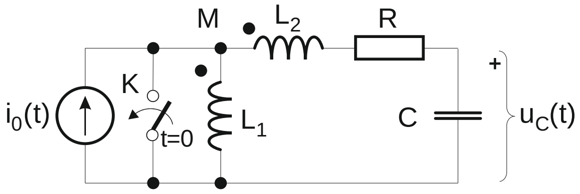
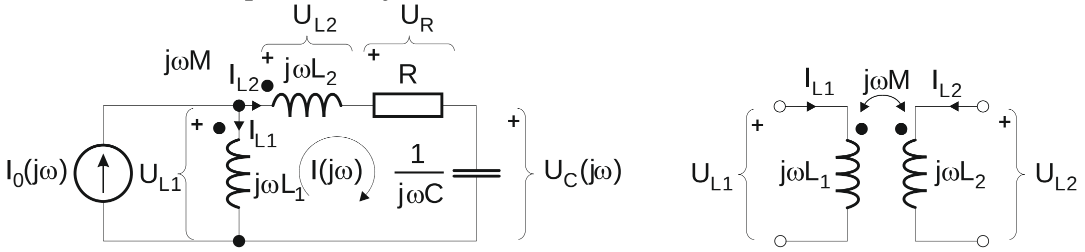
PISMENI ISPIT IZ ELEKTRIČNIH KRUGOVA 2015-2016 – Rješenja

1. Za mrežu prikazanu slikom odrediti valni oblik napona na kapacitetu $u_C(t)$ za $-\infty<t<\infty$, ako se u trenutku t=0 zatvori sklopka K. Zadane su normalizirane vrijednosti elemenata: L_1=2, L_2=2, M=1, C=1, R=1, te pobuda strujnog izvora $i_0(t)$=2sin t za $-\infty<t<\infty$ (stacionarni sinusni signal).



Rješenje:

a) za t<0 izračunavamo početne uvjete:



$$I_{L1}(j\omega)=I_0(j\omega)-I(j\omega)$$
$$I_{L2}(j\omega)=I(j\omega)$$
$$-U_{L1}(j\omega)+U_{L2}(j\omega)+U_R(j\omega)+U_C(j\omega)=0$$

$$U_C(j\omega)=I(j\omega)\cdot 1/(j\omega C); U_R(j\omega)=I(j\omega)\cdot R;$$
$$U_{L1}(j\omega)=j\omega L_1\cdot I_{L1}(j\omega)+j\omega M\cdot I_{L2}(j\omega)$$
$$U_{L2}(j\omega)=j\omega M\cdot I_{L1}(j\omega)+j\omega L_2\cdot I_{L2}(j\omega)$$

Uvrstimo izraze:

$$-j\omega L_1\cdot[I_0(j\omega)-I(j\omega)]-j\omega M\cdot I(j\omega)+j\omega M\cdot[I_0(j\omega)-I(j\omega)]+$$
$$+j\omega L_2\cdot I(j\omega)+R\cdot I(j\omega)+I(j\omega)\cdot 1/(j\omega C)=0$$

$$I_0(j\omega)[j\omega L_1-j\omega M]=I(j\omega)[j\omega L_1-j\omega M-j\omega M+j\omega L_2+R+1/(j\omega C)];$$
$$I_0(j\omega)=2\angle 0^\circ;$$

$$I(j\omega)=\frac{j\omega(L_1-M)}{j\omega(L_1+L_2-2M)+R+1/(j\omega C)}I_0(j\omega)=\frac{j(2-1)}{j(2+2-2)+1-j}I_0(j\omega)=\frac{j}{1+j}I_0(j\omega)=$$
$$=\frac{j}{1+j}\cdot\frac{1-j}{1-j}\cdot I_0(j\omega)=\frac{1}{2}(1+j)\cdot I_0(j\omega)=\frac{1}{2}(1+j)\cdot 2=1+j=\sqrt{2}\cdot e^{j\frac{\pi}{4}}$$

$$I_{L1}(j\omega)=I_0(j\omega)-I(j\omega)=I_0(j\omega)\left(1-\frac{1+j}{2}\right)=I_0(j\omega)\frac{1-j}{2}=2\frac{1-j}{2}=1-j=\sqrt{2}\cdot e^{-j\frac{\pi}{4}}$$

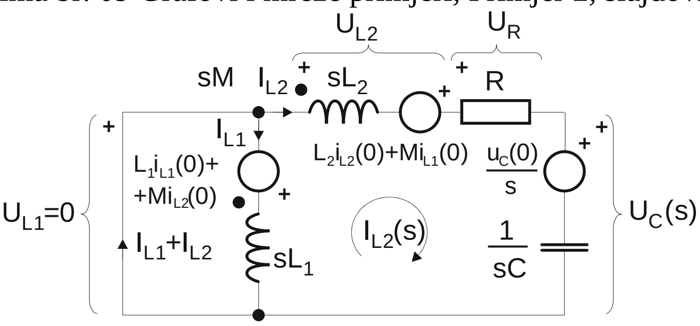
$$I_{L2}(j\omega)=I(j\omega)=1+j=\sqrt{2}\cdot e^{j\frac{\pi}{4}}$$

$$U_C(j\omega)=I(j\omega)\frac{1}{j\omega C}=-j\cdot I(j\omega)=-j\cdot(1+j)=1-j=e^{-j\frac{\pi}{2}}\cdot\sqrt{2}e^{j\frac{\pi}{4}}=\sqrt{2}e^{-j\frac{\pi}{4}}=\sqrt{2}\angle-45^\circ$$

$$i_{L1}(t)=\sqrt{2}\cdot\sin(t-\pi/4) \qquad i_{L1}(0)=\sqrt{2}\cdot\sin(-\pi/4)=\sqrt{2}\cdot\left(-\sqrt{2}/2\right)=-1[\mathrm{A}]$$
$$i_{L2}(t)=\sqrt{2}\cdot\sin(t+\pi/4) \quad\Rightarrow\quad i_{L2}(0)=\sqrt{2}\cdot\sin(\pi/4)=1[\mathrm{A}]$$
$$u_C(t)=\sqrt{2}\cdot\sin(t-\pi/4) \qquad u_C(0)=\sqrt{2}\cdot\sin(-\pi/4)=-1[\mathrm{V}]$$

(2 boda)

b) za $t \geq 0$ primijenimo Laplaceovu transformaciju na slijedeći električni krug koji ima zadane početne uvjete: $i_{L1}(0)=-1$, $i_{L2}(0)= 1$; $u_C(0)=-1$ (vezane induktivitete s početnim uvjetima vidjeti na predavanjima br. 09 Grafovi i mreže primjeri, Primjer 2, slajdovi 32 i 33):



$$sL_1 \cdot I_{L1}(s) + sM \cdot I_{L2}(s) - L_1 i_{L1}(0) - M i_{L2}(0) = 0$$

$$sM \cdot I_{L1}(s) + sL_2 \cdot I_{L2}(s) - L_2 i_{L2}(0) - M i_{L1}(0) + I_{L2}(s) \cdot \left(R + \frac{1}{sC} \right) + \frac{u_C(0)}{s} = 0$$

$$U_C(s) = I_{L2}(s) \cdot \frac{1}{sC} + \frac{u_C(0)}{s}$$

$$sL_1 \cdot I_{L1}(s) + sM \cdot I_{L2}(s) = L_1 i_{L1}(0) + M i_{L2}(0)$$

$$sM \cdot I_{L1}(s) + I_{L2}(s) \cdot \left(sL_2 + R + \frac{1}{sC} \right) = L_2 i_{L2}(0) + M i_{L1}(0) - \frac{u_C(0)}{s}$$

Uz zadane normalizirane vrijednosti elemenata $L_1=2$, $L_2=2$, $M=1$, $C=1$, $R=1$, slijedi:

$$2s \cdot I_{L1}(s) + s \cdot I_{L2}(s) = -2 + 1$$

$$s \cdot I_{L1}(s) + \left(2s + 1 + \frac{1}{s} \right) \cdot I_{L2}(s) = 2 - 1 + \frac{1}{s} \quad \Big/ \cdot 2$$

$$2s \cdot I_{L1}(s) + s \cdot I_{L2}(s) = -2 + 1$$

$$-2s \cdot I_{L1}(s) - 2\left(2s + 1 + \frac{1}{s} \right) \cdot I_{L2}(s) = -2 - \frac{2}{s} \quad \Big/ +$$

$$s \cdot I_{L2}(s) - \left(4s + 2 + \frac{2}{s} \right) \cdot I_{L2}(s) = -1 - 2 - \frac{2}{s}$$

$$\left(s - 4s - 2 - \frac{2}{s} \right) \cdot I_{L2}(s) = -3 - \frac{2}{s} \quad \Rightarrow \quad I_{L2}(s) = \frac{3 + \frac{2}{s}}{3s + 2 + \frac{2}{s}} = \frac{3s+2}{3s^2 + 2s + 2} = \frac{s + \frac{2}{3}}{s^2 + \frac{2}{3}s + \frac{2}{3}}$$

$$U_C(s) = \frac{s + \frac{2}{3}}{s^2 + \frac{2}{3}s + \frac{2}{3}} \cdot \frac{1}{s} - \frac{1}{s} = \frac{s + \frac{2}{3} - s^2 - \frac{2}{3}s - \frac{2}{3}}{s\left(s^2 + \frac{2}{3}s + \frac{2}{3} \right)} = \frac{-s^2 + \frac{1}{3}s}{s\left(s^2 + \frac{2}{3}s + \frac{2}{3} \right)} = -\frac{s - \frac{1}{3}}{s^2 + \frac{2}{3}s + \frac{2}{3}}$$ (2 boda)

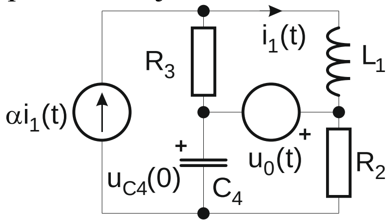
$$s^2 + \frac{2}{3}s + \frac{2}{3} = 0 \Rightarrow s_{p1,2} = -\frac{1}{3} \pm \sqrt{\left(\frac{1}{3}\right)^2 - \frac{2}{3}} = -\frac{1}{3} \pm \sqrt{\frac{1}{9} - \frac{6}{9}} = -\frac{1}{3} \pm j\frac{\sqrt{5}}{3}$$

$$U_C(s) = -\frac{s - \frac{1}{3}}{s^2 + \frac{2}{3}s + \frac{2}{3}} = -\frac{s + \frac{1}{3} - \frac{2}{3}}{\left(s + \frac{1}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2} = -\frac{s + \frac{1}{3}}{\left(s + \frac{1}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2} + \frac{\frac{2}{3} \cdot \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{3}}{\left(s + \frac{1}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2}$$

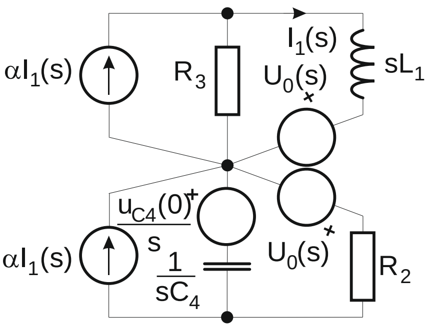
$$u_C(t) = e^{-\frac{1}{3}t}\left(-\cos\frac{\sqrt{5}}{3} + \frac{2}{\sqrt{5}} \cdot \sin\frac{\sqrt{5}}{3} \right) S(t) = e^{-0{,}3333t}\left(-\cos 0{,}74536 + 0{,}89443 \cdot \sin 0{,}74536 \right) \cdot S(t)$$

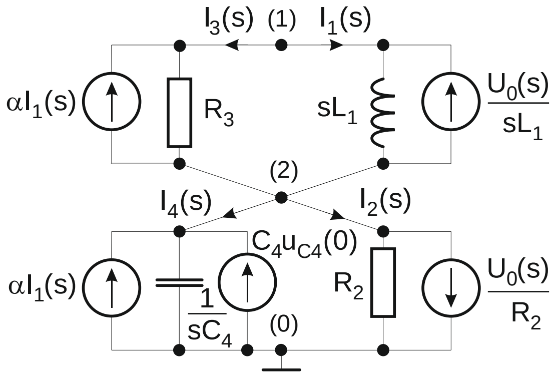
(1 bod)

2. Zadan je električni krug prema slici. Nacrtati pripadni orijentirani graf i napisati matricu incidencija **A**. Napisati strujno-naponske jednadžbe grana u matričnom obliku te ispisati matricu admitancija grana $\mathbf{Y}_b$ i vektor početnih uvjeta i nezavisnih strujnih izvora grana $\mathbf{I}_{0b}$. Matrica $\mathbf{Y}_b$ mora biti regularna. Napisati sustav jednadžbi čvorišta, odnosno odrediti matrice admitancija čvorova $\mathbf{Y}_v$ i vektor početnih uvjeta i nezavisnih izvora čvorova $\mathbf{I}_{0v}$.



Rješenje: Posmicanje strujnog i naponskog izvora i primjena Laplaceove transformacije. Pretvaranje svih izvora u strujne.





(1 bod)

Strujno-naponske jednadžbe grana (struje izražene pomoću napona):

$$I_1 = U_1\frac{1}{sL_1} - U_0\frac{1}{sL_1}$$

$$I_2 = U_2\frac{1}{R_2} + U_0\frac{1}{R_2}$$

$$I_3 = U_3\frac{1}{R_3} - \alpha\cdot I_1 = U_3\frac{1}{R_3} - \alpha\cdot\left(U_1\frac{1}{sL_1} - U_0\frac{1}{sL_1}\right) =$$

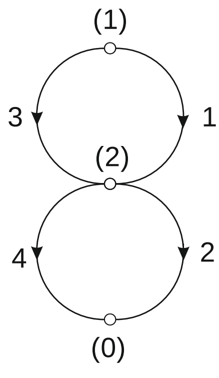
$$= -U_1\frac{\alpha}{sL_1} + U_3\frac{1}{R_3} + U_0\frac{\alpha}{sL_1}$$

$$I_4 = sC_4\cdot U_4 - I_1\cdot\alpha - C_4u_{C4}(0) =$$

$$= sC_4\cdot U_4 - \alpha\cdot\left(U_1\frac{1}{sL_1} - U_0\frac{1}{sL_1}\right) - C_4u_{C4}(0) =$$

$$= -\frac{\alpha}{sL_1}\cdot U_1 + sC_4\cdot U_4 + U_0\frac{\alpha}{sL_1} - C_4u_{C4}(0)$$

Orijentirani graf:



Matrica incidencija (nereducirana): $\mathbf{A}_a = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 1 & -1 & 1 \\ 0 & -1 & 0 & -1 \end{bmatrix}$ ili

Matrica incidencija (reducirana): $\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 1 & -1 & 1 \end{bmatrix}$ (1 bod)

Naponsko-strujne relacije grana u matričnom obliku: $\mathbf{I}_b = \mathbf{Y}_b \cdot \mathbf{U}_b + \mathbf{I}_{0b}$

$$\underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{\mathbf{I}_b} = \underbrace{\begin{bmatrix} \frac{1}{sL_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ -\frac{\alpha}{sL_1} & 0 & \frac{1}{R_3} & 0 \\ -\frac{\alpha}{sL_1} & 0 & 0 & sC_4 \end{bmatrix}}_{\mathbf{Y}_b} \underbrace{\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix}}_{\mathbf{U}_b} + \underbrace{\begin{bmatrix} -U_0 \frac{1}{sL_1} \\ U_0 \frac{1}{R_2} \\ U_0 \frac{\alpha}{sL_1} \\ U_0 \frac{\alpha}{sL_1} - C_4 u_{C4}(0) \end{bmatrix}}_{\mathbf{I}_{0b}}$$ (1 bod)

Matrica $\mathbf{Y}_b$ je regularna. Sustav jednadžbi napona čvorova u matričnom obliku $\mathbf{Y}_v \cdot \mathbf{U}_v = \mathbf{I}_{0v}$, gdje su (matrice $\mathbf{Y}_v$ i $\mathbf{I}_{0v}$):

$$\mathbf{Y}_v = \mathbf{A} \cdot \mathbf{Y}_b \cdot \mathbf{A}^T = \begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{sL_1} & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & 0 & 0 \\ -\frac{\alpha}{sL_1} & 0 & \frac{1}{R_3} & 0 \\ -\frac{\alpha}{sL_1} & 0 & 0 & sC_4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{1}{sL_1} - \frac{\alpha}{sL_1} & 0 & \frac{1}{R_3} & 0 \\ -\frac{1}{sL_1} & \frac{1}{R_2} & -\frac{1}{R_3} & sC_4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1-\alpha}{sL_1} + \frac{1}{R_3} & -\frac{1-\alpha}{sL_1} - \frac{1}{R_3} \\ -\frac{1}{sL_1} - \frac{1}{R_3} & \frac{1}{sL_1} + \frac{1}{R_2} + \frac{1}{R_3} + sC_4 \end{bmatrix}$$

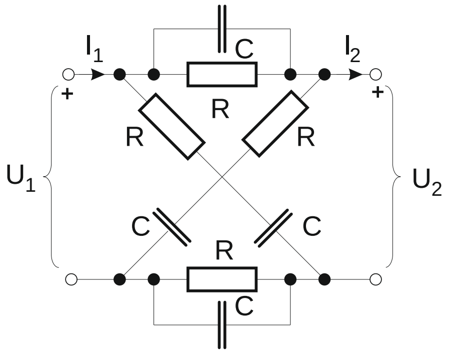
(1 bod)

$$\mathbf{I}_{0v} = -\mathbf{A} \cdot \mathbf{I}_{0b} = -\begin{bmatrix} 1 & 0 & 1 & 0 \\ -1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -U_0 \frac{1}{sL_1} \\ U_0 \frac{1}{R_2} \\ U_0 \frac{\alpha}{sL_1} \\ U_0 \frac{\alpha}{sL_1} - C_4 u_{C4}(0) \end{bmatrix} = \begin{bmatrix} U_0 \frac{1-\alpha}{sL_1} \\ -U_0 \frac{1}{sL_1} - U_0 \frac{1}{R_2} + C_4 u_{C4}(0) \end{bmatrix}$$

(1 bod)

Rješenje: $\mathbf{Y}_v \cdot \mathbf{U}_v = \mathbf{I}_{0v} \;\Rightarrow\; \mathbf{U}_v = \begin{bmatrix} U_{v1} \\ U_{v2} \end{bmatrix}$

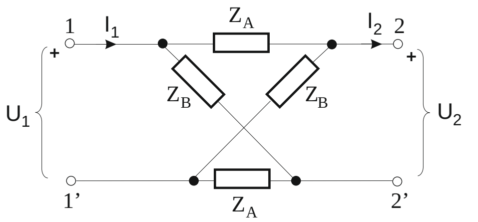
3. Za X-četveropol prikazan slikom izračunati a) prijenosne *a*-parametre, ako su zadane normalizirane vrijednosti elemenata R=1, C=1. b) Iz prijenosnih *a*-parametara izračunati *z*-parametre. c) četveropolu pridružiti ekvivalentni T-četveropol. d) Koliko iznose zrcalni parametri četveropola Z_{C1}, Z_{C2} i g ? e) Da li je četveropol: recipročan, simetričan ? Obrazložiti odgovore.

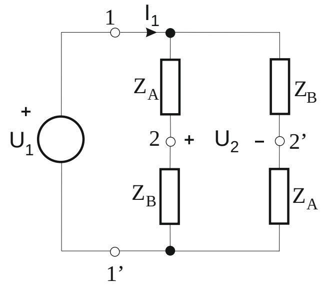


Rješenje:

a) a - parametri $\begin{array}{l} U_1 = A\cdot U_2 + B\cdot I_2 \\ I_1 = C\cdot U_2 + D\cdot I_2 \end{array} \Rightarrow$

$$\underline{I_2 = 0} \quad A = \left.\frac{U_1}{U_2}\right|_{I_2=0} \qquad C = \left.\frac{I_1}{U_2}\right|_{I_2=0}$$





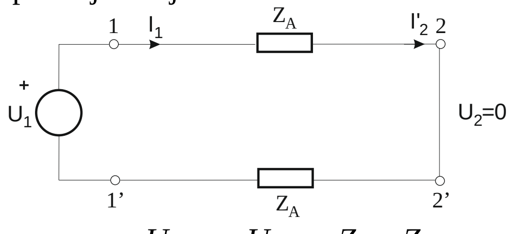
$$I_1 = \frac{U_1}{(Z_A + Z_B)/2} = \frac{2\cdot U_1}{Z_A + Z_B}; \Rightarrow U_1 = I_1 \cdot \frac{Z_A + Z_B}{2}$$

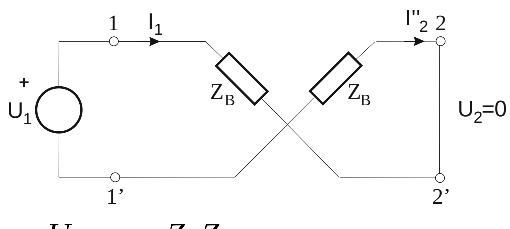
$$U_2 = U_1 \cdot \frac{Z_B}{Z_A + Z_B} - U_1 \cdot \frac{Z_A}{Z_A + Z_B} = U_1 \cdot \frac{Z_B - Z_A}{Z_A + Z_B} \quad \Rightarrow \quad A = \frac{U_1}{U_2} = \frac{Z_A + Z_B}{Z_B - Z_A}$$

$$\frac{U_1}{U_2} = \frac{I_1 \cdot (Z_A + Z_B)/2}{U_2} = \frac{Z_A + Z_B}{Z_B - Z_A} \Big/ : \frac{Z_A + Z_B}{2} \quad \Rightarrow \quad C = \frac{I_1}{U_2} = \frac{2}{Z_B - Z_A}$$

$$\underline{U_2 = 0} \quad B = \left.\frac{U_1}{I_2}\right|_{U_2=0} \qquad D = \left.\frac{I_1}{I_2}\right|_{U_2=0}$$

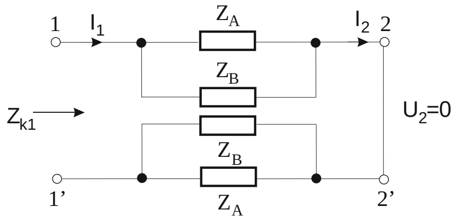
Superpozicija struja:





$$I_2 = I'_2 + I''_2 = \frac{U_1}{2\cdot Z_A} - \frac{U_1}{2\cdot Z_B} = \frac{Z_B - Z_A}{2\cdot Z_A Z_B}\cdot U_1 \quad \Rightarrow \quad B = \frac{U_1}{I_2} = 2\cdot\frac{Z_A Z_B}{Z_B - Z_A}$$

Ulazna impedancija na kratko Z_{k1}:



$$Z_{ul1} = Z_{k1} = 2\frac{Z_A Z_B}{Z_A + Z_B}; \quad U_1 = I_1 \cdot Z_{ul1} = I_1 \cdot 2\frac{Z_A Z_B}{Z_A + Z_B}$$

$$\frac{U_1}{I_2} = \frac{I_1 \cdot 2 \cdot Z_A Z_B / (Z_A + Z_B)}{I_2} = 2 \cdot \frac{Z_A Z_B}{Z_B - Z_A} \Big/ : \left(2 \cdot \frac{Z_A Z_B}{Z_A + Z_B}\right) \quad \Rightarrow \quad D = \frac{I_1}{I_2} = \frac{Z_A + Z_B}{Z_B - Z_A}$$

$$[a] = \frac{1}{Z_B - Z_A} \cdot \begin{bmatrix} Z_A + Z_B & 2Z_A Z_B \\ 2 & Z_A + Z_B \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Uz vrijednosti elemenata: $Z_A = \frac{R \cdot 1/(sC)}{R + 1/(sC)} = \frac{R}{1 + sRC} = \frac{1}{1+s}$; $Z_B = R + \frac{1}{sC} = 1 + \frac{1}{s} = \frac{s+1}{s}$

$$Z_A + Z_B = \frac{1}{1+s} + \frac{s+1}{s} = \frac{s^2+3s+1}{s(s+1)}; \quad Z_B - Z_A = \frac{s+1}{s} - \frac{1}{1+s} = \frac{(s+1)^2 - s}{s(1+s)} = \frac{s^2+s+1}{s(1+s)}$$

$$[a] = \frac{s(s+1)}{s^2+s+1} \cdot \begin{bmatrix} \frac{s^2+3s+1}{s(s+1)} & \frac{2}{s} \\ 2 & \frac{s^2+3s+1}{s(s+1)} \end{bmatrix} = \begin{bmatrix} \frac{s^2+3s+1}{s^2+s+1} & \frac{2(s+1)}{s^2+s+1} \\ \frac{2s(s+1)}{s^2+s+1} & \frac{s^2+3s+1}{s^2+s+1} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \text{ (1 bod)}$$

b) Slijede z-parametri iz a-parametara: (1 bod)

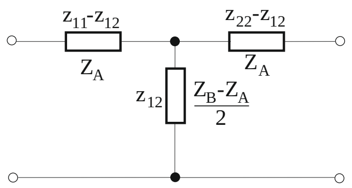
$$z_{11} = \frac{A}{C}; \; z_{12} = \frac{AD - BC}{C} = \frac{1}{C}; \; z_{21} = \frac{1}{C}; \; z_{22} = \frac{D}{C}; \; \det(a) = 1 \Rightarrow z_{12} = z_{21}; \; A = D \Rightarrow z_{11} = z_{22};$$

$$z_{11} = z_{22} = \frac{A}{C} = \frac{Z_A + Z_B}{2}; \; z_{12} = z_{21} = \frac{1}{C} = \frac{Z_B - Z_A}{2}$$

$$[z] = \frac{1}{2} \cdot \begin{bmatrix} Z_A + Z_B & -(Z_B - Z_A) \\ Z_B - Z_A & -(Z_A + Z_B) \end{bmatrix} = \frac{1}{2s(s+1)} \cdot \begin{bmatrix} s^2+3s+1 & -(s^2+s+1) \\ s^2+s+1 & -(s^2+3s+1) \end{bmatrix} = \begin{bmatrix} z_{11} & -z_{12} \\ z_{21} & -z_{22} \end{bmatrix}$$

c) Nadomjesni T – spoj: (1 bod)

$$z_{11} - z_{12} = z_{22} - z_{12} = \frac{Z_A + Z_B}{2} - \frac{Z_B - Z_A}{2} = Z_A = \frac{1}{1+s}; \quad z_{12} = z_{21} = \frac{Z_B - Z_A}{2} = \frac{s^2+s+1}{2s(s+1)}$$



d) Zrcalni parametri: Z_C, g: (1 bod)

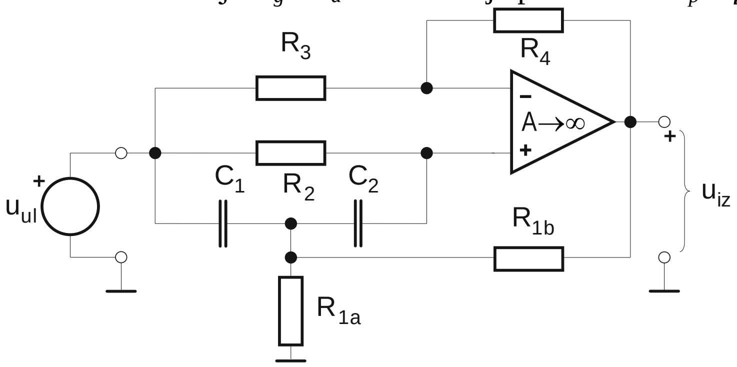
$$Z_{p1} = z_{11} = \frac{Z_A + Z_B}{2}; \quad Z_{k1} = \frac{1}{y_{11}} = 2\frac{Z_A Z_B}{Z_A + Z_B}$$

$$Z_C = Z_{C1} = Z_{C2} = \sqrt{Z_{k1} \cdot Z_{p1}} = \sqrt{Z_{k2} \cdot Z_{p2}} = \sqrt{Z_A \cdot Z_B} = \sqrt{1/(s+1) \cdot (s+1)/s} = 1/\sqrt{s};$$

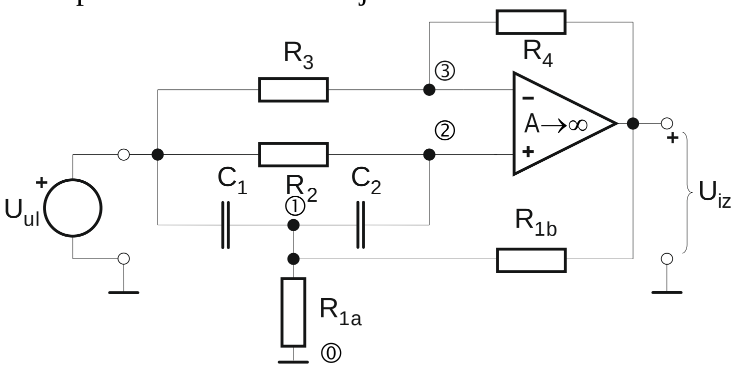
$$th\, g = \sqrt{\frac{Z_{k1}}{Z_{p1}}} = \frac{2\sqrt{Z_A \cdot Z_B}}{Z_A + Z_B} = \frac{2\sqrt{1/(s+1) \cdot (s+1)/s}}{1/(s+1) + (s+1)/s} = \frac{2}{\sqrt{s}} \cdot \frac{s(s+1)}{s + (s+1)^2} = 2\sqrt{s} \cdot \frac{s+1}{s^2+3s+1}$$

e) Očigledno je da vrijedi: A=D , det(a)=AD–CD=1, odn. z_{11}=z_{22} i z_{12}=z_{21}; mreža je simetrična i recipročna. Također vrijedi: Z_C=Z_{C1}=Z_{C2} (simetričnost). (1 bod)

4. Zadan je aktivni-RC električni filtar prikazan slikom s normaliziranim vrijednostima elemenata C_1=C_2=1; R_{1a}=3; R_{1b}=1,5; R_2=1, te R_3=1; R_4=2. Odrediti: a) njegovu naponsku prijenosnu funkciju $T(s)=U_{iz}(s)/U_{ul}(s)$. O kojem se tipu filtra radi (NP, VP, PP ili PB)? b) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja izračunati vrijednost parametara k, ω_p, ω_z, q_p. c) Prikazati raspored polova i nula u kompleksnoj ravnini. d) Nacrtati amplitudno-frekvencijsku karakteristiku. e) Koliko iznose širina pojasa propuštanja/gušenja B, te gornja i donja granična frekvencija ω_g i ω_d kao funkcije parametara ω_p i q_p ?



Rješenje: Primjenom Laplaceove transformacije**:**



Metoda napona čvorišta:

$$(1)\ U_1\left(\frac{1}{R_1}+sC_1+sC_2\right)-U_2sC_2=U_{iz}\frac{1}{R_{1b}}+U_{ul}sC_1 \Big/ \cdot R_1$$

$$(2)\ -U_1sC_2+U_2\left(\frac{1}{R_2}+sC_2\right)=U_{ul}\frac{1}{R_2} \Big/ : sC_2$$

$$(3)\ U_3\left(\frac{1}{R_3}+\frac{1}{R_4}\right)=U_{ul}\frac{1}{R_3}+U_{iz}\frac{1}{R_4} \Big/ \cdot R_3R_4$$

$$(3)\ \underline{A(U_2-U_3)=U_{iz}\ \Rightarrow\ U_2=U_3\ \ (A\to\infty)}$$

$$(1)\ U_1(1+sR_1C_1+sR_1C_2)-U_2sR_1C_2=U_{iz}\frac{R_1}{R_{1b}}+U_{ul}sR_1C_1$$

$$(2)\ -U_1+U_2\left(\frac{1}{sR_2C_2}+1\right)=U_{ul}\frac{1}{sR_2C_2}$$

$$(3)\ \underline{U_2(R_3+R_4)=U_{ul}R_4+U_{iz}R_3}$$

Uz uvrštene vrijednosti elemenata C_1=C_2=1; R_{1a}=3; R_{1b}=3/2=1,5; R_2=1, te R_3=1; R_4=2;

$$R_1=\frac{R_{1a}R_{1b}}{R_{1a}+R_{1b}}=\frac{3\cdot 3/2}{3+3/2}=1$$

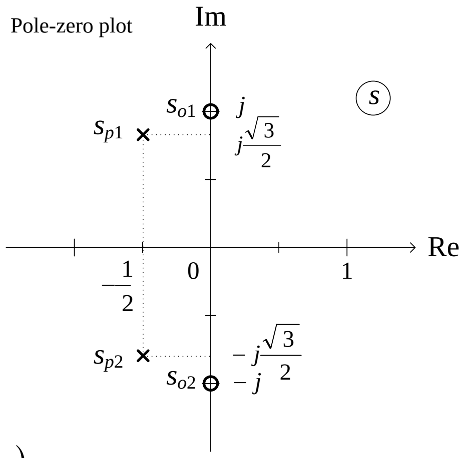
$$(1)\ U_1(1+s+s)-U_2 s=U_{iz}\frac{2}{3}+U_{ul}s$$

$$(2)\ -U_1+U_2\left(\frac{1}{s}+1\right)=U_{ul}\frac{1}{s}$$

$$(3)\ U_2(1+2)=2U_{ul}+1U_{iz}$$

$$(2)\Rightarrow U_1=-U_{ul}\frac{1}{s}+U_2\left(\frac{1}{s}+1\right)$$

$$(3)\Rightarrow U_2=2/3U_{ul}+1/3U_{iz}$$



Malo računanja: (2), (3) $\rightarrow$ (1) $\Rightarrow$

$$\left[U_2\left(\frac{1}{s}+1\right)-U_{ul}\frac{1}{s}\right](1+2s)-U_2 s=U_{iz}\frac{2}{3}+U_{ul}s \Big/ \cdot s$$

$$[U_2(s+1)-U_{ul}](1+2s)-U_2 s^2=2/3U_{iz}s+U_{ul}s^2$$

$$U_2(s+1)(1+2s)-U_2 s^2=2/3U_{iz}s+U_{ul}s^2+U_{ul}(1+2s)$$

$$U_2(s^2+3s+1)=2/3U_{iz}s+U_{ul}(s^2+2s+1)$$

$$[2/3U_{ul}+1/3U_{iz}](s^2+3s+1)=2/3U_{iz}s+U_{ul}(s^2+2s+1)$$

$$U_{iz}\left[s^2+2s+s\left(1-\frac{2/3}{1/3}\right)+1\right]=U_{ul}\left[s^2+2s+s(1-3)+1\right] \Rightarrow T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=\frac{1+s^2}{1+s+s^2}$$

b) Usporedbom s odgovarajućim općim oblikom prijenosne funkcije filtra 2. stupnja izračunati parametre k, ω_p, ω_z, q_p.

$$T(s)=\frac{U_{iz}(s)}{U_{ul}(s)}=k\cdot\frac{s^2+\omega_z^2}{s^2+(\omega_p/q_p)\cdot s+\omega_p^2}$$

$$\Rightarrow\quad \omega_p=\omega_z=1,\quad q_p=1,\quad k=1$$

-o kojem se tipu filtra radi (NP, VP, PP ili PB)?

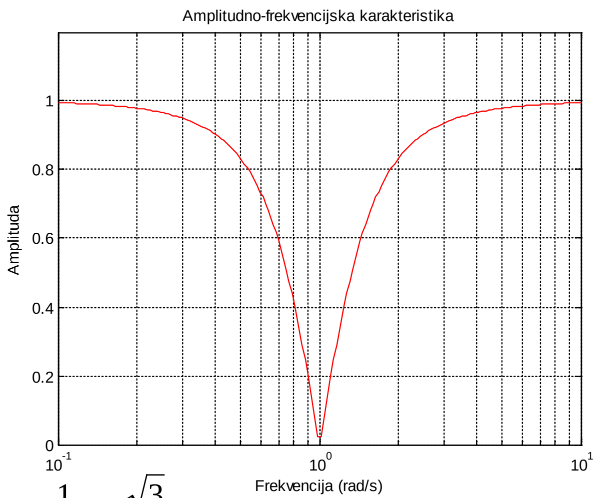
$\Rightarrow$ PB

a) + b) (3 boda)

c) raspored polova i nula u kompleksnoj ravnini:

$$T(s)=\frac{s^2+1}{s^2+s+1}$$

- nule: $s^2+1=0 \quad\Rightarrow\quad s_{o1,2}=\pm j$



- polovi: $s^2+s+1=0 \quad\Rightarrow\quad s_{p1,2}=-\frac{1}{2}\pm\sqrt{\frac{1}{4}-1}=-\frac{1}{2}\pm j\frac{\sqrt{3}}{2}=-0{,}5\pm j0{,}866$

d) amplitudno-frekvencijska karakteristika:

$$s=j\omega \Rightarrow T(j\omega)=\frac{-\omega^2+1}{-\omega^2+j\omega+1} \quad\Rightarrow\quad |T(j\omega)|=\frac{|1-\omega^2|}{\sqrt{(1-\omega^2)^2+(\omega)^2}}=\frac{|1-\omega^2|}{\sqrt{1-\omega^2+\omega^4}}$$

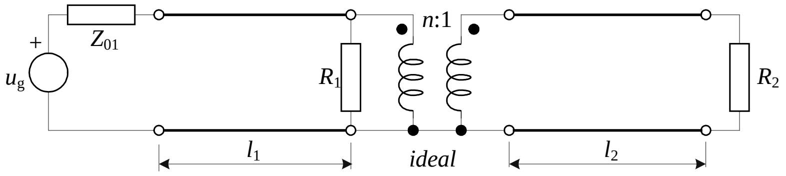
e) Širina pojasa gušenja $B=\omega_p/q_p=1$ [rad/s]

Gornja i donja granična frekvencija pojasa gušenja: $\omega_{g,d}=\omega_p\sqrt{1+\frac{1}{4q_p^2}}\pm\frac{\omega_p}{2q_p}=1\sqrt{1+\frac{1}{4}}\pm\frac{1}{2}=$

$$=\frac{\sqrt{5}}{2}\pm\frac{1}{2};\ \omega_g=\frac{\sqrt{5}+1}{2}=1{,}618;\ \omega_d=\frac{\sqrt{5}-1}{2}=0{,}618\,\text{[rad/s]}\ \Rightarrow B=\omega_g-\omega_d=1\ \text{[rad/s]}$$

c) + d) + e) (2 boda)

5. Dvije linije bez gubitaka i idealni transformator spojeni su u kaskadu prema slici. Zadano je: L_1=0,45mH/km i C_1=80nF/km, L_2=0,2mH/km, C_2=80nF/km, u_g=10 cos(2,5π 10^5 t) V, R_1 =300Ω, R_2=100Ω, l_1=3λ_1/4, l_2=λ_2/4. Odrediti: a) valne impedancije Z_{01} i Z_{02}, te koeficijente prijenosa γ_1 i γ_2 linija; b) brzine širenja vala na linijama i duljine l_1 i l_2 linija; c) omjer transformacije n da bi prva linija bila prilagođena na izlazu; d) faktor refleksije na kraju druge Γ_{i2}; i napon na kraju prve linije $u_1(l_1,t)$; e) napone na početku i na kraju druge linije: $u_2(0,t)$, $u_2(l_2,t)$.



Rješenje:

a) $Z_{01}=\sqrt{\dfrac{L_1}{C_1}}=\sqrt{\dfrac{4{,}5\cdot10^{-4}}{8\cdot10^{-8}}}=75\Omega$ $\qquad Z_{02}=\sqrt{\dfrac{L_2}{C_2}}=\sqrt{\dfrac{2\cdot10^{-4}}{8\cdot10^{-8}}}=50\Omega$ (1 bod)

$$\gamma_1=j\beta_1=j\omega_0\sqrt{L_1C_1}=j2{,}5\cdot\pi\cdot10^5\sqrt{4{,}5\cdot10^{-2}\cdot8\cdot10^{-8}}=j2{,}5\cdot\pi\cdot10^5\sqrt{36\cdot10^{-12}}=j1{,}5\pi\ \text{[rad/km]}$$

$$\gamma_2=j\beta_2=j\omega_0\sqrt{L_2C_2}=j2{,}5\cdot\pi\cdot10^5\sqrt{2\cdot10^{-4}\cdot8\cdot10^{-8}}=j\pi\ \text{[rad/km]}$$

b) $v_1=\dfrac{\omega}{\beta_1}=\dfrac{1}{\sqrt{L_1C_1}}=\dfrac{1}{\sqrt{36\cdot10^{-12}}}=166{,}7\cdot10^3\ \text{[km/s]}$

$$v_2=\frac{\omega}{\beta_2}=\frac{1}{\sqrt{L_2C_2}}=\frac{1}{\sqrt{16\cdot10^{-12}}}=250\cdot10^3\ \text{[km/s]}$$

$\lambda_1=\dfrac{2\cdot\pi}{\beta_1}=\dfrac{2\cdot\pi}{1{,}5\cdot\pi}=\dfrac{4}{3}=1{,}33\,\text{km}$ $\quad l_1=\dfrac{3\lambda_1}{4}=1\ \text{[km]};$ $\quad \lambda_2=\dfrac{2\cdot\pi}{\beta_2}=\dfrac{2\cdot\pi}{\pi}=2\,\text{km}$ $\quad l_2=\dfrac{\lambda_2}{4}=0{,}5$ [km]

(1 bod)

c) $\gamma_1\cdot l_1=j\cdot\beta_1\cdot l_1=j\dfrac{3\pi}{2}$ $\qquad \gamma_2\cdot l_2=j\cdot\beta_2\cdot l_2=j\dfrac{\pi}{2}$

$$Z_{ul2}=\frac{R_2ch(\gamma_2\cdot l_2)+Z_{02}sh(\gamma_2\cdot l_2)}{\dfrac{R_2}{Z_{02}}sh(\gamma_2\cdot l_2)+ch(\gamma_2\cdot l_2)}=\frac{R_2\cos(\beta_2\cdot l_2)+jZ_{02}\sin(\beta_2\cdot l_2)}{j\dfrac{R_2}{Z_{02}}\sin(\beta_2\cdot l_2)+\cos(\beta_2\cdot l_2)}=\frac{Z_{02}^2}{R_2}=\frac{2500}{100}=25\,\Omega$$

$$Z_{ulT}=n^2Z_{ul2}=n^2 25\,\Omega\quad \frac{R_1\cdot Z_{ulT}}{R_1+Z_{ulT}}=\frac{R_1\cdot n^2\cdot Z_{ul2}}{R_1+n^2\cdot Z_{ul2}}=75\Omega\quad\Rightarrow n^2=\frac{75\cdot R_1}{(R_1-75)\cdot Z_{ul2}}=\frac{75\cdot300}{(300-75)\cdot25}=4;$$

$n=2$ (1 bod)

d) $\Gamma_{i2}=\dfrac{R_2-Z_{02}}{R_2+Z_{02}}=\dfrac{50}{150}=\dfrac{1}{3}$

$U_1(l_1)=U(0)\cdot e^{-j\beta_1 l_1}=5\cdot e^{-j3\pi/2}=5j$ $\qquad u_1(l_1,t)=5\cos(\omega t+\pi/2)$

(1 bod)

e)

$U_2(0)=U_1(l_1)/n=2{,}5\cdot e^{-j3\pi/2}=2{,}5j$ $\qquad u_2(0,t)=2{,}5\cos(\omega t+\pi/2)$

$$U_2(l_2)=U_2(0)\cdot\cos(\beta_2\cdot l_2)-jU_2(0)\cdot\frac{Z_{02}}{Z_{ul2}}\sin(\beta_2\cdot l_2)=-jU_2(0)\cdot\frac{50}{25}=-j\cdot2{,}5\cdot j\cdot2=5$$

$u_2(l_2,t)=5\cdot\cos(\omega t)$

(1 bod)