MAS - 16.4.

#1.

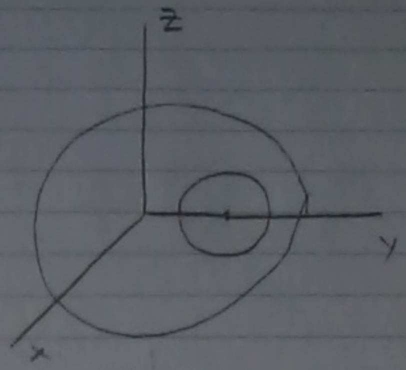
(1.) $\rho = 10^{-8}\ C/m^3$ unutar sfere $R_1 = 1m$, sa $S(0,0,0)$ 0,5

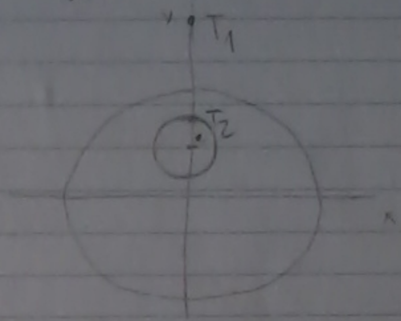
Unutar te sfere je druga, $R_2 = 0{,}25\ m$, središte u $S'(0, \frac{1}{2}R_1, 0)$

ne sadrži naboj

Potencijal u $T_1(0, 2R_1, 0)$ i

$|\vec{E}|$ u $T_2(\frac{1}{2}R_2, \frac{1}{2}R_1, 0)$





I. $\oint_S \vec{D}\, d\vec{S} = Q = \iiint_V \rho\, dV$

u ovom slučaju od velike kugle oduzeti malu

II. $\Delta\varphi = 0$ ili $\frac{\rho}{\varepsilon_0}$

nema naboja

II. $\Delta\varphi_2 = 0 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \varphi_2}{\partial r}\right) \quad / \int$

nema promjene u θ i α

$r^2 \frac{\partial \varphi_2}{\partial r} = C_1 \quad / \cdot \frac{1}{r^2} \quad / \int$

$\varphi_2 = -\frac{C_1}{r} + C_2$

$\varphi_2(\infty) = 0 \Rightarrow C_2 = 0$

Gauss: $\oint_S \vec{D}\, d\vec{S} = Q$; $\vec{E} = -\nabla\varphi$

$\vec{E} = -\frac{\partial \varphi}{\partial r}\vec{a_r}$

$\oint -\varepsilon \frac{C_1}{r^2}\vec{a_r}\vec{a_r}\, dS = Q = \rho \underbrace{\iiint_V dV}_{\frac{4}{3}r_0^3\pi}$ $\quad \vec{E} = -\frac{C_1}{r^2}\vec{a_r}$

$-\varepsilon \frac{C_1}{\cancel{r^2}} \cdot \cancel{4\pi}\cancel{r^2} = \rho \cdot \frac{\cancel{4}}{3} r_0^3 \cancel{\pi}$

$C_1 = -\frac{\rho r_0^3}{3\varepsilon}$

$r \in [r_0, \infty)$

$r_0 = R_1$

$\varphi_2 = -\frac{C_1}{r} = -\frac{\rho r_0^3}{3r\varepsilon}$

$$\vec{E}_2 = \frac{\rho r_0^3}{3\varepsilon r^2}\vec{a}_r$$

Bilješke >> Notes

$$\Delta\varphi_1 = -\frac{\rho}{\varepsilon} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi_1}{\partial r}\right) \quad / \cdot r^2 \ // \int$$

$$C_1 - \frac{\rho}{\varepsilon}\frac{r^3}{3} = r^2\frac{\partial\varphi_1}{\partial r} \quad / \cdot \frac{1}{r^2} \ // \int$$

$$\frac{C_1}{r^2} - \frac{\rho}{\varepsilon}\cdot\frac{r}{3} = \frac{\partial\varphi_1}{\partial r}$$

$$\vec{E} = -\frac{\partial\varphi_1}{\partial r}\vec{a}_r = -\frac{\rho r}{3\varepsilon} + \frac{C_1}{r}$$

$\Rightarrow C_1 = 0 \rightarrow$ jer je $\vec{E}$ u ishodištu nula

$$E_{T2} = \frac{\rho r_1}{3\varepsilon} - \frac{\rho r_2}{3\varepsilon} = \frac{\rho}{3\varepsilon} d$$

$$\frac{\partial\varphi_1}{\partial r} = -\frac{\rho}{\varepsilon}\cdot\frac{r}{3} \ // \int$$

$$\varphi_1 = -\frac{\rho}{\varepsilon}\cdot\frac{r^2}{6} + C_2$$

$$\varphi_1(r_0) = \varphi_2(r_0) \Rightarrow -\frac{\rho}{\varepsilon}\frac{r_0^2}{6} + C_2 = \frac{\rho r_0^3}{3\varepsilon r}$$

$$C_2 = \frac{\rho r_0^2}{2\varepsilon}$$

$$\varphi_1 = \frac{\rho r_0^2}{2\varepsilon} - \frac{\rho r^2}{6\varepsilon} = \boxed{\frac{\rho}{6\varepsilon}(3r_0^2 - r^2)}$$

za kuglu s nabojem ρ

$$\vec{E} = \frac{\rho r}{3\varepsilon}\vec{a}_r$$

$= \vec{a}_y \rightarrow \vec{a}_x$ [illegible]

$$\vec{E}(T_2) = \frac{\rho r}{3\varepsilon}\vec{a}_r = \frac{\rho\cdot 0{,}5R_1}{3\varepsilon}\vec{a}_r = 188$$

doprinos [illegible] doprinos [illegible]

$$\varphi(T_1) = \varphi_v - \varphi_M = \frac{\rho R_1^3}{3\varepsilon r} - \frac{\rho R_2^3}{3\varepsilon(r - 0{,}5R_1)} = \quad (r = 2R_1)$$

jer je mala pomaknuta

$$= \frac{\rho}{18\varepsilon R_1}(3R_1^3 - 4R_2^3) = 184$$

SIEMENS

(2) 2 dielektrika

!

$x - 2y + 3z = 12$ dijeli

$\varepsilon_{R1} = 4$ i $\varepsilon_{R2} = 8$

$\vec{E}_1 = 2\vec{a}_x - 3\vec{a}_y + \vec{a}_z$

$\boxed{\text{uvjeti na granici!}}$

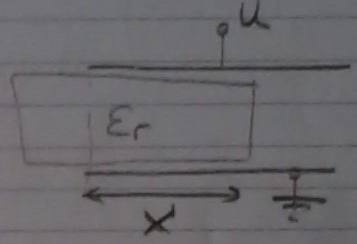
$\vec{E}_2 = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z = ?$

$$\vec{n} = \frac{\vec{i} - 2\vec{j} + 3\vec{z}}{\sqrt{1+4+9}}$$

$\vec{D}_1 = \varepsilon_1 \vec{E}_1$; $\vec{D}_2 = \varepsilon_2 \vec{E}_2$

(3.) Pločasti kond. dim. $a \times b$; $a = 0{,}1$ m ; $b = 0{,}1$ m. Razmaknute $d = 1$ mm. $\varepsilon_R = 4$

Spojen na 100 V. Diel. se djelomično izvuče do x'
Sila kojom E djeluje na diel.
(i smjer sile) ?



I : Q const $\vec{F} = -\nabla w$ $\quad \vec{F} = -\frac{\partial w}{\partial s}\vec{a}_s$

N : U const $\vec{F} = +\nabla w$

$$\vec{F} = \frac{\partial w}{\partial s}\vec{a}_s$$

$$\vec{F} = -\frac{1}{2} Q^2 \frac{\partial}{\partial s}\left(\frac{1}{C}\right)\vec{a}_s$$

$w = \frac{1}{2}CU^2 \qquad Q = CU$

$= \frac{1}{2}QU$

$= \frac{1}{2}\frac{Q^2}{C}$

$$\vec{F} = \frac{1}{2}U^2 \frac{\partial C}{\partial s}$$

za N: ε_r će se htjet ostat unutra (gura unutra)
(što više ε_r → više naboja → veća energija)

$$C_1 = \varepsilon_r \varepsilon_0 \frac{S}{d} = \varepsilon_r \varepsilon_0 \frac{ax}{d}$$

$$C_2 = \varepsilon_0 \frac{(b-x)a}{d}$$

} paralela. +

* I → izolirani sustav   Q - konst.
N → neizolirani   U - konst.

*

$$\vec{F} = \frac{1}{2} U^2 \frac{\partial}{\partial x}\left(\varepsilon_0 \varepsilon_r \frac{ax}{d} + \varepsilon_0 \frac{a(b-x)}{d}\right) \vec{a_s}$$

Bilješke >> Notes

$$= \frac{1}{2} U^2 \left(\varepsilon_0 \varepsilon_r \frac{a}{d} - \varepsilon_0 \frac{a}{d}\right) \vec{a_s} = \frac{1}{2} U^2 \varepsilon_0 \frac{a}{d}(\varepsilon_r - 1) \vec{a_s}$$

$$= 13{,}3 \ \mu N \ \vec{a_x}$$

4. $\lambda = 10$ nC/m na polukružnici $R_0 = 1$m u xy ravnini.

$\vec{E}$ u ishodištu.

$U_{AB} = ?$ A(0,0,0) B(0,0,1) m