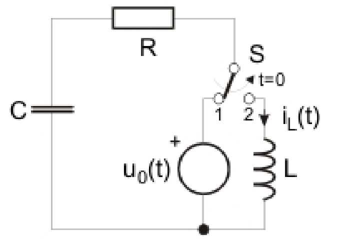
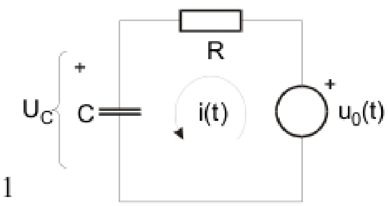
ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA

1. U trenutku t=0 sklopka S se prebaci iz položaja 1 u položaj 2. Izračunati odziv $i_L(t)$. Zadana je pobuda $u_0(t)$=2cos(2t) ($-\infty<t<+\infty$ stacionarna sinusna pobuda) i normirane vrijednosti elemenata: R=2, L=1, C=1/2.



Rješenje:

a) Za $t\leq 0$ stacionarna sinusna pobuda → fazori



$$U_0(j\omega)=I(j\omega)\cdot\left(R+\frac{1}{j\omega C}\right),\ U_C(j\omega)=I(j\omega)\cdot\frac{1}{j\omega C}$$

$$U_C(j\omega)=U_0(j\omega)\cdot\frac{1}{1+j\omega RC};\quad U_0(j\omega)=2\angle 0°$$

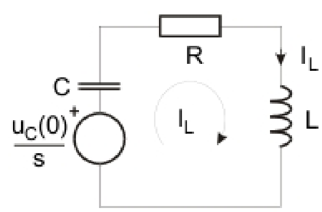
$$U_C(j\omega)=\frac{2}{1+j2\cdot 2\cdot\frac{1}{2}}=\frac{2}{1+j2}\cdot\frac{1-j2}{1-j2}=\frac{2}{5}(1-j2)=\frac{2}{\sqrt{5}}e^{-j\arctan(2)}$$

$$u_C(t)=\frac{2}{\sqrt{5}}\cos(2t-\arctan 2)\quad\Rightarrow\quad u_C(0)=\frac{2}{\sqrt{5}}\cos(\arctan 2)$$

$$\tan x=2,\quad \cos x=?\ \tan x=\frac{\sin x}{\cos x}=\frac{\sqrt{1-\cos^2 x}}{\cos x}\Rightarrow \tan^2 x=\frac{1-\cos^2 x}{\cos^2 x}\Rightarrow \cos^2 x(1+\tan^2 x)=1$$

$$\cos x=\frac{1}{\sqrt{1+\tan^2 x}},\ \cos x=\frac{1}{\sqrt{1+4}}=\frac{1}{\sqrt{5}},\Rightarrow\quad u_C(0)=\frac{2}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}}=\frac{2}{5}=0.4$$

b) Za $t>0$ Laplaceova transformacija

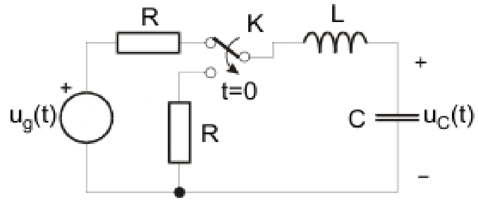


$$I_L(s)=\frac{\frac{u_C(0)}{s}}{R+sL+\frac{1}{sC}}=\frac{Cu_C(0)}{s^2LC+sRC+1}\quad\Rightarrow\quad I_L(s)=\frac{\frac{1}{2}\cdot 0.4}{\frac{s^2}{2}+s+1}=\frac{0.4}{s^2+2s+2}$$

$$I_L(s)=0.4\cdot\frac{1}{(s+1)^2+1}\quad\Rightarrow\quad \underline{i_L(t)=0.4\cdot e^{-t}\sin t\cdot S(t)}$$

Prijelazne pojave

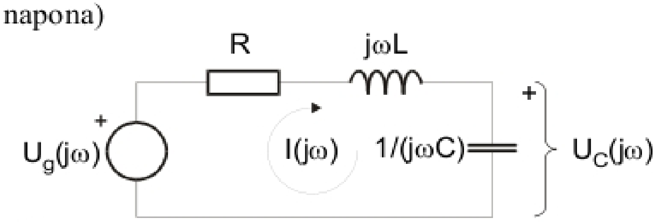
20. Za mrežu na slici odrediti napon na kapacitetu $u_C(t)$ ako se u trenutku t=0 prebaci sklopka K. Zadano je: R=4, C=1/2, L=2, $u_g(t)$=10sin(2t); $-\infty<t<\infty$ (sinusoidalno stacionarno stanje).



Rješenje:

Zadatak se rješava u dva koraka: u prvom *a*) koraku se pomoću fazora za t<0 izračuna utjecaj pobude tako da se nađu početni uvjeti: napon na kapacitetu i početna struja kroz induktivitet. U drugom *b*) koraku se za $t\geq 0$ uz poznate početne uvjete pomoću Laplaceove transformacije izračuna traženi napon na kapacitetu $u_C(t)$.

a) t<0 (fazori stuje i napona)

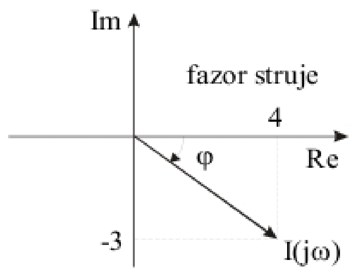


$$U_g(j\omega) = 10\angle 0°,\quad \omega = 2$$

Fazor struje u električnom krugu:

$$I(j\omega) = \frac{U_g(j\omega)}{R + j\omega L + \dfrac{1}{j\omega C}} = \frac{U_g(j\omega)}{R + j\left(\omega L - \dfrac{1}{\omega C}\right)} = \frac{10}{4 + j\left(2\cdot 2 - \dfrac{1}{2\cdot(1/2)}\right)} =$$

$$= \frac{10}{4+j3}\cdot\frac{4-j3}{4-j3} = \frac{10\cdot(4-j3)}{25} = \frac{2}{5}\cdot(4-j3)$$



$$|I(j\omega)| = \frac{10}{\sqrt{4^2+3^2}} = \frac{10}{\sqrt{25}} = \frac{10}{5} = 2$$

$$\varphi = arctg\frac{\mathrm{Im}}{\mathrm{Re}} = arctg\frac{-3}{4} = -36.87°$$

Iz fazora slijede podaci o struji u električnom krugu u vremenskoj domeni:

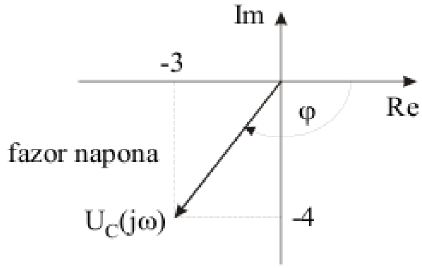
$$i(t) = 2\cdot\sin(2t - 36.87°)$$

U trenutku t=0 se tada može izračunati početna struja u el. krugu koja je ujedno i početna struja kroz induktivitet.

$$i_L(0) = i(t)_{t=0} = 2\cdot\sin(-36.87°) = 2\cdot(-0.6) = -1.2\,A$$

Fazor napon na kapacitetu C u električnom krugu je:

$$U_C(j\omega)=I(j\omega)\cdot\frac{1}{j\omega C}=\frac{10}{4+j3}\cdot\frac{1}{j2\frac{1}{2}}=\frac{10}{4+j3}\cdot\frac{1}{j}=\frac{10}{-3+j4}=\frac{2}{5}(-3-j4)$$



$$|U_C(j\omega)|=\frac{10}{\sqrt{3^2+4^2}}=\frac{10}{\sqrt{25}}=\frac{10}{5}=2$$

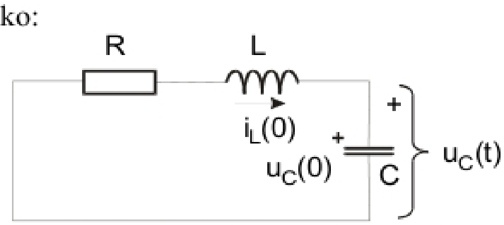
$$\varphi=arctg\frac{\text{Im}}{\text{Re}}=arctg\frac{-3}{-4}=-126.87°$$

$$u_C(t)=2\cdot\sin(2t-126.87°)$$

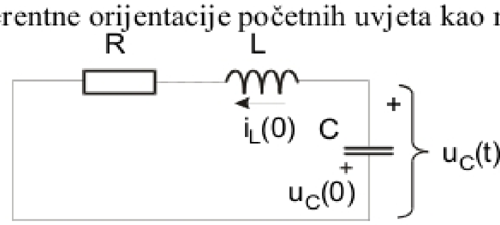
$$u_C(0)=2\cdot\sin(-126.87°)=2\cdot(-0.8)=-1.6V$$

b) $t\geq 0$ (Laplaceova transformacija)

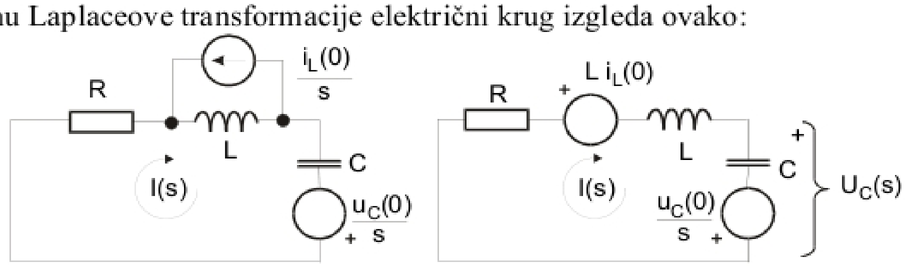
Uz poznate početne uvjete $i_L(0)=-1.2A$ i $u_C(0)=-1.6V$, te pobudu $u_g(t)=0$, (za $t\geq 0$) električni krug izgleda ovako:



Ako bismo htjeli uvrstiti pozitivne vrijednosti početnih uvjeta $i_L(0)=1.2A$ i $u_C(0)=1.6V$, tada trebamo izmjeniti referentne orijentacije početnih uvjeta kao na slijedećoj slici:



Uz primjenu Laplaceove transformacije električni krug izgleda ovako:



Jednadžba za struju za električni krug

$$I(s)\cdot\left(R+\frac{1}{sC}+sL\right)+L\cdot i_L(0)-\frac{u_C(0)}{s}=0$$

uz uvrštene vrijednosti:

$$I(s)\left(4+\frac{2}{s}+2s\right)+2\cdot 1.2-\frac{1.6}{s}=0$$

$$I(s)=\frac{\frac{1.6}{s}-2.4}{4+\frac{2}{s}+2s}=\frac{1.6-2.4s}{2s^2+4s+2}=\frac{0.8-1.2s}{s^2+2s+1}$$

Traženi napon na kapacitetu je:

$$U_C(s)=I(s)\cdot\frac{1}{sC}-\frac{u_C(0)}{s}$$

uz uvrštene vrijednosti:

$$U_C(s)=\underbrace{\frac{0.8-1.2s}{s^2+2s+1}\cdot\frac{2}{s}}_{(*)}-\frac{1.6}{s}$$

Rastav na parcijalne razlomke izraza (*):

$$(*)=\frac{1.6-2.4s}{s^2+2s+1}\cdot\frac{1}{s}=\frac{As+B}{s^2+2s+1}+\frac{C}{s}=\frac{As^2+Bs+Cs^2+2Cs+C}{(s^2+2s+1)\cdot s}=\frac{(A+C)s^2+(B+2C)s+C}{(s^2+2s+1)\cdot s}$$

$$\begin{aligned}A+C&=0\\B+2C&=-2.4\\C&=1.6\end{aligned}$$

$A=-C=-1.6$;
$B=-2C-2.4=-3.2-2.4=-5.6$.

$$(*)=\frac{-1.6s-5.6}{s^2+2s+1}+\frac{1.6}{s}$$

Konačno je:

$$U_C(s)=\underbrace{-1.6\cdot\frac{s}{(s+1)^2}-5.6\frac{1}{(s+1)^2}+\frac{1.6}{s}}_{(*)}-\frac{1.6}{s}=-1.6\cdot\frac{s}{(s+1)^2}-5.6\frac{1}{(s+1)^2}=$$

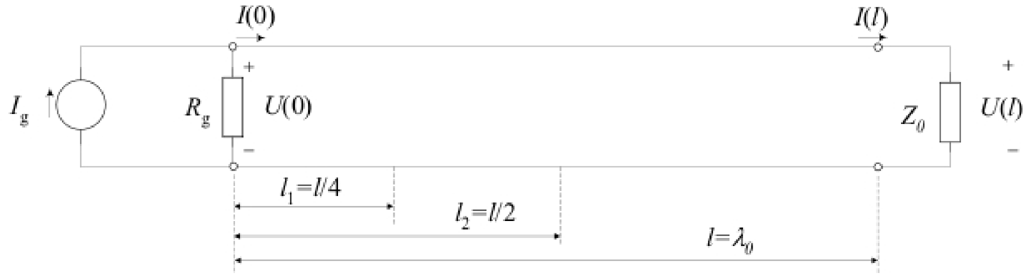
$$=-1.6\left[\frac{s+1}{(s+1)^2}-\frac{1}{(s+1)^2}\right]-5.6\frac{1}{(s+1)^2}=$$

$$=-1.6\frac{1}{s+1}+\frac{1.6}{(s+1)^2}-5.6\frac{1}{(s+1)^2}=-\frac{1.6}{s+1}-\frac{4}{(s+1)^2}$$

$$u_C(t)=(-1.6\cdot e^{-t}-4t\cdot e^{-t})\cdot S(t)$$

5. Zadana je linija bez gubitaka s L=20 μH/km i C=8 nF/km. Na ulaz linije je priključen strujni izvor $i_g(t)$ = 0,2 cos(ω_0t) paralelno s otporom R_g=10Ω. Duljina linije je $l=\lambda_0$, gdje je λ_0 valna duljina signala pri frekvenciji $\omega_0 = 5\cdot 10^6$ rad/s. Izlaz linije je zaključen karakterističnom impedancijom.

   a) Odrediti karakterističnu impedanciju Z_0 i koeficijent prijenosa γ linije.
   b) Kolika je duljina linije u km?
   c) Odrediti napon i struju na 1/4 linije.
   d) Odrediti napon i struju na 1/2 linije.



Rješenje:

a) Linija bez gubitaka $\rightarrow$ $R=0$, $G=0$ $\Rightarrow$ $Z_0=\sqrt{L/C}$, $\gamma = s\sqrt{LC}$ Ω

Stac. sinusna pobuda $\rightarrow$ $s=j\omega_0$ $\Rightarrow$ $\gamma = j\omega_0\sqrt{LC} = j\beta$

$$Z_0=\sqrt{L/C}=\sqrt{2\cdot 10^{-5}/8\cdot 10^{-9}}=\sqrt{10^4/4}=50\Omega$$

$$\gamma = j\omega_0\sqrt{LC} = j5\cdot 10^6\sqrt{20\cdot 10^{-6}\cdot 8\cdot 10^{-9}} = j5\cdot 10^6\sqrt{16\cdot 10^{-14}} = j5\cdot 10^6\cdot 4\cdot 10^{-7} = j2 \text{ km}^{-1}$$

b) $l=\lambda_0=\dfrac{2\pi}{\beta}=\dfrac{2\pi}{2}=\pi=3{,}14\text{km}$; $l_1=\lambda_0/4=\dfrac{\pi}{4}$ km ; $l_2=\lambda_0/2=\dfrac{\pi}{2}$ km

c)

$$U(x)=U(0)\cdot ch\gamma x - I(0)Z_0 sh\gamma x$$

$$I(x) = -\frac{U(0)}{Z_0}sh\gamma x + I(0)ch\gamma x$$

$$U(0)=I_g\cdot(Z_0\parallel R_g)=I_g\cdot(Z_0\cdot R_g/(Z_0+R_g))=\frac{10}{6}=1{,}666$$

$$I(0)=U(0)/Z_0=U(0)/Z_0=0{,}0333$$

$$U(x)=U(0)\cdot(ch\gamma x - sh\gamma x)=U(0)\cdot e^{-\gamma x}$$

$$I(x)=\frac{U(0)}{Z_0}(-sh\gamma x + ch\gamma x)=\frac{U(0)}{Z_0}e^{-\gamma x}$$

$$U(l/4)=U(0)\cdot(ch(\gamma l/4)-sh(\gamma l/4))=U(0)\cdot e^{-\gamma l/4}=U(0)\cdot e^{-j\pi/2}$$

$$I(l/4)=\frac{U(0)}{Z_0}(-sh(\gamma l/4)+ch(\gamma l/4))=\frac{U(0)}{Z_0}e^{-\gamma l/4}=\frac{U(0)}{Z_0}e^{-j\pi/2}$$

$$u(l/4,t)=1{,}6667\cdot\cos(\omega_0 t-90^\circ)$$

$$i(l/4,t)=0{,}0333\cdot\cos(\omega_0 t-90^\circ)$$

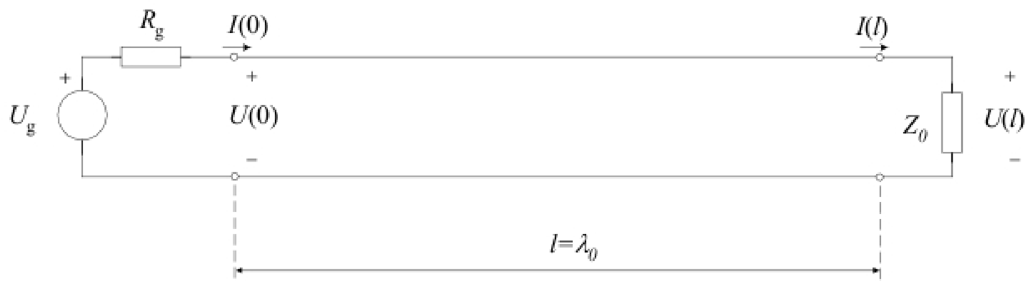
d)

$$U(l/2)=U(0)\cdot(ch(\gamma l/2)-sh(\gamma l/2))=U(0)\cdot e^{-\gamma l/2}=U(0)\cdot e^{-j\pi}$$

$$I(l/2)=\frac{U(0)}{Z_0}(-sh(\gamma l/2)+ch(\gamma l/2))=\frac{U(0)}{Z_0}e^{-\gamma l/2}=\frac{U(0)}{Z_0}e^{-j\pi}$$

$$u(l/2,t)=-1{,}6667\cdot\cos(\omega_0 t),\quad i(l/2,t)=-0{,}0333\cdot\cos(\omega_0 t)$$

5. Zadana je linija bez gubitaka s L=10 μH/km i C=100 nF/km. Na ulaz linije je priključen naponski izvor $u_g(t)$ = 2 cos(ω_0t) u seriji s otporom R_g=50Ω. Duljina linije je $l=\lambda_0$, gdje je λ_0 valna duljina signala pri frekvenciji $\omega_0=10^6$. Izlaz linije je zaključen karakterističnom impedancijom.
   a) Odrediti karakterističnu impedanciju Z_0 i koeficijent prijenosa γ linije.
   b) Kolika je duljina linije u km?
   c) Odrediti napon i struju na polovici linije.
   d) Odrediti napon i struju na kraju linije.



Rješenje:

a) Linija bez gubitaka $\rightarrow$ $R=0$, $G=0$ $\Rightarrow$ $Z_0=\sqrt{L/C}$, $\gamma=s\sqrt{LC}$

Stac. sinusna pobuda $\rightarrow$ $s=j\omega$ $\Rightarrow$ $\gamma=j\omega\sqrt{LC}=j\beta$

$$Z_0=\sqrt{L/C}=\sqrt{10^{-5}/10^{-7}}=10\Omega$$

$$\gamma=j\omega\sqrt{LC}=j10^6\sqrt{10^{-5}\cdot 10^{-7}}=j$$

b) $$l=\lambda_0=\frac{2\pi}{\beta}=\frac{2\pi}{\omega\sqrt{LC}}=2\pi=6{,}28\text{km}$$

c) $$U(x)=U(0)\cdot ch\gamma x-I(0)Z_0 sh\gamma x$$

$$I(x)=-\frac{U(0)}{Z_0}sh\gamma x+I(0)ch\gamma x$$

$$Z_{ul}=Z_0\ \Rightarrow\ U(0)=Z_0 I(0)\ \ U(0)=\frac{Z_0}{Z_0+R_g}U_g(s)=\frac{10\Omega}{10\Omega+50\Omega}U_g(s)=\frac{1}{6}U_g(s)$$

$$U(x)=U(0)\cdot(ch\gamma x-sh\gamma x)=U(0)\cdot e^{-\gamma x}$$

$$I(x)=\frac{U(0)}{Z_0}(-sh\gamma x+ch\gamma x)=\frac{U(0)}{Z_0}e^{-\gamma x}$$

$$U(l/2)=U(0)\cdot(ch(\gamma l/2)-sh(\gamma l/2))=U(0)\cdot e^{-\gamma l/2}=U(0)\cdot e^{-j\pi}=-U(0)$$

$$I(l/2)=\frac{U(0)}{Z_0}(-sh(\gamma l/2)+ch(\gamma l/2))=\frac{U(0)}{Z_0}e^{-\gamma l/2}=\frac{U(0)}{Z_0}e^{-j\pi}=-\frac{U(0)}{Z_0}$$

$$U(0)=\frac{1}{6}U_g(s)\ \ \underline{u(l/2,t)=-\frac{1}{3}\cos(\omega_0 t)},\ \underline{i(l/2,t)=-\frac{1}{30}\cos(\omega_0 t)}$$

d) $$U(l)=U(0)\cdot(ch(\gamma l)-sh(\gamma l))=U(0)\cdot e^{-\gamma l}=U(0)\cdot e^{-j2\pi}=U(0)$$

$$I(l)=\frac{U(0)}{Z_0}(-sh(\gamma l)+ch(\gamma l))=\frac{U(0)}{Z_0}e^{-\gamma l}=\frac{U(0)}{Z_0}e^{-j2\pi}=\frac{U(0)}{Z_0}$$

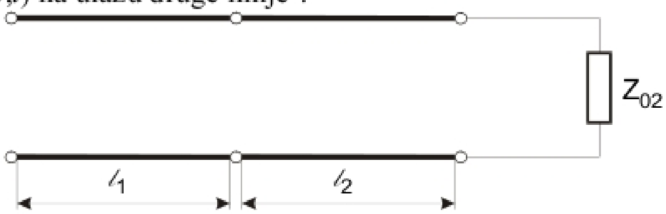
$$\underline{u(l,t)=\frac{1}{3}\cos(\omega_0 t)},\ \underline{i(l,t)=\frac{1}{30}\cos(\omega_0 t)}$$

5. Na liniju bez gubitaka duljine $l_1=\lambda_1/2$, s primarnim parametrima L_1=2 mH/km i C_1=6 nF/km, priključena je linija bez gubitaka zadana sa L_2=0,6 mH/km i C_2=40 nF/km. Druga linija je zaključena svojom karakterističnom impedancijom.

a) Koliki je faktor refleksije prve linije na spojnom mjestu ?

b) Kolika je amplituda polaznog, a kolika reflektiranog vala na spojnom mjestu, ako je napon na ulazu prve linije $u_I(0,t)=2\cos 10^4 t$?

c) Koliki je napon $u_{II}(0,t)$ na ulazu druge linije ?



Rješenje:

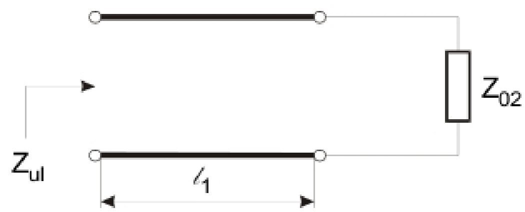
Linija bez gubitaka $\rightarrow$ $R=0$, $G=0$ $\Rightarrow$ $Z_0=\sqrt{L/C}$, $\gamma=s\sqrt{LC}$

$$Z_{01}=\sqrt{L_1/C_1}=1/\sqrt{3}\cdot 10^3\,\Omega$$

$$Z_{02}=\sqrt{L_2/C_2}=\sqrt{3/2}\cdot 10^2\,\Omega$$

a) $$\Gamma=\frac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}}=\frac{\sqrt{3/2}\cdot 10^2-1/\sqrt{3}\cdot 10^3}{\sqrt{3/2}\cdot 10^2+1/\sqrt{3}\cdot 10^3}=\frac{3-10\sqrt{2}}{3+10\sqrt{2}}=-\frac{11.1}{17.1}=-0.65$$

Za $l=\lambda/2 \Rightarrow Z_{ul}=Z_2$



$Z_{ul}=Z_{02}$

$$U_p\left(\frac{\lambda_1}{2}\right)=\frac{U(0)+Z_{01}I(0)}{2}e^{-j\beta_1\frac{\lambda_1}{2}}=\frac{U(0)+Z_{01}I(0)}{2}e^{-j\pi}=$$

b) $$=\frac{U(0)+Z_{01}\dfrac{U(0)}{Z_{ul}}}{2}e^{-j\pi}=\frac{U(0)}{2}\left(1+\frac{1}{\sqrt{3}}10^3\sqrt{\frac{2}{3}}10^{-2}\right)e^{-j\pi}=\frac{2}{2}\left(1+\frac{10\sqrt{2}}{3}\right)e^{-j\pi}=\underline{5.71405e^{-j\pi}}$$

$u_p(l_1,t)$= 5.71405 (cos 10^4 t– 180°)= – 5.71405 (cos 10^4 t)

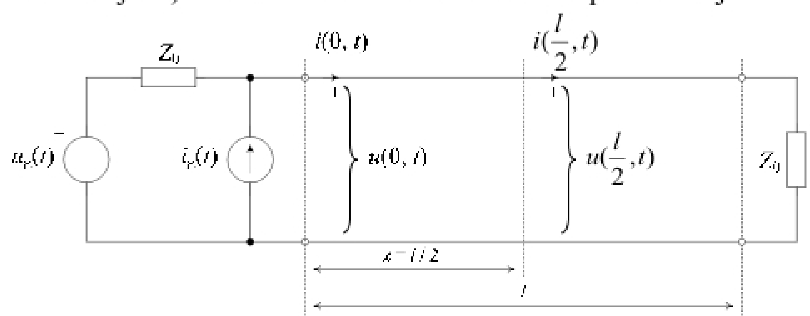
$U_r=U_p\cdot(-0.65)$=5.71405·(–0.65)=<u>– 3.71413</u>

$u_r(l_1,t)$= 3.71413 (cos 10^4 t)

c) $u_{II}(0,t)=u_p(l_1,t)+u_r(l_1,t)= U_p\,(1+\Gamma)$ (cos 10^4 t– 180°)= – 5.71405·0.35 (cos 10^4 t)=

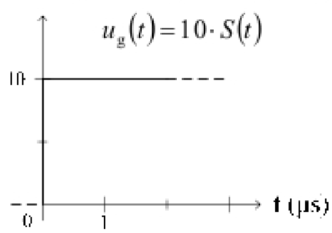
<u>$u_{II}(0,t)$= – 2.0 (cos 10^4 t)</u>

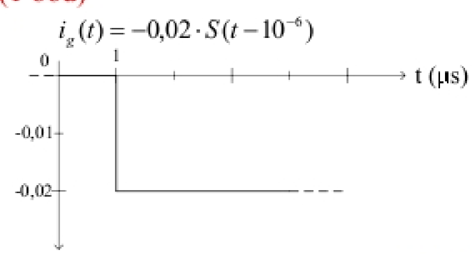
5. Linija duljine l=1 km sa primarnim parametrima: R= 1Ω/km, L= 3mH/km, G= 4μS/km i C= 12nF/km, zaključena je s obje strane svojom karakterističnom impedancijom Z_0. Na liniju su spojeni naponski izvor $u_g(t)=10S(t)$ i strujni izvor $i_g(t)=-0{,}02S(t-10^{-6})$ prema slici. *a*) Nacrtati valni oblik napona $u_g(t)$ i struje $i_g(t)$. Odrediti izraz za *b*) napon i *c*) struju na polovini linije. *d*) Nacrtati valne oblike traženih napona i struja.



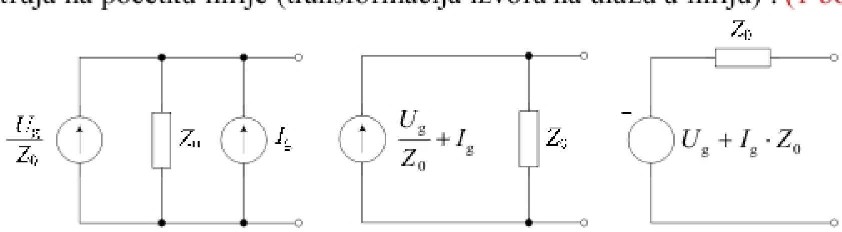
Rješenje:

a) valni oblici napona $u_g(t)$ i struje $i_g(t)$: (1 bod)

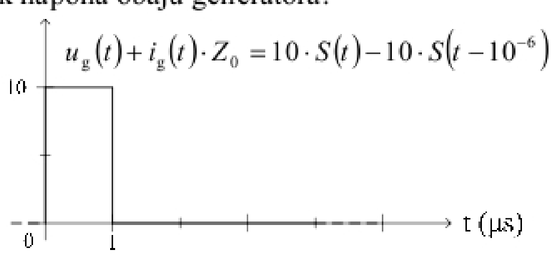




- napon i struja na početku linije (transformacija izvora na ulazu u liniju) : (1 bod)



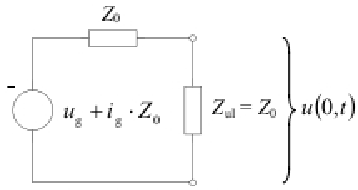
pa je ukupni valni oblik napona obaju generatora:

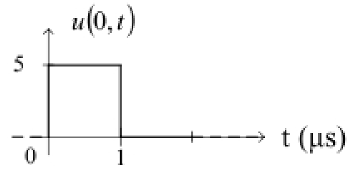


te napona i struje na ulazu linije:

$$Z_{ul} = Z_0 \Rightarrow \quad u(0,t) = \left(u_g + i_g \cdot Z_0\right)\frac{Z_0}{Z_0 + Z_0} = \frac{u_g + i_g \cdot Z_0}{2} \qquad U(0) = I(0) \cdot Z_0$$

$$i(0,t) = \frac{u(0,t)}{Z_0} \qquad I(0) = \frac{U(0)}{Z_0}$$





Prijenosne jednadžbe linije:

$$U(x) = U(0)\cdot \mathrm{ch}(\gamma x) - I(0)\cdot Z_0 \cdot \mathrm{sh}(\gamma x)$$
$$I(x) = -U(0)/Z_0 \cdot \mathrm{sh}(\gamma x) + I(0)\cdot \mathrm{ch}(\gamma x)$$

Sekundarni parametri linije:

$$\gamma = \sqrt{(R+sL)(G+sC)}$$
$$Z_0 = \sqrt{\frac{R+sL}{G+sC}}$$

Specijalni slučaj: $\frac{R}{L} = \frac{G}{C}$ $\qquad \frac{1}{3\cdot 10^{-3}} = \frac{4\cdot 10^{-6}}{12\cdot 10^{-9}}$

Linija bez distorzije:

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{3\cdot 10^{-3}}{12\cdot 10^{-9}}} = \sqrt{\frac{1}{4}\cdot 10^{6}} = \frac{1}{2}\cdot 10^{3}\,\Omega$$

$$\gamma = \sqrt{RG} + s\sqrt{LC}$$
$$= \sqrt{1\cdot 4\cdot 10^{-6}} + s\sqrt{3\cdot 10^{-3}\cdot 12\cdot 10^{-9}}$$
$$= 2\cdot 10^{-3} + 6\cdot 10^{-6}s \ /\mathrm{km}$$

$x = l/2$ polovina linije (traži se napon i struja)

$x = 0$ početak linije (zadan izvor)

$$U(x) = U(0)\cdot \mathrm{ch}(\gamma x) - U(0)\cdot \mathrm{sh}(\gamma x) = U(0)(\mathrm{ch}(\gamma x) - \mathrm{sh}(\gamma x)) = U(0)\cdot e^{-\gamma x}$$
$$I(x) = I(0)(-\mathrm{sh}(\gamma x) + \mathrm{ch}(\gamma x)) = I(0)\cdot e^{-\gamma x}$$

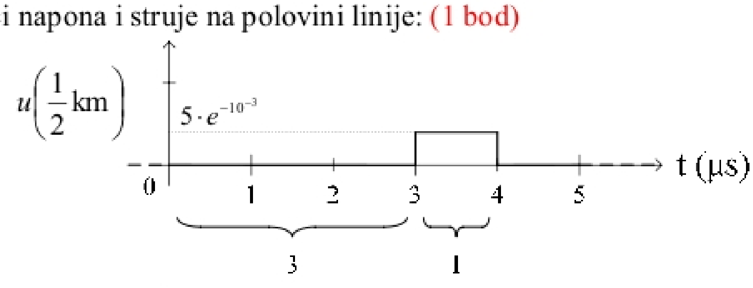
$x = 1/2\mathrm{km}$ $\qquad \gamma x = (2\cdot 10^{-3} + 6\cdot 10^{-6}s)/\mathrm{km}\cdot 1/2\mathrm{km} = 1\cdot 10^{-3} + 3\cdot 10^{-6}s$

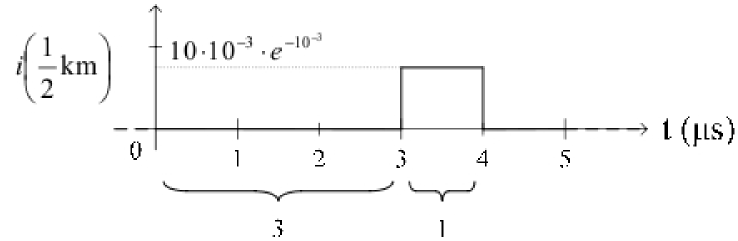
-izrazi za napon i struju na polovini linije:

b) napon: $U(1/2\mathrm{km}) = U(0)\cdot e^{-(10^{-3}+3\cdot 10^{-6}\cdot s)} = U(0)\cdot e^{-10^{-3}}\cdot e^{-3\cdot 10^{-6}s}$ (1 bod)

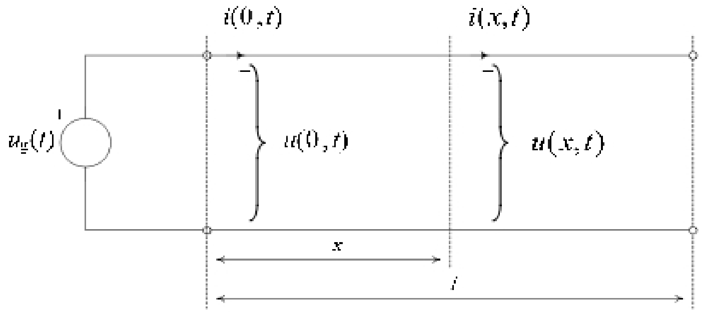
c) struja: $I(1/2\mathrm{km}) = I(0)\cdot e^{-\left(\gamma\frac{l}{2}\right)} = \frac{U(0)}{Z_0}\cdot e^{-(10^{-3}+3\cdot 10^{-6}\cdot s)} = U(0)\cdot 2\cdot 10^{-3}\cdot e^{-10^{-3}}\cdot e^{-3\cdot 10^{-6}s}$ (1 bod)

d) valni oblici napona i struje na polovini linije: (1 bod)





5. Na ulazu linije bez gubitaka s L=4mH/km, C=400nF/km, duljine $l = 2.5\lambda$ km, djeluje stacionarni sinusni izvor napona $u_g = 5\cos(2\pi \cdot f_0 \cdot t)$ uz f_0=6,25kHz. Izlaz linije je u praznom hodu. Odrediti: *a*) valnu duljinu λ signala na liniji; *b*) duljinu l linije, *c*) karakterističnu impedanciju Z_0, faktor prijenosa γ te brzinu širenja vala po liniji v; *d*) ulaznu impedanciju Z_{ul1}; *e*) napon i struju na sredini linije (x=l/2)?



Rješenje:

a) $\omega_0 = 2\pi f_0 = 39{,}2699$ rad/s

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega_0\sqrt{LC}} = \frac{2\pi}{2\pi \cdot 6{,}25 \cdot 10^3 \sqrt{4 \cdot 10^{-3} \cdot 400 \cdot 10^{-9}}} = 4\text{km} \quad \text{(1bod)}$$

b) $l = 2.5\lambda = 10\text{km}$ (1bod)

c) Za liniju bez gubitaka vrijedi:

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{4 \cdot 10^{-3}}{400 \cdot 10^{-9}}} = \frac{1}{10} \cdot 10^3\,\Omega = 100\,\Omega\,,$$

$$\gamma = j\beta;\ \ \alpha = 0\,, \qquad \beta = \frac{2\pi}{\lambda} = \frac{\pi}{2}\left[\frac{\text{rad}}{\text{km}}\right] \qquad \beta l = \beta \cdot 10 = 5\pi$$

$$v = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \cdot 10^{-3} \cdot 400 \cdot 10^{-9}}} = \frac{1}{40 \cdot 10^{-6}} = 25 \cdot 10^3\,\text{km/s} \quad \text{(1bod)}$$

d)

$$Z_{ul} = Z_0 \frac{Z_2\cosh(\gamma l) + Z_0\sinh(\gamma l)}{Z_2\sinh(\gamma l) + Z_0\cosh(\gamma l)} = Z_0\,\text{cth}(\gamma l) = Z_0 j\text{ctg}(\beta l)$$

$$\text{ctg}(\beta l) = \text{ctg}(5\pi) = \infty \quad \Rightarrow \quad Z_{ul} = Z_0 j\text{ctg}(\beta l) = \infty \quad \text{(1bod)}$$

e) Napon i struja na mjestu x=2.5 km

$$\beta x = \frac{5\pi}{2}, \qquad U(0) = 5\angle 0°, \qquad I(0) = \frac{U(0)}{Z_{ul}} = 0$$

Prijenosne jednadžbe linije:

$$U(x) = U(0)\cdot \text{ch}(\gamma x) - I(0)\cdot Z_0 \cdot \text{sh}(\gamma x) = U(0)\cdot\cos(\beta x) = 5\cdot\cos\left(\frac{\pi}{2}\right) = 0$$

$$I(x) = -\frac{U(0)}{Z_0}\cdot \text{sh}(\gamma x) + I(0)\cdot \text{ch}(\gamma x) = -j\frac{U(0)}{Z_0}\cdot\sin(\beta x) = -j\frac{5}{100}\cdot\sin\left(\frac{\pi}{2}\right) = -j0{,}05 = 0{,}05\angle -90°$$

$i(x,t) = 5\cos(2\pi f_0 t - 90°)V$ (1bod)