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Električni krugovi

Zadaci za vježbu za prvi međuispit

- verzija: v2.0.14

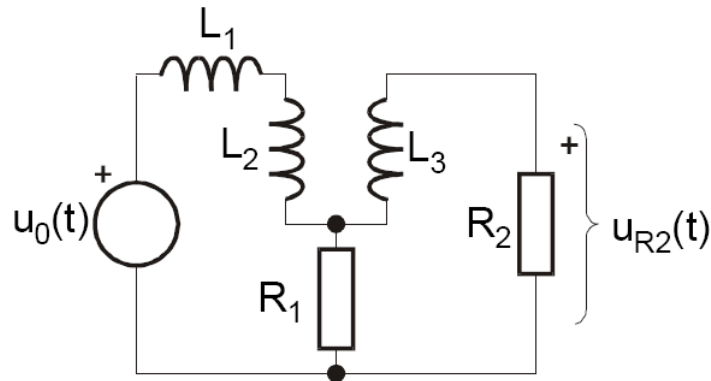
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by: Tywin



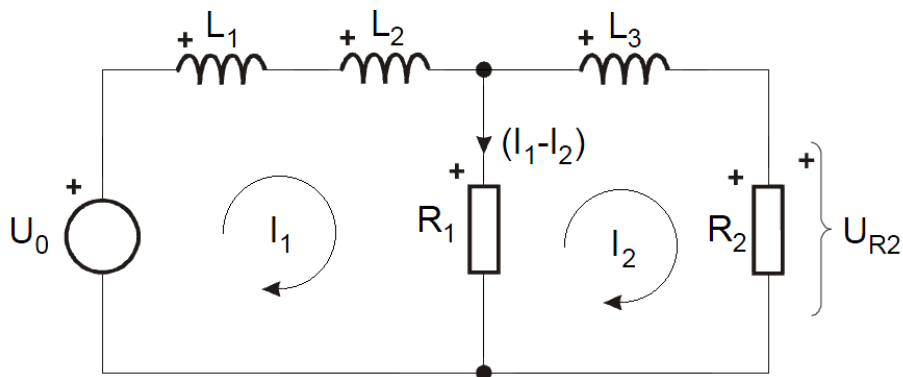
Rujan 2009.

1. Za mrežu prikazanu slikom izračunati napon $u_{R2}(t)$ ako su zadane normalizirane vrijednosti elemenata: $R_1 = 1$, $R_2 = 1$, $L_1 = 1$, $L_2 = 2$, $L_3 = 4$ te napon generatora $u_0(t) = S(t)$.



Rješenje:

Sliku ćemo malo preurediti tako da bolje vidimo što imamo. Označit ćemo si struje, proizvoljno, prema njima označiti mjesto višeg potencijala na elementima (plusom) te smjer obilaska petlje.



$$(I) \quad U_0 - I_1(sL_1 + sL_2) - (I_1 - I_2)R_1 = 0$$

$$(II) \quad (I_1 - I_2)R_1 - I_2(sL_3 + R_2) = 0$$

$$(I) \quad \frac{1}{s} - 3sI_1 - I_1 - I_2 = 0$$

$$(II) \quad I_1 - I_2 - I_2(4s + 1) = 0$$

$$(II) \quad I_1 = I_2(4s + 2)$$

$$(I) \quad \frac{1}{s} - I_1(3s + 1) - I_2 = 0$$

$$(II) \text{ u } (I) \quad \frac{1}{s} - I_2(4s + 2)(3s + 1) - I_2 = 0$$

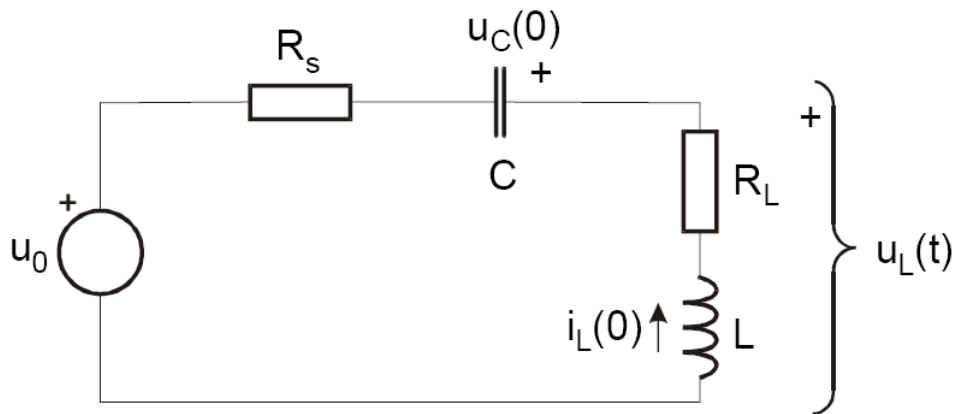
$$I_2[(4s + 2)(3s + 1) + 1] = \frac{1}{s}$$

$$I_2(12s^2 + 10s + 1) = \frac{1}{s}$$

$$I_2 = \frac{1}{s(12s^2 + 10s + 1)}$$

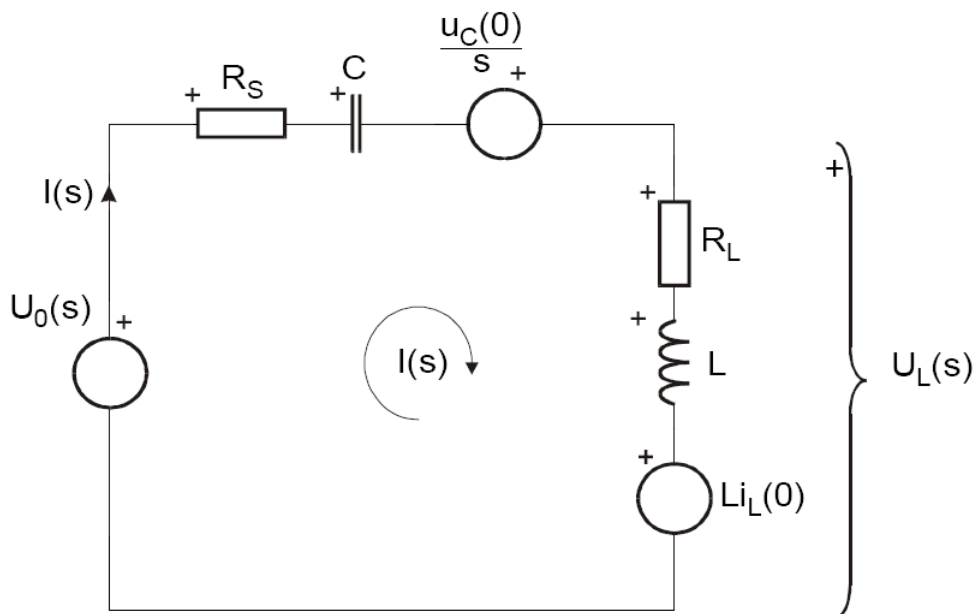
$$U_{R2}(s) = I_2 R_2 = \frac{1}{s(12s^2 + 10s + 1)}$$

2. Odrediti odziv $u_L(t)$ mreže prikazanom slikom ako je zadano: $R_s = R_L = 1$, $L = 1$, $C = 1$, $i_L(0) = 1$, $u_C(0) = 1$ i poticaj: $u_0(t) = e^{-t} S(t)$.



Rješenje:

Kao i prošli put, a tako ćemo i svaki put, prema potrebi modificirati sliku. Prebaciti je u Laplaceovo područje (početni uvjeti) te si označiti struje, smjerove obilaska, mjesta višeg potencijala i sve ostalo po potrebi.



Prvo je potrebno prebaciti izvor u Laplaceovu domenu:

$$u_0(t) = e^{-t} \cdot S(t) \xrightarrow{\mathcal{L}} U_0(s) = \frac{1}{s+1}$$

Samo je jedna petlja pa za nju pišemo:

$$U_0 - I \left(R_S + \frac{1}{sC} + R_L + sL \right) + \frac{u_C(0)}{s} - Li_L(0) = 0$$

$$I \left(2 + s + \frac{1}{s} \right) = \frac{1}{s+1} + \frac{1}{s} - 1$$

$$I \left(\frac{s^2 + 2s + 1}{s} \right) = \frac{s + s + 1 - s^2 - s}{s(s+1)}$$

$$I \frac{(s+1)^2}{s} = \frac{-s^2 + s + 1}{s(s+1)}$$

$$I = \frac{-s^2 + s + 1}{(s+1)^3}$$

A napon koji tražimo $U_L(s)$ iznosi:

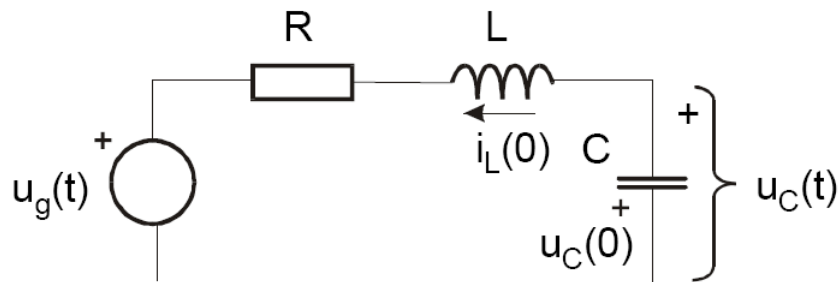
$$U_L = Li_L(0) + I(sL + R_L)$$

$$U_L = 1 + \frac{-s^2 + s + 1}{(s+1)^3} (s+1) = 1 + \frac{-s^2 + s + 1}{(s+1)^2} = 1 + \frac{-s^2 + s + 1}{s^2 + 2s + 1}$$

$$U_L = \frac{3s+2}{s^2+2s+1} = \frac{3s+2}{(s+1)^2} = \frac{3s+3-1}{(s+1)^2} = 3 \frac{s+1}{(s+1)^2} - \frac{1}{(s+1)^2}$$

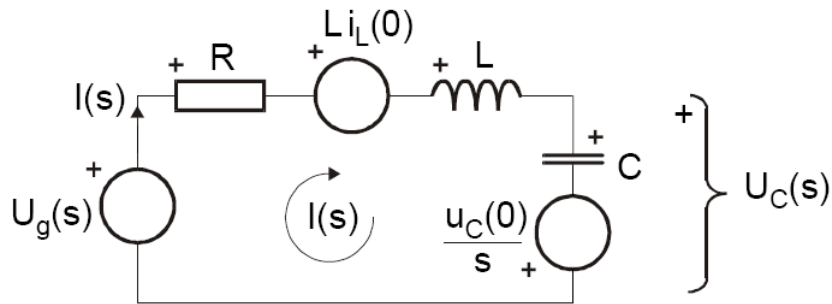
$$U_L = \frac{3}{s+1} - \frac{1}{(s+1)^2} \xrightarrow{\mathcal{L}^{-1}} \boxed{u_L(t) = (3-t) \cdot e^{-t} \cdot S(t)}$$

3. Za mrežu na slici odrediti napon na kapacitetu $u_C(t)$. Zadano je: $R = 4$, $C = 1/2$, $L = 2$, $i_L(0) = 1.2 \text{ A}$, $u_C(0) = 2.6 \text{ V}$, $u_g(t) = S(t)$.



Rješenje:

Mreža smo prilagodili i prema njoj pišemo jednačbu konture:



$$U_g - I \left(R + sL + \frac{1}{sC} \right) - Li_L(0) + \frac{u_C(0)}{s} = 0$$

$$I \left(4 + 2s + \frac{2}{s} \right) = \frac{1}{s} - 2.4 + \frac{2.6}{s}$$

$$I \frac{2s^2 + 4s + 2}{s} = \frac{-2.4s + 3.6}{s}$$

$$I = \frac{-1.2s + 1.8}{s^2 + 2s + 1} = \frac{-1.2s + 1.8}{(s + 1)^2}$$

A napon koji tražimo $U_C(s)$ iznosi:

$$U_C = -\frac{u_C(0)}{s} + I \frac{1}{sC}$$

$$U_C = -\frac{2.6}{s} + \frac{2}{s} \cdot \frac{-1.2s + 1.8}{(s + 1)^2} = -\frac{2.6}{s} + \frac{-2.4s + 3.6}{s(s + 1)^2}$$

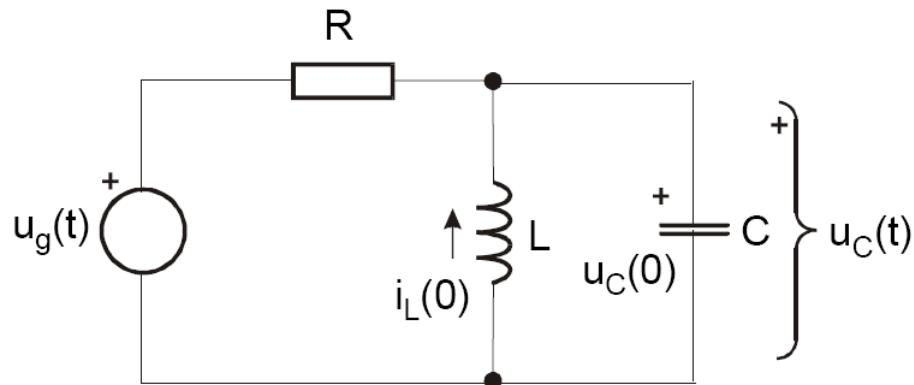
$$\left\{ \begin{array}{l} \frac{-2.4s + 3.6}{s(s + 1)^2} = \frac{A}{s} + \frac{Bs + C}{(s + 1)^2} = \frac{As^2 + 2As + A + Bs^2 + Cs}{s(s + 1)^2} \\ A + B = 0 \\ 2A + C = -2.4 \\ A = 3.6 \\ C = -9.6 ; B = -3.6 \end{array} \right\}$$

$$U_C = -\frac{2.6}{s} + \frac{3.6}{s} + \frac{-3.6s - 9.6}{(s + 1)^2} = \frac{1}{s} + \frac{-3.6s - 3.6 - 6}{(s + 1)^2}$$

$$U_C = \frac{1}{s} + \frac{-3.6s - 3.6}{(s + 1)^2} + \frac{-6}{(s + 1)^2} = \frac{1}{s} - \frac{3.6}{s + 1} - \frac{6}{(s + 1)^2}$$

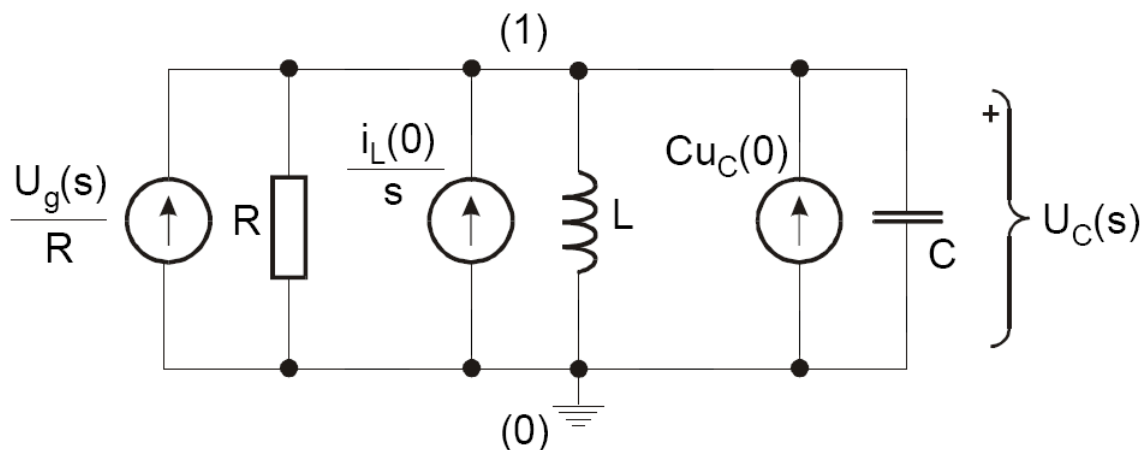
$$U_C(s) \xrightarrow{\mathcal{L}^{-1}} \boxed{u_C(t) = [1 - 3.6e^{-t} - 6te^{-t}]S(t)}$$

4. Odrediti napon $u_C(t)$ u prikazanoj mreži ako je zadano: $R = 2$, $C = 0.5$, $L = 1$, $u_C(0) = 2$, $i_L(0) = 4$, $u_g(t) = 2e^{-t}S(t)$.



Rješenje:

Prepoznamo samo dva čvora u mreži i dvije konture. Prema tome, lakše je koristiti se metodom napona čvorova pa za nju preoblikujemo mrežu i proizvoljno uzemljimo jedan čvor:



Za početak, napon izvora je:

$$u_g(t) = 2e^{-t} \cdot S(t) \xrightarrow{\mathcal{L}} U_g(s) = \frac{2}{s+1}$$

A za drugi čvor (1) pišemo jednadžbu:

$$\begin{aligned} U_1 \left[\frac{1}{R} + \frac{1}{sL} + sC \right] &= \frac{U_g}{R} + \frac{i_L(0)}{s} + Cu_C(0) \\ U_1 \left[\frac{1}{2} + \frac{1}{s} + \frac{s}{2} \right] &= \frac{2}{s+1} + \frac{4}{s} + 1 \\ U_1 \frac{s^2 + s + 2}{2s} &= \frac{s^2 + 6s + 4}{s(s+1)} \\ U_1 &= \frac{2s^2 + 12s + 8}{(s+1)(s^2 + s + 2)} = \frac{A}{s+1} + \frac{Bs + C}{s^2 + s + 2} \\ &= \frac{As^2 + As + 2A + Bs^2 + Cs + Bs + C}{(s+1)(s^2 + s + 2)} \end{aligned}$$

$$\begin{cases} A + B = 2 \\ A + B + C = 12 \\ 2A + C = 8 \\ A = -1 ; B = 3 ; C = 10 \end{cases}$$

$$U_1 = -\frac{1}{s+1} + \frac{3s+10}{s^2+s+2} = U_C(s)$$

$$U_C = -\frac{1}{s+1} + \frac{3s+10}{s^2+s+2} = -\frac{1}{s+1} + \frac{3s+10}{\left(s+\frac{1}{2}\right)^2 + \frac{7}{4}}$$

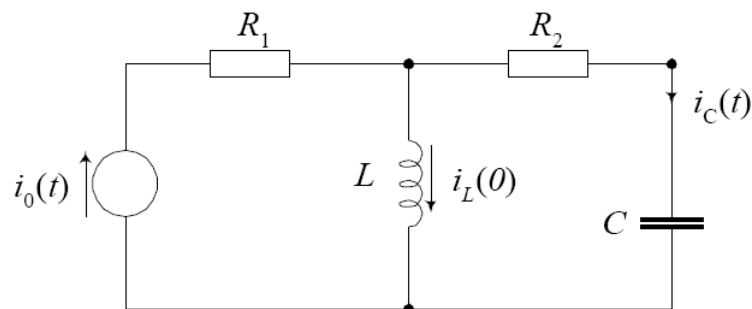
$$U_C = -\frac{1}{s+1} + \frac{3s + \frac{3}{2} + \frac{17}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2}$$

$$U_C = -\frac{1}{s+1} + \frac{3s + \frac{3}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2} + \frac{\frac{17}{2} \cdot \frac{2}{\sqrt{7}} \cdot \frac{\sqrt{7}}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2}$$

$$U_C = -\frac{1}{s+1} + 3 \frac{s + \frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2} + \frac{17}{\sqrt{7}} \frac{\frac{\sqrt{7}}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2}$$

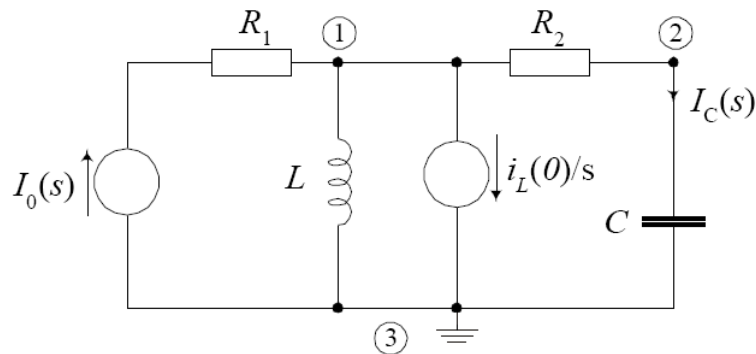
$$U_C(s) \xrightarrow{\mathcal{L}^{-1}} \boxed{u_C(t) = \left[-e^{-t} + 3e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{7}}{2}t\right) + \frac{17}{\sqrt{7}}e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{7}}{2}t\right) \right] \cdot S(t)}$$

5. Za mrežu prikazanu slikom napisati jednadžbe čvorišta. Izračunati struju $i_C(t)$, ako je zadana pobuda $i_0(t) = \delta(t)$, početna struja kroz induktivitet $i_L(0) = 1$ i normirane vrijednosti elemenata $R_1 = 1, R_2 = 1, L = 1$ i $C = 1$.



Rješenje:

Prvo, slika ☺:



Iako nije običaj postaviti si ovakvo čvorove i tako ih označiti, kao su ih oni već tako označili onda zapišimo:

$$\begin{aligned} (1) \quad & U_1 \left(\frac{1}{sL} + \frac{1}{R_2} \right) - U_2 \frac{1}{R_2} - U_3 \frac{1}{sL} = I_0 - \frac{i_L(0)}{s} \\ (2) \quad & U_2 \left(\frac{1}{R_2} + sC \right) - U_1 \frac{1}{R_2} - U_3 sC = 0 \\ (3) \quad & U_3 \left(\frac{1}{sL} + sC \right) - U_1 \frac{1}{sL} - U_2 sC = \frac{i_L(0)}{s} - I_0 \end{aligned}$$

Kako samo čvor (3) proizvoljno uzemljili, za njega nije potrebno pisati jednadžbu – pošto nam je njegov napon poznat $U_3 = 0$, koji možemo uvrstiti u preostale dvije jednadžbe. Isto tako, čvor (2) je tu tako postavljen iako ga za rješavanje nije nužno imati, ali kako bi udovoljili autoru ovog zadatka budemo ga ostavili i rješavali s njim.

$$\begin{aligned} (1) \quad & U_1 \left(\frac{1}{sL} + \frac{1}{R_2} \right) - U_2 \frac{1}{R_2} = I_0 - \frac{i_L(0)}{s} \\ (2) \quad & U_2 \left(\frac{1}{R_2} + sC \right) - U_1 \frac{1}{R_2} = 0 \end{aligned}$$

$$(1) \quad U_1 \left(\frac{1}{s} + 1 \right) - U_2 = 1 - \frac{1}{s}$$

$$(2) \quad U_2(1 + s) - U_1 = 0$$

$$(2) \quad U_1 = U_2(1 + s)$$

$$(2) \text{ u } (1) \quad U_2(1 + s) \left(\frac{1}{s} + 1 \right) - U_2 = 1 - \frac{1}{s}$$

$$U_2 \left(\frac{1}{s} + 1 + 1 + s \right) - U_2 = \frac{s - 1}{s}$$

$$U_2 \left(\frac{1}{s} + 1 + s \right) = \frac{s - 1}{s}$$

$$U_2 = \frac{s - 1}{s^2 + s + 1}$$

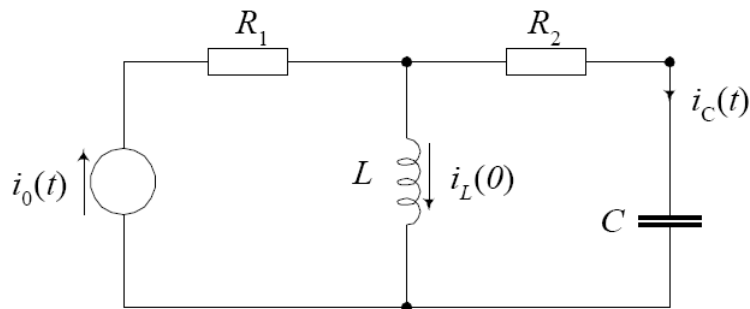
Sada možemo odrediti struju $I_C(s)$:

$$I_C = U_2 \cdot sC = \frac{s^2 - s}{s^2 + s + 1} = \frac{s^2 + s + 1 - 2s - 1}{s^2 + s + 1} = 1 - \frac{2s + 1}{s^2 + s + 1}$$

$$I_C = 1 - \frac{2s + 1}{\left(s + \frac{1}{2}\right)^2 + \frac{3}{4}} = 1 - \frac{2s + 1}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = 1 - 2 \frac{s + \frac{1}{2}}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

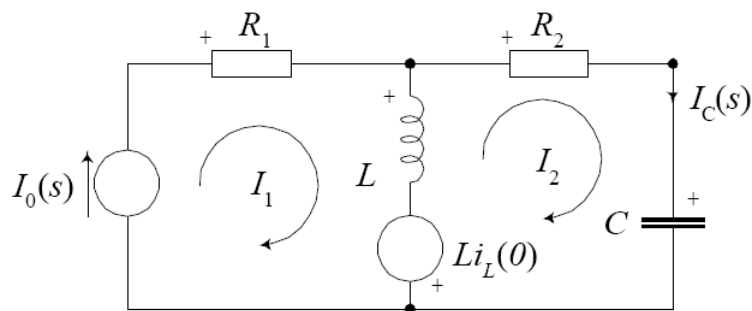
$$I_C(s) \xrightarrow{\mathcal{L}^{-1}} \boxed{i_C(t) = \delta(t) - 2e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}}{2}t\right) S(t)}$$

6. Za mrežu prikazanu slikom napisati jednačbe petlji. Izračunati struju $i_C(t)$, ako je zadana pobuda $i_0(t) = \delta(t)$, početna struja kroz induktivitet $i_L(0) = 1$ i normirane vrijednosti elemenata $R_1 = 1, R_2 = 1, L = 1$ i $C = 1$.



Rješenje:

Iako je zadatak isti kao prijašnji, mrežu ćemo prebaciti u Laplaceovo područje tako odgovara zahtjevu (jednačbama petlji):



Za svaku petlju pišemo jednačbu:

$$(1) \quad U_1 - I_1 R_1 - (I_1 - I_2)sL + Li_L(0) = 0$$

$$(2) \quad -Li_L(0) + (I_1 - I_2)sL - I_2 \left(R_2 + \frac{1}{sC}\right) = 0$$

Pri tome smo dogovorno označili da je napon strujnog izvora U_1 i da je mjesto višeg potencijala u smjeru struje I_0 . Također, taj nam strujni izvor određuje struju u toj grani pa prema tome znamo iznos struje $I_1 = I_0 = \mathcal{L}(i_0(t)) = 1$ pa jednačba (1) više nije potrebna i ostaje za riješiti samo jednačbu (2) za koju vrijedi $I_2 = I_C$.

$$-Li_L(0) + (I_0 - I_C)sL - I_C\left(R_2 + \frac{1}{sC}\right) = 0$$

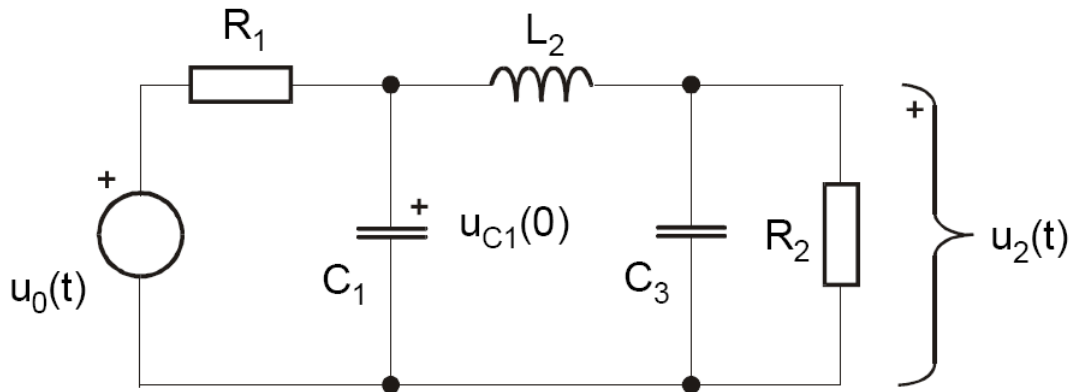
$$-1 + (1 - I_C)s - I_C\left(1 + \frac{1}{s}\right) = 0$$

$$-1 + s - sI_C - I_C - \frac{1}{s}I_C = 0$$

$$I_C\left(s + 1 + \frac{1}{s}\right) = s - 1$$

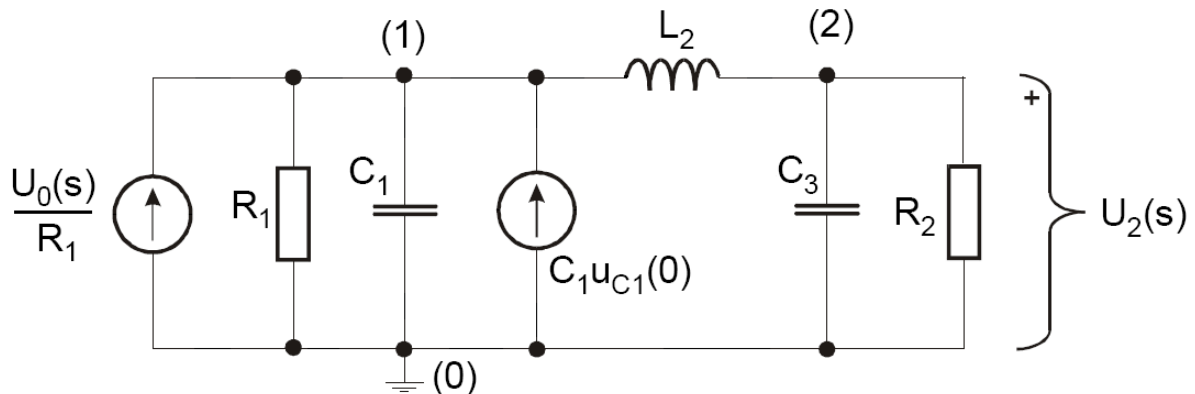
$$I_C = \frac{s^2 - s}{s^2 + s + 1} \xrightarrow{\mathcal{L}^{-1}} \boxed{i_C(t) = \delta(t) - 2e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}}{2}t\right) S(t)}$$

7. Izračunati odziv napona $u_2(t)$ na otporu R_2 za mrežu prikazanu slikom. Zadano je: pobuda $u_0(t) = \delta(t)$, početni uvjet na kondenzatoru C_1 je $u_{C1}(0) = 1$ i normalizirane vrijednosti elemenata $R_1 = R_2 = 1$, $C_1 = C_3 = 1$, $L_2 = 2$.



Rješenje:

Tri čvora – dvije nepoznanice od kojih je jedna koju tražimo, tri petlje – tri nepoznanice i još dodatni račun za traženu nepoznanicu. Dakle, metoda napona čvorova pa za nju prilagođena Laplaceova transformacija je:



$$(1) \quad U_1 \left(\frac{1}{R_1} + sC_1 + \frac{1}{sL_2} \right) - U_2 \frac{1}{sL_2} = \frac{U_0}{R_1} + C_1 u_c(0)$$

$$(2) \quad U_2 \left(\frac{1}{sL_2} + sC_3 + \frac{1}{R_2} \right) - U_1 \frac{1}{sL_2} = 0$$

$$(1) \quad U_1 \left(1 + s + \frac{1}{2s} \right) - U_2 \frac{1}{2s} = 1 + 1$$

$$(2) \quad U_2 \left(\frac{1}{2s} + s + 1 \right) - U_1 \frac{1}{2s} = 0$$

$$(2) \quad U_1 = U_2(1 + 2s^2 + 2s)$$

$$(2) \text{ u } (1) \quad U_2(2s^2 + 2s + 1) \left(\frac{2s^2 + 2s + 1}{2s} \right) - U_2 \frac{1}{2s} = 2$$

$$U_2 \left(\frac{4s^4 + 4s^3 + 2s^2 + 4s^3 + 4s^2 + 2s + 2s^2 + 2s + 1}{2s} - \frac{1}{2s} \right) = 2$$

$$U_2(4s^3 + 8s^2 + 8s + 4) = 4$$

$$U_2(s^3 + s^2 + s^2 + s + s + 1) = 1$$

$$U_2(s^2(s + 1) + s(s + 1) + (s + 1)) = 1$$

$$U_2(s + 1)(s^2 + s + 1) = 1$$

$$U_2 = \frac{1}{(s + 1)(s^2 + s + 1)} = \frac{A}{s + 1} + \frac{Bs + C}{s^2 + s + 1}$$

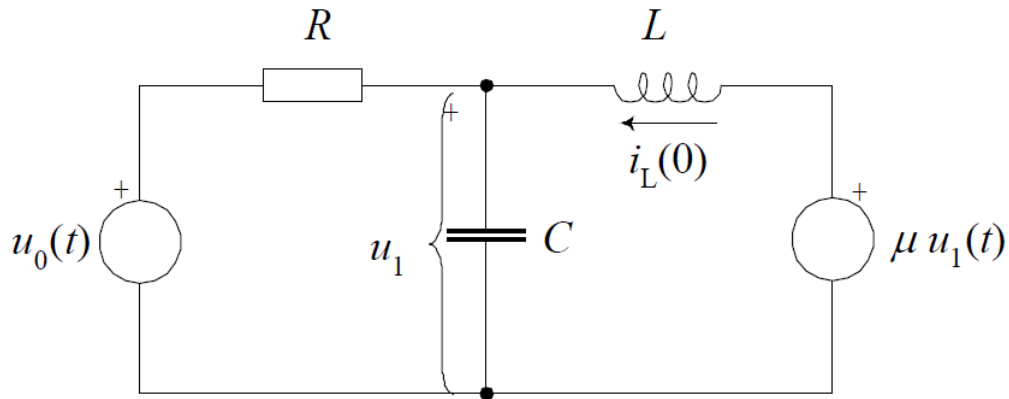
$$\left\{ \begin{array}{l} \frac{As^2 + As + A + Bs^2 + Cs + Bs + C}{(s + 1)(s^2 + s + 1)} \\ A + B = 0 \\ A + B + C = 0 \\ A + C = 1 \\ A = 1 ; B = -1 ; C = 0 \end{array} \right\}$$

$$U_2 = \frac{1}{s + 1} - \frac{s}{s^2 + s + 1} = \frac{1}{s + 1} - \frac{s + \frac{1}{2} - \frac{1}{2}}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$U_2 = \frac{1}{s + 1} - \frac{s + \frac{1}{2}}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} + \frac{1}{\sqrt{3}} \frac{\frac{\sqrt{3}}{2}}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

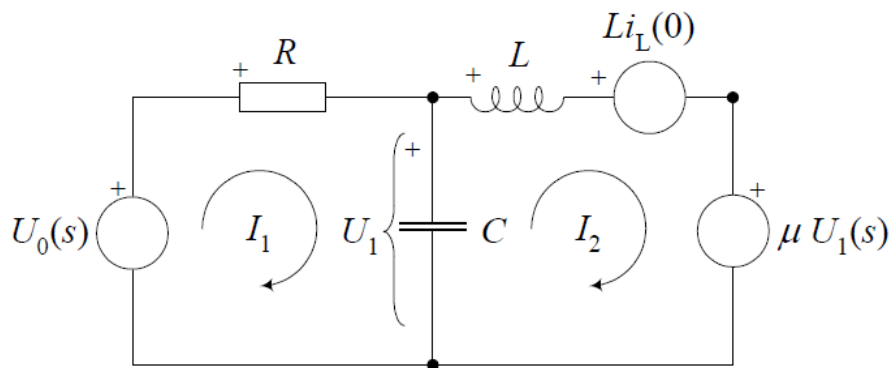
$$U_2(s) \xrightarrow{\mathcal{L}^{-1}} \boxed{u_2(t) = \left[e^{-t} - e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}}{2}t\right) + \frac{\sqrt{3}}{3} e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{3}}{2}t\right) \right] S(t)}$$

8. Za mrežu prikazanu slikom napisati jednađbe petlji. Konačni oblik jednađbi prikazati u formi matrice jednađbe. Izračunati napon $U_1(s)$, ako je zadana pobuda $u_0(t) = S(t)$, $\mu = 2/3$, početna struja kroz induktivitet $i_L(0) = 1$ i normirane vrijednosti elemenata: $R = 1, L = 1/2$ i $C = 1/3$.



Rješenje:

Mreža nakon Laplaceove transformacije, prilagođena za jednađbe petlji:



Za svaku petlju pišemo jednađbu:

$$(1) \quad U_0 - I_1 R - (I_1 - I_2) \frac{1}{sC} = 0$$

$$(2) \quad (I_1 - I_2) \frac{1}{sC} - Li_L(0) - I_2 sL - \mu U_1 = 0$$

$$(1) \quad I_1 \left(1 - \frac{3}{s}\right) - I_2 \frac{3}{s} = \frac{1}{s} \quad / \cdot s$$

$$(2) \quad I_2 \left(\frac{3}{s} + \frac{s}{2}\right) - I_1 \frac{3}{s} = -\frac{1}{2} - \frac{2}{3} U_1 \quad / \cdot 6s$$

$$(1) \quad I_1(s + 3) - 3I_2 = 1$$

$$(2) \quad I_2(3s^2 + 18) - 18I_1 = -3s - 4sU_1$$

A zapišimo i napon na kondenzatoru:

$$U_1 = (I_1 - I_2) \frac{1}{sC} \Rightarrow I_1 = U_1 sC + I_2$$

$$I_1 = U_1 \frac{s}{3} + I_2 \quad u(1) \ i(2)$$

$$(1) \quad U_1 \frac{s}{3}(s+3) + I_2(s+3) - 3I_2 = 1$$

$$(2) \quad I_2(3s^2 + 18) - 18U_1 \frac{s}{3} - 18I_2 = -3s - 4sU_1$$

$$(1) \quad U_1 \frac{s}{3}(s+3) + sI_2 = 1 \quad / \cdot 3$$

$$(2) \quad 3s^2 I_2 + 18I_2 - 6sU_1 - 18I_2 = -3s - 4sU_1$$

$$(1) \quad U_1(s^2 + 3s) + 3sI_2 = 3$$

$$(2) \quad 3sI_2 = 2U_1 - 3$$

$$(2) \text{ u } (1) \quad U_1(s^2 + 3s) + 2U_1 - 3 = 3$$

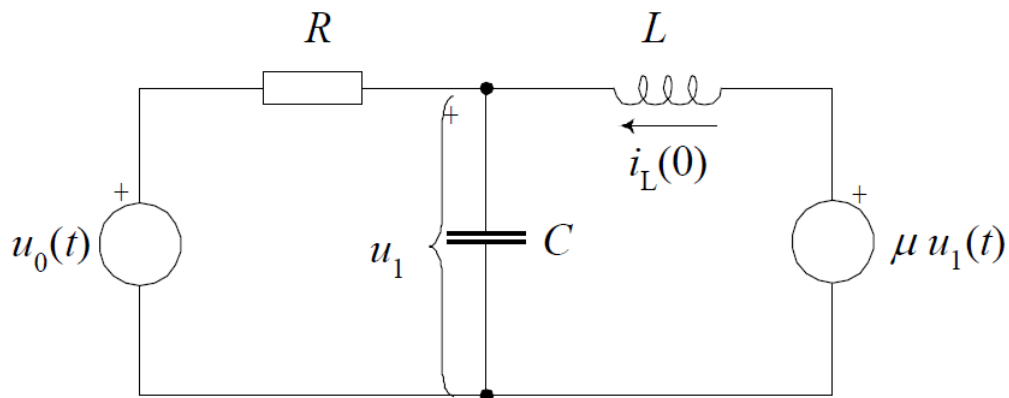
$$U_1 = \frac{6}{s^2 + 3s + 2} = \frac{6}{(s+1)(s+2)}$$

$$\left\{ \begin{array}{l} \frac{A}{s+1} + \frac{B}{s+2} = \frac{As + 2A + Bs + B}{(s+1)(s+2)} \\ A + B = 0 \\ 2A + B = 6 \\ A = 6 \quad ; \quad B = -6 \end{array} \right\}$$

$$U_1 = \frac{6}{s+1} - \frac{6}{s+2} = 6 \left(\frac{1}{s+1} - \frac{1}{s+2} \right)$$

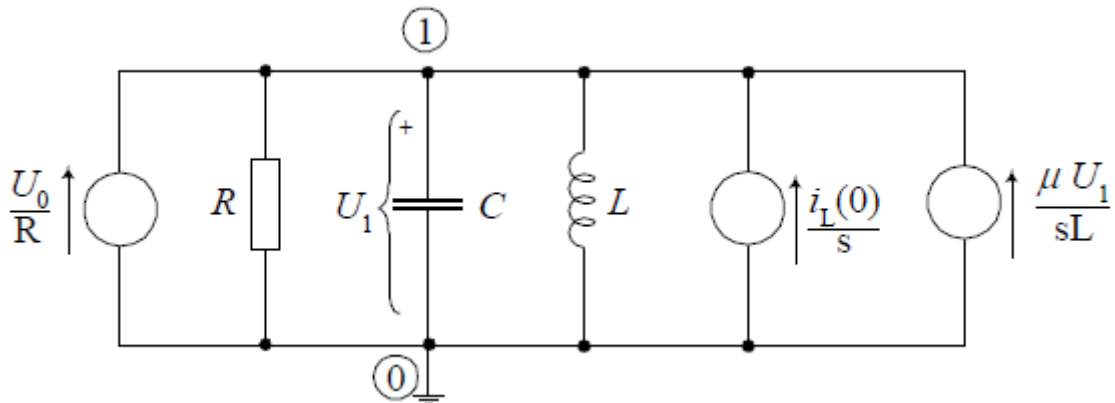
$$U_1(s) \xrightarrow{\mathcal{L}^{-1}} \boxed{u_1(t) = 6(e^{-t} - e^{-2t})S(t)}$$

9. Za mrežu prikazanu slikom napisati jednađbe petlji. Konačni oblik jednađbi prikazati u formi matrične jednađbe. Izračunati napon $U_1(s)$, ako je zadana pobuda $u_0(t) = S(t)$, $\mu = 2/3$, početna struja kroz induktivitet $i_L(0) = 1$ i normirane vrijednosti elemenata: $R = 1, L = 1/2$ i $C = 1/3$.



Rješenje:

Mreža nakon Laplaceove transformacije, prilagođena za jednačbe petlji:



Samo je jedan čvor (jer smo drugi (0) uzemljili) pa za njega pišemo jednačbu:

$$U_1 \left(\frac{1}{R} + sC + \frac{1}{sL} \right) = \frac{U_0}{R} + \frac{i_L(0)}{s} + \frac{\mu U_1}{sL}$$

$$U_1 \left(\frac{1}{R} + sC + \frac{1-\mu}{sL} \right) = \frac{U_0}{R} + \frac{i_L(0)}{s}$$

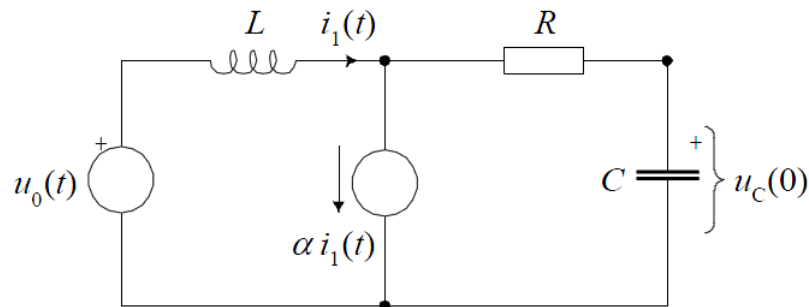
$$U_1 \left(1 + \frac{s}{3} + \frac{1}{2s} \right) = \frac{1}{s} + \frac{1}{s}$$

$$U_1 \left(1 + \frac{s}{3} + \frac{2}{3s} \right) = \frac{2}{s}$$

$$U_1 = \frac{6}{s^2 + 3s + 2} = 6 \left(\frac{1}{s+1} - \frac{1}{s+2} \right)$$

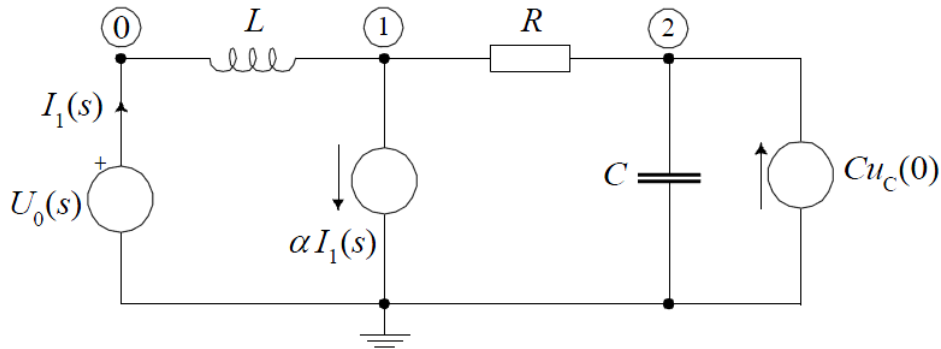
$$U_1(s) \xrightarrow{\mathcal{L}^{-1}} \boxed{u_1(t) = 6(e^{-t} - e^{-2t})S(t)}$$

- 10.** Za mrežu prikazanu slikom napisati jednačbe čvorišta. Konačni oblik jednačbi prikazati u formi matrične jednačbe. Izračunati struju $I_1(s)$, ako je zadana pobuda $u_0(t) = S(t)$, $\alpha = 1/2$, početni napon na kapacitetu $u_C(0) = 1/2$, i normirane vrijednosti elemenata $R = 2, L = 1$ i $C = 1$.



Rješenje:

Nakon Laplaceove transformacije mreže:



Sada imamo 4 čvora, što znači 3 nepoznanice. No kako je čvor (0) na naponu izvora U_0 tada njegov iznos znamo pa je jedna nepoznanica manje, a baš smo iz tog razloga stavili tom čvoru oznaku (0) – da se poklapa sa naponom izvora. No kako smo ga stavili na to mjesto znači da nekako moramo i zapisati struju izvora a to je baš ona struja koju mi tražimo i koja nam je nepoznata I_1 .

$$(0) \quad U_0 \frac{1}{sL} - U_1 \frac{1}{sL} = I_1$$

$$(1) \quad U_1 \left(\frac{1}{sL} + \frac{1}{R} \right) - U_0 \frac{1}{sL} - U_2 \frac{1}{R} = -\alpha I_1$$

$$(2) \quad U_2 \left(\frac{1}{R} + sC \right) - U_1 \frac{1}{R} = Cu_c(0)$$

$$(0) \quad \frac{1}{s^2} - \frac{U_1}{s} = I_1$$

$$(1) \quad U_1 \left(\frac{1}{s} + \frac{1}{2} \right) - \frac{1}{s^2} - \frac{U_2}{2} = -\frac{I_1}{2}$$

$$(2) \quad U_2 \left(\frac{1}{2} + s \right) - \frac{U_1}{2} = \frac{1}{2}$$

$$(0) \quad I_1 = \frac{1}{s^2} - \frac{U_1}{s}$$

$$(2) \quad U_2 \left(\frac{2s+1}{2} \right) = \frac{U_1+1}{2} \Rightarrow U_2 = \frac{U_1}{2s+1} + \frac{1}{2s+1}$$

$$(0) \text{ i } (2) \text{ u } (1) \quad U_1 \frac{s+2}{2s} - \frac{1}{s^2} - U_1 \frac{1}{2} \cdot \frac{1}{2s+1} - \frac{1}{2} \cdot \frac{1}{2s+1} = U_1 \frac{1}{2s} - \frac{1}{2s^2}$$

$$U_1 \left(\frac{s+2}{2s} - \frac{1}{2(2s+1)} - \frac{1}{2s} \right) = \frac{1}{s^2} + \frac{1}{2(2s+1)} - \frac{1}{2s^2}$$

$$U_1 \frac{2s^2 + s + 4s + 2 - s - 2s - 1}{2s(2s+1)} = \frac{4s + 2 + s^2 - 2s - 1}{2s^2(2s+1)}$$

$$U_1(2s^2 + 2s + 1) = \frac{s^2 + 2s + 1}{s}$$

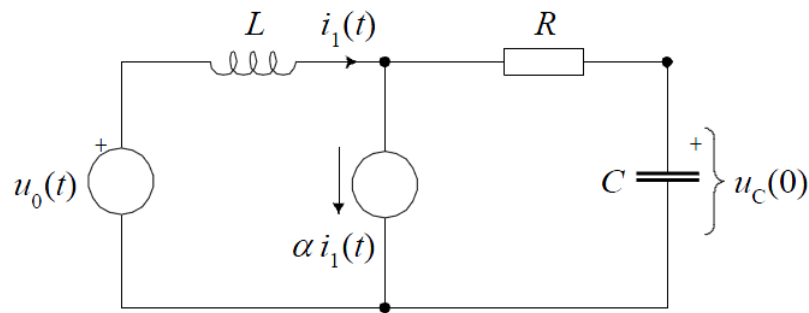
$$U_1 = \frac{s^2 + 2s + 1}{s(2s^2 + 2s + 1)} \quad u(0)$$

$$(0) \quad I_1 = \frac{1}{s^2} - \frac{s^2 + 2s + 1}{s^2(2s^2 + 2s + 1)} = \frac{2s^2 + 2s + 1 - s^2 - 2s - 1}{s^2(2s^2 + 2s + 1)}$$

$$I_1 = \frac{1}{2s^2 + 2s + 1} = \frac{1}{2} \cdot \frac{1}{s^2 + s + \frac{1}{2}} = \frac{\frac{1}{2}}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

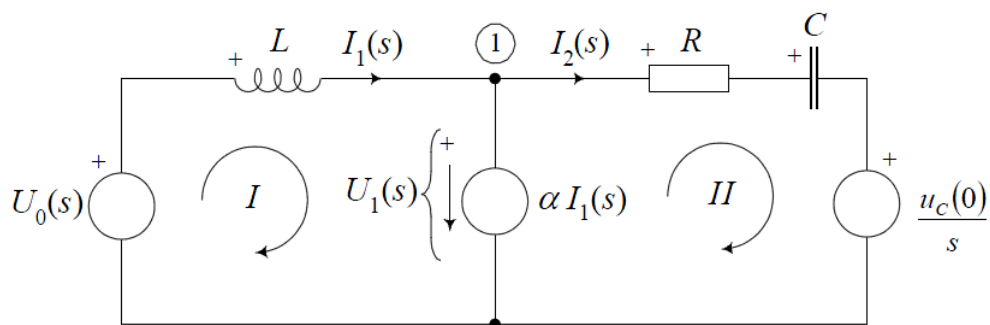
$$I_1(s) \xrightarrow{\mathcal{L}^{-1}} \boxed{i_1(t) = e^{-\frac{t}{2}} \sin\left(\frac{t}{2}\right) \cdot S(t)}$$

- 11.** Za mrežu prikazanu slikom napisati jednađbe petlji. Izračunati struju $I_1(s)$, ako je zadana pobuda $u_0(t) = S(t)$, $\alpha = 1/2$, početni napon na kapacitetu $u_C(0) = 1/2$, i normirane vrijednosti elemenata $R = 2, L = 1$ i $C = 1$.



Rješenje:

Ovaj put mrežu prilagođujemo za jednađbe petlji:



$$(I) \quad U_0 - I_1 s L - U_1 = 0$$

$$(II) \quad U_1 - I_2 \left(R + \frac{1}{sC} \right) - \frac{u_C(0)}{s} = 0$$

$$(1) \quad I_1 = \alpha I_1 + I_2$$

$$(I) + (II) \quad \frac{1}{s} - I_1 s - I_2 \left(2 + \frac{1}{s}\right) - \frac{1}{2s} = 0$$

$$(1) \quad I_2 = \frac{I_1}{2}$$

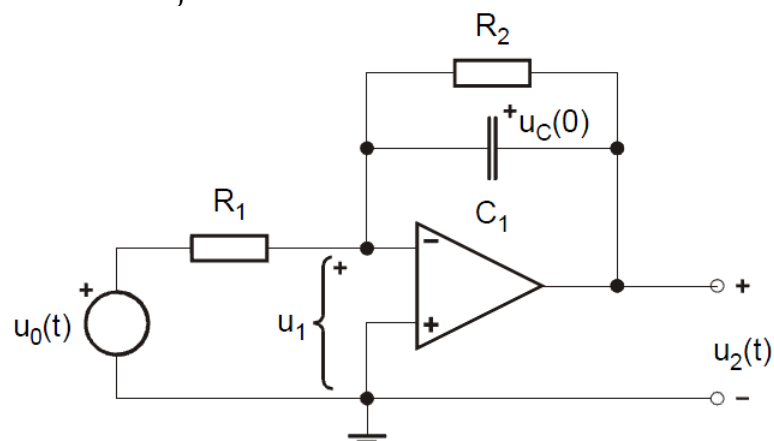
$$(1) \text{ u } (I) + (II) \quad I_1 s + \frac{I_1}{2} \left(\frac{2s+1}{s}\right) = \frac{1}{s} - \frac{1}{2s}$$

$$I_1 \left(s + \frac{2s+1}{2s}\right) = \frac{1}{2s}$$

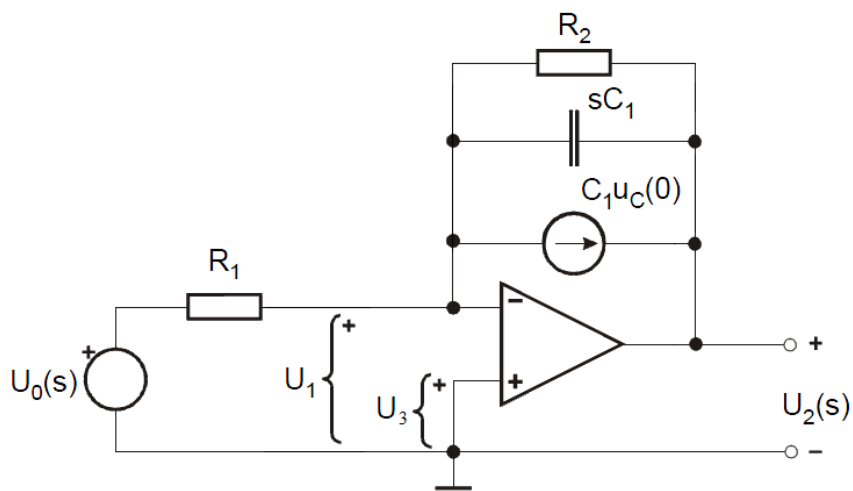
$$I_1 \cdot \frac{2s^2 + 2s + 1}{s} = \frac{1}{2s} \Rightarrow I_1 = \frac{1}{2s^2 + 2s + 1}$$

$$I_1(s) \xrightarrow{\mathcal{L}^{-1}} \boxed{i_1(t) = e^{-\frac{t}{2}} \sin\left(\frac{t}{2}\right) \cdot S(t)}$$

- 12.** Za mrežu na slici odrediti i skicirati odziv napona $u_2(t)$ ako je zadano:
 $u_0(t) = S(t)$, $R_1 = 1$, $R_2 = 1$, $C_1 = 1$, $u_C(0) = 1$. Odziv izračunati rješavanjem Laplaceove transformacije.



Rješenje:



Zapišimo prvo jednadžbu pojačala:

$$\left. \begin{array}{l} U_2 = A(U_3 - U_1) \\ A \rightarrow \infty \end{array} \right\} U_1 = U_3$$

A kako je napon U_3 uzemljen znači da vrijedi $U_1 = U_3 = 0$

Sad zapišemo jednadžbu za čvor (1), odnosno za napon U_1

$$U_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + sC_1 \right) - U_2 \left(\frac{1}{R_2} + sC_1 \right) - U_0 \frac{1}{R_1} = -Cu_c(0)$$

$$U_2 \left(\frac{1}{R_2} + sC_1 \right) = Cu_c(0) - U_0 \frac{1}{R_1}$$

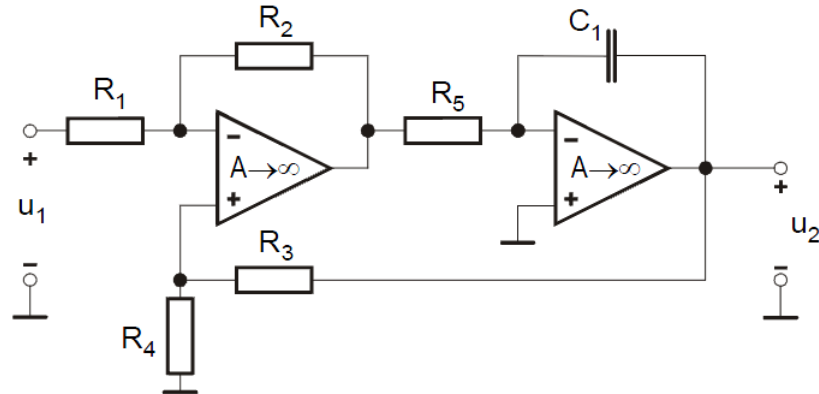
$$U_2(1 + s) = 1 - \frac{1}{s} \quad \Rightarrow \quad U_2 = \frac{s - 1}{s(s + 1)}$$

$$\left\{ \begin{array}{l} \frac{s - 1}{s(s + 1)} = \frac{A}{s} + \frac{B}{s + 1} = \frac{As + A + Bs}{s(s + 1)} \\ A + B = 1 \\ A = -1 \\ B = 2 \end{array} \right\}$$

$$U_2 = \frac{2}{s + 1} - \frac{1}{s}$$

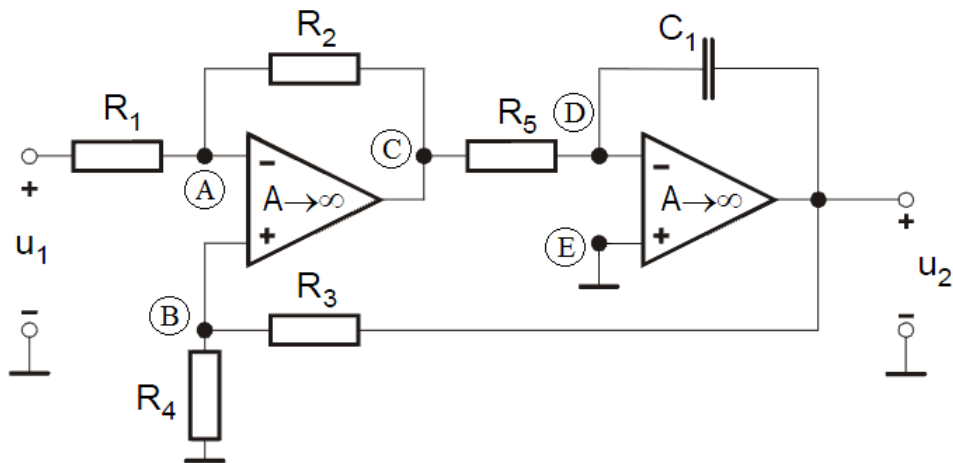
$$U_2(s) \xrightarrow{\mathcal{L}^{-1}} \boxed{u_2(t) = (2e^{-t} - 1) \cdot S(t)}$$

- 13.** Odredi odziv $U_{izl}(s)$ za mrežu prikazanu slikom ako je pobuda $U_1(s) = 1/s$.
Zadano je $R_1 = R_2 = R_3 = R_4 = R_5 = 1, C_1 = 1$.



Rješenje:

Za potrebe rješavanja, na mreži ćemo si označiti čvorove koje ćemo koristiti prilikom rješavanja. Pritom, naravno, moramo paziti da nećemo zapisati jednadžbe čvorova za sve čvorove!



$$(A) \quad U_A \left(\frac{1}{R_1} + \frac{1}{R_2} \right) - U_1 \frac{1}{R_1} - U_C \frac{1}{R_2} = 0$$

$$(B) \quad U_B \left(\frac{1}{R_4} + \frac{1}{R_3} \right) - U_2 \frac{1}{R_3} = 0$$

$$(D) \quad U_D \left(\frac{1}{R_5} + sC_1 \right) - U_C \frac{1}{R_5} - U_2 sC_1 = 0$$

$$(p1) \quad U_A = U_B$$

$$(p2) \quad U_D = U_E = 0$$

Nakon uvrštavanja brojeva te (p1) i (p2) u (A), (B) i (D):

$$(A) \quad 2U_A - \frac{1}{s} - U_C = 0$$

$$(B) \quad 2U_A - U_2 = 0 \quad \Rightarrow \quad U_A = \frac{U_2}{2}$$

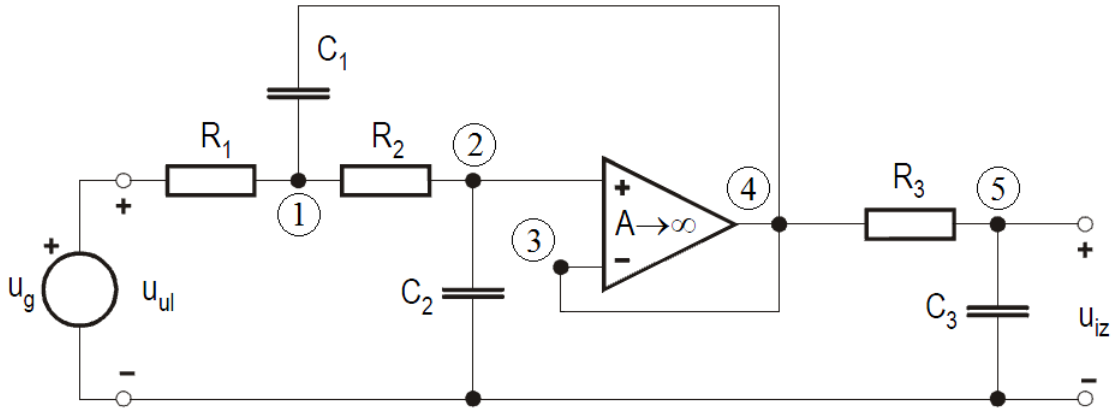
$$(D) \quad U_C + U_2 s = 0 \quad \Rightarrow \quad U_C = -U_2 s$$

$$(B) \text{ i } (D) \text{ u } (A) \quad U_2 - \frac{1}{s} + U_2 s = 0$$

$$U_2(1 + s) = \frac{1}{s}$$

$$U_2 = \frac{1}{s(s + 1)}$$

14. Za mrežu prikazanu slikom odrediti odziv napona $U_{iz}(s)$, ako je zadan poticaj $U_{ul}(s) = 1$. Zadano je $R_1 = R_2 = R_3 = 1, C_1 = 2, C_2 = 1/2, C_3 = 1$.



Rješenje:

$$(1) \quad U_1 \left(\frac{1}{R_1} + sC_1 + \frac{1}{R_2} \right) - U_{ul} \frac{1}{R_1} - U_4 sC_1 - U_2 \frac{1}{R_2} = 0$$

$$(2) \quad U_2 \left(\frac{1}{R_2} + sC_2 \right) - U_1 \frac{1}{R_2} = 0$$

$$(3) \quad U_3 = U_4$$

$$(5) \quad U_5 \left(\frac{1}{R_5} + sC_3 \right) - U_4 \frac{1}{R_3} = 0$$

$$(p1) \quad U_2 = U_3$$

Uz uvrštavanje brojeva, $U_5 = U_{iz}$ te $U_g(s) = 1$ imamo:

$$(1) \quad U_1(2 + 2s) - 1 - 2sU_4 - U_2 = 0$$

$$(2) \quad U_2 \left(1 + \frac{s}{2} \right) - U_1 = 0 \Rightarrow U_1 = U_2 \frac{s+2}{2}$$

$$(3) + (p1) \quad U_3 = U_4 = U_2$$

$$(5) \quad U_{iz}(1 + s) - U_4 = 0 \Rightarrow U_4 = U_{iz}(s + 1)$$

$$(2) \text{ u } (1) \quad U_2(s + 1)(s + 2) - U_2 - 2sU_4 = 1$$

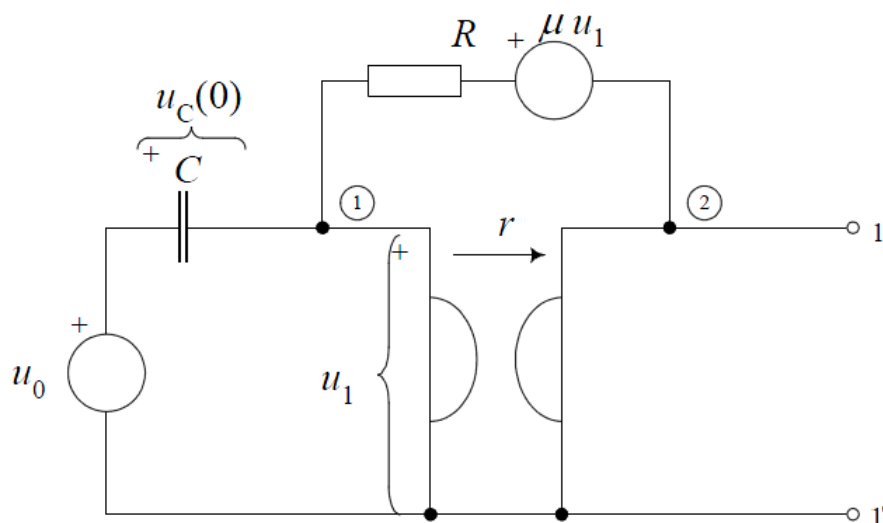
$$(3) + (p1) \text{ u } (1) \quad U_4(s^2 + 3s + 2) - U_4 - 2sU_4 = 1$$

$$U_4(s^2 + 3s + 2 - 1 - 2s) = 1$$

$$(5) \text{ u } (1) \quad U_{iz}(s + 1)(s^2 + s + 1) = 1$$

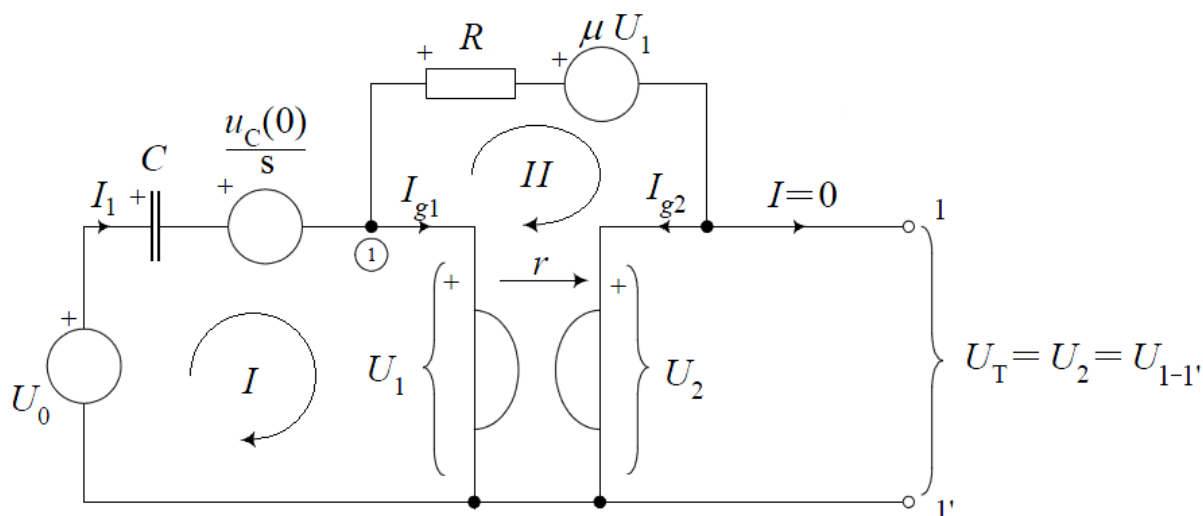
$$U_{iz} = \frac{1}{(s + 1)(s^2 + s + 1)}$$

15. Za mrežu prikazanu slikom odrediti nadomjesnu shemu po Theveninu obzirom na priključnice 1-1', primjenom jednačbi petlji, ako je pobuda $u_0(t) = S(t)$. Zadane su normirane vrijednosti elemenata: $R = 0.5$, $r = 0.5$, $\mu = 0.5$, $C = 1$ i početni napon na kapacitetu $u_C(0) = 2$.



Rješenje:

Prvo ćemo odrediti Theveninov napon pa za te potrebe mrežu označujemo na sljedeći način, a uz te oznake ćemo pisati jednačbe petlji.



$$(I) \quad U_0 - I_1 \frac{1}{sC} - \frac{u_C(0)}{s} - U_1 = 0$$

$$(II) \quad -I_{g2}R - \mu U_1 - U_2 + U_1 = 0$$

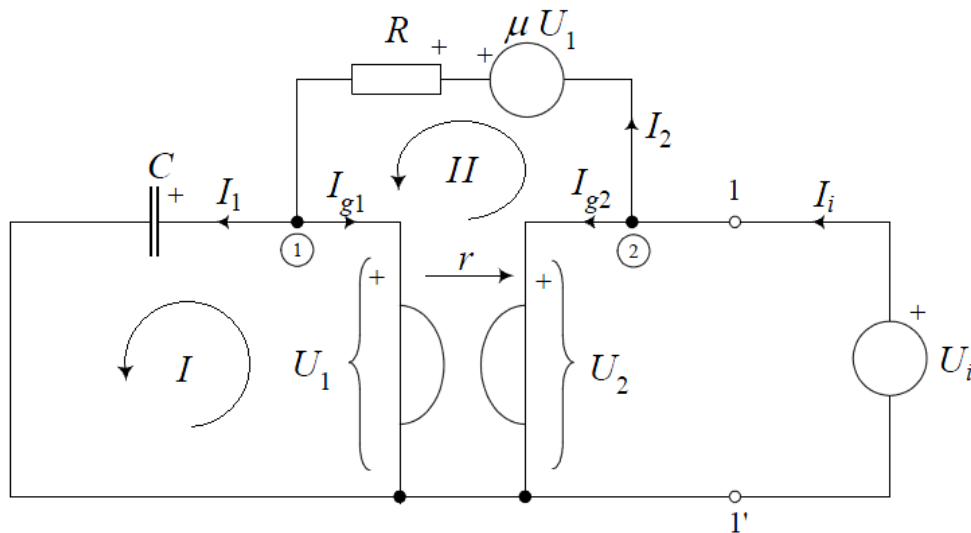
$$(1) \quad I_1 = I_{g1} + I_{g2}$$

$$(g1) \quad U_1 = r \cdot I_{g2} \Rightarrow I_{g2} = \frac{U_1}{r}$$

$$(g2) \quad U_2 = -r \cdot I_{g1} \Rightarrow I_{g1} = -\frac{U_2}{r}$$

(g1) i (g2) u (1)	$I_1 = -\frac{U_2}{r} + \frac{U_1}{r} = \frac{U_1 - U_2}{r}$
(1) u (I)	$\frac{1}{s} - \frac{U_1 - U_2}{0.5s} - \frac{2}{s} - U_1 = 0$
(II)	$-U_1 - 0.5U_1 - U_2 + U_1 = 0$
(I)	$1 - 2U_1 + 2U_2 - 2 - sU_1 = 0$
(II)	$U_1 = -2U_2$
(II) u (I)	$6U_2 + 2sU_2 = 1$
	$U_2(s) = U_T(s) = \frac{1}{2(s+3)}$

Zatim određujemo iznos Theveninove impedancije Z_T i za to primjenjujemo poznata pravila te postavljamo proizvoljno oznake:

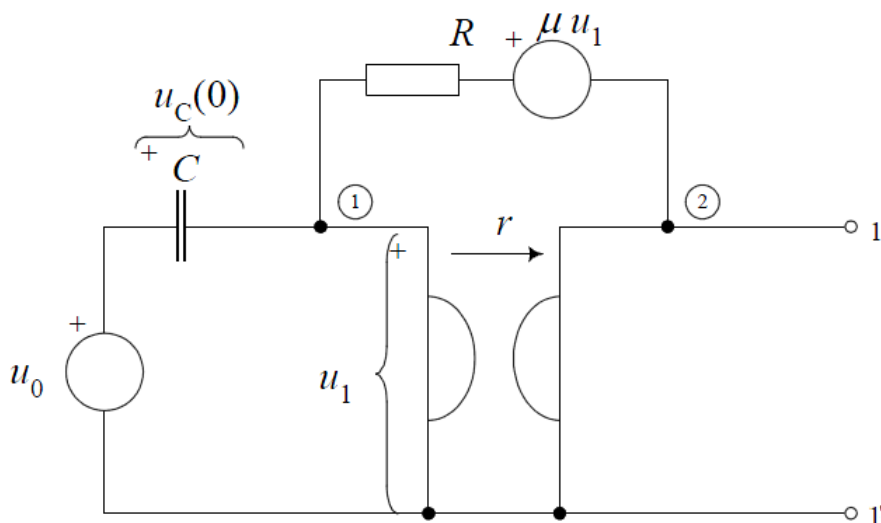


$$Z_T = \frac{U_i}{I_i} = \frac{U_2}{I_i}$$

(I)	$U_1 - I_1 \frac{1}{sC} = 0$
(II)	$U_2 + \mu U_1 - I_2 R - U_1 = 0$
(1)	$I_2 = I_{g1} + I_1 \Rightarrow I_1 = I_2 - I_{g1}$
(2)	$I_i = I_2 + I_{g2} \Rightarrow I_2 = I_i - I_{g2}$
(g1)	$U_1 = r \cdot I_{g2} \Rightarrow I_{g2} = \frac{U_1}{r}$
(g2)	$U_2 = -r \cdot I_{g1} \Rightarrow I_{g1} = -\frac{U_2}{r}$
(2) u (1)	$I_1 = I_i - I_{g1} - I_{g2}$
(1) u (I)	$U_1 - \frac{1}{sC} (I_i - I_{g1} - I_{g2}) = 0$

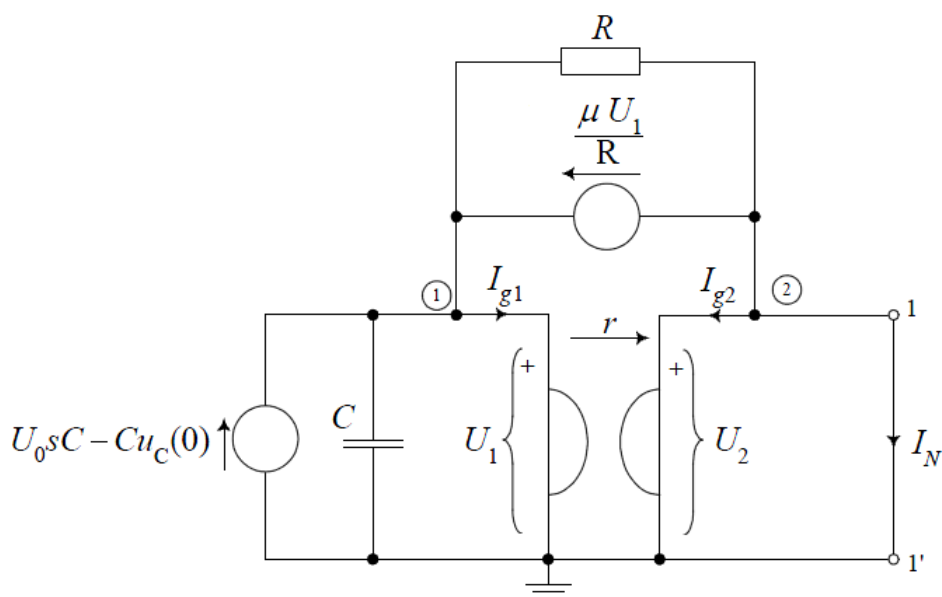
(2) u (II)	$U_2 + \mu U_1 - R(I_i - I_{g2}) - U_1 = 0$
(g1)+(g2) u (I)	$U_1 - \frac{I_i}{s} - \frac{2U_2}{s} + \frac{2U_1}{s} = 0$
(g1)+(g2) u (II)	$U_2 + 0.5U_1 - 0.5I_i + U_1 - U_1 = 0$
(I)	$U_1(s + 2) - I_i - 2U_2 = 0$
(II)	$2U_2 + U_1 - I_i = 0 \Rightarrow U_1 = I_i - 2U_2$
(II) u (I)	$(I_i - 2U_2)(s + 2) - I_i - 2U_2 = 0$
	$U_2(2 + 2s + 4) = I_i(s + 2 - 1)$
	$U_2 = I_i \frac{s + 1}{2s + 6}$
	$Z_T(s) = \frac{U_2}{I_i} = \frac{s + 1}{2(s + 3)}$

- 16.** Za mrežu prikazanu slikom odrediti nadomjesnu shemu po Nortonu obzirom na priključnice 1-1', primjenom jednačbi čvorišta, ako je pobuda $u_0(t) = S(t)$. Zadane su normirane vrijednosti elemenata: $R = 0.5$, $r = 0.5$, $\mu = 0.5$, $C = 1$ i početni napon na kapacitetu $u_C(0) = 2$.



Rješenje:

Za početak ćemo prilagoditi mrežu kako bi je što lakše mogli riješiti pomoću jednačbi čvorišta.



$$(1) \quad U_1 \left(sC + \frac{1}{R} \right) - U_2 \frac{1}{R} = U_0 sC - Cu_c(0) + \frac{\mu U_1}{R} - I_{g1}$$

$$(2) \quad U_2 \frac{1}{R} - U_1 \frac{1}{R} = -\frac{\mu U_1}{R} - I_{g2} - I_N$$

$$(g1) \quad U_1 = r \cdot I_{g2}$$

$$(g2) \quad U_2 = -r \cdot I_{g1}$$

Na izlazu 1-1' je kratki spoj pa vrijedi $U_2 = 0$ a prema (g2) to znači $I_{g1} = 0$.

$$(1) \quad U_1(s + 2) = 1 - 2 + U_1$$

$$(2) \quad 2U_1 = U_1 + I_{g2} + I_N$$

$$(g1) \quad I_{g2} = 2U_1$$

$$(1) \quad U_1 = -\frac{1}{s + 1}$$

$$(2) \quad I_N = U_1 - I_{g2}$$

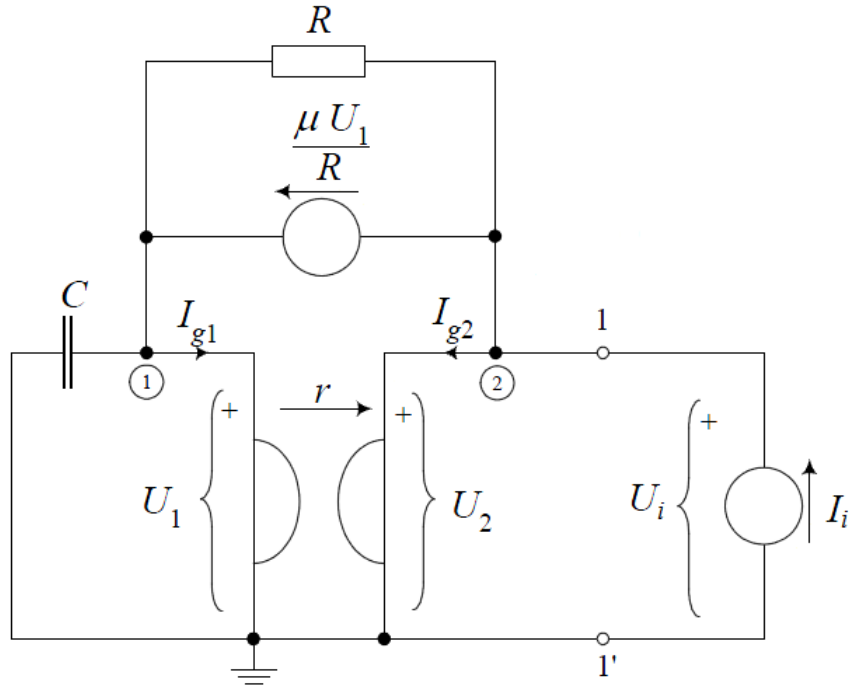
$$(1) \text{ u } (g2) \quad I_{g2} = -\frac{2}{s + 1}$$

$$(1) \text{ i } (g2) \text{ u } (2) \quad I_N = \frac{1}{s + 1}$$

Zatim određujemo iznos Nortonove impedancije, koja je po iznosu jednaka Theveninovoj impedanciji i prema tome se može računati na isti način. No, primjera radi, odrediti ćemo ju pravilu za traženje Nortonove admintancije, odnosno pomoći strujnog izvora između stezaljki 1-1'.

Gdje je Nortonova impedancija iznosa:

$$Y_N = \frac{I_i}{U_i} = \frac{I_i}{U_2}$$



$$(1) \quad U_1 \left(sC + \frac{1}{R} \right) - U_2 \frac{1}{R} = \frac{\mu U_1}{R} - I_{g1}$$

$$(2) \quad U_2 \frac{1}{R} - U_1 \frac{1}{R} = -\frac{\mu U_1}{R} - I_{g2} + I_i$$

$$(g1) \quad U_1 = r \cdot I_{g2} \Rightarrow I_{g2} = \frac{U_1}{r}$$

$$(g2) \quad U_2 = -r \cdot I_{g1} \Rightarrow I_{g1} = -\frac{U_2}{r}$$

$$(g2) \text{ u } (1) \quad U_1(s + 2) - 2U_2 = U_1 + 2U_2$$

$$(g1) \text{ u } (2) \quad 2U_2 - 2U_1 = -U_1 - 2U_1 + I_i$$

$$(1) \quad U_1(s + 1) - 4U_2 = 0$$

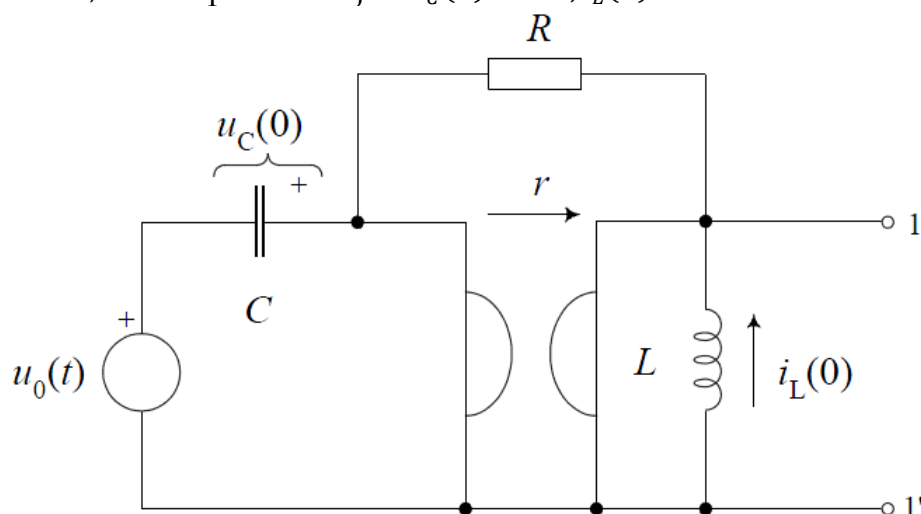
$$(2) \quad U_1 = I_i - 2U_2$$

$$(2) \text{ u } (1) \quad (I_i - 2U_2)(s + 1) - 4U_2 = 0$$

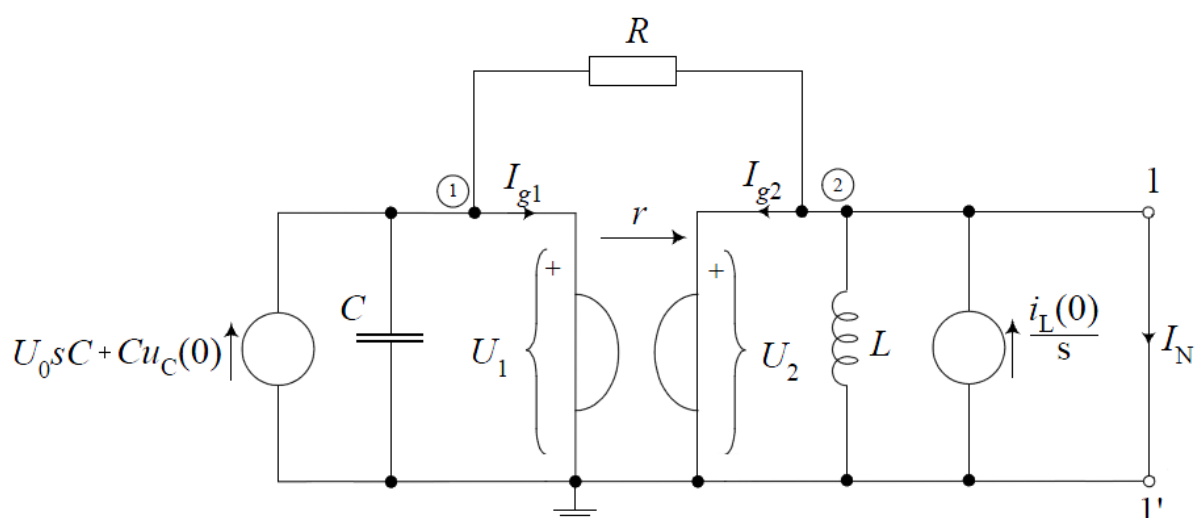
$$I_i(s + 1) = U_2(2s + 2 + 4)$$

$$Y_N(s) = \frac{I_i(s)}{U_2(s)} = \frac{2(s + 3)}{s + 1}$$

17. Za strujni krug prikazan slikom odrediti nadomjesnu shemu po Nortonu obzirom na priključnice 1-1', koristeći postupak jednažbi čvorišta, ako je pobuda $u_0(t) = \delta(t)$. Zadane su normirane vrijednosti elemenata: $R = 0.5$, $L = 1$, $C = 1$, $r = 1$ i početni uvjeti $u_C(0) = 0.5$, $i_L(0) = 1$.



Rješenje:



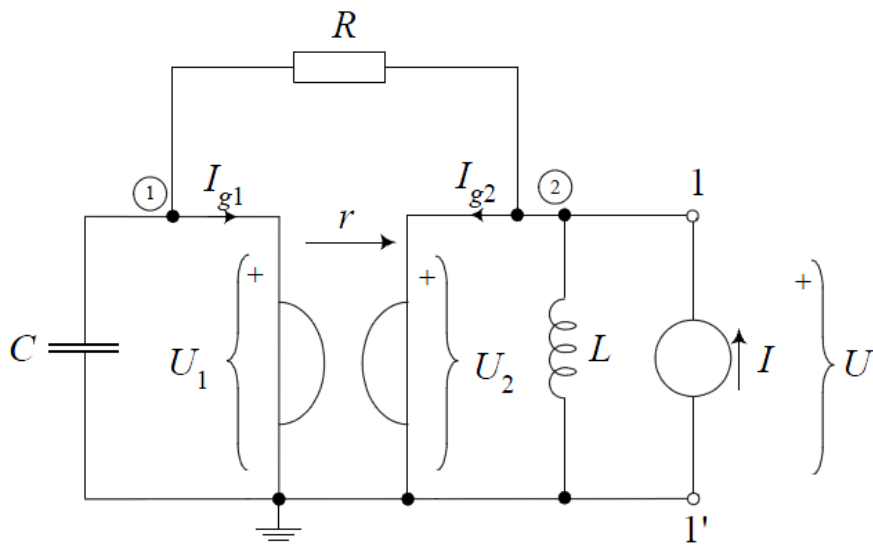
$$\begin{aligned}
 (1) \quad & U_1 \left(sC + \frac{1}{R} \right) - U_2 \frac{1}{R} = U_0 sC + C u_C(0) - I_{g1} \\
 (2) \quad & U_2 \left(\frac{1}{R} + \frac{1}{sL} \right) - U_1 \frac{1}{R} = -I_{g2} + \frac{i_L(0)}{s} - I_N \\
 (g1) \quad & U_1 = r \cdot I_{g2} \\
 (g2) \quad & U_2 = -r \cdot I_{g1}
 \end{aligned}$$

Na izlazu 1-1' je kratki spoj pa vrijedi $U_2 = 0$ a prema (g2) to znači $I_{g1} = 0$.

$$\begin{aligned}
 (1) \quad & U_1(s+2) = s + \frac{1}{2} \\
 (2) \quad & -2U_1 = -I_{g2} + \frac{1}{s} - I_N
 \end{aligned}$$

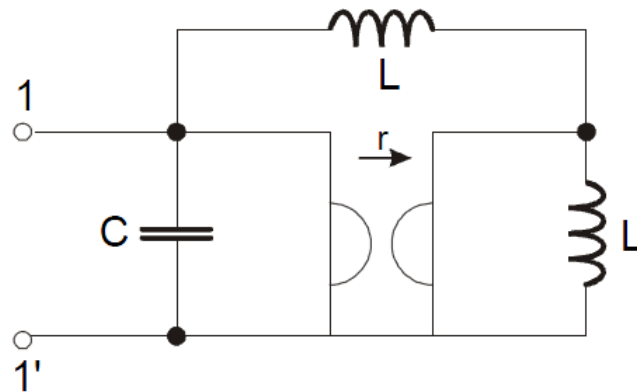
(1)	$U_1 = \frac{2s + 1}{2(s + 2)}$
(2)	$I_N = \frac{1}{s} + 2U_1 - I_{g2}$
(g1)	$I_{g2} = \frac{U_1}{r} = \frac{2s + 1}{2(s + 2)}$
<hr/>	
(1) i (g1) u (2)	$I_N = \frac{1}{s} + \frac{2s + 1}{(s + 2)} - \frac{2s + 1}{2(s + 2)}$
	$I_N = \frac{1}{s} + \frac{2s + 1}{2(s + 2)} = \frac{2s^2 + 3s + 4}{2s(s + 2)}$

Za Nortonivu admitanciju:

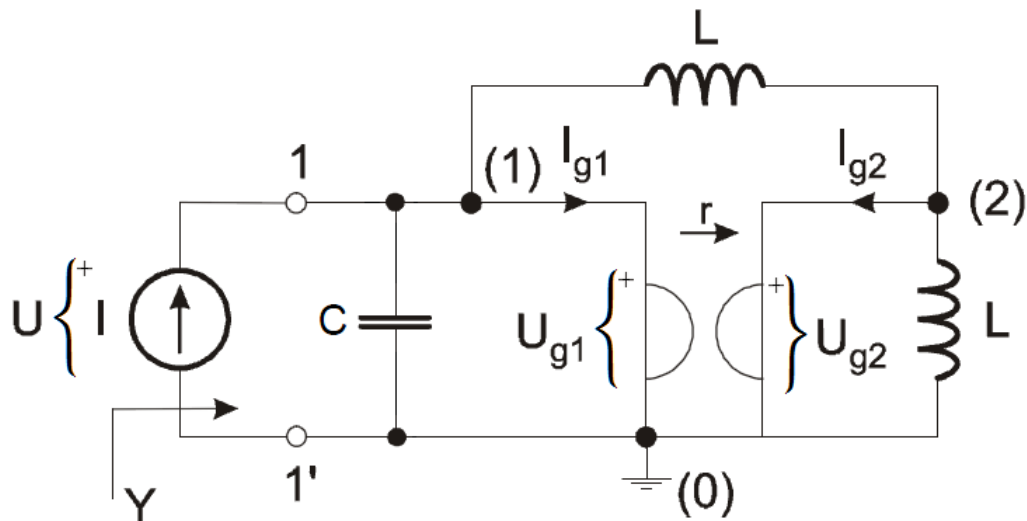


(1)	$U_1 \left(sC + \frac{1}{R} \right) - U_2 \frac{1}{R} = -I_{g1}$
(2)	$U_2 \left(\frac{1}{R} + \frac{1}{sL} \right) - U_1 \frac{1}{R} = -I_{g2} + I$
(g1)	$U_1 = r \cdot I_{g2} \Rightarrow I_{g2} = \frac{U_1}{r} = U_1$
(g2)	$U_2 = -r \cdot I_{g1} \Rightarrow I_{g1} = -\frac{U_2}{r} = -U_2$
<hr/>	
(g1)+(g2) u (1)	$U_1(s + 2) - 2U_2 = U_2 \Rightarrow U_1 = U_2 \frac{3}{s + 2}$
(g1)+(g2) u (2)	$U_2 \left(2 + \frac{1}{s} \right) - 2U_1 = -U_1 + I$
<hr/>	
(1) u (2)	$I = U_2 \frac{2s + 1}{s} - U_2 \frac{3}{s + 2} = U_2 \frac{2s^2 + 2s + 2}{s(s + 2)}$
	$Y_N(s) = \frac{I(s)}{U_2(s)} = \frac{2s^2 + 2s + 2}{s(s + 2)}$

18. Za prikazani dvopol odrediti admitanciju na priključnicama 1-1'. Zadano je $L = 1$, $C = 1$, $r = 1$.



Rješenje:



$$(1) \quad U_1 \left(sC + \frac{1}{sL} \right) - U_2 \frac{1}{sL} = I - I_{g1}$$

$$(2) \quad U_2 \left(\frac{1}{sL} + \frac{1}{sL} \right) - U_1 \frac{1}{sL} = -I_{g2}$$

$$(g1) \quad U_1 = r \cdot I_{g2} \Rightarrow I_{g2} = \frac{U_1}{r} = U_1$$

$$(g2) \quad U_2 = -r \cdot I_{g1} \Rightarrow I_{g1} = -\frac{U_2}{r} = -U_2$$

$$(g1)+(g2) \text{ u } (1) \quad U_1 \left(s + \frac{1}{s} \right) - U_2 \frac{1}{s} = I + U_2$$

$$(g1)+(g2) \text{ u } (2) \quad U_2 \left(\frac{1}{s} + \frac{1}{s} \right) - U_1 \frac{1}{s} = -U_1$$

$$(2) \quad U_2 = U_1 \frac{1-s}{2}$$

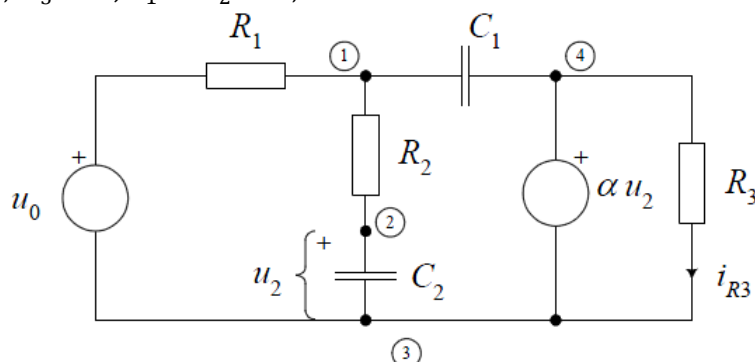
(2) u (1)

$$I = U_1 \left(\frac{s^2 + 1}{s} - \frac{1-s}{2s} - \frac{1-s}{2} \right)$$

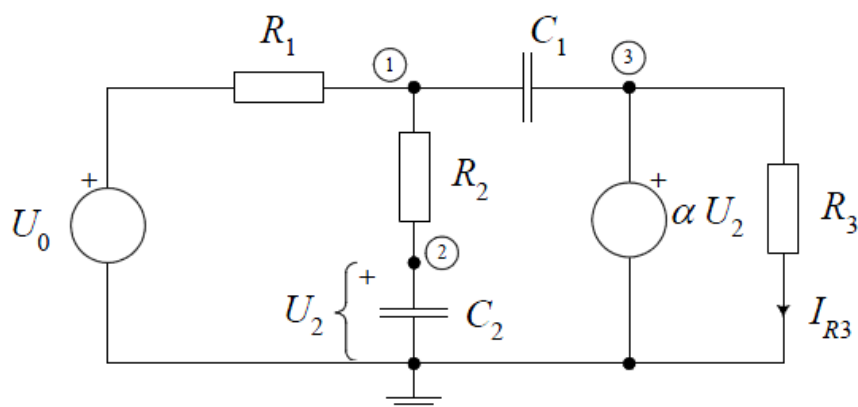
$$I = U \frac{3s^2 + 1}{2s}$$

$$Y(s) = \frac{I(s)}{U(s)} = \frac{3s^2 + 1}{2s}$$

19. Odredite odziv $i_{R3}(t)$ mreže na slici ako je pobuda $u_0(t) = \delta(t)$. Zadano je:
 $R_1 = R_2 = 1, R_3 = 2, C_1 = C_2 = 2, \alpha = 2$.



Rješenje:



$$(1) \quad U_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + sC_2 \right) - U_0 \frac{1}{R_1} - U_2 \frac{1}{R_2} - U_3 sC_1 = 0$$

$$(2) \quad U_2 \left(\frac{1}{R_2} + sC_2 \right) - U_1 \frac{1}{R_2} = 0$$

Prema mreži se vidi $U_3 = \alpha U_2$ što uvrstimo u (1) te u obje jednačbe brojeve

$$(1) \quad U_1(2 + 2s) - 1 - U_2 - 4sU_2 = 0$$

$$(2) \quad U_1 = U_2(1 + 2s)$$

$$(2) \text{ u } (1) \quad U_2((2 + 2s)(1 + 2s) - 1 - 4s) = 1$$

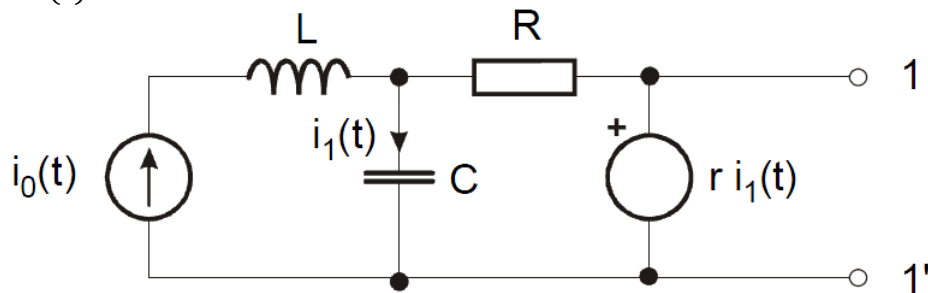
$$(1) \quad U_2 = \frac{1}{4s^2 + 2s + 1}$$

$$I_{R3} = \frac{U_3}{R_3} = \frac{\alpha U_2}{R_3} = \frac{1}{4} \cdot \frac{1}{s^2 + \frac{1}{2}s + \frac{1}{4}}$$

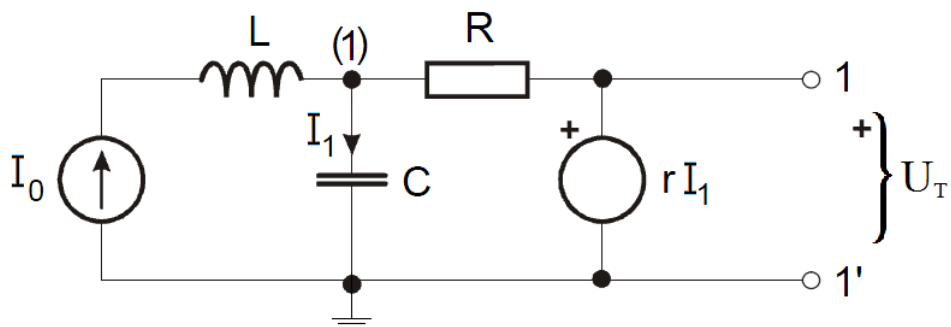
$$I_{R3} = \frac{1}{4} \cdot \frac{\frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{4}}{\left(s + \frac{1}{4}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2} = \frac{\sqrt{3}}{3} \cdot \frac{\frac{\sqrt{3}}{4}}{\left(s + \frac{1}{4}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2}$$

$$I_{R3}(s) \xrightarrow{\mathcal{L}^{-1}} \boxed{i_{R3}(t) = \frac{\sqrt{3}}{3} e^{-\frac{t}{4}} \sin\left(\frac{\sqrt{3}}{4}t\right) S(t)}$$

- 20.** Za mrežu prikazanu slikom odrediti nadomjesne parametre mreže po Theveninu $U_T(s)$ i $Z_T(s)$ na stezaljkama 1-1'. Zadano je $R = 2$, $r = 1$, $C = 1$, $i_0(t) = S(t)$.



Rješenje:



Zapišimo jednadžbu čvorišta za čvor (1):

$$U_1 \left(sC + \frac{1}{R} \right) - U_T \frac{1}{R} = I_0$$

Također vrijedi:

$$U_T = r \cdot I_1 = r \cdot \frac{U_1}{\frac{1}{sC}} = rsCU_1 = sU_1$$

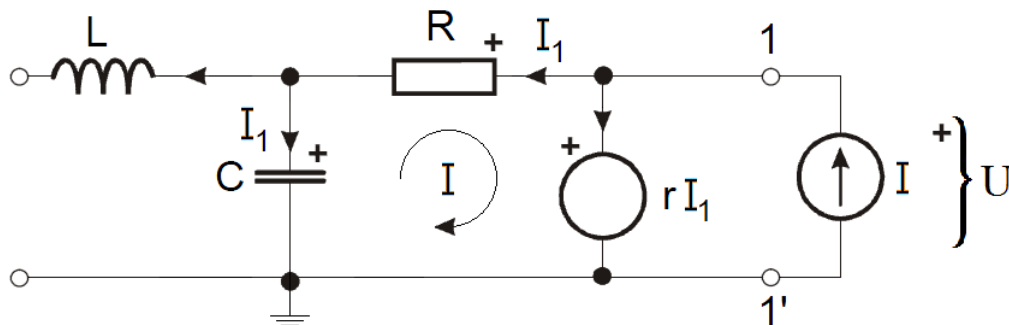
Što uvrstimo u (1)

$$U_1 \left(s + \frac{1}{2} \right) - \frac{sU_1}{2} = \frac{1}{s}$$

$$U_1 = \frac{2}{s(s+1)}$$

$$U_T(s) = sU_1(s) = \frac{2}{s+1}$$

Za Theveninovu impedanciju:



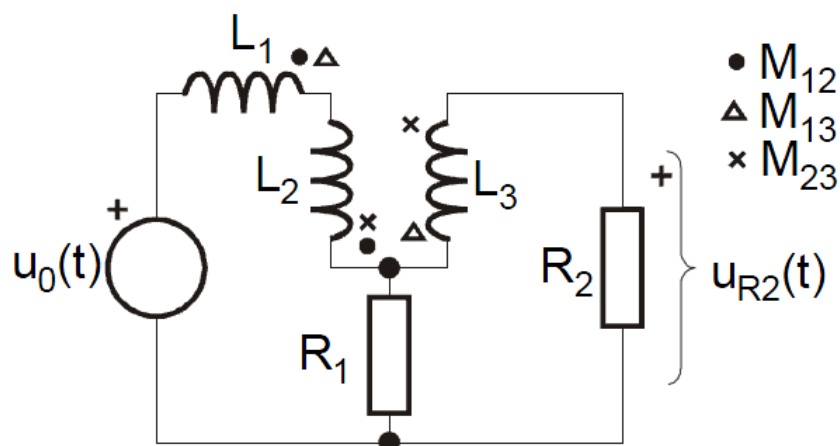
Zapišimo jednađžbu za petlju (I):

$$I_1 \left(\frac{1}{sC} + R \right) - rI_1 = 0$$

Iz čega proizlazi da je $I_1 = 0$ a Theveninova impedancija iznosi:

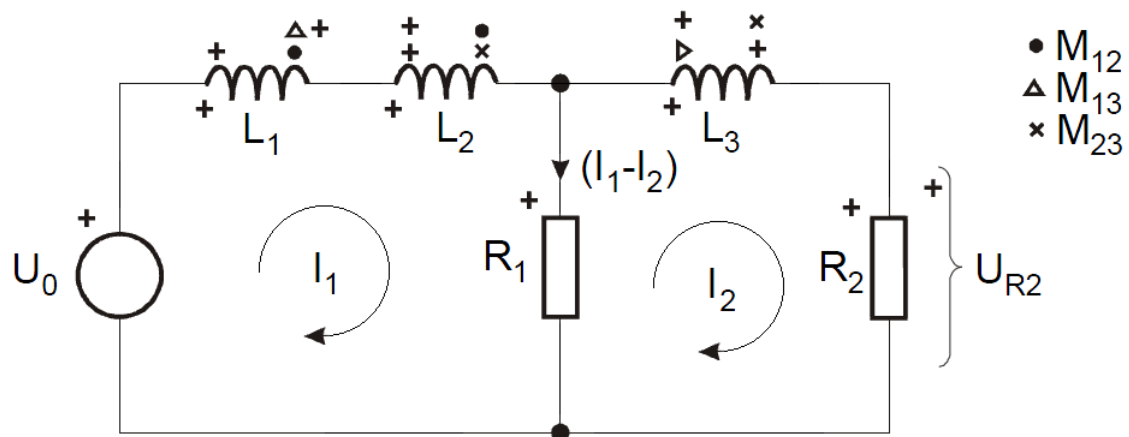
$$Z_T(s) = \frac{U}{I} = \frac{rI_1}{I} = 0$$

- 21.** Za mrežu prikazanu slikom izračunati napon $u_{R2}(t)$ ako su zadane normalizirane vrijednosti elemenata: $R_1 = 1, R_2 = 1, L_1 = 1, L_2 = 2, L_3 = 4, M_{12} = 1/2, M_{13} = 2, M_{23} = 3$ te napon generatora $u_0(t) = S(t)$.



Rješenje:

Primjena Laplaceove transformacije:



$$(1) \quad U_0 - I_1 s L_1 - I_1 s M_{12} + I_2 s M_{13} - I_1 s L_2 - I_1 s M_{12} - I_2 s M_{23} - (I_1 - I_2) R_1 = 0$$

$$(2) \quad (I_1 - I_2) R_1 - I_2 s L_3 + I_1 s M_{12} - I_1 s M_{23} - I_2 R_2 = 0$$

$$(1) \quad I_1(7s + 1) + I_2(s - 1) = \frac{1}{s}$$

$$(2) \quad I_1 \left(1 - \frac{5}{2}s\right) - I_2(4s + 2) = 0$$

I dalje se može uočiti kako je postupak rješavanja ovoga preteško da bi se nastavilo do kraja... tako da ja neću, a vi kako hoćete ;)