Elektromagnetska polja

Elektromagnetizam 1 – zadaci za vježbu

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1.

$$w = \frac{\varepsilon_0}{2}\left|\vec{E}\right|^2 = \frac{\varepsilon_0}{2} \cdot 4x^2 \cdot 10^6 = \frac{8.854 \cdot 10^{-12}}{2} \cdot 4 \cdot 16 \cdot 10^6 = 283.3\left[\frac{\mu J}{m^3}\right]$$

2.

$$\nabla \times \vec{H} = \vec{J_s} + \kappa\vec{E} + \frac{\partial\vec{D}}{\partial t}$$

$$I_{pom} = I_{prov} \rightarrow (množenje\ sa\ S) \rightarrow J_{pom} = J_{prov}$$

$$\frac{\partial D}{\partial t} = \kappa E$$

$$\varepsilon_0\varepsilon_r\frac{\partial E}{\partial t} = \kappa E$$

$$\varepsilon_0\varepsilon_r\omega \cdot 250\cos(\omega t) = \kappa \cdot 250\sin(\omega t)$$

Ako su iste amplitude:

$$\varepsilon_0\varepsilon_r\omega \cdot 250 = \kappa \cdot 250 \rightarrow \omega = \frac{\kappa}{\varepsilon_0\varepsilon_r} \rightarrow f = \frac{\kappa}{2\pi\varepsilon_0\varepsilon_r} = 59.92[GHz]$$

3.

$$\nabla \times \vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\overrightarrow{a_x}\left[\frac{\partial}{\partial z}(E_m\sin(\omega t - \beta z))\right] = -\frac{\partial\vec{B}}{\partial t}$$

$$\frac{\partial\vec{B}}{\partial t} = -\beta E_m\cos(\omega t - \beta z) \rightarrow \vec{B} = -0.6 \cdot 10^{-3}\sin(10000t - 0.6z)\,\overrightarrow{a_x}$$

$$B_m = 0.6[mT]$$

4.

$$\nabla \times \vec{E} = -\mu_0\frac{\partial\vec{H}}{\partial t}$$

$$\nabla \times \vec{E} = -\overrightarrow{a_y}\left[-\frac{\partial}{\partial z}(50\cos(\omega t - \beta z))\right] = 50\beta\sin(\omega t - \beta z) = -\mu_0\frac{\partial\vec{H}}{\partial t}$$

$$\vec{H} = \frac{50\beta}{\omega\mu_0}\cos(\omega t - \beta z)\,\overrightarrow{a_y}$$

$$\vec{N} = \vec{E} \times \vec{H} = \frac{2500\beta}{\omega\mu_0}\cos^2(\omega t - \beta z)\,\overrightarrow{a_z}$$

$$\overrightarrow{N_{sr}} = \frac{1}{T}\int_0^T \vec{E} \times \vec{H} dt\,\overrightarrow{a_z} = \frac{2500\beta}{\omega\mu_0}\frac{1}{T}\int_0^T \cos^2(\omega t - \beta z)\,dt\,\overrightarrow{a_z}$$

$$\frac{1}{T}\int_0^T \cos^2(\omega t - \beta z)\,dt = \frac{1}{2} \rightarrow vidi\ predavanja$$

$$\overrightarrow{N_{sr}} = \frac{2500\beta}{2\omega\mu_0}\overrightarrow{a_z}$$

$$P_{sr} = \iint_S N_{sr} dS = N_{sr} S = N_{sr} r^2 \pi = \frac{2500\beta}{2\omega\mu_0} r^2 \pi = 65.14[W]$$

S time da je $\beta = \omega\sqrt{\mu_0 \varepsilon_0}$.

5.

$$w = \frac{\varepsilon_0}{2} \left|\vec{E}\right|^2 = \frac{\varepsilon_0}{2} \cdot x^2 z^2 \cdot 10^6 = \frac{8.854 \cdot 10^{-12}}{2} \cdot 49 \cdot 10^6 = 0.000217 \left[\frac{J}{m^3}\right]$$

6. Isti postupak kao kod 4.

$$N_{sr} = \frac{100\beta}{2\omega\mu_0}$$

$$P_{sr} = N_{sr} r^2 \pi = \frac{100\beta}{2\omega\mu_0} r^2 \pi = 0.938[W]$$

7.

$$w = \frac{\mu_0}{2} \left|\vec{H}\right|^2 = \frac{4\pi \cdot 10^{-7}}{2} \cdot 200^2 x^2 = 0.10053 = 100.5 \left[\frac{mJ}{m^3}\right]$$

8.

$$J_{prov} = \kappa E = 6 \sin(9 \cdot 10^9 t) \left[\frac{nA}{m^2}\right]$$

$$J_m = 6 \left[\frac{nA}{m^2}\right]$$

9.

$$\vec{E}(z,t) = 10 \sin(\omega t - \beta z)\, \overrightarrow{a_x} - 15 \sin(\omega t - \beta z)\, \overrightarrow{a_y}$$

$$\vec{E}(0.75\lambda, 0) = 10 \sin(-0.75\lambda\beta)\, \overrightarrow{a_x} - 15 \sin(-0.75\lambda\beta)\, \overrightarrow{a_y}$$

Vrijedi $\lambda = \frac{2\pi}{\beta}$.

$$\vec{E}(0.75\lambda, 0) = -10 \sin(1.5\pi)\, \overrightarrow{a_x} + 15 \sin(1.5\pi)\, \overrightarrow{a_y} = 10\overrightarrow{a_x} - 15\overrightarrow{a_y}$$

$$\left|\vec{E}(0.75\lambda, 0)\right| = \sqrt{100 + 225} = 18.03 \left[\frac{V}{m}\right]$$

10. Isti postupak kao kod 4. i 6.

$$N_{sr} = \frac{150^2\beta}{2\omega\mu_0}$$

$$P_{sr} = N_{sr} ab = \frac{150^2\beta}{2\omega\mu_0} ab = 0.0134[W]$$

11.

$$w = \frac{\mu_0}{2} \left|\vec{H}\right|^2 = \frac{4\pi \cdot 10^{-7}}{2} \cdot 200^2 x^2 y^2 = 0.6283 = 628.3 \left[\frac{mJ}{m^3}\right]$$

12.

$$J_{pom} = \varepsilon_0 \varepsilon_r \frac{\partial E}{\partial t} = 1.19 \cos(9 \cdot 10^9 t) \left[\frac{\mu A}{m^2}\right]$$

$$J_m = 1.19 \left[\frac{\mu A}{m^2}\right]$$

13.

$$e_{ind} = -\frac{\partial}{\partial t}\iint_S \vec{B}\cdot\vec{n}dS = -\frac{\partial}{\partial t}\iint_S 0.5\cos(377t)\,(4\overrightarrow{a_y}+4\overrightarrow{a_z})\cdot\overrightarrow{a_z}dS = -\frac{\partial}{\partial t}\iint_S 2\cos(377t)\,dS =$$

$$= -\frac{\partial}{\partial t}(2\cos(377t)\,S) = 5.92\sin(377t)\,[V]$$

$$e_{ind,m} = 5.92[V]$$

14.

$$\vec{\imath}(t) = I_m\sin(\omega t+\varphi)\,\overrightarrow{a_y}$$

Za beskonačno dugu strujnicu indukcija je

$$\vec{B} = \frac{\mu_0 i(t)}{2\pi x}(-\overrightarrow{a_z})$$

$$e_{ind} = -\frac{\partial}{\partial t}\iint_S \vec{B}\cdot\vec{n}dS = -\frac{\partial}{\partial t}\iint_S \frac{\mu_0 i(t)}{2\pi x}(-\overrightarrow{a_z})\cdot(-\overrightarrow{a_z})dS = -\frac{\partial}{\partial t}\left(\frac{\mu_0 i(t)}{2\pi}\int_{0.05}^{0.2}\frac{dx}{x}\int_0^{0.2}dy\right) =$$

$$= -\frac{\partial}{\partial t}\left(\frac{\mu_0 i(t)}{2\pi}\cdot 0.2\cdot\ln 4\right) = -\frac{\partial}{\partial t}\left(\frac{\mu_0 I_m\sin(\omega t+\varphi)}{2\pi}\cdot 0.2\cdot\ln 4\right) = -0.554517744\cdot\frac{I_{ef}}{\frac{\sqrt{2}}{2}}\cdot 10^{-7}\cdot 2\pi f\cos(\omega t+\varphi)$$

$$e_{ind,m} = -0.554517744\cdot\frac{I_{ef}}{\frac{\sqrt{2}}{2}}\cdot 10^{-7}\cdot 2\pi f$$

$$e_{ind,m} = \frac{\sqrt{2}}{2}e_{ind,ef} \to e_{ind,ef} = \frac{e_{ind,m}}{\frac{\sqrt{2}}{2}}$$

$$e_{ind,ef} = -0.554517744\cdot I_{ef}\cdot 10^{-7}\cdot 2\pi f = -0.0017[V] \approx 2[mV]$$

15.

$$e_{ind} = -\frac{\partial\Phi}{\partial t} = -e_{ind} = -\frac{\partial}{\partial t}(\vec{B}\vec{S}) = -\frac{\partial}{\partial t}(BS\cos\alpha) = -\frac{\partial}{\partial t}(BS\cos\omega t) = BS\omega\sin\omega t = BS\omega\sin\alpha = 2828.427\cdot 10^{-9}$$

$$I_{ind} = \frac{e_{ind}}{R} = \frac{2828.427\cdot 10^{-9}}{20\cdot 10^{-3}} = 0.141[mA]$$

16.

Uvrstite u

$$\nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t}$$

i vidite da ne zadovoljava.

17.

$$\nabla\times\vec{H} = \frac{\partial\vec{D}}{\partial t}$$

$$\nabla\times\vec{H} = -\frac{E_m k_2}{k_1}\sin(\omega t - k_2 z)\,\overrightarrow{a_x}$$

$$\frac{\partial\vec{D}}{\partial t} = -\frac{E_m k_2}{k_1}\sin(\omega t - k_2 z)\,\overrightarrow{a_x} \to \vec{D} = \frac{E_m k_2}{k_1\omega}\cos(\omega t - k_2 z)\,\overrightarrow{a_x}$$

$$\vec{E} = \frac{E_m k_2}{k_1\omega\varepsilon_0}\cos(\omega t - k_2 z)\,\overrightarrow{a_x}$$

Izjednačimo sa zadano jednadžbom električnog polja i dobijemo

$$\frac{E_m k_2}{k_1 \omega \varepsilon_0} = E_m \rightarrow k_2 = k_1 \omega \varepsilon_0 \quad (*)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{E} = E_m k_2 \sin(\omega t - k_2 z)\, \overrightarrow{a_y}$$

$$-\frac{\partial \vec{B}}{\partial t} = E_m k_2 \sin(\omega t - k_2 z)\, \overrightarrow{a_y} \rightarrow \vec{B} = \frac{E_m k_2}{\omega} \sin(\omega t - k_2 z)\, \overrightarrow{a_y}$$

$$\vec{H} = \frac{E_m k_2}{\omega \mu_0} \sin(\omega t - k_2 z)\, \overrightarrow{a_y}$$

Izjednačimo sa zadanom jednadžbom jakosti magnetskog polja i dobijemo

$$\frac{E_m k_2}{\omega \mu_0} = \frac{E_m}{k_1} \rightarrow k_2 = \frac{\omega \mu_0}{k_1} \quad (**)$$

Izjednačimo (*) i (**) i slijedi

$$k_1 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

18. Samo uvrstimo u jednadžbu (*)k_1 i dobijemo

$$k_2 = \omega \varepsilon_0 \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

19.

$$\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$$

$$\overrightarrow{a_x}\left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) - \overrightarrow{a_y}\left(\frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z}\right) + \overrightarrow{a_z}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = -\omega \varepsilon_0 E_m \sin(\beta x)\cos(\omega t)\, \overrightarrow{a_z}$$

$$\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = -\omega \varepsilon_0 E_m \sin(\beta x)\cos(\omega t)$$

$$\frac{\partial H_x}{\partial y} = 0$$

$$\frac{\partial H_y}{\partial x} = -\omega \varepsilon_0 E_m \sin(\beta x)\cos(\omega t) \rightarrow H_y = -\frac{\omega \varepsilon_0}{\beta} E_m \sin(\beta x)\sin(\omega t)$$

$$\vec{H} = -\frac{\omega \varepsilon_0}{\beta} E_m \sin(\beta x)\sin(\omega t)\, \overrightarrow{a_y}$$

20.

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\beta E_m \sin(\beta x)\cos(\omega t)\, \overrightarrow{a_y} = -\frac{\partial \vec{B}}{\partial t} \rightarrow \vec{B} = -\frac{\beta}{\omega} E_m \sin(\beta x)\sin(\omega t)\, \overrightarrow{a_y}$$

$$\vec{H} = -\frac{\beta}{\omega \mu_0} E_m \sin(\beta x)\sin(\omega t)\, \overrightarrow{a_y}$$

Iz jednadžbe za $\vec{H}$ iz 19. zadatka i ove jednadžbe iznad, izjednačavanjem dobijemo $\frac{\omega}{\beta} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$.