List of integrals

List of integrals of rational functions

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} \qquad (\text{for } n \neq -1)$$

$$\int \frac{dx}{ax+b} = \frac{1}{a}\ln|ax+b|$$

$$\int x(ax+b)^n dx = \frac{a(n+1)x-b}{a^2(n+1)(n+2)}(ax+b)^{n+1} \qquad (\text{for } n \notin \{-1,-2\})$$

$$\int \frac{x\,dx}{ax+b} = \frac{x}{a} - \frac{b}{a^2}\ln|ax+b|$$

$$\int \frac{x\,dx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln|ax+b|$$

$$\int \frac{x\,dx}{(ax+b)^n} = \frac{a(1-n)x-b}{a^2(n-1)(n-2)(ax+b)^{n-1}} \qquad (\text{for } n \notin \{1,2\})$$

$$\int \frac{x^2 dx}{ax+b} = \frac{1}{a^3}\left(\frac{(ax+b)^2}{2} - 2b(ax+b) + b^2\ln|ax+b|\right)$$

$$\int \frac{x^2 dx}{(ax+b)^2} = \frac{1}{a^3}\left(ax+b-2b\ln|ax+b| - \frac{b^2}{ax+b}\right)$$

$$\int \frac{x^2 dx}{(ax+b)^3} = \frac{1}{a^3}\left(\ln|ax+b| + \frac{2b}{ax+b} - \frac{b^2}{2(ax+b)^2}\right)$$

$$\int \frac{x^2 dx}{(ax+b)^n} = \frac{1}{a^3}\left(-\frac{(ax+b)^{3-n}}{(n-3)} + \frac{2b(a+b)^{2-n}}{(n-2)} - \frac{b^2(ax+b)^{1-n}}{(n-1)}\right) \qquad (\text{for } n \notin \{1,2,3\})$$

$$\int \frac{dx}{x(ax+b)} = -\frac{1}{b}\ln\left|\frac{ax+b}{x}\right|$$

$$\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} + \frac{a}{b^2}\ln\left|\frac{ax+b}{x}\right|$$

$$\int \frac{dx}{x^2(ax+b)^2} = -a\left(\frac{1}{b^2(ax+b)} + \frac{1}{ab^2x} - \frac{2}{b^3}\ln\left|\frac{ax+b}{x}\right|\right)$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a}$$

$$\int \frac{dx}{x^2-a^2} =$$

- $-\frac{1}{a}\,\mathrm{arctanh}\frac{x}{a} = \frac{1}{2a}\ln\frac{a-x}{a+x} \qquad (\text{for } |x| < |a|)$
- $-\frac{1}{a}\,\mathrm{arccoth}\frac{x}{a} = \frac{1}{2a}\ln\frac{x-a}{x+a} \qquad (\text{for } |x| > |a|)$

$$\int \frac{(mx+n)dx}{ax^2+bx+c} = \frac{m}{2a}\ln\left|ax^2+bx+c\right| + \frac{2an-bm}{a\sqrt{4ac-b^2}}\arctan\frac{2ax+b}{\sqrt{4ac-b^2}} \qquad (\text{for } 4ac-b^2>0)$$

$$\frac{m}{2a}\ln\left|ax^2+bx+c\right| + \frac{2an-bm}{a\sqrt{b^2-4ac}}\,\mathrm{artanh}\frac{2ax+b}{\sqrt{b^2-4ac}} \qquad (\text{for } 4ac-b^2<0)$$

$$\frac{m}{2a}\ln\left|ax^2+bx+c\right| - \frac{2an-bm}{a(2ax+b)} \qquad (\text{for } 4ac-b^2=0)$$

$$\int \frac{dx}{(ax^2+bx+c)^n} = \frac{2ax+b}{(n-1)(4ac-b^2)(ax^2+bx+c)^{n-1}} + \frac{(2n-3)2a}{(n-1)(4ac-b^2)}\int \frac{dx}{(ax^2+bx+c)^{n-1}}$$

$$\int \frac{xdx}{(ax^2+bx+c)^n} = \frac{bx+2c}{(n-1)(4ac-b^2)(ax^2+bx+c)^{n-1}} - \frac{b(2n-3)}{(n-1)(4ac-b^2)}\int \frac{dx}{(ax^2+bx+c)^{n-1}}$$

$$\int \frac{dx}{x(ax^2+bx+c)} = \frac{1}{2c}\ln\left|\frac{x^2}{ax^2+bx+c}\right| - \frac{b}{2c}\int \frac{dx}{ax^2+bx+c}$$

List of integrals of irrational functions

Integrals involving $r = \sqrt{x^2 + a^2}$

$$\int r \, dx = \frac{1}{2}\left(xr + a^2 \,\ln\left(x + r\right)\right)$$

$$\int r^3 \, dx = \frac{1}{4}xr^3 + \frac{1}{8}3a^2xr + \frac{3}{8}a^4 \ln\left(x + r\right)$$

$$\int r^5 \, dx = \frac{1}{6}xr^5 + \frac{5}{24}a^2xr^3 + \frac{5}{16}a^4xr + \frac{5}{16}a^6 \ln\left(x + r\right)$$

$$\int xr \, dx = \frac{r^3}{3}$$

$$\int xr^3 \, dx = \frac{r^5}{5}$$

$$\int xr^{2n+1} \, dx = \frac{r^{2n+3}}{2n+3}$$

$$\int x^2r \, dx = \frac{xr^3}{4} - \frac{a^2xr}{8} - \frac{a^4}{8} \ln\left(x + r\right)$$

$$\int x^2r^3 \, dx = \frac{xr^5}{6} - \frac{a^2xr^3}{24} - \frac{a^4xr}{16} - \frac{a^6}{16} \ln\left(x + r\right)$$

$$\int x^3r \, dx = \frac{r^5}{5} - \frac{a^2r^3}{3}$$

$$\int x^3r^3 \, dx = \frac{r^7}{7} - \frac{a^2r^5}{5}$$

$$\int x^3r^{2n+1} \, dx = \frac{r^{2n+5}}{2n+5} - \frac{a^3r^{2n+3}}{2n+3}$$

$$\int x^4r \, dx = \frac{x^3r^3}{6} - \frac{a^2xr^3}{8} + \frac{a^4xr}{16} + \frac{a^6}{16} \ln\left(x + r\right)$$

$$\int x^4r^3 \, dx = \frac{x^3r^5}{8} - \frac{a^2xr^5}{16} + \frac{a^4xr^3}{64} + \frac{3a^6xr}{128} + \frac{3a^8}{128} \ln\left(x + r\right)$$

$$\int x^5r \, dx = \frac{r^7}{7} - \frac{2a^2r^5}{5} + \frac{a^4r^3}{3}$$

$$\int x^5r^3 \, dx = \frac{r^9}{9} - \frac{2a^2r^7}{7} + \frac{a^4r^5}{5}$$

$$\int x^5r^{2n+1} \, dx = \frac{r^{2n+7}}{2n+7} - \frac{2a^2r^{2n+5}}{2n+5} + \frac{a^4r^{2n+3}}{2n+3}$$

$$\int \frac{r \, dx}{x} = r - a\ln\left|\frac{a+r}{x}\right| = r - a\sinh^{-1}\frac{a}{x}$$

$$\int \frac{r^3\,dx}{x} = \frac{r^3}{3} + a^2 r - a^3 \ln\left|\frac{a+r}{x}\right|$$
$$\int \frac{r^5\,dx}{x} = \frac{r^5}{5} + \frac{a^2 r^3}{3} + a^4 r - a^5 \ln\left|\frac{a+r}{x}\right|$$
$$\int \frac{r^7\,dx}{x} = \frac{r^7}{7} + \frac{a^2 r^5}{5} + \frac{a^4 r^3}{3} + a^6 r - a^7 \ln\left|\frac{a+r}{x}\right|$$
$$\int \frac{dx}{r} = \sinh^{-1}\frac{x}{a} = \ln|x+r|$$
$$\int \frac{dx}{r^3} = \frac{x}{a^2 r}$$
$$\int \frac{x\,dx}{r} = r$$
$$\int \frac{x\,dx}{r^3} = -\frac{1}{r}$$
$$\int \frac{x^2\,dx}{r} = \frac{x}{2}r - \frac{a^2}{2}\sinh^{-1}\frac{x}{a} = \frac{x}{2}r - \frac{a^2}{2}\ln|x+r|$$
$$\int \frac{dx}{xr} = -\frac{1}{a}\sinh^{-1}\frac{a}{x} = -\frac{1}{a}\ln\left|\frac{a+r}{x}\right|$$

Integrals involving $s = \sqrt{x^2 - a^2}$

Assume ($x^2 > a^2$), for ($x^2 < a^2$), see next section:

$$\int xs\,dx = \frac{1}{3}s^3$$
$$\int \frac{s\,dx}{x} = s - a\cos^{-1}\left|\frac{a}{x}\right|$$
$$\int \frac{dx}{s} = \int \frac{dx}{\sqrt{x^2-a^2}} = \ln\left|\frac{x+s}{a}\right|$$

Note that $\ln\left|\frac{x+s}{a}\right| = \operatorname{sgn}(x)\cosh^{-1}\left|\frac{x}{a}\right| = \frac{1}{2}\ln\left(\frac{x+s}{x-s}\right)$, where the positive value of $\cosh^{-1}\left|\frac{x}{a}\right|$ is to be taken.

$$\int \frac{x\,dx}{s} = s$$
$$\int \frac{x\,dx}{s^3} = -\frac{1}{s}$$
$$\int \frac{x\,dx}{s^5} = -\frac{1}{3s^3}$$

$$\int \frac{x\ dx}{s^7} = -\frac{1}{5s^5}$$

$$\int \frac{x\ dx}{s^{2n+1}} = -\frac{1}{(2n-1)s^{2n-1}}$$

$$\int \frac{x^{2m}\ dx}{s^{2n+1}} = -\frac{1}{2n-1}\frac{x^{2m-1}}{s^{2n-1}} + \frac{2m-1}{2n-1}\int \frac{x^{2m-2}\ dx}{s^{2n-1}}$$

$$\int \frac{x^2\ dx}{s} = \frac{xs}{2} + \frac{a^2}{2}\ln\left|\frac{x+s}{a}\right|$$

$$\int \frac{x^2\ dx}{s^3} = -\frac{x}{s} + \ln\left|\frac{x+s}{a}\right|$$

$$\int \frac{x^4\ dx}{s} = \frac{x^3 s}{4} + \frac{3}{8}a^2 xs + \frac{3}{8}a^4\ln\left|\frac{x+s}{a}\right|$$

$$\int \frac{x^4\ dx}{s^3} = \frac{xs}{2} - \frac{a^2 x}{s} + \frac{3}{2}a^2\ln\left|\frac{x+s}{a}\right|$$

$$\int \frac{x^4\ dx}{s^5} = -\frac{x}{s} - \frac{1}{3}\frac{x^3}{s^3} + \ln\left|\frac{x+s}{a}\right|$$

$$\int \frac{x^{2m}\ dx}{s^{2n+1}} = (-1)^{n-m}\frac{1}{a^{2(n-m)}}\sum_{i=0}^{n-m-1}\frac{1}{2(m+i)+1}\binom{n-m-1}{i}\frac{x^{2(m+i)+1}}{s^{2(m+i)+1}} \qquad (n > m \geq 0)$$

$$\int \frac{dx}{s^3} = -\frac{1}{a^2}\frac{x}{s}$$

$$\int \frac{dx}{s^5} = \frac{1}{a^4}\left[\frac{x}{s} - \frac{1}{3}\frac{x^3}{s^3}\right]$$

$$\int \frac{dx}{s^7} = -\frac{1}{a^6}\left[\frac{x}{s} - \frac{2}{3}\frac{x^3}{s^3} + \frac{1}{5}\frac{x^5}{s^5}\right]$$

$$\int \frac{dx}{s^9} = \frac{1}{a^8}\left[\frac{x}{s} - \frac{3}{3}\frac{x^3}{s^3} + \frac{3}{5}\frac{x^5}{s^5} - \frac{1}{7}\frac{x^7}{s^7}\right]$$

$$\int \frac{x^2\ dx}{s^5} = -\frac{1}{a^2}\frac{x^3}{3s^3}$$

$$\int \frac{x^2\ dx}{s^7} = \frac{1}{a^4}\left[\frac{1}{3}\frac{x^3}{s^3} - \frac{1}{5}\frac{x^5}{s^5}\right]$$

$$\int \frac{x^2\ dx}{s^9} = -\frac{1}{a^6}\left[\frac{1}{3}\frac{x^3}{s^3} - \frac{2}{5}\frac{x^5}{s^5} + \frac{1}{7}\frac{x^7}{s^7}\right]$$

Integrals involving $t = \sqrt{a^2 - x^2}$

$$\int t\, dx = \frac{1}{2}\left(xt + a^2 \arcsin\frac{x}{a}\right) \qquad (|x| \le |a|)$$
$$\int xt\, dx = -\frac{1}{3}t^3 \qquad (|x| \le |a|)$$
$$\int \frac{t\, dx}{x} = t - a\ln\left|\frac{a+t}{x}\right| \qquad (|x| \le |a|)$$
$$\int \frac{dx}{t} = \arcsin\frac{x}{a} \qquad (|x| \le |a|)$$
$$\int \frac{x^2\, dx}{t} = \frac{1}{2}\left(-xt + a^2 \arcsin\frac{x}{a}\right) \qquad (|x| \le |a|)$$
$$\int t\, dx = \frac{1}{2}\left(xt - \operatorname{sgn} x\, \cosh^{-1}\left|\frac{x}{a}\right|\right) \qquad (\text{for } |x| \ge |a|)$$

Integrals involving $R = \sqrt{ax^2 + bx + c}$

$$\int \frac{dx}{R} = \frac{1}{\sqrt{a}} \ln\left|2\sqrt{a}R + 2ax + b\right| \qquad (\text{for } a > 0)$$
$$\int \frac{dx}{R} = \frac{1}{\sqrt{a}} \sinh^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \qquad (\text{for } a > 0,\ 4ac - b^2 > 0)$$
$$\int \frac{dx}{R} = \frac{1}{\sqrt{a}} \ln|2ax+b| \quad (\text{for } a > 0,\ 4ac - b^2 = 0)$$
$$\int \frac{dx}{R} = -\frac{1}{\sqrt{-a}} \arcsin\frac{2ax+b}{\sqrt{b^2-4ac}} \qquad (\text{for } a < 0,\ 4ac - b^2 < 0,\ |2ax+b| < \sqrt{b^2-4ac})$$
$$\int \frac{dx}{R^3} = \frac{4ax+2b}{(4ac-b^2)R}$$
$$\int \frac{dx}{R^5} = \frac{4ax+2b}{3(4ac-b^2)R}\left(\frac{1}{R^2} + \frac{8a}{4ac-b^2}\right)$$
$$\int \frac{dx}{R^{2n+1}} = \frac{2}{(2n-1)(4ac-b^2)}\left(\frac{2ax+b}{R^{2n-1}} + 4a(n-1)\int\frac{dx}{R^{2n-1}}\right)$$
$$\int \frac{x}{R}\, dx = \frac{R}{a} - \frac{b}{2a}\int\frac{dx}{R}$$
$$\int \frac{x}{R^3}\, dx = -\frac{2bx+4c}{(4ac-b^2)R}$$
$$\int \frac{x}{R^{2n+1}}\, dx = -\frac{1}{(2n-1)aR^{2n-1}} - \frac{b}{2a}\int\frac{dx}{R^{2n+1}}$$

$$\int \frac{dx}{xR} = -\frac{1}{\sqrt{c}} \ln \left(\frac{2\sqrt{c}R + bx + 2c}{x} \right)$$
$$\int \frac{dx}{xR} = -\frac{1}{\sqrt{c}} \sinh^{-1} \left(\frac{bx + 2c}{|x|\sqrt{4ac - b^2}} \right)$$

Integrals involving $S = \sqrt{ax+b}$

$$\int \frac{dx}{x\sqrt{ax+b}} = \frac{-2}{\sqrt{b}} \tanh^{-1} \sqrt{\frac{ax+b}{b}}$$
$$\int \frac{\sqrt{ax+b}}{x}\,dx = 2\left(\sqrt{ax+b} - \sqrt{b} \tanh^{-1} \sqrt{\frac{ax+b}{b}} \right)$$
$$\int \frac{x^n}{\sqrt{ax+b}}\,dx = \frac{2}{a(2n+1)} \left(x^n \sqrt{ax+b} - bn \int \frac{x^{n-1}}{\sqrt{ax+b}}\,dx \right)$$
$$\int x^n \sqrt{ax+b}\,dx = \frac{2}{2n+1} \left(x^{n+1}\sqrt{ax+b} + bx^n\sqrt{ax+b} - nb \int x^{n-1}\sqrt{ax+b}\,dx \right)$$

List of integrals of logarithmic functions

$$\int \ln cx \; dx = x \ln cx - x$$
$$\int \ln(ax+b) \; dx = x\ln(ax+b) - x + \frac{b}{a}\ln(ax+b)$$
$$\int (\ln x)^2 \; dx = x(\ln x)^2 - 2x\ln x + 2x$$
$$\int (\ln cx)^n \; dx = x(\ln cx)^n - n\int (\ln cx)^{n-1} dx$$
$$\int \frac{dx}{\ln x} = \ln|\ln x| + \ln x + \sum_{i=2}^{\infty} \frac{(\ln x)^i}{i \cdot i!}$$
$$\int \frac{dx}{(\ln x)^n} = -\frac{x}{(n-1)(\ln x)^{n-1}} + \frac{1}{n-1}\int \frac{dx}{(\ln x)^{n-1}} \qquad \text{(for } n \neq 1\text{)}$$
$$\int x^m \ln x \; dx = x^{m+1}\left(\frac{\ln x}{m+1} - \frac{1}{(m+1)^2} \right) \qquad \text{(for } m \neq -1\text{)}$$
$$\int x^m (\ln x)^n \; dx = \frac{x^{m+1}(\ln x)^n}{m+1} - \frac{n}{m+1}\int x^m (\ln x)^{n-1} dx \qquad \text{(for } m \neq -1\text{)}$$
$$\int \frac{(\ln x)^n \; dx}{x} = \frac{(\ln x)^{n+1}}{n+1} \qquad \text{(for } n \neq -1\text{)}$$

$$\int \frac{\ln x^n\ dx}{x} = \frac{(\ln x^n)^2}{2n} \qquad \mbox{(for } n\neq 0 \mbox{)}$$

$$\int \frac{\ln x\,dx}{x^m} = -\frac{\ln x}{(m-1)x^{m-1}}-\frac{1}{(m-1)^2 x^{m-1}} \qquad \mbox{(for } m\neq 1 \mbox{)}$$

$$\int \frac{(\ln x)^n\ dx}{x^m} = -\frac{(\ln x)^n}{(m-1)x^{m-1}}+\frac{n}{m-1}\int\frac{(\ln x)^{n-1} dx}{x^m} \qquad \mbox{(for } m\neq 1 \mbox{)}$$

$$\int \frac{x^m\ dx}{(\ln x)^n} = -\frac{x^{m+1}}{(n-1)(\ln x)^{n-1}}+\frac{m+1}{n-1}\int\frac{x^m dx}{(\ln x)^{n-1}} \qquad \mbox{(for } n\neq 1 \mbox{)}$$

$$\int \frac{dx}{x\ln x} = \ln \left|\ln x\right|$$

$$\int \frac{dx}{x^n\ln x} = \ln \left|\ln x\right| + \sum^\infty_{i=1} (-1)^i\frac{(n-1)^i(\ln x)^i}{i\cdot i!}$$

$$\int \frac{dx}{x(\ln x)^n} = -\frac{1}{(n-1)(\ln x)^{n-1}} \qquad \mbox{(for } n\neq 1 \mbox{)}$$

$$\int \ln(x^2+a^2)\ dx = x\ln(x^2+a^2)-2x+2a\tan^{-1}\frac{x}{a}$$

$$\int \frac{x}{x^2+a^2}\ln(x^2+a^2)\ dx = \frac{1}{4} \ln^2(x^2+a^2)$$

$$\int \sin (\ln x)\ dx = \frac{x}{2}(\sin (\ln x) - \cos (\ln x))$$

$$\int \cos (\ln x)\ dx = \frac{x}{2}(\sin (\ln x) + \cos (\ln x))$$

$$\int e^x \left(x \ln x - x - \frac{1}{x} \right)\ dx = e^x (x \ln x - x - \ln x)$$

$$\int \frac{1}{e^x} \left(\frac{1}{x} - \ln x \right)\ dx = \frac{\ln x}{e^x}$$

$$\int e^x \left(\frac{1}{\ln x} - \frac{1}{x\ln^2 x} \right)\ dx = \frac{e^x}{\ln x}$$

List of integrals of exponential functions

$$\int e^{cx}\ dx = \frac{1}{c} e^{cx}$$

$$\int a^{cx}\ dx = \frac{1}{c \ln a} a^{cx} \qquad \mbox{(for } a > 0,\ a \ne 1\mbox{)}$$

$$\int xe^{cx}\ dx = \frac{e^{cx}}{c^2}(cx-1)$$

$$\int x^2 e^{cx}\ dx = e^{cx}\left(\frac{x^2}{c}-\frac{2x}{c^2}+\frac{2}{c^3}\right)$$

$$\int x^n e^{cx}\, dx = \frac{1}{c} x^n e^{cx} - \frac{n}{c}\int x^{n-1} e^{cx} dx$$

$$\int \frac{e^{cx}}{x}\, dx = \ln|x| + \sum_{i=1}^{\infty} \frac{(cx)^i}{i\cdot i!}$$

$$\int \frac{e^{cx}}{x^n}\, dx = \frac{1}{n-1}\left(-\frac{e^{cx}}{x^{n-1}} + c\int \frac{e^{cx}}{x^{n-1}}\, dx\right) \qquad (\text{for } n \neq 1)$$

$$\int e^{cx} \ln x\, dx = \frac{1}{c} e^{cx} \ln|x| - \mathrm{Ei}\,(cx)$$

$$\int e^{cx} \sin bx\, dx = \frac{e^{cx}}{c^2+b^2}(c \sin bx - b\cos bx)$$

$$\int e^{cx} \cos bx\, dx = \frac{e^{cx}}{c^2+b^2}(c \cos bx + b\sin bx)$$

$$\int e^{cx} \sin^n x\, dx = \frac{e^{cx}\sin^{n-1} x}{c^2+n^2}(c \sin x - n\cos x) + \frac{n(n-1)}{c^2+n^2}\int e^{cx} \sin^{n-2} x\, dx$$

$$\int e^{cx} \cos^n x\, dx = \frac{e^{cx}\cos^{n-1} x}{c^2+n^2}(c \cos x + n\sin x) + \frac{n(n-1)}{c^2+n^2}\int e^{cx} \cos^{n-2} x\, dx$$

$$\int x e^{cx^2}\, dx = \frac{1}{2c}\, e^{cx^2}$$

$$\int x e^{-cx^2}\, dx = -\frac{1}{2c} e^{-cx^2}$$

List of integrals of trigonometric functions

Integrals of trigonometric functions containing only sin

Where c is a constant:

$$\int \sin cx \, dx = -\frac{1}{c}\cos cx$$

$$\int \sin^n cx \, dx = -\frac{\sin^{n-1} cx \cos cx}{nc} + \frac{n-1}{n}\int \sin^{n-2} cx \, dx \qquad (\text{for } n>0)$$

$$\int \sin^2 cx \, dx = \frac{x}{2} - \frac{1}{4c} sin 2cx$$

$$\int \sqrt{1-\sin x}\, dx = \int \sqrt{\operatorname{cvs} x}\, dx = 2\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\sqrt{\operatorname{cvs} x} = 2\sqrt{1+\sin x}$$

Integrals of trigonometric functions containing only cos

$$\int \cos cx \, dx = \frac{1}{c}\sin cx$$

$$\int \cos^n cx \, dx = \frac{\cos^{n-1} cx \sin cx}{nc} + \frac{n-1}{n}\int \cos^{n-2} cx \, dx \qquad (\text{for } n>0)$$

$$\int x\cos cx \, dx = \frac{\cos cx}{c^2} + \frac{x\sin cx}{c}$$

$$\int x^n \cos cx \, dx = \frac{x^n \sin cx}{c} - \frac{n}{c}\int x^{n-1}\sin cx \, dx$$

$$\int_{\frac{-a}{2}}^{\frac{a}{2}} x^2\cos^2\frac{n\pi x}{a}\, dx = \frac{a^3(n^2\pi^2-6)}{24n^2\pi^2} \qquad (\text{for } n=1,3,5...)$$

$$\int \frac{\cos cx}{x} dx = \ln|cx| + \sum_{i=1}^{\infty}(-1)^i\frac{(cx)^{2i}}{2i\cdot(2i)!}$$

$$\int \frac{\cos cx}{x^n} dx = -\frac{\cos cx}{(n-1)x^{n-1}} - \frac{c}{n-1}\int \frac{\sin cx}{x^{n-1}} dx \qquad (\text{for } n\neq 1)$$

$$\int \frac{dx}{\cos cx} = \frac{1}{c}\ln\left|\tan\left(\frac{cx}{2}+\frac{\pi}{4}\right)\right|$$

$$\int \frac{dx}{\cos^n cx} = \frac{\sin cx}{c(n-1)cos^{n-1}cx} + \frac{n-2}{n-1}\int \frac{dx}{\cos^{n-2} cx} \qquad (\text{for } n>1)$$

$$\int \frac{dx}{1+\cos cx} = \frac{1}{c}\tan\frac{cx}{2}$$

$$\int \frac{dx}{1-\cos cx} = -\frac{1}{c}\cot\frac{cx}{2}$$

$$\int \frac{x\, dx}{1+\cos cx} = \frac{x}{c}\tan\frac{cx}{2} + \frac{2}{c^2}\ln\left|\cos\frac{cx}{2}\right|$$

$$\int \frac{x\,dx}{1-\cos cx} = -\frac{x}{c}\cot\frac{cx}{2} + \frac{2}{c^2}\ln\left|\sin\frac{cx}{2}\right|$$

$$\int \frac{\cos cx\,dx}{1+\cos cx} = x - \frac{1}{c}\tan\frac{cx}{2}$$

$$\int \frac{\cos cx\,dx}{1-\cos cx} = -x - \frac{1}{c}\cot\frac{cx}{2}$$

$$\int \cos c_1x\cos c_2x\,dx = \frac{\sin(c_1-c_2)x}{2(c_1-c_2)} + \frac{\sin(c_1+c_2)x}{2(c_1+c_2)} \qquad \text{(for } |c_1| \neq |c_2|\text{)}$$

Integrals of trigonometric functions containing only tan

$$\int \tan cx\,dx = -\frac{1}{c}\ln|\cos cx| = \frac{1}{c}\ln|\sec cx|$$

$$\int \frac{dx}{\tan cx} = \frac{1}{c}\ln|\sin cx|$$

$$\int \tan^n cx\,dx = \frac{1}{c(n-1)}\tan^{n-1}cx - \int \tan^{n-2}cx\,dx \qquad \text{(for } n \neq 1\text{)}$$

$$\int \frac{dx}{\tan cx+1} = \frac{x}{2} + \frac{1}{2c}\ln|\sin cx + \cos cx|$$

$$\int \frac{dx}{\tan cx-1} = -\frac{x}{2} + \frac{1}{2c}\ln|\sin cx - \cos cx|$$

$$\int \frac{\tan cx\,dx}{\tan cx+1} = \frac{x}{2} - \frac{1}{2c}\ln|\sin cx + \cos cx|$$

$$\int \frac{\tan cx\,dx}{\tan cx-1} = \frac{x}{2} + \frac{1}{2c}\ln|\sin cx - \cos cx|$$

Integrals of trigonometric functions containing only sec

$$\int \sec cx\,dx = \frac{1}{c}\ln|\sec cx + \tan cx|$$

$$\int \sec^n cx\,dx = \frac{\sec^{n-1}cx\sin cx}{c(n-1)} + \frac{n-2}{n-1}\int \sec^{n-2}cx\,dx \qquad \text{(for } n \neq 1\text{)}$$

$$\int \frac{dx}{\sec x+1} = x - \tan\frac{x}{2}$$

Integrals of trigonometric functions containing only csc

$$\int \csc cx\,dx = -\frac{1}{c}\ln|\csc cx + \cot cx|$$

$$\int \csc^2 x\,dx = -\cot x$$

$$\int \csc^n cx\,dx = -\frac{\csc^{n-1} cx \cos cx}{c(n-1)} + \frac{n-2}{n-1}\int \csc^{n-2} cx\,dx \qquad (\text{for } n \neq 1)$$

Integrals of trigonometric functions containing both sin and cos

$$\int \frac{dx}{\cos cx \pm \sin cx} = \frac{1}{c\sqrt{2}}\ln\left|\tan\left(\frac{cx}{2} \pm \frac{\pi}{8}\right)\right|$$

$$\int \frac{dx}{(\cos cx \pm \sin cx)^2} = \frac{1}{2c}\tan\left(cx \mp \frac{\pi}{4}\right)$$

$$\int \frac{dx}{(\cos x + \sin x)^n} = \frac{1}{n-1}\left(\frac{\sin x - \cos x}{(\cos x + \sin x)^{n-1}} - 2(n-2)\int \frac{dx}{(\cos x + \sin x)^{n-2}}\right)$$

$$\int \frac{\cos cx\ dx}{\cos cx + \sin cx} = \frac{x}{2} + \frac{1}{2c}\ln|\sin cx + \cos cx|$$

$$\int \frac{\cos cx\ dx}{\cos cx - \sin cx} = \frac{x}{2} - \frac{1}{2c}\ln|\sin cx - \cos cx|$$

$$\int \frac{\sin cx\ dx}{\cos cx + \sin cx} = \frac{x}{2} - \frac{1}{2c}\ln|\sin cx + \cos cx|$$

$$\int \frac{\sin cx\ dx}{\cos cx - \sin cx} = -\frac{x}{2} - \frac{1}{2c}\ln|\sin cx - \cos cx|$$

$$\int \frac{\cos cx\ dx}{\sin cx(1 + \cos cx)} = -\frac{1}{4c}\tan^2\frac{cx}{2} + \frac{1}{2c}\ln\left|\tan\frac{cx}{2}\right|$$

$$\int \frac{\cos cx\ dx}{\sin cx(1 + - \cos cx)} = -\frac{1}{4c}\cot^2\frac{cx}{2} - \frac{1}{2c}\ln\left|\tan\frac{cx}{2}\right|$$

$$\int \frac{\sin cx\ dx}{\cos cx(1 + \sin cx)} = \frac{1}{4c}\cot^2\left(\frac{cx}{2} + \frac{\pi}{4}\right) + \frac{1}{2c}\ln\left|\tan\left(\frac{cx}{2} + \frac{\pi}{4}\right)\right|$$

$$\int \frac{\sin cx\ dx}{\cos cx(1 - \sin cx)} = \frac{1}{4c}\tan^2\left(\frac{cx}{2} + \frac{\pi}{4}\right) - \frac{1}{2c}\ln\left|\tan\left(\frac{cx}{2} + \frac{\pi}{4}\right)\right|$$

$$\int \sin cx \cos cx\ dx = \frac{1}{2c}\sin^2 cx$$

$$\int \sin c_1 x \cos c_2 x\ dx = -\frac{\cos(c_1 + c_2)x}{2(c_1 + c_2)} - \frac{\cos(c_1 - c_2)x}{2(c_1 - c_2)} \qquad (\text{for } |c_1| \neq |c_2|)$$

$$\int \sin^n cx \cos cx\ dx = \frac{1}{c(n+1)}\sin^{n+1} cx \qquad (\text{for } n \neq 1)$$

$$\int \sin cx \cos^n cx\ dx = -\frac{1}{c(n+1)}\cos^{n+1} cx \qquad (\text{for } n \neq 1)$$

$$\int \frac{dx}{\sin cx \cos cx} = \frac{1}{c} \ln |\tan cx|$$

$$\int \frac{dx}{\sin cx \cos^n cx} = \frac{1}{c(n-1)\cos^{n-1} cx} + \int \frac{dx}{\sin cx \cos^{n-2} cx} \qquad \text{(for } n \neq 1)$$

$$\int \frac{dx}{\sin^n cx \cos cx} = -\frac{1}{c(n-1)\sin^{n-1} cx} + \int \frac{dx}{\sin^{n-2} cx \cos cx} \qquad \text{(for } n \neq 1)$$

$$\int \frac{\sin cx \; dx}{\cos^n cx} = \frac{1}{c(n-1)\cos^{n-1} cx} \qquad \text{(for } n \neq 1)$$

$$\int \frac{\sin^2 cx \; dx}{\cos cx} = -\frac{1}{c} \sin cx + \frac{1}{c} \ln \left| \tan \left(\frac{\pi}{4} + \frac{cx}{2} \right) \right|$$

$$\int \frac{\sin^2 cx \; dx}{\cos^n cx} = \frac{\sin cx}{c(n-1)\cos^{n-1} cx} - \frac{1}{n-1} \int \frac{dx}{\cos^{n-2} cx} \qquad \text{(for } n \neq 1)$$

$$\int \frac{\sin^n cx \; dx}{\cos cx} = -\frac{\sin^{n-1} cx}{c(n-1)} + \int \frac{\sin^{n-2} cx \; dx}{\cos cx} \qquad \text{(for } n \neq 1)$$

$$\int \frac{\sin^n cx \; dx}{\cos^m cx} = \frac{\sin^{n+1} cx}{c(m-1)\cos^{m-1} cx} - \frac{n-m+2}{m-1} \int \frac{\sin^n cx \; dx}{\cos^{m-2} cx} \qquad \text{(for } m \neq 1)$$

$$\int \frac{\cos cx \; dx}{\sin^n cx} = -\frac{1}{c(n-1)\sin^{n-1} cx} \qquad \text{(for } n \neq 1)$$

$$\int \frac{\cos^2 cx \; dx}{\sin cx} = \frac{1}{c} \left(\cos cx + \ln \left| \tan \frac{cx}{2} \right| \right)$$

$$\int \frac{\cos^2 cx \; dx}{\sin^n cx} = -\frac{1}{n-1} \left(\frac{\cos cx}{c \sin^{n-1} cx)} + \int \frac{dx}{\sin^{n-2} cx} \right) \qquad \text{(for } n \neq 1)$$

$$\int \frac{\cos^n cx \; dx}{\sin^m cx} = -\frac{\cos^{n+1} cx}{c(m-1)\sin^{m-1} cx} - \frac{n-m-2}{m-1} \int \frac{\cos^n cx \; dx}{\sin^{m-2} cx} \qquad \text{(for } m \neq 1)$$

Integrals of trigonometric functions with symmetric limits

$$\int_{-c}^{c} \sin x \; dx = 0$$

$$\int_{-c}^{c} \cos x \; dx = 2 \int_{0}^{c} \cos x \; dx = 2 \int_{-c}^{0} \cos x \; dx$$

$$\int_{-c}^{c} \tan x \; dx = 0$$