2. KOL 2006. TMIK

(1) 1. Foster

$$Z_{RC}(s) = \frac{s^2+7s+10}{s^2+4s+3} = k_\infty + \frac{k_0}{s} + \frac{k_1}{s+1} + \frac{k_3}{s+3}$$

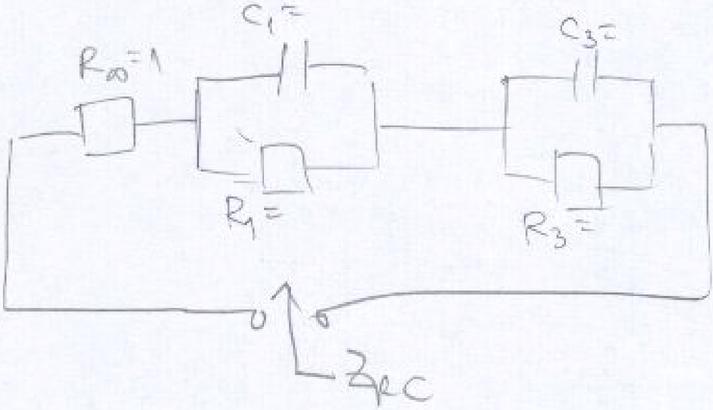
$$= R_\infty + \frac{1}{sC_0} + \frac{1}{C_1 s + \frac{1}{R_1}} + \frac{1}{C_3 s + \frac{1}{R_3}}$$

$$k_\infty = \lim_{s\to\infty} Z_{RC}(s) = 1 \qquad \Rightarrow R_\infty = 1$$

$$k_0 = \lim_{s\to 0} s Z_{RC}(s) = 0 \qquad \Rightarrow C_0 = \infty$$

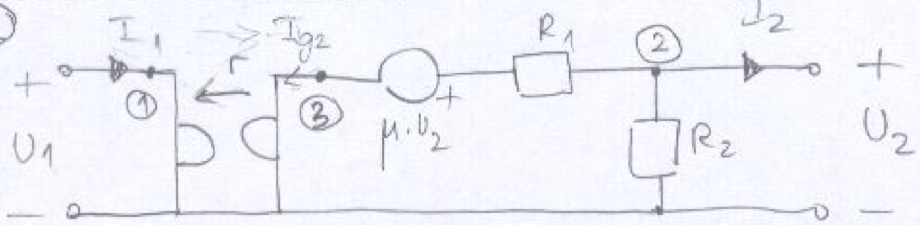
$$k_1 = \lim_{s\to -1} \frac{s^2+7s+10}{s+3} = \frac{4}{2} = 2 \qquad \Rightarrow C_1 = \frac{1}{k_1} = \frac{1}{2} \qquad R_1 = \frac{k_1}{C_1} = 2$$

$$k_3 = \lim_{s\to -3} \frac{s^2+7s+10}{s+1} = \frac{-2}{-2} = 1 \qquad \Rightarrow C_3 = \frac{1}{k_3} = 1 \qquad R_3 = \frac{k_3}{C_3} = \frac{1}{3}$$



2. KOL 2006 TMIC

②



$R_1 = 2$

$R_2 = \frac{1}{2}$

$$\begin{cases} U_1 = -r I_{g2} \\ U_3 = r I_1 \\ I_{g2} + \dfrac{U_3 + \mu U_2 - U_2}{R_1} = 0 \quad \Rightarrow \quad -I_{g2} = \dfrac{U_3 + (\mu - 1) U_2}{R_1} \\ \dfrac{U_2 - \mu U_2 - U_3}{R_1} + \dfrac{U_2}{R_2} + I_2 = 0 \end{cases}$$

$$\begin{cases} U_1 = r \cdot \dfrac{U_3 + (\mu-1) U_2}{R_1} = r \cdot \dfrac{r I_1 + (\mu - 1) \cdot U_2}{R_1} \\ I_2 + \dfrac{U_2}{R_2} + \dfrac{(1-\mu) U_2}{R_1} - \dfrac{r I_1}{R_1} = 0 \end{cases}$$

Y - parametri:

$$\begin{cases} I_1 = Y_{11} U_1 - Y_{12} U_2 \\ I_2 = Y_{21} U_1 - Y_{22} U_2 \end{cases}$$

$$Y_{11} = \frac{I_1}{U_1}\Big|_{U_2 = 0} \qquad U_1 = \frac{r^2 I_1}{R_1} \quad \Rightarrow \quad \boxed{Y_{11} = \frac{R_1}{r^2}}$$

$$Y_{21} = \frac{I_2}{U_1}\Big|_{U_2 = 0} \qquad U_1 = \frac{r^2 I_1}{R_1} \quad \Rightarrow \quad I_1 = \frac{R_1}{r^2} \cdot U_1$$

$$I_2 = r \frac{I_1}{R_1} = \frac{r}{\cancel{R_1}} \cdot \frac{\cancel{R_1}}{r^2} \cdot U_1 = \frac{U_1}{r}$$

$$\boxed{\frac{I_2}{U_1} = \frac{1}{r} = Y_{21}}$$

$$Y_{12} = -\frac{I_1}{U_2}\Big|_{U_1 = 0} \quad \Rightarrow \quad r I_1 = (1-\mu) U_2 \quad \Rightarrow \quad \frac{I_1}{U_2} = \frac{1-\mu}{r}$$

$$\boxed{Y_{12} = \frac{\mu - 1}{r}}$$

$$Y_{22} = -\frac{I_2}{U_2}\Big|_{U_1 = 0} \qquad r I_1 = (1-\mu) U_2 \quad \Rightarrow \quad I_1 = \frac{1-\mu}{r} U_2$$

$$I_2 + \left(\frac{1}{R_2} + \cancel{\frac{1-\mu}{R_1}}\right) U_2 - \cancel{\frac{r}{R_1} \cdot \frac{1-\mu}{r} \cdot U_2} = 0 \qquad I_2 = \frac{U_2}{R_2}$$

$$Y_{22} = -\frac{1}{R_2}$$

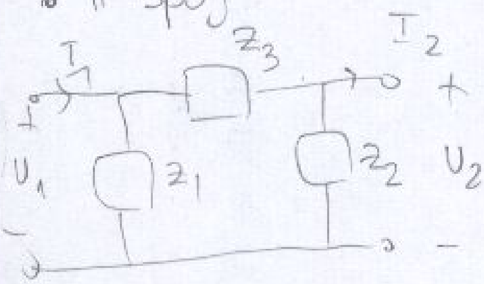
② 2. KOL 2006. 2-str

$$Y = \begin{bmatrix} \frac{R_1}{r^2} & -\frac{\mu-1}{r} \\ \frac{1}{r} & \frac{1}{R_2} \end{bmatrix} = \begin{bmatrix} \frac{2}{r^2} & \frac{1-\mu}{r} \\ \frac{1}{r} & 2 \end{bmatrix}$$

uvjet simetričnosti: $\frac{2}{r^2} = 2 \Rightarrow r = \pm 1$

uvjet recipročnosti: $\frac{1}{r} = -\frac{1-\mu}{r} \Rightarrow \boxed{\mu = 2}$

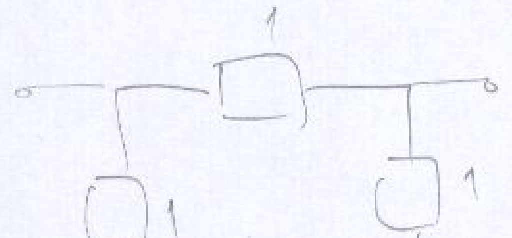
- Π spoj



$$Y = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \Rightarrow z = \begin{bmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

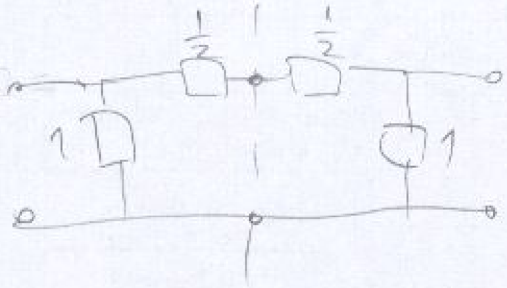
$$(z) = \begin{bmatrix} \frac{Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3} & \frac{Z_1 Z_2}{Z_1+Z_2+Z_3} \\ \frac{Z_1 Z_2}{Z_1+Z_2+Z_3} & \frac{Z_2(Z_1+Z_3)}{Z_1+Z_2+Z_3} \end{bmatrix}$$

$$\frac{Z_1(Z_2+Z_3)}{Z_1+Z_2+Z_3} = \frac{2}{3} \qquad Z_1 = Z_2 = Z_3 = 1$$

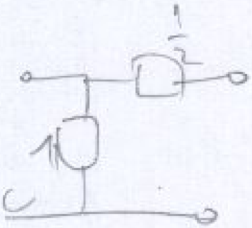


② 2. KOL 2006. TM11 3. str

X - spoj - Bartlettov teorem!

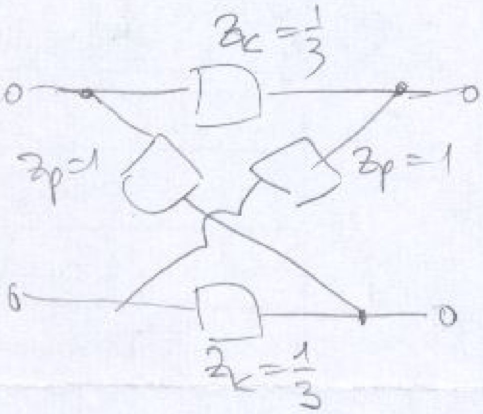


polovica

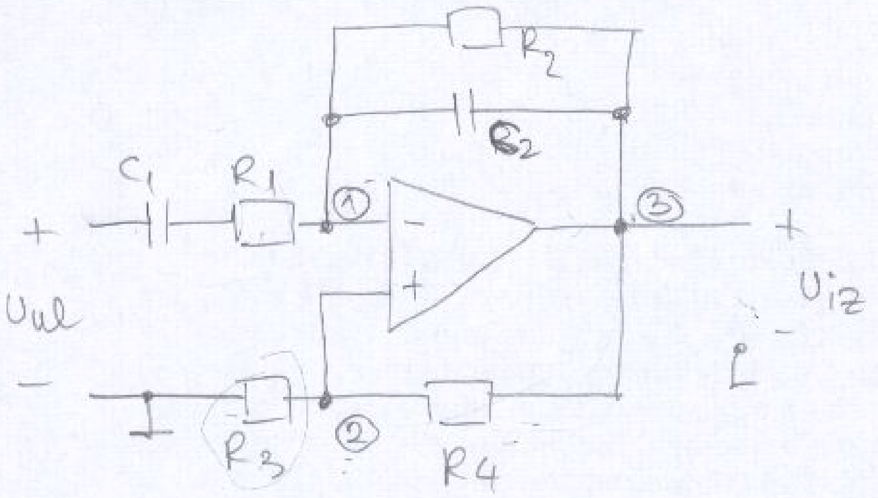


$Z_p = 1$

$Z_k = 1 \| \frac{1}{2} = \frac{\frac{1}{2}}{\frac{3}{2}} = \frac{1}{3}$



③ TMIL 2006 2. kol



$$\begin{cases} U_3 = U_{iz} \\ U_1 = U_2 \\ ① \quad \dfrac{U_1 - U_{UL}}{R_1 + \frac{1}{sC_1}} + \dfrac{U_1 - U_3}{R_2} + (U_1 - U_3)\cdot sC_2 = 0 \\ ② \quad \dfrac{U_2 - U_3}{R_4} + \dfrac{U_2}{R_3} = 0 \end{cases} \Rightarrow \frac{U_3}{R_4} = U_2\left(\frac{1}{R_3} + \frac{1}{R_4}\right) \Rightarrow \frac{U_3}{R_4} = \frac{R_3 + R_4}{R_3 R_4} U_2$$

Iz ②: $U_{iz} = \dfrac{R_3 + R_4}{R_3} \cdot U_1 \Rightarrow \boxed{U_1 = \dfrac{R_3}{R_3 + R_4} U_{iz}} = \dfrac{1}{2} U_{iz}$

to u ①:

$$\frac{\frac{1}{2}U_{iz} - U_{UL}}{\frac{1}{2} + \frac{1}{s}} + \frac{\frac{1}{2}U_{iz} - U_{iz}}{1} + \left(\frac{1}{2}U_{iz} - U_{iz}\right)\cdot s = 0$$

$$\frac{2s\cdot\left(\frac{1}{2}U_{iz} - U_{UL}\right)}{2+s} - \frac{1}{2}U_{iz}(1+s) = 0$$

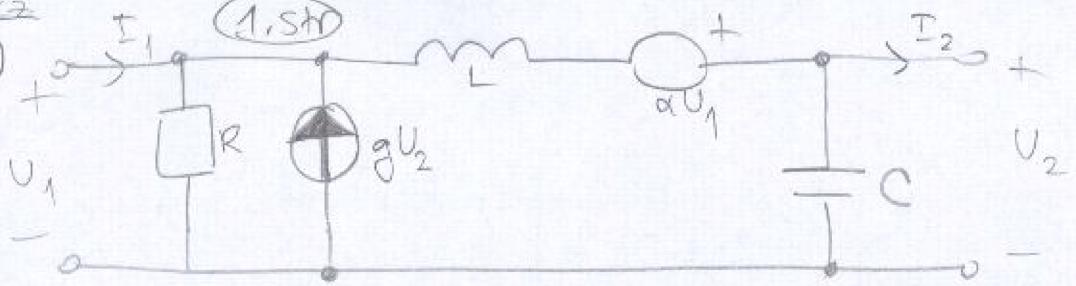
$$\left[\frac{s}{2+s} - \frac{1}{2}(1+s)\right]U_{iz} = \frac{2s}{2+s}U_{UL}$$

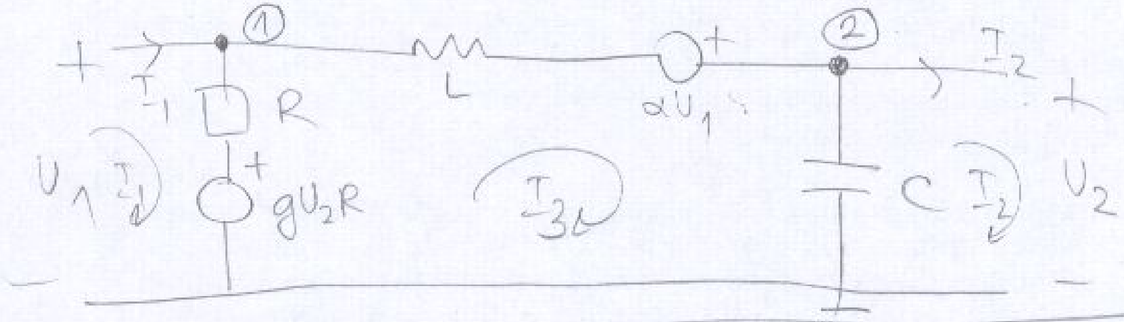
$$\frac{2s - (1+s)(2+s)}{2(2+s)}U_{iz} = \frac{2s}{2+s}U_{UL}$$

$$\frac{2s - 2 - 3s - s^2}{2} \cdot U_{iz} = 2s\cdot U_{UL}$$

$$T(s) = \frac{U_{iz}}{U_{UL}} = -\frac{4s}{s^2 + s + 2}$$

4





$R=1$

$C=1$

$L=2$

$\alpha=1$

$g=\frac{1}{4}$

$$\left\{\begin{aligned} & I_1R - I_3R = U_1 - gU_2R \\ & I_2\cdot\frac{1}{sC} - I_3\cdot\frac{1}{sC} = -U_2 \\ & -I_1R - I_2\cdot\frac{1}{sC} + I_3\left(R+sL+\frac{1}{sC}\right) = gU_2R + \alpha U_1 \end{aligned}\right\} \text{ peiyE}$$

$$\left\{\begin{aligned} & I_1R - \frac{gU_2R+\alpha U_1 + I_1R + I_2\cdot\frac{1}{sC}}{R+sL+\frac{1}{sC}}\cdot R = U_1 - gU_2R \\ & I_2\cdot\frac{1}{sC} - \frac{gU_2R+\alpha U_1+I_1R+I_2\frac{1}{sC}}{R+sL+\frac{1}{sC}}\cdot\frac{1}{sC} = -U_2 \end{aligned}\right.$$

$$\left\{\begin{aligned} & U_1 = z_{11}I_1 - z_{12}I_2 \\ & U_2 = z_{21}I_1 - z_{22}I_2 \end{aligned}\right.$$

$$z_{11} = \frac{U_1}{I_1}\bigg|_{I_2=0}$$

<u>iza 3. str.</u>

$U_2 = I_2\cdot R_L = I_2$

$I_2 = z_{21}I_1 - z_{22}I_2$

$I_2(1+z_{22}) = z_{21}I_1$

$H_i = \frac{I_2}{I_1} = \frac{z_{21}}{1+z_{22}} = \frac{4}{(2s+1)^2+4(s+1)} = \frac{4}{4s^2+8s+5}$

<u>čvorovi</u>

$$\begin{aligned} &① \quad \left\{ -I_1 + \frac{U_1-gU_2R}{R} + \frac{U_1+\alpha U_1 - U_2}{sL} = 0 \right. \\ &② \quad \left. I_2 + U_2\cdot sC + \frac{U_2-\alpha U_1 - U_1}{sL} = 0 \right. \end{aligned}$$

④ TMK 2006 2.kz 2.str.

① $-I_1 + U_1 - \frac{1}{4}U_2 + \frac{U_1}{s} - \frac{U_2}{2s} = 0$

② $I_2 + U_2 \cdot s + \frac{U_2}{2s} - \frac{U_1}{s} = 0$

$I_2 = 0 \Rightarrow U_2\left(s + \frac{1}{2s}\right) = \frac{U_1}{s} \Rightarrow$ $U_2 = \frac{U_1}{s\left(s+\frac{1}{2s}\right)} = \frac{U_1}{s^2+\frac{1}{2}} = \frac{2U_1}{2s^2+1}$

$U_1 = \frac{2s^2+1}{2}U_2$

$z_{11} = \frac{U_1}{I_1}\Big|_{I_2=0}$

$U_1 + \frac{U_1}{s} - \left(\frac{1}{4} + \frac{1}{2s}\right) \cdot \frac{2U_1}{2s^2+1} = I_1$

$U_1\left[1 + \frac{1}{s} - \frac{s+2}{\cancel{4}_2 s} \cdot \frac{\cancel{2}^1}{2s^2+1}\right] = I_1$

$U_1 \cdot \frac{2s(2s^2+1) + 2(2s^2+1) - (s+2)}{2s(2s^2+1)} = I_1$

$U_1 \cdot \frac{4s^3 + 2s + 4s^2 + \cancel{2} - s - \cancel{2}}{2s(2s^2+1)} = I_1$

$U_1 \cdot \frac{4s^2+4s+1}{2(2s^2+1)} = I_1 \Rightarrow \boxed{z_{11} = \frac{2(2s^2+1)}{(2s+1)^2}}$

$z_{21} = \frac{U_2}{I_1}\Big|_{I_2=0}$

$\frac{2s^2+1}{2} \cdot U_2 - \frac{1}{4}U_2 - \frac{U_2}{2s} + \frac{2s^2+1}{2s} \cdot U_2 = I_1$

$\frac{2s(2s^2+1) - s - 2 + 2(2s^2+1)}{4s} \cdot U_2 = I_1$

$\frac{4s^3 + 2s - s - \cancel{2} + 4s^2 + \cancel{2}}{4s} \cdot U_2 = I_1$

$\frac{4s^2+4s+1}{4} = \frac{I_1}{U_2} \Rightarrow \boxed{z_{21} = \frac{4}{(2s+1)^2}}$

TMIL 2. kz 2006

(4) 3. str.

$$I_1 = 0 \overset{U_1}{\Rightarrow} \qquad U_1\left(1+\frac{1}{s}\right) = U_2\left(\frac{1}{4}+\frac{1}{2s}\right)$$

$$U_1 \cdot \frac{s+1}{s} = U_2 \cdot \frac{s+2}{4s} \quad \Big/ \cdot \frac{4s}{s+2}$$

$$U_2 = 4\,\frac{s+1}{s+2}\,U_1$$

$$U_1 = \frac{s+2}{4(s+1)}\,U_2$$

$$z_{12} = \frac{-U_1}{I_2}\Big|_{I_1=0}$$

$$I_2 + 4\cdot\frac{s+1}{s+2}\cdot U_1\left(s+\frac{1}{2s}\right) - \frac{U_1}{s} = 0$$

$$I_2 = U_1\left(\frac{1}{s} - 4\,\frac{s+1}{s+2}\cdot\frac{2s^2+1}{2s}\right]$$

$$I_2 = U_1 \cdot \frac{2(s+2)-(4s+4)(2s^2+1)}{2s(s+2)}$$

$$I_2 = U_1\cdot\frac{2s+\cancel{4}-8s^3-4s-8s^2-\cancel{4}}{2s(s+2)} = U_1\cdot\frac{-\cancel{2s}(4s^2+4s+1)}{\cancel{2s}(s+2)}$$

$$-\frac{U_1}{I_2} = \boxed{z_{12} = \frac{s+2}{(2s+1)^2}}$$

$$z_{22} = -\frac{U_2}{I_2}\Big|_{I_1=0}$$

$$I_2 + U_2\left(s+\frac{1}{2s}\right) - \frac{s+2}{4s(s+1)}\,U_2 = 0$$

$$(2+2s)(2s^2+1)$$

$$I_2 = U_2\cdot\left(\frac{s+2}{4s(s+1)} - \frac{2s^2+1}{2s}\right) = U_2\cdot\frac{s+2-2(2s^2+1)(s+1)}{4s(s+1)}$$

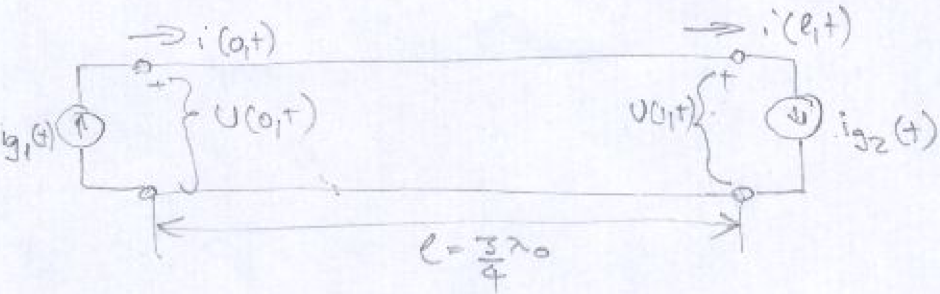
$$= U_2\cdot\frac{s+\cancel{2}-4s^2-\cancel{2}-4s^3-2s}{4s(s+1)} = -U_2\cdot\frac{4s^3+4s^2+s}{4s(s+1)} = -U_2\,\frac{(2s+1)^2}{4(s+1)}$$

$$\boxed{z_{22} = \frac{-U_2}{I_2} = \frac{4(s+1)}{(2s+1)^2}}$$

$$H_i(s) = \frac{I_2(s)}{I_1(s)} = \text{vidi 1. str.}$$

$L = 10\,\mu H/km$
$C = 40\,\mu F/km$
$l = \frac{3}{4}\lambda_0$
$\omega_0 = 10^5 \pi\, rad/s$
$i_{g1}(t) = 10\cos\omega_0 t$
$i_{g2}(t) = 20\cos\omega_0 t$

$u(0,t) = ?$
$u(l,t) = ?$



$\omega_0 = 2\pi f_0$

$f_0 = \frac{\omega_0}{2\pi} = 5\cdot 10^4\, s^{-1}$

$l = \frac{3}{4}\lambda_0 = \frac{3}{4}\frac{2\pi}{\beta} = 0.75\,km$

LINIJA BEZ GUBITAKA

$R = 0,\ G = 0$

$Z_0 = \sqrt{\frac{L}{C}} = 0.5\,\Omega$

$s \to j\omega$

$\gamma = \sqrt{(R+sL)(G+sC)} = s\sqrt{LC}$

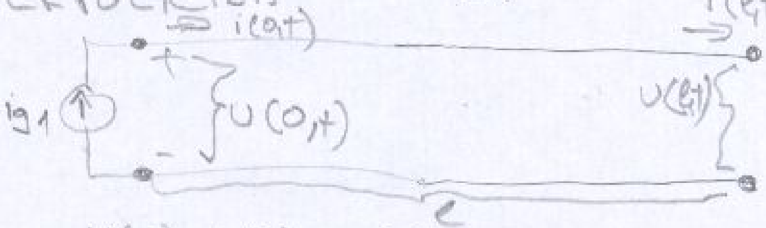
$\gamma = j\omega\sqrt{LC} = j2\pi$

$\alpha = 0$

$\beta = \omega\sqrt{LC} = 2\pi$

$\beta l = \frac{3\pi}{2}$

SUPERPOZICIJA 1#



$U(x) = U(0)\,ch\,\gamma l - I(0)\cdot Z_0\,sh\,\gamma l$

$I(x) = \frac{-U(0)}{Z_0}\,sh\,\gamma l + I(0)\,ch\,\gamma l$

$x = l$

$U(l) = U(0)\,ch\,j\beta l - I(0) Z_0\,sh\,j\beta l$

$I(l) = \frac{-U(0)}{Z_0}\,sh\,j\beta l + I(0)\,ch\,j\beta l$

$$U(0) = Z_0 \left(I(0)\, \mathrm{ch}\, j\beta l - I(l)\right) \cdot \frac{1}{\mathrm{sh}\, j\beta l}$$

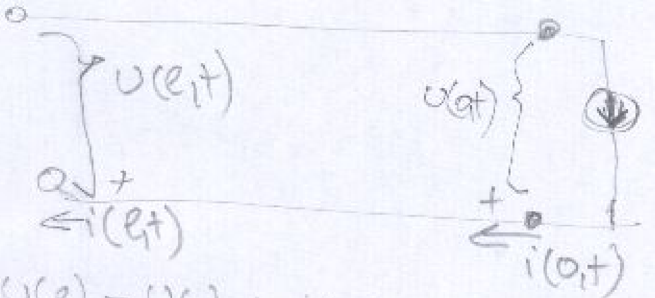
$$U(0) = Z_0 \cdot \left(I(0) \cos\beta l - I(l)\right) \cdot \frac{1}{j \sin\beta l}$$

$$U_1(0) = 0.5 \left(I_{g1} \cos\frac{3\pi}{2} - 0\right) \cdot \frac{1}{j \sin\frac{3\pi}{2}} = 0$$

$$U_1(l) = U(0)\, \mathrm{ch}\, j\beta l - I(0) Z_0\, \mathrm{sh}\, j\beta l$$

$$= -I(0) Z_0\, j \sin\frac{3\pi}{2} = j I(0) Z_0 = j5$$

SUPERPOZICIJA #2



$$U(l) = U(0)\, \mathrm{ch}\, j\beta l - I(0) Z_0\, \mathrm{sh}\, j\beta l$$

$$I(l) = -\frac{U(0)}{Z_0}\, \mathrm{sh}\, j\beta l + I(0)\, \mathrm{ch}\, j\beta l$$

$$U_2(0) = Z_0 \left(I(0)\, \mathrm{ch}\, j\beta l - I(l)\right) \cdot \frac{1}{\mathrm{sh}\, j\beta l}$$

$$= 0.5 \left(I_{g2} \cos\beta l - I(l)\right) \cdot \frac{1}{\mathrm{sh}\, j\beta l}$$

$$= 0.5 \left(20 \cos\frac{3\pi}{2} - 0\right) \cdot \frac{1}{j \sin\frac{3\pi}{2}} = 0$$

$$U_2(l) = U(0)\, \mathrm{ch}\, j\beta l - I(0) Z_0\, \mathrm{sh}\, j\beta l$$

$$= 0\, \mathrm{ch}\, j\beta l - 20 \cdot 0.5 \cdot j \sin\frac{3\pi}{2}$$

$$= j10$$

$$U(0) = U_1(0) - U_2(l) = -j10$$

$$U(l) = U_1(l) - U_2(0) = j5$$

$$u(0,t) = 10 \cos\left(\omega_0 t - \frac{\pi}{2}\right)$$

$$u(l,t) = 5 \cos\left(\omega_0 t + \frac{\pi}{2}\right)$$

zbrajanje rješenja dob. superpozicijom