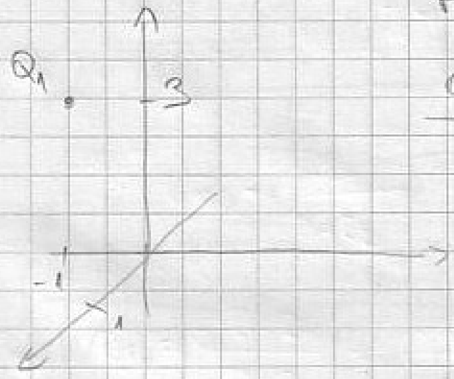
Polja MASS 27.03.2010

I sila

5.



$\vec F = 8\vec a_x - 8\vec a_y - 4\vec a_z$ [mN]

$Q_1 = 300nC$

$$\vec F_{12} = \frac{1}{4\pi\varepsilon_0} Q_1 \cdot Q_2 \cdot \frac{\vec R_{21}}{|\vec R_{21}|^3}$$

$$\vec R_{21} = \vec r_1 - \vec r_2 = -2\vec a_x + 2\vec a_y + \vec z$$

$\vec r_1 = \vec a_x - \vec a_y + 3\vec a_z$ $\qquad |\vec R_{21}| = \sqrt{4+4+1} = 3$

$\vec r_2 = 3\vec a_x - 3\vec a_y + 2\vec a_z$

$$10^{-3}\cdot(8\vec a_x - 8\vec a_y - 4\vec a_z) = \frac{1}{4\pi\varepsilon_0} Q_1 Q_2 \frac{(-2\vec a_x + 2a_y + a_y)}{3^3}$$

$$8\cdot 10^{-3} = \frac{1}{4\pi\varepsilon_0} Q_1 Q_2 \cdot \frac{-2}{27} \Rightarrow Q_2 = -40,05\mu C$$

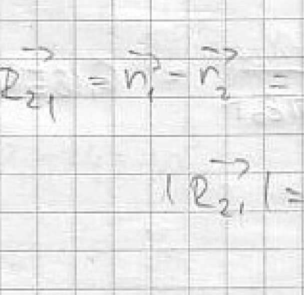
9

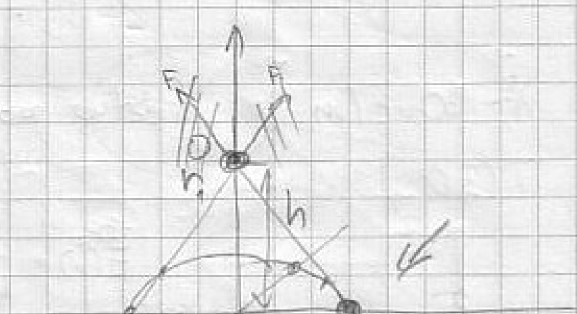
$Q_1 = 100nC$

$r = 5m$

$Q_2 = 20nC$

$h = 2m$





samo komponenta z osi
ostale se ponište

$r_2 = 5\vec a_x$

$r_1 = 2\vec a_z$

$\vec R_{21} = 2a_z - 5\vec a_x$ $\qquad F = \frac{1}{4\pi\varepsilon_0} Q_1 Q_2 \frac{(2\vec a_z - 5\vec a_x)}{(\sqrt{29})^3}$

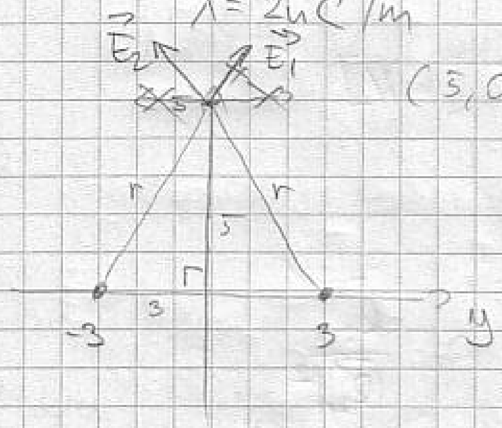
$$F_z = \frac{1}{4\pi\varepsilon_0} Q_1 Q_2 \frac{2}{(\sqrt{29})^3} = 2,3\cdot 10^{-7} N$$

$$F_{uk} = 8\cdot F_z = 1,84\mu N$$

II El. polje, jakost

① prvi pdf

$\lambda = 2\,nC/m$



$(5, 0, 10)$

$r = \sqrt{9+25} =$

$$E = \frac{\lambda}{2\pi \varepsilon r} \qquad E_1 = E_2 = \frac{\lambda}{2\pi \varepsilon_0 r} = 6.1654$$

$$\vec{E}_1 = E_1 \frac{3\vec{a}_y + 5\vec{a}_x}{\sqrt{34}}$$

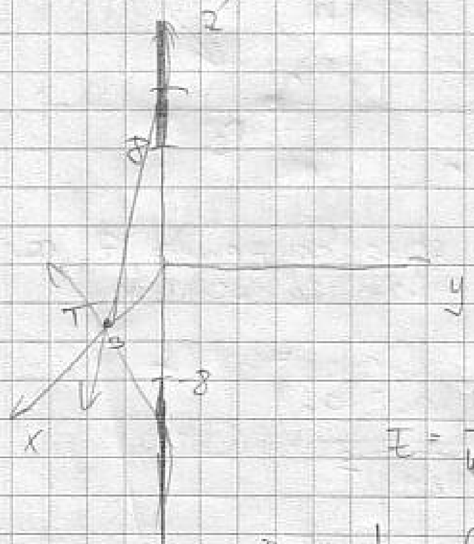
$$\vec{E}_2 = E_2 \frac{-3\vec{a}_y + 5\vec{a}_x}{\sqrt{34}}$$

$$\vec{E}_{uk} = 2E_1 \cdot \frac{5}{\sqrt{34}} \cdot \vec{a}_x = 10.57\,a_x$$

⑫

$\lambda = 30\,nC/m$ direktno po formuli zbog predida od 8 do -8

$T(3,0,0)$



$$\vec{E}(r) = \frac{1}{4\pi\varepsilon_0}\int \lambda\, dl \cdot \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

$\vec{r} = 3\vec{a}_x$ - radij vekto točke u kojoj računa polje

$\vec{r}' = z\,\vec{a}_z$ - radij vektor linije

$dl = dz$

$$E = \frac{1}{4\pi\varepsilon_0}\int \lambda\, dz\, \frac{3\vec{a}_x - z\vec{a}_z}{\left(\sqrt{9+z^2}\right)^3} =$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\int \lambda\, dz\, \frac{3}{(9+z^2)^{3/2}}$$

$$E_z = \frac{1}{4\pi\varepsilon_0}\int \lambda\, dz\, \frac{(-z)}{(9+z^2)^{3/2}} = \emptyset$$ komponente z smjeru se poništavaju

$$E_x = \frac{3\lambda}{4\pi\varepsilon_0}\left[\int_{-\infty}^{-8}\frac{dz}{(9+z^2)^{3/2}} + \int_{8}^{\infty}\frac{dz}{(9+z^2)^{3/2}}\right]$$

gledam u tablicu integrala

invarijante dija

$$r = \sqrt{z^2+a^2}$$

$$\boxed{\int\frac{dz}{r^3} = \frac{z}{a^2 r}}$$

$$\int_{-\infty}^{-8}\frac{dz}{(\sqrt{9+z^2})^3} = \frac{z}{9\sqrt{9+z^2}}\Bigg|_{-\infty}^{-8} = 7.0745\cdot10^{-3}$$

$$\int_{8}^{\infty}\frac{dz}{(\sqrt{9+z^2})^3} = 7.0745\cdot10^{-3}$$

$$E_x = 11.45\ V/m$$

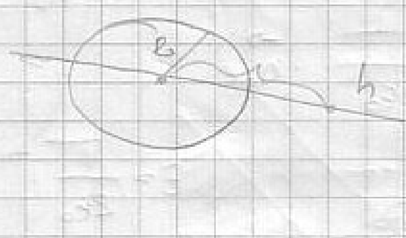
Naboj raspoređen na disku

4. $Q = 30\,nC$

$R_0 = 1m$

$h = 2m$

$$\sigma = \frac{Q}{S} = \frac{Q}{R_0^2\pi} =$$



konst. raspodjela naboja

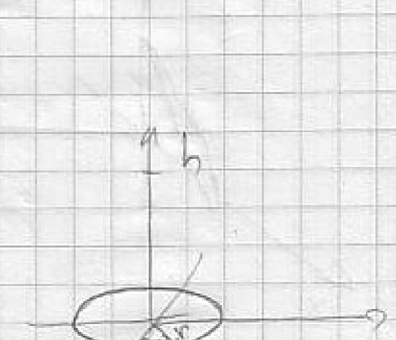
$$\varphi = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{R_0^2+h^2} - h\right) = 127.3\,V$$

(14) $\sigma = 10^{-9}\cos^2\alpha\ C/m^2$ nije konst. raspodjela naboja

$R_0 = 4m$

$h = 2m$

$$E(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\iint\sigma\, dS\,\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$



$x = r\cos\alpha$

$y = r\sin\alpha$

$dS = r\,dr\,d\alpha$

cilindrični k.s.

$\vec{r} = 2\vec{a_z}$

$\vec{r'} = x\vec{a_x} + y\vec{a_y}$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\iint \sigma \, ds \, \frac{-x\vec{a_x} - y\vec{a_y} + 2\vec{a_z}}{(x^2+y^2+4)^{3/2}}$$

x i y komp. je nula ,poništavaju se

$$\vec{E_z} = \frac{1}{4\pi\varepsilon_0}\cdot 2\int_0^{2\pi} d\alpha \int_0^4 r\,dr \, \frac{10^{-9}\cos^2\alpha}{(r^2+4)^{3/2}} =$$

$$= \frac{10^{-9}}{2\pi\varepsilon_0}\cdot 0.2764\pi = 15.61\ V/m$$

Ravnina

6. $\sigma = 0.3\,nC/m^2$

$2x - 3y + z = 6$

konst. gustoće naboja

$E = \frac{\sigma}{2\varepsilon_0}$

$\vec{n} = 2\vec{a_x} - 3\vec{a_y} + \vec{a_z}$

$\vec{n_0} = \frac{\vec{n}}{|\vec{n}|}$

$\vec{E} = \frac{\sigma}{2\varepsilon_0}\cdot\vec{n_0}$

$$E = \frac{\sigma}{2\varepsilon_0}\cdot\frac{-(2\vec{a_x} - 3\vec{a_y} + \vec{a_z})}{\sqrt{4+9+1}}$$

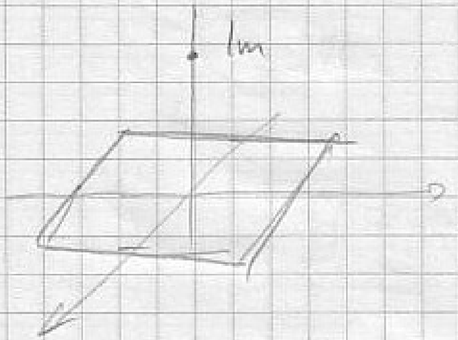
ishodište je "ispod" ravnine

zato $\vec{n}$ suprotnog smjera

$$E_x = \frac{-2\sigma}{2\varepsilon_0}\,\frac{1}{\sqrt{14}}$$

19

$$\sigma = 3(x^2+y^2+1)^{3/2} \quad nC/m^2$$



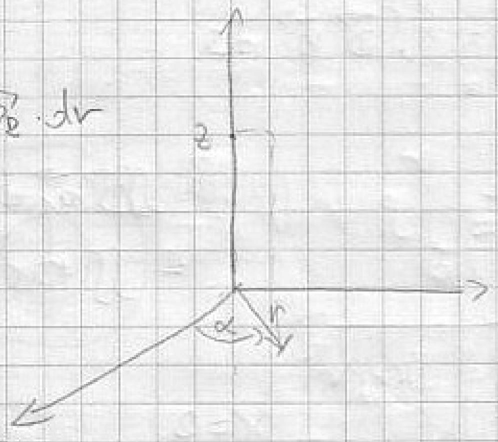
$$E = \frac{1}{4\pi\varepsilon_0}\iint \sigma\, ds\, \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

$$ds = dx\,dy$$

III Potencijal (napon) i tok

(8) $$\varphi(A) - \varphi(B) = -\int_B^A \vec{E}\,d\vec{l}$$

$$d\vec{l} = \vec{a}_r \cdot dr$$

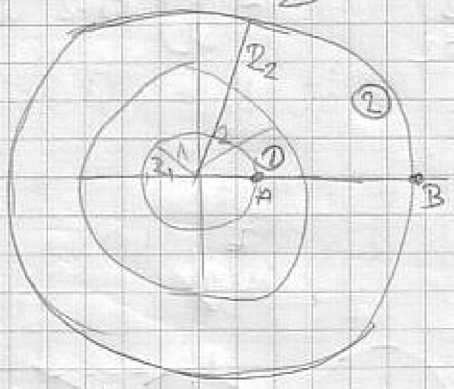


$$\vec{E}_1 = 5r^2\,\vec{a}_r, \quad 0 \le r < 2$$

$$\vec{E}_2 = \frac{2.5}{r}\,\vec{a}_r \quad r \ge 2$$

$A(1,0,0)$ (r, α, z)

$B(4,0,0)$ (r, α, z)



$$\varphi(4) - \varphi(2) = -\int_2^4 \frac{2.5}{r}\,\vec{a}_r \cdot \vec{a}_r\, dr$$

$$= -\int_2^4 \frac{2.5}{r}\,dr = -2.5(\ln 4 - \ln 2)$$

$$\varphi(2) - \varphi(1) = -\int_1^2 \frac{5}{r^2}\,dr = -\frac{5}{2}$$

$$\Rightarrow \varphi(2) = \varphi(1) - \frac{5}{2}$$

$$\varphi(4) - \varphi(1) + \frac{5}{2} = -2.5(\ln 4 - \ln 2)$$

$$\varphi(1) - \varphi(4) = 2.5(\ln 2) + \frac{3}{2}$$

(7)

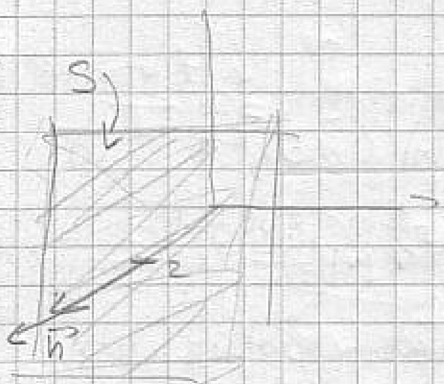
$$\vec{D} = 10x^3 \vec{a}_x$$

$$S = 2m^2$$

$$x = 2$$

$$\boxed{\Phi = \iint_S \vec{A}\, d\vec{S} = \iint \vec{A} \cdot \vec{n}\, dS}$$

$$\Phi = \iint \vec{D}\, \vec{n}\, ds \qquad \vec{D} = \varepsilon \cdot \vec{E}$$

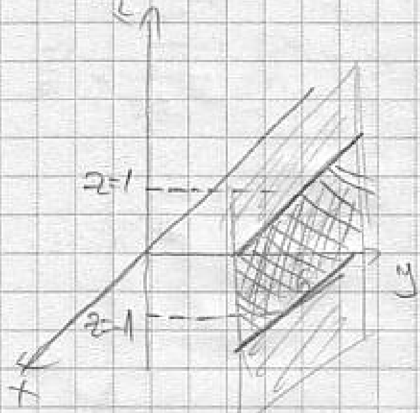


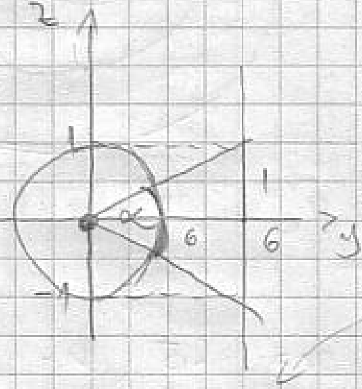
$$\Phi = \iint \vec{D}\, \vec{a}_x\, dS$$

$$= \iint 10x^3 \underbrace{\vec{a}_x \cdot \vec{a}_x}\, ds = \iint 10x^3\, ds$$

$$x = 2 \qquad = \iint 10 \cdot 2^3\, ds = 80 \iint ds = 80S = 160$$

(17)

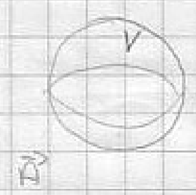




$$\alpha = 2\, tg^{-1} \frac{1}{6}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0} \frac{\vec{r}}{r} \qquad P = \frac{\alpha}{2\pi} \cdot 100\% = 5.25\%$$

IV Gauss



$$\oiint \vec{A}\,\vec{n}\,ds = \iiint_V (\nabla \vec{A})\,dV$$

$$\oiint \vec{D}\,\vec{n}\,ds = \iiint_V \rho\,dV = Q \Rightarrow \nabla \vec{D} = \rho$$

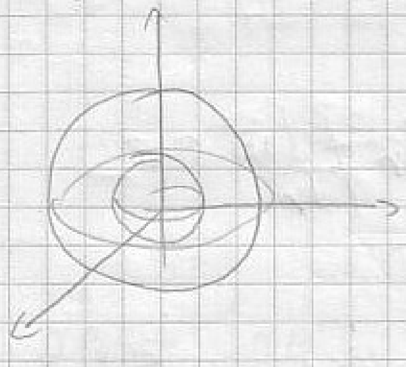
$$\nabla \vec{E} = \frac{\rho}{\varepsilon_0}$$

(3)

$R = 3m$

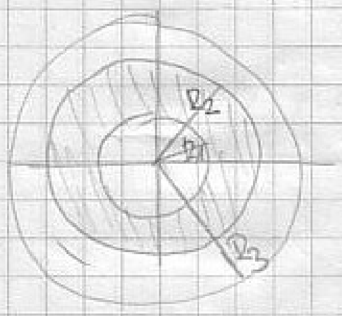
$\rho = \frac{5\sin^2\alpha}{r^2} \quad C/m^3$

$1 < r \leq 2$



sferni k.s

$dV = r^2 dr \sin\theta\, d\theta\, d\alpha$



$$\phi = Q_{uk} = \iiint \rho\,dV =$$

$$= \int_0^{2\pi} d\alpha \int_0^{\pi} \sin\theta\,d\theta \int_1^2 \frac{5}{r^2}\,r^2 dr \sin^2\alpha = 10\pi$$

(25)

$R = 1m$

$\rho = 1 - r^3$

$r = 4m$

$\vec{D} = D\,\vec{a_r}$

$$\oiint \vec{D}\,\vec{n}\,ds = \iiint \rho\,dV$$

$$\oiint D\vec{a_r} \cdot \vec{a_r}\,ds = \int_0^{2\pi} d\alpha \int_0^{\pi} \sin\theta\,d\theta \int_0^1 r^2(1-r^3)\,dr = \frac{4\pi}{6}$$

$$D \oiint ds = D\,S = D\,4\pi R^2$$

$$D\,4\pi R^2 = \frac{4\pi}{6}$$

$D = \varepsilon_0 E$

$$D = \frac{1}{6 \cdot 16} \qquad E = \frac{1}{96\,\varepsilon_0}$$

(26)
$$D = \begin{cases} \frac{5r^2}{4}, & r \leq 2 \\ \frac{20}{r^2}, & r > 2 \end{cases}$$

1°

$$\oint\!\!\oint \vec{D}_1 \vec{n} ds = \iiint \rho dV$$

$$D_1 S = \int_0^{2\pi} d\alpha \int_0^{\pi} \sin\theta d\theta \int_0^{r} r^2 \rho(r) dr$$

$$D_1 S = 4\pi \int_0^r \rho r^2 dr$$

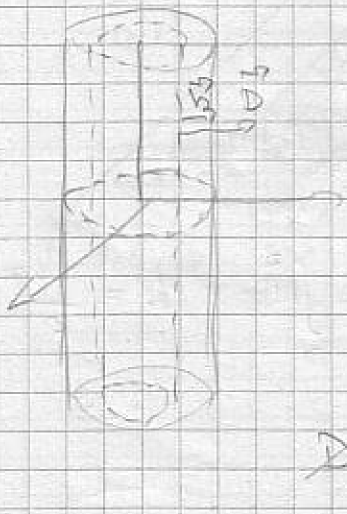
$$D_1 4\pi r^2 = -\parallel-$$

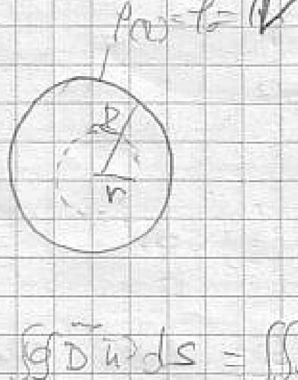
$$\frac{5r^2}{4} r^2 = \int_0^r \rho(r) r^2 dr \Big/ \frac{d}{dr}$$

$$\frac{5}{4} 4r^3 = \rho(r) r^2$$

$$\rho(r) = 5r, \ r \leq 2$$

(7)
ρ – konst
$R - 1m$
$\varphi_0(R) = 1V$





$$\oint\!\!\oint \vec{D} \vec{n} ds = \iiint \rho dV$$

$$D \cdot S = \rho \iiint dV$$

$$\cancel{D 2r\pi L = \rho R^2 \pi L}$$

$$\cancel{D =}$$

$$D 2r\cancel{\pi}\cancel{L} = \rho r^{\cancel{2}} \cancel{\pi}\cancel{L}$$

$$D = \frac{\rho}{2} r$$

$$\varepsilon_0 E = \frac{\rho}{2} r$$

$$E = \frac{\rho}{2\varepsilon_0} r$$

$$\vec{E} = \frac{\rho}{2\varepsilon_0} r \vec{a_R}$$

$$\varphi(R) - \varphi(r) = -\int_r^R \vec{E}\,\vec{a_r}\,dr = -\int_r^1 \frac{\rho}{2\varepsilon_0} r\,dr = \frac{\rho}{2\varepsilon_0}\,\frac{1}{2}r^2\Big|_r^1 =$$

$$= -\frac{\rho}{4\varepsilon_0}(1-r^2)$$

$$\varphi(r) = \varphi(R) + \frac{\rho}{4\varepsilon_0}(1-r^2) = 1 + \frac{\rho}{4\varepsilon_0}(1-r^2)$$

Energija

$$W = \frac{1}{2}\iiint_V \varepsilon_0 E^2 dV$$

$$\varphi = 2x + 4y$$

$$0 < x;\ y, z < 1$$

$$E = -\nabla\varphi = -\left(\frac{\partial \varphi_x}{\partial x}\vec{a_x} + \frac{\partial \varphi_y}{\partial y}\vec{a_y} + \frac{\partial \varphi_z}{\partial z}\vec{a_z}\right) =$$

$$= -\left(2\vec{a_x} + 4\vec{a_y}\right) = -2\vec{a_x} - 4\vec{a_y}$$

$$E^2 = \vec{E}\cdot\vec{E} = \left(\sqrt{2^2+4^2}\right)^2 = 4 + 16 = 20$$

$$W = \frac{1}{2}\varepsilon_0 \cdot 20 \cdot \iiint dv = 10\varepsilon_0 V = 10\varepsilon_0$$

Proračun polja na granici dielektrika

16

$$3x + 2y + z = 12 \qquad \varepsilon = \varepsilon_0\varepsilon_r$$

$$\vec{E_1} = 2\vec{a_x} + 5\vec{a_z} \qquad \vec{n}(\vec{D_2} - \vec{D_1}) = \sigma_s$$

$$\varepsilon_{r_1} = 3,\ \varepsilon_{r_2} = 1 \qquad \vec{n}\times(\vec{E_2} - \vec{E_1}) = 0$$

$$\vec{E_2} = ?$$

$$\vec{A} = \vec{A_n} + \vec{A_t}$$

$$D_{n_2} - D_{n_1} = 0$$

$$D_{n_2} = D_{n_1}$$

$$\varepsilon_2 E_{n_2} = \varepsilon_1 E_{n_1}$$

$$\vec{n} = \frac{(3,2,1)}{\sqrt{9+4+1}}$$

$$E_{1n} = \vec{E}_1 \cdot \vec{n} = (2\vec{a_x} + 5\vec{a_z})\left(\frac{3a_x + 2a_y + a_z}{\sqrt{14}}\right)$$

$$= \frac{1}{\sqrt{14}}(6+5) = \frac{11}{\sqrt{14}}$$

$$\vec{E}_{1n} = \frac{11}{\sqrt{14}} \cdot \vec{n}$$

$$\vec{E}_1 = \vec{E}_{1n} + \vec{E}_{1t} \Rightarrow \vec{E}_{t_1} = \vec{E}_1 - \vec{E}_{1n} = -\frac{5}{14}\vec{a_x} - \frac{11}{7}\vec{a_y} + \frac{59}{14}\vec{a_z}$$

$$E_{n_2} = \frac{\varepsilon_1}{\varepsilon_2} E_{n_1} = 3 \cdot \frac{11}{\sqrt{14}} = \frac{33}{\sqrt{14}}$$

$$\vec{E}_2 = E_{n_2} \cdot \vec{n} = \frac{33}{\sqrt{14}} \cdot \vec{n}$$

$$\vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0$$

$$\vec{n} \times \vec{E}_2 - \vec{n} \times \vec{E}_1 = 0$$

$$E_{2t} = E_{1t}$$

$$E_{2t} = -\frac{5}{14}\vec{a_x} - \frac{11}{7}\vec{a_y} + \frac{59}{14}\vec{a_z}$$

$$\vec{E}_2 = \vec{E}_{2n} + \vec{E}_{2t} = \frac{47}{7}\vec{a_x} + \frac{22}{7}\vec{a_y} + \frac{46}{7}\vec{a_z}$$

(15) Polarizacija

$$\varepsilon_r = 3.2$$

$$\vec{D} = 2 \cdot 10^{-6}\, \vec{a_x}\, [C/m^2]$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} \qquad \vec{P} = \vec{D} - \varepsilon_0 \vec{E} = \vec{D} - \varepsilon_0 \frac{D}{\varepsilon_0 \varepsilon_r} = D\left(1 - \frac{1}{\varepsilon_r}\right) = 1.37 \cdot 10^{-6} a_x$$

$$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$$

Poisson

$$\nabla \cdot \vec{D} = \rho$$

$$\vec{D} = \varepsilon \cdot \vec{E}$$

$$\vec{E} = -\nabla \cdot \varphi$$

$$\nabla(\varepsilon(\nabla \varphi)) = \rho$$

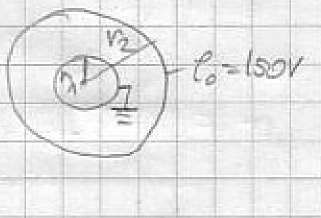
$$\boxed{\nabla(\nabla \varphi) = -\frac{\rho}{\varepsilon}}$$

$$\boxed{\Delta \varphi = -\frac{\rho}{\varepsilon}}$$

Laplace

$$\boxed{\Delta \varphi = \phi}$$

20



$$\Delta \varphi = 0$$

za cilindrični sustav

$$\Delta \varphi = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \varphi}{\partial r}\right) + \underbrace{\frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \alpha^2}}_{=0} + \underbrace{\frac{\partial^2 \varphi}{\partial z^2}}_{=0} = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \varphi}{\partial r}\right) = 0 \;/\cdot r /\int$$

$$r\frac{\partial \varphi}{\partial r} = C_1$$

$$r\frac{d\varphi}{dr} = C_1$$

$$d\varphi = \frac{dr}{r} C_1 \;/\int$$

$$\varphi = \ln(r) \cdot C_1 + C_2$$

$\varphi(r_1) = 0$

$\varphi(r_2) = 150$

$0 = \ln(r_1) C_1 + C_2$

$150 = \ln(r_2) C_1 + C_2$

$r_1 = 1mm$

$r_2 = 25mm$

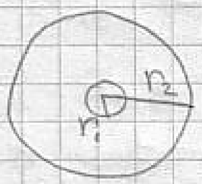
$$150 = C_1 \ln\left(\frac{r_2}{r_1}\right)$$

$$C_1 = \frac{150}{\ln\left(\frac{r_2}{r_1}\right)} = 50.071$$

$$C_2 = -\ln(r_1) \cdot C_1 = 345.88$$

$$\varphi = \ln(r) \cdot 50.071 + 345.88$$

22



$\varphi(r_1) = 0$

$\varphi(r_2) = 100V$

kugla (presjek)

$$\Delta \varphi = 0$$

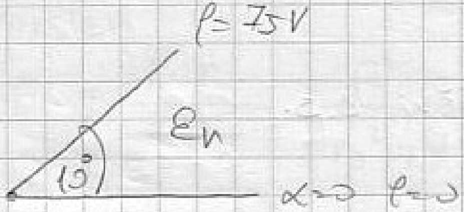
$$\Delta \varphi = \frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial \varphi}{\partial r}\right) + \underbrace{\text{član za kut } \theta \text{ i } \alpha}_{=0} = 0$$

$$r^2 \frac{d\varphi}{dr} = c_1$$

$$d\varphi = \frac{dr}{r^2} c_1$$

$$\varphi = -\frac{c_1}{r} + c_2$$

(24)



$\Delta\varphi = 0$

cilindrični sustav

$\vec{E} = -\nabla\varphi$

$\vec{D} = \varepsilon\vec{E}$

$$\Delta\varphi = \frac{1}{r^2}\frac{\partial^2\varphi}{\partial\alpha^2} = 0$$

$10° = \frac{\pi}{18}$ rad

$$\frac{d\varphi}{d\alpha} = C_1$$

$$d\varphi = d\alpha\, C_1 \quad \int$$

$$\varphi = \alpha C_1 + C_2$$

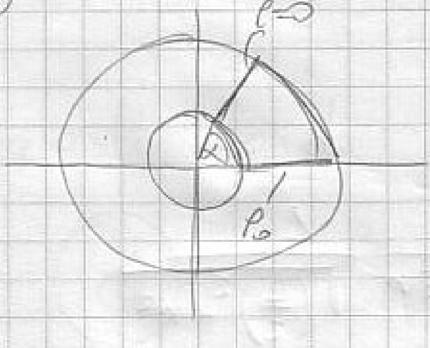
$$\boxed{\varphi = \alpha \cdot 429.72}$$

$$\nabla\varphi = \vec{a_r}\frac{\partial\varphi}{\partial r} + \vec{a_\alpha}\frac{1}{r}\frac{\partial\varphi}{\partial\alpha} + \vec{a_z}\frac{\partial\varphi}{\partial z}$$

$$E = -\vec{a_\alpha}\frac{1}{r}429.72$$

$$\vec{D} = \frac{\varepsilon_0\varepsilon_r \cdot 429.72}{r}\vec{a_\alpha}$$

(10)



$\Delta\varphi = 0$

$$\frac{1}{r^2}\frac{\partial^2\varphi}{\partial\alpha^2} = 0$$

$$\varphi = C_1\alpha + C_2$$

$$\varphi = -\frac{\varphi_0}{\beta}\cdot\alpha + \varphi_0$$

$$\vec{E} = -\nabla\varphi = -\frac{1}{r}\frac{\partial\varphi}{\partial\alpha}\vec{a_\alpha} = \frac{\varphi_0}{r\beta}\vec{a_\alpha}$$

$$C = \frac{Q}{U} \qquad W = \frac{1}{2}CU^2$$

$$W = \frac{1}{2} \iiint \varepsilon_0 E^2 dV$$

cilindrični k.s.

$$E^2 = \frac{\varphi_0^2}{r^2 \beta^2}$$

$$W = \frac{1}{2} \varepsilon_0 \int_0^{\beta} d\alpha \int_{R_1}^{R_2} r\,dr \int_0^{z} \frac{\varphi_0^2}{r^2 \beta^2} dz =$$

$$= \frac{1}{2} \varepsilon_0 \frac{\varphi_0^2}{\beta^{2}} \cancel{\beta} \cdot z \cdot \int_{R_1}^{R_2} \frac{1}{r} dr =$$

$$W = \frac{1}{\cancel{2}} \varepsilon_0 \frac{\varphi_0^2}{\beta} \cdot z \cdot \ln \frac{R_2}{R_1} = \frac{1}{\cancel{2}} C U^2$$

$$U = \varphi_0$$

$$\boxed{C = \varepsilon_0 z \ln\left(\frac{R_2}{R_1}\right) \cdot \frac{1}{\beta}}$$

Točkasti naboj

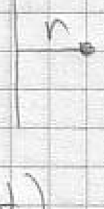
$$E = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r}$$

$$\varphi = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{r_{ref}}\right)$$

$$U_{AB} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_A} - \frac{1}{r_B}\right)$$

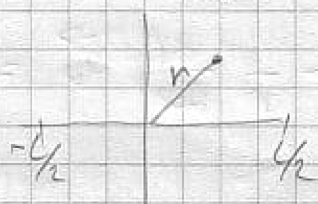
Linijski naboj

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$



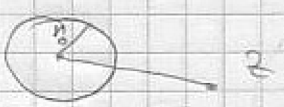
$$E = \frac{\lambda}{4\pi\varepsilon r} \frac{L}{\sqrt{\left(\frac{L}{2}\right)^2 + r^2}}$$

$$\varphi = \frac{\lambda}{2\pi\varepsilon} \ln\left(\frac{r_{ref}}{r}\right)$$



$$U_{AB} = \frac{\lambda}{2\pi\varepsilon} \ln\left(\frac{r_A}{r_B}\right)$$

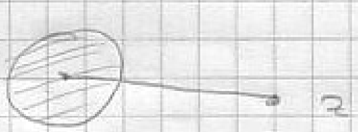
Tanki prsten



$$E_z = \frac{\lambda \cdot r_0}{2\varepsilon} \cdot \frac{z}{(r_0^2+z^2)^{3/2}}$$

$$\varphi = \frac{\lambda r_0}{2\varepsilon} \cdot \frac{1}{\sqrt{r_0^2+z^2}}$$

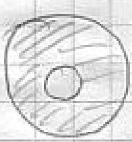
Disk



$$E_z = \frac{\sigma}{2\varepsilon}\left(1 - \frac{z}{\sqrt{R_0^2+z^2}}\right)$$

$$\varphi = \frac{\sigma}{2\varepsilon}\left(\sqrt{R_0^2+z^2} - z\right)$$

Vijenac



$$E_z = \frac{\sigma \cdot z}{2\varepsilon}\left(\frac{1}{\sqrt{R_1^2+z^2}} - \frac{1}{\sqrt{R_2^2+z^2}}\right)$$

$$\varphi = \frac{\sigma}{2\varepsilon}\left(\sqrt{R_2^2+z^2} - \sqrt{R_1^2+z^2}\right)$$

Ravnina

$$E = \frac{\sigma}{2\varepsilon}$$

R_1 – manji

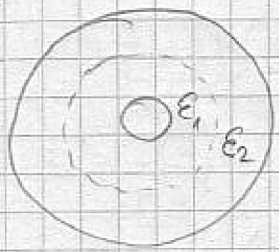
R_2 – veći

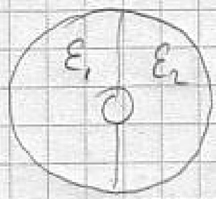
kuglasti kond. ?

$$\varphi = \frac{\ominus \varphi_0}{\frac{1}{R_1} - \frac{1}{R_2}}\left(\frac{1}{r} - \frac{1}{R_2}\right)$$

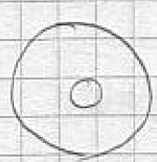
$$\vec{E} = -\nabla\varphi = \frac{R_1 R_2}{R_2 - R_1} \cdot \frac{\varphi_0}{r^2}$$

$$C = 4\pi\varepsilon \frac{R_1 R_2}{R_2 - R_1}$$

 $\rightarrow$ seri. spoj 2 kond $\quad C = \frac{C_1 C_2}{C_1 + C_2}$

 $\rightarrow$ par. spoj 2 kond $\quad C = C_1 + C_2$

cilindrični kond



$$\varphi = \varphi_0 \ln\left(\frac{R_2}{r}\right) \cdot \frac{1}{\ln\left(\frac{R_2}{R_1}\right)}$$

$$E = \vec{a}_R \frac{\varphi_0}{r \ln\left(\frac{R_2}{R_1}\right)}$$

$$C = 2\pi\varepsilon \frac{l}{\ln\left(\frac{R_2}{R_1}\right)}$$

MASS by Wolfman - 'Fala Wolf
sken by ME!