

FER2.net

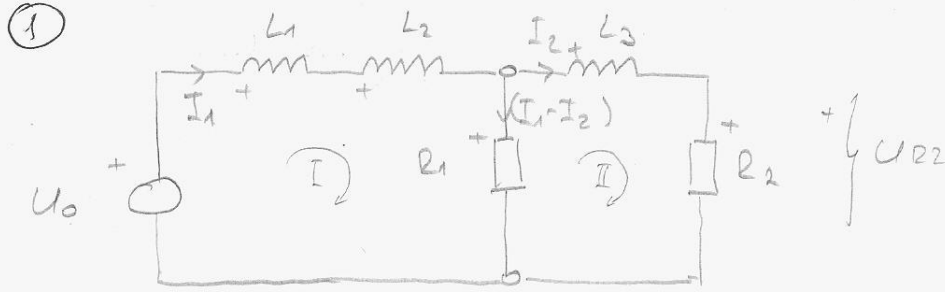
Električni krugovi

Zadaci za vježbu za prvi međuispit ak. god. 2007./2008.

- skenirani postupci rješavanja, verzija: v0.1.
- zadaci po kojima je rješavano nalaze se u istom .rar dokumentu

by: Tywin





$$(I) \quad U_0 - I_1(sL_1 + sL_2) - (I_1 - I_2)R_1 = 0$$

$$(II) \quad (I_1 - I_2)R_1 - I_2(sL_3 + R_2) = 0$$

$$(I) \quad \frac{1}{s} - 3sI_1 - I_1 + I_2 = 0$$

$$(II) \quad I_1 - I_2 - I_2(4s + 1) = 0$$

$$(II) \quad I_1 = I_2(4s + 2) \quad \text{in } (I)$$

$$\frac{1}{s} - I_1(3s + 1) - I_2 = 0$$

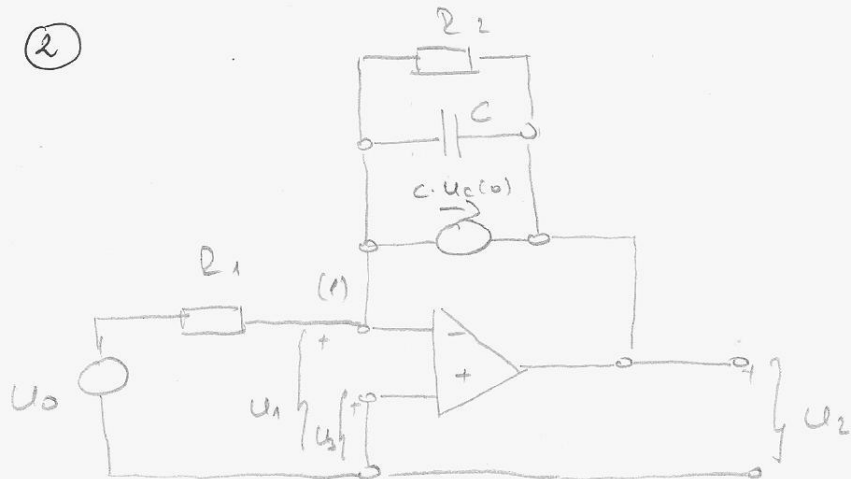
$$\frac{1}{s} - I_2(4s + 2)(3s + 1) - I_2 = 0$$

$$I_2 [12s^2 + 10s + 2 - 1] = \frac{1}{s}$$

$$I_2 = \frac{1}{s(12s^2 + 10s + 1)}$$

$$U_{R2}(s) = I_2 \cdot R_2 = \frac{1}{s(12s^2 + 10s + 1)}$$

2



JEDNADŽBA POJAČALA:

$$\left. \begin{aligned} U_2 &= A(U_3 - U_1) \\ A &\rightarrow \infty \end{aligned} \right\} U_1 = U_3 = 0$$

$$(1) \quad U_1 \left[\frac{1}{R_1} + \frac{1}{R_2} + sC \right] - U_2 \left[\frac{1}{R_2} + sC \right] - U_0 \frac{1}{R_1} = -C \cdot U_c(0)$$

$$U_2 \left[\frac{1}{R_2} + sC \right] = C U_c(0) - U_0 \frac{1}{R_1}$$

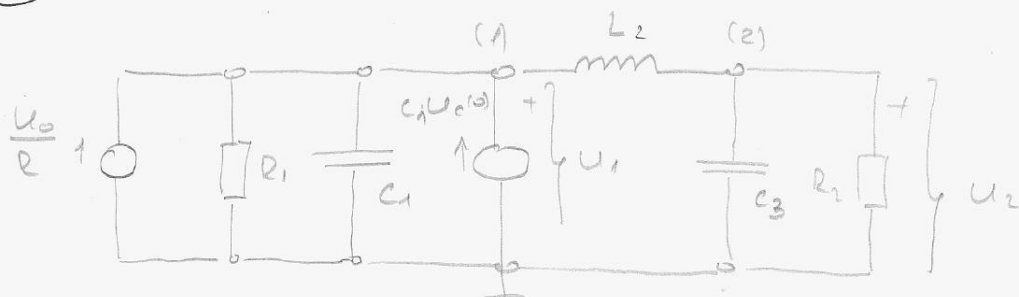
$$U_2 [1 + s] = 1 - \frac{1}{s} = \frac{s-1}{s}$$

$$U_2 = \frac{s-1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} = \frac{As + A + Bs}{s(s+1)}$$

$$\left\{ \begin{aligned} s-1 &= As + A + Bs \\ A+B &= 1 \\ A=-1 &\rightarrow B=2 \end{aligned} \right.$$

$$U_2(s) = \frac{2}{s+1} - \frac{1}{s} \rightarrow U_2(t) = (2e^{-t} - 1) \cdot 1(t)$$

(3)



$$(1) \quad U_1 \left[\frac{1}{R_1} + sC_1 + \frac{1}{sL_2} \right] - U_2 \frac{1}{sL_2} = \frac{U_0}{2} + C_1 U_C(\omega)$$

$$(2) \quad U_2 \left[sC_3 + \frac{1}{R_2} + \frac{1}{sL_2} \right] - U_1 \frac{1}{sL_2} = 0$$

$$(1) \quad U_1 \left[s + 1 + \frac{1}{2s} \right] - U_2 \frac{1}{2s} = 1 + 1$$

$$(2) \quad U_2 \left[s + 1 + \frac{1}{2s} \right] - U_1 \frac{1}{2s} = 0$$

$$(2) \quad U_1 = U_2 [2s^2 + 2s + 1] \quad u \quad (1)$$

$$(1) \quad U_2 [2s^2 + 2s + 1] \left[\frac{2s^2 + 2s + 1}{2s} \right] - U_2 \frac{1}{2s} = 2$$

$$U_2 \left[\frac{4s^4 + 4s^3 + 2s^2 + 4s^3 + 4s^2 + 4s^2 + 2s + 2s^2 + 2s + 1}{2s} - \frac{1}{2s} \right] = 2$$

$$U_2 [4s^3 + 8s^2 + 8s + 4] = 4$$

$$U_2 [4s^3 + 4s^2 + 4s^2 + 4s + 4s + 4] = 4$$

$$U_2 [4s^2(s+1) + 4s(s+1) + 4(s+1)] = 4$$

$$U_2 (s+1)(4s^2 + 4s + 4) = 4$$

$$U_2 = \frac{1}{(s+1)(s^2+s+1)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+s+1} \quad \rightarrow \text{jer nema realna rješenja}$$

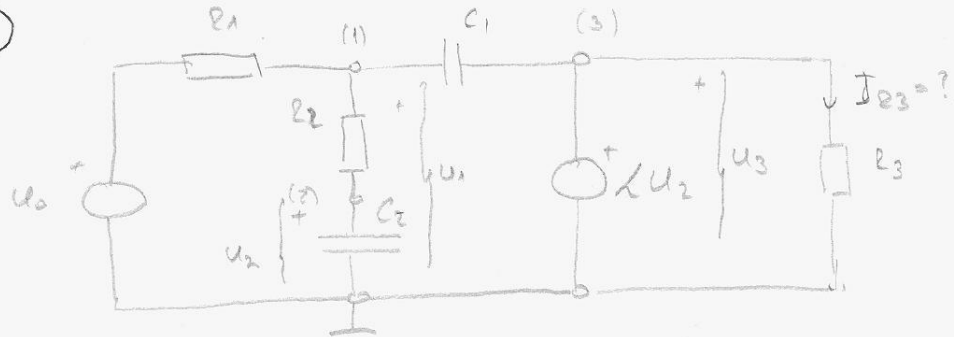
$$= \frac{As^2 + As + A + Bs^2 + Cs + Bs + C}{(s+1)(s^2+s+1)} \Rightarrow \begin{cases} A+B=0 \\ A+B+C=0 \\ A+C=1 \end{cases} \Rightarrow \begin{cases} C=0 \\ B=-1 \\ A=1 \end{cases}$$

$$U_2 = \frac{1}{s+1} - \frac{s}{s^2+s+1} = \frac{1}{s+1} - \frac{s + \frac{1}{2} - \frac{1}{2}}{(s + \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$

$$= \frac{1}{s+1} - \frac{s + \frac{1}{2}}{(s + \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} + \frac{1}{2} \cdot \frac{\sqrt{3}}{\sqrt{3}} \cdot \frac{\frac{\sqrt{3}}{2}}{(s + \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$

$$\rightarrow U_2(t) = \left[e^{-t} - e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}}{2}t\right) + e^{-\frac{t}{2}} \frac{\sqrt{3}}{3} \sin\left(\frac{\sqrt{3}}{2}t\right) \right] u(t)$$

4



$$(1) \quad u_1 \left[\frac{1}{R_1} + \frac{1}{R_2} + sC_1 \right] - u_0 \frac{1}{R_1} - u_2 \frac{1}{R_2} - u_3 sC = 0$$

$$(2) \quad u_2 \left[\frac{1}{R_2} + sC_2 \right] - u_1 \frac{1}{R_2} = 0$$

$$u_3 = L u_2 \quad u \quad (1)$$

$$(2) \quad u_1 = u_2 [1 + 2s] \quad u \quad (1)$$

$$(1) \quad u_2 (1+s) (1+1+2s) - 1 - u_2 - L u_2 2s = 0$$

$$u_2 (2s+1) (2s+2) - u_2 - 4s \cdot u_2 = 1$$

$$u_2 [4s^2 + 6s + 2 - 1 - 4s] = 1$$

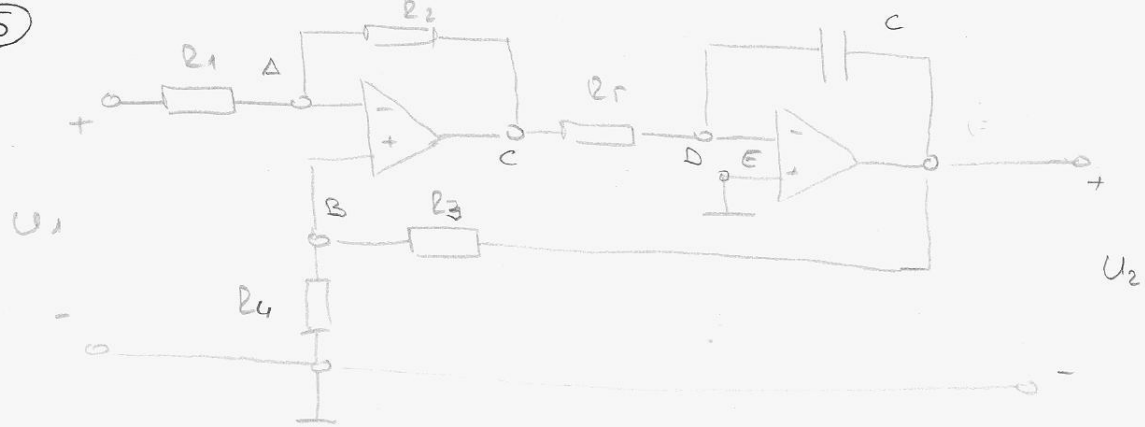
$$u_2 = \frac{1}{4s^2 + 2s + 1}$$

$$i_{R3} = \frac{u_3}{R_3} = \frac{L u_2}{R_3} = \frac{1}{4s^2 + 2s + 1} = \frac{1}{4} \cdot \frac{1}{s^2 + \frac{1}{2}s + \frac{1}{4}}$$

$$= \frac{1}{4} \cdot \frac{\frac{\sqrt{3}}{4} \cdot \frac{1}{\sqrt{3}}}{\left(s + \frac{1}{4}\right)^2 + \left(\frac{\sqrt{3}}{4}\right)^2}$$

$$\rightarrow i_{R3}(t) = \frac{\sqrt{3}}{3} e^{-\frac{t}{4}} \sin\left(\frac{\sqrt{3}}{4}t\right) \cdot s(t)$$

5



$$(A) \quad U_A \left[\frac{1}{R_1} + \frac{1}{R_2} \right] - U_1 \frac{1}{R_1} - U_C \frac{1}{R_2} = 0$$

$$(B) \quad U_B \left[\frac{1}{R_2} + \frac{1}{R_3} \right] - U_2 \frac{1}{R_3} = 0$$

$$(D) \quad U_D \left[\frac{1}{R_r} + sC \right] - U_C \frac{1}{R_r} - U_2 sC = 0$$

jednacičbe pojačala : $U_A = U_B$
 $U_D = U_E = 0$

$$(B) \quad U_A = \frac{U_2}{2} \quad u \quad (+)$$

$$(D) \quad U_C = -U_2 \cdot s \quad u \quad (-)$$

$$(A) \quad \frac{U_2}{2} [1+1] - \frac{1}{s} + U_2 \cdot s = 0$$

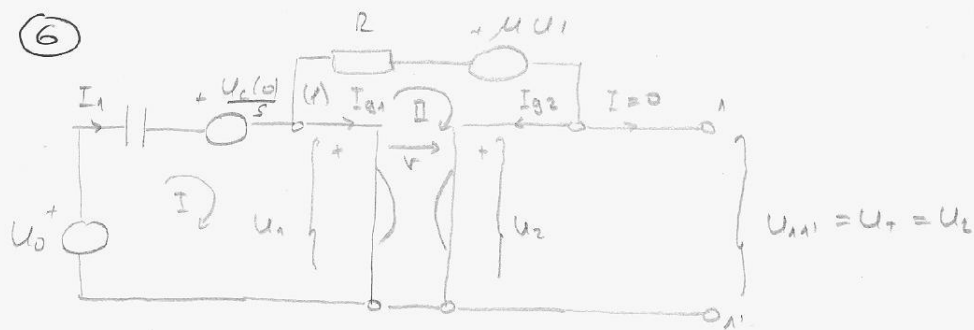
$$U_2 [1+s] = \frac{1}{s}$$

$$U_2 = \frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1} = \frac{As + A + Bs}{s(s+1)}$$

$$= \frac{1}{s} - \frac{1}{s+1}$$

$$\begin{cases} A+B=0 \\ A=1 \Rightarrow B=-1 \end{cases}$$

⑥



a) jednačine giratora uz ovako postavljene struje

i napone: $U_1 = r \cdot I_{q2}$ (g1)

$U_2 = -r \cdot I_{q1}$ (g1)

(I) $U_0 - I_1 \frac{1}{sC} - \frac{U_c(0)}{s} - U_1 = 0$

(II) $-I_{q2} R - \mu U_1 - U_2 + U_1 = 0$

(1) $I_1 = I_{q1} + I_{q2}$

(g1) $I_{q2} = \frac{U_1}{r}$ } u (1) $\Rightarrow I_1 = \frac{U_1 - U_2}{r}$ u (I)

(g2) $I_{q1} = -\frac{U_2}{r}$

(g2) u (II)

(I) $\frac{1}{s} - \frac{U_1 - U_2}{0.5s} - \frac{2}{s} - U_1 = 0 \quad / \cdot s$

(II) $-U_1 - 0.5 U_1 - U_2 + U_1 = 0$

(II) $U_1 = \frac{-U_2}{0.5} = -2 U_2$ u (I)

(I) $1 - 2U_1 + 2U_2 - 2 - U_1 s = 0$

$2U_2 - U_1(s+2) = 1$

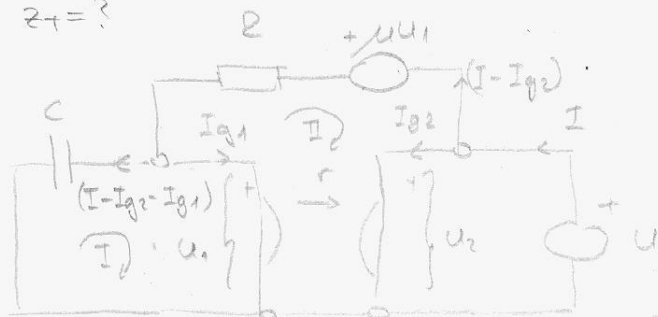
$2U_2 + 2(s+2) = 1$

$U_2 [2 + 2s + 4] = 1$

$U_2 2[s+3] = 1$

$U_2 = U_1 = \frac{1}{2} \cdot \frac{1}{s+3}$

b) $z_T = ?$



$$z_T = \frac{U}{I} = \frac{U_2}{I}$$

abbe giratora : $U_1 = r \cdot I_{q2}$ (q1)

$U_2 = -r \cdot I_{q1}$ (q2)

$$(I) \quad \frac{1}{sC} (I - I_{q2} - I_{q1}) - U_1 = 0$$

$$(II) \quad U_1 + (I - I_{q2})R - \mu U_1 - U_2 = 0$$

$$\left. \begin{array}{l} (q1) \quad I_{q2} = \frac{U_1}{r} \\ (q2) \quad I_{q1} = -\frac{U_2}{r} \end{array} \right\} \begin{array}{l} u(I) \\ r(II) \end{array}$$

$$(I) \quad \frac{1}{s} (I - 2U_1 + 2U_2) - U_1 = 0 \quad / \cdot s$$

$$(II) \quad U_1 + (I - 2U_1) \cdot 0.5 - 0.5 U_1 - U_2 = 0 \quad / \cdot 2$$

$$(I) \quad I - U_1(s+2) + 2U_2 = 0$$

$$(II) \quad I - 2U_2 - U_1 = 0 \Rightarrow U_1 = I - 2U_2 \quad u(I)$$

$$(I) \quad I - (I - 2U_2)(s+2) + 2U_2 = 0$$

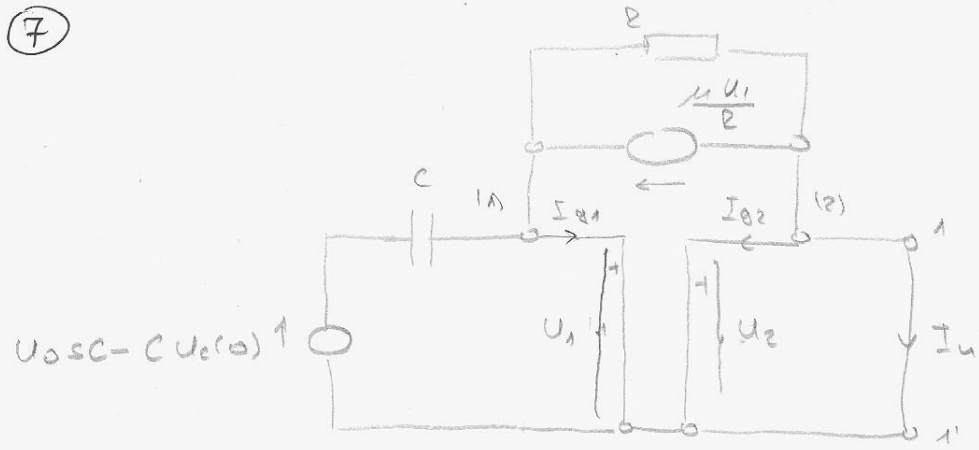
$$I - I(s+2) + 2U_2(s+2) + 2U_2 = 0$$

$$U_2(2s+4+2) = I(s+2-1)$$

$$U_2 = I \cdot \frac{s+1}{2s+6}$$

$$z_T = \frac{U_2}{I} = \frac{s+1}{2(s+3)}$$

7



jedbe giratora: $U_1 = r \cdot I_{q2} \quad (g1)$
 $U_2 = -r \cdot I_{q1} \quad (g2)$

(1) $U_1 \left[sC + \frac{1}{R} \right] - U_2 \frac{1}{R} = U_0 sC - CU_0(0) + \mu \frac{U_1}{R} - I_{q1}$

(2) $U_2 \left[\frac{1}{R} \right] - U_1 \frac{1}{R} = -\frac{\mu U_1}{R} - I_{q2} - I_n$

→ na izlazu 1-1' je kratki spoj pa $U_2 = 0$ a
 prema (g2) $I_{q1} = 0$

(1) $U_1 (s+2) = 1 - 2 + U_1 \Rightarrow U_1 = \frac{-1}{s+1}$

(2) $-2U_1 = -U_1 - I_{q2} - I_n$

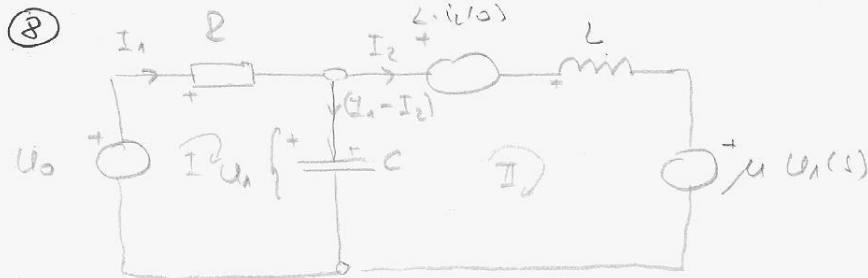
(g1) $I_{q2} = 2U_1 = \frac{-2}{s+1}$

(2) $I_n = U_1 - I_{q2} = \frac{1}{s+1}$

b) Nortonova impedancija = Theveninova impedancija

$Z_n = Z_T = \frac{1}{2} \cdot \frac{s+1}{s+3}$

$Y_n = \frac{1}{Z_n} = \frac{2(s+3)}{s+1}$



$$U_1 = \frac{1}{sC} (I_1 - I_2) \Rightarrow I_1 = U_1 \cdot sC + I_2$$

$$(I) \quad U_0 - I_1 R - (I_1 - I_2) \frac{1}{sC} = 0$$

$$(II) \quad (I_1 - I_2) \frac{1}{sC} - L \dot{i}_L(0) - I_2 L s - \mu U_1 = 0$$

$$(I) \quad I_1 \left(1 + \frac{3}{s}\right) - I_2 \cdot \frac{3}{s} = \frac{1}{s} \quad / \cdot s$$

$$(II) \quad I_2 \left(\frac{3}{s} + \frac{s}{2}\right) - I_1 \frac{3}{s} = -\frac{1}{2} - \frac{2}{3} U_1 \quad / \cdot 6s$$

$$(I) \quad I_1 (s + 3) - 3 I_2 = 1$$

$$(II) \quad I_2 (3s^2 + 18) - 18 I_1 = -3s + 4s \cdot U_1 \quad \left. \begin{array}{l} I_1 = U_1 \cdot \frac{s}{3} + I_2 \\ \text{u (I) + (II)} \end{array} \right\}$$

$$(I) \quad U_1 \frac{s}{3} (s + 3) + I_2 s = 1$$

$$(II) \quad I_2 (3s^2 + 18) - 18 \cdot U_1 \frac{s}{3} - 18 I_2 = -3s - 4s \cdot U_1$$

$$3s^2 I_2 - 28 U_1 = -3s$$

$$3s I_2 = 2 U_1 - 3 \quad \text{u (I)}$$

$$(I) \quad U_1 (s^2 + 3s) + 2 U_1 - 3 = 3$$

$$U_1 (s^2 + 3s + 2) = 6$$

$$U_1 = \frac{6}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} = \frac{4s + 2A + Bs + B}{(s+1)(s+2)}$$

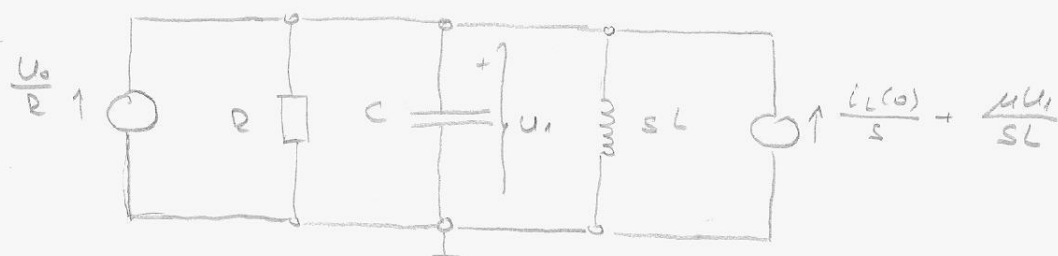
$$\left\{ \begin{array}{l} A + B = 0 \\ 2A + B = 6 \Rightarrow A = 6 \Rightarrow B = -6 \end{array} \right\}$$

$$U_1(s) = 6 \left[\frac{1}{s+1} - \frac{1}{s+2} \right]$$

$$\rightarrow U_1(t) = 6 [e^{-t} - e^{-2t}] s(t)$$

(9)

(1)



$$(1) \quad U_1 \left[\frac{1}{R} + sC + \frac{1}{sL} \right] = \frac{U_0}{R} + \frac{i_L(0)}{s} + \frac{\mu U_1}{sL}$$

$$U_1 \left[\frac{1}{R} + sC + \frac{1-\mu}{sL} \right] = \frac{U_0}{R} + \frac{i_L(0)}{s}$$

$$U_1 \left[1 + \frac{s}{3} + \frac{\frac{2}{3}}{\frac{s}{2}} \right] = \frac{1}{s} + \frac{1}{s}$$

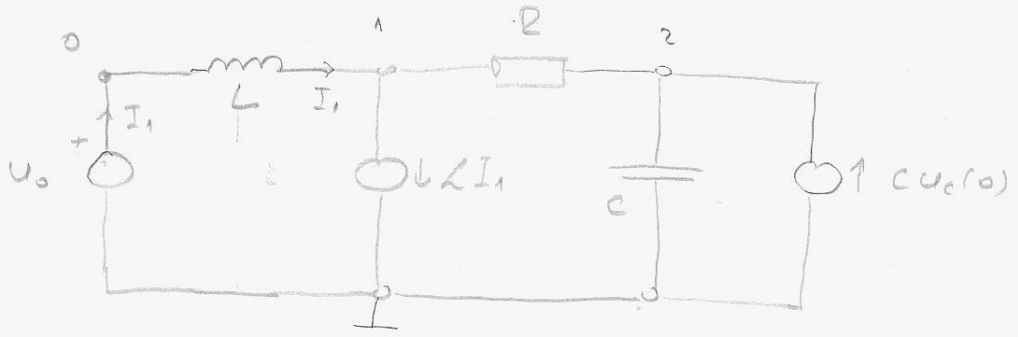
$$U_1 \left[1 + \frac{s}{3} + \frac{2}{3s} \right] = \frac{2}{s} \quad / \cdot 3s$$

$$U_1 [3s + s^2 + 2] = 6$$

$$U_1 = \frac{6}{s^2 + 3s + 2} = \frac{6}{(s+1)(s+2)} = 6 \left[\frac{1}{s+1} - \frac{1}{s+2} \right]$$

$$\rightarrow U_1(t) = 6 [e^{-t} - e^{-2t}] s(t)$$

10



$$\begin{aligned} (0) \quad & U_0 \left[\frac{1}{sL} \right] - U_1 \frac{1}{sL} = I_1 \\ (1) \quad & U_1 \left[\frac{1}{sL} + \frac{1}{R} \right] - U_0 \frac{1}{sL} - U_2 \frac{1}{R} = -\beta I_1 \\ (2) \quad & U_2 \left[\frac{1}{R} + sC \right] - U_1 \frac{1}{R} = C \cdot U_0(s) \end{aligned}$$

$$\begin{aligned} (0) \quad & \frac{1}{s^2} - \frac{U_1}{s} = I_1 \\ (1) \quad & U_1 \left[\frac{1}{s} + \frac{1}{2} \right] - \frac{1}{s^2} - \frac{U_2}{2} = -\frac{I_1}{2} \\ (2) \quad & U_2 \left[\frac{1}{2} + s \right] - \frac{U_1}{2} = \frac{1}{2} \end{aligned}$$

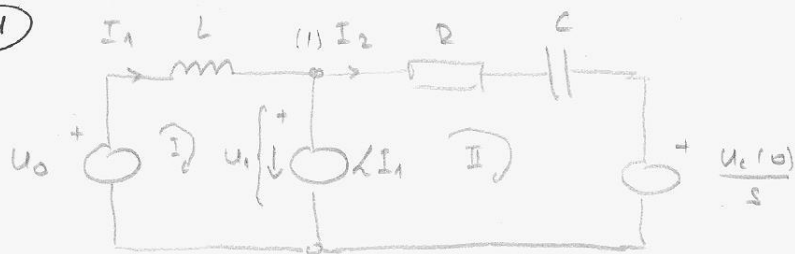
$$\begin{aligned} (0) \quad & I_1 = \frac{1}{s^2} - \frac{U_1}{s} \quad u \quad (1) \\ (2) \quad & U_2 \left[\frac{2s+1}{2} \right] = \frac{U_1+1}{2} \Rightarrow U_2 = U_1 \frac{1}{2s+1} + \frac{1}{2s+1} \quad u \quad (1) \end{aligned}$$

$$\begin{aligned} (1) \quad & U_1 \cdot \frac{s+2}{2s} - \frac{1}{s^2} - U_1 \frac{1}{2} \frac{1}{2s+1} - \frac{1}{2} \frac{1}{2s+1} = U_1 \frac{1}{2s} - \frac{1}{2s^2} \\ & U_1 \left[\frac{s+2}{2s} - \frac{1}{2(2s+1)} - \frac{1}{2s} \right] = \frac{1}{s^2} + \frac{1}{2(2s+1)} - \frac{1}{2s^2} \\ & U_1 \left[\frac{2s^2+s+4s+2-s-2s-1}{2s(2s+1)} \right] = \frac{4s+2+s^2-2s-1}{2s^2(2s+1)} \\ & U_1 \left[\frac{2s^2+2s+1}{2s(2s+1)} \right] = \frac{s^2+2s+1}{s} \Rightarrow U_1 = \frac{s^2+2s+1}{s(2s^2+2s-1)} \quad u \quad (0) \end{aligned}$$

$$\begin{aligned} (0) \quad & I_1 = \frac{1}{s^2} - \frac{s^2+2s+1}{s^2(2s^2+2s-1)} = \frac{2s^2+2s+1-s^2-2s-1}{s^2(2s^2+2s-1)} = \\ & = \frac{1}{2} \frac{1}{s^2+s+\frac{1}{2}} = \frac{\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} \end{aligned}$$

$$\Rightarrow i_1(t) = e^{-\frac{t}{2}} \sin\left(\frac{t}{2}\right) \cdot \frac{1}{2}$$

(11)



$$(I) \quad U_0 - I_1 sL - u_1 = 0$$

$$(II) \quad u_1 - I_2 \left(R + \frac{1}{sC} \right) - \frac{u_c(0)}{s} = 0$$

$$(1) \quad I_1 = I_2 + I_2$$

$$(I)+(II) \quad \frac{1}{s} - I_1 s - I_2 \left(2 + \frac{1}{s} \right) - \frac{1}{2s} = 0$$

$$(1) \quad I_2 = \frac{I_1}{2} \quad \text{in } (I)+(II)$$

$$I_1 s + \frac{I_1}{2} \left[\frac{2s+1}{s} \right] = \frac{1}{s} - \frac{1}{2s}$$

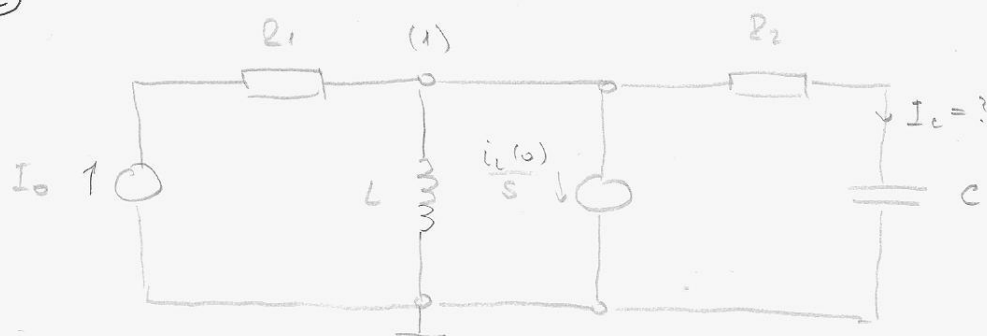
$$I_1 \left[s + \frac{2s+1}{2s} \right] = \frac{1}{2s}$$

$$I_1 \left[\frac{2s^2 + 2s + 1}{2s} \right] = \frac{1}{2s}$$

$$I_1 = \frac{1}{2s^2 + 2s + 1} = \frac{\frac{1}{2}}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2}$$

$$\Rightarrow i_1(t) = e^{-\frac{t}{2}} \sin\left(\frac{t}{2}\right) \cdot \frac{1}{2}$$

(12)



$$(1) \quad U_1 \left[\frac{1}{sL} + \frac{1}{R_2 + \frac{1}{sC}} \right] = I_0 - \frac{i_L(0)}{s}$$

$$U_1 \left[\frac{1}{s} + \frac{1}{1 + \frac{1}{s}} \right] = 1 - \frac{1}{s}$$

$$U_1 \left[\frac{1}{s} + \frac{s}{s+1} \right] = \frac{s-1}{s}$$

$$U_1 \left[\frac{s+1 + s^2}{s(s+1)} \right] = \frac{s-1}{s}$$

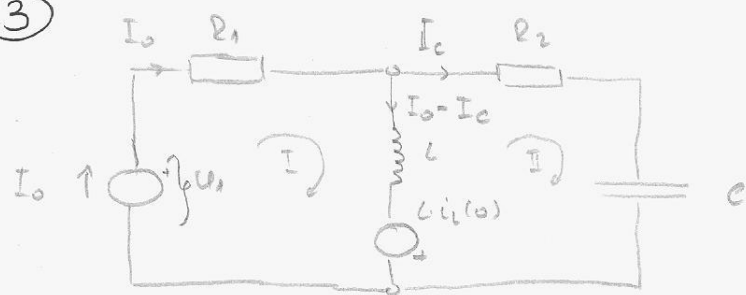
$$U_1 = \frac{s^2 - 1}{s^2 + s + 1}$$

$$I_C = \frac{U_1}{R_2 + \frac{1}{sC}} = \frac{U_1}{\frac{s+1}{s}} = \frac{(s-1)(s+1)}{\frac{s^2+s+1}{s}} = \frac{s^2 - s}{s^2 + s + 1}$$

$$= \frac{s^2 + s + 1 - 2s - 1}{s^2 + s + 1} = 1 - 2 \cdot \frac{s + \frac{1}{2}}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow i_C(t) = \delta(t) - 2 \cdot e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}}{2}t\right) \cdot \delta(t)$$

(13)



$$(I) \quad U_1 - I_0 R_1 - (I_0 - I_c) sL + L \cdot i_L(0) = 0$$

$$(II) \quad -L \cdot i_L(0) + (I_0 - I_c) sL - I_c R_2 - I_c \frac{1}{sC} = 0$$

$$(II) \quad -1 + (1 - I_c) s - I_c - I_c \frac{1}{s} = 0$$

$$-1 + s + s I_c - I_c - I_c \frac{1}{s} = 0$$

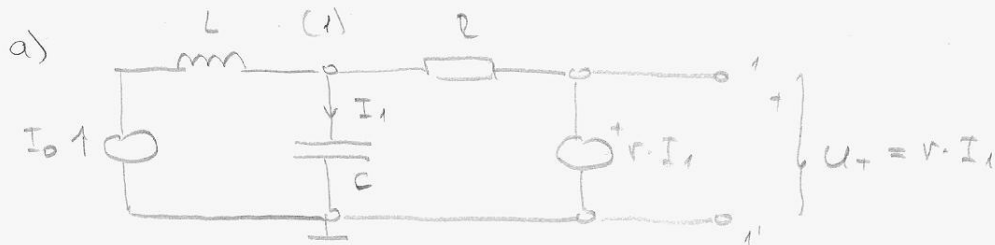
$$I_c \left[s + 1 + \frac{1}{s} \right] = s - 1 \quad / \cdot s$$

$$I_c [s^2 + s + 1] = s^2 - s$$

$$I_c = \frac{s^2 - s}{s^2 + s + 1} = \frac{s^2 + s + 1 - 2s - 1}{s^2 + s + 1} = 1 - 2 \frac{s + \frac{1}{2}}{(s + \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2}$$

$$\Rightarrow i_c(t) = \delta - 2 e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{3}}{2} t\right) \mathcal{U}(t)$$

14



$$(1) \quad U_1 \left[\frac{1}{2} + sC \right] - U_T \cdot \frac{1}{R} = I_0$$

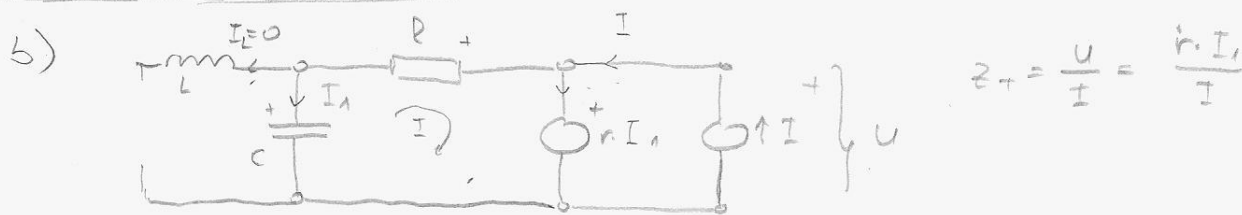
$$U_T = r \cdot I_1 = r \cdot \frac{U_1}{\frac{1}{sC}} = U_1 \cdot r s C = s U_1 \quad (1)$$

$$(1) \quad U_1 \left[\frac{1}{2} + s \right] - \frac{s U_1}{2} = \frac{1}{s}$$

$$U_1 \left[\frac{1}{2} + s - \frac{s}{2} \right] = \frac{1}{s}$$

$$U_1 \left[\frac{1+2s-s}{2} \right] = \frac{1}{s} \Rightarrow U_1 = \frac{2}{s(s+1)}$$

$$U_T = s \cdot U_1 = \frac{2}{s+1}$$



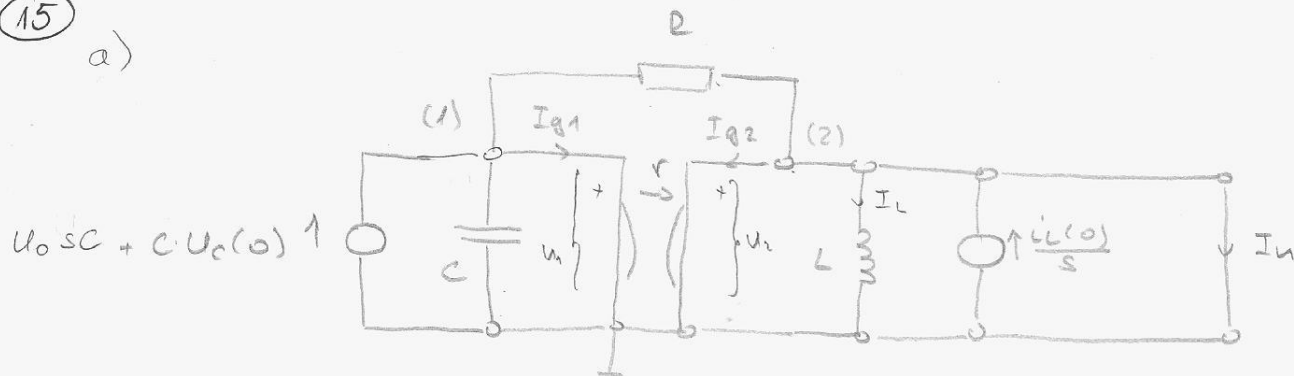
$$(I) \quad I_1 \left(R + \frac{1}{sC} \right) - r \cdot I_1 = 0$$

$$I_1 = 0$$

$$z_T = \frac{r \cdot I_1}{I} = \frac{r \cdot 0}{I} = 0$$

(15)

a)



$$(q1) \quad U_1 = r \cdot I_{q2}$$

$$(q2) \quad U_2 = -r \cdot I_{q1}$$

$$(1) \quad U_1 \left[sC + \frac{1}{r} \right] - U_2 \frac{1}{r} = U_0 s C + C \cdot U_C(0) - I_{q1}$$

$$(2) \quad U_2 \left[\frac{1}{r} + \frac{1}{sL} \right] - U_1 \frac{1}{r} = \frac{i_L(0)}{s} - I_{q2} - I_u$$

zbog kratkog spoja: $U_2 = 0 \Rightarrow I_{q1} = 0$

$$(1) \quad U_1 \left[s + 2 \right] = \dot{s} + \frac{1}{2}$$

$$(2) \quad -U_1 \cdot 2 = \frac{1}{s} - I_{q2} - I_u$$

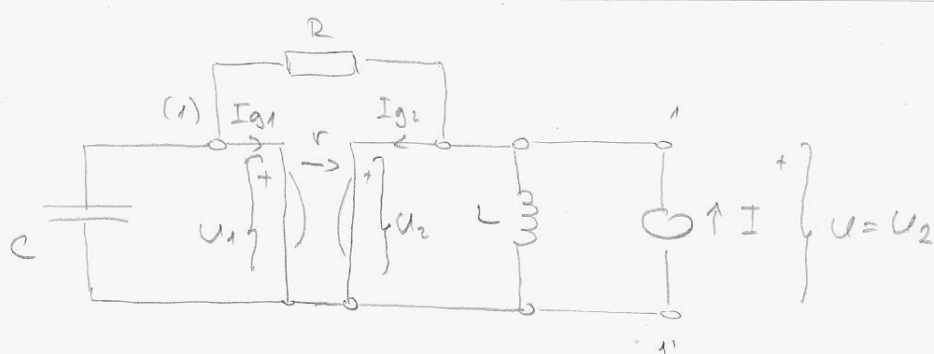
$$(1) \quad U_1 (s+2) = \frac{2s+1}{2} \Rightarrow U_1 = \frac{2s+1}{2(s+2)} \quad u(2)$$

$$(q1) \quad I_{q2} = \frac{U_1}{r} = \frac{2s+1}{2(s+2)} \quad u(1)$$

$$(2) \quad -\frac{2s+1}{s+2} = \frac{1}{s} - \frac{2s+1}{2(s+2)} - I_u$$

$$I_u = \frac{1}{s} + \frac{2s+1}{2(s+2)} = \frac{2s+4+2s^2+s}{2s(s+2)} = \frac{2s^2+3s+4}{2s(s+2)}$$

b)



$$Z_N = \frac{U}{I} = \frac{U_2}{I}$$

$$Y_N = \frac{1}{Z_N} = \frac{I}{U_2}$$

$$(q1) \quad U_1 = r \cdot I_{q2}$$

$$(q2) \quad U_2 = -r \cdot I_{q1}$$

$$(1) \quad U_1 \left[sC + \frac{1}{R} \right] - U_2 \frac{1}{R} = -I_{q1}$$

$$(2) \quad U_2 \left[\frac{1}{sL} + \frac{1}{R} \right] - U_1 \frac{1}{R} = -I_{q2} + I$$

$$(q1) \quad I_{q2} = \frac{U_1}{r} = U_1 \quad (1) \quad (2)$$

$$(q2) \quad I_{q1} = -\frac{U_2}{r} = -U_2$$

$$(1) \quad U_1 (s+2) - 2U_2 = U_2 \Rightarrow U_1 = U_2 \cdot \frac{3}{s+2}$$

$$(2) \quad U_2 \left(\frac{1}{s} + 2 \right) - 2U_1 = -U_1 + I$$

$$I = U_2 \left(\frac{2s+1}{s} \right) - U_1 \quad (1) \quad (2)$$

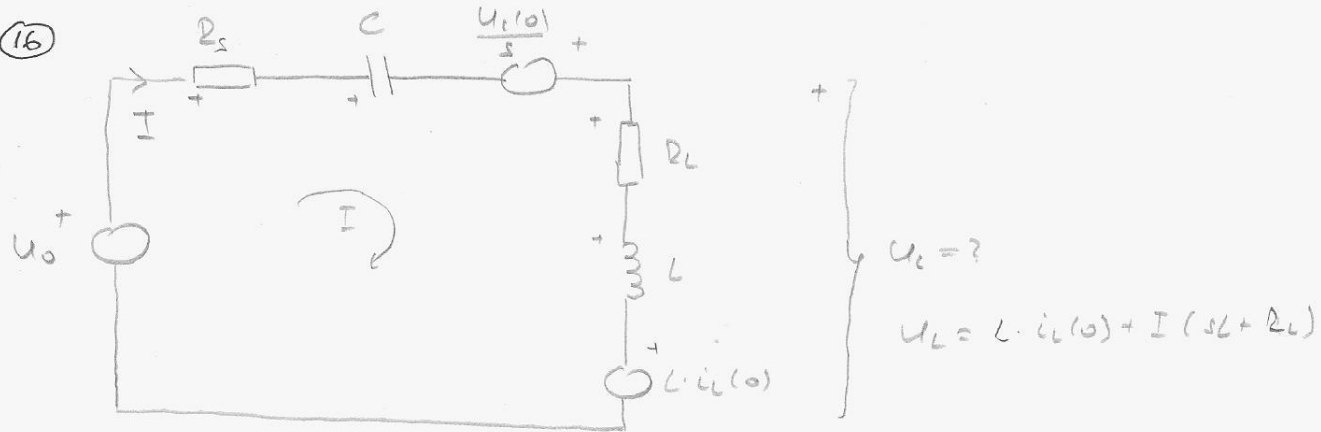
$$(2) \quad I = U_2 \frac{2s+1}{s} - U_2 \frac{3}{s+2}$$

$$= U_2 \left[\frac{2s+1}{s} - \frac{3}{s+2} \right] = U_2 \left[\frac{2s^2 + s + 4s + 2 - 3s}{s(s+2)} \right]$$

$$= U_2 \left[\frac{2s^2 + 2s + 2}{s(s+2)} \right]$$

$$Y_N = \frac{I}{U_2} = \frac{2s^2 + 2s + 2}{s(s+2)}$$

(16)



$$(I) \quad U_0 - I \left(R_s + \frac{1}{sC} + R_L + sL \right) + \frac{U_C(0)}{s} - L \cdot i_L(0) = 0$$

$$I \left(2 + s + \frac{1}{s} \right) = \frac{1}{s+1} + \frac{1}{s} - 1$$

$$I \left(\frac{s^2 + 2s + 1}{s} \right) = \frac{s + s + 1 - s^2 - s}{s(s+1)}$$

$$I \frac{(s+1)^2}{s} = \frac{-s^2 + s + 1}{s(s+1)}$$

$$I = \frac{-s^2 + s + 1}{(s+1)^3}$$

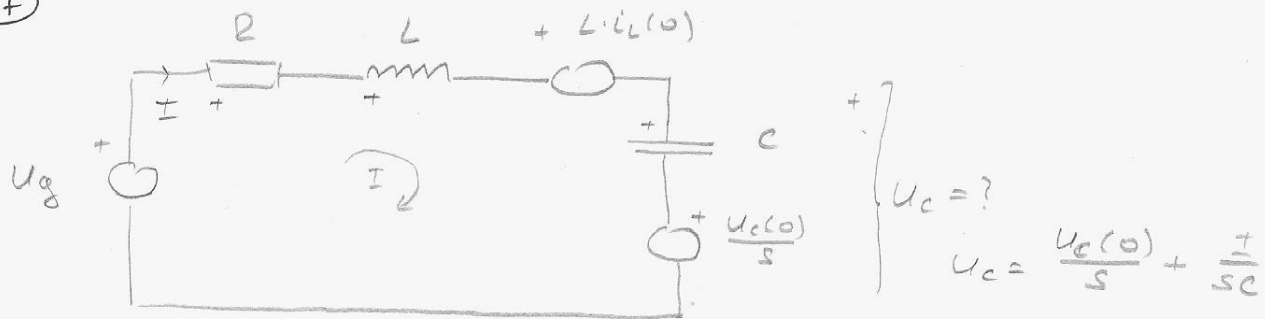
$$U_L = 1 + \frac{-s^2 + s + 1}{(s+1)^3} (s+1) = 1 - \frac{s^2 - s - 1}{s^2 + 2s + 1} = 1 - \frac{s^2 + 2s + 1 - 3s - 2}{s^2 + 2s + 1}$$

$$= 1 - \left[1 - \frac{3s + 2}{(s+1)^2} \right] = \frac{3s + 2}{(s+1)^2} = \frac{3s + 3 - 1}{(s+1)^2}$$

$$= 3 \frac{s+1}{(s+1)^2} - \frac{1}{(s+1)^2} = \frac{3}{s+1} - \frac{1}{(s+1)^2}$$

$$\rightarrow U_L(t) = (3 - t) \cdot e^{-t} \quad s(t)$$

(17)



$$(I) \quad U_g - I \left(R + sL + \frac{1}{sC} \right) - L i_L(0) - \frac{U_c(0)}{s} = 0$$

$$I \left(4 + 2s + \frac{2}{s} \right) = \frac{1}{s} - 2.4 - \frac{2.6}{s}$$

$$I \left(\frac{2s^2 + 4s + 2}{s} \right) = \frac{-2.4s - 1.6}{s}$$

$$I = - \frac{1.2s + 0.8}{s^2 + 2s + 1} = - \frac{1.2s + 0.8}{(s+1)^2}$$

$$U_c = \frac{2.6}{s} + \frac{2}{s} \cdot \left[- \frac{1.2s + 0.8}{(s+1)^2} \right]$$

$$= \frac{2.6}{s} - \frac{2.4s + 1.6}{s(s+1)^2}$$

$$\left\{ \begin{aligned} \frac{2.4s + 1.6}{s(s+1)^2} &= \frac{A}{s} + \frac{Bs + C}{(s+1)^2} = \frac{As^2 + 2As + A + Bs^2 + Cs}{s(s+1)^2} \\ A + B &= 0 \Rightarrow B = -1.6 \\ 2A + C &= 2.4 \Rightarrow C = -0.8 \\ A &= 1.6 \end{aligned} \right.$$

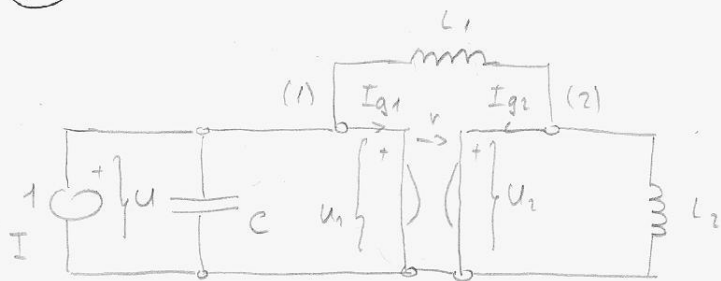
$$U_c = \frac{2.6}{s} - \left[\frac{1.6}{s} + \frac{-1.6s - 0.8}{(s+1)^2} \right]$$

$$= \frac{1}{s} + \frac{1.6s + 0.8}{(s+1)^2} = \frac{1}{s} + \frac{1.6s + 1.6 - 0.8}{(s+1)^2}$$

$$= \frac{1}{s} + \frac{1.6}{s+1} - \frac{0.8}{(s+1)^2}$$

$$\hookrightarrow U_c(t) = [1 + 1.6e^{-t} - 0.8te^{-t}] \cdot 1(t)$$

(18)



$$(q1) \quad u_1 = r \cdot I_{q2}$$

$$(q2) \quad u_2 = -r \cdot I_{q1}$$

$$Y_{UL} = \frac{I}{u} = \frac{I}{u_1}$$

$$(1) \quad u_1 \left[sC + \frac{1}{sL_1} \right] - u_2 \frac{1}{sL_1} = I - I_{q1}$$

$$(2) \quad u_2 \left[\frac{1}{sL_1} + \frac{1}{sL_2} \right] - u_1 \frac{1}{sL_1} = -I_{q2}$$

$$\begin{cases} (q1) \quad I_{q2} = \frac{u_1}{r} = u_1 \\ (q2) \quad I_{q1} = -\frac{u_2}{r} = -u_2 \end{cases} \quad u \quad (1) \quad ; \quad (2)$$

$$(1) \quad u_1 \left[\frac{s}{s} + \frac{1}{s} \right] - \frac{u_2}{s} = I + u_2$$

$$(2) \quad u_2 \left[\frac{1}{s} + \frac{1}{s} \right] - \frac{u_1}{s} = -u_1$$

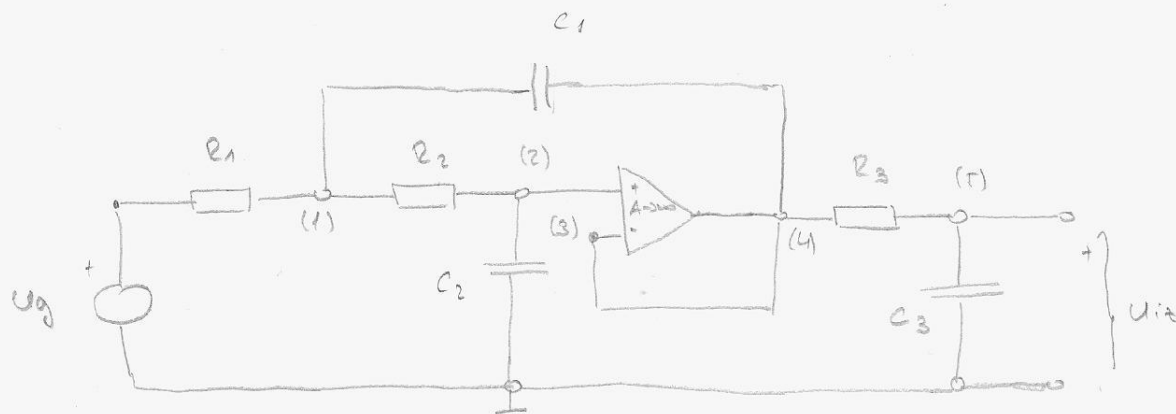
$$(2) \quad u_2 \cdot \frac{2}{s} = u_1 \left(\frac{1}{s} - 1 \right) \Rightarrow u_2 = u_1 \cdot \frac{1-s}{2} \quad u \quad (1)$$

$$(1) \quad u_1 \left(\frac{s^2+1}{s} \right) - u_2 \left(\frac{1}{s} + 1 \right) = I$$

$$\begin{aligned} I &= u_1 \cdot \frac{s^2+1}{s} - u_1 \cdot \frac{1-s}{2} \cdot \frac{s+1}{s} \\ &= u_1 \left[\frac{s^2+1}{s} + \frac{s^2-1}{2s} \right] = u_1 \cdot \frac{2s^2+2+s^2-1}{2s} = u_1 \cdot \frac{3s^2+1}{2s} \end{aligned}$$

$$Y_{UL} = \frac{I}{u_1} = \frac{3s^2+1}{2s} = \frac{3}{2}s + \frac{1}{2s}$$

(19)



$$(1) \quad U_1 \left[\frac{1}{R_1} + \frac{1}{R_2} + sC_1 \right] - U_g \cdot \frac{1}{R_1} - U_2 \frac{1}{R_2} - U_4 sC_1 = 0$$

$$(2) \quad U_2 \left[\frac{1}{R_2} + sC_2 \right] - U_1 \frac{1}{R_2} = 0$$

$$(3) \quad U_3 = U_4$$

(4) ne píšeme

$$(5) \quad U_5 \left[\frac{1}{R_3} + sC_3 \right] - U_4 \cdot \frac{1}{R_3} = 0$$

$U_2 = U_5 = U_i2$ | jednodušší po jácce - $U_2 = U_3$ (p1)

$$U_g(s) = 1$$

$$(1) \quad U_1 [2 + 2s] - 1 - U_2 - 2s U_4 = 0$$

$$(2) \quad U_2 \left[1 + \frac{s}{2} \right] - U_1 = 0 \Rightarrow U_1 = U_2 \frac{s+2}{2} \quad u(1)$$

$$(3) \quad U_3 = U_4 + (p1) \Rightarrow U_3 = U_4 = U_2$$

$$(5) \quad U_5 [1 + s] - U_4 = 0 \Rightarrow U_i2 = U_4 \cdot \frac{1}{s+1}$$

$$(1) \quad U_2 \frac{s+2}{2} \cdot 2(s+1) - U_2 - 2s U_4 = 1$$

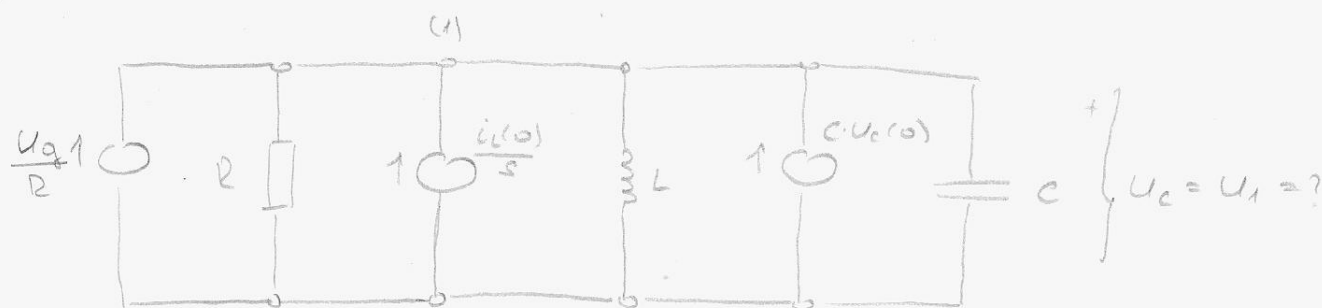
$$U_2 [s^2 + 3s + 1] - 2s U_4 = 1$$

$$(3) \quad U_2 = U_4 \quad u(1)$$

$$(1) \quad U_4 [s^2 + s + 1] = 1 \Rightarrow U_4 = \frac{1}{s^2 + s + 1} \quad u(s)$$

$$U_i2 = \frac{1}{(s+1)(s^2 + s + 1)}$$

(20)



$$(1) \quad U_1 \left[\frac{1}{2} + \frac{1}{s} + sC \right] = \frac{U_g}{2} + \frac{i_L(0)}{s} + C \cdot U_c(0)$$

$$U_g(t) = 2 \cdot e^{-t} s(t) \quad \rightarrow \quad U_g(s) = \frac{2}{s+1}$$

$$(1) \quad U_1 \left[\frac{1}{2} + \frac{1}{s} + \frac{s}{2} \right] = \frac{1}{s+1} + \frac{4}{s} + 1$$

$$U_1 \left[\frac{s^2 + s + 2}{2s} \right] = \frac{s + 4s + 4 + s^2 + s}{2(s+1)} = \frac{s^2 + 6s + 4}{2(s+1)}$$

$$U_1 = 2 \cdot \frac{s^2 + 6s + 4}{(s+1)(s^2 + s + 2)} = \frac{2s^2 + 12s + 8}{(s+1)(s^2 + s + 2)} = \frac{A}{s+1} + \frac{Bs + C}{s^2 + s + 2}$$

$$= \frac{As^2 + As + 2A + Bs^2 + Cs + Bs + C}{(s+1)(s^2 + s + 2)}$$

$$\begin{cases} A + B = 2 \\ A + B + C = 12 \Rightarrow C = 10 \\ 2A + C = 8 \Rightarrow A = -1 \quad B = 3 \end{cases}$$

$$U_1 = \frac{-1}{s+1} + \frac{3s + 10}{s^2 + s + 2} = -\frac{1}{s+1} + \frac{3s + \frac{3}{2} + \frac{17}{2}}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2}$$

$$= -\frac{1}{s+1} + 3 \frac{s + \frac{1}{2}}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2} + \frac{17}{2} \cdot \frac{2}{\sqrt{7}} \cdot \frac{\frac{\sqrt{7}}{2}}{\left(s + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2}$$

$$\rightarrow U_1(t) = \left[-e^{-t} + 3e^{-\frac{t}{2}} \cos\left(\frac{\sqrt{7}}{2}t\right) + \frac{17}{\sqrt{7}} e^{-\frac{t}{2}} \sin\left(\frac{\sqrt{7}}{2}t\right) \right] s(t)$$