ELEKTRIČNI KRUGOVI — Auditorne vježbe

1. Zadana je prijenosna funkcija $T(s)=U_{iz}(s)/U_{ul}(s)$ električnog filtra. Nacrtati raspored nula i polova u kompleksnoj s-ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$. Izračunati vrijednost faktora dobrote q_p, frekvencije ω_p i pojačanja u području propuštanja K. O kakvoj se vrsti filtra radi (NP, VP, PP ili PB)?

$$T(s)=\frac{0.6s^2}{s^2+1.2s+0.4}$$

Rješenje: izjednačimo zadanu prijenosnu funkciju sa općim oblikom prijenosne funkcije:

$$T(s)=\frac{0.6s^2}{s^2+1.2s+0.4}=\frac{K\cdot s^2}{s^2+\frac{\omega_p}{q_p}s+\omega_p^2}$$

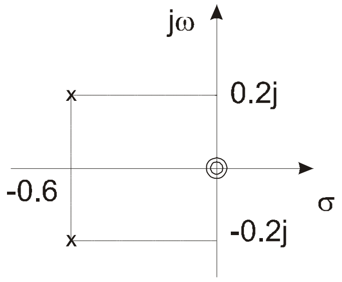
Odatle slijede parametri filtra:

$$\omega_p=\sqrt{0.4}\cong 0.63,\quad q_p=\frac{\omega_p}{2\sigma_p}=\frac{\sqrt{0.4}}{1.2}\cong 0.53,\quad K=0.6$$

Zatim slijede polovi: $s^2+1.2s+0.4=0 \Rightarrow s_{p1,2}=\dfrac{-1.2\pm\sqrt{1.2^2-4\cdot 0.4}}{2}=-0.6\pm 0.2j$

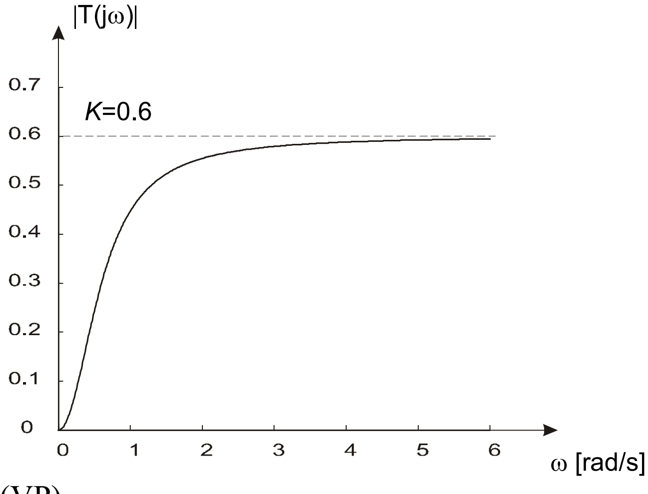
i nule: $s^2=0 \Rightarrow s_{01,2}=0$ dvostruka nula u ishodištu

raspored nula i polova u s-ravnini:



amplitudno-frekvencijska karakteristika slijedi ako uvrstimo s=jω

$$|T(j\omega)|=\left|\frac{0.6\omega^2}{-\omega^2+1.2j\omega+0.4}\right|=\frac{0.6\omega^2}{\sqrt{(0.4-\omega^2)^2+(1.2\omega)^2}}$$



ovo je visoki propust (VP)

2. Zadana je prijenosna funkcija $T(s)=U_{iz}(s)/U_{ul}(s)$ električnog filtra. Izračunati vrijednost faktora dobrote q_p, frekvencije ω_p i pojačanja u području propuštanja K. O kakvoj se vrsti filtra radi (NP, VP, PP ili PB)? Koliko iznosi širina pojasa propuštanja filtra B te gornja i donja granična frekvencija f_g i f_d ? U kakvoj vezi su granične frekvencije sa centralnom frekvencijom ? Nacrtati raspored nula i polova u kompleksnoj s-ravnini i amplitudno-frekvencijsku karakteristiku funkcije $T(s)$.

$$T(s)=\frac{31.4159\cdot s}{s^2+31.4159\cdot s+98696}$$

Rješenje: izjednačimo zadanu prijenosnu funkciju sa općim oblikom prijenosne funkcije:

$$T(s)=\frac{31.4159\cdot s}{s^2+31.4159\cdot s+98696}=\frac{K\cdot\dfrac{\omega_p}{q_p}\cdot s}{s^2+\dfrac{\omega_p}{q_p}s+\omega_p^2}$$

Odatle slijede parametri filtra:

$$\omega_p=\sqrt{98696}=314.159=2\pi\cdot 50\,[\text{rad/s}]\Rightarrow f_p=50[\text{Hz}]$$

$$q_p=\frac{\omega_p}{2\sigma_p}=\frac{314.159}{31.4159}=10$$

$$K=1$$

(gdje je $2\sigma_p$ član koji množi s u nazivniku prijenosne funkcije)

Ovo je pojasni propust (PP)

Širina pojasa propuštanja : $B=\dfrac{\omega_p}{q_p}=\dfrac{314.159}{10}=31.4159\ [\text{rad/s}]$

Gornja i donja granična frekvencija su :

$$\omega_{g,d}=\omega_p\sqrt{1+\frac{1}{4q_p^2}}\pm\frac{\omega_p}{2q_p}=314.159\sqrt{1+\frac{1}{4\cdot 100}}\pm\frac{314.159}{2\cdot 10}=314.551\pm 15.708\ [\text{rad/s}]$$

ω_g=330.259 [rad/s], ω_d=298.844 [rad/s] ili

$f_g=\omega_g/2\pi$=330.259/2π=52.5624 [Hz], $f_d=\omega_d/2\pi$=298.844 /2π=47.5625 [Hz],

$B=\omega_g-\omega_d$=330.259–298.844=31.415 [rad/s] ili

f_g-f_d=31.415/2π=5 [Hz]

centralna frekvencija $\omega_0=\omega_p=314.159=2\pi\cdot 50\,[\text{rad/s}]$ ili

f_0=50 [Hz]

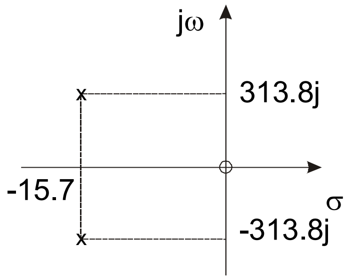
$\omega_0^2=\omega_d\cdot\omega_g\rightarrow$ ω_0 je geometrijska sredina od ω_d i ω_g ili

$f_0^2=f_d\cdot f_g\rightarrow$ f_0 je geometrijska sredina od f_d i f_g

Zatim slijede polovi: $s^2+31.4159\cdot s+98696=0\ \Rightarrow$

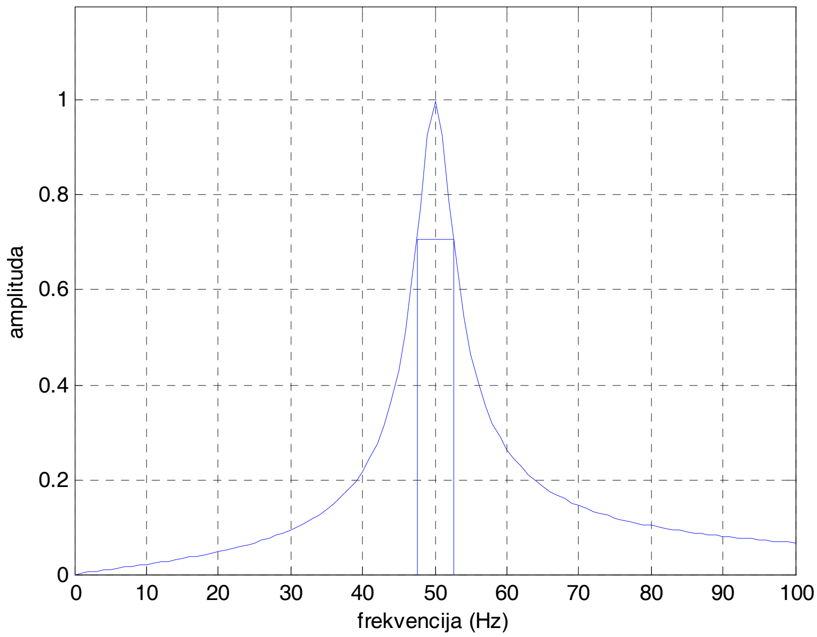
$$s_{p1,2}=\frac{-31.4159\pm\sqrt{31.4159^2-4\cdot 98696}}{2}=-15.708\pm 313.766j$$

i nule: $s=0\ \Rightarrow\ s_{01}=0,\ s_{02}=\infty$ jedna nula u ishodištu, druga u beskonačnosti raspored nula i polova u s-ravnini:



amplitudno-frekvencijska karakteristika slijedi ako uvrstimo $s=j\omega$

$$|T(j\omega)| = \frac{|31.4159 \cdot \omega|}{\sqrt{(98696 - \omega^2)^2 + (31.4159 \cdot \omega)^2}}, \ \omega=2\pi f$$



3. Koliki moraju biti α, β i γ (α, β, γ>0) da bi četveropol zadan prijenosnim parametrima bio recipročan i simetričan. Naći ekvivalentni *T*-spoj.

$$[a]=\begin{bmatrix} \frac{1+\alpha s}{1-2s} & \frac{4}{1-2s} \\ \frac{\beta s}{1-2s} & \frac{\gamma+\alpha s}{1-2s} \end{bmatrix}$$

<u>Rješenje:</u>

Uvjet simetrije: $A=D$

$A=D \Rightarrow \gamma=1$

Uvjet recipročnosti: $\Delta = AD-BC=1 \;\; (\Delta=\det[a])$

$$AD-BC=1 \quad\Rightarrow\quad \frac{(1+\alpha s)^2}{(1-2s)^2}-\frac{4\beta s}{(1-2s)^2}=1$$

$$(1+\alpha s)^2-4\beta s=(1-2s)^2$$

$$1+2\alpha s+\alpha^2 s^2-4\beta s=1-4s+4s^2$$

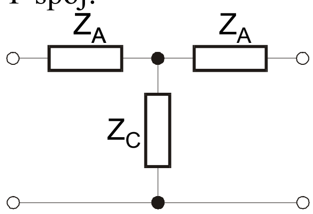
$$1+(2\alpha-4\beta)s+\alpha^2 s^2=1-4s+4s^2$$

$$\alpha^2=4 \qquad\Rightarrow\qquad \alpha=2$$

$$\underline{2\alpha-4\beta=-4} \quad\Rightarrow\quad 4-4\beta=-4 \quad\Rightarrow\quad \beta=\frac{8}{4}=2$$

$$[a]=\begin{bmatrix} \frac{1+2s}{1-2s} & \frac{4}{1-2s} \\ \frac{2s}{1-2s} & \frac{1+2s}{1-2s} \end{bmatrix}=\begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

T-spoj:



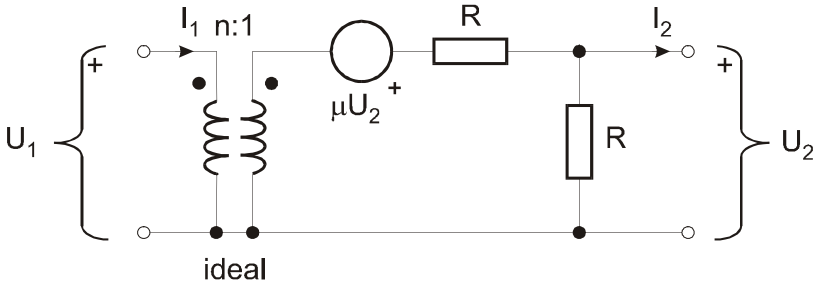
$$z_{11}=\frac{A}{C}=\frac{1+2s}{2s}$$

$$z_{12}=\frac{AD}{C}-B=\frac{(1+2s)^2}{2s(1-2s)}-\frac{4}{1-2s}=\frac{1-2s}{2s}$$

$$Z_A=z_{11}-z_{12}=2$$

$$Z_C=z_{12}=\frac{1-2s}{2s}=\frac{1}{2s}-1$$

4. Za četveropol prikazan slikom izračunati $[y]$ parametre. Odrediti vrijednost za n kako bi četveropol bio simetričan.



Rješenje:

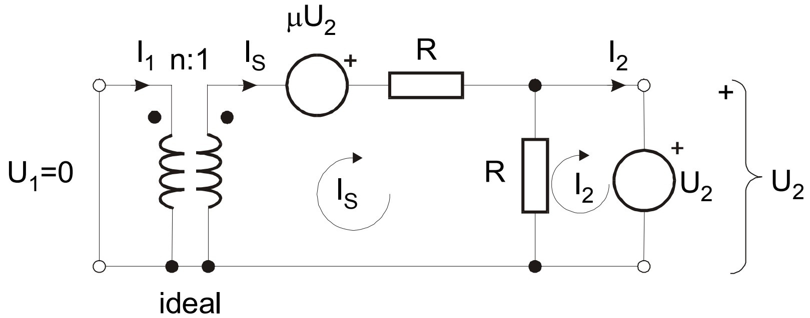
$$I_1 = y_{11} \cdot U_1 - y_{12} \cdot U_2$$
$$I_2 = y_{21} \cdot U_1 - y_{22} \cdot U_2$$

$$y_{11} = \left.\frac{I_1}{U_1}\right|_{U_2=0} \quad y_{21} = \left.\frac{I_2}{U_1}\right|_{U_2=0} \quad y_{12} = -\left.\frac{I_1}{U_2}\right|_{U_1=0} \quad y_{22} = -\left.\frac{I_2}{U_2}\right|_{U_1=0}$$

jednadžbe transformatora :

$$U_S = \frac{1}{n} \cdot U_1$$
$$\underline{I_S = n \cdot I_1}$$

a) $\underline{U_1 = 0}$ $\quad\Rightarrow\quad$ $U_S = 0$

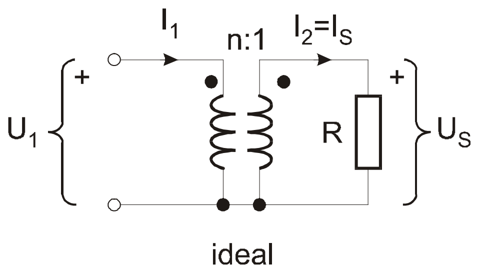


$$\underline{I_S \cdot R + U_2 = \mu \cdot U_2} \quad\Rightarrow\quad I_S \cdot R = (\mu - 1) \cdot U_2 \quad\Rightarrow\quad nI_1 \cdot R = (\mu - 1) \cdot U_2 \quad\Rightarrow\quad I_1 = -\frac{1-\mu}{n \cdot R} \cdot U_2$$

$$y_{12} = -\left.\frac{I_1}{U_2}\right|_{U_1=0} = \frac{1-\mu}{n \cdot R}$$

$$U_2 = I_S \cdot R - I_2 \cdot R \quad\Rightarrow\quad U_2 = \frac{\mu - 1}{R} \cdot U_2 \cdot R - I_2 \cdot R \quad\Rightarrow\quad y_{22} = -\left.\frac{I_2}{U_2}\right|_{U_1=0} = \frac{2-\mu}{R}$$

$\underline{U_2 = 0}$



iz jednadžbi transformatora slijedi $\Rightarrow I_1 = \frac{1}{n} \cdot I_2 \qquad I_2 = n \cdot I_1$

$$U_S = I_2 \cdot R \quad \Rightarrow \frac{1}{n} \cdot U_1 = n \cdot I_1 \cdot R \qquad \Rightarrow \quad y_{11} = \left.\frac{I_1}{U_1}\right|_{U_2=0} = \frac{1}{n^2 R}$$

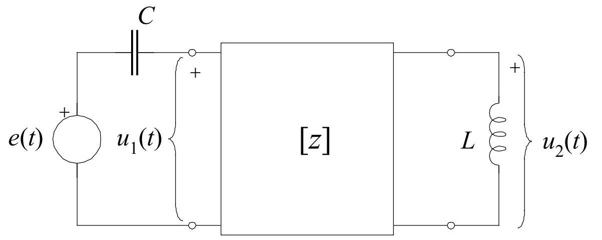
$$y_{21} = \left.\frac{I_2}{U_1}\right|_{U_2=0} = \frac{n \cdot I_1}{U_1} = n \cdot \frac{1}{n^2 R} = \frac{1}{nR}$$

$$[y] = \begin{bmatrix} \frac{1}{n^2 R} & -\frac{1-\mu}{nR} \\ \frac{1}{nR} & -\frac{2-\mu}{R} \end{bmatrix} = \begin{bmatrix} y_{11} & -y_{12} \\ y_{21} & -y_{22} \end{bmatrix}$$

$\frac{1-\mu}{nR} = \frac{1}{nR} \Rightarrow \qquad \mu = 0$ za recipročnost

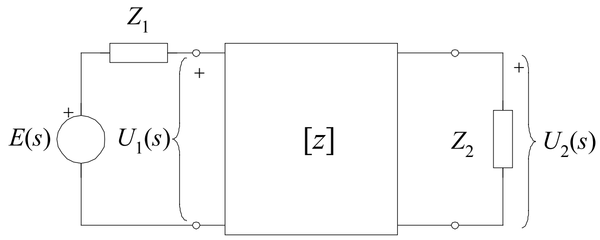
$\frac{1}{n^2 R} = \frac{2-\mu}{R} \Rightarrow \qquad n = \frac{1}{\sqrt{2-\mu}}$ za simetričnost

5. Naći naponsku prijenosnu funkciju $T(s)=U_2(s)/E(s)$ četveropola na slici. Odrediti napon $u_2(t)$ na izlazu četveropola ako je zadano $e(t)=S(t)$, $C=1$, $L=2$.



$$[\mathbf{z}]=\begin{bmatrix} 2s+1 & -2s \\ 2s & -2s \end{bmatrix}$$

Rješenje:



Prijenosna funkcija: $T(s)=\dfrac{U_2(s)}{E(s)}$

Jednadžbe četveropola

$$U_1 = z_{11}I_1 - z_{12}I_2$$
$$U_2 = z_{11}I_1 - z_{12}I_2$$

$$I_1(s)=\frac{E(s)-U_1(s)}{Z_1} \quad ; \qquad I_2(s)=\frac{U_2(s)}{Z_2}$$

$$T(s)=\frac{U_2(s)}{E(s)}=\frac{Z_2\, z_{21}}{(Z_1+z_{11})(Z_2+z_{22})-(z_{12}\, z_{21})}$$

$$T(s)=\frac{s^2}{s^2+s+1}$$

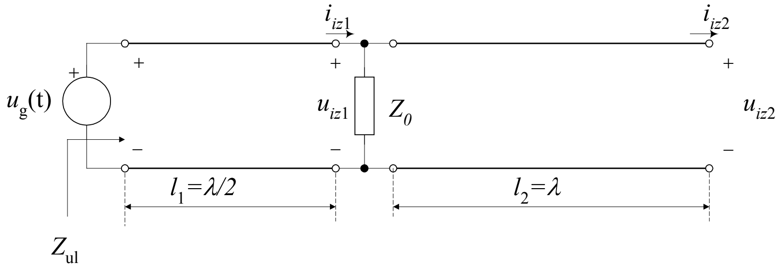
$$U_2(s)=T(s)\cdot E(s)=\frac{s^2}{s^2+s+1}\cdot\frac{1}{s}=\frac{s}{s^2+s+1}$$

$$U_2(s)=\frac{s}{s^2+s+1} \qquad s_{p1,2}=\frac{-1\pm\sqrt{1-4}}{2}=-\frac{1}{2}\pm j\frac{\sqrt{3}}{2}$$

$$U_2(s)=\frac{s+\frac{1}{2}-\frac{1}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=\frac{\left(s+\frac{1}{2}\right)}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}-\frac{1}{2}\frac{2}{\sqrt{3}}\frac{\frac{\sqrt{3}}{2}}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$u_2(t)=e^{-\frac{1}{2}t}\left(\cos\frac{\sqrt{3}}{2}t-\frac{1}{\sqrt{3}}\sin\frac{\sqrt{3}}{2}t\right)\cdot S(t)$$

6. Zadan je spoj dviju linija bez gubitaka s primarnim parametrima C=1nF/km, L=1mH/km, duljina $\ell_1=\lambda/2$ i $\ell_2=\lambda$ prema slici. Odrediti valne oblike napona i struje na krajevima linija u_{izli} i i_{izli}; (i=1,2) ako je zadano $u_g(t)=\cos(2\pi\ 10^3 t)$; $-\infty<t<\infty$.



$$Z_0=\sqrt{\frac{L}{C}}=\sqrt{\frac{1\cdot 10^{-3}}{1\cdot 10^{-9}}}=\sqrt{10^6}=10^3\,\Omega \qquad\qquad \gamma=\alpha+j\beta;\quad \alpha=0,\ \beta=\omega_0\sqrt{LC}$$

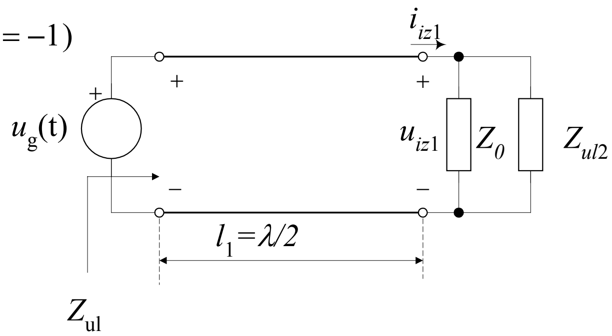
ω_0 - frekvencija sinusne pobude; $\lambda=2\pi/\beta$ - valna duljina signala

Prijenosne jednadžbe linije :

$$U(0)=U(l)\mathrm{ch}(\gamma l)+Z_0 I(l)\mathrm{sh}(\gamma l)=U(l)\cos(\beta l)+Z_0 I(l) j\sin(\beta l)$$

$$I(0)=\frac{U(l)}{Z_0}j\mathrm{sh}(\gamma l)+I(l)\mathrm{ch}(\gamma l)=\frac{U(l)}{Z_0}j\sin(\beta l)+I(l)\cos(\beta l)$$

($\mathrm{ch}\, jx=\cos x$, $\mathrm{sh}\, jx=j\sin x$, $\sin\pi=0$, $\cos\pi=-1$)



a) prva linija: $l_1=\dfrac{\lambda}{2}=\dfrac{\pi}{\beta}\ \Rightarrow\ \beta\cdot l_1=\pi$

$$U^I(0)=-U^I(l_1)$$
$$\underline{I^I(0)=-I^I(l_1)}$$
$$U^I(l_1)=(Z_0\,\|\,Z_{ul2})I^I(l_1)$$

b) druga linija: $l_2=\lambda=\dfrac{2\pi}{\beta}\ \Rightarrow\ \beta\cdot l_2=2\pi$

$$Z_{ul2}=\frac{U^{II}(0)}{I^{II}(0)}=\frac{U^{II}(l)\mathrm{ch}(\gamma l_2)+Z_0 I^{II}(l)\mathrm{sh}(\gamma l_2)}{\frac{U^{II}(l)}{Z_0}\mathrm{sh}(\gamma l_2)+I^{II}(l)\mathrm{ch}(\gamma l_2)}=\frac{Z_2\cos(\beta l_2)+jZ_0\sin(\beta l_2)}{j\frac{Z_2}{Z_0}\sin(\beta l_2)+\cos(\beta l_2)}=Z_2$$

($\mathrm{ch}\, jx=\cos x$, $\mathrm{sh}\, jx=j\sin x$, $\sin 2\pi=0$, $\cos 2\pi=1$)

$Z_2=\infty$ pa je $Z_{ul2}=\infty$

$$U^I(l_1)=Z_0 I^I(l_1)$$

$$U^I(l_1)=-U^I(0)=-1=1\cdot e^{j\pi}\ \Rightarrow\ \underline{u_l^I(t)=u_{izl1}(t)=-\cos(2\pi\cdot 10^3 t)}$$

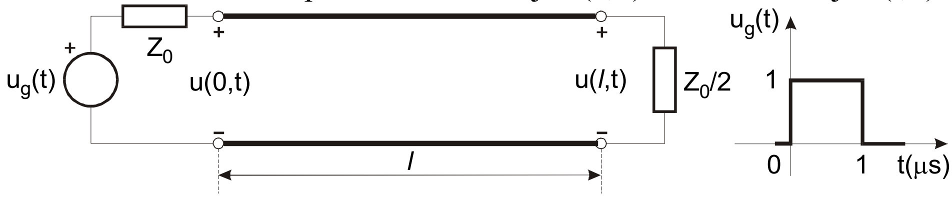
$$\underline{I^I(l_1)=\frac{U^I(l_1)}{Z_0}}=-10^{-3}=10^{-3}\cdot e^{j\pi}\ \Rightarrow\ \underline{i_l^I(t)=i_{izl1}(t)=-10^{-3}\cos(2\pi\cdot 10^3 t)}$$

$$Z_{ul}^I=\frac{U^I(0)}{I^I(0)}=Z_0$$

$$U^{II}(l_2)=U^{II}(0)\cdot\underbrace{\cos 2\pi}_{1}-I^{II}(0)\cdot Z_0 j\underbrace{\sin 2\pi}_{0}=U^{II}(0)=U^I(l_1)$$

$$\underline{u_{izl2}(t)=u_{izl1}(t)=-\cos(2\pi\cdot 10^3 t)} \qquad\qquad \underline{i_{izl2}(t)=0}$$

7. Zadana je linija s primarnim parametrima R=0.5Ω/km, L=10μH/km, G=0,2mS/km, C=40nF/km, duljine l=1km. Na liniju je spojen generator $u_g(t)$ s unutarnjim otporom jednakim zrcalnoj impedanciji linije Z_0 i valnim oblikom prema slici, a linija je zaključena s $Z_0/2$. Odrediti i nacrtati valne oblike napona na ulazu linije $u(0, t)$ i na izlazu linije $u(l, t)$.



Rješenje:

$$\frac{R}{L}=\frac{0.5\Omega}{10\mu H}=\frac{0.5}{10\cdot 10^{-6}}=5\cdot 10^{4} \qquad \frac{G}{C}=\frac{0,2mS}{40nF}=5\cdot 10^{4} \Rightarrow \frac{R}{L}=\frac{G}{C}$$

Linija bez distorzije:

$$Z_0=\sqrt{\frac{L}{C}}=\sqrt{\frac{10\cdot 10^{-6}}{40\cdot 10^{-9}}}=50\Omega \qquad \gamma=s\sqrt{LC}+\sqrt{RG}=20\cdot 10^{-6}s+0,01$$

Polazni val na izlazu: $U_p(l)=\underbrace{U(0)}_{\frac{U_g}{2}}\cdot e^{-\gamma\cdot l}=\underbrace{U(0)}_{\frac{U_g}{2}}\cdot e^{-(20\cdot 10^{-6}\cdot s)}\cdot e^{-0,01}$

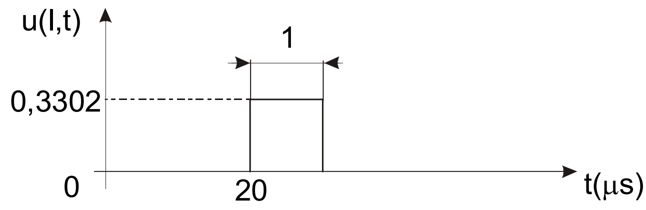
faktor refleksije na izlazu: $\Gamma_2=\dfrac{Z_2-Z_0}{Z_2+Z_0}=\dfrac{\frac{Z_0}{2}-Z_0}{\frac{Z_0}{2}+Z_0}=-\dfrac{1}{3}$

reflektirani val na izlazu: $U_r(l)=\Gamma_2\cdot U_p(l)=\Gamma_2\cdot\dfrac{U_g(s)}{2}\cdot e^{-(20\cdot 10^{-6}\cdot s)}\cdot e^{-0,01}$

ukupni napon na izlazu:

$$U_{izl}=U_p(l)+U_r(l)=(1+\Gamma_2)\cdot U_p(l)=(1+\Gamma_2)\cdot\frac{U_g(s)}{2}\cdot e^{-(20\cdot 10^{-6}\cdot s)}\cdot e^{-0,01}=\frac{2}{3}\cdot\frac{U_g(s)}{2}\cdot e^{-(20\cdot 10^{-6}\cdot s)}\cdot e^{-0,01}$$

$$\Rightarrow u_{izl}(t)=\frac{1}{3e^{0,01}}\cdot u_g(t-20\cdot 10^{-6}), \qquad e^{0,01}=1,01005$$



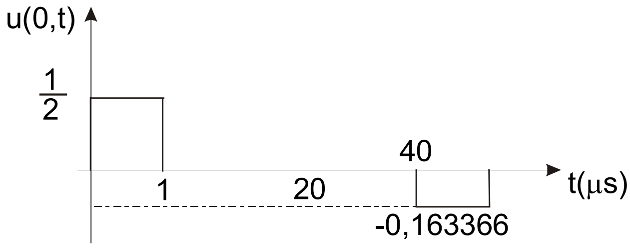
Rreflektirani val na ulazu:

$$U_r(0)=U_r(l)\cdot e^{-(20\cdot 10^{-6}\cdot s+0,01)}=\Gamma_2\cdot U_p(l)\cdot e^{-(20\cdot 10^{-6}\cdot s+0,01)}=-\frac{1}{3}\cdot\frac{U_g(s)}{2e^{0,01}}\cdot e^{-(20\cdot 10^{-6}\cdot s)}\cdot e^{-(20\cdot 10^{-6}\cdot s)}\cdot e^{-0,01}$$

$$U_r(0)=-\frac{1}{6e^{0,02}}\cdot U_g(s)\cdot e^{-(40\cdot 10^{-6}\cdot s)} \Rightarrow u_r(0,t)=-\frac{1}{6e^{0,02}}\cdot u_g(t-40\cdot 10^{-6})$$

Ukupni napon na ulazu :

$$u(0,t)=\frac{u_g(t)}{2}-\frac{1}{6e^{0,02}}\cdot u_g(t-40\cdot 10^{-6}), \qquad e^{0,02}=1,0202$$


u(0,t)
1/2
40
1
20
t(μs)
-0,163366