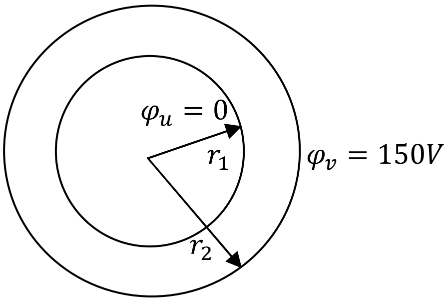
ELEKTROSTATIKA 2/3

by Vedax

20.



Cilindrični je sustav pa koristimo formulu:

$$\Delta\varphi = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) = 0$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) = 0$$

$$r\frac{\partial\varphi}{\partial r} = C_1$$

$$\frac{\partial\varphi}{\partial r} = \frac{C_1}{r}$$

$$\varphi = C_1 \ln r + C_2$$

1) unutrašnja granica:

$$0 = C_1 \ln r_1 + C_2 \rightarrow C_2 = -C_1 \ln r_1$$

2) vanjska granica:

$$150 = C_1 \ln r_2 + C_2 = C_1 \ln r_2 - C_1 \ln r_1 = C_1 \ln\frac{r_2}{r_1} \rightarrow \boldsymbol{C_1} = \frac{150}{\ln\frac{r_2}{r_1}} = \frac{150}{\ln\frac{20}{1}} = 50.07 \approx \mathbf{50.1}$$

pa je onda

$$\boldsymbol{C_2} = -C_1 \ln r_1 = -50.07 \ln 0.001 = 345.87 \approx \mathbf{345.9}$$

pa naš potencijal ima oblik:

$$\boldsymbol{\varphi} = C_1 \ln r + C_2 = \mathbf{50.1 \ln \boldsymbol{r} + 345.9}\ [\boldsymbol{V}]$$

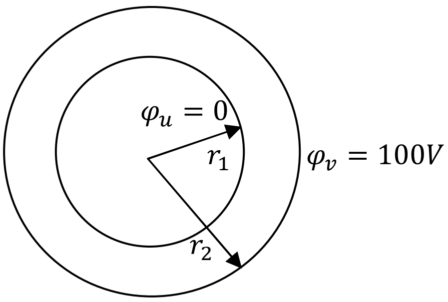
21.

Električno polje za potencijal iz prethodnog zadatka je:

$$\vec{E} = -\nabla\varphi$$

$$\vec{\boldsymbol{E}} = -\left(\frac{\partial\varphi}{\partial r}\overrightarrow{a_r} + \frac{\partial\varphi}{\partial\alpha}\overrightarrow{a_\alpha} + \frac{\partial\varphi}{\partial z}\overrightarrow{a_z}\right) = -\left(\frac{50.1}{r}\overrightarrow{a_r}\right) = \frac{\mathbf{50.1}}{\boldsymbol{r}}(-\overrightarrow{\boldsymbol{a_r}})\ [\boldsymbol{V/m}]$$

22.



S obzirom da je ovo kuglasti kondenzator, koristimo sljedeću formulu:

$$\Delta\varphi = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) = 0$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right) = 0$$

$$r^2\frac{\partial\varphi}{\partial r} = C_1$$

$$\frac{\partial\varphi}{\partial r} = \frac{C_1}{r^2}$$

$$\varphi = -\frac{C_1}{r} + C_2$$

1) unutrašnja granica:

$$0 = -\frac{C_1}{r_1} + C_2 \rightarrow C_2 = \frac{C_1}{r_1}$$

2) vanjska granica:

$$100 = -\frac{C_1}{r_2} + C_2 = -\frac{C_1}{r_2} + \frac{C_1}{r_1} = C_1\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = C_1\left(\frac{1}{0.1} - \frac{1}{2}\right) = \frac{19C_1}{2} \rightarrow \boldsymbol{C_1 = 10.53}$$

pa je onda

$$\boldsymbol{C_2} = \frac{C_1}{r_1} = \frac{10.53}{0.1} = \mathbf{105.3}$$

pa naš potencijal ima oblik:

$$\boldsymbol{\varphi} = -\frac{C_1}{r} + C_2 = -\frac{\mathbf{10.53}}{\boldsymbol{r}} + \mathbf{105.3}\ [\boldsymbol{V}]$$

23.

Električna indukcija za zadatak 22.

$$\vec{\boldsymbol{D}} = \varepsilon\vec{E} = -\varepsilon_0\left(\frac{\partial\varphi}{\partial r}\overrightarrow{a_r} + \frac{\partial\varphi}{\partial\alpha}\overrightarrow{a_\alpha} + \frac{\partial\varphi}{\partial z}\overrightarrow{a_z}\right) = -\varepsilon_0\left(\frac{10.53}{r^2}\overrightarrow{a_r}\right) = \frac{\mathbf{-9.23\cdot 10^{-11}}}{\boldsymbol{r^2}}\overrightarrow{\boldsymbol{a_r}}\ [\boldsymbol{C/m^2}]$$

25.

Opet je cilindirčni sustav, pa ćemo koristiti formulu već izvedenu u zadatku 20.

$$\varphi = C_1\ln r + C_2$$

1) unutrašnja granica:

$$0 = C_1\ln r_1 + C_2 \rightarrow C_2 = -C_1\ln r_1 = -C_1\ln 0.005 = 5.3C_1$$

2) vanjska granica:

$$\varphi_0 = C_1\ln r_2 + C_2 = C_1\ln 0.025 + 5.3C_1 = 1.611C_1 \rightarrow C_1 = 0.62\varphi_0$$

pa je onda

$$C_2 = 3.3\varphi_0$$

jednadžba za naš potencijal je

$$\varphi = C_1\ln r + C_2 = \varphi_0(0.62\ln r + 3.3)$$

S obzirom da je

$$\vec{E} = -\nabla\varphi$$

imamo:

$$-8.28\cdot 10^3\overrightarrow{a_r} = -\frac{0.62\varphi_0}{r}\overrightarrow{a_r}$$

$$\varphi_0 = \frac{8.28\cdot 10^3}{0.62}r = 13306.45r$$

S obzirom da je ovo električno polje zadano za $r = 15mm$, njega samo puknemo gore u formulu i dobijemo:

$$\boldsymbol{\varphi_0 = 199.6V \approx 200V}$$

26.

Trebamo odrediti gustoću naboja na vanjskom vodiču. Za njega vrijedi $r = 25mm$. Odnosno:

$$\vec{E} = -\frac{0.62\varphi_0}{r}\overrightarrow{a_r} = -4960\overrightarrow{a_r}$$

Jakost polja je $|\vec{E}| = E = 4960\ V/m$.

Gauss:

$$\oiint_S \vec{E}\vec{n}dS = \frac{Q}{\varepsilon_0}$$

Ostaje nam samo plašt valjka za S jer je skalarni umnožak normale i vektora električnog polja na „poklopcima" cilindra 0.

$$E\iint dS = \frac{Q}{\varepsilon_0}$$

$$E \cdot 2r\pi l = \frac{Q}{\varepsilon_0}$$

gdje je l visina(duljina) cilindra.

Obrzirom da je $\sigma = \frac{Q}{S} = \frac{Q}{2r\pi l}$, dobijemo:

$$\varepsilon_0 E = \frac{Q}{2r\pi l} = \sigma$$

$$\boldsymbol{\sigma} = 8.854 \cdot 10^{-12} \cdot 4960 = \boldsymbol{44nC/m^2}$$