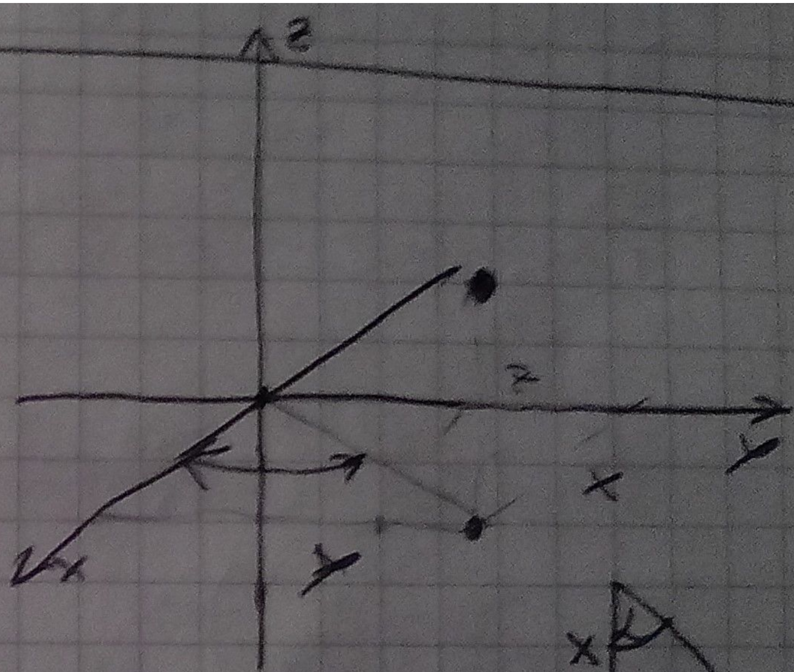
1.2) T(2,1,3)

kartezijev : T(x, y, z)
cilindrični : T(r, α, z)
sferni : T(r, θ, α)



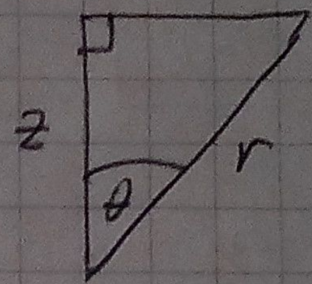
cilindrični

$r = \sqrt{x^2 + y^2} = \sqrt{2^2 + 1} = \sqrt{5} = 2{,}2360$

$\alpha = tg^{-1}\frac{y}{x} = tg^{-1}\frac{1}{2} = 26{,}56^{\circ} = 26{,}56 \cdot \frac{2\pi}{360}\ rad = 0{,}4636\ rad$

$z = 3$

<u>sferni</u> : (r, θ, α)



$$r=\sqrt{x^2+y^2+z^2}=\sqrt{2^2+1+3^2}=3{,}742$$

$\theta =$

$$\cos\theta=\frac{z}{r}=\frac{z}{\sqrt{x^2+y^2+z^2}} \Rightarrow \theta=\arccos\left(\frac{z}{\sqrt{x^2+y^2+z^2}}\right)=\arccos\left(\frac{3}{3{,}742}\right)=36{,}7^\circ$$

$\theta = 0{,}64$ rad

1. Auditorne

1.12 Vektori paralelni

$$\vec{A} = 2\hat{a}_r + 2\pi\hat{a}_\alpha + 4\hat{a}_z$$
$$\vec{B} = \alpha\hat{a}_r + \beta\hat{a}_\alpha - 2\hat{a}_z \quad \text{paralelni?}$$

uvjet: $\boxed{\vec{A} \times \vec{B} = 0} = \begin{vmatrix} \hat{a}_r & \hat{a}_\alpha & \hat{a}_z \\ 2 & 2\pi & 4 \\ \alpha & \beta & -2 \end{vmatrix} = (-4\pi - 4\beta)\cdot\hat{a}_r - \hat{a}_\alpha(-4-4\alpha) + (2\beta - 2\pi\alpha)\cdot\hat{a}_z = \vec{0}$

$\hat{a}_r: -4\pi - 4\beta = 0 \Rightarrow \boxed{\beta = -\pi}$; $\hat{a}_\alpha: -4-4\alpha = 0 \Rightarrow \boxed{\alpha = -1}$

$\hat{a}_z: 2\beta - 2\pi\alpha$

1.13 Vektori okomiti

$$\vec{A} = 2\hat{a}_x + 2\hat{a}_y + 2\hat{a}_z$$
$$\vec{B} = \alpha\hat{a}_x + 2\hat{a}_y + \hat{a}_z$$

uvjet: $\boxed{\vec{A}\cdot\vec{B} = 0}$

$$\vec{A}\cdot\vec{B} = 2\alpha + 2\cdot 2 + 2\cdot 1 = 0$$
$$2\alpha + 6 = 0$$
$$\boxed{\alpha = -3}$$

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skalar

1.14 gradijent: $f = x\cdot z - y\cdot z \quad T(2,1,1)$

$$\text{grad } f = ? = \nabla\cdot f = \left(\frac{\partial}{\partial x}\hat{a}_x + \frac{\partial}{\partial y}\hat{a}_y + \frac{\partial}{\partial z}\hat{a}_z\right)\cdot(x\cdot z - y\cdot z) =$$
$$= z\hat{a}_x - z\hat{a}_y + (x-y)\hat{a}_z =$$
$$= \boxed{1\hat{a}_x - 1\hat{a}_y + 1\hat{a}_z = \nabla f}$$

(2.61.) $m = 10^{-12}\,kg$

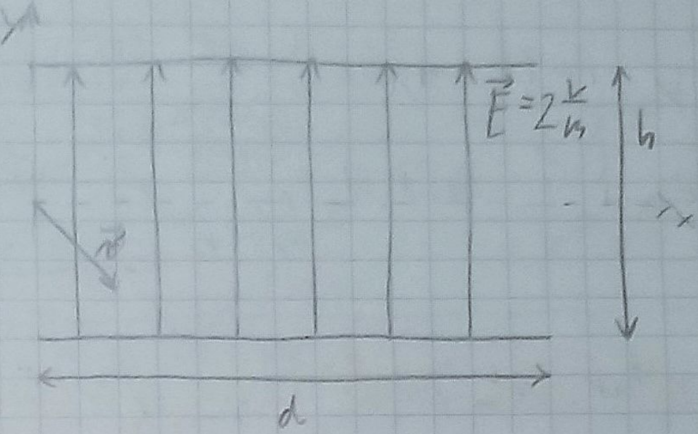
$\vec{v} = \hat{a}_x - \hat{a}_y \left[\frac{m}{s}\right]$

$\vec{E} = 2\,\frac{V}{m}$

$d = 0{,}01\,m$

$h = 0{,}2\,m$

$Q = 1\,nC$



a) $m \cdot \vec{a} = \vec{F}_L = Q(\vec{E} + \vec{v} \times \vec{B}) = q \cdot \vec{E}$

$m\left(\frac{dv_x}{dt}\hat{a}_x + \frac{dv_y}{dt}\hat{a}_y\right) = q \cdot E \cdot \hat{a}_y$

$\hat{a}_x : m\frac{dv_x}{dt} = 0 \xrightarrow{v_x(0)=1} v_x = C = 1\,;\ \underline{v_x(t) = 1}$

$\hat{a}_y : m\frac{dv_y}{dt} = q \cdot E$

$\frac{dv_y}{dt} = \frac{qE}{m} \,/\int$

$v_y = \frac{qE}{m} \cdot t + v_y(0) = \underline{\frac{qE}{m} \cdot t - 1 = v_y(t)}$

$\vec{v} = v_x\hat{a}_x + v_y\hat{a}_y = \hat{a}_x + \left(\frac{qE}{m} \cdot t - 1\right)\hat{a}_y \,/\int$

$\vec{s}(t) = t\hat{a}_x + \left(\frac{qE}{m} \cdot \frac{t^2}{2} - t\right) \cdot \hat{a}_y$ put točkastog naboja

izlaz: $s_x(t) = d = 0{,}01 = t$

$\vec{v}(0{,}01) = \hat{a}_x + \left(\frac{qE}{m} \cdot 0{,}01 - 1\right) \cdot \hat{a}_y$

$|\vec{v}(0{,}01)| = \sqrt{1^2 + \left(\frac{qE}{m} \cdot 0{,}01 - 1\right)^2} =$

$= \sqrt{1 + \left(\frac{10^{-9} \cdot 2}{10^{-12}} \cdot 0{,}01 - 1\right)^2} = \boxed{19\,\frac{m}{s} = v_{IZLAZAK}}$

b) $y_{min} = ?$ $\quad s_y(t) = \frac{qE}{m}\frac{t^2}{2} - t$

stoji $\Rightarrow v_y(t) = \frac{qE}{m}t - 1 = 0 \Rightarrow \frac{qE}{m}t = 1\,;\ t_{min} = \frac{m}{q \cdot E} = \frac{10^{-12}}{10^{-9} \cdot 2} = 0{,}0005\,s$

$s_y(t_{min}) = \frac{qE}{m} \cdot \frac{t_{min}^2}{2} - t = \frac{10^{-9} \cdot 2}{10^{-12}} \cdot \frac{0{,}0005^2}{2} - 0{,}0005 = -0{,}00025\,m$

$\boxed{s_y(t_{min}) = -0{,}25\,mm}$ $\quad$ c) $\boxed{t_{min} = 0{,}0005\,s = 0{,}5\,ms}$

c) $y_{izlaz} = ?$ $\quad s_y(0{,}01) = \frac{qE}{m}\cdot\frac{0{,}01^2}{2} - 0{,}01 =$

$$= \frac{10^{-9}\cdot 2}{10^{-12}}\cdot\frac{0{,}01^2}{2} - 0{,}01 =$$

$$= 0{,}09\,m = \boxed{9\,cm = s_y(0{,}01)}$$

(1.31.) rotor vek. polja $\quad \vec{F}(r,\alpha,z) = 5\hat{a}_r + \cos(\alpha)\,\hat{a}_\alpha - 2z\hat{a}_z$

$$\nabla\times\vec{F} = ? = \left(\frac{1}{r}\cdot\frac{\partial F_z}{\partial\alpha} - \frac{\partial F_\alpha}{\partial z}\right)\hat{a}_r + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right)\hat{a}_\alpha$$

$$+ \frac{1}{r}\left(\frac{\partial}{\partial r}(r\cdot F_\alpha) - \frac{\partial F_r}{\partial\alpha}\right)\hat{a}_z$$

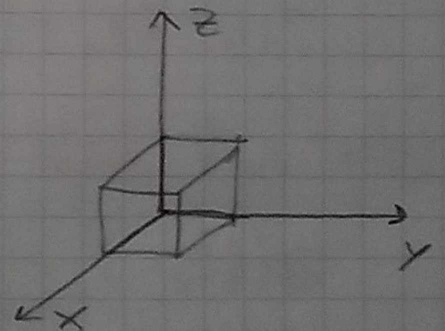
$$\vec{F} = F_r\hat{a}_r + F_\alpha\hat{a}_\alpha + F_z\hat{a}_z$$

$$\nabla\times\vec{F} = \left(\frac{1}{r}\cdot\cancel{0} - \cancel{0}\right)\hat{a}_r + \left(\cancel{0} + \cancel{0}\right)\hat{a}_\alpha + \frac{1}{r}\left(\frac{\partial}{\partial r}(r\cdot\cos\alpha) - \cancel{0}\right)\hat{a}_z =$$

$$= \frac{1}{r}\cdot\cos\alpha\cdot\hat{a}_z = \boxed{\frac{\cos\alpha}{r}\cdot\hat{a}_z = \nabla\times\vec{F}}$$

(1.27.) tok $\quad F(x,y,z) = 4x^2 y\hat{a}_x + 2z\hat{a}_y + 2y\hat{a}_z$

jedinična kocka: $0 \le x,y,z \le 1$



teorem o divergenciji $\quad \Phi = \oint\!\!\oint_S \vec{F}\cdot\vec{n}\cdot dS = \iiint_V \nabla\cdot\vec{F}\cdot dV$

$$\nabla\cdot\vec{F} = \frac{\partial}{\partial x}F_x + \frac{\partial}{\partial y}F_y + \frac{\partial}{\partial z}F_z =$$

$$= \frac{\partial}{\partial x}(4x^2y) + \frac{\partial}{\partial y}(2z) + \frac{\partial}{\partial z}(2y) = 8xy$$

$$\Phi = \iiint_V 8xy\,dx\,dy\,dz = \int_0^1 dx\int_0^1 dy\int_0^1 dz\cdot 8xy = \int_0^1 x\,dx\int_0^1 y\,dy\int_0^1 dz\cdot 8 =$$

$$= \frac{x^2}{2}\Big|_0^1\cdot\frac{y^2}{2}\Big|_0^1\cdot z\Big|_0^1\cdot 8 = \frac{1}{2}\cdot\frac{1}{2}\cdot 1\cdot 8 = \boxed{2\,Vm = \Phi}$$

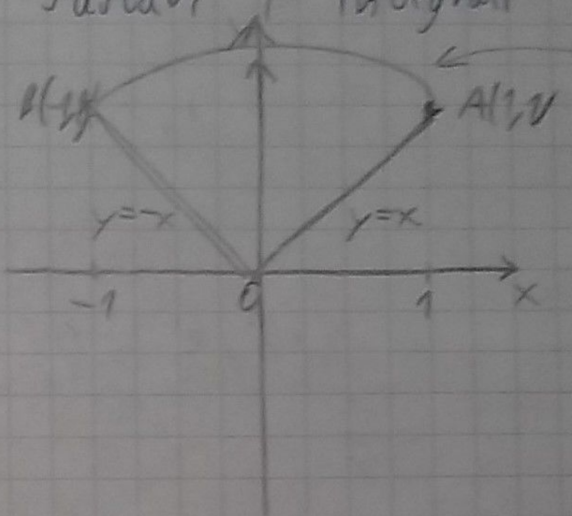
- divergencija priča $\quad \nabla F = \frac{\partial}{\partial x}F_x + \frac{\partial}{\partial y}F_y + \frac{\partial}{\partial z}F_z$

ponavljanje rustavi i integrali

granice integracije:

$$I=\iint_D f(x,y)\,dx\,dy$$

- kružni isječak OAB s centrom $O(0,0)$, $A(1,1)$, $B(-1,1)$

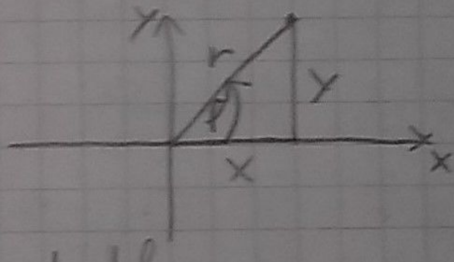


$$x^2+y^2=2$$
$$y=\sqrt{2-x^2}$$

$$I=\iint_D f(x,y)\,dx\,dy=\int_{-1}^{0}dx\int_{-x}^{\sqrt{2-x^2}}f(x,y)\,dy+\int_{0}^{1}dx\int_{x}^{\sqrt{2-x^2}}f(x,y)\,dy$$

polarne koordinate: $x=r\cos\varphi$

$y=r\sin\varphi$



$$I=\iint_D f(x,y)\,dx\,dy=\iint_{D'} f(r\cos\varphi, r\sin\varphi)\cdot r\cdot dr\,d\varphi$$

$$J=\begin{vmatrix}\frac{\partial x}{\partial r} & \frac{\partial x}{\partial \varphi}\\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi}\end{vmatrix}=\begin{vmatrix}\cos\varphi & -r\sin\varphi\\ \sin\varphi & r\cos\varphi\end{vmatrix}=r(\cos^2\varphi+\sin^2\varphi)=r$$

$$I=\int_{\frac{\pi}{4}}^{\frac{3\pi}{4}}d\varphi\int_0^{\sqrt{2}}f(r\cos\varphi, r\sin\varphi)\,r\,dr$$

$$\boxed{\text{elipsa: } \begin{matrix}x=ar\cos\varphi\\ y=br\sin\varphi\end{matrix} \quad J=abr}$$

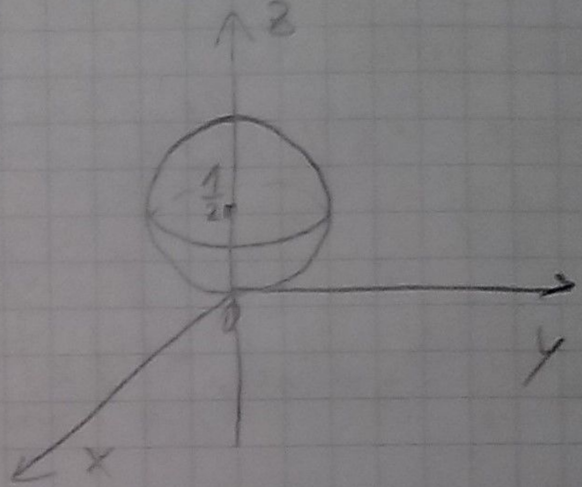
trostruki sferni + cilindrični

$$\iiint_V (x^2+y^2+z^2)\,dx\,dy\,dz \qquad V: \text{kugla } x^2+y^2+z^2 \le z \,/ -z$$

$$x^2+y^2+z^2-z \le 0$$

$$x^2+y^2+z^2-2\cdot z\cdot\frac{1}{2}+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2 \le 0$$

$$x^2+y^2+\left(z-\frac{1}{2}\right)^2 \le \frac{1}{4}$$



cilindrični:

$x = r\cos\varphi$

$y = r\sin\varphi$

$z = z$

$$J = r = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \varphi} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} & \frac{\partial y}{\partial z} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \varphi} & \frac{\partial y}{\partial z} \end{vmatrix} = \begin{vmatrix} \cos\varphi & -r\sin\varphi & 0 \\ \sin\varphi & r\cos\varphi & 0 \\ 0 & 0 & 1 \end{vmatrix} = r$$

sferni: $\qquad dV = dx\,dy\,dz = r^2\sin\theta\,dr\,d\varphi\,d\theta$

$x = r\sin\theta\cos\varphi$

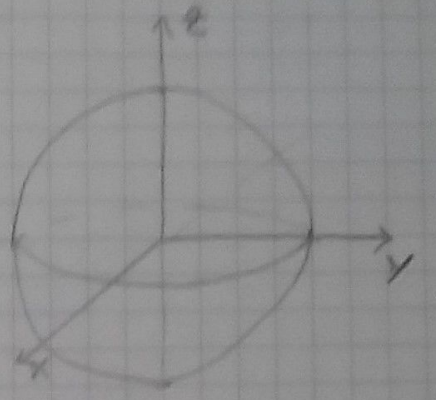
$y = r\sin\theta\sin\varphi$

$z = r\cos\theta$

$J = r^2\sin\theta$

$$\iiint_V (x^2+y^2+z^2)\,dx\,dy\,dz = \int_0^{2\pi} d\varphi \int_0^{\frac{\pi}{2}} d\theta \int dr\cdot r^2\cdot r^2\sin\theta$$

1.20. $R = 0{,}2\,m$ $\quad \rho = \frac{1}{\sqrt{x^2+y^2}} \frac{C}{m^3}$ volumna gustoća naboja



$$Q = q = \iiint_V \rho \, dV = \iiint_V \frac{1}{\sqrt{x^2+y^2}} dV$$

sferno: φ, r, θ

$x = r \sin\theta \cos\varphi$

$y = r \sin\theta \sin\varphi$

$z = r \cos\theta$

$J = r^2 \sin\theta$

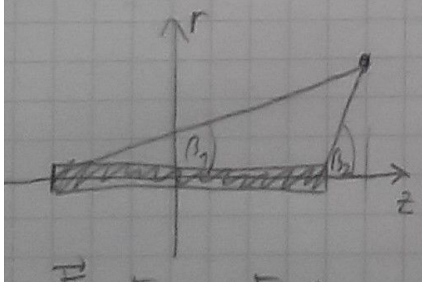
$$Q = \int_0^{2\pi} d\varphi \int_0^{0,2} dr \int_0^{\pi} d\theta \cdot r^2 \sin\theta \cdot \frac{1}{\sqrt{r^2 \sin^2\theta}} = \int_0^{2\pi} d\varphi \int_0^{0,2} dr \int_0^{\pi} d\theta \cdot r =$$

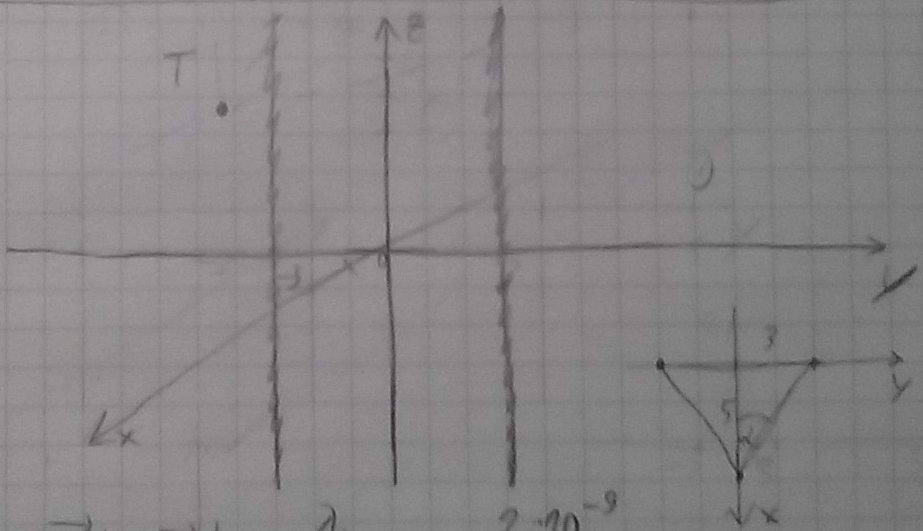
$$= \int_0^{2\pi} d\varphi \int_0^{0,2} r\,dr \int_0^{\pi} d\theta = \varphi\Big|_0^{2\pi} \cdot \frac{r^2}{2}\Big|_0^{0,2} \cdot \theta\Big|_0^{\pi} = 2\pi \cdot \frac{0{,}2^2}{2} \cdot \pi = 0{,}04\pi^2 =$$

$$\boxed{Q = 0{,}3947\,C}$$

2.1. $\lambda = 2 \frac{nC}{m}$

$E_{(5,9,10)} = ?$





$\vec{E} = E_r \hat{a}_r + E_z \hat{a}_z$

$E_r = \frac{\lambda}{4\pi\varepsilon_0 r}(\cos\beta_1 - \cos\beta_2)$

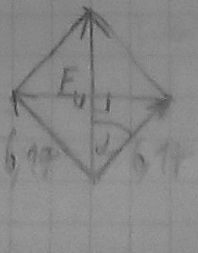
$E_z = \frac{\lambda}{4\pi\varepsilon_0 r}(-\sin\beta_1 + \sin\beta_2)$

$d \to \infty \Rightarrow E_r = \frac{\lambda}{2\pi\varepsilon_0 r}$

$E_z = 0$

$$|\vec{E}_1| = |\vec{E}_2| = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{2 \cdot 10^{-9}}{2\pi \cdot 8{,}854 \cdot 10^{-12} \cdot \sqrt{3^2+5^2}} =$$

$= 6{,}17\ V/m$



$\alpha = tg^{-1} \frac{3}{5} = 0{,}5404$

$|\vec{E}| = 2 \cdot \cos\alpha \cdot |\vec{E}_1| = \boxed{10{,}58\ V/m = |\vec{E}|}$