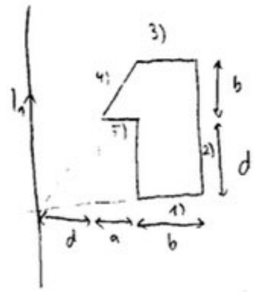
ZAVRŠNI ISPIT 2021

1. $I_1 = I_2 = 1A$

$d = 1m$

$a = 0{,}5m$

$b = 0{,}75m$



$\frac{2}{3}y - \frac{2}{3} = x - \frac{1}{3}$

$x = \frac{2}{3}y - \frac{1}{3}$

$y - 1 = \frac{3}{2}(x - \frac{1}{3})$

4) $y - d = \frac{b}{a}(x - 0)$

$y = \frac{b}{a}x - b + d$

$dy + ba - ad = bx$

$x = \frac{dy}{b} + \frac{ba}{b} - \frac{ad}{b}$

$x = \frac{2}{3}y - \frac{1}{6}$

a)

$\vec{B} = \frac{\mu I}{2\pi r} \cdot (-\hat{a}_z)$

$M = \frac{\phi}{I}$

$$\phi = \frac{\mu_0 I}{2\pi} \iint \frac{1}{r} dS = \frac{\mu_0 I}{2\pi}\left[\int_{d+a}^{d+a+b} \frac{1}{x}(b+d)dx + \int_d^{d+b} dy \int_d^{\frac{2}{3}y - \frac{1}{6}} \frac{1}{x} dx\right]$$

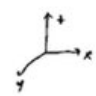
$$\phi = \frac{\mu_0 \cdot 1}{2\pi}\left[\int_{1+0{,}5}^{1+0{,}5+0{,}75} \frac{1}{x}(0{,}75+1)dx + \int_1^{1+0{,}75} dy \int_1^{\frac{2}{3}y-\frac{1}{6}} \frac{1}{x} dx\right]$$

$$\phi = 2\cdot 10^{-7} \cdot \left(0{,}7095639 + \int_1^{1{,}75} \ln\left(\frac{2}{3}y - \frac{1}{6}\right) dy\right)$$

$$\phi = 2\cdot 10^{-7} \cdot (0{,}7095639 - 0{,}2301396)$$

$$\phi = 9{,}588 \cdot 10^{-8}$$

$$M = \frac{\phi}{I} = 95{,}88 \text{ nH}$$



b) $F = I_2 \cdot \int d\vec{l} \times \vec{B}$ $\qquad I_2 = 1 \qquad B = \frac{\mu_0}{2\pi r}(-\hat{a}_z)$

1) $\vec{F_1} = \frac{\mu_0}{2\pi} \int \frac{dx\, \hat{a}_x \times (\hat{a}_z)}{x} = \frac{\mu_0}{2\pi} \int_{d+a}^{d+a+b} \frac{\hat{a}_y}{x} dx = \frac{\mu_0}{2\pi} \ln\frac{d+a+b}{d+a}$

3) $\vec{F_3} = \frac{\mu_0}{2\pi} \int_{d+a}^{d+a+b} \frac{-\hat{a}_y}{x} = -\vec{F_1}$ $\qquad \rightsquigarrow F_1 + F_3 = 0$

2) $\vec{F_2} = \frac{\mu_0}{2\pi(d+a+b)} \int_0^{b+d} dy\,(-\hat{a}_y) \times (-\hat{a}_z) = \frac{\mu_0 (b+d)}{2\pi(a+b+d)}(-\hat{a}_x)$

4) $y = \frac{b}{a}x + b \qquad dy = \frac{b}{a}dx$

$d\vec{l} = \hat{a}_x dx + \hat{a}_y dy = \left(\hat{a}_x + \hat{a}_y \frac{b}{a}\right)dx$

$d\vec{l} \times \vec{B} = \frac{\mu_0}{2\pi x}\left(\hat{a}_y - \frac{b}{d}\hat{a}_x\right)dx$

$\vec{F_4} = \frac{\mu_0}{2\pi} \int^{d+a} \frac{1}{x}\hat{a}_y - \frac{b}{a}\frac{1}{x}\hat{a}_x = \frac{\mu_0}{2\pi} \ln\frac{d+a}{d}\left(\hat{a}_x - \frac{b}{a}\hat{a}_y\right)$

5) $\vec{F_5} = \frac{\mu_0}{2\pi} \int_d^{d+a} (-\hat{a}_y)\frac{1}{x} dx = -\frac{\mu_0}{2\pi} \ln\frac{d+a}{d}\hat{a}_y$

$\vec{F_{uk}} = \sum \vec{F_i}$ $\qquad$ Y smjerovi se pokrate

(2.)

$$\begin{cases} \lambda_1 = 3m \\ \varepsilon_0, \mu_0, K=0 \end{cases}$$

$$\begin{cases} \lambda_2 = 1m \\ \varepsilon_r, \mu_r, K=0 \end{cases}$$

$E_{2m} = 120\pi \ V/m$

$H_{2m} = 0{,}5 A/m$

$\varepsilon_r, \mu_r = ?$

$Z_1 = 120\pi$

$$Z_2 = Z_1 \cdot \sqrt{\frac{\mu_r}{\varepsilon_r}} = \frac{E_{2m}}{H_{2m}} = 240\pi$$

$$120\pi \sqrt{\frac{\mu_r}{\varepsilon_r}} = 240\pi$$

$$\sqrt{\frac{\mu_r}{\varepsilon_r}} = 2$$

$$\mu_r = 4\varepsilon_r$$

$$\beta = \frac{2\pi}{\lambda}$$

$$\beta_1 = \frac{2\pi}{3} \qquad \beta_2 = 2\pi$$

$w_1 = w_2$

$$\beta_1 = w\sqrt{\mu_0 \varepsilon_0} \qquad \beta_2 = w\sqrt{\mu_0 \varepsilon_0} \cdot \sqrt{\mu_r \varepsilon_r}$$

$$\beta_2 = \beta_1 \sqrt{\mu_r \varepsilon_r}$$

$$2\pi = \frac{2\pi}{3} \sqrt{\mu_r \varepsilon_r}$$

$$\mu_r \varepsilon_r = 9$$

$$\varepsilon_r^2 = \frac{9}{4}$$

$$\boxed{\varepsilon_r = \frac{3}{2} \qquad \mu_r = 6}$$

(4.) $\vec{E} = 15 \sin(2\pi \cdot 10^7 t - 3x)\vec{a_z}$ V/m

a) ε_r, $Z = ?$

$\omega = 2\pi \cdot 10^7$

$\beta = 3$

$\mu = \mu_0$

$\beta = \omega\sqrt{\mu\varepsilon} \rightarrow \sqrt{\mu\varepsilon} = \frac{3}{2\pi \cdot 10^7}$

$\varepsilon_r = \left(\frac{3}{2\pi \cdot 10^7}\right)^2 \cdot \frac{1}{\mu_0 \varepsilon_0} = 204,8916$

$Z = Z_0 \sqrt{\frac{1}{\varepsilon_r}} = 26,337$

b) $N_{sr} = ?$

$N_{sr} = \frac{1}{2} \frac{E_0^2}{Z} = 8,5428\,W$

c) $P_{sr} = ?$ $2x + y = 5$ $S = 200cm^2$

$\vec{n} = \frac{2\hat{a_x} + \hat{a_y}}{\sqrt{5}}$

$P_{sr} = \iint_S \vec{N_{s1}} \cdot \vec{n}\, dS = \frac{1}{2} \frac{E_0^2}{Z} \frac{2}{\sqrt{5}} \cdot 200cm^2 = 0,07641\,W$

5. $z \geq 0$

1) $\varepsilon_0, \mu_0, K = 0$

$\vec{E} = 10\sin(\omega t + 3z)\,\vec{a_x}$ V/m

2) $\varepsilon = \varepsilon_0 \quad \mu = \mu_0 \quad K = 4,5 \cdot 10^7$ S/m na $z = 0$

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a) $\rho, \tau = ? \qquad f = 5\,MHz$

$Z_1 = Z_0$

$Z_2 = \frac{j\omega\mu}{\gamma} \qquad \gamma^2 = j\omega\mu(K + j\omega\varepsilon)$

$\frac{K}{\omega\varepsilon} = \frac{K}{10\pi \cdot 10^6 \cdot \varepsilon_0} = 1,62 \cdot 10^{11} \gg 1$

$\alpha = \beta = \sqrt{\frac{\omega\mu K}{2}}$

$Z_2 = \frac{j\omega\mu}{\gamma} = \sqrt{\frac{\omega\mu}{K}}\, e^{j\pi/4} = 9,366 \cdot 10^{-4}\, e^{j\pi/4}$

$\rho = \frac{Z_2 - Z_1}{Z_2 + Z_1} = 0,9999964865 \angle 179,9997987^\circ$

$\tau = 4,96881 \cdot 10^{-6} \angle 44,99989935^\circ$

b) $N_{sr} = \frac{E^2}{|Z|} \cos\varphi \qquad \varphi = \frac{\pi}{4}\ (z=0)$

$= \frac{E^2}{\sqrt{2}\,|Z|}$

$\frac{P_{g,sr}}{S} = \frac{E^2}{|Z|\sqrt{2}} = 53\,382,665\ W/m^2$