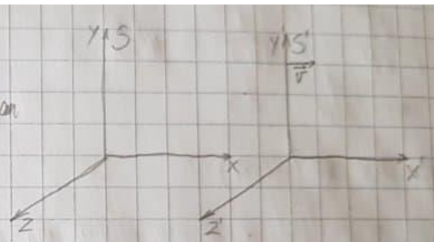
31. Skicirajte dva inercijalna referentna sustava koji se jedan u odnosu na drugi gibaju brzinom $\vec{v}$. Napišite izraze za Galileijeve i Lorentzove transformacije za tri prostorne i jednu vremensku koordinatu u tim sustavima, te za komponente brzine čestice. Detaljno objasnite razlike između tih transformacija.

GALILEJEVE TRANSFORMACIJE



→ sustav S' giba se u odnosu na sustav S stalnom brzinom $\vec{v}$ u smjeru x-osi

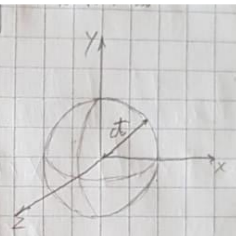
→ u $t=0$ ishodišta se preklapaju

$x = x' + vt \quad z = z' \qquad x' = x - vt \quad z = z'$

$y = y' \quad t = t' \qquad y' = y \quad t = t'$

→ događaj → mjera gdje je nešto i kada je to nešto tamo

širenje svjetlosti u dva inercijska sustava



→ kratki bljesak svjetlosti u ishodištu sustava S u trenutku $t=0$

→ sfera polumjera $r = ct$

$r^2 = x^2 + y^2 + z^2 = (ct)^2$

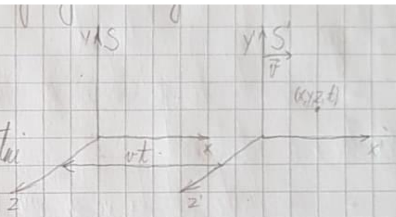
→ sustav S' se u trenutku bljeska podudara sa sustavom S

→ primjena Galilejevih transformacija na širenje svjetlosti u sustavu S'

$(x' + vt')^2 + y'^2 + z'^2 = (ct')^2$ NIJE JEDNADŽBA SFERE

→ širenje svjetlosti jednakom brzinom nije invarijantno na Galilejeve transformacije

LORENTZOVE TRANSFORMACIJE



→ za $v \ll c$ ekvivalentne Galilejevim

→ simetrične za $v \to -v$ jer su oba sustava ekvivalentni

→ trebaju biti linearne funkcije koordinata i vremena

$$x = ax' + bt'$$

→ trebaju dati zbrajanje brzina takvo da brzina svjetlosti bude uvijek c

$$x = ax' + bt' = a\left(x' + \frac{b}{a}t'\right) \Rightarrow x = \gamma(x' + vt')$$

$$x' = \gamma(x - vt)$$

$$y = y'$$

$$z = z'$$

$$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$$

$$x = ct = \gamma(x' + vt') = \gamma \cdot (ct' + vt') = \gamma t'(c+v)$$

$$x' = ct' = \gamma(x - vt) = \gamma(ct - vt) = \gamma t(c - v)$$

$$t' = \frac{\gamma t(c-v)}{c} \qquad ct = \gamma \frac{\gamma t(c-v)}{c}(c+v)$$

$$c^2 = \gamma^2(c^2 - v^2) \qquad \gamma = \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

u sustavu S svjetlost se širi $x^2 + y^2 + z^2 = (ct)^2$

$$\gamma^2(x' + vt')^2 + y'^2 + z'^2 = c^2\gamma^2\left(t' + \frac{vx'}{c^2}\right)^2$$

$$y'^2 + z'^2 = c^2\,\frac{t'^2 + \cancel{\frac{2t'vx'}{c^2}} + \frac{v^2x'^2}{c^4}}{1 - \frac{v^2}{c^2}} - \frac{x'^2 + \cancel{2x'vt'} + v^2t'^2}{1 - \frac{v^2}{c^2}}$$

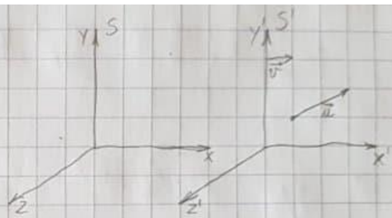
$$y'^2 + z'^2 = \frac{t'^2(c^2 - v^2) - x'^2\left(1 - \frac{v^2}{c^2}\right)}{1 - \frac{v^2}{c^2}}$$

$$y'^2 + z'^2 = \frac{t'^2(c^2 - v^2)}{\frac{c^2 - v^2}{c^2}} - x'^2$$

$x'^2 + y'^2 + z'^2 = c^2t'^2$ → širenje svjetlosti invarijantno na Lorentzove transformacije

32. Skicirajte dva inercijalna referentna sustava koji se jedan u odnosu na drugi gibaju brzinom $\vec{v}$ i česticu koja se giba brzinom $\vec{u}$ mjerenom u jednom od sustava. Izvedite Lorentzove transformacije za komponente brzine čestice.

RELATIVISTIČKO ZBRAJANJE BRZINA



$$u_x = \frac{dx}{dt} = \frac{d}{dt}\left(\gamma(x' + vt')\right) = \frac{dt'}{dt} \cdot \frac{d}{dt'}\left(\gamma(x' + vt')\right) \qquad \frac{dx'}{dt'} = u_x'$$

$$\frac{dt}{dt'} = \gamma\left(1 + \frac{vu_x'}{c^2}\right)$$

$$u_x = \frac{1}{\gamma\left(1 + \frac{vu_x'}{c^2}\right)} \cdot \gamma(u_x' + v) \qquad u_x = \frac{u_x' + v}{1 + \frac{vu_x'}{c^2}}$$

$$u_y = \frac{dy}{dt} = \frac{dy'}{dt} = \frac{dt'}{dt} \cdot \frac{dy'}{dt'} = \frac{1}{\gamma\left(1 + \frac{vu_x'}{c^2}\right)} \cdot u_y'$$

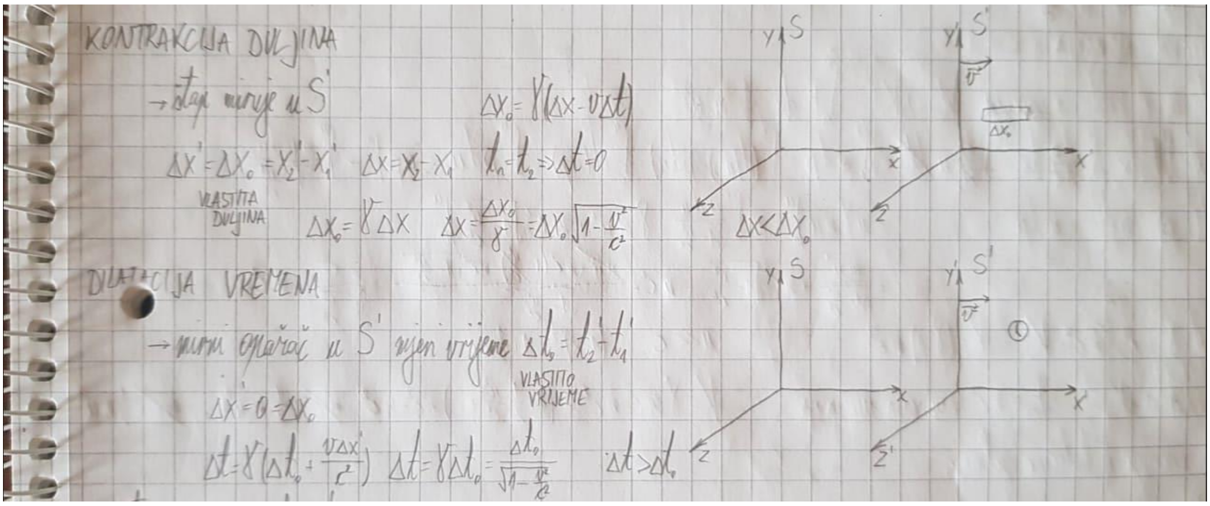
$$u_y = \frac{u_y'\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{vu_x'}{c^2}} \qquad u_z = \frac{u_z'\sqrt{1 - \frac{v^2}{c^2}}}{1 + \frac{vu_x'}{c^2}}$$

$$u_x' = \frac{u_x - v}{1 - \frac{vu_x}{c^2}} \qquad u_y' = \frac{u_y\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{vu_x}{c^2}} \qquad u_z' = \frac{u_z\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{vu_x}{c^2}}$$

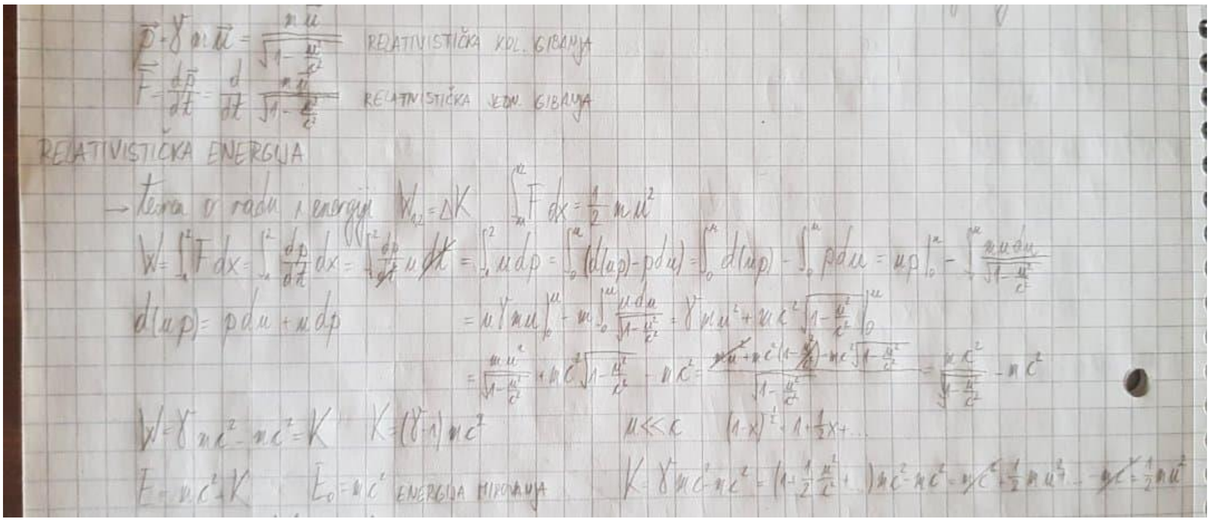
za $v \ll c$

$$u_x = u_x' + v \qquad u_y = u_y' \qquad u_z = u_z'$$

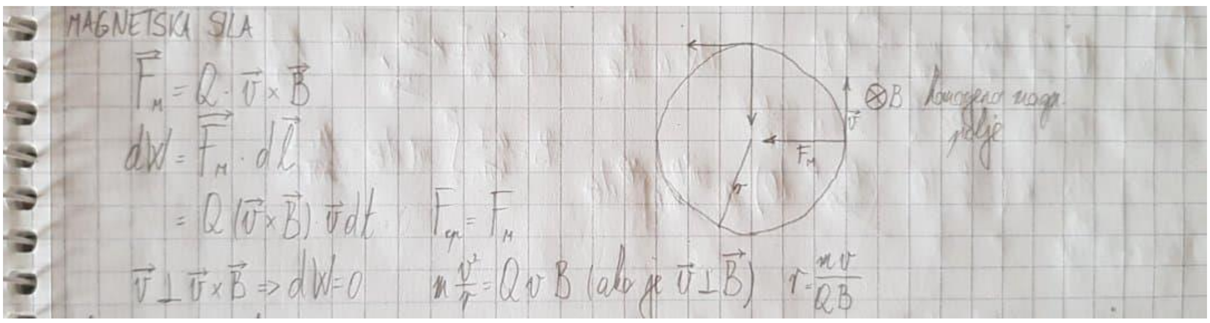
33. Uvedite pojam vlastitog vremena i vlastite duljine. Pomoću uvedenih pojmova i Lorentzovih transformacija izvedite izraze za kontrakciju duljine i dilatacije vremena.



34. Napišite izraz za relativističku količinu gibanja i relativističku energiju. Primijenite teorem o radu i kinetičkoj energiji i izvedite izraz za relativističku kinetičku energiju.

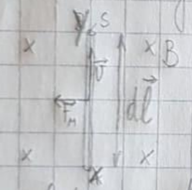


35. Pokažite da se nabijena čestica u homogenom magnetskom polju može gibati po kružnici, odredite polumjer kružnice (za zadano: m, q, v i B).



36. Izvedite izraz za silu na element vodiča kojim teče struja I, a nalazi se u magnetskom polju $\vec{B}$.

sila na vodič u magnetskom polju



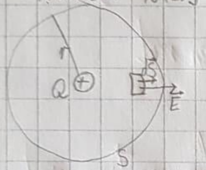
$d\vec{F} = (Q\vec{v}\times\vec{B})\,nS\,d\vec{l}$ — n – BROJ NABOJA U JEDINICI VOLUMENA

$I = QvnS \qquad d\vec{F} = I\,d\vec{l}\times\vec{B} \qquad \vec{F} = I\int_X^Y d\vec{l}\times\vec{B}$

→ gleda se najkraća udaljenost između dviju krajnjih točaka (ne ovisi o duljini)

37. Pomoću Gaussovog zakona izvedite: polje točkastog naboja, polje unutar i izvan jednoliko nabijene kugle, polje jednoliko nabijene ravne tanke žice, polje jednoliko nabijene plohe.

→ TOČKASTI NABOJ Q



$$\oint_S \vec{E}\,d\vec{S} = \oint_S E\,dS\cos 0 = \oint_S E\,dS = E\oint_S dS = E\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

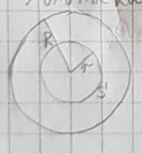
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

→ IZVAN KUGLE

$$\oint_S \vec{E}\,d\vec{S} = \oint_S E\,dS = E\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

→ UNUTAR KUGLE



$$\oint_{S'} \vec{E}\,d\vec{S} = \oint_{S'} E\,dS = E\cdot 4\pi r^2 = \frac{Q_{un}}{\varepsilon_0} = \frac{1}{\varepsilon_0}Q\frac{r^3}{R^3} \qquad \frac{Q_{un}}{Q} = \frac{r^3}{R^3}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r$$

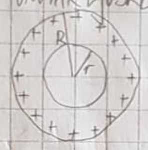
$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^3}r$$

→ IZVAN LJUSKE

$$\oint_S \vec{E}\,d\vec{S} = E\cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

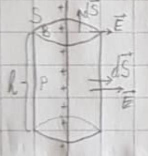
→ UNUTAR LJUSKE



$$\oint_S \vec{E}\,d\vec{S} = E\cdot 4\pi r^2 = 0$$

$$E = 0$$

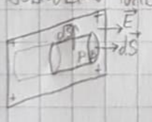
→ JEDNOLIKO NABIJENA RAVNA TANKA ŽICA



$$\oint_S \vec{E}\cdot d\vec{S} = \underbrace{\int_B \vec{E}\cdot d\vec{S}}_{0\ \vec{E}\perp d\vec{S}} + \int_P \vec{E}\cdot d\vec{S} = \frac{\lambda h}{\varepsilon_0} \qquad \lambda = \frac{Q}{h}$$ LINIJSKA GUSTOĆA NABOJA

$$\int_P E\,dS = \frac{\lambda h}{\varepsilon_0} \qquad E\int_P dS = \frac{\lambda h}{\varepsilon_0} \qquad E\cdot\cancel{h}\cdot 2r\pi = \frac{\lambda \cancel{h}}{\varepsilon_0} \qquad E = \frac{1}{2\pi\varepsilon_0}\frac{\lambda}{r}$$

→ JEDNOLIKO NABIJENA PLOHA

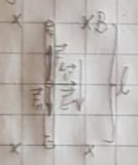


$$\oint_S \vec{E}\,d\vec{S} = \int_B \vec{E}\,d\vec{S} + \underbrace{\int_P \vec{E}\,d\vec{S}}_{0\ \vec{E}\perp d\vec{S}} = \frac{Q_{un}}{\varepsilon_0} = \frac{\sigma S}{\varepsilon_0} \qquad \sigma = \frac{Q}{S}$$ POVRŠINSKA GUSTOĆA NABOJA

$$2\int E\,dS = 2E\int dS = 2E\cancel{S} = \frac{\sigma\cancel{S}}{\varepsilon_0} \qquad E = \frac{\sigma}{2\varepsilon_0}$$

38. Izvedite izraz za elektromotornu silu pri gibanju vodiča u magnetskom polju.

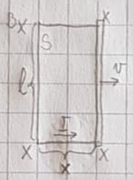
GIBANJE VODIČA U MAGNETSKOM POLJU



$\vec{F}_e = q\vec{E}$ $\quad qE = qvB$

$\vec{F}_m = q\vec{v}\times\vec{B}$ $\quad E = vB$

$\vec{F}_e = \vec{F}_m$ $\quad E = \frac{U}{l}$ $\quad U = Blv$



$|\mathcal{E}_i| = \frac{d\Phi_B}{dt}$ $\quad \Phi_B = BS = Blx$

$= \frac{d}{dt}(Blx) = Bl\frac{dx}{dt}$

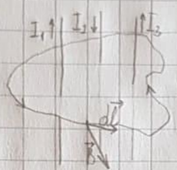
$|\mathcal{E}_i| = Blv$ $\quad$ općenito $\quad \mathcal{E}_i = \vec{l}\cdot(\vec{v}\times\vec{B})$

39. Koristeći Ampère-Maxwellov zakon izračunajte magnetsko polje beskonačnog ravnog tankog vodiča, a zatim učinite isto primjenom Biot-Savartovog zakona.

AMPEREOV ZAKON

$\oint_l \vec{B}\cdot d\vec{l} = \mu_0 I$

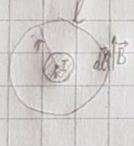
općenito $\quad \oint_l \vec{B}\cdot d\vec{l} = \mu_0 \sum_i I_i$



MAGNETSKO POLJE DUGOG RAVNOG VODIČA

→ izvan vodiča

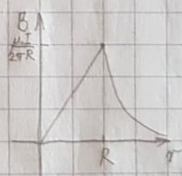
$r > R$



$\oint_l \vec{B}\cdot d\vec{l} = \mu_0 I$ $\quad B\,2r\pi = \mu_0 I$

$\oint_l B\,dl = \mu_0 I$ $\quad B = \frac{\mu_0 I}{2\pi r}$

$B\oint_l dl = \mu_0 I$



→ unutar vodiča

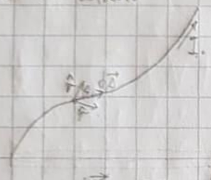
$r < R$

$\frac{I}{R^2\pi} = \frac{I_{UN}}{r^2\pi}$ $\quad I_{UN} = \frac{r^2}{R^2} I$

$\oint_l \vec{B}\cdot d\vec{l} = \mu_0 I_{UN}$ $\quad B = \frac{\mu_0 I}{2\pi R^2} r$

$B\cdot 2\pi \cancel{r} = \mu_0 \frac{r^{\cancel{2}}}{R^2} I$

BIOT-SAVARTOV ZAKON

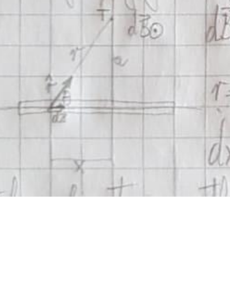


$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{s}\times\hat{r}}{r^2}$ $\quad \vec{B} = \frac{\mu_0}{4\pi}\int\frac{d\vec{s}\times\hat{r}}{r^2}$

$\mu_0 = 4\pi\cdot 10^{-7}\,\frac{Tm}{A}$

→ iznos $d\vec{B}$ ovisi o kutu između $d\vec{s}$ i $\hat{r}$ $(\sin\theta)$

magnetsko polje dugog ravnog vodiča



$d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{d\vec{x}\times\vec{r}}{r^2} = \frac{\mu_0 I}{4\pi}\frac{dx\cdot\sin\theta}{r^2}\vec{k}$ $\quad dx = a\frac{d\theta}{\sin^2\theta}$ $\quad d\vec{B} = \frac{\mu_0 I}{4\pi}\frac{a\frac{d\theta}{\sin^2\theta}\cdot\sin\theta}{\frac{a^2}{\sin^2\theta}} = \frac{\mu_0 I}{4\pi}\frac{\sin\theta\,d\theta}{a}$

$r = \frac{a}{\sin\theta}$ $\quad tg\theta = \frac{a}{-x}$ $\quad x = -a\frac{\cos\theta}{\sin\theta}$

$dx = -a\left(\frac{-\sin\theta}{\sin\theta}d\theta - \frac{\cos\theta\cos\theta}{\sin^2\theta}d\theta\right) = a\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}d\theta$

$B = \frac{\mu_0 I}{4\pi a}\int_0^\pi \sin\theta\,d\theta = \frac{\mu_0 I}{2\pi a}$

40. Krenuvši od Maxwellovih jednadžbi u vakuumu izvedite valnu jednadžbu za $\vec{E}$ ili $\vec{B}$.

$\nabla \cdot \vec{E} = 0$

$\nabla \cdot \vec{B} = 0$

a) $\nabla \times \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

b) $\nabla \times \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

a) $\nabla \times \nabla \times \vec{E} = \nabla(\underbrace{\nabla \cdot \vec{E}}_{0}) - \nabla^2 \vec{E}$

$\nabla^2 \vec{E} = \nabla \times \left(\frac{\partial \vec{B}}{\partial t}\right) = \frac{\partial}{\partial t}(\nabla \times \vec{B})$

$= \frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$

$\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$

$\nabla^2 \vec{E} - \frac{1}{v^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$

VALNA JEDNADŽBA ZA $\vec{E}$

$v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c$ (u vakuumu) $\quad v = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{n} \Rightarrow n = \sqrt{\mu_r \varepsilon_r}$

b) $\nabla \times \nabla \times \vec{B} = \nabla(\nabla \cdot \vec{B}) - \nabla^2 \vec{B}$

$\nabla^2 \vec{B} = -\nabla \times \left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) = -\mu_0 \varepsilon_0 \frac{\partial}{\partial t}(\nabla \times \vec{E})$

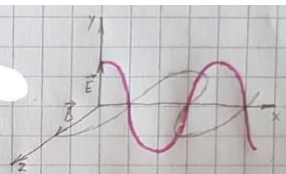
$= \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\left(\frac{\partial \vec{B}}{\partial t}\right) = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$

$\nabla^2 \vec{B} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0$

$\nabla^2 \vec{B} - \frac{1}{v^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0 \quad v = c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

VALNA JEDNADŽBA ZA $\vec{B}$

41. Napiši izraz za vektore $\vec{E}$ i $\vec{B}$ ravnog linearno polariziranog elektromagnetskog vala te pokažite da su oni rješenja odgovarajućih valnih jednadžbi. Skicirajte vektore $\vec{E}$ i $\vec{B}$ i smjer njihovog širenja.



rješenja valnih jednadžbi

$\frac{\partial^2 \vec{E}}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0$

$\frac{\partial^2 \vec{B}}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0 \Rightarrow$

$\vec{E}(x,t) = \vec{j} E_0 \cos(kx - \omega t)$

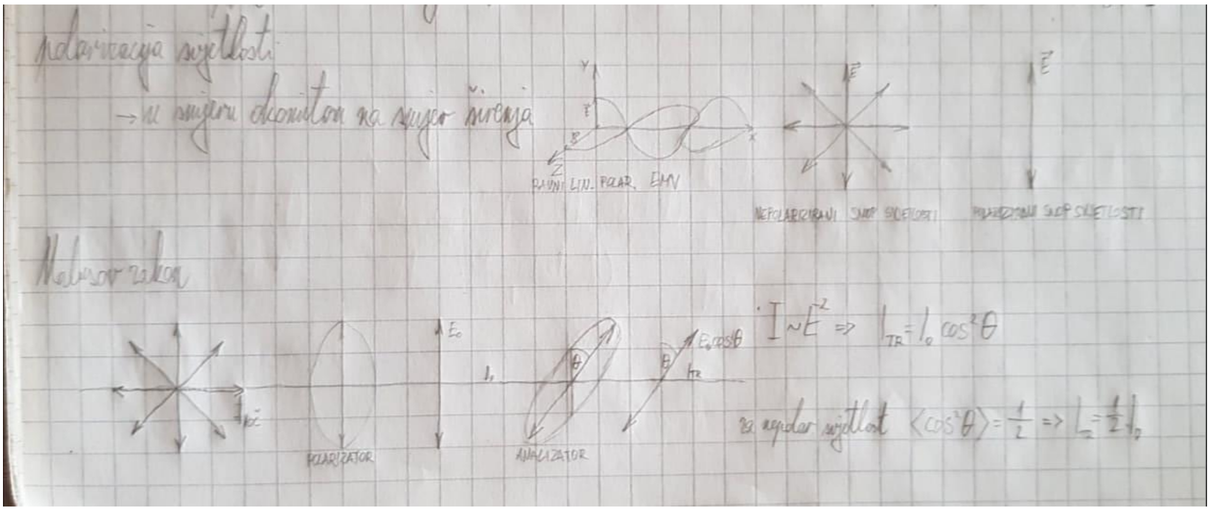
$\vec{B}(x,t) = \vec{k} B_0 \cos(kx - \omega t)$

$-k^2 E_0 \cos(kx - \omega t) - \frac{1}{c^2} \omega^2 E_0 \cos(kx - \omega t) = 0 \Rightarrow \frac{\omega}{k} = c$

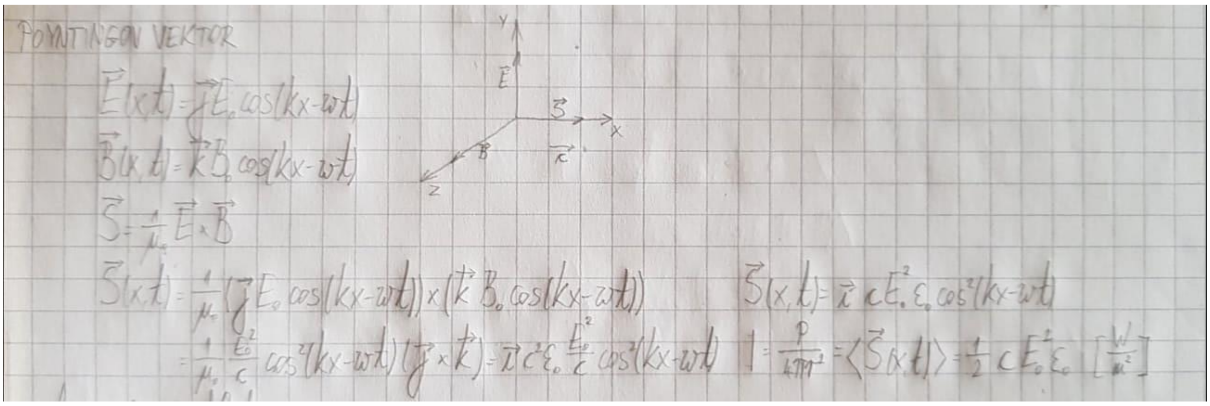
$\Rightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_0 \cos(kx - \omega t) & 0 \end{vmatrix} = \vec{k} \cdot (-k E_0 \sin(kx - \omega t)) = -k E_0 \sin(kx - \omega t) = -\omega B_0 \sin(kx - \omega t)$

$\frac{E_0}{B_0} = \frac{\omega}{k} = c$

42. Opišite polarizaciju elektromagnetskog vala (koje se polje koristite za opis, uloga polarizatora) i izvedite Malusov zakon.

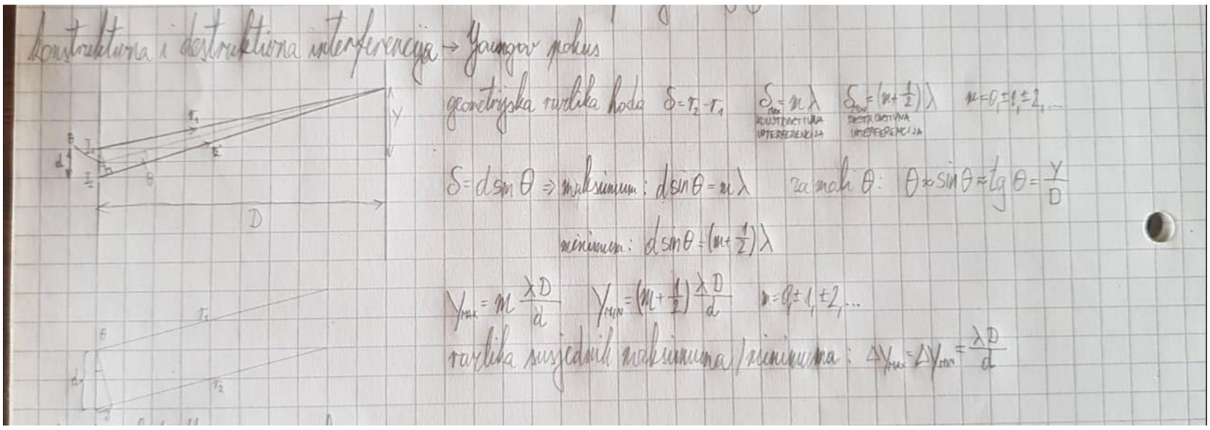


43. Napišite Poyntingov vektor ravnog vala čije je električno polje dano izrazom $\vec{E}(x,t) = E_0\vec{\jmath} \cdot \cos(\omega t - kx)$. Konačni izraz mora sadržavati smjer, iznos i jedinicu.



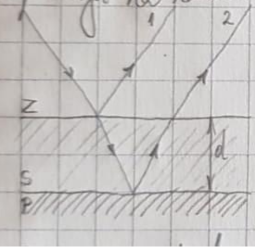
44. Izvedite izraz za položaje maksimuma intenziteta na zastoru u Youngovom pokusu.

45. Izvedite izraz za položaje minimuma intenziteta na zastoru u Youngovom pokusu.



46. Izvedite uvjete maksimuma za interferenciju pri refleksiji na tankom filmu u slučaju kada je $n_{\text{sloj}} > n_{\text{podloga}}$.

interferencija na tankom sloju, $n_s > n_p$



$n_s > n_z$

max: $2d = \frac{\lambda_n}{2}, \frac{3\lambda_n}{2}, \dots = (2m+1)\frac{\lambda_n}{2}$

1: pomak u fazi za π

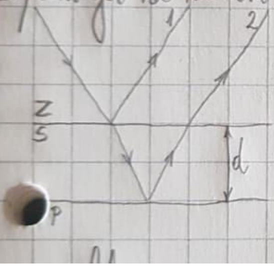
$\lambda_n = \frac{\lambda}{n}$

2: nema pomaka u fazi

MAX: $2nd = (2m+1)\frac{\lambda}{2}$ $m = 0, \pm 1, \pm 2, \dots$

47. Izvedite uvjete maksimuma za interferenciju pri refleksiji na tankom filmu u slučaju kada je $n_{\text{sloj}} < n_{\text{podloga}}$.

interferencija na tankom sloju, $n_s < n_p$



$n_s > n_z$

max: $2d = \lambda_n, 2\lambda_n, \dots = m\lambda_n$

1: pomak u fazi za π

$\lambda_n = \frac{\lambda}{n}$

2: pomak u fazi za π

MAX: $2nd = m\lambda$ $m = 0, \pm 1, \pm 2, \dots$