A. Table of Integrals

1) $\displaystyle\int x^n dx = \frac{1}{n+1}x^{n+1}$

2) $\displaystyle\int \frac{1}{x}dx = \ln x$

3) $\displaystyle\int u\,dv = uv - \int v\,du$

4) $\displaystyle\int u(x)v'(x)dx = u(x)v(x) - \int v(x)u'(x)dx$

5) $\displaystyle\int \frac{1}{ax+b}dx = \frac{1}{a}\ln(ax+b)$

6) $\displaystyle\int \frac{1}{(x+a)^2}dx = \frac{-1}{x+a}$

7) $\displaystyle\int (x+a)^n dx = (x+a)^n\left(\frac{a}{1+n} + \frac{x}{1+n}\right),\ \ n \neq -1$

8) $\displaystyle\int x(x+a)^n dx = \frac{(x+a)^{1+n}(nx+x-a)}{(n+2)(n+1)}$

9) $\displaystyle\int \frac{dx}{1+x^2} = \tan^{-1} x$

10) $\displaystyle\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}(x/a)$

11) $\displaystyle\int \frac{x\,dx}{a^2+x^2} = \frac{1}{2}\ln(a^2+x^2)$

12) $\displaystyle\int \frac{x^2 dx}{a^2+x^2} = x - a\tan^{-1}(x/a)$

13) $\displaystyle\int \frac{x^3 dx}{a^2+x^2} = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln(a^2+x^2)$

14) $\displaystyle\int (ax^2+bx+c)^{-1}dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\left\{\frac{2ax+b}{\sqrt{4ac-b^2}}\right\}$

15) $\displaystyle\int \frac{1}{(x+a)(x+b)}dx = \frac{1}{b-a}[\ln(a+x)-\ln(b+x)],\ \ a \neq b$

16) $\displaystyle\int \frac{x}{(x+a)(x+b)}dx = \frac{1}{a-b}[a\ln(a+x)-b\ln(b+x)],\ \ a \neq b$

17) $\int \frac{x}{(x+a)^2} dx = \frac{a}{a+x} + \ln(a+x), \quad a \neq b$

18) $\int \frac{x}{ax^2+bx+c} dx = \frac{\ln(ax^2+bx+c)}{2a} - \frac{b}{a\sqrt{4ac-b^2}} \tan^{-1}\left\{ \frac{2ax+b}{\sqrt{4ac-b^2}} \right\}$

19) $\int \sqrt{x-a}\,dx = \frac{2}{3}(x-a)^{3/2}$

20) $\int \frac{1}{\sqrt{x \pm a}} dx = 2\sqrt{x \pm a}$

21) $\int \frac{1}{\sqrt{a-x}} dx = 2\sqrt{a-x}$

22) $\int x\sqrt{x-a}\,dx = \frac{2}{3}a(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2}$

23) $\int \sqrt{ax+b}\,dx = \left(\frac{2b}{3a} + \frac{2x}{3} \right)\sqrt{b+ax}$

24) $\int (ax+b)^{3/2} dx = \sqrt{b+ax}\left(\frac{2b^2}{5a} + \frac{4bx}{5} + \frac{2ax^2}{5} \right)$

25) $\int \frac{x}{\sqrt{x \pm a}} dx = \frac{2}{3}(x \pm 2a)\sqrt{x \pm a}$

26) $\int \sqrt{\frac{x}{a-x}}\,dx = -\sqrt{x}\sqrt{a-x} - a\tan^{-1}\left[\frac{\sqrt{x}\sqrt{a-x}}{x-a} \right]$

27) $\int \sqrt{\frac{x}{x+a}}\,dx = \sqrt{x}\sqrt{x+a} - a\ln\left[\sqrt{x} + \sqrt{x+a} \right]$

28) $\int x\sqrt{ax+b}\,dx = \left(-\frac{4b^2}{15a^2} + \frac{2bx}{15a} + \frac{2x^2}{5} \right)\sqrt{b+ax}$

29) $\int \sqrt{x}\sqrt{ax+b}\,dx = \left(\frac{b\sqrt{x}}{4a} + \frac{x^{3/2}}{2} \right)\sqrt{b+ax} - \frac{b^2\ln\left(2\sqrt{a}\sqrt{x} + 2\sqrt{b+ax}\right)}{4a^{3/2}}$

30)
$\int x^{3/2}\sqrt{ax+b}\,dx = \left(-\frac{b^2\sqrt{x}}{8a^2} + \frac{bx^{3/2}}{12a} + \frac{x^{5/2}}{3} \right)\sqrt{b+ax}$
$$- \frac{b^3\ln\left(2\sqrt{a}\sqrt{x} + 2\sqrt{b+ax}\right)}{8a^{5/2}}$$

31) $\int \sqrt{x^2 \pm a^2}\,dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \pm \frac{1}{2}a^2 \ln\left(x + \sqrt{x^2 \pm a^2} \right)$

32) $\int \sqrt{a^2 - x^2}\,dx = \frac{1}{2}x\sqrt{a^2 - x^2} - \frac{1}{2}a^2 \tan^{-1}\left[\frac{x\sqrt{a^2-x^2}}{x^2-a^2} \right]$

33) $\int x\sqrt{x^2 \pm a^2} = \frac{1}{3}(x^2 \pm a^2)^{3/2}$

34) $\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left[x + \sqrt{x^2 \pm a^2}\right]$

35) $\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = -\tan^{-1}\left[\frac{x\sqrt{a^2 - x^2}}{x^2 - a^2}\right]$

36) $\int \frac{x}{\sqrt{x^2 \pm a^2}}\,dx = \sqrt{x^2 - a^2}$

37) $\int \frac{x}{\sqrt{a^2 - x^2}}\,dx = -\sqrt{a^2 - x^2}$

38) $\int \frac{x^2}{\sqrt{x^2 \pm a^2}}\,dx = \frac{1}{2}x\sqrt{x^2 \pm a^2} \mp \frac{1}{2}\ln\left[x + \sqrt{x^2 \pm a^2}\right]$

39) $\int \frac{x^2}{\sqrt{a^2 - x^2}}\,dx = -\frac{1}{2}x\sqrt{a - x^2} - \frac{1}{2}a^2\tan^{-1}\left[\frac{x\sqrt{a^2 - x^2}}{x^2 - a^2}\right]$

40)
$$\int \sqrt{ax^2 + bx + c} = \left(\frac{b}{4a} + \frac{x}{2}\right)\sqrt{ax^2 + bx + c}$$
$$+ \frac{4ac - b^2}{8a^{3/2}}\ln\left(\frac{2ax + b}{\sqrt{a}} + 2\sqrt{ax^2 + bc + c}\right)$$

41)
$$\int x\sqrt{ax^2 + bx + c} = \left(\frac{x^3}{3} + \frac{bx}{12a} + \frac{8ac - 3b^2}{24a^2}\right)\sqrt{ax^2 + bx + c}$$
$$- \frac{b(4ac - b^2)}{16a^{5/2}}\ln\left(\frac{2ax + b}{\sqrt{a}} + 2\sqrt{ax^2 + bc + c}\right)$$

42) $\int \frac{1}{\sqrt{ax^2 + bx + c}}\,dx = \frac{1}{\sqrt{a}}\ln\left[\frac{2ax + b}{\sqrt{a}} + 2\sqrt{ax^2 + bx + c}\right]$

43) $\int \frac{x}{\sqrt{ax^2 + bx + c}}\,dx = \frac{1}{a}\sqrt{ax^2 + bx + c} - \frac{b}{2a^{3/2}}\ln\left[\frac{2ax + b}{\sqrt{a}} + 2\sqrt{ax^2 + bx + c}\right]$

44) $\int \ln x\,dx = x\ln x - x$

45) $\int \ln(ax + b)\,dx = \frac{ax + b}{a}\ln(ax + b) - x$

46) $\int \ln(a^2 x^2 \pm b^2)\,dx = x\ln(a^2 x^2 \pm b^2) + \frac{2b}{a}\tan^{-1}\left(\frac{ax}{b}\right) - 2x$

47) $\int \ln(a^2 - b^2 x^2)\,dx = x\ln(a^2 - b^2 x^2) + \frac{2a}{b}\tan^{-1}\left(\frac{bx}{a}\right) - 2x$

48) $\int \ln(ax^2 + bx + c)dx = \frac{1}{a}\sqrt{4ac - b^2}\, \tan^{-1}\left[\frac{2ax + b}{\sqrt{4ac - b^2}}\right] - 2x + \left(\frac{b}{2a} + x\right)\ln\left(ax^2 + bx + c\right)$

49) $\int x\ln(ax + b)dx = \frac{b}{2a}x - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax + b)$

50) $\int x\ln(a^2 - b^2x^2)dx = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 - \frac{a^2}{b^2}\right)\ln(a^2 - bx^2)$

51) $\int e^{ax}dx = \frac{1}{a}e^{ax}$

52) $\int \sqrt{x}e^{ax}dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\mathrm{erf}\left(i\sqrt{ax}\right)$ where $erf(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt$

53) $\int xe^x dx = (x - 1)e^x$

54) $\int xe^{ax}dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax}$

55) $\int x^2 e^x dx = e^x(x^2 - 2x + 2)$

56) $\int x^2 e^{ax}dx = e^x\left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)$

57) $\int x^3 e^x dx = e^x(x^3 - 3x^2 + 6x - 6)$

58) $\int x^n e^{ax}dx = (-1)^n\frac{1}{a}\Gamma[1 + n, -ax]$ where $\Gamma(a, x) = \int_x^\infty t^{a-1}e^{-t}dt$

59) $\int e^{ax^2}dx = -i\frac{\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(ix\sqrt{a}\right)$

60) $\int \sin x\, dx = -\cos x$

61) $\int \sin^2 x\, dx = \frac{x}{2} - \frac{1}{4}\sin 2x$

62) $\int \sin^3 x\, dx = -\frac{3}{4}\cos x + \frac{1}{12}\cos 3x$

63) $\int \cos x\, dx = \sin x$

64) $\int \cos^2 x\, dx = \frac{x}{2} + \frac{1}{4}\sin 2x$

65) $\int \cos^3 x\, dx = \frac{3}{4}\sin x + \frac{1}{12}\sin 3x$

66) $\int \sin x\cos x\, dx = -\frac{1}{2}\cos^2 x$

67) $\int \sin^2 x \cos x dx = \frac{1}{4}\sin x - \frac{1}{12}\sin 3x$

68) $\int \sin x \cos^2 x dx = -\frac{1}{4}\cos x - \frac{1}{12}\cos 3x$

69) $\int \sin^2 x \cos^2 x dx = \frac{x}{8} - \frac{1}{32}\sin 4x$

70) $\int \tan x dx = -\ln \cos x$

71) $\int \tan^2 x dx = -x + \tan x$

72) $\int \tan^3 x dx = \ln[\cos x] + \frac{1}{2}\sec^2 x$

73) $\int \sec x dx = \ln \frac{\cos(x/2) + \sin(x/2)}{\cos(x/2) - \sin(x/2)}$

74) $\int \sec^2 x dx = \tan x$

75) $\int \sec^3 x dx = \frac{1}{2}\sec x \tan x + \ln \frac{\cos(x/2) + \sin(x/2)}{\cos(x/2) - \sin(x/2)}$

76) $\int \sec x \tan x dx = \sec x$

77) $\int \sec^2 x \tan x dx = \frac{1}{2}\sec^2 x$

78) $\int \sec^n x \tan x dx = \frac{1}{n}\sec^n x, \quad n \neq 0$

79) $\int \csc x dx = \ln \tan(x/2)$

80) $\int \csc^2 x dx = -\cot x$

81) $\int \csc^3 x dx = -\frac{1}{2}\cot x \csc x - \frac{1}{2}\ln(\cos(x/2)\sin(x/2))$

82) $\int \csc^n x \cot x dx = -\frac{1}{n}\csc^n x, \quad n \neq 0$

83) $\int \sec x \csc x dx = \ln \tan x$

84) $\int x \cos x dx = \cos x + x \sin x$

85) $\int x^2 \cos x dx = 2x \cos x + (2 - x^2)\sin x$

86) $\int x^n \cos x dx = -\frac{1}{2}(i)^{1+n}\{\Gamma(1+n, -ix) + (-1)^n \Gamma(1+n, ix)\}$

87) $\int x^n \sin x dx = -\frac{1}{2}(i)^n\{\Gamma(n+1, -ix) - (-1)^n \Gamma(n+1, -ix)\}$

88) $\int e^x \sin x dx = \frac{1}{2} e^x [\sin x - \cos x]$

89) $\int e^{bx} \sin(ax) dx = \frac{1}{b^2 + a^2} e^{bx} [b \sin ax - a \cos ax]$

90) $\int x e^x \sin x dx = \frac{1}{2} e^x [\cos x - x \cos x + x \sin x]$

91) $\int e^x \cos x dx = \frac{1}{2} e^x [\sin x + \cos x]$

92) $\int e^{bx} \cos(ax) dx = \frac{1}{b^2 + a^2} e^{bx} [a \sin ax + b \cos ax]$

93) $\int x e^x \cos x dx = \frac{1}{2} e^x [x \cos x - \sin x + x \sin x]$

94) $\int \cosh x dx = \sinh x$

95) $\int e^{ax} \cosh bx dx = \frac{e^{ax}}{a^2 - b^2} [a \cosh bx - b \sinh bx]$

96) $\int \sinh x dx = \cosh x$

97) $\int e^{ax} \sinh bx dx = \frac{e^{ax}}{a^2 - b^2} [-b \cosh bx + a \sinh bx]$

98) $\int e^x \tanh x dx = e^x - 2 \tan^{-1}(e^x)$

99) $\int \tanh ax dx = \frac{1}{a} \ln \cosh ax$

100) $\int \cos ax \cosh bx dx = \frac{1}{a^2 + b^2} [a \sin ax \cosh bx + b \cos ax \sinh bx]$

101) $\int \cos ax \sinh bx dx = \frac{1}{a^2 + b^2} [b \cos ax \cosh bx + a \sin ax \sinh bx]$

102) $\int \sin ax \cosh bx dx = \frac{1}{a^2 + b^2} [-a \cos ax \cosh bx + b \sin ax \sinh bx]$

103) $\int \sin ax \sinh bx dx = \frac{1}{a^2 + b^2} [b \cosh bx \sin ax - a \cos ax \sinh bx]$

104) $\int \sinh ax \cosh ax dx = \frac{1}{4a} [-2ax + \sinh(2ax)]$

105) $\int \sinh ax \cosh bx dx = \frac{1}{b^2 - a^2} [b \cosh bx \sinh ax - a \cosh ax \sinh bx]$