GRAFOVI: Zadaci sa rješenjima za vježbu

1. Zadana je reducirana matrica incidencija grafa u obliku:

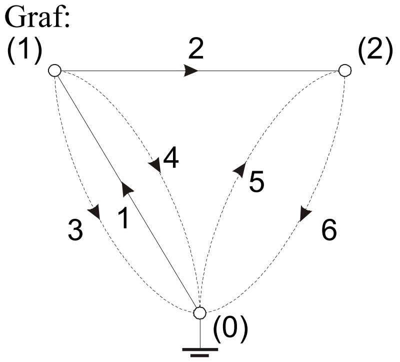
$$\mathbf{A} = \left[\mathbf{A}_t \;\; \mathbf{A}_s\right]$$

$$\mathbf{A} = \begin{array}{c} \\ (1) \\ (2) \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

pri čemu je $\mathbf{A}_t$ incidentna stablu, a $\mathbf{A}_s$ sponama. Odrediti temeljnu spojnu matricu **S** i temeljnu rastavnu matricu **Q**, ovog grafa. Nacrtati graf. Kojeg je ranga graf?

Rješenje:

Graf:



Temeljna spojna matrica **S**:

$$\mathbf{S} = \begin{array}{c} \\ I \\ II \\ III \\ IV \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cc|cccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

Temeljna rastavna matrica **Q**:

$$\mathbf{Q} = \begin{array}{c} \\ r_1 \\ r_2 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \left[\begin{array}{cc|cccc} 1 & 0 & -1 & -1 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & -1 \end{array}\right] \end{array}$$

Rang grafa $R = N_{ST} = 2$

Nulitet grafa $0 = N_{SP} = 4$

2. Zadana je reducirana matrica incidencija grafa u obliku:

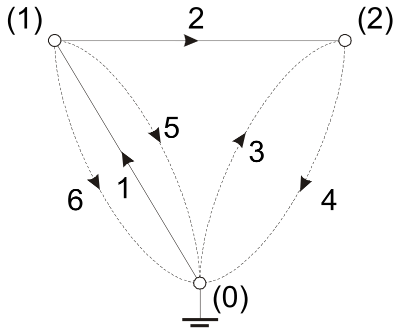
$$\mathbf{A} = \left[\mathbf{A}_t \;\; \mathbf{A}_s\right]$$

$$\mathbf{A} = \begin{array}{cc} & \begin{matrix} 1 & \;2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} (1) \\ (2) \end{matrix} & \left[\begin{array}{cc|cccc} -1 & 1 & 0 & 0 & 1 & 1 \\ 0 & -1 & -1 & 1 & 0 & 0 \end{array}\right] \end{array},$$

pri čemu je $\mathbf{A}_t$ incidentna stablu, a $\mathbf{A}_s$ sponama. Odrediti temeljnu spojnu matricu **S** i temeljnu rastavnu matricu **Q**, ovog grafa. Nacrtati graf. Kojeg je ranga graf?

<u>Rješenje:</u>

Graf:



Temeljna spojna matrica **S**:

$$\mathbf{S} = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} I \\ II \\ III \\ IV \end{matrix} & \left[\begin{array}{cc|cccc} -1 & -1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \end{array}$$

Temeljna rastavna matrica **Q**:

$$\mathbf{Q} = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} r_1 \\ r_2 \end{matrix} & \left[\begin{array}{cc|cccc} 1 & 0 & 1 & -1 & -1 & -1 \\ 0 & 1 & 1 & -1 & 0 & 0 \end{array}\right] \end{array}$$

Rang grafa $R = N_{ST} = 2$

Nulitet grafa $0 = N_{SP} = 4$

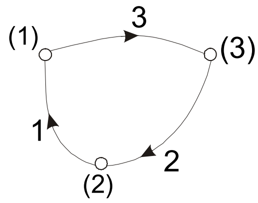
3. Nacrtati orjentirani graf i električnu mrežu koja zadovoljava slijedeće matrice:

$$\mathbf{Z}_b = \begin{bmatrix} sL_1 & 0 & 0 \\ 0 & sL_2 & 0 \\ gRsL_1 & 0 & R \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} I_g sL_1 + L_1 i_L(0) \\ 0 \\ gRI_g sL_1 + gRL_1 i_L(0) \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

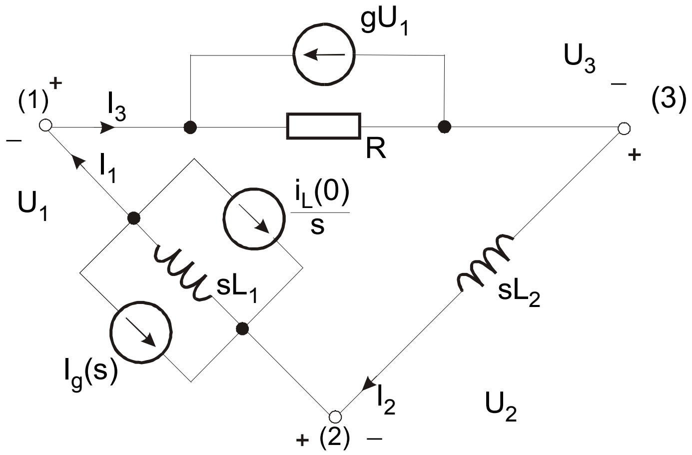
Napisati temeljni sustav jednadžbi petlji u matričnom obliku za dobivenu mrežu.

Rješenje:

orjentirani graf:



električna mreža

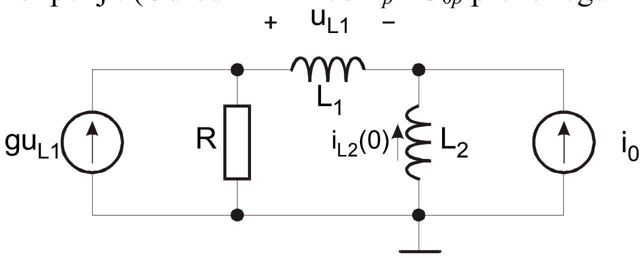


temeljni sustav jednadžbi petlji: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$

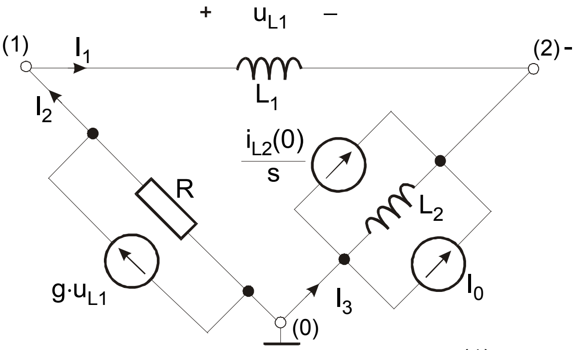
$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \left[sL_1 + gRsL_1 + sL_2 + R \right]$$

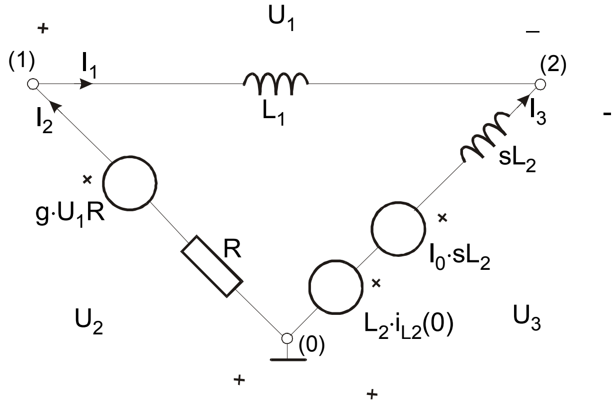
$$\mathbf{U}_{0p} = -\mathbf{S} \cdot \mathbf{U}_{0b} = -\left[I_g sL_1 + L_1 \cdot i_L(0) + gRsL_1 I_g + gRL_1 \cdot i_L(0) \right]$$

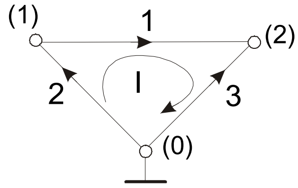
4. Zadana je mreža prema slici. Odrediti pripadni orijentirani graf, matricu incidencije i temeljni sustav jednadžbi petlji. (Odrediti matrice $\mathbf{Z}_p$ i $\mathbf{U}_{0p}$ preko regularne matrice $\mathbf{Z}_b$).



Rješenje:





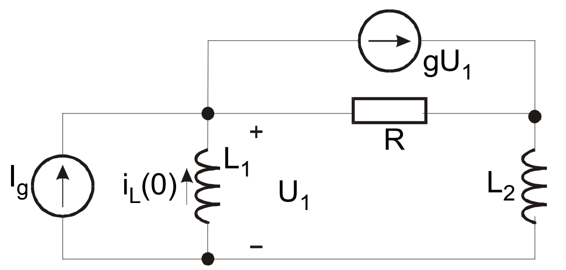


temeljni sustav jednadžbi petlji: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$

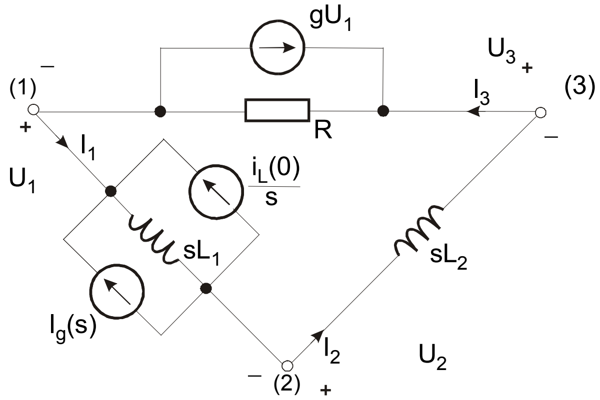
$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \left[sL_1 \cdot (1 - g \cdot R) + R + sL_2 \right]$$

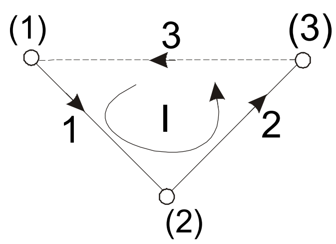
$$\mathbf{U}_{0p} = -\mathbf{S} \cdot \mathbf{U}_{0b} = \left[-I_0 \cdot sL_2 - L_2 \cdot i_{L2}(0) \right]$$

5. Za mrežu prikazanu slikom odrediti graf i matricu incidencija i napisati temeljni sustav jednadžbi petlji u matričnom obliku (matrice $\mathbf{Z}_p$ i $\mathbf{U}_{0p}$ preko matrica $\mathbf{Z}_b$ i $\mathbf{U}_{0b}$). Matrica $\mathbf{Z}_b$ mora biti regularna.



Rješenje:





$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} \qquad \mathbf{S} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$$

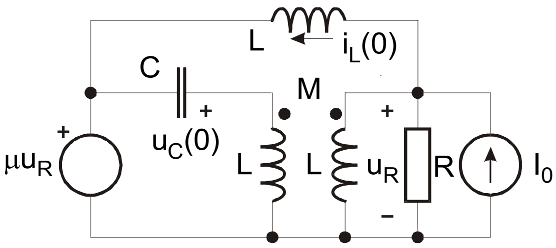
Graf (čvor 3 je referentan):

temeljni sustav jednadžbi petlji: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$

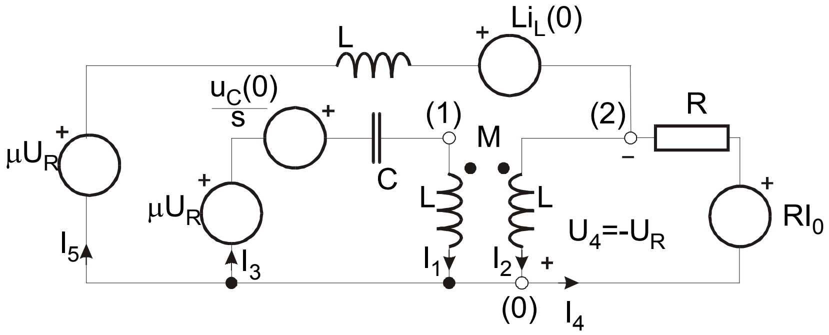
$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \left[sL_1 + gR \cdot sL_1 + sL_2 + R \right]$$

$$\mathbf{U}_{0p} = -\mathbf{S} \cdot \mathbf{U}_{0b} = -\left[I_g sL_1 + L_1 \cdot i_L(0) + gR \cdot sL_1 \cdot I_g + gRL_1 \cdot i_L(0) \right]$$

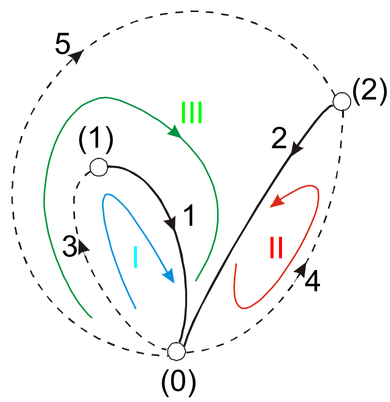
6. Mreži prikazanoj slikom pridružiti graf koji ima 2 stablene grane. Odrediti temeljni sustav jednadžbi petlji (matrice $\mathbf{Z}_p$ i $\mathbf{U}_{0p}$ preko matrica impedancija grana $\mathbf{Z}_b$ i nezavisnih izvora grana $\mathbf{U}_{0b}$). Matrica $\mathbf{Z}_b$ mora biti regularna.



Rješenje:



Orjentirani graf:



stablene grane: 1, 2 (punom linijom)
spone: 3, 4, 5 (crtkano)

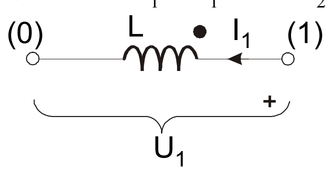
Spojna matrica:

$$\mathbf{S} = \begin{array}{c} 3 \\ 4 \\ 5 \end{array} \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{array} \right]$$

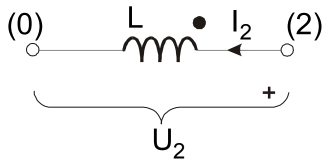
jedinicna matrica

Naponsko – strujne relacije grana:

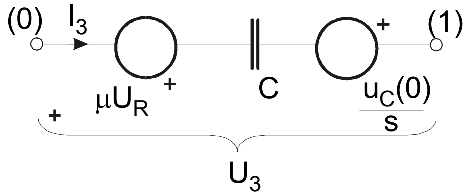
Grana 1: $U_1 = I_1 \cdot sL + I_2 \cdot sM$



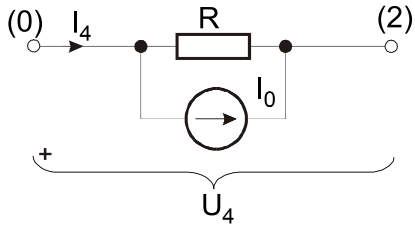
Grana 2: $U_2 = I_2 \cdot sL + I_1 \cdot sM$



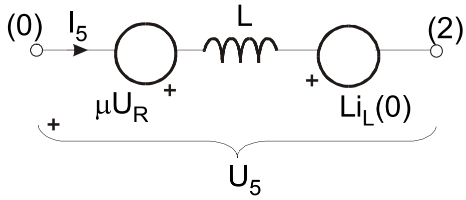
Grana 3: $U_3 = I_3 \dfrac{1}{sC} - \mu \cdot U_R - \dfrac{u_C(0)}{s}$, $U_R = -U_4$



Grana 4: $U_4 = I_4 \cdot R - I_0 \cdot R$



Grana 5: $U_5 = I_5 \cdot sL - \mu \cdot U_R + L \cdot i_L(0)$, $U_R = -U_4$



$$\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$$

$$\begin{aligned}
&U_1 = I_1 \cdot sL + I_2 \cdot sM \\
&U_2 = I_1 \cdot sM + I_2 \cdot sL \\
&U_3 = I_3 \cdot \frac{1}{sC} + \mu U_4 - \frac{u_C(0)}{s} = I_3 \cdot \frac{1}{sC} + \mu \cdot R \cdot I_4 - \mu \cdot R \cdot I_0 - \frac{u_C(0)}{s} \\
&U_4 = I_4 \cdot R - I_0 \cdot R \\
&U_5 = I_5 \cdot sL + \mu U_4 + L \cdot i_L(0) = \mu \cdot R \cdot I_4 + sL \cdot I_5 - \mu \cdot R \cdot I_0 + L \cdot i_L(0)
\end{aligned}$$

$$\mathbf{Z}_b = \begin{bmatrix} sL & sM & 0 & 0 & 0 \\ sM & sL & 0 & 0 & 0 \\ 0 & 0 & \dfrac{1}{sC} & \mu R & 0 \\ 0 & 0 & 0 & R & 0 \\ 0 & 0 & 0 & \mu R & sL \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} 0 \\ 0 \\ -\mu \cdot R \cdot I_0 - \dfrac{u_C(0)}{s} \\ -I_0 \cdot R \\ -\mu \cdot R \cdot I_0 + L \cdot i_L(0) \end{bmatrix}$$

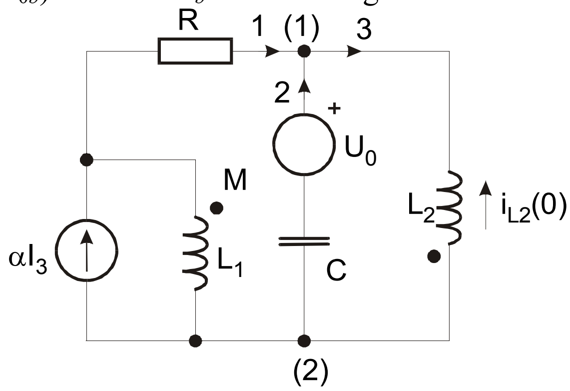
$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} sL & sM & 0 & 0 & 0 \\ sM & sL & 0 & 0 & 0 \\ 0 & 0 & \dfrac{1}{sC} & \mu R & 0 \\ 0 & 0 & 0 & R & 0 \\ 0 & 0 & 0 & \mu R & sL \end{bmatrix} \cdot \mathbf{S}^T =$$

$$=\begin{bmatrix} sL & sM & \dfrac{1}{sC} & \mu R & 0 \\ sM & sL & 0 & R & 0 \\ sM & sL & 0 & \mu R & sL \end{bmatrix}\cdot\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}=\begin{bmatrix} sL+\dfrac{1}{sC} & sM+\mu R & sM \\ sM & sL+R & sL \\ sM & sL+\mu R & 2sL \end{bmatrix}$$

$$\mathbf{U}_{0p}=-\mathbf{S}\cdot\mathbf{U}_{0b}=-\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} 0 \\ 0 \\ -\mu\cdot R\cdot I_0-\dfrac{u_C(0)}{s} \\ -I_0\cdot R \\ -\mu\cdot R\cdot I_0+L\cdot i_L(0) \end{bmatrix}=\begin{bmatrix} \mu\cdot R\cdot I_0+\dfrac{u_C(0)}{s} \\ I_0\cdot R \\ \mu\cdot R\cdot I_0-L\cdot i_L(0) \end{bmatrix}$$

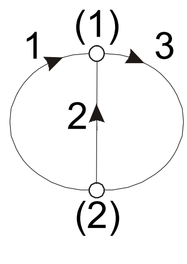
$\mathbf{I}_p=\begin{bmatrix} I_I \\ I_{II} \\ I_{III} \end{bmatrix}$, temeljni sustav jednadžbi petlji: $\mathbf{Z}_p\cdot\mathbf{I}_p=\mathbf{U}_{0p}$

7. Poštujući oznake čvorova i grana za mrežu prikazanu slikom odrediti temeljni sustav jednadžbi petlji u matričnoj formi (matrice $\mathbf{Z}_p$ i $\mathbf{U}_{0p}$ preko matrica impedancija grana $\mathbf{Z}_b$ i nezavisnih izvora grana $\mathbf{U}_{0b}$). Matrica $\mathbf{Z}_b$ mora biti regularna.



Rješenje:

$\mathbf{S}$ – temeljna spojna matrica: $\mathbf{S}=\begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$



temeljni sustav jednadžbi petlji: $\mathbf{Z}_p\cdot\mathbf{I}_p=\mathbf{U}_{0p}$

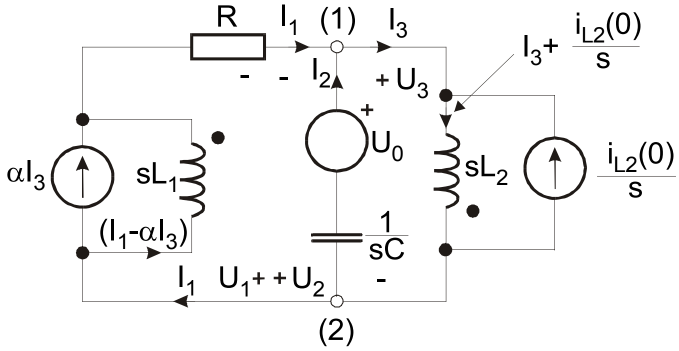
$\mathbf{Z}_p=\mathbf{S}\cdot\mathbf{Z}_b(s)\cdot\mathbf{S}^T$

$\mathbf{U}_{0p}=-\mathbf{S}\cdot\mathbf{U}_{0b}(s)$,

gdje su : $\mathbf{Z}_b(s)$ - matrica impedancija grana

$\mathbf{U}_{0b}(s)$ - vektor ekvivalentnih izvora grana

Primjena Laplaceove transformacije:



Naponsko – strujne relacije grana:

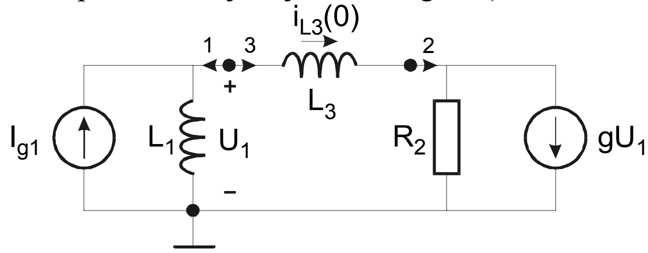
$$\underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \end{bmatrix}}_{\mathbf{U}_b} = \underbrace{\begin{bmatrix} (sL_1+R) & 0 & (-\alpha sL_1 + sM) \\ 0 & \dfrac{1}{sC} & 0 \\ sM & 0 & (sL_2 - \alpha sM) \end{bmatrix}}_{\mathbf{Z}_b} \cdot \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}}_{\mathbf{I}_b} + \underbrace{\begin{bmatrix} M \cdot i_{L2}(0) \\ -U_0 \\ L_2 \cdot i_{L2}(0) \end{bmatrix}}_{\mathbf{U}_{0b}}$$

temeljni sustav jednadžbi petlji: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$

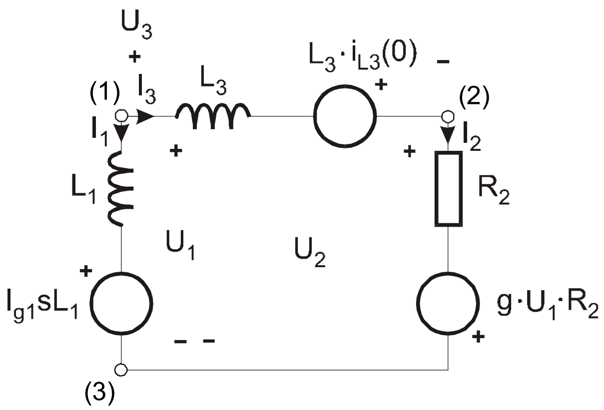
$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \begin{bmatrix} sL_1 + R + \dfrac{1}{sC} & -(1-\alpha)\cdot sL_1 - R - sM \\ -sL_1 - R - sM & (1-\alpha)\cdot sL_1 + R + (2-\alpha)\cdot sM + sL_2 \end{bmatrix}$$

$$\mathbf{U}_{0p} = -\mathbf{S} \cdot \mathbf{U}_{0b} = \begin{bmatrix} M \cdot i_{L2}(0) + U_0 \\ -M \cdot i_{L2}(0) - L_2 \cdot i_{L2}(0) \end{bmatrix}$$

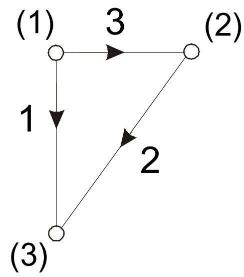
8. Za mrežu prikazanu slikom, poštujući oznake čvorova i grana, odrediti graf, matricu incidencija i napisati sustav jednadžbi čvorova u matričnom obliku preko matrice admitancija grana $\mathbf{Y}_b$ i vektora ekvivalentnih izvora grana $\mathbf{U}_{0b}$. Matrica $\mathbf{Y}_b$ mora biti regularna (<u>uputa</u>: $\mathbf{Y}_b$ treba napisati direktno iz naponsko-strujnih jednadžbi grana).



<u>Rješenje</u>:



$$\mathbf{A}=\begin{array}{c} \\ (1) \\ (2) \end{array}\!\!\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} \end{array}$$



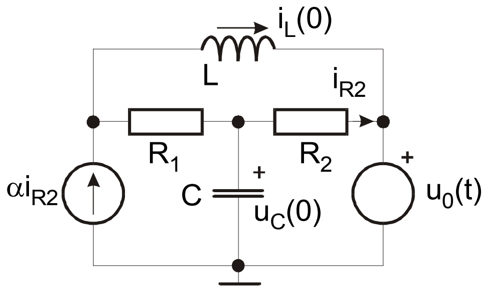
$$\mathbf{Y}_b=\mathbf{Z}_b^{-1}=\begin{bmatrix} \frac{1}{sL_1} & 0 & 0 \\ g & \frac{1}{R_2} & 0 \\ 0 & 0 & \frac{1}{sL_3} \end{bmatrix}$$

$$\mathbf{Y}_v=\mathbf{A}\cdot\mathbf{Y}_b\cdot\mathbf{A}^T=\begin{bmatrix} \frac{1}{sL_1}+\frac{1}{sL_3} & -\frac{1}{sL_3} \\ g-\frac{1}{sL_3} & \frac{1}{R_2}+\frac{1}{sL_3} \end{bmatrix}$$

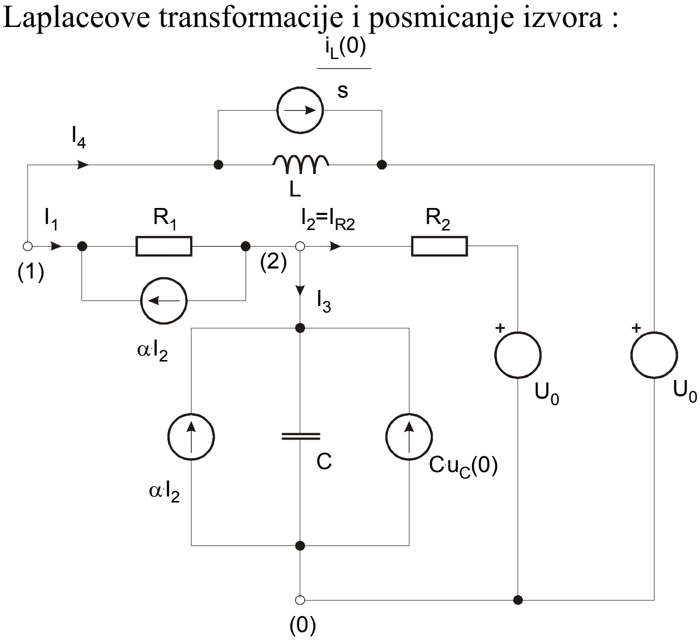
$$\mathbf{I}_v=\mathbf{A}\cdot\mathbf{Y}_b\cdot\mathbf{U}_{0b}=\begin{bmatrix} I_{g1}-\frac{i_{L3}(0)}{s} \\ g\cdot I_{g1}\cdot sL_1-g\cdot I_{g1}\cdot sL_1+\frac{i_{L3}(0)}{s} \end{bmatrix}=\begin{bmatrix} I_{g1}-\frac{i_{L3}(0)}{s} \\ \frac{i_{L3}(0)}{s} \end{bmatrix}$$

sustav jednadžbi čvorova: $\mathbf{Y}_v\cdot\mathbf{U}_v=\mathbf{I}_v$

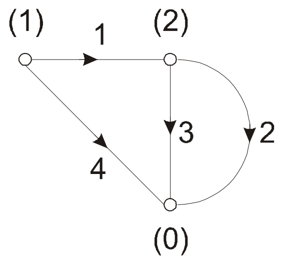
9. Za prikazanu mrežu nacrtati orijentirani graf, odrediti matricu incidencija i napisati sustav jednadžbi čvorova u matričnom obliku (matrice $\mathbf{Y}_v$ i $\mathbf{I}_v$ pomoću matrica $\mathbf{Y}_b$=$\mathbf{Z}_b^{-1}$ i $\mathbf{U}_{0b}$). Matrica $\mathbf{Y}_b$ mora biti regularna.



Rješenje: Primjena Laplaceove transformacije i posmicanje izvora :



graf:



matrica incidencija (reducirana)

$$\mathbf{A}=\begin{array}{c}\\(1)\\(2)\end{array}\begin{array}{c}(1\quad 2\quad 3\quad 4)\\ \begin{bmatrix}1 & 0 & 0 & 1\\ -1 & 1 & 1 & 0\end{bmatrix}\end{array}$$

$$\underbrace{\begin{bmatrix}U_1\\U_2\\U_3\\U_4\end{bmatrix}}_{\mathbf{U}_b}=\underbrace{\begin{bmatrix}R_1 & \alpha R_1 & 0 & 0\\ 0 & R_2 & 0 & 0\\ 0 & \dfrac{\alpha}{sC} & \dfrac{1}{sC} & 0\\ 0 & 0 & 0 & sL\end{bmatrix}}_{\mathbf{Z}_b}\cdot\underbrace{\begin{bmatrix}I_1\\I_2\\I_3\\I_4\end{bmatrix}}_{\mathbf{I}_b}+\underbrace{\begin{bmatrix}0\\U_0\\ \dfrac{u_C(0)}{s}\\ -L\cdot i_L(0)+U_0\end{bmatrix}}_{\mathbf{U}_{0b}}$$

Kako naći $\mathbf{Z}_b^{-1}(s)=\mathbf{Y}_b(s)$? Najjednostavniji način je napisati ponovo strujno-naponske relacije grana tako da su s lijeve strane izražene struje grana.

$$(2)\Rightarrow\quad I_2\cdot R_2=U_2-U_0\,/:R_2$$

$$I_2=\frac{1}{R_2}\cdot U_2-\frac{U_0}{R_2}$$

$$(4)\Rightarrow\quad U_4+L\cdot i_L(0)-U_0=I_4\cdot sL\,/:sL$$

$$I_4=\frac{1}{sL}\cdot U_4+\frac{i_L(0)}{s}-\frac{U_0}{sL}$$

$$(1)\Rightarrow\quad U_1=I_1\cdot R_1+\alpha R_1\cdot I_2=I_1\cdot R_1+\alpha R_1\left(\frac{1}{R_2}\cdot U_2-\frac{U_0}{R_2}\right)$$

$$I_1\cdot R_1=U_1-\alpha\frac{R_1}{R_2}\cdot U_2+\alpha\frac{R_1}{R_2}\cdot U_0\,\Big/:R_1$$

$$I_1=\frac{1}{R_1}\cdot U_1-\frac{\alpha}{R_2}\cdot U_2+\frac{\alpha}{R_2}\cdot U_0$$

$$(3)\Rightarrow\quad U_3=\frac{1}{sC}\cdot I_3+\frac{\alpha}{sC}\cdot I_2+\frac{u_C(0)}{s}\Big/\cdot sC$$

$$sC\cdot U_3=I_3+\alpha\cdot\left(\frac{U_2}{R_2}-\frac{U_0}{R_2}\right)+C\cdot u_C(0)$$

$$I_3=sC\cdot U_3-\frac{\alpha}{R_2}\cdot U_2+\frac{\alpha}{R_2}\cdot U_0-C\cdot u_C(0)$$

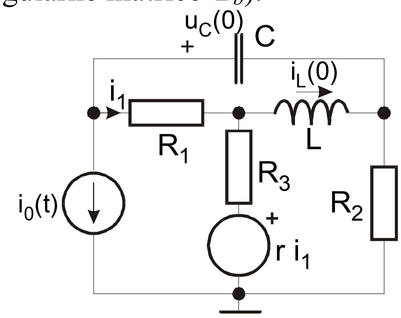
što se konačno može napisati i u matričnom obliku:

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b(s)} = \underbrace{\begin{bmatrix} \frac{1}{R_1} & -\frac{\alpha}{R_2} & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ 0 & -\frac{\alpha}{R_2} & sC & 0 \\ 0 & 0 & 0 & \frac{1}{sL} \end{bmatrix}}_{\mathbf{Y}_b(s)} \cdot \underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b(s)} + \underbrace{\begin{bmatrix} \frac{\alpha U_0}{R_2} \\ -\frac{U_0}{R_2} \\ \frac{\alpha U_0}{R_2} - C \cdot u_C(0) \\ \frac{i_L(0)}{s} - \frac{U_0}{sL} \end{bmatrix}}_{\mathbf{I}_{0b}(s)}$$

pa je sustav jednadžbi čvorova: $\mathbf{Y}_v \cdot \mathbf{U}_v = \mathbf{I}_v$,
gdje su:

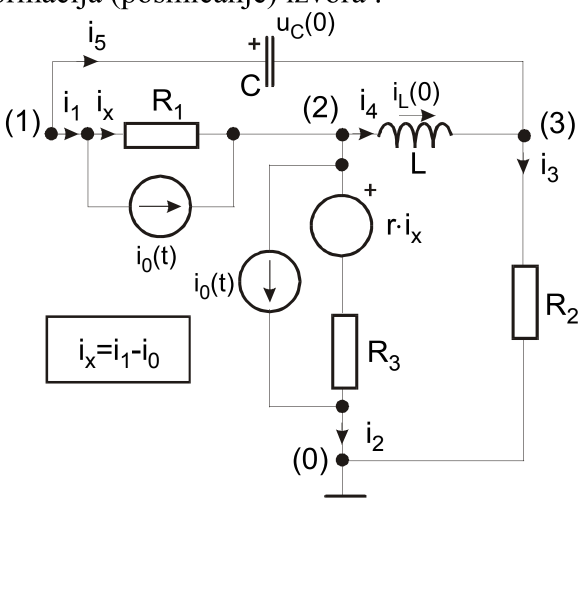
$$\mathbf{Y}_v = \mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{A}^T = \begin{bmatrix} \frac{1}{R_1} + \frac{1}{sL} & -\left(\frac{1}{R_1} + \frac{\alpha}{R_2}\right) \\ -\frac{1}{R_1} & \frac{1}{R_1} + \frac{1}{R_2} + sC \end{bmatrix} \quad \mathbf{I}_v = \mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{U}_{0b} = \begin{bmatrix} -\frac{\alpha U_0}{R_2} - \frac{i_L(0)}{s} + \frac{U_0}{sL} \\ \frac{U_0}{R_2} + C \cdot u_C(0) \end{bmatrix}$$

10. Zadana je mreža prema slici. Odrediti graf, matricu incidencija i sustav jednadžbi čvorova. (Odrediti matrice $\mathbf{Y}_v$ i $\mathbf{I}_v$ preko regularne matrice $\mathbf{Y}_b$).

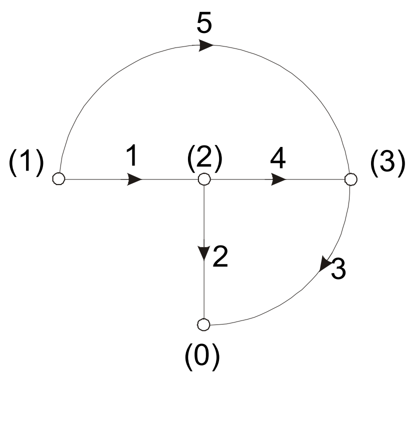


Rješenje:

Transformacija (posmicanje) izvora :



Orijentirani graf :



$$\mathbf{A}=\begin{bmatrix}1 & 0 & 0 & 0 & 1\\ -1 & 1 & 0 & 1 & 0\\ 0 & 0 & 1 & -1 & -1\end{bmatrix},\quad \underbrace{\begin{bmatrix}U_1\\U_2\\U_3\\U_4\\U_5\end{bmatrix}}_{\mathbf{U}_b}=\underbrace{\begin{bmatrix}R_1 & 0 & 0 & 0 & 0\\ r & R_3 & 0 & 0 & 0\\ 0 & 0 & R_2 & 0 & 0\\ 0 & 0 & 0 & sL & 0\\ 0 & 0 & 0 & 0 & \frac{1}{sC}\end{bmatrix}}_{\mathbf{Z}_b}\underbrace{\begin{bmatrix}I_1\\I_2\\I_3\\I_4\\I_5\end{bmatrix}}_{\mathbf{I}_b}+\underbrace{\begin{bmatrix}-I_0\cdot R_1\\ -I_0(R_3+r)\\ 0\\ -L\cdot i_L(0)\\ \frac{u_C(0)}{s}\end{bmatrix}}_{\mathbf{U}_{0b}}$$

označimo submatricu (2x2) unutar matrice $\mathbf{Z}_b$ sa

$$\mathbf{Z}'_b=\begin{bmatrix}R_1 & 0\\ r & R_3\end{bmatrix}$$

sada invertirajmo sumatricu $\mathbf{Z}'_b$

$$\mathbf{Z}'^{-1}_b=\begin{bmatrix}R_1 & 0\\ r & R_3\end{bmatrix}^{-1}=\frac{1}{R_1R_3}\begin{bmatrix}R_3 & -r\\ 0 & R_1\end{bmatrix}^T=\frac{1}{R_1R_3}\begin{bmatrix}R_3 & 0\\ -r & R_1\end{bmatrix}=\begin{bmatrix}\frac{1}{R_1} & 0\\ -\frac{r}{R_1R_3} & \frac{1}{R_3}\end{bmatrix},$$

i vratimo je na svoje mjesto unutar matrice $\mathbf{Z}_b^{-1}$. Nadalje, ako invertiramo ostale elemente na dijagonali, konačno dobivamo:

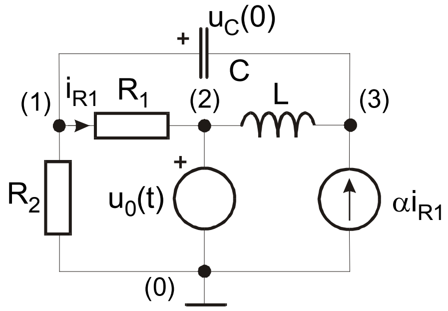
$$\mathbf{Z}_b^{-1}=\mathbf{Y}_b=\begin{bmatrix}\frac{1}{R_1} & 0 & 0 & 0 & 0\\ -\frac{r}{R_1R_3} & \frac{1}{R_3} & 0 & 0 & 0\\ 0 & 0 & \frac{1}{R_2} & 0 & 0\\ 0 & 0 & 0 & \frac{1}{sL} & 0\\ 0 & 0 & 0 & 0 & sC\end{bmatrix}$$

$$\mathbf{Y}_v=\mathbf{A}\cdot\mathbf{Z}_b^{-1}\cdot\mathbf{A}^T=\begin{bmatrix}\left(\frac{1}{R_1}+sC\right) & -\frac{1}{R_1} & -sC\\ \left(-\frac{1}{R_1}-\frac{r}{R_1R_3}\right) & \left(\frac{1}{R_1}+\frac{r}{R_1R_3}+\frac{1}{R_3}+\frac{1}{sL}\right) & -\frac{1}{sL}\\ -sC & -\frac{1}{sL} & \left(\frac{1}{R_2}+\frac{1}{sL}+sC\right)\end{bmatrix}$$

$$\mathbf{I}_v=\mathbf{A}\cdot\mathbf{Z}_b^{-1}\cdot\mathbf{U}_{0b}=\begin{bmatrix}-I_0+C\cdot u_C(0)\\ I_0R_1\left(\frac{1}{R_1}+\frac{r}{R_1R_3}\right)-I_0\frac{R_3+r}{R_3}-\frac{i_L(0)}{s}\\ \frac{i_L(0)}{s}-C\cdot u_C(0)\end{bmatrix}$$

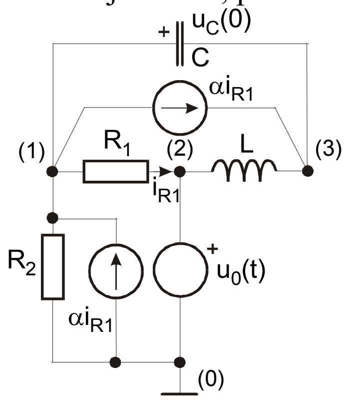
sustav jednadžbi čvorova: $\mathbf{Y}_v\cdot\mathbf{U}_v=\mathbf{I}_v$

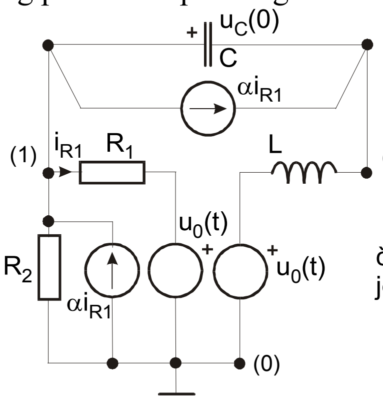
11. Za prikazanu mrežu nacrtati orijentirani graf, odrediti matricu incidencija i napisati sustav jednadžbi čvorova u matričnom obliku (matrice $\mathbf{Y}_v$ i $\mathbf{I}_v$ pomoću matrica $\mathbf{Y}_b$=$\mathbf{Z}_b^{-1}$ i $\mathbf{U}_{0b}$). Matrica $\mathbf{Z}_b$ mora biti regularna.



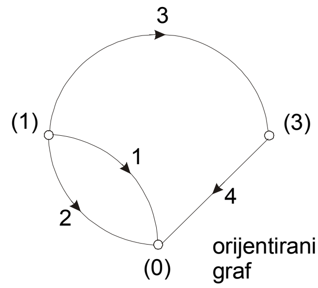
Rješenje:

Posmicanje izvora; prvo strujnog pa onda naponskog :





čvor (2) je nestao



matrica incidencija :

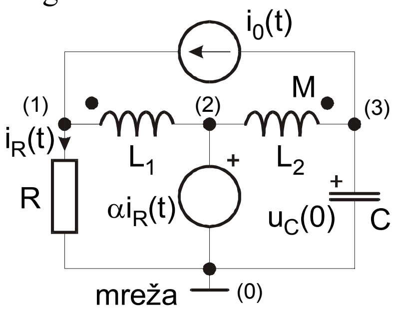
$$
\mathbf{A} = \begin{array}{c c} & \textit{grane} \\ & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} (1) \\ (3) \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} \\ \textit{čvorovi} & \end{array}
$$

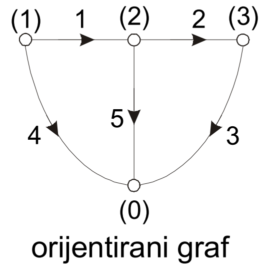
$$
\mathbf{Y}_v = \mathbf{A}\cdot\mathbf{Z}_b^{-1}\cdot\mathbf{A}^T = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}\cdot\begin{bmatrix} \frac{1}{R_1} & 0 & 0 & 0 \\ \frac{-\alpha}{R_1} & \frac{1}{R_2} & 0 & 0 \\ \frac{\alpha}{R_1} & 0 & sC & 0 \\ 0 & 0 & 0 & \frac{1}{sL} \end{bmatrix}\cdot\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} =
$$

$$
= \begin{bmatrix} \frac{1}{R_1} & \frac{1}{R_2} & sC & 0 \\ -\frac{\alpha}{R_1} & 0 & -sC & \frac{1}{sL} \end{bmatrix}\cdot\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+sC & -sC \\ -sC-\frac{\alpha}{R_1} & \frac{1}{sL}+sC \end{bmatrix}
$$

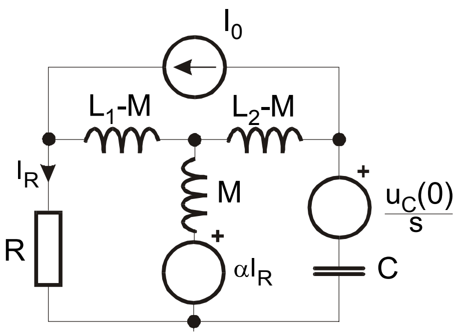
$$
\mathbf{I}_v = \mathbf{A}\cdot\mathbf{Z}_b^{-1}\cdot\mathbf{U}_{0b} = \begin{bmatrix} \frac{1}{R_1} & \frac{1}{R_2} & sC & 0 \\ -\frac{\alpha}{R_1} & 0 & -sC & \frac{1}{sL} \end{bmatrix}\cdot\begin{bmatrix} U_0(s) \\ 0 \\ \frac{u_C(0)}{s} \\ U_0(s) \end{bmatrix} = \begin{bmatrix} \frac{U_0}{R_1}+C\cdot u_C(0) \\ U_0\left(\frac{1}{sL}-\frac{\alpha}{R_1}\right)-C\cdot u_C(0) \end{bmatrix}
$$

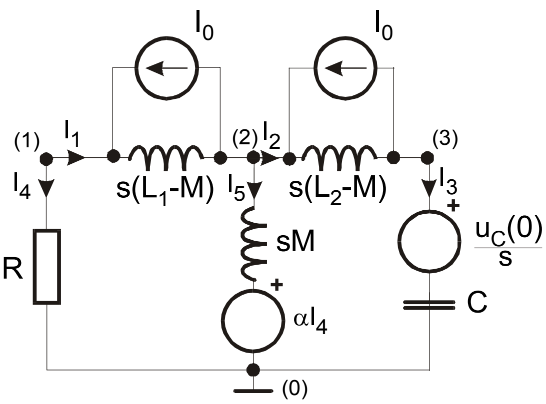
12. Zadana je mreža prema slici i njoj pripadni orijentirani graf. Odrediti matricu incidencija i sustav jednadžbi čvorova u matričnom obliku (matrice $\mathbf{Y}_v$ i $\mathbf{I}_v$ preko matrica $\mathbf{Y}_b = \mathbf{Z}_b^{-1}$ i $\mathbf{U}_{0b}$). Matrica $\mathbf{Z}_b$ mora biti regularna.





Rješenje:
transformacija mreže:





$$
\mathbf{A} = \begin{array}{c} \\ (1) \\ (2) \\ (3) \end{array}
\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix} \end{array}
$$

$$
\underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix}}_{\mathbf{U}_b} = \underbrace{\begin{bmatrix} s(L_1 - M) & 0 & 0 & 0 & 0 \\ 0 & s(L_2 - M) & 0 & 0 & 0 \\ 0 & 0 & \dfrac{1}{sC} & 0 & 0 \\ 0 & 0 & 0 & R & 0 \\ 0 & 0 & 0 & \alpha & sM \end{bmatrix}}_{\mathbf{Z}_b} \cdot \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix}}_{\mathbf{I}_b} + \underbrace{\begin{bmatrix} I_0 \cdot s(L_1 - M) \\ I_0 \cdot s(L_2 - M) \\ \dfrac{u_C(0)}{s} \\ 0 \\ 0 \end{bmatrix}}_{\mathbf{E}_{0b}}
$$

označimo submatricu (2x2) u donjem desnom uglu matrice $\mathbf{Z}_b$ sa

$$
\mathbf{Z}'_b = \begin{bmatrix} R & 0 \\ \alpha & sM \end{bmatrix}
$$

sada invertirajmo submatricu $\mathbf{Z}'_b$

$$
\mathbf{Z}'^{-1}_b = \frac{1}{\det \mathbf{Z}'_b} \cdot \widetilde{\mathbf{Z}}'_b ; \quad \det \mathbf{Z}'_b = \begin{vmatrix} R & 0 \\ \alpha & sM \end{vmatrix} = R \cdot sM ;
$$

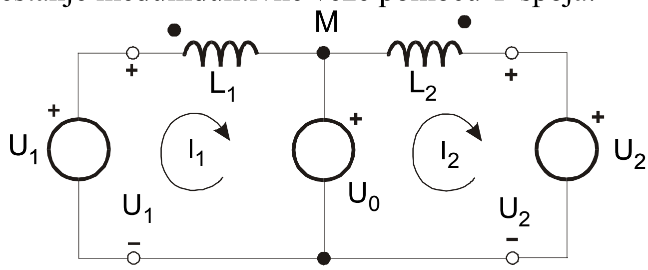
$$
\mathbf{Z}'^{-1}_b = \begin{bmatrix} R & 0 \\ \alpha & sM \end{bmatrix}^{-1} = \frac{1}{R \cdot sM} \begin{bmatrix} sM & -\alpha \\ 0 & R \end{bmatrix}^T = \frac{1}{R \cdot sM} \begin{bmatrix} sM & 0 \\ -\alpha & R \end{bmatrix} = \begin{bmatrix} \dfrac{1}{R} & 0 \\ -\dfrac{\alpha}{RsM} & \dfrac{1}{sM} \end{bmatrix}
$$

$$\mathbf{Y}_b = \mathbf{Z}_b^{-1} = \begin{bmatrix} \frac{1}{s(L_1-M)} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{s(L_2-M)} & 0 & 0 & 0 \\ 0 & 0 & sC & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{R} & 0 \\ 0 & 0 & 0 & -\frac{\alpha}{RsM} & \frac{1}{sM} \end{bmatrix}$$

$$\mathbf{Y}_v = \mathbf{A}\cdot\mathbf{Y}_b\cdot\mathbf{A}^T = \begin{bmatrix} \frac{1}{s(L_1-M)}+\frac{1}{R} & -\frac{1}{s(L_1-M)} & 0 \\ -\frac{1}{s(L_1-M)}-\frac{\alpha}{RsM} & \frac{1}{s(L_1-M)}+\frac{1}{s(L_2-M)}+\frac{1}{sM} & -\frac{1}{s(L_2-M)} \\ 0 & -\frac{1}{s(L_2-M)} & \frac{1}{s(L_2-M)}+sC \end{bmatrix}$$

$$\mathbf{I}_v = \mathbf{A}\cdot\mathbf{Y}_b\cdot\mathbf{U}_{0b} = \begin{bmatrix} I_0 \\ 0 \\ -I_0 + C\cdot u_C(0) \end{bmatrix}$$

DODATAK: Nadomještanje međuinduktivne veze pomoću T-spoja:



$$\left.\begin{aligned} I_1\cdot sL_1 - I_2\cdot sM &= -U_0 + U_1 \\ -I_1\cdot sM + I_2\cdot sL_2 &= U_0 - U_2 \end{aligned}\right. \Rightarrow$$

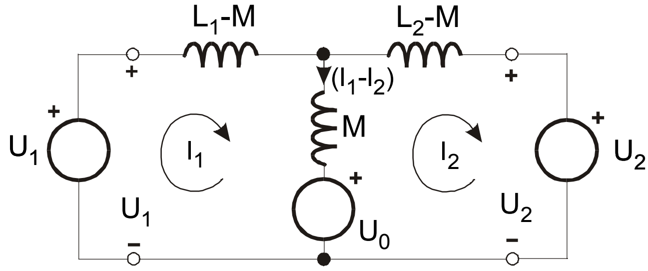
$$\begin{aligned} I_1\cdot sL_1 - I_1\cdot sM + I_1\cdot sM - I_2\cdot sM &= -U_0 + U_1 \\ -I_1\cdot sM + I_2\cdot sM - I_2\cdot sM + I_2\cdot sL_2 &= U_0 - U_2 \end{aligned}$$

---

treba grupirati sve što množi struje I_1 i I_2:

$$\begin{aligned} I_1\cdot s(L_1-M) + (I_1-I_2)\cdot sM &= -U_0 + U_1 \\ -(I_1-I_2)\cdot sM + I_2\cdot s(L_2-M) &= U_0 - U_2 \end{aligned}$$

Konačan izgled mreže:



13. Zadana je spojna matrica temeljnoga sustava petlji grafa nekog električnog kruga. Koliko grana i koliko čvorišta ima graf? Ako su grane grafa numerirane redoslijedom kojim ulaze u matricu, koje su grane stabla, a koje spone? Obrazložite odgovor.

$$\mathbf{S}=\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Rješenje:

N_b=7
N_v=4
Grane stabla: 1, 6 i 7
Spone: 2, 3, 4 i 5.

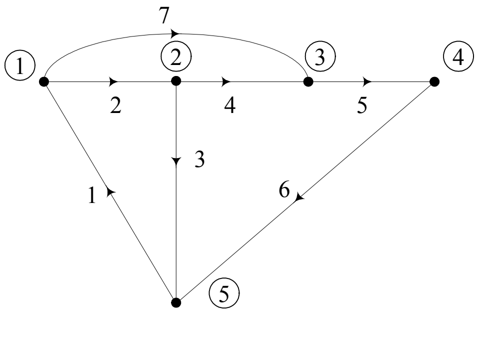
14. Zadana je rastavna matrica temeljnoga sustava petlji grafa nekog električnog kruga. Koliko grana i koliko čvorišta ima graf? Ako su grane grafa numerirane redoslijedom kojim ulaze u matricu, koje su grane stabla, a koje spone? Obrazložite odgovor.

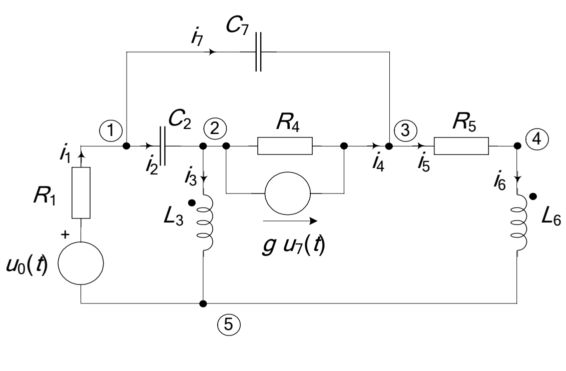
$$\mathbf{Q}=\begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Rješenje:
N_b=7
N_v=5
Grane stabla: 2, 3, 4 i 5
Spone: 1, 6 i 7.

15. Skup R_1, C_2, L_3, R_4, R_5, L_6 i C_7, je skup pasivnih elemenata mreže. Indeksi uz oznake elemenata označuju redni broj grane. Grane su redoslijedom numeracije incidentne sa sljedećim čvorištima: (5,1), (1,2), (2,5), (2,3), (3,4), (4,5), (1,3). U grani 1 otporu R_1 u seriju je priključen naponski izvor $u_0(t)$ s oznakom "+" prema čvoru 1, a u grani 4 paralelno otporu R_4, naponski ovisni strujni izvor $i_{06}(t)= gu_7(t)$ usmjeren prema čvoru 3, gdje je $u_7(t)$ napon grane 7. Induktiviteti L_3 i L_6 povezani su međuinduktivitetom M_{36} (točke na čvorištima 2 i 4). Nacrtajte graf mreže i mrežu, te odredite matricu incidencija i matricu impedancija grana.

Rješenje:





$$
\mathbf{A}_a = \begin{bmatrix} -1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & -1 & 0 \end{bmatrix}
$$

$$
\mathbf{Z}_b = \begin{bmatrix} R_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \dfrac{1}{sC_2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & sL_3 & 0 & 0 & sM_{36} & 0 \\ 0 & 0 & 0 & R_4 & 0 & 0 & \dfrac{-gR_4}{sC_7} \\ 0 & 0 & 0 & 0 & R_5 & 0 & 0 \\ 0 & 0 & sM_{36} & 0 & 0 & sL_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dfrac{1}{sC_7} \end{bmatrix}
$$

- THE END -