt{7}}{2}$$

Nule: s=0

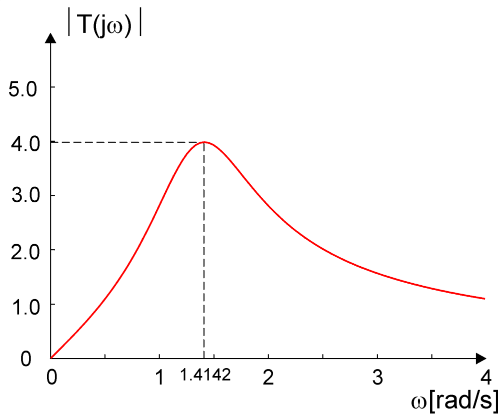
$s_{o1}=0,\quad s_{o2}=\infty$



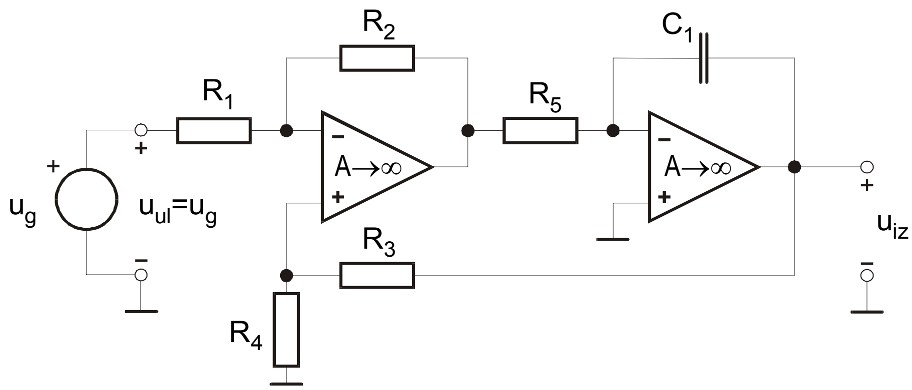
a-f karakteristika : $T(j\omega)=\dfrac{-4\cdot j\omega}{-\omega^2+j\omega+2}\Rightarrow$

$$|T(j\omega)|=\frac{4\cdot\omega}{\sqrt{(2-\omega^2)^2+\omega^2}}$$

Graf:



14. Odrediti prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$ za električni krug prikazan slikom.

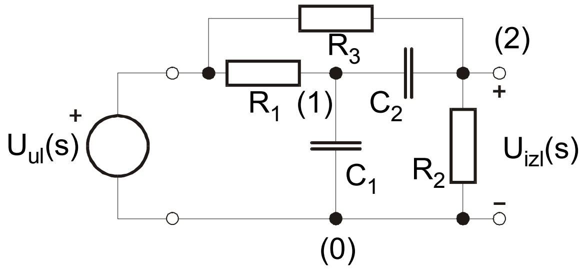


Rješenje:

$$T(s)=\frac{U_6}{U_1}=\frac{1}{s\dfrac{R_1}{R_2}R_5C_1+\dfrac{R_4}{R_3+R_4}\cdot\dfrac{R_1+R_2}{R_2}}=\frac{\dfrac{R_2}{R_1}\cdot\dfrac{1}{R_5C_1}}{s+\dfrac{R_4}{R_3+R_4}\cdot\dfrac{R_1+R_2}{R_1}\cdot\dfrac{1}{R_5\cdot C_1}}=$$

$$=\frac{R_2(R_3+R_4)}{sC_1R_1R_5(R_3+R_4)+R_4(R_1+R_2)}$$

15. Naći naponsku prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$ električnog kruga na slici. Nacrtati položaj nula i polova u s-ravnini i konstruirati amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$, ako su zadane normalizirane vrijednosti elemenata $R_1=R_2=R_3=1$, $C_1=C_2=1$.



Rješenje:

$$U_1\left(\frac{1}{R_1}+sC_1+sC_2\right)-U_2\cdot sC_2=U_{ul}\cdot\frac{1}{R_1}$$

$$-U_1\cdot sC_2+U_2\left(\frac{1}{R_2}+\frac{1}{R_3}+sC_2\right)=U_{ul}\cdot\frac{1}{R_3}$$

$$U_2=\frac{\Delta_2}{\Delta};\qquad T(s)=\frac{U_{iz}}{U_{ul}};\qquad U_{iz}=U_2$$

$$\Delta=\begin{vmatrix}\dfrac{1}{R_1}+sC_1+sC_2 & -sC_2\\ -sC_2 & \dfrac{1}{R_2}+\dfrac{1}{R_3}+sC_2\end{vmatrix}=\left(\frac{1}{R_1}+sC_1+sC_2\right)\cdot\left(\frac{1}{R_2}+\frac{1}{R_3}+sC_2\right)-(sC_2)^2$$

$$\Delta_2=\begin{vmatrix}\dfrac{1}{R_1}+sC_1+sC_2 & U_{ul}\cdot\dfrac{1}{R_1}\\ -sC_2 & U_{ul}\cdot\dfrac{1}{R_3}\end{vmatrix}=U_{ul}\cdot\left[\frac{1}{R_3}\left(\frac{1}{R_1}+sC_1+sC_2\right)+\frac{sC_2}{R_1}\right]$$

$$T(s)=\frac{U_{iz}}{U_{ul}}=\frac{\Delta_2/\Delta}{U_{ul}}=\frac{\dfrac{1}{C_1C_2R_1R_2}+s\left[\left(\dfrac{1}{C_1}+\dfrac{1}{C_2}\right)\dfrac{1}{R_3}+\dfrac{1}{R_1C_1}\right]}{\dfrac{1}{R_1C_1C_2}\left(\dfrac{1}{R_2}+\dfrac{1}{R_3}\right)+s\left[\left(\dfrac{1}{C_1}+\dfrac{1}{C_2}\right)\cdot\left(\dfrac{1}{R_2}+\dfrac{1}{R_3}\right)+\dfrac{1}{R_1C_1}\right]+s^2}$$

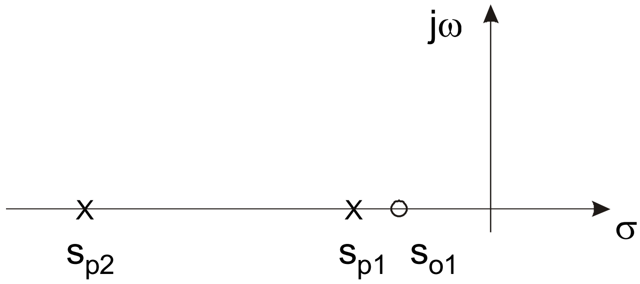
Uz uvrštene vrijednosti elemenata $R_1=R_2=R_3=1,\ C_1=C_2=1$, prijenosna funkcija glasi:

$$T(s)=\frac{1+3s}{2+5s+s^2}$$

$$\text{nule}:\quad 1+3s=0\ \Rightarrow\ s_{o1}=-\frac{1}{3},\qquad s_{o2}=\infty$$

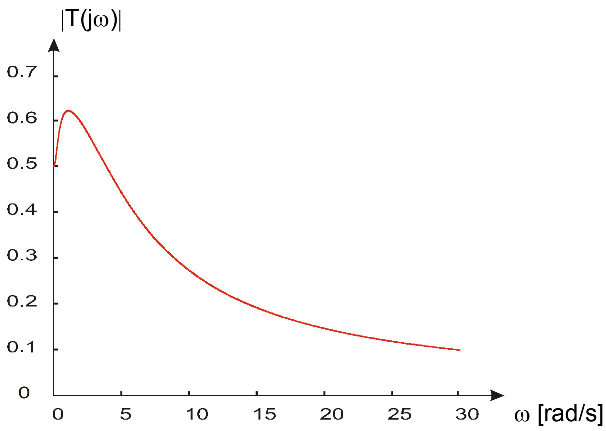
$$\text{polovi}: s^2+5s+2=0\ \Rightarrow\quad s_{p_{1,2}}=\frac{-s\pm\sqrt{25-8}}{2}=\frac{-5\pm\sqrt{17}}{2}$$

$$s_{p_1}=-0.4385\,,\quad s_{p_2}=-4.561$$

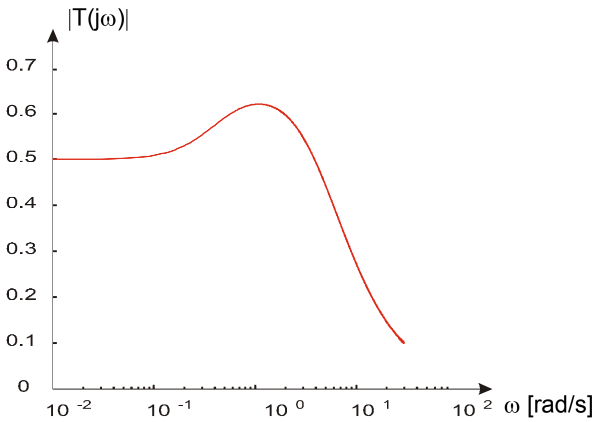


a-f karakteristika:

$$T(j\omega)=\frac{1+3j\omega}{2+5j\omega-\omega^2} \quad \Rightarrow \quad |T(j\omega)|=\frac{\sqrt{1+(3\omega)^2}}{\sqrt{(2-\omega^2)^2+(5\omega)^2}}$$

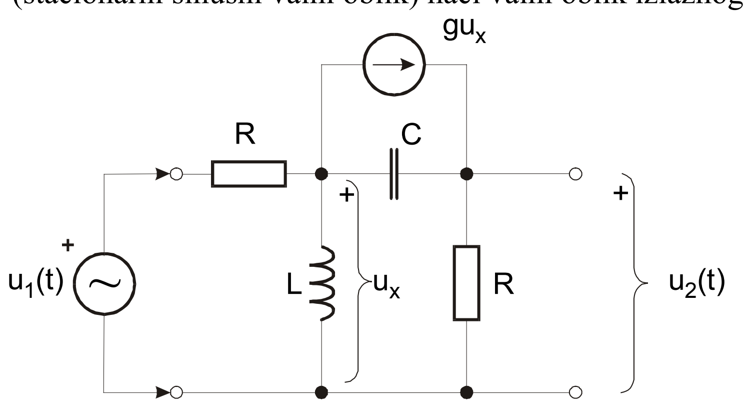


frekvencijska os je u linearnom mjerilu



frekvencijska os je u logaritamskom mjerilu

16. Za električni krug prikazan slikom naći prijenosnu funkciju napona $T(s)=\dfrac{U_2(s)}{U_1(s)}$ ako su zadane normalizirane vrijednosti elemenata: *R*=1, *C*=1, *L*=2 i parametar *g*=2. Za pobudu $u_1(t)=10\cos t$ (stacionarni sinusni valni oblik) naći valni oblik izlaznog napona $u_2(t)$.



Rješenje:

Prijenosna funkcija napona glasi: $T(s)=\dfrac{U_2(s)}{U_1(s)}=\dfrac{2s^2+4s}{4s^2+7s+1}$

Pošto je pobuda sinusoidalnog stacionarnog valnog oblika u prijenosnu funkciju $T(s)$ supstituiramo $s = j\omega$ i dobivamo:

$$T(j\omega) = \frac{-2\omega^2 + 4j\omega}{(1-4\omega^2) + 7j\omega} \qquad (*)$$

Posljednji izraz se može napisati tako da se može razdvojiti realni i imaginarni dio :

$$T(j\omega) = \frac{-2\omega^2 + 4j\omega}{(1-4\omega^2) + 7j\omega} \cdot \frac{(1-4\omega^2) - 7j\omega}{(1-4\omega^2) - 7j\omega} = \frac{2\omega^2(13+4\omega^2) + 2j\omega(2-\omega^2)}{(1-4\omega^2)^2 + (7\omega)^2} \qquad (**)$$

Frekvencija pobudnog signala je jednaka 1 stoga uvrštavamo $\omega = 1$ u gornje izraze za $T(j\omega)$.

$$T(1j) = \frac{-2+4j}{-3+7j} \quad (*); \text{ ili} \qquad T(1j) = 2\frac{17+j}{9+49} = \frac{17+j}{29} \quad (**)$$

Fazor pobudnog signala je : $U_1(j\omega)=10\angle 0°$

Fazor odzivnog signala se izračuna iz : $U_2(j\omega)= T(j\omega)\, U_1(j\omega)$, odnosno:

Amplituda izlaznog signala je: $\left|U_2(j\omega)\right| = \left|T(j\omega)\right|_{\omega=1} \cdot \left|U_1(j\omega)\right|$

Fazni kut izlaznog signala je: $\arg U_2(j\omega) = \arg T(j\omega)_{\omega=1} + \arg U_1(j\omega)$

Amplituda izlaznog signala:

(*) $\Rightarrow$

$$\left|U_2(j\omega)\right| = 10 \cdot \left.\frac{\sqrt{4\omega^4 + 16\omega^2}}{\sqrt{(1-4\omega^2)^2 + (7\omega)^2}}\right|_{\omega=1} = 10\frac{\sqrt{2^2+4^2}}{\sqrt{3^2+7^2}} = 10\frac{\sqrt{20}}{\sqrt{58}} = 5.8722 \text{ (jednostavniji način)}$$

(**) $\Rightarrow$

$$\left|U_2(j\omega)\right| = 10 \cdot \left.\frac{\sqrt{4\omega^4(13+4\omega^2)^2 + 4\omega^2(2-\omega^2)^2}}{(1-4\omega^2)^2 + (7\omega)^2}\right|_{\omega=1} = 10\frac{\sqrt{17^2+1^2}}{29} = 5.8722$$

Fazni kut izlaznog signala:

(*) $\Rightarrow$

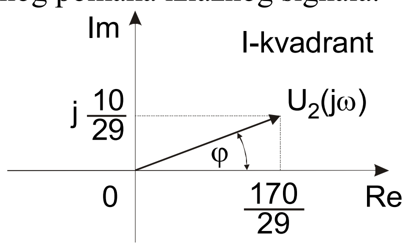
$$\varphi = \varphi_{Brojnika} - \varphi_{Nazivnika} = \left.\text{arctg}\frac{4\omega}{-2\omega^2}\right|_{\omega=1} - \left.\text{arctg}\frac{7\omega}{1-4\omega^2}\right|_{\omega=1} =$$

$$= \text{arctg}\frac{4}{-2} - \text{arctg}\frac{7}{-3} = -63.43 - (-66.8) = 3.37° \text{ (jednostavniji način)}$$

(**) $\Rightarrow$

$$\varphi = \left.\text{arctg}\frac{2\omega(2-\omega^2)}{2\omega^2(13+4\omega^2)}\right|_{\omega=1} = \text{arctg}\frac{1}{17} = 3.37°$$

Uputno je nacrtati fazor izlaznog signala $U_2(j\omega)$ kako bi se vidjelo u kojem kvadrantu se fazor nalazi i odredio točan iznos faznog pomaka izlaznog signala.



Konačno rješenje tj. valni oblik odziva je: $u_2(t)$=5.87 cos (t + 3.37°).

17. Odziv nekog električnog kruga na pobudu $x(t)=S(t)$ glasi: $y(t)=e^{-3t}\mathrm{ch}(2t)S(t)$. Odrediti funkciju mreže i fazor odziva na pobudu $x_1(t)=2\cos(3t+45^\circ)$ (pobuda je sinusoidalni stacionarni signal).

Rješenje:

$$X(s)=\frac{1}{s}$$

$$Y(s)=\frac{s+3}{(s+3)^2-4}=\frac{s+3}{s^2+6s+5}$$

$$H(s)=\frac{s(s+3)}{(s+3)^2-4}=\frac{s(s+3)}{s^2+6s+5}$$

Fazori:

$$H(\mathrm{j}\omega)=\frac{\mathrm{j}\omega(\mathrm{j}\omega+3)}{(\mathrm{j}\omega)^2+6\mathrm{j}\omega+5}$$

$$X_1(\mathrm{j}\omega)=2e^{\mathrm{j}\pi/4}$$

$$Y_1(\mathrm{j}\omega)=H(\mathrm{j}\omega)X_1(\mathrm{j}\omega)=\frac{\mathrm{j}\omega(\mathrm{j}\omega+3)}{(\mathrm{j}\omega)^2+6\mathrm{j}\omega+5}X_1(\mathrm{j}\omega)=\frac{\mathrm{j}\omega(\mathrm{j}\omega+3)}{(\mathrm{j}\omega)^2+6\mathrm{j}\omega+5}2e^{\mathrm{j}\pi/4}$$

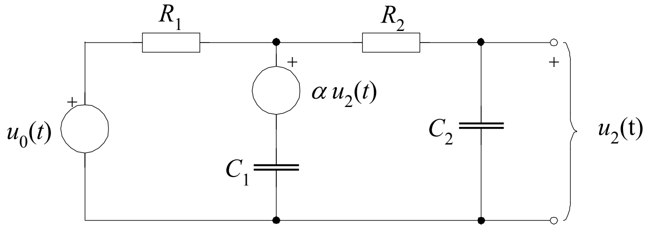
$$\omega=3$$

$$Y_1(\mathrm{j}3)=\frac{\mathrm{j}3\cdot(\mathrm{j}3+3)}{(\mathrm{j}3)^2+18\mathrm{j}+5}\cdot 2\cdot\left(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}\right)$$

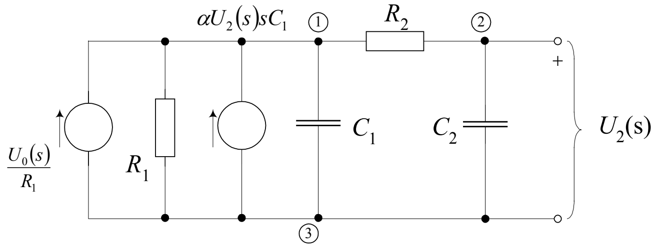
$$Y_1(\mathrm{j}3)=\frac{9\sqrt{2}}{85}\cdot(2+j9)\quad \text{(prvi kvadrant: } 0<\varphi<90^\circ\text{)}$$

$$y_1(t)=\frac{9\sqrt{2}\sqrt{85}}{85}\cos(3t+77.47^\circ)$$

18. Odrediti prijenosnu funkciju $H(s)=U_2(s)/U_0(s)$ električnog kruga prema slici. U kojim se granicama mora kretati iznos konstante α da bi polovi funkcije $H(s)$ bili realni? Zadane su normirane vrijednosti elemenata: $R_1=1$, $R_2=1$, $C_1=1$, $C_2=1$.



Rješenje:



$$H(s)=\frac{U_2}{U_0}=\frac{1}{s^2C_1C_2R_1R_2+s(C_2R_1+C_2R_2+(1-\alpha)C_1R_1)+1}$$

$$H(s)=\frac{U_2}{U_0}=\frac{1}{s^2+s(3-\alpha)+1} \quad \Rightarrow \quad s_{p1,2}=-\frac{3-\alpha}{2}\pm\sqrt{\left(\frac{3-\alpha}{2}\right)^2-1}$$

1. uvjet $\left(\frac{3-\alpha}{2}\right)^2-1\geq 0 \quad \Rightarrow \quad \left(\frac{3-\alpha}{2}\right)^2\geq 1 \Rightarrow 9-6\alpha+\alpha^2\geq 4 \Rightarrow \alpha^2-6\alpha+5\geq 0$

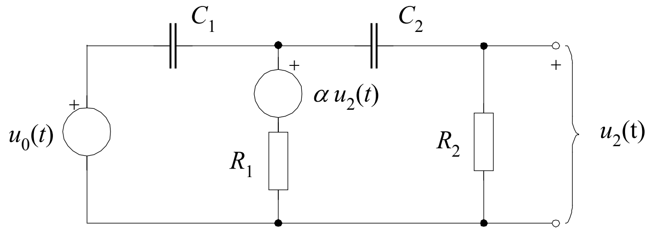
$\Rightarrow \alpha_{1,2}=3\pm\sqrt{9-5}=3\pm 2 \Rightarrow \alpha\leq 1 \,\&\, \alpha\geq 5$

2. uvjet stabilnost: polovi moraju biti u lijevoj poluravnini:

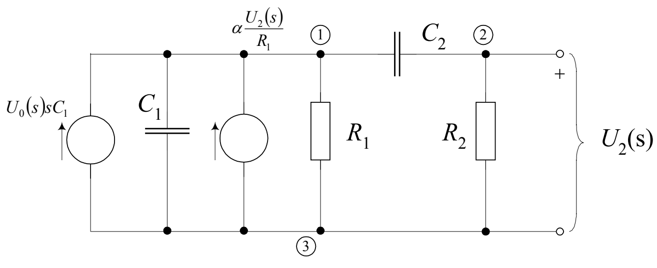
$$p_{1,2}=-\frac{3-\alpha}{2}\pm\sqrt{\left(\frac{3-\alpha}{2}\right)^2-1}\leq 0 \Rightarrow \alpha\leq 1$$

konačno α mora biti $\alpha\leq 1$.

19. Odrediti prijenosnu funkciju $H(s)=U_2(s)/U_0(s)$ električnog kruga prema slici. U kojim se granicama mora kretati iznos konstante α da bi polovi funkcije $H(s)$ bili kompleksni? Zadane su normirane vrijednosti elemenata: $R_1=1$, $R_2=1$, $C_1=1$, $C_2=1$.



Rješenje:



$$H(s)=\frac{U_2}{U_0}=\frac{s^2C_1C_2R_2R_1}{s^2C_1C_2R_2R_1+(1-\alpha)sR_2C_2+sC_1R_1+sC_2R_1+1}$$

$$H(s)=\frac{U_2}{U_0}=\frac{s^2}{s^2+s(3-\alpha)+1} \quad \Rightarrow \quad s_{p1,2}=-\frac{3-\alpha}{2}\pm\sqrt{\left(\frac{3-\alpha}{2}\right)^2-1}$$

1. uvjet $\left(\frac{3-\alpha}{2}\right)^2-1<0 \quad \Rightarrow \quad \left(\frac{3-\alpha}{2}\right)^2<1 \Rightarrow 9-6\alpha+\alpha^2<4 \Rightarrow \alpha^2-6\alpha+5<0$

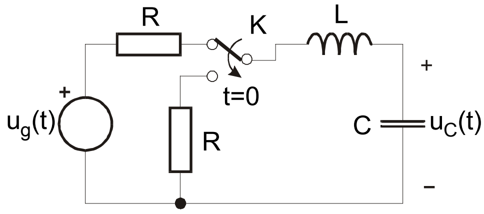
$\Rightarrow \alpha_{1,2}=3\pm\sqrt{9-5}=3\pm 2 \Rightarrow 1<\alpha<5$

2. uvjet stabilnost : $\mathrm{Re}[p_{1,2}]=-\frac{3-\alpha}{2}\leq 0 \Rightarrow \alpha\leq 3$

konačno α mora biti u granicama $1<\alpha\leq 3$.

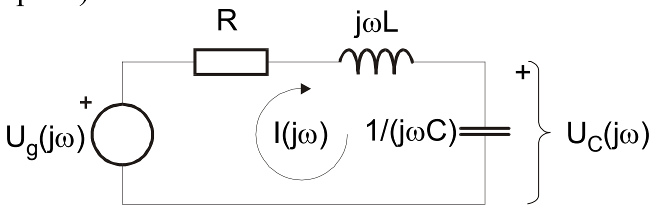
Prijelazne pojave

20. Za mrežu na slici odrediti napon na kapacitetu $u_C(t)$ ako se u trenutku t=0 prebaci sklopka K. Zadano je: R=4, C=1/2, L=2, $u_g(t)$=10sin(2t); $-\infty<t<\infty$ (sinusoidalno stacionarno stanje).



Rješenje:
Zadatak se rješava u dva koraka: u prvom a) koraku se pomoću fazora za t<0 izračuna utjecaj pobude tako da se nađu početni uvjeti: napon na kapacitetu i početna struja kroz induktivitet. U drugom b) koraku se za $t\geq 0$ uz poznate početne uvjete pomoću Laplaceove transformacije izračuna traženi napon na kapacitetu $u_C(t)$.

a) t<0 (fazori stuje i napona)

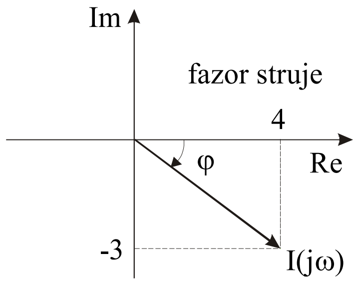


$$U_g(j\omega) = 10\angle 0°, \quad \omega = 2$$

Fazor struje u električnom krugu:

$$I(j\omega) = \frac{U_g(j\omega)}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{U_g(j\omega)}{R + j\left(\omega L - \dfrac{1}{\omega C}\right)} = \frac{10}{4 + j\left(2\cdot 2 - \dfrac{1}{2\cdot(1/2)}\right)} =$$

$$= \frac{10}{4+j3}\cdot\frac{4-j3}{4-j3} = \frac{10\cdot(4-j3)}{25} = \frac{2}{5}\cdot(4-j3)$$



$$|I(j\omega)| = \frac{10}{\sqrt{4^2+3^2}} = \frac{10}{\sqrt{25}} = \frac{10}{5} = 2$$

$$\varphi = arctg\frac{\mathrm{Im}}{\mathrm{Re}} = arctg\frac{-3}{4} = -36.87°$$

Iz fazora slijede podaci o struji u električnom krugu u vremenskoj domeni:

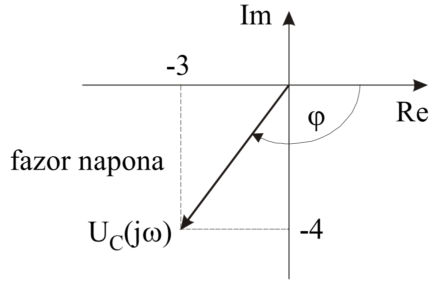
$$i(t) = 2\cdot\sin(2t - 36.87°)$$

U trenutku t=0 se tada može izračunati početna struja u el. krugu koja je ujedno i početna struja kroz induktivitet.

$$i_L(0) = i(t)_{t=0} = 2\cdot\sin(-36.87°) = 2\cdot(-0.6) = -1.2\,A$$

Fazor napon na kapacitetu C u električnom krugu je:

$$U_C(j\omega) = I(j\omega)\cdot\frac{1}{j\omega C} = \frac{10}{4+j3}\cdot\frac{1}{j2\frac{1}{2}} = \frac{10}{4+j3}\cdot\frac{1}{j} = \frac{10}{-3+j4} = \frac{2}{5}(-3-j4)$$



$$\left|U_C(j\omega)\right| = \frac{10}{\sqrt{3^2+4^2}} = \frac{10}{\sqrt{25}} = \frac{10}{5} = 2$$

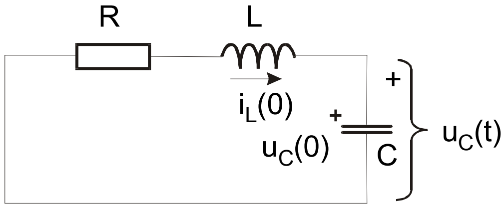
$$\varphi = arctg\frac{\mathrm{Im}}{\mathrm{Re}} = arctg\frac{-3}{-4} = -126.87°$$

$$u_C(t) = 2\cdot\sin(2t-126.87°)$$

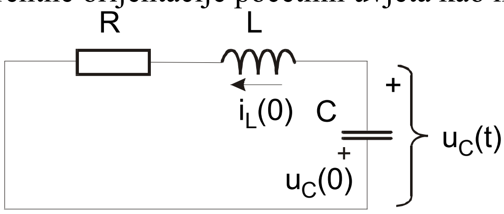
$$u_C(0) = 2\cdot\sin(-126.87°) = 2\cdot(-0.8) = -1.6V$$

b) $t \geq 0$ (Laplaceova transformacija)

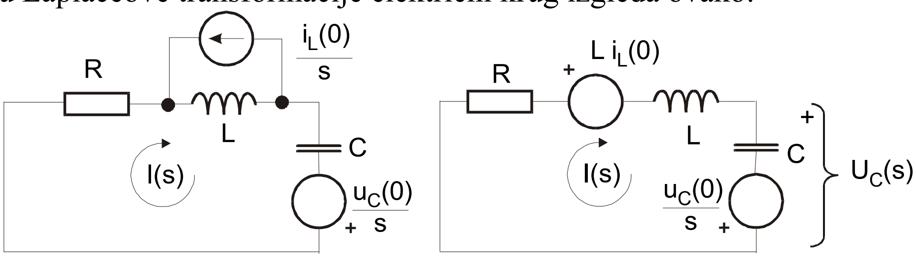
Uz poznate početne uvjete $i_L(0) = -1.2A$ i $u_C(0) = -1.6V$, te pobudu $u_g(t) = 0$, (za $t \geq 0$) električni krug izgleda ovako:



Ako bismo htjeli uvrstiti pozitivne vrijednosti početnih uvjeta $i_L(0) = 1.2A$ i $u_C(0) = 1.6V$, tada trebamo izmjeniti referentne orijentacije početnih uvjeta kao na slijedećoj slici:



Uz primjenu Laplaceove transformacije električni krug izgleda ovako:



Jednadžba za struju za električni krug

$$I(s)\cdot\left(R+\frac{1}{sC}+sL\right)+L\cdot i_L(0)-\frac{u_C(0)}{s} = 0$$

uz uvrštene vrijednosti:

$$I(s)\left(4+\frac{2}{s}+2s\right)+2\cdot 1.2-\frac{1.6}{s}=0$$

$$I(s)=\frac{\frac{1.6}{s}-2.4}{4+\frac{2}{s}+2s}=\frac{1.6-2.4s}{2s^2+4s+2}=\frac{0.8-1.2s}{s^2+2s+1}$$

Traženi napon na kapacitetu je:

$$U_C(s)=I(s)\cdot\frac{1}{sC}-\frac{u_C(0)}{s}$$

uz uvrštene vrijednosti:

$$U_C(s)=\underbrace{\frac{0.8-1.2s}{s^2+2s+1}\cdot\frac{2}{s}}_{(*)}-\frac{1.6}{s}$$

Rastav na parcijalne razlomke izraza (*):

$$(*)=\frac{1.6-2.4s}{s^2+2s+1}\cdot\frac{1}{s}=\frac{As+B}{s^2+2s+1}+\frac{C}{s}=\frac{As^2+Bs+Cs^2+2Cs+C}{(s^2+2s+1)\cdot s}=\frac{(A+C)s^2+(B+2C)s+C}{(s^2+2s+1)\cdot s}$$

$$A+C=0$$
$$B+2C=-2.4$$
$$C=1.6$$

$A=-C=-1.6$;
$B=-2C-2.4=-3.2-2.4=-5.6$.

$$(*)=\frac{-1.6s-5.6}{s^2+2s+1}+\frac{1.6}{s}$$

Konačno je:

$$U_C(s)=\underbrace{-1.6\cdot\frac{s}{(s+1)^2}-5.6\frac{1}{(s+1)^2}+\frac{1.6}{s}}_{(*)}-\frac{1.6}{s}=-1.6\cdot\frac{s}{(s+1)^2}-5.6\frac{1}{(s+1)^2}=$$

$$=-1.6\left[\frac{s+1}{(s+1)^2}-\frac{1}{(s+1)^2}\right]-5.6\frac{1}{(s+1)^2}=$$

$$=-1.6\frac{1}{s+1}+\frac{1.6}{(s+1)^2}-5.6\frac{1}{(s+1)^2}=-\frac{1.6}{s+1}-\frac{4}{(s+1)^2}$$

$$u_C(t)=(-1.6\cdot e^{-t}-4t\cdot e^{-t})\cdot S(t)$$