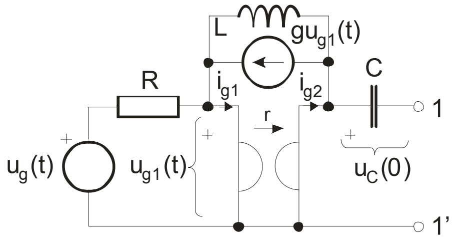
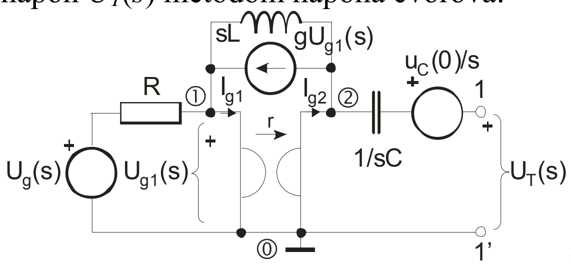
PONOVLJENI ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA 2010/11

Rješenja i bodovi (svaki zadatak je bodovan od 0 do 5 bodova):

1. Za mrežu prikazanu slikom odrediti nadomjesne parametre mreže po Theveninu $U_T(s)$ i $Z_T(s)$ s obzirom na polove 1–1'. Koristiti metodu napona čvorova u proračunu. Zadane su normalizirane vrijednosti elemenata: L=1, C=1, R=1, g=2, r=2, $u_C(0)$=1 te izvor $u_g(t)$=$S(t)$. Nacrtati i napisati: a) Slike za izračun $U_T(s)$ i $Z_T(s)$; b) Jednadžbe čvorova za izračun $U_T(s)$; c) Jednadžbe čvorova za izračun $Z_T(s)$. Uz uvrštene vrijednosti elemenata: d) Theveninov napon $U_T(s)$; e) Theveninovu impedanciju $Z_T(s)$.



Rješenje: Theveninov napon $U_T(s)$ metodom napona čvorova:



$$U_T(s)=U_2(s)-\frac{u_C(0)}{s}$$

(0,5 boda)

$$(1)\qquad U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=gU_1+\frac{U_g(s)}{R}-I_{g1}(s)\qquad U_2=-r\cdot I_{g1}\Rightarrow I_{g1}=-\frac{U_2}{r}$$

$$(2)\qquad -U_1\frac{1}{sL}+U_2\frac{1}{sL}=-gU_1(s)+I_{g2}(s)\ \textbf{(1 bod)}\qquad U_1=-r\cdot I_{g2}\Rightarrow I_{g2}=-\frac{U_1}{r}$$

$$(1)\qquad U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=gU_1+\frac{U_g(s)}{R}+\frac{U_2}{r}\ \Rightarrow U_1\left(\frac{1}{R}+\frac{1}{sL}-g\right)-U_2\left(\frac{1}{sL}+\frac{1}{r}\right)=\frac{U_g(s)}{R}$$

$$(2)\qquad -U_1\frac{1}{sL}+U_2\frac{1}{sL}=-gU_1(s)-\frac{U_1(s)}{r}$$

$$(2)\Rightarrow -U_1\left(\frac{1}{sL}-g-\frac{1}{r}\right)+U_2\frac{1}{sL}=0\ \Rightarrow\ U_1=\frac{1/sL}{1/sL-g-1/r}\cdot U_2\ \rightarrow(1)$$

$$(1),(2)\Rightarrow U_2\frac{\frac{1}{sL}}{\frac{1}{sL}-g-\frac{1}{r}}\left(\frac{1}{R}+\frac{1}{sL}-g\right)-U_2\left(\frac{1}{sL}+\frac{1}{r}\right)=\frac{U_g}{R}$$

$$U_2\frac{1}{sL}\left(\frac{1}{R}+\frac{1}{sL}-g\right)-U_2\left(\frac{1}{sL}+\frac{1}{r}\right)\left(\frac{1}{sL}-g-\frac{1}{r}\right)=\frac{U_g}{R}\left(\frac{1}{sL}-g-\frac{1}{r}\right)$$

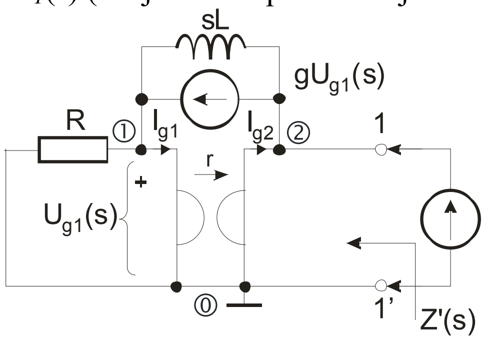
$$U_2\left(\frac{1}{sLR}+\frac{1}{(sL)^2}-\frac{g}{sL}-\frac{1}{(sL)^2}+\frac{g}{sL}+\frac{1}{rsL}-\frac{1}{rsL}+\frac{g}{r}+\frac{1}{r^2}\right)=\frac{U_g}{R}\left(\frac{1}{sL}-g-\frac{1}{r}\right)$$

$$U_2\left(\frac{1}{sLR}+\frac{g}{r}+\frac{1}{r^2}\right)=\frac{U_g}{R}\left(\frac{1}{sL}-g-\frac{1}{r}\right)$$

$$U_2(s)=\frac{\frac{1}{R}\left(\frac{1}{sL}-g-\frac{1}{r}\right)}{\frac{1}{sLR}+\frac{g}{r}+\frac{1}{r^2}}U_g(s)==\frac{\frac{1}{s}-2-\frac{1}{2}}{\frac{1}{s}+1+\frac{1}{4}}\cdot\frac{1}{s}=\frac{\frac{1}{s}-\frac{5}{2}}{\frac{1}{s}+\frac{5}{4}}\cdot\frac{1}{s}=\frac{\frac{1}{s}-\frac{5}{2}}{1+\frac{5}{4}s}=\frac{4\frac{1}{s}-10}{4+5s}=\frac{-10s+4}{s(5s+4)}$$

$$\Rightarrow\ U_T(s)=U_2(s)-\frac{u_C(0)}{s}=\frac{-10s+4}{s(5s+4)}-\frac{1}{s}=\frac{-10s+4-(5s+4)}{s(5s+4)}=\frac{-15s+8}{s(5s+4)}$$ **(1 bod)**

Theveninova impedancija $Z_T(s)$ (isključeni su početni uvjeti i neovisni izvori):



(0,5 boda)

(1) $\quad U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=gU_1-I_{g1}(s)$ $\qquad\qquad Z_T(s)=Z'(s)+\frac{1}{sC}$

(2) $\quad -U_1\frac{1}{sL}+U_2\frac{1}{sL}=-gU_1(s)+I_{g2}(s)+I(s)$ **(1 bod)**

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(1) $\quad U_1\left(\frac{1}{R}+\frac{1}{sL}\right)-U_2\frac{1}{sL}=gU_1+\frac{U_2}{r}\ \Rightarrow\ U_1\left(\frac{1}{R}+\frac{1}{sL}-g\right)-U_2\left(\frac{1}{sL}+\frac{1}{r}\right)=0$

(2) $\quad -U_1\frac{1}{sL}+U_2\frac{1}{sL}=-gU_1(s)-\frac{U_1(s)}{r}+I(s)$

$$(2)\Rightarrow\ -U_1\left(\frac{1}{sL}-g-\frac{1}{r}\right)+U_2\frac{1}{sL}=I(s)\ \Rightarrow\ U_1=\frac{\frac{1}{sL}}{\frac{1}{sL}-g-\frac{1}{r}}\cdot U_2-\frac{I(s)}{\frac{1}{sL}-g-\frac{1}{r}}\ \to(1)$$

$$(1),(2)\Rightarrow\left[\frac{\frac{1}{sL}}{\frac{1}{sL}-g-\frac{1}{r}}\cdot U_2-\frac{I(s)}{\frac{1}{sL}-g-\frac{1}{r}}\right]\left(\frac{1}{R}+\frac{1}{sL}-g\right)-U_2\left(\frac{1}{sL}+\frac{1}{r}\right)=0$$

$$\left[\frac{1}{sL}\cdot U_2-I(s)\right]\left(\frac{1}{R}+\frac{1}{sL}-g\right)=U_2\left(\frac{1}{sL}+\frac{1}{r}\right)\left(\frac{1}{sL}-g-\frac{1}{r}\right)$$

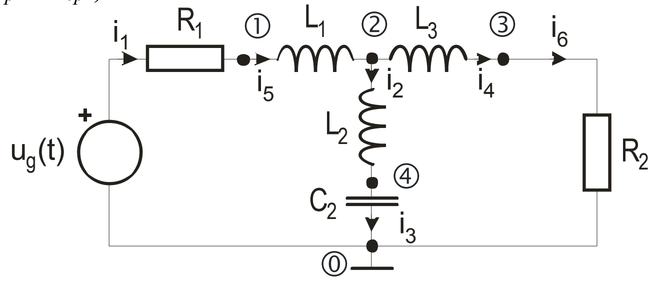
$$U_2\frac{1}{sL}\cdot\left(\frac{1}{R}+\frac{1}{sL}-g\right)-I(s)\left(\frac{1}{R}+\frac{1}{sL}-g\right)=U_2\left(\frac{1}{sL}+\frac{1}{r}\right)\left(\frac{1}{sL}-g-\frac{1}{r}\right)$$

$$U_2\frac{1}{sL}\left(\frac{1}{R}+\frac{1}{sL}-g\right)-U_2\left(\frac{1}{sL}+\frac{1}{r}\right)\left(\frac{1}{sL}-g-\frac{1}{r}\right)=I(s)\left(\frac{1}{R}+\frac{1}{sL}-g\right)$$

$$U_2\left(\frac{1}{sLR}+\frac{g}{r}+\frac{1}{r^2}\right)=I(s)\left(\frac{1}{R}+\frac{1}{sL}-g\right)\Rightarrow Z'(s)=\frac{U_2(s)}{I(s)}=\frac{\frac{1}{R}+\frac{1}{sL}-g}{\frac{1}{sLR}+\frac{g}{r}+\frac{1}{r^2}}=\frac{1+\frac{1}{s}-2}{\frac{1}{s}+1+\frac{1}{4}}=\frac{4-4s}{5s+4}$$

$$Z_T(s)=Z'(s)+\frac{1}{sC}=\frac{4s-4}{5s+4}+\frac{1}{s}=\frac{4s^2+s+4}{s(5s+4)}$$ **(1 bod)**

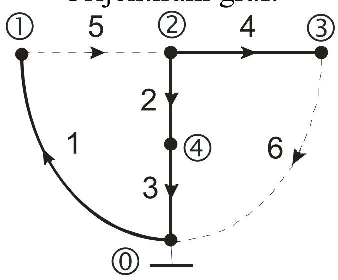
2. Za električni krug prikazan slikom i pridružene orijentacije grana i čvorove nacrtati: a) orijentirani graf, temeljni sustav petlji i temeljni sustav rezova; b) napisati matricu incidencija $\mathbf{A}_a$, temeljnu spojnu matricu **S**, temeljnu rastavnu matricu **Q**; c) matricu impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$; d) matricu admitancija grana $\mathbf{Y}_b$ i vektor početnih uvjeta i nezavisnih strujnih izvora grana $\mathbf{I}_{0b}$; e) pomoću navedenih matrica odrediti temeljni sustav jednadžbi petlji (matrice $\mathbf{Z}_p$ i $\mathbf{U}_{0p}$).



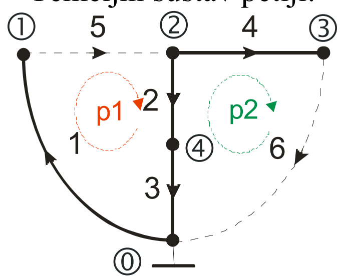
Rješenje:

a) orijentirani graf, temeljni sustav petlji i temeljni sustav rezova: **(1 bod)**

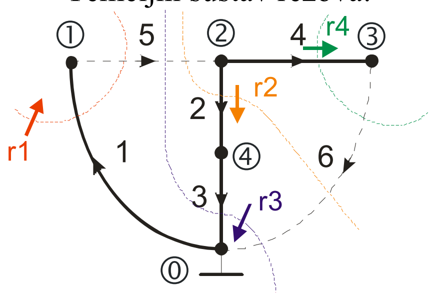
Orijentirani graf:



Temeljni sustav petlji:



Temeljni sustav rezova:



b) matrica incidencija $\mathbf{A}_a$, temeljna spojna matrica **S**, temeljna rastavna matrica **Q**: **(1 bod)**

Matrica incidencija:

$$\mathbf{A}_a = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}$$

Temeljna rastavna matrica:

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}$$

Temeljna spojna matrica:

$$\mathbf{S} = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 1 \end{bmatrix}$$

c) matrica impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$: **(1 bod)**

Naponsko – strujne relacije grana:

$$\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}\,, \text{ odn. } \mathbf{I}_b = \mathbf{Y}_b \cdot \mathbf{U}_b + \mathbf{I}_{0b}$$

$$U_1 = I_1 \cdot R_1 - U_g(s) \quad \Rightarrow \quad I_1 = \frac{1}{R_1} \cdot U_1 + \frac{U_g(s)}{R_1}$$

$$U_2 = I_2 \cdot sL_2 \qquad \Rightarrow \quad I_2 = \frac{1}{sL_2} \cdot U_2\,, \quad \text{itd.}$$

$$U_3 = I_3 \cdot \frac{1}{sC_2}\,,\ U_4 = I_4 \cdot sL_3\,,\ U_5 = I_5 \cdot sL_1\,,\ U_6 = I_6 \cdot R_2$$

Iz gornjeg sustava se mogu pročitati:

$$\mathbf{Z}_b = \begin{bmatrix} R_1 & & & & & \\ & sL_2 & & & 0 & \\ & & \frac{1}{sC_2} & & & \\ & & & sL_3 & & \\ & 0 & & & sL_1 & \\ & & & & & R_2 \end{bmatrix}, \quad \mathbf{U}_{0b} = \begin{bmatrix} -U_g(s) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

d) matrica admitancija grana $\mathbf{Y}_b$ i vektor početnih uvjeta i nezavisnih strujnih izvora grana $\mathbf{I}_{0b}$:
Jedan način je da se gornji sustav napiše tako da su s lijeve strane struje grana, a drugi način je da se invertira matrica $\mathbf{Y}_b = \mathbf{Z}_b^{-1}$. U slučaju dijagonalne matrice to je lako jer elementi na dijagonali inverzne matrice imaju recipročnu vrijednost elemenata originalne matrice. **(1 bod)**

$$\mathbf{Y}_b = \begin{bmatrix} \frac{1}{R_1} & & & & & \\ & \frac{1}{sL_2} & & & & \\ & & sC_2 & & 0 & \\ & & & \frac{1}{sL_3} & & \\ & 0 & & & \frac{1}{sL_1} & \\ & & & & & \frac{1}{R_2} \end{bmatrix}, \mathbf{I}_{0b} = \begin{bmatrix} U_g(s)/R_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

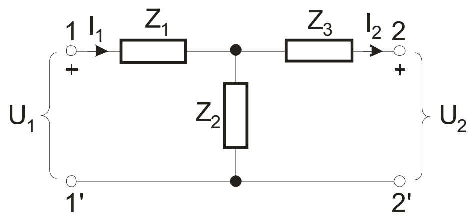
e) sustav jednadžbi petlji (matrice $\mathbf{Z}_p$ i $\mathbf{U}_{0p}$): **(1 bod)**

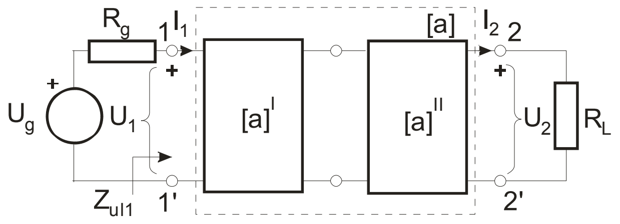
$$\mathbf{Z}_p = \mathbf{S}\cdot\mathbf{Z}_b\cdot\mathbf{S}^T = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_1 & & & & & \\ & sL_2 & & & & \\ & & \frac{1}{sC_2} & & 0 & \\ & & & sL_3 & & \\ & 0 & & & sL_1 & \\ & & & & & R_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} R_1 & sL_2 & \frac{1}{sC_2} & 0 & sL_1 & 0 \\ 0 & -sL_2 & -\frac{1}{sC_2} & sL_3 & 0 & R_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 1 & -1 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_1 + sL_2 + \frac{1}{sC_2} + sL_1 & -sL_2 - \frac{1}{sC_2} \\ -sL_2 - \frac{1}{sC_2} & sL_2 + \frac{1}{sC_2} + sL_3 + R_2 \end{bmatrix}$$

$$\mathbf{U}_{0p} = -\mathbf{S}\cdot\mathbf{U}_{0b} = -\begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} U_g(s) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -U_g(s) \\ 0 \end{bmatrix}$$

3. Za T-četveropol prikazan lijevom slikom izračunati prijenosne a-parametre. a) Napisati parametre A, B, C i D pomoću Z_1, Z_2 i Z_3 te uvrstiti slijedeće vrijednosti elemenata: $Z_1=sL_1=s$, $Z_2=R_2=1$, $Z_3=R_3=1$. b) Ako su dva ista četveropola iz prethodne točke spojena u kaskadu kao na desnoj slici, izračunati ukupne a-parametre kaskade uz uvrštene vrijednosti elemenata. c) Da li je ukupni četveropol (kaskada) recipročan, simetričan? Ako je izlazni prilaz (2–2') zaključen otporom $R_L=1$ pomoću a-parametara izračunati za kaskadu: d) ulaznu impedanciju $Z_{ul1}(s)=U_1(s)/I_1(s)$; e) ako je uz to na ulaz priključen generator ulaznog otpora $R_g=1$ izračunati prijenosnu funkciju napona kaskade $H(s)=U_2(s)/U_g(s)$.



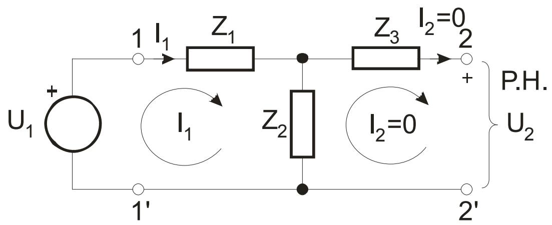


Rješenje:

a) [a]-parametri: **(1 bod)**

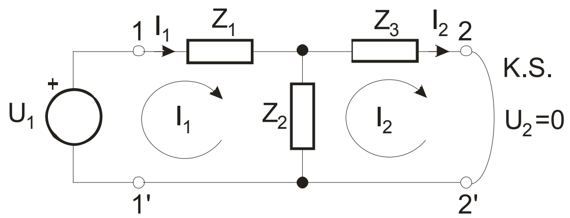
$$U_1 = A\cdot U_2 + B\cdot I_2$$
$$I_1 = C\cdot U_2 + D\cdot I_2$$

$$I_2 = 0 \qquad A=\left.\frac{U_1}{U_2}\right|_{I_2=0} \;;\quad C=\left.\frac{I_1}{U_2}\right|_{I_2=0}$$



$$\begin{aligned}&(1)\;\; U_1 = I_1(Z_1+Z_2)\\ &(2)\;\; U_2 = I_1 Z_2\end{aligned} \quad\Rightarrow\quad A=\frac{U_1}{U_2}=\frac{Z_1+Z_2}{Z_2}=1+\frac{Z_1}{Z_2};\quad C=\frac{I_1}{U_2}=\frac{1}{Z_2};$$

$$U_2 = 0 \qquad B=\left.\frac{U_1}{I_2}\right|_{U_2=0} \;;\quad D=\left.\frac{I_1}{I_2}\right|_{U_2=0}$$



$$\begin{aligned}&(1)\;\; U_1 = I_1(Z_1+Z_2) - I_2 Z_2\\ &(2)\;\; 0 = -I_1 Z_2 + I_2(Z_2+Z_3)\end{aligned} \quad\Rightarrow\quad I_1 = I_2\frac{Z_2+Z_3}{Z_2};$$

$$U_1 = I_2\frac{Z_2+Z_3}{Z_2}(Z_1+Z_2) - I_2 Z_2 = I_2\frac{(Z_2+Z_3)(Z_1+Z_2)-Z_2^2}{Z_2} \Rightarrow U_1 = I_2\frac{Z_1Z_2+Z_1Z_3+Z_2Z_3}{Z_2}$$

$$\Rightarrow\quad B=\frac{U_1}{I_2}=\frac{Z_1Z_2+Z_1Z_3+Z_2Z_3}{Z_2}=Z_1+Z_3+\frac{Z_1Z_3}{Z_2};\quad D=\frac{I_1}{I_2}=\frac{Z_2+Z_3}{Z_2}=1+\frac{Z_3}{Z_2};$$

$$[a]=\begin{bmatrix} A & B \\ C & D \end{bmatrix};\quad [a]=\begin{bmatrix} 1+\dfrac{Z_1}{Z_2} & Z_1+Z_3+\dfrac{Z_1Z_3}{Z_2} \\ \dfrac{1}{Z_2} & 1+\dfrac{Z_3}{Z_2} \end{bmatrix}=\begin{bmatrix} 1+s & 1+2s \\ 1 & 2 \end{bmatrix}$$

(ovo su parametri jednog četveropola)

b) prijenosni a-parametri kaskade dva četveropola **(1bod)**

$$[a]=[a]^{I}\cdot[a]^{II}=\begin{bmatrix} 1+s & 1+2s \\ 1 & 2 \end{bmatrix}\cdot\begin{bmatrix} 1+s & 1+2s \\ 1 & 2 \end{bmatrix}=\begin{bmatrix} (1+s)^2+1+2s & (1+s)(1+2s)+2(1+2s) \\ 1+s+2 & 1+2s+4 \end{bmatrix}$$

$$[a]=\begin{bmatrix} s^2+4s+2 & 2s^2+7s+3 \\ s+3 & 2s+5 \end{bmatrix}$$

(ovo su parametri dva četveropola spojena u kaskadu)

c) Da li je kaskada recipročna, simetrična? **(1bod)**

Za recipročnost vrijedi: $\Delta = AD - BC = 1$

$$\Delta=(s^2+4s+2)(2s+5)-(2s^2+7s+3)(s+3)=$$

$$=2s^3+8s^2+4s+5s^2+20s+10-(2s^3+7s^2+3s+6s^2+21s+9)=1$$

$\Rightarrow$ Dobiveni čeveropol je recipročan.

Za simetričnost vrijedi: A=D $\Rightarrow s^2+4s+2\neq 5+2s$

$\Rightarrow$ Dobiveni četveropol nije simetričan

Konačno iz jednadžbi

$$\underline{\begin{matrix} U_1=A\cdot U_2+B\cdot I_2 \\ I_1=C\cdot U_2+D\cdot I_2 \end{matrix}},\quad R_L=\frac{U_2}{I_2},\quad U_g=I_1R_g+U_1$$

slijede:

d) Ulazna impedancija u četveropol: **(1 bod)**

$$Z_{ul1}(s)=\frac{U_1}{I_1}=\frac{AU_2+BI_2}{CU_2+DI_2}=\frac{A\dfrac{U_2}{I_2}+B}{C\dfrac{U_2}{I_2}+D}=\frac{AR_L+B}{CR_L+D}$$

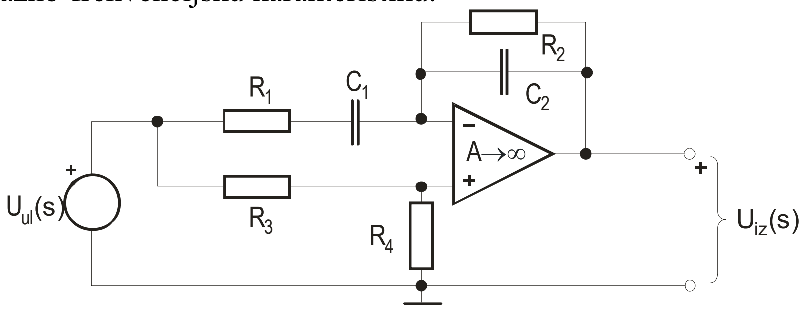
$$Z_{ul1}(s)=\frac{(s^2+4s+2)\cdot 1+(2s^2+7s+3)}{(s+3)\cdot 1+(2s+5)}=\frac{3s^2+11s+5}{3s+8}$$

e) Prijenosna funkcija napona: **(1 bod)**

$$U_g=I_1R_g+U_1=\left(CU_2+D\frac{U_2}{R_L}\right)R_g+AU_2+B\frac{U_2}{R_L}\ \Rightarrow\ H(s)=\frac{U_2}{U_g}=\frac{R_L}{AR_L+B+R_g(CR_L+D)}$$

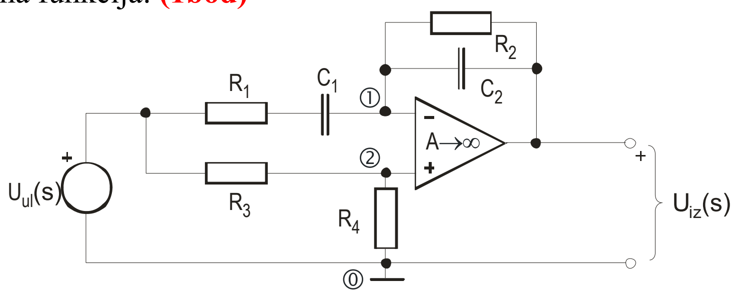
$$H(s)=\frac{1}{(s^2+4s+2)\cdot 1+2s^2+7s+3+1\cdot[(s+3)\cdot 1+2s+5]}=\frac{1}{3s^2+14s+13}$$

4. Za električni filtar (široko-pojasnu) pojasnu branu prikazanu slikom zadane su vrijednosti elemenata R_1=R_2=2kΩ, C_1=C_2=100nF i R_3=1kΩ. Izračunati: a) naponsku prijenosnu funkciju $T(s)$=$U_{iz}(s)/U_{ul}(s)$; b) otpor R_4 iz uvjeta za pojasnu branu, Q-faktor polova q_p, centralnu frekvenciju ω_0 te pojačanje u području propuštanja k; c) Kolika je širina pojasa gušenja B, te gornja i donja granična frekvencija ω_g i ω_d? Izračunati i skicirati: d) amplitudno-frekvencijsku karakteristiku $|T(j\omega)|$ u dB; e) fazno-frekvencijsku karakteristiku.



Rješenje:

a) Naponska prijenosna funkcija: **(1bod)**



Naponske jednadžbe za čvorove (1) i (2) glase:

$$(1)\quad U_1(s)\left(\frac{1}{R_1+1/sC_1}+sC_2+\frac{1}{R_2}\right)=\frac{U_{ul}(s)}{R_1+1/sC_1}+U_{iz}(s)\left(sC_2+\frac{1}{R_2}\right)$$

$$(2)\quad U_2(s)\left(\frac{1}{R_3}+\frac{1}{R_4}\right)=\frac{U_{ul}(s)}{R_3}\quad\Rightarrow\quad U_2(s)=\frac{R_4}{R_3+R_4}U_{ul}(s)$$

Zbog virtualnog kratkog spoja je $U_1(s)$=$U_2(s)$ pa vrijedi:

$$U_{ul}(s)\frac{R_4}{R_3+R_4}\left(\frac{1}{R_1+\frac{1}{sC_1}}+sC_2+\frac{1}{R_2}\right)=\frac{U_{ul}(s)}{R_1+\frac{1}{sC_1}}+U_{iz}(s)\left(sC_2+\frac{1}{R_2}\right)\Big/\cdot\left(R_1+\frac{1}{sC_1}\right)$$

$$U_{ul}(s)\frac{R_4}{R_3+R_4}\left[1+\left(sC_2+\frac{1}{R_2}\right)\left(R_1+\frac{1}{sC_1}\right)\right]=U_{ul}(s)+U_{iz}(s)\left(sC_2+\frac{1}{R_2}\right)\left(R_1+\frac{1}{sC_1}\right)$$

Nakon kraćeg računanja dobivamo:

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{R_4}{R_3+R_4}\cdot\frac{s^2C_1R_1C_2R_2+s\left(C_1R_1+C_2R_2-\frac{R_3}{R_4}C_1R_2\right)+1}{s^2C_1R_1C_2R_2+s\left(C_1R_1+C_2R_2\right)+1}$$

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{R_4}{R_3+R_4}\cdot\frac{s^2+s\frac{C_1R_1+C_2R_2-(R_3/R_4)C_1R_2}{C_1R_1C_2R_2}+\frac{1}{C_1R_1C_2R_2}}{s^2+s\frac{C_1R_1+C_2R_2}{C_1R_1C_2R_2}+\frac{1}{C_1R_1C_2R_2}}$$

b) Parametri prijenosne funkcije: q_p, ω_p i k: **(1bod)**

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=k\cdot\frac{s^2+\omega_p^2}{s^2+(\omega_p/q_p)\cdot s+\omega_p^2} \quad\Rightarrow\quad \omega_p=\frac{1}{\sqrt{C_1R_1C_2R_2}}, \qquad k=\frac{R_4}{R_3+R_4},$$

$$\frac{\omega_p}{q_p}=\frac{C_1R_1+C_2R_2}{C_1R_1C_2R_2} \quad\Rightarrow\quad q_p=\omega_p\cdot\frac{C_1R_1C_2R_2}{C_1R_1+C_2R_2}=\frac{\sqrt{C_1R_1C_2R_2}}{C_1R_1+C_2R_2}$$

Uvjet za pojasnu branu je da srednji član u brojniku koji množi s bude jednak nuli (slijedi R_4):

$$C_1R_1+C_2R_2-(R_3/R_4)C_1R_2=0 \;\Rightarrow\; C_1R_1+C_2R_2=(R_3/R_4)C_1R_2 \quad\Rightarrow\quad R_4=R_3\cdot\frac{C_1R_2}{C_1R_1+C_2R_2}$$

Uz zadane vrijednosti: $C_1=C_2=C=100\text{nF}$, $R_1=R_2=R=1500\Omega$ i $R_3=1\text{k}\Omega$ slijedi:

$$\omega_p=\frac{1}{RC}=\frac{1}{100\cdot10^{-9}\cdot2000}=5\cdot10^3\,\text{rad/s},\; q_p=\frac{1}{2},\; R_4=R_3\cdot\frac{1}{2}=500\Omega,\; k=\frac{R_3/2}{R_3+R_3/2}=\frac{1/2}{3/2}=\frac{1}{3}$$

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{1}{3}\cdot\frac{s^2+2.5\cdot10^7}{s^2+10^4\cdot s+2.5\cdot10^7}$$

Napomena: za ovaj električni krug te zadanu centralnu frekvenciju $\omega_0=5\cdot10^3$ rad/s, širina pojasa B ne može biti manja od 10^4 rad/s, odn. normirana širina pojasa (omjer) B/ω_0 ne može biti manja od 2. Zato se ovaj električni filtar zove širokopojasni jer se njime ne može realizirati uži pojas gušenja B od navedenoga (i time mu je ograničeno područje primjene).

c) Širina pojasa gušenja B, te gornja i donja granična frekvencija ω_g i ω_d (koriste se isti izrazi kao za pojasno-propusni filtar): **(1bod)**

Širina pojasa gušenja $B=\dfrac{\omega_p}{q_p}=\dfrac{5\cdot10^3}{1/2}=10^4$ [rad/s]

Gornja i donja granična frekvencija pojasa gušenja su:

$$\omega_{g,d}=\omega_p\sqrt{1+\frac{1}{4q_p^2}}\pm\frac{\omega_p}{2q_p}=5\cdot10^3\sqrt{1+\frac{1}{4\cdot0.25}}\pm\frac{5\cdot10^3}{1}=5\cdot10^3(\sqrt{2}\pm1)\text{ [rad/s]}$$

ω_g=12071 [rad/s], ω_d=2071 [rad/s], $B=\omega_g-\omega_d$=12071–2017=10 000= 10^4 [rad/s]

d) Amplitudno-frekvencijska karakteristika: **(1bod)**

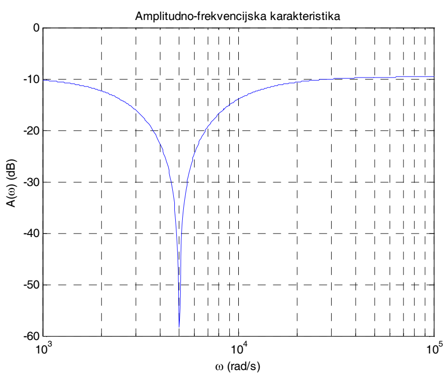
$$\Rightarrow\; T(j\omega)=\frac{N(j\omega)}{D(j\omega)}=\frac{1}{3}\cdot\frac{2.5\cdot10^7-\omega^2}{-\omega^2+10^4\cdot j\omega+2.5\cdot10^7} \quad\Rightarrow\quad |T(j\omega)|=\frac{1}{3}\cdot\frac{\left|2.5\cdot10^7-\omega^2\right|}{\sqrt{(2.5\cdot10^7-\omega^2)^2+(10^4\cdot\omega)^2}}$$

$$\Rightarrow\; A(\omega)[\text{dB}]=20\log|T(j\omega)|=20\log\frac{1}{3}\cdot\frac{\left|2.5\cdot10^7-\omega^2\right|}{\sqrt{(2.5\cdot10^7-\omega^2)^2+(10^4\cdot\omega)^2}}$$

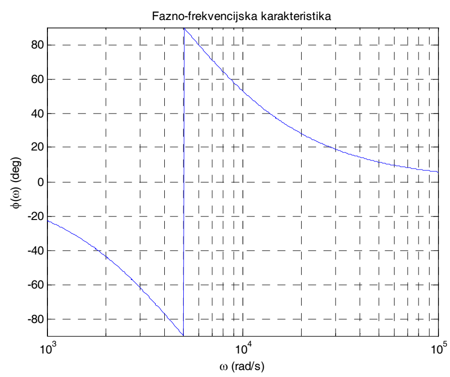
e) Fazno-frekvencijska karakteristika: **(1bod)**

$$\varphi(\omega)=\arctan\frac{\text{Im}[T(j\omega)]}{\text{Re}[T(j\omega)]}=\arctan\frac{\text{Im}[N(j\omega)]}{\text{Re}[N(j\omega)]}-\arctan\frac{\text{Im}[D(j\omega)]}{\text{Re}[D(j\omega)]}=\pi\cdot S(\omega-5\cdot10^3)-\arctan\frac{10^4\cdot\omega}{2.5\cdot10^7-\omega^2}$$

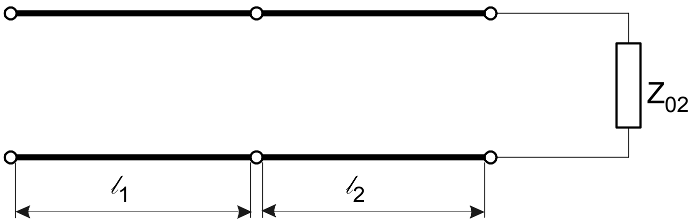
Amplitudno-frekvencijska karakteristika u Matlabu



Fazno-frekvencijska karakteristika u Matlabu



5. Na ulazu linije bez gubitaka duljine $l_1=\lambda_1/2$, s primarnim parametrima L_1=1mH/km i C_1=400nF/km, djeluje napon $u_I(0,t)=2\cos(10^4 t)$, a na izlaz je priključena linija bez gubitaka zadana sa L_2=2,25 mH/km i C_2=400 nF/km, koja je zaključena svojom karakterističnom impedancijom. Izračunati: a) valne impedancije obiju linija Z_{01} i Z_{02}; b) faktor refleksije prve linije na spojnom mjestu; c) polazni i reflektirani val struje na spojnom mjestu; d) struju $i_{II}(0,t)$ na ulazu druge linije; e) vrijednost otpora R, kojeg treba spojiti paralelno ulazu druge linije, da bi prva linija bila prilagođena.



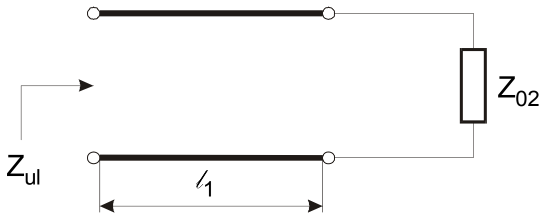
Rješenje:

a) $$Z_{01}=\sqrt{L_1/C_1}=\sqrt{10^{-3}/4\cdot10^{-7}}=50\Omega$$ **(1 bod)**

$$Z_{02}=\sqrt{L_2/C_2}=\sqrt{2{,}25\cdot10^{-3}/4\cdot10^{-7}}=75\Omega$$

b) $$\Gamma_2=\frac{Z_{02}-Z_{01}}{Z_{02}+Z_{01}}=\frac{25}{125}=\frac{1}{5}=0.2$$ **(1 bod)**

Za $l=\lambda/2 \Rightarrow Z_{ul}=Z_2$



$Z_{ul}=Z_{02}$

c) $$U_p\left(\frac{\lambda_1}{2}\right)=A_1e^{-j\beta_1\frac{\lambda_1}{2}}=\frac{U(0)+Z_{01}I(0)}{2}e^{-j\beta_1\frac{\lambda_1}{2}}=\frac{U(0)+Z_{01}I(0)}{2}e^{-j\pi}=$$

$$=\frac{U(0)+Z_{01}\dfrac{U(0)}{Z_{ul}}}{2}e^{-j\pi}=\frac{U(0)}{2}\left(1+\frac{50}{75}\right)e^{-j\pi}=-1.667\text{V}$$

$$I_p(\lambda_1/2)=\frac{U_p(\lambda_1/2)}{Z_{01}}=\frac{-1{,}667}{50}\text{A}=-0{,}03333\text{A}$$

$i_p(l_1,t)$= 0.03333(cos 10^4 t– 180°)= – 0.03333 (cos 10^4 t)

$$U_r=\Gamma_2\cdot U_p=\Gamma_2\cdot A_1e^{-j\beta_1\frac{\lambda_1}{2}}=-0.2\cdot1.667=\underline{-0.3333\text{ V}}$$

$$I_r=-\frac{U_r}{Z_{01}}=-\frac{\Gamma_2\cdot U_p}{Z_{01}}=-\frac{\Gamma_2\cdot A_1e^{-j\beta_1\frac{\lambda_1}{2}}}{Z_{01}}=-\frac{\Gamma_2\cdot A_1e^{-j\beta_1\frac{\lambda_1}{2}}}{Z_{01}}=0.006667A$$

$i_r(l_1,t)$= 0.03333 (cos 10^4 t) **(1 bod)**

d) $i_{II}(0,t)=i_p(l_1,t)+i_r(l_1,t)$= – 0.026667(cos 10^4 t) **(1 bod)**

e) $$Z_{01}=\frac{Z_{02}\cdot R}{Z_{02}+R}\quad\Rightarrow\quad R=\frac{Z_{02}\cdot Z_{01}}{Z_{02}-Z_{01}}=150\Omega$$ **(1 bod)**