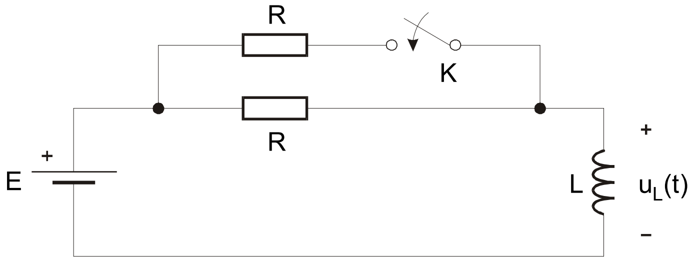
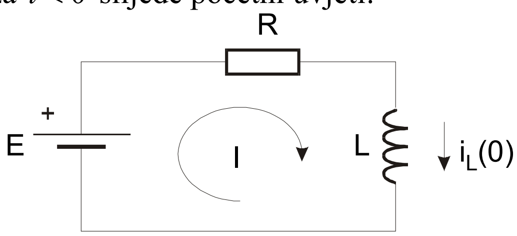
PISMENI ISPIT IZ ELEKTRIČNIH KRUGOVA Jesen 2015 – Rješenja

1. U trenutku t=0 zatvara se sklopka K u prikazanoj mreži. Odrediti i skicirati valni oblik napona $u_L(t)$ ako je zadano: $E = 10[\mathrm{V}]$; $L = 1$; $R = 2$.



Rješenje:

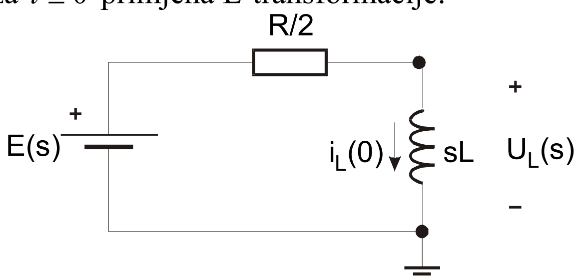
a) za $t < 0$ slijede početni uvjeti:



$$i(t) = \frac{E}{R} = \frac{10}{2} = 5[\mathrm{A}]$$

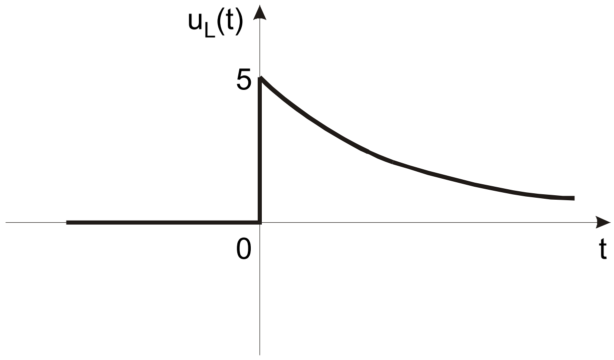
$$i_L(0) = 5[\mathrm{A}]$$

b) za $t \geq 0$ primjena L-transformacije:

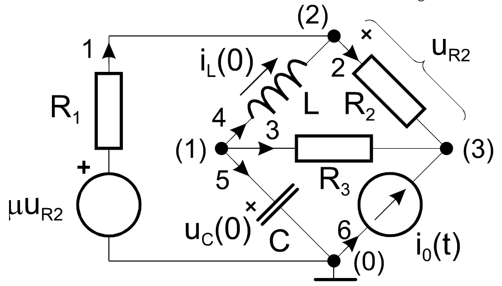


$$e(t) = E = 10[\mathrm{V}] \circ\!\!-\!\!\circ E(s) = \frac{10}{s};\; U_L(s)\left(\frac{2}{R} + \frac{1}{sL}\right) = \frac{E(s)}{R/2} - \frac{i_L(0)}{s}$$

$$U_L(s) = \frac{\frac{10}{s} - \frac{5}{s}}{1 + \frac{1}{s}} = \frac{10-5}{1+s} = \frac{5}{1+s};\; U_L(s) = \frac{5}{1+s} \;\Rightarrow\; u_L(t) = 5 \cdot e^{-t} \cdot S(t)$$

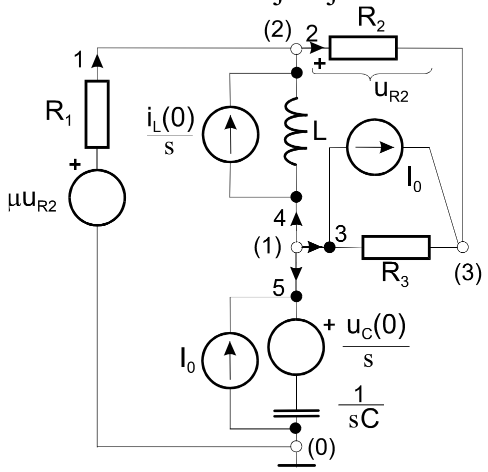


2. Za prikazanu mrežu topološkom analizom napisati sustav jednadžbi čvorova u matričnom obliku (matrice $\mathbf{Y}_v$ i $\mathbf{I}_{0v}$ pomoću matrica $\mathbf{Y}_b=\mathbf{Z}_b^{-1}$ i $\mathbf{I}_{0b}$). Nacrtati orijentirani graf (držati se oznaka čvorova i grana te pritom voditi računa da matrica $\mathbf{Z}_b$ mora biti regularna).

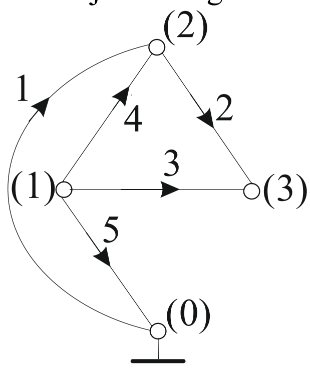


Rješenje: sustav jednadžbi čvorova glasi: $\mathbf{Y}_v \cdot \mathbf{U}_v = \mathbf{I}_v$ rješenje vektor napona čvorišta $\mathbf{U}_v$.

Mreža u frekvencijskoj domeni:



Orijentirani graf:



Matrica incidencija:

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ -1 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 \end{bmatrix}$$

Naponsko–strujne relacije grana glase:

$$U_1 = -\mu \cdot U_{R2} + I_1 \cdot R_1 = -\mu \cdot R_2 \cdot I_2 + I_1 \cdot R_1$$

$$U_2 = I_2 \cdot R_2$$

$$U_3 = I_3 \cdot R_3 - I_0 \cdot R_3$$

$$U_4 = I_4 \cdot sL - L \cdot i_L(0)$$

$$U_5 = I_5 \cdot \frac{1}{sC} + I_0 \cdot \frac{1}{sC} + \frac{u_C(0)}{s}$$

a u matričnom obliku glase:

$$\underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix}}_{\mathbf{U}_b} = \underbrace{\begin{bmatrix} R_1 & -\mu R_2 & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 & 0 \\ 0 & 0 & R_3 & 0 & 0 \\ 0 & 0 & 0 & sL & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{sC} \end{bmatrix}}_{\mathbf{Z}_b} \cdot \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix}}_{\mathbf{I}_b} + \underbrace{\begin{bmatrix} 0 \\ 0 \\ -I_0 \cdot R_3 \\ -L \cdot i_L(0) \\ I_0 \cdot \frac{1}{sC} + \frac{u_C(0)}{s} \end{bmatrix}}_{\mathbf{U}_{0b}}$$

Zavisni izvori (i međuinduktiviteti) se upisuju u matricu $\mathbf{Z}_b$, a nezavisni izvori i početni uvjeti u vektor $\mathbf{U}_{0b}$. Nađimo inverziju matrice $\mathbf{Z}_b$, tj. $\mathbf{Y}_b = \mathbf{Z}_b^{-1}$.

$$\begin{bmatrix} R_1 & -\mu R_2 \\ 0 & R_2 \end{bmatrix}^{-1} = \frac{1}{R_1 R_2}\begin{bmatrix} R_2 & 0 \\ \mu R_2 & R_1 \end{bmatrix}^T = \frac{1}{R_1 R_2}\begin{bmatrix} R_2 & \mu R_2 \\ 0 & R_1 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1} & \frac{\mu}{R_1} \\ 0 & \frac{1}{R_2} \end{bmatrix}$$

$$\mathbf{Y}_b = \begin{bmatrix} \frac{1}{R_1} & \frac{\mu}{R_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{R_3} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{sL} & 0 \\ 0 & 0 & 0 & 0 & sC \end{bmatrix}$$

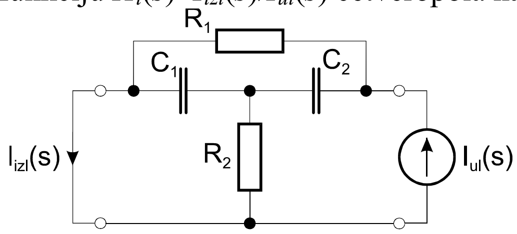
$$\mathbf{Y}_v = \mathbf{A}\cdot\mathbf{Y}_b\cdot\mathbf{A}^T = \begin{bmatrix} 0 & 0 & 1 & 1 & 1 \\ -1 & 1 & 0 & -1 & 0 \\ 0 & -1 & -1 & 0 & 0 \end{bmatrix}\cdot\begin{bmatrix} \frac{1}{R_1} & \frac{\mu}{R_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{R_3} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{sL} & 0 \\ 0 & 0 & 0 & 0 & sC \end{bmatrix}\cdot\mathbf{A}^T =$$

$$= \begin{bmatrix} 0 & 0 & \frac{1}{R_3} & \frac{1}{sL} & sC \\ -\frac{1}{R_1} & -\frac{\mu}{R_1}+\frac{1}{R_2} & 0 & -\frac{1}{sL} & 0 \\ 0 & -\frac{1}{R_2} & -\frac{1}{R_3} & 0 & 0 \end{bmatrix}\cdot\begin{bmatrix} 0 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} =$$

$$= \begin{bmatrix} \left(\frac{1}{R_3}+\frac{1}{sL}+sC\right) & -\frac{1}{sL} & -\frac{1}{R_3} \\ -\frac{1}{sL} & \left(\frac{1}{R_1}-\frac{\mu}{R_1}+\frac{1}{R_2}+\frac{1}{sL}\right) & \frac{\mu}{R_1}-\frac{1}{R_2} \\ -\frac{1}{R_3} & -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3} \end{bmatrix}$$

$$\mathbf{I}_v = \mathbf{A}\cdot\mathbf{Y}_b\cdot\mathbf{U}_{0b} = \begin{bmatrix} 0 & 0 & \frac{1}{R_3} & \frac{1}{sL} & sC \\ -\frac{1}{R_1} & -\frac{\mu}{R_1}+\frac{1}{R_2} & 0 & -\frac{1}{sL} & 0 \\ 0 & -\frac{1}{R_2} & -\frac{1}{R_3} & 0 & 0 \end{bmatrix}\cdot\begin{bmatrix} 0 \\ 0 \\ -I_0\cdot R_3 \\ -L\cdot i_L(0) \\ I_0\cdot\frac{1}{sC}+\frac{u_C(0)}{s} \end{bmatrix} = \begin{bmatrix} -\frac{i_L(0)}{s}+C\cdot u_C(0) \\ \frac{i_L(0)}{s} \\ I_0 \end{bmatrix}$$

3. Naći strujnu prijenosnu funkciju $H_i(s)=I_{izl}(s)/I_{ul}(s)$ četveropola na slici.



<u>Rješenje:</u> Jednadžbe čvorišta: (zadatak se može riješiti i pomoću [y]-parametara)

$$(1)\quad U_1\cdot\left(\frac{1}{R_2}+sC_1+sC_2\right)-U_2\cdot sC_2-U_3\cdot sC_1=0$$

$$(2)\quad -U_1\cdot sC_2+U_2\cdot\left(\frac{1}{R_1}+sC_2\right)-U_3\cdot\frac{1}{R_1}=I_{ul}(s)$$

$$(3)\quad -U_1\cdot sC_1-U_2\cdot\frac{1}{R_1}+U_3\cdot\left(\frac{1}{R_1}+sC_1\right)=-I_{izl}(s)$$

$U_3=0$

$$(1)\quad U_1\cdot\left(\frac{1}{R_2}+sC_1+sC_2\right)-U_2\cdot sC_2=0$$

$$(2)\quad -U_1\cdot sC_2+U_2\cdot\left(\frac{1}{R_1}+sC_2\right)=I_{ul}(s)$$

$$(3)\quad U_1\cdot sC_1+U_2\cdot\frac{1}{R_1}=I_{izl}(s)$$

$(1),(3)\ \Rightarrow\ U_1,U_2\ \rightarrow\ (2)$

$$\Delta=\begin{vmatrix}\frac{1}{R_2}+sC_1+sC_2 & -sC_2\\ sC_1 & \frac{1}{R_1}\end{vmatrix}=\frac{1}{R_1}\cdot\left(\frac{1}{R_2}+sC_1+sC_2\right)+s^2C_1C_2$$

$$\Delta_1=\begin{vmatrix}0 & -sC_2\\ I_{izl} & \frac{1}{R_1}\end{vmatrix}=sC_2\cdot I_{izl};\quad \Delta_2=\begin{vmatrix}\frac{1}{R_2}+sC_1+sC_2 & 0\\ sC_1 & I_{izl}\end{vmatrix}=\left(\frac{1}{R_2}+sC_1+sC_2\right)\cdot I_{izl}$$

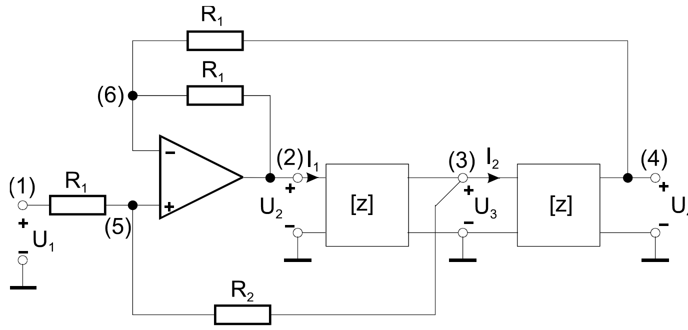
$$U_1=\frac{\Delta_1}{\Delta};\quad U_2=\frac{\Delta_2}{\Delta}\quad\rightarrow\quad(2):\quad -\frac{\Delta_1}{\Delta}\cdot sC_2+\frac{\Delta_2}{\Delta}\cdot\left(\frac{1}{R_1}+sC_2\right)=I_{ul}(s)$$

$$I_{ul}=\frac{-(sC_2)^2+\left(\frac{1}{R_2}+sC_1+sC_2\right)\cdot\left(\frac{1}{R_1}+sC_2\right)}{\Delta}\cdot I_{izl}$$

$$H_i(s)=\frac{I_{izl}(s)}{I_{ul}(s)}=\frac{\frac{1}{R_1\cdot R_2}+s\frac{C_1}{R_1}+s\frac{C_2}{R_1}+s^2C_1C_2}{-(sC_2)^2+\frac{1}{R_1\cdot R_2}+s\frac{C_1}{R_1}+s\frac{C_2}{R_1}+s\frac{C_2}{R_2}+s^2C_1C_2+(sC_2)^2}\Bigg/\cdot\ \frac{R_1R_2}{R_1R_2}$$

$$H_i(s)=\frac{s^2R_1R_2C_1C_2+s(C_1R_2+C_2R_2)+1}{s^2R_1R_2C_1C_2+s(C_1R_2+C_2R_2+R_1C_2)+1}$$

4. Za mrežu prikazanu slikom naći prijenosnu funkciju $T(s)=U_4(s)/U_1(s)$ ako je zadano $R_1=1$; $R_2=0{,}414$; $A\to\infty$; $C=1$. Za pobudu $u_1(t)=10\cos(0{,}9\,t)$ naći valni oblik napona $u_4(t)$.

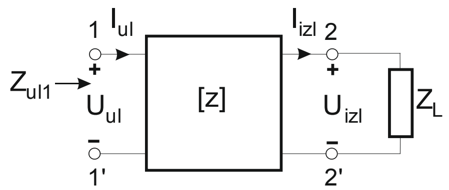


$$[z]=\begin{bmatrix} R_1 & 0 \\ -\dfrac{1}{sC} & 0 \end{bmatrix}$$

Rješenje: a) najprije odredimo naponsku prijenosnu funkciju i ulaznu impedanciju četveropola (koji je zadan [z] ili [a] parametrima):

[z]-parametri:

$$U_{ul}=z_{11}I_{ul}-z_{12}I_{izl}$$
$$U_{izl}=z_{21}I_{ul}-z_{22}I_{izl}$$



$$H_u(s)=\frac{U_{izl}}{U_{ul}}=\frac{Z_L z_{21}}{\Delta z+z_{11}Z_L}=\left|Z_L=\infty\right|=\frac{z_{21}}{z_{11}}\ \text{ili}\ H_u(s)=\frac{U_{izl}}{U_{ul}}=\frac{Z_L z_{21}}{\Delta z+z_{11}Z_L}=\begin{vmatrix} z_{12}=0 \\ z_{22}=0\end{vmatrix}=\frac{z_{21}}{z_{11}};$$

gdje su $\Delta z=z_{22}z_{11}-z_{12}z_{21}$ i $U_{izl}=Z_L\cdot I_{izl}$.

$$Z_{ul1}(s)=\frac{U_{ul}}{I_{ul}}=z_{11}-\frac{z_{12}z_{21}}{z_{22}+Z_L}=\left|Z_L=\infty\right|=z_{11}\ \text{ili}\ Z_{ul1}(s)=\frac{U_{ul}}{I_{ul}}=z_{11}-\frac{z_{12}z_{21}}{z_{22}+Z_L}=\begin{vmatrix} z_{12}=0 \\ z_{22}=0\end{vmatrix}=z_{11}$$

Moguće je četveropole zadati i prijenosnim [a] parametrima:

$$U_1=A\cdot U_2+B\cdot I_2$$
$$I_1=C\cdot U_2+D\cdot I_2$$

$$A=\frac{z_{11}}{z_{21}},B=\frac{z_{11}z_{22}}{z_{21}}-z_{12},C=\frac{1}{z_{21}},D=\frac{z_{22}}{z_{21}}\quad\Rightarrow\quad [a]=\begin{bmatrix} -sR_1C & 0 \\ -sC & 0\end{bmatrix}$$

$$H_u(s)=\frac{U_{izl}}{U_{ul}}=\frac{Z_L}{AZ_L+B}=\left|Z_L=\infty\ \text{ili}\ \begin{matrix} B=0 \\ D=0\end{matrix}\right|=\frac{1}{A}=\frac{z_{21}}{z_{11}}$$

odn. $$Z_{ul1}(s)=\frac{U_{ul}}{I_{ul}}=\frac{AZ_L+B}{CZ_L+D}=\left|Z_L=\infty\ \text{ili}\ \begin{matrix} B=0 \\ D=0\end{matrix}\right|=\frac{A}{C}=z_{11}$$

Stoga je prijenosna funkcija: $H_u(s)=\dfrac{U_{izl}}{U_{ul}}=\dfrac{z_{21}}{z_{11}}=\dfrac{-\dfrac{1}{sC}}{R_1}=-\dfrac{1}{s}$,

a ulazna impedancija: $Z_{ul1}(s)=\dfrac{U_{ul}}{I_{ul}}=z_{11}=R_1$

Postavimo jednadžbe čvorišta:

(5) $$U_5\left(\frac{1}{R_1}+\frac{1}{R_2}\right)-U_3\frac{1}{R_2}=\frac{U_1}{R_1}$$

(6) $$U_6\left(\frac{1}{R_1}+\frac{1}{R_1}\right)-U_2\frac{1}{R_1}-U_4\frac{1}{R_1}=0$$

(2) $$U_2=A(U_5-U_6)\Rightarrow U_5-U_6=\frac{U_2}{A};A\to\infty\Rightarrow U_5=U_6$$

(3) $$U_3=-\frac{1}{sR_1C}U_2$$

(4) $$U_4=-\frac{1}{sR_1C}U_3$$

Nakon malo sređivanja i uvrštavanja slijedi:

$$\begin{aligned}&U_5\cdot 3{,}4154-U_3\cdot 2{,}4154=U_1\\&U_6\cdot 2-U_2-U_4=0\\&U_3=-\frac{1}{s}U_2\\&U_4=-\frac{1}{s}U_3\end{aligned}$$

$$\begin{aligned}&U_5\cdot 3{,}4154-U_3\cdot 2{,}4154=U_1\\&2U_5-U_2-U_4=0\\&U_2=-sU_3\\&U_3=-sU_4\end{aligned}$$

$$\begin{aligned}&U_5\cdot 3{,}4154-U_3\cdot 2{,}4154=U_1\\&2U_5+s\cdot U_3-U_4=0\\&U_3=-sU_4\end{aligned}$$

$$\begin{aligned}&U_5\cdot 3{,}4154+U_4\cdot s\cdot 2{,}4154=U_1\\&2U_5-U_4\cdot s^2-U_4=0\Rightarrow U_5=\frac{1}{2}(s^2+1)U_4\end{aligned}$$

$$\left[\frac{1}{2}(s^2+1)\cdot 3{,}4154+s\cdot 2{,}4154\right]U_4=U_1$$

$$T(s)=\frac{U_4(s)}{U_1(s)}=\frac{1}{1{,}7077s^2+1{,}7077+s\cdot 2{,}4154}$$

$$\boxed{T(s)=\frac{U_4(s)}{U_1(s)}=\frac{0{,}5858}{s^2+1{,}4142s+1}}$$ uz uvršteno $s=j\omega$ slijedi

$$T(j\omega)=\frac{0{,}5858}{(1-\omega^2)+j\sqrt{2}\omega}=\frac{0{,}5858}{(1-\omega^2)^2+2\omega^2}\left[(1-\omega^2)-j\sqrt{2}\omega\right]$$

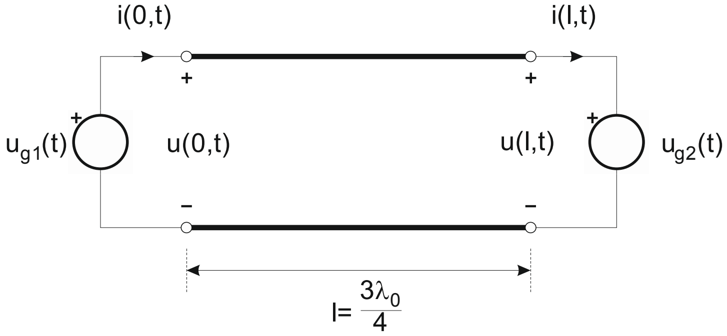
$$|T(j\omega)|=\left.\frac{0{,}5858}{\sqrt{(1-\omega^2)^2+(\sqrt{2}\cdot\omega)^2}}\right|_{\omega=0,9}=\frac{0{,}5858}{\sqrt{(1-0{,}9^2)^2+2\cdot 0{,}9^2}}=0{,}455204$$

$$\angle T(j\omega)=\arctan\frac{\mathrm{Im}\{T(j\omega)\}}{\mathrm{Re}\{T(j\omega)\}}=\arctan\left.\frac{-\sqrt{2}\omega}{1-\omega^2}\right|_{\omega=0,9}=\arctan\frac{-1{,}2728}{0{,}19}=-1{,}4226\ \mathrm{rad}=-81{,}5^\circ$$

$$u_1(t)=10\cos(0{,}9t)\qquad\Rightarrow\qquad U_1(j\omega)=10\angle 0^0;\quad \omega=0{,}9\ \mathrm{[rad/s]}$$

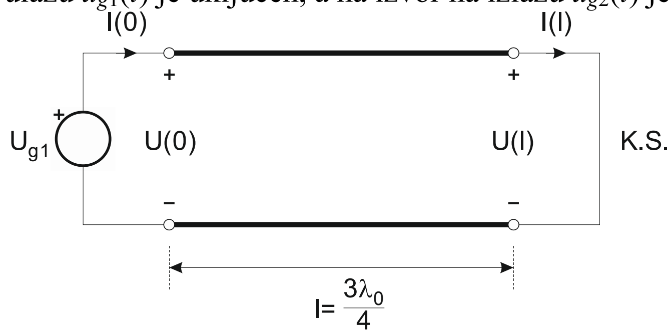
$$U_4(j\omega)=U_1(j\omega)\cdot T(j\omega)=10\angle 0^\circ\cdot 0{,}455204\angle-81.5^\circ\quad\Rightarrow\quad \underline{u_4(t)=4{,}552\cos(0{,}9t-81{,}5^\circ)}$$

5. Zadana je linija bez gubitaka s primarnim parametrima $L = 10\,\mu\text{H/km}; C = 40\,\mu\text{F/km}$ i duljine $l = (3/4)\cdot\lambda_0$ na frekvenciji generatora $\omega_0 = 10^5$ rad/s. Na ulaz linije je spojen naponski izvor $u_{g1}(t) = 10\cos(\omega_0 t)$, a na izlaz linije spojen je naponski izvor $u_{g2}(t) = 20\cos(\omega_0 t)$. Treba izračunati valne oblike struje na ulazu linije $i(x=0,t)$ i na izlazu $i(x=l,t)$. Kolika je duljina linije *l*?



Rješenje: Metodom superpozicije:

a) naponski izvor na ulazu $u_{g1}(t)$ je uključen, a na izvor na izlazu $u_{g2}(t)$ je isključen.



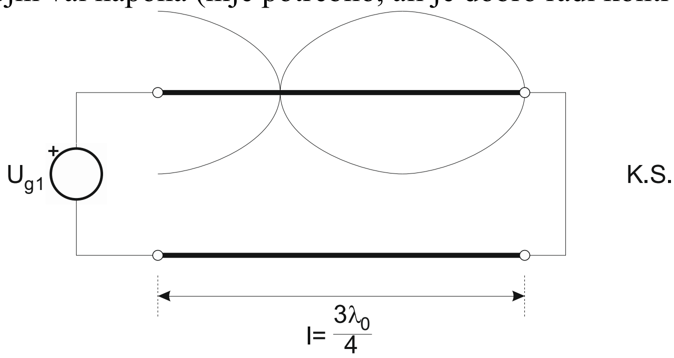
$$U(l) = R_2 \cdot I(l),\ (R_2 = 0,\ \text{KS}) \Rightarrow U(l) = 0;$$

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{10\cdot 10^{-6}}{40\cdot 10^{-6}}} = \frac{1}{2}[\Omega];$$

$$\gamma = \alpha + j\beta;\ \alpha = 0;\ \beta = \omega_0\sqrt{LC}$$

$$g = \gamma\cdot l = j\omega_0\sqrt{LC}\cdot\frac{3}{4}\cdot\frac{2\pi}{\omega_0\sqrt{LC}} = j\frac{3\pi}{2};\ \lambda_0 = \frac{2\pi}{\beta}$$

Možemo skicirati stojni val napona (nije potrebno, ali je dobro radi kontrole):



Najprije izračunajmo $I(0)$:

Prijenosne jednadžbe linije:

$$U(0) = U(l)\cdot \operatorname{ch} g + I(l)\cdot Z_0 \cdot \operatorname{sh} g$$

$$I(0) = U(l)\frac{\operatorname{sh} g}{Z_0} + I(l)\cdot \operatorname{ch} g$$

Vrijedi da je $\begin{array}{l}\mathrm{ch}(g)=\mathrm{ch}(j\beta\cdot l)=\cos(\beta\cdot l)\\ \mathrm{sh}(g)=\mathrm{sh}(j\beta\cdot l)=j\sin(\beta\cdot l)\end{array}$ $\Rightarrow$ $\begin{array}{l}\mathrm{ch}\left(j\dfrac{3\pi}{2}\right)=\cos\left(\dfrac{3\pi}{2}\right)=0\\ \mathrm{sh}\left(j\dfrac{3\pi}{2}\right)=j\sin\left(\dfrac{3\pi}{2}\right)=-j\end{array}$

$$U(0)=-jZ_0I(l)$$
$$I(0)=\frac{-j}{Z_0}U(l)$$

Odavdje se odmah vidi da je uz $U(l)=0$ (KS) $\Rightarrow \boxed{I(0)=0}$

Ili na drugi način:

$$Z_{ul}=\frac{U(0)}{I(0)}=\frac{R_2\cdot\mathrm{ch}\,g+Z_0\cdot\mathrm{sh}\,g}{\dfrac{R_2}{Z_0}\cdot\mathrm{sh}\,g+\mathrm{ch}\,g}=\frac{R_2\cdot\cos\dfrac{3\pi}{2}+Z_0\cdot j\cdot\sin\dfrac{3\pi}{2}}{\dfrac{R_2}{Z_0}\cdot j\cdot\sin\dfrac{3\pi}{2}+\cos\dfrac{3\pi}{2}}=\frac{Z_0\cdot(-j)}{(-j)\cdot\dfrac{R_2}{Z_0}}=\frac{Z_0^{\,2}}{R_2}$$

uz $R_2=0$, (KS) $\Rightarrow Z_{ul}=\infty\Rightarrow I(0)=0$

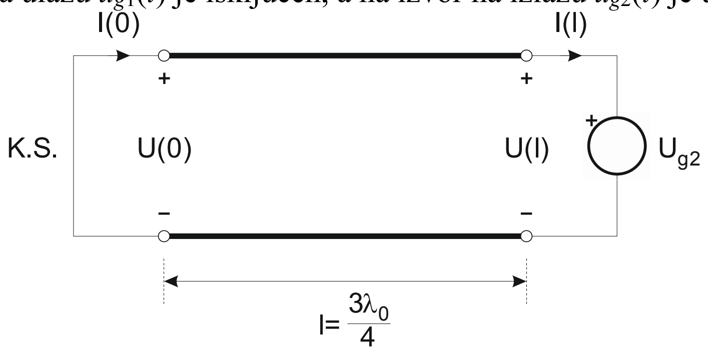
<u>Zatim izračunajmo *I*(*l*):</u>

Prijenosne jednadžbe linije:

$$\begin{aligned}U(l)&=U(0)\cdot\mathrm{ch}\,g-I(0)\cdot Z_0\cdot\mathrm{sh}\,g\\ I(l)&=-U(0)\cdot\frac{\mathrm{sh}\,g}{Z_0}+I(0)\cdot\mathrm{ch}\,g\end{aligned}\quad\Rightarrow\quad\begin{aligned}U(l)&=j\cdot I(0)\cdot Z_0\\ I(l)&=j\cdot\frac{U(0)}{Z_0}\end{aligned}$$

Odavdje se odmah vidi da je uz $U_{g1}(j\omega)=10\angle0^\circ=U(0)$ $\Rightarrow \boxed{I(l)=20\angle 90^\circ}$

b) naponski izvor na ulazu $u_{g1}(t)$ je isključen, a na izvor na izlazu $u_{g2}(t)$ je uključen.



$$U_{g2}(j\omega)=20\angle0^\circ=U(l)$$
$$U(0)=jI(l)\cdot Z_0\Rightarrow U(0)=0\Rightarrow\boxed{I(l)=0}$$
$$I(0)=\frac{-j}{Z_0}U(l)\Rightarrow\boxed{I(0)=40\angle-90^\circ}$$

Zbroj slučajeva a) i b) daje struje:

$$i(0,t)=40\cos(\omega_0t-90^\circ)$$
$$i(l,t)=20\cos(\omega_0t+90^\circ)$$

Duljina linije je:

$$l=\frac{3}{4}\cdot\lambda_0=\frac{3}{4}\cdot\frac{2\pi}{\omega_0\sqrt{LC}}=\frac{2\cdot3\pi/4}{10^5\cdot\sqrt{10\cdot10^{-6}\cdot40\cdot10^{-6}}}=\frac{3\pi/2}{10^5\cdot20\cdot10^{-6}}=\frac{3}{4}\pi[\mathrm{km}]$$