FER2.net

Elektromagnetska polja

1. M.I. ak. god. 2006./2007.

- skenirani postupci rješavanja, version: 2.0
- navedena rješenja su potvrđena službenom obavješću

by: Tywin

Napomena: sve navedene formule mogu se naći u materijalima. Korištene su:
Formule FER1 OE1
Formule za MI-te by I V A N

Electromagnetska polja
1. M.I. 2006./2007.

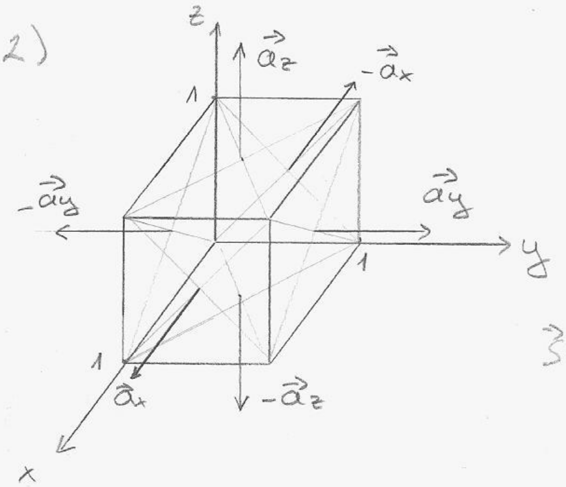
I. $\vec{E} = \frac{A}{x^2+1}\,\vec{a}_x$

1) $\rho(1m, 2m, 3m) = ?$

$$\rho = \varepsilon_0 \nabla \vec{E} = \varepsilon_0 \cdot \frac{\partial}{\partial x}\left(\frac{A}{x^2+1}\right) = \varepsilon_0 A \frac{-2x}{(x^2+1)^2}$$

$$\rho\big|_{x=1} = -\frac{A}{2}\varepsilon_0 \quad \boxed{A}$$

2)



rastav površine S na 6 manjih sa njihovim normalama:

$$S = \begin{cases} x=0,\ 0\le y\le 1,\ 0\le z\le 1 \Rightarrow \vec{n} = -\vec{a}_x \\ x=1,\ 0\le y\le 1,\ 0\le z\le 1 \Rightarrow \vec{n} = \vec{a}_x \\ y=0,\ 0\le x\le 1,\ 0\le z\le 1 \Rightarrow \vec{n} = -\vec{a}_y \\ y=1,\ 0\le x\le 1,\ 0\le z\le 1 \Rightarrow \vec{n} = \vec{a}_y \\ z=0,\ 0\le x\le 1,\ 0\le y\le 1 \Rightarrow \vec{n} = -\vec{a}_z \\ z=0,\ 0\le x\le 1,\ 0\le y\le 1 \Rightarrow \vec{n} = \vec{a}_z \end{cases}$$

$$\vec{E}\cdot\vec{n} = \frac{A}{(x^2+1)}\bigg|_{x=1} + \frac{-A}{x^2+1}\bigg|_{x=0}$$

$$Q = \varepsilon \oiint_S \vec{E}\,\vec{n}\,dS = \varepsilon \iint_{\Omega yz} \frac{A}{x^2+1}\bigg|_{x=1} dy\,dz + \varepsilon\iint_{\Omega yz} \frac{-A}{(x^2+1)}\bigg|_{x=0} dy\,dz$$

$$= \varepsilon_0 \frac{A}{2}\, y\big|_0^1\, z\big|_0^1 - \varepsilon_0 A\, y\big|_0^1\, z\big|_0^1 = \varepsilon_0 \frac{A}{2} - \varepsilon_0 A$$

$$= -\varepsilon_0 \frac{A}{2} \quad \boxed{E}$$

3) $\varphi(3m, 3m, 3m) = ?$

$$\varphi(\vec{r}) = -\int_{r_{ref}}^{r} \vec{E}(\vec{r})\,d\vec{l} = \left\{ \vec{E}(\vec{r}) = \frac{A}{x^2+1}\vec{a}_x;\ \begin{matrix} \vec{l} = x\vec{a}_x + y\vec{a}_y + z\vec{a}_z \\ d\vec{l} = dx\,\vec{a}_x + dy\,\vec{a}_y + dz\,\vec{a}_z \end{matrix} \right\} =$$

$$= -\int_{\infty}^{r} \frac{A}{x^2+1}dx = -A\,\mathrm{arctg}\Big|_{x=\infty}^{3} = -A\left(1,249 - \frac{\pi}{2}\right) = 0,32 \quad \boxed{A}$$

4) radi se o zatvorenoj krivulji pa je ukupni rad jednak nuli $W = 0$ $\boxed{E}$

II $r_1 = 2$ cm

$r_2 = 4$ cm

$\varphi_0 = 200$ V

5) $E(r_1) = ?$

$$\varphi(2\,cm) = 200 = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{0{,}02} - \frac{1}{0{,}04}\right) = \frac{25Q}{4\pi\varepsilon_0}$$

$$Q = 32\pi\varepsilon_0$$

$$E(2\,cm) = \frac{Q}{4\pi\varepsilon_0 (0{,}02)^2} = 20\ kV/m \quad \boxed{C}$$

6) by: quartz, skripta: „Elektrostatika" str. 70.

$$F = \frac{\varepsilon E^2 S}{2} = \frac{1}{2} Q E = 8{,}896\ \mu F \quad \boxed{E}$$

7) $$C = 4\pi\varepsilon_0 \frac{r_1 r_2}{r_2 - r_1} = 4{,}45\ pF \quad \boxed{D}$$

8) $$W = \frac{C \cdot U^2}{2} = 89\ nJ \quad \boxed{B}$$

III) $\varepsilon_{r1} = \varepsilon_{r3} = 1 \quad \varepsilon_{r2} = 3$

$S = 0{,}03\ m^2$

$d = 2$ mm

$\varphi_0 = 100$ V

$$C_1 = C_3 = \varepsilon_0 \cdot \frac{S}{d} = 132{,}81\ pF$$

$$C_2 = \varepsilon_r \cdot C_1 = 398{,}43\ pF$$

$$\Rightarrow \frac{1}{C_{uk}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$U = 100$ V ;

$$C_{uk} = \frac{C_1 C_2 C_3}{C_1 C_2 + C_1 C_3 + C_2 C_3} = 56{,}919\ pF$$

$$Q = U \cdot C_{uk} = 5{,}692\ nC$$

$$U_1 = \frac{Q}{C_1} = U_3 = 42{,}86\ V$$

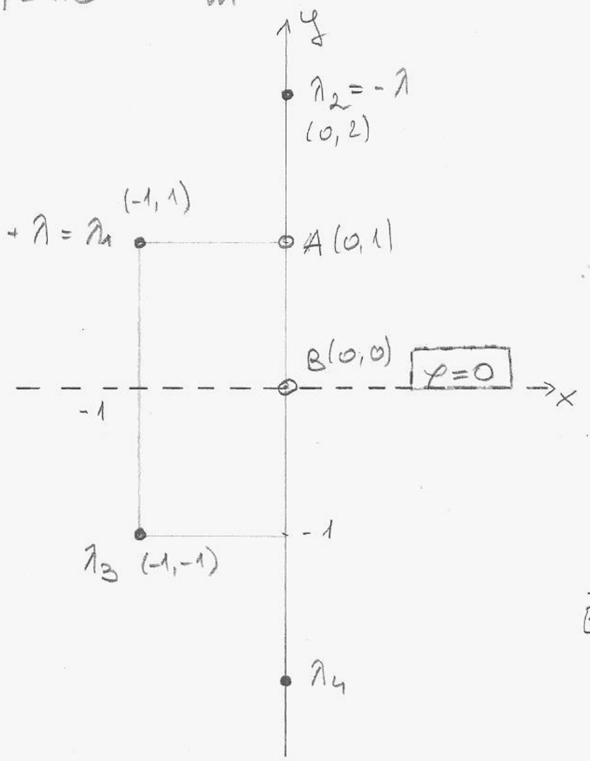
$$U_2 = 14{,}28$$

9) $E_2 = \frac{U_2}{d} = 7{,}14\ kV/m \quad \boxed{C}$ 10) $E_3 = \frac{U_3}{d} = 21{,}43\ kV/m \quad \boxed{E}$

11) $\varphi(d) = U - U_1 = 57{,}14\ V \quad \boxed{D}$

12) $\varphi(d) = 57{,}14 - U_2 = 42{,}86\ V \quad \boxed{E}$

IV $|\lambda| = 10^{-9}$ C/m



→ metodom odslikavanja nastaju λ_3 i λ_4 takvi da vrijedi

$$\lambda_3 = -\lambda_1 = -\lambda$$

$$\lambda_4 = -\lambda_2 = +\lambda$$

naboj λ iz točke $T(x_T, y_T)$ stvara električno polje u točki $A(x_A, y_A)$ iznosa i smjera:

$$\vec{E}(A) = \frac{\lambda}{2\pi\varepsilon} \cdot \frac{(x_A - x_T)\vec{a}_x + (y_A - y_T)\vec{a}_y}{(x_A - x_T)^2 + (y_A - y_T)^2}$$

14) $\vec{E}(A) = \vec{E}_1(A) + \vec{E}_2(A) + \vec{E}_3(A) + \vec{E}_4(A)$

$$\vec{E}_1(A) = \frac{\lambda_1}{2\pi\varepsilon_0}\vec{a}_x = 179{,}85\,\vec{a}_x \text{ V/m}$$

$$\vec{E}_2(A) = \frac{\lambda_2}{2\pi\varepsilon_0}\cdot\vec{a}_y = -179{,}85\,\vec{a}_y \text{ V/m}$$

$$\vec{E}_3(A) = \frac{\lambda_3}{2\pi\varepsilon_0}\left(\frac{\vec{a}_x + 2\vec{a}_y}{5}\right) = -179{,}85\left(\frac{\vec{a}_x}{5} + \frac{2\vec{a}_y}{5}\right) = -35{,}97\,\vec{a}_x - 71{,}94\,\vec{a}_y \text{ V/m}$$

$$\vec{E}_4(A) = \frac{\lambda_4}{2\pi\varepsilon_0}\,\frac{3\vec{a}_y}{9} = 59{,}95\,\vec{a}_y \text{ V/m}$$

$$\vec{E}(A) = 143{,}88\,\vec{a}_x + 167{,}86\,\vec{a}_y \Rightarrow E(A) = 221 \text{ V/m} \quad \boxed{E}$$

13) $q = 10$ nC

$$F(A) = q \cdot E(A) = 2{,}21\ \mu N \quad \boxed{A}$$

(15) $\vec{E}_1(B) = \frac{\lambda_1}{2\pi\varepsilon_0} \cdot \frac{\vec{a}_x - \vec{a}_y}{2} = 89,93\ \vec{a}_x - 89,93\ \vec{a}_y$

$\vec{E}_2(B) = \frac{\lambda_2}{2\pi\varepsilon_0} \cdot \frac{-2\vec{a}_y}{4} = 89,93\ \vec{a}_y$

$\vec{E}_3(B) = \frac{\lambda_3}{2\pi\varepsilon_0} \cdot \frac{-\vec{a}_x - \vec{a}_y}{2} = -89,93\ \vec{a}_x - 89,93\ \vec{a}_y$

$\vec{E}_4(B) = \frac{\lambda_4}{2\pi\varepsilon_0} \cdot \frac{2\vec{a}_y}{4} = 89,93\ \vec{a}_y$

$\vec{E}(B) = 0 \quad \boxed{A}$

(16) $\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r^2}\ \vec{r}$

$U_{AB} = -\int_B^A \vec{E}\, d\vec{l} = \int_A^B E\, dr = \frac{\lambda}{2\pi\varepsilon_0} \cdot \ln r \Big|_{r=A}^{r=B} = \frac{\lambda}{2\pi\varepsilon_0} \ln \frac{r_B}{r_A}$

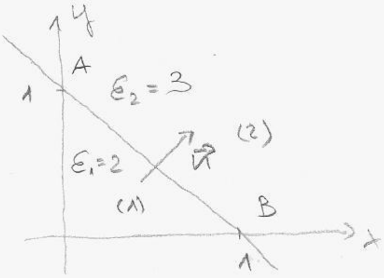
r_B = udaljenost točke A do točke B = 1

r_A = udaljenost točke A do naboja λ

$U_{AB} = \frac{\lambda_1}{2\pi\varepsilon_0} \cdot \ln\frac{1}{1} + \frac{\lambda_2}{2\pi\varepsilon_0} \ln\frac{1}{1} + \frac{\lambda_3}{2\pi\varepsilon_0} \ln\frac{1}{\sqrt{5}} + \frac{\lambda_4}{2\pi\varepsilon_0} \ln\frac{1}{\sqrt{3}}$

$= \frac{\lambda}{2\pi\varepsilon_0}\left[\ln 1 - \ln 1 - \ln\frac{1}{\sqrt{5}} + \ln\frac{1}{\sqrt{3}}\right] = -53\ V \quad \boxed{D}$

V. $\vec{E}_1 = 2\vec{a}_x - 5\vec{a}_y + \vec{a}_z$



$$\cancel{E_{x2} = E_{x1} \cdot \frac{\varepsilon_1}{\varepsilon_2} = 2 \cdot \frac{2}{3} = \frac{4}{3}}$$

$$\vec{n} = \frac{\sqrt{2}}{2}\vec{a}_x + \frac{\sqrt{2}}{2} a_y$$

$$\vec{E}_2 = E_{2x}\vec{a}_x + E_{2y}\vec{a}_y + E_{2z}\vec{a}_z$$

(1) $n \times [\vec{E}_2 - \vec{E}_1] = \vec{0}$

(2) $n \cdot (\vec{D}_2 - \vec{D}_1) = 0$

(1) $\frac{1}{\sqrt{2}}(\vec{a}_x + \vec{a}_y) \times [(E_{2x} - 2)\vec{a}_x + (E_{2y} + 5)\vec{a}_y + (E_{2z} - 1)\vec{a}_z] = 0$

$$\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ E_x - 2 & E_y + 5 & E_z - 1 \end{vmatrix} = \vec{a}_x \frac{1}{\sqrt{2}}(E_z - 1) - \vec{a}_y \frac{1}{\sqrt{2}}(E_z - 1) + \vec{a}_z \frac{1}{\sqrt{2}}(E_y + 5 - E_x + 2)$$

(2) $\frac{1}{\sqrt{2}}(\vec{a}_x + \vec{a}_y)[\vec{a}_x(3E_x - 4) + \vec{a}_y(3E_y + 10) + \vec{a}_z(3E_z - 2)] = 0$

iz (1): $E_z - 1 = 0 \Rightarrow E_z = 1$ $\boxed{19.\ B}$

$E_y + 5 - E_x + 2 = 0$

$\Rightarrow \vec{E}_2 = \frac{5}{2}\vec{a}_x - \frac{9}{2}\vec{a}_y + \vec{a}_z$

iz (2): $\frac{1}{2}[3E_x - 4 + 3E_y + 10] = 0$

$$\left.\begin{array}{l} E_y - E_x = -7 \ /\cdot 3 \\ 3E_x + 3E_y = -6 \end{array}\right| + \ \Bigg\} \ 6E_y = -27$$

$E_y = -\frac{9}{2}$ $\boxed{18.\ C}$

$E_x = E_y + 7 = \frac{5}{2}$ $\boxed{17.\ A}$

20) $U_{AB} = ?$

$$U_{AB} = -\int_B^A E(\vec{r}) \cdot d\vec{l} = E_T \cdot \overline{AB}$$

$\overline{AB} = \sqrt{2}$

$\vec{r}_{ab} = \vec{AB}_0 = \frac{\vec{a}_x - \vec{a}_y}{\sqrt{2}}$ → jedinični vektor od A prema B

$E_{T1} = \vec{E}_1 \cdot \vec{r}_{ab} = \frac{7}{\sqrt{2}}$ $\quad E_{T2} = \vec{E}_2 \cdot \vec{r}_{ab} = \frac{7}{\sqrt{2}}$ $\quad E_T = E_{T1} = E_{T2}$

$U_{AB} = E_T \cdot \overline{AB} = 7\ \text{V}$ $\boxed{B}$