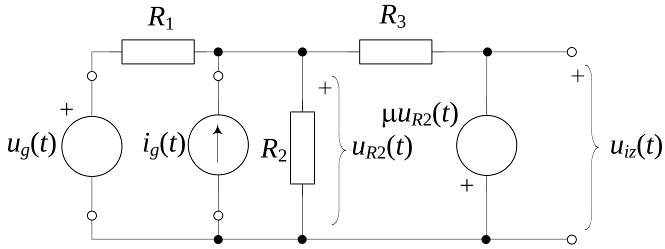
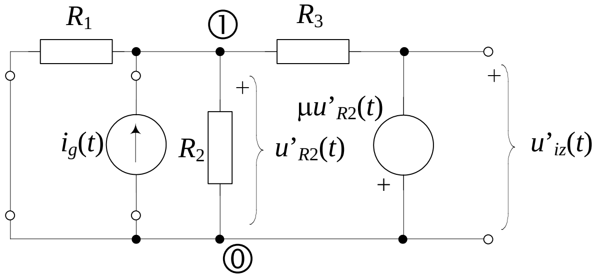
ZAVRŠNI ISPIT IZ ELEKTRIČNIH KRUGOVA 2014-2015 – Rješenja

1. Za električni krug prikazan slikom primjenom metode superpozicije izračunati valni oblik napona $u_{iz}(t)$ kao odziv, ako je zadana pobuda $u_g(t)=6S(t)$ i $i_g(t)=3\delta(t)$. Zadane su normalizirane vrijednosti elemenata $R_1=1$, $R_2=1$, $R_3=2$ i a) $\mu=1$; b) $\mu=\infty$.



Rješenje: Primjena metode superpozicije.

a) Isključen je naponski izvor: $u_g(t)=0$ (umjesto isključenog naponskog izvora je kratki spoj). Ovisni izvor (NONI) s parametrom μ (naponsko pojačanje) ostaje uključen.



Jednadžbe čvorišta (samo jedan čvor)

$$(1)\ u'_1(t)\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)=i_g(t)+\frac{u'_{iz}(t)}{R_3}$$

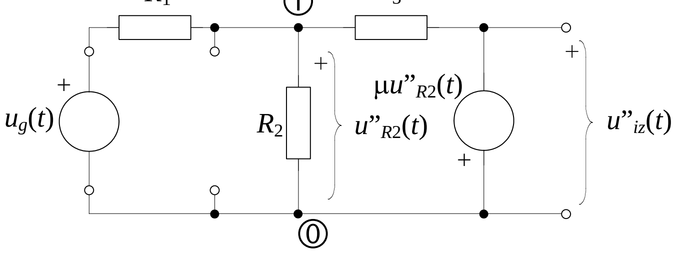
$$(2)\ u'_{iz}(t)=-\mu u'_{R2}(t)=-\mu u'_1(t)\ \Rightarrow\ u'_1(t)=-\frac{u'_{iz}(t)}{\mu}$$

$$\Rightarrow\qquad -\frac{u'_{iz}(t)}{\mu}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)=i_g(t)+\frac{u'_{iz}(t)}{R_3}$$

$$\frac{u'_{iz}(t)}{\mu}\frac{1}{R_1}+\frac{u'_{iz}(t)}{\mu}\frac{1}{R_2}+\frac{u'_{iz}(t)}{R_3}\left(1+\frac{1}{\mu}\right)=-i_g(t)$$

$$u'_{iz}(t)=\frac{-i_g(t)}{\frac{1}{\mu R_1}+\frac{1}{\mu R_2}+\frac{1}{R_3}\left(1+\frac{1}{\mu}\right)}$$

b) Isključen je strujni izvor: $i_g(t)=0$ (umjesto isključenog strujnog izvora je prazni hod). Ovisni izvor (NONI) s parametrom μ (naponsko pojačanje) ostaje uključen.



Jednadžbe čvorišta (samo jedan čvor)

$$(1)\ \ u''_1(t)\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)=\frac{u_g(t)}{R_1}+\frac{u''_{iz}(t)}{R_3}$$

$$(2)\ \ u''_{iz}(t)=-\mu u''_{R2}(t)=-\mu u''_1(t)\ \Rightarrow\ u''_1(t)=-\frac{u''_{iz}(t)}{\mu}$$

$$\Rightarrow\qquad -\frac{u''_{iz}(t)}{\mu}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)=\frac{u_g(t)}{R_1}+\frac{u''_{iz}(t)}{R_3}$$

$$\frac{u''_{iz}(t)}{\mu}\frac{1}{R_1}+\frac{u''_{iz}(t)}{\mu}\frac{1}{R_2}+\frac{u''_{iz}(t)}{R_3}\left(1+\frac{1}{\mu}\right)=-\frac{u_g(t)}{R_1}$$

$$u''_{iz}(t)=\frac{-\dfrac{u_g(t)}{R_1}}{\dfrac{1}{\mu R_1}+\dfrac{1}{\mu R_2}+\dfrac{1}{R_3}\left(1+\dfrac{1}{\mu}\right)}$$

(3 boda)

c) Superpozicija:

$$u_{iz}(t)=u'_{iz}(t)+u''_{iz}(t)=\frac{-i_g(t)-\dfrac{u_g(t)}{R_1}}{\dfrac{1}{\mu R_1}+\dfrac{1}{\mu R_2}+\dfrac{1}{R_3}\left(1+\dfrac{1}{\mu}\right)}$$

(1 bod)

Uz uvrštene vrijednosti elemenata R_1=1, R_2=1, R_3=2 i μ=1 slijedi:

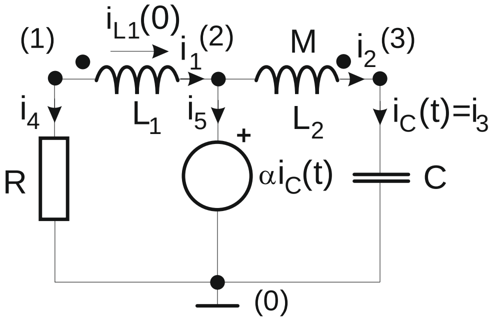
$$u_{iz}(t)=u'_{iz}(t)+u''_{iz}(t)=\frac{-i_g(t)-\dfrac{u_g(t)}{R_1}}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}\left(1+\dfrac{1}{1}\right)}=\frac{-i_g(t)-u_g(t)}{3}=-\delta(t)-2S(t)$$

Uz uvrštene vrijednosti elemenata R_1=1, R_2=1, R_3=2 i μ=∞ slijedi:

$$u_{iz}(t)=u'_{iz}(t)+u''_{iz}(t)=\frac{-i_g(t)-\dfrac{u_g(t)}{R_1}}{\dfrac{1}{R_3}}=-R_3 i_g(t)-\frac{R_3}{R_1}u_g(t)=-6\delta(t)-12S(t)$$

(1 bod)

2. Zadan je električni krug prema slici. Poštujući oznake grana i čvorišta nacrtati pripadni orijentirani graf. Napisati spojnu matricu **S**. Napisati naponsko-strujne jednadžbe grana u matričnom obliku te ispisati matricu impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$. Matrica $\mathbf{Z}_b$ mora biti regularna. Napisati temeljni sustav jednadžbi petlji, odnosno odrediti matrice impedancija petlji $\mathbf{Z}_p$ i vektor početnih uvjeta i nezavisnih izvora petlji $\mathbf{U}_{0p}$.



Rješenje: Naponsko-strujne relacije grana, uz primjenu Laplaceove transformacije:

$$u_1(t)=L_1\frac{di_1(t)}{dt}-M\frac{di_2(t)}{dt}$$

$$u_2(t)=-M\frac{di_1(t)}{dt}+L_2\frac{di_2(t)}{dt} \qquad U_1(s)=sL_1I_1(s)-L_1i_1(0)-sMI_2(s)+Mi_2(0)$$

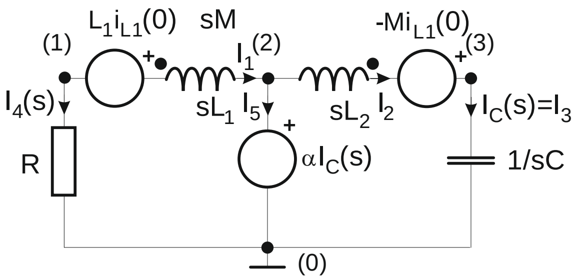
$$U_2(s)=-sMI_1(s)+Mi_1(0)+sL_2I_2(s)-L_2i_2(0)$$

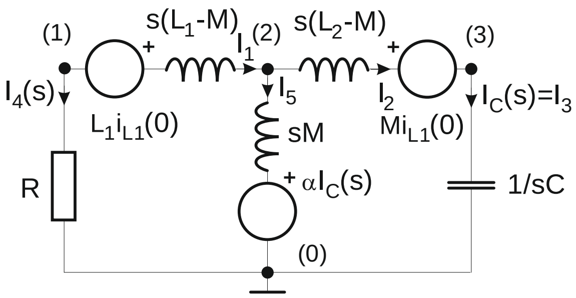
$$u_3(t)=\frac{1}{C}\int_0^t i_3(\tau)d\tau+u_3(0) \qquad U_3(s)=\frac{1}{sC}I_3(s)+\frac{u_3(0)}{s}$$

$$u_4(t)=R\cdot i_4(t) \qquad U_4(s)=R\cdot I_4(s)$$

$$u_5(t)=\alpha\cdot i_3(t) \qquad U_5(s)=\alpha\cdot I_3(s)$$

Od početnih uvjeta postoji jedino $i_{L1}(0)=i_1(0)$, a $i_{L2}(0)=0$ i $u_C(0)=u_3(0)=0$. Stoga električni krug u frekvencijskoj domeni i nakon transformacije mreže (da se izgubi M) izgleda ovako:





Konačno naponsko-strujne relacije grana glase:

$$U_1(s)=s(L_1-M)I_1(s)-L_1i_{L1}(0)$$

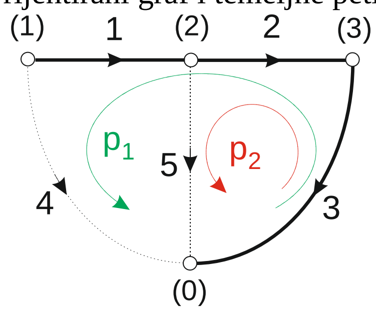
$$U_2(s)=s(L_2-M)I_2(s)+Mi_{L1}(0)$$

$$U_3(s)=\frac{1}{sC}I_3(s)$$

$$U_4(s)=R\cdot I_4(s)$$

$$U_5(s)=\alpha\cdot I_3(s)+sMI_5(s)$$

Orijentirani graf i temeljne petlje



(2 boda)

$$\text{Spojna matrica:}\quad \mathbf{S}=\begin{array}{c} \\ (p_1) \\ (p_2) \end{array}\!\!\!\begin{array}{c} \overbrace{\begin{array}{ccccc}1 & 2 & 3 & 4 & 5\end{array}}^{grane} \\ \begin{bmatrix}-1 & -1 & -1 & 1 & 0\\ -1 & -1 & -1 & 0 & 1\end{bmatrix}\end{array}$$

$$\uparrow$$

temeljne petlje

Naponsko-strujne relacije grana u matričnom obliku: $\mathbf{U}_b=\mathbf{Z}_b\cdot\mathbf{I}_b+\mathbf{U}_{0b}$

$$\underbrace{\begin{bmatrix}U_1\\U_2\\U_3\\U_4\\U_5\end{bmatrix}}_{\mathbf{U}_b}=\underbrace{\begin{bmatrix}s(L_1-M) & 0 & 0 & 0 & 0\\ 0 & s(L_2-M) & 0 & 0 & 0\\ 0 & 0 & \frac{1}{sC} & 0 & 0\\ 0 & 0 & 0 & R & 0\\ 0 & 0 & \alpha & 0 & sM\end{bmatrix}}_{\mathbf{Z}_b}\cdot\underbrace{\begin{bmatrix}I_1\\I_2\\I_3\\I_4\\I_5\end{bmatrix}}_{\mathbf{I}_b}+\underbrace{\begin{bmatrix}-L_1i_{L1}(0)\\Mi_{L1}(0)\\0\\0\\0\end{bmatrix}}_{\mathbf{U}_{0b}}$$ **(1 bod)**

Matrica $\mathbf{Z}_b$ je regularna. Temeljni sustav jednadžbi petlji u matričnom obliku: $\mathbf{Z}_p\cdot\mathbf{I}_p=\mathbf{U}_{0p}$, gdje su (matrice $\mathbf{Z}_p$ i $\mathbf{U}_{0p}$):

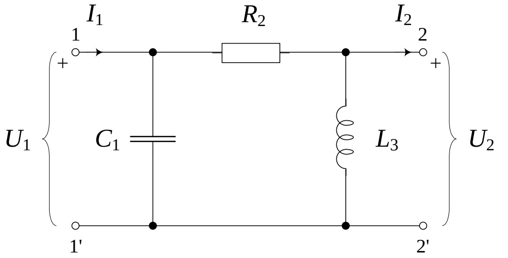
$$\mathbf{Z}_p=\mathbf{S}\cdot\mathbf{Z}_b\cdot\mathbf{S}^T=\begin{bmatrix}-1 & -1 & -1 & 1 & 0\\ -1 & -1 & -1 & 0 & 1\end{bmatrix}\begin{bmatrix}s(L_1-M) & 0 & 0 & 0 & 0\\ 0 & s(L_2-M) & 0 & 0 & 0\\ 0 & 0 & \frac{1}{sC} & 0 & 0\\ 0 & 0 & 0 & R & 0\\ 0 & 0 & \alpha & 0 & sM\end{bmatrix}\begin{bmatrix}-1 & -1\\ -1 & -1\\ -1 & -1\\ 1 & 0\\ 0 & 1\end{bmatrix}=$$

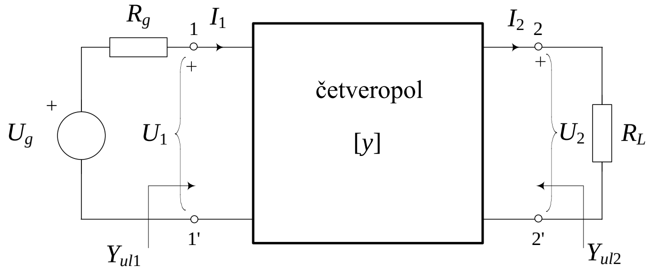
$$=\begin{bmatrix}-s(L_1-M) & -s(L_2-M) & -\frac{1}{sC} & R & 0\\ -s(L_1-M) & -s(L_2-M) & -\frac{1}{sC} & 0 & sM\end{bmatrix}\begin{bmatrix}-1 & -1\\ -1 & -1\\ -1 & -1\\ 1 & 0\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}s(L_1+L_2-2M)+\frac{1}{sC}+R & s(L_1+L_2-2M)\\ s(L_1+L_2-2M) & s(L_1+L_2-M)\end{bmatrix}$$ **(1 bod)**

$$\mathbf{U}_{0p}=-\mathbf{S}\cdot\mathbf{U}_{0b}=-\begin{bmatrix}-1 & -1 & -1 & 1 & 0\\ -1 & -1 & -1 & 0 & 1\end{bmatrix}\begin{bmatrix}-L_1i_{L1}(0)\\Mi_{L1}(0)\\0\\0\\0\end{bmatrix}=\begin{bmatrix}-L_1i_{L1}(0)+Mi_{L1}(0)\\-L_1i_{L1}(0)+Mi_{L1}(0)\end{bmatrix}$$ **(1 bod)**

3. Za Π-četveropol prikazan lijevom slikom izračunati *y*-parametre. a) Napisati *y*-parametre pomoću C_1, R_2 i L_3 te uvrstiti normirane vrijednosti elemenata: C_1=1, R_2=1/2, L_3=1. Četveropol je spojen u električni krug na desnoj slici. b) Ako je izlazni prilaz (2–2') zaključen otporom R_L=1 pomoću *y*-parametara izračunati: ulaznu admitanciju $Y_{ul1}(s)=I_1(s)/U_1(s)$ gledano sa priključnica 1–1'; c) ako je R_g=1 izračunati izlaznu admitanciju $Y_{ul2}(s)=-I_2(s)/U_2(s)$ gledano sa priključnica 2–2'; i d) izračunati prijenosnu funkciju napona $H(s)=U_2(s)/U_g(s)$.



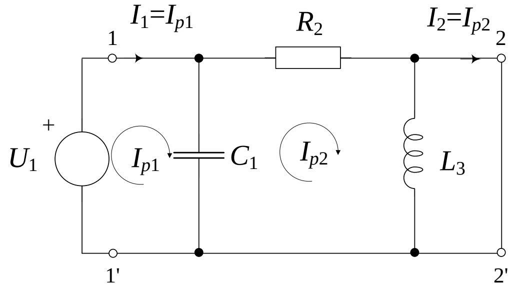


Rješenje:

a) [*y*]-parametri:

$$I_1 = y_{11} \cdot U_1 - y_{12} \cdot U_2$$
$$I_2 = y_{21} \cdot U_1 - y_{22} \cdot U_2$$

$$U_2 = 0 \qquad y_{11} = \left.\frac{I_1}{U_1}\right|_{U_2=0}; \quad y_{21} = \left.\frac{I_2}{U_1}\right|_{U_2=0}$$



$U_2 = 0$
(K.S.)

$$(1)\ U_1 = I_{p1}\frac{1}{sC_1} - I_{p2}\frac{1}{sC_1}$$
$$(2)\ 0 = -I_{p1}\frac{1}{sC_1} + I_{p2}\left(\frac{1}{sC_1} + R_2\right)$$

$$(2)\ \Rightarrow\ I_{p1} = I_{p2}\left(1 + sC_1R_2\right)\ \rightarrow\ (1)\ \Rightarrow$$

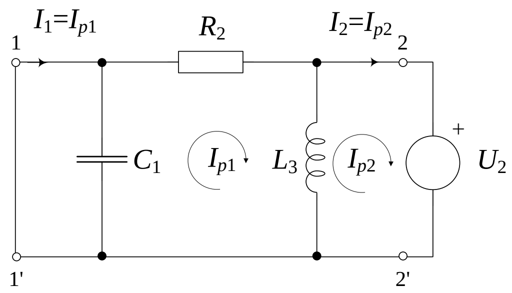
$$U_1 = I_{p2}\left(1 + sC_1R_2\right)\frac{1}{sC_1} - I_{p2}\frac{1}{sC_1} = I_{p2}\left[R_2 + \frac{1}{sC_1} - \frac{1}{sC_1}\right] = I_{p2}R_2\,;$$

$$U_1 = I_{p1}\frac{1}{sC_1} - \frac{I_{p1}}{1 + sC_1R_2}\cdot\frac{1}{sC_1} = I_{p1}\left[\frac{1}{sC_1} - \frac{1}{1 + sC_1R_2}\cdot\frac{1}{sC_1}\right] = I_{p1}\frac{R_2}{1 + sC_1R_2} = I_{p1}\frac{1}{1/R_2 + sC_1}\,;$$

$$y_{11} = \left.\frac{I_1}{U_1}\right|_{U_2=0} = \frac{1}{R_2} + sC_1; \quad y_{21} = \left.\frac{I_2}{U_1}\right|_{U_2=0} = \frac{1}{R_2}$$

$$U_1 = 0 \qquad y_{12} = -\left.\frac{I_1}{U_2}\right|_{U_1=0}; \ y_{22} = -\left.\frac{I_2}{U_2}\right|_{U_1=0}$$

$U_1 = 0$
(K.S.)



$$\begin{array}{l}(1)\ \ 0 = I_{p1}(sL_3 + R_2) - I_{p2}sL_3 \\ (2)\ \ -U_2 = -I_{p1}sL_3 + I_{p2}sL_3 \\ \hline \end{array} \quad (1) \Rightarrow \ I_{p2} = I_{p1}\left(\frac{R_2 + sL_3}{sL_3}\right) \ \rightarrow (2) \Rightarrow$$

$$-U_2 = -I_{p1}sL_3 + I_{p1}\left(\frac{R_2 + sL_3}{sL_3}\right)sL_3 = I_{p1}R_2$$

$$-U_2 = -I_{p2}\frac{sL_3}{R_2 + sL_3}sL_3 + I_{p2}sL_3 = I_{p2}\left[sL_3 - \frac{sL_3}{R_2 + sL_3}sL_3\right] = I_{p2}\frac{R_2 sL_3}{R_2 + sL_3}$$

$$y_{12} = -\frac{I_1}{U_2}\bigg|_{U_1=0} = \frac{1}{R_2};\ y_{22} = -\frac{I_2}{U_2}\bigg|_{U_1=0} = \frac{1}{R_2} + \frac{1}{sL_3}$$

$$[y] = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix} = \begin{bmatrix} \frac{1}{R_2} + sC_1 & -\frac{1}{R_2} \\ \frac{1}{R_2} & -\left(\frac{1}{R_2} + \frac{1}{sL_3}\right) \end{bmatrix} = \begin{bmatrix} 2+s & -2 \\ 2 & -\left(2 + \frac{1}{s}\right) \end{bmatrix}$$ **(2 boda)**

b) Ulazna admitancija u četveropol:

$$Y_{ul1}(s) = \frac{I_1}{U_1} = y_{11} - \frac{y_{12}y_{21}}{Y_L + y_{22}}; \quad R_L = \frac{1}{Y_L} = \frac{U_2}{I_2}$$

$$= 2 + s - \frac{2 \cdot 2}{1 + 2 + 1/s} = 2 + s - \frac{4s}{3s+1} = \frac{(s+2)(3s+1) - 4s}{3s+1} = \frac{3s^2 + 3s + 2}{3s+1}$$

(1 bod)

c) Izlazna admitancija iz četveropola:

$$Y_{ul2}(s) = -\frac{I_2}{U_2} = y_{22} - \frac{y_{12}y_{21}}{Y_g + y_{11}}; \quad R_g = \frac{1}{Y_g} = -\frac{U_1}{I_1}$$

$$= 2 + \frac{1}{s} - \frac{2 \cdot 2}{1 + 2 + s} = \frac{2s+1}{s} - \frac{4}{s+3} = \frac{(2s+1)(s+3) - 4s}{s(s+3)} = \frac{2s^2 + 3s + 3}{s^2 + 3s};$$

(1 bod)

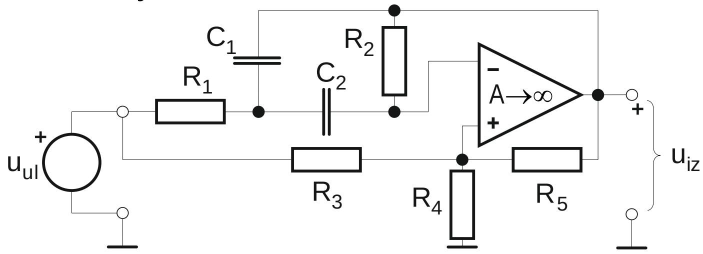
d) Prijenosna funkcija napona: uz $Y_g = 1/R_g = 1; Y_L = 1/R_L = 1$ slijedi:

$$H(s) = \frac{U_2}{U_g} = \frac{y_{21}Y_g}{(y_{11} + Y_g)(y_{22} + Y_L) - y_{12}y_{21}} = \frac{2 \cdot 1}{(2 + s + 1)(2 + 1/s + 1) - 4} = \frac{2}{(3+s)(3+1/s) - 4}$$

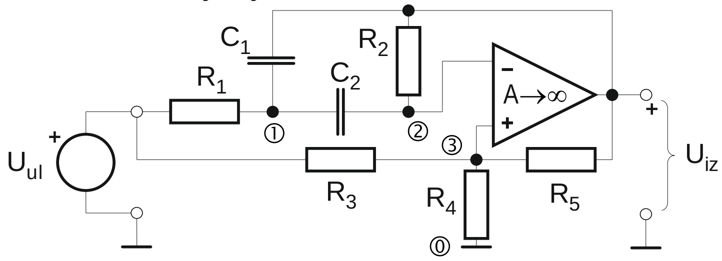
$$H(s) = \frac{U_2}{U_g} = \frac{2}{3/s + 3s + 6} = \frac{2}{3} \cdot \frac{1}{1/s + s + 2} = \frac{2}{3} \cdot \frac{s}{s^2 + 2s + 1} = \frac{2}{3} \cdot \frac{s}{(s+1)^2}$$

(1 bod)

4. Zadan je aktivni-RC električni filtar prikazan slikom s normaliziranim vrijednostima elemenata C_1=1, C_2=1, R_1=1, R_2=1, te R_3=R_4=R_5=1. a) Izračunati njegovu naponsku prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$. O kojem se tipu filtra radi (NP, VP, PP ili PB)? b) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja izračunati vrijednost parametara k, ω_p, q_p. c) Prikazati raspored polova i nula u kompleksnoj ravnini. d) Nacrtati amplitudno-frekvencijsku karakteristiku.



Rješenje: Laplaceova transformacija i jednadžbe čvorišta:



Metoda napona čvorišta:

$$(1)\ U_1\left(\frac{1}{R_1}+sC_1+sC_2\right)-U_2sC_2=U_{iz}sC_1+U_{ul}\frac{1}{R_1}\Big/\cdot R_1$$

$$(2)\ -U_1sC_2+U_2\left(\frac{1}{R_2}+sC_2\right)=U_{iz}\frac{1}{R_2}\Big/\cdot R_2$$

$$(3)\ U_3\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)=U_{ul}\frac{1}{R_3}+U_{iz}\frac{1}{R_5}\Big/\cdot R_3R_4R_5$$

$$(4)\ A(U_3-U_2)=U_{iz}\ \Rightarrow\ U_3=U_2\ \ (A\to\infty)$$

$$(1)\ U_1(1+sR_1C_1+sR_1C_2)-U_2sR_1C_2=U_{iz}sR_1C_1+U_{ul}$$

$$(2)\ -U_1sR_2C_2+U_2(1+sR_2C_2)=U_{iz}$$

$$(3)\ U_3(R_4R_5+R_3R_5+R_3R_4)=U_{ul}R_4R_5+U_{iz}R_3R_4$$

$$(4)\ U_3=U_2$$

$$(2)\Rightarrow\ U_1=U_2\left(\frac{1}{sR_2C_2}+1\right)-\frac{1}{sR_2C_2}U_{iz}\ \to (1)\Rightarrow$$

$$\left[U_2\left(\frac{1}{sR_2C_2}+1\right)-\frac{1}{sR_2C_2}U_{iz}\right](1+sR_1C_1+sR_1C_2)-U_2sR_1C_2=U_{iz}sR_1C_1+U_{ul}$$

$$U_2\left(\frac{1}{sR_2C_2}+1\right)(1+sR_1C_1+sR_1C_2)-U_2sR_1C_2=$$

$$=\frac{1}{sR_2C_2}U_{iz}(1+sR_1C_1+sR_1C_2)+U_{iz}sR_1C_1+U_{ul}\Big/\cdot sR_2C_2$$

$$U_2(sR_2C_2+1)(1+sR_1C_1+sR_1C_2)-U_2sR_1C_2sR_2C_2=$$

$$=U_{iz}(1+sR_1C_1+sR_1C_2)+U_{iz}sR_1C_1sR_2C_2+U_{ul}sR_2C_2$$

$$U_2(1+sR_2C_2+sR_2C_2sR_1C_1+sR_1C_1+sR_1C_2)=$$
$$=U_{iz}(1+sR_1C_1+sR_1C_2+sR_1C_1sR_2C_2)+U_{ul}sR_2C_2$$

$$(3)\Rightarrow U_3=\frac{R_4R_5}{R_4R_5+R_3R_5+R_3R_4}U_{ul}+\frac{R_3R_4}{R_4R_5+R_3R_5+R_3R_4}U_{iz}=\alpha U_{ul}+\beta U_{iz}$$

$$(4)\Rightarrow U_3=U_2$$

$$(\alpha U_{ul}+\beta U_{iz})(1+sR_2C_2+sR_2C_2sR_1C_1+sR_1C_1+sR_1C_2)=$$
$$=U_{iz}(1+sR_1C_1+sR_1C_2+sR_1C_1sR_2C_2)+U_{ul}sR_2C_2$$

$$\alpha U_{ul}(1+sR_2C_2+sR_2C_2sR_1C_1+sR_1C_1+sR_1C_2)-U_{ul}sR_2C_2=$$
$$=-\beta U_{iz}(1+sR_2C_2+sR_2C_2sR_1C_1+sR_1C_1+sR_1C_2)+U_{iz}(1+sR_1C_1+sR_1C_2+sR_1C_1sR_2C_2)$$

$$U_{ul}\left[1+sR_2C_2\left(1-\frac{1}{\alpha}\right)+sR_2C_2sR_1C_1+sR_1C_1+sR_1C_2\right]=$$
$$=\frac{1-\beta}{\alpha}U_{iz}\left(1+s\frac{-\beta}{1-\beta}R_2C_2+sR_2C_2sR_1C_1+sR_1C_1+sR_1C_2\right)$$

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{\alpha}{1-\beta}\cdot\frac{1+s\left[R_1(C_1+C_2)-\frac{1-\alpha}{\alpha}R_2C_2\right]+s^2R_2C_2R_1C_1}{1+s\left[R_1(C_1+C_2)-\frac{\beta}{1-\beta}R_2C_2\right]+s^2R_1C_1R_2C_2}$$

Uz uvrštene vrijednosti elemenata C_1=1, C_2=1, R_1=1, R_2=1, R_3=R_4=R_5=1:

$$\Rightarrow \alpha=1/;\beta=1/3.$$

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{1/3}{2/3}\cdot\frac{1+s[1\cdot(1+1)-(2/3)/(1/3)\cdot1\cdot1]+s^2}{1+s[1\cdot(1+1)-(1/3)/(2/3)\cdot1\cdot1]+s^2}=0{,}5\cdot\frac{1+s^2}{1+1{,}5\cdot s+s^2}$$ **(2 boda)**

b) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja izračunati parametre k, ω_p, q_p.

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=k\cdot\frac{s^2+\omega_p^2}{s^2+(\omega_p/q_p)s+\omega_p^2}\quad\Rightarrow\quad\omega_p=1,\quad q_p=\frac{2}{3}=0{,}667,\quad k=\frac{1}{2}$$ **(1 bod)**

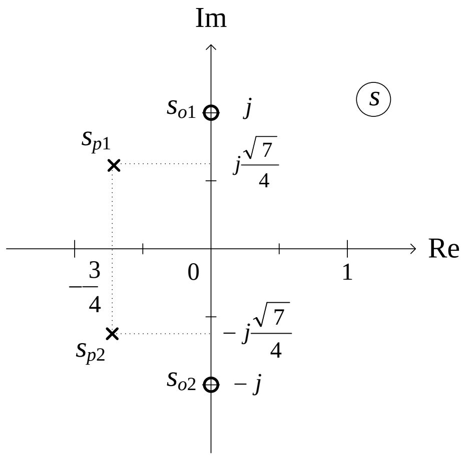
-o kojem se tipu filtra radi (NP, VP, PP ili PB)? $\Rightarrow$ PB

c) raspored polova i nula u kompleksnoj ravnini: **(1 bod)**

$$T(s)=0{,}5\frac{s^2+1}{s^2+1{,}5s+1}$$

nule $\quad s^2+1=0\quad\Rightarrow\quad s_{o1,2}=\pm j$

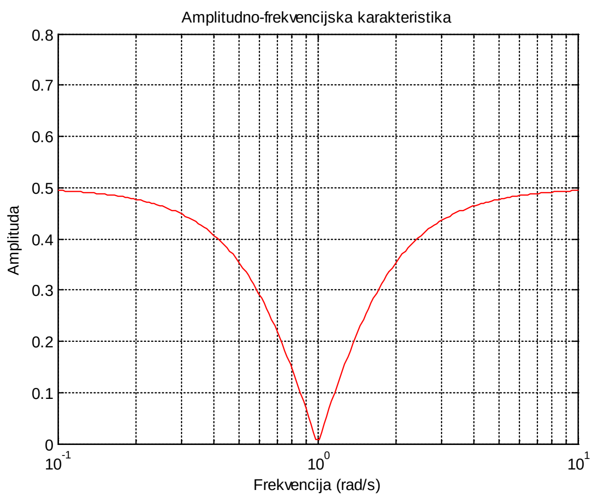
polovi $\quad s^2+\frac{3}{2}s+1=0\quad\Rightarrow\quad s_{p1,2}=-\frac{3}{4}\pm\sqrt{\frac{9}{16}-1}=-\frac{3}{4}\pm j\frac{\sqrt{7}}{4}=-0{,}75\pm j0{,}6614$



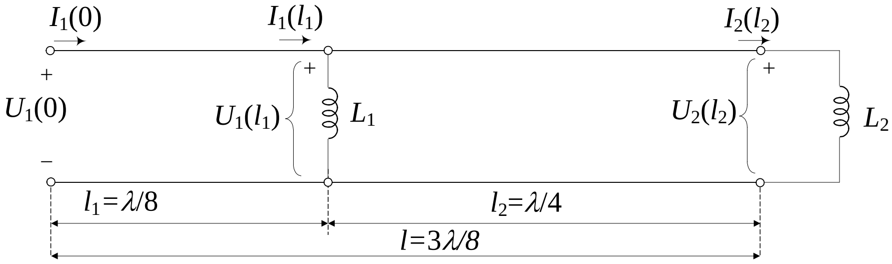
d) amplitudno-frekvencijska karakteristika: **(1 bod)**

$s=j\omega \Rightarrow$

$$T(j\omega)=\frac{1}{2}\frac{-\omega^2+1}{-\omega^2+j\frac{3}{2}\omega+1} \quad \Rightarrow \quad |T(j\omega)|=\frac{1}{2}\frac{|1-\omega^2|}{\sqrt{(1-\omega^2)^2+\left(\frac{3}{2}\omega\right)^2}}$$



5. Na ulazu kaskadnoga spoja dviju linija bez gubitaka s istim primarnim prametrima L=450μH/km i C=80nF/km, duljina $l_1=\lambda/8$ i $l_2=\lambda/4$, djeluje napon $u(0,t)=4\cdot\cos(\omega_0\, t)$. Na kraj prve linije priključen je induktivitet L_1=15mH, a na kraj druge L_2=3,75mH. Odrediti: a) karakterističnu impedanciju Z_0 i brzinu propagacije signala po linijama v; b) izraz za ulaznu impedanciju druge linije Z_{ul2}; c) frekvenciju ω_0 za koju je struja na kraju prve linije $i_1(l_1,t)$ jednaka nuli i koeficijent prijenosa γ; d) napon na kraju prve linije $u_1(l_1,t)$ i na kraju druge linije: $u_2(l_2,t)$; e) struju na kraju druge linije $i_2(l_2, t)$ i duljine linija l_1 i l_2.

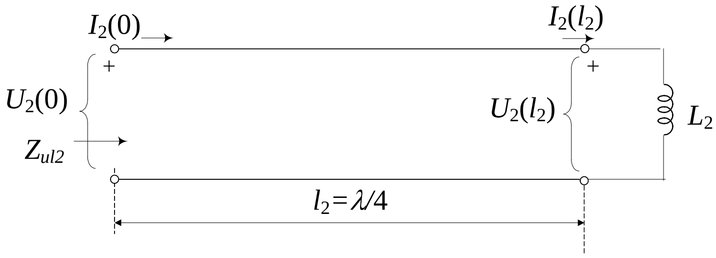


Rješenje: Liniju ćemo analizirati kao dvije linije s istim primarnim parametrima, spojene u kaskadu

a) $\boxed{Z_0=\sqrt{\frac{L}{C}}=\sqrt{\frac{450\cdot10^{-6}}{80\cdot10^{-9}}}=75\Omega}$; $\boxed{v=\frac{1}{\sqrt{LC}}=\frac{1}{6\cdot10^{-6}}=166{,}7\cdot10^{3}\,\text{km/s}}$; **(1 bod)**

b) Ulazna impedancija druge linije. **(1 bod)**

$$Z_{ul2}=\frac{U_2(0)}{I_2(0)}=\frac{U_2(l_2)ch(\gamma l_2)+I_2(l_2)Z_0 sh(\gamma l_2)}{\frac{U_2(l_2)}{Z_0}sh(\gamma l_2)+I_2(l_2)Z_0 ch(\gamma l_2)}$$



$$\gamma\cdot l_2=j\beta\cdot l_2=j\beta\frac{\lambda}{4}=j\frac{\pi}{2}\qquad \sin(\beta l_2)=1\qquad \cos(\beta l_2)=0\qquad U_2(l_2)=I_2(l_2)Z_2$$

$$\boxed{Z_{ul2}=\frac{U_2(l_2)\cos(\beta l_2)+jI_2(l_2)Z_0\sin(\beta l_2)}{j\frac{U_2(l_2)}{Z_0}\sin(\beta l_2)+I_2(l_2)Z_0\cos(\beta l_2)}=Z_0^2\frac{I_2(l_2)}{U_2(l_2)}=\frac{Z_0^2}{Z_2}=\frac{Z_0^2}{j\omega L_2}}$$

c) Struja na kraju prve linije: **(1 bod)**

$$I_1(l_1)=U_1(l_1)\left(\frac{1}{j\omega L_1}+\frac{1}{Z_{ul2}}\right)=U_1(l_1)\left(\frac{1}{j\omega L_1}+\frac{j\omega L_2}{Z_0^2}\right)=U_1(l_1)\frac{Z_0^2-\omega_0^2 L_1 L_2}{j\omega L_1}=0$$

$$Z_0^2-\omega_0^2 L_1 L_2=0\qquad\Rightarrow\qquad \boxed{\omega_0=\frac{Z_0}{\sqrt{L_2 L_1}}=\frac{75}{\sqrt{3{,}75\cdot10^{-3}\cdot15\cdot10^{-3}}}=10^4[rad/s]}$$

$$\boxed{\gamma=j\beta=j\omega_0\sqrt{LC}=j6\cdot10^{-2}/\text{km}}$$

d) Napon na kraju prve linije: $\gamma\cdot l_1=j\beta\cdot l_1=j\beta\frac{\lambda}{8}=j\frac{\pi}{4}\Rightarrow\ \sin(\beta l_1)=\cos(\beta l_1)=\frac{1}{\sqrt{2}}$

$$U_1(l_1)=U_1(0)\cos(\beta l_1)-jI_1(0)Z_0\sin(\beta l_1)$$

$$Z_{ul1}=\frac{U_1(l_1)\cos(\beta l_1)+jI_1(l_1)Z_0\sin(\beta l_1)}{j\frac{U_1(l_1)}{Z_0}\sin(\beta l_1)+I_1(l_1)Z_0\cos(\beta l_1)}=-jZ_0\frac{\cos(\beta l_1)}{\sin(\beta l_1)}=-jZ_0$$

$$I_1(0)=\frac{U_1(0)}{Z_{ul1}}=\frac{U_1(0)}{-jZ_0}=j\frac{U_1(0)}{Z_0}=j\frac{4}{75}=53{,}333e^{j\frac{\pi}{2}}[mA]$$

$$U_1(l_1)=U_1(0)\cos(\beta l_1)-j\left(j\frac{U_1(0)}{Z_0}\right)Z_0\sin(\beta l_1)=U_1(0)(\cos(\beta l_1)+\sin(\beta l_1))=\sqrt{2}\cdot U_1(0)$$

$$\boxed{u_1(l_1,t)=4\sqrt{2}\cdot\cos(\omega_0 t)}$$ **(1 bod)**

Napon na kraju druge linije:

$$U_2(l_2)=U_2(0)\cos(\beta l_2)-jI_2(0)Z_0\sin(\beta l_2)$$

$$I_2(0)=\frac{U_2(0)}{Z_{ul2}}=\frac{U_2(0)}{Z_0^2}j\omega L_2 \qquad \omega L_2=10^4\cdot 3{,}75\cdot 10^{-3}=37{,}5\Omega$$

$$U_2(l_2)=U_2(0)\cos(\beta l_2)-j\frac{U_2(0)}{Z_0^2}j\omega L_2 Z_0\sin(\beta l_2)=\frac{\omega L_2}{Z_0}U_1(l_1)=\frac{1}{2}U_1(l_1)=\frac{\sqrt{2}}{2}U_1(0)$$

$$\boxed{u_2(l_2,t)=2\sqrt{2}\cdot\cos(\omega_0 t)}$$

e) $$I_2(l_2)=-jU_2(0)\frac{\sin(\beta l_2)}{Z_0}+I_2(0)\cos(\beta l_2)=\frac{-jU_2(0)}{Z_0}=-j75{,}42\cdot 10^{-3}$$

$$\boxed{i_2(l_2,t)=75{,}42\cdot\cos\left(\omega_0 t-\frac{\pi}{2}\right)[mA]}$$

$$\boxed{l_1=\frac{1}{8}\lambda=\frac{2\pi}{8\beta}=\frac{\pi}{4\omega_0\sqrt{LC}}=\frac{\pi}{4\cdot 10^4\cdot 6\cdot 10^{-6}}=\frac{\pi}{24\cdot 10^{-2}}=13{,}08[km]}$$

$$\boxed{l_2=\frac{1}{4}\lambda=\frac{\pi}{2\beta}=\frac{\pi}{2\omega_0\sqrt{LC}}=\frac{\pi}{2\cdot 10^4\cdot 6\cdot 10^{-6}}=\frac{\pi}{12\cdot 10^{-2}}=26{,}16[km]}$$ **(1 bod)**