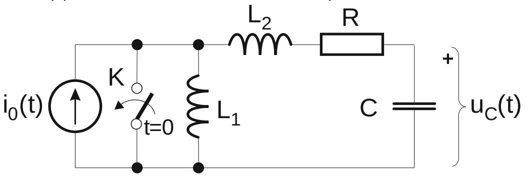
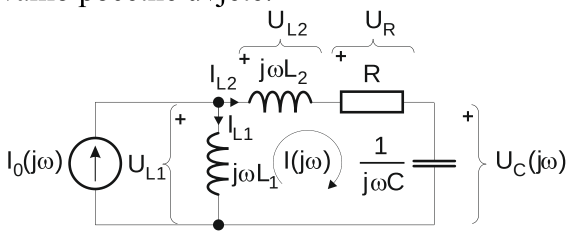
PISMENI ISPIT IZ ELEKTRIČNIH KRUGOVA 2015-2016 – Rješenja

1. Za mrežu prikazanu slikom odrediti valni oblik napona na kapacitetu $u_C(t)$ za $-\infty<t<\infty$, ako se u trenutku $t=0$ zatvori sklopka K. Zadane su normalizirane vrijednosti elemenata: $L_1=2$, $L_2=2$, $C=1$, $R=1$, te pobuda strujnog izvora $i_0(t)=2\sin t$ za $-\infty<t<\infty$ (stacionarni sinusni signal).



Rješenje: a) za $t<0$ izračunavamo početne uvjete:



$$I_{L1}(j\omega)=I_0(j\omega)-I(j\omega) \qquad U_C(j\omega)=I(j\omega)\cdot 1/(j\omega C); U_R(j\omega)=I(j\omega)\cdot R;$$
$$I_{L2}(j\omega)=I(j\omega) \qquad U_{L1}(j\omega)=j\omega L_1\cdot I_{L1}(j\omega)$$
$$-U_{L1}(j\omega)+U_{L2}(j\omega)+U_R(j\omega)+U_C(j\omega)=0 \qquad U_{L2}(j\omega)=+j\omega L_2\cdot I_{L2}(j\omega)$$

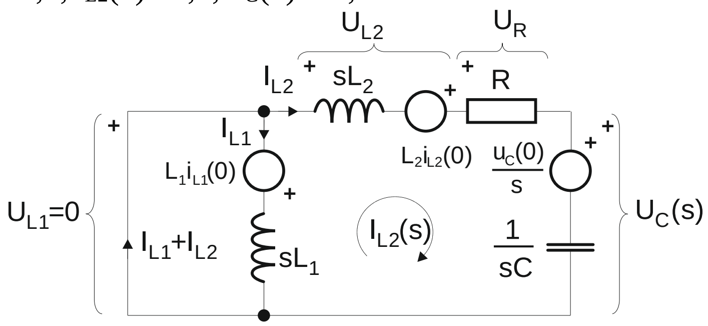
Uvrstimo izraze:

$$-j\omega L_1\cdot[I_0(j\omega)-I(j\omega)]+j\omega L_2\cdot I(j\omega)+R\cdot I(j\omega)+I(j\omega)\cdot 1/(j\omega C)=0$$
$$I_0(j\omega)j\omega L_1=I(j\omega)[j\omega L_1+j\omega L_2+R+1/(j\omega C)];\ I_0(j\omega)=2\angle 0^\circ;$$
$$I(j\omega)=\frac{j\omega L_1}{j\omega(L_1+L_2)+R+1/(j\omega C)}I_0(j\omega)=\frac{j2}{j(2+2)+1-j}I_0(j\omega)=\frac{2j}{1+3j}I_0(j\omega)=$$
$$=\frac{2j}{1+3j}\cdot\frac{1-3j}{1-3j}\cdot I_0(j\omega)=\frac{1}{5}(3+j)\cdot I_0(j\omega)=\frac{1}{5}(3+j)\cdot 2=\frac{2}{5}(3+j)=2\sqrt{\frac{2}{5}}\cdot e^{j\arctan(1/3)}$$
$$I_{L1}(j\omega)=I_0(j\omega)-I(j\omega)=I_0(j\omega)\left(1-\frac{3+j}{5}\right)=I_0(j\omega)\frac{2-j}{5}$$
$$I_{L1}(j\omega)=2\frac{2-j}{5}=\frac{2}{5}(2-j)=\frac{2\sqrt{5}}{5}e^{-j\arctan(1/2)}$$
$$I_{L2}(j\omega)=I(j\omega)=\frac{2}{5}(3+j)=2\sqrt{\frac{2}{5}}\cdot e^{j\arctan(1/3)}$$
$$U_C(j\omega)=I(j\omega)\frac{1}{j\omega C}=-j\cdot I(j\omega)=-j\cdot\frac{2}{5}(3+j)=e^{-j\frac{\pi}{2}}\cdot 2\sqrt{\frac{2}{5}}\cdot e^{j\arctan(1/3)}=$$
$$=2\sqrt{\frac{2}{5}}\cdot e^{j(\arctan(1/3)-\pi/2)}=2\sqrt{\frac{2}{5}}\angle(\arctan(1/3)-\pi/2)$$

$$i_{L1}(t)=\frac{2}{\sqrt{5}}\cdot\sin(t-\arctan(1/2)) \qquad i_{L1}(0)=\frac{2}{\sqrt{5}}\cdot\sin(-\arctan(1/2))=-0{,}4[\mathrm{A}]$$
$$i_{L2}(t)=2\sqrt{\frac{2}{5}}\cdot\sin(t+\arctan(1/3)) \quad\Rightarrow\quad i_{L2}(0)=2\sqrt{\frac{2}{5}}\cdot\sin(\arctan(1/3))=0{,}4[\mathrm{A}]$$
$$u_C(t)=2\sqrt{\frac{2}{5}}\cdot\sin(t+\arctan(1/3)-\pi/2) \qquad u_C(0)=2\sqrt{\frac{2}{5}}\cdot\sin(\arctan(1/3)-\pi/2)=-1{,}2[\mathrm{V}]$$

(2 boda)

b) za $t \geq 0$ primijenimo Laplaceovu transformaciju na slijedeći električni krug koji ima zadane početne uvjete: $i_{L1}(0)=-0{,}4$; $i_{L2}(0)= 0{,}4$; $u_C(0)=-1{,}2$:



$$sL_1 \cdot I_{L1}(s) - L_1 i_{L1}(0) = 0 \;\Rightarrow\; sL_1 \cdot I_{L1}(s) = L_1 i_{L1}(0) \Rightarrow I_{L1}(s) = i_{L1}(0)/s \;\;\text{(ne treba)}$$

$$sL_2 \cdot I_{L2}(s) - L_2 i_{L2}(0) + I_{L2}(s) \cdot \left(R + \frac{1}{sC} \right) + \frac{u_C(0)}{s} = 0$$

$$\boxed{I_{L2}(s) \cdot \left(sL_2 + R + 1/(sC) \right) = L_2 i_{L2}(0) - u_C(0)/s} \Rightarrow I_{L2}(s)$$

$$\boxed{U_C(s) = I_{L2}(s) \cdot \frac{1}{sC} + \frac{u_C(0)}{s}}$$

Uz zadane normalizirane vrijednosti elemenata $L_1=2$, $L_2=2$, $C=1$, $R=1$, slijedi:

$$\left(2s + 1 + \frac{1}{s} \right) \cdot I_{L2}(s) = 2 \cdot 0{,}4 + \frac{1{,}2}{s} \Big/ \cdot s$$

$$U_C(s) = I_{L2}(s) \cdot \frac{1}{sC} + \frac{u_C(0)}{s} \;\Rightarrow$$

$$\left(2s^2 + s + 1 \right) \cdot I_{L2}(s) = 0{,}8 \cdot s + 1{,}2$$

$$I_{L2}(s) = \frac{0{,}8 \cdot s + 1{,}2}{2s^2 + s + 1}$$

$$U_C(s) = \frac{0{,}8 \cdot s + 1{,}2}{2s^2 + s + 1} \cdot \frac{1}{s} - \frac{1{,}2}{s} = \frac{0{,}8 \cdot s + 1{,}2 - 1{,}2\left(2s^2 + s + 1\right)}{s\left(2s^2 + s + 1\right)} = \frac{0{,}8 \cdot s + 1{,}2 - 2{,}4s^2 - 1{,}2s - 1{,}2}{s\left(2s^2 + s + 1\right)}$$

$$U_C(s) = -\frac{2{,}4s^2 + 0{,}4s}{s\left(2s^2 + s + 1\right)} = -\frac{2{,}4s + 0{,}4}{2s^2 + s + 1} = -\frac{1{,}2s + 0{,}2}{s^2 + (1/2)s + 1/2}$$

(2 boda)

c) odziv u vremenskoj domeni:

$$s^2 + \frac{1}{2}s + \frac{1}{2} = 0 \Rightarrow s_{p1,2} = -\frac{1}{4} \pm \sqrt{\left(\frac{1}{4}\right)^2 - \frac{1}{2}} = -\frac{1}{4} \pm \sqrt{\frac{1}{16} - \frac{8}{16}} = -\frac{1}{4} \pm j\frac{\sqrt{7}}{4}$$

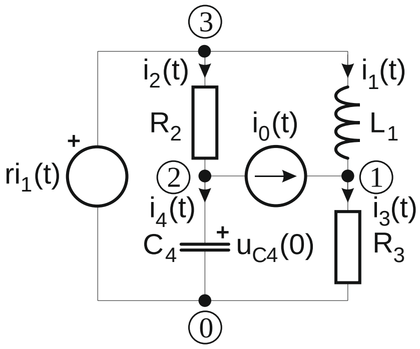
$$U_C(s) = -\frac{1{,}2\left(s + \frac{1}{4}\right) - \frac{1{,}2}{4} + \frac{0{,}8}{4}}{s^2 + (1/2)s + 1/2} = -1{,}2\frac{\left(s + \frac{1}{4}\right)}{\left(s + \frac{1}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2} + 0{,}1 \cdot \frac{4}{\sqrt{7}} \cdot \frac{\left(\frac{\sqrt{7}}{4}\right)}{\left(s + \frac{1}{4}\right)^2 + \left(\frac{\sqrt{7}}{4}\right)^2}$$

$$u_C(t) = e^{-\frac{1}{4}t} \left[-\frac{6}{5}\cos\left(\frac{\sqrt{7}}{4}t\right) + \frac{0{,}4}{\sqrt{7}} \cdot \sin\left(\frac{\sqrt{7}}{4}t\right) \right] \cdot S(t)$$

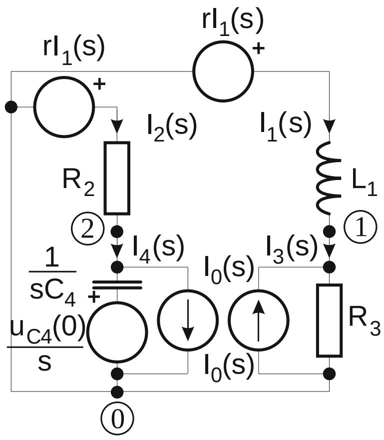
$$u_C(t) = e^{-0{,}25 t}\left[-1{,}2 \cdot \cos(0{,}6614t) + 0{,}1512 \cdot \sin(0{,}6614t)\right] \cdot S(t)$$

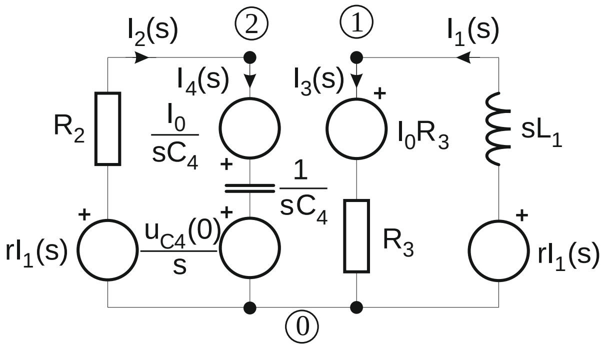
(1 bod)

2. Zadan je električni krug prema slici. Poštujući oznake čvorišta i grana, nacrtati pripadni orijentirani graf i napisati spojnu matricu **S**. Napisati naponsko-strujne jednadžbe grana u matričnom obliku te ispisati matricu impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$. Matrica $\mathbf{Z}_b$ mora biti regularna. Napisati temeljni sustav jednadžbi petlji, odnosno odrediti matrice impedancija petlji $\mathbf{Z}_p$ i vektor početnih uvjeta i nezavisnih izvora petlji $\mathbf{U}_{0p}$.



<u>Rješenje:</u> Posmicanje strujnog i naponskog izvora (zato da bismo dobili regularnu matricu $\mathbf{Z}_b$) i zatim primjena Laplaceove transformacije. Čvor ③ je nestao i jedna petlja je nestala. Pretvaranje svih izvora u naponske.





(1 bod)

Naponsko-strujne jednadžbe grana:

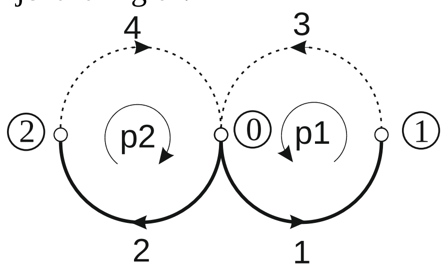
$$U_1 = sL_1 \cdot I_1 - r \cdot I_1$$

$$U_2 = -r \cdot I_1 + R_2 \cdot I_2$$

$$U_3 = R_3 \cdot I_3 + I_0 \cdot R_3$$

$$U_4 = \frac{1}{sC_4} \cdot I_4 - \frac{1}{sC_4} \cdot I_0 + \frac{u_{C4}(0)}{s}$$

Orijentirani graf:



Spojna matrica: $\mathbf{S} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ **(1 bod)**

Naponsko-strujne relacije grana u matričnom obliku: $\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$

$$\underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b} = \underbrace{\begin{bmatrix} sL_1 - r & 0 & 0 & 0 \\ -r & R_2 & 0 & 0 \\ 0 & 0 & R_3 & 0 \\ 0 & 0 & 0 & \dfrac{1}{sC_4} \end{bmatrix}}_{\mathbf{Z}_b} \cdot \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ I_0 R_3 \\ -I_0 \dfrac{1}{sC_4} + \dfrac{u_{C4}(0)}{s} \end{bmatrix}}_{\mathbf{U}_{0b}}$$ **(1 bod)**

Matrica $\mathbf{Z}_b$ je regularna. Temeljni sustav jednadžbi petlji u matričnom obliku: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$, gdje su

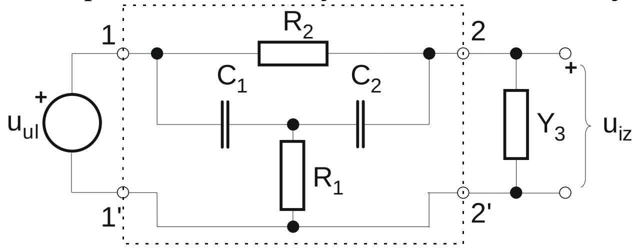
$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} sL_1 - r & 0 & 0 & 0 \\ -r & R_2 & 0 & 0 \\ 0 & 0 & R_3 & 0 \\ 0 & 0 & 0 & \dfrac{1}{sC_4} \end{bmatrix} \cdot \mathbf{S}^T =$$

$$= \begin{bmatrix} sL_1 - r & 0 & R_3 & 0 \\ -r & R_2 & 0 & \dfrac{1}{sC_4} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} sL_1 - r + R_3 & 0 \\ -r & R_2 + \dfrac{1}{sC_4} \end{bmatrix}$$ **(1 bod)**

$$\mathbf{U}_{0p} = -\mathbf{S} \cdot \mathbf{U}_{0b} = -\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ I_0 R_3 \\ -I_0 \dfrac{1}{sC_4} + \dfrac{u_{C4}(0)}{s} \end{bmatrix} = \begin{bmatrix} -I_0 R_3 \\ I_0 \dfrac{1}{sC_4} - \dfrac{u_{C4}(0)}{s} \end{bmatrix}$$ **(1 bod)**

Rješenje: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p} \;\Rightarrow\; \mathbf{I}_p = \begin{bmatrix} I_{p1} \\ I_{p2} \end{bmatrix}$

3. Za četveropol prikazan slikom s obzirom na polove 2–2' izračunati: a) y-parametre, ako su zadane normalizirane vrijednosti elemenata $R_1=R_2=1$, $C_1=C_2=2$. b) Iz y-parametara izračunati prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$ ako je kao admitancija Y_3 spojen kapacitet $C_3=3$. c) Izračunati istim postupkom prijenosnu funkciju napona $T(s)=U_{iz}(s)/U_{ul}(s)$ ako je kao admitancija Y_3 spojen otpor $R_3=1/3$. d) Nacrtati raspored nula i polova u slučajevima b) i c). e) Iz rasporeda nula i polova kvalitativno skicirati amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$ u slučajevima b) i c).



Rješenje: Nađimo najprije y-parametre četveropola. Ukupni četveropol se može promatrati kao kombinacija dva četveropola u paralelu.

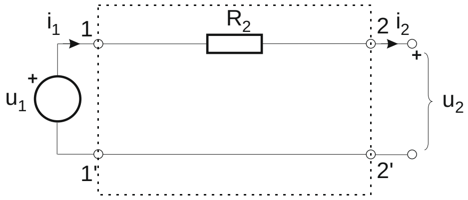
a) Za četveropol uz referentne oznake struja i napona slijede y-parametri:

Strujne jednadžbe četveropola

$$I_1 = y_{11}\cdot U_1 - y_{12}\cdot U_2$$
$$I_2 = y_{21}\cdot U_1 - y_{22}\cdot U_2$$

Naponske jednadžbe četveropola

$$U_1 = z_{11}\cdot I_1 - z_{12}\cdot I_2$$
$$U_2 = z_{21}\cdot I_1 - z_{22}\cdot I_2$$

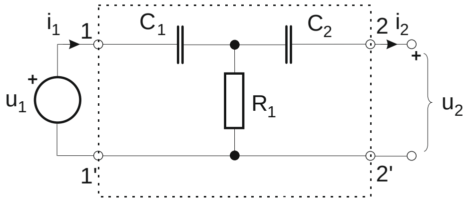


$U_2 = 0, I_2 = I_1 \Rightarrow$ $\quad$ $U_1 = 0, I_2 = I_1 \Rightarrow$

$y_{11} = I_1/U_1 = 1/R_2$ $\quad$ $y_{12} = -I_1/U_2 = 1/R_2$

$y_{21} = I_2/U_1 = 1/R_2$ $\quad$ $y_{22} = -I_2/U_2 = 1/R_2$

$$\Rightarrow [y]^I = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$



$I_2 = 0, U_2 = U_{R1} = I_1 R_1 \Rightarrow$ $\quad$ $I_1 = 0, U_1 = U_{R1} = -I_2 R_1 \Rightarrow$

$z_{11} = U_1 / I_1 = R_1 + 1/sC_1$ $\quad$ $z_{12} = -U_1 / I_2 = R_1$

$z_{21} = U_2 / I_1 = R_1$ $\quad$ $z_{22} = -U_2 / I_2 = R_1 + 1/sC_2$

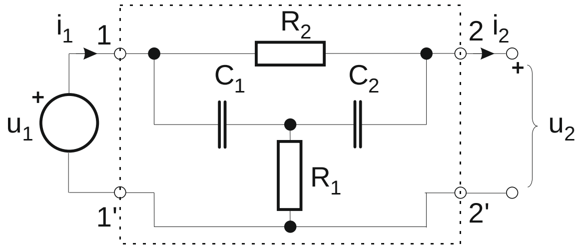
$$\Rightarrow [z]^{II} = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix} = \begin{bmatrix} 1+1/(2s) & -1 \\ 1 & -[1+1/(2s)] \end{bmatrix}$$

$[y] = [z]^{-1}$;

$$y_{11} = \frac{z_{22}}{|\Delta z|} = \frac{s(s+1/2)}{s+1/4} = \frac{4s^2+2s}{4s+1}; \; y_{12} = \frac{z_{12}}{|\Delta z|} = \frac{s^2}{s+1/4} = \frac{4s^2}{4s+1}; \; y_{21} = \frac{z_{21}}{|\Delta z|} = y_{12};$$

$$y_{22} = \frac{z_{11}}{|\Delta z|} = y_{11}; \; |\Delta z| = [1+1/(2s)]^2 - 1 = [1+1/s+1/(4s^2)] - 1 = 1/s + 1/(4s^2) = (s+1/4)/s^2$$

Ukupni četveropol:



$$[y]=[y]^I+[y]^{II}=\begin{bmatrix}\dfrac{4s^2+2s}{4s+1}+1 & -\left(\dfrac{4s^2}{4s+1}+1\right)\\ \dfrac{4s^2}{4s+1}+1 & -\left(\dfrac{4s^2+2s}{4s+1}+1\right)\end{bmatrix}=\begin{bmatrix}\dfrac{4s^2+6s+1}{4s+1} & -\dfrac{4s^2+4s+1}{4s+1}\\ \dfrac{4s^2+4s+1}{4s+1} & -\dfrac{4s^2+6s+1}{4s+1}\end{bmatrix}$$ **(2 boda)**

b) Prijenosna funkcija napona sa zaključenjem $Y_3(s)=\dfrac{1}{R_3}=3$

$$H(s)=\frac{U_2(s)}{U_1(s)}=\frac{y_{21}}{y_{22}+Y_3}$$

$$H(s)=\frac{U_2(s)}{U_1(s)}=\frac{\dfrac{4s^2}{4s+1}+1}{\dfrac{4s^2+2s}{4s+1}+1+3}=\frac{4s^2+4s+1}{4s^2+2s+4(4s+1)}=\frac{4s^2+4s+1}{4s^2+18s+4}=\frac{s^2+s+1/4}{s^2+(9/2)s+1}$$

Vidljivo je $H(0)=1/4;\ H(\infty)=1$

d1) raspored nula i polova za slučaj b):

Nule: $4s^2+4s+1=(2s+1)^2=0\Rightarrow s_{o1,2}=-1/2$

Polovi: $s^2+(9/2)s+1=0\Rightarrow s_{p1,2}=-\dfrac{9}{4}\pm j\sqrt{\dfrac{81}{16}-\dfrac{16}{16}}=-\dfrac{9\pm\sqrt{65}}{4}$

ili $s_{p1}=-4{,}26556; s_{p2}=-0{,}234436$ **(b) + d1) =1 bod)**

c) Prijenosna funkcija napona sa zaključenjem $Y_3(s)=sC_3=3s$

$$H(s)=\frac{U_2(s)}{U_1(s)}=\frac{\dfrac{4s^2}{4s+1}+1}{\dfrac{4s^2+2s}{4s+1}+1+3s}=\frac{4s^2+4s+1}{4s^2+2s+(4s+1)(3s+1)}=\frac{4s^2+4s+1}{16s^2+9s+1}=\frac{1}{4}\cdot\frac{s^2+s+\frac{1}{4}}{s^2+\frac{9}{16}s+\frac{1}{16}}$$ Vidlj

ivo je $H(0)=1;\ H(\infty)=1/4$

d2) raspored nula i polova za slučaj c):

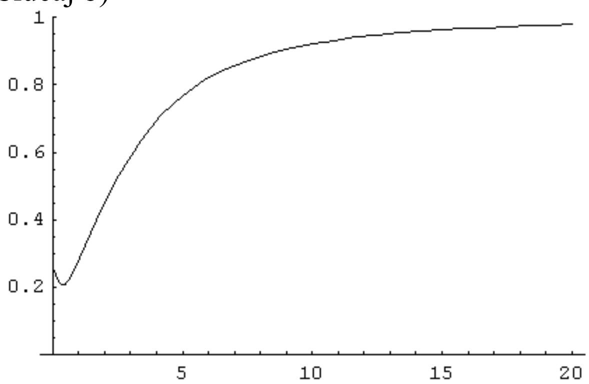
Nule: $4s^2+4s+1=(2s+1)^2=0\Rightarrow s_{o1,2}=-1/2$

Polovi: $s^2+\dfrac{9}{16}s+\dfrac{1}{16}=0\Rightarrow s_{p1,2}=-\dfrac{9}{32}\pm\sqrt{\left(\dfrac{9}{32}\right)^2-\dfrac{1}{16}}=-\dfrac{9\pm\sqrt{17}}{32}$

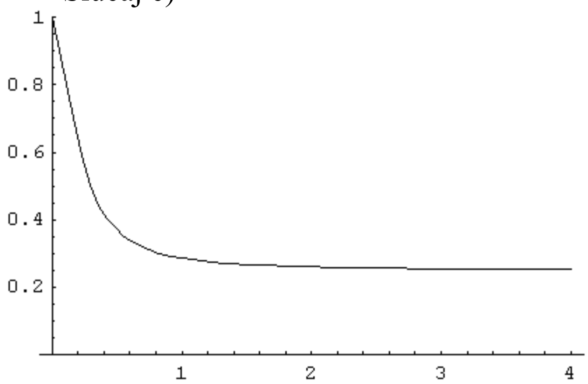
ili $s_{p1}=-0{,}4101; s_{p2}=-0{,}1524$ **(c) + d2) =1 bod)**

e) amplitudno-frekvencijske karakteristike **(1 bod)**

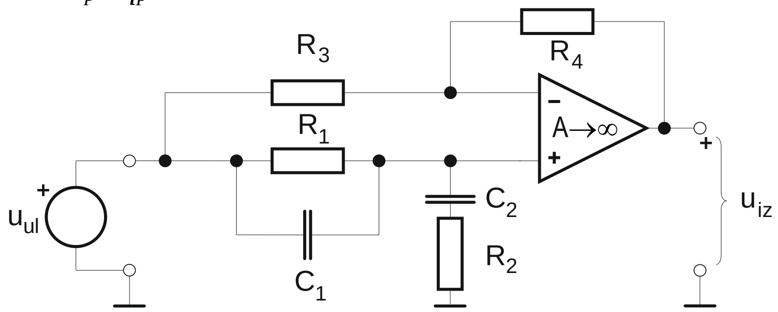
Slučaj b)



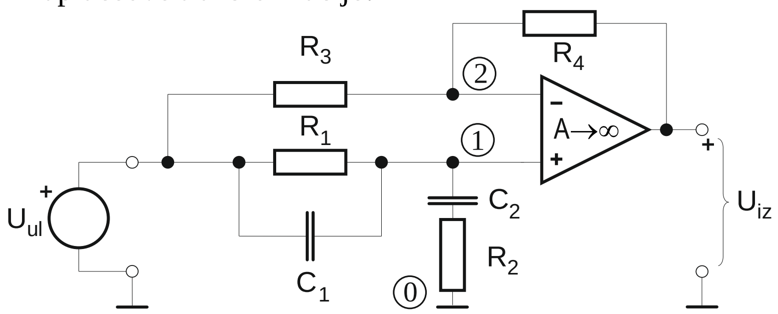
Slučaj c)



4. Zadan je aktivni-RC električni filtar prikazan slikom s normaliziranim vrijednostima elemenata C_1=C_2=1; R_1=R_2=1 te R_3=1; R_4=2. Odrediti: a) naponsku prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$. O kojem se tipu filtra radi (NP, VP, PP ili PB)? b) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja izračunati vrijednost parametara k, ω_p, ω_z, q_p. c) Prikazati raspored polova i nula u kompleksnoj ravnini. d) Nacrtati amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$. e) Koliko iznose širina pojasa propuštanja/gušenja B, te gornja i donja granična frekvencija ω_g i ω_d kao funkcije parametara ω_p i q_p ?



Rješenje: Primjenom Laplaceove transformacije:



Metoda napona čvorišta:

$$(1)\ U_1\left(\frac{1}{R_1}+sC_1+\frac{1}{R_2+\frac{1}{sC_2}}\right)=U_{ul}\left(\frac{1}{R_1}+sC_1\right)\Big/\cdot R_1$$

$$(2)\ U_2\left(\frac{1}{R_3}+\frac{1}{R_4}\right)=U_{ul}\frac{1}{R_3}+U_{iz}\frac{1}{R_4}\Big/\cdot R_3R_4$$

$$(3)\ A\left(U_1-U_2\right)=U_{iz}\ \Rightarrow\ U_1=U_2\ \ (A\to\infty)$$

$$(1)\ U_1\left(1+sR_1C_1+R_1\frac{sC_2}{sR_2C_2+1}\right)=U_{ul}\left(1+sR_1C_1\right)\Big/\cdot(sR_2C_2+1)$$

$$(2)\ U_1\left(R_3+R_4\right)=U_{ul}R_4+U_{iz}R_3/:\left(R_3+R_4\right)$$

$$(1)\ U_1\left[(sR_1C_1+1)(sR_2C_2+1)+sR_1C_2\right]=U_{ul}(sR_1C_1+1)(sR_2C_2+1)$$

$$(2)\ U_1=U_{ul}\frac{R_4}{R_3+R_4}+U_{iz}\frac{R_3}{R_3+R_4}$$

Uvedimo oznaku:

$$\beta=\frac{R_3}{R_3+R_4};\ 1-\beta=\frac{R_4}{R_3+R_4};\ (2)\ \Rightarrow\ U_1=(1-\beta)U_{ul}+\beta U_{iz}$$

Malo računanja:

$$(2)\to(1)\Rightarrow$$

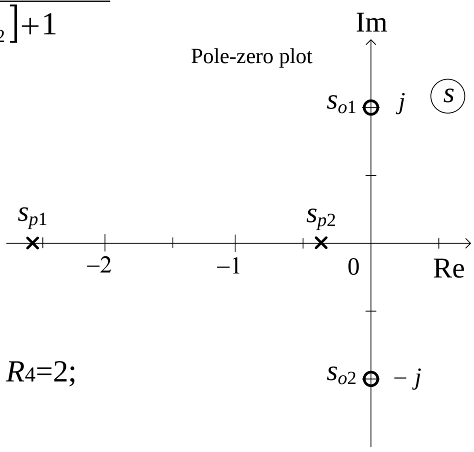
$$\left[(1-\beta)U_{ul}+\beta U_{iz}\right]\cdot\left[(sR_1C_1+1)(sR_2C_2+1)+sR_1C_2\right]=U_{ul}(sR_1C_1+1)(sR_2C_2+1)$$

$$(1-\beta)U_{ul}\cdot(sR_1C_1+1)(sR_2C_2+1)+(1-\beta)U_{ul}\cdot sR_1C_2-U_{ul}(sR_1C_1+1)(sR_2C_2+1)=$$
$$=-\beta U_{iz}(sR_1C_1+1)(sR_2C_2+1)-\beta U_{iz}sR_1C_2$$
$$U_{ul}\cdot\left[-\beta(sR_1C_1+1)(sR_2C_2+1)+(1-\beta)sR_1C_2\right]=-\beta U_{iz}\cdot\left[(sR_1C_1+1)(sR_2C_2+1)+sR_1C_2\right]$$
$$/:(-\beta)$$
$$U_{ul}\cdot\left[(sR_1C_1+1)(sR_2C_2+1)+\frac{1-\beta}{-\beta}sR_1C_2\right]=U_{iz}\left[(sR_1C_1+1)(sR_2C_2+1)+sR_1C_2\right]$$
$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{s^2R_1C_1R_2C_2+s\left[R_1C_1+R_2C_2+R_1C_2\left(1-\frac{1}{\beta}\right)\right]+1}{s^2R_1C_1R_2C_2+s\left[R_1C_1+R_2C_2+R_1C_2\right]+1}$$

Vratimo natrag oznaku: $1-\frac{1}{\beta}=1-\frac{R_3+R_4}{R_3}=-\frac{R_4}{R_3}$

Konačno je:

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{s^2+s\frac{R_1C_1+R_2C_2-R_1C_2R_4/R_3}{R_1C_1R_2C_2}+\frac{1}{R_1C_1R_2C_2}}{s^2+s\frac{R_1C_1+R_2C_2+R_1C_2}{R_1C_1R_2C_2}+\frac{1}{R_1C_1R_2C_2}}$$



Uz uvrštene vrijednosti elemenata $C_1=C_2=1$; $R_1=R_2=1$ te $R_3=1$; $R_4=2$;

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{s^2+1}{s^2+3s+1}$$

b) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja izračunati parametre k, ω_p, ω_z, q_p.

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=k\cdot\frac{s^2+\omega_z^2}{s^2+(\omega_p/q_p)\cdot s+\omega_p^2}\Rightarrow \begin{array}{l}\omega_p=\omega_z=1,\\ q_p=1/3,\ k=1\end{array}$$

-o kojem se tipu filtra radi (NP, VP, PP ili PB)? $\Rightarrow$ PB

a) + b) (3 boda)

c) raspored polova i nula u kompleksnoj ravnini:

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{s^2+1}{s^2+3s+1}$$

- nule: $s^2+1=0 \quad\Rightarrow\quad s_{o1,2}=\pm j$

- polovi: $s^2+3s+1=0 \Rightarrow s_{p1,2}=-\frac{3\pm\sqrt{5}}{2}$; $s_{p1}=-2{,}61803; s_{p2}=-0{,}381966$

d) amplitudno-frekvencijska karakteristika:

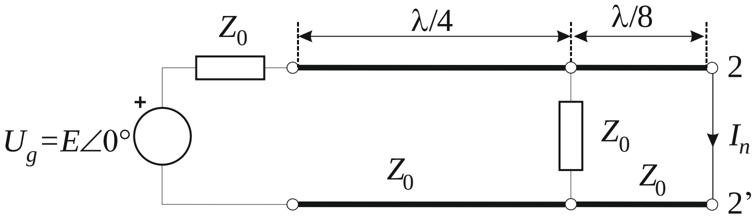
$$s=j\omega \Rightarrow T(j\omega)=\frac{-\omega^2+1}{-\omega^2+j3\omega+1} \Rightarrow |T(j\omega)|=\frac{|1-\omega^2|}{\sqrt{(1-\omega^2)^2+(3\omega)^2}}$$

e) Širina pojasa gušenja $B=\omega_p/q_p$=3 [rad/s]. Koristi se isti izraz kao za širinu pojasa propuštanja. Gornja i donja granična frekvencija pojasa gušenja su:

$$\omega_{g,d}=\omega_p\sqrt{1+\frac{1}{4q_p^2}}\pm\frac{\omega_p}{2q_p} \Rightarrow \omega_g=\sqrt{1+\frac{9}{4}}+\frac{3}{2}=3{,}30278;\ \omega_d=\sqrt{1+\frac{9}{4}}-\frac{3}{2}=0{,}30278\text{ [rad/s]}$$

$\Rightarrow B=\omega_g-\omega_d$ =3 [rad/s] **c) + d) + e) (2 boda)**

5. Na slici je zadan sustav linija bez gubitaka istih primarnih parametara. Karakteristične impedancije linija su jednake Z_0. Napon generatora na ulazu je stacionarni sinusni signal $u_g(t)=E\cdot\cos(\omega t)$; $-\infty<t<\infty$ (prilaz 2–2' druge linije je kratko spojen). Odrediti: a) ulaznu impedanciju u drugu liniju; b) ulaznu impedanciju u prvu liniju; c) fazore napona i struje na izlazu prve linije; d) fazor struje ($\mathbf{I}_n$) na izlazu druge linije (napon je $\mathbf{U}_{2-2'}=0$); e) valni oblik struje $i_n(t)$ na izlazu 2–2' u vremenskoj domeni.



Rješenje: Zadatak rješavamo pomoću fazora jer se radi o stacionarnoj sinusnoj pobudi.
Ulazna impedancija linije lako se računa iz prijenosnih jednadžbi uz $x=l$:

$$U(0)=U(x)\cdot ch\gamma x+I(x)Z_0 sh\gamma x$$

$$I(0)=\frac{U(x)}{Z_0}sh\gamma x+I(x)ch\gamma x$$

$$Z_{ul}=\frac{U(0)}{I(0)}=\frac{Z_2 ch(\gamma l)+Z_0 sh(\gamma l)}{\frac{Z_2}{Z_0}sh(\gamma l)+ch(\gamma l)};\ Z_2=\frac{U(l)}{I(l)}$$

Z_2 je impedancija kojom je linija zaključena na izlazu.

Linija bez gubitaka $Z_0=\sqrt{\dfrac{L}{C}}$; $\gamma=j\beta=j\omega\sqrt{LC};\alpha=0$; $\lambda=\dfrac{2\pi}{\beta}$.

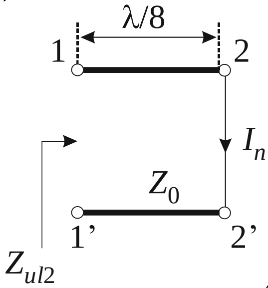
Ako je duljina linije: $l_1=\dfrac{\lambda}{4}$ tada je $\gamma\cdot l_1=j\beta\dfrac{\lambda}{4}=j\beta\dfrac{2\pi}{\beta}\dfrac{1}{4}=j\dfrac{\pi}{2}$.

Ako je duljina linije: $l_2=\dfrac{\lambda}{8}$ tada je $\gamma\cdot l_2=j\beta\dfrac{\lambda}{8}=j\beta\dfrac{2\pi}{\beta}\dfrac{1}{8}=j\dfrac{\pi}{4}$.

Za $l_1=\lambda/4$: $sh(\gamma\cdot l_1)=sh(j\pi/2)=j\sin(\pi/2)=j$; $ch(\gamma\cdot l_1)=ch(j\pi/2)=\cos(\pi/2)=0$;

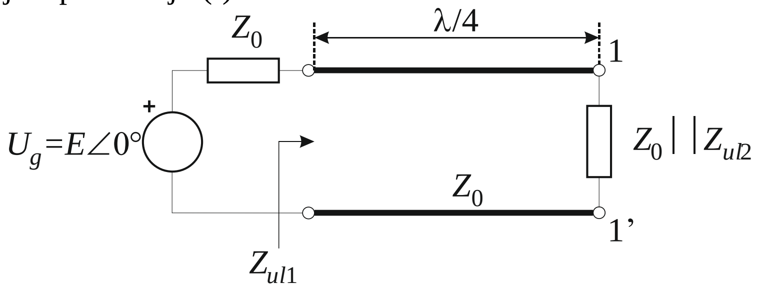
Za $l_2=\dfrac{\lambda}{8}$: $sh(\gamma\cdot l_2)=sh(j\pi/4)=j\sin(\pi/4)=j\dfrac{\sqrt{2}}{2}$; $ch(\gamma\cdot l_2)=ch(j\pi/4)=\cos(\pi/4)=\dfrac{\sqrt{2}}{2}$.

a) Ulazna impedancija u drugu liniju (II)



$$Z_{ul2}=\left.\frac{U^{\mathrm{II}}(0)}{I^{\mathrm{II}}(0)}\right|_{Z_2=0,\ l_2=\lambda/8}=\frac{Z_0 sh(j\pi/4)}{ch(j\pi/4)}=\frac{Z_0 j\sin(\pi/4)}{\cos(\pi/4)}=\frac{Z_0 j(\sqrt{2}/2)}{(\sqrt{2}/2)}=jZ_0.\ \textbf{(1 bod)}$$

b) Ulazna impedancija u prvu liniju (I)



$$Z_{ul1}=\left.\frac{U(0)}{I(0)}\right|_{Z_2=Z_0\mathrm{II}Z_{ul2},\ l_1=\lambda/4}=\frac{Z_2 ch(j\pi/2)+Z_0 sh(j\pi/2)}{(Z_2/Z_0)sh(j\pi/2)+ch(j\pi/2)}=\frac{Z_0^2}{Z_2}=\frac{Z_0^2}{Z_0\mathrm{II}Z_{ul2}}=Z_0(1-j)$$ **(1 bod)**

gdje je

$$Z_0\mathrm{II}Z_{ul2}=\frac{Z_0^2 j}{Z_0(1+j)}=Z_0\frac{j}{1+j}=Z_0\frac{1}{1-j}.$$

Napon i struja na ulazu prve linije:

$$U^{\mathrm{I}}(0)=U_g\frac{Z_{ul1}}{Z_0+Z_{ul1}}=U_g\frac{1-j}{2-j};\quad I^{\mathrm{I}}(0)=\frac{U_g}{Z_0+Z_{ul1}}=\frac{U_g}{Z_0(2-j)}$$

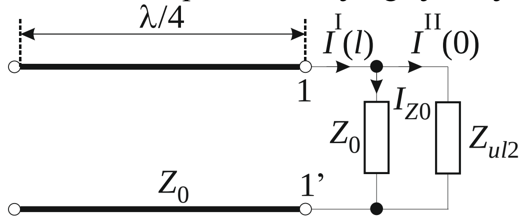
c) Napon i struja na izlazu prve linije (I) (slijede iz prijenosnih jednadžbi uz $x=l_1$):

$$\begin{aligned}U(x)&=U(0)\cdot ch\gamma x-I(0)Z_0 sh\gamma x\\ I(x)&=-\frac{U(0)}{Z_0}sh\gamma x+I(0)ch\gamma x\end{aligned}\quad\Rightarrow\quad\begin{aligned}U^{\mathrm{I}}(l)&=-I^{\mathrm{I}}(0)Z_0 sh(j\pi/2)=-jZ_0 I^{\mathrm{I}}(0)=U_g\cdot\frac{-j}{2-j}\\ I^{\mathrm{I}}(l)&=-\frac{U^{\mathrm{I}}(0)}{Z_0}sh\,(j\pi/2)=-j\frac{U^{\mathrm{I}}(0)}{Z_0}=U_g\cdot\frac{-(1+j)}{Z_0(2-j)}\end{aligned}$$

(1 bod)

Napon na ulazu u drugu liniju jednak je naponu na izlazu prve linije $U^{\mathrm{II}}(0)=U^{\mathrm{I}}(l)$.

Struja na ulazu u drugu liniju se izračunava pomoću strujnog djelitelja:



$$I^{\mathrm{II}}(0)=\frac{Z_0}{Z_0+Z_{ul2}}\cdot I^{\mathrm{I}}(l)=\frac{Z_0}{Z_0(1+j)}\cdot U_g\frac{-(1+j)}{Z_0(2-j)}=\frac{-U_g}{Z_0(2-j)}.$$

d) Napon i struja na izlazu druge linije (II)

$$U^{\mathrm{II}}(l)=U^{\mathrm{II}}(0)ch(j\pi/4)-I^{\mathrm{II}}(0)Z_0 sh(j\pi/4)=U^{\mathrm{II}}(0)\frac{\sqrt{2}}{2}-jZ_0 I^{\mathrm{II}}(0)\frac{\sqrt{2}}{2}$$

$$I^{\mathrm{II}}(l)=-\frac{U^{\mathrm{II}}(0)}{Z_0}sh\,(j\pi/4)+I^{\mathrm{II}}(0)ch(j\pi/4)=-j\frac{U^{\mathrm{II}}(0)}{Z_0}\frac{\sqrt{2}}{2}-I^{\mathrm{II}}(0)\frac{\sqrt{2}}{2}$$

Uz uvrštene vrijednosti $U^{\mathrm{II}}(0)$ i $I^{\mathrm{II}}(0)$

$$U^{\mathrm{II}}(l)=\frac{\sqrt{2}}{2}U_g\cdot\left[\frac{-j}{2-j}+jZ_0\frac{1}{Z_0(2-j)}\right]=0$$

$$I^{\mathrm{II}}(l)=\frac{\sqrt{2}}{2}U_g\cdot\left[\frac{-1}{Z_0(2-j)}+\frac{-1}{Z_0(2-j)}\right]=\frac{\sqrt{2}}{2}U_g\cdot\frac{-2}{Z_0(2-j)}$$

Odnosno

$$I^{\mathrm{II}}(l)=\sqrt{2}\cdot U_g\cdot\frac{-1}{Z_0(2-j)}=\sqrt{2}\cdot U_g\cdot\frac{-j}{Z_0(1+2j)}=\frac{U_g}{Z_0}\cdot\sqrt{\frac{2}{5}}\cdot e^{-\frac{\pi}{2}-\arctan(2)}$$ **(1 bod)**

Konačno je fazor struje $\mathbf{I}_n=I^{\mathrm{II}}(l)=\dfrac{\sqrt{2}E}{Z_0\sqrt{5}}\angle-\dfrac{\pi}{2}-\arctan(2)$.

e) U vremenskoj domeni je struja $i_n(t)=\dfrac{E}{Z_0}\sqrt{\dfrac{2}{5}}\cos\left[\omega t-\dfrac{\pi}{2}-\arctan(2)\right]$; $-\infty<t<\infty$. **(1 bod)**