— 2014. —

1) Dva magnetska materijala razdvaja ravnina $x=0$. U poluprostoru $x<0$ je $\mu_{r1}=15$, a u poluprostoru $x>0$ je $\mu_{r2}=20$. Ako su zadani mag. ind. u $x>0$ kao $\vec{B_2}=\vec{a_x}-0.5\vec{a_y}+\vec{a_z}$ [T] i strujni oblog na granici $\vec{K}=\frac{0.1}{\mu_0}\vec{a_y}-\frac{0.2}{\mu_0}\vec{a_z}$ [A/m], odredite $\vec{B_1}$ u prostoru $x<0$.

$\vec{n_{12}}=\vec{a_x}$

$\vec{B_1}=x\vec{a_x}+y\vec{a_y}+z\vec{a_z}$ $\qquad \vec{H_1}=\frac{x}{\mu_0\mu_{r1}}\vec{a_x}+\frac{y}{\mu_0\mu_{r1}}\vec{a_y}+\frac{z}{\mu_0\mu_{r1}}\vec{a_z}$

$\vec{n_{12}}\cdot(\vec{B_2}-\vec{B_1})=0$

$\vec{a_x}\cdot\left[\vec{a_x}(1-x)+\vec{a_y}(-0.5-y)+\vec{a_z}(1-z)\right]=0 \rightarrow 1-x=0$

$\boxed{x=1}$

$$\vec{n_{12}}\times(\vec{H_2}-\vec{H_1})=\begin{vmatrix}\vec{a_x} & \vec{a_y} & \vec{a_z}\\ 1 & 0 & 0\\ \frac{1}{\mu_0\mu_{r2}}-\frac{x}{\mu_0\mu_{r1}} & \frac{1}{\mu_0}\left(\frac{-0.5}{\mu_{r2}}-\frac{y}{\mu_{r1}}\right) & \frac{1}{\mu_0}\left(\frac{1}{\mu_{r2}}-\frac{z}{\mu_{r1}}\right)\end{vmatrix}=$$

$$=-\vec{a_y}\cdot\frac{1}{\mu_0}\left(\frac{1}{\mu_{r2}}-\frac{z}{\mu_{r1}}\right)+\vec{a_z}\cdot\frac{1}{\mu_0}\left(\frac{-0.5}{\mu_{r2}}-\frac{y}{\mu_{r1}}\right)=\frac{0.1}{\mu_0}\vec{a_y}-\frac{0.2}{\mu_0}\vec{a_z}$$

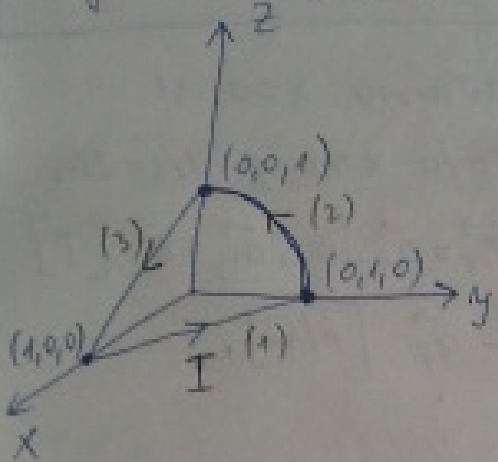
$-\frac{1}{\mu_0}\left(\frac{1}{20}-\frac{z}{15}\right)=\frac{0.1}{\mu_0}$ $\qquad \frac{-0.5}{20}-\frac{y}{15}=-0.2$

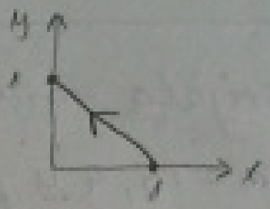
$-\frac{z}{15}=-0.1-\frac{1}{20}$ $\qquad \frac{y}{15}=-\frac{0.5}{20}+0.2$

$z=\frac{9}{4}=2.25$ $\qquad y=\frac{21}{8}=2.625$

$\vec{B_1}=\vec{a_x}+\frac{21}{8}\vec{a_y}+\frac{9}{4}\vec{a_z}$

2) Strujnicom prema slici teče struja $I = 1$ A. Odredite vektor jakosti mag. polja u ishodištu koord. sustava.




A(1,0)

B(0,1)

$$\frac{x-1}{0-1} = \frac{y-0}{1-0}$$

$x = -t+1 \quad dx = -dt$

$y = t \quad dy = dt$

strujnica 1:

$\vec{r} = 0$, $\vec{r}\,' = x\vec{a_x} + y\vec{a_y}$, $\vec{R} = -x\vec{a_x} - y\vec{a_y}$, $\vec{dl} = dx\,\vec{a_x} + dy\,\vec{a_y}$

$$\vec{dl} \times \vec{R} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ dx & dy & 0 \\ -x & -y & 0 \end{vmatrix} = \vec{a_z} \cdot (x\,dy - y\,dx)$$

$$\vec{B} = \frac{\mu_0 \cdot I}{4\pi} \int \frac{x\,dy - y\,dx}{(\sqrt{x^2+y^2})^3}\,\vec{a_z} = \frac{\mu_0 \cdot I}{4\pi}\left[\int \frac{(-t+1)\cdot dt + t \cdot dt}{(\underbrace{\sqrt{(1-t)^2+t^2}}_{1-2t+t^2+t^2})^3}\right] =$$

$$= \frac{\mu_0 \cdot I}{4\pi} \int_0^1 \frac{dt}{(\sqrt{2t^2-2t+1})^3} = \frac{\mu_0 \cdot I}{4\pi} \cdot 2 = \underline{\frac{\mu_0 \cdot I}{2\pi}\,\vec{a_z}}$$

strujnica 2:

$\vec{r}\,' = y\vec{a_y} + z\vec{a_z} \qquad \vec{R} = -y\vec{a_y} - z\vec{a_z}$

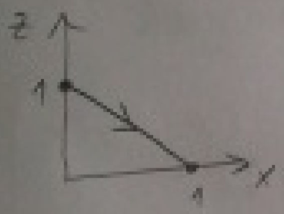
$$\vec{dl} \times \vec{R} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 0 & dy & dz \\ 0 & -y & -z \end{vmatrix} = \vec{a_x} \cdot (y\,dz - z\,dy)$$

$$\vec{B} = \frac{\mu_0 \cdot I}{4\pi} \int \frac{y\,dz - z\,dy}{\left(\sqrt{y^2+z^2}\right)^3}\,\vec{a_x} = \frac{\mu_0 \cdot I}{4\pi}\cdot\int \frac{r\cos\alpha \cdot r\cos\alpha\,d\alpha + r\sin\alpha \cdot r\sin\alpha\,d\alpha}{\left(\sqrt{r^2}\right)^3}$$

$$= \frac{\mu_0 I}{4\pi}\cdot\int_0^{\pi/2} \frac{\cancel{r^2}\,d\alpha}{r^{\cancel{3}}} = \frac{\mu_0 \cdot I}{4\pi\cdot 1}\cdot\frac{\pi}{2} = \underline{\frac{\mu_0 \cdot I}{8}\,\vec{a_x}}$$

stranica 3: $\vec{r}\,' = x\,\vec{a_x} + z\,\vec{a_z}$

$$d\vec{l}\times\vec{R} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ dx & 0 & dz \\ -x & 0 & -z \end{vmatrix} = -\vec{a_y}\cdot(-z\,dx + x\,dz) = \vec{a_y}\,(z\,dx - x\,dz)$$



A(0,1)
B(1,0)

$$\frac{x-0}{1-0} = \frac{z-1}{0-1} \rightarrow \begin{matrix} x = t & dx = dt \\ z = -t+1 & dz = -dt \end{matrix}$$

$$\vec{B} = \frac{\mu_0 \cdot I}{4\pi}\int \frac{z\,dx - x\,dz}{\left(\sqrt{x^2+z^2}\right)^3} = \frac{\mu_0 \cdot I}{4\pi}\int_0^1 \frac{(-t+1)\,dt - t\cdot(-dt)}{\left(\sqrt{t^2+t^2-2t+1}\right)^3} =$$

$$= \frac{\mu_0 I}{4\pi}\int_0^1 \frac{dt}{\left(\sqrt{2t^2-2t+1}\right)^3} = \frac{\mu_0 \cdot I}{4\pi}\cdot 2 = \underline{\frac{\mu_0 \cdot I}{2\pi}\,\vec{a_y}}$$

$$\vec{B}_{uk} = \frac{\mu_0 I}{8}\,\vec{a_x} + \frac{\mu_0 I}{2\pi}\,\vec{a_y} + \frac{\mu_0 I}{2\pi}\,\vec{a_z}$$

$$\vec{H}_{uk} = \frac{1}{8}\,\vec{a_x} + \frac{1}{2\pi}\,\vec{a_y} + \frac{1}{2\pi}\,\vec{a_z} = \underline{0.125\,\vec{a_x} + 0.159\,\vec{a_y} + 0.159\,\vec{a_z}}$$

3) Magnetsko polje ravnog EM vala koji se prostire u sredstvu bez gubitaka zadano je izrazom. Odredite vektor električnog polja, te srednju snagu kroz kvadrat stranice $a=0.1$ m čija je površina okomita na pravac prostiranja vala.

$\mu=\mu_0$, $\omega=10^7\ s^{-1}$, $\vec{H}=(-\vec{a_x}+\vec{a_y})\cos[\omega t-0.1\pi(x+y+z)]$ (A/m)

$$\vec{\beta}\cdot\vec{r}=0.1\pi(x+y+z) \rightarrow \begin{array}{l}\vec{r}=x\vec{a_x}+y\vec{a_y}+z\vec{a_z}\\ \vec{\beta}=0.1\pi(\vec{a_x}+\vec{a_y}+\vec{a_z})\\ \hookrightarrow \beta_x=\beta_y=\beta_z\end{array}$$

$$\vec{H_0}=(-\vec{a_x}+\vec{a_y})$$

$$\vec{H_0}=\frac{1}{\omega\mu}\vec{\beta}\times\vec{E_0}, \quad \vec{\beta}\times\vec{E_0}=\begin{vmatrix}\vec{a_x} & \vec{a_y} & \vec{a_z}\\ \beta_x & \beta_y & \beta_z\\ E_x & E_y & E_z\end{vmatrix}$$

$$\vec{H_0}=\frac{1}{\omega\mu}\left[\vec{a_x}(\beta_y E_z-\beta_z E_y)-\vec{a_y}(\beta_x E_z-\beta_z E_x)\right]$$

$$① \quad \frac{1}{\omega\mu}(\beta_y E_z-\beta_z E_y)=-1$$

$$② \quad \frac{1}{\omega\mu}(\beta_z E_x-\beta_x E_z)=1$$

$$\vec{E_0}\cdot\vec{\beta}=0=E_x\beta_x+E_y\beta_y+E_z\beta_z=0$$

$$\beta_x(E_x+E_y+E_z)=0$$

$$E_x+E_y+E_z=0$$

① $\beta x E_z - \beta x E_y = -\omega\mu$

$$E_z = \frac{-\omega\mu}{\beta x} + E_y$$

② $\beta x E_x - \beta x E_z = \omega\mu$

$$E_x = \frac{\omega\mu}{\beta x} + E_z = \frac{\omega\mu}{\beta x} - \frac{\omega\mu}{\beta x} + E_y \rightarrow E_x = E_y$$

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$$E_x + E_y + E_z = 0$$

$$2E_y + E_y - \frac{\omega\mu}{\beta x} = 0$$

$$3E_y = \frac{\omega\mu}{\beta x} \rightarrow E_y = \frac{\omega\mu}{3\beta x} = \frac{10^7 \cdot \mu_0}{3 \cdot 0.1\pi} = \underline{13.33 \text{ V/m}} = E_x$$

$$E_z = \underline{-26.67 \text{ V/m}}$$

$$\vec{E_0} = E_x \vec{a_x} + E_y \vec{a_y} + E_z \vec{a_z}$$

$$\underline{\vec{E} = \left(13.33\,\vec{a_x} + 13.33\,\vec{a_y} - 26.67\,\vec{a_z}\right) \cdot \cos\left[\omega t - 0.1\pi(x+y+z)\right] \text{ V/m}}$$

$$\underline{\vec{H_0}}^* = \left(-\vec{a_x} + \vec{a_y}\right) e^{j0.1\pi(x+y+z)}$$

$$\underline{\vec{E_0}} = \left(13.33\,\vec{a_x} + 13.33\,\vec{a_y} - 26.67\,\vec{a_z}\right) e^{-j0.1\pi(x+y+z)}$$

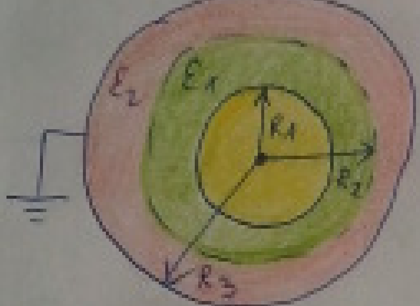
$$\vec{N_{sr}} = \frac{1}{2}\left(\underline{\vec{E_0}} \times \underline{\vec{H_0}}^*\right) = \frac{1}{2} \cdot \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 13.33 & 13.33 & -26.67 \\ -1 & 1 & 0 \end{vmatrix} = \frac{1}{2}\left[\vec{a_x} \cdot 26.67 - \vec{a_y} \cdot (-26.67) + \vec{a_z}(2 \cdot 13.33)\right]$$

$$\vec{N_{sr}} = 13.33\left(\vec{a_x} + \vec{a_y} + \vec{a_z}\right)$$

$$P_{sr} = \int \vec{N_{sr}} \cdot \vec{n}\, dS = 13.33 \cdot 3 \cdot 0.1^2 = \underline{0.4 \text{ W}}$$

→ val se prostire u sva 3 smjera

4) Kuglasti dvoslojni kondenzator zadan je slikom i priključen je na izvor stalnog napona $U = 100\,V$. Zadano je $\varepsilon_{r1} = 2$, $\varepsilon_{r2} = 4$, $R_2 = 0.1\,m$, $R_3 = 0.15\,m$. Odredite $R_1$ tako da maksimalno el. polje u sredstvu $\varepsilon_{r1}$ postigne minimalni mogući iznos. Kolika je pritom energija (ukupna) pohranjena u izolaciji kondenzatora?




$$\Delta \varphi = 0$$

$$\frac{1}{r^2} \cdot \frac{\partial}{\partial r}\left(r^2 \frac{\partial \varphi}{\partial r}\right) = 0 \quad / \cdot r^2 / \int$$

$$r^2 \frac{\partial \varphi}{\partial r} = C_1$$

$$\varphi(r) = -\frac{C_1}{r} + C_2$$

$$\varphi(r = R_1) = -\frac{C_1}{R_1} + C_2 = 100 \longrightarrow -\frac{C_1}{R_1} + \frac{C_1}{R_3} = 100 \rightarrow C_1 = \frac{100}{\frac{1}{R_3} - \frac{1}{R_1}}$$

$$\varphi(r = R_3) = -\frac{C_1}{R_3} + C_2 = 0 \rightarrow C_2 = \frac{C_1}{R_3}$$

$$C_1 = \frac{R_1 R_3 \cdot 100}{R_1 - R_3} = \frac{15 R_1}{R_1 - 0.15}$$

$$C_2 = \frac{1}{0.15} \cdot \frac{15 R_1}{R_1 - 0.15} = \frac{100 R_1}{R_1 - 0.15}$$

$$\varphi(r) = -\frac{1}{r}\cdot\frac{15R_1}{R_1-0.15} + \frac{100R_1}{R_1-0.15}$$

$$\vec{E} = -\operatorname{grad}\varphi$$

$$\vec{E} = -\left(-\frac{-1}{r^2}\cdot\frac{15R_1}{R_1-0.15}\right)\vec{a_r} = -\frac{1}{r^2}\cdot\frac{15R_1}{R_1-0.15}\vec{a_r}$$

$$E(r=R_1) = -\frac{1}{R_1^{\cancel{2}}}\cdot\frac{15\cancel{R_1}}{R_1-0.15} = \frac{-15}{R_1^2-0.15R_1} \quad \Big/ \frac{d}{dR_1}$$

$$\frac{dE}{dR_1} = 0 = -15\cdot\frac{-(2R_1-0.15)}{(R_1^2-0.15R_1)^2} = 0 \rightarrow 2R_1 = 0.15$$

$$\underline{R_1 = 0.075\ m}$$

$$W = \frac{\varepsilon}{2}\int_V E^2 dV = \frac{\varepsilon_0\varepsilon_{r1}}{2}\int_{V_1}\left(-\frac{1}{r^2}\cdot\frac{15R_1}{R_1-0.15}\right)^2 dV + \frac{\varepsilon_0\varepsilon_{r2}}{2}\int_{V_2}\left(-\frac{1}{r^2}\cdot\frac{15R_1}{R_1-0.15}\right)^2 dV$$

$$W = \frac{\varepsilon_0\varepsilon_{r1}}{2}\cdot 225\cdot\int_{V_1}\frac{dV}{r^4} + \frac{\varepsilon_0\varepsilon_{r2}}{2}\cdot 225\int_{V_2}\frac{dV}{r^4}$$

$$= \varepsilon_0\cdot 225\cdot\int_{R_1}^{R_2}\frac{r^2dr}{r^4}\int_0^{\pi}\sin\vartheta\, d\vartheta\int_0^{2\pi}d\alpha + \varepsilon_0\cdot 450\cdot\int_{R_2}^{R_3}\frac{r^2dr}{r^4}\int_0^{\pi}\sin\vartheta\, d\vartheta\int_0^{2\pi}d\alpha$$

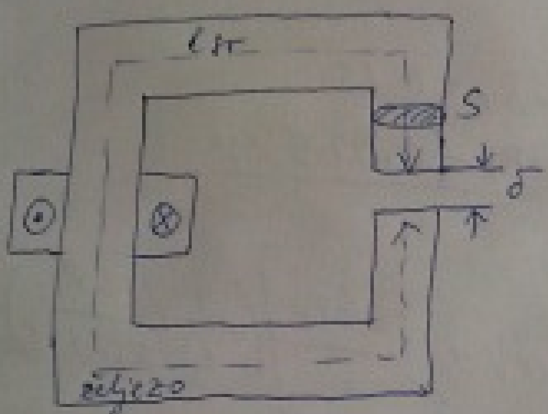
$$W = \varepsilon_0\cdot 4\pi\cdot\left(225\cdot\frac{-1}{r}\Big|_{R_1}^{R_2} + 450\cdot\frac{-1}{r}\Big|_{R_2}^{R_3}\right)$$

$$W = \varepsilon_0\cdot 4\pi\cdot(-1950+1500) = \underline{-50.1\ nJ}$$

5) Za mag. krug prema slici odredite mag. ind. i energiju mag. polja u zračnom rasporu. Krivulja magnetiziranja zadana je grafički. Zadatak riješite grafoanalitičkom metodom. Zadano je:

$S = 4\ cm^2$ $\quad \delta = 0.27\ mm$ $\quad N = 280$

$l_{sr} = 20\ cm$ $\quad I = 1\ A$




$$\Theta = N \cdot I = 280 = H_{Fe} \cdot l_{Fe} + H_\delta \cdot \delta$$

$$B_{Fe} \cdot \cancel{S_{Fe}} = B_\delta \cdot \cancel{S_\delta}$$

$$B_{Fe} = B_\delta$$

$$H_\delta = \frac{B_\delta}{\mu_0} \quad , \quad H_{Fe} = \frac{H_\delta}{\mu_r}$$

$$\Theta = \frac{H_\delta}{\mu_r} \cdot l_{sr} + H_\delta \cdot \delta = H_\delta \left( \frac{l_{sr}}{\mu_r} + \delta \right)$$

$B_{Fe} = 1.1\ T$

$H_{Fe} = 200\ A/m$

$$B = \mu_0 \mu_r H$$

$$\mu_r = \frac{B}{\mu_0 H} = 4376.761$$

$$B_\delta = \mu_0 \cdot H_\delta = \mu_0 \cdot \frac{\Theta}{\frac{l_{sr}}{\mu_r} + \delta} = \underline{1.1145\ T}$$

$W = \frac{1}{2\mu_0} \cdot B_\delta^2 \cdot \underbrace{4 \cdot 10^{-4}}_{V} \cdot$ **0.27*10^(-3) =0.052 J**