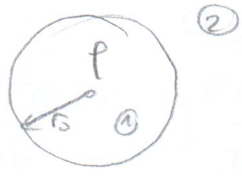
14/15 kugla $r = 0.1\,m$ je ispunjena nabojem gustoće $\rho = \frac{r}{4}\left[\frac{\mu C}{m^3}\right]$
odrediti potencijal u prostoru

$\boxed{\text{pr. 3.1.5.}}$



u prostoru izvan kugle, potencijal:

(2) Laplace: $$\Delta\varphi_2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi_2}{\partial r}\right) = 0$$

$$\varphi_2 = -\frac{C_{12}}{r} + C_{22} \qquad \bar{E} = -\bar{a}_r\frac{\partial\varphi_2}{\partial r} = -\bar{a}_r\frac{C_{12}}{r^2}$$

$$\varphi_2(\infty) = 0, \quad C_{22} = 0$$

druga konstanta: Gaussov zakon

$$\oint \bar{D}_2\bar{n}\,dS = Q$$

$$-\varepsilon_0\frac{C_{12}}{r^2}4\pi r^2 = Q = \rho\frac{4}{3}r_0^3\pi \qquad C_{12} = -\frac{\rho r_0^3}{3\varepsilon_0}$$

$$C_{12} = -\frac{r}{4}\cdot\frac{r_0^3}{3\varepsilon_0}$$

$$\varphi_2 = -\frac{1}{r}\cdot\left(\frac{r}{4}\frac{r_0^3}{3\varepsilon_0}\right) = \frac{r_0^3}{12\varepsilon_0}$$

(1) Poisson: $$\Delta\varphi_1 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi_1}{\partial r}\right) = -\frac{\rho}{\varepsilon_0} = -\frac{r}{4\varepsilon_0}$$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi_1}{\partial r}\right) = -\frac{r^3}{4\varepsilon_0} \quad / \int$$

$$r^2\frac{\partial\varphi_1}{\partial r} = -\int\frac{r^3}{4\varepsilon_0}$$

$$\frac{\partial\varphi_1}{\partial r} = -\frac{1}{r^2}\left(\frac{r^4}{16\varepsilon_0} + C_{11}\right) \quad \int$$

$$\varphi_1 = -\int\left(\frac{1}{\cancel{r^2}}\cdot\frac{r^{\cancel{4}\,2}}{16\varepsilon_0} - \frac{C_{11}}{r^2}\right)dr$$

$$\varphi_1 = -\left(\frac{r^3}{3\cdot 16\varepsilon_0} + \frac{C_{11}}{r}\right) + C_{21}$$

$$\varphi_1 = -\frac{r^3}{3\cdot 16\varepsilon_0} + \frac{r_0^3}{16\varepsilon_0}$$

konstante:

a) $\varphi_1(r \to 0) \neq \infty$

$C_{11} = 0$

b) na granici (1) i (2) za $r = r_0$ vrijedi

$\varphi_1(r_0) = \varphi_2(r_0)$

$$-\frac{r_0^3}{3\cdot 16\varepsilon_0} + C_{21} = \frac{r_0^3}{12\varepsilon_0}$$

$$C_{21} = \frac{r_0^3}{\varepsilon_0}\left(\frac{1}{12} - \frac{1}{3\cdot 16}\right)$$

$$C_{21} = \frac{r_0^3}{16\varepsilon_0}$$

Pr. 3.1.5.

Izvan kugle

$$\Delta \varphi_2 = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \varphi_2}{\partial r}\right) = 0 \quad / \int$$

$$r^2 \frac{\partial \varphi_2}{\partial r} = c_1$$

$$\frac{\partial \varphi_2}{\partial r} = \frac{c_1}{r^2} \quad / \int$$

$$\varphi_2 = -\frac{c_1}{r} + c_2$$

$$\varphi_2(\infty) = 0 \qquad 0 = 0 + c_2 \rightarrow c_2 = 0$$

unutar kugle

$$\Delta \varphi_1 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \varphi_1}{\partial r}\right) = -\frac{\rho}{\varepsilon_0}$$

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial \varphi_1}{\partial r}\right) = -\frac{r^2 \rho}{\varepsilon_0} \quad / \int$$

$$r^2 \frac{\partial \varphi_1}{\partial r} = -\frac{r^3 \rho}{3\varepsilon_0} + c_3$$

$$\frac{\partial \varphi_1}{\partial r} = -\frac{\rho r}{3\varepsilon_0} + \frac{c_3}{r^2} \quad / \int$$

$$\varphi_1 = -\frac{\rho r^2}{6\varepsilon_0} - \frac{c_3}{r} + c_4$$

$$\varphi_1(r \to 0) \neq \infty \qquad \rightarrow c_3 = 0$$

J 2015/16 pr. 3.1.6.

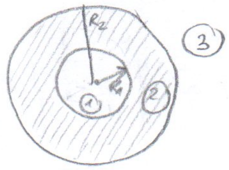
4. Gustoća naboja u sfernom sustavu je zadana s

$$\rho = \begin{cases} 0, & r < R_1 \\ \rho_0\left(1-\frac{2}{r^5}\right), & R_1 \le r \le R_2 \\ 0, & r \ge R_2 \end{cases}$$

$R_1 = 2\ cm$
$R_2 = 4\ cm$

$\bar{E}(r = 2{,}5\ cm)$ i $\varphi(0) = ?$
potencijal

$\downarrow$
2



3 $\Delta\varphi_3 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi_3}{\partial r}\right) = 0$ $\qquad \bar{E}_3 = -\bar{a}_r\frac{C_{13}}{r^2}$

$\varphi_3 = -\frac{C_{13}}{r} + C_{23}$ $\qquad \varphi_3(\infty) = 0 \qquad C_{23} = 0$

Gauss $\oint_S \bar{D}_3\,\bar{n}\,dS = Q = \int_V \rho\,dV$ $\qquad dV = 4\pi r^2 dr$, $\bar{D}_3 = \varepsilon\bar{E}$

$$-\varepsilon\frac{C_{13}}{\cancel{r^2}}\cancel{4\pi r^2} = \int_{r=R_1}^{R_2}\rho_0\left(1-\frac{2}{r^5}\right)\cancel{4\pi} r^2 dr$$

$$C_{13} = -\frac{\rho_0}{\varepsilon}\int_{R_1}^{R_2}\left(r^2 - \frac{2}{r^3}\right)dr$$

$$= -\frac{\rho_0}{\varepsilon}\left(\frac{r^3}{3} + \frac{\cancel{2}}{\cancel{2}r^2}\right)\Bigg|_{R_1}^{R_2} =$$

2 Poisson $\Delta\varphi_2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi_2}{\partial r}\right) = -\frac{\rho}{\varepsilon}$ $\qquad r^2\left(1-\frac{2}{r^5}\right) = r^2 - \frac{2}{r^3}$

$$\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi_2}{\partial r}\right) = -\frac{\rho_0}{\varepsilon}\left(r^2 - \frac{2}{r^3}\right)$$

C_{12}:
Gauss

$$\varepsilon\left(\frac{\rho_0}{\varepsilon}\left(\frac{r}{3}+\frac{1}{r^4}\right)+\frac{C_{12}}{r^2}\right)\cancel{4\pi} r^2 = \int_{R_1}^{r}\left(\rho_0\left(1-\frac{2}{r^5}\right)\right)\cancel{4\pi} r^2 dr$$

$$r^2\frac{\partial\varphi_2}{\partial r} = -\frac{\rho_0}{\varepsilon}\left(\frac{r^3}{3} + \frac{2}{2r^2}\right) + C_{12}$$

$$\frac{\partial\varphi_2}{\partial r} = -\frac{\rho_0}{\varepsilon}\left(\frac{r}{3} + \frac{1}{r^4}\right) + \frac{C_{12}}{r^2} \Big/ \int$$

$$\varphi_2 = -\frac{\rho_0}{\varepsilon}\left(\frac{r^2}{6} - \frac{1}{3r^3}\right) - \frac{C_{12}}{r} + C_{22}$$

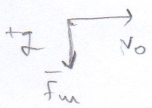
$$\bar{E}_2 = -\bar{a}_r\frac{\partial\varphi_2}{\partial r} = \bar{a}_r\frac{\rho_0}{\varepsilon}\left(\frac{r}{3} + \frac{1}{r^4}\right) + \frac{C_{12}}{r^2}$$

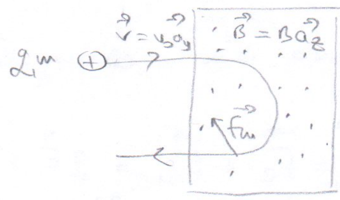
prsti B desna r
palac sila

"čestica mase m i $+q$ upada u područje gdje vlada $\vec{B}$ pri brzini $\vec{v}$ odredite putanju čestice"

$\odot B$

$$\bar{F}_m = q(\vec{v}\times\vec{B})$$





$\vec{v} = \vec{a}_y v_0$ $\qquad m\bar{a} = \bar{F}_m$

$\vec{B} = \vec{a}_z B$ $\qquad m\dfrac{d\bar{v}}{dt} = q(\bar{v}\times\vec{B})$

$$\bar{v} = \bar{a}_x v_x + \bar{a}_y v_y$$

$$m\left\{\bar{a}_x\frac{dv_x}{dt} + \bar{a}_y\frac{dv_y}{dt}\right\} = q\left\{(\bar{a}_x v_x + \bar{a}_y v_y)\times\bar{a}_z B\right\}$$

$$= qB(-\bar{a}_y v_x + \bar{a}_x v_y)$$

$$m\frac{dv_x}{dt} = qBv_y$$

$$m\frac{dv_y}{dt} = -qBv_x \;/\frac{d}{dt} \Rightarrow m\frac{d^2v_y}{dt^2} = -qB\frac{dv_x}{dt}$$

$$\Rightarrow \frac{dv_x}{dt} = -\frac{m}{qB}\frac{d^2v_y}{dt^2}$$

$$-\frac{m^2}{qB}\frac{d^2v_y}{dt^2} = qBv_y \;/ : \frac{m^2}{qB}$$

harmonijska fja $\leftarrow$ $\dfrac{d^2v_y}{dt^2} + \dfrac{q^2B^2}{m^2}v_y = 0$ $\qquad \omega_c^2 = \dfrac{q^2B^2}{m^2}$

$$\Downarrow$$

$$v_y = A'\sin(\omega_c t) + B'\cos(\omega_c t)$$

$$v_x = -\frac{m}{qB}\cdot\frac{dv_y}{dt}$$

$$v_x = -\frac{1}{\omega_c}\left(A'\omega_c\cos(\omega_c t) - \omega_c B'\sin(\omega_c t)\right)$$

$$v_x = -A'\cos\omega_c t + B'\sin\omega_c t$$

$$v_x(t=0) = -A' = 0 \Rightarrow A' = 0$$

$$v_y(t=0) = B' = v_0$$

$v_x = v_0\sin(\omega_c t)$; $\quad v_x = \dfrac{dx}{dt}$

$v_y = v_0\cos(\omega_c t)$; $\quad v_y = \dfrac{dy}{dt}$

$$x = \int v_x dt = -\frac{v_0}{\omega_c}\cos(\omega_c t) + c'$$

$$x(t=0) = -\frac{v_0}{\omega_c} + c' \Rightarrow c' = \frac{v_0}{\omega_c}$$

$x = \dfrac{v_0}{\omega_c}(1-\cos(\omega_c t))$ $\qquad y = \dfrac{v_0}{\omega_c}\sin(\omega_c t)$

$y = \int v_y dt = \dfrac{v_0}{\omega_c}\sin(\omega_c t) + D'$; $y(t=0) = D' =$

$x(t) = x_0 - R\cos(\omega_c t)$; $\quad x_0 = R = \dfrac{v_0}{\omega_c}$

$y(t) = R\sin(\omega_c t)$

kružnica polumjera R $\qquad t=0, x=0, y=0$

središte $(x_0, 0)$ $\qquad \omega_c t_1 = \dfrac{\pi}{2}$; $x=R, y=R$

$\omega_c t_2 = \pi$; $x = 2R, y = 0$

Maxwell (1862.)

- elmag polje se širi u obliku elmag. valova

vektorska polja $\bar{a}_x$ ($\vec{i}$) $\bar{a}_y$ ($\vec{j}$) $\bar{a}_z$ ($\vec{k}$)   vektorske veličine $\vec{F}, \bar{F}, \mathbf{F}$

Lorentzova sila – el.mag. sila na naboj q koji se giba brzinom v

$$\vec{F}_{em} = \vec{F}_e + \vec{F}_m = q\vec{E} + q(\vec{E} + \vec{v}\times\vec{B})$$

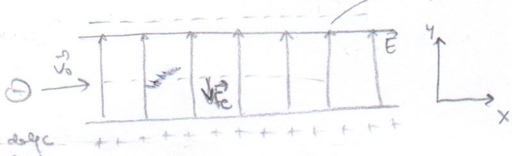
vektorski produkt!

$$\vec{F}_e = q\cdot\vec{E} \qquad \vec{F}_m = q(\vec{v}\times\vec{B})$$

$(\vec{E}, \vec{B})$ – fizikalno temeljne veličine
– uz njih opisujemo mag polje

1.

smjer dogovoren (smjer $\vec{E}$ je jednak smjeru sile na q+)



naboj starta u 0 pa se pomiče prema dolje pod utjecajem el. sile $\vec{F}_e \downarrow$

$\vec{F}_e$ prema dolje jer je to elektron

(1)

$$m\vec{a} = \vec{F}_e = q\vec{E} = -e\vec{E} \quad ; \quad \bar{E} = \bar{a}_y E$$

$$m\frac{d\vec{v}}{dt} = -e\vec{E} \quad ;$$

$$\vec{v} = \vec{a}_x v_x + \vec{a}_y v_y$$

$$\vec{v} = \bar{a}_x \frac{dx}{dt} + \bar{a}_y \frac{dy}{dt}$$

$$\bar{a} = \frac{dv}{dt} = \bar{a}_x \frac{dv_x}{dt} + \bar{a}_y \frac{dv_y}{dt}$$

$$a_x m \frac{dv_x}{dt} + a_y m \frac{dv_y}{dt} = -\bar{a}_y \cdot eE$$

$$\downarrow$$

$$m\frac{dv_x}{dt} = 0 \qquad \frac{dv_x}{dt} = 0 \Rightarrow v_x = C = konst.$$

$$v_x(t=0) = v_{x0} = v_0$$

$$v_x = v_0$$

$$\frac{dx}{dt} = v_0 \;/\int$$

$$x = v_0 t + x_0 \qquad x = v_0 t + C \;\; (C \to x_0)$$

$$x(t=0) = x_0 = 0$$

$$\boxed{x = v_0 t}$$

(2)

$$m\frac{dv_y}{dt} = -eE$$

$$\frac{dv_y}{dt} = -\frac{eE}{m} \;/\int_t$$

$$v_y = -\frac{eE}{m}t + C_1$$

$$v_y(t=0) = C_1 = 0$$

$$v_y = -\frac{eE}{m}t$$

$$v_y = \frac{dy}{dt} = -\frac{eE}{m}\cdot t \;/\int_t$$

$$y = -\frac{eE}{m}t^2 + y_0$$

$$y(t=0) = y_0 = 0$$

$$y = -\frac{eE}{m}\cdot\frac{t^2}{2}$$

(3)

$$x = v_0 t \Rightarrow t = \frac{x}{v_0}$$

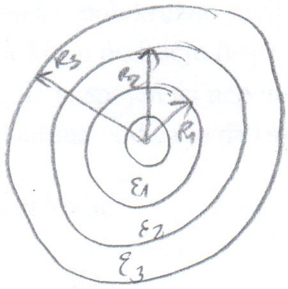
$$y = -\frac{eE}{2m}\cdot\frac{x^2}{v_0^2}$$

$$y = -\frac{eE}{2mv_0^2}\cdot x^2$$

J 2014

4. Zadan je troslojni kuglasti kondenzator prema slici. Odrediti ε_{r1} i ε_{r2} pri kojima su njegove apsolutne vrijednosti jakosti el. polja u sva 3 sloja jednake.

$R_1 = R$
$R_2 = 2R$
$R_3 = 3R$



$$\oiint_S \bar{D}\bar{n}\, dS = \int_V \int \varsigma\, dV = Q$$

$E_1 = E_2 = E_3$

$$\oint_S \varepsilon_0 \varepsilon_r E\, n\, dS = \varepsilon_0 \varepsilon_r E \oiint_S dS = \varepsilon_0 \varepsilon_r E\, 4r^2\pi = Q$$

$$E = \frac{Q}{4r^2\pi\varepsilon}$$

(1) $r = R_1 = R$

$\varepsilon_r = \varepsilon_{r1}$

$$E_1 = \frac{Q}{4R^2\pi\varepsilon_0\varepsilon_{r1}}$$

(2) $r = R_2 = 2R$

$\varepsilon_r = \varepsilon_{r2}$

$$E_2 = \frac{Q}{4\cdot 4R^2\pi\varepsilon_0\varepsilon_{r2}} = \frac{Q}{16R^2\pi\varepsilon_0\varepsilon_{r2}}$$

(3) $r = R_3 = 3R$

$\varepsilon_r = \varepsilon_{r3}$

$$E_3 = \frac{Q}{4\cdot 9R^2\pi\varepsilon_0\varepsilon_{r3}} = \frac{Q}{36R^2\pi\varepsilon_0\varepsilon_{r3}}$$

$E_2 = E_3$

$$\frac{1}{16\varepsilon_{r2}} = \frac{1}{36\varepsilon_{r3}}$$

$$\varepsilon_{r3} = \frac{16}{36}\varepsilon_{r2}$$

$E_1 = E_3$

$$\frac{1}{4\varepsilon_{r1}} = \frac{1}{36\varepsilon_{r3}}$$

$$\varepsilon_{r3} = \frac{4}{36}\varepsilon_{r1}$$