FER2.net

Elektromagnetska polja

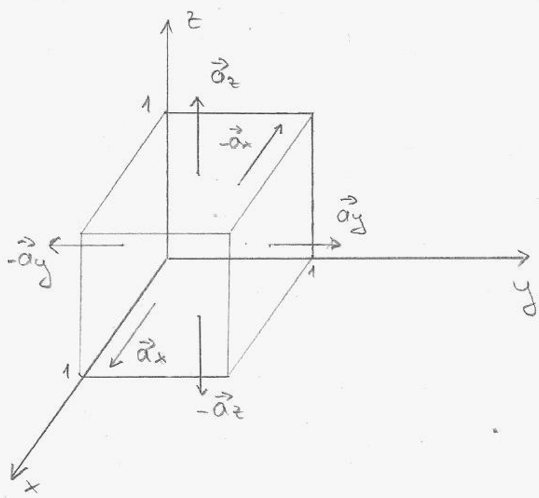
1. M.I. ak. god. 2007./2008.

- skenirani postupci rješavanja, version: 2.0
- navedena rješenja su potvrđena službenom obaviješću

by: Tywin

Napomena: sve navedene formule mogu se naći u materijalima. Korištene su:
Formule FER1 OE1
Formule za MI-te by I V A N

Elektromagnetska polja
1. M.I. 2007./2008.



I $\varphi = \frac{A}{y^3+a}$

$A=1, \quad a=1$

$\vec{E} = -\nabla\varphi = -\frac{\partial}{\partial y}\left(\frac{A}{y^3+1}\right)\vec{a}_y$

$= \frac{3Ay^2}{(y^3+a)^2}\vec{a}_y$

1) $|E(1m,1m,1m)| = \frac{3}{4} = 0,75 \ V/m$ $\boxed{D}$

$\rho = \varepsilon_0 \nabla \vec{E} = \varepsilon_0 \cdot \frac{\partial}{\partial y}\left[\frac{3Ay^2}{(y^3+a)^2}\right] = \varepsilon_0 \cdot \frac{6Ay(y^3+a)^2 - 18Ay^4(y^3+a)}{(y^3+a)^4}$

$= \varepsilon_0 \frac{6Ay(y^3+a) - 18Ay^4}{(y^3+a)^4}$

2) $\rho(2m,2m,2m) = -0,247\,\varepsilon_0$ $\boxed{F}$

- normale površine: $\underset{x=1}{\vec{a}_x}; \underset{x=0}{-\vec{a}_x}; \underset{y=1}{\vec{a}_y}; \underset{y=0}{-\vec{a}_y}; \underset{z=1}{\vec{a}_z}; \underset{z=0}{-\vec{a}_y}$

$Q = \varepsilon \oint\!\!\oint_S \vec{E}\cdot\vec{n}\, dS$

$\vec{E}\cdot\vec{n} = \frac{3Ay^2}{(y^3+a)^2}\Big|_{y=1} - \frac{3Ay^2}{(y^2+a)}\Big|_{y=0}$

$Q = \varepsilon \iint_{\Omega xz} \frac{3Ay^2}{(y^3+a)^2}\Big|_{y=1} dx\,dz + \varepsilon \iint_{\Omega xz} \frac{-3Ay^2}{(y^3+a)^2}\Big|_{y=0} dx\,dz$

$= \varepsilon \iint_{\Omega xz} \frac{3A}{(a+1)^2} dx\,dy = \varepsilon \frac{3A}{(a+1)^2} \times \Big|_0^1 z\Big|_0^1 = \varepsilon \frac{3A}{(a+1)^2} = 0,75\,\varepsilon_0$ $\boxed{C}$

4) Rad po zatvorenoj krivulji u električnom polju uvijek je jednak nuli

$W = 0$ $\boxed{A}$

II $E_{2M} = 100$ V

→ cilindrični, tj. valjkasti kondenzatori

$$\left.\begin{matrix} R_1 = 1 \text{ cm} \\ R_2 = 3 \text{ cm} \\ R_3 = 5 \text{ cm} \end{matrix}\right\} \begin{matrix} \varepsilon_{r1} = 3 \\ \varepsilon_{r2} = 5 \end{matrix}$$

$$E_{2M} = \frac{\lambda}{2\pi \varepsilon_0 \varepsilon_{r2} R_2}$$

5) $$E_{1M} = \frac{\lambda}{2\pi \varepsilon_0 \varepsilon_{r1} R_1} = E_{2M} \cdot \frac{\varepsilon_{r2} R_2}{\varepsilon_{r1} R_1} = 500 \ \text{V/m} \quad \boxed{D}$$

6) $$\lambda = E_{2M} \cdot 2\pi \varepsilon_0 \varepsilon_{r2} R_2 = 834{,}05 \ \frac{pC}{m} \quad \boxed{C}$$

7) $$U_{ab} = \frac{\lambda}{2\pi \varepsilon_0 \varepsilon_{r1}} \ln \frac{R_2}{R_1} = 5{,}493 \ \text{V}$$

8) $$C = 2\pi \varepsilon_0 \varepsilon_r \frac{l}{\ln \frac{R_2}{R_1}} \quad \boxed{C}$$

$$C_1' = \frac{C_1}{l} = 2\pi \varepsilon_0 \varepsilon_{r1} \cdot \frac{1}{\ln \frac{R_2}{R_1}} = 151.836 \ \frac{pF}{m}$$

$$C_2' = \frac{C_2}{l} = 2\pi \varepsilon_0 \varepsilon_{r2} \frac{1}{\ln \frac{R_3}{R_2}} = 544{,}248 \ \frac{pF}{m}$$

$$C'_{uk} = \frac{C_1' \cdot C_2'}{C_1' + C_2'} = 118{,}716 \ \frac{pF}{m} \quad \boxed{B}$$

III

$d_1 = 1\ cm \quad \varepsilon_{r1} = 2 \quad \Rightarrow \quad C_1 = 88,54\ pF$

$d_2 = 2\ cm \quad \varepsilon_{r2} = 3 \quad \Rightarrow \quad C_2 = 66,405\ pF$

$d_3 = 3\ cm \quad \varepsilon_{r3} = 5 \quad \Rightarrow \quad C_3 = 73,78\ pF$

$S = 0,05\ m^2$

$U = 100\ V$

$C = \varepsilon_0 \varepsilon_r \frac{S}{d}$

12) $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \Rightarrow C = 25,05\ pF$ $\boxed{C}$

$Q = C \cdot U = 2,505\ nC$

$U_1 = \frac{Q}{C_1} = 28,3\ V$

$U_2 = \frac{Q}{C_2} = 37,72\ V$

$U_3 = \frac{Q}{C_3} = 33,95\ V$

9) $E_B = \frac{U_2}{d_2} = 1886\ V/m$ $\boxed{A}$

10) $\varphi(x = d_1) = U - U_1 = 71,7\ V$ $\boxed{F}$

11) $D_C = \varepsilon_0 \varepsilon_{r3} E_3 = \varepsilon_0 \varepsilon_{r3} \cdot \frac{U_3}{d_3} = 50,099\ \frac{nC}{m^2}$ $\boxed{D}$

$\overline{\text{IV}}$ $L = 1\ m$

$\lambda = 10^{-9}\ C/m$

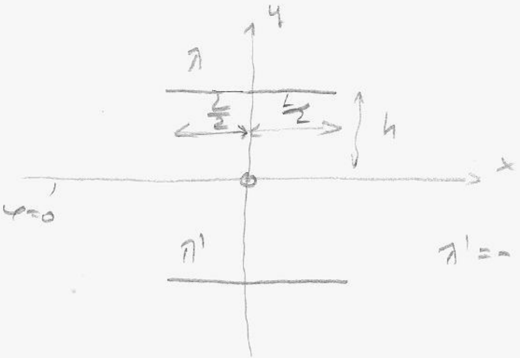
$r_0 = 1\ cm$

$h = 1\ m$

$\vec{r} = \vec{0}$

$\vec{r}_1' = x\vec{a}_x + \vec{a}_y$

$\vec{r}_2' = x\vec{a}_x - \vec{a}_y$



$\lambda' = -\lambda = -1\ nC/m$

$\vec{R}_1 = \vec{r} - \vec{r}_1' = -x\,\vec{a}_x - \vec{a}_y$

$\vec{R}_2 = \vec{r} - \vec{r}_2 = -x\,\vec{a}_x + \vec{a}_y$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 = \frac{\lambda}{4\pi\varepsilon_0}\int \frac{\vec{R}_1}{R_1^3}\,dl + \frac{\lambda'}{4\pi\varepsilon_0}\int \frac{\vec{R}_2}{R_2^3}\,dl$$

13)

$$E_y = \frac{\lambda}{2\pi\varepsilon_0}\int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{-1}{\sqrt{x^2+1}^{\,3}}\,dx$$

$$= \frac{-\lambda}{2\pi\varepsilon_0}\cdot\frac{x}{\sqrt{x^2+1}}\Bigg|_{x=-0,5}^{0,5} = \frac{-\lambda}{2\pi\varepsilon_0\sqrt{1,25}} = -16,09\ V/m \quad \boxed{D}$$

14) $\varphi(A) = 0\ V$ $\boxed{A}$

↳ tako je zadano zadatkom!

15.) ne znam! ali rješenje je: $78,33\ V$ $\boxed{A}$

16) $\lambda' = -10^{-9}\ C/m$

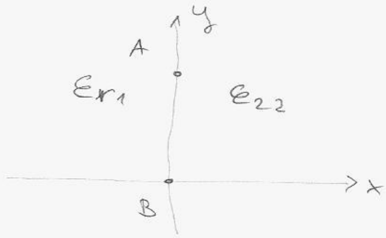
$Q = -1\ nC$ $\boxed{D}$

V $\varepsilon_{r1} = 2$

$\varepsilon_{r2} = 3$

$\vec{E}_1 = 3\vec{a}_x + 2\vec{a}_y + 3\vec{a}_z$

$\vec{n} = \vec{a}_x$



$$\vec{n} \times (\vec{E}_2 - \vec{E}_1) = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 1 & 0 & 0 \\ E_x - 3 & E_y - 2 & E_z - 3 \end{vmatrix} =$$

$$= -\vec{a}_y (E_z - 3) + \vec{a}_z (E_y - 2) = \vec{0}$$

uvjet na granici

$E_z = 3 \qquad E_y = 2$

$\vec{n} (\vec{D}_2 - \vec{D}_1) = 0$

$= \varepsilon_{r2} E_x - \varepsilon_{r1} 3 \Rightarrow E_x = 2$

17) $\vec{E}_2 = 2\vec{a}_x + 2\vec{a}_y + 3\vec{a}_z$ $\boxed{A}$

$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$; $\quad \vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$

$\vec{P} = \varepsilon_0 E (\varepsilon_r - 1)$

18) $\vec{P}_1 = 26,6\,\vec{a}_x + 17,7\,\vec{a}_y + 26,6\,\vec{a}_z$ $\boxed{E}$

19) $\vec{P}_2 = 35,4\,\vec{a}_x + 35,4\,\vec{a}_y + 53,1\,\vec{a}_z$ $\boxed{C}$

20) $\vec{e}_{AB} = -\vec{a}_y$

$E_{1T} = \vec{E}_1 \cdot \vec{e}_{AB} = -2$

$E_{2T} = \vec{E}_2 \cdot \vec{e}_{AB} = -2$

$\overline{AB} = 1$ m

$U_{AB} = E_{1T} \cdot \overline{AB} = -2$ V