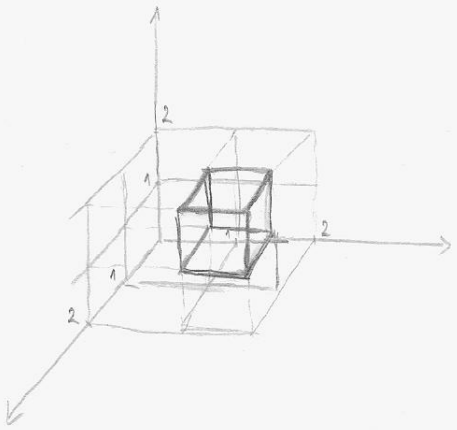
I U vakuumu vrijedi statičko magnetsko polje zadano jednadžbom:

$$\vec{B} = \frac{Ax}{b^2+y}\,\vec{a}_x + C\ln(b^2+y)\,\vec{a}_y$$

$$\boxed{A = 3}$$

$$\boxed{b = 3}$$



1) Odredi konstantu C

$$\nabla\cdot\vec{B} = 0 \Rightarrow \oint \vec{B}\,\vec{n}\,ds = 0$$

$$\nabla\cdot\vec{B} = \frac{\partial}{\partial x}\left[\frac{3x}{9+y}\right] + \frac{\partial}{\partial y}\left[C\cdot\ln(9+y)\right] = 0$$

$$\frac{3}{9+y} + C\cdot\frac{1}{9+y} = 0 \Rightarrow C = -3 \qquad \boxed{D}$$

2) Odredi gustoću struje u točki $(1,3,1)$

$$\nabla\times\vec{B} = \mu_0\vec{J}$$

$$\vec{J} = \frac{1}{\mu_0}(\nabla\times\vec{B}) = \frac{1}{\mu_0}\begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{3x}{9+y} & -3\ln(9+y) & 0 \end{vmatrix}$$

$$= \frac{1}{\mu_0}\left[\vec{a}_x\cdot 0 - \vec{a}_y\cdot 0 + \vec{a}_z\left[0 - \frac{\partial}{\partial y}\left(\frac{3x}{9+y}\right)\right]\right]$$

$$= \vec{a}_z\cdot\frac{3x}{\mu_0(9+y)^2}\Bigg|_{(1,3,1)} = \vec{a}_z\frac{1}{\mu_0 48} \qquad \boxed{F}$$

3) Odredi ukupnu struju kroz kvadrat definiran dijagonalom $(0,0,0)$ i $(1,1,0)$

$$\vec{J} = \vec{a}_z \frac{3x}{\mu_0 (9+y)^2}$$

$$I = \iint \vec{J}\, \vec{n}\, ds$$

$$\vec{n}\, ds = \vec{a}_z\, dx\, dy$$

$$I = \iint \frac{3x}{\mu_0 (9+y)^2} dx\, dy$$

$$= \frac{3}{\mu_0} \int_0^1 x\, dx \int_0^1 \frac{dy}{(9+y)^2} = \frac{3}{\mu_0} \left. \frac{x^2}{2} \right|_{x=0}^{1} \left. \frac{1}{-(9+y)} \right|_{y=0}^{1}$$

$$= \frac{3}{\mu_0} \cdot \frac{1}{2} \cdot \left(\frac{1}{-10} + \frac{1}{9} \right) = \frac{1}{60\mu_0}$$ **C**

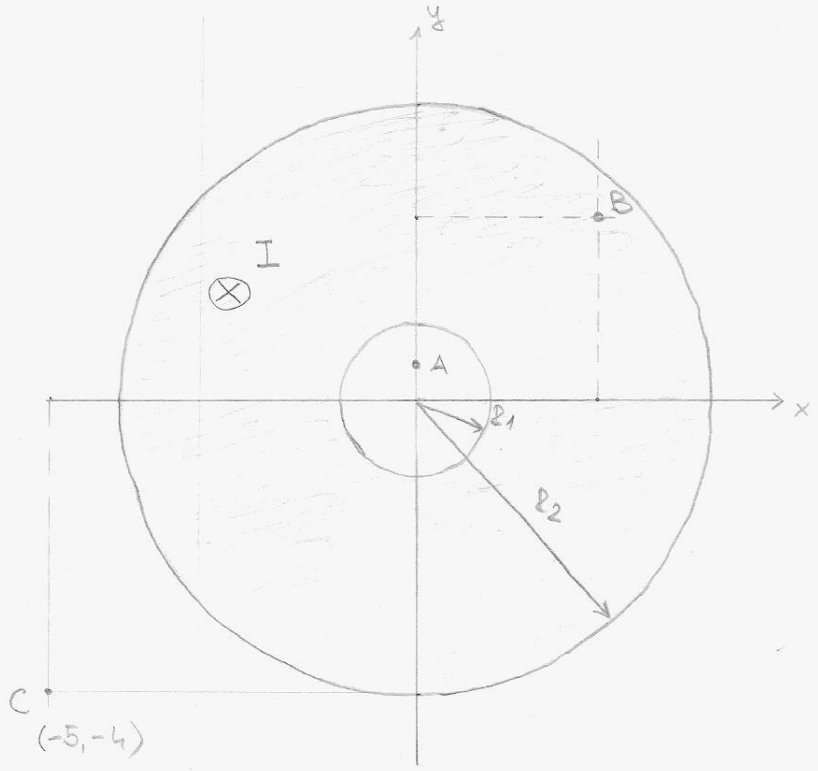
4) Odredi ukupni tok zadanog polja kroz površinu kocke zadane prostornom dijagonalom $(1,1,1)$ i $(2,2,2)$

$$\Phi = \iint \vec{B}\, \vec{n}\, ds$$

→ radi se o zatvorenoj površini pa je

$$\Phi = \oiint \vec{B}\, \vec{n}\, ds = 0$$ **A**

II Zadan je beskonačno dugi cijevni vodič prema slici. Kroz vodič teče struja I jednoliko raspoređena po presjeku. Zadano je: $R_1 = 1\ m$, $R_2 = 4\ m$, $I = 15\ A$



5) Odredi vektor jakosti magnetskog polja u točki $A\left(\frac{R_1}{2}, 0\right)$

→ vektor jakosti mag. polja $\vec{H}$ jednak je po cijeloj kružnici pa se koristi izraz

$$\oint_c \vec{H}\, d\vec{c} = \Sigma I$$

→ kako je po kružnici jednak to prelazi u izraz

$$\vec{H} \cdot 2r\pi = \Sigma I$$

$$\Sigma = 0 \Rightarrow \vec{H} = 0$$

6) isto u točki $B\left(\frac{R_1 + R_2}{2}, \frac{R_1 + R_2}{2}\right)$

$B(2.5,\ 2.5) \Rightarrow r = 2.5\sqrt{2}$

$$\vec{H} = \frac{I}{2\pi r} \cdot \frac{r^2 - R_1^2}{R_2^2 - R_1^2} = \text{???}$$

7.) isto u točki $C(-(R_1+R_2), -R_2)$

$C(-5,-4) \Rightarrow r = \sqrt{41}$

$$\vec{H} = \frac{I}{2\pi r} \cdot \underbrace{\left[\frac{-4\vec{a}_x - 5\vec{a}_y}{\sqrt{41}}\right]}_{\text{jedinični vektor smjera}}$$

$$\vec{H} = -233\,\vec{a}_x - 0.291\,\vec{a}_y \quad \boxed{E}$$

8.) Odredi energiju pohranjenu u magnetskom polju u. prostoru oko vodiča između R_2 i $2R_2$ duljine 1 m

$\ell = 1$ m

$$W = \frac{1}{2}\iiint H \cdot B \cdot dV = \frac{\mu_0}{2}\iiint H^2\, dV$$

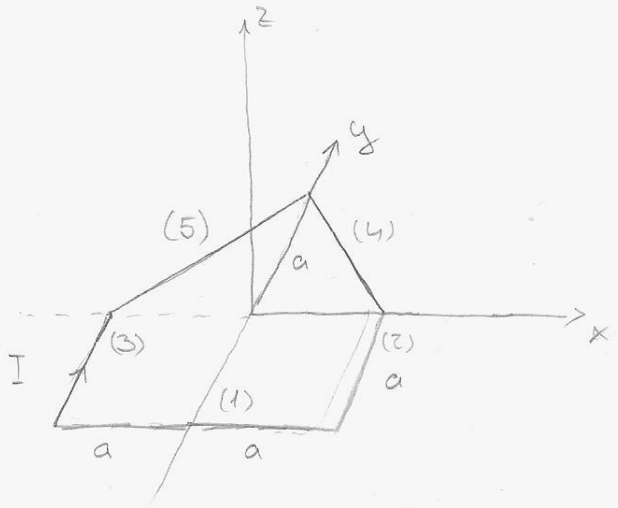
$$dV = r\, d\varphi\, dr\, dz = r \cdot 2\pi\, dr\, dz = r \cdot 2\pi \cdot \ell \cdot dr$$

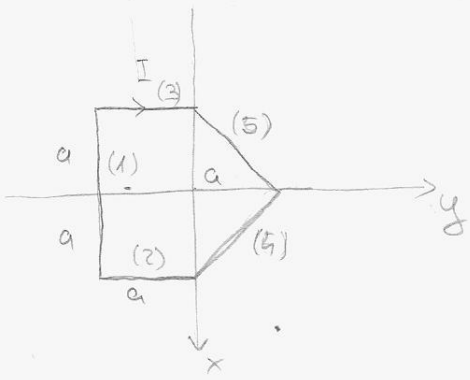
$$H = \frac{I}{2\pi r}$$

$$W = \frac{\mu_0}{2}\int \frac{I^2}{4\pi^2 r^2} \cdot r \cdot 2\pi \cdot \ell\, dr$$

$$= \frac{\mu_0 I^2 \ell}{4\pi}\int_{R_2}^{2R_2} \frac{dr}{r} = \frac{\mu_0 I^2}{4\pi} \ln\frac{2R_2}{R_2} = 15.6\ \mu\text{J} \quad \boxed{A}$$

III Strujna je petlja zadana slikom uz $a = 1\,m$, $I = 11\,A$





g) Odredite apsolutnu vrijednost vektora jakosti polja u točki (0, 0, 0)

→ sva polja u točki (0,0,0) su u smjeru $-\vec{a}_z$!

$$H_1 = \frac{I}{4\pi a}(\sin 45° + \sin 45°) = \frac{I\sqrt{2}}{4\pi a}$$

$$H_2 = H_3 = \frac{I}{4\pi a}(\sin 0° + \sin 45°) = \frac{I\cdot\sqrt{2}}{8\pi a}$$

$$H_4 = H_5 = \frac{I}{4\pi \frac{a}{\sqrt{2}}}(\sin 45° + \sin 45°) = \frac{I}{2\pi a}$$

$$H = H_1 + H_2 + H_3 + H_4 + H_5 = \frac{I}{8\pi a}(2\sqrt{2} + \sqrt{2} + \sqrt{2} + 4 + 4)$$

$$= \frac{I}{2\pi a}(\sqrt{2} + 2) = 5.977\ A/m \quad \boxed{F}$$

Odredite komponente vektora jakosti magnetskog polja u točki $(0,0,1)$ u smjeru: 10) x-osi
11) y-osi
12) z-osi

(1)

$$H_1 = \frac{I}{4\pi a\sqrt{2}}(2\cdot\sin\alpha)\cdot$$

$$\vec{H}_1 = \frac{I}{2\pi a\sqrt{6}}\cdot\left(\frac{\vec{a}_y - \vec{a}_z}{\sqrt{2}}\right) = \frac{I}{4\pi a\sqrt{3}}(\vec{a}_y - \vec{a}_z)$$

$$H_2 = H_3 = \frac{I}{4\pi a\sqrt{2}}(\sin 0 + \sin\alpha) = \frac{I}{4\pi a\sqrt{6}}$$

$$\vec{H}_2 = \frac{I}{4\pi a\sqrt{6}}\left(\frac{-\vec{a}_x - \vec{a}_z}{\sqrt{2}}\right) = \frac{I}{8\pi a\sqrt{3}}(-\vec{a}_x - \vec{a}_z)$$

$$\vec{H}_3 = \frac{I}{4\pi a\sqrt{6}}\left(\frac{\vec{a}_x - \vec{a}_z}{\sqrt{3}}\right) = \frac{I}{8\pi a\sqrt{3}}(\vec{a}_x - \vec{a}_z)$$

$$H_4 = H_5 = \frac{I}{4\pi a\frac{\sqrt{3}}{\sqrt{2}}}(\sin 30° + \sin 30°) = \frac{I\sqrt{2}}{4\pi a\sqrt{3}}$$

$$\vec{H}_4 = \frac{I\sqrt{2}}{4\pi a\sqrt{3}}\left(\frac{-\vec{a}_x - \vec{a}_y - \vec{a}_z}{\sqrt{3}}\right)$$

$$= \frac{I\sqrt{2}}{12\pi a}(-\vec{a}_x - \vec{a}_y - \vec{a}_z)$$

$$\vec{H}_5 = \frac{I\sqrt{2}}{12\pi a}(\vec{a}_x - \vec{a}_y - \vec{a}_z)$$

$$\vec{H} = \vec{H}_1 + \vec{H}_2 + \vec{H}_3 + \vec{H}_4 + \vec{H}_5$$

$$= \vec{a}_x\cdot 0 + \vec{a}_y\left[\frac{I}{4\pi a\sqrt{3}} - \frac{I\sqrt{2}}{6\pi a}\right] + \vec{a}_z\left[-\frac{I}{4\pi a\sqrt{3}} - \frac{I}{4\pi a\sqrt{3}} - \frac{I\sqrt{2}}{6\pi a}\right]$$

$$= \vec{a}_x\cdot 0 - \vec{a}_y\, 0.32 - \vec{a}_z\, 1.836$$

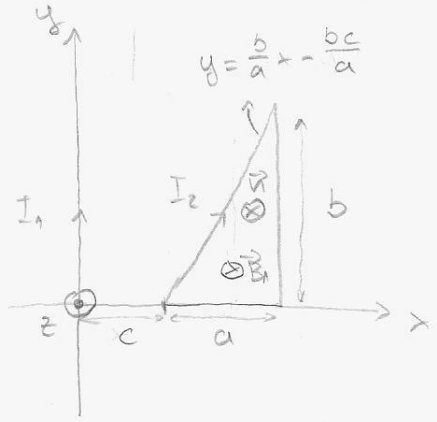
10) $H_x = 0$ A/m [C]

11) $H_y = -0.32$ A/m [A]

12) $H_z = -1.84$ A/m [F]

IV Trokutna petlja protjecana strujom $I_2 = 5$ A nalazi se u ravnini s beskonačno dugom strujnicom i protjecane strujom $I_1 = 3$ A prema slici. Zadano je $a = 2$ m $b = 7$ m, $c = 2$ m

13.) Odredi međuinduktivitet petlje i strujnice.



$$\vec{B}_1 = \frac{\mu_0 I_1}{2\pi x}(-\vec{a}_z)$$

$$y = \frac{b}{a}x - \frac{bc}{a}$$

$$x = \frac{a}{b}y + c$$

$$\Phi_{12} = \iint_{S_2} \vec{B}_1 \vec{n}\, dS$$

$$\vec{n}\, dS = -\vec{a}_z\, dx\, dy$$

$$\Phi_{12} = \iint_{S_2} \frac{\mu_0 I_1}{2\pi x}\, dx\, dy = \frac{\mu_0 I_1}{2\pi} \int_c^{c+a} \frac{dx}{x} \int_0^{\frac{b}{a}x - \frac{bc}{a}} dy$$

$$= \frac{\mu_0 I_1}{2\pi} \int_c^{c+a} \frac{dx}{x}\left(\frac{b}{a}x - \frac{bc}{a}\right) = \frac{\mu_0 I_1}{2\pi} \int_c^{c+a} \left(\frac{b}{a} - \frac{bc}{a}\frac{1}{x}\right) dx$$

$$= \frac{\mu_0 I_1}{2\pi}\left[b - \frac{bc}{a}\ln\frac{c+a}{c}\right] = 1.2888\ \text{mWb}$$

$$M = \frac{\Phi_{12}}{I_2} = 257.76\ \text{nH}$$

→ nema ponuđeno, točan prizvat: 429.6 nH

14) Odredi silu na petlju u smjeru osi x.

$$d\vec{l}_2 = dx\,\vec{a}_x + dy\,\vec{a}_y$$

$$\vec{B}_1 = \frac{-I_1\mu_0}{2\pi x}\vec{a}_z$$

$$d\vec{l}_2 \times \vec{B}_1 = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ dx & dy & 0 \\ 0 & 0 & \frac{-\mu_0 I_1}{2\pi x} \end{vmatrix} = \vec{a}_x\left[\frac{-\mu_0 I_1}{2\pi x}dy\right] - \vec{a}_y\left[\frac{-\mu_0 I_1}{2\pi x}dx\right]$$

$$\vec{F}_{12} = I_2 \int d\vec{\ell}_2 \times \vec{B}_1$$

$$= \frac{-\mu_0 I_1 I_2}{2\pi} \vec{a}_x \int \frac{dy}{x} + \frac{\mu_0 I_1 I_2}{2\pi} \vec{a}_y \int \frac{dx}{x}$$

$$F_x = \frac{-\mu_0 I_1 I_2}{2\pi} \left[\int_0^b \frac{dy}{\frac{a}{b}y + c} + \int_b^0 \frac{1}{c+a}\, dy \right]$$

$$= \frac{-\mu_0 I_1 I_2}{2\pi} \left[\frac{b}{a} \cdot \ln\left(\frac{a}{b}y + c\right)\Big|_{y=0}^{b} - \frac{b}{c+a} \right]$$

$$= \frac{-\mu_0 I_1 I_2}{2\pi} \left[\frac{b}{a} \ln \frac{a+c}{c} - \frac{b}{c+a} \right]$$

$$= -2.028\ \mu N$$ **E**

15)
$$F_y = \frac{\mu_0 I_1 I_2}{2\pi} \left[\int_c^{c+a} \frac{dx}{x} + \int_{c+a}^{a} \frac{dx}{x} \right] = 0$$ **C**

16) Odredi iznos energije međudjelovanja petlje i strujnice

$$W = M \cdot I_1 \cdot I_2 = 3.87\ mJ$$

→ ovdje kolega nije prepisao rješenja ponuđena.

V Na feromagnetskom prstenu sa zračnim rasporom $\delta = 3\,mm$, duljine feromagnetskog materijala $\ell_m = 400\,mm$, površine poprečnog presjeka $S = 8\,cm^2$ namotana je zavojnica sa $N = 800$ zavoja.

17.) Ako je magnetska indukcija u zračnom rasporu $B = 1.1\,T$ odredi struju kroz zavojnicu.

$B = B_\delta = B_m = 1.1\,T$

$H_m \rightarrow$ sa grafa $= 200\ A/m$

$H_\delta = \frac{B_\delta}{\mu_0} = 875352\ A/m$

$I = \frac{H_m \cdot \ell_m + H_\delta \cdot \delta}{N} = 3.38\,A$ **D**

18.) Odredite magnetsku energiju u zračnom rasporu

$W_\delta = \frac{1}{2} \iiint_V H_\delta \cdot B_\delta\, dV = \frac{H_\delta B_\delta \cdot \delta \cdot S}{2} = 1.155\,J$ **A**

19) odredite magnetsku energiju u feromagnetskom materijalu ako je on jednolično magnetiziran

$W_m = \frac{1}{2} \iiint_V H_m B_m\, dV = \frac{H_m B_m \cdot \ell_m \cdot S}{2} = 0.0352\,J$ **D**

20) Odredite induktivitet zavojnice

$W = \frac{1}{2} I^2 L \Rightarrow L = \frac{2W}{I^2} = \frac{2(W_\delta + W_m)}{I^2} = 0.208\,H$ **D**