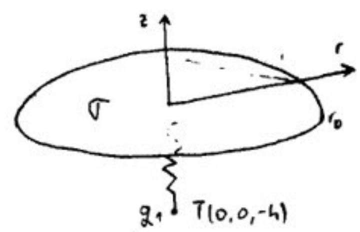
MEĐUISPIT 2020

1. m

točkasti naboj q_1

$z=0$ okrugli disk r_0, σ

$$\sigma = \frac{-q_1 h}{2\pi \left(r^2+h^2\right)^{1,5}}$$



a) $\vec{E} = ?$

$T(0,0,-h)$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \iint \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}\,\sigma\, ds = \frac{1}{4\pi\varepsilon_0} \int_0^{2\pi}\int_0^{r_0} -\frac{q_1 \cdot h}{2\pi (r^2+h^2)^{3/2}} \cdot \frac{-d\,\hat{a}_z - r\hat{a}_r}{(r^2+h^2)^{3/2}}\, r\, dr\, d\alpha$$

$$= \frac{1}{4\pi\varepsilon_0} \int_0^{r_0}\int_0^{2\pi} \frac{q_1 h}{2\pi (r^2+h^2)^3} \cdot (\underbrace{r\hat{a}_r}_{=0 \text{ jer je } \int_0^{2\pi} \vec{a}_r\, d\alpha = 0} + h\hat{a}_z)\, r\, d\alpha\, dr$$

$$= \frac{1}{4\pi\varepsilon_0} \int_0^{r_0}\int_0^{2\pi} \frac{q_1 h^2 r}{2\pi (r^2+h^2)^3}\, d\alpha\, dr\, \hat{a}_z$$

$$= \frac{1}{4\pi\varepsilon_0}\, q_1 h^2 \int_0^{r_0} \frac{r}{(r^2+h^2)^3}\, dr\, \hat{a}_z$$

$$= \frac{q_1 h^2}{4\pi\varepsilon_0} \cdot \frac{1}{4} \frac{1}{(r^2+h^2)^2} \Big|_0^{r_0} \hat{a}_z$$

$$= \frac{q_1 h^2}{16\pi\varepsilon_0} \left(\frac{1}{h^4} - \frac{1}{(r_0^2+h^2)^2}\right) \hat{a}_z$$

$$E_z = \underline{\frac{q_1 h^2}{16\pi\varepsilon_0} \left(\frac{1}{h^4} - \frac{1}{(r_0^2+h^2)^2}\right)}$$

$$F_z = q_1 \cdot E_z = \underline{\frac{q_1^2 h^2}{16\pi\varepsilon_0} \left(\frac{1}{h^4} - \frac{1}{(r_0^2+h^2)^2}\right)}$$

b) $m = ?$

$\vec{r}_0 \to \infty$

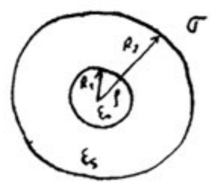
$$\vec{F} = \frac{q_1^2}{16\pi\varepsilon_0 h^2} = mg \implies \boxed{m = \frac{q_1^2}{16\pi\varepsilon_0 h^2 g}}$$

② $\rho = -kr \quad , r < R_1$

ε_s

σ

$E = ?$



$$\varepsilon E \cdot 2r\pi h = \int_0^h \int_0^{2\pi} \int_0^r -kr \cdot r \, dr \, d\alpha \, dh$$

$r < R_1$

$$\varepsilon E \cdot \cancel{2}r\cancel{\pi}\cancel{h} = -k \cdot \cancel{h} \cdot \cancel{2\pi} \cdot \frac{r^3}{3}$$

$$\boxed{E = -k \cdot \frac{r^2}{3\varepsilon_0} \quad , r < R_1}$$

$R_2 > r > R_1$

$$\varepsilon E \; 2r\pi h = -k \cdot h \; 2\pi \; \frac{R_1^3}{3}$$

$$\boxed{E = -k \frac{R_1^3}{3\varepsilon_s r} \quad , \quad R_2 > r > R_1}$$

$r > R_2$

$$\varepsilon E \cdot 2r\pi h = -k h \cdot 2\pi \cdot \frac{R_1^3}{3} + \underbrace{\iint \sigma dS}_{\sigma \iint_S dS = \sigma \cdot 2R_2\pi \cdot h}$$

$$\varepsilon E \cdot \cancel{2}r\cancel{\pi}\cancel{h} = -k\cancel{h}\cancel{2\pi} \; \frac{R_1^3}{3} + \sigma \cdot \cancel{2}R_2\cancel{\pi}\cancel{h}$$

$$\boxed{E = -\frac{kR_1^3}{3r\varepsilon_0} + \frac{\sigma R_2}{r\varepsilon_0} \quad , \quad r > R_2}$$

3) $-x - 2y + z = 0$

$\mu_{r1} = 6$

$\vec{B_1} = 3\vec{a_x} + 3\vec{a_y} - 2\vec{a_z}$

$\mu_{r2} = 2$

$\vec{B_2} = ?$

$\vec{n_{12}} \times (\vec{H_2} - \vec{H_1}) = 0$

$\vec{n_{12}} \cdot (\vec{B_2} - \vec{B_1}) = 0$

$$\vec{n_{12}} = \frac{-\vec{a_x} - 2\vec{a_y} + \vec{a_z}}{\sqrt{6}}$$

$\vec{n_{12}} \cdot (\vec{B_2} - \vec{B_1}) = -(B_{2x} - 3) - 2(B_{2y} - 3) + (B_{2z} + 2) = 0$

$\vec{n_{12}} \times (\vec{H_2} - \vec{H_1}) = 0$

$$\begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ -1 & -2 & 1 \\ H_{2x}-H_{1x} & H_{2y}-H_{1y} & H_{2z}-H_{1z} \end{vmatrix} = \hat{a}_x(-2H_{2z} + 2H_{1z} - H_{2y} + H_{1y}) - \hat{a}_y(-H_{2z} + H_{1z} - H_{2x} + H_{1x}) + \hat{a}_z(-H_{2y} + H_{1y} + 2H_{2x} - 2H_{1x}) = 0$$

$-2H_{2z} - H_{2y} = -2H_{1z} - H_{1y}$ +

$H_{2z} + H_{2x} = H_{1z} + H_{1x} \quad / \cdot 2$

$2H_{2x} - H_{2y} = 2H_{1x} - H_{2y}$

$2H_{2x} - H_{2y} = 2H_{1x} - H_{2y}$

Nisu nezavisne jednadžbe → biramo dvije ; gornju od skalarnog umnoška

$2H_{2z} + H_{2y} = 2H_{1z} + H_{1y}$

$H_{2z} + H_{2x} = H_{1z} + H_{1x}$

$-B_{2x} - 2B_{2y} + B_{2z} = -3 - 6 - 2 = -11\,T \qquad B = \mu H \rightarrow H = \frac{B}{\mu}$

$2\frac{B_{2z}}{2} + \frac{B_{2y}}{2} = 2 \cdot \frac{B_{1z}}{6} + \frac{B_{1y}}{6} \Big/ \cdot 6$

$\frac{B_{2z}}{2} + \frac{B_{2x}}{2} = \frac{B_{1z}}{6} + \frac{B_{1x}}{6} \Big/ \cdot 6$

$B_{2x} + 2B_{2y} - B_{2z} = 11$

$$3B_{2y} + 6B_{2z} = 2B_{1z} + B_{1y} = -1$$

$$3B_{2x} + 3B_{2z} = B_{1z} + B_{1x} = 1$$

$$B_{2x} + 2B_{2y} - B_{2z} = 11 \qquad /\cdot 3 \qquad /\cdot 6$$

$$6B_{2x} + 6B_{2y} = 34$$

$$6B_{2x} + 15B_{2y} = 65$$

$$9B_{2y} = 31$$

$$B_{2y} = \frac{31}{9}$$

$$B_{2x} = \frac{17}{3} - \frac{31}{9} = \frac{20}{9}$$

$$B_{2z} = \frac{1}{3} - B_{2x} = -\frac{17}{9}$$

$$\boxed{\vec{B_2} = \frac{20}{9}\hat{a_x} + \frac{31}{9}\hat{a_y} - \frac{17}{9}\hat{a_z}}$$

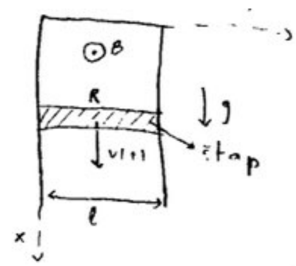
(4) l, m, R

$t=0 \quad v=0$

$v(t) = ?$

$i(t) = ?$

$v_{max} = ?$



Smjer struje

$$I = \frac{Blv}{R}$$

$$ma = mg - BIl$$

$$m\frac{dv}{dt} = mg - \frac{B^2l^2}{R}v \quad / : m$$

$$\frac{dv}{dt} + \underbrace{\frac{B^2l^2}{Rm}}_{k} v = g \quad / \cdot \mathcal{L}$$

laplaceova transformacija uz $v(0) = 0$

$$s \cdot V + kV = \frac{g}{s} \qquad K = \frac{B^2l^2}{Rm}$$

$$V = \frac{g}{s(s+k)} = \frac{g}{k}\left(\frac{1}{s} - \frac{1}{s+k}\right) \multimap \frac{g}{k}\left(1 - e^{-kt}\right)$$

$$\boxed{v(t) = \frac{gRm}{B^2l^2} \cdot \left(1 - e^{-\frac{B^2l^2}{Rm}t}\right)}$$

$$\boxed{I(t) = \frac{Blv}{R} = \frac{gm}{Bl}\left(1 - e^{-\frac{B^2l^2}{Rm}t}\right)}$$

$$v_{max} \rightarrow v'(t) = 0$$

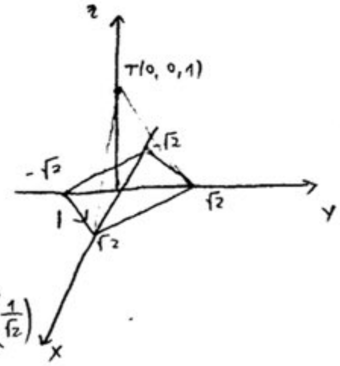
$$v'(t) = \left[\frac{g}{k}\left(1 - e^{-kt}\right)\right]' = \frac{g}{k} \cdot k e^{-kt} = g e^{-kt} \qquad t \rightarrow \infty$$

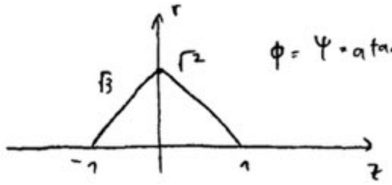
$$\boxed{v_{max} = v(\infty) = \frac{gRm}{B^2l^2}}$$

(5.) $I = 1A$

$T(0, 0, 1)$

$\vec{B} = ?$





$\phi = \Psi = atan\left(\frac{1}{\sqrt{2}}\right)$

$$\vec{B_1} = \frac{I\mu_0}{4\pi r}(\sin\Psi + \sin\Psi) = \frac{1}{4\pi\frac{\sqrt{2}}{2}} \cdot 2\sin\phi \; \hat{a}_{\alpha_1} = \frac{1}{2\sqrt{2}} \sin\phi \; \hat{a}_z$$

$$\hat{a}_\alpha = \hat{a}_z \times \hat{a}_r$$

$$\hat{a}_{z_1} = \frac{-\hat{a}_x + \hat{a}_y}{\sqrt{2}} \qquad \hat{a}_{r_1} = -\frac{1}{2}\hat{a}_x - \frac{1}{2}\hat{a}_y + \frac{1}{2}\hat{a}_z$$

$$\hat{a}_{\alpha_1} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{2\sqrt{2}}\hat{a}_x + \frac{1}{2\sqrt{2}}\hat{a}_y + \frac{1}{\sqrt{2}}\hat{a}_z$$

$$|\vec{B_1}| = |\vec{B_2}| = |\vec{B_3}| = |\vec{B_4}|$$

$$\hat{a}_{\alpha_2} = -\frac{1}{2\sqrt{2}}\hat{a}_x + \frac{1}{2\sqrt{2}}\hat{a}_y + \frac{1}{\sqrt{2}}\hat{a}_z$$

$$\hat{a}_{\alpha_3} = -\frac{1}{2\sqrt{2}}\hat{a}_x - \frac{1}{2\sqrt{2}}\hat{a}_y + \frac{1}{\sqrt{2}}\hat{a}_z$$

$$\hat{a}_{\alpha_4} = \frac{1}{2\sqrt{2}}\hat{a}_x - \frac{1}{2\sqrt{2}}\hat{a}_y + \frac{1}{\sqrt{2}}\hat{a}_z$$

$$\Sigma B = \frac{1}{4\pi} \Sigma \hat{a}_{\alpha i} = \frac{\mu_0}{\cancel{2}\sqrt{2}\pi} \sin\phi \; \frac{\cancel{4}}{\cancel{\sqrt{2}}}\hat{a}_z = \frac{\mu_0}{\pi} \sin\left(atan\left(\frac{1}{\sqrt{2}}\right)\right)\hat{a}_z$$

$$\boxed{\vec{B} = 2,31 \cdot 10^{-7} \; \hat{a}_z}$$