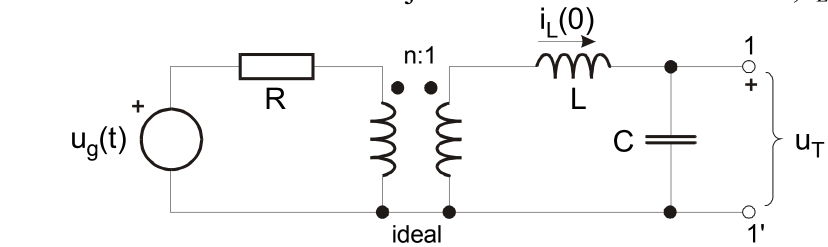
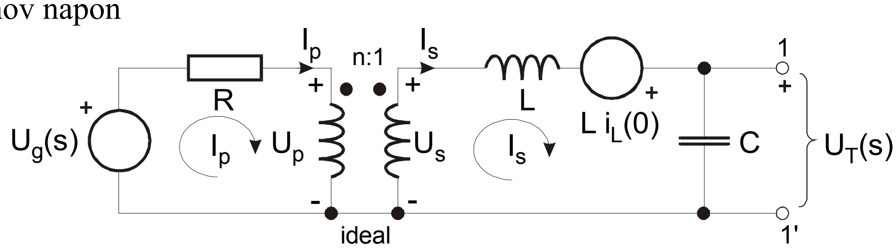
PONOVLJENI ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA

1. Za električni krug prikazan slikom odrediti $U_T(s)$ i $Z_T(s)$ nadomjesne sheme po Teveninu s obzirom na stezaljke 1-1'. Zadane su normalizirane vrijednosti elemenata: $L=C=R=1$, $i_L(0)=1$, $n=2$, $u_g(t)=S(t)$.



Rješenje:

a) Theveninov napon



Jednadžbe transformatora:

$$U_p = n \cdot U_s \quad \Rightarrow \quad U_s = \frac{U_p}{n}$$

$$I_p = \frac{1}{n} \cdot I_s \quad \Rightarrow \quad I_s = nI_p$$

Jednadžbe petlji :

(1) $\quad U_g(s) = I_p \cdot R + U_p(s)$

(2) $\quad U_s = I_s sL + I_s \frac{1}{sC} - Li_L(0)$

(3) $\quad U_T(s) = I_s \cdot \frac{1}{sC}$

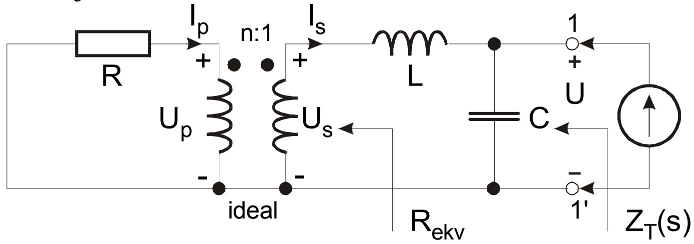
$$(1) \Rightarrow \quad U_g(s) = \frac{1}{n} I_s R + nU_s; \quad (2) \rightarrow (1) \Rightarrow \quad U_g(s) = \frac{1}{n} I_s R + n\left(I_s sL + I_s \frac{1}{sC} - Li_L(0)\right) \Big/ : n$$

$$\Rightarrow \quad \frac{U_g(s)}{n} + Li_L(0) = I_s\left(\frac{R}{n^2} + sL + \frac{1}{sC}\right); \quad \Rightarrow \quad I_s = \frac{\frac{U_g(s)}{n} + Li_L(0)}{\frac{R}{n^2} + sL + \frac{1}{sC}}$$

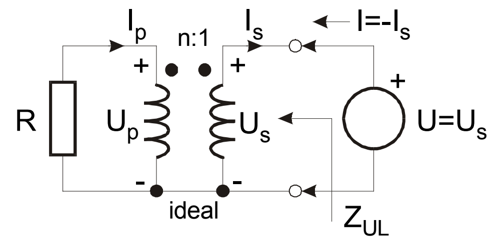
$$\Rightarrow \quad U_T(s) = I_s \cdot \frac{1}{sC} = \frac{\frac{U_g(s)}{n} + Li_L(0)}{\frac{R}{n^2} + sL + \frac{1}{sC}} \cdot \frac{1}{sC} = \frac{\frac{U_g(s)}{n} + Li_L(0)}{s^2 LC + sC\frac{R}{n^2} + 1}$$

$$U_T(s) = \frac{\frac{1}{2s} + 1}{s^2 + \frac{1}{4}s + 1} = \frac{\frac{2}{s} + 4}{4s^2 + s + 4} = \frac{2(2s+1)}{s(4s^2 + s + 4)}$$

b) Teveninova impedancija

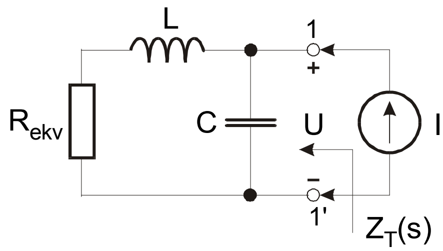


Izračunajmo najprije R_{ekv} :



$$Z_{ul} = \frac{U}{I} = \frac{U_s}{-I_s} = -\frac{\frac{U_p}{n}}{nI_p} = \frac{1}{n^2}\cdot\left(-\frac{U_p}{I_p}\right); \qquad -\frac{U_p}{I_p} = R; \quad \Rightarrow \quad Z_{ul} = \frac{R}{n^2} = R_{ekv}; \quad R_{ekv} = \frac{R}{n^2} = \frac{1}{4}$$

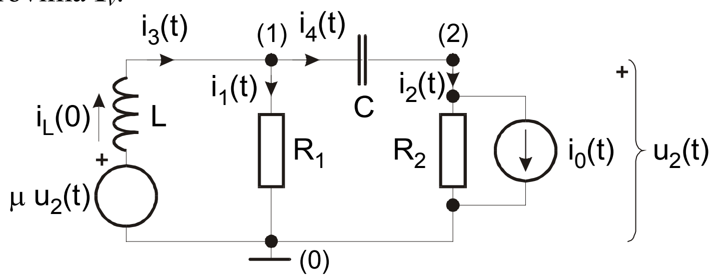
Teveninova impedancija :



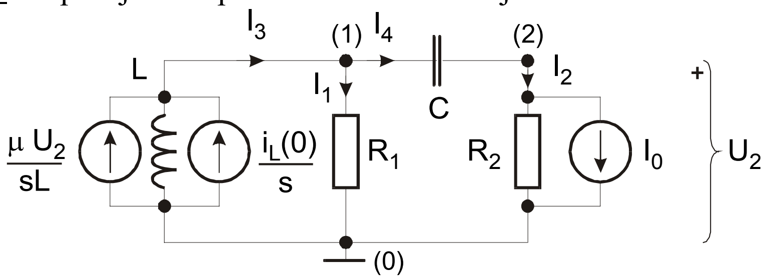
$$Z_T = \frac{U}{I} = \frac{\frac{1}{sC}\left(R_{ekv} + sL\right)}{\frac{1}{sC} + R_{ekv} + sL} = \frac{R_{ekv} + sL}{1 + sCR_{ekv} + s^2CL} = \frac{s + \frac{1}{4}}{s^2 + \frac{1}{4}s + 1} = \frac{4s+1}{4s^2+s+4}$$

$$Z_T(s) = \frac{4s+1}{4s^2+s+4}$$

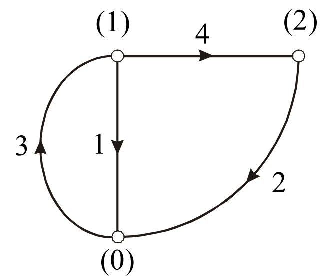
2. Zadan je električni krug prema slici. Poštujući oznake grana i čvorišta nacrtati pripadni orijentirani graf. Napisati reduciranu matricu incidencija **A**. Napisati naponsko-strujne jednadžbe grana u matričnom obliku te ispisati matricu admitancija grana $\mathbf{Y}_b$ i vektor naponskih izvora grana $\mathbf{U}_{0b}$. Napisati sustav jednadžbi čvorova, odnosno odrediti matrice admitancija čvorova $\mathbf{Y}_v$ i vektor strujnih izvora u čvorovima $\mathbf{I}_v$.



Rješenje: Uz primjenu Laplaceove transformacije :



Električni krug



orijentirani graf

Matrica incidencija (reducirana):

$$\mathbf{A} = \begin{array}{c} \\ (1) \\ (2) \end{array} \overset{\overbrace{\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array}}^{grane}}{\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}}$$

$\uparrow$

cvorovi

Naponsko strujne relacije grana (naponi izraženi pomoću struja):

$$\begin{aligned} U_1 &= I_1 R_1 & \qquad U_1 &= I_1 R_1 \\ U_2 &= (I_2 - I_0) R_2 & \qquad U_2 &= I_2 R_2 - I_0 R_2 \\ U_3 &= I_3 sL - L i_L(0) - \mu U_2 & \Rightarrow \qquad U_3 &= -\mu I_2 R_2 + I_3 sL - L i_L(0) + \mu I_0 R_2 \\ U_4 &= I_4 \frac{1}{sC} & \qquad U_4 &= I_4 \frac{1}{sC} \end{aligned}$$

Naponsko-strujne relacije grana u matričnom obliku:

$$\underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b} = \underbrace{\begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & -\mu R_2 & sL & 0 \\ 0 & 0 & 0 & \dfrac{1}{sC} \end{bmatrix}}_{\mathbf{Z}_b} \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b} + \underbrace{\begin{bmatrix} 0 \\ -I_0 R_2 \\ \mu I_0 R_2 - L i_L(0) \\ 0 \end{bmatrix}}_{\mathbf{U}_{0b}}$$

Naponsko strujne relacije grana (struje izražene pomoću napona):

$$I_1 = U_1 \frac{1}{R_1}$$

$$I_2 = U_2 \frac{1}{R_2} + I_0$$

$$I_3 = U_2 \frac{\mu}{sL} + U_3 \frac{1}{sL} + \frac{i_L(0)}{s}$$

$$\underline{I_4 = U_4 sC}$$

Naponsko-strujne relacije grana u matričnom obliku:

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b} = \underbrace{\begin{bmatrix} \frac{1}{R_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ 0 & \frac{\mu}{sL} & \frac{1}{sL} & 0 \\ 0 & 0 & 0 & sC \end{bmatrix}}_{\mathbf{Y}_b} \underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b} + \underbrace{\begin{bmatrix} 0 \\ I_0 \\ \frac{i_L(0)}{s} \\ 0 \end{bmatrix}}_{\mathbf{I}_{0b}}$$

Sustav jednadžbi čvorova u matričnom obliku:

$$\mathbf{Y}_v = \mathbf{A}\mathbf{Y}_b\mathbf{A}^T = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} \frac{1}{R_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ 0 & \frac{\mu}{sL} & \frac{1}{sL} & 0 \\ 0 & 0 & 0 & sC \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 1 & -1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{1}{R_1} & -\frac{\mu}{sL} & -\frac{1}{sL} & sC \\ 0 & \frac{1}{R_2} & 0 & -sC \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{sL} + sC & -\frac{\mu}{sL} - sC \\ -sC & \frac{1}{R_2} + sC \end{bmatrix}$$

$$\mathbf{I}_v = \mathbf{A}\mathbf{Y}_b\mathbf{U}_{0b} = \begin{bmatrix} \frac{1}{R_1} & -\frac{\mu}{sL} & -\frac{1}{sL} & sC \\ 0 & \frac{1}{R_2} & 0 & -sC \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -I_0 R_2 \\ \mu R_2 I_0 - L i_L(0) \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{i_L(0)}{s} \\ -I_0 \end{bmatrix}$$

sustav jednadžbi čvorova: $\mathbf{Y}_v \cdot \mathbf{U}_v = \mathbf{I}_v$

Drugi način izračunavanja matrice admitancija grana $\mathbf{Y}_b$ invertiranjem matrice impedancija grana $\mathbf{Z}_b$.

$$\mathbf{Z}_b = \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ 0 & -\mu R_2 & sL & 0 \\ 0 & 0 & 0 & \frac{1}{sC} \end{bmatrix}$$

Označimo submatricu (2x2) unutar matrice $\mathbf{Z}_b$ sa

$$\mathbf{Z'}_b = \begin{bmatrix} R_2 & 0 \\ -\mu R_2 & sL \end{bmatrix}$$

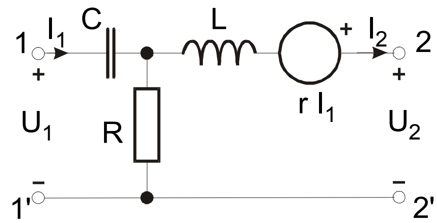
sada invertirajmo sumatricu $\mathbf{Z'}_b$

$$\mathbf{Z'}_b^{-1} = \begin{bmatrix} R_2 & 0 \\ -\mu R_2 & sL \end{bmatrix}^{-1} = \frac{1}{R_2 sL}\begin{bmatrix} sL & -(-\mu R_2) \\ 0 & R_2 \end{bmatrix}^T = \frac{1}{R_2 sL}\begin{bmatrix} sL & 0 \\ \mu R_2 & R_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_2} & 0 \\ \frac{\mu}{sL} & \frac{1}{sL} \end{bmatrix},$$

i vratimo je na svoje mjesto unutar matrice $\mathbf{Z}_b^{-1}$. Nadalje, ako invertiramo ostale elemente na dijagonali, konačno dobivamo:

$$\mathbf{Y}_b = \mathbf{Z}_b^{-1} = \begin{bmatrix} \frac{1}{R_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ 0 & \frac{\mu}{sL} & \frac{1}{sL} & 0 \\ 0 & 0 & 0 & sC \end{bmatrix}$$

3. Za četveropol prikazan slikom izračunati *y*-parametre i napisati matricu *y*-parametara. Zadane su normalizirane vrijednosti elemenata R=1, L=2, C=1 i r=2. Da li je četveropol: *a*) recipročan, *b*) simetričan ? Obrazložiti odgovor.



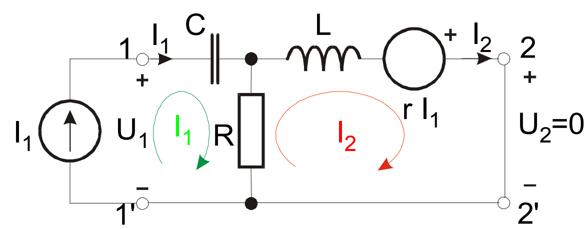
Rješenje:

[*y*]-parametri:

$I_1 = y_{11}U_1 - y_{12}U_2$

$I_2 = y_{21}U_1 - y_{22}U_2$

a) U_2=0



$$(1) \qquad I_1\left(R+\frac{1}{sC}\right)-I_2R=U_1$$

$$(2) \qquad -I_1R+I_2(R+sL)=rI_1$$

$$(2)\Rightarrow I_1(R+r)=I_2(R+sL)$$

$$(1)\Rightarrow I_1\left(R+\frac{1}{sC}\right)-I_1\frac{R+r}{R+sL}R=U_1$$

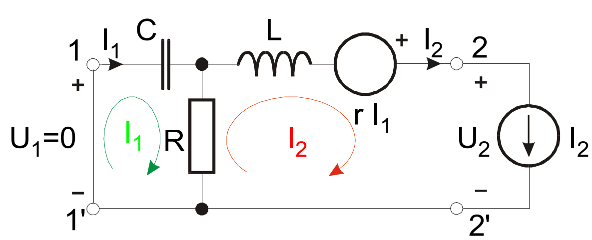
$$y_{11}=\left.\frac{I_1}{U_1}\right|_{U_2=0}=\frac{1}{\left(R+\frac{1}{sC}\right)-\frac{R+r}{R+sL}R}=\frac{R+sL}{\left(R+\frac{1}{sC}\right)(R+sL)-(R+r)R}=$$

$$=\frac{1+2s}{\left(1+\frac{1}{s}\right)(1+2s)-(1+2)1}=\frac{1+2s}{1+2s+\frac{1}{s}+2-3}=\frac{1+2s}{2s+\frac{1}{s}}=\frac{s(1+2s)}{2s^2+1}$$

$$(1)\Rightarrow I_2\frac{R+sL}{R+r}\left(R+\frac{1}{sC}\right)-I_2R=U_1$$

$$y_{21}=\left.\frac{I_2}{U_1}\right|_{U_2=0}=\frac{1}{\frac{R+sL}{R+r}\left(R+\frac{1}{sC}\right)-R}=\frac{R+r}{\left(R+\frac{1}{sC}\right)(R+sL)-(R+r)R}=\frac{1+2}{2s+\frac{1}{s}}=\frac{3s}{2s^2+1}$$

b) U_1=0



$$(1)\qquad I_1\left(R+\frac{1}{sC}\right)-I_2R=0$$

$$(2)\qquad -I_1(R+r)+I_2(R+sL)=-U_2$$

$$(1)\Rightarrow I_1\left(R+\frac{1}{sC}\right)=I_2R$$

$$(2)\Rightarrow -I_1(R+r)+I_1\frac{R+\frac{1}{sC}}{R}(R+sL)=-U_2$$

$$y_{12}=-\left.\frac{I_1}{U_2}\right|_{U_1=0}=\frac{1}{\frac{R+\frac{1}{sC}}{R}(R+sL)-(R+r)}=\frac{R}{\left(R+\frac{1}{sC}\right)(R+sL)-(R+r)R}=$$

$$=\frac{1}{2s+\frac{1}{s}}=\frac{s}{2s^2+1}$$

$$(2)\Rightarrow -I_2\frac{R}{R+\frac{1}{sC}}(R+r)+I_2(R+sL)=-U_2$$

$$y_{22}=-\left.\frac{I_2}{U_2}\right|_{U_1=0}=\frac{1}{(R+sL)-\frac{R}{R+\frac{1}{sC}}(R+r)}=\frac{\left(R+\frac{1}{sC}\right)}{\left(R+\frac{1}{sC}\right)(R+sL)-(R+r)R}=$$

$$=\frac{1+\frac{1}{s}}{2s+\frac{1}{s}}=\frac{s+1}{2s^2+1}$$

Konačno rješenje glasi: $[y]=\begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22}\end{bmatrix}$

$$[y]=\frac{1}{\left(R+\frac{1}{sC}\right)(R+sL)-(R+r)R}\begin{bmatrix} R+sL & -R \\ R+r & -\left(R+\frac{1}{sC}\right)\end{bmatrix}=$$

$$=\frac{1}{sLR+\frac{R}{sC}+\frac{L}{C}-rR}\cdot\begin{bmatrix} R+sL & -R \\ R+r & -\left(R+\frac{1}{sC}\right)\end{bmatrix}=\frac{1}{2s+\frac{1}{s}}\cdot\begin{bmatrix} 1+2s & -1 \\ 3 & -\left(1+\frac{1}{s}\right)\end{bmatrix}=$$

$$=\frac{s}{2s^2+1}\cdot\begin{bmatrix} 1+2s & -1 \\ 3 & -\frac{s+1}{s}\end{bmatrix}.$$

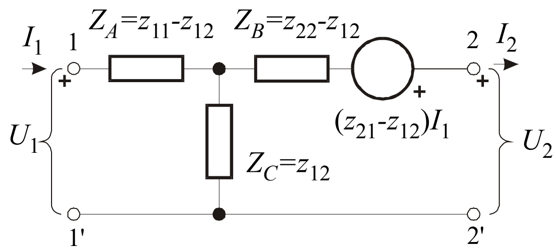
Odgovori na pitanja:

a) Četveropol nije recipročan jer sadrži strujno ovisni naponski izvor i parametri y_{12} i y_{21} su stoga različiti;

b) Četveropol nije električki simetričan jer se parametri y_{11} i y_{22} razlikuju.

Drugi jednostavniji način rješavanja zadatka:

Na slajdovima 14. predavanja "Četveropoli" nalazi se ekvivalentni T-spoj nerecipročnog četveropola (za koji vrijedi $z_{12} \neq z_{21}$):



Opisan je jednadžbama: $z_{11}=Z_A+Z_C$; $z_{12}=Z_C$; $z_{22}=Z_B+Z_C$; $z_{21}-z_{12}=r$; $z_{21}=z_{12}+r$; gdje su z_{11}, z_{12}, z_{21}, z_{22}, z-parametri, a r je parametar strujno-ovisnog naponskog izvora zadanog u zadatku.

Vidljivo je da je: $Z_A=1/(sC)$, $Z_B=sL$, $Z_C=R$, odavde slijede z-parametri: $z_{11}=1/(sC)+R$; $z_{12}=R$; $z_{22}=sL+R$; $z_{21}=R+r$. Odnosno u matričnom obliku:

$$[z]=\begin{bmatrix} R+\dfrac{1}{sC} & -R \\ R+r & -(R+sL) \end{bmatrix}$$

Ako invertiramo matricu sa z-parametrima lako izračunamo y-parametre:

$\Delta z=-\left(R+\dfrac{1}{sC}\right)(R+sL)+R(R+r)$ je determinanta matrice sa z-parametrima

$$[y]=[z]^{-1}=\frac{1}{\Delta z}\begin{bmatrix} -(R+sL) & -(R+r) \\ -(-R) & R+\dfrac{1}{sC} \end{bmatrix}^T=\frac{1}{\Delta z}\begin{bmatrix} -(R+sL) & R \\ -(R+r) & R+\dfrac{1}{sC} \end{bmatrix}$$

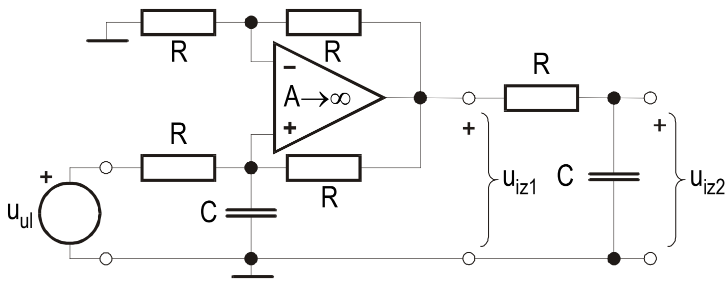
$$=\frac{1}{-\Delta z}\begin{bmatrix} R+sL & -R \\ R+r & -\left(R+\dfrac{1}{sC}\right) \end{bmatrix}$$

$$[y]=\frac{1}{\left(R+\dfrac{1}{sC}\right)(R+sL)-R(R+r)}\begin{bmatrix} R+sL & -R \\ R+r & -\left(R+\dfrac{1}{sC}\right) \end{bmatrix}$$

Odnosno uz uvrštene vrijednosti elemenata:

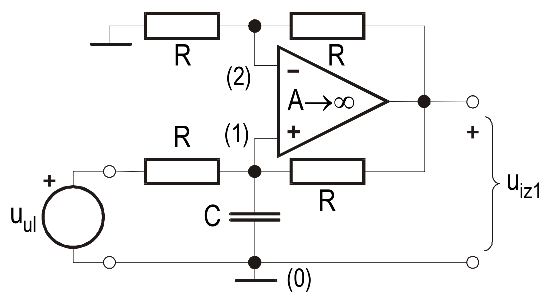
$$[y]=\frac{s}{2s^2+1}\cdot\begin{bmatrix} 1+2s & -1 \\ 3 & -\dfrac{s+1}{s} \end{bmatrix}=\begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix}$$

4. Za električni krug prikazan slikom odrediti naponske prijenosne funkcije $T_1(s)=U_{iz1}(s)/U_{ul}(s)$ i $T(s)=U_{iz2}(s)/U_{ul}(s)$. Nacrtati raspored polova u kompleksnoj ravnini te amplitudno-frekvencijsku karakteristiku ukupne prijenosne funkcije $T(s)$. Zadane su normalizirane vrijednosti R=1, C=1.



Rješenje:

a) prva naponska prijenosna funkcija $T_1(s)= U_{iz1}(s)/U_{ul}(s)$:
Jednadžbe čvorova:



(1) $$U_1\left(\frac{1}{R}+\frac{1}{R}+sC\right)-U_{ul}\frac{1}{R}=\frac{U_{iz1}}{R}$$

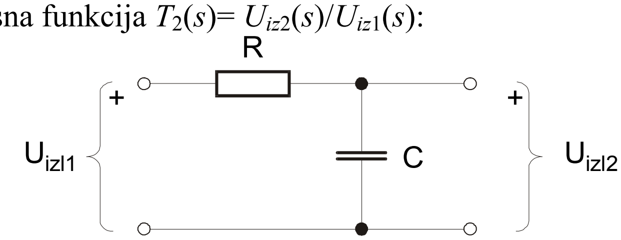
(2) $$U_2\left(\frac{1}{R}+\frac{1}{R}\right)-U_{iz1}\frac{1}{R}=0 \quad\Rightarrow\quad U_2\frac{2}{R}=U_{iz1}\frac{1}{R} \quad\Rightarrow\quad U_2=\frac{U_{iz1}}{2}$$

(3) $$U_{iz1}=A(U_1-U_2)/:A,\quad A\to\infty \quad\Rightarrow\quad U_1=U_2$$

(1) $$\frac{U_{iz1}}{2}\left(\frac{2}{R}+sC\right)-U_{ul}\frac{1}{R}=\frac{U_{iz1}}{R}\Big/\cdot R \quad\Rightarrow\quad U_{iz1}+U_{iz1}\frac{sRC}{2}-U_{iz1}=U_{ul}$$

$$\Rightarrow\quad U_{iz1}=\frac{1}{\frac{sRC}{2}}U_{ul} \quad\Rightarrow\quad T_1(s)=\frac{U_{iz1}}{U_{ul}}=\frac{2}{sRC}$$

b) druga naponska prijenosna funkcija $T_2(s)= U_{iz2}(s)/U_{iz1}(s)$:



$$T_2(s)=\frac{U_{iz2}}{U_{iz1}}=\frac{\frac{1}{sC}}{R+\frac{1}{sC}}=\frac{\frac{1}{RC}}{s+\frac{1}{RC}}$$

c) ukupna naponska prijenosna funkcija $T(s)=\dfrac{U_{iz2}}{U_{ul}}=T_1(s)\cdot T_2(s)=\dfrac{U_{iz1}}{U_{ul}}\cdot\dfrac{U_{iz2}}{U_{iz1}}=\dfrac{2}{sRC}\cdot\dfrac{\frac{1}{RC}}{s+\frac{1}{RC}}$

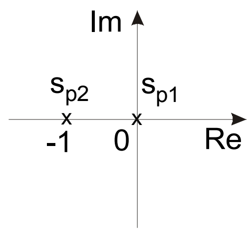
Uz uvrštene normalizirane vrijednosti elemenata R=1 i C=1 :

$$T(s)=\frac{U_{iz2}}{U_{ul}}=\frac{2}{s}\cdot\frac{1}{s+1}=\frac{2}{s^2+s}$$

Raspored nula i polova u s-ravnini:

Polovi : $s(s+1)=0\Rightarrow s_{p1}=0, s_{p2}=-1$

Nule: $s_{o1,2}=\infty$



Amplitudno-frekvencijska karakteristika:

$$T(j\omega)=\frac{U_{iz2}}{U_{ul}}=\frac{2}{-\omega^2+j\omega}\quad\Rightarrow\quad\left|T(j\omega)\right|=\frac{U_{iz2}}{U_{ul}}=\frac{2}{\sqrt{\omega^4+\omega^2}}=\frac{2}{\omega\sqrt{1+\omega^2}}$$

Crtanje a-f karakteristike točku po točku :

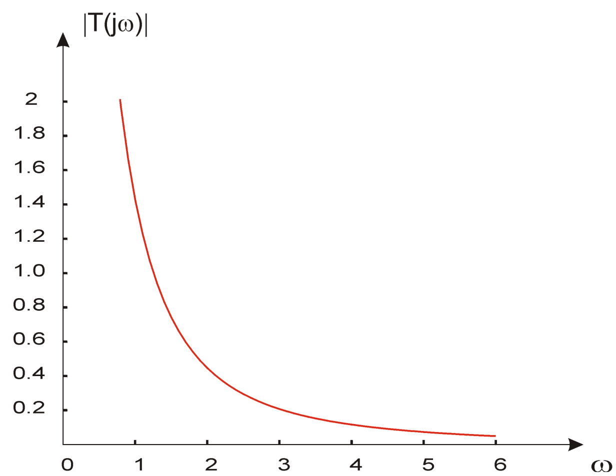
$\omega=0\quad\Rightarrow\quad\left|T(j\omega)\right|=\infty$

$\omega=1\quad\Rightarrow\quad\left|T(j\omega)\right|=\dfrac{2}{\sqrt{2}}=\sqrt{2}=1.41$

$\omega=2\quad\Rightarrow\quad\left|T(j\omega)\right|=\dfrac{2}{2\sqrt{1+4}}=\dfrac{1}{\sqrt{5}}=0.4472$

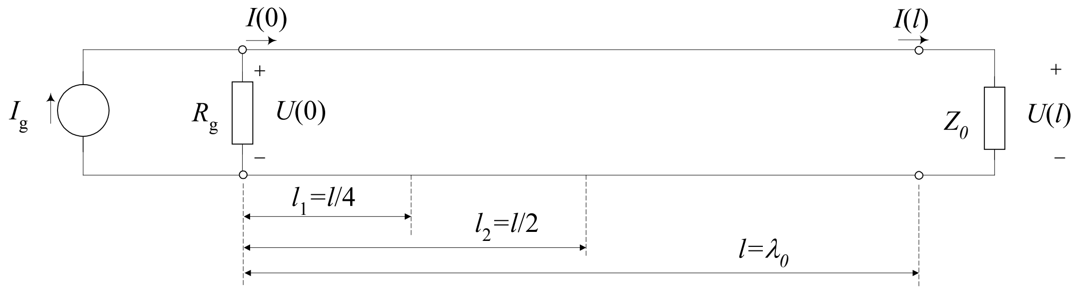
$\omega=3\quad\Rightarrow\quad\left|T(j\omega)\right|=\dfrac{2}{3\sqrt{10}}=0.2108$

$\omega=\infty\quad\Rightarrow\quad\left|T(j\omega)\right|=0$



5. Zadana je linija bez gubitaka s L=20 μH/km i C=8 nF/km. Na ulaz linije je priključen strujni izvor $i_g(t)$ = 0,2 cos(ω_0t) paralelno s otporom R_g=10Ω. Duljina linije je $l=\lambda_0$, gdje je λ_0 valna duljina signala pri frekvenciji $\omega_0 = 5\cdot 10^6$ rad/s. Izlaz linije je zaključen karakterističnom impedancijom.

a) Odrediti karakterističnu impedanciju Z_0 i koeficijent prijenosa γ linije.
b) Kolika je duljina linije u km?
c) Odrediti napon i struju na 1/4 linije.
d) Odrediti napon i struju na 1/2 linije.



Rješenje:

a) Linija bez gubitaka $\rightarrow\; R=0,\; G=0 \quad\Rightarrow\quad Z_0=\sqrt{L/C},\; \gamma = s\sqrt{LC}\;\Omega$

Stac. sinusna pobuda $\rightarrow\; s=j\omega_0 \quad\Rightarrow\quad \gamma = j\omega_0\sqrt{LC}=j\beta$

$$Z_0=\sqrt{L/C}=\sqrt{2\cdot 10^{-5}/8\cdot 10^{-9}}=\sqrt{10^4/4}=50\Omega$$

$$\gamma = j\omega_0\sqrt{LC}=j5\cdot 10^6\sqrt{20\cdot 10^{-6}\cdot 8\cdot 10^{-9}}=j5\cdot 10^6\sqrt{16\cdot 10^{-14}}=j5\cdot 10^6\cdot 4\cdot 10^{-7}=j2\ \text{km}^{-1}$$

b) $l=\lambda_0=\dfrac{2\pi}{\beta}=\dfrac{2\pi}{2}=\pi=3{,}14\text{km}$; $\quad l_1=\lambda_0/4=\dfrac{\pi}{4}\text{km}$; $\quad l_2=\lambda_0/2=\dfrac{\pi}{2}\text{km}$

c) $$U(x)=U(0)\cdot ch\gamma x - I(0)Z_0 sh\gamma x$$

$$I(x)=-\frac{U(0)}{Z_0}sh\gamma x + I(0)ch\gamma x$$

$$U(0)=I_g\cdot(Z_0\parallel R_g)=I_g\cdot(Z_0\cdot R_g/(Z_0+R_g))=\frac{10}{6}=1{,}666$$

$$I(0)=U(0)/Z_0=U(0)/Z_0=0{,}0333$$

$$U(x)=U(0)\cdot(ch\gamma x - sh\gamma x)=U(0)\cdot e^{-\gamma x}$$

$$I(x)=\frac{U(0)}{Z_0}(-sh\gamma x + ch\gamma x)=\frac{U(0)}{Z_0}e^{-\gamma x}$$

$$U(l/4)=U(0)\cdot(ch(\gamma l/4)-sh(\gamma l/4))=U(0)\cdot e^{-\gamma l/4}=U(0)\cdot e^{-j\pi/2}$$

$$I(l/4)=\frac{U(0)}{Z_0}(-sh(\gamma l/4)+ch(\gamma l/4))=\frac{U(0)}{Z_0}e^{-\gamma l/4}=\frac{U(0)}{Z_0}e^{-j\pi/2}$$

$$u(l/4,t)=1{,}6667\cdot\cos(\omega_0 t-90^\circ)$$

$$i(l/4,t)=0{,}0333\cdot\cos(\omega_0 t-90^\circ)$$

d) $$U(l/2)=U(0)\cdot(ch(\gamma l/2)-sh(\gamma l/2))=U(0)\cdot e^{-\gamma l/2}=U(0)\cdot e^{-j\pi}$$

$$I(l/2)=\frac{U(0)}{Z_0}(-sh(\gamma l/2)+ch(\gamma l/2))=\frac{U(0)}{Z_0}e^{-\gamma l/2}=\frac{U(0)}{Z_0}e^{-j\pi}$$

$$u(l/2,t)=-1{,}6667\cdot\cos(\omega_0 t),\qquad i(l/2,t)=-0{,}0333\cdot\cos(\omega_0 t)$$