

## OTPOR

$$\begin{array}{c} i(t) \\ \text{---} R \text{---} \\ + \end{array} \quad u(t) \quad i(t) = \frac{u(t)}{R} \quad u(t) = R \cdot i(t)$$

## KAPACITET

$$\begin{array}{c} i(t) \\ \text{---} C \text{---} \\ + \end{array} \quad u(t) \quad i(t) = C \frac{du(t)}{dt} \quad u(t) = \frac{1}{C} \int_{-\infty}^t i(\tau) d\tau = u_C(0) + \frac{1}{C} \int_0^t i(\tau) d\tau$$

## INDUKTIVITET

$$\begin{array}{c} i(t) \\ \text{---} L \text{---} \\ + \end{array} \quad u(t) \quad i(t) = \frac{1}{L} \int_{-\infty}^t u(\tau) d\tau = i_L(0) + \frac{1}{L} \int_0^t u(\tau) d\tau \quad u(t) = L \frac{di(t)}{dt}$$

**Transformator** → dva induktiviteta koji su međuinaktivno vezani

$$\begin{array}{c} \text{Ideal.} \\ \begin{array}{c} i_1(t) \\ \text{---} L_1 \text{---} \\ + \end{array} \quad \begin{array}{c} i_2(t) \\ \text{---} L_2 \text{---} \\ + \end{array} \\ \text{---} M \text{---} \\ u_1(t) \quad u_2(t) \end{array} \quad \begin{array}{l} u_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \\ u_2(t) = +M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \end{array}$$

$$\begin{array}{c} \text{Ideal.} \\ \begin{array}{c} i_1(t) \\ \text{---} L_1 \text{---} \\ + \end{array} \quad \begin{array}{c} i_2(t) \\ \text{---} L_2 \text{---} \\ + \end{array} \\ \text{---} M \text{---} \\ u_1(t) \quad u_2(t) \end{array} \quad \begin{array}{l} u_1(t) = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt} \\ u_2(t) = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt} \end{array}$$

$$\begin{array}{c} \text{Ideal.} \\ \begin{array}{c} i_1(t) \\ \text{---} n:1 \text{---} \\ + \end{array} \quad \begin{array}{c} i_2(t) \\ \text{---} 1:n \text{---} \\ + \end{array} \\ u_1(t) \quad u_2(t) \end{array} \quad \begin{array}{l} u_1(t) = n \cdot u_2(t) \\ i_1(t) = -\frac{1}{n} \cdot i_2(t) \end{array}$$

$$\begin{array}{c} \text{Ideal.} \\ \begin{array}{c} i_1(t) \\ \text{---} n:1 \text{---} \\ + \end{array} \quad \begin{array}{c} i_2(t) \\ \text{---} 1:n \text{---} \\ + \end{array} \\ u_1(t) \quad u_2(t) \end{array} \quad \begin{array}{l} u_1(t) = -n \cdot u_2(t) \\ i_1(t) = \frac{1}{n} \cdot i_2(t) \end{array}$$

**Girator** je četveropol određen simbolom

$$\begin{array}{c} i_1(t) \\ \text{---} r \text{---} \\ + \end{array} \quad \begin{array}{c} i_2(t) \\ \text{---} r \text{---} \\ + \end{array} \\ u_1(t) \quad u_2(t) \end{array} \quad \begin{array}{l} u_1(t) = r \cdot i_2(t) \\ u_2(t) = -r \cdot i_1(t) \end{array}$$

**Negativni konvertor**

$$\begin{array}{c} I_1 \\ \text{---} \text{NC} \text{---} \\ + \end{array} \quad \begin{array}{c} I_2 \\ \text{---} \text{NC} \text{---} \\ + \end{array} \\ U_1 \quad U_2 \end{array} \quad \begin{array}{l} u_1(t) = k_1 \cdot u_2(t) \\ i_2(t) = k_2 \cdot i_1(t) \\ k = k_1 \cdot k_2 \end{array}$$

**Operacijsko pojačalo** → element sa 3 prilaza.

$$\begin{array}{c} i_2(t) \\ \text{---} \text{OpAmp} \text{---} \\ + \end{array} \quad \begin{array}{c} i_3(t) \\ \text{---} \text{OpAmp} \text{---} \\ + \end{array} \\ u_2(t) \quad u_3(t) \end{array} \quad u_2 - u_1 = \frac{u_3}{A} \rightarrow 0$$

## Kapacitet

$$\begin{array}{c} i(t) \\ \text{---} C \text{---} \\ + \end{array} \quad u_C(0) \quad U(s) \quad U(s) = \frac{1}{sC} I(s) + \frac{u_C(0)}{s}$$

$$\begin{array}{c} i(t) \\ \text{---} C \text{---} \\ + \end{array} \quad u_C(0) \quad U(s) \quad I(s) = sCU(s) - Cu_C(0)$$

## Induktivitet

$$\begin{array}{c} i(t) \\ \text{---} L \text{---} \\ + \end{array} \quad i_L(0) \quad U(s) \quad I(s) = \frac{1}{sL} U(s) + \frac{i_L(0)}{s}$$

$$\begin{array}{c} i(t) \\ \text{---} L \text{---} \\ + \end{array} \quad i_L(0) \quad U(s) \quad U(s) = sLI(s) - Li_L(0)$$

→ transformator mijenja vrijednost otpora

$$\begin{array}{c} I_1 \\ \text{---} n:1 \text{---} \\ + \end{array} \quad \begin{array}{c} I_2 \\ \text{---} 1:n \text{---} \\ + \end{array} \\ U_1 \quad U_2 \end{array} \quad R_{ulr} = \frac{U_1}{I_1} = -n^2 \frac{U_2}{I_2} = n^2 R$$

→ girator invertira vrijednost otpora

$$\begin{array}{c} I_1 \\ \text{---} r \text{---} \\ + \end{array} \quad \begin{array}{c} I_2 \\ \text{---} r \text{---} \\ + \end{array} \\ U_1 \quad U_2 \end{array} \quad R_{ulG} = \frac{U_1}{I_1} = -r^2 \frac{I_2}{U_2} = \frac{r^2}{R}$$

→ negativni konvertor mjenja predznak

$$\begin{array}{c} I_1 \\ \text{---} \text{NC} \text{---} \\ + \end{array} \quad \begin{array}{c} I_2 \\ \text{---} \text{NC} \text{---} \\ + \end{array} \\ U_1 \quad U_2 \end{array} \quad R_{ulNK} = \frac{U_1}{I_1} = \frac{k_1 \cdot U_2}{k_2^{-1} \cdot I_2} = -k_1 k_2 R$$

$$\mathbf{U}_g = \mathbf{Z}_p \cdot \mathbf{I}_p$$

$\mathbf{U}_g \rightarrow$  vektor naponskih izvora i početnih veličina u petljama

$$\mathbf{U}_p = \begin{bmatrix} U_{g1}(s) + \frac{u_{C5}(0)}{s} - L_2 i_{L2}(0) \\ L_2 i_{L2}(0) \\ -\frac{u_{C5}(0)}{s} \end{bmatrix}$$

$\mathbf{Z}_p$  kvadratna matrica  $\rightarrow$  **matrica impedancija petlji** :

$$\mathbf{Z}_p = \begin{bmatrix} R_1 + sL_2 + \frac{1}{sC_5} & -sL_2 & -\frac{1}{sC_5} \\ -sL_2 & R_3 + R_4 + sL_2 & -R_4 \\ -\frac{1}{sC_5} & -R_4 & R_4 + R_6 + \frac{1}{sC_5} \end{bmatrix}$$

$\mathbf{I}_p \rightarrow$  vektor struja petlji

$$\mathbf{I}_p = \begin{bmatrix} I_{p1} \\ I_{p2} \\ I_{p3} \end{bmatrix}$$

$$\mathbf{I}_g = \mathbf{Y}_v \cdot \mathbf{U}_v$$

$\mathbf{I}_g \rightarrow$  vektor strujnih izvora

$$\mathbf{I}_g = \begin{bmatrix} \frac{U_{g1}(s)}{R_1} + \frac{i_{L2}(0)}{s} \\ -\frac{i_{L2}(0)}{s} - C_5 u_{C5}(0) \\ 0 \end{bmatrix}$$

$\mathbf{Y}_v$  kvadratna matrica  $\rightarrow$  **matrica admitancija čvorišta**.

$$\mathbf{Y}_v = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{sL_2} + \frac{1}{R_3} & -\frac{1}{sL_2} & -\frac{1}{R_3} \\ -\frac{1}{sL_2} & \frac{1}{sL_2} + \frac{1}{R_4} + sC_5 & -\frac{1}{R_4} \\ -\frac{1}{R_3} & -\frac{1}{R_4} & \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_6} \end{bmatrix}$$

$\mathbf{U}_v \rightarrow$  vektor napona čvorišta

$$\mathbf{U}_v = \begin{bmatrix} U_{v1} \\ U_{v2} \\ U_{v3} \end{bmatrix}$$

**Tablica  $\mathcal{L}$  transformacije**

$$1 \text{ --- } \bullet \frac{1}{s}$$

$$t \text{ --- } \bullet \frac{1}{s^2}$$

$$e^{-at} \text{ --- } \bullet \frac{1}{s+a}$$

$$\frac{1}{b-a}(e^{-at} - e^{-bt}) \text{ --- } \bullet \frac{1}{(s+a)(s+b)}$$

$$\frac{1}{a-b}(ae^{-at} - be^{-bt}) \text{ --- } \bullet \frac{s}{(s+a)(s+b)}$$

$$\frac{1}{a}e^{-bt} \sin(at) \text{ --- } \bullet \frac{1}{(s+b)^2 + a^2}$$

$$e^{-bt}(\cos(at) - \frac{b}{a} \sin(at)) \text{ --- } \bullet \frac{s}{(s+b)^2 + a^2}$$

$$\sin \omega t \text{ --- } \bullet \frac{\omega}{s^2 + \omega^2},$$

$$\text{sh } \omega t \text{ --- } \bullet \frac{\omega}{s^2 - \omega^2},$$

$$\cos \omega t \text{ --- } \bullet \frac{s}{s^2 + \omega^2},$$

$$\text{ch } \omega t \text{ --- } \bullet \frac{s}{s^2 - \omega^2}.$$

■ Element glavne dijagonale

■  $\rightarrow$  suma impedancija u promatranoj petlji.

■ Elementi izvan glavne dijagonale

■  $\rightarrow$  impedancije, zajedničke dvjema petljama.

■ Elementi izvan glavne dijagonale imaju negativan predznak  
 $\rightarrow$  posljedica odabira istoga smjera za sve struje petlji.

■ Element glavne dijagonale matrice

■  $\rightarrow$  suma admitancija grana vezanih na promatrano čvorište.

■ Elementi izvan glavne dijagonale

■  $\rightarrow$  admitancije grana, spojenih na dva promatrana čvorišta.

■ Negativni predznaci elemenata izvan glavne dijagonale  
 $\rightarrow$  posljedica odabira orijentacija napona čvorišta.

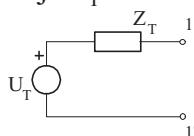
- **Kirchhoffov zakon za struje (KZS)** i
- Algebarska suma struja, koje se sastaju u jednom čvorištu mreže s koncentriranim elementima u svakom je trenutku jednaka nuli.
- Strujama orijentiranim od čvorišta pridružiti
  - **pozitivan predznak**
- Strujama orijentiranim prema čvorištu pridružiti
  - **negativan predznak.**
- za mrežu s  $N_v$  čvorišta broj linearno neovisnih jednažbi KZS jednak  $N_v - 1$ .

- **Kirchhoffov zakon za napone (KZN).**
- Za svaku mrežu moguće je napisati onoliko jednažbi KZN koliko ta mreža sadrži zatvorenih kontura.
- Za mrežu s  $N_v$  čvorišta i  $N_b$  grana, broj linearno neovisnih jednažbi KZN jednak je

$$N_b - (N_v - 1) = N_b - N_v + 1$$

## Theveninov teorem

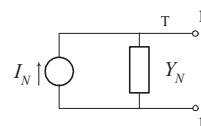
- Svakoj linearnoj, vremenski nepromjenljivoj, aktivnoj mreži s jednim prilazom moguće je odrediti ekvivalentni dvopol koji se sastoji od jednog neovisnog naponskog izvora  $U_T$  i jedne serijski spojene impedancije  $Z_T$



- Napon  $U_T$  naponskog izvora jednak je naponu na otvorenim priključnicama promatrane mreže.
- Impedancija  $Z_T$  je jednaka impedanciji gledanoj sa prilaza mreže uz
- ugašene sve neovisne izvore (ovisni izvori ostaju u krugu) i
- uz početne uvjete na kapacitetima i induktivitetima jednake nuli.

## Nortonov teorem

- Svakoj linearnoj, vremenski nepromjenljivoj, aktivnoj mreži s jednim prilazom moguće je odrediti ekvivalentni dvopol koji se sastoji od jednog neovisnog strujnog izvora  $I_N$  i jedne paralelno spojene admitancije  $Y_N$



- Struja  $I_N$  strujnog izvora jednaka je struji kroz kratko spojene priključnice promatrane mreže
- Admitancija  $Y_N$  je jednaka admitanciji gledanoj sa prilaza mreže uz ugašene sve neovisne izvore i uz početne uvjete na kapacitetima i induktivitetima jednake nuli.

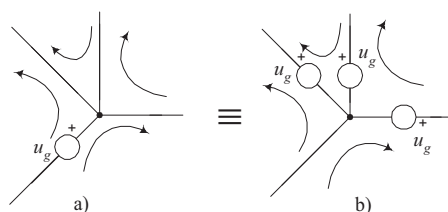
## Teorem superpozicije

- Odziv  $y(t)$  linearnog kruga na istovremeno djelovanje  $n$  različitih pobuda  $x_1, x_2, x_3, \dots, x_n$ , koje kad se primijene pojedinačno daju odzive  $y_1, y_2, y_3, \dots, y_n$ , jednak je sumi svih tih odziva, tj.

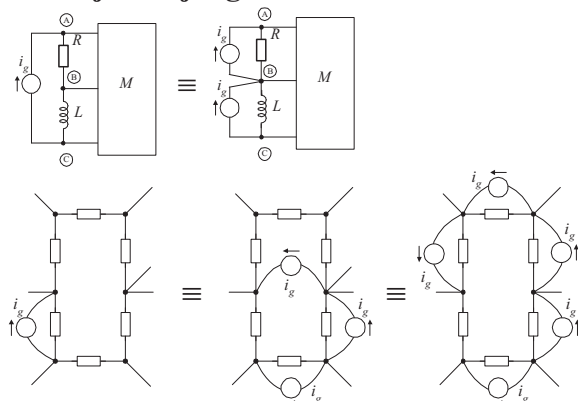
$$y(t) = \sum_{k=1}^n y_k(t)$$

- Pobude mogu biti neovisni naponski ili strujni izvori, kao i početni naponi na kapacitetima i struje u induktivitetima.

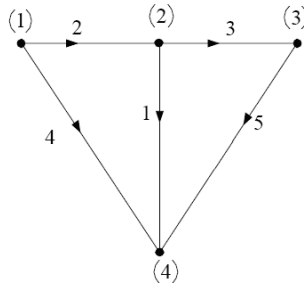
### posmicanjem naponskoga izvora.



### Posmicanje strujnoga izvora

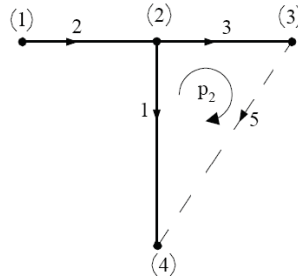
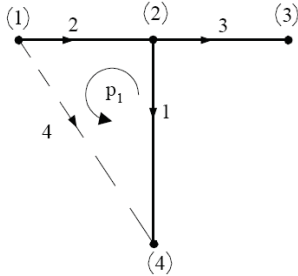


- $b \rightarrow$  grana (odnosno grane)
- $v \rightarrow$  čvor (čvorovi)
- $p \rightarrow$  petlja (petlje)
- $t \rightarrow$  grana stabla (grane stabla)
- $s \rightarrow$  spona (spone)
- $r \rightarrow$  rez (rezovi)
- $N \rightarrow$  broj nečega (npr.  $N_b$  – broj grana)
- $N_t = N_r = N_v - 1$
- $N_s = N_p = N_b - N_v + 1$

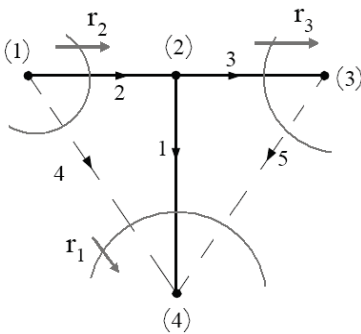


$$A = \begin{bmatrix} g_1 & g_2 & g_3 & g_4 & g_5 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 \end{bmatrix} \begin{matrix} \text{č1} \\ \text{č2} \\ \text{č3} \end{matrix} \quad \text{ulazi: -1}$$

- $A_a \rightarrow$  matrica incidencije; veličine je  $N_v \times N_b$
- $A \rightarrow$  reducirana matrica incidencije; veličine je  $N_t \times N_b$
- $S \rightarrow$  spojna matrica; veličine je  $N_s \times N_b$
- $Q \rightarrow$  rastavna matrica; veličine je  $N_t \times N_b$



$$S = \begin{bmatrix} g_1 & g_2 & g_3 & g_4 & g_5 \\ -1 & -1 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{matrix} p_1 \\ p_2 \end{matrix} \quad \text{u smjeru spona: 1}$$



$$Q = \begin{bmatrix} g_1 & g_2 & g_3 & g_4 & g_5 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix} \begin{matrix} r_1 \\ r_2 \\ r_3 \end{matrix} \quad \text{u smjeru reza: 1}$$

Nepoznanice su:

- $U_b \rightarrow$  vektor stupac duljine  $N_b$  gdje svaki član matrice predstavlja jedan napon grane
- $U_v \rightarrow$  vektor stupac duljine  $N_v$  gdje svaki član matrice predstavlja jedan napon čvora
- $U_r \rightarrow$  vektor stupac duljine  $N_r$  gdje svaki član matrice predstavlja jedan napon reza
- $I_b \rightarrow$  vektor stupac duljine  $N_b$  gdje svaki član matrice predstavlja jednu struju grane
- $I_p \rightarrow$  vektor stupac duljine  $N_p$  gdje svaki član matrice predstavlja jednu struju petlje

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{a \cdot d - b \cdot c} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Poznate matrice:

- $U_{0b} \rightarrow$  vektor stupac duljine  $N_b$  gdje svaki član matrice predstavlja vrijednost napona izvora i početnih uvjeta u grani
- $U_{0p} \rightarrow$  vektor stupac duljine  $N_p$  gdje svaki član matrice predstavlja vrijednost napona izvora i početnih uvjeta u petlji
- $I_{0b} \rightarrow$  vektor stupac duljine  $N_b$  gdje svaki član matrice predstavlja vrijednost struje izvora i početnih uvjeta u grani
- $I_{0v} \rightarrow$  vektor stupac duljine  $N_v$  gdje svaki član matrice predstavlja vrijednost struje izvora i početnih uvjeta čvora
- $I_{0r} \rightarrow$  vektor stupac duljine  $N_r$  gdje svaki član matrice predstavlja vrijednost struje izvora i početnih uvjeta reza
- $Z_b \rightarrow$  kvadratna matrica veličine  $N_b \times N_b$ ; sadrži impedancije grana
- $Z_p \rightarrow$  kvadratna matrica veličine  $N_p \times N_p$ ; sadrži impedancije petlji
- $Y_b \rightarrow$  kvadratna matrica veličine  $N_b \times N_b$ ; sadrži admitancije grana
- $Y_v \rightarrow$  kvadratna matrica veličine  $N_v \times N_v$ ; sadrži admitancije čvorova
- $Y_r \rightarrow$  kvadratna matrica veličine  $N_r \times N_r$ ; sadrži admitancije rezova

Jednadžbe strujno naponskih relacija:

$$U_b = Z_b \cdot I_b + U_{0b}$$

$$I_b = Y_b \cdot U_b + I_{0b}$$

- Jednadžbe petlji  $\rightarrow Z_p \cdot I_p = U_{0p}$
- Jednadžbe čvorova  $\rightarrow Y_v \cdot U_v = I_{0v}$
- Jednadžbe rezova  $\rightarrow Y_r \cdot U_r = I_{0r}$

$$Z_p = S \cdot Z_b \cdot S^T$$

$$Y_v = A \cdot Y_b \cdot A^T$$

$$Y_r = Q \cdot Y_b \cdot Q^T$$

$$U_{0p} = -S \cdot U_{0b}$$

$$I_{0v} = -A \cdot I_{0b}$$

$$I_{0r} = -Q \cdot I_{0b}$$

$$H(s) = \frac{\text{odziv}}{\text{pobuda}}$$

$$\alpha(\omega) = 20 \log |H(j\omega)|$$

$$H(s) = |H(s)| e^{j\angle H(s)}$$

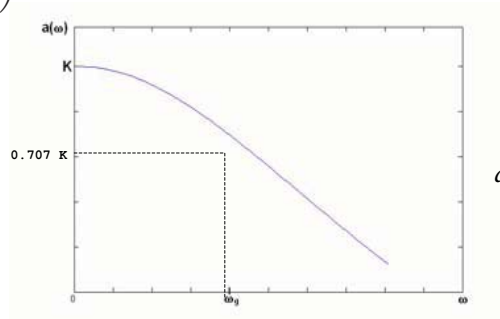
$$|H(j\omega)| = \sqrt{(\operatorname{Re}[H(j\omega)])^2 + (\operatorname{Im}[H(j\omega)])^2}$$

$$\angle H(j\omega) = \arctan \left( \frac{\operatorname{Im}[H(j\omega)]}{\operatorname{Re}[H(j\omega)]} \right)$$

### 1) Niskopropusni (NP)

$$H(s) = K \cdot \frac{\omega_g}{s + \omega_g}$$

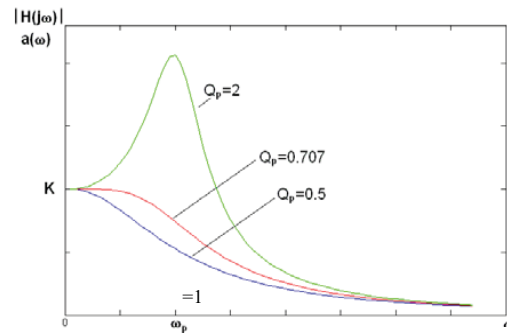
$$a(\omega) = \frac{K}{\sqrt{1 + \left(\frac{\omega}{\omega_g}\right)^2}}$$



$$a(\omega_g) = \frac{K}{\sqrt{2}} = 0,707K$$

$$H(s) = K \frac{\omega_p^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

$$a(\omega) = K \frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_p}\right)^2\right)^2 + \left(\frac{1}{Q_p} \frac{\omega}{\omega_p}\right)^2}}$$



### 2) Visokopropusni (VP)

$$H_{VP}(s) = \frac{K \cdot s}{s + \omega_g}$$

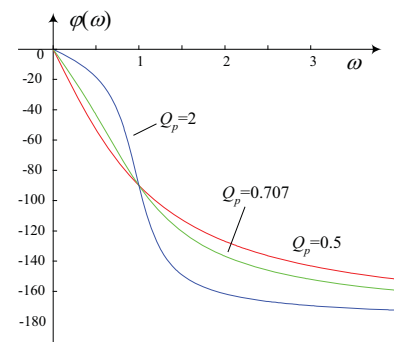
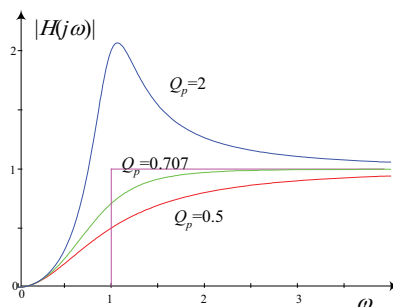
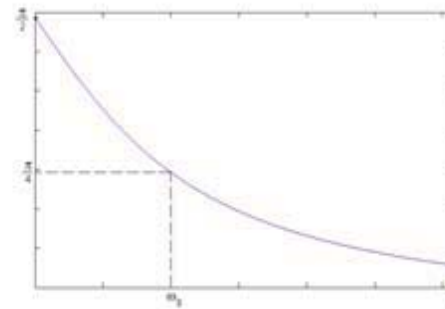
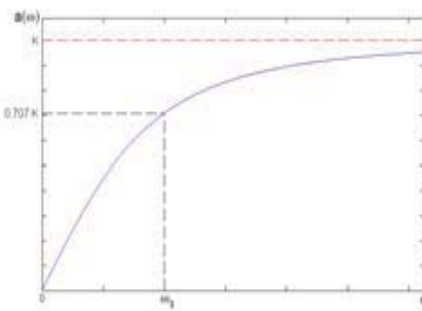
$$a(\omega) = K \frac{|\omega|}{\sqrt{\omega^2 + \omega_g^2}}$$

$$\phi(\omega) = \frac{\pi}{2} - \arctg\left(\frac{\omega}{\omega_g}\right)$$

$$H(s) = K \cdot \frac{s^2}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

$$a(\omega) = |H(j\omega)| = K \frac{\omega^2}{\sqrt{(\omega_p^2 - \omega^2)^2 + \left(\frac{\omega\omega_p}{Q_p}\right)^2}}$$

$$\phi(\omega) = \pi - \arctg \frac{\omega_p \cdot \omega / Q_p}{\omega_p^2 - \omega^2}$$

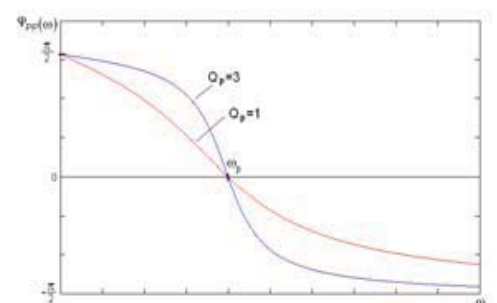
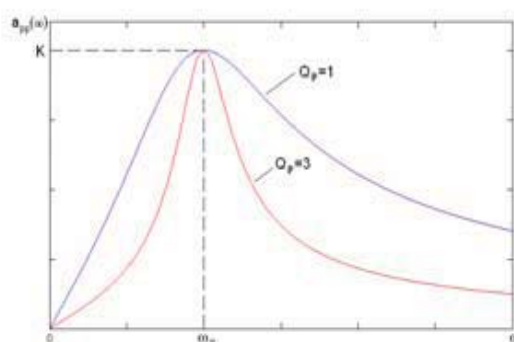


### 3) Pojasno propusni (PP)

$$H_{PP}(s) = K \cdot \frac{s \cdot \frac{\omega_p}{Q_p}}{s^2 + \frac{\omega_p}{Q_p}s + \omega_p^2}$$

$$a_{pp}(\omega) = \frac{K}{\sqrt{1 + Q_p^2 \left(\frac{\omega}{\omega_p} - \frac{\omega_p}{\omega}\right)^2}}$$

$$\phi_{pp}(\omega) = -\arctg \left[ Q \left( \frac{\omega}{\omega_p} - \frac{\omega_p}{\omega} \right) \right]$$

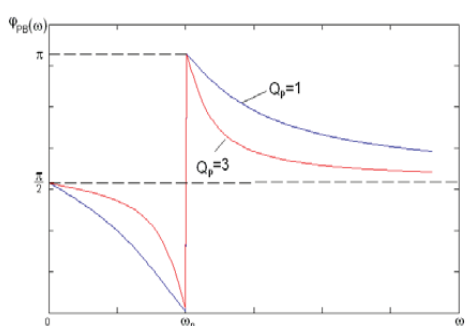
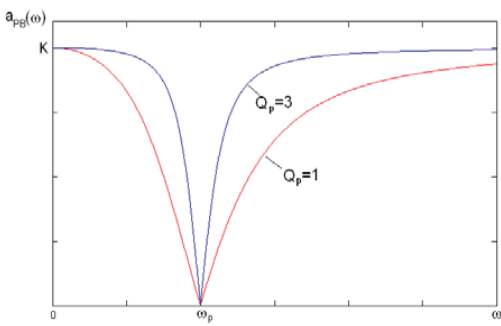


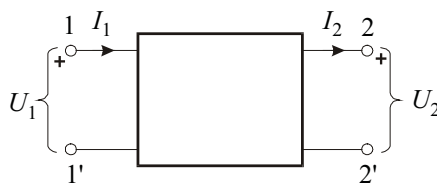
4) Pojasna brana (PB)

$$H_{PB}(s)=K\cdot\frac{s^2+\omega_p^2}{s^2+\frac{\omega_p}{Q_p}s+\omega_p^2}$$

$$a_{PB}(\omega)=K\cdot\frac{Q_p\left|\frac{\omega_p}{\omega}-\frac{\omega}{\omega_p}\right|}{\sqrt{1+Q_p^2\left(\frac{\omega_p}{\omega}-\frac{\omega}{\omega_p}\right)^2}}$$

$$\varphi_{PB}(\omega)=\pi S(\omega-\omega_p)-\arctg\left(Q_p\left(\frac{\omega_p}{\omega}-\frac{\omega}{\omega_p}\right)\right)$$





### Strujne jednačbe četveropola

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$$

### Naponske jednačbe četveropola

$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

### Prijenosne jednačbe četveropola

$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix}$$

■ y-parametri → iz četveropola na kratko    z-parametri → iz četveropola na prazno    α-parametri → iz 2-2' na prazno i na kratko

$$y_{11} = \left. \frac{I_1}{U_1} \right|_{U_2=0}$$

$$y_{21} = \left. \frac{I_2}{U_1} \right|_{U_2=0}$$

$$z_{11} = \left. \frac{U_1}{I_1} \right|_{I_2=0}$$

$$z_{21} = \left. \frac{U_2}{I_1} \right|_{I_2=0}$$

$$A = \left. \frac{U_1}{U_2} \right|_{I_2=0}$$

$$C = \left. \frac{I_1}{U_2} \right|_{I_2=0}$$

$$y_{12} = -\left. \frac{I_1}{U_2} \right|_{U_1=0}$$

$$y_{22} = -\left. \frac{I_2}{U_2} \right|_{U_1=0}$$

$$z_{12} = -\left. \frac{U_1}{I_2} \right|_{I_1=0}$$

$$z_{22} = -\left. \frac{U_2}{I_2} \right|_{I_1=0}$$

$$B = \left. \frac{U_1}{I_2} \right|_{U_2=0}$$

$$D = \left. \frac{I_1}{I_2} \right|_{U_2=0}$$

$$[z] = [y]^{-1}$$

### Hibridne jednačbe četveropola

$$\begin{bmatrix} U_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ U_2 \end{bmatrix}$$

### Hibridne jednačbe četveropola

$$\begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$$

h-parametri → iz 1-1' na prazno i 2-2' na kratko    g-parametri → iz 2-2' na prazno i 1-1' na kratko

$$h_{11} = \left. \frac{U_1}{I_1} \right|_{U_2=0}$$

$$h_{21} = \left. \frac{I_2}{I_1} \right|_{U_2=0}$$

$$g_{11} = \left. \frac{I_1}{U_1} \right|_{I_2=0}$$

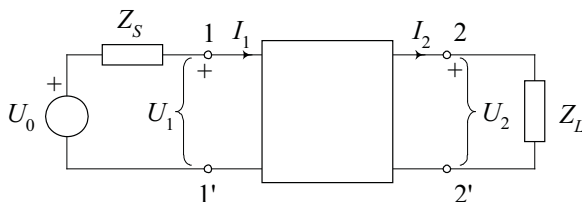
$$g_{21} = \left. \frac{U_2}{U_1} \right|_{I_2=0}$$

$$h_{12} = \left. \frac{U_1}{U_2} \right|_{I_1=0}$$

$$h_{22} = \left. \frac{I_2}{U_2} \right|_{I_1=0}$$

$$g_{12} = \left. \frac{I_1}{I_2} \right|_{U_1=0}$$

$$g_{22} = \left. \frac{U_2}{I_2} \right|_{U_1=0}$$



- Prijenosnu funkciju napona  $H_u(s) = U_2(s)/U_1(s)$
- Prijenosnu funkciju struje  $H_i(s) = I_2(s)/I_1(s)$
- Ekvivalentnu ulaznu impedanciju  $Z_u(s) = U_1(s)/I_1(s)$
- Ekvivalentnu izlaznu impedanciju  $Z_i(s) = -U_2(s)/I_2(s)|_{U_0=0}$

Prijenosne funkcije izražene z-parametrima    Prijenosne funkcije izražene y-parametrima    Prijenosne funkcije izražene prijenosnim parametrima

$$H_i(s) = \frac{I_2}{I_1} = \frac{z_{21}}{Z_L + z_{22}}$$

$$H_i(s) = \frac{I_2}{I_1} = \frac{Y_L z_{21}}{y_{11}(y_{22} + Y_L) - y_{12}y_{21}}$$

$$= \frac{Y_L y_{21}}{\Delta_y + y_{11}Y_L}$$

$$H_i(s) = \frac{I_2}{I_1} = \frac{1}{CZ_L + D}$$

$$H_u(s) = \frac{U_2}{U_1} = \frac{Z_L z_{21}}{z_{11}(z_{22} + Z_L) - z_{12}z_{21}}$$

$$= \frac{Z_L z_{21}}{\Delta_z + z_{11}Z_L}$$

$$H_u(s) = \frac{U_2}{U_1} = \frac{y_{21}}{Y_L + y_{22}}$$

$$H_u(s) = \frac{U_2}{U_1} = \frac{Z_L}{AZ_L + B}$$

$$H(s) = \frac{U_2}{U_0} = \frac{Z_L}{AZ_L + B + Z_s(CZ_L + D)}$$

$$H(s) = \frac{U_2}{U_0} = \frac{Z_L z_{21}}{(z_{11} + Z_s)(z_{22} + Z_L) - z_{12}z_{21}}$$

$$Y_{ul1} = y_{11} - \frac{y_{12}y_{21}}{Y_2 + y_{22}}$$

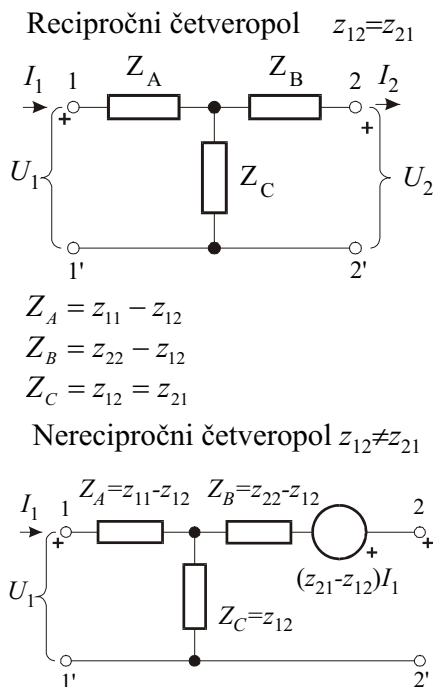
$$Z_{ul} = \frac{U_1}{I_1} = \frac{AZ_2 + B}{CZ_2 + D}$$

$$Z_{ul1} = \frac{U_1}{I_1} = z_{11} - z_{12} \cdot \frac{z_{21}}{Z_2 + z_{22}}$$

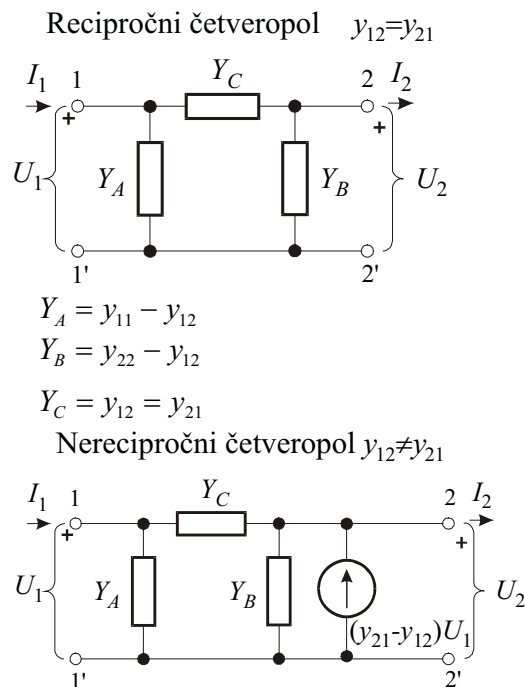
$$Y_{ul2} = y_{22} - \frac{y_{12}y_{21}}{Y_1 + y_{11}}$$

$$Z_{ul2} = -\frac{U_2}{I_2} = \frac{DZ_1 + B}{CZ_1 + A}$$

## Ekvivalentni četveropol u T-spoju



## Ekvivalentni četveropol u Π-spoju



## simetričan četveropol

$$z_{11} = z_{22}$$

$$y_{11} = y_{22}$$

$$A = D$$

$$\begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} = -1$$

$$\begin{vmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{vmatrix} = 1$$

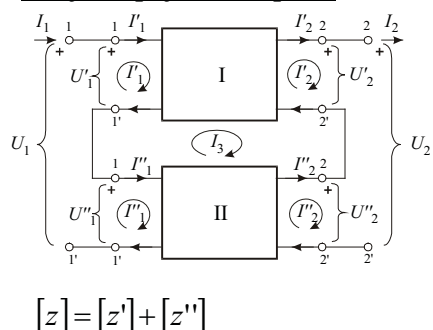
## recipročan četveropol

$$z_{12}=z_{21}$$

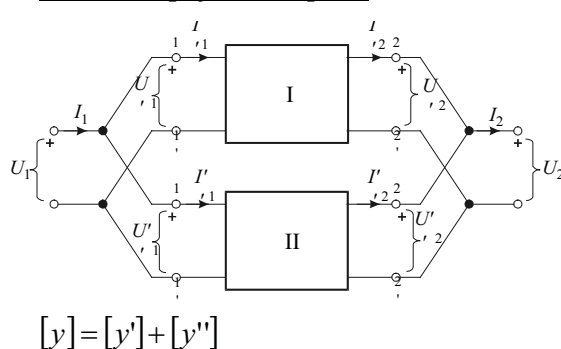
$$y_{12}=y_{21}$$

$$\Delta_A = \begin{vmatrix} A & B \\ C & D \end{vmatrix} = AD - BC = 1$$

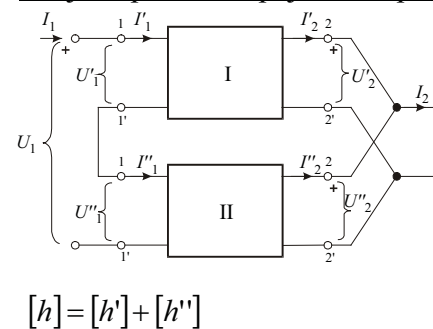
## Serijski spoj četveropola:



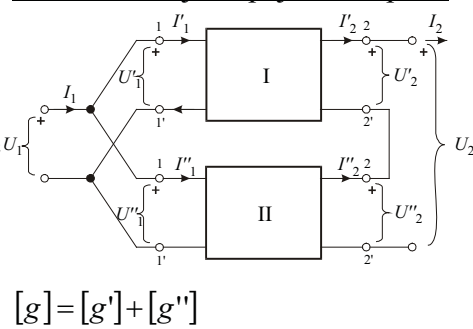
## Paralelni spoj četveropola:



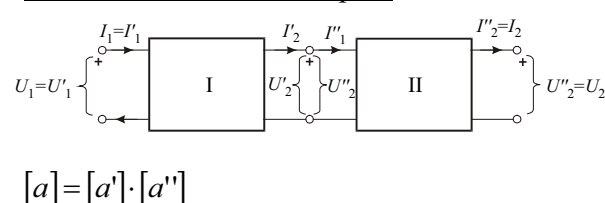
## Serijsko-paralelni spoj četveropola:



## Paralelno-serijski spoj četveropola:



## Lanac ili kaskada četveropola:





$$Z_0 = \sqrt{\frac{R + sL}{G + sC}}$$

$$\gamma = \sqrt{(R + sL)(G + sC)}$$

$$g = \gamma \cdot l$$

sinusoidalna pobuda:

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

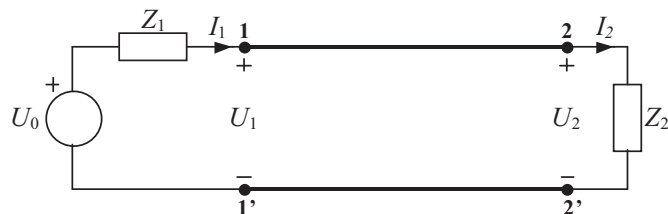
$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$U(l) = U(0) \operatorname{ch} g - I(0) Z_0 \operatorname{sh} g$$

$$I(l) = -U(0) \frac{\operatorname{sh} g}{Z_0} + I(0) \operatorname{ch} g$$

$$U(0) = U(l) \operatorname{ch} g + I(l) Z_0 \operatorname{sh} g$$

$$I(0) = U(l) \frac{\operatorname{sh} g}{Z_0} + I(l) \operatorname{ch} g$$



$$Z_{ul} = \frac{U_1}{I_1} = \frac{U_2 \operatorname{ch} g + I_2 Z_0 \operatorname{sh} g}{U_2 \frac{\operatorname{sh} g}{Z_0} + I_2 \operatorname{ch} g}$$

$$U_2 = I_2 \cdot Z_2$$

$$Z_{ul} = Z_0 \frac{Z_2 \operatorname{ch} g + Z_0 \operatorname{sh} g}{Z_2 \operatorname{sh} g + Z_0 \operatorname{ch} g}$$

$$\Gamma_2 = \frac{Z_2 - Z_0}{Z_2 + Z_0} \quad \Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$|\Gamma_2| = \left| \frac{U_{odb}}{U_{pol}} \right|$$

$$u(x, t) = U(x) e^{st}$$

$$i(x, t) = I(x) e^{st}$$

$$U(x) = U(0) \operatorname{ch} \gamma x - I(0) Z_0 \operatorname{sh} \gamma x$$

$$I(x) = -U(0) \frac{\operatorname{sh} \gamma x}{Z_0} + I(0) \operatorname{ch} \gamma x$$

$x$  - udaljenost od početka linije

$\gamma$  i  $Z_0$  - sekundarni parametri linije

$U(0)$  i  $I(0)$  - napon i struja na početku linije

sinusoidalni izvor frekvencije  $\omega_0$

$$\text{valna duljina } \lambda_0: \quad \lambda_0 = \frac{2\pi}{\beta} = \frac{2\pi}{\omega_0 \sqrt{LC}}$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \alpha + j\beta$$

$$g = (\alpha + j\beta)l = a + jb$$

$\alpha$  - faktor gušenja linije po jedinici duljine

$\beta$  - faktor faze linije po jedinici duljine

$$\beta = \omega \sqrt{LC}$$

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

$$g = \gamma \cdot l = \gamma \cdot k \cdot \lambda = j\omega \sqrt{LC} \cdot k \cdot \frac{2\pi}{\omega \sqrt{LC}} = j2\pi k$$

### 1. LINIJA BEZ GUBITAKA

$$R = G = 0$$

$$Z_0 = \sqrt{\frac{L}{C}} \quad \gamma = s\sqrt{LC}$$

Za  $s = j\omega \rightarrow$  sinusna pobuda

$$\gamma = j\omega \sqrt{LC} = j\beta \quad \alpha = 0$$

### 2. LINIJA BEZ DISTORZIJE

$$RC = GL$$

$$Z_0 = \sqrt{\frac{L}{C}} \quad \gamma = \sqrt{RG} + s\sqrt{LC}$$

Za  $s = j\omega \rightarrow$  sinusna pobuda

$$\gamma = \sqrt{RC} + j\omega \sqrt{LC}$$

$$\alpha = R \sqrt{\frac{C}{L}} = \sqrt{RG} \quad \beta = \omega \sqrt{LC}$$

$$v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$$

### 3. RC-LINIJA

$$G = 0 \quad L = 0$$

$$Z_0 = \sqrt{\frac{R}{sC}}$$

$$Z_0 = \sqrt{\frac{R}{j\omega C}} = \sqrt{\frac{R}{\omega C}} \cdot e^{-j45^\circ}$$

$$\gamma = \sqrt{R \cdot j\omega C} = \sqrt{\omega RC} e^{j45^\circ} = \sqrt{\frac{\omega RC}{2}} + j\sqrt{\frac{\omega RC}{2}}$$

$$\alpha = \beta = \sqrt{\frac{\omega RC}{2}}$$

### 4. LINIJA S MALIM GUBICIMA

$$\omega L \gg R \quad \omega C \gg G$$

$$Z_0 = \sqrt{\frac{L}{C}} \cdot e^{-j\left(\frac{R}{2\omega L} - \frac{G}{2\omega C}\right)}$$

$$\gamma \cong \left( \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}} \right) + j\omega \sqrt{LC}$$

$$\alpha = \frac{R}{2} \sqrt{\frac{C}{L}} + \frac{G}{2} \sqrt{\frac{L}{C}}$$

$$\beta = \omega \sqrt{LC}$$