(1)

Potencijal između 2 koncentrične sferne plohe zadan je jednadžbom:

$$\boxed{\varphi = \frac{a \cdot e^{-br}}{r}} \qquad R_1 \le r \le R_2$$

Odrediti ukupni naboj između ploha $(\varepsilon = \varepsilon_0)$

$Q = ?$

(1) $\boxed{\Delta\varphi = -\frac{\rho}{\varepsilon_0} \Rightarrow}$ $\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\varphi}{dr}\right) = -\frac{\rho}{\varepsilon_0}$

$$\frac{1}{r^2}\frac{d}{dr}\left(\cancel{r^2} \cdot a \frac{-b\, e^{-br} \cdot r - e^{-br} \cdot 1}{\cancel{r^2}}\right) = -\frac{\rho}{\varepsilon_0}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(a \cdot -b e^{-br} \cdot r - e^{-br} \cdot 1\right) = -\frac{\rho}{\varepsilon_0}$$

$$\varepsilon_0 \cdot a \frac{1}{r^2}\frac{d}{dr}\left(e^{-br}(br+1)\right) = \rho$$

$$\rho_0 = -\varepsilon_0 a b^2 \cdot \frac{e^{-br}}{r}$$

(2) $Q = \int_V \rho\, dV \Rightarrow dV = 4\pi r^2 dr$

$$Q = -\varepsilon_0 a b^2 4\pi \int_{R_1}^{R_2} \frac{e^{-br}}{\cancel{r}} \cdot r^2\, dr$$

$$Q = -\varepsilon_0 a b^2 4\pi \int_{R_1}^{R_2} e^{-br} \cdot r\, dr \Leftarrow \text{elementarni}$$

$$Q = 4\pi\varepsilon_0 a \left\{ e^{-bR_2}(1+bR_2) - e^{-bR_1}(1+bR_2) \right\}$$

(2.) Uvjeti na granici (1) $\varepsilon_{r1} = 2$ i sredstva (2) $\varepsilon_r = 5$ granica

ravnina $y + 2z - 2 = 0$.

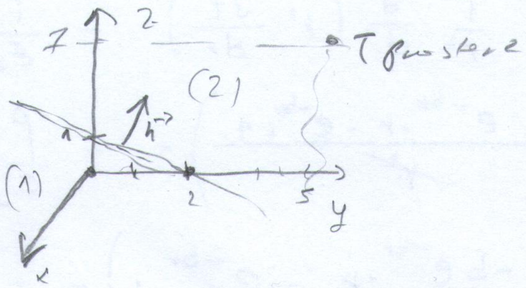
Sredstvo (1) obuhvaća ishodište.

E u (2) ako je $E(1) = \vec{a_x} - 2\vec{a_y} - 3\vec{a_z}$ $\left[\frac{V}{m}\right]$. Na granici dva sredstva

nema naboja. Odrediti gustoću energije u $(3;5;7)$

$\varepsilon_{r1} = 2 \qquad y + 2z - 2 = 0$

$\varepsilon_{r2} = 5 \qquad \vec{E_1} = \vec{a_x} - 2\vec{a_y} - 3\vec{a_z}$



$$\vec{n} = \frac{\vec{a_y} + 2\vec{a_z}}{\sqrt{1^2 + 2^2}} = \frac{\vec{a_y} + 2\vec{a_z}}{\sqrt{5}}$$

$$\left.\begin{aligned} &\vec{n}\,(D_2 - D_1) = 0 \\ &\vec{n} \times (\vec{E_2} - \vec{E_1}) = 0 \end{aligned}\right\} \text{ riješi se, cap } \quad \left|\ \begin{aligned} &\vec{D_{1n}} = \vec{D_{2n}} \\ &\vec{E_{1t}} = \vec{E_{2t}} \end{aligned}\right.$$

$$E_{1n} = \vec{E_1} \cdot \vec{n} = (\vec{a_x} - 2\vec{a_y} - 3\vec{a_z}) \left(\frac{\vec{a_y} + 2\vec{a_z}}{\sqrt{5}}\right) = \frac{1}{\sqrt{5}} \cdot (-2 - 6) = -\frac{8}{\sqrt{5}}$$

$$\vec{E_{1n}} = E_{1n} \cdot \vec{n} = -\frac{8}{5}(\vec{a_y} + 2\vec{a_z}), \quad \underline{\vec{E_{1t}}} = \vec{E_1} - \vec{E_{1n}} = \frac{5\vec{a_x} - 2\vec{a_y} + \vec{a_z}}{5} = \underline{\vec{E_{2t}}}$$

$$D_{1n} = D_{2n} \Rightarrow \varepsilon_1 E_{1n} = \varepsilon_2 E_{2n} \Rightarrow E_{2n} = \frac{\varepsilon_1}{\varepsilon_2} \cdot E_{1n} = \frac{2}{5} \cdot -\frac{8}{\sqrt{5}} = \frac{-16}{5\sqrt{5}}$$

$$\vec{E_{2n}} = \vec{n} \cdot E_{2n} = \frac{-16}{5\sqrt{5}} \cdot \frac{\vec{a_y} + 2\vec{a_z}}{\sqrt{5}} = \frac{-16}{25}(\vec{a_y} + 2\vec{a_z})$$

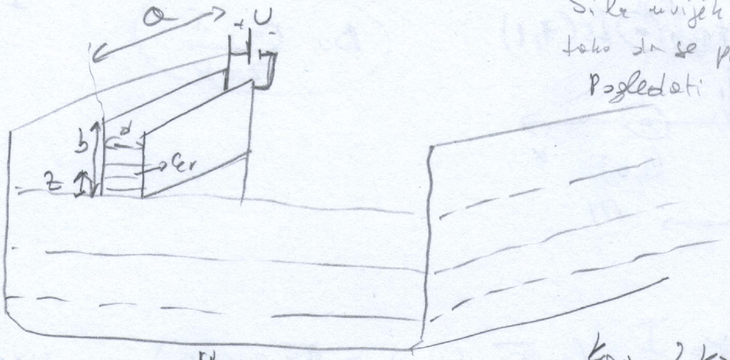
$$\vec{E_2} = \vec{E_{2n}} + \vec{E_{2t}} = -\frac{16}{25}(\vec{a_y} + 2\vec{a_z}) + \frac{25\vec{a_x} - 10\vec{a_y} + 5\vec{a_z}}{25} = \frac{25\vec{a_x} - 26\vec{a_y} - 27\vec{a_z}}{25}$$

Gustoća energije $w = \frac{1}{2}\varepsilon E^2$. Točka je u prostoru 2

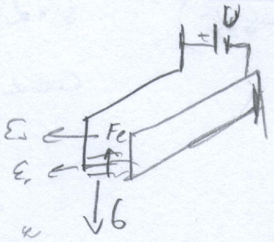
$$w = \frac{1}{2}\varepsilon_2 E_2^2 \qquad \overline{E_2}(3,5,7) = \overline{E_2} = \text{konst}$$

$$w = \frac{1}{2} \cdot 5 \cdot \frac{\sqrt{25^2 + 26^2 + 27^2}}{25} = 4{,}50\,\mathrm{J}\,\frac{V^2}{m^2} \qquad E_2 = \frac{\sqrt{25^2 + 26^2 + 27^2}}{25}$$

③ Plo. kondenzator širine ploča a i razmaka između ploča d uronjen je u dielektrično ulje prema slici. Odredite visinu z na koju će se podići dielektrično ulje gustoće ρ_m relativne dielektričnosti ε_r, ako kondenzator spojimo na izvor U



Sila uvijek djeluje tako da se povećava. Pogledati vrpinke o ovisnosti sile



Kao 2 kondenzatora spojena u paralelu

$$\vec{F_e} = \vec{F_G}$$

donji, popunjen, ostatak

$$C = C_1 + C_2 = \varepsilon_0 \cdot \varepsilon_r \cdot \frac{z \cdot a}{d} + \varepsilon_0 \cdot \frac{(b-z)\cdot a}{d}$$

$$C = \varepsilon_0 \frac{a}{d}\left(\varepsilon_r z + (b-z)\right) \quad / \frac{d}{dz}$$

BITNO

neki kondenzator

$$\vec{F_z} = +\vec{a_z}\, \frac{1}{2} U^2 \frac{dC}{dz} = \frac{1}{2} U^2 \cdot \varepsilon_0 \cdot \frac{a}{d} \cdot (\varepsilon_r - 1) \cdot \vec{a_z}$$

↑ povećanje za kapaciteta

$$\varepsilon_0 \frac{aU^2}{2d} \cdot (\varepsilon_r - 1) = mg$$

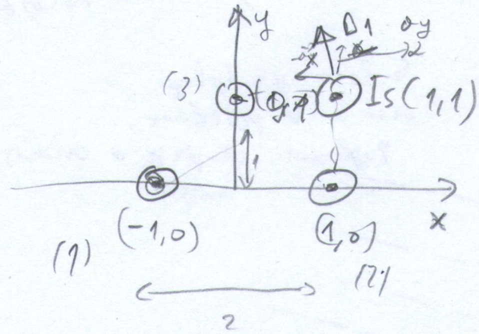
$$\rho = \frac{m}{V}$$

$$\varepsilon_0 \frac{aU^2}{2d}(\varepsilon_r - 1) = \rho \cdot V \cdot g$$

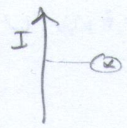
$$\varepsilon_0 \frac{aU^2}{2d}(\varepsilon_r - 1) = \rho_m \cdot d \cdot z \cdot a \cdot g$$

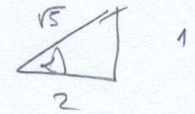
$$\boxed{z = \frac{\varepsilon_0 U^2 (\varepsilon_r - 1)}{2d^2 \rho_m g}}$$

(4.) Tri su beskonačno duga vodiča zanemarivog presjeka postavljena u pozicije prema slici. Ako vodičima teče struja I odredite silu po jedinici duljine na signalni vodič koji teče struja I_s



$$B = \frac{\mu_0 I}{2\pi r}$$





Vodič 1: $\quad \vec{B}_1 = \frac{\mu_0 I}{2\pi\sqrt{5}}\left(-\vec{a_x}\cdot\sin\alpha + \vec{a_y}\cos\alpha\right)$ $\qquad \sin\alpha = \frac{1}{\sqrt{5}}$

$\cos\alpha = \frac{2}{\sqrt{5}}$

Vodič 2: samo $-\vec{a_x}$ $\quad \vec{B}_2 = \frac{\mu_0 I}{2\pi\cdot 1}\cdot -\vec{a_x}$

Vodič 3: samo $\vec{a_y}$ $\quad \vec{B}_3 = \frac{\mu_0 I}{2\pi 1}\cdot\vec{a_y}$

$$\vec{B} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 = \frac{\mu_0 I}{10\pi}\left(-6\vec{a_x} + 7\vec{a_y}\right)$$

$\vec{F} = I_s(\vec{l_s}\times\vec{B}) \Rightarrow \vec{l_s} = \vec{a_z}\cdot l_s$; $F_s' = \frac{F_s}{l_s} =$

vektorski of $\quad l_s \times \frac{\mu_0 I}{10\pi}\left(-6\vec{a_x} + 7\vec{a_y}\right) = I_s\frac{\mu_0 I}{10\pi}\cdot\left(-6\vec{a_y} - 7\vec{a_x}\right)$

privlačna sila

(5.) $\varphi = \frac{A}{\sqrt{x^2+y^2+z^2}}$ $\quad$ A = konst. $\qquad \Phi$ kroz $\vec{D}$, oplošje kocke ? $\quad$ 2

Središte u ishodištu!

$\oint_S \vec{D}\vec{n}\,dS = ?$ $\qquad \vec{E} = -\nabla\varphi = -\vec{a_x}\frac{\partial\varphi}{\partial x} - \vec{a_y}\frac{\partial\varphi}{\partial y} - \vec{a_z}\frac{\partial\varphi}{\partial z}$

$$\vec{E} = A\frac{1}{\sqrt{x^2+y^2+z^2}^{\,3}}\left(\vec{a_x}\cdot x + \vec{a_y}\cdot y + \vec{a_z}\cdot z\right)$$

$$D = \varepsilon_0\cdot\vec{E}$$

gornja horizontalna ploha $\vec{n} = \vec{a_z}$, $dS = dx\cdot dy$ $\qquad \vec{D}\cdot\vec{n} = \frac{\varepsilon_0\cdot A\,1}{\sqrt{x^2+y^2+1}}$ $\qquad \oint_S \vec{D}\vec{n}\,dS$

$\Phi_1 = \int_{x=-1}^{1}\int_{y=-1}^{1}\frac{\varepsilon_0 A}{(\sqrt{x^2+y^2+1})^{3/2}}\,dx\,dy$ $\qquad \Phi_1 = 2\varepsilon_0 A\int_{y=-1}^{1}\frac{dy}{(y^2+1)\sqrt{y^2+2}}$ $\qquad y^2+1 = u$, $2y\,dy = du$

$\Phi_1 = 2\varepsilon_0\cdot A\cdot\arccos\left(\frac{1}{u}\right)$ sad granice $\qquad$ integriraj sobane kosne znaš „trik"

$\Phi_1 = \frac{2\pi}{3}\varepsilon_0 A$

KRAĆE

$\boxed{\Phi = 6\,\Phi_1 = 4\pi\varepsilon_0 A}$

$\varphi = \frac{A}{\sqrt{x^2+y^2+z^2}}$; $\sqrt{x^2+y^2+z^2} = r$ $\qquad \vec{E} = -\nabla\varphi = -\vec{a_r}\cdot\frac{d\varphi}{dr} = \vec{a_r}\frac{A}{r^2}$ ← polje točkastog naboja

$E = \frac{\vec{a_r}}{r^2} = \frac{Q}{4\pi\varepsilon_0 r^2} \rightarrow Q_{obuh} = 4\pi\varepsilon_0 A$ $\qquad \boxed{\oint \vec{D}\vec{n}\,dS = Q = 4\pi\varepsilon_0 A}$