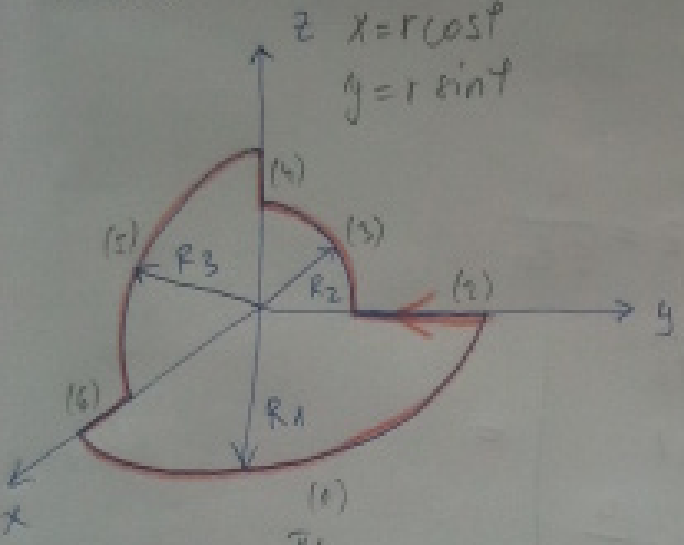
LJETNI ISPITNI ROK
– 2014/15 –

1.) Odredite jakost mag. polja u ishodištu koord. sust. za konturu na slici.



(1) $\vec{r} = -x\vec{a_x} - y\vec{a_y}$

$$\vec{H_1} = \frac{I}{4\pi}\int \frac{(x\,dy - y\,dx)\,\vec{a_z}}{(\sqrt{x^2+y^2})^3}$$

$$\vec{H_1} = \frac{I}{4\pi}\int \frac{(r\cos\varphi \cdot r\cos\varphi - r\sin\varphi(-r\sin\varphi))\,d\varphi\,\vec{a_z}}{(\sqrt{r^2})^3}$$

$$\vec{H_1} = \frac{I}{4\pi R_1}\int_0^{\pi/2} d\varphi\,\vec{a_z} = \frac{I}{4\pi R_1}\cdot\frac{\pi}{2} = \frac{I}{8R_1}\vec{a_z}$$

(3) $\vec{r} = -y\vec{a_y} - z\vec{a_z}$ $\quad y = r\cos\varphi \quad z = r\sin\varphi$

$$\vec{H_3} = \frac{I}{4\pi R_2}\int_0^{\pi/2} d\varphi\,\vec{a_x} = \frac{I}{8R_2}\vec{a_x}$$

(5) $\vec{r} = -x\vec{a_x} - z\vec{a_z}$ $\quad x = r\cos\varphi \quad z = r\sin\varphi$

$$H_5 = \frac{-I}{4\pi R_3}\int_{\pi/2}^{0} d\varphi\,\vec{a_y} = \frac{I}{8R_3}\vec{a_y}$$

(2) $\vec{r} = -y\vec{a_y}$, $d\vec{l} \times \vec{r} = 0 \Rightarrow H_2 = H_4 = H_6 = 0$

$$\vec{H}_{ukl} = \frac{I}{8}\left(\frac{\vec{a_x}}{R_2} + \frac{\vec{a_y}}{R_3} + \frac{\vec{a_z}}{R_1}\right)$$

Zadan je $\vec{D_1} = 2\vec{a_x} - 2\vec{a_y} + 4\vec{a_z}$, $\varepsilon_{r_1} = 2$, $z < 0$.
Odredite $\vec{E_2}$ u $z > 0$, $\varepsilon_{r_2} = 5$ i napon U_{AB} između točaka $A(0,0,1)$ i $B(0,-1,2)$.

$\vec{E_2} = x\vec{a_x} + y\vec{a_y} + z\vec{a_z}$

$\vec{D_2} = \varepsilon_0(5x\vec{a_x} + 5y\vec{a_y} + 5z\vec{a_z})$

$\vec{n_{12}} = -\vec{a_z}$

$\vec{n_{12}} \cdot (\vec{D_2} - \vec{D_1}) = -(5z\varepsilon_0 - 4) = 0 \rightarrow z = \frac{4}{5\varepsilon_0}$

$$\vec{n_{12}} \times (\vec{E_2} - \vec{E_1}) = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 0 & 0 & -1 \\ x - \frac{2}{2\varepsilon_0} & y + \frac{2}{2\varepsilon_0} & z - \frac{4}{2\varepsilon_0} \end{vmatrix} =$$

$$= \vec{a_x}\left(y + \frac{1}{\varepsilon_0}\right) - \vec{a_y}\left(x - \frac{1}{\varepsilon_0}\right) = 0 \rightarrow y = -\frac{1}{\varepsilon_0}, \; x = \frac{1}{\varepsilon_0}$$

$$\vec{E_2} = \frac{1}{\varepsilon_0}\left(\vec{a_x} - \vec{a_y} + \frac{4}{5}\vec{a_z}\right) \quad V/m$$

U_{AB} za $z > 0 \rightarrow \vec{E_2}$

$$U_{AB} = \int \vec{E_2}\, d\vec{l} = \frac{1}{\varepsilon_0}\left[\int_0^0 dx - \int_0^{-1} dy + \frac{4}{5}\int_1^2 dz\right] = \frac{1}{\varepsilon_0} \cdot \frac{9}{5} = \frac{1.8}{\varepsilon_0} \; V$$

3) Potencijal je u dijelu prostora $0 \le x \le a$, $-\infty \le y \le \infty$, $-\infty \le z \le \infty$ zadan jednadžbom. Odredite jakost el. polja i gustoću naboja u dijelu prostora u kojem je zadan potencijal.

$$\varphi(x,y,z) = \frac{\varphi_0}{\pi} e^{-\frac{\pi y}{a}} \sin\left(\frac{\pi x}{a}\right)$$

$$\vec{E} = -\operatorname{grad}\varphi = -\frac{\varphi_0}{\pi}\left[-e^{-\frac{\pi y}{a}} \cdot \frac{\pi}{a}\cos\left(\frac{\pi x}{a}\right)\vec{a_x} - \frac{\pi}{a} e^{-\frac{\pi y}{a}} \sin\left(\frac{\pi x}{a}\right)\vec{a_y}\right]$$

$$= \frac{\varphi_0}{a} e^{-\frac{\pi y}{a}}\left[\cos\left(\frac{\pi x}{a}\right)\vec{a_x} - \sin\left(\frac{\pi x}{a}\right)\vec{a_y}\right] \quad [V/m]$$

$$\Delta\varphi = -\frac{\rho_s}{\varepsilon_0}$$

$$\frac{\partial}{\partial x}\left(\frac{\varphi_0}{a} e^{-\frac{\pi y}{a}} \cos\left(\frac{\pi x}{a}\right)\right) + \frac{\partial}{\partial y}\left(-\frac{\varphi_0}{a} e^{-\frac{\pi y}{a}} \sin\left(\frac{\pi x}{a}\right)\right) = -\frac{\rho_s}{\varepsilon_0}$$

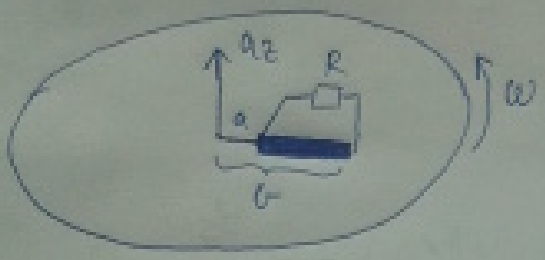
$$-\frac{\varphi_0 \pi}{a^2} e^{-\frac{\pi y}{a}} \sin\left(\frac{\pi x}{a}\right) + \frac{\varphi_0 \pi}{a^2} e^{-\frac{\pi y}{a}} \sin\left(\frac{\pi x}{a}\right) = -\frac{\rho_s}{\varepsilon_0}$$

$$\rho_s = 0$$

) Na kružnom disku prema slici nalaze se kratki vodič $a \le r \le b$ i otpornik otpora $R = 2\ \Omega$ spojeni u strujni krug. Kružni disk rotira brzinom od 120 okretaja u minuti. Odredite inducirani napon na krajevima vodiča i razvijenu snagu na otporniku. Indukcija na kružnom disku je jednolika i iznosi $\vec{B} = 0.6\, a_z^{\rightarrow}\ [T]$, $a = 2\ cm$, $b = 5\ cm$

$$f = \frac{120}{60} = 2$$

$$\omega = 4\pi\ rad/s$$



$$U_{ind} = \int_l (\vec{v} \times \vec{B})\, d\vec{l}$$

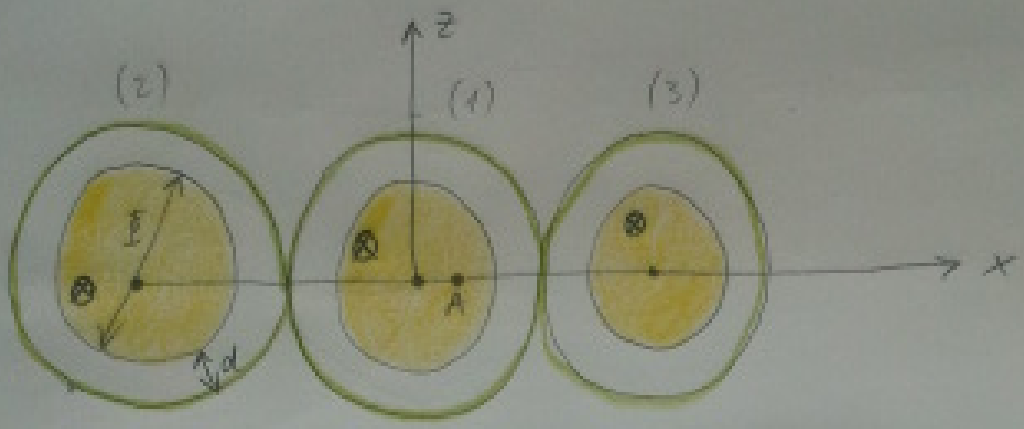
$$v = \omega r\, a_{\varphi}^{\rightarrow} \rightarrow \vec{v} \times \vec{B} = \omega r\, a_{\varphi}^{\rightarrow} \times 0.6\, a_z^{\rightarrow} = 0.6\,\omega r\, a_r^{\rightarrow}$$

$$U_{ind} = \int_l 0.6\,\omega r\, a_r^{\rightarrow} \cdot a_r^{\rightarrow} dr = 0.6\,\omega \int_a^b r\, dr = 0.6 \cdot \frac{\omega}{2}(b^2 - a^2)$$

$$U_{ind} = 0.6 \cdot \frac{4\pi}{2}(5^2 - 2^2) \cdot 10^{-4} = \underline{7.92\ mV}$$

$$P = U \cdot I = \frac{U^2}{R} = \frac{(7.92 \cdot 10^{-3})^2}{2} = \underline{3.14 \cdot 10^{-5}\ W}$$

5) Zadana su tri jednaka elektroenergetska kabela promjera vodiča $\Phi = 10\ cm$ i debljine izolacije $d = 2.5\ cm$ kojima teku jednake istosmjerne struje $I = 120\ A$. Odredite jakost mag. polja u točki $A(2.5\ cm, 0, 0)$ prema slici.



$$\oint_C \vec{H}\, d\vec{l} = I$$

$$H_1 \cdot 2r\pi = I_{obuh.} \qquad \frac{I_{obuh.}}{r^2\pi} = \frac{I}{\left(\frac{\Phi}{2}\right)^2\pi}$$

$$H_1 \cdot 2r\pi = I \cdot \frac{r^2}{\Phi^2} \rightarrow H_1 = I \cdot \frac{r}{2\pi\left(\frac{\Phi}{2}\right)^2} = 120 \cdot \frac{0.025}{2\pi \cdot (0.05)^2}$$

$$H_1 = 190.986\ A/m$$

$$H_2 \cdot 2r\pi = I \rightarrow H_2 = \frac{I}{2r\pi} = \frac{120}{2\pi \cdot (0.1 + 2 \cdot 0.025 + 0.025)}$$

$$H_2 = 109.135\ A/m$$

$$H_3 \cdot 2r\pi = I \Rightarrow H_3 = \frac{I}{2\pi\,(0.05 + 2 \cdot 0.025 + 0.05 - 0.025)} = 152.789$$

$$\vec{H}_{uk} = (H_1 + H_2 - H_3)(-\vec{a}_z) = \underline{-147.332\ \vec{a}_z\ A/m}$$

→ prema pravilu desne ruke → H_3 u smjeru točke A ima smjer $+\vec{a}_z$

→ H_1 i H_2 imaju smjer $-\vec{a}_z$