TEMELJNI POSTULATI ELEKTROMAGNETIZMA

Zakon Lorentzove sile
$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$

Definicije $\vec{E}$ i $\vec{B}$

$\vec{E} = \lim_{q \to 0}\left(\frac{\vec{F}}{q}\right)$; $\vec{v} = 0$ $\quad \vec{v} \times \vec{B} = \frac{\vec{F}}{q} - \vec{E}$

Gustoća naboja: $\rho = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V} = \frac{\mathrm{d}Q}{\mathrm{d}V} \quad \frac{\mathrm{C}}{\mathrm{m}^3}$

Struja: $I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{\mathrm{d}Q}{\mathrm{d}t}$

Gustoća struje:
$\vec{J} = \vec{a}_v \lim_{\Delta S \to 0} \frac{\Delta I}{\Delta S} = \vec{a}_v \frac{\mathrm{d}I}{\mathrm{d}S} \quad \frac{\mathrm{A}}{\mathrm{m}^2}$

Veza gustoća sa nabojima i strujama
$Q(t) = \iiint_V \rho(\vec{r}, t)\mathrm{d}V \quad ; \quad I(t) = \iint_S \vec{J}(\vec{r}, t) \cdot \vec{n}\,\mathrm{d}S$

Plošni naboj $\sigma = \lim_{\Delta S \to 0} \frac{\Delta Q}{\Delta S} = \frac{\mathrm{d}Q}{\mathrm{d}S}$

$\sigma = \lim_{\substack{\rho \to \infty \\ \Delta l \to 0}} (\rho\,\Delta l)$

Ukupni naboj $Q = \iint_S \sigma\,\mathrm{d}S$

Plošna struja
$\vec{K} = \lim_{\Delta s \to 0}\left[\vec{a}_I \frac{\Delta I}{\Delta s}\right] = \vec{a}_I \frac{\mathrm{d}I}{\mathrm{d}s}$
$\vec{K} = \lim_{\substack{J \to \infty \\ \Delta l \to 0}} \left(\vec{J}\,\Delta l\right)$

Ukupna struja $I = \int_c \vec{K} \cdot \vec{a}_I\,\mathrm{d}s$

Linijski naboj
$\lambda = \lim_{\Delta l \to 0} \frac{\Delta Q}{\Delta l} = \frac{\mathrm{d}Q}{\mathrm{d}l}$
$\lambda = \lim_{\substack{\rho \to \infty \\ \Delta S \to 0}} (\rho\,\Delta S)$

Ukupni naboj $Q = \int_l \lambda\,\mathrm{d}l$

Linijska struja
$\vec{J}\mathrm{d}V = i\,\mathrm{d}\vec{l}$
$i = \lim_{\substack{J \to \infty \\ \Delta S \to 0}} \left(\vec{J} \cdot \vec{n}\,\mathrm{d}S\right)$

Točkasti naboj
$\rho(\vec{r}) = q\,\delta(\vec{r} - \vec{r}\,') \;;\; \iiint_V \rho(\vec{r})\,\mathrm{d}V = \iiint_V q\,\delta(\vec{r} - \vec{r}\,')\,\mathrm{d}V = q$

RJEŠAVANJE STATIĆKIH ELEKTRIĆNIH POLJA

Gaussov zakon u diferencijalnom obliku:
$\nabla \cdot \vec{D} = \rho_s \;;\; \vec{D} = \varepsilon\vec{E} \;;\; \vec{E} = -\nabla\varphi \;\Rightarrow\; \nabla \cdot (\nabla\varphi) = -\frac{\rho_s}{\varepsilon}$

Poissonova jednadžba:
$\Delta\varphi = -\frac{\rho_s}{\varepsilon}$

Laplaceova jednadžba $(\rho_s = 0)$:
$\Delta\varphi = 0$

Rješenja Laplaceove i Poissonove jednadžbe su jedinstvena

STATICKO ELEKTRICNO POLJE U VAKUUMU

Coulombov zakon

$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0} \frac{Q_1 Q_2}{\left|\vec{R}_{12}\right|^2} \vec{a}_{\vec{R}}$

Jakost polja točkastog naboja
$Q_1 = Q$, $Q_2 = Q_p$:

$\vec{E} = \lim_{Q_p \to 0} \frac{\vec{F}}{Q_p} = \frac{Q}{4\pi\varepsilon_0\left|\vec{R}\right|^2} \frac{\vec{R}}{\left|\vec{R}\right|} = \frac{Q}{4\pi\varepsilon_0} \frac{\vec{r} - \vec{r}\,'}{\left|\vec{r} - \vec{r}\,'\right|^3}$

Električno polje skupine točkastih naboja
$\vec{E}(\vec{r}) = \sum_{i=1}^{N} \vec{E}_i(\vec{r}) = \sum_{i=1}^{N} \frac{Q_i}{4\pi\varepsilon_0} \frac{\vec{r} - \vec{r}_i\,'}{\left|\vec{r} - \vec{r}_i\,'\right|^3}$

Električno polje kontinuiranih raspodjela naboja

Volumni naboj
$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \iiint_V \mathrm{d}Q \frac{\vec{r} - \vec{r}\,'}{\left|\vec{r} - \vec{r}\,'\right|^3} = \frac{1}{4\pi\varepsilon_0} \iiint_V \frac{\vec{r} - \vec{r}\,'}{\left|\vec{r} - \vec{r}\,'\right|^3} \rho(\vec{r}\,')\mathrm{d}V$

Plošni naboj
$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \iint_S \frac{\vec{r} - \vec{r}\,'}{\left|\vec{r} - \vec{r}\,'\right|^3} \sigma(\vec{r}\,')\mathrm{d}S$

Linijski naboj
$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_l \frac{\vec{r} - \vec{r}\,'}{\left|\vec{r} - \vec{r}\,'\right|^3} \lambda(\vec{r}\,')\mathrm{d}l$

Primjena Gaussova teorema → Gaussov zakon
$\oiint_S \vec{E} \cdot \vec{n}\,\mathrm{d}S = \iiint_V \nabla \cdot \vec{E}\,\mathrm{d}V = \frac{1}{\varepsilon_0} \iiint_V \rho_s\,\mathrm{d}V \Rightarrow$
$\Rightarrow \varepsilon_0 \nabla \cdot \vec{E} = \rho_s$

Električno polje $\vec{E}$ na točkasti naboj q djeluje silom $\vec{F} = q\vec{E}$
Uvodimo (skalarni) električni potencijal φ u točki P
$\varphi(\mathrm{P}) = \frac{A(\mathrm{P})}{q} = -\int_{\mathrm{T_{referentno}}}^{\mathrm{P}} \vec{E} \cdot \mathrm{d}\vec{l}$

Potencijal točkastog naboja q na udaljenosti r je:
$\varphi(r) = \frac{q}{4\pi\varepsilon_0 r}$

Potencijal skupine N točkastih naboja u točkama
$\vec{r}_i\,' \;;\; i = 1, \ldots N$
$\varphi(\vec{r}) = \sum_{i=1}^{N} \varphi(\vec{r}) = \sum_{i=1}^{N} \frac{q_i}{4\pi\varepsilon_0} \frac{1}{\left|\vec{r} - \vec{r}_i\,'\right|}$

Potencijal kontinuiranih raspodjela naboja

- Volumni naboj $\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \iiint_V \frac{\mathrm{d}q}{\left|\vec{r} - \vec{r}\,'\right|} = \frac{1}{4\pi\varepsilon_0} \iiint_V \frac{\rho(\vec{r}\,')\mathrm{d}V}{\left|\vec{r} - \vec{r}\,'\right|}$
- Plošni naboj $\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \iint_S \frac{\sigma(\vec{r}\,')\mathrm{d}S}{\left|\vec{r} - \vec{r}\,'\right|}$
- Linijski naboj $\varphi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \int_l \frac{\lambda(\vec{r}\,')\mathrm{d}l}{\left|\vec{r} - \vec{r}\,'\right|}$

Potencijal i polje točkastog naboja
$\varphi = \frac{1}{4\pi\varepsilon_0\left|\vec{r}\right|} \quad ; \quad \vec{E} = \frac{1}{4\pi\varepsilon_0\left|\vec{r}\right|^2} \frac{\vec{r}}{\left|\vec{r}\right|}$

Definicija potencijala
$\mathrm{d}\varphi = -\vec{E} \cdot \mathrm{d}\vec{l} = -\left|\vec{E}\right|\left|\mathrm{d}\vec{l}\right|\cos\alpha = -E_l\,\mathrm{d}l$

Uvrstimo $\vec{E} = -\nabla\varphi$ u izraz za potencijal između točaka A i B
$\varphi(\mathrm{A}) - \varphi(\mathrm{B}) = -\int_{\mathrm{B}}^{\mathrm{A}} \vec{E} \cdot \mathrm{d}\vec{l} = \int_{\mathrm{B}}^{\mathrm{A}} \nabla\varphi \cdot \mathrm{d}\vec{l}$

Stokesov teorem
$\iint_S (\nabla \times F) \cdot \vec{n}\,\mathrm{d}S = \oint_c \vec{F} \cdot \mathrm{d}\vec{l}$

Primjena Stokesova teorema
$\oint_c \vec{E} \cdot \mathrm{d}\vec{l} = \iint_S \left(\nabla \times \vec{E}\right) \cdot \vec{n}\,\mathrm{d}S = 0 \quad \Rightarrow \nabla \times \vec{E} = 0$

MATERIJALI U ELEKTRIČNOM POLJU

Sila na slobodni elektron: $\vec{F} = -e\vec{E}$

Brzina strujanja: $\vec{v} = \mu\vec{E}$

Gustoća naboja: $\rho_s = -Ne$

Gustoća struje: $\vec{J}_s = \rho_s\vec{v} = -Ne\vec{v} = -Ne\mu\vec{E} = \kappa\vec{E}$

Definiramo polarizaciju:

$$\vec{P}(\vec{r}) = \lim_{\substack{\Delta V \to 0 \\ \text{oko } r}} \frac{\sum_{\Delta V} \vec{p}_i}{\Delta V} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}V} \Rightarrow \mathrm{d}\vec{p} = \vec{P}\mathrm{d}V$$

Naboj na dS je $\sigma_p\,\mathrm{d}S = \vec{P}\cdot\vec{n}\,\mathrm{d}S$

Naboj koji izlazi iz volumena V je: $Q_{izl} = \oiint_S \vec{P}\cdot\vec{n}\,\mathrm{d}S$

Tijelo ostaje električki neutralno pa vrijedi:

$$Q_{pol} = \iiint_V \rho_p \mathrm{d}V = -Q_{izl} = -\oiint_S \vec{P}\cdot\vec{n}\,\mathrm{d}S \quad \Rightarrow \quad \rho_p = -\nabla\cdot\vec{P}$$

Uvodimo $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$

Gaussov zakon u materijalima:

$$\nabla\cdot\vec{D} = \rho_s \quad \Rightarrow \quad \oiint_S \vec{D}\cdot\vec{n}\,\mathrm{d}S = \iiint_V \rho_s \mathrm{d}V$$

Neka je sredstvo 1 vodič nabijen nabojem plošne gustoće σ_s a sredstvo 2 dielektrik s ε

U vodiču nema polja D_1=0 pa je:

$$\vec{n}(\vec{D}_2 - \vec{D}_1) = \sigma_s \Rightarrow \vec{n}\vec{D}_2 = \sigma_s \Rightarrow D_{2n} = \sigma_s \Rightarrow E_{2n} = \frac{\sigma_s}{\varepsilon}$$

$$\vec{n}\times(\vec{E}_2 - \vec{E}_1) = 0 \Rightarrow \vec{n}\times\vec{E}_2 = 0 \Rightarrow E_{2t} = 0$$

Na površini nabijenog vodiča postoji samo normalna komponenta polja

ENERGIJA I KAPACITET

Za sustav N točkastih naboja vrijedi:

$$W = \frac{1}{2}\sum_{i=1}^{N}\varphi_i Q_i \quad ; \quad \varphi_i = \sum_{\substack{k=1 \\ k\neq i}}^{N}\varphi_{ki}$$

Energiju prostorne raspodjele naboja $\rho(\vec{r}\,')$ određujemo superpozicijom diferencijalno malih točkastih naboja $\rho(\vec{r}\,')\mathrm{d}V$:

$$W = \frac{1}{2}\iiint_V \varphi(\vec{r}\,')\rho(\vec{r}\,')\mathrm{d}V$$

N idealno vodljivih tijela

$$W_e = \frac{1}{2}\sum_{k=1}^{N} Q_k\varphi_k$$

$$W = \frac{1}{2}\iiint_V \varphi\,\rho\,\mathrm{d}V = \frac{1}{2}\iiint_V \nabla\cdot(\varphi\vec{D})\mathrm{d}V + \frac{1}{2}\iiint_V (\vec{D}\cdot\vec{E})\mathrm{d}V$$

Primjena Gaussova teorema o divergenciji rezultira s

$$W = \frac{1}{2}\oiint_S (\varphi\vec{D})\cdot\vec{n}\,\mathrm{d}S + \frac{1}{2}\iiint_V (\vec{D}\cdot\vec{E})\mathrm{d}V$$

Ako V obuhvaća cijeli prostor polja vrijedi:

$$W = \frac{1}{2}\iiint_V (\vec{D}\cdot\vec{E})\mathrm{d}V = \frac{1}{2}\iiint_V \varepsilon|\vec{E}|^2\mathrm{d}V$$

Gustoća energije električnog polja je:

$$w_e = \frac{\varepsilon|\vec{E}|^2}{2}$$

Kapacitet $C = \frac{Q}{U}$

Pohrana električne energije

$$W_e = \frac{1}{2}QU = \frac{1}{2}CU^2 = \frac{1}{2}\frac{Q^2}{C}$$