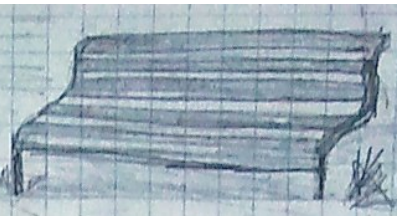
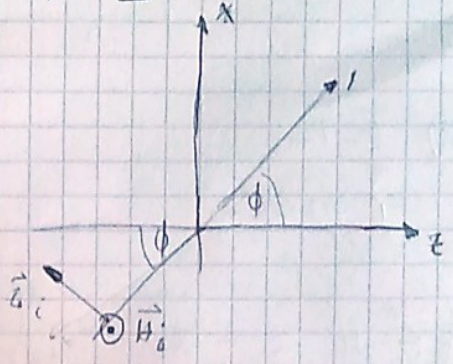
EMP 3. auditorne vježbe 2. ciklus



2003./2011., IPZ1

① EM val putuje pod kutem ϕ prema osi z u sredstvu. Električno polje

$$\vec{H}_i = H_0 \, 2e\left[e^{j(2\pi \cdot 10^8 t - \underbrace{\vec{p}(x+\sqrt{3}z)}_{\vec{p}\cdot\vec{r}})} \right] \vec{a}_y \quad \left[\frac{A}{m}\right]$$



1° $f = 10^8$ Hz

2° $\beta_x x + \beta_z z$

$\vec{p} = (\vec{a}_x + \sqrt{3}\vec{a}_z)\,\vec{1}$

$p = \sqrt{1^2+(\sqrt{3})^2}\,\vec{1} = 6.283 \; \frac{rad}{m}$

$$\boxed{\lambda = \frac{2\pi}{p} = 1\,m}$$

3° $c = \lambda f = 10^8$ m/s

4° $\phi = ?$

$tg\phi = \frac{p_z}{p_x} = 1/\sqrt{3} \Rightarrow \phi = \pi/6$

② Izvodu z cilindrično vodiče mag. ind. može se aproksimirati s:

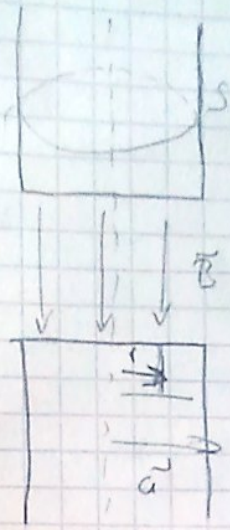
$$B = \begin{cases} -B_0 \frac{at}{a^2 r^2}\vec{a}_\phi, & t>0 \\ 0, & t<0 \end{cases}$$

$a = 0.1$ m

$B_0 = 1$ T

$\chi = 0$

$\varepsilon_r = 1$



1°) $t = 1ms$

$r = 0.05m$

$$\oint_{\gamma} \vec{E}\, d\vec{l} = -\frac{\partial \Phi}{\partial t} = -\frac{\partial}{\partial t}\iint_{S} \vec{B}\,\vec{n}\, ds$$

$\vec{B}$ sono uscenti

$B = f(r)$

$ds = 2\pi r\, dr$

$$E_i \, 2\pi r = \frac{\partial}{\partial t}\int_{y=0}^{r} \frac{B_0 a t}{a^2+y^2}\, 2\pi y\, dy$$

$$= \frac{\partial}{\partial t}\left[B_0 a t 2\pi \frac{1}{2} \ln \frac{a^2+r^2}{a^2} \right]$$

$$\vec{E}_i = \frac{B_0 a}{2r} \ln\left(\frac{a^2+r^2}{a^2}\right) \longrightarrow$$

$t = 1ms$ $\quad E = 0,22\ V/m$
$0,5m$

2° $t = 3ms$
$0,1m$ $\quad E = 0,35\ V/m$

3° $t = 1ms$ $\quad E = 0 V/m$
$0,05m$

3. Odredite iznos mag. polja koje prolazi osi magneta t = 1 ms

$$\phi = \iint \vec{B}\,\vec{n}\,ds = \int_{y=a}^{a} \frac{B_0 a t}{a^2+y^2}\, 2\pi y\, dy$$

$$= \frac{B_0 a t}{2}\, 2\pi \ln \frac{2a^2}{a^2}$$

$$\phi\Big|_{t=1ms} = 218\ \mu Wb$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\frac{\partial}{\partial r}\left(r E_\varphi \right) = -B_0 a \frac{r}{r^2+a^2}$$

konst. int.

$$E_\varphi = -\frac{B_0 a}{2r} \ln(r^2+a^2) + \frac{K}{r}$$

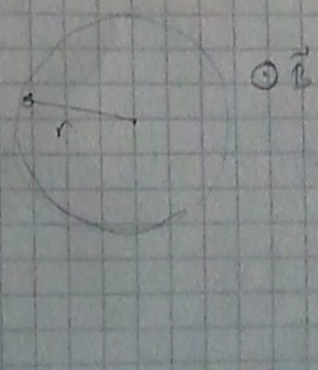
4. Kružni piston tan. pop. presjeka, radijusa r prema slici nalazi se u polju ind. $\vec{B}$. Pistone se zagrijavanjem s vremenom mijenja prema jedn.

$r = r_0 + k_1 t \qquad r_0 = 1m, \ k_1 = 0.001\ m/s$

Vrem. promjena mag. ind. određena je $B = 1 + k_2 t$ [T]

$k_2 = 0.005$ [T/s]

Mag. tok kroz petlju u t = 50 s



$$\phi = B \cdot S = (1+k_2 t)(r_0 + k_1 t)^2 \pi$$

$$\phi(t=50s) = 4.33\ Wb$$

2° U_{ob} u $t=15$ s uslijed promjene mag. polja u vremenu

$\hookrightarrow e = \left| (r_0 + k_1 t)^2 \pi \cdot \frac{\partial B}{\partial t} \right| = \left| (r_0 + k_1 t)^2 \pi \cdot k_2 \right| = 16.2\,mV$

3° U_{ob} u petlji $t = 15$ s zbog promjene radiusa

$$e = \left| B \frac{\partial S}{\partial t} \right| = \left| (1 + k_2 t) \cdot 2 (r_0 + k_1 t) \cdot k_1 \pi \right| = 6.9\,mV$$

4° $R = 0.1\,\Omega$ → otpor petlje $I(t = 100\,s) = ?$

$$i = \frac{e}{R} = \dots = 0.29\,A$$

$$e = \left| \frac{\partial \varphi}{\partial t} \right| = \left| k_2 (r_0 + k_1 t)^2 \pi + (1 + k_2 t) 2 (r_0 + k_1 t) \pi \, k_1 \right|$$

(5.) Ravni val u sredstvu

$\mu_r = 1$

$\varepsilon_r = 3$

$\vec{H} = \vec{a}_y \, 10 \cos(\omega t - 2x) \; A/m$

4° $+x$ – os

$\vec{p} = +2\vec{a}_x$

1°) $\lambda = \frac{2\pi}{2\,rad/m} = \pi\,[m]$

$\beta = 2\,rad/m$

2°) $c = ?$

$$c = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{c_0}{\sqrt{\mu_r \varepsilon_r}} = \frac{c_0}{\sqrt{3}} = \sqrt{3} \cdot 10^8\,m/s$$

3° $E(t = 15\,ns)$, $x = 0.6\,m$

$Z = \sqrt{\frac{\mu}{\varepsilon}}$ → karakt. imp. $= Z_0 \frac{1}{\sqrt{\varepsilon_r}} = 217.66\,\Omega$

$Z_0 = 120\pi \; (377)$

$E = |\vec{H} \cdot Z| = |-1429| = 1429 \; \frac{A}{m}$

5) Ne. ravni val. ... val.

zad:
$\lambda_0 = 2\pi m$

kad se širi u sredstvu nep. znač. $\lambda = \lambda/4$ $\frac{E}{H} = 2 \cdot \frac{E_0}{H_0}$

1° ε_r, μ_r

$c = \lambda f$ ⇒ $c = \frac{c_0}{4}$
konst.

$$\frac{1}{\sqrt{\mu_0 \varepsilon_0}} \frac{1}{\sqrt{\mu_r \varepsilon_r}} = \frac{c_0}{4} \Rightarrow \mu_r \varepsilon_r = 16$$

$$\sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = 2 \cdot \sqrt{\frac{\mu_0}{\varepsilon_0}} \Rightarrow \mu_r = 4 \varepsilon_r$$

$\mu_r \varepsilon_r = 16$
$\mu_r = 4 \varepsilon_r$

$\varepsilon_r = 2 \Rightarrow \mu_r = 8$

2°
$c_0 = \lambda_0 f$

$3 \cdot 10^8 \frac{m}{s} = 2\pi f \Rightarrow \omega = 3 \cdot 10^8 \frac{rad}{s}$

3°
$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{2\pi} \cdot 4 = 4 \frac{rad}{m}$

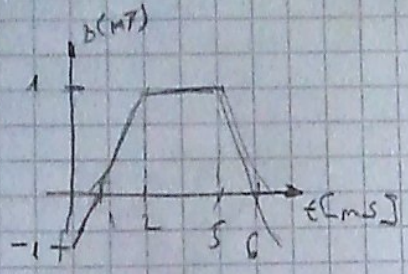
↳ fazna konst.

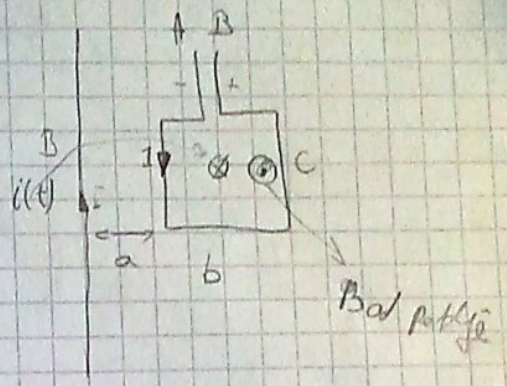
Na udaljenosti 2m od beskonačno dugog vodiča kroz koje teče struja I nalazi se petlja

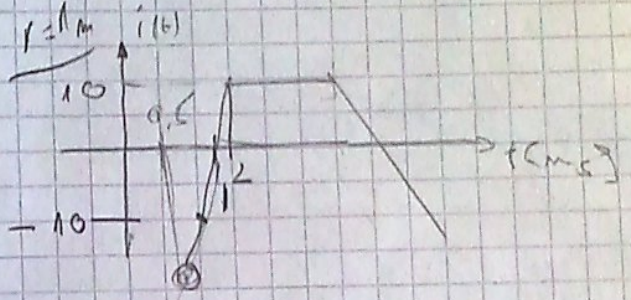
$a = 1m$

$b = 1m$

$c = 2m$







$$B = \frac{\mu_0 i(t)}{2\pi r}$$

$$\phi = \int_S \frac{\mu_0 i(t)}{2\pi r} c\,dr$$

$$= c \int_{r=a}^{a+b} \frac{\mu_0 i(t)}{2\pi r}\,dr \rightarrow \phi = \frac{\mu_0 i(t)}{2\pi} \ln\left(\frac{a+b}{a}\right)$$

1°) $U_{ab}\ (t = 0.5\,ms)$

$$\frac{\partial i}{\partial t} = \frac{20 \cdot 10^{-3}}{2 \cdot 10^{-3}} = 10\,A$$

$$|e| = \left|\frac{\partial \phi}{\partial t}\right| = 1.62\,V$$

Pazi!
Struja ide od "-" prema "+"
(struja struja)

$$\Rightarrow U_{ab} = -1.62\,V$$

$t = 3ms$ i

$U_{ab} = 0 \rightarrow$ nema promjene

$t = 5ms$

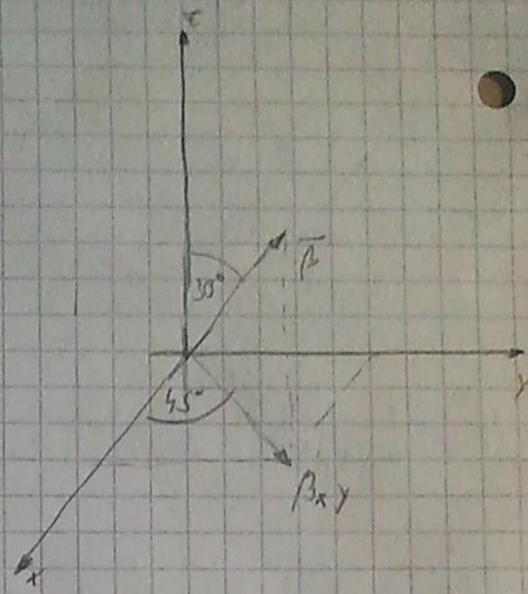
sada predznak se mijenja

(7)

Svijet širenja vala 6 = 10 MHz

+30° s z-osi $\mu_r = \mu_0$

x_y 45° s x-osi



$\varphi = -\frac{\pi}{6}$

E - nema z

$E(t = 10^{-6} s)\Big|_{T(0,0,0)} = 10\cos\left(\omega t - \frac{\pi}{6}\right)$ V/m

$E_{0x} > 0$

$E_{0x}, E_{0y}, H_{0x}, H_{0y} = ?$

$$\vec{\beta} = \left(\sin 30^\circ \cdot \cos 45^\circ \vec{a}_x + \sin 30^\circ \cdot \sin 45^\circ \vec{a}_y + \cos 30^\circ \vec{a}_z\right)\beta_0$$

($\sqrt{2}/4$, $\sqrt{2}/4$, β_z)

$$\beta = \frac{\omega}{c} = \frac{2\pi \cdot 10^6}{3 \cdot 10^8} = 0.209 \text{ rad/m}$$

$\beta \cdot \vec{E} = 0$ (okomiti su) $\vec{\beta} \perp \vec{E}$

$E_0 = 10 \quad E_{0x}^2 + E_{0y}^2 = 10^2$

$E_{0z} = 0$ (zadano)

$\frac{\sqrt{2}}{4} E_{0x} + \frac{\sqrt{2}}{4} E_{0y} = 0$

$E_{0x} = -E_{0y}$

$E_{0x} = \sqrt{50}$ V/m

$E_{0y} = -\sqrt{50}$ V/m

$$\vec{E} = \left(\sqrt{50}\,\vec{a}_x - \sqrt{50}\,\vec{a}_y\right)\cos\left(\omega t - \frac{\sqrt{2}}{4}\beta_0 x - \frac{\sqrt{2}}{4}\beta_0 y - \frac{\sqrt{3}}{2}\beta_0 z - \frac{\pi}{6}\right)$$

$$\vec{H} = \frac{1}{\omega\mu}(\vec{\beta} \times \vec{E})$$

$$\vec{\beta} \times \vec{E} = \left(-\frac{\sqrt{100}}{4}\vec{a}_z - \frac{\sqrt{100}}{4}\vec{a}_z + \frac{\sqrt{150}}{2}\vec{a}_y + \frac{\sqrt{150}}{2}\vec{a}_x\right)\beta_0$$

$$\omega\mu = 78.96$$

$$\vec{H}_{ox} = \beta_0 \frac{\sqrt{150}}{2} \frac{1}{\omega\mu}\vec{a}_x = 0.016\,\vec{a}_x \ \frac{A}{m}$$

$$\vec{H}_{oy} = \beta_0 \frac{\sqrt{150}}{2} \frac{1}{\omega\mu}\vec{a}_y = 0.016\,\vec{a}_y \ A/m$$