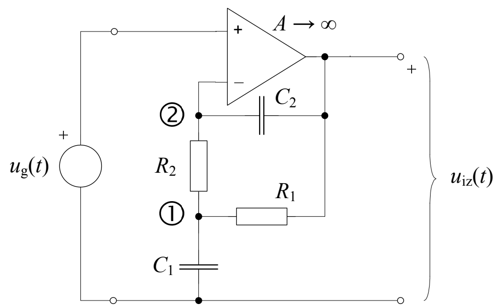
PONOVLJENI ZAVRŠNI ISPIT IZ PREDMETA ELEKTRIČNI KRUGOVI 2008/09

Rješenja i bodovi (svaki zadatak je bodovan od 0 do 5 bodova):

1. Za električni krug na slici zadana je pobuda $u_g(t)=\delta(t)$ i normalizirane vrijednosti elemenata: $R_1=R_2=1$, $C_1=2$, $C_2=1/2$. Odrediti: *a*) jednadžbe čvorišta; *b*) prijenosnu funkciju: $T(s)=U_{iz}(s)/U_g(s)$; c) polove i nule $T(s)$ i odziv $U_{iz}(s)$; *d*) odziv $u_{iz}(t)$.



Rješenje: *a*) jednadžbe čvorišta

$$1)\ U_1\left(sC_1+\frac{1}{R_1}+\frac{1}{R_2}\right)-U_2\left(\frac{1}{R_2}\right)-\frac{U_{iz}(s)}{R_1}=0$$

$$2)\ U_2\left(sC_2+\frac{1}{R_2}\right)-U_1\left(\frac{1}{R_2}\right)-U_{iz}(s)sC_2=0 \qquad\qquad U_2=U_g$$

b) prijenosna funkcija: $T(s)=U_{iz}(s)/U_g(s)$

$$2)\quad\rightarrow\quad U_1=U_g(sR_2C_2+1)-U_{iz}sR_2C_2$$

$$\left[U_g(sR_2C_2+1)-U_{iz}sR_2C_2\right]\left(sC_1+\frac{1}{R_1}+\frac{1}{R_2}\right)=U_g\frac{1}{R_2}+\frac{U_{iz}}{R_1}\Big/\cdot R_1$$

$$\left(U_g(sR_2C_2+1)-U_{iz}sR_2C_2\right)\left(sR_1C_1+1+\frac{R_1}{R_2}\right)=U_g\frac{R_1}{R_2}+U_{iz}$$

$$-U_{iz}\left[sR_2C_2\left(sR_1C_1+1+\frac{R_1}{R_2}\right)+1\right]=U_g\left[\frac{R_1}{R_2}-(sR_2C_2+1)\left(sR_1C_1+1+\frac{R_1}{R_2}\right)\right]$$

$$T(s)=\frac{U_{iz}(s)}{U_g(s)}=-\frac{-s^2R_1R_2C_1C_2-sR_1C_2-sR_2C_2-sR_1C_1-1}{s^2R_1R_2C_1C_2+sR_1C_2+sR_2C_2+1}=$$

$$=\frac{s^2R_1R_2C_1C_2+s(R_1C_2+R_2C_2+R_1C_1)+1}{s^2R_1R_2C_1C_2+s(R_1+R_2)C_2+1}=\frac{s^2+3s+1}{s^2+s+1}=1+\frac{2s}{s^2+s+1}\ \text{(2 boda)}$$

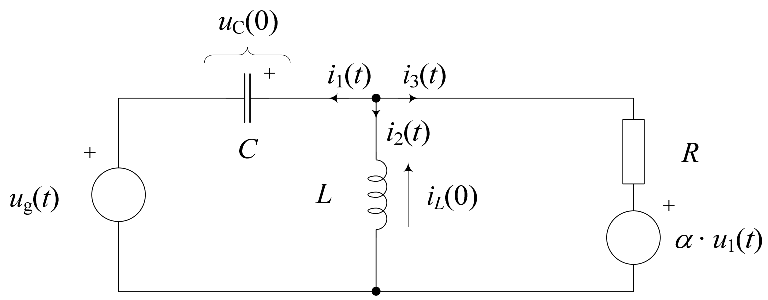
c) polovi i nule $T(s)$ i odziv $U_{iz}(s)$

-nule: $s_{o1,2}=-\frac{3}{2}\pm\sqrt{\frac{9}{4}-1}=-\frac{3}{2}\pm\frac{\sqrt{5}}{2}$; polovi: $s_{p1,2}=-\frac{1}{2}\pm\sqrt{\frac{1}{4}-1}=-\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$ (1 bod)

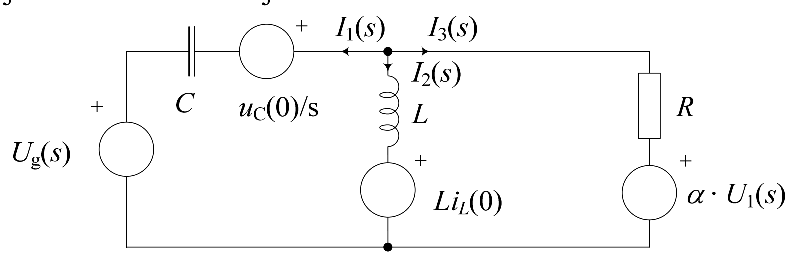
-odziv $U_{iz}(s)$: $U_{iz}(s)=T(s)\cdot U_g(s)=T(s)\cdot 1$ (1 bod)

d) odziv $u_{iz}(t)$: $u_{iz}(t)=\delta(t)+\left(2e^{-\frac{1}{2}t}\cos\frac{\sqrt{3}}{2}t-\frac{2}{\sqrt{3}}e^{-\frac{1}{2}t}\sin\frac{\sqrt{3}}{2}t\right)\cdot S(t)$ (1 bod)

2. Za električni krug prikazan slikom odrediti: *a*) orijentirani graf i temeljni sustav petlji; *b*) spojnu matricu **S**; *c*) naponsko-strujne jednadžbe grana; *d*) temeljni sustav jednadžbi petlji u matričnom obliku (matrice $\mathbf{Z}_p$ i $\mathbf{U}_{0p}$ preko matrica $\mathbf{Z}_b$ i $\mathbf{U}_{0b}$). Matrica $\mathbf{Z}_b$ mora biti regularna.



Rješenje: Primjena $\mathcal{L}$-transformacije



c) Naponsko-strujne jednadžbe grana: (2 boda)

$$U_1 = I_1 \cdot \frac{1}{sC} + \frac{u_C(0)}{s} + U_g(s)$$

$$U_2 = I_2 \cdot sL + Li_L(0)$$

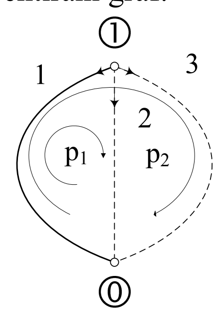
$$U_3 = I_3 \cdot R + \alpha \cdot U_1(s) =$$

$$= I_3 \cdot R + \alpha \cdot I_1 \cdot \frac{1}{sC} + \alpha \frac{u_C(0)}{s} + \alpha U_g(s)$$

U matričnom obliku: $\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$

$$\underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}}_{\mathbf{U}_b} = \underbrace{\begin{bmatrix} \frac{1}{sC} & 0 & 0 \\ 0 & sL & 0 \\ \frac{\alpha}{sC} & 0 & R \end{bmatrix}}_{\mathbf{Z}_b} \cdot \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{\mathbf{I}_b} + \underbrace{\begin{bmatrix} \frac{u_C(0)}{s} + U_g(s) \\ Li_L(0) \\ \alpha \frac{u_C(0)}{s} + \alpha U_g(s) \end{bmatrix}}_{\mathbf{U}_{0b}}$$

a) Orijentirani graf:



b) Spojna matrica:

$$\mathbf{S} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

(1bod)

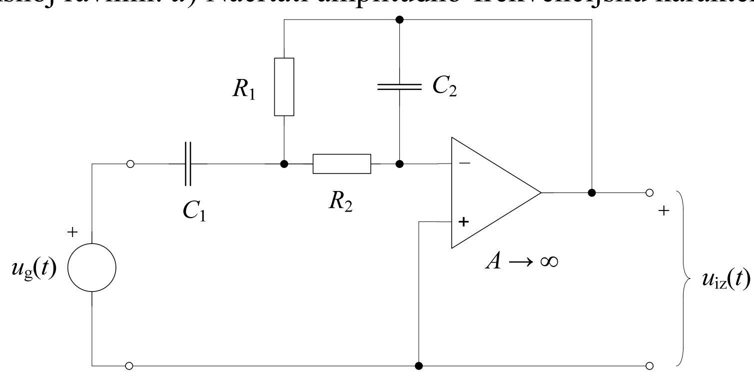
d) Temeljni sustav jednadžbi petlji u matričnom obliku $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$, gdje su:

$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{sC} & 0 & 0 \\ 0 & sL & 0 \\ \frac{\alpha}{sC} & 0 & R \end{bmatrix} \cdot \begin{bmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} =$$

$$=\begin{bmatrix} -\frac{1}{sC} & sL & 0 \\ \frac{\alpha-1}{sC} & 0 & R \end{bmatrix}\cdot\begin{bmatrix} -1 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} \frac{1}{sC}+sL & \frac{1}{sC} \\ \frac{1-\alpha}{sC} & \frac{1-\alpha}{sC}+R \end{bmatrix} \quad \text{(1bod)}$$

$$\mathbf{U}_{0p}=-\mathbf{S}\cdot\mathbf{U}_{0b}=-\begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} \frac{u_C(0)}{s}+U_g(s) \\ Li_L(0) \\ \alpha\frac{u_C(0)}{s}+\alpha U_g(s) \end{bmatrix}=\begin{bmatrix} \frac{u_C(0)}{s}+U_g(s)-Li_L(0) \\ \left(\frac{u_C(0)}{s}+U_g(s)\right)(1-\alpha) \end{bmatrix} \quad \text{(1bod)}$$

3. Zadan je aktivni filtar prikazan slikom i njegova prijenosna funkcija $T(s)=U_{iz}(s)/U_g(s)$. *a*) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja odrediti parametre k, ω_0, Q. O kojem se tipu filtra radi (NP, VP, PP ili PB)? *b*) Ako su zadane normalizirane vrijednosti parametara $\omega_0=1$ i $Q=2$ te ako je $R_1=R_2=1$, izračunati pojačanje k i normalizirane vrijednosti kapaciteta C_1 i C_2. *c*) Prikazati raspored polova i nula u kompleksnoj ravnini. *d*) Nacrtati amplitudno-frekvencijsku karakteristiku.



$$T(s)=\frac{U_{iz}(s)}{U_g(s)}=-\frac{sR_1C_1}{s^2R_1R_2C_1C_2+s(C_2R_1+C_2R_2)+1}$$

Rješenje:

$$a)\ T(s)=\frac{-k\cdot\frac{\omega_0}{Q}\cdot s}{s^2+\frac{\omega_0}{Q}\cdot s+\omega_0^{\ 2}} \qquad \text{Opći oblik PP}$$

(uvršteno je $-k$ kako bi pojačanje k bilo pozitivno, a utjecaj "–" predznaka se uzima u obzir kod izračuna fazno-frekvencijske karakteristike)

-o kojem se tipu filtra radi (NP, VP, PP ili PB)? $\Rightarrow$ PP (pojasni propust)

-parametri k, ω_0, Q: (1 bod)

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_1C_2}} \qquad \frac{\omega_0}{Q}=\frac{C_2(R_1+R_2)}{R_1R_2C_1C_2} \qquad k\cdot\frac{\omega_0}{Q}=\frac{1}{R_2C_2}$$

$$\Rightarrow Q=\frac{\omega_0}{\frac{C_2(R_1+R_2)}{R_1R_2C_1C_2}}=\frac{\sqrt{R_1R_2C_1C_2}}{C_2(R_1+R_2)},\quad k=\frac{1}{R_2C_2}\cdot\frac{Q}{\omega_0}=\frac{1}{R_2C_2}\cdot\frac{R_1R_2C_1C_2}{C_2(R_1+R_2)}=\frac{C_1R_1}{C_2(R_1+R_2)}$$

b) ako su zadane vrijednosti parametara ω_0=1 i Q=2 te ako je R_1=R_2=1, izračunati pojačanje k i normalizirane vrijednosti kapaciteta C_1 i C_2. (2 boda)

uz $R_1=R_2=R=1 \Rightarrow \omega_0 = \dfrac{1}{R\sqrt{C_1C_2}}$; $Q = \dfrac{1}{2}\sqrt{\dfrac{C_1}{C_2}}$; $\Rightarrow \dfrac{C_1}{C_2} = (2Q)^2 = (2\cdot 2)^2 = 16$;

$$k = \frac{C_1}{2C_2} = \frac{16}{2} = 8\,;\; R = \frac{1}{\omega_0\sqrt{C_1C_2}} = \frac{1}{\omega_0 C_2\sqrt{16}} = \frac{1}{4C_2} \Rightarrow C_2 = \frac{1}{4R} = \frac{1}{4}, C_1 = 16C_2 = \frac{16}{4} = 4$$

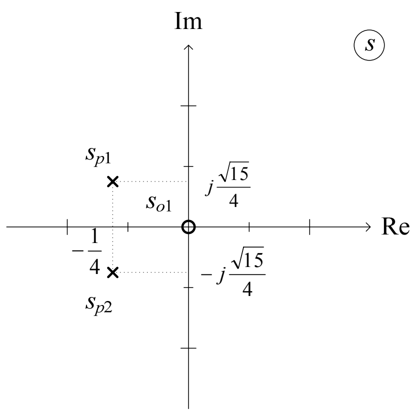
c) raspored polova i nula u kompleksnoj ravnini: (1bod)

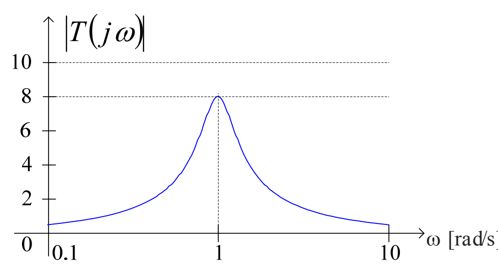
$$T(s) = \frac{-8\cdot\frac{1}{2}\cdot s}{s^2 + \frac{1}{2}\cdot s + 1} \qquad \text{nule} \qquad s_{o1} = 0, s_{o1} = \infty$$

$$\text{polovi} \quad s^2 + \frac{1}{2}\cdot s + 1 = 0 \qquad \Rightarrow \qquad s_{p1,2} = -\frac{1}{4} \pm \sqrt{\frac{1}{16} - 1} = -\frac{1}{4} \pm j\frac{\sqrt{15}}{4}$$

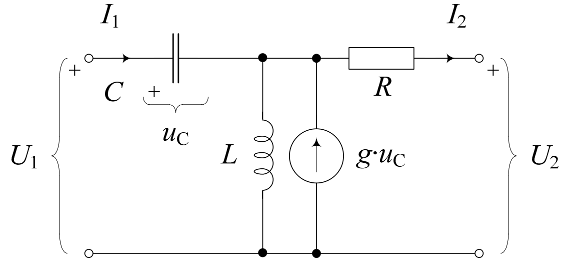
c) amplitudno-frekvencijska karakteristika: (1bod)

$$T(j\omega) = \frac{-\omega\cdot 4}{-\omega^2 + j\omega\cdot\frac{1}{2} + 1} \qquad \Rightarrow \qquad |T(j\omega)| = \frac{\left|8\cdot\frac{1}{2}\cdot\omega\right|}{\sqrt{(1-\omega^2)^2 + \left(\omega\cdot\frac{1}{2}\right)^2}}$$





4. Za četveropol prikazan slikom izračunati: *a*) prijenosne [*a*]-parametre direktno pomoću definicijskih jednadžbi, ako su zadane normalizirane vrijednosti elemenata R=2, L=1, C=1, g=1. *b*) Da li je četveropol recipročan i simetričan? Obrazložiti odgovor.



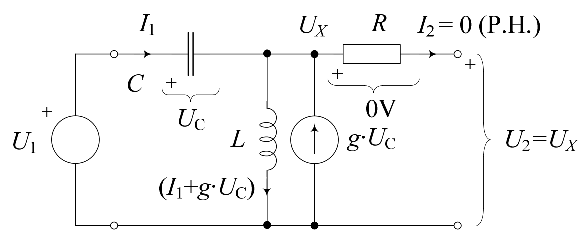
Rješenje:

a) izračun [*a*] parametara:

$$U_1 = A \cdot U_2 + B \cdot I_2$$

$$I_1 = C \cdot U_2 + D \cdot I_2$$

$$I_2 = 0 \qquad A = \left.\frac{U_1}{U_2}\right|_{I_2=0} \; ; \quad C = \left.\frac{I_1}{U_2}\right|_{I_2=0}$$



$$U_1 = I_1 \cdot \frac{1}{sC} + (I_1 + g \cdot U_C) sL$$

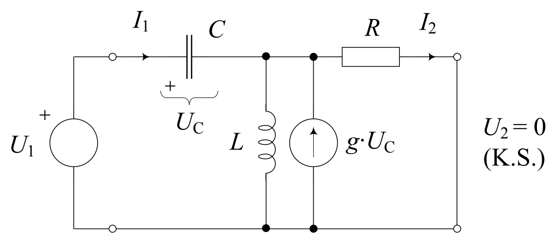
$$U_C = I_1 \cdot \frac{1}{sC} \qquad \Rightarrow \quad U_1 = I_1 \cdot \left[\frac{1}{sC} + \left(1 + g \cdot \frac{1}{sC}\right) sL\right]$$

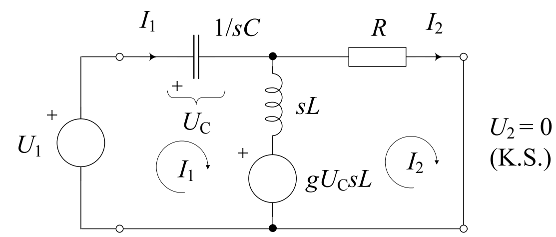
$$U_2 = U_X = (I_1 + g \cdot U_C) sL = I_1 \cdot \left(1 + g \cdot \frac{1}{sC}\right) sL$$

$$A = \left.\frac{U_1}{U_2}\right|_{I_2=0} = \frac{I_1 \cdot \left[\frac{1}{sC} + \left(1 + g \cdot \frac{1}{sC}\right) sL\right]}{I_1 \cdot \left(1 + g \cdot \frac{1}{sC}\right) sL} = \frac{1 + (sC + g) sL}{(sC + g) sL}$$

$$C = \left.\frac{I_1}{U_2}\right|_{I_2=0} = \frac{1}{\left(1 + g \cdot \frac{1}{sC}\right) sL} = \frac{sC}{(sC + g) sL}$$

$$U_2 = 0 \qquad B = \left.\frac{U_1}{I_2}\right|_{U_2=0} \; ; \quad D = \left.\frac{I_1}{I_2}\right|_{U_2=0}$$





$$U_1 - gU_C sL = I_1 \cdot \left(\frac{1}{sC} + sL\right) - I_2 \cdot sL;\;\; U_C = I_1 \cdot \frac{1}{sC}$$

$$gU_C sL = -I_1 \cdot sL + I_2 \cdot (sL + R)$$

$$U_1 = I_1 \cdot \left(\frac{1}{sC} + g \cdot \frac{1}{sC} sL + sL\right) - I_2 \cdot sL$$

$$0 = -I_1 \cdot \left(sL + g\frac{1}{sC} sL\right) + I_2 \cdot (sL + R)$$

$$2) \;\; \Rightarrow \;\; I_1 \cdot \left(1 + g\frac{1}{sC}\right) sL = I_2 \cdot (sL + R) \qquad D = \left.\frac{I_1}{I_2}\right|_{U_2 = 0} = \frac{sL + R}{\left(1 + g\frac{1}{sC}\right) sL} = \frac{sC(sL + R)}{(sC + g) sL}$$

$2) \to 1) \;\; \Rightarrow$

$$U_1 = I_1 \cdot \left[\frac{1}{sC} + sL\left(1 + g \cdot \frac{1}{sC}\right)\right] - I_2 \cdot sL = \frac{sL + R}{\left(1 + g\frac{1}{sC}\right) sL} I_2 \left[\frac{1}{sC} + sL\left(1 + g \cdot \frac{1}{sC}\right)\right] - I_2 \cdot sL$$

$$B = \left.\frac{U_1}{I_2}\right|_{U_2 = 0} = \frac{sL + R}{\left(1 + g\frac{1}{sC}\right) sL} \frac{1}{sC} + sL + R - sL = \frac{sL + R}{(sC + g) sL} + R = \frac{sL + R + R(sC + g) sL}{(sC + g) sL}$$

$$[a] = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{(sC + g) sL} \begin{bmatrix} 1 + (sC + g) sL & sL + R + R(sC + g) sL \\ sC & sC(sL + R) \end{bmatrix}$$

Uz uvrštene vrijednosti:

$$[a] = \frac{1}{(s + 1)s} \begin{bmatrix} 1 + (s + 1)s & s + 2 + 2(s + 1)s \\ s & s(s + 2) \end{bmatrix} = \frac{1}{(s + 1)s} \begin{bmatrix} s^2 + s + 1 & 2s^2 + 3s + 2 \\ s & s(s + 2) \end{bmatrix}$$

(4 boda: po jedan bod za svaki točan parametar)

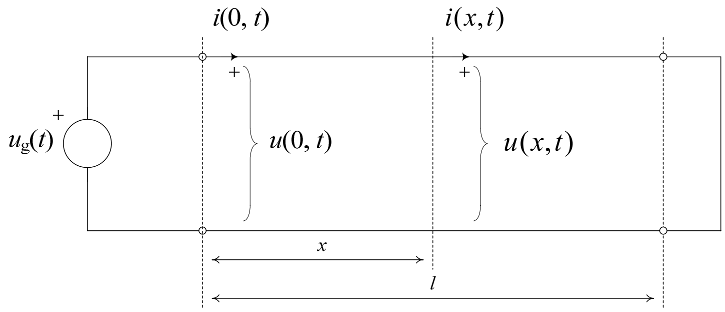
b) Da li je četveropol recipročan i simetričan? Za recipročnost vrijedi: $\Delta = AD - BC = 1$

$$\Delta = \frac{(s^2 + s + 1)(s^2 + 2s) - s(2s^2 + 3s + 2)}{(s^2 + s)^2} = \frac{s^4 + 2s^3 + s^3 + 2s^2 + s^2 + 2s - (2s^3 + 3s^2 + 2s)}{s^2 (s + 1)^2} =$$

$$= \frac{s^4 + s^3}{s^2 (s + 1)^2} = \frac{s^2 + s}{(s + 1)^2} = \frac{s(s + 1)}{(s + 1)^2} = \frac{s}{s + 1} \neq 1$$ nije recipročan (jer sadrži ovisni izvor).

Za simetričnost vrijedi: A=D $\Rightarrow \dfrac{s^2 + s + 1}{s(s + 1)} \neq \dfrac{s(s + 2)}{s(s + 1)}$ četveropol nije simetričan (1bod)

5. Zadana je linija bez gubitaka s L=2mH/km, C=800nF/km, duljine l=1 km. Na početku linije je stacionarni sinusni izvor napona $u_g(t)=10\sin(39{,}27\cdot 10^3 t)$; $-\infty<t<\infty$, a izlaz linije je u kratkome spoju. *a*) Kolika je duljina λ vala na liniji i koliko valnih duljina je na liniji? *b*) Kolika je karakteristična impedancija Z_0, faktor prijenosa linije γ te brzina širenja vala po liniji v? *c*) Kolika je ulazna impedancija u liniju Z_{ul}? *d*) Koliki su napon i struja na mjestu x=750m od početka linije?



Rješenje:

a) $$\lambda=\frac{2\pi}{\beta}=\frac{2\pi}{\omega_1\sqrt{LC}}=\frac{2\pi}{39{,}27\cdot 10^3\cdot\sqrt{2\cdot 10^{-3}\cdot 800\cdot 10^{-9}}}=\frac{2\pi}{39{,}27\cdot 10^3\cdot 40\cdot 10^{-6}}=4\text{km}$$

$$\Rightarrow \ell=1km=\frac{\lambda}{4}\quad (1\text{bod})$$

b) Za liniju bez gubitaka vrijedi:

$$Z_0=\sqrt{\frac{L}{C}}=\sqrt{\frac{2\cdot 10^{-3}}{800\cdot 10^{-9}}}=\frac{1}{20}\cdot 10^3\,\Omega=50\Omega,\quad \gamma=j\beta;\ \alpha=0,$$

$$\beta\ell=\beta\frac{\lambda}{4}=\beta\frac{1}{4}\frac{2\pi}{\beta}=\frac{\pi}{2}$$

$$v=\frac{\omega}{\beta}=\frac{\omega}{\omega\sqrt{LC}}=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{2\cdot 10^{-3}\cdot 800\cdot 10^{-9}}}=\frac{1}{40\cdot 10^{-6}}=25\cdot 10^3\,\text{km/s}\quad (1\text{bod})$$

c) $$Z_{ul}=\frac{Z_2\cosh(\gamma\ell)+Z_0\sinh(\gamma\ell)}{\frac{Z_2}{Z_0}\sinh(\gamma\ell)+\cosh(\gamma\ell)}=Z_0\tanh(\gamma\ell)=Z_0 j\tan(\beta\ell)$$

$$\tan(\beta\ell)=\tan(\pi/2)=\infty\quad\Rightarrow\quad Z_{ul}=Z_0 j\tan(\beta\ell)=\infty\quad (1\text{bod})$$

$$U(0)=10\angle 0°,\ I(0)=\frac{U(0)}{Z_{ul}}=0$$

d) Koliki su napon i struja na mjestu x=750m od početka linije ?

$$x=750\text{m}=\frac{3}{4}1km=\frac{3}{4}\ell=\frac{3}{4}\frac{\lambda}{4}=\frac{3\lambda}{8}\quad\Rightarrow\quad \beta x=\beta\frac{3\lambda}{8}=\beta\frac{3}{8}\frac{2\pi}{\beta}=\frac{3\pi}{4}$$

Prijenosne jednadžbe linije:

$$U(x)=U(0)\cdot\text{ch}(\gamma x)-I(0)\cdot Z_0\cdot\text{sh}(\gamma x)$$

$$I(x)=-U(0)/Z_0\cdot\text{sh}(\gamma x)+I(0)\cdot\text{ch}(\gamma x)$$

$I(0)=0 \Rightarrow U(x) = U(0)\cdot\cosh(\gamma x) = U(0)\cdot\cosh(j\beta x) = U(0)\cdot\cos(\beta x) = U(0)\cdot\cos\left(\frac{3\pi}{4}\right)$

$=U(0)\cdot\left(-\frac{\sqrt{2}}{2}\right) = -7{,}07\ \text{V}$ $\underline{u(750,t) = 7{,}07\sin(39{,}27\cdot 10^3 t)V}$ (1bod)

$I(x) = -\frac{U(0)}{Z_0}\sinh(\gamma x) = -\frac{U(0)}{Z_0}\sinh(j\beta x) = -j\frac{U(0)}{Z_0}\sin(\beta x) = -j\frac{U(0)}{Z_0}\sin\left(\frac{3\pi}{4}\right) =$

$= -j\frac{U(0)}{50}\left(\frac{\sqrt{2}}{2}\right) = -j\frac{7{,}07}{50} = -j0{,}141\ \text{A}$ $\underline{i(750,t) = 141\cdot 10^{-3}\sin(39{,}27\cdot 10^3 t - 90°)A}$

(1bod)