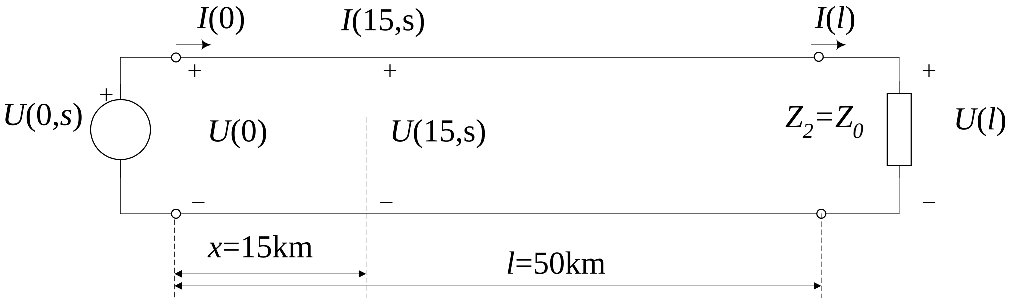
1. Zadana je linija duljine l=50 km. Primarni parametri linije su R=5,4 Ω/km, L=2mH/km, G=1μS/km i C=6nF/km. Odrediti iznos napona i struje na 15 km od početka linije ako je linija zaključena impedancijom $Z_2=Z_0$, a napon na ulazu linije je $u(0,t)$= 10 cos(5 10^3t).



Rješenje: Linija je prilagođena $\rightarrow$ $Z_2 = Z_0$ $\Rightarrow$ $Z_{ul1} = Z_0$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{5{,}4 + j10}{10^{-6}(1 + j30)}} = \sqrt{10^5(3{,}39 - j1{,}69)} = 10^2(5{,}99 - j1{,}41)$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = 10^{-3}\sqrt{(5{,}4 + j10)(1 + j30)} = 0{,}0048 + j0{,}0178$$

$$U(0) = U(x)\cdot ch\gamma x + I(x)Z_0 sh\gamma x$$

$$I(0) = \frac{U(x)}{Z_0} sh\gamma x + I(x) ch\gamma x$$

$$U(x) = U(0)\cdot ch\gamma x - I(0)Z_0 sh\gamma x$$

$$I(x) = -\frac{U(0)}{Z_0} sh\gamma x + I(0) ch\gamma x$$

$$U(0) = Z_0 I(0)$$

$$U(x) = U(0)\cdot(ch\gamma x - sh\gamma x) = U(0)\cdot e^{-\gamma x}$$

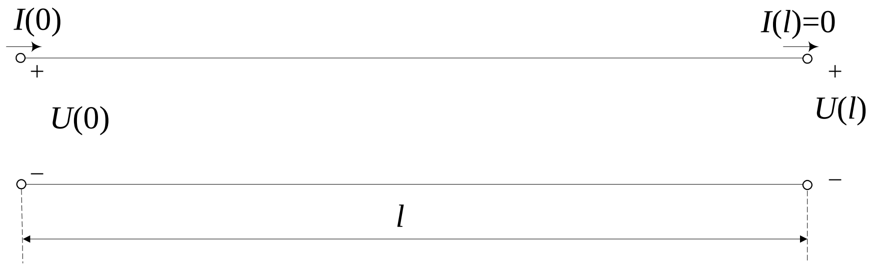
$$I(x) = \frac{U(0)}{Z_0}(-sh\gamma x + ch\gamma x) = \frac{U(0)}{Z_0} e^{-\gamma x}$$

$$U(15) = 10\cdot e^{-\gamma 15} = 8{,}071 - j2{,}458 = 9{,}3\cdot e^{-j15^\circ}[V]$$

$$I(15) = \frac{10}{Z_0} e^{-\gamma 15} = 0{,}015 - j0{,}00055 = 15{,}1\cdot e^{-j2^\circ}[mA]$$

$$u(15,t) = 9{,}3\cdot\cos(\omega t - 15^\circ)[V]$$
$$i(15,t) = 15{,}1\cdot\cos(\omega t - 2^\circ)[mA]$$

2. Zadana je linija bez gubitaka s L=4 mH/km i C=8 nF/km.
   a. Koliko najmanje mora biti duga ova linija da kod $\omega=10^6$ rad/s ulazna impedancija bude jednaka nuli, kad je suprotni kraj otvoren?
   b. Koliki su $u(0,t)$, $u(l,t)$ i $i(l,t)$ na toj liniji ako je $i(0,t) = 5\cos(10^6\, t)$?
   c. Odrediti brzinu širenja signala po liniji,
   d. Odrediti valnu duljinu signala.



Rješenje: Linija bez gubitaka $\rightarrow$ $R=0$, $G=0$ $\quad\Rightarrow\quad$ $Z_0=\sqrt{L/C}$, $\gamma = s\sqrt{LC}$

Stac. sinusna pobuda $\rightarrow$ $s=j\omega$ $\quad\Rightarrow\quad$ $\gamma = j\omega\sqrt{LC} = j\beta$

a) $I(l)=0$

$$U(0)=U(l)\cdot ch\gamma\, l + I(l)Z_0 sh\gamma\, l = U(l)\cdot ch\gamma\, l$$

$$I(0)=\frac{U(l)}{Z_0}sh\gamma\, l + I(l)ch\gamma\, l = \frac{U(l)}{Z_0}sh\gamma\, l$$

$$Z_{ul1}=\frac{U(0)}{I(0)}=Z_0\frac{\mathrm{ch}(\gamma\, l)}{\mathrm{sh}(\gamma\, l)}=0 \qquad \Rightarrow \qquad \mathrm{ch}(\gamma\cdot l)=0$$

$$\mathrm{ch}(\gamma\cdot l)=\mathrm{ch}(j\beta\cdot l)=\cos(\beta\cdot l)=\cos\left(\omega\sqrt{LC}\cdot l\right)=0 \;\Rightarrow\; \omega\sqrt{LC}l=\pi/2 \;\Rightarrow\; l=\frac{\pi}{2\omega\sqrt{LC}}=\frac{\pi}{8\sqrt{2}}\,\mathrm{km}$$

b)
$$u(0,t)=0$$
$$i(0,t)=5\cos\left(10^6 t\right)$$
$$i(l,t)=0$$

$$U(l)=U(0)\cdot ch(\gamma\, l)-I(0)Z_0 sh(\gamma\, l)=-I(0)Z_0\mathrm{sh}(j\beta l)=-I(0)Z_0\, j\sin(\pi/2)$$

$$Z_0=\sqrt{\frac{L}{C}}=\frac{10^3}{\sqrt{2}}$$

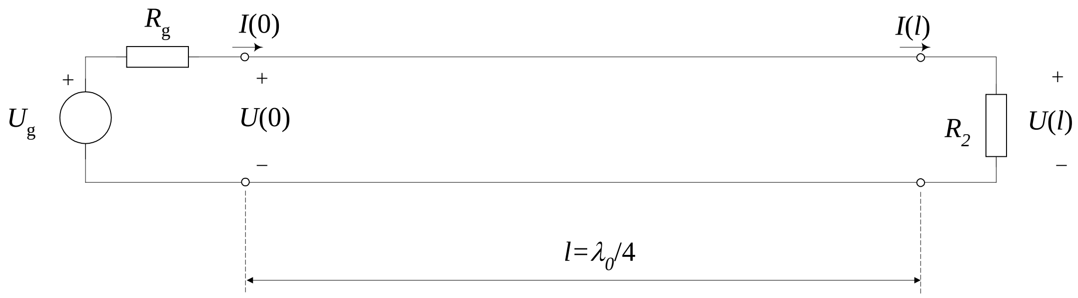
$$U(l)=-jI(0)Z_0=-j\frac{5\cdot 10^3}{\sqrt{2}}=\frac{5\cdot 10^3}{\sqrt{2}}e^{-j\pi/2}$$

$$u(l,t)=\frac{5\cdot 10^3}{\sqrt{2}}\cos\left(10^6 t-\frac{\pi}{2}\right)$$

c)
$$v=\frac{\omega}{\beta}=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{4\cdot 10^{-3}\cdot 8\cdot 10^{-9}}}=\frac{10^6}{\sqrt{32}}=176.776{,}69\,\mathrm{km/s}$$

d)
$$\lambda=\frac{2\pi}{\beta}=\frac{2\pi}{\omega\sqrt{LC}}=\frac{2\pi}{10^6\cdot\sqrt{4\cdot 10^{-3}\cdot 8\cdot 10^{-9}}}=\frac{2\pi}{\sqrt{32}}=1{,}1107\,\mathrm{km}$$

3. Zadana je linija bez gubitaka s L=0,8 mH/km, C=80 nF/km i $l=\lambda_0/4$ kod ω_0. Na ulaz linije priključen je generator napona $u_g(t)$ s unutarnjim otporom R_g, a na kraju linije je otpor R_2=1kΩ. Na frekvenciji $\omega_0=10^5$ rad/s ulazna impedancija je prilagođena na Rg.
   a. Koliki je Rg?
   b. Koliko je duga linija?
   c. Odrediti $u(l,t)$ i $i(l,t)$ na toj liniji ako je $u_g(t)$ = 4 cos(ω_0t).



Rješenje:

Linija bez gubitaka $\rightarrow$ $R=0$, $G=0$ $\quad\Rightarrow\quad$ $Z_0=\sqrt{L/C}$

$\gamma = s\sqrt{LC}$ Stac. sinusna pobuda $\rightarrow$ $s=j\omega$ $\quad\Rightarrow\quad$ $\gamma = j\omega\sqrt{LC}=j\beta$

a) $U(l)=R_2 I(l)$

$$U(0)=U(l)\cdot ch(\gamma l)+I(l)Z_0 sh(\gamma l)=I(l)\cdot\left(R_2 ch(\gamma l)+Z_0 sh(\gamma l)\right)$$

$$I(0)=\frac{U(l)}{Z_0}sh(\gamma l)+I(l)ch(\gamma l)=I(l)\left(\frac{R_2}{Z_0}sh(\gamma l)+ch(\gamma l)\right)$$

$$Z_{ul1}=\frac{R_2 ch(\gamma l)+Z_0 sh(\gamma l)}{\frac{R_2}{Z_0}sh(\gamma l)+ch(\gamma l)}$$

$$l=\frac{\lambda_0}{4}=\frac{2\pi}{\beta}\cdot\frac{1}{4}=\frac{\pi}{2\beta}$$

$$\gamma\cdot l=j\beta\cdot l=j\frac{\pi}{2}$$

$$sh(\gamma l)=sh\left(j\frac{\pi}{2}\right)=j\sin\left(\frac{\pi}{2}\right)=j$$

$$ch(\gamma l)=ch\left(j\frac{\pi}{2}\right)=j\cos\left(\frac{\pi}{2}\right)=0$$

$$R_g=Z_{ul1}=\frac{R_2\cos(\pi/2)+Z_0 j\sin(\pi/2)}{\frac{R_2}{Z_0}j\sin(\pi/2)+\cos(\pi/2)}=\frac{Z_0^2}{R_2}=\frac{L}{CR_2}=\frac{8\cdot10^{-4}}{8\cdot10^{-8}\cdot10^3}=10\,\Omega$$

b) $$l=\frac{\lambda_0}{4}=\frac{2\pi}{\beta}\cdot\frac{1}{4}=\frac{\pi}{2\omega_0\sqrt{LC}}=\frac{\pi}{2\cdot10^5\sqrt{8\cdot10^{-4}\cdot8\cdot10^{-8}}}=\frac{\pi}{1,6}=1,9634\text{ km}$$

c)

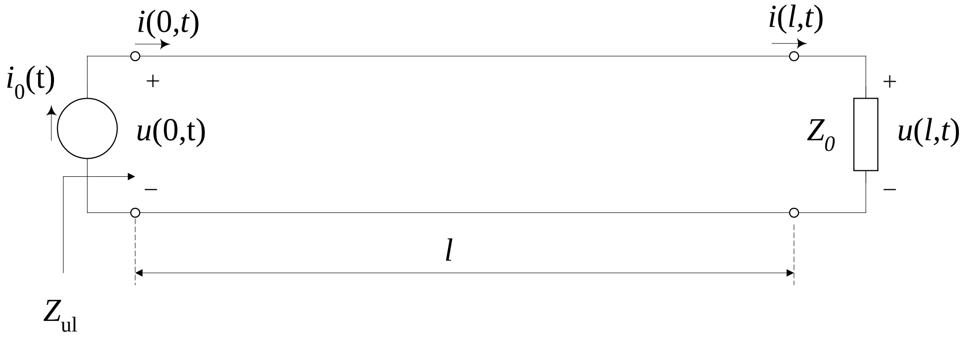
$$U(l)=U(0)\cdot ch(\gamma l)-I(0)Z_0 sh(\gamma l)=-jI(0)\cdot Z_0=-j\frac{U_g}{R_g+Z_{ul1}}\cdot Z_0=-j\frac{U_g}{2R_g}\cdot Z_0=-j20=20e^{-j\frac{\pi}{2}}$$

$$I(l)=-\frac{U(0)}{Z_0}\cdot sh(\gamma l)-I(0)ch(\gamma l)=-j\frac{U(0)}{Z_0}=-j\frac{U_g}{Z_0}\cdot\frac{Z_{ul1}}{R_g+Z_{ul1}}=-j\frac{U_g}{2Z_0}=-j0,02=0,02e^{-j\frac{\pi}{2}}$$

$$u(l,t)=20\cos\left(\omega t-90^\circ\right)\text{V}$$

$$i(l,t)=0,02\cos\left(\omega t-90^\circ\right)\text{A}$$

4. Zadana je linija duljine l=100 km. Primarni parametri linije su R=1 Ω/km, L=3mH/km, G=3μS/km i C=9nF/km. Odrediti izraz za napon na izlazu linije ako je linija zaključena svojom karakterističnom impedancijom Z_0, a na ulazu linije je strujni izvor $i(t)=\delta(t)$.



Rješenje:

Linija bez distorzije: $\rightarrow\ R/L = G/C \ \Rightarrow \quad Z_0 = \sqrt{L/C} = 578\,\Omega,$

$$\gamma = \sqrt{RG} + s\sqrt{LC} = \sqrt{3}\cdot 10^{-3} + s3\sqrt{3}\cdot 10^{-6}$$

Linija je prilagođena $\rightarrow\ Z_2 = Z_0$

$$U(l,s) = U(0,s)\cdot ch(\gamma\, l) - I(0,s) Z_0 sh(\gamma\, l) = I(0,s)\cdot Z_0\left(ch(\gamma\, l) - sh(\gamma\, l)\right) = I(0,s)\cdot Z_0 e^{-\gamma\, l}$$

$$U(l,s) = I(0,s)\cdot Z_0 e^{-\gamma\, l} = \frac{10^3}{\sqrt{3}} e^{-\left(0{,}1\sqrt{3} + s3\sqrt{3}\cdot 10^{-4}\right)} = \frac{10^3}{\sqrt{3}} e^{-0{,}1\sqrt{3}} \cdot e^{-s3\sqrt{3}\cdot 10^{-4}}$$

$$u(l,t) = \frac{10^3}{\sqrt{3}} e^{-0{,}1\sqrt{3}} \delta\left(t - 3\sqrt{3}\cdot 10^{-4}\right) = 485\cdot\delta\left(t - 3\sqrt{3}\cdot 10^{-4}\right)$$