1. Jednadžbe statičkog strujnog polja i uvjeti na granici dvaju vodiča.

I

$$\boxed{\operatorname{div}\vec{J} = \nabla\vec{J} = -\frac{\partial\rho}{\partial t}}$$ – jednadžba kontinuiteta

$$\boxed{\vec{J} = \varkappa\vec{E}}$$

$$\nabla\vec{J} = \varkappa\nabla\vec{E} = -\frac{\partial\rho}{\partial t}$$

$$\vec{\nabla}E = -\frac{1}{\varkappa}\frac{\partial\rho}{\partial t}$$

$$\nabla\vec{D} = \rho_s \rightarrow \nabla\vec{E} = \frac{\rho}{\varepsilon}$$

$$\frac{\rho}{\varepsilon} = -\frac{1}{\varkappa}\frac{\partial\rho}{\partial t} \rightarrow \frac{\partial\rho}{\partial t} + \frac{\varkappa}{\varepsilon}\rho = 0 \rightarrow \rho(t) = \rho_0\cdot e^{-\left(\frac{\varkappa}{\varepsilon}\right)t} = \rho_0\cdot e^{-\frac{t}{\tau_r}}$$

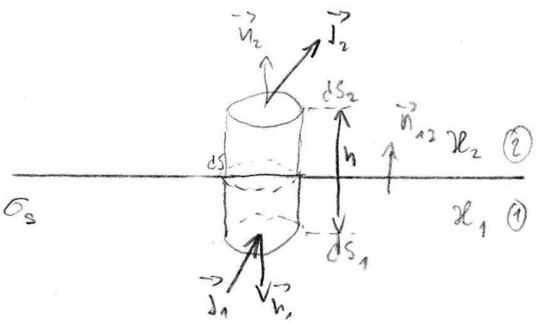
$$\nabla\vec{J} = \nabla(\varkappa\vec{E}) = \nabla(-\varkappa\nabla\varphi) = 0 \rightarrow \boxed{\Delta\varphi = 0}$$

$$\boxed{I = \int_S \vec{J}\vec{n}\,dS}$$ – strujni tok

$$\boxed{G = \frac{I}{U_{ab}} = \frac{\int_S \vec{J}\vec{n}\,dS}{-\int_b^a \vec{E}\,dl}}$$ – vodljivost

$$\boxed{\vec{E} = -\nabla\varphi}$$

II



$$\oint_S \vec{J}\vec{n}\,dS = -\frac{d}{dt}\int_V \rho_s\,dV$$

$$\vec{J}_1\vec{n}_1\,dS_1 + \vec{J}_2\vec{n}_2\,dS_2 + (\text{doprinosi struji kroz plašt}) = -\frac{d}{dt}(\rho_s h\,dS)$$

$$\lim_{h\to 0}\{-\vec{J}_1\vec{n}_{12}\,dS + J_2 n_{12}\,dS + (D.S.K.P)\} = -\frac{d}{dt}\lim_{h\to 0}(\rho_s h\,dS), \quad h\to 0 \Rightarrow (D.S.K.P)\to 0$$

$$\vec{n}_{12}(J_2 - J_1) = -\frac{d\sigma_s}{dt}$$

ako je $\sigma_s = 0$ ili $\frac{d\sigma_s}{dt} = 0 \Rightarrow \vec{n}_{12}(\vec{J}_2 - \vec{J}_1) = 0 \Rightarrow J_{1n} = J_{2n} \Rightarrow \varkappa_1 E_{1n} = \varkappa_2 E_{2n} \Rightarrow$

$$\Rightarrow \frac{E_{1n}}{E_{2n}} = \frac{\varkappa_2}{\varkappa_1} \qquad (*)$$

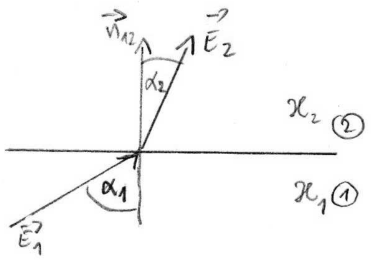
$$E_{1t} = E_{2t} \Rightarrow \vec{n}_{12} \times (E_2 - E_1) = 0$$

$$\Downarrow$$

$$\frac{J_{1t}}{\varkappa_1} = \frac{J_{2t}}{\varkappa_2} \Rightarrow \frac{J_{1t}}{J_{2t}} = \frac{\varkappa_1}{\varkappa_2} \qquad (**)$$

$$\boxed{\begin{aligned} &\vec{n}_{12}(\vec{J}_2 - \vec{J}_1) = -\frac{\partial \sigma_s}{\partial t} = 0 \\ &\vec{n}_{12} \times (\vec{E}_2 - E_1) = 0 \end{aligned}}$$

Iz $(*)$ i $(**)$ slijedi:



$$tg\,\alpha_1 = \frac{E_{1t}}{E_{1n}} \quad ; \quad tg\,\alpha_2 = \frac{E_{2t}}{E_{2n}}$$

$$\boxed{\frac{tg\,\alpha_1}{tg\,\alpha_2} = \frac{\frac{E_{1t}}{E_{1n}}}{\frac{E_{2t}}{E_{2n}}} = \frac{E_{1t} \cdot E_{2n}}{E_{2t} \cdot E_{1n}} = \frac{\varkappa_1}{\varkappa_2}}$$

ne mijenjaju se na granici

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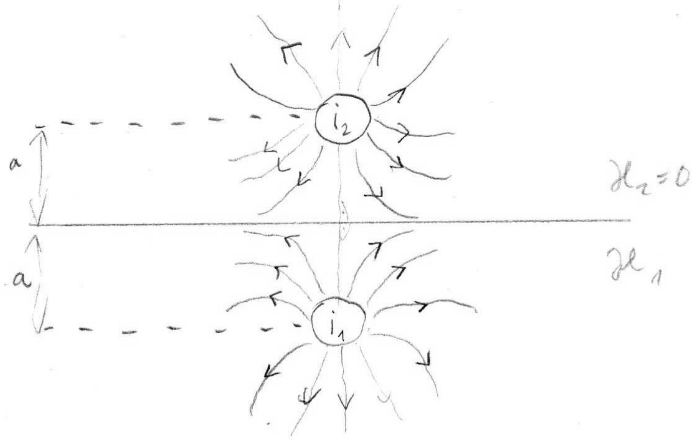
② Analogija statičkog strujnog polja i statičkog električnog polja i odslikavanje u statičkom strujnom polju.

(I) S.S.P = istosmjerna struja $\Rightarrow \nabla \vec{J} = 0$

Homogeni dielektrik bez naboja ($\rho_s = 0$)	Vodljivi materijal, stac. strujanje $\partial \rho_s / \partial t = 0$
Gauss: $\nabla \vec{D} = 0$	Kontinuitet: $\nabla \vec{J} = 0$
$\vec{D} = \varepsilon \vec{E}$	$J = \varkappa \vec{E}$
$\Delta \varphi = 0$	$\Delta \varphi = 0$
Električni tok: $\phi_e = \int_S \vec{D} \vec{n} \, ds$	Strujni tok: $I = \int_S \vec{J} \vec{n} \, ds$
$C = \frac{Q}{U_{ab}} = \frac{\int_S \vec{D} n \, ds}{-\int_b^a \vec{E} \, dl}$	$G = \frac{I}{U_{ab}} = \frac{\int_S \vec{J} \vec{n} \, ds}{-\int_b^a \vec{E} \, dl}$

$\vec{D}$	$\vec{J}$
$\vec{E}$	$\vec{E}$
ε	$\varkappa$
φ	φ
ϕ_e	I
C	G

II



$J_{2n}=0$

$J_{1n}=J_{2n} \Rightarrow J_{1n}=0,\ J_{2n}=0 \Rightarrow E_{1n}=E_{2n}=0,\ \frac{\partial\varphi_1}{\partial n}=\frac{\partial\varphi_2}{\partial n}=0$

$i_1=i_2$

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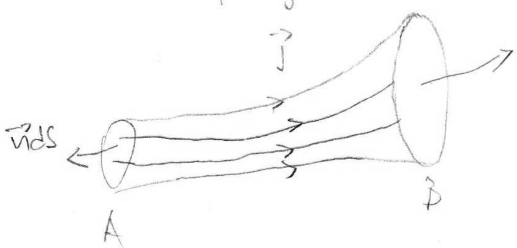
3. Gubici snage u vodiču u statičkom strujnom polju

Nastaju zbog otpora.

$$R=\frac{U_{AB}}{I}=\frac{-\int_b^a \vec E\, d\vec l}{\int_S \vec J \vec n\, dS}=\frac{-\int_b^a \vec E\, d\vec l}{\varkappa\int_S \vec E\vec n\, dS}$$

Jednoliki presjek: $R=\frac{1}{\varkappa}\frac{l}{S}=\rho\frac{l}{S}$

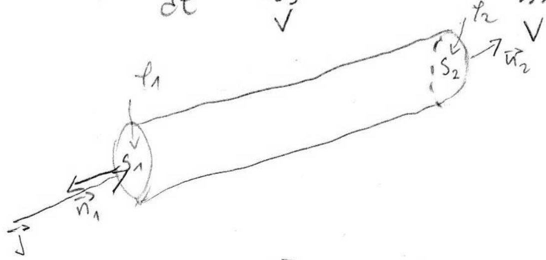


$dq=\vec J\vec n\, dS\, dt$

$\varphi_A-\varphi_B=-\vec E\, d\vec s$

$dW=dq[\varphi_A-\varphi_B]=\vec J\vec n\, dS\, dt\, \vec E\, d\vec s=\vec J\vec E\, dV\, dt$

$P=\frac{dW}{dt}=\iiint_V \vec J\vec E\, dV=-\iiint_V \nabla\cdot(\varphi\vec J)\, dV=-\oint\!\!\oint_S \varphi\vec J\vec n\, dS$ $\quad(\nabla\varphi\vec J=\varphi\nabla\vec J+(\nabla\varphi)\vec J=-\vec E\vec J)$



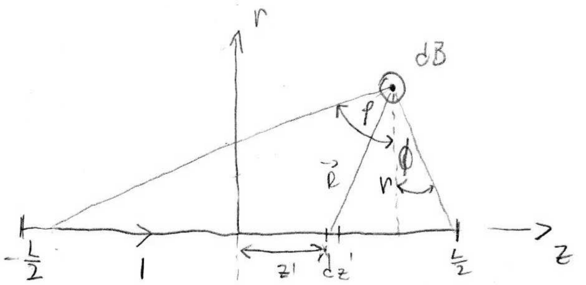
$P=-\oint\!\!\oint \varphi\vec J\vec n\, dS=I\varphi_1-I\varphi_2=UI=I^2R=\frac{U^2}{R}$

(4) Biot-Savartov zakon i magnetska indukcija kratke ravne strujnice.

$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{R}}{R^3} \qquad \vec{R} = \vec{r} - \vec{r}\,'$$

$\vec{r}\,'$ – položaj dijela strujnice

$\vec{r}$ – točka u kojoj računamo B

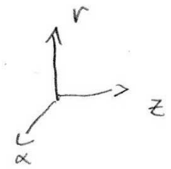


$$\vec{r} = r\vec{a}_r + z\vec{a}_z$$

$$\vec{r}\,' = z'\vec{a}_z$$

$$\vec{R} = r\vec{a}_r + (z-z')\vec{a}_z$$

$$d\vec{l} = dz'\,\vec{a}_z \qquad d\vec{l}\times\vec{R} = r\,dz'\,\vec{a}_\alpha$$



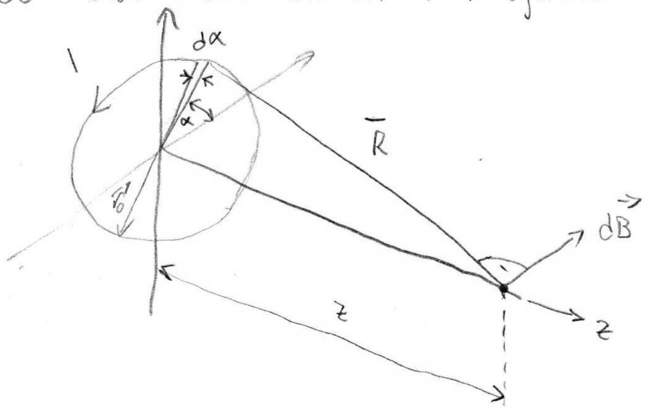
$$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{R}}{R^3} = \frac{\mu_0 I}{4\pi}\cdot\frac{r\,dz'\,\vec{a}_\alpha}{[r^2+(z-z')^2]^{\frac{3}{2}}}$$

$$\vec{B} = \frac{\mu_0 I \vec{a}_\alpha r}{4\pi}\int_{-\frac{L}{2}}^{\frac{L}{2}}\frac{dz'}{[r^2+(z-z')^2]^{\frac{3}{2}}} = \vec{a}_\alpha\frac{\mu_0 I}{4\pi r}\left[\frac{\frac{L}{2}+z}{\sqrt{(\frac{L}{2}+z)^2+r^2}} + \frac{\frac{L}{2}-z}{\sqrt{(\frac{L}{2}-z)^2+r^2}}\right] =$$

$$= \vec{a}_\alpha\frac{\mu_0 I}{4\pi r}\left(\sin\rho + \sin\phi\right)$$

$$L\to\infty \implies \vec{B} = \vec{a}_\alpha\frac{\mu_0 I}{2\pi r}$$

(5.) Biot-Savartov zakon i magnetska indukcija na osi kružne strujnice.



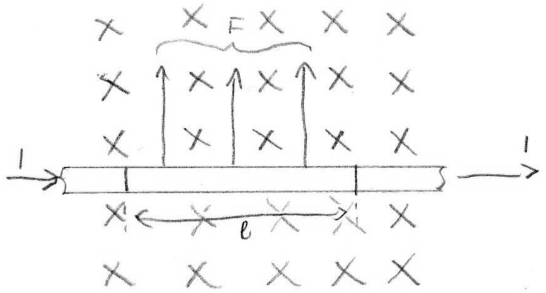
$$\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{l}\times\vec{R}}{R^3}$$

Ostaje samo komponenta u smjeru $\vec{a}_z$ jer se ostale poništavaju.

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int_0^{2\pi}\frac{r_0\vec{a}_z}{(r_0^2+z^2)^{\frac{3}{2}}}\cdot r_0\,d\alpha = \frac{\mu_0 I}{4\pi}\frac{r_0^2\vec{a}_z}{(r_0^2+z^2)^{\frac{3}{2}}}\int_0^{2\pi}d\alpha =$$

$$= \frac{\mu_0 I}{2}\cdot\frac{r_0^2}{(r_0^2+z^2)^{\frac{3}{2}}}\vec{a}_z$$

(6) Sila na strujni element u magnetskom polju



$\vec{f} = q(\vec{v} \times \vec{B})$ – sila na pojedini naboj

$\vec{F} = N\vec{f} = n l S q (\vec{v} \times \vec{B})$

$dQ = Nq = n l S q = n v dt S q \qquad (l = v dt)$

$I = \frac{dQ}{dt} = n v S q$

$\vec{F} = I(\vec{l} \times \vec{B})$

$d\vec{F} = I(d\vec{l} \times \vec{B})$

$$\boxed{\vec{F} = \int_l d\vec{F} = \int_l I(d\vec{l} \times \vec{B})}$$

$i\, d\vec{l} = \vec{J}\, dV$

$$\boxed{\vec{F} = \iiint_V (\vec{J} \times \vec{B})\, dV}$$

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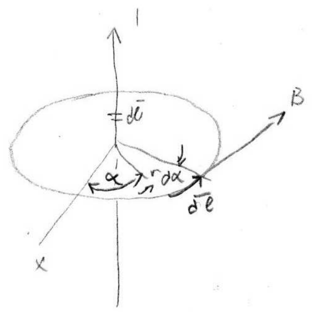
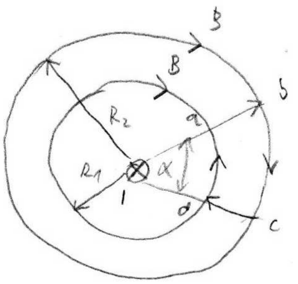
(7) Jednadžbe statičkog magnetskog polja u diferencijalnom i integralnom obliku.

Ampereov zakon





$d\vec{l}_{a-b} = \vec{a}_r dr$

$d\vec{l}_{b-c} = \vec{a}_\alpha R_2 d\alpha$

$d\vec{l}_{c-d} = -\vec{a}_r dr$

$d\vec{l}_{d-a} = -\vec{a}_\alpha R_1 d\alpha$

$\vec{B} = \vec{a}_\alpha \frac{\mu_0 I}{2\pi r} = \text{konst.}$

$d\vec{l} = \vec{a}_\alpha r d\alpha$

$\oint_c \vec{B} d\vec{l} = \oint_c \vec{a}_\alpha \frac{\mu_0 I}{2\pi r} \vec{a}_\alpha r d\alpha = \frac{\mu_0 I}{2\pi} \oint_c d\alpha = \frac{\mu_0 I}{2\pi} \cdot 2\pi = \mu_0 I$

$$\oint_c \vec{B}\,d\vec{l} = \int_{R_1}^{R_2} \vec{a}_\alpha \frac{\mu_0 \vec{I}}{2\pi \vec{r}} \vec{a}_r\,dr + \int_b^c \vec{a}_\alpha \frac{\mu_0 \vec{I}}{2\pi \vec{R_2}} \vec{a}_\alpha \vec{R_2}\,d\alpha - \int_{R_1}^{R_2} \vec{a}_\alpha \frac{\mu_0 \vec{I}}{2\pi \vec{r}} \vec{a}_r\,dr - \int_d^a \vec{a}_\alpha \frac{\mu_0 \vec{I}}{2\pi \vec{R_1}} \vec{R_1}\,d\alpha$$

$$= 0 + \frac{\mu_0 \vec{I}}{2\pi} \int_b^c d\alpha - 0 - \frac{\mu_0 \vec{I}}{2\pi} \int_d^a d\alpha = \frac{\mu_0 \vec{I}}{2\pi} \alpha - \frac{\mu_0 \vec{I}}{2\pi} \alpha = 0$$

$$\oint \vec{B}\,d\vec{l} = \mu_0 \Sigma I$$

$$\boxed{\oint \vec{B}\,d\vec{l} = \mu_0 \int_S \vec{J_s}\,\vec{n}\,dS}$$

$$\oint_c \vec{B}\,d\vec{l} = \int_S (\nabla \times \vec{B}) \cdot \vec{n} \cdot dS = \mu_0 \int_S \vec{J_s}\,\vec{n}\,dS \rightarrow \boxed{\nabla \times \vec{B} = \mu_0 \vec{J_s}}$$

Gaussov zakon

$$\boxed{\oint_S \vec{B}\,\vec{n}\,dS = 0}$$

$$\oint \vec{B}\,\vec{n}\,dS = \iiint_V \nabla \vec{B}\,dV = 0$$

$$\boxed{\operatorname{div} B = 0}$$