Table with the del operator in cylindrical, spherical and parabolic cylindrical coordinates

Operation	Cartesian coordinates (x,y,z)	Cylindrical coordinates (ρ,φ,z)	Spherical coordinates (r,θ,φ)
Definition of coordinates	$\begin{aligned} \rho &= \sqrt{x^2+y^2} \\ \phi &= \arctan(y/x) \\ z &= z \end{aligned}$	$\begin{aligned} x &= \rho\cos\phi \\ y &= \rho\sin\phi \\ z &= z \end{aligned}$	$\begin{aligned} x &= r\sin\theta\cos\phi \\ y &= r\sin\theta\sin\phi \\ z &= r\cos\theta \end{aligned}$
	$\begin{aligned} r &= \sqrt{x^2+y^2+z^2} \\ \theta &= \arccos(z/r) \\ \phi &= \arctan(y/x) \end{aligned}$	$\begin{aligned} r &= \sqrt{\rho^2+z^2} \\ \theta &= \arctan(\rho/z) \\ \phi &= \phi \end{aligned}$	$\begin{aligned} \rho &= r\sin(\theta) \\ \phi &= \phi \\ z &= r\cos(\theta) \end{aligned}$
Definition of unit vectors	$\begin{aligned} \hat{\boldsymbol{\rho}} &= \frac{x}{\sqrt{x^2+y^2}}\hat{\mathbf{x}} + \frac{y}{\sqrt{x^2+y^2}}\hat{\mathbf{y}} \\ \hat{\boldsymbol{\phi}} &= -\frac{y}{\sqrt{x^2+y^2}}\hat{\mathbf{x}} + \frac{x}{\sqrt{x^2+y^2}}\hat{\mathbf{y}} \\ \hat{\mathbf{z}} &= \hat{\mathbf{z}} \end{aligned}$	$\begin{aligned} \hat{\mathbf{x}} &= \cos\phi\hat{\boldsymbol{\rho}} - \sin\phi\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} &= \sin\phi\hat{\boldsymbol{\rho}} + \cos\phi\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} &= \hat{\mathbf{z}} \end{aligned}$	$\begin{aligned} \hat{\mathbf{x}} &= \sin\theta\cos\phi\hat{\boldsymbol{r}} + \cos\theta\cos\phi\hat{\boldsymbol{\theta}} - \sin\phi\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{y}} &= \sin\theta\sin\phi\hat{\boldsymbol{r}} + \cos\theta\sin\phi\hat{\boldsymbol{\theta}} + \cos\phi\hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} &= \cos\theta\hat{\boldsymbol{r}} - \sin\theta\hat{\boldsymbol{\theta}} \end{aligned}$
	$\begin{aligned} \hat{\mathbf{r}} &= \frac{x^3+y^2}{\sqrt{x^2+y^2+z^2}\sqrt{x^2+y^2}}\hat{\mathbf{x}} + \frac{x^2+y^3}{\sqrt{x^2+y^2+z^2}\sqrt{x^2+y^2}}\hat{\mathbf{y}} + \frac{z}{\sqrt{x^2+y^2+z^2}}\hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} &= \frac{xz}{\sqrt{x^2+y^2+z^2}\sqrt{x^2+y^2}}\hat{\mathbf{x}} + \frac{yz}{\sqrt{x^2+y^2+z^2}\sqrt{x^2+y^2}}\hat{\mathbf{y}} - \frac{\sqrt{x^2+y^2}}{\sqrt{x^2+y^2+z^2}}\hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} &= -\frac{y}{\sqrt{x^2+y^2}}\hat{\mathbf{x}} + \frac{x}{\sqrt{x^2+y^2}}\hat{\mathbf{y}} \end{aligned}$	$\begin{aligned} \hat{\mathbf{r}} &= \frac{\rho}{\sqrt{\rho^2+z^2}}\hat{\boldsymbol{\rho}} + \frac{z}{\sqrt{\rho^2+z^2}}\hat{\mathbf{z}} \\ \hat{\boldsymbol{\theta}} &= \frac{z}{\sqrt{\rho^2+z^2}}\hat{\boldsymbol{\rho}} - \frac{\rho}{\sqrt{\rho^2+z^2}}\hat{\mathbf{z}} \\ \hat{\boldsymbol{\phi}} &= \hat{\boldsymbol{\phi}} \end{aligned}$	$\begin{aligned} \hat{\boldsymbol{\rho}} &= \sin\theta\hat{\mathbf{r}} + \cos\theta\hat{\boldsymbol{\theta}} \\ \hat{\boldsymbol{\phi}} &= \hat{\boldsymbol{\phi}} \\ \hat{\mathbf{z}} &= \cos\theta\hat{\mathbf{r}} - \sin\theta\hat{\boldsymbol{\theta}} \end{aligned}$
A vector field $\mathbf{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_\rho\hat{\boldsymbol{\rho}} + A_\phi\hat{\boldsymbol{\phi}} + A_z\hat{\boldsymbol{z}}$	$A_r\hat{\boldsymbol{r}} + A_\theta\hat{\boldsymbol{\theta}} + A_\phi\hat{\boldsymbol{\phi}}$
Gradient ∇f	$\frac{\partial f}{\partial x}\hat{\mathbf{x}} + \frac{\partial f}{\partial y}\hat{\mathbf{y}} + \frac{\partial f}{\partial z}\hat{\mathbf{z}}$	$\frac{\partial f}{\partial \rho}\hat{\boldsymbol{\rho}} + \frac{1}{\rho}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}} + \frac{\partial f}{\partial z}\hat{\boldsymbol{z}}$	$\frac{\partial f}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\boldsymbol{\phi}}$
Divergence $\nabla\cdot\mathbf{A}$	$\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$	$\frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(A_\theta\sin\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$

Curl $\nabla \times \mathbf{A}$	$\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathbf{x}} \quad +$ $\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathbf{y}} \quad +$ $\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathbf{z}}$	$\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right)\hat{\boldsymbol{\rho}} \quad +$ $\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right)\hat{\boldsymbol{\phi}} \quad +$ $\frac{1}{\rho}\left(\frac{\partial (\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi}\right)\hat{\boldsymbol{z}}$	$\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial \theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial \phi}\right)\hat{\boldsymbol{r}} \quad +$ $\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial}{\partial r}(rA_\phi)\right)\hat{\boldsymbol{\theta}} \quad +$ $\frac{1}{r}\left(\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial \theta}\right)\hat{\boldsymbol{\phi}}$
Laplace operator $\Delta f = \nabla^2 f$	$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial f}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$
Vector Laplacian $\Delta \mathbf{A} = \nabla^2 \mathbf{A}$	$\Delta A_x \hat{\mathbf{x}} + \Delta A_y \hat{\mathbf{y}} + \Delta A_z \hat{\mathbf{z}}$	$\left(\Delta A_\rho - \frac{A_\rho}{\rho^2} - \frac{2}{\rho^2}\frac{\partial A_\phi}{\partial \phi}\right)\hat{\boldsymbol{\rho}} \quad +$ $\left(\Delta A_\phi - \frac{A_\phi}{\rho^2} + \frac{2}{\rho^2}\frac{\partial A_\rho}{\partial \phi}\right)\hat{\boldsymbol{\phi}} \quad +$ $(\Delta A_z)\hat{\boldsymbol{z}}$	$\left(\Delta A_r - \frac{2A_r}{r^2} - \frac{2}{r^2 \sin\theta}\frac{\partial (A_\theta \sin\theta)}{\partial \theta} - \frac{2}{r^2 \sin\theta}\frac{\partial A_\phi}{\partial \phi}\right)\hat{\boldsymbol{r}} \quad +$ $\left(\Delta A_\theta - \frac{A_\theta}{r^2 \sin^2\theta} + \frac{2}{r^2}\frac{\partial A_r}{\partial \theta} - \frac{2\cos\theta}{r^2 \sin^2\theta}\frac{\partial A_\phi}{\partial \phi}\right)\hat{\boldsymbol{\theta}} \quad +$ $\left(\Delta A_\phi - \frac{A_\phi}{r^2 \sin^2\theta} + \frac{2}{r^2 \sin\theta}\frac{\partial A_r}{\partial \phi} + \frac{2\cos\theta}{r^2 \sin^2\theta}\frac{\partial A_\theta}{\partial \phi}\right)\hat{\boldsymbol{\phi}}$
Material derivative ш $(\mathbf{A} \cdot \nabla)\mathbf{B}$	$\left(A_x\frac{\partial B_x}{\partial x} + A_y\frac{\partial B_x}{\partial y} + A_z\frac{\partial B_x}{\partial z}\right)\hat{\boldsymbol{x}} \quad +$ $\left(A_x\frac{\partial B_y}{\partial x} + A_y\frac{\partial B_y}{\partial y} + A_z\frac{\partial B_y}{\partial z}\right)\hat{\boldsymbol{y}} \quad +$ $\left(A_x\frac{\partial B_z}{\partial x} + A_y\frac{\partial B_z}{\partial y} + A_z\frac{\partial B_z}{\partial z}\right)\hat{\boldsymbol{z}}$	$\left(A_\rho\frac{\partial B_\rho}{\partial \rho} + \frac{A_\phi}{\rho}\frac{\partial B_\rho}{\partial \phi} + A_z\frac{\partial B_\rho}{\partial z} - \frac{A_\phi B_\phi}{\rho}\right)\hat{\boldsymbol{\rho}} +$ $\left(A_\rho\frac{\partial B_\phi}{\partial \rho} + \frac{A_\phi}{\rho}\frac{\partial B_\phi}{\partial \phi} + A_z\frac{\partial B_\phi}{\partial z} + \frac{A_\phi B_\rho}{\rho}\right)\hat{\boldsymbol{\phi}} +$ $\left(A_\rho\frac{\partial B_z}{\partial \rho} + \frac{A_\phi}{\rho}\frac{\partial B_z}{\partial \rho} + A_z\frac{\partial B_z}{\partial z}\right)\hat{\boldsymbol{z}}$	$\left(A_r\frac{\partial B_r}{\partial r} + \frac{A_\theta}{r}\frac{\partial B_r}{\partial \theta} + \frac{A_\phi}{r\sin(\theta)}\frac{\partial B_r}{\partial \phi} - \frac{A_\theta B_\theta + A_\phi B_\phi}{r}\right)\hat{\boldsymbol{r}} +$ $\left(A_r\frac{\partial B_\phi}{\partial r} + \frac{A_\theta}{r}\frac{\partial B_\theta}{\partial \theta} + \frac{A_\phi}{r\sin(\theta)}\frac{\partial B_\theta}{\partial \phi} - \frac{A_\theta B_r}{r} - \frac{A_\phi B_\phi \cot(\theta)}{r}\right)\hat{\boldsymbol{\theta}} +$ $\left(A_r\frac{\partial B_\phi}{\partial r} + \frac{A_\theta}{r}\frac{\partial B_\phi}{\partial \theta} + \frac{A_\phi}{r\sin(\theta)}\frac{\partial B_\phi}{\partial \phi} + \frac{A_\theta B_r}{r} + \frac{A_\phi B_\theta \cot(\theta)}{r}\right)\hat{\boldsymbol{\phi}}$
Differential displacement	$d\mathbf{l} = dx\hat{\mathbf{x}} + dy\hat{\mathbf{y}} + dz\hat{\mathbf{z}}$	$d\mathbf{l} = d\rho\hat{\boldsymbol{\rho}} + \rho d\phi\hat{\boldsymbol{\phi}} + dz\hat{\boldsymbol{z}}$	$d\mathbf{l} = dr\hat{\mathbf{r}} + rd\theta\hat{\boldsymbol{\theta}} + r\sin\theta d\phi\hat{\boldsymbol{\phi}}$
Differential normal area	$d\mathbf{S} = \quad dy\,dz\,\hat{\mathbf{x}} +$ $dx\,dz\,\hat{\mathbf{y}} +$ $dx\,dy\,\hat{\mathbf{z}}$	$d\mathbf{S} = \quad \rho\,d\phi\,dz\,\hat{\boldsymbol{\rho}} +$ $d\rho\,dz\,\hat{\boldsymbol{\phi}} +$ $\rho\,d\rho d\phi\,\hat{\mathbf{z}}$	$d\mathbf{S} = \quad r^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{r}} +$ $r\sin\theta\,dr\,d\phi\,\hat{\boldsymbol{\theta}} +$ $r\,dr\,d\theta\,\hat{\boldsymbol{\phi}}$
Differential volume	$dV = dx\,dy\,dz$	$dV = \rho\,d\rho\,d\phi\,dz$	$dV = r^2\sin\theta\,dr\,d\theta\,d\phi$

Non-trivial calculation rules:

1. $\text{div grad } f = \nabla \cdot (\nabla f) = \nabla^2 f = \Delta f$ (Laplacian)
2. $\text{curl grad } f = \nabla \times (\nabla f) = \mathbf{0}$
3. $\text{div curl } \mathbf{A} = \nabla \cdot (\nabla \times \mathbf{A}) = 0$
4. $\text{curl curl } \mathbf{A} = \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$ (using Lagrange's formula for the cross product)
5. $\Delta fg = f\Delta g + 2\nabla f \cdot \nabla g + g\Delta f$