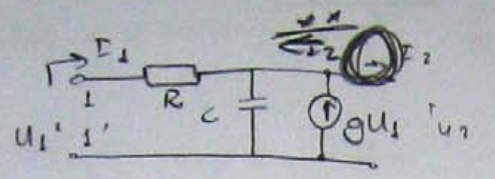
2.I.2006

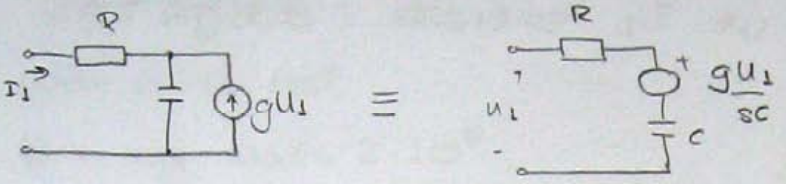
③



$U_1 = I_1 z_{11} - I_2 z_{12}$

$U_2 = I_1 z_{21} - I_2 z_{22}$

$$z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0}$$



$$U_1 = I_1 R + \frac{gU_1}{sC} + \frac{I_1}{sC}$$

$$U_1 = \frac{I_1\left(R + \frac{1}{sC}\right)}{1 - \frac{g}{sC}}$$

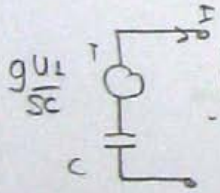
$$z_{11} = \frac{R + \frac{1}{sC}}{1 - \frac{g}{sC}} \qquad z_{21} = \left.\frac{U_2}{I_1}\right|_{I_2=0}$$

$$U_2 = I_1 \cdot \frac{1}{sC} + \frac{gU_1}{sC} = \frac{I_1}{sC} + \frac{g}{sC} \cdot \frac{I_1\left(R + \frac{1}{sC}\right)}{1 - \frac{g}{sC}}$$

$$z_{21} = \frac{1}{sC} + \frac{g}{sC} \cdot \frac{R + \frac{1}{sC}}{1 - \frac{g}{sC}} = \frac{1 + Rg}{sC - g}$$

$I_1 = 0$

$U_1 = U_2$



$$z_{12} = -\frac{U_1}{I_2} \qquad z_{21} = -\frac{U_2}{I_2}$$

$$U_2 = \frac{-I_2}{sC} = \frac{gU_1}{sC} \rightarrow U_2 = \frac{-I_2}{sC - g}$$

$$z_{22} = \frac{1}{sC - g} \qquad z_{12} = \frac{1}{sC - g}$$

+ ako **

$$[Z] = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix}$$

- nije recipročan jer ima neuzajamni element (C).
- simetričan: $z_{11} \neq z_{22}$ → ne! pa nije simetričan

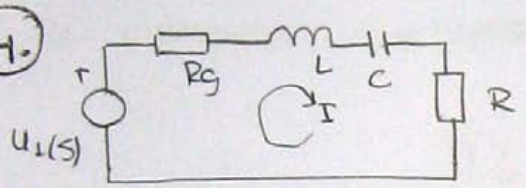
prijenosna funkcija napona

$$T(s) = \frac{U_2(s)}{U_1(s)} = \frac{I_1 z_{21} - I_2 z_{22}}{I_1 z_{11} - I_2 z_{12}}$$

otvoren: $I_2 = 0$, članovi uz I_2 skrate i dobija se

$$T(s) = \frac{I_1 z_{21}}{I_1 z_{11}} = \frac{z_{21}}{z_{11}}$$

(4.)



$L = 1\,mH$
$R_g = 50\Omega$
$C = 100\,nF$
$R = 150\Omega$

$$I\left(R_g + sL + \frac{1}{sC} + R\right) = U_1$$

$$I_1 = \frac{U_1}{100 + 10^{-3}s + \frac{10^7}{s}} = \frac{U_1}{\frac{10^{-3}s^2 + 200s + 10^7}{s}}$$

$$U_2 = \frac{sU_1}{10^{-3}s^2 + 200s + 10^7} \cdot 150$$

$$T(s) = \frac{U_2}{U_1} = \frac{150s}{10^{-3}s^2 + 200s + 10^7} = \frac{150s}{10^{-3}\left(s^2 + 2\cdot 10^5 s + 10^{10}\right)} =$$

$$= \frac{1.5 \cdot 10^5 s}{s^2 + 2\cdot 10^5 s + 10^{10}} = \frac{K\frac{\omega_p}{q_p}s}{s^2 + \frac{\omega_p}{q_p}s + \omega_p^2}$$

$$\frac{\omega_p}{q_p} = 2\cdot 10^5 \qquad \omega_p^2 = 10^{10} \Rightarrow \quad \omega_p = 10^5$$

$$q_p = 1.5$$

$$K = 0.75$$

$$\omega_{g,d} = \omega_p \sqrt{1 + \frac{1}{4q_p^2}} \mp \frac{\omega_p}{2q_p}$$

$$\omega_{g,d} = \omega_p \mp \frac{\omega_p}{2q_p} \quad \leftarrow \text{ako } \boxed{q_p > 10}$$

$\omega_g = 2.41 \cdot 10^5$

$\omega_d = 0.41 \cdot 10^5$

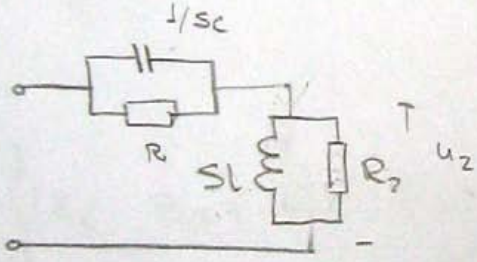
$B = \omega_g - \omega_d = 2 \cdot 10^5$

4- pomoću naponskog djelila

$$T(s) = \frac{R}{R + sC + \frac{1}{sC} + R_g}$$

koji nam treba zbirj svih ostalih

za serijski spoj



$$Z_1 = R_1 \| \frac{1}{sC}$$

$$Z_2 = R_2 \| sL$$

$$T(s) = \frac{Z_2}{Z_1 + Z_2}$$

za paralelu

$$T(s) = \frac{1.5 \cdot 10^5 s}{s^2 + 2 \cdot 10^5 s + 10^{10}}$$

$$T(j\omega) = \frac{1.5 \cdot 10^5 s}{10^{10} - \omega^2 + 2 \cdot 10^5 j\omega}$$

$$|T(j\omega)| = \frac{1.5 \cdot 10^5}{\sqrt{(10^{10} - \omega^2)^2 + (2 \cdot 10^5 \omega)^2}}$$

⑤ $L = 10\,nH/km$

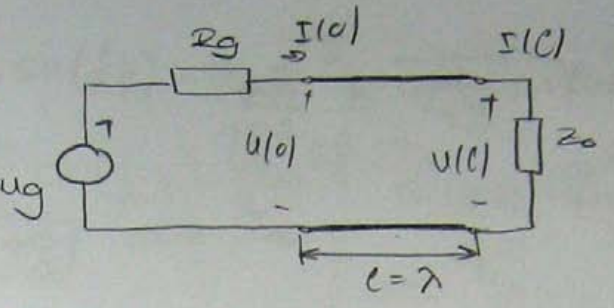
$C = 100\,nF/km$

$u_g = 2\cos(\omega_0 t)$

$R_g = 50\,\Omega$

$\ell = \lambda$

$\omega_0 = 10^6$



bez distorzije:

$\frac{R}{L} = \frac{G}{C}$

$Z_0 = \sqrt{\frac{L}{C}}$

$\gamma = \sqrt{RG} + s\sqrt{LC}$

bez gubitaka:

$R = 0 \quad G = 0$

$Z_0 = \sqrt{\frac{L}{C}}$

$\gamma = s\sqrt{LC}$

$\lambda = \frac{2\pi}{\beta}$

a) $Z_0 = \sqrt{\frac{L}{C}} = 10\,\Omega$

$\gamma = s\sqrt{LC}$

$\gamma = j\omega_0\sqrt{LC} = j \qquad \gamma = j\beta$

b) $\ell = \lambda = \frac{2\pi}{\beta} = 2\pi = 6.28\,km$

c) $U(x) = U_0 ch(\gamma x) - Z_0 I_0 sh(\gamma x)$

$I(x) = -\frac{U_0}{Z_0} sh(\gamma x) + I_0 ch(\gamma x)$

kada četveropol završava sa Z_0

- kad imamo Z_0 onda je ulazna impedancija jednaka Z_0

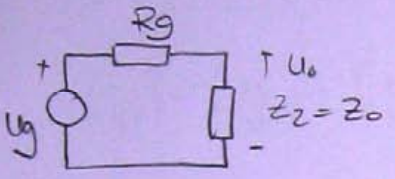
$$U\left(\frac{\ell}{2}\right) = U_0 ch\left(\frac{\ell}{2}\gamma\right) - U_0 sh\left(\frac{\ell}{2}\gamma\right) = U_0 e^{-\gamma\frac{\ell}{2}}$$

$$I\left(\frac{\ell}{2}\right) = \frac{U_0}{Z_0} sh\left(\frac{\ell}{2}\gamma\right) - \frac{U_0}{Z_0} ch\left(\frac{\ell}{2}\gamma\right) = \frac{U_0}{Z_0} e^{-\gamma\frac{\ell}{2}}$$

$$ch x = \frac{e^x + e^{-x}}{2}$$

$$sh x = \frac{e^x - e^{-x}}{2}$$

$$ch x - sh x = 2e^{-x} = e^{-x2}$$



$$U_0 = \frac{Z_0}{Z_0 + R_g} U_g = \frac{1}{6} U_g$$

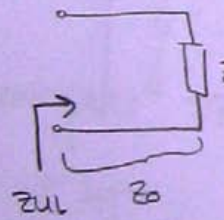
$$U\left(\frac{\ell}{2}\right) = \frac{1}{6} U_g e^{-\gamma\pi} = \frac{1}{6} e^{-j\pi} \cdot U_g = 2e^{-j0} \cdot \frac{1}{6} e^{-j\pi} =$$

$$= \frac{1}{3} e^{-\pi} = \frac{1}{3}\pi = \frac{1}{3}\cos(\omega_0 t + \pi)$$

$$U = A\cos(\omega t + \varphi)$$

$$U = A\angle\varphi = Ae^{-j\varphi}$$

$$A\cos\varphi + jA\sin\varphi$$



$$Z_{ul} = \frac{Z_2 ch(\gamma\ell) + Z_0 sh(\gamma\ell)}{\frac{Z_2}{Z_0} sh(\gamma\ell) + ch(\gamma\ell)}$$

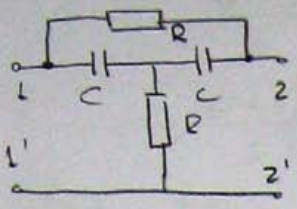
$$U_g = 2\cos\omega t \rightarrow 2\angle 0°$$

$$= 2e^{-j0}$$

$$I\left(\frac{\ell}{2}\right) = \frac{U_0}{Z_0} e^{-\gamma\frac{\ell}{2}} = \frac{U_0}{Z_0} e^{-j\pi} = \frac{1}{6} U_g \cdot \frac{1}{10} e^{-j\pi} = \frac{1}{60} e^{-j\pi} \cdot 2e^{-j0} =$$

$$= \frac{1}{30} e^{-j\pi} = \frac{1}{30}\cos\pi + j\underbrace{\frac{1}{30}\sin\pi}_{0} = -\frac{1}{30}$$

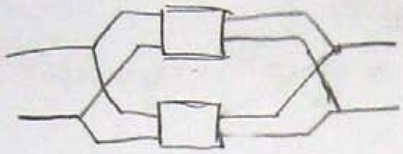
zadatci za vježbu (ZI) 7.



$[Y] = ?$

- 2 paralelno spojeni četveropoli, vrijedi:

$$[y] = [y'] + [y'']$$



$$I_1 = y_{11} U_1 - y_{12} U_2$$

$$I_2 = y_{21} U_1 - y_{22} U_2$$

---

① $U_2 = 0$

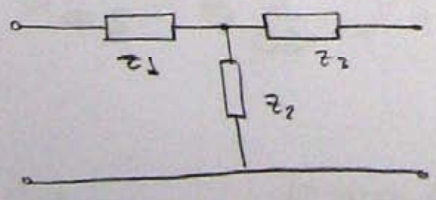
$$y_{11} = \left.\frac{I_1}{U_1}\right|_{U_2=0} \qquad y_{21} = \left.\frac{I_2}{U_1}\right|_{U_2=0}$$

② $U_1 = 0$

$$y_{12} = -\left.\frac{I_1}{U_2}\right|_{U_1=0} = \frac{1}{R}$$

$$y_{22} = -\left.\frac{I_2}{U_2}\right|_{U_1=0} = \frac{1}{R}$$

$$[y'] = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{R} & -\frac{1}{R} \\ \frac{1}{R} & -\frac{1}{R} \end{bmatrix}$$



$$\begin{bmatrix} \frac{1}{Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}} & \frac{-1}{Z_1 + Z_3 + \frac{Z_1 Z_3}{Z_2}} \\ \frac{1}{Z_1 + R_3 + \frac{Z_1 Z_3}{Z_2}} & \frac{1}{Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2}} \end{bmatrix}$$

$Z_1 = \frac{1}{sC}$

$Z_2 = R$

$Z_3 = \frac{1}{sC}$

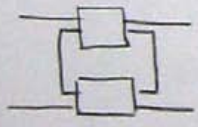
$$y'' = \begin{bmatrix} \frac{sC(sRC+1)}{2sRC+1} & \frac{s^2C^2R}{1+2sRC} \\ \frac{s^2C^2R}{-1+2sRC} & \frac{sC(1+sRC)}{1+2sRC} \end{bmatrix}$$

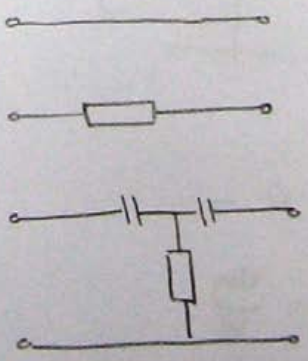
$$[y] = [y'] + [y''] = \begin{bmatrix} \frac{s^2+s}{2s+1} + 1 & -\frac{s^2}{2s+1} - 1 \\ \frac{s^2}{2s+1} + 1 & -\frac{s^2+s}{2s+1} - 1 \end{bmatrix}$$

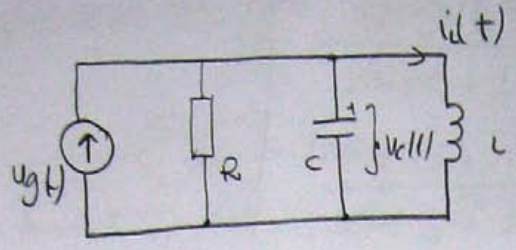
$$T(s) = \frac{y_{21}}{y_{11}+y_{22}} = \frac{U_2}{U_1} = \frac{(s+1)^2}{s^2+2s+1}$$

- y1 kratimo jer se traži da je prazan hod na izlazu (dakle da nis nije spojeno)

- sljedeći spoj četveropola (isti zadatak)

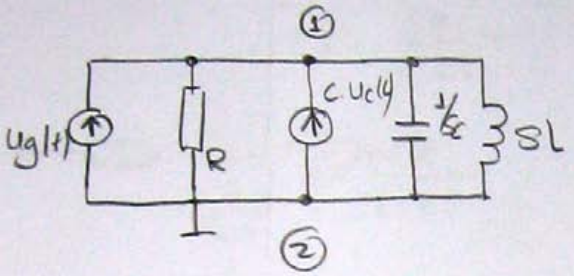






$C = 2$
$R = \frac{1}{2}$
$L = \frac{1}{4}$

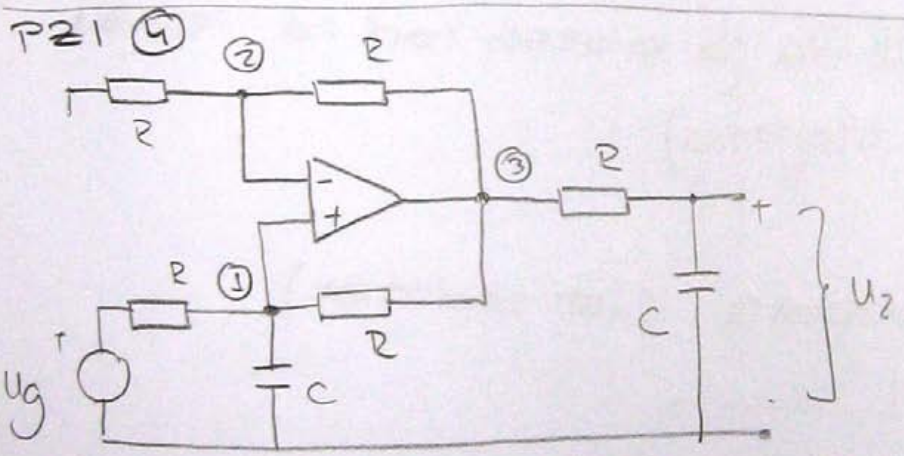
$u_c(0) = 0$
$i_1(0) = 0$
$I_g = e^{-2t} S(t)$



$$U_L\left(\frac{1}{R} + SC + \frac{1}{SL}\right) = I_g + C\,u_c(0)$$

$$I_L = \frac{U_1}{SL}$$

PZ1 (4)



$R = C = 1$

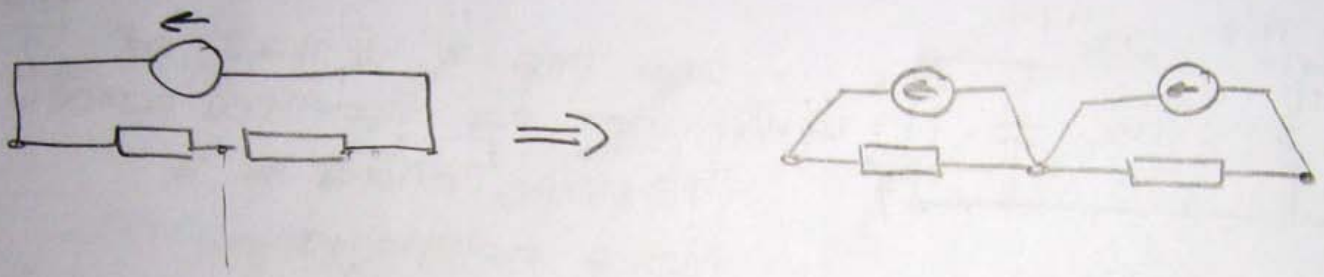
(2) $\quad U_2\left(\frac{1}{R} + \frac{1}{R}\right) - U_3 \cdot \frac{1}{R} = 0$

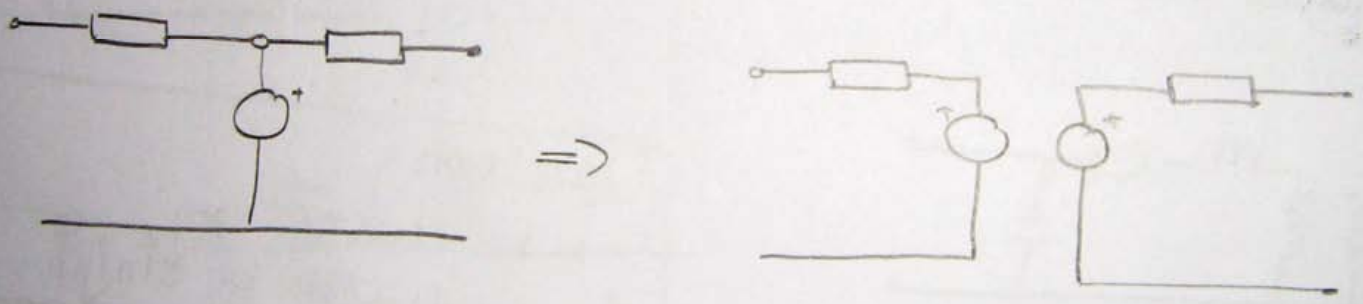
(1) $\quad U_1\left(\frac{1}{R} + \frac{1}{R} + SC\right) - U_3 \frac{1}{R} = \frac{U_g}{R}$

(4) $\quad U_4\left(\frac{1}{R} + SC\right) - U_3 \frac{1}{R} = 0$

$U_3 = A(U_2 - U_1) \quad A \to \infty \quad U_2 = U_1$

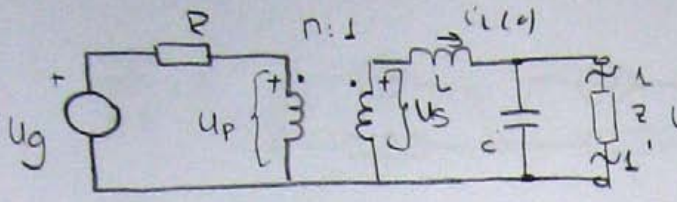
- posmicanje izvora





PZ1

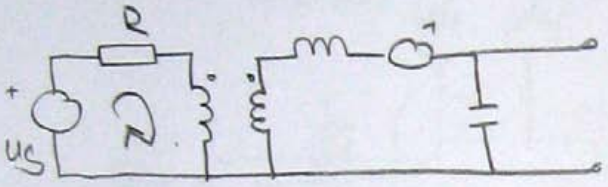
Thevenin



ako ima Z, a traži se $U_L = U_1$ napon na otporniku pomoću Thevenina, onda se Z mora odspojit

U_p ⇒ napon primara

U_s ⇒ napon sekundara



$$U_g - I_p R_1 - U_p = 0$$

$$U_s - I_s SL + s i_L(0) + I_s \frac{1}{SC} = 0$$

$$\boxed{U_p : U_s = n : 1} \rightarrow U_p = nU_s$$

$$\boxed{I_p : I_s = 1 : n} \rightarrow I_p = \frac{I_s}{n}$$

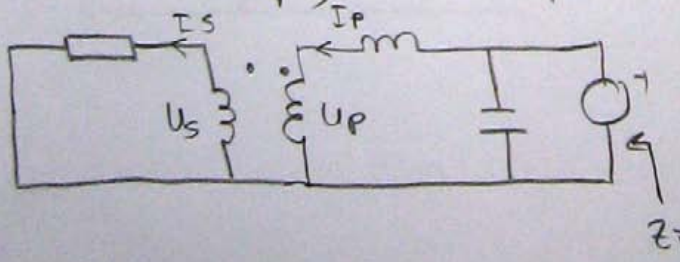
Is dobijemo . . .

Northon

1-1' - kratki spoj, tražili bi struju kroz kondenzator

na isti način se otpor traži, strujni izvor odspoji naponski kratko spojit početni uvjeti = 0

Thevenirov otpor: transformirati mrežu u pasivnu, sve strujni (nezavisni) izvori se odspoje, a naponski kratko spoje, te se početni uvjeti ugase



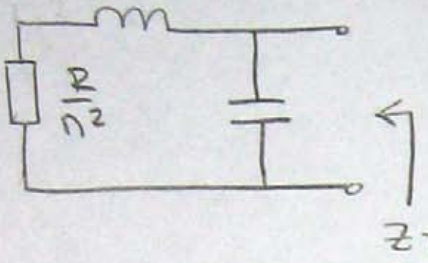
$$U_s : U_p = n : 1 \rightarrow U_s = nU_p$$

$$I_s : I_p = 1 : n \rightarrow I_s = \frac{I_p}{n}$$

$$U_s = I_s R$$

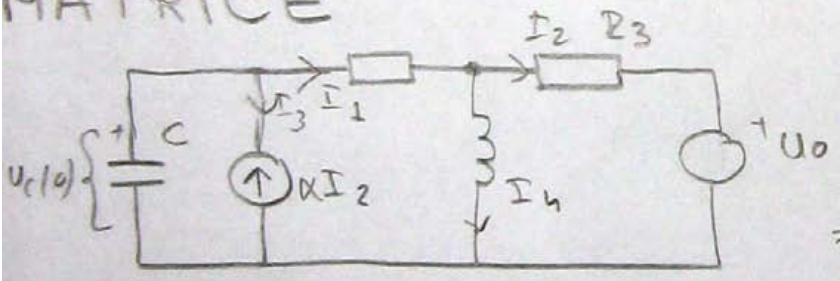
$$n U_p = \frac{I_p}{n} R$$

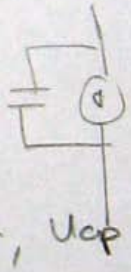
$$\frac{U_p}{I_p} = \frac{R}{n^2}$$



$$Z_T = \frac{\frac{1}{sC}\left(\frac{R}{n^2} + sL\right)}{\frac{R}{n^2} + sL + \frac{1}{sC}}$$

MATRICE





ZD, U0b, ZT, U0p

grana 1 ; grana 2 su grana stabla