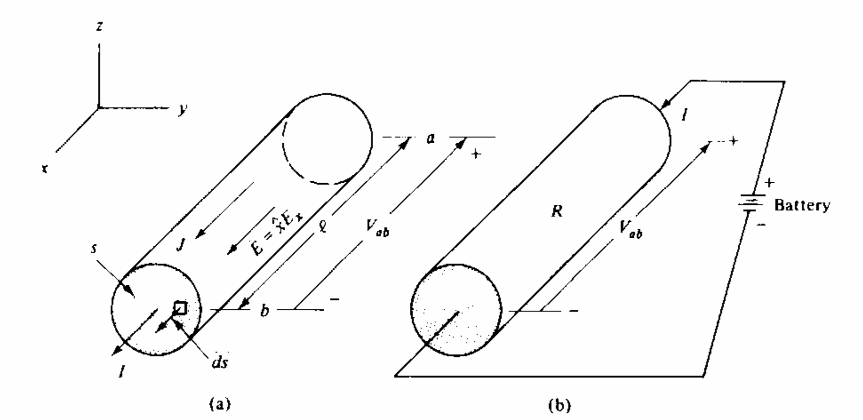
Odredimo elektricni otpor vodica duljine *l*, jednakog poprecnog presjeka *S*, nacinjenog od materijala provodnosti _. Neka je elektricno polje u vodicu homogeno duž vodica i po njegovom presjeku i usmjereno u *x* smjeru: $\boldsymbol{E} = \boldsymbol{a}_x E_x$, prema slici a:



$$I = \int_S \boldsymbol{J} \cdot \boldsymbol{n} \cdot \mathrm{d}S = \int_S \kappa \boldsymbol{E} \cdot \boldsymbol{n} \cdot \mathrm{d}S = \int_S \kappa \cdot \boldsymbol{a}_x E_x \cdot \boldsymbol{a}_x \cdot \mathrm{d}S = \kappa E_x \cdot S$$

jer je normala na površinu presjeka vodica $\boldsymbol{n} = \boldsymbol{a}_x$.

$$U_{ab} = \varphi(a) - \varphi(b) = -\int_b^a \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\int_b^a \boldsymbol{a}_x E_x \cdot \boldsymbol{a}_x \mathrm{d}x = E_x \cdot \int_a^b \mathrm{d}x = E_x \cdot l$$

Odavde je jakost električnog polja:

$$E_x = \frac{U_{ab}}{l}$$

$$I = \kappa E_x \cdot S = \kappa \frac{U_{ab}}{l} S \qquad R = \frac{U_{ab}}{I} = \frac{1}{\kappa}\frac{l}{S} \qquad U_{ab} = I \cdot R$$

vodic u zatvorenom elektricnom krugu kako je prikazano na slici 15.1.b) Sa slike vidimo da je viši potencijal kraja *a* vodica i da elektricna struja kroz vodic tece od tocke višeg prema tocki nižeg potencijala.

$$R = \frac{U_{ab}}{I} = \frac{-\int_b^a \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l}}{\int_S \boldsymbol{J} \cdot \boldsymbol{n} \cdot \mathrm{d}S} = \frac{-\int_b^a \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l}}{\int_S \kappa \boldsymbol{E} \cdot \boldsymbol{n} \cdot \mathrm{d}S}$$

Ponašanje vodica u statickim uvjetima

Postojanje elektricnog naboja unutar vodica uzrokovat ce elektricno polje u vodicu, a naboj i njegovo gibanje u vodicu (elektricna struja) moraju zadovoljavati jednadžbu kontinuiteta.

$$\mathrm{div}\, \boldsymbol{J} = \nabla \cdot \boldsymbol{J} = -\frac{\partial \rho}{\partial t}$$

Ako je vodic linearan, homogen i izotropan, uvrštenjem Ohmovog zakona u jednadžbu kontinuiteta dobije se

$$\boldsymbol{J} = \kappa \cdot \boldsymbol{E} \qquad \nabla \cdot \boldsymbol{E} = -\frac{1}{\kappa}\frac{\partial \rho}{\partial t} \qquad \mathrm{div}\, \boldsymbol{D} = \nabla \cdot \boldsymbol{D} = \rho_s \quad \Rightarrow \quad \nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon}$$

$$\frac{\rho}{\varepsilon} = -\frac{1}{\kappa}\frac{\partial \rho}{\partial t} \quad \Rightarrow \quad \frac{\partial \rho}{\partial t} + \frac{\kappa}{\varepsilon}\rho = 0 \qquad \rho(t) = \rho_0 \cdot e^{-\left(\frac{\kappa}{\varepsilon}\right)t} = \rho_0 \cdot e^{-\frac{t}{\tau_r}}$$

gdje je _r vremenska konstanta relaksacije naboja

$$\tau_r = \frac{\varepsilon}{\kappa} \quad (\mathrm{s})$$

Analogija statickog strujnog polja i statickog elektricnog polja
jednadžba kontinuiteta mora glasiti

$$\mathrm{div}\, \boldsymbol{J} = \nabla \cdot \boldsymbol{J} = -\frac{\partial \rho}{\partial t} = 0$$

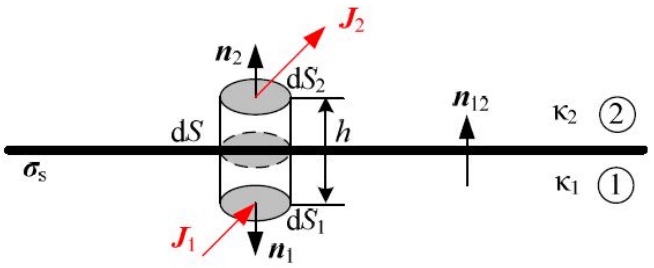
Tablica 15.1. Analogije jednadžbi statičkog električnog polja i statičkog strujnog polja

Homogeni dielektrik bez naboja: $\rho_s = 0$		Vodljivi materijal (stacionarno strujanje): $\partial \rho_s / \partial t = 0$	
Gaussov zakon:	$\nabla \cdot \boldsymbol{D} = 0$	Jednadžba kontinuiteta:	$\nabla \cdot \boldsymbol{J} = 0$
	$\boldsymbol{D} = \varepsilon \cdot \boldsymbol{E}$		$\boldsymbol{J} = \kappa \cdot \boldsymbol{E}$
	$\boldsymbol{E} = -\nabla \cdot \varphi$		$\boldsymbol{E} = -\nabla \cdot \varphi$
Laplaceova jednadžba:	$\Delta \varphi = 0$	Laplaceova jednadžba:	$\Delta \varphi = 0$
Električni tok (vektora $\boldsymbol{D}$):	$\phi_e = \int_S \boldsymbol{D} \cdot \boldsymbol{n} \cdot \mathrm{d}S$	Strujni tok (vektora $\boldsymbol{J}$):	$I = \int_S \boldsymbol{J} \cdot \boldsymbol{n} \cdot \mathrm{d}S$
Kapacitet:	$C = \frac{Q}{U_{ab}} = \frac{\int_S \boldsymbol{D} \cdot \boldsymbol{n} \cdot \mathrm{d}S}{-\int_b^a \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l}}$	Vodljivost:	$G = \frac{I}{U_{ab}} = \frac{\int_S \boldsymbol{J} \cdot \boldsymbol{n} \cdot \mathrm{d}S}{-\int_b^a \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l}}$

Tablica 15.2. Analogne veličine statičkog električnog polja i statičkog strujnog polja

Homogeni dielektrik bez naboja: $\rho_s = 0$	Vodljivi materijal (stacionarno strujanje): $\partial \rho_s / \partial t = 0$
$\boldsymbol{D}$	$\boldsymbol{J}$
$\boldsymbol{E}$	$\boldsymbol{E}$
ε	κ
φ	φ
Φ_e	I
C	G

Odredivanje vektora gustoce struje $\boldsymbol{J}$ na granici



$\boldsymbol{n}12 = -\boldsymbol{n}1 = \boldsymbol{n}2$, te d$S$ = dS1 = dS2

$$\boldsymbol{n}_{12}(\boldsymbol{J}_2-\boldsymbol{J}_1)=-\frac{\mathrm{d}\sigma_s}{\mathrm{d}t}$$

Normalna komponenta vektora gustoce struje ***J*** mijenja se na granici za iznos vremenske promjene gustoce plošnog slobodnog naboja na granici. Ako na granici nema slobodnog naboja _s = 0 ili se on ne mijenja u vremenu d_s/d*t* = 0, onda su normalne komponente vektora gustoce struje ***J*** s obje strane granice jednake

$$\boldsymbol{n}_{12}(\boldsymbol{J}_2-\boldsymbol{J}_1)=0 \quad \Rightarrow \quad J_{1n}=J_{2n}$$

$$J_{1n}=J_{2n} \quad \Rightarrow \quad \kappa_1 E_{1n}=\kappa_2 E_{2n} \quad \Rightarrow \quad \frac{E_{1n}}{E_{2n}}=\frac{\kappa_2}{\kappa_1}$$

Za promjenu jakosti elektricnog polja vrijede isti uvjeti kao za staticko elektricno polje

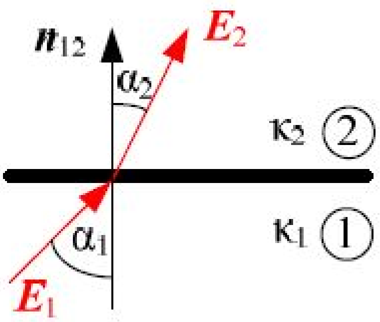
$$E_{1t}=E_{2t} \quad \Rightarrow \quad \boldsymbol{n}_{12}\ \mathrm{x}\ (\boldsymbol{E}_2-\boldsymbol{E}_1)=0$$

$$E_{1t}=E_{2t} \quad \Rightarrow \quad \frac{J_{1t}}{\kappa_1}=\frac{J_{2t}}{\kappa_2} \quad \Rightarrow \quad \frac{J_{1t}}{J_{2t}}=\frac{\kappa_1}{\kappa_2}$$

Na granici dva vodica razlicitih provodnosti u slucaju statickog strujanja

$$\boldsymbol{n}_{12}(\boldsymbol{J}_2-\boldsymbol{J}_1)=-\frac{\partial\sigma_s}{\partial t}=0 \quad ; \quad \boldsymbol{n}_{12}\ \mathrm{x}\ (\boldsymbol{E}_2-\boldsymbol{E}_1)=0$$

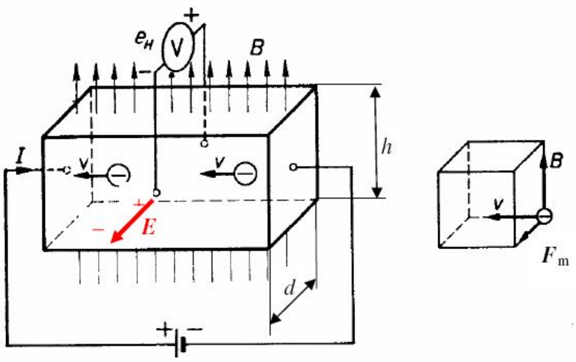
zakon loma strujnice na granici



$$\mathrm{tg}\alpha_1=\frac{E_{1t}}{E_{1n}} \quad ; \quad \mathrm{tg}\alpha_2=\frac{E_{2t}}{E_{2n}}$$

$$\frac{\mathrm{tg}\alpha_1}{\mathrm{tg}\alpha_2}=\frac{\dfrac{E_{1t}}{E_{1n}}}{\dfrac{E_{2t}}{E_{2n}}}=\frac{E_{1t}\cdot E_{2n}}{E_{2t}\cdot E_{1n}}=\frac{\kappa_1}{\kappa_2}$$

Hallov ucinak

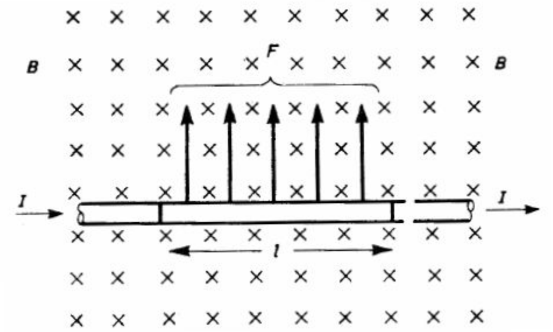


Razdvojeni naboji stvaraju elektricno polje i razliku potencijala (*Hallov napon U*H) izmedu prednje i stražnje površine vodica. Uz pretpostavku da je to elektricno polje homogeno njegova je jakost $E = U_H/d$

Elektricno polje koje stvaraju razdvojeni naboji djeluje na slobodne elektrone elektricnom silom ***F***e koja je $\boldsymbol{F}_e = q\cdot\boldsymbol{E}$ i suprotno je usmjerena od magnetske sile ***F***m

Otklanjanje slobodnih elektrona u vodicu trajat ce sve dotle dok kolicina razdvojenih naboja ne stvori toliku jakost elektricnog polja i time elektricnu silu koja ce uravnotežiti magnetsku silu $\boldsymbol{F_e} = \boldsymbol{F}$m ; $qvB = q^{*}\, U_H/d$ iz cega slijedi da je Hallov napon $U_H = vdB$

MAGNETSKA SILA NA VODIC PROTJECAN STRUJOM



$$f = q(\boldsymbol{v} \times \boldsymbol{B}) \qquad \boldsymbol{F} = N \cdot \boldsymbol{f} = n \cdot l \cdot S \cdot q(\boldsymbol{v} \times \boldsymbol{B})$$

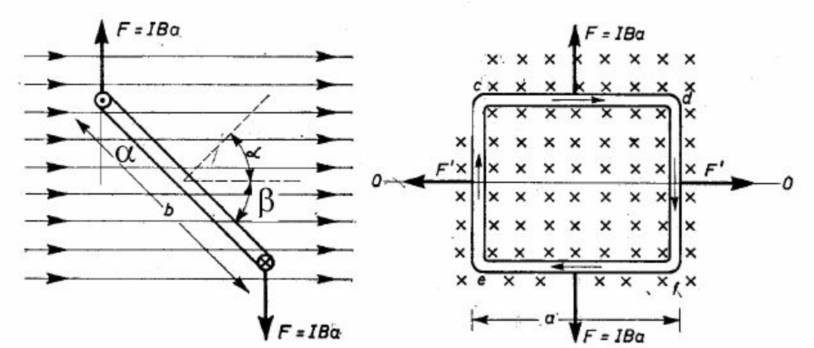
$$\mathrm{d}Q = N \cdot q = n \cdot l \cdot S \cdot q = n \cdot v \cdot \mathrm{d}t \cdot S \cdot q \qquad I = \frac{\mathrm{d}Q}{\mathrm{d}t} = n \cdot v \cdot S \cdot q$$

$$\boldsymbol{F} = I(\boldsymbol{l} \times \boldsymbol{B})$$

Ako magnetsko polje nije homogeno ili ako vodic nije ravni, onda prvo izracunamo
diferencijalnu silu d***F*** na diferencijalno mali dio vodica d***l*** za kojeg možemo smatrati da se
nalazi u magnetskom polju iste indukcije a ukupnu silu na vodic dobijemo integracijom diferencijalne sile uzduž vodica

$$\mathrm{d}\boldsymbol{F} = I(\mathrm{d}\boldsymbol{l} \times \boldsymbol{B}) \qquad \boldsymbol{F} = \int_l \mathrm{d}\boldsymbol{F} = \int_l I(\mathrm{d}\boldsymbol{l} \times \boldsymbol{B})$$

Magnetska sila i zakretni moment na strujnu petlju



Stranice petlje *cd* i *ef* okomite su na magnetsko polje pa na njih djeluju sile jednakoga iznosa, koje su zbog suprotnih smjerova struje suprotno usmjerene:

$F = I \cdot B{\cdot}a$

Stranice *ce* i *df* s vektorom magnetske indukcije zatvaraju kut . Sile na njih ce takoder biti jednakoga iznosa, suprotno usmjerene:

$F' = I \cdot B{\cdot}b \cdot \sin\beta$

Zbroj svih sila jednak je nuli, jer se sile u parovima poništavaju. Medutim, na stranice *cd* i *ef* djeluje par sila *F* zakretnim momentom na kraku duljine *b*_sin:

$M = F \cdot b \cdot \sin\alpha = I \cdot B{\cdot}a \cdot b \cdot \sin\alpha = I \cdot B \cdot S \cdot \sin\alpha$

Zakretni moment djeluje tako da nastoji postaviti ravninu petlje okomito na smjer magnetskog polja (= 0). U tom položaju zakretni moment je *M* = 0 a kroz strujnu petlju prolazi maksimalni tok magnetskog polja. Najveci zakretni moment nastaje kad je ravnina petlje u smjeru magnetskog polja (= 90°)

Gaussov zakon

U pravocrtnom koordinatnom sustavu (x, y, z) divergencija vektora $\boldsymbol{B}$ je:

$$\mathrm{div}\,\boldsymbol{B} = \nabla \cdot \boldsymbol{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = 0$$

U cilindričnom koordinatnom sustavu (r, α, z) divergencija vektora $\boldsymbol{B}$ je:

$$\mathrm{div}\,\boldsymbol{B} = \nabla \cdot \boldsymbol{B} = \frac{1}{r}\left[\frac{\partial}{\partial r}\left(r \cdot B_r\right)\right] + \frac{1}{r}\frac{\partial B_\alpha}{\partial \alpha} + \frac{\partial B_z}{\partial z} = 0$$

U sfernom koordinatnom sustavu (r, ϑ, α) divergencija vektora $\boldsymbol{B}$ je:

$$\mathrm{div}\,\boldsymbol{B} = \nabla \cdot \boldsymbol{B} = \frac{1}{r^2}\left[\frac{\partial}{\partial r}\left(r^2 \cdot B_r\right)\right] + \frac{1}{r \cdot \sin\vartheta}\left[\frac{\partial}{\partial \vartheta}\left(B_\vartheta \cdot \sin\vartheta\right)\right] + \frac{1}{r \cdot \sin\vartheta}\frac{\partial B_\alpha}{\partial \alpha} = 0$$

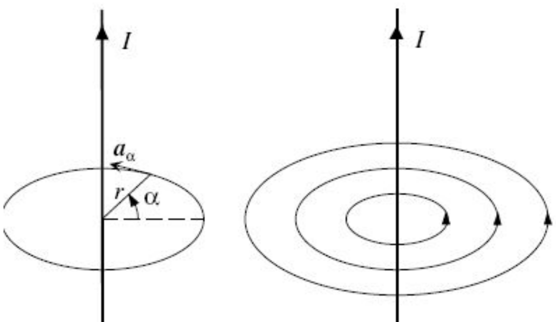
BIOT-SAVARTOV ZAKON

$$\mathrm{d}\boldsymbol{B} = \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}\boldsymbol{l} \times \boldsymbol{R}}{R^3} \qquad \mu_0 = 4\pi \cdot 10^{-7} \quad \mathrm{Vs/Am} = \mathrm{H/m}$$

$$\boldsymbol{B} = \int_l \mathrm{d}\boldsymbol{B} = \int_l \frac{\mu_0 I}{4\pi}\frac{\mathrm{d}\boldsymbol{l} \times \boldsymbol{R}}{R^3} = \frac{\mu_0}{4\pi}\int_l I\,\frac{\mathrm{d}\boldsymbol{l} \times \boldsymbol{R}}{R^3}$$

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J} \times \boldsymbol{R}}{R^3}\mathrm{d}V \qquad \boldsymbol{B} = \frac{\mu_0}{4\pi}\int_S \frac{\boldsymbol{K} \times \boldsymbol{R}}{R^3}\mathrm{d}S$$

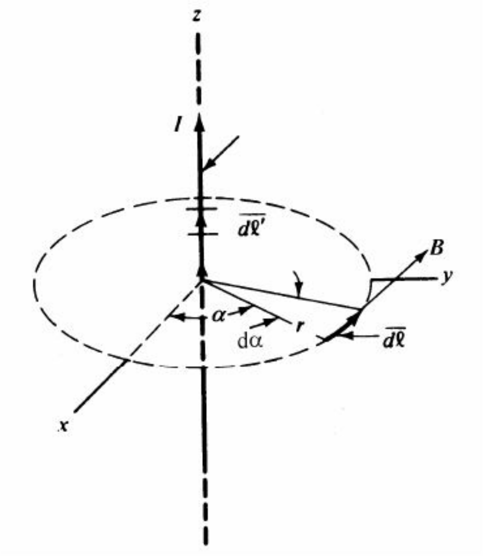
Magnetsko polje ravne duge strujnice



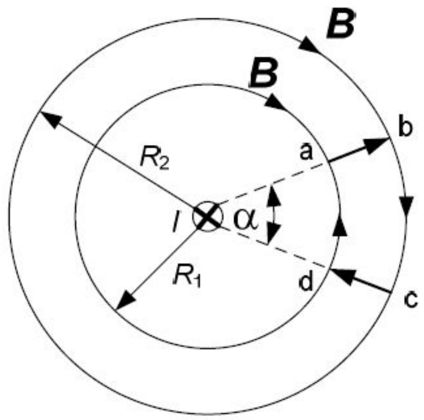
$$\boldsymbol{B} = \boldsymbol{a}_\alpha \frac{\mu_0 I}{2\pi r}$$

$\boldsymbol{a}$(alfa) - jedinicni tangencijalni vektor u cilindricnom koordinatnom sustavu

AMPÈREOV ZAKON KRUŽNOG PROTJECANJA



a)



b)

$$\boldsymbol{B} = \boldsymbol{a}_\alpha \frac{\mu_0 I}{2\pi r} = \text{konst.} \qquad \mathrm{d}\boldsymbol{l} = \boldsymbol{a}_\alpha r \mathrm{d}\alpha$$

$$\oint_c \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = \oint_c \boldsymbol{a}_\alpha \frac{\mu_0 I}{2\pi r} \boldsymbol{a}_\alpha r \mathrm{d}\alpha = \frac{\mu_0 I}{2\pi} \oint_c \mathrm{d}\alpha = \frac{\mu_0 I}{2\pi} 2\pi = \mu_0 I$$

izracunavanje za krivulju *a-b-c-d* pokraj ravne, beskonacno duge strujnice
prema slici 21.1.b)

$$\oint_c \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = \int_{r=R_1}^{R_2} \boldsymbol{a}_\alpha \frac{\mu_0 I}{2\pi r} \boldsymbol{a}_r \mathrm{d}r + \int_b^c \boldsymbol{a}_\alpha \frac{\mu_0 I}{2\pi R_2} \boldsymbol{a}_\alpha R_2 \mathrm{d}\alpha - \int_{r=R_1}^{R_2} \boldsymbol{a}_\alpha \frac{\mu_0 I}{2\pi r} \boldsymbol{a}_r \mathrm{d}r - \int_d^a \boldsymbol{a}_\alpha \frac{\mu_0 I}{2\pi R_1} \boldsymbol{a}_\alpha R_1 \mathrm{d}\alpha$$

$$\oint_c \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = 0 + \frac{\mu_0 I}{2\pi} \int_b^c \mathrm{d}\alpha - 0 - \frac{\mu_0 I}{2\pi} \int_d^a \mathrm{d}\alpha = \frac{\mu_0 I}{2\pi}\alpha - \frac{\mu_0 I}{2\pi}\alpha = 0$$

Diferencijalni oblik Ampèreovog zakona

$$\oint_c \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l} = \int_S (\nabla \times \boldsymbol{A}) \cdot \boldsymbol{n} \cdot \mathrm{d}S \qquad \oint_c \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = \int_S (\nabla \times \boldsymbol{B}) \cdot \boldsymbol{n} \cdot \mathrm{d}S = \mu_0 \int_S \boldsymbol{J}_s \cdot \boldsymbol{n} \cdot \mathrm{d}S$$

$$\nabla \times \boldsymbol{B} = \mathrm{rot}\, \boldsymbol{B} = \mu_0 \boldsymbol{J}_s$$

U pravocrtnom koordinatnom sustavu (x, y, z) rotacija vektora $\boldsymbol{B}$ je:

$$\nabla \times \boldsymbol{B} = \begin{vmatrix} \boldsymbol{a}_x & \boldsymbol{a}_y & \boldsymbol{a}_z \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = \boldsymbol{a}_x\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) + \boldsymbol{a}_y\left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right) + \boldsymbol{a}_z\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right) = \mu_0 \boldsymbol{J}_s$$

U cilindričnom koordinatnom sustavu (r, α, z) rotacija vektora $\boldsymbol{B}$ je:

$$\nabla \times \boldsymbol{B} = \boldsymbol{a}_r\left(\frac{1}{r}\frac{\partial B_z}{\partial \alpha} - \frac{\partial B_\alpha}{\partial z}\right) + \boldsymbol{a}_\alpha\left(\frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}\right) + \boldsymbol{a}_z \frac{1}{r}\left[\frac{\partial}{\partial r}(rB_\alpha) - \frac{\partial B_r}{\partial \alpha}\right] = \mu_0 \boldsymbol{J}_s$$

U sfernom koordinatnom sustavu (r, ϑ, α) rotacija vektora $\boldsymbol{B}$ je:

$$\nabla \times \boldsymbol{B} = \boldsymbol{a}_r \frac{1}{r \cdot \sin\vartheta}\left[\frac{\partial}{\partial \vartheta}(\sin\vartheta \cdot B_\alpha) - \frac{\partial B_\vartheta}{\partial \alpha}\right] + \boldsymbol{a}_\vartheta \frac{1}{r}\left[\frac{1}{\sin\vartheta}\frac{\partial B_r}{\partial \alpha} - \frac{\partial}{\partial r}(r \cdot B_\alpha)\right] +$$

$$+ \boldsymbol{a}_\alpha \frac{1}{r}\left[\frac{\partial}{\partial r}(r \cdot B_\vartheta) - \frac{\partial B_r}{\partial \vartheta}\right] = \mu_0 \boldsymbol{J}_s$$

MATERIJALI U MAGNETSKOM POLJU

$\boldsymbol{m} = \boldsymbol{n} \cdot i \cdot S$ --> normala $\boldsymbol{n}$ odredena pravilom desne ruke

Utjecaj magnetizacije na magnetsko polje

Ukupna magnetska indukcija u materijalu je: $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_M$

Indukcija $\boldsymbol{B}_0$ stvorena vanjskom strujom kroz zavojnicu je: $\boldsymbol{B}_0 = \boldsymbol{B} - \boldsymbol{B}_M$

$$\oint_c \boldsymbol{B}_0 \cdot \mathrm{d}\boldsymbol{l} = \mu_0 \int_S \boldsymbol{J}_s \cdot \boldsymbol{n} \cdot \mathrm{d}S \quad ; \quad \oint_c (\boldsymbol{B} - \boldsymbol{B}_M) \cdot \mathrm{d}\boldsymbol{l} = \mu_0 \int_S \boldsymbol{J}_s \cdot \boldsymbol{n} \cdot \mathrm{d}S$$

$$\oint_c \left(\frac{\boldsymbol{B} - \boldsymbol{B}_M}{\mu_0}\right) \cdot \mathrm{d}\boldsymbol{l} = \int_S \boldsymbol{J}_s \cdot \boldsymbol{n} \cdot \mathrm{d}S$$

$$\boldsymbol{H} = \frac{\boldsymbol{B}_0}{\mu_0} = \frac{\boldsymbol{B} - \boldsymbol{B}_M}{\mu_0} = \frac{\boldsymbol{B}}{\mu_0} - \boldsymbol{M} \quad ; \quad \boldsymbol{M} = \frac{\boldsymbol{B}_M}{\mu_0}$$

$$\oint_c \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = \int_S \boldsymbol{J}_s \cdot \boldsymbol{n} \cdot \mathrm{d}S$$

Ukupna magnetska indukcija u materijalu je onda $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_M$; $\boldsymbol{B} = \mu_0 \boldsymbol{H} + \mu_0 \boldsymbol{M}$

$\boldsymbol{M} = \chi_m \cdot \boldsymbol{H}$ ––> χ_m - *magnetska susceptibilnost, M* - vektor gustoce magnetiziranja

$\mu_r = 1 + \chi_m$

$$\mathrm{d}\boldsymbol{B} = \frac{\mu \cdot I}{4\pi} \frac{\mathrm{d}\boldsymbol{l} \times \boldsymbol{R}}{R^3} \quad ; \quad \mathrm{d}\boldsymbol{H} = \frac{I}{4\pi} \frac{\mathrm{d}\boldsymbol{l} \times \boldsymbol{R}}{R^3}$$

$$\oint_c \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{l} = \mu \int_S \boldsymbol{J}_s \cdot \boldsymbol{n} \cdot \mathrm{d}S \quad ; \quad \oint_c \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = \int_S \boldsymbol{J}_s \cdot \boldsymbol{n} \cdot \mathrm{d}S$$

$$\nabla \times \boldsymbol{B} = \mathrm{rot}\, \boldsymbol{B} = \mu \boldsymbol{J}_s \quad ; \quad \nabla \times \boldsymbol{H} = \mathrm{rot}\, \boldsymbol{H} = \boldsymbol{J}_s$$

Tablica 22.1. Krivulja magnetiziranja i permeabilnost željeza

H (A/m)	B (T)	μ x 10^7 (H/m)	μ_r	M (A/m)	B_M (T)
0	0	3 100	247	0	0
10	0,0042	4 200	334	3 332	0,00419
20	0,01	5 000	398	7 938	0,00997
50	0,043	8 600	684	34 168	0,0429
100	0,67	67 000	5 332	533 069	0,6699
150	1,01	67 333	5 358	803 582	1,0098
200	1,18	59 000	4 695	938 814	1,1797
500	1,44	28 800	2 292	1 145 416	1,4394
1 000	1,58	15 800	1 257	1 256 324	1,5787
10 000	1,72	1 720	137	1 358 732	1,707
100 000	2,26	226	18	1 698 451	2,134
800 000	3,15	39	3,1	1 796 690	2,145

Ako se magnetski materijal magnetizira jednako u svim smjerovima, takav materijal se naziva *izotropnim* i u njemu su vektor jakosti magnetskog polja ***H*** i vektor magnetizacije ***M*** u istome smjeru, tj. magnetska susceptibilnost _m je u svim smjerovima konstanta. U izotropnom materijalu vrijedi

$$M_x = \chi_m \cdot H_x \quad ; \quad M_y = \chi_m \cdot H_y \quad ; \quad M_z = \chi_m \cdot H_z$$

$$B_x = \mu \cdot H_x \quad ; \quad B_y = \mu \cdot H_y \quad ; \quad B_z = \mu \cdot H_z$$

Za materijal koji nije izotropan (*anizotropan*), vektor jakosti magnetskog polja ***H*** i vektor magnetizacije ***M*** nisu u istome smjeru. U takvim materijalima magnetska susceptibilnost _m je razlicita u razlicitim smjerovima, pa vrijedi

$$M_x = \chi_{m11} \cdot H_x + \chi_{m12} \cdot H_y + \chi_{m13} \cdot H_z$$

$$M_y = \chi_{m21} \cdot H_x + \chi_{m22} \cdot H_y + \chi_{m23} \cdot H_z$$

$$M_z = \chi_{m31} \cdot H_x + \chi_{m32} \cdot H_y + \chi_{m33} \cdot H_z$$

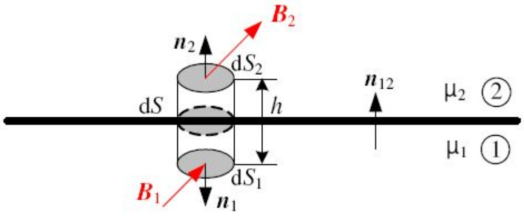
Vektori magnetske indukcije po smjerovima su

$$B_x = \mu_{11} \cdot H_x + \mu_{12} \cdot H_y + \mu_{13} \cdot H_z$$

$$B_y = \mu_{21} \cdot H_x + \mu_{22} \cdot H_y + \mu_{23} \cdot H_z$$

$$B_z = \mu_{31} \cdot H_x + \mu_{32} \cdot H_y + \mu_{33} \cdot H_z$$

UVJETI NA GRANICI



$$\oint_S \boldsymbol{B} \cdot \boldsymbol{n} \cdot dS = 0$$

$$\boldsymbol{B}_1 \cdot \boldsymbol{n}_1 \cdot dS_1 + \boldsymbol{B}_2 \cdot \boldsymbol{n}_2 \cdot dS_2 + (\text{doprinos toku kroz plašt}) = 0$$

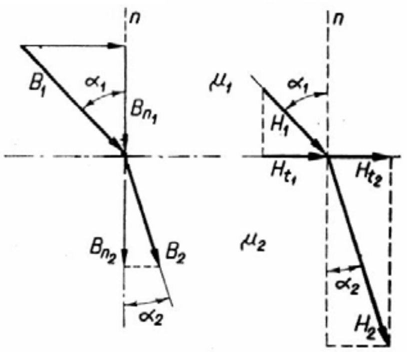
$$\boldsymbol{n}_{12} \left(\boldsymbol{B}_2 - \boldsymbol{B}_1\right) = 0$$

$$B_{1n} = B_{2n}$$

$$\mu_1 H_{1n} = \mu_2 H_{2n} \quad \Rightarrow \quad \frac{H_{1n}}{H_{2n}} = \frac{\mu_2}{\mu_1}$$

$$\boldsymbol{n}_{12} \times \left(\boldsymbol{H}_2 - \boldsymbol{H}_1\right) = \boldsymbol{K}_S$$

$$H_{1t} = H_{2t} \quad \Rightarrow \quad \frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2} \quad \Rightarrow \quad \frac{B_{1t}}{B_{2t}} = \frac{\mu_1}{\mu_2}$$



$$\mathrm{tg}\alpha_1 = \frac{H_{1t}}{H_{1n}} \quad ; \quad \mathrm{tg}\alpha_2 = \frac{H_{2t}}{H_{2n}}$$

$$\frac{\mathrm{tg}\alpha_1}{\mathrm{tg}\alpha_2} = \frac{\dfrac{H_{1t}}{H_{1n}}}{\dfrac{H_{2t}}{H_{2n}}} = \frac{H_{1t} \cdot H_{2n}}{H_{2t} \cdot H_{1n}} = \frac{\mu_1}{\mu_2}$$

Uvjeti na granici feromagnetski materijal – zrak

Ako magnetsko polje izlazi iz feromagnetskog materijala (sredstvo „1“ s μ_1 ») u zrak (sredstvo „2“ s $\mu_2 = \mu_0$), tada je

$$\mathrm{tg}\alpha_2 = \mathrm{tg}\alpha_1 \frac{\mu_2}{\mu_1} \approx 0 \quad \Rightarrow \quad \alpha_2 \approx 0$$

Ako magnetsko polje ulazi iz zraka (sredstvo „1“ s $\mu_1 = \mu_0$) u feromagnetski materijal (sredstvo „2“ s μ_2 ») u tada je

$$\mathrm{tg}\alpha_2 = \mathrm{tg}\alpha_1 \frac{\mu_2}{\mu_1} \rangle\rangle \quad \Rightarrow \quad \alpha_2 \approx 90^\circ$$

MAGNETSKI KRUGOVI

Ako sav magnetski tok u magnetskom krugu prolazi kroz krug, tada ce na bilo kojem presjeku S, na kojem je normala $\boldsymbol{n}$, magnetski tok biti

$$\phi = \int_S \boldsymbol{B} \cdot \boldsymbol{n} \cdot dS \qquad\qquad I = \int_S \boldsymbol{J} \cdot \boldsymbol{n} \cdot dS$$

$$\oint_c \boldsymbol{H} \cdot d\boldsymbol{l} = NI$$

$$\oint \boldsymbol{E} \cdot d\boldsymbol{l} = U$$

Omjer magnetske pobude i magnetskog toka naziva se *magnetski otpor* ili *reluktancija*

$$R_m = \frac{\Theta}{\phi} = \frac{\oint_c \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l}}{\int_S \boldsymbol{B} \cdot \boldsymbol{n} \cdot \mathrm{d}S} = \frac{\oint_c \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l}}{\int_S \mu \boldsymbol{H} \cdot \boldsymbol{n} \cdot \mathrm{d}S}$$

Tablica 24.1. Analogije jednadžbi i veličina magnetskog i električnog kruga

Magnetski krug	Električni krug
Magnetski tok (vektora $\boldsymbol{B}$): $\phi_m = \int_S \boldsymbol{B} \cdot \boldsymbol{n} \cdot \mathrm{d}S$	Strujni tok (vektora $\boldsymbol{J}$): $I = \int_S \boldsymbol{J} \cdot \boldsymbol{n} \cdot \mathrm{d}S$
Magnetska indukcija $\boldsymbol{B}$: $\boldsymbol{B} = \mu \boldsymbol{H}$	Gustoća struje $\boldsymbol{J}$: $\boldsymbol{J} = \kappa \boldsymbol{E}$
Jakost magnetskog polja $\boldsymbol{H}$	Jakost električnog polja $\boldsymbol{E}$
Magnetska permeabilnost μ	Električna provodnost κ
Magnetska pobuda: $NI = \oint_c \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l}$	Napon: $U = \oint_c \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l}$
Magnetski otpor: $R_m = \frac{\Theta}{\phi} = \frac{\oint_c \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l}}{\int_S \boldsymbol{B} \cdot \boldsymbol{n} \cdot \mathrm{d}S} = \frac{\oint_c \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l}}{\int_S \mu \boldsymbol{H} \cdot \boldsymbol{n} \cdot \mathrm{d}S}$	Električni otpor: $R = \frac{U}{I} = \frac{\oint_c \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l}}{\int_S \boldsymbol{J} \cdot \boldsymbol{n} \cdot \mathrm{d}S} = \frac{\oint_c \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l}}{\int_S \kappa \boldsymbol{E} \cdot \boldsymbol{n} \cdot \mathrm{d}S}$
Magnetska vodljivost: $\Lambda = \frac{\phi}{\Theta} = \frac{1}{R_m} = \frac{\int_S \boldsymbol{B} \cdot \boldsymbol{n} \cdot \mathrm{d}S}{\oint_c \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l}}$	Električna vodljivost: $G = \frac{I}{U} = \frac{1}{R} = \frac{\int_S \boldsymbol{J} \cdot \boldsymbol{n} \cdot \mathrm{d}S}{\oint_c \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l}}$

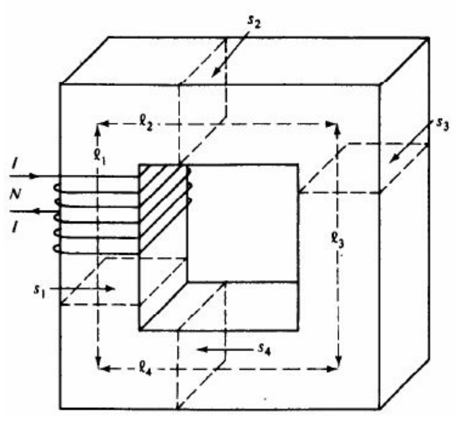
Rješavanje magnetskih krugova

$\phi = B_{sr} S_{sr}$

$H_{sr} \cdot l_{sr} = NI = \Theta$

$$R_m = \frac{\Theta}{\phi} = \frac{H_{sr} \cdot l_{sr}}{B_{sr} \cdot S_{sr}} = \frac{1}{\mu} \frac{l_{sr}}{S_{sr}}$$

Serijski magnetski krug



$\phi = B_1 S_1 = B_2 S_2 = B_3 S_3 = B_4 S_4$

$H_1 \cdot l_1 + H_2 \cdot l_2 + H_3 \cdot l_3 + H_4 \cdot l_4 = NI = \Theta$

$R_m = R_{m1} + R_{m2} + R_{m3} + R_{m4}$

$\Theta = U_{m1} + U_{m2} + U_{m3} + U_{m4}$

Paralelni magnetski krug

$\phi = B_3 S_3$

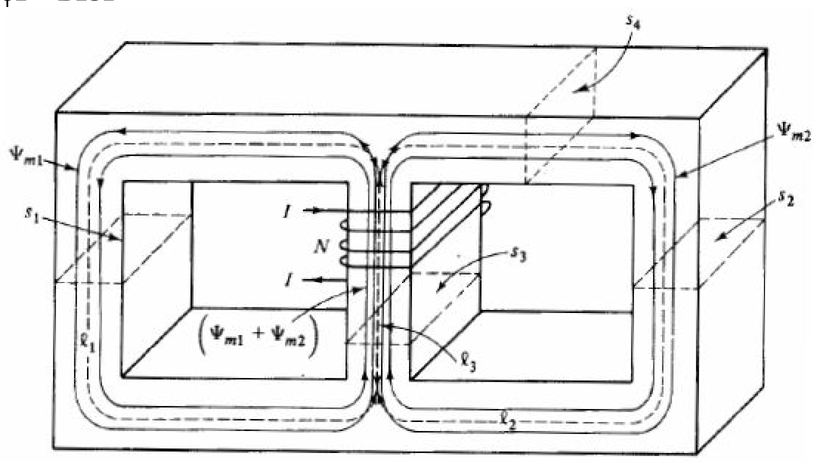
$\phi = \phi_1 + \phi_2$

$H1 \cdot l1 + H3 \cdot l\,3 = NI = \Theta$; $H2 \cdot l\,2 + H3 \cdot l\,3 = NI = \Theta$

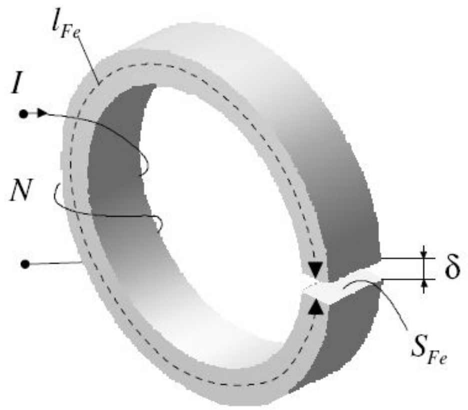
$$\phi_1 = \frac{U_{m1}}{R_{m1}} \quad ; \quad \phi_2 = \frac{U_{m2}}{R_{m2}} \quad ; \quad \frac{\phi_1}{\phi_2} = \frac{R_{m2}}{R_{m1}}$$

$$U_{m1} = H_1 \cdot l_1 = U_{m2} = H_2 \cdot l_2 \quad \Rightarrow \quad H_2 = \frac{H_1 \cdot l_1}{l_2}$$

$\phi 2 = B2S2$



Magnetski krug sa zracnim rasporom



$\phi\ Fe = \phi\delta$; $BFe \cdot SFe = B\delta \cdot S\delta$ _ $BFe = B\delta$

$$H_\delta = \frac{B_\delta}{\mu_0} \quad ; \quad H_{Fe} = \frac{B_{Fe}}{\mu_0 \cdot \mu_r} = \frac{H_\delta}{\mu_r}$$

$$H_{Fe} \cdot l_{Fe} + H_\delta \cdot \delta = NI = \Theta$$

$$H_{Fe} \cdot l_{Fe} + \frac{B_{Fe}}{\mu_0} \cdot \delta = NI$$

$$B_{Fe} = \frac{\mu_0}{\delta}\left(NI - H_{Fe} \cdot l_{Fe}\right)$$

VEKTORSKI MAGNETSKI POTENCIJAL

U pravocrtnom koordinatnom sustavu (x, y, z) magnetska indukcija $\boldsymbol{B}$ izražena preko rotacije vektorskog magnetskog potencijala $\boldsymbol{A}$ je:

$$\boldsymbol{B} = \operatorname{rot} \boldsymbol{A} = \boldsymbol{a}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \boldsymbol{a}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \boldsymbol{a}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right) \tag{25.2}$$

U cilindričnom koordinatnom sustavu (r, α, z) magnetska indukcija $\boldsymbol{B}$ izražena preko rotacije vektorskog magnetskog potencijala $\boldsymbol{A}$ je:

$$\boldsymbol{B} = \operatorname{rot} \boldsymbol{A} = \boldsymbol{a}_r \left(\frac{1}{r} \frac{\partial A_z}{\partial \alpha} - \frac{\partial A_\alpha}{\partial z} \right) + \boldsymbol{a}_\alpha \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \boldsymbol{a}_z \frac{1}{r} \left[\frac{\partial}{\partial r} (r \cdot A_\alpha) - \frac{\partial A_r}{\partial \alpha} \right] \tag{25.3}$$

U sfernom koordinatnom sustavu (r, ϑ, α) magnetska indukcija $\boldsymbol{B}$ izražena preko rotacije vektorskog magnetskog potencijala $\boldsymbol{A}$ je:

$$\begin{aligned} \boldsymbol{B} = \operatorname{rot} \boldsymbol{A} = \boldsymbol{a}_r \frac{1}{r \sin\vartheta} \left[\frac{\partial}{\partial \vartheta} (sin\, \vartheta \cdot A_\alpha) - \frac{\partial A_\vartheta}{\partial \alpha} \right] + \boldsymbol{a}_\vartheta \frac{1}{r} \left[\frac{1}{\sin\vartheta} \frac{\partial A_r}{\partial \alpha} - \frac{\partial}{\partial r} (r \cdot A_\alpha) \right] + \\ + \boldsymbol{a}_\alpha \frac{1}{r} \left[\frac{\partial}{\partial r} (r \cdot A_\vartheta) - \frac{\partial A_r}{\partial \vartheta} \right] \end{aligned} \tag{25.4}$$

Magnetski tok kroz površinu S, na kojem je normala $\boldsymbol{n}$ prema (18.1) je:

$$\phi = \int_S \boldsymbol{B} \cdot \boldsymbol{n} \cdot \mathrm{d}S$$

Ako umjesto magnetske indukcije $\boldsymbol{B}$ uvrstimo (25.1) i primjenimo Stokesov teorem:

$$\int_S \operatorname{rot} \boldsymbol{F} \cdot \boldsymbol{n} \cdot dS = \oint_c \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{l}$$

gdje je c kontura koja obrubljuje površinu S, a $\mathrm{d}\boldsymbol{l}$ je element konture, dobije se:

$$\phi = \int_S \boldsymbol{B} \cdot \boldsymbol{n} \cdot \mathrm{d}S = \int_S \operatorname{rot} \boldsymbol{A} \cdot \boldsymbol{n} \cdot \mathrm{d}S = \oint_c \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{l}$$

U pravocrtnom koordinatnom sustavu (x, y, z) Poissonova jednadžba (25.9) je:

$$\Delta \boldsymbol{A} = \boldsymbol{a}_x \Delta A_x + \boldsymbol{a}_y \Delta A_y + \boldsymbol{a}_z \Delta A_z = -\mu \cdot \boldsymbol{J}_s \qquad (25$$

U cilindričnom koordinatnom sustavu (r, α, z) Poissonova jednadžba (25.9) je:

$$\Delta \boldsymbol{A} = \boldsymbol{a}_r \left[\Delta A_r - \frac{1}{r^2}\left(A_r + 2\frac{\partial A_\alpha}{\partial \alpha} \right) \right] + \boldsymbol{a}_\alpha \left[\Delta A_\alpha - \frac{1}{r^2}\left(A_\alpha - 2\frac{\partial A_r}{\partial \alpha} \right) \right] + \boldsymbol{a}_z \Delta A_z = -\mu \cdot \boldsymbol{J}_s \qquad (25$$

U sfernom koordinatnom sustavu (r, ϑ, α) Poissonova jednadžba (25.9) je:

$$\begin{aligned} \Delta \boldsymbol{A} = \boldsymbol{a}_r & \left[\Delta A_r - \frac{2}{r^2}\left(A_r + \mathrm{ctg}\,\vartheta \cdot A_\vartheta + \frac{1}{\sin\vartheta}\frac{\partial A_\alpha}{\partial \alpha} + \frac{\partial A_\vartheta}{\partial \vartheta} \right) \right] + \\ & + \boldsymbol{a}_\vartheta \left[\Delta A_\vartheta - \frac{1}{r^2}\left(\frac{1}{\sin^2\vartheta} A_\vartheta - 2\frac{\partial A_r}{\partial \vartheta} + \frac{2\cos\vartheta}{\sin^2\vartheta}\frac{\partial A_\alpha}{\partial \alpha} \right) \right] + \\ & + \boldsymbol{a}_\alpha \left[\Delta A_\alpha - \frac{1}{r^2}\left(\frac{1}{\sin^2\vartheta} A_\alpha - \frac{2}{\sin\vartheta}\frac{\partial A_r}{\partial \alpha} - \frac{2\cos\vartheta}{\sin^2\vartheta}\frac{\partial A_\vartheta}{\partial \alpha} \right) \right] = -\mu \cdot \boldsymbol{J}_s \end{aligned} \qquad (25$$

Poissonova jednadžba za vektorski magnetski potencijal u pravocrtnom koordinatnom sustavu (25.11) svodi se na tri skalarne diferencijalne jednadžbe po komponentama (x, y, z):
$\Delta A_x = -\mu \cdot J_x$; $\Delta A_y = -\mu \cdot J_y$; $\Delta A_z = -\mu \cdot J_z$

Jednadžbe vektorskog magnetskog potencijala u tocki promatranja ***r*** s

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu}{4\pi}\int_V \frac{\boldsymbol{J}(\boldsymbol{r}')\cdot \mathrm{d}V}{|\boldsymbol{r}-\boldsymbol{r}'|} \qquad \boldsymbol{A}(\boldsymbol{r}) = \frac{\mu}{4\pi}\int_S \frac{\boldsymbol{K}(\boldsymbol{r}')\cdot \mathrm{d}S}{|\boldsymbol{r}-\boldsymbol{r}'|} \qquad \boldsymbol{A}(\boldsymbol{r}) = \frac{\mu \cdot I}{4\pi}\int_l \frac{\mathrm{d}\boldsymbol{l}}{|\boldsymbol{r}-\boldsymbol{r}'|}$$

INDUKTIVITET

$$L = \frac{\phi}{I} \qquad \psi = N \cdot \phi \qquad L = \frac{\psi}{I} = \frac{N \cdot \phi}{I}$$

$$L = \frac{\psi}{I} = \frac{N \cdot \phi}{I} = \frac{N \cdot N \cdot \phi_1}{I} = N^2 \frac{\phi_1}{I} = N^2 \cdot L_1$$

Međuniduktivitet

Omjer magnetskog toka obuhvacenog krugom „2“ _21 i struje I_1 strujnog kruga „1“ koja je proizvela taj magnetski tok naziva se *koeficijent meduindukcije* ili *meduinduktivitet* M_{21} izmedu krugova „1“ i „2“

$$M_{21} = \frac{\phi_{21}}{I_1}$$

Ako je petlja strujnog kruga „1“ nacinjena s N_1 zavoja tada ce njezin ukupni magnetski tok _1u kao i dio tog toka koji prolazi kroz petlju „2“ _21u biti N_1 puta veci

$$\phi_{1u} = N_1 \cdot \phi_1 \quad ; \quad \phi_{21u} = N_1 \cdot \phi_{21} = k_1 \cdot N_1 \cdot \phi_1 = k_1 \cdot \phi_{1u}$$

Ako je petlja strujnog kruga „2“ nacinjena s N_2 zavoja tada ce ukupni magnetski tok kojeg ona obuhvaca _21u biti N_2 puta veci:

$$\psi_{21u} = N_2 \cdot \phi_{21u} = N_2 \cdot N_1 \cdot \phi_{21} \qquad M_{21} = \frac{\psi_{21u}}{I_1} = \frac{N_2 \cdot N_1 \cdot \phi_{21}}{I_1}$$

Analogno, vrijedi za meduinduktivitet izmedu krugova „2“ i „1“

$$M_{12} = \frac{\psi_{12u}}{I_2} = \frac{N_1 \cdot N_2 \cdot \phi_{12}}{I_2} = \frac{N_1 \cdot N_2 \cdot k_2 \cdot \phi_2}{I_2} = N_1 \cdot k_2 \frac{N_2 \cdot \phi_2}{I_2} = k_2 \cdot N_1 \frac{L_2}{N_2}$$

$$M_{21} = \frac{\psi_{21u}}{I_1} = \frac{N_2 \cdot N_1 \cdot \phi_{21}}{I_1} = \frac{N_2 \cdot N_1 \cdot k_1 \cdot \phi_1}{I_1} = N_2 \cdot k_1 \frac{N_1 \cdot \phi_1}{I_1} = k_1 \cdot N_2 \frac{L_1}{N_1}$$

Koeficijent k naziva se *koeficijent magnetske sprege* i uvijek je <=1.

$$M = k\sqrt{L_1 L_2} \le \sqrt{L_1 L_2}$$

Ako magnetsko polje stvara strujnica „1“ konture l_1, protjecana strujom I_1, njezin vektorski magnetski potencijal je

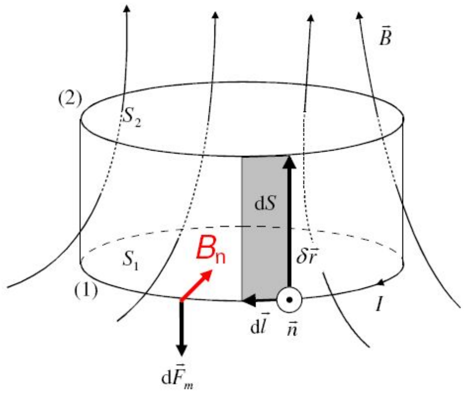
$$\mathbf{A}_1(\mathbf{r}) = \frac{\mu \cdot I_1}{4\pi} \oint_{l_1} \frac{\mathrm{d}\mathbf{l}_1}{|\mathbf{r} - \mathbf{r}'|}$$

Obuhvaceni magnetski tok strujnicom „2“ konture l_2

$$\psi_{21} = \oint_{l_2} \mathbf{A}_1 \cdot \mathrm{d}\mathbf{l}_2 = \frac{\mu \cdot I_1}{4\pi} \oint_{l_2} \oint_{l_1} \frac{\mathrm{d}\mathbf{l}_1 \cdot \mathrm{d}\mathbf{l}_2}{|\mathbf{r} - \mathbf{r}'|} \qquad M_{21} = M = \frac{\psi_{21}}{I_1} = \frac{\mu}{4\pi} \oint_{l_2} \oint_{l_1} \frac{\mathrm{d}\mathbf{l}_1 \cdot \mathrm{d}\mathbf{l}_2}{|\mathbf{r} - \mathbf{r}'|}$$

meduinduktivitet dvaju medusobno okomitih dijelova strujnica, zbog d$\mathbf{l}_1$_d$\mathbf{l}_2$ = 0, jednak nuli

ENERGIJA STATICKOG MAGNETSKOG POLJA



Na diferencijalni dio strujnice d$\boldsymbol{l}$ djeluje magnetska sila

$$\mathrm{d}\boldsymbol{F}_m = I\left(\mathrm{d}\boldsymbol{l} \times \boldsymbol{B}\right)$$

Aksijalnu silu, suprotnu od smjera pomaka, stvara komponenta magnetske indukcije B_n okomita na struju. Diferencijalni rad kojeg obavi vanjska sila d$\boldsymbol{F}_v$ pri pomaku strujnice za $\delta\boldsymbol{r}$ pretvori se u povecanje energije strujnice

$$\delta W = \mathrm{d}\boldsymbol{F}_v \cdot \delta\boldsymbol{r} = -\mathrm{d}\boldsymbol{F}_m \cdot \delta\boldsymbol{r} = -I\left(\mathrm{d}\boldsymbol{l} \times \boldsymbol{B}\right)\delta\boldsymbol{r}$$

$$\delta W = -I \cdot \boldsymbol{B}\left(\delta\boldsymbol{r} \times \mathrm{d}\boldsymbol{l}\right) = -I \cdot \boldsymbol{B} \cdot \boldsymbol{n} \cdot \mathrm{d}S \quad ; \quad \delta\boldsymbol{r} \times \mathrm{d}\boldsymbol{l} = \boldsymbol{n} \cdot \mathrm{d}S$$

ukupno povecanje energije cijele strujnice pri pomaku iz položaja „1“ u položaj „2“ je

$$\mathrm{d}W = I \cdot \mathrm{d}\phi_{pl} \qquad \mathrm{d}\phi_{pl} = \phi_2 - \phi_1 \qquad W = I \cdot \phi_2 = I \cdot \phi$$

Magnetska energija sustava strujnica

ukupna energija W potrebna da se formira sustav dviju strujnica

$$W = \frac{1}{2}\left(\phi_{12} \cdot I_2 + \phi_{21} \cdot I_1\right)$$

Za skupinu od n strujnica

$$W = \frac{1}{2}\sum_{i=1}^{n} I_i \cdot \sum_{j=1}^{n} \phi_{ji} = \frac{1}{2}\sum_{i=1}^{n} I_i \cdot \phi_i$$

Magnetska energija prostorno raspodijeljene struje

Ako prostorom kroz n vodica teku struje gustoce $\boldsymbol{J}_i$, ukupni obuhvaceni magnetski tok kroz $i$ti vodic je

$$\phi_i = \sum_{j=1}^{n} \oint_{l_i} \boldsymbol{A}_j \cdot \mathrm{d}\boldsymbol{l}_i \qquad I_i = \int_{S_i} \boldsymbol{J}_i \cdot \boldsymbol{n}_i \cdot \mathrm{d}S_i$$

Ukupna magnetska energija je

$$W = \frac{1}{2}\sum_{i=1}^{n} I_i \cdot \phi_i = \frac{1}{2}\sum_{i=1}^{n} \int_{S_i} \boldsymbol{J}_i \cdot \boldsymbol{n}_i \cdot \mathrm{d}S_i \left(\sum_{j=1}^{n} \oint_{l_i} \boldsymbol{A}_j \cdot \mathrm{d}\boldsymbol{l}_i \right)$$

$$\boldsymbol{n}_i \cdot \mathrm{d}l_i = \mathrm{d}\boldsymbol{l}_i \quad ; \quad \mathrm{d}S_i \cdot \mathrm{d}l_i = \mathrm{d}V_i$$

$$W = \frac{1}{2}\int_V \boldsymbol{J} \cdot \boldsymbol{A} \cdot \mathrm{d}V$$

A **-** ukupni vektorski magnetski potencijal kojeg na mjestu diferencijalnog volumena d*V* stvaraju sve struje u prostoru

Magnetska energija prikazana preko velicina polja

$$W = \frac{1}{2}\int_V \boldsymbol{J} \cdot \boldsymbol{A} \cdot \mathrm{d}V = \frac{1}{2}\int_V \boldsymbol{B} \cdot \boldsymbol{H} \cdot \mathrm{d}V + \frac{1}{2}\int_V \nabla \cdot (\boldsymbol{H} \times \boldsymbol{A}) \cdot \mathrm{d}V$$

$$W = \frac{1}{2}\int_V \boldsymbol{B} \cdot \boldsymbol{H} \cdot \mathrm{d}V + \frac{1}{2}\oint_S (\boldsymbol{H} \times \boldsymbol{A}) \cdot \boldsymbol{n} \cdot \mathrm{d}S$$

$$W = \frac{1}{2}\int_V \boldsymbol{B} \cdot \boldsymbol{H} \cdot \mathrm{d}V$$

volumen *V* obuhvaca cijeli prostor do u beskonacnost

Ako je magnetski materijal linearan, homogen i izotropan, vrijedi ***B*** = μ***H***, odnosno ***H*** = ***B***/μ

$$W = \frac{1}{2}\mu\int_V H^2 \mathrm{d}V = \frac{1}{2\mu}\int_V B^2 \mathrm{d}V$$

$$\mathrm{d}W = i \cdot \mathrm{d}\phi$$

$$\mathrm{d}\phi = \int_S \mathrm{d}\boldsymbol{B} \cdot \boldsymbol{n} \cdot \mathrm{d}S \quad ; \quad i = \oint_l \boldsymbol{H} \cdot d\boldsymbol{l} \quad ; \quad \boldsymbol{n} \cdot \mathrm{d}l = \mathrm{d}\boldsymbol{l} \quad ; \mathrm{d}S \cdot \mathrm{d}l = \mathrm{d}V$$

prirast magnetske energije je:

$$\mathrm{d}W = \int_V \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{B} \cdot \mathrm{d}V$$

Ukupna magnetska energija je

$$W = \int_V \left(\int_{B=0}^{B} \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{B} \right) \mathrm{d}V$$

gustoca energije magnetskog polja

$$w = \int_{B=0}^{B} \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{B}$$

Za linearan, homogen i izotropan magnetski materijal (μ = konst.), gustoca magnetske energije je

$$w = \int_{B=0}^{B} \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{B} = \frac{1}{\mu}\int_{B=0}^{B} B \cdot \mathrm{d}B = \frac{1}{2\mu}B^2 = \frac{1}{2}\mu \cdot H^2 \quad (\mathrm{J/m^3})$$

Ako linearni materijal magnetiziramo tako da povecavamo indukciju od ništice do iznosa *B*, d*B* > 0, pa je gustoca magnetske energije > 0, tj. izvor predaje energiju magnetskom polju. Ako linearni materijal onda demagnetiziramo tako da smanjujemo indukciju od iznosa *B* do ništice, d*B* < 0, pa je gustoca magnetske energije < 0, tj. magnetsko polje vraca energiju izvoru.

Magnetska energija strujnog kruga

$$\phi = L \cdot I \qquad W = \frac{1}{2}I \cdot \phi = \frac{1}{2}L \cdot I^2 = \frac{1}{2L}\phi^2$$

Magnetska energija dva strujna kruga

$$\phi_1 = L_1 I_1 \pm M I_2 \quad ; \quad \phi_2 = L_2 I_2 \pm M I_1$$

prvi clanovi predstavljaju magnetske tokove stvorene vlastitim strujama, a drugi clanovi magnetske tokove proizvedene strujom drugog strujnog kruga. Predznak „+“ je za tzv. *suglasnu vezu* u kojoj se ti magnetski tokovi podudaraju, a predznak „–“ je za tzv. *nesuglasnu vezu* u kojoj se ti magnetski tokovi ne podudaraju

$$W = \frac{1}{2} I_1 \cdot \phi_1 + \frac{1}{2} I_2 \cdot \phi_2 = \frac{1}{2} I_1 (L_1 I_1 \pm M I_2) + \frac{1}{2} I_2 (L_2 I_2 \pm M I_1)$$

$$W = \frac{1}{2} L_1 \cdot I_1^2 + \frac{1}{2} L_2 \cdot I_2^2 \pm M I_1 I_2$$

$$W = \frac{1}{2} I_1 \cdot \phi_1 + \frac{1}{2} I_2 \cdot \phi_2 = \frac{1}{2} I_1 (\phi_{11} \pm \phi_{12}) + \frac{1}{2} I_2 (\phi_{22} \pm \phi_{21})$$

$$I_1 = \frac{\phi_{11}}{L_1} \quad ; \quad I_1 = \frac{\phi_{21}}{M} \quad ; \quad I_2 = \frac{\phi_{22}}{L_2} \quad ; \quad I_2 = \frac{\phi_{12}}{M}$$

$$W = \frac{1}{2} \frac{\phi_{11}^2}{L_1} \pm \frac{1}{2} \frac{\phi_{21}}{M} \phi_{12} + \frac{1}{2} \frac{\phi_{22}^2}{L_2} \pm \frac{1}{2} \frac{\phi_{12}}{M} \phi_{21}$$

$$W = \frac{1}{2} \frac{\phi_{11}^2}{L_1} + \frac{1}{2} \frac{\phi_{22}^2}{L_2} \pm \frac{\phi_{12} \cdot \phi_{21}}{M}$$

SILE U STATICKOM MAGNETSKOM POLJU

sila na strujnicu duljine *l*, protjecanu strujom *I* koja se nalazi u magnetskom polju indukcije ***B***:

$$\boldsymbol{F} = \int_l I (\mathrm{d}\boldsymbol{l} \times \boldsymbol{B}) \qquad i \cdot \mathrm{d}\boldsymbol{l} = \boldsymbol{J} \cdot \mathrm{d}V \qquad \boldsymbol{F} = \int_V (\boldsymbol{J} \times \boldsymbol{B}) \mathrm{d}V$$

Odredivanje sila pomocu energije

silu u smjeru osi *s* u strujnom krugu induktiviteta *L*. Za izolirani strujni krug

$$F_s = -\left\{\frac{\partial W_m}{\partial s}\right\}_{\phi=\text{konst.}} = -\frac{\partial}{\partial s}\left\{\frac{1}{2}\frac{\phi^2}{L}\right\}_{\phi=\text{konst.}} = -\frac{1}{2}\phi^2 \frac{\partial}{\partial s}\left(\frac{1}{L}\right)$$

Za neizolirani strujni krug

$$F_s = \left\{\frac{\partial W_m}{\partial s}\right\}_{I=\text{konst.}} = \frac{\partial}{\partial s}\left\{\frac{1}{2} L I^2\right\}_{I=\text{konst.}} = \frac{1}{2} I^2 \frac{\partial L}{\partial s}$$

u sustavu dva strujna kruga s meduinduktivitetom *M*. Za izolirani sustav

$$F_s = -\left\{\frac{\partial W_m}{\partial s}\right\}_{\phi=\text{konst.}} = -\frac{\partial}{\partial s}\left\{\pm \frac{\phi_{12} \cdot \phi_{21}}{M}\right\}_{\phi=\text{konst.}} = \mp \phi_{12} \cdot \phi_{21} \frac{\partial}{\partial s}\left(\frac{1}{M}\right)$$

Za neizolirani sustav

$$F_s = \left\{\frac{\partial W_m}{\partial s}\right\}_{I=\text{konst.}} = \frac{\partial}{\partial s}\{\pm M I_1 \cdot I_2\}_{I=\text{konst.}} = \pm I_1 \cdot I_2 \frac{\partial M}{\partial s}$$