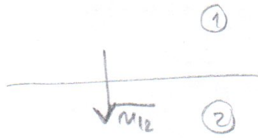
pr. 2.2.1



GRANICE

$$\bar{D} = \varepsilon \bar{E}$$
$$\bar{B} = \mu \bar{H}$$

$$\bar{n}_{12} \, (\bar{B}_2 - \bar{B}_1) = 0 \qquad \bar{n}_{12} \, (\bar{D}_2 - \bar{D}_1) = \sigma$$

$$\bar{n}_{12} \times (\bar{H}_2 - \bar{H}_1) = \bar{K} \qquad \bar{n}_{12} \times (\bar{E}_2 - \bar{E}_1) = 0$$

str. 50 2.2 LJ 14/15

1/15 $\bar{D}_1 = 2\bar{a}_x - 2\bar{a}_y + 4\bar{a}_z$ [C/m²] , $\varepsilon_{r1} = 2$ $z<0$ granica je ravnina $z=0$

$\bar{E}_2 = ?$ $\varepsilon_{r2} = 5$ $z>0$ $\bar{n}_{12} = \bar{a}_z$

$U_{AB} = ?$ A(0,0,1), B(0,-1,2)

UAB

D ≠ E

$$\bar{D} = \varepsilon \bar{E} \qquad \bar{E}_1 = \frac{\bar{D}_1}{\varepsilon_1}$$

$$\bar{E}_2 = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$$

$$\bar{D}_2 = \varepsilon_0 \varepsilon_{r2} \bar{E}_2 = \varepsilon_0 (5x\bar{a}_x + 5y\bar{a}_y + 5z\bar{a}_z)$$

(1) $\boxed{\bar{n}_{12}(\bar{D}_2 - \bar{D}_1) = \sigma}$

(2) $\boxed{\bar{n}_{12} \times (\bar{E}_2 - \bar{E}_1) = 0}$

(1) $\bar{n}_{12} \cdot (\bar{D}_2 - \bar{D}_1) = 0$ → nema naboja

$$\bar{a}_z \left[(5\varepsilon_0 x - 2)\bar{a}_x + (5\varepsilon_0 y + 2)\bar{a}_y + (5\varepsilon_0 z - 4)\bar{a}_z\right] = 0$$

$$5\varepsilon_0 z - 4 = 0$$

$$z = \frac{4}{5\varepsilon_0}$$

(2)
$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 0 & 0 & 1 \\ x - \frac{2}{\varepsilon_1} & y + \frac{2}{\varepsilon_1} & z - \frac{4}{\varepsilon_1} \end{vmatrix} = 0$$

$$\bar{a}_x\left(-y - \frac{2}{\varepsilon_1}\right) - \bar{a}_y\left(-x + \frac{2}{\varepsilon_1}\right) = 0$$

$$-y - \frac{2}{\varepsilon_1} = 0 \qquad x - \frac{2}{\varepsilon_1} = 0$$

$$y = -\frac{2}{\varepsilon_1} \qquad x = \frac{2}{\varepsilon_1}$$

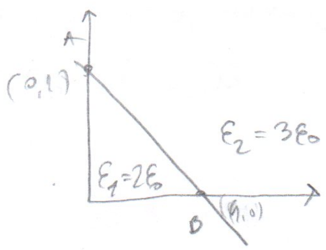
$$y = -\frac{2}{2\varepsilon_0} = -\frac{1}{\varepsilon_0} \qquad x = \frac{1}{\varepsilon_0}$$

$$\bar{E}_2 = \frac{1}{\varepsilon_0}\bar{a}_x - \frac{1}{\varepsilon_0}\bar{a}_y + \frac{4}{5\varepsilon_0}\bar{a}_z \quad \text{V/m}$$

b) $U_{AB} = ?$ A i B se nalaze u $z>0$ → $\bar{E}_2$ A(0,0,1) B(0,-1,2)

$$U_{AB} = \int \bar{E}_2\, d\bar{l} = \frac{1}{\varepsilon_0}\left[\left(\int_0^0 dx - \int_0^{-1} dy + \frac{4}{5}\int_1^2 dz\right]\right. = \frac{1}{\varepsilon_0}\left(1 + \frac{4}{5}\right) = \frac{9}{5\varepsilon_0}\text{ V}$$

П/-07



$$\bar{E_1} = 2\bar{a_x} - 5\bar{a_y} + \bar{a_z}$$

$$\bar{E_2} = \frac{5}{2}\bar{a_x} - \frac{9}{2}\bar{a_y} + \bar{a_z}$$

$U_{AB} = ?$

A (0,1,0)
B (1,0,0)

$$\bar{r}_{AB} = \frac{(1-0)\bar{a_x} + (0-1)\bar{a_y}}{\sqrt{2}}$$

$$\bar{r}_{AB} = \frac{\bar{a_x} - \bar{a_y}}{\sqrt{2}}$$

$$E_{1T} = \bar{E_1} \cdot \bar{r}_{AB} = \frac{2}{\sqrt{2}} + \frac{5}{\sqrt{2}} = \frac{7}{\sqrt{2}}$$

$$E_{2T} = \bar{E_2} \cdot \bar{r}_{AB} = \frac{5}{2\sqrt{2}} + \frac{9}{2\sqrt{2}} = \frac{7}{\sqrt{2}}$$

$$E_T = \frac{7}{\sqrt{2}}$$

$$U_{AB} = \varphi_A - \varphi_B = -\int_B^A \bar{E}\, d\bar{l} = E_T \cdot |\bar{AB}| = \frac{7}{\sqrt{2}} \cdot \sqrt{2} = 7V$$

13/14

3. $z>0 \quad K_1=0, \; \varepsilon_{r1}=1, \; \mu_{r1}=4 \qquad \bar{B}_1 = B_0(2\bar{a}_x + 4\bar{a}_y + 5\bar{a}_z)$

$z<0 \quad K_2=0, \; \varepsilon_{r2}=1, \; \mu_{r2}=2 \qquad \bar{B}_2 = ?$

$\bar{K} = \frac{B_0}{\mu_0}(\bar{a}_x - 2\bar{a}_y) \qquad \bar{n} = -\bar{a}_z$

$z>0$ ①

$z<0$ ② $\quad \downarrow \bar{n}_{12}$

$\bar{n}_{12}(\bar{B}_2 - \bar{B}_1) = 0 \quad (1) \qquad \bar{B}_2 = B_0(x\bar{a}_x + y\bar{a}_y + z\bar{a}_z)$

$\bar{B}$ & $\bar{H}$

$\bar{n}_{12} \times (\bar{H}_2 - \bar{H}_1) = \bar{K}$

(1) $-\bar{a}_z \cdot B_0((x-2)\bar{a}_x + (y-4)\bar{a}_y + (z-5)\bar{a}_z) = 0$

$-(z-5) = 0$

$z = 5$

(2) $\bar{n}_{12} \times (\bar{H}_2 - \bar{H}_1) = \bar{K} \qquad \bar{H}_2 = \frac{\bar{B}_2}{\mu_2} = \frac{B_0}{2\mu_0}(x\bar{a}_x + y\bar{a}_y + z\bar{a}_z)$

$\bar{H}_1 = \frac{\bar{B}_1}{\mu_1} = \frac{B_0}{4\mu_0}(2\bar{a}_x + 4\bar{a}_y + 5\bar{a}_z)$

$$\cancel{\frac{B_0}{\mu_0}} \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 0 & 0 & -1 \\ \frac{1}{2}x - \frac{2}{4} & \frac{y}{2} - 1 & \frac{z}{2} - \frac{5}{4} \end{vmatrix} = \cancel{\frac{B_0}{\mu_0}}(\bar{a}_x - 2\bar{a}_y)$$

$\bar{a}_x\left(\frac{y}{2} - 1\right) - \bar{a}_y\left(\frac{1}{2}x - \frac{1}{2}\right) = \bar{a}_x - 2\bar{a}_y$

$\frac{y}{2} - 1 = 1 \qquad -\frac{1}{2}x + \frac{1}{2} = -2 \; / \cdot 2$

$y = 4 \qquad -x + 1 = -4$

$x = 5$

$\bar{B}_2 = B_0(5\bar{a}_x + 4\bar{a}_y + 5\bar{a}_z)$ A/m

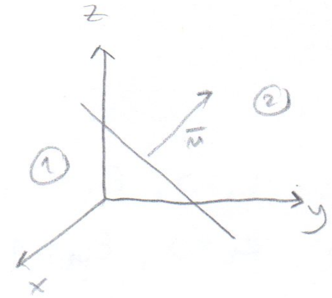
$\bar{H}_2 = \frac{\bar{B}_2}{2\mu_0}$

MI 15/16

$\varepsilon_{r1} = 2$ $\quad \bar E_1 = \bar a_x - 2\bar a_y - 3\bar a_z$ V/m

$\varepsilon_{r2} = 5$ $\quad \bar E_2$ i w u $(3,5,7) = ?$

razdvojeni ravninom $y + 2z - 2 = 0$



$$\bar n_{12} = \frac{\bar a_y + 2\bar a_z}{\sqrt5}$$

$$\bar D_1 = \varepsilon \bar E_1$$

$$\bar D_1 = 2\varepsilon_0(\bar a_x - 2\bar a_y - 3\bar a_z)$$

(1) $\bar n_{12}(\bar D_2 - \bar D_1) = 0$

$\bar n_{12} \times (\bar E_2 - \bar E_1) = 0$

$$\bar E_2 = x\bar a_x + y\bar a_y + z\bar a_z$$

$$\bar D_2 = 5\varepsilon_0(x\bar a_x + y\bar a_y + z\bar a_z)$$

① $\bar D_{1n} = \bar D_{2n}$

② $\bar E_{1t} = \bar E_{2t}$

$$E_{1n} = \bar E_1 \cdot \bar n = \frac{-2-6}{\sqrt5} = -\frac{8}{\sqrt5}$$

$\bar D_{1n} = \bar D_{2n}$

$\varepsilon_1 E_{1n} = \varepsilon_2 E_{2n}$

$$\bar E_{1n} = E_{1n} \cdot \bar n = -\frac{8}{5}(\bar a_y + 2\bar a_z)$$

$$E_{2n} = \frac{\varepsilon_1}{\varepsilon_2} E_{1n} = -\frac{16\sqrt5}{25}$$

$$\bar E_{1t} = \bar E_1 - \bar E_{1n} = \frac{5\bar a_x - 2\bar a_y + \bar a_z}{5} = \bar E_{2t}$$

$$\bar E_{2n} = E_{2n} \cdot \bar n = -\frac{16}{25}(\bar a_y + 2\bar a_z)$$

$$\bar E_2 = \bar E_{2n} + \bar E_{2t} = \frac{25\bar a_x - 26\bar a_y - 27\bar a_z}{25}$$

$(3,5,7)$ je u ②

$\bar E_2(3,5,7) = \bar E_2$ (jer nema x, y, z nigdje)

$$E_2 = \sqrt{\left(\frac{25}{25}\right)^2 + \left(\frac{26}{25}\right)^2 + \left(\frac{27}{25}\right)^2} = 1.8$$

gustoća energije: $w = \frac{1}{2}\varepsilon_2 E_2^2$

$$W = \frac{1}{2}\iiint_V (\bar D \cdot \bar E)\, dV = \frac{1}{2}\int_V \varepsilon E^2\, dV$$

0/14

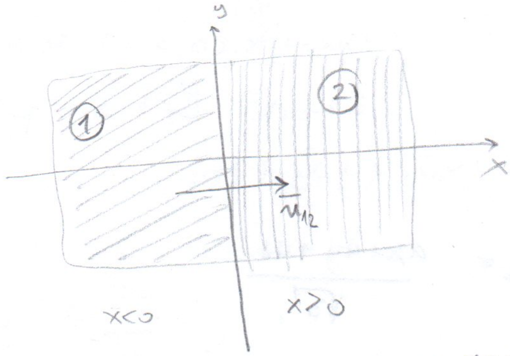
pr. 2.2.1. (str. 50) LJ 13/14

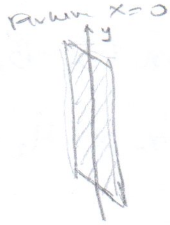
Dva mag. materijala razdvaja ravnina $x=0$

u $x<0$ $\mu_{r1} = 15$

$x>0$ $\mu_{r2} = 20$ $\quad \bar{B}_2 = \bar{a}_x - 0{,}5\,\bar{a}_y + \bar{a}_z$

$\bar{K} = \bar{a}_y \dfrac{0{,}1}{\mu_0} - \bar{a}_z \dfrac{0{,}2}{\mu_0}$ $\qquad \bar{B}_1 = ?$





$\bar{n}_{12} = \bar{a}_x$ $\qquad \bar{H}_2 = \dfrac{1}{\mu_0\mu_{r1}}\bar{B}_2 = \dfrac{1}{20\mu_0}\bar{B}_2$

$\bar{B}_1 = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$ $\qquad \bar{H}_1 = \dfrac{1}{\mu_0\mu_{r2}}\bar{B}_1 = \dfrac{1}{15\mu_0}\bar{B}_1$

1) $\bar{n}_{12}\cdot(\bar{B}_2 - \bar{B}_1) = 0$

2) $\bar{n}_{12}\times(\bar{H}_2 - \bar{H}_1) = \bar{K}$

B4H

1) $\bar{n}_{12}\cdot(\bar{B}_2 - \bar{B}_1) = 0$

$\bar{a}_x\left((1-x)\bar{a}_x + (-0{,}5-y)\bar{a}_y + (1-z)\bar{a}_z\right) = 0$

$1 - x = 0 \Rightarrow \boxed{x = 1}$

2) $\bar{n}_{12}\times(\bar{H}_2 - \bar{H}_1) = \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 1 & 0 & 0 \\ \frac{1}{20\mu_0} - \frac{x}{15\mu_0} & \frac{-0{,}5}{20\mu_0} - \frac{y}{15\mu_0} & \frac{1}{20\mu_0} - \frac{z}{15\mu_0} \end{vmatrix} = \bar{K}$

$-\bar{a}_y\left(\dfrac{1}{20\mu_0} - \dfrac{z}{15\mu_0}\right) + \bar{a}_z\left(-\dfrac{0{,}5}{20\mu_0} - \dfrac{y}{15\mu_0}\right) = \bar{a}_y\dfrac{0{,}1}{\mu_0} - \bar{a}_z\dfrac{0{,}2}{\mu_0}$

$\dfrac{z}{15\mu_0} - \dfrac{1}{20\mu_0} = \dfrac{0{,}1}{\mu_0}$ $\qquad -\dfrac{0{,}5}{20\mu_0} - \dfrac{y}{15\mu_0} = -\dfrac{0{,}2}{\mu_0}$

$\dfrac{z}{15} - \dfrac{1}{20} = 0{,}1$ $\qquad \dfrac{0{,}5}{20} + \dfrac{y}{15} = 0{,}2$

$z = \dfrac{9}{4} = 2.25$ $\qquad y = \dfrac{21}{8} = 2{,}625$

$\bar{B}_1 = \bar{a}_x + 2.625\,\bar{a}_y + 2{,}25\,\bar{a}_z$

ZI 16/17 Dva mag. materijala razdvaja ravnina $x+y+z=\sqrt{3}$
Ishodište (0,0,0) se nalazi u sredstvu $\mu_1 = 4$, $\bar{B}_1 = \bar{a}_x - 0.5\,\bar{a}_y$
Odrediti $\bar{B}_2$ ako je $\mu_{r2} = 1$

B & H

→ SAMO NORMALA NA ZADANU RAVNINU

$\bar{B}_2 = x\bar{a}_x + y\bar{a}_y + z\bar{a}_z$

$\bar{n}_{12} = \dfrac{a_x + a_y + a_z}{\sqrt{3}}$

① $\bar{n}(\bar{B}_2 - \bar{B}_1) = 0$

② $\bar{n} \times (\bar{H}_2 - \bar{H}_1) = \bar{K}$

① $\bar{n}(\bar{B}_2 - \bar{B}_1) = 0$

$$\frac{\bar{a}_x + \bar{a}_y + \bar{a}_z}{\sqrt{3}}\left((x-1)\,\bar{a}_x + (y+0.5)\,\bar{a}_y + z\,\bar{a}_z\right) = 0 \,/\sqrt{3}$$

$$x - 1 + y + 0.5 + z = 0$$

②
$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\mu_0\mu_{r2}}x - \frac{1}{\mu_0\mu_{r1}} & \frac{y}{\mu_0\mu_{r2}} + \frac{0.5}{\mu_0\mu_{r1}} & \frac{z}{\mu_0\mu_{r2}} \end{vmatrix} \begin{matrix} /\cdot\sqrt{3} \\ = \\ /\cdot\mu_0 \end{matrix} \; 0 \qquad \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 1 & 1 & 1 \\ \frac{x}{1} - \frac{1}{4} & \frac{y}{1} + \frac{0.5}{4} & \frac{z}{1} \end{vmatrix} = 0$$

$$\bar{a}_x\left(z - y - \frac{0.5}{4}\right) - \bar{a}_y\left(z - x + \frac{1}{4}\right) + \bar{a}_z\left(y + \frac{0.5}{4} - x + \frac{1}{4}\right) = 0$$

$$\bar{a}_x(z - y - 0.125) - \bar{a}_y(z - x + 0.25) + \bar{a}_z(y - x + 0.375) = 0$$

$z - y - 0.125 = 0 \qquad -z + x - 0.25 = 0 \qquad y - x + 0.375 = 0$

$y = z - 0.125 \qquad x = z + 0.25$

u ①
$$z + 0.25 - 1 + z - 0.125 + 0.5 + z = 0$$
$$3z = \frac{3}{8}$$
$$z = \frac{1}{8} \;\rightarrow\; x = z + 0.25 = \frac{3}{8} \qquad y = z - 0.125 = 0$$

$$\bar{B}_2 = \frac{3}{8}\,\bar{a}_x + \frac{1}{8}\,\bar{a}_y$$