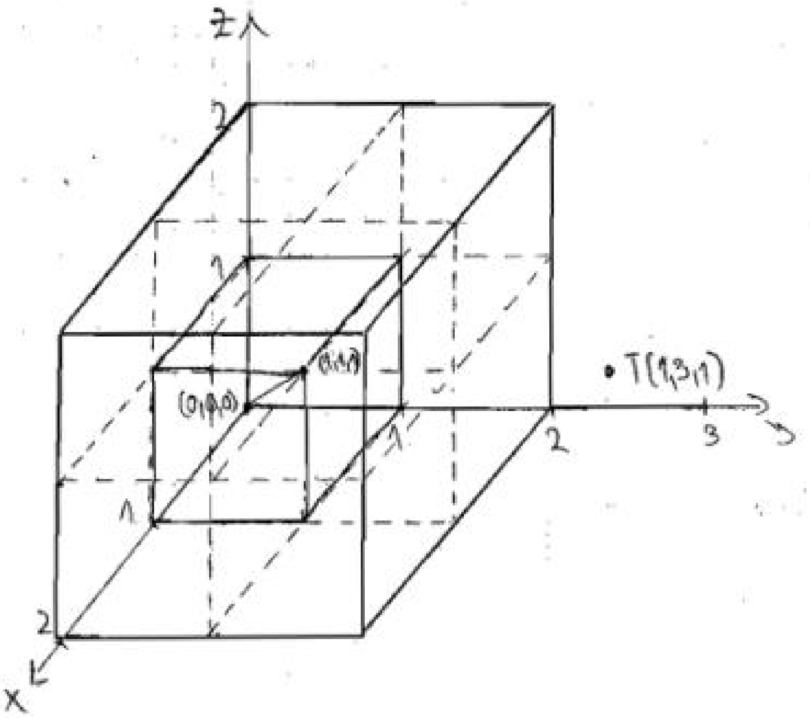
2. MI 2007/2008

I.

U VAKUUMU VLADA STATIČKO MAGNETSKO POLJE ZADANO JEDNADŽBOM:

$$\vec{B} = \frac{Ax}{b^2+y}\vec{a}_x + c\ln(b^2+y)\,\vec{a}_y \quad ; \quad A=3 \quad ; \quad b=3$$



① ODREDITE KONSTANTU c !

$$\boxed{\nabla\vec{B}=0 \Rightarrow \oint_S \vec{B}\,\vec{n}\,dS=0}$$ - GAUSSOV ZAKON ZA MAGNETSKO POLJE !

- MAGNETSKA INDUKCIJA U PRAVOKUTNOM KOORDINATNOM SUSTAVU (x,y,z):

$$\boxed{\vec{B} = \mathrm{rot}\,\vec{A} = \vec{a}_x\left(\frac{\partial A_z}{\partial y}-\frac{\partial A_y}{\partial z}\right)+\vec{a}_y\left(\frac{\partial A_x}{\partial z}-\frac{\partial A_z}{\partial x}\right)+\vec{a}_z\left(\frac{\partial A_y}{\partial x}-\frac{\partial A_x}{\partial y}\right)}$$

$$\boxed{\nabla\vec{B} = \frac{\partial B_x}{\partial x}+\frac{\partial B_y}{\partial y}+\frac{\partial B_z}{\partial z}}$$ - DIVERGENCIJA U PRAVOKUTNOM KOORDINATNOM SUSTAVU !

$\nabla \vec{B} = 0$

$$\nabla \vec{B} = \frac{\partial}{\partial x}\left[\frac{3x}{9+y}\right] + \frac{\partial}{\partial y}\left[c \cdot \ln(9+y)\right] = 0$$

$$\frac{3}{9+y} + c \cdot \frac{1}{9+y} = 0 \Rightarrow \boxed{c = -3}$$

② ODREDI GUSTOĆU STRUJE U TOČKI (1, 3, 1)!

$\boxed{\nabla \times \vec{B} = \mu_0 \vec{J}}$ - TEMELJNI ZAKON STATIČKOG MAGNETSKOG POLJA!

$$\vec{J} = \frac{1}{\mu_0} \nabla \times \vec{B}$$

$$\vec{J} = \frac{1}{\mu_0} \cdot \begin{bmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{3x}{9+y} & -3\ln(9+y) & 0 \end{bmatrix}$$

$$\vec{J} = \frac{1}{\mu_0}\left[\vec{a_x} \cdot 0 - \vec{a_y} \cdot 0 + \vec{a_z} \cdot \left(0 - \frac{\partial}{\partial y}\left(\frac{3x}{9+y}\right)\right)\right]$$

$$\boxed{\vec{J} = \frac{1}{\mu_0} \cdot \frac{3x}{(9+y)^2} \vec{a_z}}$$

$\boxed{\vec{J}\Big|_{T(1,3,1)} = \vec{a_z} \frac{1}{\mu_0 48}}$ - GUSTOĆA STRUJE U TOČKI T(1,3,1)!

③ ODREDITE UKUPNU STRUJU KROZ KVADRAT DEFINIRAN DIJAGONALOM $T_1(0,0,0)$ i $T_2(1,1,0) \Rightarrow d = \overline{T_1T_2}$!

- GUSTOĆA STRUJE: $\boxed{\gamma = \frac{I}{dS}}$ [A/m²]

- JAKOST STRUJE:

$$\boxed{I = \gamma \, dS} \quad [A]$$

$S \Rightarrow$ KVADRAT S DIJAGONALOM $d = \overline{T_1T_2}$!

$$\boxed{I = \iint_S \vec{\gamma}\, \vec{n}\, dS} \Rightarrow \vec{n}\, dS = \vec{a_z}\, dx\, dy$$

$$I = \iint_S \vec{\gamma}\, \vec{n}\, dS = \iint_{\Omega_{xy}} \frac{1}{\mu_0} \cdot \frac{3x}{(9+y^2)} \vec{a_z} \cdot \vec{a_z}\, dx\, dy$$

$$= \frac{1}{\mu_0} \int_0^1 \int_0^1 \frac{3x}{(9+y)^2}\, dx\, dy = \frac{1}{\mu_0} \int_0^1 3x\, dx \int_0^1 \frac{1}{(9+y)^2}\, dy = \boxed{\frac{1}{60\mu_0}} \, [A]$$

④ ODREDI UKUPNI TOK ZADANOG POLJA KROZ POVRŠINU KOCKE ZADANE PROSTORNOM DIJAGONALOM $T_1(1,1,1)$ i $T_2(2,2,2)$!

$$\boxed{\Phi = \oiint \vec{B}\, \vec{n}\, dS} \text{ - MAGNETSKI TOK !}$$

- KOCKA ZADANA PROSTORNOM DIJAGONALOM $d = \overline{T_1T_2}$ JE ZATVORENA POVRŠINA, A UKUPNI MAGNETSKI TOK KROZ ZATVORENU POVRŠINU JE NULA !

$$\boxed{\Phi = \oiint \vec{B}\, \vec{n}\, dS = 0 \quad ; \text{ S - ZATVORENA !}}$$

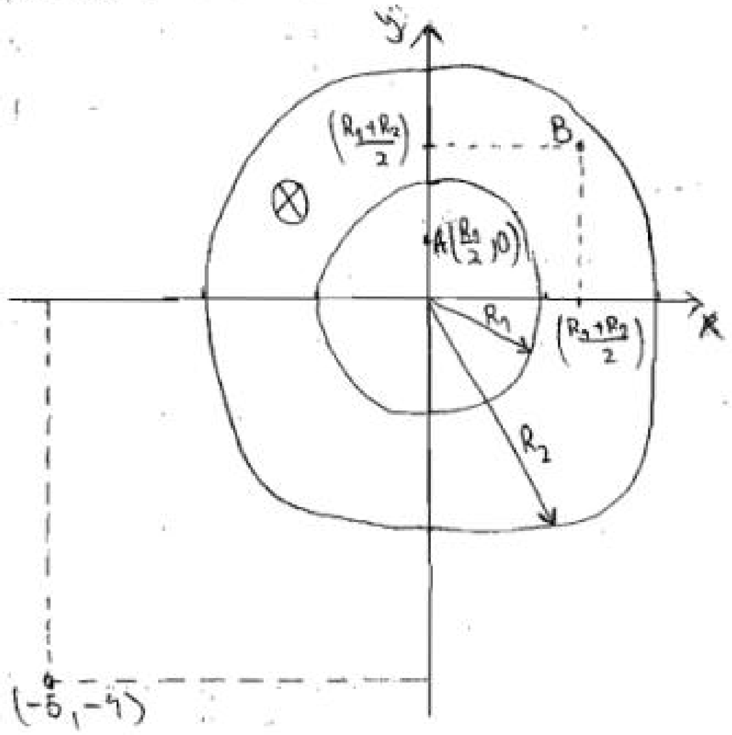
II. ZADAN JE BESKONAČNO DUGI CIJEVNI VODIĆ PREMA SLICI. KROZ VODIĆ TEČE STRUJA I JEDNOLIKO RASPOREĐENA PO PRESJEKU.

- ZADANO JE:

$R_1 = 1\ [m]$

$R_2 = 4\ [m]$

$I = 15\ [A]$



(5) ODREDI VEKTOR JAKOSTI MAGNETSKOG POLJA U TOČKI $A\left(\frac{R_1}{2},0\right)$!

- VEKTOR JAKOSTI MAGNETSKOG POLJA $\vec{H}$ JEDNAK JE PO CIJELOJ KRUŽNICI PA SE KORISTI IZRAZ:

$$\boxed{\oint_{K} \vec{H}\, d\vec{l} = \Sigma I}$$ - ZAKON PROTJECANJA !

K ... KRUŽNICA $\Rightarrow \oint_{K} d\vec{l} = 2r\pi$; $r = \frac{R_1}{2}$

$\vec{H} \cdot 2 \cdot \frac{R_1}{2}\pi = \Sigma I$; $\vec{H} R_1 \pi = \Sigma I$

$$\boxed{\Sigma I = 0}$$ - ZATVORENA KRIVULJA - KRUŽNICA !

$$\boxed{\vec{H} = 0}$$

⑥ ODREDI VEKTOR JAKOSTI MAGNETSKOG POLJA $\vec{H}$ U TOČKI:

$B\left(\frac{R_1+R_2}{2} ; \frac{R_1+R_2}{2}\right)$!

$$B\left(\frac{1+4}{2} ; \frac{1+4}{2}\right) \Rightarrow B(2.5, 2.5) \Rightarrow r = \sqrt{2.5^2+2.5^2}$$

$$r = 2.5\sqrt{2}\ [m]$$

– BIOT-SAVARTOV ZAKON:

$$\boxed{dH = \frac{I}{4\pi} \frac{|dl \times R|}{R^3}} \Rightarrow \boxed{H = \int dH = \frac{I}{4\pi} \int \frac{|dl \times R|}{R^3}}$$

– MAGNETSKO POLJE BESKONAČNO DUGOG RAVNOG VODIČA:

$$\boxed{|\vec{H}| = \frac{I}{2r\pi}} \quad \boxed{\vec{H} = |\vec{H}| \cdot d\vec{l}}$$

– U ZADANOM SLUČAJU: $\vec{H} \parallel d\vec{l} \Rightarrow \boxed{\vec{H} d\vec{l} = Hl}$

$$\oint_{\kappa} \vec{H}\, d\vec{l} = \frac{1}{\mu_0} \int_S \gamma\, \vec{m}\, dS = \frac{1}{\mu_0} \gamma \cdot S' = \frac{1}{\mu_0} \cdot \frac{\mu_0 I}{r_0^2 \pi} \cdot S'$$

$$Hl = \frac{I}{r_0^2 \pi} \cdot r^2 \pi = \frac{I}{r_0^2} \cdot r^2$$

$$Hl = H \cdot 2\pi r$$

$$\frac{I}{r_0^2} r^2 = H \cdot 2\pi r \Rightarrow H = \frac{I}{2\pi r_0^2} \cdot \frac{r^2}{r} \Rightarrow \boxed{H = \frac{I}{2\pi r_0^2} \cdot r}$$

$$r_0 = 2.5\sqrt{2}\ [m]; \quad r = 1\ [m]$$

$$H = \frac{15}{2\pi \cdot (2.5\sqrt{2})^2} \cdot 1^2 = \frac{15}{2\pi \cdot 12.5} = 0.1909\ [A/m]$$

(7) ODREDI VEKTOR JAKOSTI U TOČKI $C(-(R_1+R_2), -R_2)$!

$C(-(1+4), -4) \Rightarrow C(-5, -4)$

$$\boxed{\vec{H} = |\vec{H}| \cdot |d\vec{l}|}$$

$$\boxed{|\vec{H}| = \frac{I}{2r\pi}} \; ; \; \boxed{d\vec{l} = l_x \vec{a_x} + l_y \vec{a_y}} \; ; \; \boxed{|d\vec{l}| = \frac{d\vec{l}}{r}}$$

$l_x = -5$; $l_y = -4$

$$r = \sqrt{(l_x)^2 + (l_y)^2} = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$$

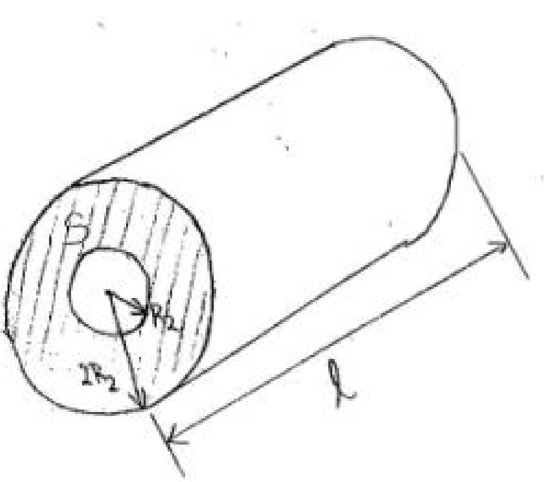
$$\vec{H} = \frac{15}{2\pi \cdot \sqrt{41}} \cdot \left(\frac{-5\vec{a_x} - 4\vec{a_y}}{\sqrt{41}} \right)$$

$$\boxed{\vec{H} = -0.233\,\vec{a_x} - 0.291\,\vec{a_y}}$$

(8) ODREDI ENERGIJU POHRANJENU U MAGNETSKOM POLJU U PROSTORU OKO VODIČA IZMEĐU R_2 i $2R_2$ DULJINE 1m!

- ENERGIJA MAGNETSKOG POLJA:

$$\boxed{W = \int_V \vec{B}\vec{H}\, dV} \qquad \boxed{\vec{B} = \mu_0 \vec{H}} \Rightarrow \boxed{W = \frac{\mu_0}{2} \iiint_V H^2\, dV}$$



$$\iiint_V dV = \iiint r \, d\varphi \, dr \, dz$$

$$d\varphi = 2\pi \quad ; \quad dz = dl = 1 \, [m]$$

$$\iiint dV = 2\pi \cdot 1 \cdot \int r \, dr$$

$$W = \frac{\mu_0}{2} \cdot 2\pi \int_{R_2}^{2R_2} \left(\frac{I}{2r\pi} \right)^2 r \, dr =$$

$$= \frac{\mu_0}{2} \cdot 2\pi \cdot \frac{I^2}{4\pi^2} \int_{R_2}^{2R_2} \frac{1}{r^2} \cdot r \, dr =$$

$$= \frac{\mu_0 I^2}{4\pi} \int_4^8 \frac{1}{r} \, dr = \frac{\mu_0 I^2}{4\pi} \ln(r) \Big|_4^8 =$$

$$= \frac{\mu_0 I^2}{4\pi} \ln\left(\frac{8}{4} \right) = \frac{4\pi \cdot 10^{-7} \cdot 15^2}{4\pi} \ln(2) = \boxed{15.6 \, [\mu J]}$$

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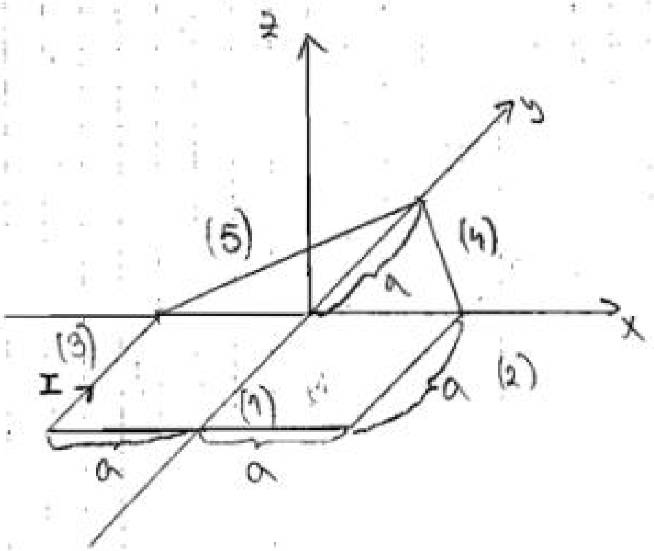
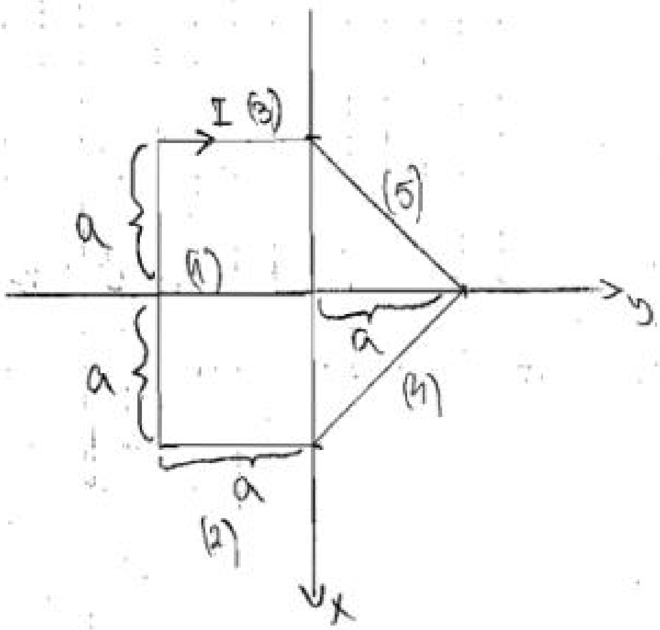
III. STRUJNA JE PETLJA ZADANA SLIKOM UZ $a = 1\,[m]$, $I = 11\,[A]$



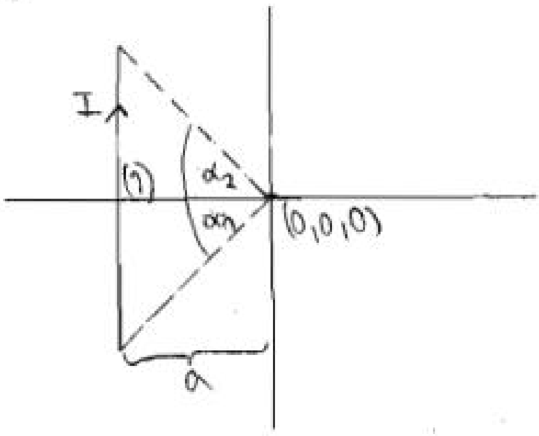


5) ODREDITE APSOLUTNU VRIJEDNOST VEKTORA JAKOSTI MAGNETSKOG POLJA U TOČKI (0,0,0).

- MAGNETSKO POLJE RAVNOG VODIČA KONAČNE DULJINE:

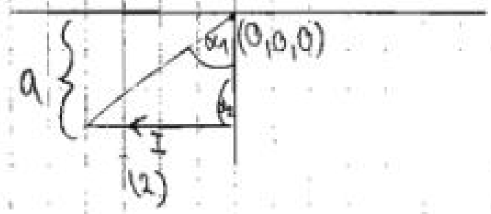
$$\boxed{|\vec{H}| = \frac{I}{4\pi d}\left(\sin\alpha_1 - \sin\alpha_2\right)}$$

- PODRUČJE (1):



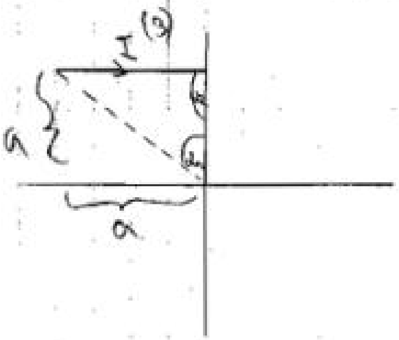
$$|\vec{H}_1| = \frac{I}{4\pi a}\left(\sin 45^\circ + \sin 45^\circ\right) = \boxed{\frac{I\sqrt{2}}{4\pi a}}$$

- PODRUČJE (2):



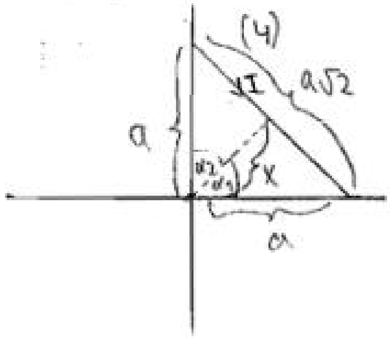
$$|\vec{H}_2| = \frac{I}{4\pi a}\left(\sin 45^\circ + \sin 0^\circ\right) = \frac{I\sqrt{2}}{8\pi a}$$

- PODRUČJE (3):



$$|\vec{H}_3| = \frac{I}{4\pi a}\left(\sin 45^\circ + \sin 0^\circ\right) = \frac{I\sqrt{2}}{8\pi a}$$

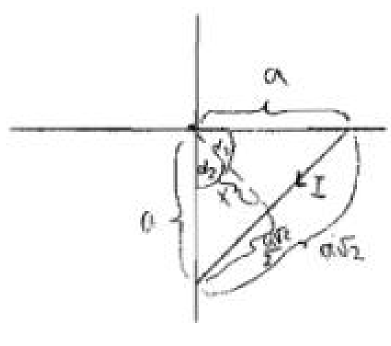
- PODRUČJE (4):



$$x = \sqrt{a^2 - \frac{a^2 2}{4}} = \sqrt{\frac{a^2}{2}} = \frac{a}{\sqrt{2}}$$

$$|\vec{H}_4| = \frac{I}{4\pi \frac{a}{\sqrt{2}}}\left(\sin 45^\circ + \sin 45^\circ\right) = \frac{I}{2\pi a}$$

- PODRUČJE (5):



$$x = \frac{a}{\sqrt{2}}$$

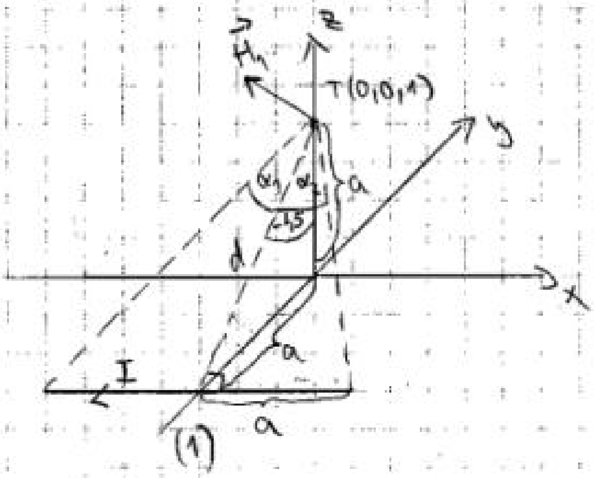
$$|\vec{H}_5| = \frac{I\sqrt{2}}{8\pi a}\left(\sin 45^\circ + \sin 45^\circ\right) = \frac{I}{2\pi a}$$

- APSOLUTNA VRIJEDNOST VEKTORA JAKOSTI MAGNETSKOG POLJA:

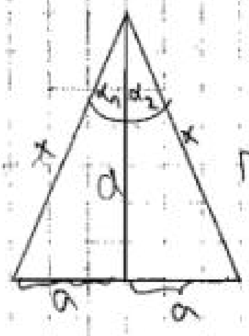
$$|\vec{H}| = |\vec{H}_1| + |\vec{H}_2| + |\vec{H}_3| + |\vec{H}_4| + |\vec{H}_5| = \boxed{5.977\ [A/m]}$$

(10) ODREDITE KOMPONENTU VEKTORA JAKOSTI MAGNETSKOG POLJA U TOČKI $(0,0,1)\,[m]$ U SMJERU X-OSI ?

- PODRUČJE (1):



$$d=\sqrt{a^2+a^2}=\sqrt{2a^2}=a\sqrt{2}$$



$$x=\sqrt{(a\sqrt{2})^2+a^2}=a\sqrt{3}$$

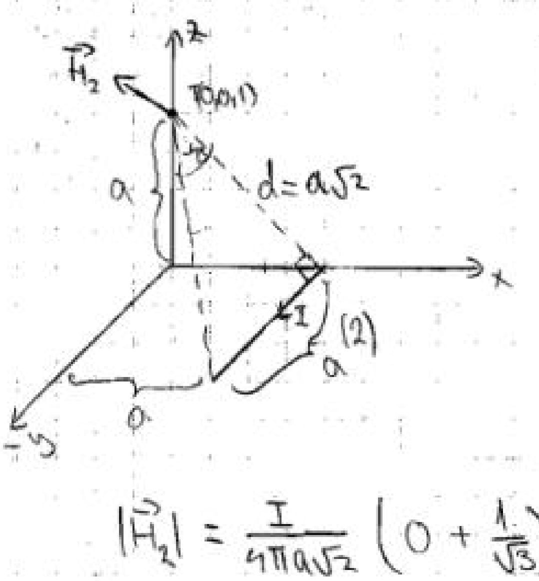
$$\sin\alpha_{1,2}=\frac{a}{a\sqrt{3}}=\frac{1}{\sqrt{3}}$$

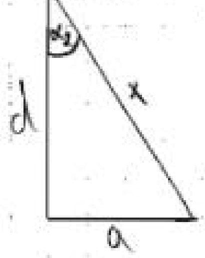
$$|\vec{H}_1|=\frac{I}{4\pi a\sqrt{2}}\left(\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}}\right)=\frac{I}{2\pi a\sqrt{6}}$$

$$\vec{H}_1=|\vec{H}_1|\cdot\vec{a}_\alpha \quad ; \quad \vec{a}_\alpha=-\sin(45^\circ)\vec{a}_y+\sin(-45^\circ)\vec{a}_z=\frac{1}{\sqrt{2}}\vec{a}_y-\frac{1}{\sqrt{2}}\vec{a}_z$$

$$\vec{H}_1=\frac{I}{2\pi a\sqrt{6}}\left(\frac{\vec{a}_y-\vec{a}_z}{\sqrt{2}}\right)=\boxed{\frac{I}{4\pi a\sqrt{3}}\left(\vec{a}_y-\vec{a}_z\right)}$$

- PODRUČJE (2):





$$\sin\alpha_1=0$$

$$\sin\alpha_2=\frac{a}{x}$$

$$x=\sqrt{(a\sqrt{2})^2+a^2}=\sqrt{2a^2+a^2}=a\sqrt{3}$$

$$\sin\alpha_2=\frac{a}{a\sqrt{3}}=\frac{1}{\sqrt{3}}$$

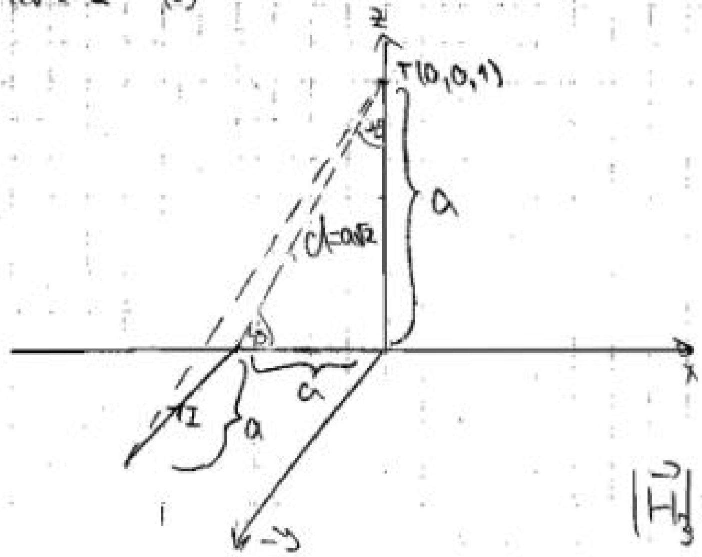
$$|\vec{H}_2|=\frac{I}{4\pi a\sqrt{2}}\left(0+\frac{1}{\sqrt{3}}\right)=\frac{I}{4\pi a\sqrt{6}}$$

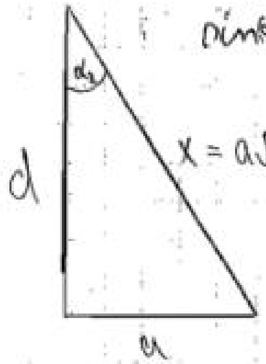
$$\vec{H}_2=|\vec{H}_2|\cdot\vec{a}_\alpha \quad ; \quad \vec{a}_\alpha=\sin(-45^\circ)\vec{a}_x+\sin(-45^\circ)\vec{a}_y$$

$$=-\frac{1}{\sqrt{2}}\vec{a}_x-\frac{1}{\sqrt{2}}\vec{a}_y$$

$$\boxed{\vec{H}_2=\frac{I}{8\pi a\sqrt{3}}\left(-\vec{a}_x-\vec{a}_z\right)}$$

- PODRUČJE (3):





$\sin(\alpha_1) = 0$

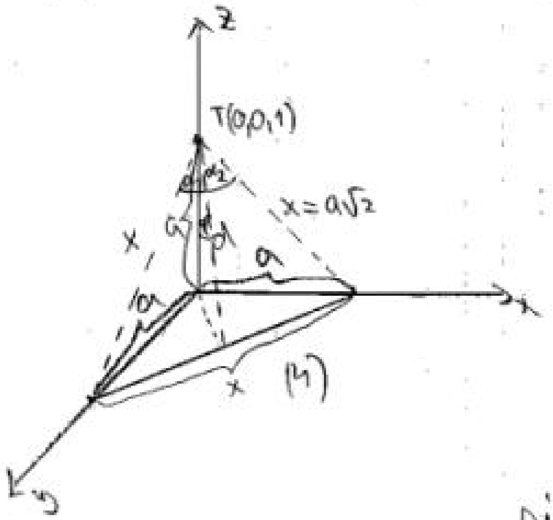
$x = a\sqrt{3} \Rightarrow \sin\alpha_2 = \frac{a}{a\sqrt{3}} = \frac{1}{\sqrt{3}}$

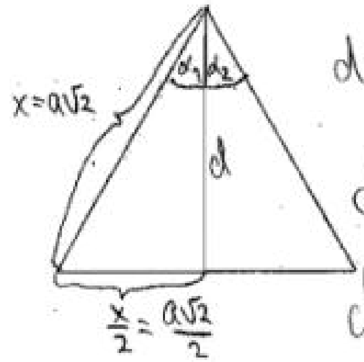
$$|\vec{H_3}| = \frac{I}{4\pi a\sqrt{2}}\left(0 + \frac{1}{\sqrt{3}}\right) = \frac{I}{4\pi a\sqrt{6}}$$

$$\vec{H_3} = |\vec{H_3}| \cdot \vec{a_\alpha} \quad ; \quad \vec{a_\alpha} = \sin(45^\circ)\,\vec{a_x} + \sin(-45^\circ)\,\vec{a_z}$$
$$= \frac{1}{\sqrt{2}}\vec{a_x} - \frac{1}{\sqrt{2}}\vec{a_z}$$

$$\boxed{\vec{H_3} = \frac{I}{8\pi a\sqrt{3}}\left(\vec{a_x} - \vec{a_z}\right)}$$

- PODRUČJE (4):





$\frac{x}{2} = \frac{a\sqrt{2}}{2}$

$$d = \sqrt{x^2 - \left(\frac{x}{2}\right)^2}$$

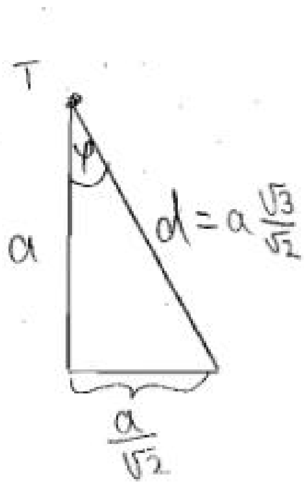
$$d = \sqrt{(a\sqrt{2})^2 - \left(\frac{a\sqrt{2}}{2}\right)^2}$$

$$d = \sqrt{2a^2 - \frac{1}{2}a^2}$$

$$d = a\sqrt{\frac{3}{2}}$$

$$\sin\alpha_1 = \sin\alpha_2 = \frac{\frac{x}{2}}{x} = \frac{x}{2x} = \frac{1}{2}$$

$$|\vec{H_4}| = \frac{I}{4\pi a\sqrt{\frac{3}{2}}}\left(\frac{1}{2} + \frac{1}{2}\right) = \frac{I}{4\pi a\frac{\sqrt{3}}{\sqrt{2}}} = \frac{I\sqrt{2}}{4\pi a\sqrt{3}}$$

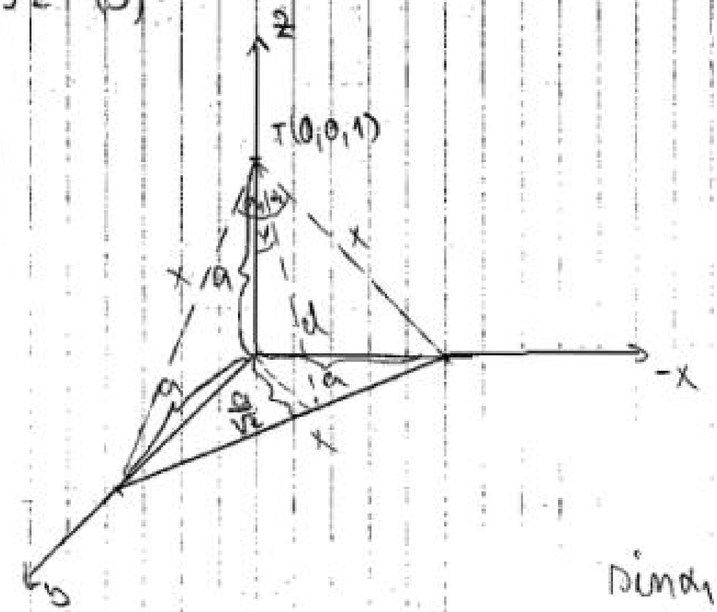


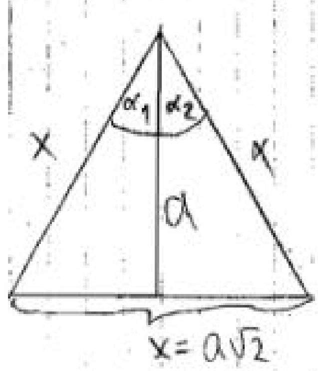
$$-\sin(\varphi) = \frac{\frac{a}{\sqrt{2}}}{\frac{a\sqrt{3}}{\sqrt{2}}} = \frac{1}{\sqrt{3}} \Rightarrow \sin(\varphi) = -\frac{1}{\sqrt{3}}$$

$$\vec{H_4} = |\vec{H_4}| \cdot \vec{a_\alpha} \quad ; \quad \vec{a_\alpha} = -\frac{1}{\sqrt{3}}\vec{a_x} - \frac{1}{\sqrt{3}}\vec{a_y} - \frac{1}{\sqrt{3}}\vec{a_z}$$

$$\boxed{\vec{H_4} = \frac{I\sqrt{2}}{12\pi a}\left(-\vec{a_x} - \vec{a_y} - \vec{a_z}\right)}$$

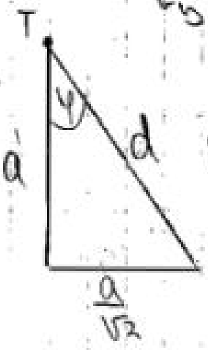
- PODRUČJE (5):





$$d = a\sqrt{\frac{3}{2}}$$

$$\sin\alpha_1 = \sin\alpha_2 = \frac{1}{2}$$



$\sin\psi = \frac{1}{\sqrt{3}}$

$$|\vec{H}_5| = \frac{I\sqrt{2}}{4\pi a\sqrt{3}}\left(\frac{1}{2}+\frac{1}{2}\right) = \frac{I\sqrt{2}}{4\pi a\sqrt{3}}$$

$$\vec{H}_5 = |\vec{H}_5|\,\vec{a}_\alpha \;;\quad \vec{a}_\alpha = -\sin\psi\,\vec{a}_x + \sin\psi\,\vec{a}_y + \sin\psi\,\vec{a}_z$$

$$\vec{a}_\alpha = \frac{1}{\sqrt{3}}\vec{a}_x - \frac{1}{\sqrt{3}}\vec{a}_y - \frac{1}{\sqrt{3}}\vec{a}_z$$

$$\boxed{\vec{H}_5 = \frac{I\sqrt{2}}{12\pi a}\left(\vec{a}_x - \vec{a}_y - \vec{a}_z\right)}$$

- UKUPNI VEKTOR JAKOSTI MAGNETSKOG POLJA U TOČKI (0,0,1):

$$\vec{H} = \vec{H}_1 + \vec{H}_2 + \vec{H}_3 + \vec{H}_4 + \vec{H}_5$$

$$\boxed{\vec{H} = \vec{a}_x \cdot 0 - \vec{a}_y\, 0.32 - \vec{a}_z\, 1.836}$$

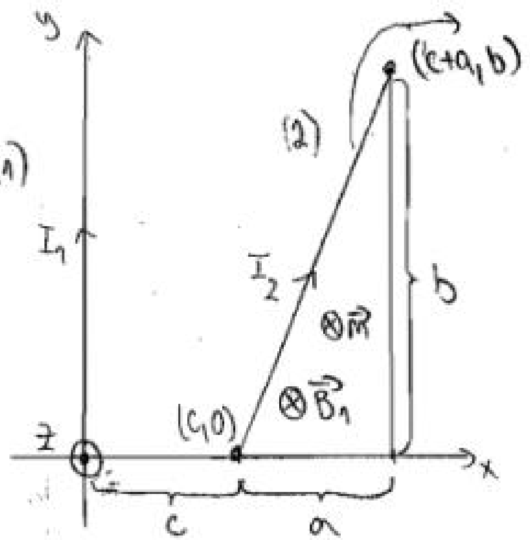
$\boxed{H_x = 0}$ [A/m]

(11) $\boxed{H_y = -0.32}$ [A/m]

(12) $\boxed{H_z = -1.836}$ [A/m]

IV.

TROKUTNA PETLJA PROTJECANA STRUJOM $I_2 = 5$ [A], NALAZI SE U RAVNINI S BESKONAČNO DUGOM STRUJNICOM KOJA JE PROTJECANA STRUJOM $I_1 = 3$ [A] PREMA SLICI. ZADANO JE $a = 2$ [m]; $b = 7$ [m]; $c = 2$ [m].



$y = kx + l$

- JEDNADŽBA PRAVCA KROZ DVIJE TOČKE:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{b - 0}{c + a - c}(x - c)$$

$$\boxed{y = \frac{b}{a}x - \frac{bc}{a}} \Rightarrow \boxed{x = \frac{a}{b}y + c}$$

(13) ODREDI MEĐUINDUKTIVITET PETLJE I STRUJNICE !

- MAGNETSKO POLJE KOJE STVARA STRUJNICA (1) PROTJECANA STRUJOM I_1 PROIZVODI MAGNETSKI TOK Φ_{21} U STRUJNICI (2), OMJER TOKA Φ_{21} I STRUJE I_1 NAZIVA SE KOEFICIJENT MEĐUINDUKCIJE ILI MEĐUINDUKTIVITET M_{21} IZMEĐU STRUJNICA (1) i (2) !

$$\boxed{M_{21} = \frac{\Phi_{21}}{I_1}}$$

- MAGNETSKI TOK KROZ POVRŠINU "S":

$$\boxed{\Phi = \iint_S \vec{B}\,\vec{n}\,dS \;;\quad \Phi = \iint_S \mathrm{rot}\,\vec{A}\,\vec{n}\,dS \;;\quad \Phi = \oint_{\mathcal{K}} \vec{A}\,d\vec{l}}$$

- MAGNETSKA INDUKCIJA BESKONAČNO DUGOG VODIČA NA UDALJENOSTI r OD ISTOG:

$$\boxed{B = \frac{\mu_0 I}{2\pi r}} \text{ [T]}$$

- MAGNETSKI TOK:

$$\boxed{\Phi = \iint_S \vec{B}\,\vec{n}\,dS}$$

- PODRUČJE: $S = \Omega_{xy}$

$$dS = dx\,dy$$

- NORMALA - U SMJERU MAGNETSKOG POLJA: $\vec{n} = -\vec{a_z}$

- PROMJENE x, y PO PODRUČJU S:

$r = x \in [c, c+a]$

$y \in [0, \frac{b}{a}x - \frac{bc}{a}]$

$$\vec{B_1} = \frac{\mu_0 I_1}{2\pi r} \cdot (-\vec{a_z})$$

$$\Phi_{21} = \iint_{\Omega_{xy}} \frac{\mu_0 I_1}{2\pi x} \underbrace{(-\vec{a_z})(-\vec{a_z})}_{1} dx\,dy = \frac{\mu_0 I_1}{2\pi} \int_c^{c+a} \frac{1}{x} dx \int_0^{\frac{b}{a}x - \frac{bc}{a}} dy$$

$$\Phi_{21} = \frac{\mu_0 I_1}{2\pi} \int_c^{c+a} \frac{1}{x} \cdot y \Big|_0^{\frac{b}{a}x - \frac{bc}{a}} dx = \frac{\mu_0 I_1}{2\pi} \int_c^{c+a} \frac{1}{x}\left(\frac{b}{a}x - \frac{bc}{a}\right) dx$$

$$\Phi_{21} = \frac{\mu_0 I_1}{2\pi}\left[b - \frac{bc}{a}\ln\frac{c+a}{c}\right] \quad ; \quad b=7;\ a=2;\ c=2;\ I_1=3;\ \mu_0=4\pi\cdot 10^{-7}$$

$$\boxed{\Phi_{12} = 1.2888\ [\mu Wb]}$$

- MEĐU INDUKTIVITET:

$$M_{21} = \frac{\Phi_{21}}{I_2} = \frac{1.2888 \cdot 10^{-3}}{5} = 257.76 \cdot 10^{-9}\ [H] = \boxed{257.76\ [nH]}$$

(15) ODREDI SILU NA PETLJU U SMJERU OSI X !

- MAGNETSKA SILA NA VODIĆ PROTJECAN STRUJOM:

$$\boxed{\vec{F} = I(\vec{l} \times \vec{B}) \ [N]}$$

- AKO POLJE NIJE HOMOGENO ILI AKO VODIĆ NIJE RAVNI ONDA SE PRVO IZRAČUNA DIFERENCIJALNA SILA $d\vec{F}$ NA DIFERENCIJALNO MALOM DIJELU VODIĆA $d\vec{l}$ ZA KOJEG MOŽEMO SMATRATI DA SE NALAZI U MAGNETSKOM POLJU ISTE INDUKCIJE A UKUPNU SILU NA VODIĆ DOBIJEMO INTEGRACIJOM DIFERENCIJALNE SILE UZDUŽ VODIĆA !:

$$\boxed{\vec{F} = \int_l d\vec{F} = \int_l I(d\vec{l} \times \vec{B}) \ [N]}$$

- MAGNETSKA INDUKCIJA:

$$\vec{B} = \frac{\mu_0 I_1}{2\pi r} \cdot \vec{m} \quad ; \quad r = x \quad ; \quad \vec{m} = -\vec{a_z}$$

$$\vec{B}_1 = \frac{-\mu_0 I_1}{2\pi x} \vec{a_z}$$

- DIFERENCIJAL VODIĆA U RAVNINI xy:

$$d\vec{l} = dx\,\vec{a_x} + dy\,\vec{a_y}$$

- VEKTORSKI UMNOŽAK:

$$d\vec{l} \times \vec{B}_1 = \begin{bmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ dx & dy & 0 \\ 0 & 0 & \frac{-\mu_0 I}{2\pi x} \end{bmatrix} = \vec{a_x}\left[\frac{-\mu_0 I_1}{2\pi x} dy\right] - \vec{a_y}\left[\frac{-\mu_0 I_1}{2\pi x} dx\right]$$

- DIFERENCIJALNA SILA $d\vec{F}$ NA DIO VODIĆA $d\vec{l}$

$$d\vec{F}_{12} = I_2(d\vec{l} \times \vec{B}_1) = \vec{a_x}\left[\frac{-\mu_0 I_1 I_2}{2\pi x} dy\right] - \vec{a_y}\left[\frac{-\mu_0 I_1 I_2}{2\pi x} dx\right]$$

- UKUPNA SILA NA VODIČ DUŽINE l:

$$\vec{F}_{12} = \int_l d\vec{F}_{12}$$

$$\vec{F}_{12} = \int_l \left[\vec{a}_x\left(\frac{-\mu_0 I_1 I_2}{2\pi x}dy\right) - \vec{a}_y\left(\frac{-\mu_0 I_1 I_2}{2\pi x}dx\right)\right]$$

$$= \underbrace{\int_{l_y} \vec{a}_x\left[\frac{-\mu_0 I_1 I_2}{2\pi x}\right]dy}_{\vec{F}_x} - \underbrace{\int_{l_x} \vec{a}_y\left[\frac{-\mu_0 I_1 I_2}{2\pi x}\right]dx}_{\vec{F}_y}$$

$$\vec{F}_x = \vec{a}_x \frac{-\mu_0 I_1 I_2}{2\pi}\int_{l_y}\frac{1}{x}\,dy \quad ; \quad \left[\begin{array}{l} l_y \Rightarrow [0,b] \Rightarrow x = \frac{a}{b}y + c \\ l_y \Rightarrow [b,0] \Rightarrow x = c+a \end{array}\right]$$

$$= \vec{a}_x \frac{-\mu_0 I_1 I_2}{2\pi}\cdot\left[\int_0^b \frac{1}{\frac{a}{b}y+c}\,dy + \int_b^0 \frac{1}{c+a}\,dy\right] =$$

$$= \vec{a}_x \frac{-\mu_0 I_1 I_2}{2\pi}\left[\frac{b}{a}\ln\left(\frac{a}{b}y+c\right)\Big|_0^b + \frac{1}{c+a}\,y\Big|_b^0\right]$$

$$= \vec{a}_x \frac{-\mu_0 I_1 I_2}{2\pi}\left[\frac{b}{a}\ln\left(\frac{a+c}{c}\right) - \frac{b}{c+a}\right]$$

$$F_x = \frac{-\mu_0 I_1 I_2}{2\pi}\left[\frac{b}{a}\ln\left(\frac{a+c}{c}\right) - \frac{b}{c+a}\right]$$

$$\boxed{F_x = -2.028\ [\mu N]}$$

(15) ODREDI SILU NA PETLJU U SMJERU OSI "y" :

$$\vec{F}_y = -\int_{l_x} \vec{a}_y \left[-\frac{\mu_0 I_1 I_2}{2\pi x}\right] dx$$

$$\vec{F}_y = \vec{a}_y \frac{\mu_0 I_1 I_2}{2\pi} \int_{l_x} \frac{1}{x} dx \quad ; \quad \begin{bmatrix} l_x \Rightarrow [c, c+a] \\ l_x \Rightarrow [c+a, c] \end{bmatrix}$$

$$\vec{F}_y = \vec{a}_y \frac{\mu_0 I_1 I_2}{2\pi} \left[\int_{c}^{c+a} \frac{1}{x} dx + \int_{c+a}^{c} \frac{1}{x} dx\right] = \boxed{0} \text{ [N]}$$

- KOMPONENTA SILE $\vec{F}$ U SMJERU OSI "y" JE OKOMITA NA DIFERENCIJALNI DIO VODIČA $d\vec{l}_x$ U PETLJI, PA JE ZATO NJENO DJELOVANJE NA PETLJU NULA :

$$\boxed{\vec{F}_y \perp d\vec{l}_x \Rightarrow F_y = 0}$$

(16) ODREDI IZNOS ENERGIJE MEĐUDJELOVANJA PETLJE I STRUJNICE :

$$\boxed{W_{21} = M_{21} \cdot I_1 \cdot I_2 = 3.87 \text{ [mJ]}}$$

V. NA FEROMAGNETSKOM PRSTENU SA ZRAČNIM RASPOROM $\delta = 3$ [mm], DUŽINE FEROMAGNETSKOG MATERIJALA $l_m = 400$ [mm], POVRŠINE POPREČNOG PRESJEKA $S = 8$ [cm^2], NAMOTANA JE ZAVOJNICA SA $N = 800$ [ZAVOJA].

- KRIVULJA MAGNETIZIRANJA I SLIKA PRSTENA ISTO KAO U V. ZAD IZ 2.MI. 2006/2007 !

(17) AKO JE MAGNETSKA INDUKCIJA U ZRAČNOM RASPORU $B = 1.1$ [T], ODREDI STRUJU KROZ ZAVOJNICU !

$$\boxed{\oint_K \vec{H}\, d\vec{l} = \sum NI}$$ - ZAKON PROTJECANJA !

$$\boxed{\sum I = I_m + I_\delta}$$

$$\boxed{I = \frac{H \cdot d}{N}} \text{ [A]} \Rightarrow I_m = \frac{H_m l_m}{N} \text{ [A]}; \quad I_\delta = \frac{H_\delta \cdot \delta}{N} \text{ [A]}$$

- IZ GRAFA ZA $B = 1.1$ [T] $\Rightarrow H_m = 200$ [A/m]

$$H_\delta = \frac{B_\delta}{\mu_0} \left[\frac{A}{m}\right]; \quad \boxed{B = B_m = B_\delta} \quad ; \quad \delta = 3 \text{ [mm]} = 3 \cdot 10^{-3} \text{ [m]}$$

$$l_m = 400 \text{ [mm]} = 0.4 \text{ [m]}$$

$$H_\delta = \frac{1.1}{4\pi \cdot 10^{-7}} = 875352 \text{ [A/m]}$$

$$\sum I = \frac{H_m \cdot l_m}{N} + \frac{H_\delta \cdot \delta}{N} = \boxed{3.38 \text{ [A]}}$$

18) ODREDITE MAGNETSKU ENERGIJU U ZRAČNOM RASPORU !

- MAGNETSKA ENERGIJA :

$$\boxed{W = \frac{1}{2} \iiint_V B \cdot H \cdot dV \ [J]}$$

$$W_\delta = \frac{1}{2} \iiint_V B_\delta H_\delta \, dV \ [J]$$

$$\iiint_V dV = S \cdot \delta = 8 \cdot 10^{-4} \cdot 3 \cdot 10^{-3} = 24 \cdot 10^{-7} \ [m^3]$$

$$W_\delta = \frac{1}{2} \, 1.1 \cdot 875352 \cdot 24 \cdot 10^{-7} = \boxed{1.55 \ [J]}$$

19) ODREDITE MAGNETSKU ENERGIJU U FEROMAGNETSKOM MATERIJALU AKO JE ON JEDNOLIKO MAGNETIZIRAN !

$$W_m = \frac{1}{2} \iiint_V B_m H_m \, dV \ [J]$$

$$\iiint_V dV = l_m \cdot S = 0.4 \cdot 8 \cdot 10^{-4} = 3.2 \cdot 10^{-4} \ [m^3]$$

$$W_m = \frac{1}{2} \cdot 1.1 \cdot 200 \cdot 3.2 \cdot 10^{-4} = \boxed{0.0352 \ [J]}$$

20) ODREDITE INDUKTIVITET ZAVOJNICE !

$$\boxed{W = \frac{1}{2} L \cdot I^2 \ [J]}$$ - SKRIPTA MAGNETOSTATIKA str. 17. !

$$L = \frac{2W}{I^2} = \frac{2 \cdot (W_m + W_\delta)}{I^2} = \boxed{0.208 \ [H]}$$