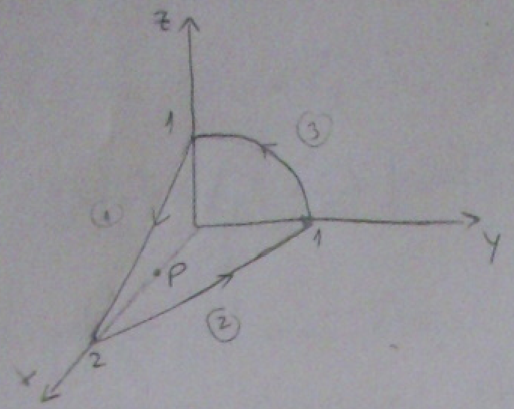
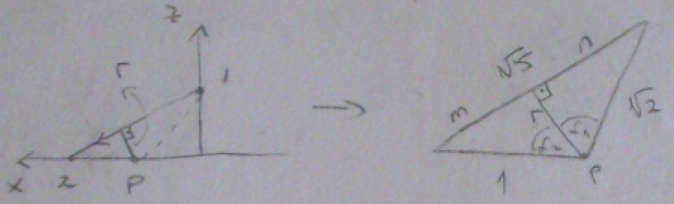
(1) strujnicom teče struja $I = 2A$, $P(1m, 0, 0)$



a)

strujnica (1)



$m + n = \sqrt{5}$

$r = \sqrt{1^2 - m^2} = \sqrt{2 - n^2}$

$1 - (\sqrt{5} - n)^2 = 2 - n^2$

$1 - 5 + 2\sqrt{5}n - \cancel{n^2} = 2 - \cancel{n^2}$

$2\sqrt{5}n = 6 \longrightarrow n = \frac{3}{\sqrt{5}}, \; m = \frac{2\sqrt{5}}{5}$

$r = \sqrt{2 - n^2} = 0.447$

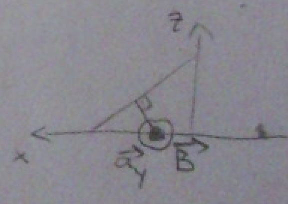
mag. polje vodiča konačne duljine:

$|\vec{H}_1| = \frac{I}{4\pi r}(\sin\alpha_1 + \sin\alpha_2)$, $I = 2A$, $r = 0.447$, $\sin\alpha_1 = \frac{n}{\sqrt{2}} = 0.9487$

$\sin\alpha_2 = \frac{m}{1} = 0.8944$

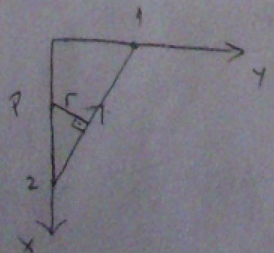
$|\vec{H}_1| = 0.656$

$\vec{H}_1 = |\vec{H}_1| \cdot \vec{a}_x$ → vektor smjera mag. polja



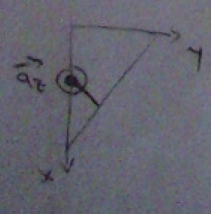
$\boxed{\vec{H}_1 = 0.656\,\vec{a}_y}$

strujnica (2)



r i α_1, α_2 su isti : $r = 0.447$, $\sin\alpha_1 = 0.9487$

$\sin\alpha_2 = 0.8944$



$\vec{H}_2 = 0.656\,\vec{a}_z$

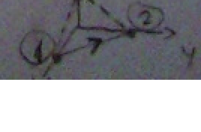
općenito : $\vec{a}_L$ je jednak normali $\vec{n}$ na ravninu dobivenu između 3 točke:

(1) točka koja je početak segmenta strujnice

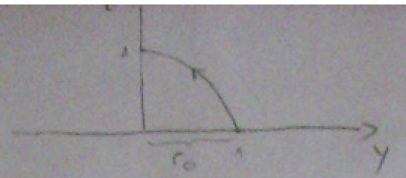
(2) točka koja je kraj -||- -||-

(3) točka u kojoj računamo mag. polje

npr.



strujnica (3)



$r_0 = 1$

$x_P = 1$, koordinata točke P

analogno prema Berberoviću 20.str :

$$\vec{H} = \frac{I}{4\pi}\int \frac{\vec{a_r}\cdot x_P + \vec{a_x}\cdot r_0}{(r_0^2 + x_P^2)^{3/2}}\, r_0\, d\alpha \quad , \quad \vec{a_r} = \vec{a_y}\cos\alpha + \vec{a_z}\sin\alpha$$

$$\vec{H_3} = \frac{I}{4\pi}\int_0^{\pi/2} \frac{\vec{a_y}\cdot x_P \cdot \cos\alpha + \vec{a_z}\, x_P \sin\alpha + \vec{a_x}\cdot r_0}{(r_0^2 + x_P^2)^{3/2}}\, r_0\, d\alpha$$

$$= \frac{2}{4\pi}\left(\int_0^{\pi/2} \frac{\vec{a_y}\cdot\cos\alpha}{2\sqrt{2}}\, d\alpha + \int_0^{\pi/2}\frac{\vec{a_z}\sin\alpha}{2\sqrt{2}}\, d\alpha + \int_0^{\pi/2}\vec{a_x}\, d\alpha\right)$$

$$= \frac{1}{4\sqrt{2}\pi}\left(\vec{a_y} + \vec{a_z} + \frac{\pi}{2}\vec{a_x}\right)$$

$$\boxed{\vec{H_3} = 0.05627\,\vec{a_y} + 0.05627\,\vec{a_z} + 0.0884\,\vec{a_x}}$$

$$\vec{H} = \vec{H_1} + \vec{H_2} + \vec{H_3} = \underbrace{0.0884\,\vec{a_x}}_{H_x} + \underbrace{0.712\,\vec{a_y}}_{H_y} + \underbrace{0.712\,\vec{a_z}}_{H_z}$$

b) odredi $\nabla\cdot\vec{H}$ u točki (2,3,1)

$$\frac{\partial}{\partial x}(H_x) + \frac{\partial}{\partial y}(H_y) + \frac{\partial}{\partial z}(H_z) = 0$$

(2) vektorski mag. potencijal u cilindričnom koordinatnom sustavu u sredstvu $\mu = \mu_0$ je:

$$\vec{A} = \frac{z}{r}\left(\frac{1}{4}\sin(2r) - \frac{r}{2}\cos(2r)\right)\vec{a_r} + \frac{z^2}{K}\sin(2r)\,\vec{a_z}$$

a) primjenom Coulombovog baždarenja odredite konstantu K :

-> Berberović, 56. str.

$$\nabla\cdot\vec{A} = 0$$

divergencija cilindričnog sustava : $\nabla\cdot\vec{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\alpha}{\partial\alpha} + \frac{\partial A_z}{\partial z}$

$$\nabla\cdot\vec{A} = \frac{1}{r}\cdot\frac{\partial}{\partial r}\left(\frac{z}{4}\sin(2r) - \frac{zr}{2}\cos(2r)\right) + \frac{\partial}{\partial z}\cdot\frac{z^2}{K}\sin(2r)$$

$$= \frac{1}{r}\cdot\left(\frac{z}{2}\cos(2r) - \frac{z}{2}\cos(2r) + zr\sin(2r)\right) + \frac{2z}{K}\sin(2r)$$

-> formule nisu iste kao kod Kartezijevog koordinatnog sustava

$$z\sin(2r) + \frac{2z}{k}\sin(2r) = 0 \quad / : \sin(2r)$$

$$z + \frac{2z}{k} = 0 \quad / : z$$

$$k = -2$$

b) odredi $\vec{B}$ u točki $\left(r = \frac{\pi}{8}, \alpha = \frac{\pi}{4}, z = 2\right)$

$$\vec{B} = \nabla \times \vec{A}$$

rotacija cilindričnog sustava:

$$\nabla \times \vec{A} = \underbrace{\left(\frac{1}{r}\cdot\frac{\partial A_z}{\partial \alpha} - \frac{\partial A_\alpha}{\partial z}\right)}_{0}\vec{a_r} + \left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right)\vec{a_\alpha} + \frac{1}{r}\underbrace{\left(\frac{\partial (rA_\alpha)}{\partial r} - \frac{\partial A_r}{\partial \alpha}\right)}_{0}\vec{a_z}$$

$$= \left[\frac{\partial}{\partial z}\left(\frac{z}{4r}\sin(2r) - \frac{z}{2}\cos(2r)\right) - \frac{\partial}{\partial r}\cdot\left(-\frac{z^2}{2}\sin(2r)\right)\right]\vec{a_\alpha}$$

$$\vec{B} = \left(\frac{\sin(2r)}{4r} - \frac{\cos(2r)}{2} + z^2\cos(2r)\right)\vec{a_\alpha} \quad , \quad r = \frac{\pi}{8}, \quad z = 2$$

$$\vec{B} = 2.925\,\vec{a_\alpha}\,[T] \longrightarrow |\vec{B}| = 2.925\,T$$

c) odredi $\vec{J}$ u točki $\left(r = \frac{\pi}{8}, \alpha = \frac{\pi}{4}, z = 2\right)$

$$\nabla \times \vec{B} = -\mu_0 \vec{J} \quad , \quad \vec{B} = \left(\frac{\sin(2r)}{4r} - \frac{\cos(2r)}{2} + z^2\cos(2r)\right)\vec{a_\alpha}$$

$\nabla \times \vec{B}$ = rotacija cilindr. sustava.....

$$= -\frac{\partial}{\partial z}A_\alpha\,\vec{a_r} + \frac{1}{r}\cdot\frac{\partial}{\partial r}(r\cdot A_\alpha)\cdot\vec{a_z}$$

$$= -\vec{a_r}\cdot\left(2z\cos(2r)\right) + \vec{a_z}\cdot\frac{1}{r}\cdot\left(\cancel{\frac{\cos(2r)}{2}} - \cancel{\frac{\cos(2r)}{2}} + r\sin(2r) + z^2\cos(2r) - 2z^2 r\sin(2r)\right)$$

$$= \vec{a_r}\left(-2z\cos(2r)\right) + \vec{a_z}\left(\sin(2r) + \frac{z^2\cos(2r)}{r} - 2z^2\sin(2r)\right)$$

$$-\mu_0\vec{J} = -2.828\,\vec{a_r} + 2.25\,\vec{a_z} \quad , \quad \mu_0 = 12.566\cdot 10^{-7}$$

$$\vec{J} = -2250517\,\vec{a_r} + 1790545.9\,\vec{a_z}\left[\frac{A}{m^2}\right] = -2.25\,\vec{a_r} + 1.7905\,\vec{a_z}\left[\frac{A}{mm^2}\right]$$

$$|\vec{J}| = \sqrt{2.25^2 + 0.704^2} = 2.88\,\frac{A}{mm^2}$$

(3) dva magnetska materijala razdvaja ravnina $y+z=1$. u poluprostoru (1) koji sadrži ishodište nalazi se materijal relativne permeabilnosti $\mu_{r1}=5$, a u poluprostoru (2) je $\mu_{r2}=2$

$$\vec{B_1} = 5\vec{a_x} + 3\vec{a_y} + 2\vec{a_z} \rightarrow \vec{B_1} = \mu_0 \mu_{r1} \cdot \vec{H_1} \rightarrow \vec{H_1} = \frac{\vec{a_x}}{\mu_0} + \frac{3\vec{a_y}}{5\mu_0} + \frac{2\vec{a_z}}{5\mu_0}$$

a) odredi vektor jakosti mag. polja u sredstvu (2)

normala na ravninu $y+z=1$ $(Ax+By+Bz-D=0)$

$$\vec{n_{12}} = \frac{A\vec{a_x} + B\vec{a_y} + C\vec{a_z}}{\sqrt{A^2+B^2+C^2}}$$

$$\vec{n_{12}} = \frac{\vec{a_y}+\vec{a_z}}{\sqrt{2}}$$

uvjeti:

$$\vec{n_{12}} \cdot (\vec{B_2} - \vec{B_1}) = 0 \rightarrow \vec{n_{12}} \cdot (\mu_0 \mu_{r2} \vec{H_2} - \mu_0 \mu_{r1} \vec{H_1}) = 0$$

$$\vec{n_{12}} \times (\vec{H_2} - \vec{H_1}) = 0$$ (plošna struja gustoće K je 0)

$$\vec{n_{12}} \times (\vec{H_2} - \vec{H_1}) = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ H_{2x}-\frac{1}{\mu_0} & H_{2y}-\frac{3}{5\mu_0} & H_{2z}-\frac{2}{5\mu_0} \end{vmatrix} = 0$$

$$= \vec{a_x} \cdot \frac{1}{\sqrt{2}}\left(H_{2z} - \frac{2}{5\mu_0} - H_{2y} + \frac{3}{5\mu_0}\right) - \vec{a_y}\left(\frac{-1}{\sqrt{2}} \cdot \left(H_{2x} - \frac{1}{\mu_0}\right)\right) + \vec{a_z} \cdot \left(-\frac{1}{\sqrt{2}} \cdot \left(H_{2x} - \frac{1}{\mu_0}\right)\right)$$

$$\vec{a_x} \cdot \frac{1}{\sqrt{2}}\left(H_{2z} - H_{2y} + \frac{1}{5\mu_0}\right) + \vec{a_y} \cdot \frac{1}{\sqrt{2}}\left(H_{2x} - \frac{1}{\mu_0}\right) - \vec{a_z} \cdot \frac{1}{\sqrt{2}}\left(H_{2x} - \frac{1}{\mu_0}\right) = 0$$

(1) $H_{2z} - H_{2y} + \frac{1}{5\mu_0} = 0$ $\qquad H_{2x} = \frac{1}{\mu_0} \rightarrow \boxed{H_{2x} = 795.8 \text{ kA/m}}$

$$\vec{n_{12}} \left(\mu_{r2} \vec{H_2} - \mu_{r1} \vec{H_1}\right) = 0$$

$$\frac{1}{\sqrt{2}}\left(\vec{a_y} + \vec{a_z}\right)\left[\cancel{\left(2H_{2x} - \frac{5}{\mu_0}\right)\vec{a_x}} + \left(2H_{2y} - \frac{3}{\mu_0}\right)\vec{a_y} + \left(2H_{2z} - \frac{2}{\mu_0}\right)\vec{a_z}\right] = 0$$

$$\frac{1}{\sqrt{2}}\left(2H_{2y} - \frac{3}{\mu_0}\right) + \frac{1}{\sqrt{2}}\left(2H_{2z} - \frac{2}{\mu_0}\right) = 0 \quad / \cdot \frac{\sqrt{2}}{2}$$

(2) $H_{2y} - \frac{3}{2\mu_0} + H_{2z} - \frac{1}{\mu_0} = 0$

dvije jednadžbe s dvije nepoznanice:

$$\left.\begin{array}{l} H_{2z} - H_{2y} + \frac{1}{5\mu_0} = 0 \\ H_{2y} + H_{2z} - \frac{5}{2\mu_0} = 0 \end{array}\right\} +$$

$$2H_{2z} = \frac{1}{\mu_0}\left(\frac{5}{2} - \frac{1}{5}\right) \Rightarrow \boxed{H_{2z} = 915.168\ kA/m}$$

$$H_{2y} = H_{2z} + \frac{1}{5\mu_0} \Rightarrow \boxed{H_{2y} = 1074.3\ kA/m}$$

$$\vec{H} = \vec{H}_{2x} + \vec{H}_{2y} + \vec{H}_{2z}$$

b) odredi koliki treba biti iznos strujnog obloga K na granici sredstava da bi ukupni vektor mag. indukcije u sredstvu (2) bio ortogonalan na graničnu plohu!

-> $\vec{B}_2$ će biti ortogonalan ako mu je $B_{2x} = 0$ (jer normala na ravninu ima komponente $\vec{a_y}$ i $\vec{a_z}$) -> primjer 2.2.3, 52.str., Berberović

$$\vec{K} = K_y\vec{a_y} + K_z\vec{a_z} \quad ???$$

$$\vec{n}_{12} \times (\vec{H}_2 - \vec{H}_1) = \vec{K}$$

$$\vec{n}_{12} \times \left(\frac{\vec{B}_2}{\mu_0\mu_{r2}} - \frac{\vec{B}_1}{\mu_0\mu_{r1}}\right) = \vec{K}$$

$$\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{B_{2x}}{2\mu_0} - \frac{1}{\mu_0} & \frac{B_{2y}}{2\mu_0} - \frac{3}{5\mu_0} & \frac{B_{2z}}{2\mu_0} - \frac{2}{5\mu_0} \end{vmatrix} = -\vec{a_y}\cdot\frac{1}{\mu_0\sqrt{2}} + \vec{a_z}\cdot\frac{1}{\mu_0\sqrt{2}}$$

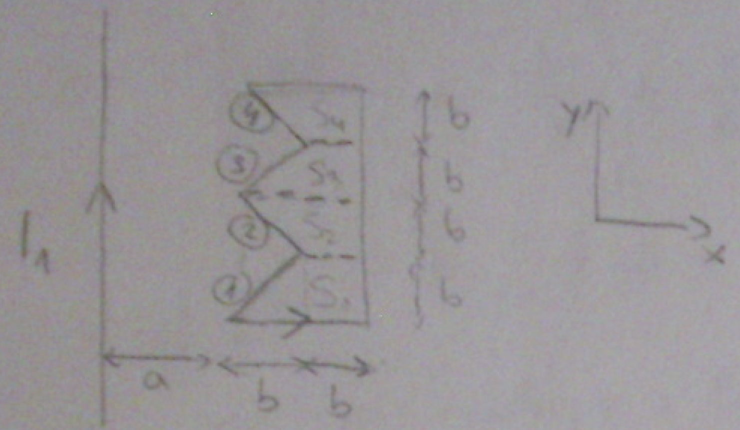
$$-\vec{a_y}\frac{1}{\mu_0\sqrt{2}} + \vec{a_z}\cdot\frac{1}{\mu_0\sqrt{2}} = K_y\vec{a_y} + K_z\vec{a_z}$$

$$K_y = -\frac{1}{\mu_0\sqrt{2}}, \quad K_z = \frac{1}{\mu_0\sqrt{2}}$$

$$|\vec{K}| = \sqrt{K_z^2 + K_y^2} = 795.8\ kA/m \quad ???$$

(4) U blizini beskonačno dugog vodiča zanemarivog presjeka kojim teče struja

$I_1 = 2A$ nalazi se petlja $I_2 = 1A$, $a = 1m$, $b = 1m$

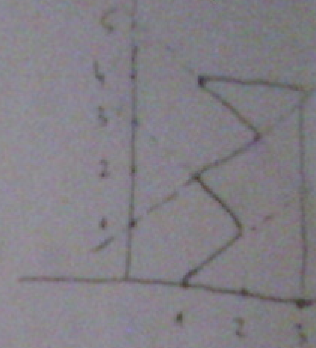


pravac (1) $y = x - 1$

(2) $y = -x + 3$

(3) $y = x + 1$

(4) $y = -x + 5$



a) odredi međuinduktivitet petlje i dugog vodiča: 0.57 μH

mag. polje beskonačno dugog vodiča na udaljenosti r od vodiča: $\vec{B} = \frac{\mu_0 I_1}{2\pi r}$

$$\Phi_{12} = \iint \vec{B}\,\vec{n}\,dS \qquad , dS = dx\,dy$$

$$\int_a^b \ln(x+y)\,dx = \left(\ln\frac{(b+y)^{b+y}}{(a+y)^{a+y}}\right) - b + a$$

$$\int_a^b \ln(y-x)\,dx = \left(\ln\frac{(y-b)^{b-y}}{(y-a)^{a-y}}\right) - b + a$$

$$\Phi_{12} = \iint_{S_1} \vec{B}_1\vec{n} + \iint_{S_2} \vec{B}_2\vec{n} + \iint_{S_3} \vec{B}_3\vec{n} + \iint_{S_4} \vec{B}_4\vec{n}$$

$$= \int_0^1 dy \int_{y+1}^3 \frac{\mu_0 I_1}{2\pi x}dx + \int_1^2 dy \int_{3-y}^3 \frac{\mu_0 I_1}{2\pi x}dx + \int_2^3 dy \int_{y-1}^3 \frac{\mu_0 I_1}{2\pi x}dx + \int_3^4 dy \int_{5-y}^3 \frac{\mu_0 I_1}{2\pi x}dx$$

$$= \frac{\mu_0 I_1}{2\pi}\left[\int_0^1 (\ln 3 - \ln(y+1))dy + \int_1^2 (\ln 3 - \ln(3-y))dy + \int_2^3 (\ln 3 - \ln(y-1))dy + \int_3^4 (\ln 3 - \ln(5-y))dy\right]$$

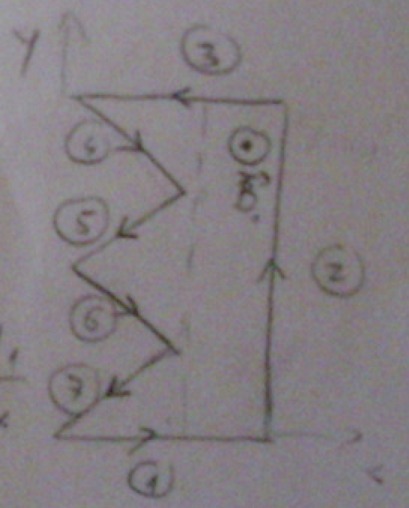
$$\ldots \int_{2} dy \int_{y-1} \frac{\ldots}{2\pi x}\,dx + \int_{3} dy \int_{5-y} \frac{\ldots}{2\pi x}\,dx$$

$$= \frac{\mu_0 I_1}{2\pi}\left[\int_0^1 \left(\ln 3 - \ln(y+1)\right)dy + \int_1^2 \left(\ln 3 - \ln(3-y)\right)dy + \int_2^3 \left(\ln 3 - \ln(y-1)\right)dy + \int_3^4 \left(\ln 3 - \ln(5-y)\right)dy\right]$$

$$= \frac{\mu_0 I_1}{2\pi}\left[4\ln 3 - \ln\frac{2^2}{1} + 1 - \ln\frac{1}{2^{-2}} + 2 - 1 - \ln\frac{2^2}{1} + 3 - 2 - \ln\frac{1}{2^{-2}} + 4 - 3\right]$$

$$\Phi_{12} = \frac{\mu_0 I_1}{2\pi}\cdot 2.849 = 1.1395\cdot 10^{-6}\ \mathrm{Wb}$$

$$\boxed{M_{12} = \frac{\Phi_{12}}{I_1} = 0.57\ \mu H}$$



b) odredi iznos sile kojom vodič djeluje na petlju

$$\vec{F} = I_2 \cdot \int_l d\vec{l} \times \vec{B}_1 \quad , \quad d\vec{l} = dx\,\vec{a}_x + dy\,\vec{a}_y$$

$$d\vec{l} \times \vec{B}_1 = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ dx & dy & 0 \\ 0 & 0 & \frac{\mu_0 I_1}{2\pi x} \end{vmatrix} = \vec{a}_x \cdot dy \cdot \frac{\mu_0 I_1}{2\pi x} - \vec{a}_y \cdot dx \cdot \frac{\mu_0 I_1}{2\pi x}$$

→ po dx se strujnice poništavaju (1 i 3, 4 5 6 7)

→ ostaje dy ($\vec{a}_x$)

↳ postoji samo komponenta sile $\vec{F}_x = F \cdot \vec{a}_x$

$$F = I_2 \cdot \int_l \frac{\mu_0 I_1}{2\pi x}\, dy \quad , \; x \text{ je udaljenost strujnica}$$

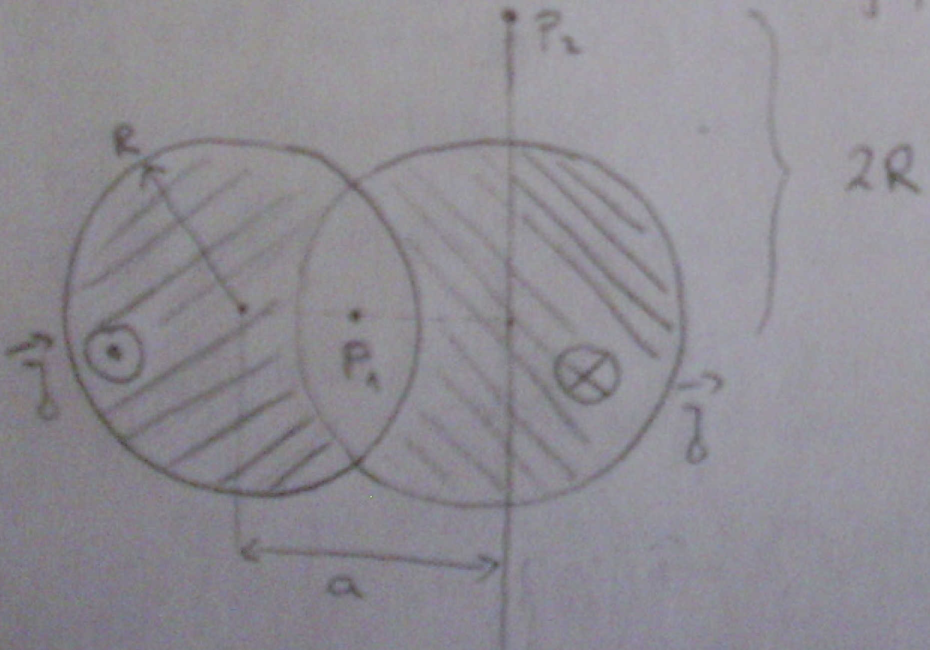
→ strujnice ②, ④, ⑤, ⑥, ⑦

$$F = \frac{\mu_0 I_2 I_1}{2\pi}\left[\int_0^{4b} \frac{dy}{a+4b+b} + \int_{4b}^{3b} \frac{dy}{5-y} + \int_{3b}^{2b} \frac{dy}{y-1} + \int_{2b}^{b} \frac{dy}{3-y} + \int_{b}^{0} \frac{dy}{y+1}\right]$$

$$= \frac{\mu_0 I_2 I_1}{2\pi}\left[\frac{4}{3} - \ln\frac{5-3b}{5-4b} + \ln\frac{2b-1}{3b-1} - \ln\frac{3-b}{3-2b} + \ln\frac{1}{1+b}\right]$$

$F = -0.576\,\mu N$

⑤ vodom teče struja gustoće $J = 0.5\ A/mm^2$ u suprotnim smjerovima. U srednjem dijelu koji sadrži točku P_1 je ostavljena rupa za potrebe hlađenja tekućim dušikom (tu ne teče struja), $R = 1\ mm$, $a = 1{,}25\ mm$



$$\oint \vec{H}\, d\vec{l} = \iint_S \vec{J}\, \vec{n}\, dS$$

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$