# 1. Međuispit iz Elektromagnetskih polja

01.04.2009.

Ime i prezime____________ Matični broj________ INAČICA D

Ispit se sastoji od pet cjelina, u kojima se točan odgovor na svako pitanje nezavisno boduje, te se sastoji od ukupno 20 pitanja. Ukoliko želite odgovoriti na neko pitanje, zacrnite odgovor na obrascu za test. Svaki točan odgovor donosi 1 bod, dok se neodgovorena pitanja ne boduju. Netočan odgovor donosi -0.2 boda. Napišite ime na svim papirima s postupcima i predajte ih na kraju ispita zajedno s primjerkom testa u košuljici, dok se Obrazac za test posebno predaje.

I .Statički električni naboj je raspoređen po sfernoj ljusci unutrašnjeg radijusa $R_1 = 2$ i vanjskog radijusa $R_2 = 3$. Gustoća naboja u prostoru određena je jednadžbom:

$\rho = a + b \cdot r \quad R_1 \le r \le R_2$.

Odredite ukoliko su $a$ i $b$ konstante:

1. Jakost električnog polja u prostoru $r < R_1$.

$\boxed{A)\ E = 0}$ $B)\ E = \frac{a}{r^2}$ $C)\ E = \frac{a+br}{r^2}$ $D)\ E = \frac{b}{r}$ $E)\ E = a + br$ $F)\ E = 2a$

2. Jakost električnog polja u prostoru $R_1 < r < R_2$.

$A)\ E = \frac{1}{\varepsilon_0 r}\left[\frac{a}{3}\left(r^3 - 1\right) + \frac{b}{4}\left(r^4 - 1\right)\right]$ $B)\ E = \frac{1}{\varepsilon_0 r^2}\left[\frac{a}{3}\left(r^3 - 1\right) + \frac{b}{4}\left(r^4 - 1\right)\right]$

$\boxed{C)\ E = \frac{1}{\varepsilon_0 r^2}\left[\frac{a}{3}\left(r^3 - 8\right) + \frac{b}{4}\left(r^4 - 16\right)\right]}$ $D)\ E = \frac{1}{\varepsilon_0 r^2}\left[\frac{a}{3}\left(r^3 - 3\right) + \frac{b}{4}\left(r^4 - 6\right)\right]$

$E)\ E = \frac{1}{\varepsilon_0 r^3}\left[\frac{a}{3}\left(r^3 - 1\right) + \frac{b}{4}\left(r^4 - 1\right)\right]$ $F)\ E = \frac{1}{\varepsilon_0 r^3}\left[\frac{a}{3}\left(r^3 - 8\right) + \frac{b}{4}\left(r^4 - 16\right)\right]$

3. Jakost električnog polja u prostoru $r > R_2$.

$A)\ E = \frac{1}{\varepsilon_0 r^2}\left[\frac{5a}{3} + \frac{15b}{4}\right]$ $\boxed{B)\ E = \frac{1}{\varepsilon_0 r^2}\left[\frac{19a}{3} + \frac{65b}{4}\right]}$

$C)\ E = \frac{1}{\varepsilon_0 r^2}\left[-\frac{5a}{3} + \frac{15b}{4}\right]$ $D)\ E = \frac{1}{\varepsilon_0 r^2}\left[\frac{11a}{3} + \frac{29b}{4}\right]$

$E)\ E = \frac{1}{\varepsilon_0 r^2}\left[-\frac{a}{3} - \frac{15b}{4}\right]$ $F)\ E = \frac{1}{\varepsilon_0 r^2}\left[\frac{7a}{3} + \frac{15b}{4}\right]$

4. Potencijal u središtu sferne ljuske.

$A)\ \varphi = \frac{1}{\varepsilon_0}\left[\frac{5a}{3} + \frac{b}{4}\right]$ $B)\ \varphi = \frac{1}{\varepsilon_0}\left[\frac{5a}{3} - \frac{b}{4}\right]$ $C)\ \varphi = \frac{1}{\varepsilon_0}\left[\frac{a}{3} + \frac{b}{4}\right]$

$D)\ \varphi = \frac{1}{\varepsilon_0}\left[\frac{5a}{3} + \frac{b}{4}\right]$ $E)\ \varphi = \frac{1}{\varepsilon_0}\left[\frac{3a}{2} + \frac{7b}{3}\right]$ $\boxed{F)\ \varphi = \frac{1}{\varepsilon_0}\left[\frac{5a}{2} + \frac{19b}{3}\right]}$

II. Kuglasti je kondenzator unutrašnjeg radijusa $R_1$, a vanjskog radijusa $R_2$ ispunjen s dva dielektrika, tako da jednu polovicu ispunjava dielektrik dielektričnosti $\varepsilon_1 = 1$, a drugu dielektrik dielektričnosti $\varepsilon_2 = 3$ prema slici. Ako je na unutrašnjoj elektrodi raspoređen ukupni naboj $+Q$, a vanjska je elektroda uzemljena odredite:

5. Vektor gustoće električnog tijeka u prostoru dielektričnosti $\varepsilon_1$.

$A)\ D = \frac{Q}{2\pi r^2}$ $B)\ D = \frac{Q}{4\pi r^2}$ $C)\ D = \frac{Q}{6\pi r^2}$ $\boxed{D)\ D = \frac{Q}{8\pi r^2}}$ $E)\ D = \frac{Q}{10\pi r^2}$ $F)\ D = \frac{Q}{12\pi r^2}$

6. Vektor gustoće električnog tijeka u prostoru dielektričnosti $\varepsilon_2$.

$A)\ D=\dfrac{Q}{2\pi r^2}$ $B)\ D=\dfrac{Q}{3\pi r^2}$ $C)\ D=\dfrac{5Q}{6\pi r^2}$ $D)\ D=\dfrac{Q}{8\pi r^2}$ $\boxed{E)\ D=\dfrac{3Q}{8\pi r^2}}$ $F)\ D=\dfrac{7Q}{12\pi r^2}$

7. Napon na kondenzatoru

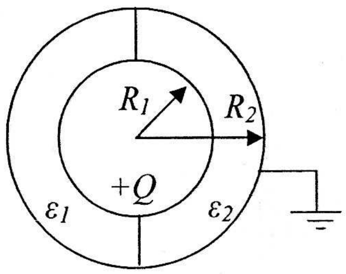
$A)U=\dfrac{Q}{6\pi}\dfrac{R_2-R_1}{R_1R_2}$ $\boxed{B)U=\dfrac{Q}{8\pi}\dfrac{R_2-R_1}{R_1R_2}}$ $C)U=\dfrac{Q}{12\pi}\dfrac{R_2-R_1}{R_1R_2}$

$D)U=\dfrac{Q}{18\pi}\dfrac{R_2-R_1}{R_1R_2}$ $E)U=\dfrac{5Q}{6\pi}\dfrac{R_2-R_1}{R_1R_2}$ $F)U=\dfrac{3Q}{8\pi}\dfrac{R_2-R_1}{R_1R_2}$

8. Kapacitet kondenzatora

$A)C=12\pi\dfrac{R_2R_1}{R_2-R_1}$ $\boxed{B)C=8\pi\dfrac{R_2R_1}{R_2-R_1}}$ $C)C=\pi\dfrac{R_2R_1}{R_2-R_1}$

$D)C=6\pi\dfrac{R_2R_1}{R_2-R_1}$ $E)C=3\pi\dfrac{R_2R_1}{R_2-R_1}$ $F)C=15\pi\dfrac{R_2R_1}{R_2-R_1}$




III. Beskonačno dugi metalni cilindar radijusa $R$ = 0,02m nabijen nabojem površinske gustoće $\sigma$ =5 µC / m$^2$ smješten je na visinu h =2m iznad uzemljene vodljive plohe prema slici.

9. Odredite jakost električnog polja u točki A.

$A)\ E_A=\dfrac{5\cdot10^{-7}}{\varepsilon_0}$ $B)\ E_A=\dfrac{2\cdot10^{-7}}{\varepsilon_0}$ $\boxed{C)\ E_A=\dfrac{1\cdot10^{-7}}{\varepsilon_0}}$ $D)\ E_A=\dfrac{1\cdot10^{-8}}{\varepsilon_0}$ $E)\ E_A=\dfrac{2\cdot10^{-8}}{\varepsilon_0}$ $F)\ E_A=\dfrac{4\cdot10^{-8}}{\varepsilon_0}$

10. Odredite jakost električnog polja u točki B.

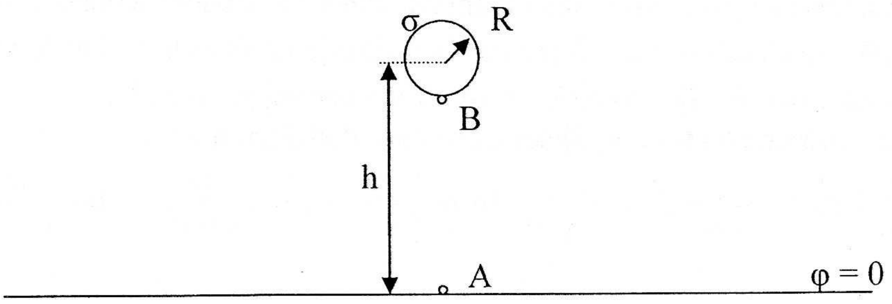
$\boxed{A)\ E_B=\dfrac{5\cdot10^{-6}}{\varepsilon_0}}$ $B)\ E_B=\dfrac{2\cdot10^{-6}}{\varepsilon_0}$ $C)\ E_B=\dfrac{1\cdot10^{-6}}{\varepsilon_0}$ $D)\ E_B=\dfrac{1\cdot10^{-5}}{\varepsilon_0}$ $E)\ E_B=\dfrac{2\cdot10^{-5}}{\varepsilon_0}$ $F)\ E_B=\dfrac{4\cdot10^{-5}}{\varepsilon_0}$

11. Odredite potencijal cilindra.

$A)\ \varphi=\dfrac{727\cdot10^{-9}}{\varepsilon_0}$ $B)\ \varphi=\dfrac{615\cdot10^{-9}}{\varepsilon_0}$ $\boxed{C)\ \varphi=\dfrac{529\cdot10^{-9}}{\varepsilon_0}}$ $D)\ \varphi=\dfrac{217\cdot10^{-9}}{\varepsilon_0}$ $E)\ \varphi=\dfrac{92\cdot10^{-9}}{\varepsilon_0}$ $F)\ \varphi=\dfrac{11\cdot10^{-9}}{\varepsilon_0}$

12. Odredite kapacitet sustava cilindra i plohe po jedinici duljine.

$A)C'=0,57\varepsilon_0$ $B)C'=0,97\varepsilon_0$ $C)C'=1,05\varepsilon_0$ $\boxed{D)C'=1,19\varepsilon_0}$ $E)C'=1,37\varepsilon_0$ $F)C'=2,17\varepsilon_0$



IV. Električno je polje u prostoru 1 zadano jednadžbom $\vec{E}_1 = 3\vec{a}_x - 4\vec{a}_y + 5a_z$ V/m.

13. Odredite komponentu vektora gustoće električnog tijeka u prostoru 2 u smjeru osi x $D_{2x}$.

$A)\ D_{2x} = 2\varepsilon_0\ \ B)\ D_{2x} = 3\varepsilon_0\ \ C)\ D_{2x} = 4\varepsilon_0\ \ D)\ D_{2x} = 5\varepsilon_0\ \ E)\ D_{2x} = 10\varepsilon_0\ \ \boxed{F)\ D_{2x} = 15\varepsilon_0}$

14. Odredite apsolutnu vrijednost vektora gustoće električnog tijeka u prostoru 2 $\left|\vec{D}_2\right|$.

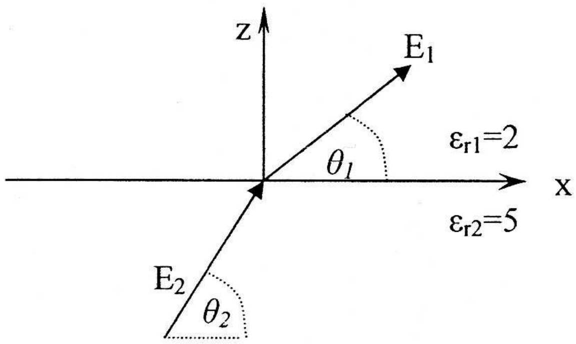
$A)\ \left|\vec{D}_2\right| = 2\sqrt{11}\varepsilon_0\ \ \boxed{B)\ \left|\vec{D}_2\right| = 5\sqrt{29}\varepsilon_0}\ \ C)\ \left|\vec{D}_2\right| = 5\sqrt{17}\varepsilon_0\ \ D)\ \left|\vec{D}_2\right| = 4\sqrt{17}\varepsilon_0\ \ E)\ \left|\vec{D}_2\right| = 3\sqrt{29}\varepsilon_0\ \ F)\ \left|\vec{D}_2\right| = \sqrt{11}\varepsilon_0$

15. Odredite kut $\theta_1$ koji vektor $\vec{E}_1$ zatvara s ravninom x-y.

$A)\theta_1 = 67{,}1°\ \ B)\theta_1 = 54{,}2°\ \ C)\theta_1 = 50{,}1°\ \ \boxed{D)\theta_1 = 45°}\ \ E)\theta_1 = 29°\ \ F)\theta_1 = 21{,}8°$

16. Odredite kut $\theta_2$ koji vektor $\vec{E}_2$ zatvara s ravninom x-y.

$A)\theta_2 = 67{,}1°\ \ B)\theta_2 = 54{,}2°\ \ C)\theta_2 = 50{,}1°\ \ D)\theta_2 = 45°\ \ E)\theta_2 = 29°\ \ \boxed{F)\theta_2 = 21{,}8°}$




V. Vektor gustoće električnog tijeka u statičkom električnom polju u prostoru je zadan jednadžbom:

$$\vec{D} = \vec{a}_y \left( \frac{A}{x+1} + \frac{15}{(y+2)^3} \right)$$

Odredi:

17. Odredite konstantu A

$A)\ A = -2\ \ B)\ A = -1\ \ \boxed{C)\ A = 0}\ \ D)\ A = 1\ \ E)\ A = 2\ \ F)\ A = 3$

18. Odredite izraz za naboj u prostoru

$A)\ \rho = \frac{-5}{(y+2)^4}\ \ B)\ \rho = \frac{-10}{(y+2)^4}\ \ C)\ \rho = \frac{-15}{(y+2)^4}\ \ D)\ \rho = \frac{-25}{(y+2)^4}\ \ E)\ \rho = \frac{-30}{(y+2)^4}\ \ \boxed{F)\ \rho = \frac{-45}{(y+2)^4}}$

19. Odredite električni tok vektora $\vec{D}$ kroz kvadrat (0,0,0), (1m,0,0), (1m,0,1m), (0,0,1m)

$A)\ \varphi = \frac{3}{8}\ \ \boxed{B)\ \varphi = \frac{15}{8}}\ \ C)\ \varphi = \frac{7}{8}\ \ D)\ \varphi = \frac{13}{8}\ \ E)\ \varphi = \frac{1}{8}\ \ F)\ \varphi = \frac{15}{4}$

20. Odredite jakost električnog polja u ishodištu.

$A)\ E = \frac{3}{8\varepsilon_0}\ \ \boxed{B)\ E = \frac{15}{8\varepsilon_0}}\ \ C)\ E = \frac{7}{8\varepsilon_0}\ \ D)\ E = \frac{13}{8\varepsilon_0}\ \ E)\ E = \frac{1}{8\varepsilon_0}\ \ F)\ E = \frac{15}{4\varepsilon_0}$

**I** Zbog centralne simetrije polje je radijalno: $\vec{E} = E(r)\vec{a}_r \quad \vec{n} = \vec{a}_r \quad d\vec{l} = \vec{a}_r dr \quad \vec{E}\vec{a}_r = E(r) = E$

**1.)** primjenjuje se gaussov zakon na prostor unutar ljuske, gdje nema naboja:

$$\oint\!\!\!\oint_S \vec{E}\vec{n}dS = \frac{1}{\varepsilon_0}\iiint_V \rho dV = 0 \Rightarrow E = 0$$

**2.)** primjenjuje se gaussov zakon na prostor $R_1 < r < R_2$

$$\oint\!\!\!\oint_S \vec{E}\vec{n}dS = \frac{1}{\varepsilon_0}\iiint_V \rho dV \qquad dV = 4\pi r^2 dr \qquad \rho = a + br$$

$$E\oint\!\!\!\oint_S dS = \frac{1}{\varepsilon_0}\int_{R_1}^{r}(a+br)\,4\pi r^2 dr \qquad E4\pi r^2 = \frac{1}{\varepsilon_0}4\pi\left(a\int_{R_1}^{r} r^2 dr + b\int_{R_1}^{r} r^3 dr\right)$$

$$E = \frac{1}{\varepsilon_0 r^2}\left[\frac{a}{3}(r^3-8) + \frac{b}{4}(r^4-16)\right]$$

**3.)** primjenjuje se gaussov zakon na prostor $r > R_2$

$$\oint\!\!\!\oint_S \vec{E}\vec{n}dS = \frac{1}{\varepsilon_0}\iiint_V \rho dV \qquad dV = 4\pi r^2 dr \qquad \rho = a + br$$

$$E\oint\!\!\!\oint_S dS = \frac{1}{\varepsilon_0}\int_{R_1}^{R_2}(a+br)\,4\pi r^2 dr \qquad E4\pi r^2 = \frac{1}{\varepsilon_0}4\pi\left(a\int_{R_1}^{R_2} r^2 dr + b\int_{R_1}^{R_2} r^3 dr\right)$$

$$E = \frac{1}{\varepsilon_0 r^2}\left[\frac{19a}{3} + \frac{65b}{4}\right]$$

**4.)** Potencijal unutar sferne ljuske je konstantan jer je polje nula. $\vec{E} = -grad\varphi \quad \vec{E} = 0 \quad \varphi = konst.$

$$\varphi_{R_2} - \varphi_{R_{ref}} = -\int_{R_{ref}}^{R_2}\vec{E}d\vec{l} \qquad \varphi_{R_2} - 0 = -\int_{\infty}^{R_2}\vec{E}d\vec{l} = -\int_{\infty}^{R_2}\frac{1}{\varepsilon_0 r^2}\left[\frac{19a}{3}+\frac{65b}{4}\right]dr = \frac{1}{\varepsilon_0}\left[\frac{19a}{3}+\frac{65b}{4}\right]\frac{1}{R_2}$$

$$\varphi_{R_2} = \frac{1}{\varepsilon_0}\left[\frac{19a}{9}+\frac{65b}{12}\right] \qquad \varphi_{R_1} - \varphi_{R_2} = \int_{R_1}^{R_2}\vec{E}d\vec{l} = \int_{R_1}^{R_2}\frac{1}{\varepsilon_0 r^2}\left[\frac{a}{3}(r^3-8)+\frac{b}{4}(r^4-16)\right]dr$$

$$\varphi_0 = \varphi_{R_1} = \varphi_{R_2} + \frac{1}{\varepsilon_0}\int_2^3\left(\frac{a}{3}r - \frac{8a}{3r^2} + \frac{b}{4}r^2 - \frac{16b}{4r^2}\right)dr = \frac{1}{\varepsilon_0}\left[\frac{19a}{9}+\frac{65b}{12}\right] + \frac{1}{\varepsilon_0}\left[\frac{7a}{18}+\frac{11b}{12}\right] = \frac{1}{\varepsilon_0}\left[\frac{5a}{2}+\frac{19b}{3}\right]$$

**II** Zbog sferne simetrije polje je radijalno pa je zato i tangencijalno na granicu između dielektrika. Kako su tangencijalne komponente električnog polja jednake u oba dielektrika, a samo one i postoje (polje je tangencijalno na granicu => normalne komponente su nula) polje je jednako u oba dielektrika (napon i razmak između elektroda je isti u oba dielektika). Gustoća naboja nije ista u oba dielektrika, naboj se raspodjeli proporcionalno dielektričnim konstantama. Vektor gustoće električnog toka je različit.

$$\oiint_S \vec{E}\vec{n}dS = \iiint_V \frac{1}{\varepsilon}\rho dV = \iint_S \frac{1}{\varepsilon}\sigma dS = \sum_i \frac{Q_i}{\varepsilon_i} \qquad \sigma = \frac{dQ}{dS} \qquad \sigma = \frac{Q}{S}$$

$$Q = Q_1 + Q_2 \qquad \frac{Q_1}{Q_2} = \frac{\varepsilon_1}{\varepsilon_2} \qquad S_1 = S_2 \qquad D = \varepsilon E$$

**5.**) $$\oiint_S \vec{E}\vec{n}dS = \iint_S \frac{1}{\varepsilon}\sigma dS \qquad E4\pi r^2 = \frac{\sigma_1 S_1}{\varepsilon_1} + \frac{\sigma_2 S_2}{\varepsilon_2} = \frac{Q_1}{\varepsilon_1} + \frac{Q_2}{\varepsilon_2} \qquad Q = Q_1 + Q_2 \qquad \frac{Q_1}{Q_2} = \frac{\varepsilon_1}{\varepsilon_2}$$

$$Q_1 = \frac{1}{4}Q \qquad Q_2 = \frac{3}{4}Q \qquad E4\pi r^2 = \frac{1}{4}\frac{Q}{\varepsilon_1} + \frac{3}{4}\frac{Q}{\varepsilon_2} \implies E = \frac{Q}{8\pi\varepsilon_0 r^2} \qquad D = \varepsilon E \qquad D_1 = \frac{Q}{8\pi r^2}$$

ili

$$\oiint_S \vec{D}\vec{n}dS = \iiint_V dQ = Q \quad \rightarrow \quad D_1 S_1 + D_2 S_2 = Q \qquad \varepsilon_1 E_1 S_1 + \varepsilon_2 E_2 S_2 = Q \qquad S_1 = S_2 = S \qquad E_1 = E_2 = E$$

$$ES = \frac{Q}{\varepsilon_1 + \varepsilon_2} \qquad E2\pi r^2 = \frac{Q}{4\varepsilon_0} \qquad E = \frac{Q}{8\pi\varepsilon_0 r^2} \qquad D = \varepsilon E$$

$S_1$,$S_2$ → polovica sfere

**6.**) $$D_2 = \varepsilon_2 E_2 = \varepsilon_2 E = 3\varepsilon_0 \frac{Q}{8\pi\varepsilon_0 r^2} = \frac{3Q}{8\pi r^2}$$

**7.**) $$U_{12} = \varphi_1 - \varphi_2 = \int_1^2 \vec{E}d\vec{l} \qquad U = \int_{R_1}^{R_2} \frac{Q}{8\pi\varepsilon_0 r^2}dr = \frac{Q}{8\pi\varepsilon_0}\left(\frac{-1}{r}\right)\Bigg|_{R_1}^{R_2} = \frac{Q}{8\pi\varepsilon_0}\frac{R_2 - R_1}{R_1 R_2}$$

**8.**) $$C = \frac{Q}{U} = 8\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}$$

* napomena → u službenim rješenjima 7. i 8. fali $\varepsilon_0$

**III** Beskonačno dugi metalni cilindar nabijen nabojem površinske gustoće σ modelira se linijskim nabojem λ smještenim u sredini cilindra. Koristi se metoda odslikavanja, pa se račun provodi sa nabojem +λ koji se nalazi 2m iznad zemlje i nabojem –λ koji se nalazi 2m ispod zemlje. Polje i potencijal linijskog naboja su:

$$E(r) = \frac{\lambda}{2\pi\varepsilon r} \qquad \varphi(r) = \frac{\lambda}{2\pi\varepsilon}\ln\frac{r_{ref}}{r} \qquad \sigma = \frac{dQ}{dS} \qquad \lambda = \frac{dQ}{dl} \qquad dS = 2\pi R dl \qquad \lambda = \sigma 2\pi R$$

**9.**) $E_A = E_{+\lambda} + E_{-\lambda} = 2E = 2\dfrac{\sigma 2\pi R}{2\pi\varepsilon 2} = \dfrac{1\cdot 10^{-7}}{\varepsilon_0}$

**10.**) $E_B = E_{+\lambda} + E_{-\lambda} = \dfrac{\sigma 2\pi R}{2\pi\varepsilon R} + \dfrac{\sigma 2\pi R}{2\pi\varepsilon 3{,}98} = \dfrac{5{,}025\cdot 10^{-6}}{\varepsilon_0}$

**11.**) $\varphi = \varphi_{+\lambda} + \varphi_{-\lambda} = \dfrac{\sigma 2\pi R}{2\pi\varepsilon}\ln\dfrac{h}{R} + \dfrac{-\sigma 2\pi R}{2\pi\varepsilon}\ln\dfrac{h}{3{,}98} = \dfrac{529{,}33\cdot 10^{-9}}{\varepsilon_0}$

$r_{ref}$ = udaljenost referentne točke potencijala φ=0 (zemlje) do naboja koji stvara potencijal
r = udaljenost točke promatranja do naboja koji stvara potencijal

**12.**) $\dfrac{C}{l} = \dfrac{Q}{Ul} = \dfrac{\lambda l}{Ul} = \dfrac{\lambda}{\varphi} = \dfrac{\sigma 2\pi R\varepsilon_0}{529{,}33\cdot 10^{-9}} = 1{,}18775\varepsilon_0$

**IV** $\vec{n}(\vec{D}_2 - \vec{D}_1) = \sigma_s \qquad \vec{n}\times(\vec{E}_2 - \vec{E}_1) = 0$ => tangencijalne komponente električnog polja su jednake, normalne komponente toka su jednake (D=εE)

$$\vec{n} = \vec{a}_z \qquad \sigma_s = 0$$

$$\vec{E}_{1T} = 3\vec{a}_x - 4\vec{a}_y = \vec{E}_{2T} \qquad \varepsilon_1\vec{E}_{1N} = 2\varepsilon_0\cdot 5\vec{a}_z = \varepsilon_2\vec{E}_{2N} \;\Rightarrow\; \vec{E}_{2N} = \frac{2\cdot 5\vec{a}_z}{5} = 2\vec{a}_z$$

$$\vec{E}_2 = \vec{E}_{2T} + \vec{E}_{2N} = 3\vec{a}_x - 4\vec{a}_y + 2\vec{a}_z \qquad \vec{D}_2 = \varepsilon_2\vec{E}_2$$

ili:

$$\vec{a}_z(\varepsilon_2 E_{2x}\vec{a}_x + \varepsilon_2 E_{2y}\vec{a}_y + \varepsilon_2 E_{2z}\vec{a}_z - \varepsilon_1 E_{1x}\vec{a}_x - \varepsilon_1 E_{1y}\vec{a}_y - \varepsilon_1 E_{1z}\vec{a}_z) = 0$$

$$\varepsilon_2 E_{2z} = \varepsilon_1 E_{1z} \qquad E_{2z} = \frac{\varepsilon_1}{\varepsilon_2}E_{1z} = \frac{2}{5}5 = 2$$

$$\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ 0 & 0 & 1 \\ E_{2x}-3 & E_{2y}+4 & E_{2z}-5 \end{vmatrix} = \vec{a}_x(-1)(E_{2y}+4) - \vec{a}_y(-1)(E_{2x}-3) + \vec{a}_z\cdot 0 = 0$$

$$E_{2y} = -4 \qquad E_{2x} = 3 \qquad \vec{E}_2 = 3\vec{a}_x - 4\vec{a}_y + 2\vec{a}_z \qquad \vec{D}_2 = \varepsilon_2\vec{E}_2 = \varepsilon_0(15\vec{a}_x - 20\vec{a}_y + 10\vec{a}_z)$$

**13.)** $D_{2x} = 15\varepsilon_0$

**14.)** $|\vec{D}_2| = \varepsilon_0\sqrt{15^2 + 20^2 + 10^2} = 5\sqrt{29}\varepsilon_0$

**15.**) $\theta_1 = \sin^{-1}\dfrac{E_{1z}}{|\vec{E}_1|} = 45°$

**16.**) $\theta_2 = \sin^{-1}\dfrac{E_{2z}}{|\vec{E}_2|} = 21{,}8° \qquad \dfrac{\operatorname{tg}\alpha_1}{\operatorname{tg}\alpha_2} = \dfrac{\varepsilon_1}{\varepsilon_2} \qquad \theta_{1,2} = 90° - \alpha_{1,2}$

**V** U statičkom električnom polju vrijedi zakon o konzervativnosti električnog polja:

$$\oint_c \vec{E} d\vec{l} = 0 \qquad \nabla \times \vec{E} = rot\vec{E} = \vec{0}$$

**17.)** $\vec{D} = \varepsilon \vec{E} \qquad rot\vec{E} = \vec{0}$

$$rot\vec{E} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} = \vec{a}_x \cdot (-1) \cdot \frac{\partial}{\partial z}\left(\frac{A}{x+1} + \frac{15}{(y+2)^3}\right) - \vec{a}_y \cdot 0 + \vec{a}_z \frac{\partial}{\partial x}\left(\frac{A}{x+1} + \frac{15}{(y+2)^3}\right) = 0$$

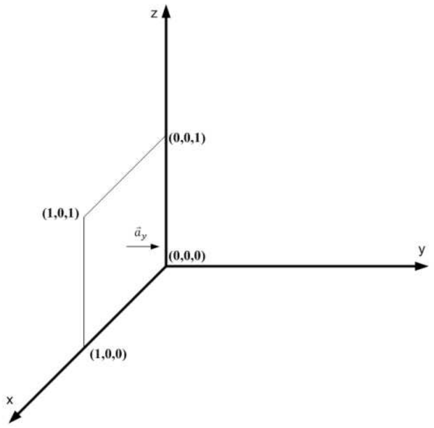
$$\vec{a}_x \cdot 0 + \vec{a}_z \cdot \left(\frac{-A}{(x+1)^3}\right) = 0 \quad \Rightarrow \quad A = 0$$

**18.)** $\varepsilon \nabla \vec{\mathrm{E}} = \rho \qquad \nabla \vec{\mathrm{D}} = \rho$

$$\left(\vec{a}_x \frac{\partial}{\partial x} + \vec{a}_y \frac{\partial}{\partial y} + \vec{a}_z \frac{\partial}{\partial z}\right) \cdot \left(\vec{a}_y \frac{15}{(y+2)^3}\right) = \frac{-45}{(y+2)^4} = \rho$$

**19.)** $\vec{n} = \vec{a}_y \qquad S = 1\ m^2 \qquad \vec{D} \cdot \vec{n} = \frac{15}{(y+2)^3}$

$$\varphi = \iint_S \vec{D}\vec{n} dS = \iint_S \frac{15}{(y+2)^3} dxdz = \frac{15}{(0+2)^3} \int_0^1 dx \int_0^1 dz = \frac{15}{8} S = \frac{15}{8}$$




**20.)**

$$\vec{E}(x,y,z) = \frac{\vec{D}(x,y,z)}{\varepsilon_0} = \frac{1}{\varepsilon_0} \vec{a}_y \left(\frac{15}{(y+2)^3}\right) \qquad E(x,y,z) = \frac{1}{\varepsilon_0}\left(\frac{15}{(y+2)^3}\right) \qquad E(0,0,0) = \frac{15}{8\varepsilon_0}$$