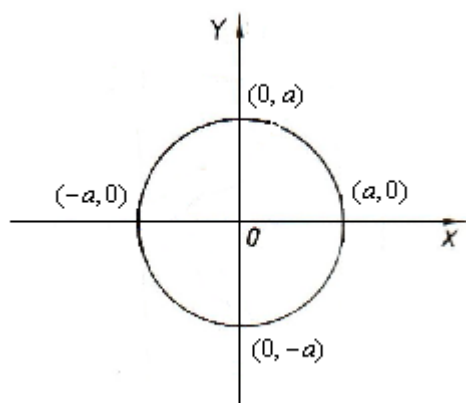


Dodatak - Primjeri krivulja u ravnini

1. Krivulje drugog reda

Kružnica

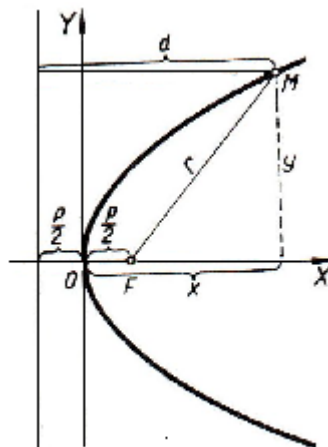


Implicitna jednačina: $x^2 + y^2 = a^2$

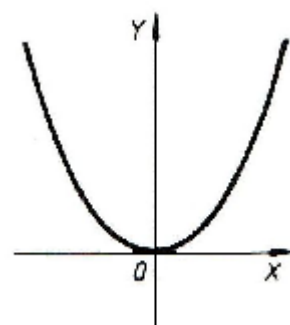
Parametarske jednačine:

$$\begin{cases} x = a \cos t \\ y = a \sin t \end{cases}$$

Parabola

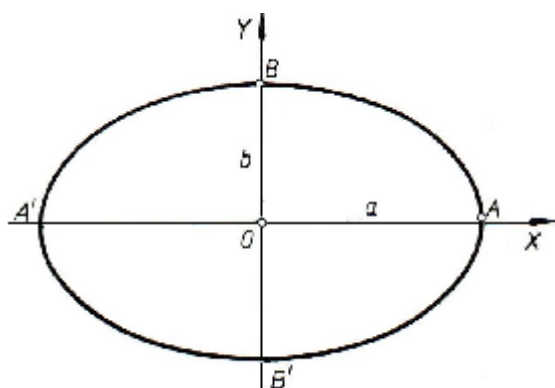


$$y^2 = 2px$$



$$y = ax^2$$

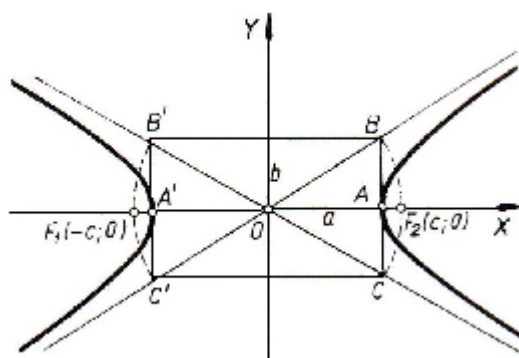
Elipsa



Implicitna jednačina: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Parametarske jednačine: $\begin{cases} x = a \cos t \\ y = b \sin t \end{cases}$

Hiperbola

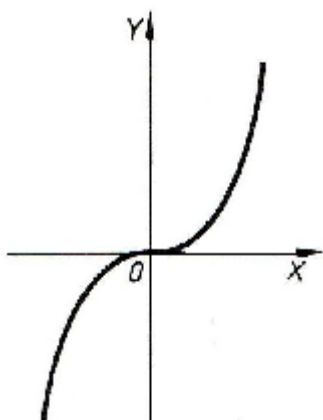


$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$\begin{cases} x = a \cosh t \\ y = b \sinh t \end{cases}$ (za desnu granu)

2. Krivulje trećeg reda

Kubna parabola

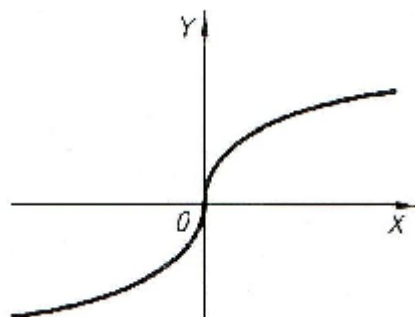


Implicitna jednačina: $y = ax^3$

Parametarske jednačine:
$$\begin{cases} x = t \\ y = at^3 \end{cases}$$

U ishodištu: točka infleksije

Kubna parabola

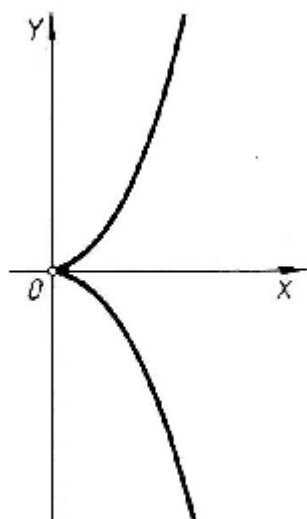


$y = ax^{1/3}$

$$\begin{cases} x = t^3 \\ y = at \end{cases}$$

točka infleksije

Semikubna parabola

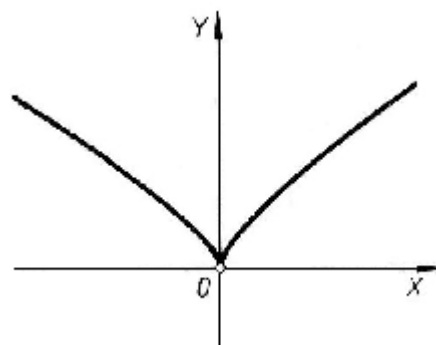


Implicitna jednačina: $y = ax^{3/2}$

Parametarske jednačine:
$$\begin{cases} x = t^2 \\ y = at^3 \end{cases}$$

U ishodištu: šiljak (singularna točka)

Neilova parabola

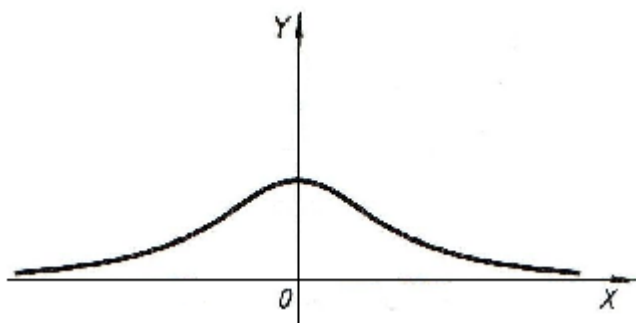


$y = ax^{2/3}$

$$\begin{cases} x = t^3 \\ y = at^2 \end{cases}$$

šiljak (singularna točka)

"Versiera" Marije Agnesi

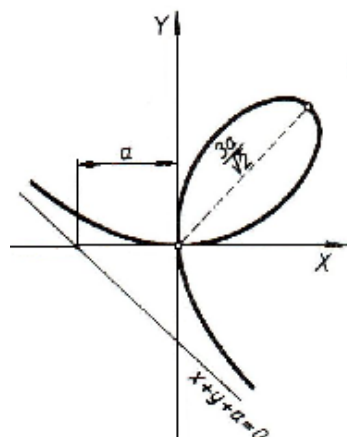


Implicitna jednačina: $y = \frac{a^3}{a^2 + x^2}$

Parametarske jednačine:

Točke infleksije: $\left(\pm \frac{a}{\sqrt{3}}, \frac{3a}{4}\right)$

Descartesov list

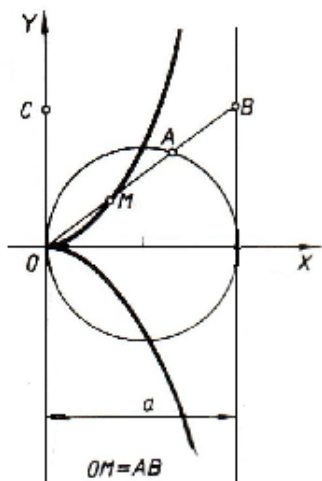


$$x^3 + y^3 = 3axy$$

$$\begin{cases} x = \frac{3at}{1+t^3} \\ y = \frac{3at^2}{1+t^3} \end{cases}$$

U ishodištu: dvostruka (singularna) točka

Dioklova cisoida

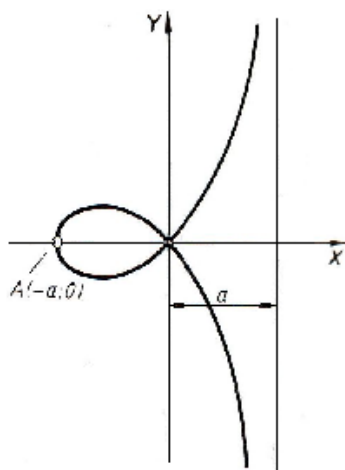


Implicitna jednačina: $y^2 = \frac{x^3}{a-x}$

$$\begin{cases} x = \frac{at^2}{t^2+1} \\ y = \frac{at^3}{t^2+1} \end{cases}$$

U ishodištu: šiljak (singularna točka)

Strofoida



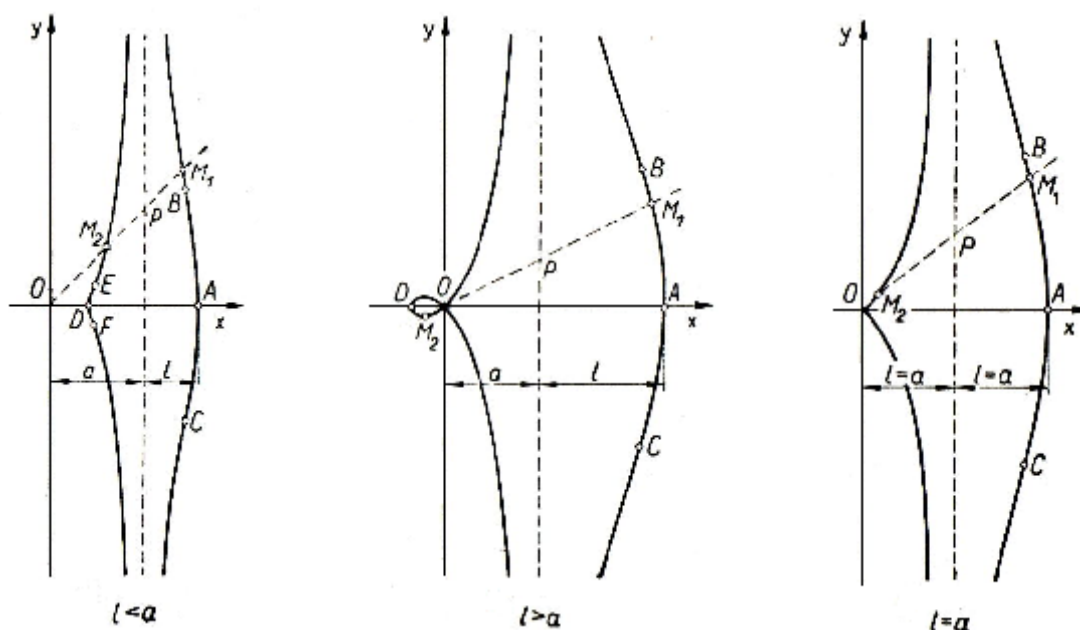
$$y^2 = \frac{x^2 \cdot (a+x)}{a-x}$$

$$\begin{cases} x = \frac{a(t^2-1)}{t^2+1} \\ y = \frac{at(t^2-1)}{t^2+1} \end{cases}$$

dvostruka (singularna) točka.

2. Krivulje četvrtog reda

Nikomedova konhoida



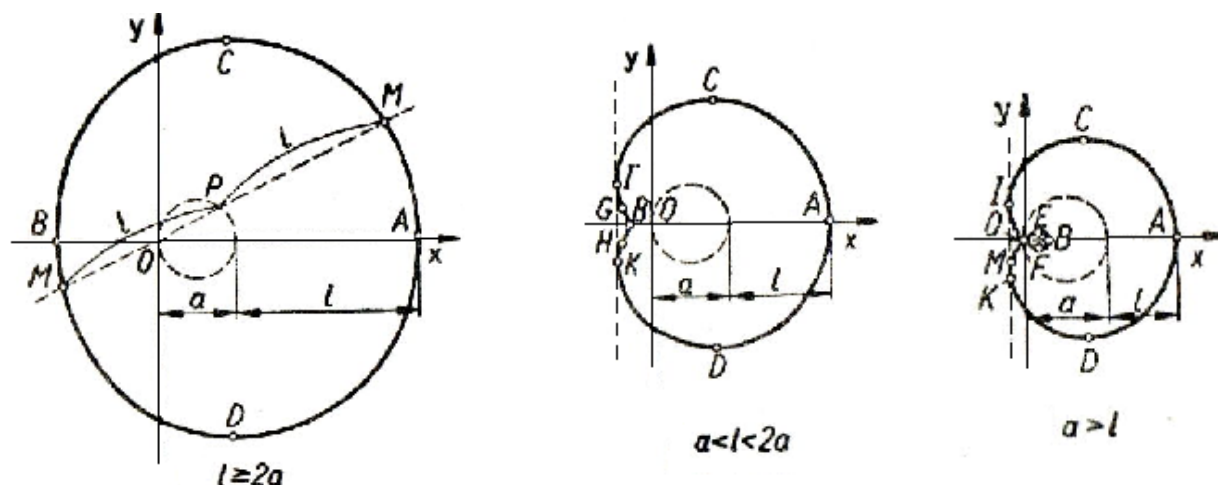
Implicitna jednačina: $(x-a)^2 \cdot (x^2 + y^2) - l^2 x^2 = 0$

Parametarske jednačine:
$$\begin{cases} x = a + l \cos \varphi \\ y = a \tan \varphi + l \sin \varphi \end{cases}$$

Polarna jednačina: $\rho = \frac{a}{\cos \varphi} \pm l$

U ishodištu (singularne točke):
izolirana točka ako je $l < a$,
čvorna točka ako je $l > a$,
šiljak ako je $l = a$.

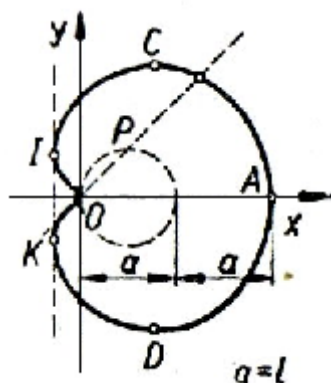
Pascalov puž (konhoida kružnice)



Implicitna jednačina: $(x^2 + y^2 - ax)^2 = l^2 (x^2 + y^2)$. Polarna jednačina: $\rho = a \cos \varphi + l$

Parametarske jednačine: $x = a \cos^2 \varphi + l \cos \varphi$, $y = a \cos \varphi \sin \varphi + l \sin \varphi$

Kardioida (poseban slučaj Pascalovog puža za $a = l$)

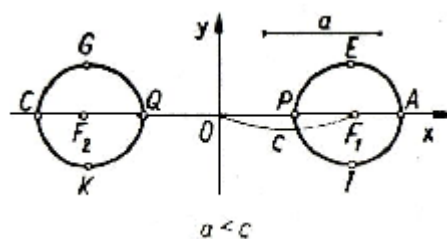
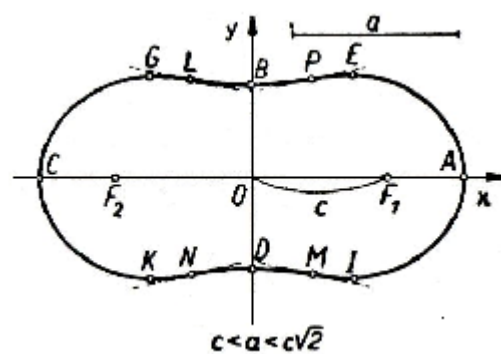
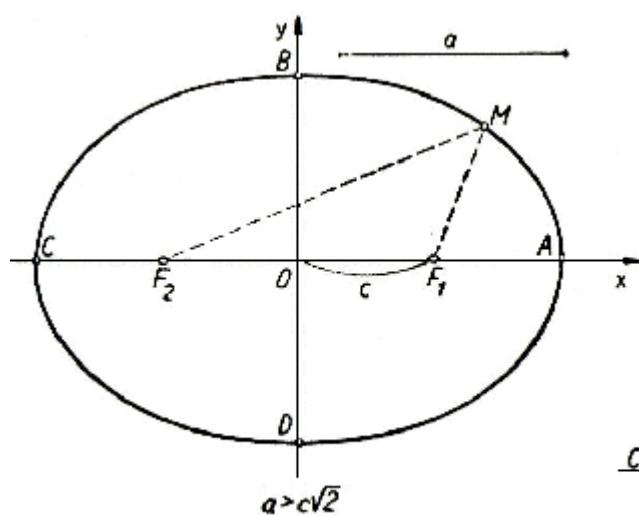


Implicitna jednačina: $(x^2 + y^2)^2 - 2ax(x^2 + y^2) = a^2y^2$

Parametarske jednačine:
$$\begin{cases} x = a \cos \varphi (1 + \cos \varphi) \\ y = a \sin \varphi (1 + \cos \varphi) \end{cases}$$

Polarna jednačina: $\rho = a(1 + \cos \varphi)$

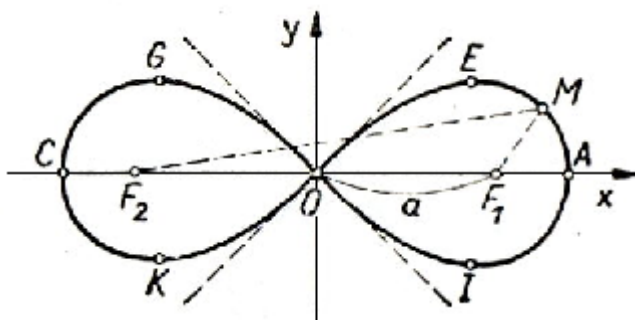
Cassinijevi ovali




Implicitna jednačina: $(x^2 + y^2)^2 - 2c^2(x^2 - y^2) = a^4 - c^4$

Polarna jednačina: $\rho^2 = c^2 \cos 2\varphi \pm \sqrt{c^4 \cos^2 2\varphi + (a^4 - c^4)}$

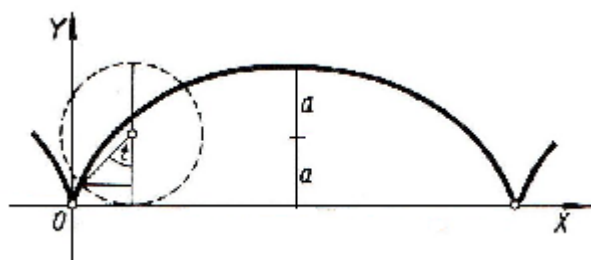
Lemniskata (poseban slučaj Cassinijevih ovala za $a = c$)



Implicitna jednačina: $(x^2 + y^2)^2 - 2a^2(x^2 - y^2) = 0$. Polarna jednačina: $\rho = a\sqrt{2\cos 2\varphi}$.

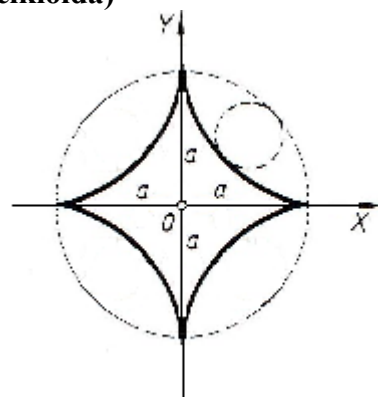
 Navedimo još neke krivulje:

Cikloida



Parametarske jednačine:
$$\begin{cases} x = a(t - \sin t) \\ y = a(1 - \cos t) \end{cases}$$

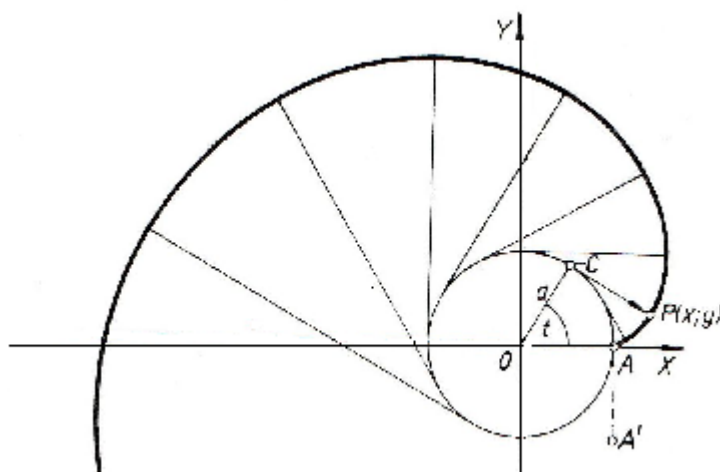
Astroida (hipocikloida)



$$\begin{cases} x = a \cos^3 t \\ y = a \sin^3 t \end{cases}$$

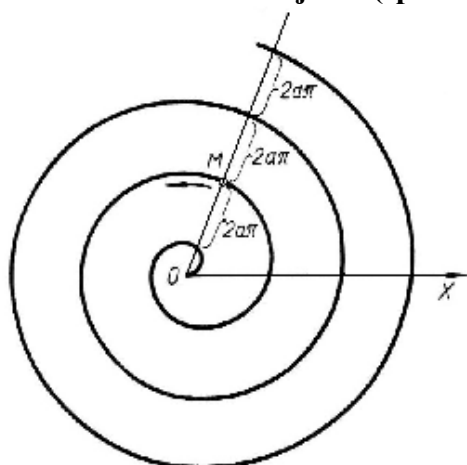
Implicitna jednačina: $x^{2/3} + y^{2/3} = a^{2/3}$

Evolventa kružnice



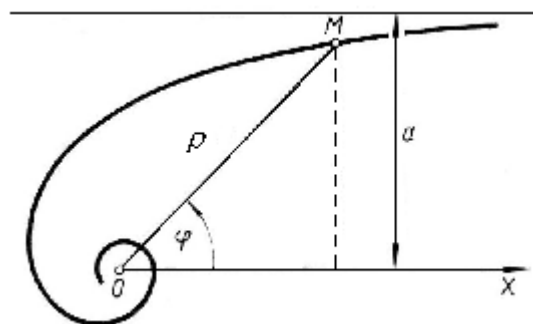
Parametarske jednačine: $x = a(\cos t + t \sin t), \quad y = a(\sin t - t \cos t)$

Arhimedova zavojnica (spirala)



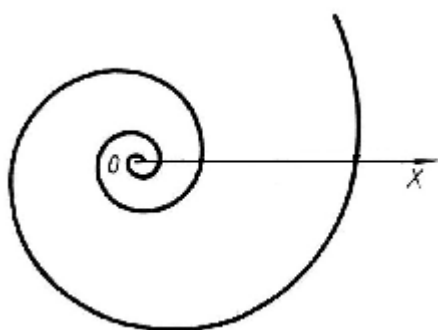
Polarna jednađžba: $\rho = a \cdot \varphi$, ($\rho \geq 0$).

Hiperbolna zavojnica



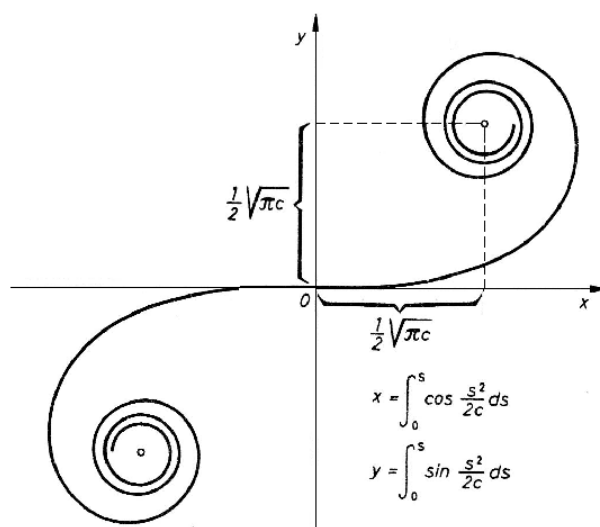
$$\rho = \frac{a}{\varphi}, \quad (\rho > 0).$$

Logaritamska zavojnica (spirala)

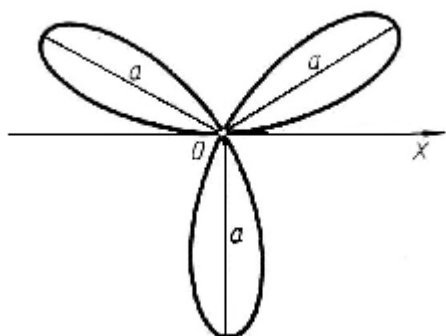


Polarna jednađžba: $\rho = e^{a \cdot \varphi}$.

Klotoida

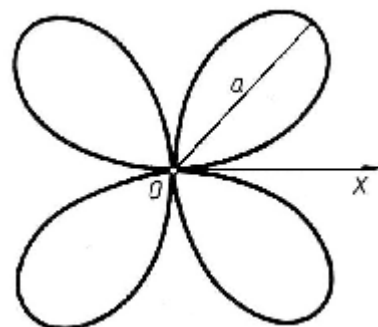


Ruža sa tri latice



Polarna jednađžba: $\rho = a \sin 3\varphi$, ($\rho \geq 0$).

Ruža sa četiri latice



$$\rho = a \sin 2\varphi$$