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Elektromagnetska polja

2. M.I. ak. god. 2006./2007.

- skenirani postupci rješavanja, verzija1.0.1.
- navedena rješenja su potvrđena službenom obaviješću

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2. M.I. 2006./2007.
Elektromagnetska polja

I

$$\vec{A} = \frac{1}{x+2y+3}\,\vec{a}_z$$

1) $B(1,2,3) = ?$

$$\vec{B} = \nabla \times \vec{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & \frac{1}{x+2y+3} \end{vmatrix} =$$

$$= \vec{a}_x \frac{\partial}{\partial y}\left(\frac{1}{x+2y+3}\right) - \vec{a}_y \frac{\partial}{\partial x}\left(\frac{1}{x+2y+3}\right) + \vec{a}_z \cdot \emptyset$$

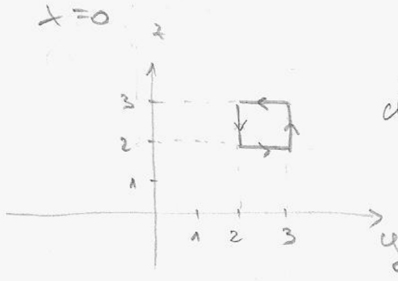
$$= \frac{-2}{(x+2y+3)^2}\,\vec{a}_x + \frac{1}{(x+2y+3)^2}\,\vec{a}_y \Big|_{(1,2,3)}$$

$$= \frac{-2}{64}\,\vec{a}_x + \frac{1}{64}\,\vec{a}_z$$

$$B = \sqrt{\left(\frac{1}{32}\right)^2 + \left(\frac{1}{64}\right)^2} = 34{,}94\ \text{mT} \qquad \boxed{E}$$

2) $\phi = \oint_c \vec{A} \cdot d\vec{l}$

$x = 0$



$$d\vec{l} \begin{cases} dy\,\vec{a}_y\ ; & z = 2 \quad 2 \le y \le 3\ ;\ x = 0 \\ dz\,\vec{a}_z\ ; & y = 3 \quad 2 \le z \le 3\ ;\ x = 0 \\ dy\,\vec{a}_y\ ; & z = 3 \quad 3 \ge y \ge 2\ ;\ x = 0 \\ dz\,\vec{a}_z\ ; & y = 2 \quad 3 \ge z \ge 2\ ;\ x = 0 \end{cases}$$

$$\phi = \int_2^3 \frac{1}{x+2y+3}\Big|_{y=3;\,x=0} dz + \int_3^2 \frac{1}{x+2y+3}\Big|_{y=2;\,x=0} dz$$

$$= \left(\frac{1}{9} - \frac{1}{7}\right) \cdot \int_2^3 dz = -31{,}75\ \text{mWb} \qquad \boxed{C}$$

3) $J(3,2,1) = ?$

$$\nabla \times \vec{B} = \mu_0 \vec{J} \Rightarrow \vec{J} = \frac{1}{\mu_0} \nabla \times \vec{B}$$

$$= \frac{1}{\mu_0} \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{-2}{(x+2y+3)^2} & \frac{1}{(x+2y+3)^2} & 0 \end{vmatrix}$$

$$= \frac{1}{\mu_0} \left(\vec{a}_x \cdot 0 - \vec{a}_y \cdot 0 + \vec{a}_z \left[\frac{\partial}{\partial x} \frac{1}{(x+2y+3)^2} + \frac{\partial}{\partial y} \frac{2}{(x+2y+3)^2} \right] \right)$$

$$= \frac{1}{\mu_0} \vec{a}_z \left[\frac{-2}{(x+2y+3)^3} + \frac{-8}{(x+2y+3)^3} \right] = \frac{-10}{\mu_0 (x+2y+3)^3} \vec{a}_z$$

$J(3,2,1) = 7958 \ A/m^2$ $\boxed{B}$

4) $I = \iint \vec{J} \vec{n} \, dS = \frac{-10}{\mu_0} \iint \frac{1}{(x+2y+3)^3} dx \, dy$

$$= -\frac{10}{\mu_0} \int_2^3 dy \int_2^3 \frac{1}{(x+2y+3)^3} dx$$

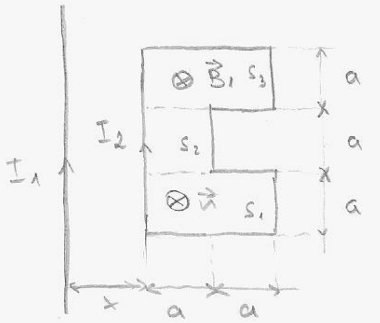
$$= -\frac{10}{\mu_0} \int_2^3 dy \cdot \frac{-1}{2(x+2y+3)^2} \Big|_{x=2}^{3}$$

$$= \frac{5}{\mu_0} \int_2^3 \frac{1}{(2y+6)^2} - \frac{1}{(2y+5)^2} dy$$

$$= \frac{5}{\mu_0 2} \left[\frac{1}{2y+5} - \frac{1}{2y+6} \Big|_{y=2}^{3} \right] = \frac{15}{2\mu_0} \left[\frac{1}{11} - \frac{1}{12} - \frac{1}{9} + \frac{1}{10} \right]$$

$= -7033{,}4 \ A$ $\boxed{D}$

II. POLOŽAJ (A)



$B = \frac{\mu_0 I}{2\pi r}$ → mag. polje beskonačno dugog vodiča na udaljenosti r od istog

$L_{12} = \frac{\phi_{12}}{I_2}$

$\phi_{12} = \iint_{S_{petlje}} \vec{B}_1 \vec{n}\, dS$

5.)

$\vec{n}$ → biramo u smjeru mag polja tj. „u papir"

S → dijelimo na 3 djela

$$\phi_{12} = \frac{I_1 \mu_0}{2\pi r}\left[\iint_{S_1+S_3} \frac{dS}{x} + \iint_{S_2} \frac{dS}{x}\right]$$

$$= \frac{I_1 \mu_0}{2\pi}\left[2\iint_{S_1} \frac{dS}{x} + \iint_{S_2} \frac{dS}{x}\right] = \frac{I \mu_0}{2\pi}\left[2\int_x^{x+2a} \frac{a\,dx}{x} + \int_x^{x+a} \frac{a\,dx}{x}\right]$$

$$= \frac{I_1 \mu_0}{2\pi} a\left[2 \ln\frac{x+2a}{x} + \ln\frac{x+a}{x}\right]$$

$L_{12} = 115{,}615\ \mu H$ $\boxed{D}$

6) $\vec{dl}_2 = dx\,\vec{a}_x + dy\,\vec{a}_y$

$\vec{B}_1 = -\frac{\mu_0 I_1}{2\pi x}\vec{a}_z$

$$\vec{dl}_2 \times \vec{B} \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ dx & dy & 0 \\ 0 & 0 & -\frac{\mu_0 I_1}{2\pi x} \end{vmatrix} = -\frac{\mu_0 I_1}{2\pi x} dy\,\vec{a}_x + \frac{\mu_0 I_1}{2\pi x} dx\,\vec{a}_y$$

$\vec{F} = I_2 \int \vec{dl}_2 \times \vec{B}$

$$F_x = \frac{-\mu_0 I_1 I_2}{2\pi}\left[\int_0^{3a} \frac{1}{x} dy + \int_{3a}^{2a} \frac{1}{x+2a} dy + \int_{2a}^{a} \frac{1}{x+a} dy + \int_a^0 \frac{1}{x+2a} dy\right]$$

$$= \frac{-\mu_0 I_1 I_2}{2\pi}\left[3 - \frac{1}{3} + \frac{1}{2} - \frac{1}{3}\right] = 366{,}\dot{6}\ \mu N$$ $\boxed{C}$

7) $\vec{F}_y = \frac{-\mu_0 I_1 I_2}{2\pi}\left[\int_x^{x+2a} \frac{1}{x}dx + \cancel{\int_{x+2a}^{x+a} \frac{1}{x}dx} + \cancel{\int_{x+a}^{x+2a} \frac{1}{x}dx} + \int_{x+2a}^{x} \frac{1}{x}dx\right]$

$= 0$ $\boxed{E}$

8) → zbog oblika strujnice i petlje u oba položaja imaju jednake energije. Stoga je razlika u energiji međudjelovanja.

$W_{12} = L_{12} \cdot I_1 \cdot I_2$

$W_{12}(A) = 115,615 \text{ nJ}$

položaj (B):

$$\phi_{12} = \iint_{S_{petlje}} \vec{B}_1 \vec{n}\, dS = \iint_S B\, dS$$

$$= \frac{\mu_0 I_1}{2\pi}\left[\int_x^{x+3a} \frac{a\,dx}{x} + \int_x^{x+a} \frac{a\,dx}{x} + \int_{x+2a}^{x+3a} \frac{a\,dx}{x}\right]$$

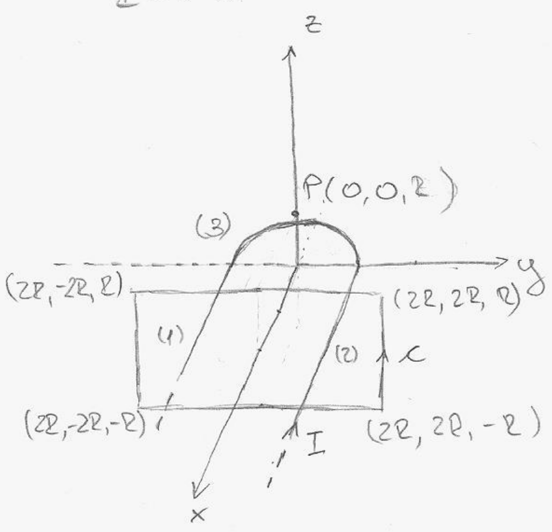
$$= \frac{\mu_0 I_1}{2\pi} a \cdot \left[\ln\frac{x+3a}{x} + \ln\frac{x+a}{x} + \ln\frac{x+3a}{x+2a}\right]$$

$W_{12}(B) = 94,685 \text{ nJ}$

$|\Delta W| = |W_{12}(A) - W_{12}(B)| = 20,93 \text{ nJ}$ $\boxed{D}$

III. $I = 10\ A$

$R = 1\ m$



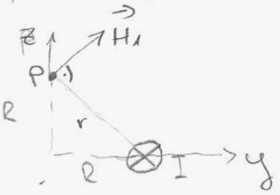
g) $\vec{H}(P) = ?$

za H_1 :

Ω_{xz}:

$$H_1 = \frac{I}{4\pi r}\left(\sin 0° + \sin 90°\right) = \frac{I}{4\pi r}$$

Ω_{zy}



$r = R\sqrt{2}$

$$\vec{H}_1 = \frac{\vec{a}_y + \vec{a}_z}{\sqrt{2}} \cdot H_1$$

pa je
$$\vec{H}_1 = \frac{I}{4\pi R\sqrt{2}} \cdot \left(\frac{\vec{a}_y + \vec{a}_z}{\sqrt{2}}\right)$$

$$= \frac{I}{8\pi R}\left(\vec{a}_y + \vec{a}_z\right)$$

→ analogno tome za H_2 :

$$\vec{H}_2 = \frac{I}{8\pi R}\left(-\vec{a}_y + \vec{a}_z\right)$$

za H_3 :

$$\vec{H}_3 = \frac{I \cdot R \cdot R\, \vec{a}_x}{4\pi\sqrt{R^2+R^2}^3}\int_{\pi/2}^{3\pi/2} \cos\alpha\, d\alpha + \frac{I \cdot R \cdot R \cdot \vec{a}_y}{4\pi\sqrt{R^2+R^2}^3}\int_{\pi/2}^{3\pi/2} \sin\alpha\, d\alpha + \frac{I\, R^2\, \vec{a}_z}{4\pi\sqrt{R^2+R^2}^3}\int_{\pi/2}^{3\pi/2} d\alpha =$$

$$= \frac{-I}{4\sqrt{2}\pi R}\,\vec{a}_x + \emptyset\,\vec{a}_y + \frac{I}{8\sqrt{2}R}\,\vec{a}_z$$

$$\vec{H} = \vec{H}_1 + \vec{H}_2 + \vec{H}_3 = -0.563\,\vec{a}_x + \emptyset\,\vec{a}_y + 1.68\,\vec{a}_z$$

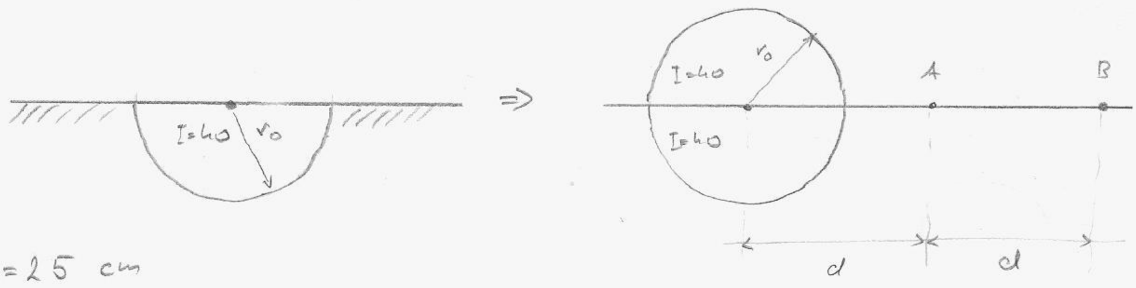
9) $H_x = -0.56$ $\boxed{C}$

10) $H_y = 0$ $\boxed{A}$

11) $H_z = 1.68$ $\boxed{B}$

12)
$$\oint_c \vec{H}\,d\vec{l} = \Sigma I = I - I = 0 \quad \boxed{A}$$

IV



$r_0 = 25$ cm

$d = 1$ m

$\rho = 100$ Ωm

14) $$\varphi = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r} - \frac{1}{r_{ref}}\right)$$

$r_{ref} = \infty$

$$\varphi_k = \underbrace{\frac{I}{4\pi K}\left(\frac{1}{r_0} - \frac{1}{r_{ref}}\right)}_{\text{„originalna" polukugla}} + \underbrace{\frac{I}{4\pi K}\left(\frac{1}{r_0} - \frac{1}{r_{ref}}\right)}_{\text{preslikavanjem dodana polukugla}} \qquad \text{uz } K = \frac{1}{\rho}$$

$$\varphi_k = \frac{I \cdot \rho}{2\pi r_0} = 2546.5 \text{ V} \qquad \boxed{D}$$

13) $$R = \frac{\varphi_k}{I} = 63.66 \qquad \boxed{E}$$

15) $$U_{ab} = \frac{I\rho}{2\pi}\left(\frac{1}{d} - \frac{1}{2d}\right) = 318.3 \text{ V}$$

$$\frac{U_{ab}}{\varphi_k} = 0.125 \qquad \boxed{C}$$

16) $\varphi_k \sim \rho$; $U_{ab} \sim \rho$

$$\frac{U_{ab}}{\varphi_k} \nsim \rho \Rightarrow 0.125 \qquad \boxed{C}$$

V.

$S = 4 \cdot 10^{-4}\ m^2$

$\delta = 3 \cdot 10^{-3}\ m$

$l_m = 250 \cdot 10^{-3}\ m$

$N = 800$ zavoja

$W_\delta = 0{,}5777\ J$

17) $W_\delta = \frac{\mu_0}{2} \iiint_V H_\delta^2\, dV = \frac{\mu_0}{2} H_\delta^2\, S \cdot \delta$

$H_\delta = \sqrt{\frac{2 W_\delta}{\mu_0 S \delta}} = 875{,}328\ kA/m$ $\boxed{E}$

18) $B_\delta = B_m = B = \mu_0 H_\delta = \mu_0 \mu_r H_m$

$B = \mu_0 H_\delta = 1{,}1\ T$

$\rightarrow$ sa krivulje za $1{,}1T \Rightarrow H_m = 200\ A/m$ $\boxed{E}$

19) $\oint H\, dl = \Sigma I$

$H_m l_m + H_\delta \delta = N \cdot I$

$I = \frac{H_m \cdot l_m + H_\delta \cdot \delta}{N} = 3{,}345\ A$ $\boxed{C}$

20) $W_m = \frac{1}{2} \iiint_V H_m \cdot B_m\, dV$

$= \frac{1}{2} H_m B_m l_m \cdot S = 11\ mJ$ $\boxed{B}$