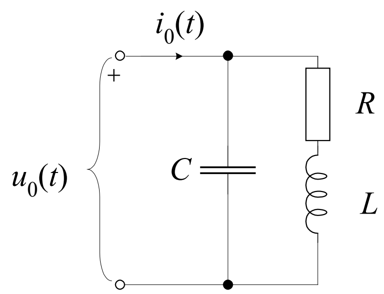
PRVI MEĐUISPIT IZ ELEKTRIČNIH KRUGOVA

1. Na priključnice dvopola sastavljenog od paralelnoga spoja kapaciteta C=1 nF i serijske kombinacije otpora R=1000 Ω i induktiviteta L=1 mH, djeluje strujni izvor $i_0(t)=\delta(t)$. Normirati elemente dvopola na frekvenciju $\omega_0=10^6$ rad/s i na otpor R_0=1000 Ω. Odrediti napon $u(t)$ na priključnicama tog dvopola.

<u>Rješenje:</u> Normiranje elemenata zatim primjena Laplaceove transformacije



$$R_n = \frac{R}{R_0} = \frac{1000}{1000} = 1$$

$$Z_{C_n} = \frac{1}{sCR_0} = \frac{1}{\frac{s}{\omega_0}\underbrace{\omega_0 CR_0}_{C_n}} \qquad \Rightarrow \qquad C_n = \omega_0 CR_0 = 10^6 \cdot 10^{-9} \cdot 10^3 = 1$$

$$Z_{L_n} = \frac{sL}{R_0} = \frac{s}{\omega_0} \cdot \underbrace{\frac{\omega_0 L}{R_0}}_{L_n} \qquad \Rightarrow \qquad L_n = \frac{\omega_0 L}{R_0} = \frac{10^6 \cdot 10^{-3}}{10^3} = 1$$

$$Z_n(s) = \frac{\frac{1}{sC}(R+sL)}{\frac{1}{sC}+R+sL} = \frac{R+sL}{1+sCR+s^2LC}$$

Uz uvrštene normirane vrijednosti elemenata impedancija dvopola glasi:

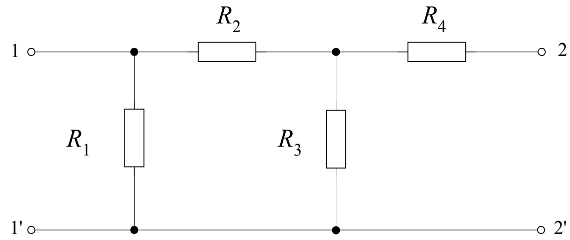
$$Z_n(s) = \frac{1+s}{s^2+s+1} = \frac{s+\frac{1}{2}+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}}$$

Odnosno:

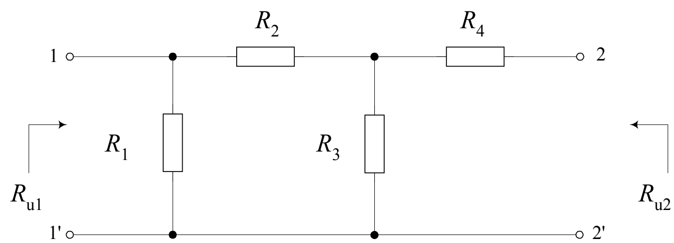
$$U_0(s) = I_0(s)\cdot Z_n(s) = \frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}} + \frac{\frac{1}{2}\cdot\frac{\sqrt{3}}{2}\cdot\frac{2}{\sqrt{3}}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}} = \frac{s+\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}} + \frac{1}{\sqrt{3}}\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2+\frac{3}{4}}$$

pa je $u_0(t) = \mathscr{L}^{-1}\left[U_0(s)\right] = e^{-\frac{t}{2}}\left(\cos\frac{\sqrt{3}}{2}t + \frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t\right)S(t)$

2. Za krug prikazan slikom odrediti otpor R_1 tako da ukupni otpor gledan sa priključnica 1-1' bude jednak otporu gledanome s priključnica 2-2'. Zadano je: R_2= 40 Ω, R_3=30 Ω i R_4=20 Ω.



Rješenje:



$$R_{u1} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2 + R_3}} = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} = \frac{R_1 \cdot 70}{R_1 + 70}$$

$$R_{u2} = R_4 + \frac{1}{\frac{1}{R_3} + \frac{1}{R_1 + R_2}} = R_4 + \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3} = 20 + \frac{30(R_1 + 40)}{R_1 + 70}$$

$$R_{u1} = R_{u2}$$

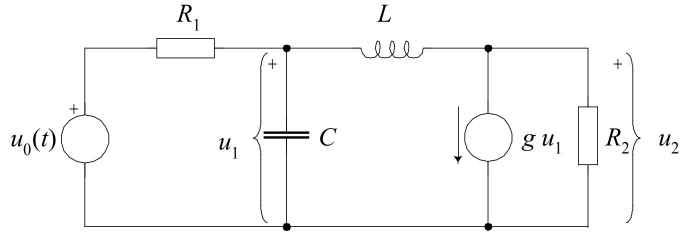
$$\frac{R_1 \cdot 70}{R_1 + 70} = 20 + \frac{30(R_1 + 40)}{R_1 + 70}$$

$$\frac{R_1 \cdot 70}{R_1 + 70} = \frac{20(R_1 + 70) + 30(R_1 + 40)}{R_1 + 70}$$

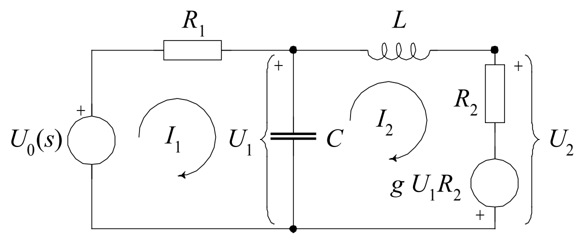
$$R_1 \cdot 70 - R_1 \cdot 50 = 20 \cdot 70 + 30 \cdot 40$$

$$R_1 = \frac{20 \cdot 70 + 30 \cdot 40}{20} = 130\,\Omega$$

3. Za krug prikazan slikom napisati jednadžbe petlji. Konačni oblik jednadžbi prikazati u formi matrične jednadžbe. Izračunati napon $U_2(s)$, ako je zadana pobuda $u_0(t)= 3S(t)$, konstanta g=1, a normirane vrijednosti elemenata su: $R_1= R_2=1$, L=1 i C=1. Početni uvjeti su jednaki nuli.



Rješenje: Primjena Laplaceove transformacije



$$(1)\qquad -U_0(s)+I_1(s)R_1+\left(I_1(s)-I_2(s)\right)\frac{1}{sC}=0$$

$$(2)\qquad -\left(I_1(s)-I_2(s)\right)\frac{1}{sC}+sLI_2(s)+R_2I_2(s)-gU_1(s)R_2=0$$

$$U_1(s)=\left(I_1(s)-I_2(s)\right)\frac{1}{sC}$$

$$(1)\qquad U_0=\left(R_1+\frac{1}{sC}\right)I_1-\frac{1}{sC}I_2$$

$$(2)\qquad 0=-\left(1+gR_2\right)\frac{1}{sC}I_1+\left(\left(1+gR_2\right)\frac{1}{sC}+sL+R_2\right)I_2$$

$$\begin{bmatrix} R_1+\frac{1}{sC} & -\frac{1}{sC} \\ -\left(1+gR_2\right)\frac{1}{sC} & \left(1+gR_2\right)\frac{1}{sC}+sL+R_2 \end{bmatrix}\cdot\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}=\begin{bmatrix} U_0 \\ 0 \end{bmatrix} \qquad \Rightarrow \qquad I_2=\frac{\Delta_2}{\Delta}$$

$$\Delta=\begin{vmatrix} R_1+\frac{1}{sC} & -\frac{1}{sC} \\ -\left(1+gR_2\right)\frac{1}{sC} & \left(1+gR_2\right)\frac{1}{sC}+sL+R_2 \end{vmatrix}=R_1\left(\frac{1+gR_2}{sC}+sL+R_2\right)+\frac{1}{sC}\left(sL+R_2\right)$$

$$\Delta_1=\begin{vmatrix} U_0 & -\frac{1}{sC} \\ 0 & \left(1+gR_2\right)\frac{1}{sC}+sL+R_2 \end{vmatrix}=U_0\left(\frac{1+gR_2}{sC}+sL+R_2\right)$$

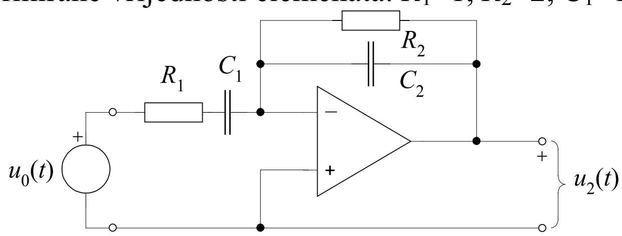
$$\Delta_2 = \begin{vmatrix} R_1 + \dfrac{1}{sC} & U_0 \\ -\left(1 + gR_2\right)\dfrac{1}{sC} & 0 \end{vmatrix} = \frac{1 + gR_2}{sC} U_0$$

$$U_2 = R_2 I_2 - \frac{gR_2}{sC}\left(I_1 - I_2\right) = \frac{-gR_2\Delta_1}{sC\Delta} + \left(R_2 + \frac{gR_2}{sC}\right)\frac{\Delta_2}{\Delta} = U_0 \frac{-\dfrac{gR_2}{sC} sL + \dfrac{R_2}{sC}}{R_1\left(\dfrac{1 + gR_2}{sC} + sL + R_2\right) + \dfrac{1}{sC}\left(sL + R_2\right)}$$

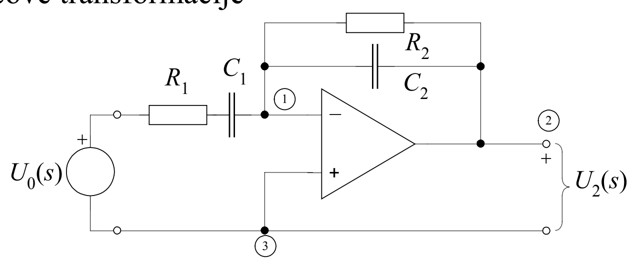
$$U_2 = U_0 \frac{-gsLR_2 + R_2}{R_1\left(1 + gR_2 + sC\left(sL + R_2\right)\right) + sL + R_2}$$

$$U_2 = \frac{3}{s} \cdot \frac{-s + 1}{s^2 + 2s + 3}$$

4. Za mrežu prikazanu slikom odrediti napon na izlazu operacijskog pojačala $u_2(t)$, ako je pobuda $u_0(t)=S(t)$. Zadane su normirane vrijednosti elemenata: $R_1=1$, $R_2=2$, $C_1=1$, $C_2=1$, $A\to\infty$.



<u>Rješenje:</u> Primjena Laplaceove transformacije



(1) $$\frac{U_0}{\dfrac{1}{sC_1}+R_1}=U_1\left(\frac{1}{\dfrac{1}{sC_1}+R_1}+sC_2+\frac{1}{R_2}\right)-U_2\left(sC_2+\frac{1}{R_2}\right)$$

$$U_1=0,\quad \text{jer}\ \ A\to\infty$$

(1) $\Rightarrow$ $$U_2=-\frac{U_0}{\left(\dfrac{1}{sC_1}+R_1\right)\left(sC_2+\dfrac{1}{R_2}\right)}=-\frac{U_0}{\dfrac{R_1}{s}\left(s+\dfrac{1}{R_1C_1}\right)C_2\left(s+\dfrac{1}{R_2C_2}\right)}$$

$$U_2=-\frac{\dfrac{1}{R_1C_2}s}{\left(s+\dfrac{1}{R_1C_1}\right)\left(s+\dfrac{1}{R_2C_2}\right)}U_0=-\frac{1\cdot s\cdot\dfrac{1}{s}}{\left(s+\dfrac{1}{2}\right)(s+1)}=\frac{-1}{\left(s+\dfrac{1}{2}\right)(s+1)}$$

Rastav na parcijalne razlomke:

$$U_2(s)=\frac{-1}{\left(s+\dfrac{1}{2}\right)(s+1)}=\frac{A}{s+\dfrac{1}{2}}+\frac{B}{s+1}=\frac{A(s+1)+B\left(s+\dfrac{1}{2}\right)}{\left(s+\dfrac{1}{2}\right)(s+1)}$$

$$A+B=0\ \Rightarrow\ B=-A$$

$$A+\frac{B}{2}=-1\ \Rightarrow\ A-\frac{A}{2}=-1$$

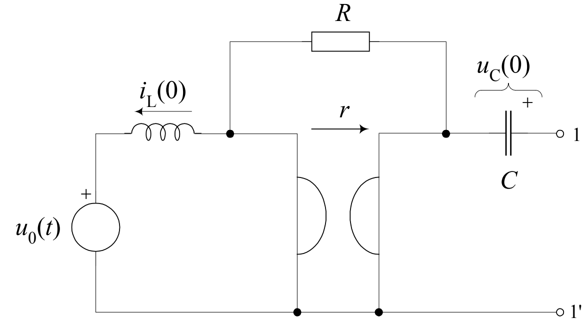
$$\frac{A}{2}=-1\ \Rightarrow\ A=-2$$

$$\underline{B=-A=2}$$

$$U_2(s)=\frac{-2}{s+\dfrac{1}{2}}+\frac{2}{s+1}$$

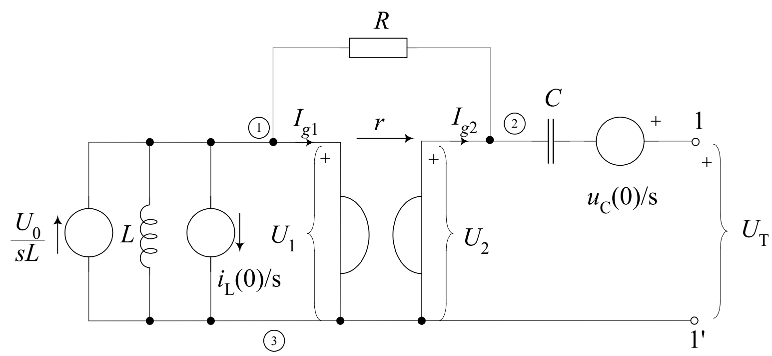
Konačno je: $u_2(t)=\left(-2e^{-\frac{t}{2}}+2e^{-t}\right)\cdot S(t)$

5. Za mrežu prikazanu slikom odrediti nadomjesnu shemu po Theveninu obzirom na priključnice 1-1', koristeći postupak jednadžbi čvorišta, ako je pobuda $u_0(t)=\delta(t)$. Zadane su normirane vrijednosti elemenata: R=0.5, L=1, C=1, r=1 i početni uvjeti $u_C(0)=1$, $i_L(0)=0.5$.



Rješenje: Primjena Laplaceove transformacije

a) Teveninov napon $U_T(s)$:



(1) $$\frac{U_0}{sL}-\frac{i_L(0)}{s}-I_{g1}=U_1\left(\frac{1}{sL}+\frac{1}{R}\right)-U_2\frac{1}{R} \qquad I_{g1}=-U_2\frac{1}{r}$$

(2) $$I_{g2}=-U_1\frac{1}{R}+U_2\frac{1}{R} \qquad I_{g2}=-U_1\frac{1}{r}$$

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(1) $$\frac{U_0}{sL}-\frac{i_L(0)}{s}=U_1\left(\frac{1}{sL}+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right)$$

(2) $$0=-U_1\left(\frac{1}{R}-\frac{1}{r}\right)+U_2\frac{1}{R}$$

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(2) $$\Rightarrow \qquad U_1=\frac{U_2\frac{1}{R}}{\frac{1}{R}-\frac{1}{r}}$$

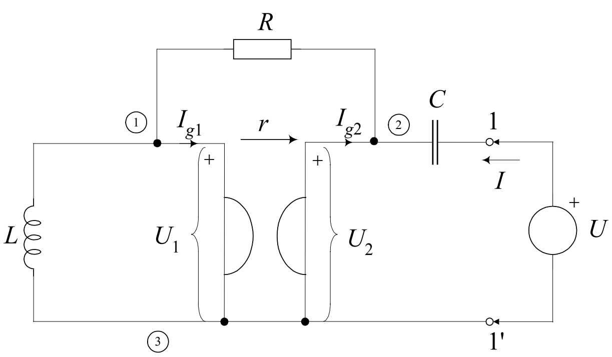
(1) $$\frac{U_0}{sL}-\frac{i_L(0)}{s}=U_2\frac{\frac{1}{R}}{\frac{1}{R}-\frac{1}{r}}\left(\frac{1}{sL}+\frac{1}{R}\right)-U_2\left(\frac{1}{R}+\frac{1}{r}\right) \qquad \Big/\cdot\left(\frac{1}{R}-\frac{1}{r}\right)$$

$$U_2 = \frac{\left(\frac{1}{R}-\frac{1}{r}\right)\left(\frac{U_0}{sL}-\frac{i_L(0)}{s}\right)}{\frac{1}{sRL}+\frac{1}{r^2}} = \frac{\left(r^2-Rr\right)\left(U_0-Li_L(0)\right)}{r^2+sRL}$$

$$U_T(s) = U_2(s) + \frac{u_C(0)}{s} = \frac{\left(r^2-Rr\right)\left(U_0-Li_L(0)\right)}{r^2+sRL} + \frac{u_C(0)}{s} = \frac{(1-0.5)(1-0.5)}{1+0.5s} + \frac{1}{s}$$

$$U_T(s) = \frac{0.5}{s+2} + \frac{1}{s} = \frac{1.5s+2}{s(s+2)}$$

a) Teveninova impedancija $Z_T(s)$:



$$Z_T(s) = \frac{U}{I}, \qquad I = (U-U_2)sC \qquad \Rightarrow \qquad Z_T(s) = \frac{U_2}{I} + \frac{1}{sC}$$

(1) $\quad -I_{g1} = U_1\left(\frac{1}{sL}+\frac{1}{R}\right) - U_2\frac{1}{R} \qquad\qquad I_{g1} = -U_2\frac{1}{r}$

(2) $\quad I_{g2} + I = -U_1\frac{1}{R} + U_2\frac{1}{R} \qquad\qquad I_{g2} = -U_1\frac{1}{r}$

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(1) $\quad 0 = U_1\left(\frac{1}{sL}+\frac{1}{R}\right) - U_2\left(\frac{1}{R}+\frac{1}{r}\right) \qquad \Rightarrow \qquad U_1 = \frac{\frac{1}{R}+\frac{1}{r}}{\frac{1}{sL}+\frac{1}{R}}U_2$

(2) $\quad I = -U_1\left(\frac{1}{R}-\frac{1}{r}\right) + U_2\frac{1}{R}$

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(2) $\quad I = -\frac{\frac{1}{R}+\frac{1}{r}}{\frac{1}{sL}+\frac{1}{R}}\left(\frac{1}{R}-\frac{1}{r}\right)U_2 + U_2\frac{1}{R} = \frac{\frac{1}{sLR}+\frac{1}{r^2}}{\frac{1}{sL}+\frac{1}{R}}U_2 = \frac{r^2+sLR}{r^2(sL+R)}U_2$

$$Z_T(s) = \frac{U_2}{I} + \frac{1}{sC} = \frac{r^2(sL+R)}{r^2+sLR} + \frac{1}{sC} = \frac{1(s+0.5)}{1+0.5s} + \frac{1}{s} = \frac{2s+1}{s+2} + \frac{1}{s}$$

$$Z_T(s) = \frac{2(s^2+s+1)}{s(s+2)}$$