NABOJI U NEJEDNOLIKOM GIBANJU – ELEKTRODINAMIKA

- **Faradayev zakon**: $e=\oint_c \vec{E}_{ind}\cdot d\vec{l}=-\frac{d}{dt}\iint_S \vec{B}\cdot\vec{n}\,dS$
- Stokesov teorem
$$\iint_S(\nabla\times\vec{E}_{ind})\cdot\vec{n}\,dS=-\iint_S\frac{\partial\vec{B}}{\partial t}\cdot\vec{n}\,dS+\iint_S\nabla\times(\vec{v}\times\vec{B})\cdot\vec{n}\,dS$$
- Transformator
$$u_1=N_1\frac{d\Phi}{dt}\quad;\quad u_2=N_2\frac{d\Phi}{dt}\qquad i_2=i_1\frac{N_1}{N_2}$$
$$\frac{u_1}{u_2}=\frac{N_1}{N_2}\quad;\quad u_2=u_1\frac{N_2}{N_1}\qquad p_1=u_1i_1$$
$$N_1i_1=N_2i_2\quad;\quad\frac{i_1}{i_2}=\frac{N_2}{N_1}\qquad p_2=u_2i_2=u_1\frac{N_2}{N_1}i_1\frac{N_1}{N_2}=p_1$$

Inducirani napon uslјed samoindukcije i međuindukcije

- Ako petlja ima *N* zavoja inducirana elektromotorna sila zbog promjene vlastitog magnetskog toka je:
$$e=\oint_c\vec{E}_{ind}\cdot d\vec{l}=-N\frac{d\Phi}{dt}=-\frac{d\psi}{dt}=-L\frac{di}{dt}$$
- Ako petlja (1) ima N_1 zavoja, zbog promjene magnetskog toka stvorenog strujom i_2 u susjednoj petlji (2) inducirana elektromotorna sila je:
$$e_1=\oint_{c_1}\vec{E}_{ind1}\cdot d\vec{l}_1=-N_1\frac{d\Phi_{12}}{dt}=-\frac{d\psi_{12}}{dt}=-L_{12}\frac{di_2}{dt}$$

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- Ampèreov kružni zakon
$$\nabla\times\vec{H}=\vec{J}\quad\Rightarrow\quad\oint_{c_S}\vec{H}\cdot d\vec{l}=\iint_S\vec{J}\cdot\vec{n}\,dS$$

Maxwellove jednadžbe

- Coulombov zakon: $\nabla\cdot\vec{D}=\rho_S\quad;\quad\oiint_S\vec{D}\cdot\vec{n}\,dS=\iiint_V\rho_S\,dV$
- Faradayev zakon: $\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\quad;\quad\oint_c\vec{E}\cdot d\vec{l}=-\frac{d}{dt}\iint_S\vec{B}\cdot\vec{n}\,dS$
- Gaussov zakon za magnetsko polje:
$$\nabla\cdot\vec{B}=0\quad;\quad\oiint_S\vec{B}\cdot\vec{n}\,dS=0$$
- Ukupna utrošena snaga jest: $P=\iiint_V\vec{J}\cdot\vec{E}\,dV$

- Uvodimo Poyntingov vektor $\vec{N}=\vec{E}\times\vec{H}$

- Interpretacija
$$\iiint_V(\nabla\cdot\vec{N})\,dV=\oiint_S\vec{N}\cdot\vec{n}\,dS$$
kao tijeka snage iz volumena *V* ograničenog plohom *S* uvijek daje točan odgovor

Potencijali u dielektriku bez izvora

- Neograničeni prostor ispunjen materijalom u mirovanju: $\rho_s=0$, $\vec{J}_s=0$, $\varepsilon,\mu,\kappa=0$
- Maxwellove jednadžbe:
$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\qquad\nabla\cdot\vec{D}=0$$
$$\nabla\times\vec{H}=\frac{\partial\vec{D}}{\partial t}\qquad\nabla\cdot\vec{B}=0$$

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Potencijali u vodljivom materijalu s neovisnim izvorima

- Neovisni izvori $\vec{J}_s$ i ρ_s
- Električno polje uzrokuje struju $\kappa\vec{E}$
- Maxwellove jednadžbe:
$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\qquad\nabla\cdot\vec{D}=\rho_s$$
$$\nabla\times\vec{H}=\vec{J}_s+\kappa\vec{E}+\frac{\partial\vec{D}}{\partial t}\qquad\nabla\cdot\vec{B}=0$$

Integralno predstavljanje potencijala

- Zadana raspodjela izvora
- Želimo naći rješenje u neograničenom prostoru
- Skalarni električni potencijal točkastog naboja
  - Miruje u ishodištu
  - Zadana vremenska promjena $q_s=q(t)$
  - Za sve $r>0$ vrijedi
$$\Delta\varphi-\mu\varepsilon\frac{\partial^2\varphi}{\partial t^2}=\Delta\varphi-\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}=0\quad;\quad c^2=\frac{1}{\mu\varepsilon}$$
- U sfernom koordinatnom sustavu je:
$$\Delta\varphi=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right)+\frac{1}{r^2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial\varphi}{\partial\vartheta}\right)+\frac{1}{r^2\sin^2\vartheta}\frac{\partial^2\varphi}{\partial\alpha^2}$$
- Pa jednadžba za φ prelazi u
$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\varphi}{\partial r}\right)-\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}=0\quad\Rightarrow\quad\varphi=\frac{g\left(t-\frac{r}{c}\right)}{r}+\frac{h\left(t+\frac{r}{c}\right)}{r}$$
- Član $g\left(t-\frac{r}{c}\right)$ je izlazeći val koji vremenski zaostaje (*retardirani potencijal*)
- Član $h\left(t+\frac{r}{c}\right)$ je ulazeći val koji vremenski prethodi (*avanzirani potencijal*)

Sinusno promjenjiva polja

- Sve veličine polja su vremenski sinusno promjenjive:
$$\vec{E}(\vec{r},t)=E_x(\vec{r})\cos[\omega_0t+\psi_x(\vec{r})]\vec{a}_x+E_y(\vec{r})\cos[\omega_0t+\psi_y(\vec{r})]\vec{a}_y+E_z(\vec{r})\cos[\omega_0t+\psi_z(\vec{r})]\vec{a}_z$$
- Vrijedi: $A\cos(\omega t+\psi)=\mathrm{Re}\{Ae^{j\psi}e^{j\omega t}\}=\mathrm{Re}\{\underline{A}e^{j\omega t}\}$
- $\underline{A}$ je kompleksni broj, *FAZOR*, pa je:
$$\vec{E}(\vec{r},t)=\mathrm{Re}\left\{\left[E_x(\vec{r})e^{j\psi_x(\vec{r})}\vec{a}_x+E_y(\vec{r})e^{j\psi_y(\vec{r})}\vec{a}_y+E_z(\vec{r})e^{j\psi_z(\vec{r})}\vec{a}_z\right]e^{j\omega_0t}\right\}=\mathrm{Re}\{\underline{\vec{E}}(\vec{r})e^{j\omega_0t}\}$$
- Preslikavanje iz vremenske u kompleksnu domenu
- Maxwellove jednadžbe u fazorskoj domeni
$$\nabla\times\underline{\vec{E}}=-j\omega\underline{\vec{B}}\qquad\nabla\cdot\underline{\vec{D}}=\underline{\rho}_s$$
$$\nabla\times\underline{\vec{H}}=\underline{\vec{J}}+j\omega\underline{\vec{D}}\qquad\nabla\cdot\underline{\vec{B}}=0$$
- EM potencijali u fazorskoj domeni
$$\underline{\vec{B}}=\nabla\times\underline{\vec{A}}\quad;\quad\underline{\vec{E}}=-\nabla\underline{\varphi}-j\omega\underline{\vec{A}}$$

Energija i snaga u sinusno promjenjivim poljima

- Zanimaju nas srednje, a ne trenutne vrijednosti
$$\vec{E}=E_x\cos(\omega_0t+\psi_{E_x})\vec{a}_x+E_y\cos(\omega_0t+\psi_{E_y})\vec{a}_y+E_z\cos(\omega_0t+\psi_{E_z})\vec{a}_z$$
$$\vec{H}=H_x\cos(\omega_0t+\psi_{H_x})\vec{a}_x+H_y\cos(\omega_0t+\psi_{H_y})\vec{a}_y+H_z\cos(\omega_0t+\psi_{H_z})\vec{a}_z$$
  - gdje su E_x, E_y, E_z, H_x, H_y i H_z maksimalne vrijednosti
- Srednja vrijednost Poyntingova vektora:
$$\vec{N}_{sr}=\frac{1}{T}\int_0^T(\vec{E}\times\vec{H})\,dt$$

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- Slijedi:

$$\vec{N}_{sr} = \mathrm{Re}\left\{\frac{1}{2}\left(\underline{\vec{E}} \times \underline{\vec{H}}^*\right)\right\}$$

- $\underline{\vec{H}}^*$ je konjugirano kompleksni fazor $\underline{\vec{H}}$

$$\underline{\vec{H}}^* = H_x e^{-j\psi_{H_x}} \vec{a}_x + H_y e^{-j\psi_{H_y}} \vec{a}_y + H_z e^{-j\psi_{H_z}} \vec{a}_z$$

- Srednji gubici jesu:

$$P_{g,sr} = \frac{1}{2\kappa} \iiint_V \left|\underline{\vec{J}}\right|^2 dV = \frac{1}{2} \iiint_V \underline{\vec{E}} \cdot \underline{\vec{J}}^* dV \quad \text{jer je:} \quad \frac{1}{T}\int_0^T \cos^2(\omega t + \psi) dt = \frac{1}{2}$$

- Srednja energija pohranjena u magnetskom polju jest:

$$W_{m,sr} = \frac{\mu}{2}\frac{1}{T}\int_0^T dt \iiint_V \left| H_x \cos\left(\omega t + \psi_{H_x}\right)\vec{a}_x + H_y \cos\left(\omega t + \psi_{H_y}\right)\vec{a}_y + H_z \cos\left(\omega t + \psi_{H_z}\right)\vec{a}_z \right|^2 dV$$

$$= \frac{\mu}{4} \iiint_V \left|\underline{\vec{H}}\right|^2 dV = \frac{\mu}{4} \iiint_V \underline{\vec{H}} \cdot \underline{\vec{H}}^* dV$$

- Srednja energija pohranjena u električnom polju jest:

$$W_{e,sr} = \frac{\varepsilon}{2}\frac{1}{T}\int_0^T dt \iiint_V \left| E_x \cos\left(\omega t + \psi_{E_x}\right)\vec{a}_x + E_y \cos\left(\omega t + \psi_{E_y}\right)\vec{a}_y + E_z \cos\left(\omega t + \psi_{E_z}\right)\vec{a}_z \right|^2 dV =$$

$$= \frac{\varepsilon}{4} \iiint_V \left|\underline{\vec{E}}\right|^2 dV = \frac{\varepsilon}{4} \iiint_V \underline{\vec{E}} \cdot \underline{\vec{E}}^* dV$$

- Kompleksna relacija za Poyntingov teorem

$$P_N = \frac{1}{2} \oint\!\!\!\oint_S \underline{\vec{N}} \cdot \vec{n}\, dS = -P_{g,sr} + j2\omega\left(W_{e,sr} - W_{m,sr}\right)$$

- P_N nazivamo prividnom srednjom snagom

ELEKTROMAGNETSKI VALOVI U SREDSTVIMA BEZ GUBITAKA

- U određenom trenutku t_0 je prostorni period ponavljanja

$$\lambda = \frac{2\pi}{\omega\sqrt{\mu\varepsilon}}$$

- Argumente kosinusa zovemo faze. Promatrač se mora gibati brzinom $c = \frac{1}{\sqrt{\mu\varepsilon}}$ da bi slijedio istu fazu.

Tu brzinu zovemo i fazna brzina $v_f = c = \frac{1}{\sqrt{\mu\varepsilon}}$

- Slijedi: $v_f = \lambda f$
- Veličinu $\beta = \omega\sqrt{\mu\varepsilon}$ zovemo fazna konstanta. Ona je mjera brzine promjene faze sa udaljenošću u određenom trenutku! Vrijedi: $\lambda = \frac{2\pi}{\beta} \;\; ; \;\; v_f = \frac{\omega}{\beta}$
- Vektor jakosti električnog polja ravnog vala koji se širi u smjeru $\vec{\beta}$ jest:

$$\vec{E} = \vec{E}_0 \cos\left(\omega t - \vec{\beta}\cdot\vec{r} + \varphi\right)$$

- U fazorskom obliku pišemo $\underline{\vec{E}} = \vec{E}_0 e^{j\varphi} e^{-j\vec{\beta}\cdot\vec{r}} = \underline{\vec{E}}_0 e^{-j\vec{\beta}\cdot\vec{r}}$
- Budući da je E u poprečnoj (transverzalnoj) ravnini slijedi: $\vec{E}\cdot\vec{\beta} = \vec{E}_0\cdot\vec{\beta} = \underline{\vec{E}}_0\cdot\vec{\beta} = 0$
- Analogno: $\vec{H} = \vec{H}_0 \cos\left(\omega t - \vec{\beta}\cdot\vec{r} + \varphi\right) \;\; ; \;\; \underline{\vec{H}} = \vec{H}_0 e^{j\varphi} e^{-j\vec{\beta}\cdot\vec{r}} = \underline{\vec{H}}_0 e^{-j\vec{\beta}\cdot\vec{r}}$
- H je u transverzalnoj ravnini: $\vec{H}\cdot\vec{\beta} = \vec{H}_0\cdot\vec{\beta} = \underline{\vec{H}}_0\cdot\vec{\beta} = 0$
- Vrijedi:

$$\frac{E_0}{H_0} = Z = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \frac{\omega\mu}{\beta} \;\Rightarrow\; \vec{H}_0 = \frac{1}{\omega\mu}\vec{\beta}\times\vec{E}_0 \;\; ; \;\; \vec{H} = \frac{1}{\omega\mu}\vec{\beta}\times\vec{E}$$

- Valne dužine i fazne brzine u smjerovima osi su

$$\lambda = \frac{2\pi}{\beta} \;\Rightarrow\; \lambda_x = \frac{2\pi}{\beta_x},\ \lambda_y = \frac{2\pi}{\beta_y},\ \lambda_z = \frac{2\pi}{\beta_z}$$

$$v_f = \frac{\omega}{\beta} \;\Rightarrow\; v_{fx} = \frac{\omega}{\beta_x},\ v_{fy} = \frac{\omega}{\beta_y},\ v_{fz} = \frac{\omega}{\beta_z}$$

ELEKTROMAGNETSKI VALOVI U REALNIM DIELEKTRICIMA I VODIČIMA

- α je prigušna konstanta, β je fazna konstanta, a γ je konstanta prostiranja

$$\gamma^2 = -\omega^2\mu\varepsilon + j\omega\mu\kappa \;\Rightarrow$$

$$\alpha = \frac{\omega}{\sqrt{2}c}\sqrt{\sqrt{1+\left(\frac{\kappa}{\omega\varepsilon}\right)^2} - 1} \;\; ; \;\; \beta = \frac{\omega}{\sqrt{2}c}\sqrt{\sqrt{1+\left(\frac{\kappa}{\omega\varepsilon}\right)^2} + 1} \;\; ; \;\; c = \frac{1}{\sqrt{\mu\varepsilon}}$$

- Budući da dva člana u rješenju predstavljaju direktni i inverzni val pišemo $\underline{E}_x(z) = \underline{E}_x^+ e^{-\gamma z} + \underline{E}_x^- e^{\gamma z}$
- Odgovarajuće rješenje za H jest:

$$\nabla\times\underline{\vec{E}} = \frac{\partial\underline{\vec{E}}}{\partial z}\vec{a}_y = -j\omega\mu\underline{\vec{H}} \;\Rightarrow\; \underline{\vec{H}} = \underline{H}_y(z)\vec{a}_y = \frac{1}{Z}\left(\underline{E}_x^+ e^{-\gamma z} - \underline{E}_x^- e^{\gamma z}\right)\vec{a}_y$$

- Z je valna impedancija

$$Z = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\alpha + j\beta} = \frac{j\omega\mu(\alpha - j\beta)}{\alpha^2+\beta^2} = \frac{\omega\mu}{\sqrt{\alpha^2+\beta^2}} e^{j\arctan\frac{\alpha}{\beta}}$$

- Električno i magnetsko polje nisu u fazi, magnetsko polje zaostaje za $0^\circ \le \varphi \le 45^\circ$
- Fazna brzina:

$$v_f = \frac{\omega}{\beta} = \pm\sqrt{2}c\left(\sqrt{1+\left(\frac{\kappa}{\omega\varepsilon}\right)^2} + 1\right)^{-\frac{1}{2}}$$

- Dubina prodiranja – udaljenost na kojoj se amplituda polja vala priguši na 1/e odnosno na 36,8 % početne vrijednosti

$$d = \frac{1}{\alpha} = \frac{\sqrt{2}c}{\omega}\left(\sqrt{1+\left(\frac{\kappa}{\omega\varepsilon}\right)^2} - 1\right)^{-\frac{1}{2}}$$

- Srednja snaga:

$$N_{\mathrm{Re},sr} = \frac{1}{2}\frac{E_0^2}{|Z|} e^{-2\alpha z}\cos\varphi$$ gdje je E_0 amplitudna vrijednost

- Razmotrimo dva posebna slučaja:
  - Dobri vodiči: $\frac{\kappa}{\omega\varepsilon} >> 1 \;\Rightarrow\; \sqrt{1+\left(\frac{\kappa}{\omega\varepsilon}\right)^2} \approx \frac{\kappa}{\omega\varepsilon}$

$$\alpha = \beta = \sqrt{\frac{\omega\mu\kappa}{2}} \;\; ; \;\; \gamma = (1+j)\sqrt{\frac{\omega\mu\kappa}{2}} \;\; ; \;\; Z = (1+j)\sqrt{\frac{\omega\mu}{2\kappa}} = \sqrt{\frac{\omega\mu}{\kappa}} e^{j\frac{\pi}{4}}$$

$$v_f = \frac{\omega}{\beta} = \sqrt{\frac{2\omega}{\mu\kappa}} \;\; ; \;\; \lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{\frac{\omega\mu\kappa}{2}}}$$

$$\vec{N}_{sr} = \frac{1}{2}\left(\underline{\vec{E}}\times\underline{\vec{H}}^*\right) = \frac{1}{2}\left|\underline{H}_0\right|^2 e^{-2\alpha z}(1+j)\sqrt{\frac{\omega\mu}{2\kappa}}\,\vec{a}_z$$

  - Dobri izolatori: $\frac{\kappa}{\omega\varepsilon} << 1 \;\Rightarrow\; \sqrt{1+\left(\frac{\kappa}{\omega\varepsilon}\right)^2} \approx 1 + \frac{1}{2}\left(\frac{\kappa}{\omega\varepsilon}\right)^2$

$$\alpha = \frac{\kappa}{2}\sqrt{\frac{\mu}{\varepsilon}} \;\; ; \;\; \beta = \frac{\omega}{c} \;\; ; \;\; \gamma = \frac{\kappa}{2}\sqrt{\frac{\mu}{\varepsilon}} + j\frac{\omega}{c} \;\; ; \;\; Z = \sqrt{\frac{\mu}{\varepsilon}}$$

$$v_f = c \;\; ; \;\; \lambda = \frac{c}{f}$$

- Površinski učinak (skin effect)
  - Val koji polazi od površine dobrog vodiča se vrlo brzo priguši, pa je polje kvazistatičko
  - Dubina prodiranja $d = \frac{1}{\alpha} = \sqrt{\frac{2}{\omega\mu\kappa}}$