3.1-3

a)   1) $y(n) - 0.8y(n-1) + 0.2y(n-2) = u(n)$

$$y(-1) = 0 \qquad\qquad u(n) = \delta(n)$$
$$y(-2) = 0$$

$y(n) = u(n) + 0.8y(n-1) - 0.2y(n-2)$ $\qquad\qquad h(-2) = h(-1) = 0$

$h(n) = \delta(n) + 0.8h(n-1) - 0.2h(n-2)$

$h(0) = \delta(0) + 0.8h(-1) - 0.2h(-2)$

$h(0) = 1$

$$g^2 - 0.8g + 0.2 = 0$$

$$g_{1,2} = \frac{0.8 \pm j0.4}{2} \qquad\qquad \operatorname{arctg}\frac{0.2}{0.4} = \operatorname{arctg}\frac{1}{2} = 26.56°$$

$$g_1 = 0.4 + j0.2$$
$$g_2 = 0.4 - j0.2 \qquad\qquad h(n) = |g|^n \big(A\cos(n\varphi) + B\sin(n\varphi)\big) =$$
$$= 0.447^n \big(A\cos(26.56° n) + B\sin(26.56° n)\big)$$

$h(0) = \boxed{A = 1}$

$h(-1) = 0.447^{-1}\big(1 \cdot \cos(-26.56°) - B\sin(26.56°)\big) = \dfrac{1}{0.447}\big(0.894 - B\,0.447\big) = 2 - B = 0$

$$\boxed{B = 2}$$

$h(n) = 0.447^n \big(\cos(26.56° n) + 2\sin(26.56° n)\big)$ — impulsni odziv

$u(m) = \mu(m)$

$y(m) - 0.8y(m-1) + 0.2y(m-2) = u(m)$

$Y_h = 0.447^m(A\cos(26.56°m) + B\sin(26.56°m))$

$Y_p = K$

$K - 0.8K + 0.2K = 1$
$$0.4K = 1$$
$$K = 2.5$$

$y(m) = 0.447^m A\cos(26.56°m) + 0.447^m B\sin(26.56°m) + 2.5$

$y(0) = 1 = A + 2.5$
$$\boxed{A = -1.5}$$

$y(-1) = 0 = 0.447^{-1}A\cos(-26.56°) + 0.447^{-1}B\sin(-26.56°) + 2.5$

$0 = \dfrac{0.894}{0.447}A - \dfrac{0.447}{0.447}B + 2.5$

$0 = 2A - B + 2.5$
$B = 2.5 + 2 \cdot (-1.5) = -0.5$
$$\boxed{B = -0.5}$$

$y(m) = 0.447^m(-1.5\cos(26.56°m) - 0.5\sin(26.56°m))$ — odezv na jednotkový

skok

2) $\quad y(n) - \sqrt{3}\, y(n-1) + y(n-2) = u(n)$

$\quad u(n) = \delta(n)$ $\qquad\qquad\qquad h(-1) = h(-2) = 0$

$h(n) - \sqrt{3}\, h(n-1) + h(n-2) = \delta(n)$

$g^2 - \sqrt{3}\, g + 1 = 0$

$g_{1,2} = \dfrac{\sqrt{3} \pm j}{2} = \dfrac{\sqrt{3}}{2} \pm \dfrac{1}{2} j$ $\qquad\qquad$ $\text{arctg}\; \dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \text{arctg}\, \dfrac{1}{\sqrt{3}} = \dfrac{\pi}{6}$

$h(n) = A\cos\left(\dfrac{\pi}{6} n\right) + B \sin\left(\dfrac{\pi}{6} n\right)$

$h(-1) = A \cdot \dfrac{\sqrt{3}}{2} - B\dfrac{1}{2} = 0$ $\qquad\qquad$ $\dfrac{1}{2} B = \dfrac{\sqrt{3}}{2}$

$\boxed{B = \sqrt{3}}$

$h(0) = \delta(0) + \sqrt{3}\, h(-1) - h(-2) = 1$

$h(0) = \boxed{A = 1}$

$\qquad\qquad h(n) = \cos\left(\dfrac{\pi}{6} n\right) + \sqrt{3}\, \sin\left(\dfrac{\pi}{6} n\right)$ $\qquad$ — impulsni odziv

$u(n) = \mu(n)$

$y_h = A\cos\left(\dfrac{\pi}{6} n\right) + B\sin\left(\dfrac{\pi}{6} n\right)$

$y_p = K$

$\qquad\qquad K - \sqrt{3}\, K + K = 1$

$\qquad\qquad (2 - \sqrt{3}) K = 1$

$\qquad\qquad\qquad K = \dfrac{1}{2 - \sqrt{3}}$

$y(n) = A\cos\left(\dfrac{\pi}{6} n\right) + B\sin\left(\dfrac{\pi}{6} n\right) + \dfrac{1}{2 - \sqrt{3}}$

$y(-1) = 0 = A\dfrac{\sqrt{3}}{2} - \dfrac{1}{2} B + \dfrac{1}{2 - \sqrt{3}}$ $\qquad\qquad$ $\dfrac{1}{2} B = 2 + \sqrt{3} + \dfrac{\sqrt{3}}{2}(-1 - \sqrt{3})$

$y(0) = 1 = A + \dfrac{1}{2 - \sqrt{3}}$ $\qquad$ $\boxed{A = -1 - \sqrt{3}}$ $\qquad$ $B = 4 + 2\sqrt{3} + \sqrt{3}(-1 - \sqrt{3}) = 4 + 2\sqrt{3} - \sqrt{3} - 3$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\boxed{B = 1 + \sqrt{3}}$

$y(n) = (-1 - \sqrt{3}) \cos\left(\dfrac{\pi}{6} n\right) + (1 + \sqrt{3}) \sin\left(\dfrac{\pi}{6} n\right)$ $\qquad$ — odziv na jediničnu skok

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4)

$y(n) - 2y(n-1) + 5y(n-2) = u(n)$

$u(n) = \delta(n)$

$g^2 - 2g + 5 = 0$

$$g_{1,2} = \frac{2 \pm j4}{2} = 1 \pm j2 \qquad arctg \frac{2}{1} = 63.43°$$

$$h(n) = (\sqrt{5})^n \left(A \cos(63.43° n) + B \sin(63.43° n) \right)$$

$h(0) = 1$
$h(-1) = 0$

$h(0) = \boxed{A = 1}$

$$h(-1) = \frac{1}{\sqrt{5}} \left(A \cos(-63.43°) - B \sin(63.43°) \right) = 0$$

$$\frac{1}{\sqrt{5}} \left(0.447 - B \cdot 0.894 \right) = 0$$

$$0.199 - B \cdot 0.399 = 0$$

$$\boxed{B = 0.5}$$

$$h(n) = (\sqrt{5})^n \left(\cos(63.43° n) + 0.5 \sin(63.43° n) \right) \qquad \text{— impulsni odziv}$$

$u(n) = \mu(n)$

$$y_h(n) = (\sqrt{5})^n \left(A \cos(63.43° n) + B \sin(63.43° n) \right)$$

$y_p(n) = K$

$K - 2K + 5K = 1 \qquad\qquad y_p(n) = \frac{1}{4}$

$K = \frac{1}{4}$

$$y(n) = (\sqrt{5})^n \left(A \cos(63.43° n) + B \sin(63.43° n) \right) + \frac{1}{4}$$

$y(0) = 1$

$y(-1) = 0$

$y(0) = A + \frac{1}{4} = 1 \qquad \boxed{A = \frac{3}{4}}$

$$y(-1) = \frac{1}{\sqrt{5}} \left(\frac{3}{4} \cos(63.43°) - B \sin(63.43°) \right) + \frac{1}{4} = 0$$

$$0.15 - B \cdot 0.399 = -\frac{1}{4} \qquad\qquad \boxed{B = 1}$$

$$y(n) = (\sqrt{5})^n \left(\frac{3}{4} \cos(63.43° n) + \sin(63.43° n) \right) + \frac{1}{4} \qquad \text{— odziv na jedinični skok}$$

- linearnost -

$$u(t) = \alpha u_1(t) + \beta u_2(t)$$

$$y_1(t) = \frac{d}{dt} u_1(t)$$

$$y_2(t) = \frac{d}{dt} u_2(t)$$

$$y(t) = \frac{d}{dt}\left[\alpha u_1(t) + \beta u_2(t)\right] = \frac{d}{dt}\alpha u_1(t) + \frac{d}{dt}\beta u_2(t) = \alpha\frac{d}{dt}u_1(t) + \beta\frac{d}{dt}u_2(t) =$$

$$= \alpha y_1(t) + \beta y_2(t)$$

— sustav je linearan

— derivator je memorijski sustav

3.2-2 a)

1) $y''(t) + 2y'(t) + 25 y(t) = u(t)$

$$s^2 + 2s + 25 = 0$$

$$s_{1,2} = \frac{-2 \pm \sqrt{4-100}}{2} = \frac{-2 \pm j4\sqrt{6}}{2} = -1 \pm j2\sqrt{6}$$

$$y_h(t) = e^{-t}\left(A\cos(2\sqrt{6}t) + B\sin(2\sqrt{6}t)\right) \qquad h_A(0^+) = 0$$

$$h_A(t) = e^{-t}\left(A\cos(2\sqrt{6}t) + B\sin(2\sqrt{6}t)\right) \qquad h_A'(0^+) = 1$$

$$h_A'(t) = -Ae^{-t}\cos(2\sqrt{6}t) + Ae^{-t}(-\sin(2\sqrt{6}t))\cdot 2\sqrt{6} - Be^{-t}\sin(2\sqrt{6}t) + Be^{-t}\cos(2\sqrt{6}t)\cdot 2\sqrt{6}$$

$$h_A'(0^+) = -A + 2\sqrt{6}\,B = 1 \qquad\qquad B = \frac{1}{2\sqrt{6}}\cdot\frac{\sqrt{6}}{\sqrt{6}} = \boxed{\frac{\sqrt{6}}{12}}$$

$$h_A(0^+) = \boxed{A = 0}$$

$$\begin{array}{ll} a_0 = 1 & b_0 = 0 \\ a_1 = 2 & b_1 = 0 \\ a_2 = 25 & b_2 = 1 \end{array} \qquad\qquad h_A(t) = e^{-t}\frac{\sqrt{6}}{12}\sin(2\sqrt{6}t)$$

$$N = 2 \qquad M = 0$$

$$h(t) = \underbrace{b_0\delta(t)}_{0} + \sum_{m=0}^{M}\left(b_{N-m}D^m\right)h_A(t) = \sum_{m=0}^{0}\left(b_{2-m}D^m\right)h_A(t) = b_2 D^0 h_A(t) = 1\cdot h_A(t)$$

$$h(t) = e^{-t}\frac{\sqrt{6}}{12}\sin(2\sqrt{6}t)$$

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2) $y''(t) + 23y(t) = u(t)$

$$s^2 + 23 = 0$$

$$s^2 = -23$$

$$s_1 = j\sqrt{23} \qquad s_2 = -j\sqrt{23}$$

$$Y_h(t) = A\cos(\sqrt{23}\,t) + B\sin(\sqrt{23}\,t)$$

$$h_A'(0^+) = 1$$
$$h_A(0^+) = 0$$

$$h_A(t) = A\cos(\sqrt{23}\,t) + B\sin(\sqrt{23}\,t)$$

$$h_A'(t) = -A\sin(\sqrt{23}\,t)\cdot\sqrt{23} + B\cos(\sqrt{23}\,t)\cdot\sqrt{23}$$

$$h_A(0^+) = \boxed{A = 0}$$

$$h_A'(0^+) = B\sqrt{23} = 1$$

$$\boxed{B = \frac{\sqrt{23}}{23}}$$

$$h_A(t) = \frac{\sqrt{23}}{23}\sin(\sqrt{23}\,t)$$

$$\begin{array}{lll} a_0 = 1 & b_0 = 0 & N = 2 \\ a_1 = 0 & b_1 = 0 & M = 0 \\ a_2 = 23 & b_2 = 1 & \end{array}$$

$$h(t) = \underbrace{b_0\delta(t)}_{0} + \sum_{m=0}^{M}(b_{N-m}D^m)h_A(t) = \sum_{m=0}^{0}(b_{2-m}D^m)h_A(t) = b_2 D^0 h_A(t) =$$

$$= 1\cdot h_A(t)$$

$$h(t) = \frac{\sqrt{23}}{23}\sin(\sqrt{23}\,t) \qquad t > 0$$

3) $y''(t) = u(t)$

$$s^2 = 0$$

$$s_{1,2} = 0$$

$$h_A'(0^+) = 1$$
$$h_A(0^+) = 0$$

$$Y_h(t) = (C_1 + C_2 t)e^{0\cdot t}$$

$$h_A(t) = C_1 + C_2 t \qquad h_A(0^+) = \boxed{C_1 = 0}$$

$$h_A'(t) = C_2 \qquad h_A'(0^+) = \boxed{C_2 = 1}$$

$$h_A(t) = t \,, \quad t > 0$$

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