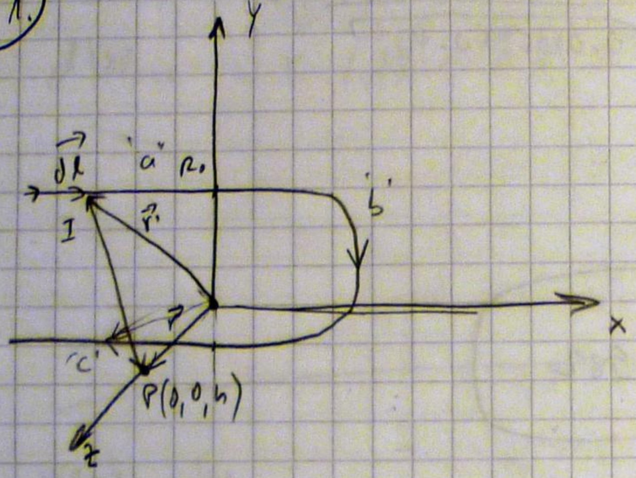
AUDITORNE 21. 4. 2011.

1.



$$d\vec{H} = \frac{I}{4\pi} \int \frac{\vec{dl} \times \vec{R}}{R^3}$$

u 3 segmenta

a:)

$$\vec{r} = \vec{a_z} \cdot h$$

$$\vec{R} = \vec{r} - \vec{r}' =$$

$$\vec{r}' = -x'\vec{a_x} + \vec{a_y} R_0$$

$$= \vec{a_x} \cdot x' - \vec{a_y} \cdot R_0 + \vec{a_z} \cdot h$$

$$\vec{dl} = \vec{a_x} \cdot dx'$$

$$\vec{dl} \times \vec{R} = (-\vec{a_z} R_0 - \vec{a_y} h)\, dx'$$

$$|\vec{R}| = \sqrt{x'^2 + R_0^2 + h^2}$$

$$\frac{-\infty}{\infty} \longrightarrow +\infty \text{ u } -1$$

$$\vec{H}_a = \frac{I}{4\pi} \int_{-\infty}^{0} \frac{(-\vec{a_z} R_0 - \vec{a_y} h)}{\left(\sqrt{x'^2 + R_0^2 + h^2}\right)^3}\, dx' =$$

$$= \frac{I}{4\pi} (-\vec{a_z} R_0 - \vec{a_y} h) \cdot \frac{x'}{(R_0^2 + h^2)\sqrt{x'^2 + R_0^2 + h^2}} \Bigg|_{-\infty}^{0} =$$

$$= -\frac{I}{4\pi} (\vec{a_z} R_0 + \vec{a_y} h) \cdot \left(+ \frac{1}{R_0^2 + h^2} \right) =$$

$$= -\frac{I}{4\pi (R_0^2 + h^2)} (\vec{a_z} R_0 + \vec{a_y} h)$$

c:) $\vec{r} = \vec{a_z}\cdot h$

$\vec{r'} = -\vec{a_x}\, x - \vec{a_y}\, R_0$

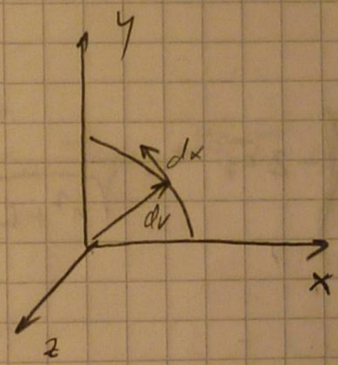
$d\vec{l} = -\vec{a_x}\cdot dx$

$\vec{R} = \vec{r} - \vec{r'} = \vec{a_z}\cdot h + \vec{a_x}\cdot x + \vec{a_y}\cdot R_0$

$d\vec{l} \times \vec{R} = \left(-\vec{a_z}\, R_0 + \vec{a_y}\cdot h\right)\cdot dx$

$$\vec{H}_c = \frac{I}{4\pi}\int_{-\infty}^{0}\frac{\left(-\vec{a_z}\,R_0 + \vec{a_y}\cdot h\right)dx}{\left(x^2+R_0^2+h^2\right)^{\frac{3}{2}}} = \frac{I\left(-\vec{a_z}\,R_0+\vec{a_y}\,h\right)}{4\pi}\cdot\frac{x}{\left(R_0^2+h^2\right)\sqrt{x^2+R_0^2+h^2}}\Bigg|_{-\infty}^{0}$$

$$= \frac{I}{4\pi}\left(\vec{a_z}\,R_0 + \vec{a_y}\,h\right)\cdot\frac{1}{R_0+h^2}$$



b:) $d\vec{l} = -\vec{a_\alpha}\cdot R_0\cdot d\alpha$

$\vec{r'} = \vec{a_r}\cdot R_0$

$\vec{r} = \vec{a_z}\cdot h$

$\vec{R} = \vec{r} - \vec{r'} = \vec{a_z}\cdot h - \vec{a_r}\cdot R_0$

$d\vec{l}\times\vec{R} = -\vec{a_\alpha}\,R_0\,d\alpha \times \left(\vec{a_z}\,h - \vec{a_r}\,R_0\right) =$

$= \vec{a_\alpha}\cdot R_0\cdot d\alpha \times \left(\vec{a_r}\cdot R_0 - \vec{a_z}\cdot h\right) =$

$= R_0\,d\alpha\left(-\vec{a_z}\,R_0 - \vec{a_r}\cdot h\right) =$

$= R_0\,d\alpha\cdot\left[-\vec{a_z}\,R_0 - \left(\vec{a_x}\cos\alpha + \vec{a_y}\sin\alpha\right)\cdot h\right]$

$$\vec{H}_b = \frac{I}{4\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{R_0\,d\alpha\left(-\vec{a_z}R_0 - \left(\vec{a_x}\cos\alpha\, h + \vec{a_y}\sin\alpha\, h\right)\right)}{\left(h^2+R_0^2\right)^{\frac{3}{2}}} =$$

$$= \frac{I R_0}{4\pi\left(h^2+R_0^2\right)^{\frac{3}{2}}}\cdot\left[-\vec{a_z}\cdot R_0\cdot\pi - \vec{a_x}\cdot\sin\alpha\cdot h\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \vec{a_y}\cos\alpha\cdot h\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\right] =$$

$$= \frac{I}{4\pi}\frac{R_0}{(1+R_0^2)^{\frac{3}{2}}}\left(-\vec{a}_z \cdot R_0\pi - \vec{a}_x h \cdot 2\right)$$

$$\vec{H}_{kon} = \vec{H}_a + \vec{H}_b + \vec{H}_c =$$

$$= \frac{I}{4\pi}\Bigg[\frac{1}{R_0^2+h^2}\left(-\vec{a}_z \cdot R_0 - \vec{a}_y \cdot h - \vec{a}_z R_0 + \vec{a}_y \cdot h +\right.$$

$$\left. + \frac{R_0}{(R_0^2+h^2)^{\frac{3}{2}}} \cdot \left(-\vec{a}_z R_0 \pi - \vec{a}_x \cdot 2h\right)\right] =$$

$$= \frac{I \cdot R_0}{4\pi(R_0^2+h^2)}\left(-2\vec{a}_z + \frac{1}{\sqrt{h^2+R_0^2}} \cdot \left(-\vec{a}_z R_0 \cdot \pi - \vec{a}_x \cdot 2h\right)\right)$$

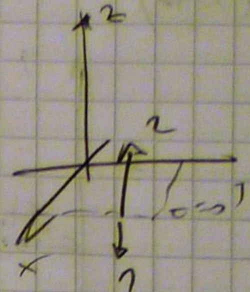
$$\vec{H}_{(h=0)} = \frac{I \cdot \cancel{R_0}}{4\pi R_0^{\cancel{2}}}\left(-2\vec{a}_z + \frac{1}{\cancel{R_0}}\left(-\vec{a}_z \cancel{R_0}\pi\right)\right) =$$

$$= \vec{a}_z \frac{I}{4\pi R_0}(-2-\pi)$$

(2.) (3.8 iz moodle zbirke)

$\vec{K} = 9\vec{a}_y \ \frac{A}{m} \qquad z=0$

$\vec{n}_{12} = \vec{a}_z$



1° $z<0$ $\mu_{R1} = 4$

2° $z>0$ $\mu_{R2} = 3$

$$\vec{H}_2 = 14.5\,\vec{a}_x + 8\,\vec{a}_z \ \frac{A}{m}$$

$$\boxed{\begin{aligned} \vec{n}_{12} \cdot (\vec{B}_2 - \vec{B}_1) &= 0 \\ \vec{n}_{12} \times (\vec{H}_2 - \vec{H}_1) &= \vec{K} \end{aligned}}$$

$\vec{B} = \mu \cdot \vec{H}$

$$H_1 = H_{1x}\,\vec{a}_x + H_{1y}\,\vec{a}_y + H_{1z}\cdot\vec{a}_z$$

$$\vec{a}_z \cdot \mu_0\left(14.5\cdot\mu_{R2}\,\vec{a}_x + 8\mu_{R2}\,\vec{a}_z - \mu_{R1}H_{1x}\,\vec{a}_x - \mu_{R1}\cdot H_{1y}\,\vec{a}_y - \mu_{R1}\cdot H_{1z}\cdot\vec{a}_z\right) = 0$$

$$8\mu_{R2} = \mu_{R1}\cdot H_{1z}$$

$$H_{1z} = \frac{\cancel{8}^{2}\cdot 3}{\cancel{4}} = 6$$

$$\vec{a}_z \times \left(14.5\,\vec{a}_x + \cancel{8\,\vec{a}_z} - H_{1x}\,\vec{a}_x - H_{1y}\,\vec{a}_y - \cancel{6\,\vec{a}_z}\right) = 9a_y$$

$$+\left(14.5\,\vec{a}_y - H_{1x}\,\vec{a}_y\right) + \vec{a}_x H_{1y} = 9\,\vec{a}_y$$

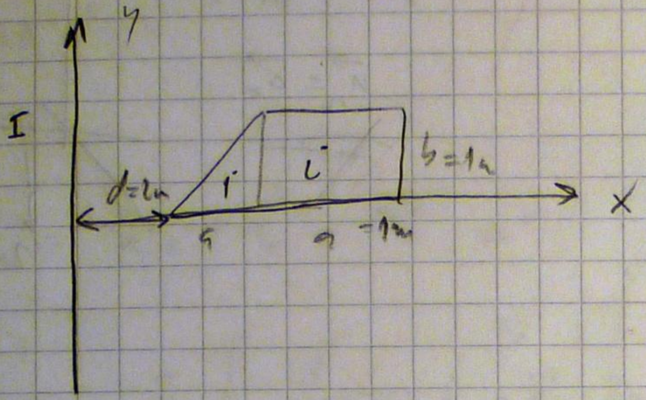
$$H_{1y} = 0$$

$$\underline{H_{1x} = 5.5} \ \frac{A}{m}$$

3. (3.40)

$M = ?$ → međuinduktivitet

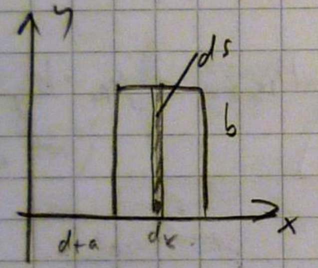
$$M = \frac{\Phi}{I}$$



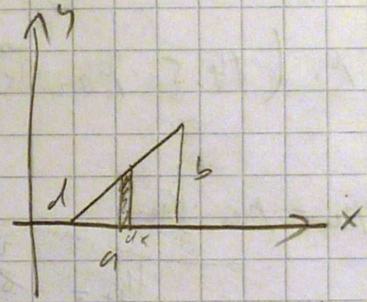
$$B = \frac{\mu_0 I}{2x\pi}$$

$$\Phi = \overbrace{\iint_S \vec{B}\cdot\vec{n}\, dS}^{?} = \iint_S B\, dS = \iint_{S_I} \frac{\mu_0 I}{2x\pi}\, dS + \iint_{S_{II}} \frac{\mu_0 I}{2x\pi}\, dS =$$

$=$



$dS = dx \cdot b$



$dS = y \cdot dx = \frac{b}{a}(x-d)\,dx$

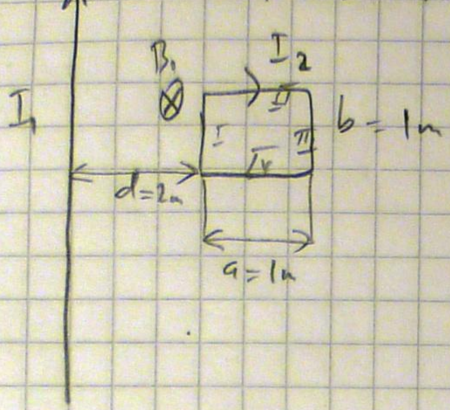
$$\Phi = \frac{\mu_0 I}{2\pi} \cdot \frac{b}{a} \int_d^{d+a} \frac{x-d}{x}\, dx + \frac{\mu_0 I \cdot b}{2\pi} \int_{d+a}^{d+2a} \frac{1}{x}\, dx =$$

$$= \frac{\mu_0 I}{2\pi} \cdot \frac{b}{a} \left(x \Big|_d^{d+a} - d \ln x \Big|_d^{d+a} \right) + \frac{\mu_0 I b}{2\pi} \ln x \Big|_{d+a}^{d+2a}$$

$$\Rightarrow \underline{M = \frac{\Phi}{I} = 95.35\ \text{mH}}$$

(4) (3.41)

$I_1 = I_2 = 1A$



$$F = I \cdot \int d\vec{l} \times \vec{B}$$

$$F_I = I_2 \cdot \int d\vec{l}_1 \times \vec{B} =$$

$$d\vec{l}_1 = \vec{a_y}\, dy$$

$$\vec{B} = -\vec{a}_z \frac{\mu_0 I}{2x\pi}$$

$$= I_2 \cdot \int_{y=0}^{b} -\vec{a_x} \frac{\mu_0 I_1}{2d\pi} = -\vec{a_x} \frac{\mu_0 I_1 I_2 b}{2d\pi}$$

$$F_{III} = \vec{a_x} \frac{\mu_0 I_1 I_2 b}{2(d+a)\pi}$$

$$F_{II} = I_2 \cdot \int_{x=d}^{d+a} \vec{a_y} \frac{\mu_0 I_1}{2\pi x} dx = \frac{\mu_0 I_1 I_2}{2\pi} \ln \frac{d+a}{d}$$

$F_{ukupno} = 33.3\,N$