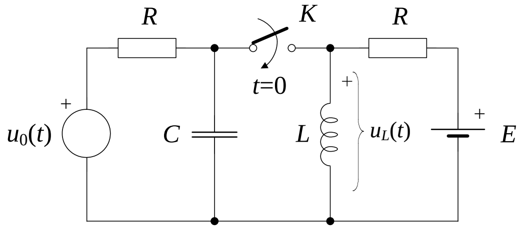
PISMENI ISPIT IZ ELEKTRIČNIH KRUGOVA 2014-2015 – Rješenja

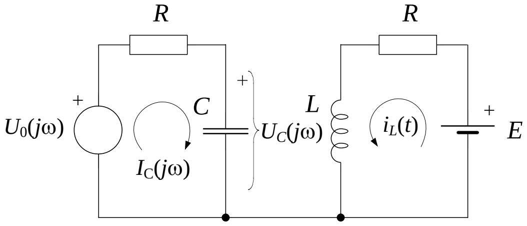
1. Za mrežu prikazanu slikom odrediti valni oblik napona na induktivitetu $u_L(t)$ za $-\infty<t<\infty$, ako se u trenutku $t=0$ zatvori sklopka K. Zadane su normalizirane vrijednosti elemenata: $L=2$, $C=1/2$, $R=1$, napon baterije $E=2$ za $-\infty<t<\infty$, te pobuda naponskog izvora:

$$u_0(t)=\begin{cases}10\sqrt{2}\sin(t+\pi/4), & \text{za } t<0;\\ 0, & \text{za } t\geq 0.\end{cases}$$



<u>Rješenje:</u>

a) za $t<0$ izračunavamo početne uvjete:



$$i_L(t)=\frac{E}{R}=\frac{2}{1}=2 \quad\Rightarrow\quad i_L(0)=2[\text{A}]$$

$$U_C(j\omega)=U_0(j\omega)\cdot\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}=U_0(j\omega)\cdot\frac{1}{1+j\omega RC}$$

$$U_0(j\omega)=10\sqrt{2}\cdot e^{j\frac{\pi}{4}}=10\sqrt{2}\cdot\left(\cos\frac{\pi}{4}+j\sin\frac{\pi}{4}\right)=10\sqrt{2}\cdot\left(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}\right)=10\cdot(1+j)$$

$$U_C(j\omega)=10\cdot(1+j)\cdot\frac{1}{1+j\cdot 1\cdot 1\cdot\frac{1}{2}}=20\cdot\frac{1+j}{2+j}\cdot\frac{2-j}{2-j}=20\cdot\frac{2+2j-j+1}{4+1}=4\cdot(3+j)$$

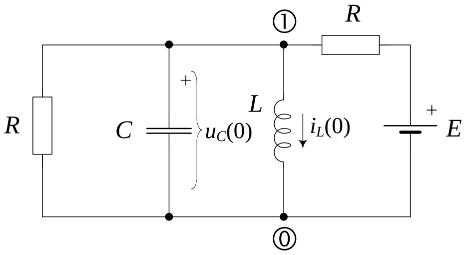
$$|U_C(j\omega)|=4\cdot\sqrt{9+1}=4\cdot\sqrt{10}=12{,}649$$

$$\varphi_C(\omega)=\arctan\left(\frac{\text{Im}[U_C(j\omega)]}{\text{Re}[U_C(j\omega)]}\right)=\arctan\left(\frac{1}{3}\right)=18{,}436°$$

$$u_C(t)=12{,}649\cdot\sin(t+18{,}436°)$$

$$u_C(0)=12{,}649\cdot\sin(18{,}436°)=4[\text{V}]$$ **(2 boda)**

b) za $t\geq 0$ primijenimo Laplaceovu transformaciju na slijedeći električni krug koji ima zadane početne uvjete:



(1) $$U_1(s)\left(sC+\frac{1}{sL}+\frac{1}{R}+\frac{1}{R}\right)=\frac{E/R}{s}+C\cdot u_C(0)-\frac{i_L(0)}{s};$$

$$U_1(s)=U_L(s)$$

Uz zadane normalizirane vrijednosti elemenata L=2, C=1/2, R=1, E=2 slijedi:

$$U_1(s)=\frac{\frac{E/R}{s}+C\cdot u_C(0)-\frac{i_L(0)}{s}}{sC+\frac{1}{sL}+\frac{1}{R}+\frac{1}{R}}=\frac{\frac{2}{s}+\frac{1}{2}\cdot 4-\frac{2}{s}}{\frac{s}{2}+\frac{1}{2s}+1+1}=\frac{2}{\frac{s}{2}+\frac{1}{2s}+2}=\frac{4s}{s^2+4s+1}$$ **(2 boda)**

$$s^2+4s+1=0\Rightarrow s_{p1,2}=\frac{-4\pm\sqrt{16-4}}{2}=-2\pm\sqrt{3}$$

$$\Rightarrow s_{p1}=-2+\sqrt{3}=-0{,}2679;\quad s_{p2}=-2-\sqrt{3}=-3{,}73205$$

$$U_L(s)=\frac{4s}{s^2+4s+1}=\frac{4s}{(s-2-\sqrt{3})(s-2+\sqrt{3})}=\frac{A}{s-2-\sqrt{3}}+\frac{B}{s-2+\sqrt{3}}$$

$$U_L(s)=\frac{4s}{s^2+4s+1}=\frac{A(s-2+\sqrt{3})+B(s-2-\sqrt{3})}{(s-2-\sqrt{3})(s-2+\sqrt{3})}=\frac{As+2A-A\sqrt{3}+Bs+2B+B\sqrt{3}}{s^2+4s+1}$$

$$\begin{aligned}&A+B=4\\&2(A+B)+(B-A)\sqrt{3}=0\end{aligned}\qquad\qquad \begin{aligned}&A+B=4\\&A-B=\frac{8}{\sqrt{3}}\end{aligned}$$

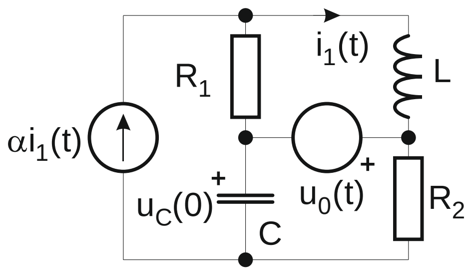
$$2A=4+\frac{8}{\sqrt{3}}\Rightarrow A=2+\frac{4}{\sqrt{3}}=4{,}3094$$

$$B=4-A=2-\frac{4}{\sqrt{3}}=-0{,}3094$$

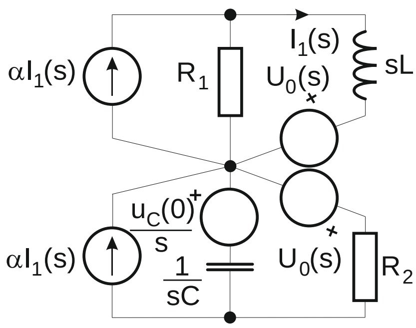
$$U_L(s)=\frac{4s}{s^2+4s+1}=\frac{2+4/\sqrt{3}}{s-2-\sqrt{3}}+\frac{2-4/\sqrt{3}}{s-2+\sqrt{3}}=\frac{4{,}3094}{s+0{,}2679}-\frac{0{,}3094}{s+3{,}73205}$$

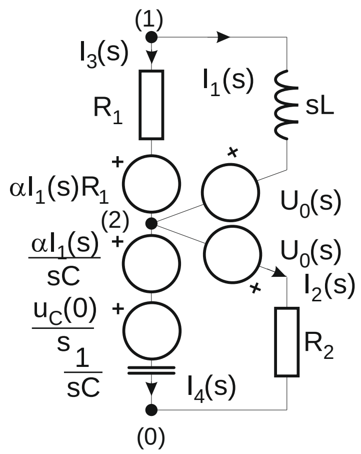
$$u_L(t)=\left(4{,}3094\cdot e^{-0{,}2679\cdot t}-0{,}3094\cdot e^{-3{,}73205\cdot t}\right)\cdot S(t)$$ **(1 bod)**

2. Zadan je električni krug prema slici. Nacrtati pripadni orijentirani graf i napisati spojnu matricu **S**. Napisati naponsko-strujne jednadžbe grana u matričnom obliku te ispisati matricu impedancija grana $\mathbf{Z}_b$ i vektor početnih uvjeta i nezavisnih izvora grana $\mathbf{U}_{0b}$. Matrica $\mathbf{Z}_b$ mora biti regularna. Napisati temeljni sustav jednadžbi petlji, odnosno odrediti matrice impedancija petlji $\mathbf{Z}_p$ i vektor početnih uvjeta i nezavisnih izvora petlji $\mathbf{U}_{0p}$.



Rješenje: Posmicanje strujnog i naponskog izvora i primjena Laplaceove transformacije



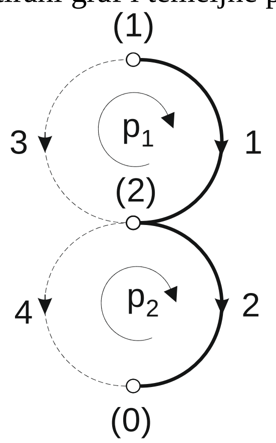


(1 bod)

Naponsko-strujne jednadžbe grana:

$$U_1 = I_1 \cdot sL + U_0$$
$$U_2 = I_2 \cdot R_2 - U_0$$
$$U_3 = I_1 \cdot \alpha R_1 + I_3 \cdot R_1$$
$$U_4 = I_1 \cdot \alpha \frac{1}{sC} + I_4 \cdot \frac{1}{sC} + \frac{u_C(0)}{s}$$

Orijentirani graf i temeljne petlje:



Spojna matrica:

$$\mathbf{S} = \begin{array}{c} \\ (p_1) \\ (p_2) \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \end{array}$$

(1 bod)

Naponsko-strujne relacije grana u matričnom obliku: $\mathbf{U}_b = \mathbf{Z}_b \cdot \mathbf{I}_b + \mathbf{U}_{0b}$

$$\underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b} = \underbrace{\begin{bmatrix} sL & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ \alpha R_1 & 0 & R_1 & 0 \\ \dfrac{\alpha}{sC} & 0 & 0 & \dfrac{1}{sC} \end{bmatrix}}_{\mathbf{Z}_b} \cdot \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b} + \underbrace{\begin{bmatrix} U_0 \\ -U_0 \\ 0 \\ \dfrac{u_C(0)}{s} \end{bmatrix}}_{\mathbf{U}_{0b}}$$ **(1 bod)**

Matrica $\mathbf{Z}_b$ je regularna. Temeljni sustav jednadžbi petlji u matričnom obliku: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p}$, gdje su (matrice $\mathbf{Z}_p$ i $\mathbf{U}_{0p}$):

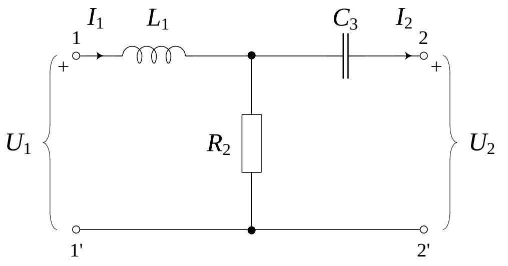
$$\mathbf{Z}_p = \mathbf{S} \cdot \mathbf{Z}_b \cdot \mathbf{S}^T = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} sL & 0 & 0 & 0 \\ 0 & R_2 & 0 & 0 \\ \alpha R_1 & 0 & R_1 & 0 \\ \dfrac{\alpha}{sC} & 0 & 0 & \dfrac{1}{sC} \end{bmatrix} \cdot \mathbf{S}^T$$

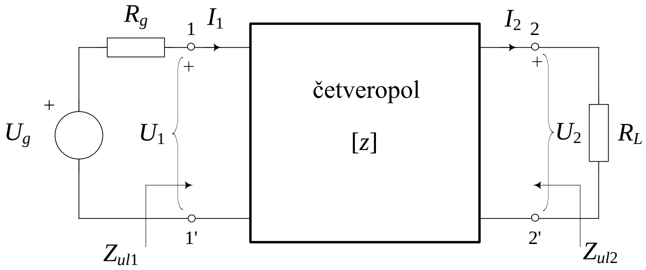
$$= \begin{bmatrix} sL - \alpha R_1 & 0 & -R_1 & 0 \\ -\dfrac{\alpha}{sC} & R_2 & 0 & -\dfrac{1}{sC} \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} sL + (1-\alpha) R_1 & 0 \\ -\dfrac{\alpha}{sC} & R_2 + \dfrac{1}{sC} \end{bmatrix}$$ **(1 bod)**

$$\mathbf{U}_{0p} = -\mathbf{S} \cdot \mathbf{U}_{0b} = -\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} U_0 \\ -U_0 \\ 0 \\ \dfrac{u_C(0)}{s} \end{bmatrix} = \begin{bmatrix} -U_0 \\ U_0 + \dfrac{u_C(0)}{s} \end{bmatrix}$$ **(1 bod)**

Rješenje: $\mathbf{Z}_p \cdot \mathbf{I}_p = \mathbf{U}_{0p} \;\; \Rightarrow \;\; \mathbf{I}_p = \begin{bmatrix} I_{p1} \\ I_{p2} \end{bmatrix}$

3. Za T-četveropol prikazan lijevom slikom izračunati z-parametre. a) Napisati z-parametre pomoću C_1, R_2 i L_3 te uvrstiti normirane vrijednosti elemenata: L_1=1, R_2=2, C_3=1. Četveropol je spojen u električni krug na desnoj slici. b) Ako je izlazni prilaz (2–2') zaključen otporom R_L=1 pomoću z-parametara izračunati: ulaznu impedanciju $Z_{ul1}(s)=U_1(s)/I_1(s)$ gledano sa priključnica 1–1'; c) ako je R_g=1 izračunati izlaznu impedanciju $Z_{ul2}(s)=-U_2(s)/I_2(s)$ gledano sa priključnica 2–2'; i d) izračunati prijenosnu funkciju napona $H(s)=U_2(s)/U_g(s)$.



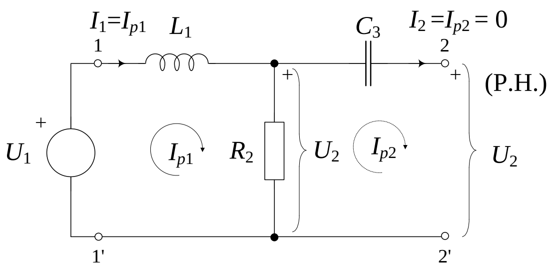


Rješenje:

a) [z]-parametri:

$$U_1 = z_{11} \cdot I_1 - z_{12} \cdot I_2$$
$$U_2 = z_{21} \cdot Z_1 - z_{22} \cdot I_2$$

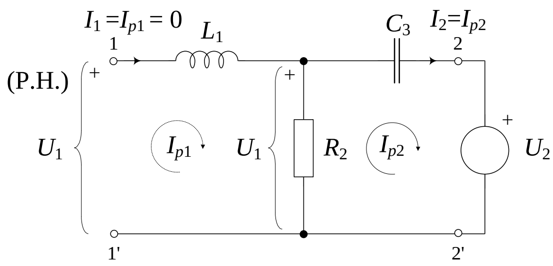
$$I_2 = 0 \quad z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0} \quad ; \quad z_{21} = \left.\frac{U_2}{I_1}\right|_{I_2=0}$$



$$(1)\ U_1 = I_{p1}(sL_1 + R_2)$$
$$(2)\ U_2 = I_{p1}R_2$$

$$(1) \Rightarrow z_{11} = \left.\frac{U_1}{I_1}\right|_{I_2=0} = sL_1 + R_2 \quad (2) \Rightarrow z_{21} = \left.\frac{U_2}{I_1}\right|_{I_2=0} = R_2$$

$$I_1 = 0 \quad z_{12} = -\left.\frac{U_1}{I_2}\right|_{I_1=0} ; \ z_{22} = -\left.\frac{U_2}{I_2}\right|_{I_1=0}$$



$$(1)\ U_1 = -I_{p2}R_2$$
$$(2)\ U_2 = -I_{p2}\left(R_2 + \frac{1}{sC_3}\right)$$

$$(1) \Rightarrow z_{12} = \left.\frac{U_1}{I_2}\right|_{I_1=0} = R_2 \quad (2) \Rightarrow z_{22} = -\left.\frac{U_2}{I_2}\right|_{I_1=0} = R_2 + \frac{1}{sC_3}$$

$$[z] = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix} = \begin{bmatrix} sL_1 + R_2 & -R_2 \\ R_2 & -\left(R_2 + \frac{1}{sC_3}\right) \end{bmatrix} = \begin{bmatrix} 2+s & -2 \\ 2 & -\left(2 + \frac{1}{s}\right) \end{bmatrix}$$ **(2 boda)**

b) Ulazna impedancija u četveropol:

$$Z_{ul1}(s) = \frac{U_1}{I_1} = z_{11} - \frac{z_{12}z_{21}}{Z_L + z_{22}}; \quad R_L = \frac{U_2}{I_2}$$

$$= 2 + s - \frac{2\cdot 2}{1+2+1/s} = 2 + s - \frac{4s}{3s+1} = \frac{(s+2)(3s+1)-4s}{3s+1} = \frac{3s^2+3s+2}{3s+1}$$

(1 bod)

c) Izlazna impedancija iz četveropola:

$$Z_{ul2}(s) = -\frac{U_2}{I_2} = z_{22} - \frac{z_{12}z_{21}}{Z_g + z_{11}}; \quad U_g = 0; \quad R_g = -\frac{U_1}{I_1}$$

$$= 2 + \frac{1}{s} - \frac{2\cdot 2}{1+2+s} = \frac{2s+1}{s} - \frac{4}{s+3} = \frac{(2s+1)(s+3)-4s}{s(s+3)} = \frac{2s^2+3s+3}{s^2+3s};$$

(1 bod)

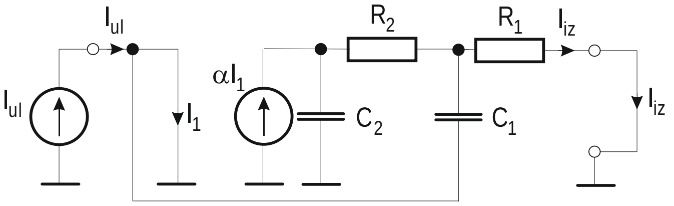
d) Prijenosna funkcija napona: uz $Z_g = R_g = 1$; $Z_L = R_L = 1$ slijedi:

$$H(s) = \frac{U_2}{U_g} = \frac{z_{21}Z_g}{(z_{11}+Z_g)(z_{22}+Z_L) - z_{12}z_{21}} = \frac{2\cdot 1}{(2+s+1)(2+1/s+1)-4} = \frac{2}{(3+s)(3+1/s)-4}$$

$$H(s) = \frac{U_2}{U_g} = \frac{2}{3/s+3s+6} = \frac{2}{3}\cdot\frac{1}{1/s+s+2} = \frac{2}{3}\cdot\frac{s}{s^2+2s+1} = \frac{2}{3}\cdot\frac{s}{(s+1)^2}$$

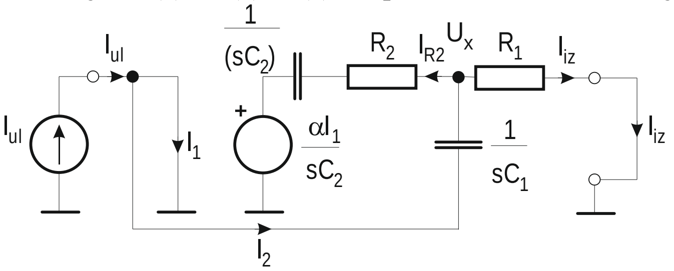
(1 bod)

4. Za četveropol prikazan slikom s normaliziranim vrijednostima elemenata: $R_1=1/\sqrt{2}$, $R_2=\sqrt{2}$, $C_1=1$, $C_2=1$, $\alpha=2$: a) izračunati strujnu prijenosnu funkciju $H_i(s)=I_{iz}(s)/I_{ul}(s)$. O kojem se tipu prijenosne funkcije radi (NP, VP, PP ili PB)? b) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja izračunati vrijednost parametara k, ω_p, q_p. c) Prikazati raspored polova i nula u kompleksnoj ravnini. d) Nacrtati amplitudno-frekvencijsku karakteristiku.



Rješenje:

a) Strujna prijenosnu funkciju $H_i(s)=I_{iz}(s)/I_{ul}(s)$. Laplaceova transformacija:



$$(1) \qquad I_{ul}=I_1+I_2 \qquad \Rightarrow I_1=I_{ul}-I_2;\;\; I_2=I_{R2}+I_{iz}$$

$$(2) \qquad I_2=\frac{U_x-\alpha\cdot I_1\cdot\frac{1}{sC_2}}{R_2+\frac{1}{sC_2}}+I_{iz} \Bigg/ \cdot\left(R_2+\frac{1}{sC_2}\right)$$

$$\left.\begin{aligned}&(3) \qquad I_{iz}=\frac{U_x}{R_1} \qquad \Rightarrow U_x=R_1\cdot I_{iz}\\ &(4) \qquad I_2=-U_x\cdot sC_1\end{aligned}\right\}\Rightarrow I_2=-I_{iz}\cdot sR_1C_1$$

$$(2)\Rightarrow I_2\left(R_2+\frac{1}{sC_2}\right)=U_x-\alpha\cdot I_1\cdot\frac{1}{sC_2}+I_{iz}\cdot\left(R_2+\frac{1}{sC_2}\right)$$

$$-I_{iz}\cdot sR_1C_1\left(R_2+\frac{1}{sC_2}\right)=R_1\cdot I_{iz}-\alpha\cdot(I_{ul}-I_2)\frac{1}{sC_2}+I_{iz}\left(R_2+\frac{1}{sC_2}\right)\Bigg/\cdot sC_2$$

$$-I_{iz}\cdot sR_1C_1(sR_2C_2+1)=sR_1C_2\cdot I_{iz}-\alpha\cdot I_{ul}+\alpha\cdot I_2+I_{iz}(sR_2C_2+1)$$

$$-I_{iz}(s^2R_1C_1R_2C_2+sR_1C_1)=I_{iz}\cdot sR_1C_2-I_{iz}\cdot\alpha\cdot sR_1C_1-\alpha\cdot I_{ul}+I_{iz}(sR_2C_2+1)$$

$$-I_{iz}(s)\cdot\left[sR_1C_1-\alpha\cdot R_1C_1+sR_2C_2+1+s^2R_1C_1R_2C_2+sR_1C_1\right]=-\alpha\cdot I_{ul}(s)$$

$$H_i(s)=\frac{I_{iz}(s)}{I_{ul}(s)}=\frac{\alpha}{s^2R_1C_1R_2C_2+s\left[R_1C_2+R_2C_2+R_1C_1\cdot(1-\alpha)\right]+1}$$

$$H_i(s)=\frac{I_{iz}(s)}{I_{ul}(s)}=\frac{2}{s^2+\left[\frac{1}{\sqrt{2}}+\sqrt{2}+\frac{1}{\sqrt{2}}(1-2)\right]\cdot s+1}=\frac{2}{s^2+\sqrt{2}\cdot s+1} \quad \textbf{(2 boda)}$$

b) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja izračunati parametre k, ω_p, q_p.

$$H_i(s)=\frac{I_{iz}(s)}{I_{ul}(s)}=k\cdot\frac{\omega_p^2}{s^2+\frac{\omega_p}{q_p}s+\omega_p^2}\quad\Rightarrow\quad\omega_p=1;\ \ q_p=\frac{1}{\sqrt{2}}=0{,}707;\quad k=2.$$ **(1 bod)**

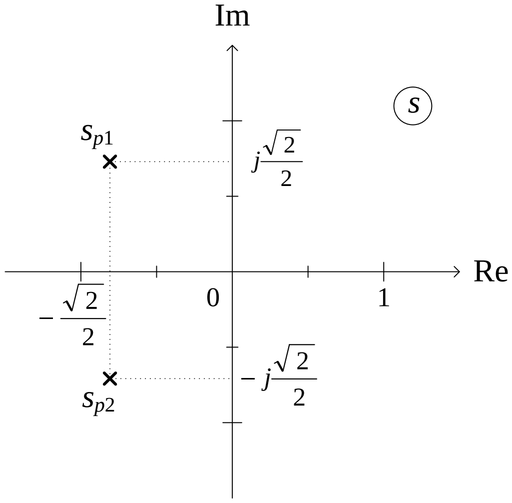
-o kojem se tipu prijenosne funkcije radi (NP, VP, PP ili PB)? $\Rightarrow$ NP

c) Raspored polova i nula u kompleksnoj ravnini: **(1 bod)**

Polovi: $s^2+\sqrt{2}\cdot s+1=0$

$$s_{p1,2}=\frac{-\sqrt{2}\pm\sqrt{2-4}}{2}=\frac{-\sqrt{2}\pm j\sqrt{2}}{2}=-\frac{\sqrt{2}}{2}\pm j\frac{\sqrt{2}}{2}=-0{,}707\pm j0{,}707\,.$$

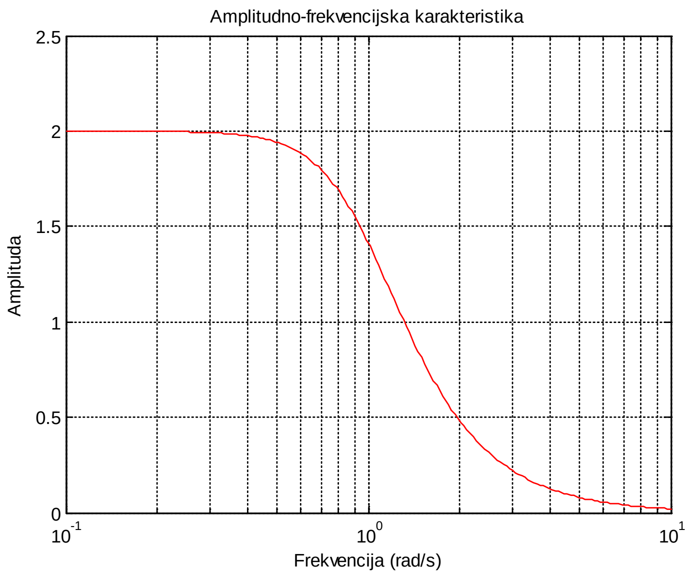
Nule: dvije u beskonačnosti:



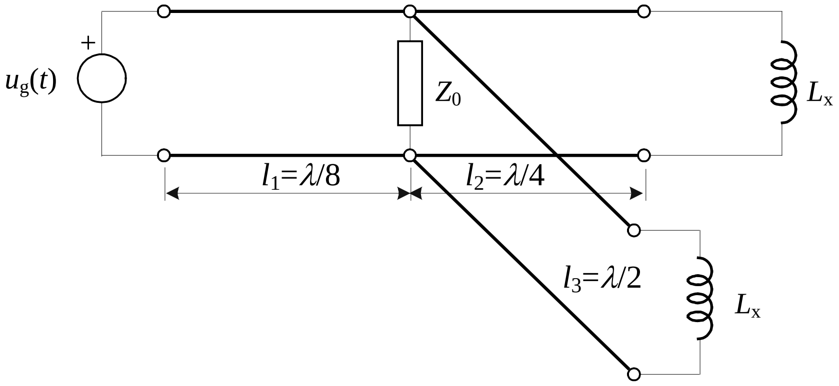
d) Amplitudno-frekvencijska karakteristika: **(1 bod)**

$$s{=}j\omega\ \Rightarrow\ H_i(j\omega)=2\cdot\frac{1}{-\omega^2+j\sqrt{2}\omega+1}$$

$$|H_i(j\omega)|=2\cdot\frac{1}{\sqrt{(1-\omega^2)^2+(\sqrt{2}\omega)^2}}=2\cdot\frac{1}{\sqrt{\omega^4-2\omega^2+1+2\omega^2}}=2\cdot\frac{1}{\sqrt{\omega^4+1}}$$



5. Tri linije bez gubitaka s istim primarnim parametrima L=200μH/km i C=80nF/km, spojene su prema slici. Duljine linija su: $l_1=\lambda/8$, $l_2=\lambda/4$ i $l_3=\lambda/2$. Zadano je: $u_g(t)=2\cos(1{,}25\cdot\pi\cdot10^5 t)$ [V]. Odrediti: a) valnu impedanciju, brzinu širenja vala na linijama i koeficijent prijenosa linija; b) ulazne impedancije druge i treće linije: Z_{ul2} i Z_{ul3}; c) induktivitet L_x, za kojeg je prva linija prilagođena na izlazu, te duljine linija: l_1, l_2 i l_3; d) faktore refleksije na krajevima druge i treće linije: Γ_{i2} i Γ_{i3}; e) napone na kraju svake linije: $u_1(l_1,t)$, $u_2(l_2,t)$, $u_3(l_3,t)$.



Rješenje:

a) $Z_0=\sqrt{\dfrac{L}{C}}=\sqrt{\dfrac{200\cdot10^{-6}}{80\cdot10^{-9}}}=50\Omega$

$$\gamma=j\beta=j\omega_0\sqrt{LC}=j1{,}25\cdot\pi\cdot10^5\sqrt{2\cdot10^{-4}\cdot8\cdot10^{-8}}=j\frac{\pi}{2}\ \text{[rad/km]}$$

$$v=\frac{\omega_0}{\beta}=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{2\cdot10^{-4}\cdot8\cdot10^{-8}}}=\frac{10^6}{4}=25\cdot10^4\ \text{[km/s]}$$

(1 bod)

b) $\gamma\cdot l_2=j\cdot\beta\cdot l_2=j\dfrac{\pi}{2}$ $\qquad\gamma\cdot l_3=j\cdot\beta\cdot l_3=j\pi$

$$Z_{ul2}=\frac{j\omega_0L_x ch(\gamma\cdot l_2)+Z_0 sh(\gamma\cdot l_2)}{\frac{j\omega_0L_x}{Z_0}sh(\gamma\cdot l_2)+ch(\gamma\cdot l_2)}=\frac{j\omega_0L_x\cos(\beta\cdot l_2)+jZ_0\sin(\beta\cdot l_2)}{j\frac{j\omega_0L_x}{Z_0}\sin(\beta\cdot l_2)+\cos(\beta\cdot l_2)}=\frac{Z_0^2}{j\omega_0L_x}$$

$$Z_{ul3}=\frac{j\omega_0L_x ch(\gamma\cdot l_2)+Z_0 sh(\gamma\cdot l_2)}{\frac{j\omega_0L_x}{Z_0}sh(\gamma\cdot l_2)+ch(\gamma\cdot l_2)}=\frac{j\omega_0L_x\cos(\beta\cdot l_2)+jZ_0\sin(\beta\cdot l_2)}{j\frac{j\omega_0L_x}{Z_0}\sin(\beta\cdot l_2)+\cos(\beta\cdot l_2)}=j\omega_0L_x$$

(1 bod)

c) Ukupna impedancija na izlazu prve linije: $\quad Z_{uk}=Z_0\parallel Z_{ul2}\parallel Z_{ul3}=\dfrac{1}{Y_0+Y_{ul2}+Y_{ul3}}=\dfrac{1}{Y_0}$

$$\Rightarrow\ Y_{ul2}+Y_{ul3}=0\ \Rightarrow\ \frac{j\omega_0L_x}{Z_0^2}+\frac{1}{j\omega_0L_x}=0$$

$$\Rightarrow L_x=\frac{Z_0}{\omega_0}=\frac{50}{1{,}25\cdot\pi\cdot10^5}=12{,}73\cdot10^{-6}H$$

$$\lambda=\frac{2\cdot\pi}{\beta}=\frac{2\cdot\pi}{0{,}5\cdot\pi}=4\ \text{[km]}$$

$$l_1=\frac{\lambda}{8}=500\ \text{[m]};\quad l_2=\frac{\lambda}{4}=1\ \text{[km]};\quad l_3=\frac{\lambda}{2}=2\ \text{[km]}$$

(1 bod)

d) $\Gamma_{i2} = \Gamma_{i3} = \dfrac{j\omega_0 L_x - Z_0}{j\omega_0 L_x + Z_0} = \dfrac{j-1}{j+1} = \dfrac{(j-1)^2}{2} = j$

(1 bod)

e) $\gamma \cdot l_1 = j\beta \dfrac{\lambda}{8} = j\beta \dfrac{2\pi}{8 \cdot \beta} = j\dfrac{\pi}{4}$ $\quad U_1(l_1) = U(0) \cdot e^{-j\beta l_1} = 2 \cdot e^{-j\pi/4}$ $\quad u_1(l_1,t) = 2\cos\left(\omega_0 t - \dfrac{\pi}{4}\right)$

$$U_2(l_2) = U_1(l_1) \cdot \cos(\beta \cdot l_2) - jU_1(l_1) \cdot \frac{Z_0}{Z_{ul2}} \sin(\beta \cdot l_2) = -jU_1(l_1) \cdot \frac{j\omega_0 L_2}{Z_0} = 2 \cdot e^{-j\pi/4}$$

$$u_1(l_2,t) = 2\cos\left(\omega t - \frac{\pi}{4}\right)$$

$$U_3(l_3) = U(l_1) \cdot \cos(\beta \cdot l_3) - jU(l_1) \cdot \frac{Z_0}{Z_{ul3}} \sin(\beta \cdot l_3) = -U(l_1) = -2 \cdot e^{-j\pi/4}$$

$$u_1(l_3,t) = -2\cos(\omega t - \pi/4)$$

(1 bod)