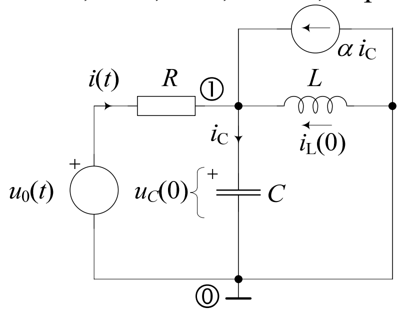
PISMENI ISPIT IZ ELEKTRIČNIH KRUGOVA – Rješenja – 2011

1. Za električni krug prikazan slikom odrediti odziv $i(t)$ ako je zadan poticaj $u_0(t)=\cos(t)S(t)$. Zadani su normalizirani elementi $R=1$, $C=1$, $L=2$, $\alpha=1/2$, te početni uvjeti $u_C(0)=1$, $i_L(0)=1/2$.



Rješenje:

Metoda napona čvorova:

$$(1)\ U_1\cdot\left(\frac{1}{R}+sC+\frac{1}{sL}\right)=\frac{U_0}{R}+\frac{i_L(0)}{s}+\alpha I_C+Cu_C(0) \quad \text{(1 bod)}$$

$$U_1=I_C\frac{1}{sC}+\frac{u_C(0)}{s}\Rightarrow I_C=sC\left(U_1-\frac{u_C(0)}{s}\right) \quad \text{(1 bod)}$$

Uz uvrštene vrijednosti elemenata: $I_C=s\left(U_1-\frac{1}{s}\right)=sU_1-1$

$$U_1\cdot\left(1+s+\frac{1}{2s}\right)=\frac{s}{s^2+1}+\frac{1}{2s}+\frac{s}{2}U_1-\frac{1}{2}+1$$

$$U_1\cdot\left(1+s+\frac{1}{2s}-\frac{s}{2}\right)=\frac{s}{s^2+1}+\frac{1}{2s}+\frac{1}{2}$$

$$U_1\cdot\left(1+\frac{s}{2}+\frac{1}{2s}\right)=\frac{2s^2+s^2+1+s^3+s}{2s(s^2+1)}$$

$$U_1\frac{1+2s+s^2}{2s}=\frac{s^3+3s^2+s+1}{2s(s^2+1)}\quad\Rightarrow\quad U_1=\frac{s^3+3s^2+s+1}{(s+1)^2(s^2+1)} \quad \text{(1 bod)}$$

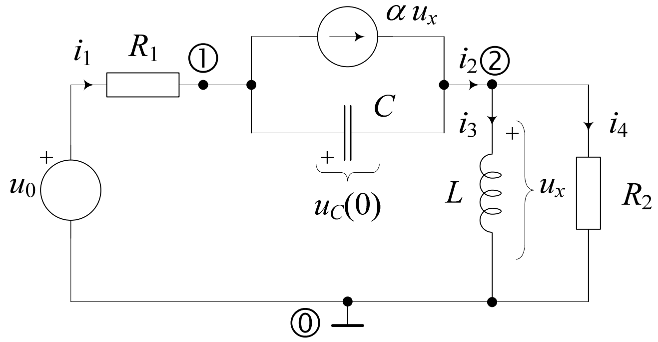
$$U_1=U_0-IR\Rightarrow I=U_0-U_1$$

$$I=\frac{s}{s^2+1}-\frac{s^3+3s^2+s+1}{(s+1)^2(s^2+1)}=\frac{s(s^2+2s+1)-(s^3+3s^2+s+1)}{(s+1)^2(s^2+1)}$$

$$=\frac{-(s^2+1)}{(s+1)^2(s^2+1)}=\frac{-1}{(s+1)^2} \quad \text{(1 bod)}$$

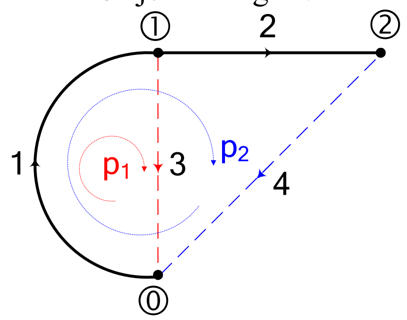
$$I(s)=\frac{-1}{(s+1)^2}\Rightarrow i(t)=-te^{-t}S(t) \quad \text{(1 bod)}$$

2. Za električni krug prikazan slikom i pridruženim orijentacijama grana te čvorovima (grane stabla: 1, 2) treba odrediti temeljni sustav jednadžbi petlji primjenom grafova. Napisati: a) spojnu matricu **S**, b) matricu impedancija grana $\mathbf{Z}_b$, c) vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$, d) matricu impedancija petlji $\mathbf{Z}_p$ i e) vektor početnih uvjeta i nezavisnih izvora petlji $\mathbf{U}_{0p}$.



Rješenje:

Orijentirani graf:



Spojna matrica:

$$\mathbf{S} = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

(1 bod)

Naponsko – strujne relacije grana: $\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$

(1) $U_1 = R_1 \cdot I_1 - U_0$

(2) $U_2 = \dfrac{1}{sC} \cdot I_2 - \alpha \dfrac{L}{C} I_3 + \dfrac{u_C(0)}{s}$

(3) $U_3 = sL \cdot I_3$

(4) $U_4 = R_2 \cdot I_4$

$$\mathbf{Z}_b = \begin{bmatrix} R_1 & 0 & 0 & 0 \\ 0 & \dfrac{1}{sC} & -\alpha \dfrac{L}{C} & 0 \\ 0 & 0 & sL & 0 \\ 0 & 0 & 0 & R_2 \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} -U_0(s) \\ \dfrac{u_C(0)}{s} \\ 0 \\ 0 \end{bmatrix}$$

(1 bod) (1 bod)

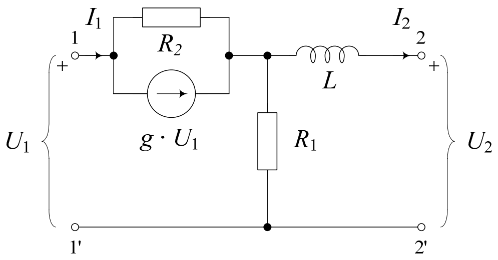
Matrica $\mathbf{Z}_b$ je regularna.

Temeljni sustav jednadžbi petlji u matričnom obliku: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$

$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \begin{bmatrix} R_1 + \dfrac{1}{sC} + sL - \alpha \dfrac{L}{C} & R_1 + \dfrac{1}{sC} \\ R_1 + \dfrac{1}{sC} - \alpha \dfrac{L}{C} & R_1 + \dfrac{1}{sC} + R_2 \end{bmatrix} \text{ (1 bod)}$$

$$\mathbf{U}_{0p} = -\mathbf{S} \cdot \mathbf{U}_{0b} = \begin{bmatrix} U_0(s) - \dfrac{u_C(0)}{s} \\ U_0(s) - \dfrac{u_C(0)}{s} \end{bmatrix} \text{ (1 bod)}$$

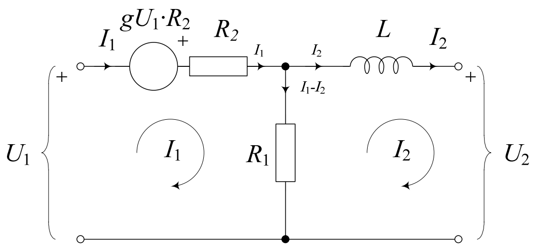
3. Za četveropol na slici: *a*) izračunati [**z**]-parametre i napisati ih u matričnom obliku. *b*) Iz [**z**]-parametara izračunati [**y**]-parametre. Ako je na na ulaz 1–1' spojen naponski izvor $u_1(t)=S(t)$ te na izlaz 2–2' spojena impedancija $Z_2=1/sC$ izračunati: *c*) prijenosnu funkciju napona $H_u(s)=U_2(s)/U_1(s)$ i izlazni napon $U_2(s)$, *d*) ulaznu impedanciju $Z_{ul1}(s)=U_1(s)/I_1(s)$ i ulaznu struju $I_1(s)$. Zadano je L=1, C=1, R_1=1, R_2=2, g=3.



Rješenje:

a) izračun [**z**] parametara pomoću jednadžbi petlji:

$$U_1 = z_{11}\cdot I_1 - z_{12}\cdot I_2$$
$$U_2 = z_{21}\cdot I_1 - z_{22}\cdot I_2$$



$$(1)\ \ I_1\cdot(R_1+R_2)-I_2\cdot R_1 = U_1 + gU_1\cdot R_2$$
$$(2)\ \ -I_1\cdot R_1 + I_2(sL+R_1) = -U_2$$

$$(1)\ \ I_1\cdot(R_1+R_2)-I_2\cdot R_1 = U_1(1+g\cdot R_2)/:(1+g\cdot R_2)$$
$$(2)\ \ -I_1\cdot R_1 + I_2(sL+R_1) = -U_2/\cdot(-1)$$

$$(1)\ \ I_1\cdot\frac{R_1+R_2}{1+g\cdot R_2} - I_2\cdot\frac{R_1}{1+g\cdot R_2} = U_1$$
$$(2)\ \ I_1\cdot R_1 - I_2(sL+R_1) = U_2$$

Matrica [**z**]-parametara (uz uvrštene vrijednosti elemenata L=1, C=1, R_1=1, R_2=2, g=3):

$$[\mathbf{z}] = \begin{bmatrix} \dfrac{R_1+R_2}{1+g\cdot R_2} & -\dfrac{R_1}{1+g\cdot R_2} \\ R_1 & -(sL+R_1) \end{bmatrix} = \begin{bmatrix} \dfrac{3}{7} & -\dfrac{1}{7} \\ 1 & -(s+1) \end{bmatrix}$$

(do sada: maksimum 2 boda – ako su svi parametri točni)

b) Matrica [**y**]-parametara: $[\mathbf{y}] = [\mathbf{z}]^{-1}$ (1 bod)

$$\det[\mathbf{z}] = \begin{vmatrix} \dfrac{3}{7} & -\dfrac{1}{7} \\ 1 & -(s+1) \end{vmatrix} = -\frac{3}{7}\cdot(s+1)+\frac{1}{7} = \frac{-3s-3+1}{7} = \frac{-3s-2}{7} = -\frac{1}{7}(3s+2)$$

$$[\mathbf{y}]=\begin{bmatrix}\frac{3}{7} & -\frac{1}{7}\\ 1 & -(s+1)\end{bmatrix}^{-1}=\frac{-7}{3s+2}\begin{bmatrix}-(s+1) & -1\\ \frac{1}{7} & \frac{3}{7}\end{bmatrix}^{T}=\frac{-7}{3s+2}\begin{bmatrix}-(s+1) & \frac{1}{7}\\ -1 & \frac{3}{7}\end{bmatrix}=\begin{bmatrix}\frac{7(s+1)}{3s+2} & -\frac{1}{3s+2}\\ \frac{7}{3s+2} & -\frac{3}{3s+2}\end{bmatrix}$$

c) Prijenosna funkcija napona i odziv na Step: (1 bod)

$$H_u(s)=\frac{U_2(s)}{U_1(s)}=\frac{Z_L z_{21}}{z_{11}(z_{22}+Z_L)-z_{12}z_{21}}=\frac{\frac{1}{s}\cdot 1}{\frac{3}{7}\left(s+1+\frac{1}{s}\right)-\frac{1}{7}}=\frac{7}{3\left(s^2+s+1\right)-s}$$

$$=\frac{7}{3\left(s^2+s+1\right)-s}=\frac{7}{3s^2+2s+3}$$

ili

$$H_u(s)=\frac{U_2(s)}{U_1(s)}=\frac{y_{21}}{y_{22}+Y_L}=\frac{\frac{7}{3s+2}}{\frac{3}{3s+2}+s}=\frac{7}{3+s(3s+2)}=\frac{7}{3s^2+2s+3}$$

$$U_1(s)=\frac{1}{s}\Rightarrow U_2(s)=H_u(s)U_1(s)=\frac{1}{s}\cdot\frac{7}{3s^2+2s+3}=\frac{7}{s(3s^2+2s+3)}$$

d) Izračunati ulaznu impedanciju $Z_{ul1}(s)$ i ulaznu struju $I_1(s)$: (1 bod)

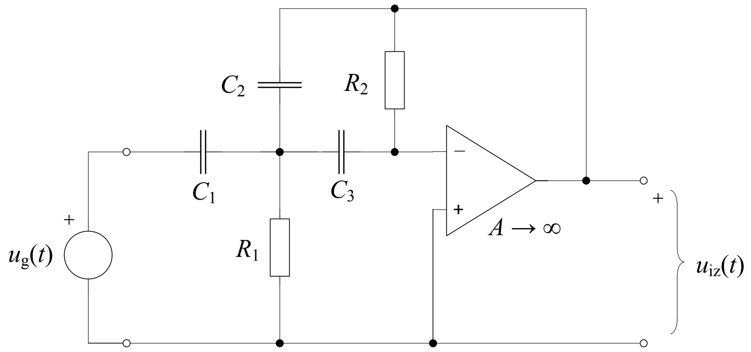
$$Z_{ul1}(s)=\frac{U_1(s)}{I_1(s)}=z_{11}-\frac{z_{12}z_{21}}{z_{22}+Z_L}=\frac{3}{7}-\frac{\frac{1}{7}}{s+1+\frac{1}{s}}=\frac{3}{7}-\frac{s}{7(s^2+s+1)}=\frac{3(s^2+s+1)-s}{7(s^2+s+1)}$$

$$\frac{3(s^2+s+1)-s}{7(s^2+s+1)}=\frac{3s^2+2s+3}{7(s^2+s+1)}\quad\Rightarrow\quad I_1(s)=\frac{U_1(s)}{Z_{ul1}(s)}=\frac{1}{s}\cdot\frac{7(s^2+s+1)}{3s^2+2s+3}$$

ili

$$Y_{ul1}(s)=\frac{I_1(s)}{U_1(s)}=y_{11}-\frac{y_{12}y_{21}}{y_{22}+Y_L}\quad\Rightarrow\quad I_1(s)=U_1(s)\cdot Y_{ul1}(s)=\frac{1}{s}\cdot\frac{7(s^2+s+1)}{3s^2+2s+3}$$

4. Zadan je aktivni filtar prikazan slikom i njegova prijenosna funkcija $T(s)=U_{iz}(s)/U_g(s)$. *a*) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja odrediti parametre k, ω_0, Q. O kojem se tipu filtra radi (NP, VP, PP ili PB)? *b*) Ako su zadane normalizirane vrijednosti parametara $\omega_0=1$, $Q=5$ i $|k|=1$ te ako je $C_2=4C_3=1$, izračunati normalizirane vrijednosti kapaciteta C_1 i otpora R_1 i R_2. *c*) Prikazati raspored polova i nula u kompleksnoj ravnini. *d*) Nacrtati amplitudno-frekvencijsku karakteristiku.



$$T(s)=\frac{U_{iz}(s)}{U_g(s)}=-\frac{s^2R_1R_2C_1C_3}{s^2R_1R_2C_2C_3+sR_1(C_1+C_2+C_3)+1}$$

Rješenje:

a) $T(s)=\dfrac{k\cdot s^2}{s^2+\dfrac{\omega_0}{Q}\cdot s+\omega_0{}^2}$ Opći oblik VP (visoki propust)

(uobičajeno je kod el. filtara da je pojačanje k zadano s apsolutnom vrijednosti)
prepišimo $T(s)$ tako da najvišu potenciju od s u nazivniku množi jedinica

$$T(s)=\frac{U_{iz}(s)}{U_g(s)}=\frac{-\dfrac{C_1}{C_2}\cdot s^2}{s^2+s\cdot\dfrac{R_1(C_1+C_2+C_3)}{R_1R_2C_2C_3}+\dfrac{1}{R_1R_2C_2C_3}}$$

-o kojem se tipu filtra radi (NP, VP, PP ili PB)? $\Rightarrow$ NP (niski propust)
-parametri k, ω_0, Q: (1 bod)

$$\omega_0=\frac{1}{\sqrt{R_1R_2C_2C_3}} \qquad \frac{\omega_0}{Q}=\frac{R_1(C_1+C_2+C_3)}{R_1R_2C_2C_3}=\frac{C_1+C_2+C_3}{R_2C_2C_3}$$

$$\Rightarrow \qquad Q=\frac{R_1R_2C_2C_3}{R_1(C_1+C_2+C_3)}\omega_0=\frac{\sqrt{R_1R_2C_2C_3}}{R_1(C_1+C_2+C_3)}, \qquad k=\frac{C_1}{C_2}$$

b) ako su zadane vrijednosti parametara $\omega_0=1$, $Q=5$ i $|k|=1$ te ako je $C_2=4C_3=1$ (tj. $C_2=1$, $C_3=1/4$), izračunati normalizirane vrijednosti kapaciteta C_1 i otpora R_1 i R_2. (2 boda)

$$\text{uz } C_2=C_3/4=1 \Rightarrow \omega_0=\frac{2}{\sqrt{R_1R_2}}=1;\Rightarrow R_1R_2=4 \Rightarrow R_1=\frac{4}{R_2}$$

$$Q=\frac{\sqrt{R_1R_2C_2C_3}}{R_1(C_1+C_2+C_3)}=\frac{\frac{1}{2}\sqrt{R_1R_2}}{R_1(C_1+1+1/4)}=\frac{2}{9}\cdot\sqrt{\frac{R_2}{R_1}};\ Q=\frac{2}{9}\cdot\sqrt{\frac{R_2}{R_1}}\Rightarrow\frac{R_2}{R_1}=\left(\frac{9}{2}\right)^2Q^2$$

$$k=\frac{C_1}{C_2}=1\Rightarrow C_1=C_2=1;\Rightarrow\frac{R_2^2}{4}=\left(\frac{9}{2}\right)^2Q^2 \ \Rightarrow\ R_2^2=4\left(\frac{9}{2}\right)^2Q^2\Rightarrow\ R_2=2\frac{9}{2}Q=9Q$$

$$R_2=9Q=9\cdot 5=45, \qquad R_1=4/R_2=4/45=0.08888$$

$$\boxed{R_1 = 4/45 = 0.088888,\ R_2 = 45;\ C_1 = C_2 = 1;\ C_3 = 1/4 = 0.25.}$$

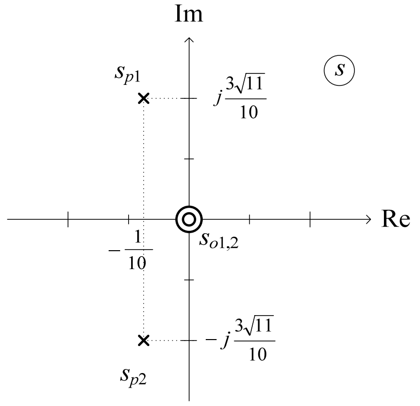
c) raspored polova i nula u kompleksnoj ravnini: (1bod)

$$T(s) = \frac{s^2}{s^2 + \frac{1}{5}\cdot s + 1} = \frac{s^2}{s^2 + 0.2\cdot s + 1}$$

nule $s_{o1,2} = 0$ (dvije nule su u ishodištu)

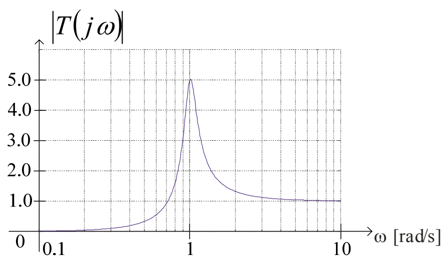
polovi $s^2 + 0.2\cdot s + 1 = 0 \quad \Rightarrow$

$$s_{p1,2} = \frac{-0.2 \pm \sqrt{0.2^2 - 4}}{2} = -\frac{1}{10} \pm j\frac{3\sqrt{11}}{10} = -0.1 \pm j0.994978$$

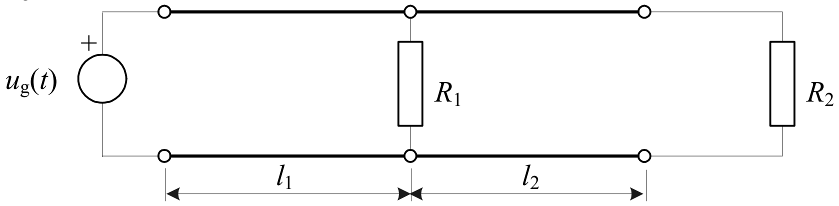


d) amplitudno-frekvencijska karakteristika: (1bod)

$$T(j\omega) = \frac{-\omega^2}{-\omega^2 + j\omega\cdot 0.2 + 1} \quad \Rightarrow \quad |T(j\omega)| = \frac{\omega^2}{\sqrt{(1-\omega^2)^2 + (\omega\cdot 0.2)^2}}$$



5. Na ulazu linije bez gubitaka, duljine $l_1=\lambda_1/2$, s primarnim parametrima L_1=4,5 mH/km i C_1=0.8μF/km, djeluje napon $u_g(t)=2\cos(\omega_0 t)$. Na izlaz je priključen je otpor R_1 paralelno s linijom bez gubitaka duljine $l_2=\pi/8$ km, zadanom s L_2=200 μH/km i C_2=0.08 μF/km, zaključenom otporom R_2=25Ω. Izračunati: a) valne impedancije obiju linija Z_{01} i Z_{02}, te frekvenciju ω_0 signala ako je $l_2=\lambda_2/4$; b) duljinu prve linije l_1, te koeficijente prijenosa γ_1 i γ_2; c) ulaznu impedanciju Z_{ul2} druge linije i vrijednost otpora R_1 da bi prva linija bila prilagođena na svome izlazu; d) napon $u_{II}(0,t)$ na ulazu druge linije; e) faktor refleksije i napon $u_{II}(l_2,t)$ na izlazu druge linije.



Rješenje:

a) $Z_{01}=\sqrt{L_1/C_1}=\sqrt{4{,}5\cdot10^{-3}/8\cdot10^{-7}}=75\Omega$, $Z_{02}=\sqrt{L_2/C_2}=\sqrt{2\cdot10^{-4}/8\cdot10^{-8}}=50\Omega$

$$l_2=\frac{\lambda_2}{4}=\frac{2\pi}{4\beta_2}=\frac{\pi}{2\omega_0\sqrt{L_2C_2}}=\frac{\pi}{8}km \qquad \omega_0=\frac{\pi}{2l_2\sqrt{L_2C_2}}=\frac{4}{\sqrt{2\cdot10^{-4}\cdot8\cdot10^{-8}}}=10^6\,\text{rad/s}$$

(1 bod)

b) $$l_1=\frac{\lambda_1}{2}=\frac{2\pi}{2\beta_1}=\frac{\pi}{\omega\sqrt{L_1C_1}}=\frac{\pi}{10^6\sqrt{4{,}5\cdot10^{-3}\cdot8\cdot10^{-7}}}=\frac{\pi}{60}\text{km}$$

$$\gamma_1=j\beta_1=j\omega\sqrt{L_1\cdot C_1}=j10^6\sqrt{4{,}5\cdot10^{-3}\cdot8\cdot10^{-7}}=j60\frac{\text{rad}}{\text{km}} \quad\Rightarrow\quad \beta_1l_1=\beta_1\frac{\lambda_1}{2}=\beta_1\frac{2\pi}{2\beta_1}=\pi$$

$$\gamma_2=j\beta_2=j\omega\sqrt{L_2\cdot C_2}=j10^6\sqrt{2\cdot10^{-4}\cdot8\cdot10^{-8}}=4\frac{\text{rad}}{\text{km}} \quad\Rightarrow\quad \beta_2l_2=\beta_2\frac{\lambda_2}{4}=\beta_2\frac{2\pi}{4\beta_2}=\frac{\pi}{2}$$

(1 bod)

c) $$Z_{ul2}=\frac{R_2ch(\gamma_2l_2)+Z_{02}sh(\gamma_2l_2)}{\frac{R_2}{Z_{02}}sh(\gamma_2l_2)+ch(\gamma_2l_2)}=Z_{02}\frac{R_2\cos(\beta_2l_2)+jZ_{02}\sin(\beta_2l_2)}{R_2j\sin(\beta_2l_2)+Z_{02}\cos(\beta_2l_2)}=\frac{Z_{02}^2}{R_2}=\frac{2500}{25}=100\Omega$$

$$Z_{ul2}=Z_{01}=\frac{Z_{02}^2}{R_2} \qquad \frac{1}{Z_{01}}=\frac{1}{Z_{ul2}}+\frac{1}{R_1}=\frac{R_2}{Z_{02}^2}+\frac{1}{R_1}\Rightarrow\frac{1}{R_1}=\frac{1}{Z_{01}}-\frac{1}{Z_{ul2}}=\frac{1}{75}-\frac{1}{100}=\frac{1}{300}$$

$$R_1=300\Omega$$

(1 bod)

d) $$U_2(0)=U_1(0)e^{-j\beta_1l_1}=U_1(0)e^{-j\pi}=-2 \qquad u_2(0,t)=-2\cos(\omega t)$$

(1 bod)

e) $$\Gamma_2=\frac{R_2-Z_{02}}{R_2+Z_{02}}=\frac{25-50}{25+50}=-\frac{25}{75}=-0{,}3333 \qquad I_2(0)=\frac{U_2(0)}{Z_{ul2}}=\frac{2}{100}=0{,}02A$$

$$U_2(l)=U_2(0)ch(j\beta_2l_2)-I_2(0)Z_{02}sh(j\beta_2l_2)=-U_1(0)\left(\cos\left(\frac{\pi}{2}\right)-j\frac{R_2}{Z_{02}}\sin\left(\frac{\pi}{2}\right)\right)=0{,}5jU_1(0)$$

$$u_2(l,t)=\cos\left(\omega t+\frac{\pi}{2}\right)$$

(1 bod)