ELEKTROMAGNETSKA POLJA

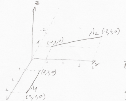
by Tguin

-> WIKIPEDIA -> DEL IN CYLINDRICAL AND SPHERICAL COORDINATES
-> BY IVAN
-> OGROMNA TABLICA INTEGRALA

$$\vec{E} = \frac{\vec{D}}{\varepsilon_0 \varepsilon_r}$$

$\vec{D}$ -> gustoća električnog toka

$\varepsilon_0 = 8.854 \cdot 10^{-12}$



$\lambda_1 = 10\,nC/m$
$\lambda_2 = 15\,nC/m$ } konst.

$\vec{E}(0,0,5) = ?$

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_l \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|} \lambda(\vec{r})\, dl$$

$\vec{r} = 0\vec{a_x} + 0\vec{a_y} + 5\vec{a_z}$

$\vec{r} = 5\vec{a_z}$

$\vec{r}_1' = x\vec{a_x} + 1\vec{a_y}$

$\vec{r}_2' = x\vec{a_x} + y\vec{a_y}$

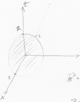
$$\vec{E_1} = \frac{10^{-8}}{4\pi\varepsilon_0} \int_1^3 \frac{-x\vec{a_x} - 1\vec{a_y} + 5\vec{a_z}}{\sqrt{x^2+1^2+5^2}^{\,3}}\, dx$$

mijenja se samo po x-u

$$\vec{E_2} = \frac{15 \cdot 10^{-9}}{4\pi\varepsilon_0} \int_l \frac{-x\vec{a_x} - y\vec{a_y} + 5\vec{a_z}}{\sqrt{x^2+y^2+5^2}^{\,3}}\, dl$$

$$\left.\begin{matrix}(-1,1,0)\\(-2,4,0)\end{matrix}\right\} \frac{x+1}{-1} = \frac{y-1}{3} = \frac{z-0}{0}$$

$$\left.\begin{matrix}x = -t-1\\ y = 3t+1\\ z = 0\end{matrix}\right\} dl = \sqrt{(x')^2+(y')^2+(z')^2} \cdot dt$$



disk u XZ

$r = 2\,m$

$\sigma = \frac{4}{r^2}\,nC/m^2$

$\vec{E}(0,10,0) = ?$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\iint_S \frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}\,\sigma(\vec{r}\,')\,ds$$

$\vec{r} = 10\vec{a}_y$

$\vec{r}\,' = x\vec{a}_x + z\vec{a}_z$

vidi se iz slike (na točku na y osi)

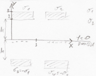
$$E = \frac{1}{4\pi\varepsilon_0}\iint_S \frac{\cancel{-x\vec{a}_x} + 10\vec{a}_y \cancel{- z\vec{a}_z}}{\sqrt{x^2+100+z^2}^{\,3}} \cdot \frac{4}{r^2}\,dx\,dz$$

$z = r\cos\varphi$

$x = r\sin\varphi$

$y = y$

$dx\,dz = r\,dr\,d\varphi$



$\sigma_1 = 10\,nC/m^2$

$\sigma_2 = -20\,nC/m^2$

$\vec{r} = 3\vec{a}_x$

$\vec{r}\,' = x\vec{a}_x + y\vec{a}_y$

$$\vec{E}_1 = \frac{10^{-8}}{4\pi\varepsilon}\int_2^3 dy \int_0^3 \frac{(3-x)\vec{a}_x - y\vec{a}_y}{\sqrt{(3-x)^2+y^2}^{\,3}}\,dx \quad \left| \begin{matrix} t = 3-x \\ dt = -dx \end{matrix} \right|$$

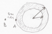
GAUSSOV ZAKON

$$\nabla \cdot \vec{D} = \rho_s$$

*na cijeloj površini polje istog iznosa

$$\oint_S \vec{D}\vec{n}\, ds = \iiint_V \rho_s\, dV$$

$$\varepsilon_0 \varepsilon_r \cdot \vec{E} \oint_S ds = \sum Q$$



$R_1 = 2m$
$R_2 = 3m$
$\rho = \frac{10}{r^2}\ \mu C/m^3$

1) $R < R_1$, $\sum Q = 0$

$E = 0$

2) $R_1 < R < R_2$,

$$\varepsilon_0 \varepsilon_r E \cdot 4R^2\pi = \iiint \rho\, r^2 \sin\theta\, dr\, d\theta\, d\phi$$

POVRŠINA SFERE

$$= \int_0^{\pi} \sin\theta\, d\theta \int_0^{2\pi} d\phi \int_2^{R} \frac{10^{-5}}{r^2} \cdot r^2\, dr$$

$$\boxed{E = \frac{R-2}{\varepsilon_0 \varepsilon_r R^2} \cdot 10^{-5}}$$

3) $R > R_2$,

$$\varepsilon_0 \varepsilon_r E \cdot 4R^2\pi = 4\pi \int_2^3 10^{-5}\, dr$$

$$\boxed{E = \frac{10^{-5}}{\varepsilon_0 \varepsilon_r R^2}}$$

$$\vec{E} = \frac{10^{-5}}{\varepsilon_0 \varepsilon_r R^2} \cdot \vec{a_r}$$

$U_{AB} = ?$

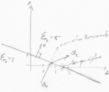
$U_{AB} = \varphi_A - \varphi_B$

$$U_{AB} = -\int_B^A \vec{E}\, d\vec{l}$$

SAMO AKO JE $\vec{E}$ FUNKCIJA

$d\vec{l} = dr\, \vec{a_r}$

$$U_{AB} = -\int_5^3 \frac{10^{-5}}{\varepsilon_0 r^2}\, dr$$



$\vec{E_1} = 2\vec{a_x} - 3\vec{a_y} + 2\vec{a_z}$

$\vec{E_2} = ?$

$\vec{E_2} = E_x\vec{a_x} + E_y\vec{a_y} + E_z\vec{a_z}$

$\vec{D_1} = \varepsilon_0\varepsilon_r\vec{E_1}$

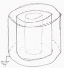
$\vec{D_2} = \varepsilon_0\varepsilon_r\vec{E_2}$

$\vec{n}(\vec{D_2} - \vec{D_1}) = \sigma = 0$ → u većini slučajeva

$\vec{n} \times (\vec{E_2} - \vec{E_1}) = 0$

$\vec{n}$ → jedinični, gleda iz područja 1 u 2

$$\left.\begin{matrix}(1,0,0)\\(0,0,3)\\(1,1,0)\end{matrix}\right\} \vec{n} = \frac{3\vec{a_x} + 1\vec{a_z}}{\sqrt{10}}$$



* na granici postoji samo tangencijalna komponenta

$Q = 20\,nC$

$R_1 = 0.1\,m$

$R_2 = 0.3\,m$

$R_3 = 0.8\,m$

$\varepsilon_{r_1} = 2$

$\varepsilon_{r_2} = 5$

\# samo normala

$E_1 \neq E_2$

$D_1 = D_2$

$C' = ?$ [kapacitet po jedinici duljine]

$C' = \frac{C}{l} = \frac{Q}{Ul}$

$\oiint_S \vec{E}\,d\vec{s} = E_P \cdot S_P + \cancel{E_B \cdot S_B} \to 0$

$\varepsilon_0\varepsilon_r E \cdot 2R\pi \cdot l = Q$

$D = \frac{Q}{2R\pi l}$

$E_1 = E_2$

$\frac{Q_1}{2\varepsilon_0\varepsilon_{r_1}R\pi l} = \frac{Q_2}{2\varepsilon_0\varepsilon_{r_2}R\pi l}$

$Q_1 + Q_2 = Q$

$\frac{Q_1}{\varepsilon_{r_1}} = \frac{Q_2}{\varepsilon_{r_2}}$

$U_1 = -\int_{0.8}^{0.3} E_1\,dR$

$\vec{E} = -\nabla \varphi$

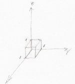
$\varphi = \frac{2}{x^2+4}$

$E(1,1,1)$

$\vec{E} = -\frac{\partial}{\partial x}\left(\frac{2}{x^2+4}\right)\vec{a_x}$ $\qquad$ $\mathrm{rot}\, E = 0$

$= \frac{4x}{(x^2+4)^2}$

$E(1,1,1) = \frac{4}{25}$



$\nabla \cdot \vec{D} = \rho_s \Rightarrow \oiint \vec{D}\, d\vec{s} = \iiint_V \rho\, dV$

$\rho_s =$

RAD

$q = 2nC$ iz $B(0,0,0)$ u $A(1,1,1)$

$W = W_{EA} - W_{EB}$

$W_{EA} = -\int_{\infty}^{A} q\, \vec{E}\, d\vec{l}$

$\boxed{W = -\int_{B}^{A} q\, \vec{E}\, d\vec{l}}$ $\qquad$ $d\vec{l} = dx\vec{a_x} + dy\vec{a_y} + dz\vec{a_z}$

$W = -\int_{0}^{1} q\, \vec{E}\, d\vec{x}$

$W = \left[\varphi_{(1,1,1)} - \varphi_{(0,0,0)}\right] \cdot q$