Zadatak

Primjer sustava drugog reda opisanog jednadžbom diferencija:
y(n)+a1*y(n-1)+a2*y(n-2) = b0*u(n)

a1= -1/2, a2=1/4, b0=1

uz početne uvjete y(-1)=2, y(-2)=8

pobuđen sa u(n)=cos(2*pi/3*n + pi/3)

Potrebno je analitički odrediti :

a) odziv nepobuđenog sustava na početna stanja,
b) prisilni odziv sustava,
c) odziv mirnog sustava vlastitim frekvencijama,
d) ukupni odziv mirnog sustava,
e) prirodni odziv sustava,
f) ukupni odziv sustava,

Napisati Matalb program koji racuna i prikazuje sve analiticki izvedene odzive za n=0,1,2,...30

Napisati Matlab program koji odredjuje odziv metodom korak po korak.

Usporediti analitička i numerička rješenja.

Općenit oblik jed. dif. za sustav II reda

$$y(n+2) + a_1 y(n+1) + a_2 y(n) = b_0 u(n+2) + b_1 u(n+1) + b_2 u(n) \quad (1)$$

$n' = n+2$

$$y(n') + a_1 y(n'-1) + a_2 y(n'-2) = b_0 u(n') + b_1 u(n'-1) + b_2 u(n'-2)$$

homogeni dio rješenja $\quad u(n') = \emptyset \quad \forall n'$

$$y(n') + a_1 y(n'-1) + a_2 y(n'-2) = \emptyset$$

Pretpostavimo rj. oblika $\quad c \cdot q^{n'} = y(n')$

$$c \cdot q^{n'} + a_1 \cdot c \cdot q^{n'-1} + a_2 \cdot c \cdot q^{n'-2} = \emptyset$$

$$c \cdot q^{n-2} \underbrace{(q^2 + a_1 q + a_2)}_{\text{karakteristični polinom}} = \emptyset$$

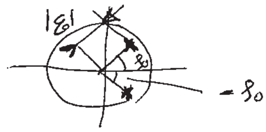
$$\underbrace{q^2 + a_1 q + a_2 = 0}_{\text{karakteristična jednadžba}}$$

Potražimo rješenje:

$$q_{12} = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2}}{2}$$

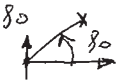
Odaberimo primjer sa konj. kompl. parom $q_1 = q_0$, $q_2 = q_0^*$

$$q_1 = |q_0| \cdot e^{j\vartheta_0} \qquad q_2 = |q_0| \cdot e^{-j\vartheta_0}$$



radijus $|q_0|$

kut ϑ_0



Kako glase koeficijenti ~~karakter.~~ polinoma za ovakav par karakterističnih frekvencija?

Prikažimo polinom kao produkt korijenskih faktora

$$(q - q_1)(q - q_2) = \left(q - |q_0| \cdot e^{j\vartheta_0}\right)\left(q - |q_0| \cdot e^{-j\vartheta_0}\right)$$

$$= q^2 - q|q_0| \cdot \underbrace{\left(e^{j\vartheta_0} + e^{-j\vartheta_0}\right)}_{2\cos\vartheta_0} + |q_0|^2 \cdot \underbrace{e^{j\vartheta_0} \cdot e^{-j\vartheta_0}}_{1}$$

$$= q^2 \underbrace{- 2|q_0| \cdot \cos\vartheta_0}_{a_1} \cdot q + \underbrace{|q_0|^2}_{a_2}$$

Npr. odaberimo $|z_0| = 1/2$, $\vartheta_0 = \frac{\pi}{3} \Rightarrow \cos\vartheta_0 = \frac{1}{2}$

$$a_1 = -2|z_0|\cdot\cos\vartheta_0 = -2\cdot\frac{1}{2}\cdot\frac{1}{2} = -\frac{1}{2}$$

$$a_2 = |z_0|^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Dakle karakteristični polinom (jednadžba) glasi:

$$z^2 - \frac{1}{2}z + \frac{1}{4} = 0$$

Za provjeru nađimo rješenje:

par konjug.-kompl. karakterističnih frekvencija

$$z_{12} = \frac{\frac{1}{2} \pm \sqrt{\frac{1}{4} - 1}}{2} = \frac{\frac{1}{2} \pm \frac{1}{2}j\sqrt{3}}{2} = \underbrace{\frac{1}{2}}_{|z_0|}\underbrace{\left(\frac{1}{2} \pm \frac{\sqrt{3}}{2}j\right)}_{e^{j\pm\pi/3}}$$

$$e^{j\pm\pi/3} = \left(\cos\left(\frac{\pi}{3}\right) \pm j\sin\left(\frac{\pi}{3}\right)\right)$$

$$\frac{1}{2} \pm j\frac{\sqrt{3}}{2} \quad \checkmark \text{ OK}$$

Kada smo odredili karakteristične (ili vlastite) frekvencije sustava z_1, z_2 možemo odrediti odziv nepobuđenog sustava (početna stanja $\neq 0$, ali $u(n) = 0 \ \forall n$)

Pretpostavimo rješenje oblika

$$y_H(n) = C_1\cdot z_1^n + C_2\cdot z_2^n$$

$$y(n) = \underbrace{-a_1 y(n-1) - a_2 y(n-2)}$$

Moramo poznavati početne uvjete

$\left.\begin{matrix} y(n-1) \\ y(n-2) \end{matrix}\right\}$ npr. za $n=\emptyset$

$$y(\emptyset) = -a_1 y(-1) - a_2 y(-2)$$

$$y(1) = -a_1 y(\emptyset) - a_2 y(-1)$$

npr. poznamo $\left.\begin{matrix} y(-1) \\ y(-2) \end{matrix}\right\}$ treba nam $y(\emptyset)$ & $y(1)$

$$y(\emptyset) = \overbrace{-a_1 y(-1) - a_2 y(-2)}$$

$$y(1) = a_1^2 y(-1) + a_1 a_2 y(-2) - a_2 y(-1)$$

$$= (a_1^2 - a_2) y(-1) + a_1 a_2 y(-2)$$

Iz homogenog rješenja

Za homogeno rješenje mogli smo odabrati i bilo koji drugi par n-ova uključujući $n=-1$, $n=-2$
↓
svi daju isto rješenje za c_1/c_2

$$c_1 q_1^n + c_2 q_2^n = /n=\emptyset/ = c_1 + c_2 = y(\emptyset)$$

$$= /n=1/ = c_1 q_1 + c_2 q_2 = y(1)$$

Dobivamo 2 jedn. sa 2 nepoz. (c_1, c_2)

$$c_1 + c_2 = -a_1 y(-1) - a_2 y(-2)$$

$$c_1 q_1 + c_2 q_2 = (a_1^2 - a_2) y(-1) + a_1 a_2 y(-2)$$

$a_1 = -\frac{1}{2}$

$a_2 = \frac{1}{4}$

$$c_1 + c_2 = \frac{1}{2} y(-1) - \frac{1}{4} y(-2)$$

$$c_1 q_1 + c_2 q_2 = \left(\frac{1}{4} - \frac{1}{4}\right) y(-1) - \frac{1}{2} \cdot \frac{1}{4} y(-2) = -\frac{1}{8} y(-2)$$

Npr. odaberimo početne uvjete $y(-1) = 2$
$y(-2) = 8$

$$c_1 + c_2 = \frac{1}{2}\cdot 2 - \frac{1}{4}\cdot 8 = 1 - 2 = -1 \qquad c_1 = -c_2 - 1$$

$$c_1 q_1 + c_2 q_2 = -\frac{1}{8} y(-2) = -1$$

$$q_1(-c_2 - 1) + c_2 q_2 = -1$$

$$c_2(q_2 - q_1) = q_1 - 1$$

$$c_2 = \frac{q_1 - 1}{q_2 - q_1}$$

$$q_1 = \frac{1}{4} + \frac{\sqrt{3}}{4} j$$

$$q_2 = \frac{1}{4} - \frac{\sqrt{3}}{4} j$$

$$q_2 - q_1 = -\frac{\sqrt{3}}{2} j$$

$$c_2 = \frac{-\frac{3}{4} + \frac{\sqrt{3}}{4} j}{-\frac{\sqrt{3}}{2} j} \cdot \frac{\frac{2}{\sqrt{3}} j}{\frac{2}{\sqrt{3}} j} = -\frac{6}{4\sqrt{3}} j - \frac{1}{2}$$

$$= -\frac{1}{2} - \frac{\sqrt{3}}{2} j$$

$$c_1 = -c_2 - 1 = \frac{1}{2} + \frac{\sqrt{3}}{2} j - 1 = -\frac{1}{2} + \frac{\sqrt{3}}{2} j = c_2^*$$

Provjera

$$c_1 q_1 + c_2 q_2 = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} j\right)\left(\frac{1}{4} + \frac{\sqrt{3}}{4} j\right) + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} j\right)\left(\frac{1}{4} - \frac{\sqrt{3}}{4} j\right)$$

$$= \left(-\frac{1}{8} - \frac{3}{8} + j\cancel{\left(\frac{\sqrt{3}}{8} - \frac{\sqrt{3}}{8}\right)}\right) + \left(-\frac{1}{8} - \frac{3}{8} + j\cancel{\left(-\frac{\sqrt{3}}{8} + \frac{\sqrt{3}}{8}\right)}\right) = -\frac{1}{2} - \frac{1}{2} = -1$$

Dakle: rješenje glasi:

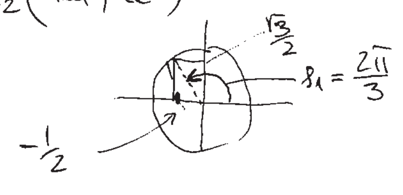
$$y_H(n) = c_1 q_1^n + c_2 q_2^n = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2} j\right)\left(\frac{1}{4} + \frac{\sqrt{3}}{4} j\right)^n + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2} j\right)\left(\frac{1}{4} - \frac{\sqrt{3}}{4} j\right)^n$$

ili u polarnom obliku:

$$|c_1| = |c_2| = \frac{1}{4} + \frac{3}{4} = 1 \qquad c_1 = |c_1| \cdot e^{j\vartheta_1} \qquad c_2 = |c_2| \cdot e^{j\vartheta_2}$$

$$c_1 = 1 \cdot e^{j\frac{2\pi}{3}} \qquad c_2 = c_1^* = 1 \cdot e^{-j\frac{2\pi}{3}}$$

$$\vartheta_1 = \operatorname{atan}_2(\mathrm{Im}, \mathrm{Re})$$



$$q_1 = |q_0| \cdot e^{j\vartheta_0} \qquad q_2 = q_1^* = |q_0| \cdot e^{-j\vartheta_0}$$

$$|q_0| = \frac{1}{2}, \quad \vartheta_0 = \frac{\pi}{3}$$

Dakle u polarnom obliku:

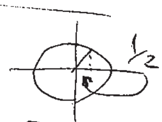
$$y_H(n) = e^{j\frac{2\pi}{3}}\left(\frac{1}{2}e^{j\frac{\pi}{3}}\right)^n + e^{-j\frac{2\pi}{3}}\left(\frac{1}{2}\cdot e^{-j\frac{\pi}{3}}\right)^n$$

$$= \left(\frac{1}{2}\right)^n \left[e^{j\left(n\cdot\frac{\pi}{3}+\frac{2\pi}{3}\right)} + e^{-j\left(n\cdot\frac{\pi}{3}+\frac{2\pi}{3}\right)}\right]$$

$$= \left(\frac{1}{2}\right)^n \cdot 2\cos\left(n\frac{\pi}{3}+\frac{2\pi}{3}\right)$$

$$= 2^{(1-n)}\cdot\cos\left(n\frac{\pi}{3}+\frac{2\pi}{3}\right)$$

Provjerimo početne uvjete:



$$y_H(-1) = 2^{(1-(-1))}\cdot\cos\left(-\frac{\pi}{3}+\frac{2\pi}{3}\right) = 4\cdot\cos\left(\frac{\pi}{3}\right) = 4\cdot\frac{1}{2} = 2 \quad \checkmark$$

$$y_H(-2) = 2^{(1+2)}\cdot\cos\left(-\frac{2\pi}{3}+\frac{2\pi}{3}\right) = 8\cdot\cos(0) = 8 \quad \checkmark$$

Metoda korak po korak ... nađimo $y(0)$, $y(1)$

$$y(0) = -a_1 y(-1) - a_2 y(-2) = \Big/ \begin{matrix} a_1 = -\frac{1}{2} \\ a_2 = \frac{1}{4} \end{matrix} \Big/ = \frac{1}{2}y(-1) - \frac{1}{4}y(-2)$$

$$= \frac{1}{2}\cdot 2 - \frac{1}{4}\cdot 8 = -1$$

$$y(1) = -a_1 y(0) - a_2 y(-1) = \Big/ -\|- \Big/ = \frac{1}{2}\cdot(-1) - \frac{1}{4}\cdot 2$$

$$= -\frac{1}{2} - \frac{1}{2} = -1$$

Provjerimo opće rješenje za $n=0, 1$

$$y_H(0) = 2^1\cdot\underbrace{\cos\left(\frac{2\pi}{3}\right)}_{-\frac{1}{2}} = -1 \quad \checkmark \text{ OK}$$

$$y_H(1) = 2^0\cdot\cos\left(\frac{\pi}{3}+\frac{2\pi}{3}\right) = 1\cdot(-1) = -1 \quad \checkmark \text{ OK}$$

Pobudimo ovaj sustav drugog reda sa pobudom harmonijskog oblika

$$u[n] = A\cos\left(\frac{2\pi}{3}n + \frac{\pi}{3}\right) \qquad \text{upr. neka } A=1$$

$$\omega_0 = \frac{2\pi}{3} \qquad \Theta = \frac{\pi}{3}$$

općenito:

$$u[n] = A\cdot\cos(\omega_0 n + \Theta)$$

možemo je prikazati i u exp. obliku (suma dvije kompl. exp.)

$$u[n] = \frac{A}{2}\cdot e^{j\Theta}\cdot e^{+j\omega n} + \frac{A}{2}\cdot e^{-j\Theta} e^{-j\omega n}, \text{ jer}$$

$$= \frac{A}{\cancel{2}}\Big(\underbrace{e^{j(\Theta+\omega n)} + e^{-j(\Theta+\omega n)}}_{\cancel{2}\cos(\omega n+\Theta)}\Big) = A\cdot\cos(\omega n+\Theta)$$

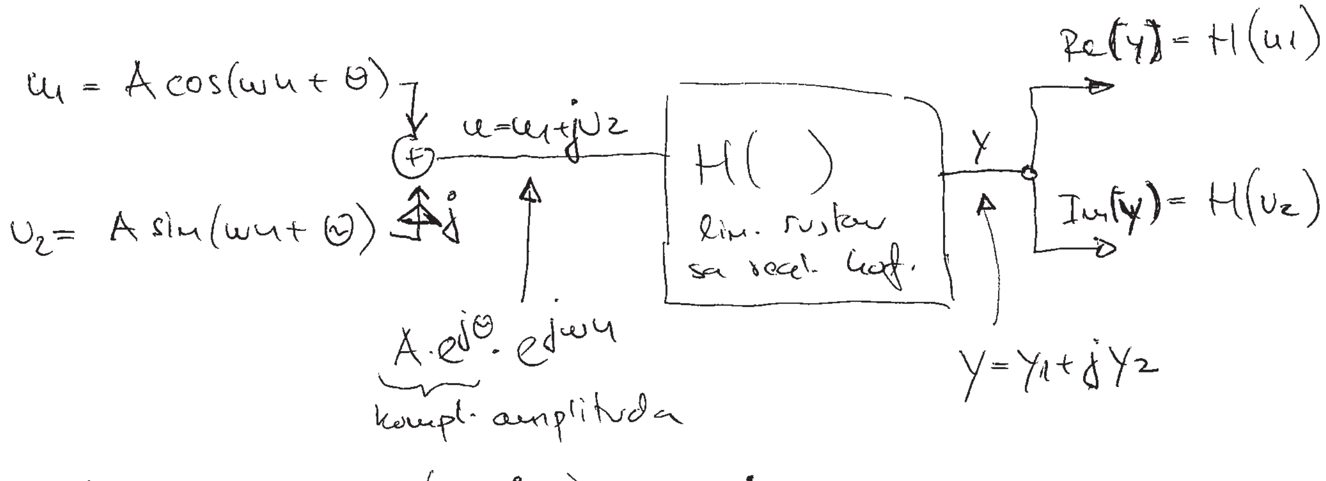
ili

$$u[n] = \mathrm{Re}\left(A\cdot e^{j\Theta}\cdot e^{j\omega n}\right) =$$

$$= \mathrm{Re}\left(A\cdot e^{j(\omega n+\Theta)}\right) =$$

$$= \mathrm{Re}\left(A\cdot\cos(\omega n+\Theta) + jA\sin(\omega n+\Theta)\right)$$

$$= A\cdot\cos(\omega n+\Theta)$$



Odziv na pobudu $H(u_1 + ju_2) = y_1 + jy_2$

Zbog linearnosti ako je u_1 realan i u_2 realan i H realan tada je $\mathrm{Re}(y) = y_1$ a $\mathrm{Im}(y) = y_2$

Pri određivanju partikularnog rješenja metodom neodređenog koef. uz pobudu oblika:

$$u[n] = \underbrace{A}_{}\cdot e^{j\omega_0 n}$$

A... kompl. amplituda

moramo pretpostaviti rješenje istog <u>oblika</u> ali druge kompl. amplitude K

$$y_p[n] = K\cdot e^{j\omega_0 n}$$

K... obuhvaća i realnu amplitudu $|K|$ i fazni kut (početni kut)

Uvrstimo pretpostavljeno rj. u jedn. dif.

$$y(n) + a_1 y(n-1) + a_2 y(n-2) = b_0 u(n)$$

/upr. $b_1 = b_2 = \emptyset$/

/$a_1 = -\frac{1}{2}$ $a_2 = \frac{1}{4}$/

/neka je $b_0 = 1$/

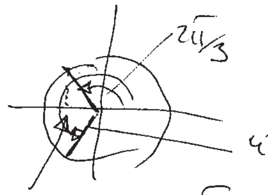
$$y(n) - \frac{1}{2} y(n-1) + \frac{1}{4} y(n-2) = u(n)$$

$$K\cdot e^{j\omega_0 n} - \frac{1}{2} K\cdot e^{j\omega_0(n-1)} + \frac{1}{4} K e^{j\omega_0(n-2)} = A\cdot e^{j\omega_0 n}$$

$$K\cdot \cancel{e^{j\omega_0 n}}\left(1 - \frac{1}{2}\cdot e^{-j\omega_0} + \frac{1}{4}\cdot e^{-2j\omega_0}\right) = A\cdot \cancel{e^{j\omega_0 n}}$$

$$K = \frac{A}{\left(1 - \frac{1}{2}e^{-j\omega_0} + \frac{1}{4}\cdot e^{-2j\omega_0}\right)}$$

Drugi primjer... za $\omega_0 = 2\pi/3$, $A = 1\cdot e^{j\pi/3}$



$$K = \frac{A}{\left(1-\frac{1}{2}\cdot e^{-j\omega_0}+\frac{1}{4}e^{-2j\omega_0}\right)}$$

$$e^{-j\frac{4\pi}{3}} = \cos\left(\frac{4\pi}{3}\right) - j\sin\left(\frac{4\pi}{3}\right) = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$e^{-j\frac{2\pi}{3}} = \cos\left(\frac{2\pi}{3}\right) - j\sin\left(\frac{2\pi}{3}\right) = -\frac{1}{2} - j\cdot\frac{\sqrt{3}}{2}$$

$$K = \frac{e^{j\pi/3}}{1-\frac{1}{2}\left(-\frac{1}{2}-j\frac{\sqrt{3}}{2}\right)+\frac{1}{4}\left(-\frac{1}{2}+j\frac{\sqrt{3}}{2}\right)}$$

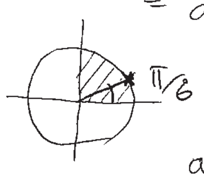
$$K = \frac{e^{j\pi/3}}{1+\frac{1}{4}+j\frac{\sqrt{3}}{4}-\frac{1}{8}+j\frac{\sqrt{3}}{8}} = \frac{e^{j\pi/3}}{\underbrace{\frac{9}{8}+j\frac{3\sqrt{3}}{8}}_{\frac{3\sqrt{3}}{4}\cdot e^{j\pi/6}}} \quad \text{ok}$$

$$|\,|^2 = \frac{9^2}{8^2} + \frac{3^2\cdot 3}{8^2} = \frac{81+27}{64} = \frac{108}{64} = \frac{54}{32} = \frac{27}{16}$$

$$|\,| = \frac{3\sqrt{3}}{4}$$

$$\arg = \mathrm{atan}_2(\mathrm{im},\mathrm{re}) = \mathrm{atan}_2(\underbrace{3\sqrt{3}}_{>0},\underbrace{9}_{>0})$$

I kvadrant



$$\mathrm{atan}\left(\frac{3\sqrt{3}}{9}\right) = \mathrm{atan}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

$$K = \frac{4}{3\sqrt{3}}e^{j\pi/3}\cdot e^{-j\pi/6} = \frac{4}{3\sqrt{3}}\cdot e^{j\frac{\pi}{6}} = \frac{4\sqrt{3}}{9}\cdot e^{j\frac{\pi}{6}}$$

Dakle uz pretpostavljeno rješenje oblika

$$y_p[n] = K\cdot e^{j\omega_0 n}$$

dobivamo

$$y_p[n] = \frac{4\sqrt{3}}{9}\, e^{j\frac{\pi}{6}}\cdot e^{j\frac{2\pi}{3}n}$$

modul — početna faza (za $n=0$) — frekvencija odziva

na pobudu oblika:

$$u[n] = 1\cdot e^{j\pi/3}\cdot e^{j\frac{2\pi}{3}n}$$

frekvencija pobudnog signala

moraju biti jednake zbog linearnosti sustava!

$$y_p[n] = \mathrm{Re}[y_p[n]] + j\,\mathrm{Im}(y_p[n]) = y_{pr}[n] + j\,y_{pi}[n]$$

su odziv na

$$u[n] = \mathrm{Re}[u[n]] + j\,\mathrm{Im}(u[n]) = u_r[n] + j\,u_i[n]$$

pri tome $y_{pr}(n)$ je odziv na $u_r[n]$ a $y_{pi}[n]$ je odziv na $u_i[n]$

$$u_r[n] = \cos\left(\frac{2\pi}{3}n + \frac{\pi}{3}\right)$$

$$y_{pr}[n] = \mathrm{Re}\left(\frac{4\sqrt{3}}{9} \cdot e^{j\pi/6} \cdot e^{j\frac{2\pi}{3}n}\right)$$

$$= \frac{4\sqrt{3}}{9} \cdot \mathrm{Re}\left(e^{j\left(\frac{2\pi}{3}n + \frac{\pi}{6}\right)}\right)$$

$$= \frac{4\sqrt{3}}{9} \cdot \cos\left(\frac{2\pi}{3}n + \pi/6\right)$$

Tako nas je interesirao odziv na $u_r[n]$, primjenom kompleksne eksp. automatski smo dobili i odziv na $u_i[n] = \sin\left(\frac{2\pi}{3}n + \pi/3\right)$ kao imaginarni dio odziva $\mathrm{Im}(y_p[n])$

$$y_{pi}[n] = \mathrm{Im}\left(\frac{4\sqrt{3}}{9} \cdot e^{j\pi/6} \cdot e^{j\frac{2\pi}{3}n}\right)$$

$$= \frac{4\sqrt{3}}{9} \cdot \sin\left(\frac{2\pi}{3}n + \pi/6\right)$$

Alternativan način rješavanja kod kojeg izbjegavamo rad sa kompleksnim brojevima je da pretpostavimo realno rješenje u obliku:

$$y_{pr}[n] = k_1 \cdot \cos(\omega_0 n) + k_2 \cdot \sin(\omega_0 n)$$

$\Rightarrow$ u konstantama k_1, k_2 sadržana je informacija o realnoj amplitudi i početnoj fazi izlazne kosinusoide:

$$y_{pr}[n] = \underbrace{A_y}_{\text{realna ampl.}} \cdot \cos(\omega_0 n + \underbrace{\Theta_0}_{\text{početna faza}})$$

$$y_{pr}[n] = A_y\left(\cos(\omega_0 n)\cos(\Theta_0) - \sin(\omega_0 n) \cdot \sin(\Theta_0)\right)$$

$$= \underbrace{(A_y \cdot \cos(\Theta_0))}_{K_1} \cdot \cos(\omega_0 n) + \underbrace{(-A_y \cdot \sin(\Theta_0))}_{K_2} \cdot \sin(\omega_0 n)$$

$$k_1 = A_y \cdot \cos(\Theta_0)$$

$$k_2 = -A_y \cdot \sin(\Theta_0)$$

kako odrediti A_y i Θ_0 iz k_1 & k_2 ?

$$
\begin{aligned}
k_1^2 + k_2^2 &= A_y^2 \cdot \cos^2(\Theta_0) + A_y^2 \cdot \sin^2(\Theta_0) \\
&= A_y^2 (\cos^2 \Theta_0 + \sin^2 \Theta_0) \\
&= A_y^2
\end{aligned}
$$

$$\Rightarrow A_y = \sqrt{k_1^2 + k_2^2} \qquad A_y \in R$$

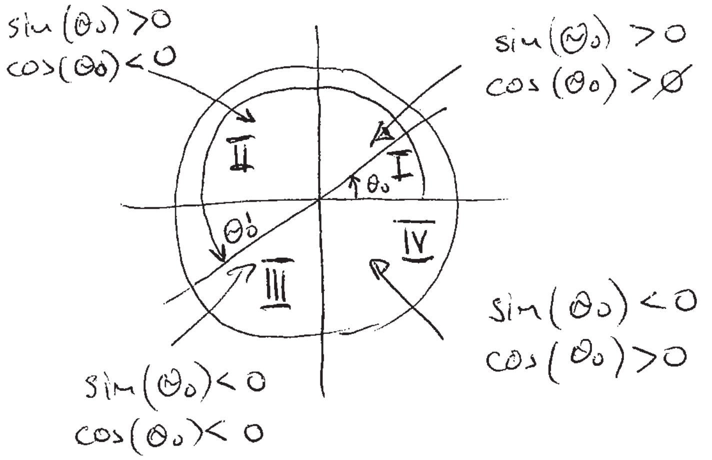
kada znamo A_y treba odrediti Θ_0 :

$$\cos(\Theta_0) = \frac{k_1}{A_y} \quad , \quad \sin(\Theta_0) = -\frac{k_2}{A_y}$$

$$tg(\Theta_0) = \frac{\sin(\Theta_0)}{\cos(\Theta_0)} \Rightarrow \Theta_0 = atan\left(\frac{\sin(\Theta_0)}{\cos(\Theta_0)}\right)$$

Moramo koristiti četvero-kvadrantnu atan funkciju.

Obični atan daje samo dvo-kvadrantno rješenje



Za obični atan kutevi Θ_0 u I kvadrantu i Θ_0' u III kvadrantu su ekvivalentni

$$\Theta_0' = \Theta_0 + \pi \qquad \Theta_0 \in (0, \tfrac{\pi}{2})$$

$$tg(\Theta_0') = \frac{\sin(\Theta_0 + \pi)}{\cos(\Theta_0 + \pi)} = \frac{\sin(\Theta_0) \cdot \underbrace{\cos(\pi)}_{-1} + \cos(\Theta_0) \cdot \underbrace{\sin(\pi)}_{0}}{\cos(\Theta_0) \cdot \underbrace{\cos(\pi)}_{-1} - \sin(\Theta_0) \cdot \underbrace{\sin(\pi)}_{0}}$$

$$= \frac{-\sin(\Theta_0)}{-\cos(\Theta_0)} = tg(\Theta_0)$$

Jednaka "ekvivalencija" vrijedi za kuteve u II i IV kvadrantu za $\Theta_0 \in (0, -\frac{\pi}{2})$ i $\Theta' = \Theta_0 + \pi$

kako su predznaci sin & cos suprotni imamo istu vrijednost tg funkcije

Kako odrediti četvero-kvadrantno rješenje ako znamo vrijednost sinusa i kosinusa? (11)

1. Izračunamo obični $atan\left(\frac{sin}{cos}\right) = \Phi_{I,\underline{IV}}$

2. Uz pretpostavku da je kodomena običnog atan-a $[-\pi/2, \pi/2]$ tada automatski dobivamo traženo rješenje u slučaju da je Θ_0 u I ili $\underline{IV}$ kvadrantu.

3. Međutim, ako je $\cos < 0$ tada je sigurno rješenje u $\underline{II}$ ili $\underline{III}$ kvadrantu ... dakle pravo rješenje je tada

$$\Phi_{\pm} \begin{cases} \Phi_{I,\underline{IV}} & \text{ako} \quad \cos \geq 0 \\ \Phi_{I,\underline{IV}} + \pi & \text{inače} \end{cases}$$

4. Ako želimo da konačna kodomena četvero-kvadratnog atan-a bude $[0, 2\pi)$ tada je još potrebno rješenja iz $\underline{IV}$ kvadranta pretvoriti u pozitivna dodavanjem 2π

$$\Phi = \begin{cases} \Phi & \text{ako} \quad \Phi \geq 0 \\ 2\pi + \Phi & \text{ako} \quad \Phi < 0 \end{cases}$$

Uvrstimo sada pretpostavljeno rješenje u jed. dif.

$$y_{pr}[n] = k_1 \cdot \cos(\omega_0 n) + k_2 \sin(\omega_0 n) \qquad \omega_0 = \frac{2\pi}{3}$$

$$u_r[n] = \cos\left(\frac{2\pi}{3}n + \frac{\pi}{3}\right)$$

$$y[n] + a_1 y[n-1] + a_2 y[n-2] = u_r[n]$$

$$= \cos\frac{2\pi}{3}n \cos\frac{\pi}{3} - \sin\frac{2\pi}{3}n \sin\frac{\pi}{3}$$

$$= \frac{1}{2}\cos\left(\frac{2\pi}{3}n\right) - \frac{\sqrt{3}}{2}\sin\left(\frac{2\pi}{3}n\right)$$

$$k_1 \cdot \cos(\omega_0 n) + k_2 \cdot \sin(\omega_0 n) +$$

$$a_1 k_1 \cos(\omega_0(n-1)) + a_1 k_2 \sin(\omega_0(n-1)) +$$

$$a_2 k_1 \cos(\omega_0(n-2)) + a_2 k_2 \sin(\omega_0(n-2)) = \frac{1}{2}\cos\left(\frac{2\pi}{3}n\right) - \frac{\sqrt{3}}{2}\sin\left(\frac{2\pi}{3}n\right)$$

$$\left.\begin{aligned} \cos(\omega_0(n-k)) &= \cos(\omega_0 n)\cos(k\omega_0) + \sin(\omega_0 n)\sin(k\omega_0) \\ \sin(\omega_0(n-k)) &= \sin(\omega_0 n)\cos(k\omega_0) - \cos(\omega_0 n)\sin(k\omega_0) \end{aligned}\right\} \text{ za } \omega_0 = \frac{2\pi}{3}$$

$k=1$ $\quad \cos(\omega_0(n-1)) = \cos(\omega_0 n) \cdot \left(-\frac{1}{2}\right) + \sin(\omega_0 n) \cdot \frac{\sqrt{3}}{2}$

$k=2$ $\quad \cos(\omega_0(n-2)) = \cos(\omega_0 n) \cdot \left(-\frac{1}{2}\right) + \sin(\omega_0 n)\left(-\frac{\sqrt{3}}{2}\right)$

$k=1$ $\quad \sin(\omega_0(n-1)) = \sin(\omega_0 n) \cdot \left(-\frac{1}{2}\right) - \cos(\omega_0 n)\frac{\sqrt{3}}{2}$

$k=2$ $\quad \sin(\omega_0(n-2)) = \sin(\omega_0 n) \cdot \left(-\frac{1}{2}\right) - \cos(\omega_0 n) \cdot \left(-\frac{\sqrt{3}}{2}\right)$

Grupirajmo sve članove sa lijeve strane koji sadrže titranje istog oblika, ali različitih amplituda (... dakle posebno $\cos(\omega_0 n)$, posebno $\sin \omega_0 n$)

$$\left(k_1 + a_1 k_1\left(-\frac{1}{2}\right) + a_1 k_2\left(-\frac{\sqrt{3}}{2}\right) + a_2 k_1 \left(-\frac{1}{2}\right) + a_2 k_2\left(\frac{\sqrt{3}}{2}\right)\right)\cos(\omega_0 n) +$$

$$\left(k_2 + a_1 k_1\left(\frac{\sqrt{3}}{2}\right) + a_1 k_2\left(-\frac{1}{2}\right) + a_2 k_1 \cdot \left(-\frac{\sqrt{3}}{2}\right) + a_2 k_2\left(-\frac{1}{2}\right)\right)\sin(\omega_0 n) =$$

$$\left/ \begin{aligned} a_1 &= -1/2 \\ a_2 &= 1/4 \end{aligned} \right/ = \left(k_1 + k_1 \cdot \frac{1}{4} + k_2 \cdot \frac{\sqrt{3}}{4} + k_1\left(-\frac{1}{8}\right) + k_2\frac{\sqrt{3}}{8}\right) \cdot \cos(\omega_0 n) +$$

$$\left(k_2 - k_1 \cdot \frac{\sqrt{3}}{4} + k_2 \cdot \frac{1}{4} + k_1\left(-\frac{\sqrt{3}}{8}\right) + k_2\left(-\frac{1}{8}\right)\right) \cdot \sin(\omega_0 n) =$$

$$= \underbrace{\frac{1}{2}\cos\left(\overset{\omega_0}{\frac{2\pi}{3}}n\right) - \frac{\sqrt{3}}{2}\sin\left(\overset{\omega_0}{\frac{2\pi}{3}}n\right)}_{\text{desna strana}} \Rightarrow$$

Dobivamo sustav od dvije jednadžbe sa dvije nepoz.

$$K_1\left(1+\frac{1}{4}-\frac{1}{8}\right)+K_2\left(\frac{\sqrt3}{4}+\frac{\sqrt3}{8}\right)=\frac{1}{2} \quad /\cdot 8$$

$$K_1\left(-\frac{\sqrt3}{4}-\frac{\sqrt3}{8}\right)+K_2\left(1+\frac{1}{4}-\frac{1}{8}\right)=-\frac{\sqrt3}{2} \quad /\cdot 8$$

$$9K_1+3\sqrt3 K_2 = 4 \Rightarrow K_1=\frac{4}{9}-\frac{3\sqrt3}{9}\cdot K_2$$

$$-3\sqrt3 K_1 + 9K_2 = -4\sqrt3$$

$$-\frac{12\sqrt3}{9}+\frac{27}{9}K_2+9K_2=-4\sqrt3$$

$$12K_2=-4\sqrt3+\frac{4}{3}\sqrt3=-\frac{8}{3}\sqrt3$$

$$\boxed{K_2=-\frac{2}{9}\sqrt3}$$

$$K_1=\frac{4}{9}+\cancel{\frac{3\sqrt3}{9}}\cdot\frac{8}{3}\sqrt3\cdot\frac{1}{12}$$

$$\boxed{K_1=\frac{4}{9}+\frac{2}{9}=\frac{6}{9}=\frac{2}{3}}$$

Dakle odziv glasi:

$$y_{pr}(n)=\frac{2}{3}\cos(\omega_0 n)-\frac{2\sqrt3}{9}\cdot\sin(\omega_0 n)$$

$$=A_y\cdot\cos(\omega_0 n+\Theta_0)$$

$$A_y=\sqrt{\left(\frac{2}{3}\right)^2+\left(\frac{2\sqrt3}{9}\right)^2}=\sqrt{\frac{4}{9}+\frac{4\cdot3}{81}}=\sqrt{\frac{4\cdot9+4\cdot3}{81}}=\sqrt{\frac{4\cdot4\cdot3}{81}}=\frac{4}{9}\sqrt3$$

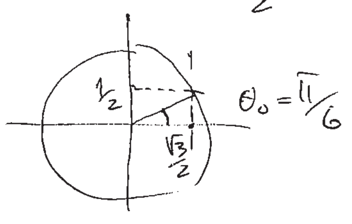
Odredimo početni kut odziva Θ_0

$$K_1=\frac{2}{3}=A_y\cdot\cos(\Theta_0) \Rightarrow \cos(\Theta_0)=\frac{K_1}{A_y}=\frac{\frac{2}{3}}{\frac{2}{3}\cancel{\frac{4}{9}}\sqrt3}=\frac{3}{2\sqrt3}=\frac{\sqrt3}{2}$$

$$K_2=-\frac{2\sqrt3}{9}=-A_y\cdot\sin(\Theta_0) \Rightarrow \sin(\Theta_0)=\frac{K_2}{-A_y}=\frac{-\frac{2\sqrt3}{\cancel{9}}}{-\frac{4\sqrt3}{\cancel{9}}}=\frac{1}{2}$$

$$\left.\begin{aligned}\cos\Theta_0&=\frac{\sqrt3}{2}>0\\ \sin\Theta_0&=\frac{1}{2}>0\end{aligned}\right\}$$ prvi kvadrant

$$\Theta_0=\operatorname{atan}\left(\frac{\sin\Theta_0}{\cos(\Theta_0)}\right)=\operatorname{atan}\left(\frac{\frac{1}{2}}{\frac{\sqrt3}{2}}\right)=$$



$$=\operatorname{atan}\left(\frac{1}{\sqrt3}\right)=\frac{\pi}{6}$$

Dakle rješenje glasi:

$$\boxed{\begin{aligned} y_{p0}[n] &= A_{y0} \cdot \cos(\omega_0 n + \theta_0) \\ &= \frac{4}{9}\sqrt{3} \cdot \cos\left(\frac{2\pi}{3} n + \frac{\pi}{6}\right) \end{aligned}}$$

Dobili smo isto rj. kao i metodom sa kompleksnom eksponencijalom, ali postupkom koji je nešto duži i složeniji jer zahtjeva raspisivanje sinusa i kosinusa, kao i rješavanje sustava jed. sa dvije nepoznanice K_1, K_2, te konačno pretvorbu rješenja $K_1 \cos + K_2 \sin$ u $A \cdot \cos(\omega_0 n + \theta_0)$

Sada uz poznato partikularno rj. možemo odrediti odziv mirnog sustava (onog čija su početna stanja $= \emptyset$)

Taj odziv $y_m[n]$ mora sadržavati i titranje vlastitim frekvencijama q_1, q_2 jer sustav „nije spreman" na pobudu koja se pojavila u trenutku $n = \emptyset$, jer su njegova stanja jednaka nuli. Da je pobuda djelovala od $n = -\infty$ na dalje, tada bi stanja stabilnog sustava u trenutku $n = \emptyset$ bila sukladna pobudi, jer bi prijelazna pojava davno već utrnula (nestala) te bi se sustav nastavio se odazivati frekvencijama pobude.

Međutim kako pobuda djeluje tek od $n = \emptyset$ na dalje.. jer $u[n] = (\quad) \cdot \mu(n)$ dolazi do prijelazne pojave, pa pišemo

$$y_m[n] = c_3 q_1^n + c_4 q_2^n + \underbrace{y_{p}[n]}_{\text{ovo smo upravo odredili}}$$

Zbog činjenice da je sustav miran znamo da je

$y_m[n] = \emptyset$ za $n < \emptyset$, a zbog kauzalnosti pobudnog signala znamo da je $u[n] = \emptyset$ također za $n < \emptyset$.

Partikularno rješenje $y_p[n]$ vrijedi samo za $n \geq 0$, pa stoga u svrhu određivanja nepoznatih koef. c_3 i c_4 moramo odabrati bilo koja dva uzorka odziva $y_m[n]$ za $n \geq \emptyset$, npr. $n = \emptyset$ i $n = 1$. ili npr. $n = 3$ i $n = 7$.

Međutim $y_m[n]$ nije poznat za te n-ove, te ju trebamo odrediti metodom korak po korak.

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Znamo da je $y_m(-2) = y_m(-1) = 0$

$y_{pr}[n] = \frac{4\sqrt{3}}{9}\cos\left(\frac{2\pi}{3}n + \frac{\pi}{6}\right)$

$u_r[n] = \cos\left(\frac{2\pi}{3}n + \frac{\pi}{3}\right)$

Iz jed. dif.

$u_r(0) = \cos\frac{\pi}{3} = \frac{1}{2}$

$u_r(1) = \cos(\pi) = -1$

$$y_m(0) = -a_1 y_m(-1) - a_2 y(-2) + u_r(0)$$
$$= \frac{1}{2} y_m(-1) - \frac{1}{4} y_m(-2) + \frac{1}{2}$$
$$= \frac{1}{2}\cdot 0 - \frac{1}{4}\cdot 0 + \frac{1}{2} = \frac{1}{2}$$

$$y_m(1) = \frac{1}{2} y_m(0) - \frac{1}{4} y_m(-1) + u_r(1)$$
$$= \frac{1}{2}\cdot\frac{1}{2} - \frac{1}{4}\cdot 0 + (-1) = -\frac{3}{4}$$

Iz pretpostavljenog općeg rješenja za $y_m(n)$ za $n = 0$ i 1 slijedi:

za $n=0$ $\quad y_m(0) = C_3 \cdot z_1^0 + C_4 \cdot z_2^0 + y_{pr}(0)$

za $n=1$ $\quad y_m(1) = C_3 \cdot z_1^1 + C_4 z_2^1 + y_{pr}(1)$

$y_{pr}[0] = \frac{\cancel{4}^{2}\cancel{\sqrt{3}}}{\cancel{9}_3}\cdot\frac{\cancel{\sqrt{3}}}{\cancel{2}} = \frac{2}{3}$

$y_{pr}[1] = \frac{4\sqrt{3}}{9}\cdot\cos\left(\frac{5\pi}{6}\right) = -\frac{2}{3}$

$$\left.\begin{aligned} y_m(0) &= \frac{1}{2} = C_3 + C_4 + \frac{2}{3} \\ y_m(1) &= -\frac{3}{4} = C_3 z_1 + C_4 z_2 - \frac{2}{3}\end{aligned}\right\}$$

$C_3 + C_4 = -\frac{1}{6}$

$C_3 z_1 + C_4 z_2 = -\frac{1}{12}$

Dvije jedn. sa dvije nep.

$C_3 = -C_4 - \frac{1}{6}$

$-C_4 z_1 - \frac{1}{6} z_1 + C_4 z_2 = -\frac{1}{12}$

$C_4(z_2 - z_1) = -\frac{1}{12} + \frac{1}{6} z_1$

$$\left.\begin{aligned} z_1 &= \frac{1}{2} + j\frac{\sqrt{3}}{2} \\ z_2 &= \frac{1}{2} - j\frac{\sqrt{3}}{2}\end{aligned}\right\}$$

$z_2 - z_1 = -j\sqrt{3}/2$

$-1 + 2z_1 = \cancel{-1} + \cancel{1} + j\frac{\sqrt{3}}{2}$
$= j\frac{\sqrt{3}}{2} - \frac{1}{2}$

$$C_4 = \frac{1}{12}\cdot\frac{(-1+2z_1)}{z_2 - z_1} =$$
$$= \frac{1}{12}\cdot\frac{-\frac{1}{2} + j\frac{\sqrt{3}}{2}}{\cancel{-j\frac{\sqrt{3}}{2}}}\cdot\frac{\frac{2}{\sqrt{3}}j}{\cancel{\frac{2}{\sqrt{3}}j}} =$$
$$= \frac{1}{12}\left(-\frac{1}{\sqrt{3}}j - 1\right)$$
$$= -\frac{1}{12}\cdot\frac{2}{\sqrt{3}}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}j\right)$$
$$= -\frac{\sqrt{3}}{18}\cdot e^{j\frac{\pi}{6}}$$

$$C_3 = C_4^* = -\frac{\sqrt{3}}{18}\cdot e^{-j\frac{\pi}{6}}$$

Provjera: (17)

$$q_1 = \frac{1}{2} \cdot e^{j\pi/3} \qquad c_4 = -\frac{\sqrt{3}}{18} \cdot e^{j\pi/6}$$

$$q_2 = \frac{1}{2} \cdot e^{-j\pi/3} \qquad c_3 = -\frac{\sqrt{3}}{18} \cdot e^{-j\pi/6}$$

$$q_1 \cdot c_3 + q_2 \cdot c_4 = \frac{1}{2} \cdot \left(-\frac{\sqrt{3}}{18}\right) \cdot \left(e^{j\frac{\pi}{3}} \cdot e^{-j\pi/6} + e^{-j\frac{\pi}{3}} \cdot e^{j\pi/6}\right)$$

$$= \frac{1}{2}\left(\frac{-\sqrt{3}}{18}\right) \cdot \left(e^{j\pi/6} + e^{-j\pi/6}\right) = \left(-\frac{\sqrt{3}}{18}\right) \cdot \cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{\cancel{18}_{6}} \cdot \frac{\sqrt{3}}{2} = -\frac{1}{12} \text{ ok}$$

$$c_3 + c_4 = -\frac{\sqrt{3}}{18}\left(e^{j\pi/6} + e^{-j\pi/6}\right) = -\frac{\sqrt{3}}{18} \cdot 2\cos\left(\frac{\pi}{6}\right) = -\frac{\cancel{\sqrt{3}}}{\cancel{18}_{6}} \cdot \cancel{2} \cdot \frac{\cancel{\sqrt{3}}}{\cancel{2}} = -\frac{1}{6} \text{ ok}$$

Dobivamo odziv mirnog sustava vlastitim frekvencijama:

$$y_{m\text{-}vlast}[n] = c_3 \cdot q_1^n + c_4 \cdot q_2^n = -\frac{\sqrt{3}}{18} \cdot |q_0|^n \cdot \left(e^{-j\pi/6} \cdot e^{j\frac{\pi}{3} \cdot n} + e^{j\frac{\pi}{6}} \cdot e^{-j\frac{\pi}{3}n}\right)$$

$$= -\frac{\sqrt{3}}{9} \cdot |q_0|^n \cdot \cos\left(\frac{\pi}{3}n - \frac{\pi}{6}\right) \text{ ok}$$

$$= \frac{\sqrt{3}}{9} \cdot |q_0|^n \cdot \cos\left(\frac{\pi}{3}n + \frac{5\pi}{6}\right)$$

$$= \frac{\sqrt{3}}{9} \cdot 2^{-n} \cdot \cos\left(\frac{\pi}{3}n + \frac{5\pi}{6}\right)$$

Za vježbu odredite početna stanja $y(-1)$ & $y(-2)$ takva da odziv mirnog sustava vlastitim frekvencijama bude jednak odzivu nepobuđenog sustava, ali suprotnog predznaka tj: $y_{m\text{-}vlast}[n] = -y_0[n]$

$$\Rightarrow \quad c_1 = -c_3$$
$$c_2 = -c_4$$

Konačno dobivamo izraz za odziv mirnog sustava

(18)

$$y_m[n] = \underbrace{\frac{\sqrt{3}}{9} 2^{-n} \cdot \cos\left(\frac{\pi}{3} n + \frac{5\pi}{6}\right)}_{} + \underbrace{\frac{4\sqrt{3}}{9} \cdot \cos\left(\frac{2\pi}{3} n + \frac{\pi}{6}\right)}_{}$$

titranje vlastitim frekvencijama zbog neslada početnog stanja sustava (nul-stanja) i stanja koje bi odgovaralo sve-vremenskoj pobudi koja djeluje od $-\infty$

titranje frekvencijama pobude

$\boxed{\Rightarrow \text{prisilni odziv}}$

Odziv nepobuđenog sustava sa početnim stanjima:

$y(-1) = 2$ & $y(-2) = 8$ bio je:

$$y_0[n] = 2^{1-n} \cdot \cos\left(n \cdot \frac{\pi}{3} + \frac{2\pi}{3}\right)$$

ista frekvencija kao i

ali druga početna faza i druga amplituda !!

Označimo ovaj dio odziva sa $y_{m\_vlast}[n]$

$y_{m\_vast}[n]$ i $y_0[n]$ se mogu „udružiti" u jedan signal frekvencije $f_0 = \pi/3$, a neke nove amplitude i početne faze. Ovo udruživanje je najlakše provesti zbrajanjem c-koeficijenata

$\boxed{\text{prirodni odziv}}$

$$y_{m\_vlast}[n] + y_0[n] = (c_1+c_3)\, z_1^n + (c_2+c_4)\, z_2^n = y_{prir}[n]$$

$$\left.\begin{array}{l} c_1 = c_2^* \\ c_3 = c_4^* \end{array}\right\} \Rightarrow (c_1+c_3) = (c_2+c_4)^*$$

od prije sa strane 4 znamo da je

$$c_1 = -\frac{1}{2} + \frac{\sqrt{3}}{2} j$$

a na strani 16 imamo c_3 & c_4

$$c_3 = -\frac{\sqrt{3}}{18} \cdot e^{-j\pi/6} = -\frac{\sqrt{3}}{18}\left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right) = -\frac{1}{12} + j\frac{\sqrt{3}}{36}$$

Dakle $c_1 + c_3 = -\frac{1}{2} + \frac{\sqrt{3}}{2} j - \frac{1}{12} + j\frac{\sqrt{3}}{36} = \frac{-7}{12} + \frac{19}{36} j\sqrt{3}$ OK

$$|c_1 + c_3| = \sqrt{\frac{49}{144} + \underbrace{\frac{19 \cdot 19 \cdot \cancel{3}}{36 \cdot \cancel{36}_{12}}}_{3 \cdot 144}} = \sqrt{\frac{3 \cdot 49 + 19 \cdot 19}{3 \cdot 144}} = \sqrt{\frac{508}{3 \cdot 144}} = \frac{2\sqrt{127}}{12 \cdot \sqrt{3}} = \frac{1}{6}\sqrt{\frac{127}{3}}$$

$$\measuredangle(c_1 + c_3) = \text{atan}_2(\text{Im}, \text{Re}) = \text{atan}_2\left(\frac{19\sqrt{3}}{36}, -\frac{7}{12}\right)$$

$$= 0.6808\,\pi$$

$$|c_2 + c_4| = |c_1 + c_3| = \frac{1}{6} \cdot \sqrt{\frac{127}{3}}$$

$$\measuredangle(c_2 + c_4) = -\measuredangle(c_1 + c_3)$$

$$y_{prir}(n) = |c_1 + c_3| \cdot e^{j\measuredangle(c_1+c_3)} \cdot |z_0|^n \cdot e^{j\vartheta_0 n} + |c_2 + c_4| \cdot e^{j\measuredangle(c_2+c_4)} |z_0|^n \cdot e^{-j\vartheta_0 n}$$

$$= \frac{1}{\cancel{6}_3}\sqrt{\frac{127}{3}} \cdot 2^{-n} \cdot \cancel{2} \cos(\vartheta_0 n + \measuredangle(c_1 + c_3))$$

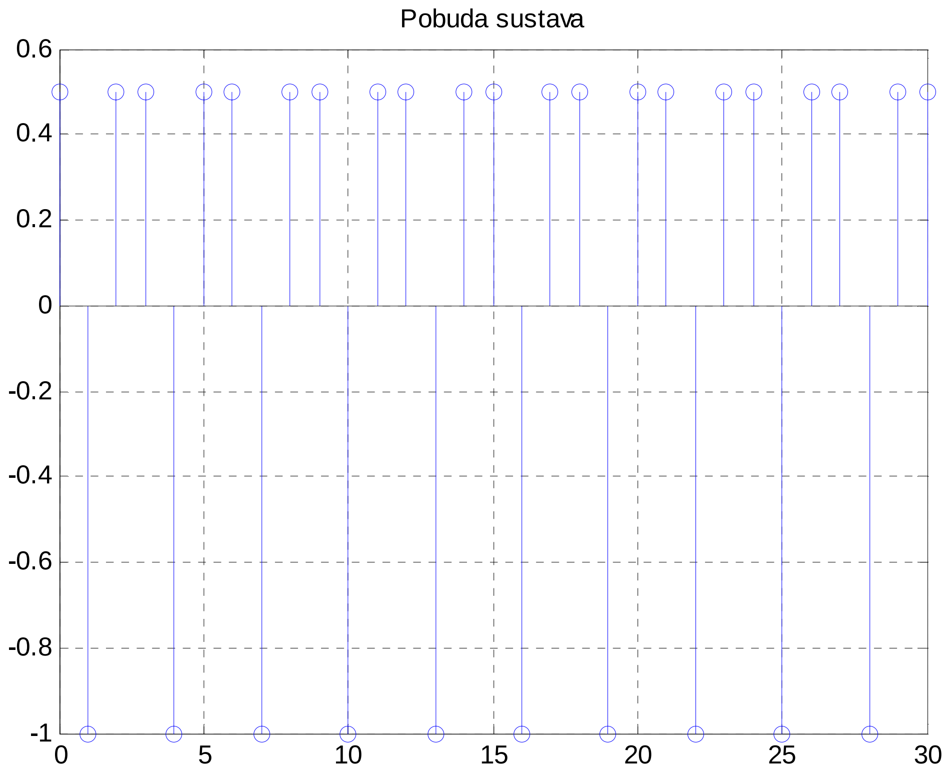
$$= \underbrace{\frac{1}{3}\sqrt{\frac{127}{3}} \cdot 2^{-n} \cos\left(\vartheta_0 n + \text{atan}_2\left(\frac{19\sqrt{3}}{36}, \frac{-7}{12}\right)\right)}_{}$$ OK

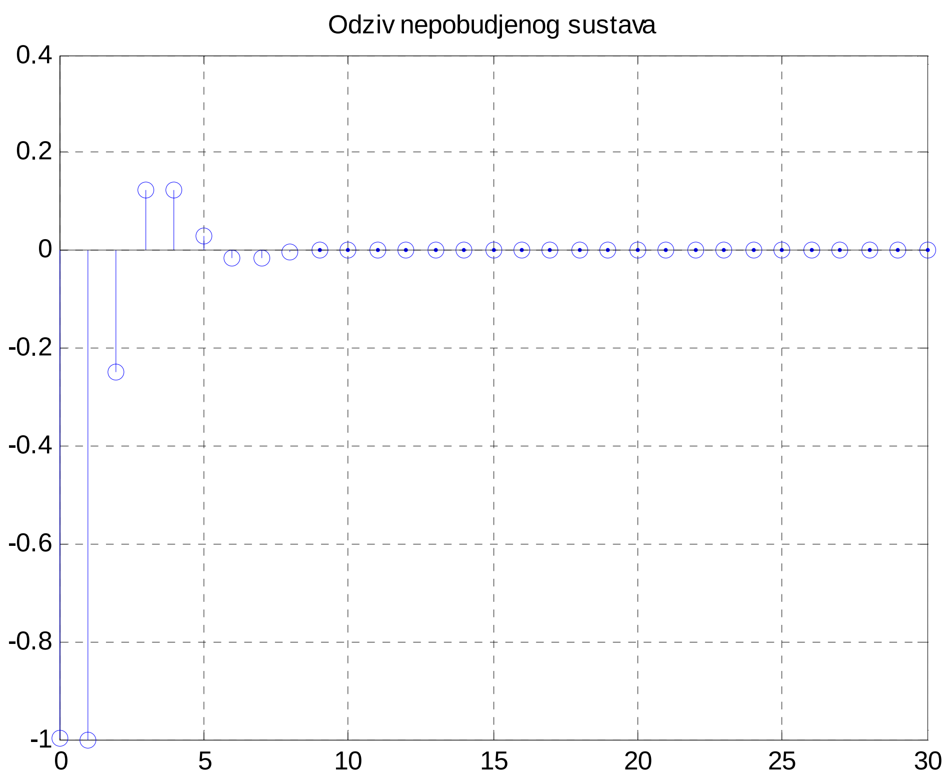
ukupno titranje vlastitim frekvencijama
ili prirodni odziv sustava

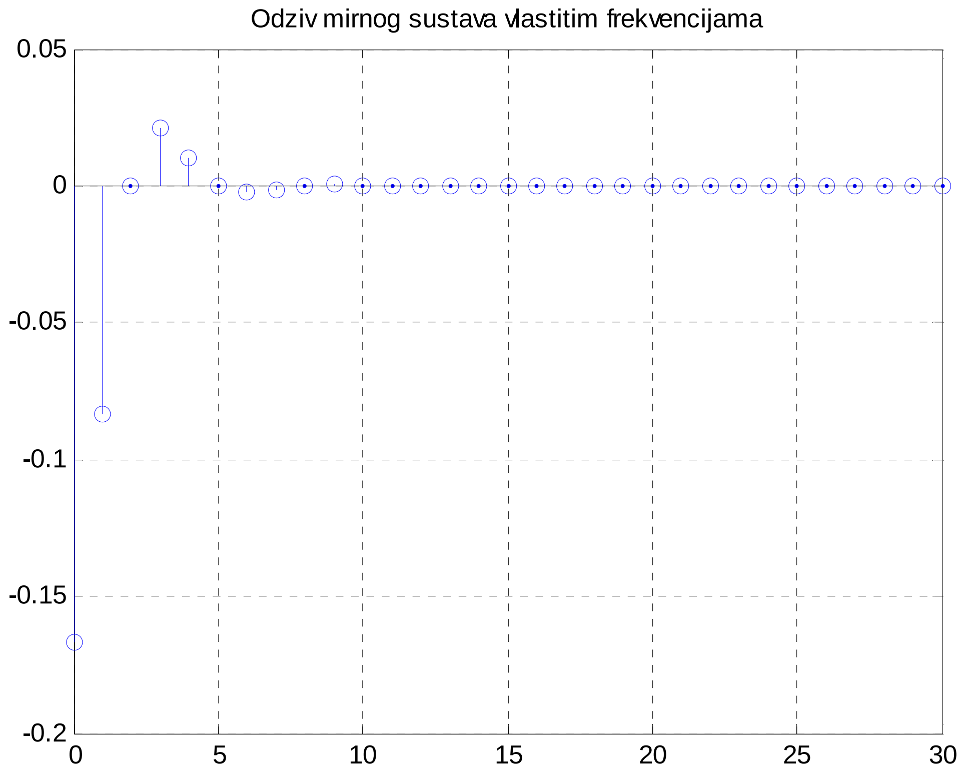
Traženi Matlab program

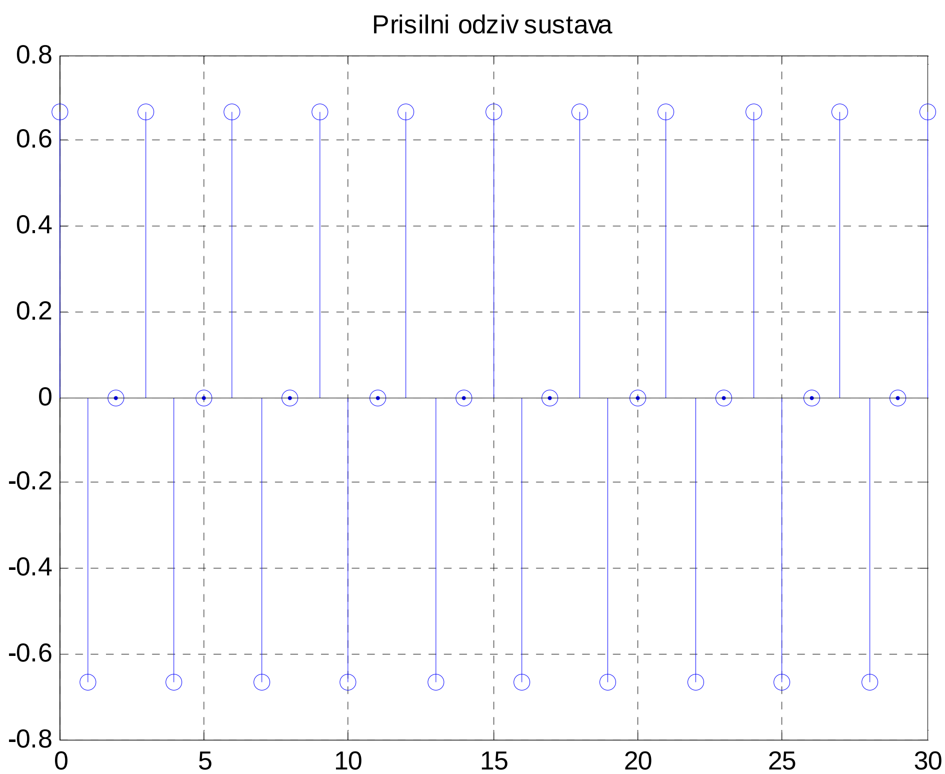
```
clear;
n_max=30;
% vremenska os
n=[0:n_max];

% Odziv mirnog sustava vlastitim frekvencijama
ym_vlast=sqrt(3)/9*(2.^(-n)).*cos(pi/3*n+5*pi/6);

% Odziv nepobudjenog sustava vlastitim frekvencijama
y0=(2.^(1-n)).*cos(pi/3*n+2*pi/3);

% Pocetna faza ukupnog titranja vlastitim frekvencijama
an=atan2(19*sqrt(3)/36,-7/12);

% Ukupno titranje vlastitim frekvencijama
y_prir=sqrt(127/3)/6*(2.^(1-n)).*cos(pi/3*n+an);

% pokazi da je y_prir jednak sumi ym_vlast i y0
disp(max(abs(y_prir-(y0+ym_vlast))))

% partikularno rjesenje ... titranje frekvencijama pobude
y_par=4*sqrt(3)/9*cos(2*pi/3*n+pi/6);

% totalni odziv (suma titranja vlastitim frekvencijama i
% prisilnog odziva)
ytot=y_prir+y_par;

% Ukupni odziv mirnog sustava
y_mir=ym_vlast+y_par;

% Pobuda sustava
u=1*cos(2*pi/3*n+pi/3);

% Inicijaliziraj vektor odziva za metodu korak po korak
y=0*u;

% Odredi odziv metodom korak po korak
y_nm_2=8;     % pocetna vrijednost y(n-2) za n=0 ... y(-2)
y_nm_1=2;     % pocetna vrijednost y(n-1) za n=0 ... y(-1)

% koeficijenti jedn. dif. uz y(n-1) i y(n-2)
a1=-1/2;
a2=1/4;

% koef. jedn. dif. uz u(n)
b0=1;

for nn=0:n_max,
   % Zbog Matlaba koji ne pozna indekse polja koji su manji od
   % jedan ... vremenski indeks n=0 pretvaramo u indeks polja 1
   nM=nn+1;
   % Jednadzba diferencije ...
   ykk(nM)= -a1*y_nm_1 -a2*y_nm_2 +b0*u(nM);

   % za novi prolaz petlje (n+1)
   y_nm_2=y_nm_1;     % y((n+1)-2) = y(n-1)
   y_nm_1=ykk(nM);    % y((n+1)-1) = y(n)

end;

% Usporedi analiticko rjesenje i ono dobiveno metodom korak po korak
disp(max(abs(ykk-ytot)))
```

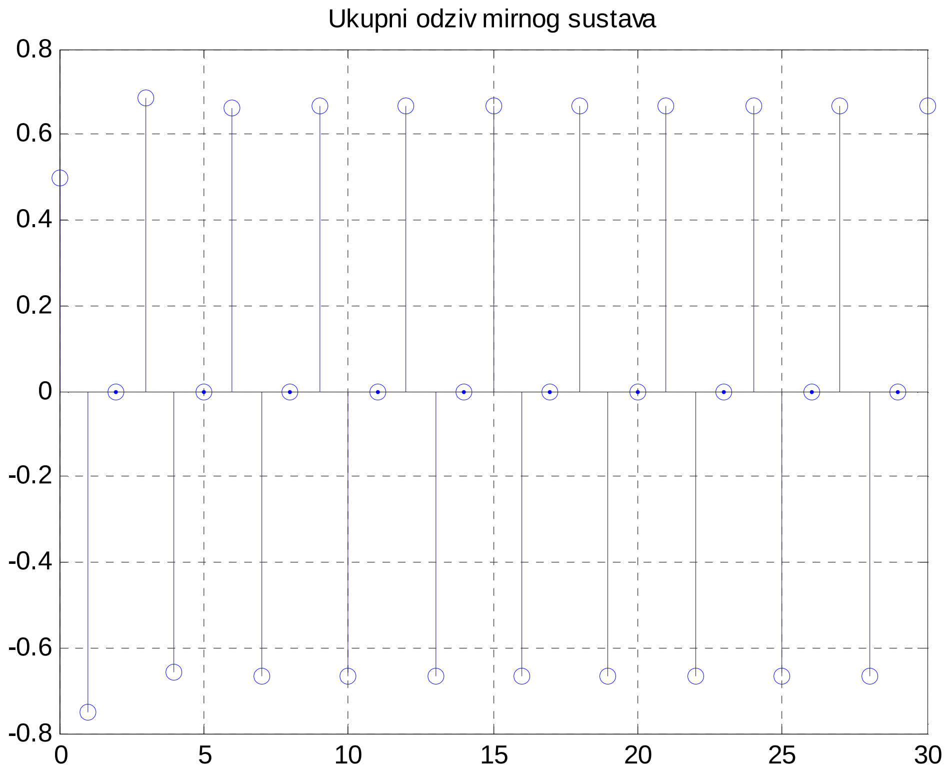
```
figure(1);
stem(n,u)
title('Pobuda sustava');
grid;

figure(2);
stem(n,y0)
title('Odziv nepobudjenog sustava');
grid;

figure(3);
stem(n,ym_vlast)
title('Odziv mirnog sustava vlastitim frekvencijama');
grid;

figure(4);
stem(n,y_par)
title('Prisilni odziv sustava');
grid;

figure(5);
stem(n,y_prir)
title('Prirodni odziv sustava');
grid;

figure(6);
stem(n,y_mir)
title('Ukupni odziv mirnog sustava');
grid;

figure(7);
stem(n,ytot)
title('Ukupni odziv sustava - analiticko rjesenje');
grid;

figure(8);
stem(n,ykk)
title('Ukupni odziv sustava - korak po korak');
grid;
```

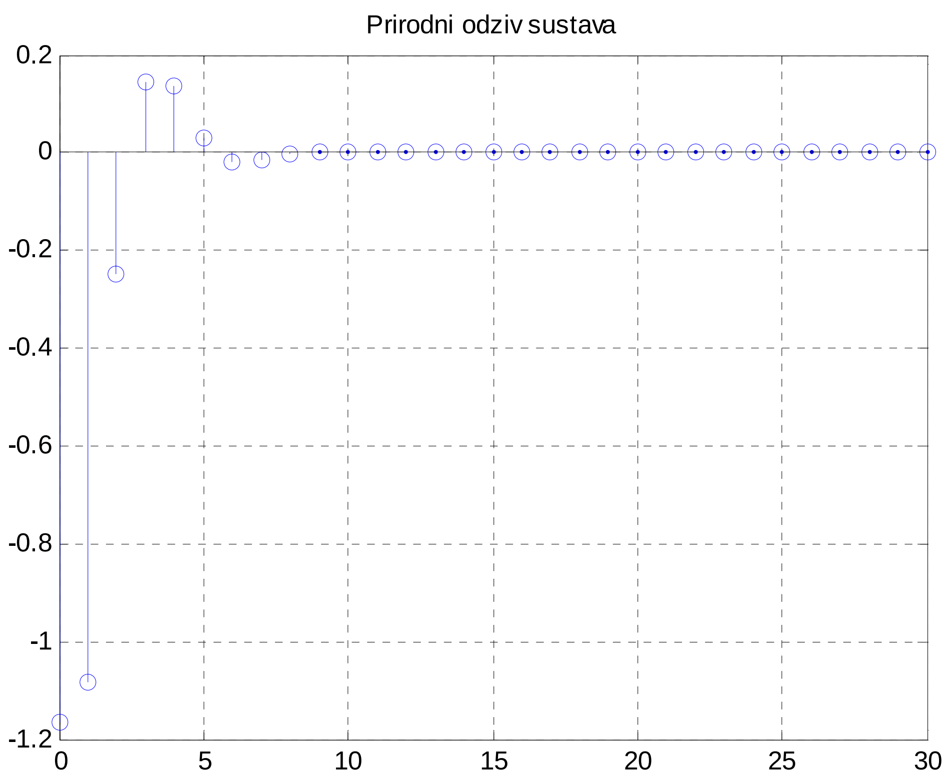

Pobuda sustava
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
20
25
30

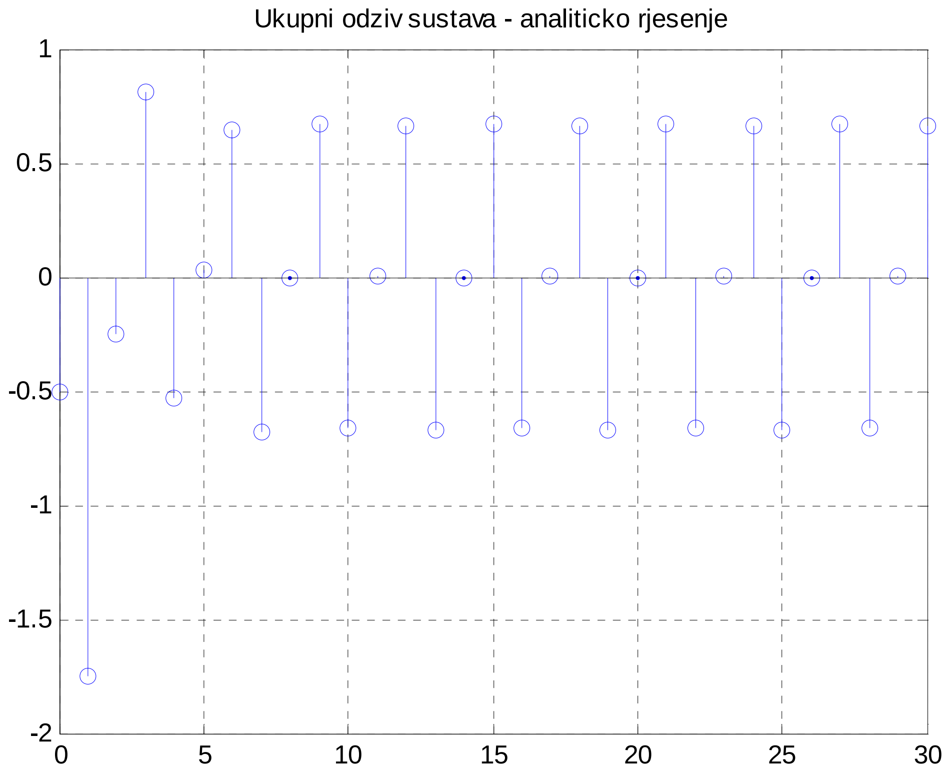

Odziv nepobudjenog sustava
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
20
25
30

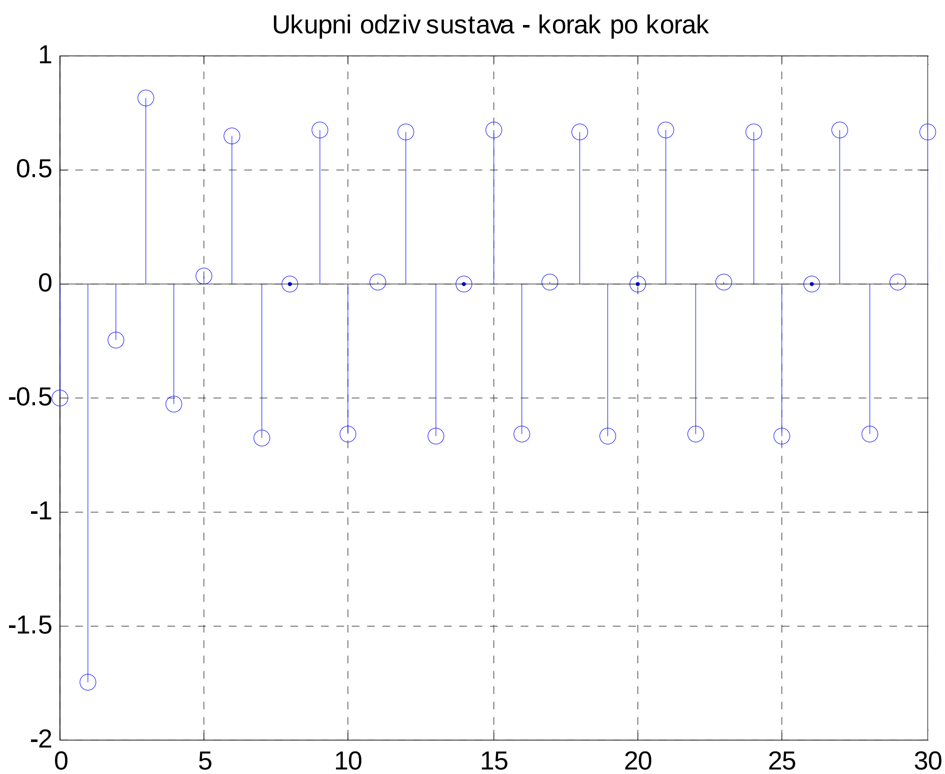

Odziv mirnog sustava vlastitim frekvencijama
0.05
0
-0.05
-0.1
-0.15
-0.2
0
5
10
15
20
25
30


Prisilni odziv sustava
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
5
10
15
20
25
30


Ukupni odziv mirnog sustava
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
0
5
10
15
20
25
30


Prirodni odziv sustava
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
5
10
15
20
25
30


Ukupni odziv sustava - analiticko rjesenje
1
0.5
0
-0.5
-1
-1.5
-2
0
5
10
15
20
25
30


Ukupni odziv sustava - korak po korak
1
0.5
0
-0.5
-1
-1.5
-2
0
5
10
15
20
25
30