Signali i sustavi - Rješenja zadataka za vježbu (II. kolokvij)

1.

$$\begin{bmatrix} \dfrac{di_L}{dt} \\ \dfrac{du_C}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{R_1}{L} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_L \\ u_C \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} & \dfrac{R_1}{L} \\ 0 & \dfrac{1}{C} \end{bmatrix} \begin{bmatrix} U \\ I \end{bmatrix}$$

$$\begin{bmatrix} U_{R_1} \\ U_{R_2} \end{bmatrix} = \begin{bmatrix} R_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} i_L \\ u_C \end{bmatrix} + \begin{bmatrix} 0 & -R_1 \\ 0 & -R_2 \end{bmatrix} \begin{bmatrix} U \\ I \end{bmatrix}$$

2.

$$\begin{bmatrix} \dfrac{di_{L_1}}{dt} \\ \dfrac{di_{L_2}}{dt} \\ \dfrac{dU_{C_1}}{dt} \end{bmatrix} = \begin{bmatrix} \dfrac{-R_2}{L_1} & 0 & \dfrac{1}{L_1} \\ 0 & 0 & 0 \\ \dfrac{-1}{C_1} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_{L_1} \\ i_{L_2} \\ U_{C_1} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L_1} & \dfrac{-R_2}{L_1} \\ \dfrac{1}{L_2} & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} U \\ I \end{bmatrix}$$

$$\begin{bmatrix} U_{R_1} \\ U_{R_2} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ R_2 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} i_{L_1} \\ i_{L_2} \\ U_{C_1} \end{bmatrix} + \begin{bmatrix} 0 & R_1 \\ 0 & R_2 \end{bmatrix} \cdot \begin{bmatrix} U \\ I \end{bmatrix}$$

3. Frekvencijske karakteristike:

$$A(\omega) = \frac{1}{\sqrt{\omega^4 - 2\omega^2 + 9}}$$

$$\varphi(\omega) = -arctg\frac{2\omega}{3-\omega^2}$$

Odziv na pobudu:

$$y_u(t) = \left(\frac{23}{12}\cos\sqrt{2}t + \frac{13\sqrt{2}}{12}\sin\sqrt{2}t\right)e^{-t} + \frac{\sqrt{2}}{12}\cos\left(3t + \frac{\pi}{4}\right)$$

4. $L = 2$; $i_L(0) = 1$

Odziv *nepobuđenog* sustava: $i_n(t) = e^{-\frac{t}{2}}$

Odziv *mirnog* (*mrtvog*) sustava: $i_m(t) = e^{-\frac{t}{2}} + \sin\frac{t}{2} - \cos\frac{t}{2}$

5. Odziv *nepobuđenog* sustava: $y_n(t) = e^{3t} + e^{-2t}$

Odziv *mirnog* sustava: $y_m = \frac{1}{5}e^{3t} - \frac{1}{5}e^{-2t} - te^{-2t}$

Ukupni odziv sustava: $y_u(t) = \frac{6}{5}e^{3t} + \frac{4}{5}e^{-2t} - te^{-2t}$

6. *Ukupni* odziv sustava: $y_u(t) = \frac{1}{2}e^{3t} + \frac{5}{3}e^{2t} - \frac{5}{2}te^{2t}$

Odziv *mirnog* sustava: $y_m(t) = \frac{5}{2}e^{3t} - \frac{5}{2}e^{2t} - \frac{5}{2}te^{2t}$

Početna stanja: $y(0) = \frac{13}{6}, \quad y'(0) = \frac{7}{3}$

Kontinuirani sustav je *nestabilan* jer mu se polovi nalaze u desnoj poluravnini.

7. Sustav je opisan diferencijalnom jednadžbom: $y'' - y' - 6y = u(t)$

Odziv *nepobuđenog* sustava:

$$y_{nepob.}(t) = \frac{9}{10}e^{3t} + \frac{11}{10}e^{-2t}$$

Odziv *mirnog* (*mrtvog*) sustava:

$$y_{mirni} = \frac{1}{50}e^{3t} - \frac{1}{50}e^{-2t} - \frac{1}{10}te^{-2t}$$

Ukupni odziv sustava:

$$y_{ukupni}(t) = \frac{23}{25}e^{3t} + \frac{27}{25}e^{-2t} - \frac{1}{10}te^{-2t}$$

8.

$$h[n] = 3\left(\frac{1}{2}\right)^n - 2\delta[n] = \begin{cases} 1 & n = 0 \\ 3\left(\frac{1}{2}\right)^n & n \geq 1 \end{cases}$$

$$y[n] = \left(\frac{1}{2}\right)^n \cdot (1 + 3n)$$

9.

$$u[n] = -\delta[n] + 2^n s[n]$$

10. Partikularno rješenje jednadžbe diferencija:

$$y_p[n] = \frac{1}{2}\sin\frac{n\pi}{6} + \left(\frac{2-\sqrt{3}}{2}\right)\cos\frac{n\pi}{6}$$

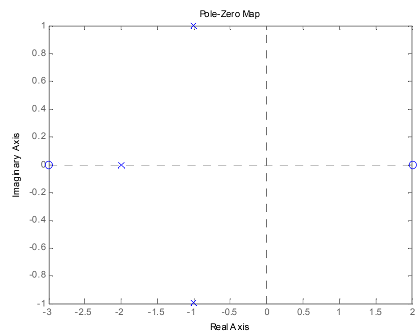
Ukupni odziv:

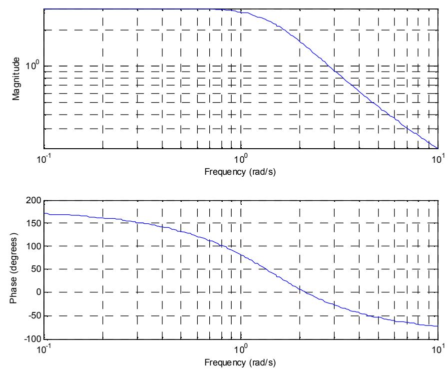
$$y[n] = \left(\frac{-2+\sqrt{3}}{2}\right)\cdot(-1)^n + \frac{1}{2}\sin\frac{n\pi}{6} + \left(\frac{2-\sqrt{3}}{2}\right)\cos\frac{n\pi}{6} \qquad n \geq 0$$

11. $y[n] = C_1 + C_2 n + n^2 + \frac{1}{6}n^3 + \frac{1}{9}4^n$

12. $y[n] = C_1\cos\frac{n\pi}{3} + C_2\sin\frac{n\pi}{3} + \frac{2}{\sqrt{3}}\sum_{i=0}^{n}\left(\frac{1}{(n-i-2)!}\sin\frac{(i+1)\pi}{3}\right)$

13. $H(s) = \frac{2s^2+2s-12}{s^3+4s^2+6s+4} = \frac{2(s+3)(s-2)}{(s+2)(s+1+j)(s+1-j)}$





14. $H(s) = \frac{z-1}{4z^3-z} = \frac{z-1}{z(2z+1)(2z-1)}$