1. Jednadžbe statičkog strujnog polja i uvjeti na granici dvaju vodiča.

I

$$\boxed{\operatorname{div}\vec{J} = \nabla\vec{J} = -\frac{\partial\rho}{\partial t}}$$ – jednadžba kontinuiteta

$$\boxed{\vec{J} = \varkappa\vec{E}}$$

$$\nabla\vec{J} = \varkappa\nabla\vec{E} = -\frac{\partial\rho}{\partial t}$$

$$\vec{\nabla}E = -\frac{1}{\varkappa}\frac{\partial\rho}{\partial t}$$

$$\nabla\vec{D} = \rho_s \longrightarrow \nabla\vec{E} = \frac{\rho}{\varepsilon}$$

$$\frac{\rho}{\varepsilon} = -\frac{1}{\varkappa}\frac{\partial\rho}{\partial t} \longrightarrow \frac{\partial\rho}{\partial t} + \frac{\varkappa}{\varepsilon}\rho = 0 \longrightarrow \rho(t) = \rho_0\cdot e^{-\left(\frac{\varkappa}{\varepsilon}\right)t} = \rho_0\cdot e^{-\frac{t}{\tau_r}}$$

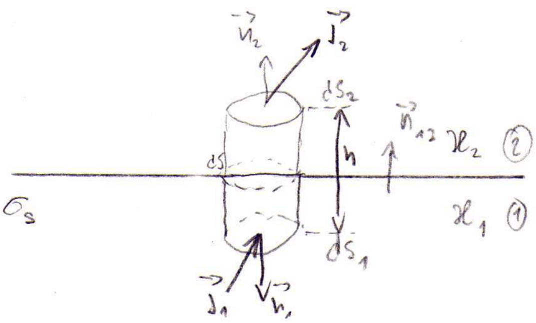
$$\nabla\vec{J} = \nabla(\varkappa\vec{E}) = \nabla(-\varkappa\nabla\varphi) = 0 \rightarrow \boxed{\Delta\varphi = 0}$$

$$\boxed{I = \int_S \vec{J}\vec{n}\,dS}$$ – strujni tok

$$\boxed{G = \frac{1}{U_{ab}} = \frac{\int_S \vec{J}\vec{n}\,dS}{-\int_b^a \vec{E}\,dl}}$$ – vodljivost

$$\boxed{\vec{E} = -\nabla\varphi}$$

II



$$\oint_S \vec{J}\vec{n}\,dS = -\frac{d}{dt}\int_V \rho_s\,dV$$

$$\vec{J}_1\vec{n}_1\,dS_1 + \vec{J}_2\vec{n}_2\,dS_2 + (\text{doprinosi struji kroz plašt}) = -\frac{d}{dt}(\rho_s h\,dS)$$

$$\lim_{h\to 0}\{-\vec{J}_1\vec{n}_{12}\,dS + J_2 n_{12}\,dS + (D.S.K.P)\} = -\frac{d}{dt}\lim_{h\to 0}(\rho_s h\,dS)\ ,\quad h\to 0 \Rightarrow (D.S.K.P)\to 0$$

$$\vec{n}_{12}(J_2 - J_1) = -\frac{d\sigma_s}{dt}$$

ako je $\sigma_s = 0$ ili $\frac{d\sigma_s}{dt} = 0 \Rightarrow \vec{n}_{12}(\vec{J}_2 - J_1) = 0 \Rightarrow J_{1n} = J_{2n} \Rightarrow \varkappa_1 E_{1n} = \varkappa_2 E_{2n} \Rightarrow$

$$\Rightarrow \frac{E_{1n}}{E_{2n}} = \frac{\varkappa_2}{\varkappa_1} \qquad (*)$$

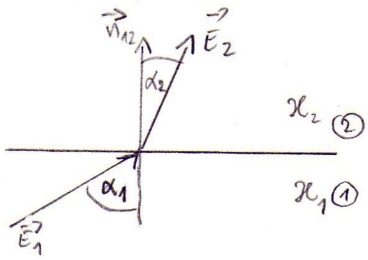
$$E_{1t} = E_{2t} \Rightarrow \vec{n}_{12} \times (E_2 - E_1) = 0$$

$$\Downarrow$$

$$\frac{J_{1t}}{\varkappa_1} = \frac{J_{2t}}{\varkappa_2} \Rightarrow \frac{J_{1t}}{J_{2t}} = \frac{\varkappa_1}{\varkappa_2} \qquad (**)$$

$$\boxed{\begin{aligned} &\vec{n}_{12}(\vec{J}_2 - \vec{J}_1) = -\frac{\partial \sigma_s}{\partial t} = 0 \\ &\vec{n}_{12} \times (\vec{E}_2 - E_1) = 0 \end{aligned}}$$

Iz $(*)$ i $(**)$ slijedi:



$$tg\,\alpha_1 = \frac{E_{1t}}{E_{1n}} \; ; \; tg\,\alpha_2 = \frac{E_{2t}}{E_{2n}}$$

$$\boxed{\frac{tg\,\alpha_1}{tg\,\alpha_2} = \frac{\frac{E_{1t}}{E_{1n}}}{\frac{E_{2t}}{E_{2n}}} = \frac{E_{1t} \cdot E_{2n}}{E_{2t} \cdot E_{1n}} = \frac{\varkappa_1}{\varkappa_2}}$$

ne mijenjaju se na granici

---

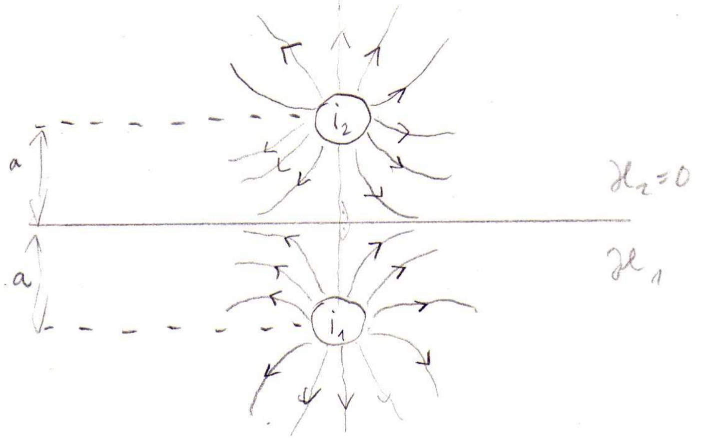
② Analogija statičkog strujnog polja i statičkog električnog polja i odslikavanje u statičkom strujnom polju.

(I) S.S.P = istosmjerna struja $\Rightarrow \nabla \vec{J} = 0$

Homogeni dielektrik bez naboja ($\rho_s = 0$)	Vodljivi materijal, stac. strujanje $\partial\rho_s / \partial t = 0$
Gauss: $\nabla \vec{D} = 0$	Kontinuitet: $\nabla \vec{J} = 0$
$\vec{D} = \varepsilon \vec{E}$	$J = \varkappa \vec{E}$
$\Delta\varphi = 0$	$\Delta\varphi = 0$
Električni tok: $\phi_e = \int_S \vec{D}\vec{n}\,ds$	Strujni tok: $I = \int_S \vec{J}\vec{n}\,ds$
$C = \frac{Q}{U_{ab}} = \frac{\int_S \vec{D} n\,ds}{-\int_b^a \vec{E}\,dl}$	$G = \frac{I}{U_{ab}} = \frac{\int_S \vec{J}\vec{n}\,ds}{-\int_b^a \vec{E}\,dl}$

$\vec{D}$	$\vec{J}$
$\vec{E}$	$\vec{E}$
ε	$\varkappa$
φ	φ
ϕ_e	I
C	G

II



$J_{2n} = 0$

$$J_{1n} = J_{2n} \Rightarrow J_{1n} = 0,\ J_{2n} = 0 \Rightarrow E_{1n} = E_{2n} = 0,\ \frac{\partial \varphi_1}{\partial n} = \frac{\partial \varphi_2}{\partial n} = 0$$

$i_1 = i_2$

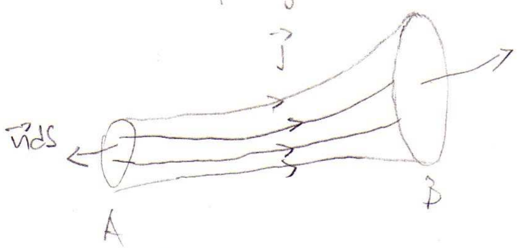
---

3. Gubici snage u vodiču u statičkom strujnom polju

Nastaju zbog otpora.

$$R = \frac{U_{AB}}{I} = \frac{-\int_b^a \vec{E}\,d\vec{l}}{\int_S \vec{J}\vec{n}\,dS} = \frac{-\int_b^a \vec{E}\,d\vec{l}}{\varkappa \int_S \vec{E}\vec{n}\,dS}$$

Jednoliki presjek: $R = \frac{1}{\varkappa}\frac{l}{S} = \rho\frac{l}{S}$

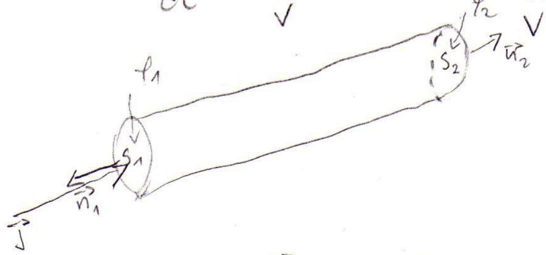


$dq = \vec{J}\vec{n}\,dS\,dt$

$\varphi_A - \varphi_B = -\vec{E}\,d\vec{s}$

$dW = dq\,[\varphi_A - \varphi_B] = \vec{J}\vec{n}\,dS\,dt\,\vec{E}\,d\vec{s} = \vec{J}\vec{E}\,dV\,dt$

$$P = \frac{dW}{dt} = \iiint_V \vec{J}\vec{E}\,dV = -\iiint_V \nabla\cdot(\varphi\vec{J})\,dV = -\oint_S \varphi\vec{J}\vec{n}\,dS \quad (\nabla\varphi\vec{J} = \varphi\nabla\vec{J} + (\nabla\varphi)\vec{J} = -\vec{E}\vec{J})$$



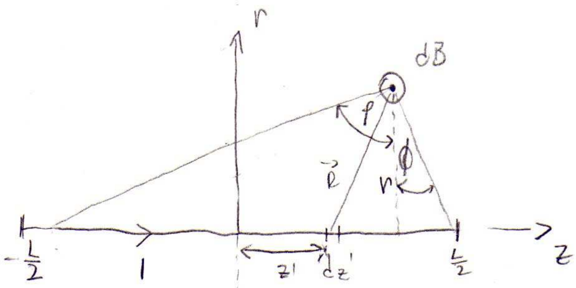
$$P = -\oint \varphi\vec{J}\vec{n}\,dS = I\varphi_1 - I\varphi_2 = UI = I^2R = \frac{U^2}{R}$$

4. Biot-Savartov zakon i magnetska indukcija kratke ravne strujnice.

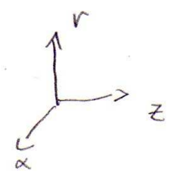
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{R}}{R^3} \qquad \vec{R} = \vec{r} - \vec{r}\,'$$

$\vec{r}\,'$ – položaj dijela strujnice

$\vec{r}$ – točka u kojoj računamo B



$$\vec{r} = r\vec{a}_r + z\vec{a}_z$$



$$\vec{r}\,' = z'\vec{a}_z$$

$$\vec{R} = r\vec{a}_r + (z - z')\vec{a}_z$$

$$d\vec{l} = dz'\,\vec{a}_z \qquad d\vec{l} \times \vec{R} = r\,dz'\,\vec{a}_\alpha$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \vec{R}}{R^3} = \frac{\mu_0 I}{4\pi} \cdot \frac{r\,dz'\,\vec{a}_\alpha}{\left[r^2 + (z - z')^2\right]^{\frac{3}{2}}}$$

$$\vec{B} = \frac{\mu_0 I \vec{a}_\alpha r}{4\pi} \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{dz'}{\left[r^2 + (z - z')^2\right]^{\frac{3}{2}}} = \vec{a}_\alpha \frac{\mu_0 I}{4\pi r} \left[\frac{\frac{L}{2} + z}{\sqrt{\left(\frac{L}{2} + z\right)^2 + r^2}} + \frac{\frac{L}{2} - z}{\sqrt{\left(\frac{L}{2} - z\right)^2 + r^2}} \right] =$$

$$= \vec{a}_\alpha \frac{\mu_0 I}{4\pi r} \left(\sin\rho + \sin\phi \right)$$

$$L \to \infty \implies \vec{B} = \vec{a}_\alpha \frac{\mu_0 I}{2\pi r}$$

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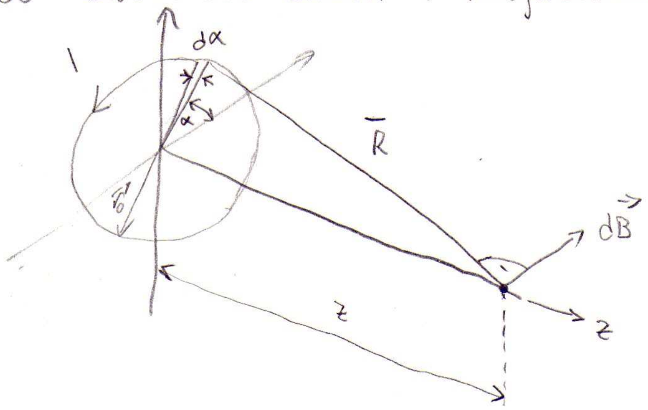
5. Biot-Savartov zakon i magnetska indukcija na osi kružne strujnice.



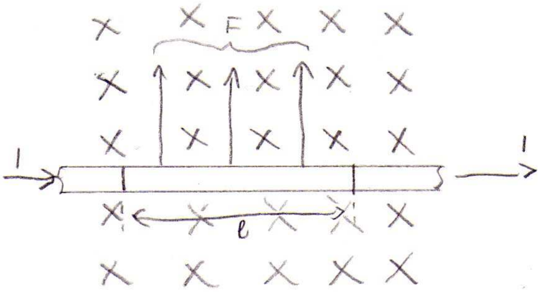
$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{l} \times \vec{R}}{R^3}$$

Ostaje samo komponenta u smjeru $\vec{a}_z$ jer se ostale poništavaju.

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int_0^{2\pi} \frac{r_0 \vec{a}_z}{(r_0^2 + z^2)^{\frac{3}{2}}} \cdot r_0\,d\alpha = \frac{\mu_0 I}{4\pi} \frac{r_0^2 \vec{a}_z}{(r_0^2 + z^2)^{\frac{3}{2}}} \int_0^{2\pi} d\alpha =$$

$$= \frac{\mu_0 I}{2} \cdot \frac{r_0^2}{(r_0^2 + z^2)^{\frac{3}{2}}} \vec{a}_z$$

(6) Sila na strujni element u magnetskom polju



$\vec{f} = q(\vec{v} \times \vec{B})$ – sila na pojedini naboj

$\vec{F} = N\vec{f} = n l S q (\vec{v} \times \vec{B})$

$dQ = Nq = n l S q = n v dt S q \qquad (l = v dt)$

$I = \frac{dQ}{dt} = n v S q$

$\vec{F} = I(\vec{l} \times \vec{B})$

$d\vec{F} = I(d\vec{l} \times \vec{B})$

$$\boxed{\vec{F} = \int_l d\vec{F} = \int_l I(d\vec{l} \times \vec{B})}$$

$i\, d\vec{l} = \vec{J}\, dV$

$$\boxed{\vec{F} = \iiint_V (\vec{J} \times \vec{B})\, dV}$$

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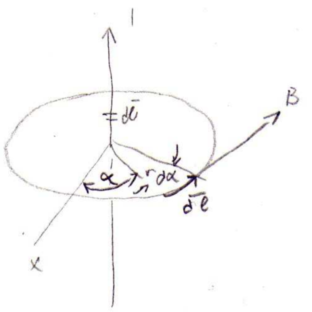
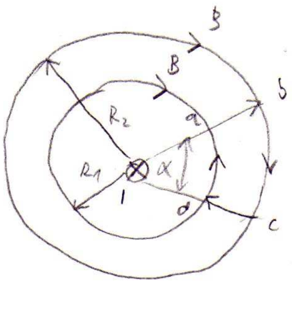
(7) Jednadžbe statičkog magnetskog polja u diferencijalnom i integralnom obliku.

Ampereov zakon





$d\vec{l}_{a-b} = \vec{a}_r dr$

$d\vec{l}_{b-c} = \vec{a}_\alpha R_2 d\alpha$

$d\vec{l}_{c-d} = -\vec{a}_r dr$

$dl_{d-a} = -\vec{a}_\alpha R_1 d\alpha$

$\vec{B} = \vec{a}_\alpha \frac{\mu_0 I}{2\pi r} = \text{konst.}$

$d\vec{l} = \vec{a}_\alpha r d\alpha$

$\oint_c \vec{B} d\vec{l} = \oint_c \vec{a}_\alpha \frac{\mu_0 I}{2\pi r} \vec{a}_\alpha r d\alpha = \frac{\mu_0 I}{2\pi} \oint_c d\alpha = \frac{\mu_0 I}{2\pi} \cdot 2\pi = \mu_0 I$

$$\oint_c \vec{B}\, d\vec{l} = \int_{R_1}^{R_2} \vec{a}_\alpha \frac{\mu_0 \vec{I}}{2\pi \vec{r}} \vec{a}_r\, dr + \int_b^c \vec{a}_\alpha \frac{\mu_0 \vec{I}}{2\pi \vec{R_2}} \vec{a}_\alpha \vec{R_2}\, d\alpha - \int_{R_1}^{R_2} \vec{a}_\alpha \frac{\mu_0 \vec{I}}{2\pi \vec{r}} \vec{a}_r\, dr - \int_d^a \vec{a}_\alpha \frac{\mu_0 \vec{I}}{2\pi \vec{R_1}} \vec{R_1}\, d\alpha$$

$$= 0 + \frac{\mu_0 \vec{I}}{2\pi} \int_b^c d\alpha - 0 - \frac{\mu_0 \vec{I}}{2\pi} \int_d^a d\alpha = \frac{\mu_0 \vec{I}}{2\pi} \alpha - \frac{\mu_0 \vec{I}}{2\pi} \alpha = 0$$

$$\oint \vec{B}\, d\vec{l} = \mu_0 \Sigma I$$

$$\boxed{\oint \vec{B}\, d\vec{l} = \mu_0 \int_S \vec{J_s} \vec{n}\, dS}$$

$$\oint_c \vec{B}\, d\vec{l} = \int_S (\nabla \times \vec{B}) \cdot \vec{n} \cdot dS = \mu_0 \int_S \vec{J_s} \vec{n}\, dS \rightarrow \boxed{\nabla \times \vec{B} = \mu_0 \vec{J_s}}$$

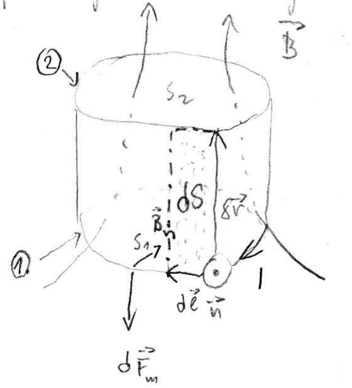
Gaussov zakon

$$\boxed{\oint_S \vec{B} \vec{n}\, dS = 0}$$

$$\oint \vec{B} \vec{n}\, dS = \iiint_V \nabla \vec{B}\, dV = 0$$

$$\boxed{\mathrm{div} B = 0}$$

8. Energija pohranjena u magnetskom polju izražena pomoću magnetskog toka.



$$d\vec{F}_m = I(d\vec{\ell} \times \vec{B})$$

$\delta W = d\vec{F}_v \cdot \delta\vec{r} = -d\vec{F}_m \cdot \delta\vec{r} = -I(d\vec{\ell} \times \vec{B})\,\delta\vec{r}$ – povećanje energije strujnice zbog pomaka silom suprotnom od magnetske

$\delta W = -I \cdot \vec{B}\,(\delta\vec{r} \times d\vec{\ell}) = -I\vec{B}\vec{n}\,dS$ $\quad [\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{C} \cdot (\vec{A} \times \vec{B})]$

$\delta\phi_{pl} = \vec{B}(-\vec{n})dS$ – tok kroz dif. dio plašta valjka opisanog pomakom iz položaja 1 u položaj 2

$$\delta W = -I\vec{B}\vec{n}\,dS = I\,\delta\phi_{pl}$$

$dW = I\,d\phi_{pl}$ $\quad d\phi_{pl}$ – ukupni tok kroz plašt $\quad (d\phi_{pl} = \phi_{2} - \phi_{1})$

Ako strujnicu unesemo u polje iz prostora u kojem nema polja: $\quad W = I(\phi_2 - \phi_1) = I(\phi_2 - 0) = I\phi$

$\boxed{W = I\phi}$ – mag. energija strujnice u mag. polju

9. Magnetska energija sustava strujnih petlji izražena pomoću vektorskog magnetskog potencijala.

Dvije strujnice

$W = \phi_{12} \cdot I_2$ $\quad$ (strujnica 1 fiksna, strujnicu 2 unosimo u polje strujnice 1; ϕ_{12} - tok stvoren strujnicom 1, a obuhvaćen strujnicom 2)

$W = \phi_{21} \cdot I_1$ $\quad$ (S_2 fiksna, S_1 unosimo)

$W = \frac{1}{2}(\phi_{12} \cdot I_2 + \phi_{21} \cdot I_1)$ – energija potrebna za formiranje sustava strujnica

n strujnica

$$W = \frac{1}{2}\sum_{i=1}^{n} I_i \cdot \sum_{j=1}^{n} \phi_{ji} = \frac{1}{2}\sum_{i=1}^{n} I_i \cdot \phi_i$$ (ϕ_i tok svih strujnica osim i-te obuhvaćen i-tom strujnicom)

Prostor, n vodiča sa gustoćama struja $\vec{J}_i$

$\phi_i = \sum_{j=1}^{n} \oint \vec{A}_j d\vec{l}_i$ – tok kroz i-ti vodič

- l_i – kontura i-tog vodiča
- $\vec{A}_j$ – vekt. mag. pot. stvoren j-tim vodičem

$I_i = \int_{S_i} \vec{J}_i \vec{n}_i \, dS_i$ – struja kroz i-ti vodič

- S_i – površina presjeka
- $\vec{n}_i$ – normala na površinu i-tog vodiča

$$W = \frac{1}{2}\sum_{i=1}^{n} I_i \cdot \phi_i = \frac{1}{2}\sum_{i=1}^{n} \int \vec{J}_i \vec{n}_i dS_i \left(\sum_{j=1}^{n} \oint \vec{A}_j d\vec{l}_i\right)$$

$\vec{n}_i \cdot d\vec{l}_i = dl_i$ – zbog kolinearnosti

$dS_i \cdot dl_i = dV_i$ – dV_i - dif. volumen i-tog vodiča

Prijelaz sa diskretnih vodiča na cijeli prostor u kojem se nalaze:

$W = \frac{1}{2}\int_V \vec{J}\vec{A} \, dV$ – $\vec{A}$ – ukupni vekt. mag. pot. na mjestu dV

(10) Magnetska energija sustava strujnih petlji izražena pomoću vektora magnetskog polja.

$$\vec{J}\vec{A} = \vec{B}\vec{H} + \nabla(\vec{H} \times \vec{A})$$

$$W = \frac{1}{2}\int_V \vec{J}\vec{A} \, dV = \frac{1}{2}\int_V \vec{B}\vec{H} \, dV + \frac{1}{2}\int_V \nabla(\vec{H} \times \vec{A}) \, dV$$

primjenimo Gaussov teorem: $W = \frac{1}{2}\int_V \vec{B}\vec{H} \, dV + \underbrace{\frac{1}{2}\oint_S (\vec{H} \times \vec{A})\vec{n} \, dS}_{\text{ako obuhvatimo cijeli prostor izraz teži k nuli}}$

$$W = \frac{1}{2}\int_V \vec{B}\vec{H} \, dV$$

Materijal linearan, homogen i izotropan: $\vec{B} = \mu \vec{H}$

$$W = \frac{1}{2}\mu \int_V H^2 dV = \frac{1}{2\mu}\int_V B^2 dV$$

(11) Magnetska energija u nelinearnim materijalima i gubici zbog histereze.

Linearan materijal, magnetsko polje stvaramo strujom od 0 do I, mag. ind. raste od 0 do B.

$$dW = i \cdot d\phi$$

$$d\phi = \int_S d\vec{B}\,\vec{n} \cdot dS \; ; \quad i = \oint \vec{H}\, d\vec{l} \; ; \quad \vec{n}\, d\vec{l} = dl \; ; \quad dS \cdot dl = dV$$

$$dW = \int_V \vec{H}\, d\vec{B}\, dV$$

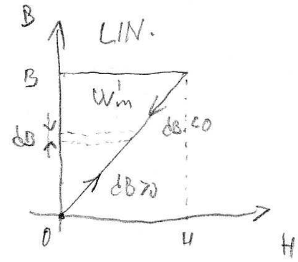
$$W = \int_V \left(\int_{B=0}^{B} \vec{H}\, d\vec{B} \right) dV$$

$$w = \int_{B=0}^{B} \vec{H}\, d\vec{B}$$ – gustoća energije mag. polja

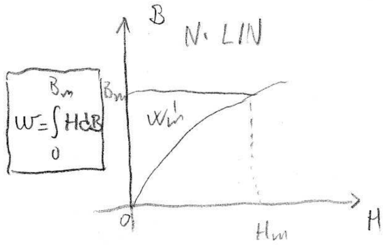
Lin. materijal magnetiziramo povećavajući indukciju, a zatim razmagnetiziramo smanjujući indukciju:

$$w = \int_{B=0}^{B} \vec{H}\, d\vec{B} = -\int_{B}^{B=0} \vec{H}\, d\vec{B}$$ → energija se u cijelosti vraća!

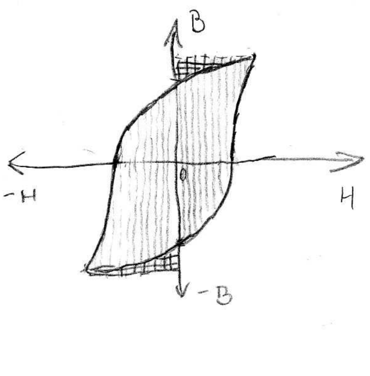
$w = \frac{B_m H_m}{2}$



$w = \int_0^{B_m} H dB$



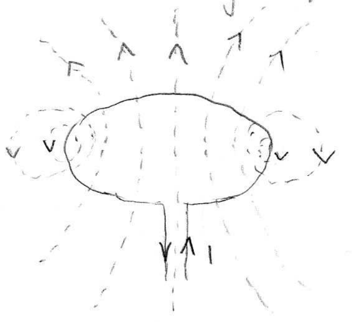
Kod feromagnetskih materijala je uložena energija veća od one koju dobivamo razmagnetiziranjem. Izmjeničnim magnetiziranjem dobivamo petlju histereze čija površina je proporcionalna gubicima. Do gubitaka dolazi zbog trenja domena u feromagnetskom materijalu.



– uložena energija

– vraćena energija

(12.) Induktivitet strujne petlje.



Obuhvaćeni tok: $\phi = L \cdot I$

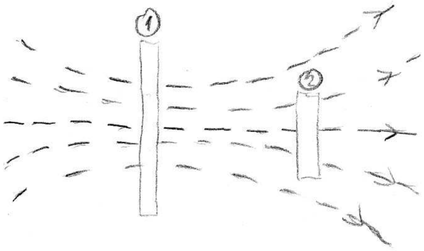
$$L = \frac{\phi}{I}$$ – induktivitet ili koeficijent samoindukcije $\left[\frac{Vs}{A} = \Omega s = H\right]$

Zavojnica – ulančani tok: $\psi = N\phi$

$$L = \frac{\psi}{I} = \frac{N \cdot \phi}{I} = \frac{N \cdot N \cdot \phi_1}{I} = N^2 \frac{\phi_1}{I} = N^2 L_1$$

- ϕ_1 – tok stvoren jednim zavojem
- L_1 – induktivitet jednog zavoja

(13.) Međuinduktivitet



ϕ_1 – tok stvoren krugom 1

ϕ_{21} – tok stvoren krugom 1 obuhvaćen krugom 2

$$\phi_{21} = k_1 \cdot \phi_1 \; ; \; k_1 \leq 1$$

$$M_{21} = \frac{\phi_{21}}{I_1}$$

N_1, N_2 broj zavoja krugova

$$\phi_{1u} = N_1 \cdot \phi_1 \; ; \; \phi_{21u} = N_1 \cdot \phi_{21} = k_1 \cdot N_1 \cdot \phi_1 = k_1 \cdot \phi_{1u}$$

$$\psi_{21u} = N_2 \cdot \phi_{21u} = N_2 N_1 \phi_{21}$$

$$M_{21} = \frac{\psi_{21u}}{I_1} = \frac{N_2 \cdot N_1 \cdot \phi_{21}}{I_1}$$

$M_{12} = M_{21}$

(14) Odnos međuinduktiviteta i samoinduktiviteta dviju strujnih petlji.

$$M_{12} = \frac{\Psi_{12u}}{I_2} = \frac{N_1 N_2 \phi_{12}}{I_2} = \frac{N_1 N_2 k_2 \phi_2}{I_2} = N_1 \cdot k_2 \frac{N_2 \cdot \phi_2}{I_2} = k_2 \cdot N_1 \frac{L_2}{N_2} \quad (*)$$

$$M_{21} = k_1 \cdot N_2 \frac{L_1}{N_1} \quad (**)$$

$$L_1 = \frac{\Psi_{1u}}{I_1} = \frac{N_1 \cdot \phi_{1u}}{I_1} = \frac{N_1 \cdot N_1 \cdot \phi_1}{I_1} \quad ; \quad L_2 = \frac{\Psi_{2u}}{I_2} = \frac{N_2 \cdot \phi_{2u}}{I_2} = \frac{N_2 \cdot N_2 \cdot \phi_2}{I_2}$$

$$(*) \cdot (**) = M_{12} \cdot M_{21} = M^2 = k_1 k_2 L_1 L_2 = k^2 L_1 L_2$$

$$M = k\sqrt{L_1 L_2}$$

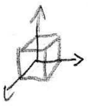
(15.) (16.) (17.) (18.) Magnetostatika 35 – 43.

(19.) Vidi (7). Pretpostavljam da računamo sa gustoćom struje ($\oint \vec{B} \, d\vec{l} = \mu_0 \int_S \vec{J}_s \vec{n} \, ds$)

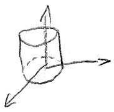
(20.) Vektorski magnetski potencijal, dif. jednadžba i proračun tokova u mag. polju.

$$\operatorname{div} \vec{B} = \nabla \vec{B} = 0$$

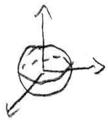
$$\vec{B} = \operatorname{rot} \vec{A} = \nabla \times \vec{A}$$



$$\vec{B} = \vec{a}_x \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) + \vec{a}_y \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) + \vec{a}_z \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)$$



$$\vec{B} = \vec{a}_r \left(\frac{1}{r} \frac{\partial A_z}{\partial \alpha} - \frac{\partial A_\alpha}{\partial z} \right) + \vec{a}_\alpha \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r} \right) + \vec{a}_z \frac{1}{r} \left(\frac{\partial}{\partial r} (r \cdot A_\alpha) - \frac{\partial A_r}{\partial \alpha} \right)$$



$$\vec{B} = \vec{a}_r \frac{1}{r \sin\theta} \left(\frac{\partial}{\partial \theta} (\sin\theta \cdot A_\alpha) - \frac{\partial A_\theta}{\partial \alpha} \right) + \vec{a}_\theta \frac{1}{r} \left(\frac{1}{\sin\theta} \frac{\partial A_r}{\partial \alpha} - \frac{\partial}{\partial r} (r \cdot A_\alpha) \right) + \vec{a}_\alpha \frac{1}{r} \left(\frac{\partial}{\partial r} (r \cdot A_\theta) - \frac{\partial A_r}{\partial \theta} \right)$$

$\operatorname{div} \vec{A} = \nabla \cdot A = 0$ – Coulombovo baždarenje – osigurava jedinstvenost vekt.mag. pot.

$$\phi = \int_S \vec{B} \vec{n} \, dS \xrightarrow[\text{TEOREM}]{\text{STOKESOV}} \phi = \int_S \operatorname{rot} \vec{A} \vec{n} \, ds = \oint_C \vec{A} \cdot d\vec{l}$$

DIF. JEDNADŽBA

$$\nabla \times \vec{H} = \operatorname{rot} \vec{H} = \vec{J}_s$$

$$\vec{H} = \frac{\vec{B}}{\mu} = \frac{\operatorname{rot} \vec{A}}{\mu}$$

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \vec{A} \right) = J_s$$

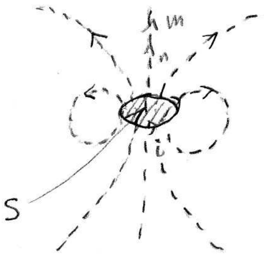
$\nabla \times \nabla \times \vec{A} = \mu \vec{J}_s$ (homogeni, linearni, izotropni materijal)

$\nabla \times \nabla \times \vec{A} = \nabla(\nabla A) - \Delta A = \mu J_s$

$\Delta \vec{A} = -\mu \vec{J}_s$ – Poissonova jednadžba

Ako u prostoru nema slobodnih struja ($J_s = 0$) : $\Delta \vec{A} = 0$ – Laplaceova jednadžba

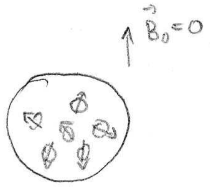
21. Magnetizacija i amperske struje



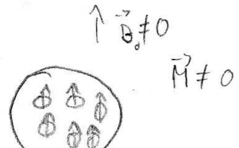
Magnetsko polje strujne petlje

$\vec{m} = \vec{n} i S$ – magnetski moment

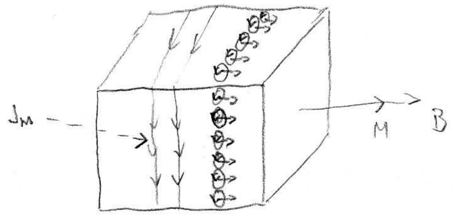
Gibanje naboja = struja



Materijal bez narinutog polja



Materijal u vanjskom polju



Materijal u homogenom mag. polju – domene su orijentirane u istom smjeru, mikroskopske strujne petlje stvaraju mag. momente. U unutrašnjosti se struje poništavaju, a na površini ostaje struja magnetizacije (amperska struja) gustoće J_m.

22. Jakost magnetskog polja i ponašanje materijala u magnetskom polju.

$\vec{B} = \vec{B}_0 + \vec{B}_M$ – $\vec{B}_0$ – vanjsko polje iz (21)

– $\vec{B}_M$ – od amperskih struja

$\vec{B}_0 = \vec{B} - \vec{B}_M$

$\oint_C \vec{B}_0 \, d\vec{l} = \mu_0 \int_S \vec{J}_s \vec{n} \, dS$; $\oint_C (\vec{B} - \vec{B}_M) \, d\vec{l} = \mu_0 \int_S \vec{J}_s \vec{n} \, dS$

$\oint_C \left(\frac{\vec{B} - \vec{B}_M}{\mu_0} \right) d\vec{l} = \int_S \vec{J}_s \vec{n} \, dS$

$H = \frac{B_0}{\mu_0} = \frac{B - B_M}{\mu_0} = \frac{B}{\mu_0} - M$; $M = \frac{B_M}{\mu_0}$ (vektor gustoće magnetiziranja, $\left[\frac{A}{m}\right]$)

$$\oint_C \vec{H} d\vec{l} = \int_S \vec{J_s} \vec{n} dS$$

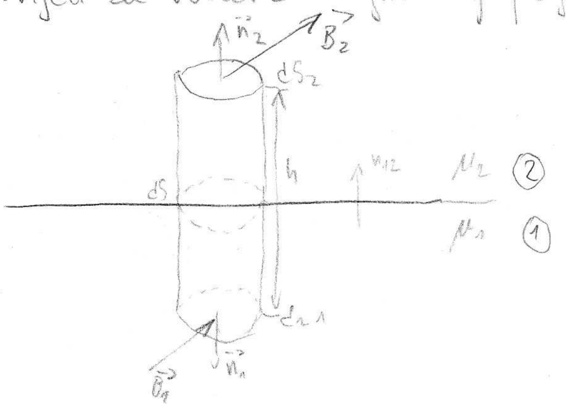
$$\vec{B} = \vec{B_0} + \vec{B_M} = \mu_0 \vec{H} + \mu_0 \vec{M} = \mu_0 (\vec{H} + \vec{M}) = \mu_0 \vec{H} (1 + \chi_m) = \mu_0 \mu_r \vec{H} = \mu \vec{H}$$

Dijamagnetski materijali
- μ_r je nešto manji od 1 (npr. 0,99998)
- postoji vektor magnetizacije $\vec{M}$ suprotan vanjskom polju
- djelovanje je slabo izraženo
- bizmut, vodik, zlato, ugljik...

Paramagnetski materijali
- μ_r je nesto veći od 1 (npr. 1,0013)
- $\vec{M}$ u smjeru polja
- djelovanje jače od dijamagnetskog, ali i dalje neznatno
- kisik, aluminij, olovo...

Feromagnetski materijali
- veliki μ_r
- moguća je permanentna magnetizacija
- nelinearni - kod snažnih polja ulaze u zasićenje
- željezo, kobalt, nikal, određene legure (ne nužno feromagnetskih materijala)
- više u skripti 26 - 30 !

(23) Uvjeti za vektore magnetskog polja na granici dva materijala.



$$\oint_S \vec{B} \vec{n} dS = 0$$

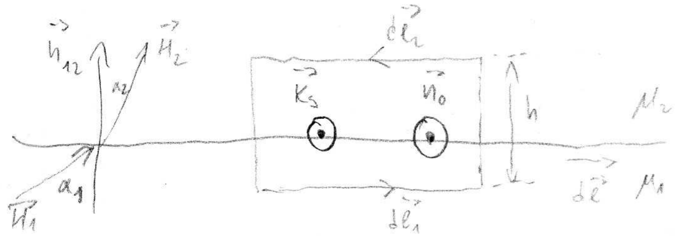
$$\vec{B_1} \vec{n_1} \cdot dS_1 + \vec{B_2} \cdot \vec{n_2} \cdot dS_2 + (\text{DOPRINOS TOKU KROZ PLAŠT}) = 0$$

$h \to 0$

$$\lim_{h \to 0} \{ -\vec{B_1} \vec{n_{12}} dS + \vec{B_2} \vec{n_{12}} dS + \underbrace{(\text{D.T.K.P.})}_{\to 0} \} = 0 \quad / : dS$$

$$\vec{n_{12}} (\vec{B_2} - \vec{B_1}) = 0$$

$$B_{1n} = B_{2n} \longrightarrow \mu_1 H_{1n} = \mu_2 H_{2n} \longrightarrow \frac{H_{1n}}{H_{2n}} = \frac{\mu_2}{\mu_1}$$



$$d\vec{l} = \vec{t}\,dl = d\vec{l}_1 = -d\vec{l}_2$$

$t \rightarrow$ jedinični tangencijalni vektor

Primjenjujemo amperov zakon:

$$\oint_l \vec{H}\,d\vec{l} = \int_S \vec{J}_s \vec{n} \cdot ds$$

$\vec{H}_1 d\vec{l}_1 + \vec{H}_2 d\vec{l}_2 +$ (DOPRINOSI NA STRANICAMA h) $= \vec{J}_s \cdot \vec{n}_0 \cdot h \cdot dl$

PO PRAVILU DESNE RUKE

$$\lim_{h \to 0} \{ \vec{H}_1 \vec{t}\,dl - \vec{H}_2 \vec{t}\,dl + \underbrace{(D.N.S.\ h)}_{\to 0} \} = \lim_{h \to 0} \{ \vec{J}_s \vec{n}_0 h \cdot dl \} \quad /:dl$$

$$(\vec{H}_1 - \vec{H}_2)\,\vec{t} = \vec{n}_0 \lim_{h \to 0} \{ \vec{J}_s h \} = \vec{n}_0 \cdot \vec{K}_s$$

$$\vec{t} = -(\vec{n}_0 \times \vec{n}_{12})$$

$$(\vec{H}_2 - \vec{H}_1)(\vec{n}_0 \times \vec{n}_{12}) = \vec{n}_0 \vec{K}_s$$

$$n_0 \{ \vec{n}_{12} \times (\vec{H}_2 - \vec{H}_1) \} = \vec{n}_0 \cdot \vec{K}_s \quad /:\vec{n}_0$$

$$\vec{n}_{12} \times (\vec{H}_2 - \vec{H}_1) = \vec{K}_s$$

$$\vec{H}_{1t} = \vec{H}_{2t} \rightarrow \frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2} \longrightarrow \frac{B_{1t}}{B_{2t}} = \frac{\mu_1}{\mu_2}$$

24. - 30. Prilično detaljno u skripti za labose, nema smisla skraćivati...