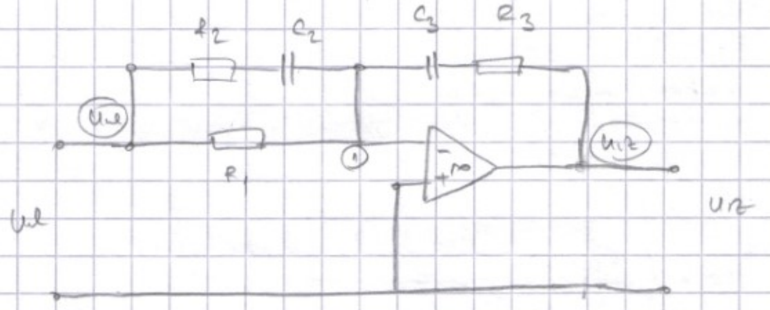
20.01.2012

3. MASS

PRIJENOSNE FUNKCIJE



$$H(s) = \frac{U_{iz}(s)}{U_{ul}(s)} = ?$$

$R_1 = R_2 = R_3 = 1$
$C_1 = C_2 = 1$

$$-U_{iz}\left(\frac{1}{R_3 + \frac{1}{sC_3}}\right) - U_{ul}\left(\frac{1}{R_1} + \frac{1}{R_2 + \frac{1}{sC_2}}\right) = 0$$

$$-U_{iz}\left(\frac{sC_3}{R_3 sC_3 + 1}\right) = U_{ul}\left(\frac{1}{R_1} + \frac{sC_2}{R_2 sC_2 + 1}\right)$$

$$-U_{iz}\left(\frac{sC_3}{R_3 sC_3 + 1}\right) = U_{ul}\left(\frac{R_2 sC_2 + 1 + R_1 sC_2}{R_1(R_2 sC_2 + 1)}\right)$$

$$U_{iz} = U_{ul} \cdot \frac{-(R_2 sC_3 + 1)(R_2 C_2 s + 1 + sR_1C_2)}{sC_3(R_1R_2C_2 s + R_1)}$$

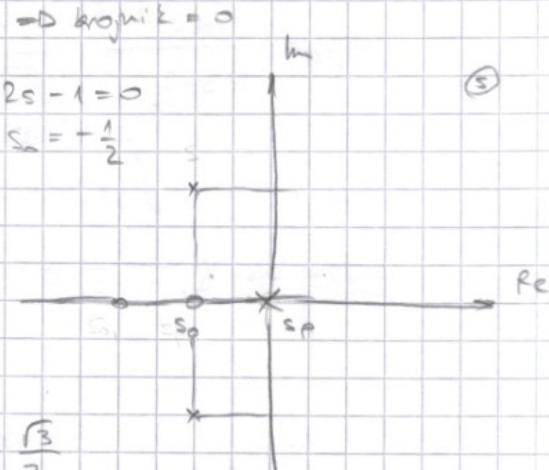
$$U_{iz} = U_{ul}\left[\frac{-\cancel{(s+1)}(2s+1)}{s\cancel{(s+1)}}\right] = U_{ul} \cdot \underbrace{\frac{-(2s+1)}{s}}_{H(s)}$$

polovi $\Rightarrow$ nazivnik $= 0$

nule $\Rightarrow$ brojnik $= 0$

$s_p = 0$

$-2s - 1 = 0$
$s_o = -\frac{1}{2}$



$\underline{N_p = N_o}$

$$H(s) = \frac{s+1}{s^2+s+1}$$

$s_{o_1} = -1$
$s_{o_2} = \infty$

$$s_{p_{12}} = \frac{-1 \pm \sqrt{-3}}{2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$$

STABILNOST

STABILAN – polovi su na lijevom dijelu grafa

GRANIČNO STABILAN – polovi su na y-osi

NESTABILAN – polovi su na desnom dijelu grafa

$$H(s) = \frac{s+1}{(s-1)^2 + \frac{\sqrt{3}}{2}} \multimap \left[e^{-t} \cos\left(\frac{\sqrt{3}}{2} t\right)\right] S(t)$$

ODZIV U STACIONARNOM STANJU (STABILAN SUSTAV)

$$u_{ul}(t) = 3\cos(2t + 15^\circ)$$

$\omega = 2$

$$H(s) = \frac{u_{iz}(s)}{u_{ul}(s)} \Rightarrow A(\omega)\angle \varphi(\omega) \Leftrightarrow A(\omega)\, e^{j\varphi(\omega)}$$

$$s = j\omega = 2j$$

$$H(s) = \frac{s+1}{s^2 + s + 1}$$

$$H(j\omega) = \frac{j2+1}{1-4+2j} = \frac{1+2j}{-3+2j}$$

- AMPLITUDA:

$$|H(j2)| = \frac{|1+2j|}{|-3+2j|} = \frac{\sqrt{1+2^2}}{\sqrt{3^2+2^2}}$$

- KUT:

$$\varphi(\omega) = \operatorname{arctg}\frac{2}{1} - \operatorname{arctg}\frac{2}{-3} \pm 180^\circ$$

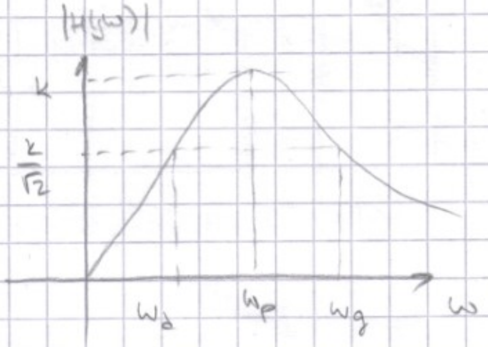
$\operatorname{arctg}\frac{Im}{Re}$

KARAKTERISTIKE :

$$H(s) = \frac{2s}{s^2+4s+4} \Rightarrow H(j\omega) = \frac{2j\omega}{-\omega^2+4j\omega+4}$$

- amplitudno - frekvencijske karakteristike

$$|H(j\omega)| = \frac{\sqrt{(2\omega)^2}}{\sqrt{(4-\omega^2)^2+(4\omega)^2}}$$



$k = \frac{1}{2}$

$\omega_p = 2$

$Q_p = \frac{1}{2}$

$$\omega_{d,g} = \omega_p\sqrt{1+\frac{1}{4Q_p^2}} \pm \frac{\omega_p}{2Q_p}$$

$$H(s) = k \cdot \frac{s \cdot \frac{\omega_p}{Q_p}}{s^2+\frac{\omega_p}{Q_p}s+\omega_p^2}$$

- fazno - frekvencijske karakteristike

$$\varphi(\omega) = \operatorname{arctg}\frac{\omega}{2} - \operatorname{arctg}\frac{4\omega}{4-\omega^2}$$

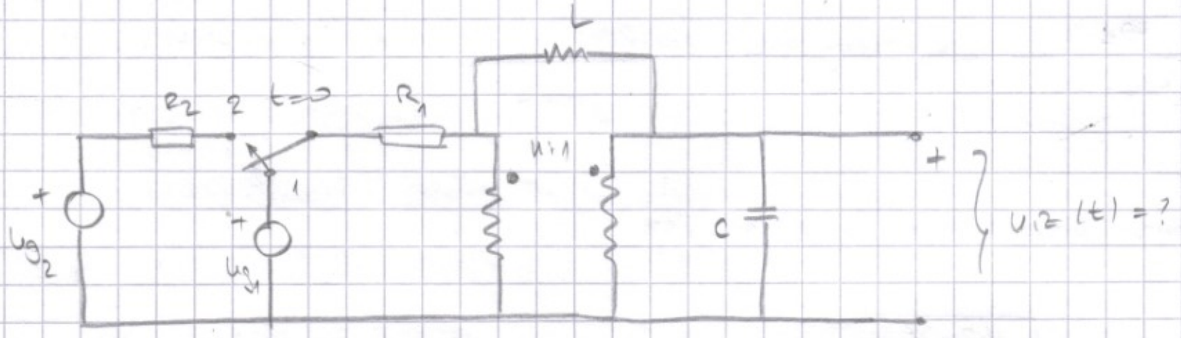
$4-\omega^2 = 0$

$\omega = \pm 2$

$\times$ za $\omega \in \langle -\infty, -2 \rangle \cup \langle 2, \infty \rangle$

$$= \operatorname{arctg}\frac{\omega}{2} - \left(\operatorname{arctg}\frac{4\omega}{4-\omega^2} \pm 180^\circ\right)$$

SKLOPKA

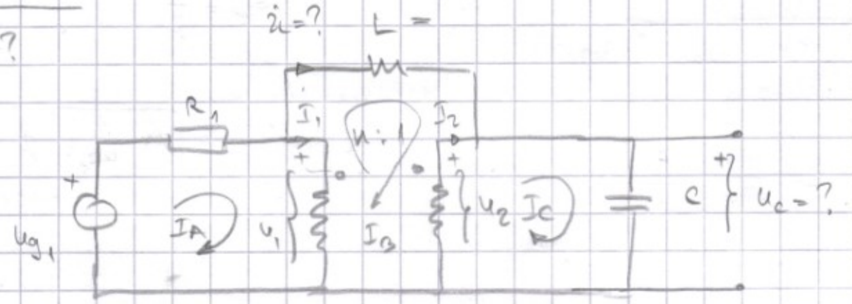


$u_{g_1}(t) = 4\sin t$

$n = 2$
$C = 1$
$L = 1$
$R_1 = 2$
$R_2 = 1$
$u_{g_2} = 2$

$u_{iz} = ?$

$t < 0$



$s = j\omega$

$Z_C = \frac{1}{sC} = \frac{1}{j\omega C} = \frac{-j}{\omega C} = -jX_C$

$Z_L = sL = j\omega L = jX_L$

$u_{g_1} = 4\angle 0^\circ$

1) $\dot{I}_A \cdot R = \dot{U}_{g_1} - \dot{U}_1$

2) $\dot{I}_B \cdot (jX_L) = \dot{U}_1 - \dot{U}_2$

3) $\dot{I}_C \cdot (-jX_C) = \dot{U}_2$

$\dot{U}_1 = n \cdot \dot{U}_2$

$I_1 = \frac{1}{n} \cdot I_2$

$\dot{I}_1 = \dot{I}_A - \dot{I}_B$

$\dot{I}_2 = \dot{I}_C - \dot{I}_B$

$\dot{I}_L = \dot{I}_B = 1\angle 36{,}87^\circ$

$\dot{U}_C = \dot{U}_2 = \frac{1}{2}\angle -64{,}42^\circ$

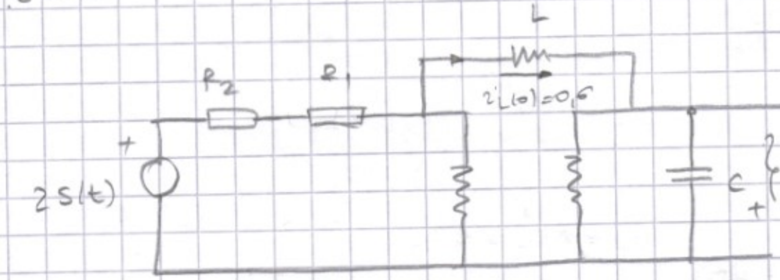
$i_L(t) = 1 \sin(t + 36{,}87^\circ)$

$i_L(0) = 1 \sin(36{,}87^\circ) = \boxed{0{,}6}$

$u_C(t) = \frac{1}{2} \sin(t - 64{,}42^\circ)$

$u_C(0) = -\frac{1}{4\sqrt{5}} = \boxed{-0{,}2}$ → na blef

$t > 0$



da su dobili $-0{,}6$

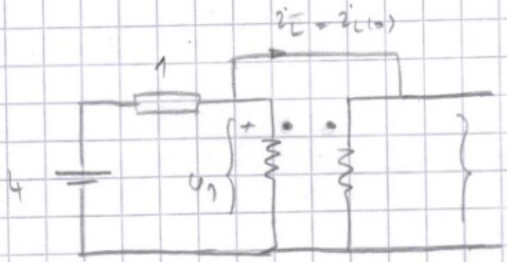
$i_L(0) = -0{,}6$

i_L

$i_L(0) = 0{,}6$

Da je bilo $u_{g1}(t) = 4$

$\omega = 0 \rightarrow X_L = 0$
$X_C = \infty$



ČETVEROPOLI

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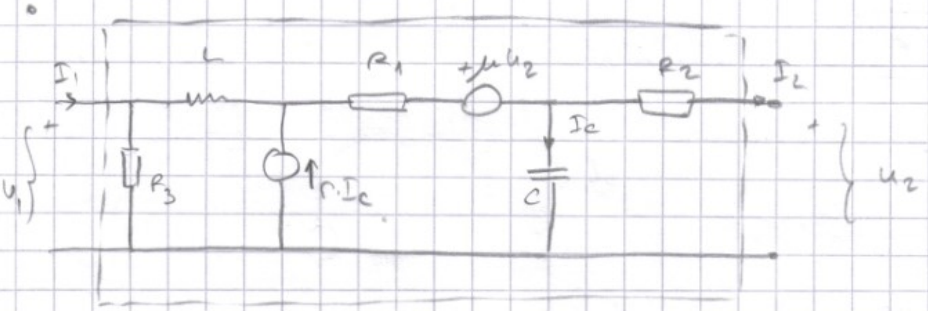
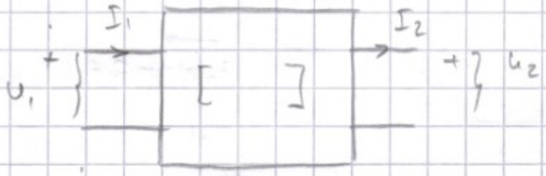


$\Downarrow$



$$\begin{bmatrix} U_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} \Rightarrow \begin{aligned} U_1 &= A U_2 + B I_2 \\ I_1 &= C U_2 + D I_2 \end{aligned}$$

$I_2 = 0 \Rightarrow \quad A = \dfrac{U_1}{U_2} \qquad C = \dfrac{I_1}{U_2}$

$U_2 = 0 \Rightarrow \quad B = \dfrac{U_1}{I_2} \qquad D = \dfrac{I_1}{I_2}$

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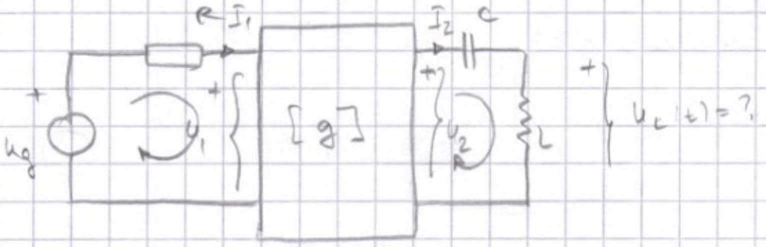


$u_g(t) = S(t)$

$R = L = C = 1$

$$[g] = \begin{bmatrix} 2+s & -3s \\ 1/2s & 1-s \end{bmatrix} \qquad \begin{bmatrix} I_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} U_1 \\ I_2 \end{bmatrix}$$

$\Longrightarrow$

$$H(s) = \frac{U_L(s)}{U_g(s)} = ?$$

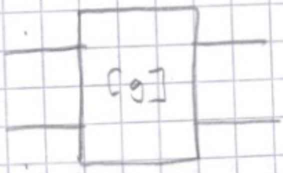
$$I_1 = (2+s)U_1 + (-3s)I_2$$

$$U_2 = \frac{1}{2s}U_1 + (1-s)I_2$$

$$U_g - I_1R_1 - U_1 = 0$$

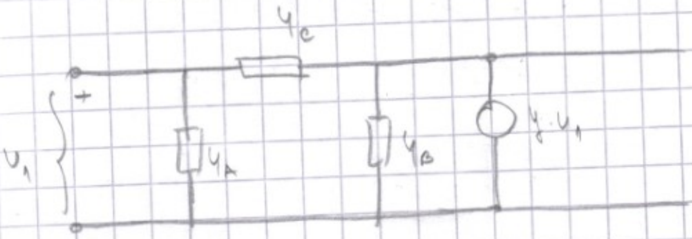
$$U_2 - I_2\cdot\frac{1}{sC} - I_2 sL = 0$$

$$U_L = I_2 sL$$



$$[g] = \begin{bmatrix} 2+s & -3s \\ 1/2s & 1-s \end{bmatrix}$$

$$\Downarrow$$



$$y_C = y_{12} \qquad y = y_{21} - y_{12}$$

$$y_A = y_{11} - y_{12}$$

$$y_B = y_{22} - y_{12}$$

$$[y] = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix}$$

$$I_1 = y_{11}U_1 - y_{12}U_2$$

$$I_2 = y_{21}U_1 - y_{22}U_2$$