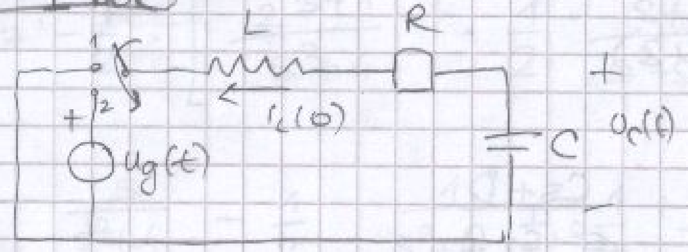
PISMENI 1 ROK

(1)



$L = R = 1$

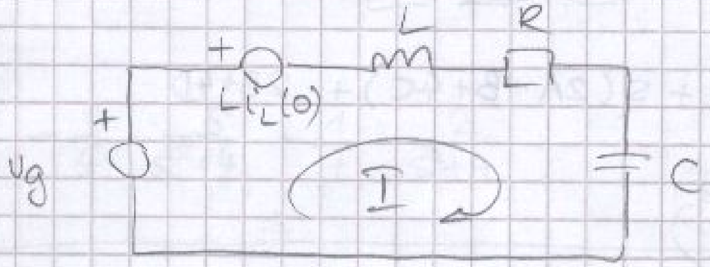
$C = \frac{1}{2}$

$i_L(0) = \frac{1}{2}$

$u_g(t) = \sin 2t$

$u_g(s) = \frac{2}{s^2+4}$

u $t = 0$:



$$u_g(s) - L i_L(0) = I \cdot \left(sL + R + \frac{1}{sC}\right)$$

$$u_c(s) = I \cdot \frac{1}{sC}$$

$$I(s) = \frac{u_g(s) - L i_L(0)}{sL + R + \frac{1}{sC}} = \frac{\frac{2}{s^2+4} - \frac{1}{2}}{s + 1 + \frac{2}{s}}$$

$$I(s) = \frac{\frac{4 - s^2 - 4}{2(s^2+4)}}{\frac{s^2+s+2}{s}} = -\frac{s^3}{2(s^2+4)(s^2+s+2)}$$

$$u_c(s) = I(s) \cdot \frac{1}{sC} = -\frac{\overset{s^2}{\cancel{s^3}}}{2(s^2+4)(s^2+s+2)} \cdot \frac{1}{s} \to \frac{2}{s}$$

$$u_c(s) = -\frac{s^2}{\cancel{2}(s^2+4)(s^2+s+2)}$$

$$U_c(s) = -\frac{1}{2} \cdot \frac{s^2}{(s^2+4)(s^2+s+2)}$$

$$\frac{s^2}{(s^2+4)(s^2+s+2)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+s+2}$$

$$s^2 = (As+B)(s^2+s+2) + (Cs+D)(s^2+4)$$

$$s^2 = As^3 + \boxed{As^2} + \underline{\underline{2As}} + \boxed{Bs^2} + \underline{\underline{Bs}} + 2B + Cs^3 + \underline{\underline{4Cs}} + \boxed{Ds^2} + 4D$$

$$s^2 = s^3(A+C) + s^2(A+B+D) + + s(2A+B+4C) + 2B+4D$$

$$\begin{cases} A+C=0 & \Rightarrow A=-C \\ A+B+D=1 & \\ 2A+B+4C=0 & \\ \underline{2B+4D=0} & \Rightarrow B=-2D \Rightarrow D=-\frac{1}{2}B \end{cases}$$

$$B+2C=0 \Rightarrow B=-2C=2A$$

$$A+2A-\frac{1}{2}\cdot 2A = 1$$

$$2A=1 \qquad A=\frac{1}{2}$$

$$C=-\frac{1}{2}$$

$$B=1$$

$$D=-\frac{1}{2}$$

provjera:

$$\frac{\frac{1}{2}s+1}{s^2+4} + \frac{-\frac{1}{2}s-\frac{1}{2}}{s^2+s+2} =$$

$$= \frac{(s^2+s+2)(\frac{1}{2}s+1) - (s^2+4)(\frac{1}{2}s+\frac{1}{2})}{(s^2+4)(s^2+s+2)} =$$

$$= \frac{\cancel{\frac{1}{2}s^3} + s^2 + \cancel{\frac{1}{2}s^2} + \cancel{s} + \cancel{s} + \cancel{2} - \cancel{\frac{1}{2}s^3} - \cancel{\frac{1}{2}s^2} - \cancel{2s} - \cancel{2}}{(\ )\ (\ )} = \frac{s^2}{(\ )(\ )}$$ ✓

$$U_c(s) = -\frac{1}{2} \cdot \left[\frac{\frac{1}{2}s+1}{s^2+4} - \frac{1}{2} \frac{s+1}{s^2+s+2} \right]$$

$$= -\frac{1}{4} \frac{s}{s^2+4} - \frac{1}{2} \cdot \frac{1}{s^2+4} + \frac{1}{4} \frac{s+1}{\left(s+\frac{1}{2}\right)^2 + \frac{7}{4}}$$

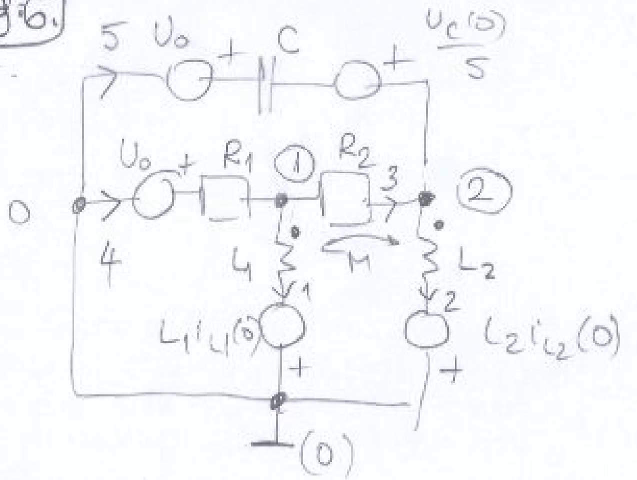
$$= -\frac{1}{4} \cdot \frac{s}{s^2+4} - \frac{1}{4} \cdot \frac{2}{s^2+4} + \frac{1}{4} \cdot \frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \frac{7}{4}} + \frac{1}{4} \cdot \frac{\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \frac{7}{4}}$$

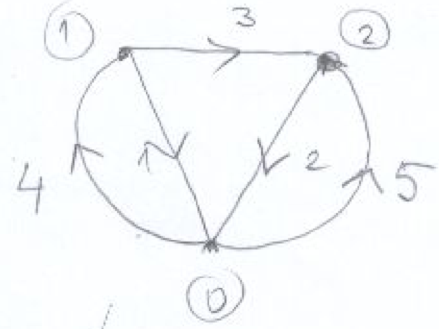
$$= -\frac{1}{4} \cdot \frac{s}{s^2+4} - \frac{1}{4} \cdot \frac{2}{s^2+4} + \frac{1}{4} \cdot \frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2} + \frac{1}{8} \cdot \frac{2}{\sqrt{7}} \cdot \frac{\frac{\sqrt{7}}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2}$$

$$\boxed{u_c(t) = -\frac{1}{4} \cdot \cos 2t - \frac{1}{4} \sin 2t + \frac{1}{4} \cdot e^{-\frac{1}{2}t} \cdot \cos \frac{\sqrt{7}}{2} t + \frac{1}{4\sqrt{7}} \cdot e^{-\frac{1}{2}t} \sin \frac{\sqrt{7}}{2} t}$$

② 19.6.

1,5?





$$u_1 = u_{10} = sL_1 I_1 + sM \cdot I_2 - L_1 i_{L1}(0)$$

$$u_2 = u_{20} = sL_2 I_2 + sM I_1 - L_2 i_{L2}(0)$$

$$u_3 = u_{12} = I_3 R_2$$

$$\cancel{u_4 = u_{01} = I_4 \frac{1}{sC} - U_0 - \frac{u_C(0)}{s}}$$

$$u_4 = u_{01} = I_4 R_1 - U_0$$

$$u_5 = u_{02} = I_5 \cdot \frac{1}{sC} - U_0 - \frac{u_C(0)}{s}$$

grane

$$V_r = \begin{array}{cc} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \text{čvorovi} \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & -1 \end{bmatrix} \end{array}$$

$$U_b = Z_b I_b + E_b$$

$$Z_b = \begin{bmatrix} sL_1 & sM & 0 & 0 & 0 \\ sM & sL_2 & 0 & 0 & 0 \\ 0 & 0 & R_2 & 0 & 0 \\ 0 & 0 & 0 & R_1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{sC} \end{bmatrix} \qquad E_b = \begin{bmatrix} -L_1 i_{L1}(0) \\ -L_2 i_{L2}(0) \\ 0 \\ -U_0 \\ -U_0 - \frac{u_C(0)}{s} \end{bmatrix}$$

$$Z_b = \begin{bmatrix} s & s & 0 & 0 & 0 \\ s & 2s & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{s} \end{bmatrix}$$

$$\begin{bmatrix} s & s \\ s & 2s \end{bmatrix} = s \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right) \sim \left[\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & 2 & -1 \\ 0 & 1 & -1 & 1 \end{array}\right]$$

$$Y_b = Z_b^{-1} = \begin{bmatrix} \frac{2}{s} & -\frac{1}{s} & 0 & 0 & 0 \\ -\frac{1}{s} & \frac{1}{s} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & s \end{bmatrix}$$

② 2.str TMIL ROK 19.06.2006

$$Y_n = V_r \cdot Z_b^{-1} \cdot V_r^T$$

$$\begin{bmatrix} 1 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} \frac{2}{s} & -\frac{1}{s} & 0 & 0 & 0 \\ -\frac{1}{s} & \frac{1}{s} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & s \end{bmatrix} = \begin{bmatrix} \frac{2}{s} & -\frac{1}{s} & 1 & -1 & 0 \\ -\frac{1}{s} & \frac{1}{s} & -1 & 0 & -s \end{bmatrix}$$

$$Y_n = \begin{bmatrix} \frac{2}{s} & -\frac{1}{s} & 1 & -1 & 0 \\ -\frac{1}{s} & \frac{1}{s} & -1 & 0 & -s \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & -1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} \frac{2}{s}+2 & -1/s-1 \\ -\frac{1}{s}-1 & \frac{1}{s}+1+s \end{bmatrix}$$

$$I_n = V_r \cdot Z_b^{-1} \cdot E_b = \begin{bmatrix} \frac{2}{s} & -1/s & 1 & -1 & 0 \\ -\frac{1}{s} & \frac{1}{s} & -1 & 0 & -s \end{bmatrix} \cdot \begin{bmatrix} -L_1 i_{L_1}(0) \\ -L_2 i_{L_2}(0) \\ 0 \\ -U_0 \\ -U_0 - \frac{u_C(0)}{s} \end{bmatrix}$$

$$I_n = \begin{bmatrix} -\frac{2}{s} L_1 i_{L_1}(0) + U_0 + \frac{1}{s} L_2 i_{L_2}(0) \\ \frac{1}{s}\left[L_1 i_{L_1}(0) - L_2 i_{L_2}(0)\right] + s U_0 + u_C(0) \end{bmatrix}$$

(3) 196

$$Z_{RC}(s) = \frac{s^2+7s+10}{s^2+4s+3}$$

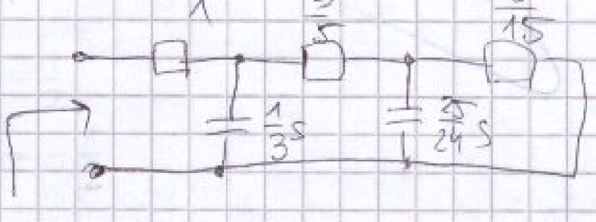
$$(s^2+7s+10) : (s^2+4s+3) = 1 = R$$
$$-s^2-4s-3$$
$$3s+7$$

$$(s^2+4s+3) : (3s+7) = \frac{1}{3}s = sC$$
$$-s^2-\frac{7}{3}s$$
$$\frac{5}{3}s+3$$

$$(3s+7) : \left(\frac{5}{3}s+3\right) = \frac{9}{5} = R$$
$$-3s-\frac{27}{5}$$
$$\frac{8}{5}$$

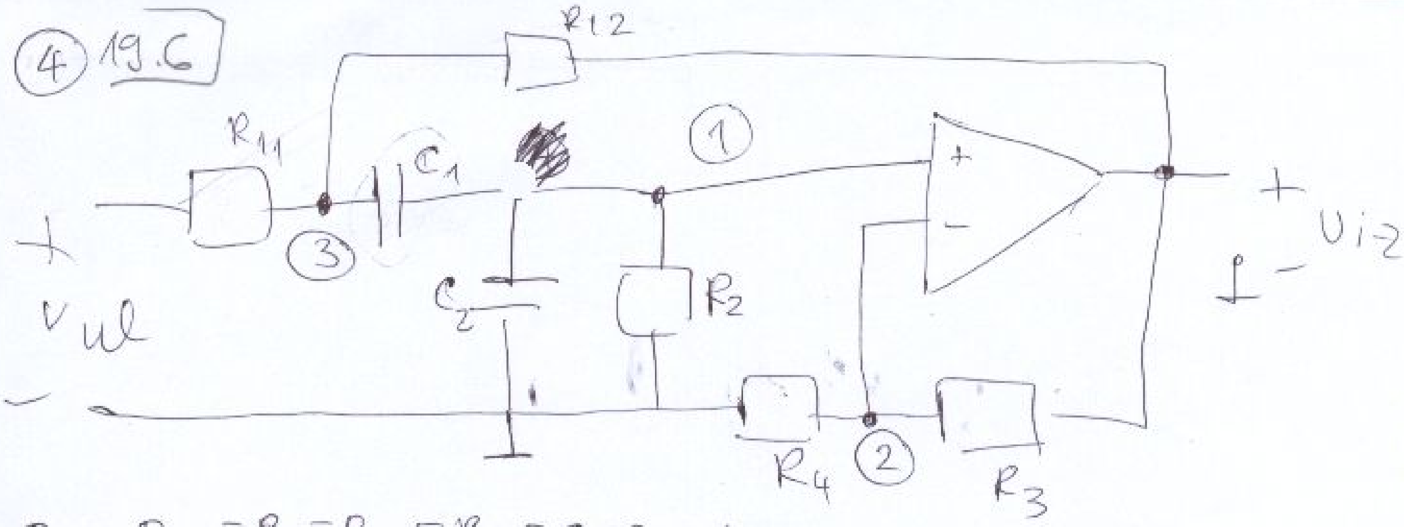
$$\left(\frac{5}{3}s+3\right) : \frac{8}{5} = \frac{25}{24}s = sC$$
$$-\frac{5}{3}s$$
$$3$$

$$\frac{8}{5} : 3 = \frac{8}{15} = R$$



$Z_{RC}(s)$

④ 19.6



$R_{11}=R_{12}=R_2=R_3=R_4=C_1=C_2=1$

1a $\quad U_1=U_2$

(1) $(U_1-U_3)\cdot sC_1+U_1\cdot sC_2+\frac{U_1}{R_2}=0$

(2) $\frac{U_2}{R_4}+\frac{U_2-U_{iz}}{R_3}=0 \Rightarrow \boxed{U_2=\frac{R_4}{R_3+R_4}\cdot U_{iz}=\frac{1}{2}U_{iz}}$

(3) $\frac{U_3-U_{UL}}{R_{11}}+\frac{U_3-U_{iz}}{R_{12}}+(U_3-U_1)\cdot sC_1+\cancel{U_3(sC_1+sC_2)}=0$

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iz (1) $U_3\cdot sC_1=U_1\cdot\left(sC_1+sC_2+\frac{1}{R_2}\right)$

$$\boxed{U_3=U_1\cdot\frac{2s+1}{s}}$$

u (3) $U_1\cdot\frac{2s+1}{s}-U_{UL}+U_1\cdot\frac{2s+1}{s}-U_{iz}+U_1\left(\frac{2s+1}{s}-1\right)\cdot s=0$

$\cancel{2}\cdot\frac{1}{\cancel{2}}U_{iz}\cdot\frac{2s+1}{s}-U_{UL}-U_{iz}+\frac{s+1}{\cancel{s}}\cdot\frac{1}{2}U_{iz}\cdot\cancel{s}=0$

$$U_{iz}\left(\frac{2s+1}{s}-1+\frac{s+1}{2}\right)=U_{UL}$$

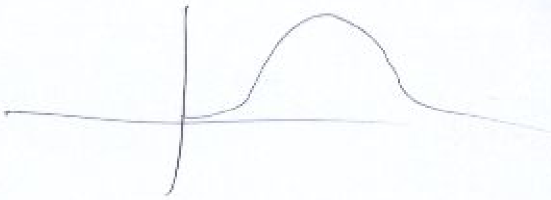
$$U_{iz}\,\frac{4s+2-2s+s^2+s}{2s}=U_{UL}$$

$$U_{iz}\cdot\frac{s^2+3s+2}{2s}=U_{UL}$$

$$T(s)=\frac{U_{iz}}{U_{ul}}=\frac{2s}{s^2+3s+2}=\frac{2s}{(s+1)(s+2)}$$

4) 126 NULE $s=0$ $s=\infty$

POLOVI $s=-1$ $s=-2$



$s=j\omega$

$$T(j\omega) = \frac{2j\omega}{2-\omega^2+3j\omega}$$

$$|T(j\omega)| = \frac{2\omega}{\sqrt{(2-\omega^2)^2+9\omega^2}} = \frac{2\omega}{\sqrt{\omega^4+5\omega^2+4}}$$