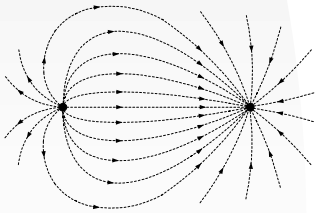
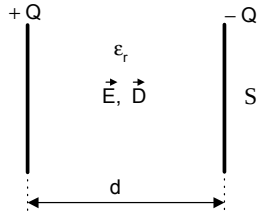
Elektrostatika

- Električno polje na granici dva dielektrika.
- Pločasti kondenzator.
- Cilindrični kondenzator.
- Kuglasti kondenzator.



1. zadatak

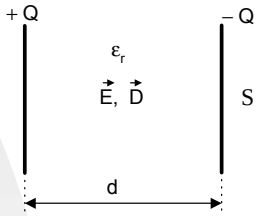
Dvije metalne ploče sa zrakom kao izolatorom bile su spojene na izvor napona U, a zatim odspojene od njega. Nakon toga je razmak ploča povećan na dvostruki iznos, a zrak je zamijenjen tinjcem (ε_r = 6). Odredite što se događa s električnim poljem, naponom između ploča, kapacitetom kondenzatora, nabojem na pločama i energijom u kondenzatoru.

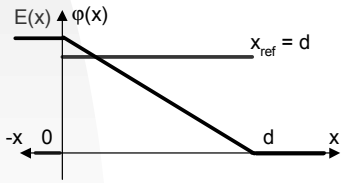


Početna stranica

Uvodni pojmovi

- Za pločasti kondenzator vrijedi:





$$D = \sigma = \frac{Q}{S}$$

$$\vec{D} = \varepsilon \cdot \vec{E} = \varepsilon_0 \cdot \varepsilon_r \cdot \vec{E}$$

$$U = \frac{Q}{C}$$

$$U = E \cdot d$$

$$C = \varepsilon_0 \cdot \varepsilon_r \cdot \frac{S}{d}$$

$$W = \frac{Q^2}{2 \cdot C} = \frac{Q \cdot U}{2} = \frac{E \cdot D}{2} \cdot S \cdot d$$

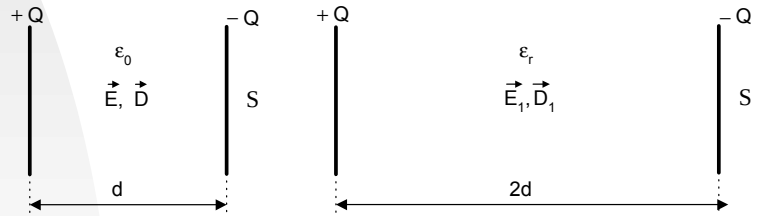
El. polje je konstantno.

Potencijal je linearna funkcija.

Početna stranica

Rješenje zadatka

- Na ploče kondenzatora je bio spojen napon U i ploče su se nabile nabojem Q.
- Nakon toga je kondenzator odspojen, povećan je razmak među pločama i ubačen je dielektrik.



- Budući da je kondenzator odspojen od izvora napajanja nakon ubacivanja izolatora vrijedi:

$$Q = konst.$$

Početna stranica

- Vektor dielektričnog pomaka D:

$$D = \frac{Q}{S};\ D_1 = \frac{Q}{S} \quad \Rightarrow \quad D = D_1$$

- El. polje E:

$$E = \frac{D}{\varepsilon_0};\ E_1 = \frac{D_1}{\varepsilon_0 \cdot \varepsilon_r} \quad \Rightarrow \quad \frac{E_1}{E} = \frac{\frac{D_1}{\varepsilon_0 \cdot \varepsilon_r}}{\frac{D}{\varepsilon_0}} = \frac{1}{\varepsilon_r} = \frac{1}{6}$$

- Napon U:

$$U = E \cdot d\ ;U_1 = E_1 \cdot 2d \ \Rightarrow \quad \frac{U_1}{U} = \frac{E_1 \cdot 2d}{E \cdot d} = \frac{1}{3}$$

- Kapacitet C:

$$C = \varepsilon_0 \cdot \frac{S}{d};\ C_1 = \varepsilon_0 \cdot \varepsilon_r \cdot \frac{S}{2d} \quad \Rightarrow \quad \frac{C_1}{C} = \frac{\varepsilon_0 \cdot \varepsilon_r \cdot \frac{S}{2d}}{\varepsilon_0 \cdot \frac{S}{d}} = \frac{\varepsilon_r}{2} = 3$$

- Energija W:

$$W = \frac{Q \cdot U}{2};\ W_1 = \frac{Q \cdot U_1}{2} \quad \Rightarrow \quad \frac{W_1}{W} = \frac{\frac{Q \cdot U_1}{2}}{\frac{Q \cdot U}{2}} = \frac{U_1}{U} = \frac{1}{3}$$

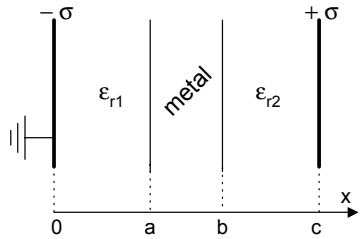
Početna stranica

2. zadatak

Na slici su prikazane dvije ploče nabijene nabojem površinske gustoće σ između kojih se nalaze dva sloja dielektrika uz njih te sloj metala u sredini.

a) Skicirajte funkcije jakosti polja E(x) i potencijala φ(x).

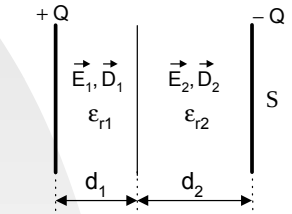
b) Izvedite izraze za funkciju potencijala φ(x) za 0 < x < c uz pretpostavku da su poznati σ, a, b, c te $\varepsilon_1 > \varepsilon_2$.

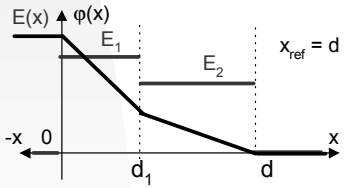


Početna stranica

Uvodni pojmovi

- Pločasti kondenzator s dva dielektrika (serija):





$$D_1 = D_2$$

$$E_1 \neq E_2$$

$$E_1 = \frac{Q}{\varepsilon_0 \cdot \varepsilon_{r1} \cdot S};\ E_2 = \frac{Q}{\varepsilon_0 \cdot \varepsilon_{r2} \cdot S}$$

$$C_1 = \varepsilon_0 \cdot \varepsilon_{r1} \cdot \frac{S}{d_1};\ C_2 = \varepsilon_0 \cdot \varepsilon_{r2} \cdot \frac{S}{d_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \ \Rightarrow \ C = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

$$U_1 = E_1 \cdot d_1;\ U_2 = E_2 \cdot d_2$$

$$U = U_1 + U_2$$

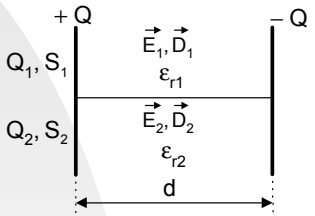
$$W = W_1 + W_2$$

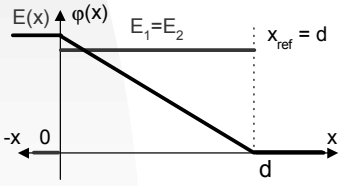
Za $\varepsilon_{r1} < \varepsilon_{r2}$ el. polje i potencijal izgledaju kao na slici:

Početna stranica

Uvodni pojmovi

- Pločasti kondenzator s dva dielektrika (paralela):





$$E_1 = E_2 = E$$

$$D_1 \neq D_2$$

$$Q = Q_1 + Q_2$$

$$D_1 = \frac{Q_1}{S_1} = \varepsilon_0 \cdot \varepsilon_{r1} \cdot E;\ D_2 = \frac{Q_2}{S_2} = \varepsilon_0 \cdot \varepsilon_{r2} \cdot E$$

$$C_1 = \varepsilon_0 \cdot \varepsilon_{r1} \cdot \frac{S_1}{d};\ C_2 = \varepsilon_0 \cdot \varepsilon_{r2} \cdot \frac{S_2}{d}$$

$$C = C_1 + C_2$$

$$U_1 = U_2 = U = E_1 \cdot d = E_2 \cdot d$$

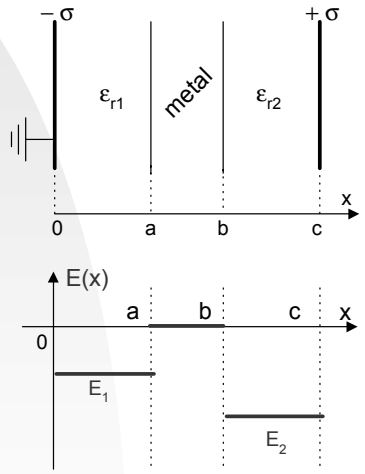
$$W = W_1 + W_2$$

El. polje i potencijal izgledaju kao na slici:

Početna stranica

Rješenje zadatka

- Prvo određujemo el. polje.



El. polje u prvom dielektriku iznosi:

$$E_1 = -\frac{D_1}{\varepsilon} = -\frac{\sigma}{\varepsilon_0 \cdot \varepsilon_{r1}}$$

El. polje u metalu:

$$E_{metal} = 0$$

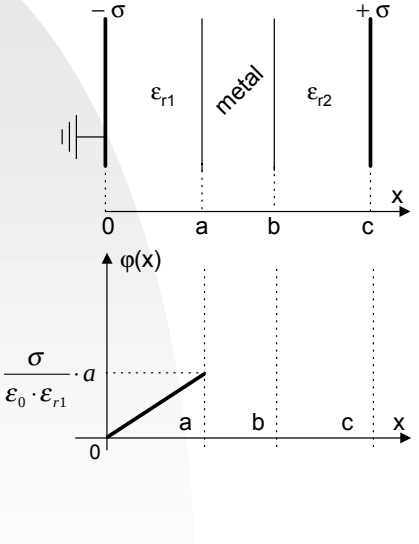
El. polje u drugom dielektriku iznosi:

$$E_2 = -\frac{D_2}{\varepsilon} = -\frac{\sigma}{\varepsilon_0 \cdot \varepsilon_{r2}}$$

Početna stranica

- El. potencijal određuje se na sljedeći način:

$$\varphi(x) = -\int_{xref}^{x} E(x)dx$$



Ref. točka je u ishodištu:

$$\varphi(0) = 0$$

Za 0 < x < a:

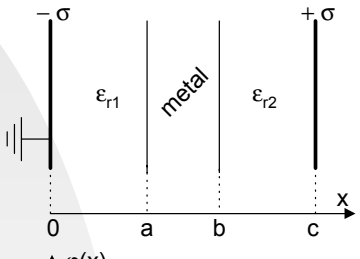
$$\varphi(x) = -\int_0^x E(x)dx = -\int_0^x -\frac{\sigma}{\varepsilon_0 \cdot \varepsilon_{r1}}dx$$

$$\varphi(x) = \frac{\sigma}{\varepsilon_0 \cdot \varepsilon_{r1}} \cdot x\Big|_0^x = \frac{\sigma}{\varepsilon_0 \cdot \varepsilon_{r1}} \cdot x$$

Za x = a:

$$\varphi(x) = \frac{\sigma}{\varepsilon_0 \cdot \varepsilon_{r1}} \cdot a$$

Početna stranica



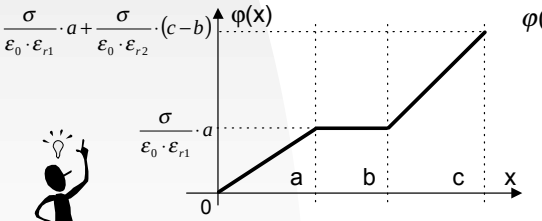
Za a < x < b:

$$\varphi(x) = -\int_0^a E_1(x)dx - \int_a^x E_{metal}(x)dx$$

$$\varphi(x) = \varphi(a) - \int_a^x 0dx = \varphi(a)$$

$$\varphi(b) = \varphi(a) = \frac{\sigma}{\varepsilon_0 \cdot \varepsilon_{r1}} \cdot a$$

Za b < x < c:



$$\varphi(x) = -\int_0^a E_1(x)dx - \int_a^b E_{metal}(x)dx - \int_b^x E_2(x)dx$$

$$\varphi(x) = \varphi(a) + \frac{\sigma}{\varepsilon_0 \cdot \varepsilon_{r2}} \cdot x\Big|_b^x$$

$$\varphi(x) = \frac{\sigma}{\varepsilon_0 \cdot \varepsilon_{r2}} \cdot x + \frac{\sigma}{\varepsilon_0 \cdot \varepsilon_{r1}} \cdot a - \frac{\sigma}{\varepsilon_0 \cdot \varepsilon_{r2}} \cdot b$$

Za x = c:

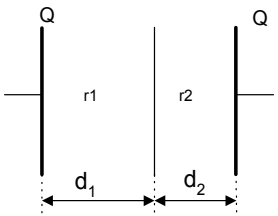
$$\varphi(x) = \frac{\sigma}{\varepsilon_0 \cdot \varepsilon_{r1}} \cdot a + \frac{\sigma}{\varepsilon_0 \cdot \varepsilon_{r2}} \cdot (c - b)$$

Početna stranica

3. zadatak

Pločasti kondenzator sadrži dva sloja dielektrika prema slici. Odredite maksimalnu vrijednost napona U pri kojem neće doći do proboja, ako je zadano:

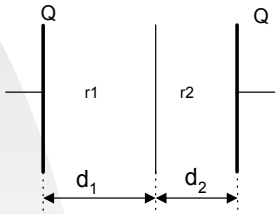
- E_{1p} = 10 [kV/m]
- E_{2p} = 20 [kV/m]
- d_1 = 7 [mm]
- d_2 = 3 [mm]
- ε_{r1} = 5
- ε_{r2} = 2



Početna stranica

Rješenje zadatka

- Probojno polje označava maksimalno el. polje kod kojeg u određenom dielektriku neće doći do proboja.



Za serijski spojene kondenzatore vrijedi:

$$D_1 = D_2$$

$$E_1 \neq E_2$$

- Ako pretpostavimo da će el. polje u prvom dielektriku imati svoju maksimalnu vrijednost vrijedi:

$$E_1 = E_{1p} = 10\,[\mathrm{kV/m}]$$

$$\varepsilon_0 \cdot \varepsilon_{r1} \cdot E_{1p} = \varepsilon_0 \cdot \varepsilon_{r2} \cdot E_2 \quad \Rightarrow \quad E_2 = \frac{\varepsilon_{r1}}{\varepsilon_{r2}} \cdot E_{1p} = 25\,[\mathrm{kV/m}] > E_{2p}$$

- Ovaj slučaj ne zadovoljava, jer iako ne dolazi do proboja u prvom dielektriku u drugom dolazi.

Početna stranica

- Uz pretpostavku da je u drugom dielektriku maksimalno polje vrijedi:

$$E_2 = E_{2p} = 20\,[\mathrm{kV/m}]$$

$$\varepsilon_0 \cdot \varepsilon_{r1} \cdot E_1 = \varepsilon_0 \cdot \varepsilon_{r2} \cdot E_{2p} \quad \Rightarrow \quad E_1 = \frac{\varepsilon_{r2}}{\varepsilon_{r1}} \cdot E_{2p} = 8\,[\mathrm{kV/m}] < E_{1p}$$

- Znači el. polja u prvom i drugom dielektriku iznose:

$$E_1 = 8\,[\mathrm{kV/m}]$$

$$E_2 = E_{2p} = 20\,[\mathrm{kV/m}]$$

- Maksimalni napon onda iznosi:

$$U_{\max} = U_1 + U_2 = E_1 \cdot d_1 + E_2 \cdot d_2$$

$$U_{\max} = 8 \cdot 10^3 \cdot 7 \cdot 10^{-3} + 20 \cdot 10^3 \cdot 3 \cdot 10^{-3}$$

$$\boxed{U_{\max} = 116\,[\mathrm{V}]}$$

Početna stranica

4. zadatak

Za koaksijalni kabel s polietilenskom izolacijom kao na slici (negativan linijski naboj na unutrašnjem vodiču) potrebno je odrediti:

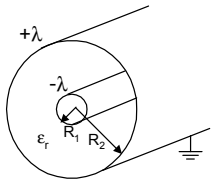
a) potencijal unutarnjeg vodiča

b) ako el. polje u polietilenu ne smije prijeći vrijednost od $3 \cdot 10^7$ [V/m] koliki je maksimalni napon koji se smije prikljuciti izmedu vodica kabela

c) kapacitet, ako je zadana du□ina kabela l

Zadano:

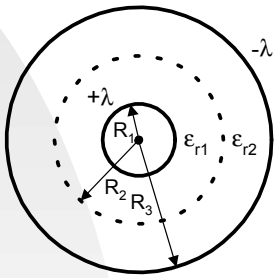
- $\varepsilon_r = 2.3$
- $\lambda = 1.15 \cdot 10^{-8}$ [As/m]
- $2 \cdot R_1 = 2.6$ [mm]
- $2 \cdot R_2 = 9.5$ [mm]
- $E_{max} = 30$ [MV/m]
- $l = 500$ [m]



Početna stranica

Uvodni pojmovi

- Cilindrični kondenzator s dva dielektrika (serija):

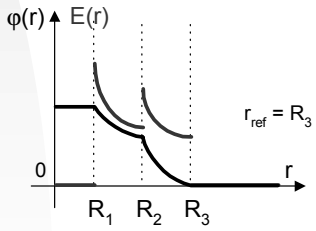


$$E_1(r) = \frac{\lambda}{2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r1} \cdot r};\; E_2(r) = \frac{\lambda}{2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r2} \cdot r}$$

$$D_1(R_2) = D_2(R_2)$$

$$C_1 = \frac{2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r1} \cdot l}{ln\frac{R_2}{R_1}};\; C_2 = \frac{2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r2} \cdot l}{ln\frac{R_3}{R_2}}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \;\Rightarrow\; C = \frac{C_1 \cdot C_2}{C_1 + C_2}$$



$$U_{R1R2} = \frac{\lambda}{2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r1}} ln\frac{R_2}{R_1}$$

$$U_{R2R3} = \frac{\lambda}{2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r2}} ln\frac{R_3}{R_2}$$

$$W = W_1 + W_2$$

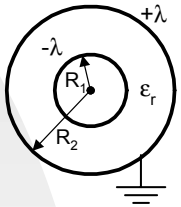
El. polje i potencijal izgledaju kao na slici:

Početna stranica

Rješenje zadatka

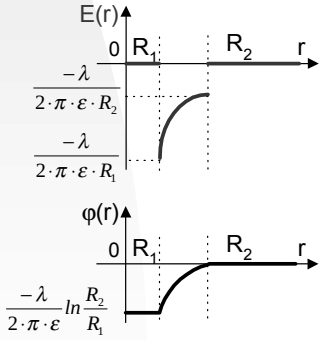
- U kondenzatoru ($R_1 < r < R_2$) se el. polje mijenja kao :

$$E(r) = \frac{-\lambda}{2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_r \cdot r}$$



Potencijal se određuje u odnosu na ref. točku koja se nalazi na R_2:

$$\varphi(r) = \frac{-\lambda}{2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_r} ln \frac{r_{ref}}{r}$$



Potencijal unutarnjeg vodiča:

$$\varphi(R_1) = \frac{-\lambda}{2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_r} ln \frac{R_2}{R_1}$$

$$\varphi(R_1) = \frac{-1.15 \cdot 10^{-8}}{2 \cdot \pi \cdot 8.854 \cdot 10^{-12} \cdot 2.3} ln \frac{4.75 \cdot 10^{-3}}{1.3 \cdot 10^{-3}}$$

$$\boxed{\varphi(R_1) = -116 \,[\mathrm{V}]}$$

El. polje i potencijal izgledaju kao na slici:

Početna stranica

- Maksimalni napon će se postići u slučaju kada el. polje ima maksimalni iznos. Da ne bi došlo do proboja dielektrika to max. polje je na mjestu R_1:

$$E_{\max}(R_1) = \frac{\lambda_{\max}}{2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_r \cdot R_1}$$

- Maksimalni napon je jednak:

$$U_{R1R2} = \frac{-|\lambda_{\max}|}{2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_r} ln \frac{R_2}{R_1}$$

$$|\lambda_{\max}| = E_{\max}(R_1) \cdot 2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_r \cdot R_1 \;\Rightarrow\; U_{R1R2} = -E_{\max}(R_1) \cdot R_1 \cdot ln \frac{R_2}{R_1}$$

$$U_{R1R2} = -30 \cdot 10^6 \cdot 1.3 \cdot 10^{-3} \cdot ln \frac{4.75 \cdot 10^{-3}}{1.3 \cdot 10^{-3}}$$

$$\boxed{U_{R1R2} = -51 \,[\mathrm{kV}]}$$

- Kapacitet kondenzatora:

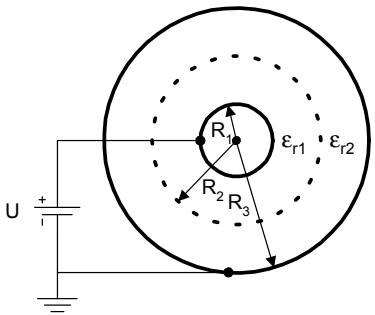
$$C = \frac{2 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_r \cdot l}{ln \frac{R_2}{R_1}} = \frac{2 \cdot \pi \cdot 8.854 \cdot 10^{-12} \cdot 2.3 \cdot 500}{ln \frac{4.75 \cdot 10^{-3}}{1.3 \cdot 10^{-3}}} = 49 \,[\mathrm{nF}]$$

Početna stranica

5. zadatak

Kuglasti kondenzator s dva sloja dielektrika priključen je na napon U prema slici. Odredite polumjer granične površine (R_2) da bi na oba sloja vladao jednak napon. Koliki se najveći napon može priključiti na takav kondenzator a da ne dođe do proboja. Nacrtajte dijagrame promjene potencijala i iznosa vektora jakosti el. polja u zavisnosti o udaljenosti r od središta kondenzatora, φ(r), E(r), s karakteristicnim vrijednostima polja i potencijala za taj slucaj. Zadano:

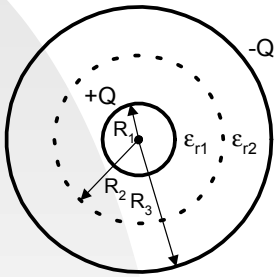
- $\varepsilon_{r1} = 4$
- $\varepsilon_{r2} = 2$
- $R_1 = 1$ [cm]
- $R_3 = 6$ [cm]
- $E_{1P} = 200$ [kV/m]
- $E_{2P} = 75$ [kV/m]



Početna stranica

Uvodni pojmovi

- Kuglasti kondenzator s dva dielektrika (serija):

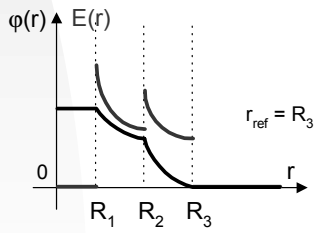


$$E_1(r) = \frac{Q}{4 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r1} \cdot r^2};\; E_2(r) = \frac{Q}{4 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r2} \cdot r^2}$$

$$D_1(R_2) = D_2(R_2)$$

$$C_1 = 4 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r1} \cdot \frac{R_1 \cdot R_2}{R_2 - R_1};\; C_2 = 4 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r2} \cdot \frac{R_2 \cdot R_3}{R_3 - R_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \;\Rightarrow\; C = \frac{C_1 \cdot C_2}{C_1 + C_2}$$



$$U_{R1R2} = \frac{Q}{4 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r1}} \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$U_{R2R3} = \frac{Q}{4 \cdot \pi \cdot \varepsilon_0 \cdot \varepsilon_{r2}} \left(\frac{1}{R_2} - \frac{1}{R_3} \right)$$

$$W = W_1 + W_2$$

El. polje i potencijal izgledaju kao na slici:

Početna stranica

Rješenje zadatka

- Napon na prvom i drugom dielektriku su jednaki:

$$U_{R1R2} = U_{R2R3}$$

$$\frac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r1}}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}}\left(\frac{1}{R_2}-\frac{1}{R_3}\right)$$

$$\frac{1}{\varepsilon_{r1}}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{1}{\varepsilon_{r2}}\left(\frac{1}{R_2}-\frac{1}{R_3}\right)$$

$$R_2 = \frac{(\varepsilon_{r1}+\varepsilon_{r2})\cdot R_1\cdot R_3}{\varepsilon_{r1}\cdot R_1+\varepsilon_{r2}\cdot R_3}$$

$$R_2 = \frac{(4+2)\cdot 1\cdot 10^{-2}\cdot 6\cdot 10^{-2}}{4\cdot 1\cdot 10^{-2}+2\cdot 6\cdot 10^{-2}}$$

$$\boxed{R_2 = 2.25\,[\mathrm{cm}]}$$

- Maksimalno polje u prvom dielektriku je na mjestu R_1, a u drugom na mjestu R_2.

Početna stranica

- Pretpostavimo da je u prvom dielektriku el. polje jednako probojnom polju:

$$E_{1m}(R_1) = E_{1\mathrm{P}} = \frac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r1}\cdot R_1^{\,2}}$$

- Uz takvo polje provjeravamo koliko je polje na granici (R_2) u drugom dielektriku:

$$D_1(R_2) = D_2(R_2)$$

$$\varepsilon_0\cdot\varepsilon_{r1}\cdot E_1(R_2) = \varepsilon_0\cdot\varepsilon_{r2}\cdot E_2(R_2)$$

$$E_2(R_2) = E_1(R_2)\cdot\frac{\varepsilon_{r1}}{\varepsilon_{r2}}$$

- El. polje u prvom dielektriku na granici (R_2) iznosi:

$$E_1(R_2) = \frac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r1}\cdot R_2^{\,2}} = E_{1\mathrm{P}}\cdot\frac{R_1^2}{R_2^2}$$

- Uvrštavanjem poznatih vrijednosti el. polje drugom dielektriku na granici iznosi:

$$E_2(R_2) = E_{1\mathrm{P}}\cdot\frac{R_1^2}{R_2^2}\cdot\frac{\varepsilon_{r1}}{\varepsilon_{r2}} = 200\cdot\left(\frac{2.25}{1}\right)^2\cdot\frac{4}{2} = 79[\mathrm{kV/m}] > E_{2\mathrm{P}}$$

Početna stranica

- Uz maksimalno polje u prvom dielektriku, u drugom bi došlo do proboja.
- Ukoliko je pak u drugom dielektriku polje jednako probojnom u prvom dielektriku polje iznosi:

$$E_{2m}(R_2) = E_{2\mathrm{P}} = \frac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}\cdot R_2^{\,2}}$$

$$\varepsilon_0\cdot\varepsilon_{r1}\cdot E_1(R_2) = \varepsilon_0\cdot\varepsilon_{r2}\cdot E_2(R_2)$$

$$E_1(R_2) = E_{2\mathrm{P}}\cdot\frac{\varepsilon_{r2}}{\varepsilon_{r1}}$$

$$E_1(R_1) = E_{2\mathrm{P}}\cdot\frac{\varepsilon_{r2}}{\varepsilon_{r1}}\cdot\frac{R_2^{\,2}}{R_1^{\,2}} = 75\cdot\frac{2}{4}\cdot\left(\frac{2.25}{1}\right)^2 = 190\,[\mathrm{kV/m}] < E_{1\mathrm{P}}$$

- Maksimalni napon određuje se:

$$U_{\max} = U_{R1R2} + U_{R2R3} = \frac{Q_{\max}}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r1}}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{Q_{\max}}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}}\left(\frac{1}{R_2}-\frac{1}{R_3}\right)$$

- Količina naboja na kuglama može se odrediti kao:

$$Q_{\max} = 4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}\cdot R_2^{\,2}\cdot E_{2\mathrm{P}}$$

Početna stranica

$$U_{\max} = \frac{\varepsilon_{r2}\cdot R_2^{\,2}\cdot E_{2\mathrm{P}}}{\varepsilon_{r1}}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{\varepsilon_{r2}\cdot R_2^{\,2}\cdot E_{2\mathrm{P}}}{\varepsilon_{r2}}\left(\frac{1}{R_2}-\frac{1}{R_3}\right)$$

$$U_{\max} = \frac{\varepsilon_{r2}\cdot R_2^{\,2}\cdot E_{2\mathrm{P}}}{\varepsilon_{r1}}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+R_2^{\,2}\cdot E_{2\mathrm{P}}\cdot\left(\frac{1}{R_2}-\frac{1}{R_3}\right)$$

- Uvrštenjem poznatih vrijednosti dobijemo maksimalni napon:

$$\boxed{U_{\max} = 2.1\,[\mathrm{kV}]}$$

- Za el. polje znamo sljedeće:

$$E_1(R_1) = 190\,[\mathrm{kV/m}]$$

$$E_2(R_2) = 75\,[\mathrm{kV/m}]$$

$$E_1(R_2) = E_{2\mathrm{P}}\cdot\frac{\varepsilon_{r2}}{\varepsilon_{r1}} = 75\cdot\frac{2}{4} = 37.5\,[\mathrm{kV/m}]$$

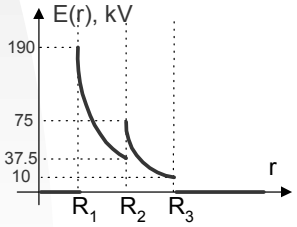
$$E_2(R_3) = E_{2\mathrm{P}}\cdot\left(\frac{R_2}{R_3}\right)^2 = 75\cdot\left(\frac{2.25}{6}\right)^2 = 10.5\,[\mathrm{kV/m}]$$

Početna stranica

- Pomoću izračunatih vrijednosti polja mogu se odrediti funkcije promjene el. polja, E(r):

$$E(r)=\begin{cases}0;\ za\ 0<r<R_1\\ \dfrac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r1}\cdot r^2};\ za\ R_1<r<R_2\\ \dfrac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}\cdot r^2};\ za\ R_2<r<R_3\\ 0;\ za\ r>R_3\end{cases}$$

- Dijagram promjene jakosti el. polja:



Budući da Q nije zadan, on se može odrediti kao:

$$Q=4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}\cdot R_2^{\,2}\cdot E_{2P}$$

Početna stranica

- Dijagram potencijala određujemo uz referentnu točku na udaljenosti R_3 (pogledati sliku).

$$\varphi_{ref}=\varphi(R_3)=0$$

- Potencijal u drugom dielektriku, za $R_2 < r < R_3$, se mijenja kao:

$$\varphi(r)=U_{rR3}+\varphi(R_3)=\frac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}\cdot r}-\frac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}\cdot R_3}+0$$

$$\varphi(R_2)=\frac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}\cdot R_2}\cdot\left(\frac{1}{R_2}-\frac{1}{R_3}\right)=\frac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}}\cdot\left(\frac{1}{R_2}-\frac{1}{R_3}\right)=1.05\,[\mathrm{kV}]$$

- Potencijal u prvom dielektriku:

$$\varphi(r)=U_{rR2}+\varphi(R_2)=U_{rR2}+U_{R2R3}+\varphi(R_3)$$

$$\varphi(r)=\frac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r1}\cdot r}-\frac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r1}\cdot R_2}+\frac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}}\left(\frac{1}{R_2}-\frac{1}{R_3}\right)$$

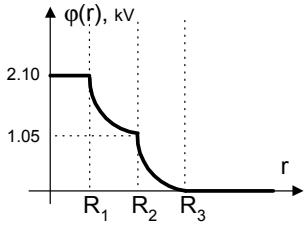
$$\varphi(R_1)=\frac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r1}}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}}\left(\frac{1}{R_2}-\frac{1}{R_3}\right)=2.1\,[\mathrm{kV}]$$

Početna stranica

- Potencijal, φ(r):

$$\varphi(r)=\begin{cases}\dfrac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r1}}\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)+\dfrac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}}\left(\dfrac{1}{R_2}-\dfrac{1}{R_3}\right);\ za\ 0<r<R_1\\ \dfrac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r1}\cdot r}-\dfrac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r1}\cdot R_2}+\dfrac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}}\left(\dfrac{1}{R_2}-\dfrac{1}{R_3}\right);\ za\ R_1<r<R_2\\ \dfrac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}\cdot r}-\dfrac{Q}{4\cdot\pi\cdot\varepsilon_0\cdot\varepsilon_{r2}\cdot R_3};\ za\ R_2<r<R_3\\ 0;\ za\ r>R_3\end{cases}$$

- Dijagram promjene potencijala:



Početna stranica