nd

Magnetostatika – 1 (1. – 12. zadatak)

Postupci rješavanja

1. Imamo zadano $\vec{A} = 2.5\overrightarrow{a_\theta} + 5\overrightarrow{a_\alpha}$ i točku u prostoru $T = (r, \theta, \alpha) = (2, \frac{\pi}{6}, 0)$. Trebamo naći iznos magnetske indukcije u smjeru $\overrightarrow{a_\alpha}$. Izraz koji povezuje magnetsku indukciju i $\vec{A}$ jest

$$\vec{B} = \nabla \times \vec{A}$$

Moramo paziti na to da se ovdje radi o sfernim koordinatama, pa je rotacija u sfernim koordinatama jednaka:

$$\vec{B} = \frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}(A_\theta \sin\theta) - \frac{\partial A_\theta}{\partial\alpha}\right)\overrightarrow{a_r} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\alpha} - \frac{\partial}{\partial r}(rA_\alpha)\right)\overrightarrow{a_\theta} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\alpha) - \frac{\partial A_r}{\partial\theta}\right)\overrightarrow{a_\alpha}$$

Nama treba B_α pa nam treba samo izraz $\frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\alpha) - \frac{\partial A_r}{\partial\theta}\right)$.

$$\boldsymbol{B_\alpha} = \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right) = \frac{1}{r}\left(\frac{\partial}{\partial r}(2.5r) - \frac{\partial}{\partial\theta}(0)\right) = \frac{1}{r}\cdot 2.5 = \frac{1}{2}\cdot 2.5 = \mathbf{1.25}$$

2. Imamo zadanu gustoću struje, i površinu *S* kroz koju ta struja prolazi. Trebamo naći jakost struje.

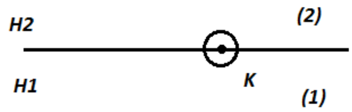
$$I = \iint_S \vec{J}\cdot\vec{n}dS = \iint_S 100y\sin(2y)\,dydz = \int_{-0.25\pi}^{0.25\pi} 100y\sin(2y)\,dy \int_{-0.01}^{0.01} dz =$$

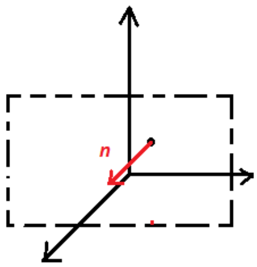
$$= 0.02\int_{-0.25\pi}^{0.25\pi} 100y\sin(2y)\,dy$$

Ovaj integral riješimo parcijalnom integracijom ili pomoću Wolfram Alphe i dobijemo rezultat.

$$\boldsymbol{I = 1A}$$

3. Ovdje se radi o uvjetima na granici.





$$\vec{n} \times \left(\overrightarrow{H_2} - \overrightarrow{H_1}\right) = \vec{K}$$

$$\overrightarrow{a_x} \times \left(H_{2x}\overrightarrow{a_x} + \left(H_{2y} - 10\right)\overrightarrow{a_y} + H_{2z}\overrightarrow{a_z}\right) = 6.5\overrightarrow{a_z}$$

$$\left(H_{2y} - 10\right)\overrightarrow{a_z} - H_{2z}\overrightarrow{a_y} = 6.5\overrightarrow{a_z}$$

$$H_{2y} - 10 = 6.5 \rightarrow H_{2y} = 16.5$$

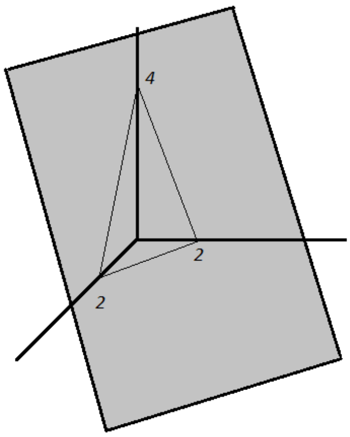
$$H_{2z} = H_{2x} = 0$$

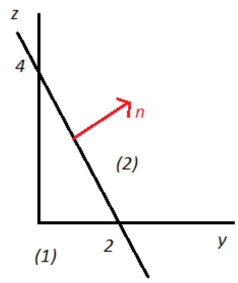
Pa je jakost polja u području 2 jednaka $\left|\overrightarrow{\boldsymbol{H_2}}\right| = \mathbf{16.5}$.

4. Zadatak sličan kao i 1., samo što ovdje imamo obični *xyz* sustav.

$$\overrightarrow{\boldsymbol{B}} = \nabla \times \vec{A} = \begin{vmatrix} \overrightarrow{a_x} & \overrightarrow{a_y} & \overrightarrow{a_z} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \cos x \sin y & \sin x \cos y & 0 \end{vmatrix} = \mathbf{0}$$

5. (12.) – stavio sam i 12. zadatak ovdje jer je isti postupak, samo drugi brojevi.





Granica:

$$4x + 4y + 2z = 8 \rightarrow \frac{x}{2} + \frac{y}{2} + \frac{z}{4} = 1 \rightarrow 2x + 2y + z - 4 = 0$$

Normala:

$$\vec{n} = \frac{2\overrightarrow{a_x} + 2\overrightarrow{a_y} + \overrightarrow{a_z}}{\sqrt{2^2 + 2^2 + 1^2}} = \frac{2}{3}\overrightarrow{a_x} + \frac{2}{3}\overrightarrow{a_y} + \frac{1}{3}\overrightarrow{a_z}$$

1. uvjet na granici:

$$\vec{n} \cdot \left(\overrightarrow{B_2} - \overrightarrow{B_1}\right) = 0$$

$$\vec{n} \cdot \left(\mu_0\mu_{r2}\overrightarrow{H_2} - \mu_0\mu_{r1}\overrightarrow{H_1}\right) = 0$$

$$\left(\frac{2}{3}\overrightarrow{a_x} + \frac{2}{3}\overrightarrow{a_y} + \frac{1}{3}\overrightarrow{a_z}\right)\left((3\mu_0 H_{2x} - 8)\overrightarrow{a_x} + \left(3\mu_0 H_{2y} + 4\right)\overrightarrow{a_y} + 3\mu_0 H_{2z}\overrightarrow{a_z}\right) = 0$$

Kad sve to pomnožimo dobijemo jednadžbu (1):

$$2\mu_0 H_{2x} + 2\mu_0 H_{2y} + \mu_0 H_{2z} = \frac{8}{3}$$

2. uvjet na granici (ovdje je $\vec{K} = \vec{0}$):

$$\vec{n} \times \left(\overrightarrow{H_2} - \overrightarrow{H_1}\right) = \vec{K}$$

$$\begin{vmatrix} \overrightarrow{a_x} & \overrightarrow{a_y} & \overrightarrow{a_z} \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ H_{2x} - \frac{2}{\mu_0} & H_{2y} + \frac{1}{\mu_0} & H_{2z} \end{vmatrix} = \vec{0}$$

Kad malo ovo porješavamo, dobijemo 3 jednadžbe od kojih sam ja uzeo dvije i izvukao sljedeće:

$$(2) \quad H_{2y} = 2H_{2z} - \frac{1}{\mu_0}$$

$$(2) \quad H_{2x} = 2H_{2z} + \frac{2}{\mu_0}$$

Ovo uvrstimo u (1) i dobijemo $H_{2z} = \frac{2}{27\mu_0}$, pa je $H_{2y} = -\frac{23}{27\mu_0}$. Iz ovoga slijedi da je

$$\boldsymbol{B_{2y}} = \mu_0\mu_{r2}H_{2y} = -\mu_0 \cdot 3 \cdot \frac{23}{27\mu_0} = \mathbf{-2.56}$$

6. Ovdje koristimo jednadžbu za rotaciju u cilindričnim koordinatama:

$$\vec{B} = \left(\frac{1}{r}\frac{\partial A_z}{\partial \alpha} - \frac{\partial A_\alpha}{\partial z}\right)\overrightarrow{a_r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\overrightarrow{a_\alpha} + \frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\alpha) - \frac{\partial A_r}{\partial \alpha}\right)\overrightarrow{a_z}$$

Nama se traži indukcija u smjeru $\overrightarrow{a_r}$:

$$B_r = \frac{1}{r}\frac{\partial A_z}{\partial \alpha} - \frac{\partial A_\alpha}{\partial z} = \frac{1}{r}\frac{\partial}{\partial \alpha}(0) - \frac{\partial}{\partial z}\left(e^{-2z}(\sin(0.5\alpha)\right) = 2e^{-2z}\sin(0.5\alpha)$$

Kada uvrstimo vrijednosti dobijemo:

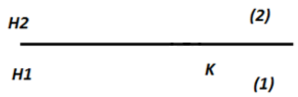
$$\boldsymbol{B_r} = \frac{1}{e} = \boldsymbol{0.37}$$

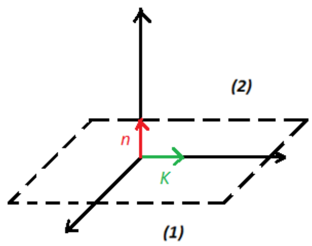
7. Slično kao 2. zadatak:

$$\boldsymbol{I} = \iint_S \vec{J}\cdot\vec{n}dS = \iint_S 100|x|dxdz = 2\int_0^{0.1} 100xdx \int_{-0.02}^{0.02} dz =$$

$$= 0.04 = \boldsymbol{4mA}$$

8.





Trebamo odrediti tangencijalnu komponentu jakosti magnetskog polja u području 1.

1. uvjet na granici:

$$\vec{n}\cdot\left(\overrightarrow{B_2} - \overrightarrow{B_1}\right) = 0$$

$$\overrightarrow{a_z}\cdot\left(\mu_0\mu_{r2}\overrightarrow{H_2} - \mu_0\mu_{r1}\overrightarrow{H_1}\right) = 0$$

$$\overrightarrow{a_z}\cdot\left(3\mu_0\overrightarrow{H_2} - 4\mu_0\overrightarrow{H_1}\right) = 0$$

$$\overrightarrow{a_z}\cdot\left(3\mu_0(14.5\overrightarrow{a_x} + 8\overrightarrow{a_z}) - 4\mu_0\left(H_{1x}\overrightarrow{a_x} + H_{1y}\overrightarrow{a_y} + H_{1z}\overrightarrow{a_z}\right)\right) = 0$$

$$24\mu_0 - 4\mu_0 H_{1z} = 0 \rightarrow H_{1z} = 6$$

2. uvjet na granici:

$$\vec{n} \times (\overrightarrow{H_2} - \overrightarrow{H_1}) = \vec{K}$$

$$\overrightarrow{a_z} \times \left((14.5\overrightarrow{a_x} + 8\overrightarrow{a_z}) - (H_{1x}\overrightarrow{a_x} + H_{1y}\overrightarrow{a_y} + H_{1z}\overrightarrow{a_z})\right) = \vec{K}$$

$$\overrightarrow{a_z} \times \left((14.5 - H_{1x})\overrightarrow{a_x} - H_{1y}\overrightarrow{a_y} + (8 - H_{1z})\overrightarrow{a_z}\right) = 9\overrightarrow{a_y}$$

$$(14.5 - H_{1x})\overrightarrow{a_y} - H_{1y}\overrightarrow{a_x} = 9\overrightarrow{a_y}$$

$$14.5 - H_{1x} = 9 \rightarrow H_{1x} = 5.5$$

$$H_{1y} = 0$$

Pa je iz ovoga svega $\overrightarrow{H_1} = 5.5\overrightarrow{a_x} + 6\overrightarrow{a_z}$.

Normalna komponenta je iz ovoga:

$$H_{1n} = \overrightarrow{H_1} \cdot \vec{n} = 6 \rightarrow \overrightarrow{H_{1n}} = H_1 \cdot \vec{n} = 6\overrightarrow{a_z}$$

Pa je tangencijalna komponenta:

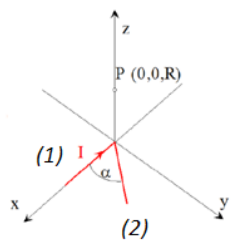
$$\overrightarrow{H_{1t}} = \overrightarrow{H_1} - \overrightarrow{H_{1n}} = 5.5\overrightarrow{a_x}$$

$$\boldsymbol{H_{1t} = 5.5}$$

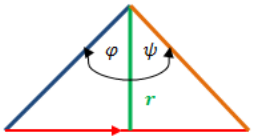
9. Kod ovakvih zadataka, uvijek podijelimo strujnicu po segmentima. Tu nam vrijedi jedna zgodna formula koja kaže sljedeće:

$$\vec{H} = \frac{I}{4\pi r}(\sin\varphi + \sin\psi)\overrightarrow{a_H}$$

gdje je $\overrightarrow{a_H}$ vektor smjera jakosti magnetskog polja (određen smjerom magnetske indukcije i pravila desne ruke).

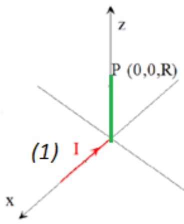


Iz sljedeće slike vidjet ćemo što predstavlja što u navedenoj formuli.

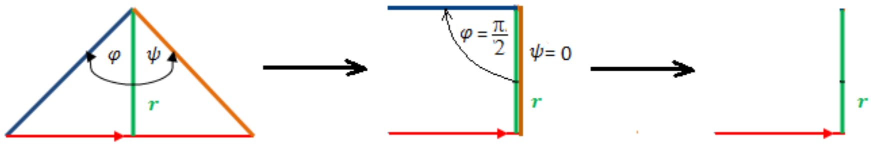


Slika 1.

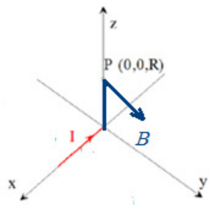
Uvijek poštujete oznake kao na slici, i smjer kretanja kuteva. Primijenimo sada tu sliku na naš prvi segment (1).



Kada ćemo iz slike 1. dobiti sliku za segment (1)? Pa kada nam kut φ ide prema $\frac{\pi}{2}$, a kut ψ prema 0.



Još namo samo fali smjer jakosti magnetskog polja u točki *P*. To ćemo odrediti preko magnetske indukcije i pravila desne ruke.

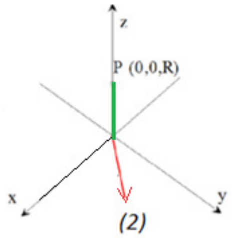


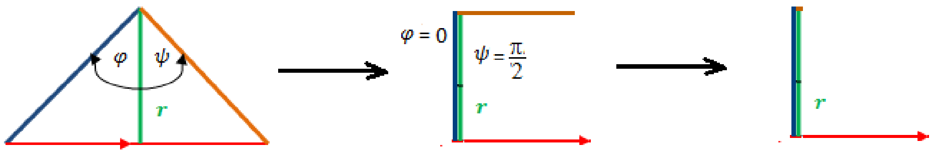
Vidimo da je smjer u smjeru osi *y*, odnosno $\overrightarrow{a_y}$.

Pa je jakost magnetskog polja za 1. segment jednaka:

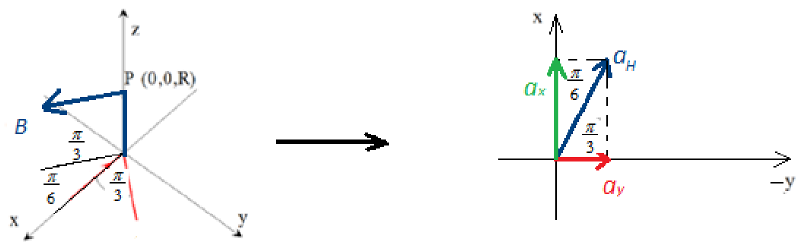
$$\overrightarrow{H_1} = \frac{I}{4\pi r}(\sin\varphi + \sin\psi)\overrightarrow{a_H} = \frac{10}{4\pi}\left(\sin\frac{\pi}{2} + \sin 0\right)\overrightarrow{a_y} = \frac{10}{4\pi}\overrightarrow{a_y}$$

Napravimo sad istu stvar za 2. segment.





Još samo da odredimo smjer jakosti polja.



$$\cos\frac{\pi}{6}=\frac{a_x}{a_H}=\frac{a_x}{1}\rightarrow a_x=\frac{\sqrt{3}}{2}$$

$$\cos\frac{\pi}{3}=\frac{a_y}{a_H}=\frac{a_y}{1}\rightarrow a_y=\frac{1}{2}$$

$$\overrightarrow{a_H}=a_x\overrightarrow{a_x}-a_y\overrightarrow{a_y}=\frac{\sqrt{3}}{2}\overrightarrow{a_x}-\frac{1}{2}\overrightarrow{a_y}\;(minus\ zato\ jer\ je\ u\ negativnom\ smjeru\ y-osi)$$

Pa je jakost polja za drugi segment:

$$\overrightarrow{H_2}=\frac{I}{4\pi r}(\sin\varphi+\sin\psi)\overrightarrow{a_H}=\frac{10}{4\pi}\left(\sin 0+\sin\frac{\pi}{2}\right)\overrightarrow{a_H}=\frac{10}{4\pi}\left(\frac{\sqrt{3}}{2}\overrightarrow{a_x}-\frac{1}{2}\overrightarrow{a_y}\right)$$

Te ja ukupna jakost polja u točki *P* jednaka:

$$\vec{H}=\overrightarrow{H_1}+\overrightarrow{H_2}=\frac{10}{4\pi}\overrightarrow{a_y}+\frac{10}{4\pi}\left(\frac{\sqrt{3}}{2}\overrightarrow{a_x}-\frac{1}{2}\overrightarrow{a_y}\right)=\frac{5\sqrt{3}}{4\pi}\overrightarrow{a_x}+\frac{5}{4\pi}\overrightarrow{a_y}$$

Pa je jakost polja u smjeru osi *x* jednaka $\frac{\mathbf{5\sqrt{3}}}{\mathbf{4\pi}} = \mathbf{0.69}$.

10. Isti kao 9., samo je drugačiji kut α. I samo to promijenimo pa dobijemo:

$$\overrightarrow{H_2} = \frac{I}{4\pi r}(\sin\varphi + \sin\psi)\overrightarrow{a_H} = \frac{10}{4\pi}\left(\sin 0 + \sin\frac{\pi}{2}\right)\overrightarrow{a_H} = \frac{10}{4\pi}\left(\frac{\sqrt{2}}{2}\overrightarrow{a_x} - \frac{\sqrt{2}}{2}\overrightarrow{a_y}\right)$$

$$\vec{H} = \overrightarrow{H_1} + \overrightarrow{H_2} = \frac{10}{4\pi}\overrightarrow{a_y} + \frac{10}{4\pi}\left(\frac{\sqrt{2}}{2}\overrightarrow{a_x} - \frac{\sqrt{2}}{2}\overrightarrow{a_y}\right) = \frac{5\sqrt{3}}{4\pi}\overrightarrow{a_x} + \frac{10}{4\pi}\left(\frac{2-\sqrt{2}}{2}\right)\overrightarrow{a_y}$$

Pa je jakost polja u smjeru osi *y* jednaka $\frac{\mathbf{10}}{\mathbf{4\pi}}\left(\frac{\mathbf{2-\sqrt{2}}}{\mathbf{2}}\right) = \mathbf{0.23}$.

11. Ovaj zadatak je već riješen pod **8.**, samo što se tamo tražila tangencijalna, a ovdje se traži okomita (normalna) komponenta.

$$\boldsymbol{H_{1n} = 6}$$

12. Pogledaj zadatak **5.** Isti su, samo drugi brojevi.