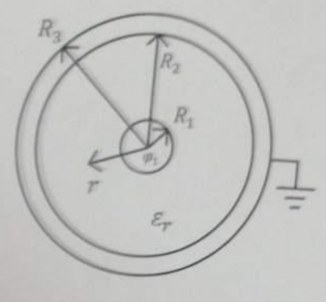
Završni ispit iz Elektromagnetskih polja

20.6.2017.

1. Koaksijalni kabel zanemarivog otpora protjecan je strujom $I = 1\,A$. Unutarnji vodič kabela nalazi se na potencijalu $\varphi_1 = 100\,V$, a vanjski je na potencijalu 0. Odredite iznos Poyntingova vektora u izolaciji vodiča za $r = 0{,}5\,cm$ (prema slici). Zadano je $R_1 = 0{,}25\,cm, R_2 = 0{,}7\,cm, R_3 = 0{,}85\,cm, \varepsilon_r = 2{,}1$.

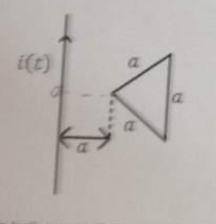


2. U prostoru sa značajkama $\varepsilon_r = 7$ i $\mu_r = 1$ zadano je električno polje:

$$\vec{E} = 100e^{-\alpha t}\cos(\omega t - \beta z)\vec{a}_x$$

gdje su α i β konstante. Odredite vektor magnetskog polja $\vec{H}$ pomoću Faradayevog zakona.

3. Odredite inducirani napon u trokutnoj petlji prema slici koja se nalazi u polju beskonačno duge ravne strujnice protjecane strujom $i(t) = I_0 e^{-t}\sin(\omega t)$.



4. Jakost električnog polja ravnog vala koji se prostire u sredstvu bez gubitaka zadana je s:

$$\vec{E} = 100\vec{a}_z\cos(\omega t - x - 2y)\ \left[\frac{V}{m}\right].$$

Svojstva materijala su $\varepsilon_r = 4$, $\mu_r = 1$. Odredite izraz za vektor jakosti magnetskog polja $\vec{H}$ te kružnu frekvenciju ω. er=4 ur=1

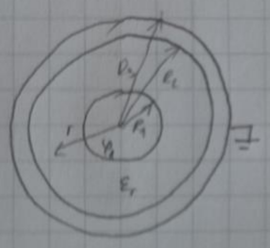
5. Dva magnetska materijala razdvaja ravnina $x + y + z = \sqrt{3}$. Ishodište $O(0,0,0)$ se nalazi u sredstvu s relativnom permeabilnosti $\mu_{r1} = 4$, gdje je magnetska indukcija zadana s
B1=1ax-0.5ay $\vec{B_1} = \vec{a}_x - 0{,}5\vec{a}_y$. Odredite magnetsku indukciju $\vec{B_2}$ u sredstvu s relativnom magnetskom permeabilnosti $\mu_{r2} = 1$.

ur2=1 ur1=4

1. $I = 1A$ $\varphi_1 = 100V$

$r = 0.5cm$ $R_1 = 0.25cm$

$R_2 = 0.9cm$ $R_3 = 0.85cm$



$\vec{N} = ?$

Berberović – 6.3.1

$$\vec{E} = \vec{a_r}\frac{\varphi_0}{r\ln\left(\frac{R_2}{R_1}\right)} \qquad \vec{H} = \vec{a_\varphi}\frac{I}{2\pi r}$$

$$\vec{N} = \vec{E}\times\vec{H} = \vec{a_z}\frac{\varphi_0 I}{2\pi\ln\left(\frac{R_2}{R_1}\right)}\frac{1}{r^2} = \vec{a_z}\cdot 618305.9$$

2. $\varepsilon_r = 7$, $\mu_r = 1$

$\vec{E} = 100e^{-\alpha t}\cos(\omega t - \beta z)\vec{a_x}$

$\vec{H} = ?$

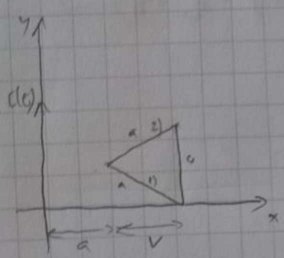
$$\nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{E} = \frac{\partial E_x}{\partial z}\vec{a_y} = \vec{a_y}\,100\beta e^{-\alpha t}\sin(\omega t - \beta z)$$

$$\vec{H} = -\frac{1}{\mu_0}\int_t(\nabla\times\vec{E})dt = -\frac{100\beta}{\mu_0}\int_t e^{-\alpha t}\sin(\omega t - \beta z)dt =$$

$$= \vec{a_y}\frac{100\beta}{\mu_0}\frac{e^{-\alpha t}}{\alpha^2+\omega^2}\left(\alpha\sin(\omega t-\beta z) + \omega\cos(\omega t - \beta z)\right)$$

3. $i(t) = I_0 e^{-t}\sin(\omega t)$



$U_{ind} = ?$

$$V = \frac{a\sqrt{3}}{2}$$

1) $y = \dfrac{\frac{a}{2} - 0}{a - a - \frac{a\sqrt{3}}{2}}\left(x - \left(a + \frac{a\sqrt{3}}{2}\right)\right)$ 2.) $y - \dfrac{a}{2} = \dfrac{a - \frac{a}{2}}{-a + a + \frac{a\sqrt{3}}{2}}(x - a)$

$$y_1 = -\frac{\sqrt{3}}{3}x + \frac{3+2\sqrt{3}}{6} \qquad y_2 = \frac{\sqrt{3}}{3}x + \frac{3-2\sqrt{3}}{6}$$

$$\vec{B} = -\vec{a_z}\frac{\mu_0 I}{2\pi x} \qquad \vec{n} = -\vec{a_z}$$

$$\phi = \iint_S \vec{B}\,\vec{n}\,dS = \frac{\mu_0 I}{2\pi}\int_a^{a+\frac{a\sqrt{3}}{2}}\frac{1}{x}\int_{y_1}^{y_2}dy = \frac{\mu_0 I}{2\pi}\int_a^{a+\frac{a\sqrt{3}}{2}}\frac{1}{x}\left(\frac{2\sqrt{3}}{3}x - \frac{2\sqrt{3}}{3}a\right)dx =$$

$$= \frac{2\sqrt{3}}{3}\frac{\mu_0 I}{2\pi}\left[\cancel{a} + \frac{a\sqrt{3}}{2} - \cancel{a} - a\ln\left(\frac{a + a\frac{\sqrt{3}}{2}}{a}\right)\right] =$$

$$= \frac{2\sqrt{3}}{3}a\frac{\mu_0 I_0 e^{-t}\sin(\omega t)}{2\pi}\left(\frac{\sqrt{3}}{2} - \ln\left(\frac{2+\sqrt{3}}{2}\right)\right)$$

$$e = -\frac{d\phi}{dt} = \frac{2\sqrt{3}}{3}a\frac{\mu_0 I_0}{2\pi}\left(\frac{\sqrt{3}}{2} - \ln\left(\frac{2+\sqrt{3}}{2}\right)\right)\left(e^{-t}\sin(\omega t) - e^{-t}\cos(\omega t)\right)$$

4. $\varepsilon_r = 4 \quad \mu_r = 1$

$\vec{E} = \vec{a_z}\, 100 \cos(\omega t - x - 2y)$

$\omega = ? \quad \vec{H} = ?$

$\vec{\beta} = \beta_x \vec{a_x} + \beta_y \vec{a_y} + \beta_z \vec{a_z} = \vec{a_x} + 2\vec{a_y}$

$|\vec{\beta}| = \sqrt{5}$

$$\omega = \frac{\beta}{\sqrt{\mu_0 \varepsilon_0 \mu_r \varepsilon_r}} = 3.35 \cdot 10^8 \ s^{-1}$$

$$\vec{H} = \frac{1}{\mu\omega} \vec{\beta} \times \vec{E} = \frac{1}{\mu\omega} \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 1 & 2 & 0 \\ 0 & 0 & E \end{vmatrix} =$$

= **(0.475<u>ax</u>-0.238<u>ay</u>)*cos(wt-x-2y)**

5. $x + y + z = \sqrt{3}$

$\mu_{r1} = 4 \quad \mu_{r2} = 1$

$\vec{B_1} = \vec{a_x} - 0.5\vec{a_y}$

$\vec{B_2} = ?$

$$\vec{n} = \frac{1}{\sqrt{3}} (\vec{a_x} + \vec{a_y} + \vec{a_z})$$

I.) $\vec{n} (\vec{B_2} - \vec{B_1}) = 0$

$B_{2x} - 1 + B_{2y} + 0.5 + B_{2z} = 0 \qquad (*)$

II.) $\vec{n} \times (\vec{H_2} - \vec{H_1}) = \vec{n} \times \left(\frac{\vec{B_2}}{\mu_2} - \frac{\vec{B_1}}{\mu_1} \right) = 0$

$$\frac{1}{\sqrt{3}} \cdot \frac{1}{\mu_0} \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 1 & 1 & 1 \\ B_{2x} - \frac{1}{4} & B_{2y} + \frac{1}{8} & B_{2z} \end{vmatrix} = 0$$

$$\vec{a_x} \left(B_{2z} - B_{2y} - \frac{1}{8} \right) + \vec{a_y} \left(B_{2x} - \frac{1}{4} - B_{2z} \right) + \vec{a_z} \left(B_{2y} + \frac{1}{8} - B_{2x} + \frac{1}{4} \right) = 0$$

$B_{2z} - B_{2y} = \frac{1}{8} \qquad (**)$

$B_{2x} - B_{2z} = \frac{1}{4} \qquad (***)$

$B_{2x} = \frac{3}{8} \qquad B_{2y} = 0 \qquad B_{2z} = \frac{1}{8}$

$$\vec{B} = \frac{3}{8} \vec{a_x} + \frac{1}{8} \vec{a_y}$$