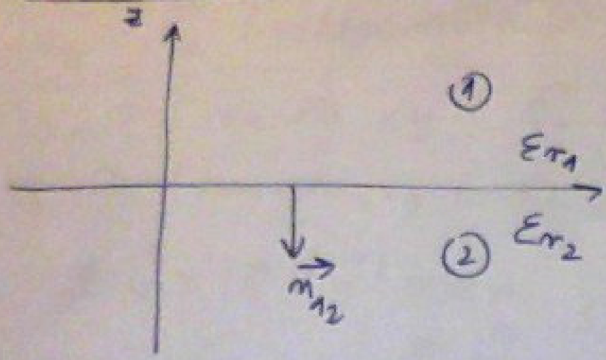
(13) $\vec{E}_1 = 2\vec{a}_x - 3\vec{a}_y + 5\vec{a}_z$ [V/m] , $\varepsilon_{r1} = 2$, $z > 0$

$\vec{E}_2 = ?$, $\varepsilon_{r2} = 5$, $z < 0$



$\vec{n}_{12} = -\vec{a}_z$

$\vec{D}_2 = \varepsilon_0 \varepsilon_{r2} \vec{E}_2$

$\vec{D}_1 = \varepsilon_0 \varepsilon_{r1} \vec{E}_1$

$\vec{E}_2 = E_{2x}\vec{a}_x + E_{2y}\vec{a}_y + E_{2z}\vec{a}$

1) $\vec{n}_{12} \cdot (\vec{D}_2 - \vec{D}_1) = 0$

$-\vec{a}_z \cdot (\varepsilon_0 \varepsilon_{r2}(E_{x2}\vec{a}_x + E_{y2}\vec{a}_y + E_{z2}\vec{a}_z) - \varepsilon_0\varepsilon_{r1}(2\vec{a}_x - 3\vec{a}_y + 5$

$\vec{n}_{12} = -\vec{a}_z$

$\vec{D}_2 = \varepsilon_0 \varepsilon_{r2} \vec{E}_2$

$\vec{D}_1 = \varepsilon_0 \varepsilon_{r1} \vec{E}_1$

$\vec{E}_2 = E_{2x}\vec{a}_x + E_{2y}\vec{a}_y + E_{2z}\vec{a}_z$

1) $\vec{n}_{12} \cdot (\vec{D}_2 - \vec{D}_1) = 0$

$-\vec{a}_z \cdot (\varepsilon_0 \varepsilon_{r2}(E_{x2}\vec{a}_x + E_{y2}\vec{a}_y + E_{z2}\vec{a}_z) - \varepsilon_0\varepsilon_{r1}(2\vec{a}_x - 3\vec{a}_y + 5\vec{a}_z))$

$-\vec{a}_z \cdot [\vec{a}_x(\varepsilon_0\varepsilon_{r2}E_{x2} - \varepsilon_0\varepsilon_{r1}\cdot 2) + \vec{a}_y(\varepsilon_0\varepsilon_{r2}E_{y2} + \varepsilon_0\varepsilon_{r1}\cdot 3) +$

$+\vec{a}_z(\varepsilon_0\varepsilon_{r2}E_{z2} - \varepsilon_0\varepsilon_{r1}\cdot 5)] = 0$

$-\varepsilon_0\varepsilon_{r2}E_{z2} + \varepsilon_0\varepsilon_{r1}\cdot 5 = 0$

$$\underline{\underline{E_{z2}}} = \frac{5\cancel{\varepsilon_0}\varepsilon_{r1}}{\cancel{\varepsilon_0}\varepsilon_{r2}} = \cancel{5}\cdot\frac{2}{\cancel{5}} = \underline{2}$$

2) $\vec{n}_{12} \times (\vec{E}_2 - \vec{E}_1) = \vec{0}$

$(\ldots - 2\vec{a}_x + 3\vec{a}_y - 5\vec{a}_z) = \vec{0}$

$$\varepsilon_0 \varepsilon_{r_2} E_{z_2} + \varepsilon_0 \varepsilon_{r_1} \cdot 5 = 0$$

$$\underline{\underline{E_{z_2}}} = \frac{5 \cancel{\varepsilon_0} \varepsilon_{r_1}}{\cancel{\varepsilon_0} \varepsilon_{r_2}} = \cancel{5} \cdot \frac{2}{\cancel{5}} = \underline{\underline{2}}$$

2) $\vec{n}_{12} \times (\vec{E_2} - \vec{E_1}) = \vec{0}$

$$-\vec{a_z} \times (E_{x2}\vec{a_x} + E_{y2}\vec{a_y} + E_{z2}\vec{a_z} - 2\vec{a_x} + 3\vec{a_y} - 5\vec{a_z}) =$$

$$\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 0 & 0 & -1 \\ (E_{x_2}-2) & (E_{y_2}+3) & (E_{z_2}-5) \end{vmatrix} = \vec{0}$$

$$\vec{a_x}(E_{y2}+3) - \vec{a_y}(E_{x_2}-2) + \vec{a_z} \cdot 0 = 0$$

$E_{y2}+3=0 \Rightarrow \underline{\underline{E_{y_2}=-3}}$ $E_{x_2}-2=0 \Rightarrow \underline{\underline{E_{x_2}=}}$

$$\boxed{\vec{E_2} = 2\vec{a_x} - 3\vec{a_y} + 2\vec{a_z} \ [V/m]}$$

14. $\vec{E_1} = 2\vec{a_x} - 3\vec{a_y} + 5\vec{a_z}$

$\vec{E_2} = 2\vec{a_x} - 3\vec{a_y} + 2\vec{a_z}$

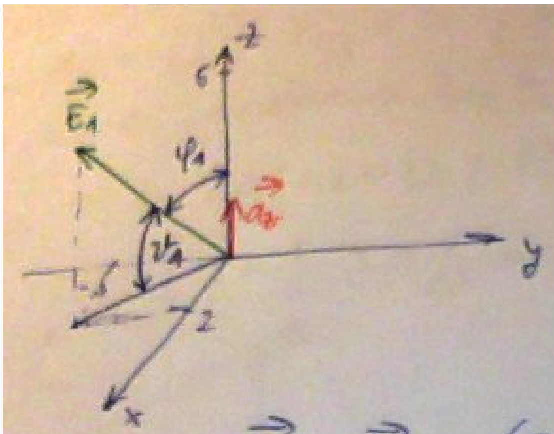
$\vec{a_{xy}} = \vec{a_z}$ (vektor normale ravnine xy)

$\vartheta_1 = ?$ (izmedu xy i $\vec{E_1}$)

$\vartheta_2 = ?$ (izmedu xy i $\vec{E_2}$)

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\varphi$$

$$\cos\varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$



$$\cos\varphi_1 = \frac{\vec{E_1}\cdot\vec{a_z}}{|\vec{E_1}||\vec{a_z}|} = \frac{(2\vec{a_x} - 3\vec{a_y} + 5\vec{a_z})\cdot\vec{a_z}}{\sqrt{4+9+25}\cdot\sqrt{1}} = \frac{5}{\sqrt{38}}$$

$$\varphi_1 = 35.8^\circ$$

$$\boxed{\vartheta_1} = 90^\circ - \varphi_1 = \boxed{54.2^\circ}$$

$$\cos\varphi_2 = \frac{\vec{E_2}\cdot\vec{a_z}}{|\vec{E_2}||\vec{a_z}|} = \frac{2}{\sqrt{17}} \Rightarrow \varphi_2 = 60.98^\circ \approx 61^\circ$$

$$\boxed{\vartheta_2} = 90^\circ - \varphi_2 = \boxed{29^\circ}$$

(15) $\vec{P} = \varepsilon_0 \chi_e \vec{E} = (\varepsilon_r - 1)\,\varepsilon_0 \vec{E} =$

$= (\varepsilon_r - 1)\cdot \cancel{\varepsilon_0} \cdot \frac{\vec{D}}{\cancel{\varepsilon_0}\varepsilon_r} = \vec{D}\left(1 - \frac{1}{\varepsilon_r}\right) =$

$= \vec{D}\left(1 - \frac{1}{3.2}\right) = 0.6875\,\vec{D}$

$\vec{P} = 0.6875 \cdot 2 \cdot 10^{-6}\,\vec{a_x}$

$\boxed{\vec{P} = 1.375 \cdot 10^{-6}\,\vec{a_x}}$

(16) → granica $3x + 2y + z = 12\ m$

izlodište → (1) → $\varepsilon_{r_1} = 3$, $\vec{E_1} = 2\vec{a_x} + 5\vec{a_z}$ [V/m]

(2) slobodni prostor → $\varepsilon_{r_2} = 1$, $\vec{E_2} = ?$

granica: $3x + 2y + z = 12 \;/:12$

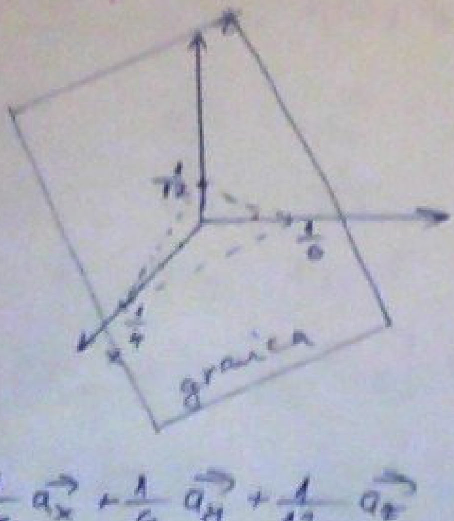
(16) → granica $3x + 2y + z = 12\ m$

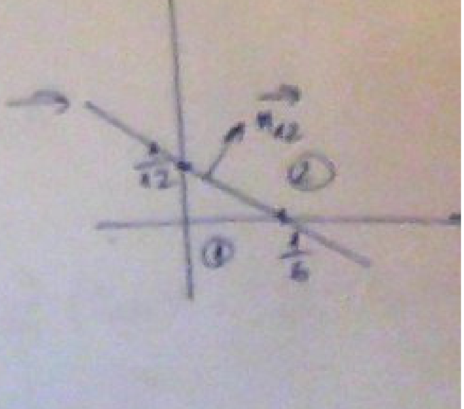
izlodište → (1) → $\varepsilon_{r_1} = 3$, $\vec{E_1} = 2\vec{a_x} + 5\vec{a_z}$ [V/m]

(2) slobodni prostor → $\varepsilon_{r_2} = 1$, $\vec{E_2} = ?$

granica: $3x + 2y + z = 12 \;/:12$

$\frac{1}{4}x + \frac{1}{6}y + \frac{1}{12}z = 1$

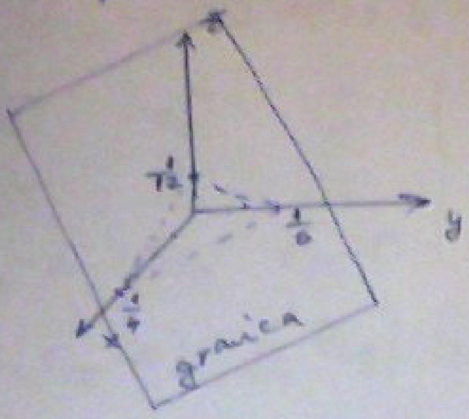


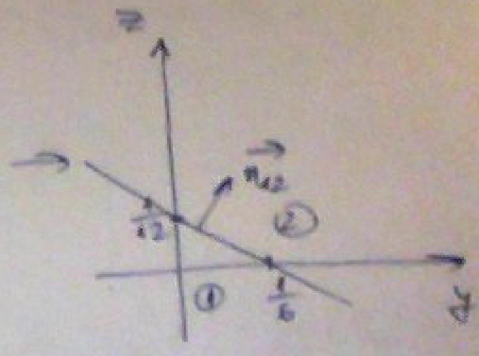


$\vec{n} = \frac{1}{4}\vec{a_x} + \frac{1}{6}\vec{a_y} + \frac{1}{12}\vec{a_z}$

Rj: $3x + 2y + z = 12 \; / \cdot 12$

$$\frac{1}{4}x + \frac{1}{6}y + \frac{1}{12}z = 1$$





$$\vec{n} = \frac{1}{4}\vec{a_x} + \frac{1}{6}\vec{a_y} + \frac{1}{12}\vec{a_z}$$

$$\vec{n_{12}} = \frac{\vec{n}}{|\vec{n}|} = \dots = \frac{3}{\sqrt{14}}\vec{a_x} + \frac{2}{\sqrt{14}}\vec{a_y} + \frac{1}{\sqrt{14}}\vec{a_z}$$

1) $\vec{n_{12}} \cdot (\vec{D_2} - \vec{D_1}) = 0 \quad \rightarrow \quad \vec{D_2} = \varepsilon_0 \varepsilon_{r_2} \vec{E_2}$, $\quad \vec{D_1} = \varepsilon_0 \varepsilon_{r_1} \vec{E_1}$

$$\left(\frac{3}{\sqrt{14}}\vec{a_x} + \frac{2}{\sqrt{14}}\vec{a_y} + \frac{1}{\sqrt{14}}\vec{a_z}\right) \cdot \Big(\vec{a_x}(\varepsilon_0 \varepsilon_{r_2} E_{2x} - \varepsilon_0 \varepsilon_{r}\ldots$$

$$+ \vec{a_z}\left(\varepsilon_0 \varepsilon_{r_2} E_{2z} - \varepsilon_0 \varepsilon_{r_1} \cdot 5\right)\Big) = 0$$

$$\frac{3}{\sqrt{14}}\left(\varepsilon_0 \varepsilon_{r_2} E_{2x} - \varepsilon_0 \varepsilon_{r_1} \cdot 2\right) + \frac{2}{\sqrt{14}}\varepsilon_0 \varepsilon_{r_2} E_{2y} + \frac{1}{\sqrt{14}}(\ldots$$

$$3\left(\varepsilon_{r_2} E_{2x} - \varepsilon_{r_1} \cdot 2\right) + 2\varepsilon_{r_2} E_{2y} + \varepsilon_{r_2} E_{2z} - \ldots$$

$$3\left(E_{2x} - 3 \cdot 2\right) + 2E_{2y} + E_{2z} - 5 \cdot 3 = 0$$

$$3E_{2x} - 18 + 2E_{2y} + E_{2z} - 15 = 0$$

$\vec{D_1}) = 0 \quad \rightarrow \quad \vec{D_2} = \varepsilon_0 \varepsilon_{r_2} \vec{E_2}$

$\vec{D_1} = \varepsilon_0 \varepsilon_{r_1} \vec{E_1}$

$+ \frac{1}{\sqrt{14}} \vec{a_z} \Big) \cdot \Big(\vec{a_x} (\varepsilon_0 \varepsilon_{r_2} E_{2x} - \varepsilon_0 \varepsilon_{r_1} \cdot 2) + \vec{a_y} \cdot \varepsilon_0 \varepsilon_{r_2} E_{2y} +$

$E_{2z} - \varepsilon_0 \varepsilon_{r_1} \cdot 5)\Big)\Big) = 0$

$\varepsilon_0 \varepsilon_{r_1} \cdot 2) + \frac{2}{\sqrt{14}} \varepsilon_0 \varepsilon_{r_2} E_{2y} + \frac{1}{\sqrt{14}} (\varepsilon_0 \varepsilon_{r_2} E_{2z} - \varepsilon_0 \varepsilon_{r_1} \cdot 5) = 0$

$\cdot 2) + 2 \varepsilon_{r_2} E_{2y} + \varepsilon_{r_2} E_{2z} - \varepsilon_{r_1} \cdot 5 = 0$

$2) + 2E_{2y} + E_{2z} - 5 \cdot 3 = 0$

$- 18 + 2E_{2y} + E_{2z} - 15 = 0$

$3E_{2x} + 2E_{2y} + E_{2z} = \ldots$

$\vec{a_{n_{12}}} \times (\vec{E_2} - \vec{E_1}) = \vec{0}$

$\vec{a_{n_{12}}} \times \left[(E_{2x} - 2)\vec{a_x} + (E_{2y} \vec{a_y}) + (E_{2z} - 5)\vec{a_z} \right] = \vec{0}$

$$\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{3}{\sqrt{14}} & \frac{2}{\sqrt{14}} & \frac{1}{\sqrt{14}} \\ E_{2x} - 2 & E_{2y} & E_{2z} - 5 \end{vmatrix} = \vec{0}$$

$\vec{a_x} \left[\frac{2}{\sqrt{14}} (E_{2z} - 5) - \frac{1}{\sqrt{14}} E_{2y} \right] - \vec{a_y} \left[\frac{3}{\sqrt{14}} (E_{2z} - 5) - \frac{1}{\sqrt{14}} (E_{2x} - 2) \right]$

$+ \vec{a_z} \left(\frac{3}{\sqrt{14}} E_{2y} - \frac{2}{\sqrt{14}} (E_{2x} - 2) \right) = \vec{0}$

a) $2E_{2z} - 10 - E_{2y} = 0 \Rightarrow 2E_{2z} - E_{2y} = 10 \Rightarrow \underline{E_{2y} = 2E_{2z}} \ldots$

b) $3E_{2z} - 15 - E_{2x} + 2 = 0 \Rightarrow E_{2x} = \underline{\underline{3E_{2z} - 13}}$

c) $3E_{2y} - 2E_{2x} = -4 \rightarrow$ υπάρχου 1)

$$\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{3}{\sqrt{14}} & \frac{2}{\sqrt{14}} & \frac{1}{\sqrt{14}} \\ E_{2x}-2 & E_{2y} & E_{2z}-5 \end{vmatrix} = \vec{0}$$

$$\vec{a_x}\left[\frac{2}{\sqrt{14}}(E_{2z}-5) - \frac{1}{\sqrt{14}}E_{2y}\right] - \vec{a_y}\left[\frac{3}{\sqrt{14}}(E_{2z}-5) - \frac{1}{\sqrt{14}}(E_{2x}-2)\right] +$$

$$+\vec{a_z}\left(\frac{3}{\sqrt{14}}E_{2y} - \frac{2}{\sqrt{14}}(E_{2x}-2)\right) = \vec{0}$$

a) $2E_{2z}-10-E_{2y}=0 \Rightarrow 2E_{2z}-E_{2y}=10 \Rightarrow E_{2y}=\underline{2E_{2z}-1}$

b) $3E_{2z}-15-E_{2x}+2=0 \Rightarrow E_{2x} = \underline{3E_{2z}-13}$

c) $3E_{2y}-2E_{2x}=-4$

→ ubacimo u 1)

$$3(3E_{2z}-13)+2(2E_{2z}-10)+E_{2z}=33$$

$$\boxed{E_{2z}=6.58} \to \boxed{E_{2y}=3.16} \to \boxed{E_{2x}=6.74}$$

$$\boxed{\vec{E_2} = 6.74\vec{a_x} + 3.16\vec{a_y} + 6.58\vec{a_z}}$$

(17) granica $3x+z=5$

ishodište → (1) → $\vec{D_1} = (4.5\vec{a_x} + 3.2\vec{a_z})\cdot 10^{-7}\ C/m^2$, $\varepsilon_{r1}=4.3$

(2) $\varepsilon_{r2} = 1.8$, $|\vec{D_2}| = ?$

---

granica: $3x+z=5 \ /:5$

$$\frac{3}{5}x + \frac{1}{5}z = 1 \Rightarrow \vec{n} = \frac{3}{5}\vec{a_x} + \frac{1}{5}\vec{a_z}$$

$$\vec{n_{12}} = \frac{\vec{n}}{|\vec{n}|} = \dots = \frac{3}{\sqrt{10}}\vec{a_x} + \frac{1}{\sqrt{10}}\vec{a_z}$$

1) $\vec{n_{12}}\ (\vec{D_2} - \vec{D_1}) = 0$

$$\left(\frac{3}{\sqrt{10}}\vec{a_x} + \frac{1}{\sqrt{10}}\vec{a_z}\right)\left((D_{2x}-4.5)\vec{a_x} + D_{2y}\vec{a_y} + (D_{2z}-3.2)\vec{a_z}\right) = 0$$

$-13.5 + D_{2z} - 3.2 = 0$

$3D_{2x} + D_{2z} = 0$

$\times (\vec{E_2} - \vec{E_1}) = \vec{0}$

1) $\vec{n_{12}}\ (\vec{D_2} - \vec{D_1}) = 0$

$$\left(\frac{3}{\sqrt{10}}\vec{a_x} + \frac{1}{\sqrt{10}}\vec{a_z}\right)\left((D_{2x}-4.5)\vec{a_x} + D_{2y}\vec{a_y} + (D_{2z}-3.2)\vec{a_z}\right) = 0$$

$3D_{2x} - 13.5 + D_{2z} - 3.2 = 0$

$3D_{2x} + D_{2z} = 0$

2) $\vec{n_{12}} \times (\vec{E_2} - \vec{E_1}) = \vec{0}$

$$\vec{n_{12}} \times \left(\frac{\vec{D_2}}{\varepsilon_0\varepsilon_{r2}} - \frac{\vec{D_1}}{\varepsilon_0\varepsilon_{r1}}\right) = \vec{0}$$

$$\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{3}{\sqrt{10}} & 0 & \frac{1}{\sqrt{10}} \\ \frac{D_{2x}}{\varepsilon_0\varepsilon_{r2}} - \frac{4.5}{\varepsilon_0\varepsilon_{r1}} & \frac{D_{2y}}{\varepsilon_0\varepsilon_{r2}} & \frac{D_{2z}}{\varepsilon_0\varepsilon_{r2}} - \frac{3.2}{\varepsilon_0\varepsilon_{r1}} \end{vmatrix} = \vec{0}$$

$$\vec{a_x}\left(-\frac{1}{\sqrt{10}}\frac{D_{2y}}{\dots}\right) - \vec{a_y}\left(\frac{3}{\sqrt{10}}\left(\frac{D_{2z}}{\varepsilon_0\varepsilon_{r2}} - \frac{3.2}{\varepsilon_0\varepsilon_{r1}}\right) - \frac{1}{\sqrt{10}}\left(\frac{D_{2x}}{\varepsilon_0\varepsilon_{r2}} - \dots\right.\right.$$

$$\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{3}{\sqrt{10}} & 0 & \frac{1}{\sqrt{10}} \\ \frac{D_{2x}}{\varepsilon_0 \varepsilon_{r2}} - \frac{4.5}{\varepsilon_0 \varepsilon_{r1}} & \frac{D_{2y}}{\varepsilon_0 \varepsilon_{r2}} & \frac{D_{2z}}{\varepsilon_0 \varepsilon_{r2}} - \frac{3.2}{\varepsilon_0 \varepsilon_{r1}} \end{vmatrix} = \vec{0}$$

$$\vec{a_x}\left(-\frac{1}{\sqrt{10}} \frac{D_{2y}}{\varepsilon_0 \varepsilon_{r2}}\right) - \vec{a_y}\left(\frac{3}{\sqrt{10}}\left(\frac{D_{2z}}{\varepsilon_0 \varepsilon_{r2}} - \frac{3.2}{\varepsilon_0 \varepsilon_{r1}}\right) - \frac{1}{\sqrt{10}}\left(\frac{D_{2x}}{\varepsilon_0 \varepsilon_{r2}} - \frac{4.5}{\varepsilon_0 \varepsilon_{r1}}\right)\right)$$

$$+ \vec{a_z}\left(\frac{3}{\sqrt{10}} \frac{D_{2y}}{\varepsilon_0 \varepsilon_{r2}}\right) = \vec{0}$$

$$D_{2y} = 0$$

$$\frac{3D_{2z}}{\varepsilon_0 \varepsilon_{r2}} - \frac{9.6}{\varepsilon_0 \varepsilon_{r1}} - \frac{D_{2x}}{\varepsilon_0 \varepsilon_{r2}} + \frac{4.5}{\varepsilon_0 \varepsilon_{r1}} = 0$$

$$\frac{3D_{2z}}{\varepsilon_{r2}} - \frac{9.6}{\varepsilon_{r1}} - \frac{D_{2x}}{\varepsilon_{r2}} + \frac{4.5}{\varepsilon_{r1}} = 0$$

$$\frac{3D_{2z}}{1.8} - \frac{9.6}{4.3} - \frac{D_{2x}}{1.8} + \frac{4.5}{4.3} = 0 \quad / \cdot 7.74$$

$$12.9 D_{2z} - 17.28 - 4.3 D_{2x} + 8.1 = 0$$

$$-4.3 D_{2x} + 12.9 D_{2z} = 9.18$$

1) i 2) daju: (malo uvrštavaja)

$$D_{2x} = 4.7965$$

$$D_{2z} = 2.3105$$

$$\vec{D_2} = 4.7965 \vec{a_x} + 2.3105 \vec{a_z}$$

ovo je s početka

$$|\vec{D_2}| = \sqrt{4.7965^2 \ldots}$$

$$-4.3 D_{2x} + 12.9 D_{2z} = 9.18$$

1) i 2) daju: (malo uvrštavanja)

$$D_{2x} = 4.7965$$

$$D_{2z} = 2.3105$$

$$\vec{D}_2 = 4.7965 \vec{a_x} + 2.3105 \vec{a_z}$$

ovo je s potka

$$\boxed{|\vec{D}_2|} = \sqrt{4.7965^2 + 2.3105^2} = \boxed{5.32 \cdot 10^{-7}\ C/m^2}$$