EMP- Moss-2. ciklus

13.5.08 ①

Osnovne formule:

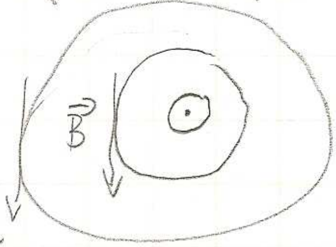
vektorski magnetski potencijal → $\vec{A} = \dfrac{1}{x+2y+3} a_z$

u papir ⊗   iz papira ⊙

Oko vodiča se stvara polje → Polje se određuje pravilom desne ruke

↳ palac - smjer struje

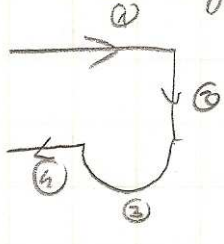
↳ prsti smjer polja



gustoća mag toka ili mag. ind

$$\vec{B} = \mu \frac{I}{4\pi r}(\sin\alpha + \sin\beta)$$

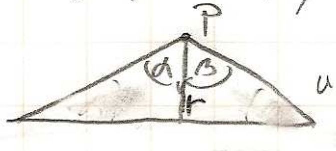
Obično vodič izgleda nekako ovako



Ovdje ga rastavimo na dijelove i svaki posebno računamo

jakost mag. polja → $H = \mu B$

$$\mu = \mu_0 \mu_r \ , \ \mu_0 = 4\pi \cdot 10^{-7} \ , \ \mu_{0\,ZRAK} = 1$$

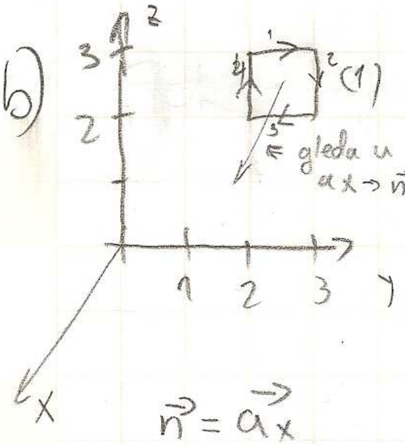


u točki P $H = \dfrac{1}{4\pi r}(\sin\alpha + \sin\beta)$

1. MI $\vec{A} = \frac{1}{x+2y+3}\vec{a_z}$

a) $\vec{B} = \nabla \times \vec{A} = \text{rot } A = \dots = \frac{-2}{(x+2y+3)^2}\vec{a_x} + \frac{1}{(x+2y+3)^2}\vec{a_y}$

b)



$\Leftarrow$ gleda u smjeru $a_x \to \vec{n} = \vec{a_x}$

$$\phi = \iint \vec{B}\,\vec{n}\,dS = \oint \vec{A}\,d\vec{l}$$

sigurnije jer je moguće da smo B krivo dobili

$\vec{n} = \vec{a_x}$

$dS = dy\,dz$ jer je u y-z ravnini

$$\iint \vec{B}\,\vec{n}\,dS = \iint \frac{-2}{(x+2y+3)^2}\,dy\,dz$$

Kad se množi sa normalom ostaje samo a_x komp.

po y u a_y smjeru

$$dl = \begin{cases} z=2, & 2 \le y \le 3, & dy\,\vec{a_y} \\ y=3, & 2 \le z \le 3, & dz\,\vec{a_z} \\ z=3, & 2 \le y \le 3, & -dy\,\vec{a_y} \\ y=3, & 2 \le z \le 3, & -dz\,\vec{a_z} \end{cases}$$

ili $z=3,\ 3 \le y \le 2,\ dy\,\vec{a_y}$

$$\int_2^3 -\frac{1}{\underset{0}{x} + 2\underset{2}{y} + 3}\,dz$$

I sad se zbraji svaki dio petlje pošto u $\vec{A}$ imamo samo a_z-komponentu svugdje gdje je a_y je nula. Tj 'prežive' samo 1. i 3. dio petlje

c) $\nabla \times \vec{B} = \mu_0 \vec{J}$

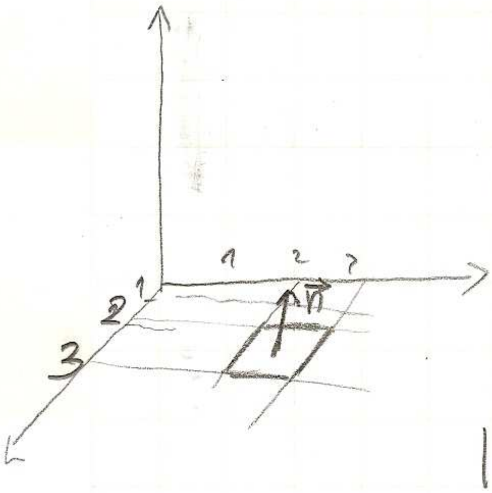
$\Delta \vec{A} = -\mu_0 \vec{J}$ $\to$ pokušaj izbjeć

$$\vec{J} = \frac{-10}{\mu_0 (x+2y+3)^3} a_z$$

i sad uvrsti!

d) $\underline{I = \iint_S \vec{J}\,\vec{n}\,dS}$

Ukoliko se dobije negativan broj, to nije krivo, jer je to samo drugi smjer obilaska



$\vec{n} = \vec{a_z}$ - u plus a_z smjeru

$$I = -\frac{10}{\mu_0}\iint \frac{1}{(x+2y+3)^3}\,dS$$

$dS = dx\,dy$ - jer je samo u x-y sust.

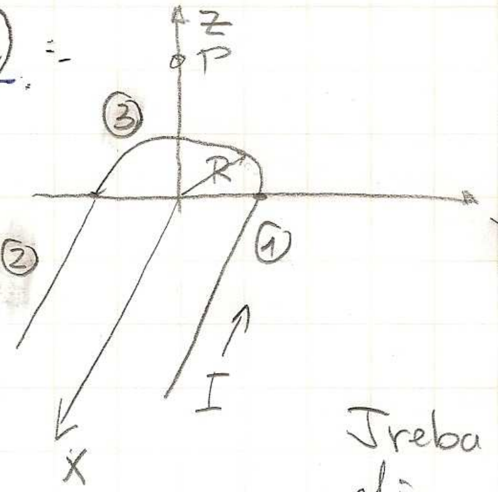
$$I = -\frac{10}{\mu_0}\int_2^3 dx \int_2^3 \frac{1}{(x+2y+3)^3}\,dy$$

→ Koristiti veliku tablicu integrala!!

→ Ako se dobije nešto što nije u tablici vrlo vjerovatno je KRIVO!!

ako je slučajno normala u $\vec{a_z}$ smjeru, a J je u ay smjeru onda je STRUJA $\underline{= 0}$

3. M12 :



Ova cijela petlja leži samo u x-y ravnini.
Da leži još i u z ravnini onda su veliki problemi →
→ vjerovatno neće biti

Treba uočiti ravne dijelove i polukružni dio

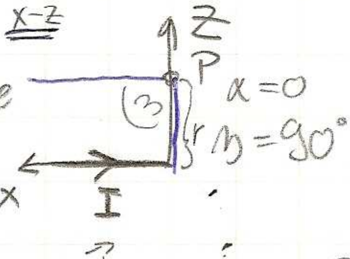
$$\vec{H} = \vec{H}_1 + \vec{H}_2 + \vec{H}_3$$

$$H = H_x \vec{a}_x + H_y \vec{a}_y + H_z \vec{a}_z$$

Ako nas traže u smjeru x osi to je rješenje i tako analogno za y i z osi.

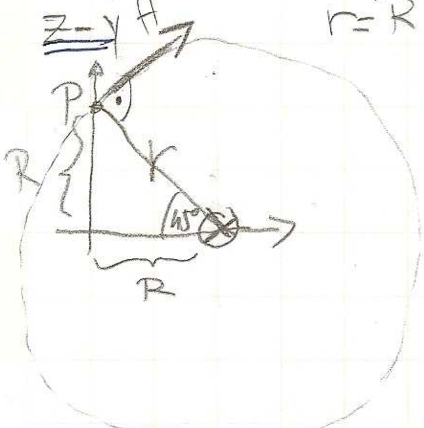
$$H = \frac{1}{4\pi r}(\sin\alpha + \sin\beta)$$

Malo mašte



$\alpha = 0$
$\beta = 90°$

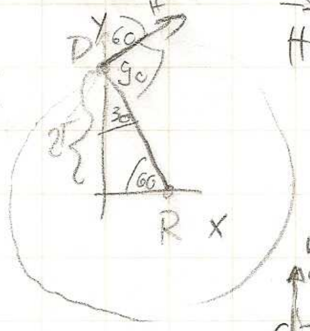
$$H_1 = \frac{I}{4\pi r}(\sin 0° + \sin 90°) = \frac{I}{4\pi r}$$



$r = R\sqrt{2}$

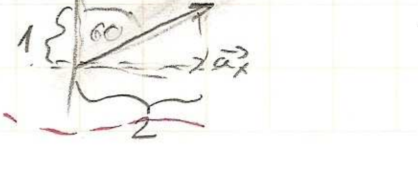
$$H = \frac{I}{4\pi R\sqrt{2}}\left(\frac{\vec{a}_y + \vec{a}_z}{\sqrt{2}}\right)$$

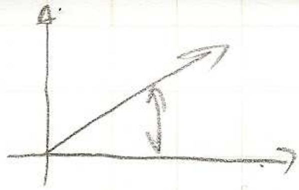
Npr.



$$\vec{H} = H\left(\frac{\vec{a}_y + 2\vec{a}_x}{\sqrt{1^2+2^2}}\right)$$

Gleda se tangenta na ○ : $\vec{H}$





$$\vec{a} = \cos\alpha\,\vec{a_x} + \cos\beta\,\vec{a_b}$$

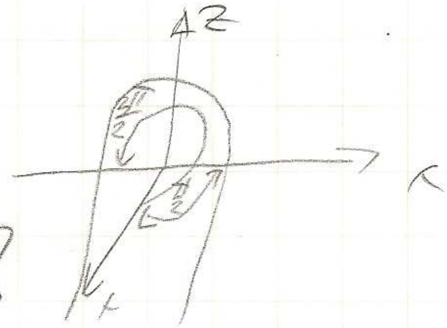
za dio kruga $H_3 = \dfrac{I}{4\pi}\displaystyle\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \dfrac{z\cdot\cos\alpha\,\vec{a_x} + \sin\alpha\,\vec{a_y} + R\vec{a_z}}{\sqrt{z^2+R^2}^{\,3}}\,R\,d\alpha$

z = visina točke (R)

granice integracije

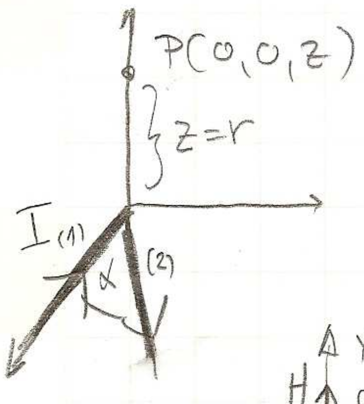
$\frac{\pi}{2}, \frac{3\pi}{2}$

uvijek koristiti pozitivne!



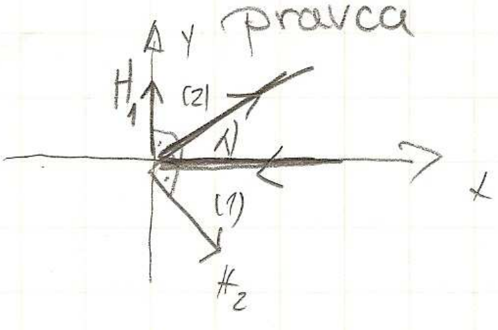
(3.12) $\oint \vec{H}\,d\vec{l} = \sum I = 0$

• IZ DZ (prvi od zadnja tri) 12.



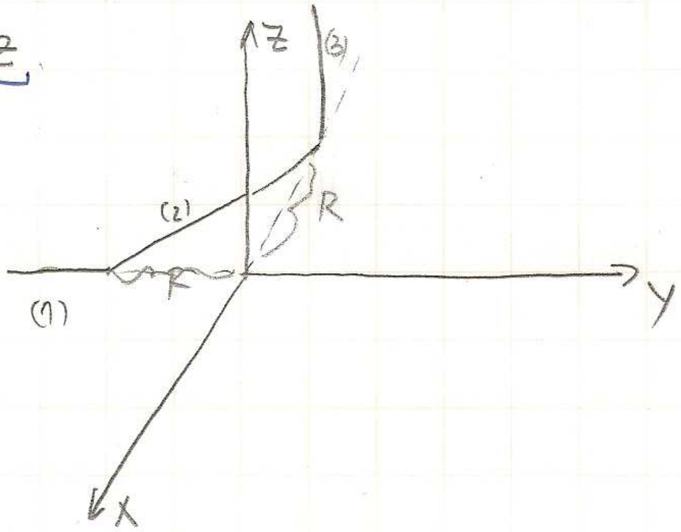
$$H = \frac{I}{4\pi z}$$

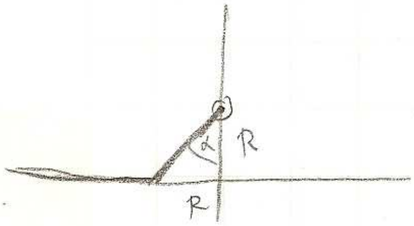
z smjera nema jer je točka točno iznad pravca



pomoću trigonometrije odredit smjera

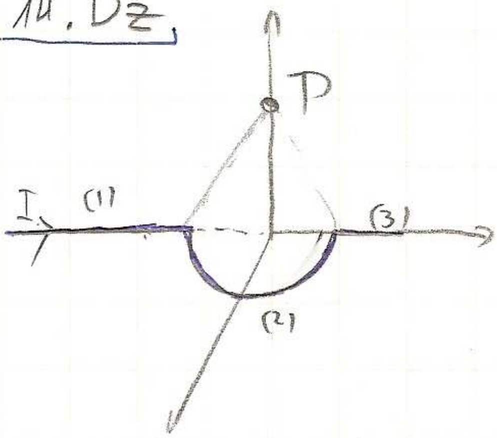
• 13. DZ





$$H = \frac{I}{4\pi R}\left(\sin 90^\circ + \sin(-\alpha)\right)$$
$$= \frac{I}{4\pi R}\left(1 - \sin 45^\circ\right)$$

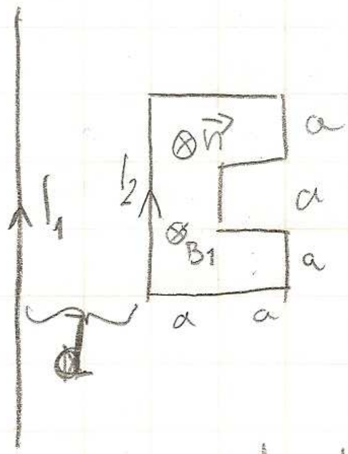
• 14. DZ



$$H_{P2} = \underbrace{\frac{I}{2\pi R}}_{\text{beskr. dug vodič}} - \underbrace{\frac{I}{4\pi R}\left(\sin 45^\circ + \sin 45^\circ\right)}_{\text{manje dio (2)}}$$

• 2. M12

• a)



$I_1 = I_2 = 1A$

$d = a = 20cm$

$$\underbrace{B = \frac{\mu_0 I_0}{2\pi r}}$$ → udaljenost

meduind. $\underbrace{M = \frac{\Phi_{12}}{I_2}} =$

$$\underbrace{\Phi_{12} = \iint_{S_2} \vec{B}_1 \vec{n}_1 dS = \iint_{S_2} B\, ds = \iint \frac{\mu_0 I}{2\pi x} dx dy} =$$

pošto su B i n u istom smjeru možemo maknut vektore (→)

$$= \Phi_{12} = \frac{\mu_0 I}{2\pi}\left[\int_0^a dy \int_d^{d+2a} \frac{dx}{x} + \int_0^a dy \int_d^{d+a} \frac{dx}{x} + \int_0^a dy \int_d^{d+2a} \frac{dx}{x}\right] =$$

$$= \frac{\mu_0 I}{2\pi} a \left[\ln\frac{d+2a}{d} + \ln\frac{d+a}{d} + \ln\frac{d+2a}{d}\right] =$$

$\underline{0{,}2m}[2\ln 3 + \ln 2]$

$\Phi_{12} = \ldots \rightarrow \underline{M = \frac{\Phi_{12}}{I_2}}$

•

M = oko 0 jer ne možemo odredit površine

b) $\vec{F} = \vec{I}_2 \cdot \int d\vec{l}_2 \times \vec{B}_1$

smjer je u $-a_z$ smjeru

$$\vec{B}_1 = -\frac{\mu_0 I_1}{2\pi x}\vec{a}_z$$

$$\vec{l}_2 = x\vec{a}_x + y\vec{a}_y$$

$$d\vec{l}_2 = dx\,\vec{a}_x + dy\,\vec{a}_y$$

$$d\vec{l}_2 \times \vec{B}_1 = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ dx & dy & 0 \\ 0 & 0 & \frac{-\mu I_1}{2\pi x} \end{vmatrix} = \frac{-\mu_0 I_1}{2\pi x}dx\,\vec{a}_y + \frac{\mu_0 I_1}{2\pi x}dy\,\vec{a}_x$$

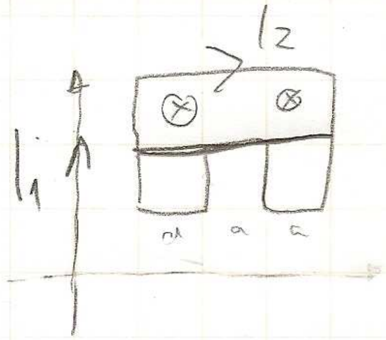
$$\vec{F} = I_2\left[\int -\frac{I_1\mu_0}{2\pi x}dy\,\vec{a}_x + \int \frac{I_1\mu_0}{2\pi x}dx\,\vec{a}_y\right]$$

$$F_x = \frac{I_1 I_2 \mu_0}{2\pi}\left[\int_0^{3a}\frac{1}{d}dy + \int_{3a}^{2a}\frac{1}{d+2a}dy + \int_{2a}^{a}\frac{1}{b+a}dy + \int_a^0\frac{1}{d+2a}dy\right]$$

$$F_y = \frac{I_1 I_2 \mu_0}{2\pi}\left[\int_{d}^{d+2a}\frac{dx}{x} + \int_{d+2a}^{d+a}\frac{dx}{x} + \cancel{\int_{d+a}^{d+2a}\frac{dx}{x}} + \cancel{\int_{d+2a}^{a}\frac{dx}{x}}\right]$$

$F_y = 0$

• d)



$$\phi_{12} = \iint_{S_2} B_1 \, ds = \frac{I_1 \mu_0}{2\pi}\left[a\int_{d}^{d+3a}\frac{dx}{x} + a\int_{d}^{d+a}\frac{dx}{x} + a\int_{d+2a}^{d+3a}\frac{dx}{x}\right]$$

$ds = dx\,dy$

$\phi = \ldots$

$$W = \frac{LI^2}{2} = \frac{L_1 I_1^2}{2} + \frac{L_2 I_2^2}{2} + M I_1 I_2$$

$$M(B) = \frac{\phi_{12}(B)}{I_2}$$

• 5.) M12

$\phi = konst$

prsten $\Rightarrow S = konst$

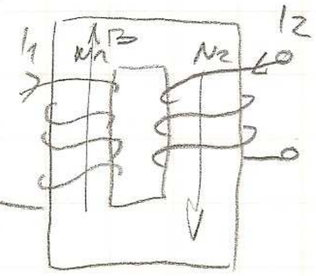
$$\phi = \frac{\Theta}{R_M}$$

$$\Theta = N \cdot I$$

$$R_m = \frac{l}{\mu S}$$

$$\mu = \mu_0 \mu_r$$

$$\phi = \frac{N \cdot I}{\frac{l}{\mu_0 \mu_r S}}$$



Primjer neki

$\Theta_1 = N_1 I_1 \quad \Theta_2 = N_2 I_2$

$\Theta = \Theta_1 \pm \Theta_2$

PRAVILO D. RUKE

$S = 4\ cm^2$

$\delta = 3\ mm$

$l_m = 250\ mm$

$N = 800$, $W = 0{,}5777$

$H_\delta, H_m, I, W_\delta = ?$

$$\phi = \frac{NI}{\frac{l_{sr}}{\mu_0 \mu_r S} + \frac{\delta}{\mu_0 S}} \quad \Big/ : S$$

$$B = \frac{NI}{\frac{l_{sr}}{\mu_0 \mu_r} + \frac{\delta}{\mu_0}}$$

$$W = \frac{1}{2} \iiint H B \, dV = \frac{\overset{?}{H_m} B_m}{2} l_{sr} \cdot S$$

$$B_\delta = \mu_0 H_\delta$$

$$W_\delta = \frac{\mu_0}{2} \iiint H^2 \, dV$$

$$= \frac{\mu_0}{2} H_\delta^2 \cdot \delta \cdot S$$

↳ izvadimo

$B_\delta = \mu_0 H_\delta = B_m = B$ – B je isti na cijelom djelu

$= 1,1 T$

$H_m \rightarrow$ sa grafa $(B = 1,1)$

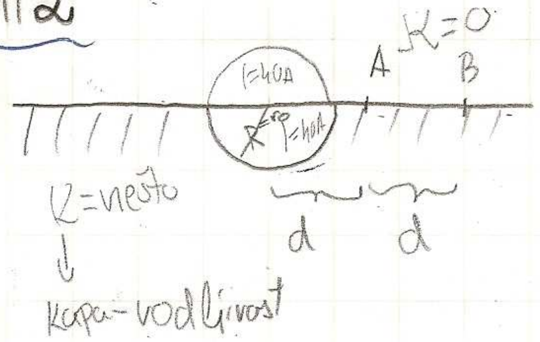
$H_m = 200$ Am

$$B_m = \mu_0 \mu_r \cdot H_m$$

↳ AK PITAJU

$$I = \frac{H_m l_{sr} + H_\delta \delta}{N}$$

4. MI2



$$\underbrace{R = \rho \cdot \frac{l}{S} = \frac{1}{K} \cdot \frac{l}{S}}$$

U zadacima se traži da se odrede neki potencijali

b) $\underbrace{\varphi_{kugle} = \frac{Q}{4\pi\varepsilon}\left(\frac{1}{r} - \frac{1}{r_{ref}}\right)}$

$\varphi(r_{ref}) = 0$, $r_{ref} = \infty$

$$\frac{Q}{\varepsilon_0} = \frac{I}{K} = I \cdot \rho$$

$$\frac{\lambda}{\varepsilon} = \frac{I/l}{K}$$

$$\varphi_K(r_0) = \frac{I \cdot \rho}{4\pi} \cdot \frac{1}{r_0} + \frac{I \cdot \rho}{4\pi} \cdot \frac{1}{r_0} = \frac{I \cdot \rho}{2\pi R}$$

a) $\underbrace{R_{rasp.\,ZEM.} = \frac{\varphi_K}{I}}$

Uvjeti na granici

- $\vec{K} = 6{,}5\,\vec{a_z}\ [A m^{-1}]$

$x = 0$

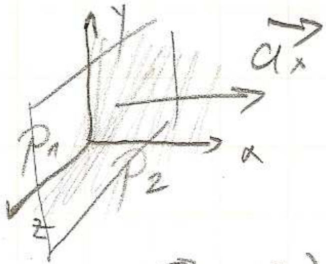
$P_1 \Rightarrow \quad x < 0$

$P_2 \Rightarrow \quad x > 0$

$H_1 = 10\,\vec{a_y}$

$\vec{n} \times (\vec{H_2} - \vec{H_1}) = \vec{K}$ $\qquad H_2 = H_x\vec{a_x} + H_y\vec{a_y} + H_z\vec{a_z}$

$\vec{n} \cdot (\vec{B_2} - \vec{B_1}) = 0$



$n = \vec{a_x}$

$\vec{n} \times (\vec{H_2} - \vec{H_1}) = \vec{K}$

$$\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 1 & 0 & 0 \\ H_x & H_y - 10 & H_z \end{vmatrix} = 6{,}5\,\vec{a_z}$$

$0\,\vec{a_x} - \vec{a_y} H_z + \vec{a_z}(H_y - 10) = 6{,}5\,\vec{a_z}$

$(H_y - 10) = 6{,}5$

$H_y = 16{,}5$

$H_z = 0$

$\underbrace{B_{T_1} = \vec{n} \cdot B_1}_{\Rightarrow \vec{n} = a_x} = 0 = B_x$

9. $\mu_{r1} = 5$

$6x + 4y + 3z = 12$

$$\vec{n} = \frac{6\vec{a_x} + 4\vec{a_y} + 3\vec{a_z}}{\sqrt{6^2 + 4^2 + 3^2}}$$

$\mu_{r2} = 3$

$H = H_x\vec{a_x} + H_y\vec{a_y} + H_z\vec{a_z}$

$B = \mu_0 \mu_r \vec{H}$