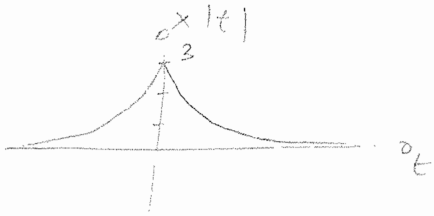
1. $x(t) = 3e^{-2|t|}$ $\boxed{1}$

$$= \begin{cases} 3e^{-2t} & 0 \le t < \infty \\ 3e^{2t} & -\infty < t < 0 \end{cases}$$



a) signal je kontinuiran aperiodičan → treba koristiti vremenski kont. Fourierovu transformaciju (CTFT) spektar će biti kontinuiran aperiodičan.

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

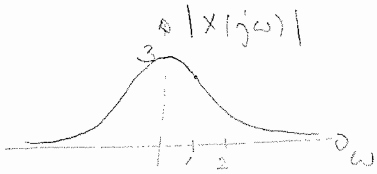
$$= \int_{-\infty}^{0} 3e^{2t} e^{-j\omega t} dt + \int_{0}^{\infty} 3e^{-2t} e^{-j\omega t} dt$$

$$= 3 \int_{-\infty}^{0} e^{(2-j\omega)t} dt + 3 \int_{0}^{\infty} e^{(-2-j\omega)t} dt$$

$$= 3 \left. \frac{e^{(2-j\omega)t}}{2-j\omega} \right|_{-\infty}^{0} + 3 \left. \frac{e^{-(2+j\omega)t}}{-2-j\omega} \right|_{0}^{\infty}$$

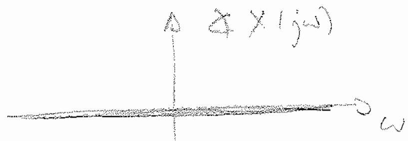
$$= \frac{3}{2-j\omega}(1-0) + \frac{3}{-2-j\omega}(0-1) = \frac{3}{2-j\omega} + \frac{3}{2+j\omega} = \frac{6+3j\omega+6-3j\omega}{4+\omega^2}$$

$$X(j\omega) = \frac{12}{4+\omega^2}$$



b) $$|X(j\omega)| = \left| \frac{12}{4+\omega^2} \right| = \frac{12}{4+\omega^2}$$

$$\measuredangle X(j\omega) = \operatorname{arctg} \frac{0}{\frac{12}{4+\omega^2}} = 0$$



c) $$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \frac{12}{4+\omega^2} \right|^2 d\omega = \frac{12^2}{2\pi} \int_{-\infty}^{\infty} \frac{1}{(4+\omega^2)^2} d\omega = \frac{144}{2\pi} \left(\frac{\omega}{2 \cdot 4 (4+\omega^2)} + \frac{1}{2 \cdot 8} \operatorname{arctg} \frac{\omega}{2} \right)$$

$$= \frac{72}{\pi} \left(\frac{\omega}{32+8\omega^2} + \frac{1}{16} \operatorname{arctg} \frac{\omega}{2} \right) \Bigg|_{-\infty}^{\infty} = \frac{72}{\pi} \left(0 + \frac{1}{16} \cdot \frac{\pi}{2} - \frac{1}{16} \left(-\frac{\pi}{2}\right) \right)$$

$$= \frac{\cancel{72}^{9}}{\cancel{\pi}} \cdot \frac{1}{\cancel{16}_{2}} \cdot \cancel{\pi} = \frac{9}{2}$$

d) $$x(t) \rightarrow X(j\omega) = \frac{12}{4+\omega^2}$$

$$X_1(j\omega) = \int_{-\infty}^{\infty} x(3t+2) e^{-j\omega t} dt = \left| \begin{array}{l} a = 3t+2 \\ da = 3dt \\ t = \frac{1}{3}(a-2) \end{array} \right| = \frac{1}{3} \int_{-\infty}^{\infty} x(a) e^{-j\frac{\omega}{3}(a-2)} da$$

$$= \frac{1}{3} e^{j\frac{2\omega}{3}} \int_{-\infty}^{\infty} x(a) e^{-j\frac{\omega}{3}a} da = \frac{1}{3} e^{j\frac{2\omega}{3}} X\left(j\frac{\omega}{3}\right)$$

$$= \frac{1}{3} e^{j\frac{2\omega}{3}} \frac{12}{4+\frac{\omega^2}{9}} = e^{j\frac{2\omega}{3}} \frac{36}{36+\omega^2}$$

2. $x(t) = 1 + \sin(26\pi t) + \cos(40\pi t)$

A

a) TEOREM OČITAVANJA:
Vremenski kont. signal $x(t)$, $\forall t \in \mathbb{R}$, s frekvencijama ne većim od f_{max}, može biti egzaktno rekonstruisan iz svojih očitaka $x(n) = x(nT)$, $\forall n \in \mathbb{Z}$, ako je očitavanje provedeno frekv. $f_S = \frac{1}{T}$ koja je veća od $2 \cdot f_{max}$.

b) $\sin 26\pi t$
$26\pi T = 2k\pi$
$T_1 = \frac{1}{13} k$
$f_1 = 13$

$\sin 40\pi t$
$40\pi T = 2k\pi$
$T_2 = \frac{1}{20} k$
$f_2 = 20$

$f_2 > f_1$

da ne bi bilo aliasing $f_S > 2 f_2 \rightarrow f_S > 40\ Hz$
$T_S < \frac{1}{40}\ s$

preklapanje spektra ima za $T_S \geq \frac{1}{40} s$
$f_S \leq 40\ Hz$

c) $T_S = \frac{1}{6}$
$x(nT_S) = 1 + \sin(26\pi n T_S) + \cos(40\pi n T_S)$
$= 1 + \sin(\frac{13}{3}\pi n) + \cos(\frac{20}{3}\pi n)$

d) $\frac{13}{3}\pi N = 2k\pi$
$N = \frac{6}{13} k$
$N_1 = 6$

$\frac{20}{3}\pi N = 2k\pi$
$N = \frac{6}{20} k = \frac{3}{10} k$
$N_2 = 3$

$N = 6$

signal je periodičan sa $N = 6$

e) periodičan diskretan signal $\rightarrow$ DTFS
$x(n) = 1 + \frac{1}{2j}\left(e^{+j\frac{13}{3}\pi n} - e^{-j\frac{13}{3}\pi n}\right) + \frac{1}{2}\left(e^{j\frac{20}{3}\pi n} + e^{-j\frac{20}{3}\pi n}\right)$

$= 1 + \frac{1}{2j} e^{j\frac{2\pi}{6} n \cdot 13} - \frac{1}{2j} e^{-j\frac{2\pi}{6} n \cdot 13} + \frac{1}{2} e^{j\frac{2\pi}{6} \cdot 20 n} + \frac{1}{2} e^{-j\frac{2\pi}{6} \cdot 20 n}$

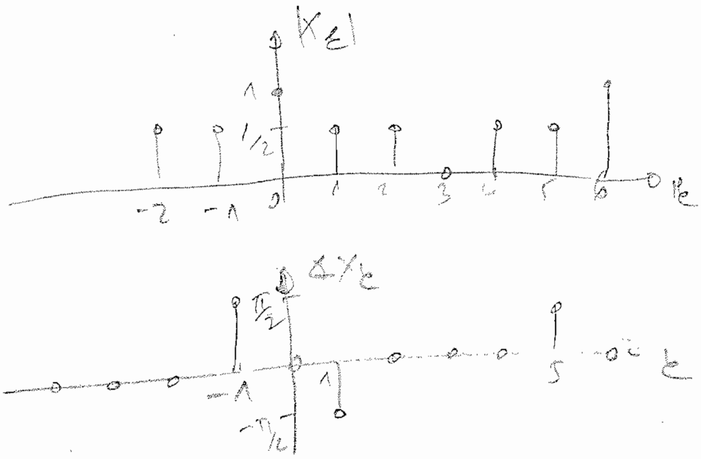
$= 1 + \frac{1}{2j} e^{j\frac{2\pi}{6} n \cdot 1} - \frac{1}{2j} e^{-j\frac{2\pi}{6} \cdot n \cdot 1} + \frac{1}{2} e^{j\frac{2\pi}{6} n \cdot 2} + \frac{1}{2} e^{-j\frac{2\pi}{6} \cdot n \cdot 2}$

$= 1 + \frac{1}{2j} e^{j\frac{2\pi}{6} n \cdot 1} - \frac{1}{2j} e^{j\frac{2\pi}{6} n \cdot 5} + \frac{1}{2} e^{j\frac{2\pi}{6} n \cdot 2} + \frac{1}{2} e^{j\frac{2\pi}{6} \cdot n \cdot 4}$

$X_0 = 1$
$X_1 = \frac{1}{2j} = \frac{-j}{2} = \frac{1}{2} e^{-j\frac{\pi}{2}}$

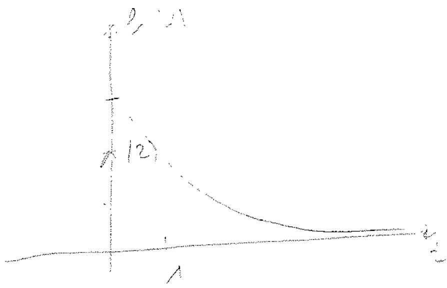
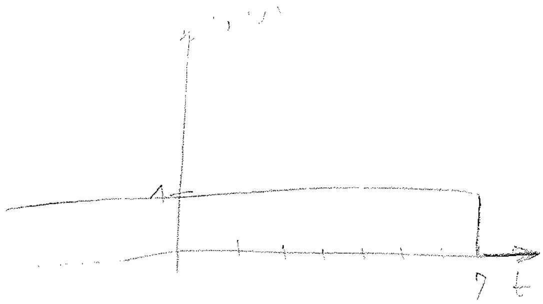
$X_5 = -\frac{1}{2j} = \frac{j}{2} = \frac{1}{2} e^{j\frac{\pi}{2}}$
$X_2 = \frac{1}{2}$
$X_4 = \frac{1}{2}$

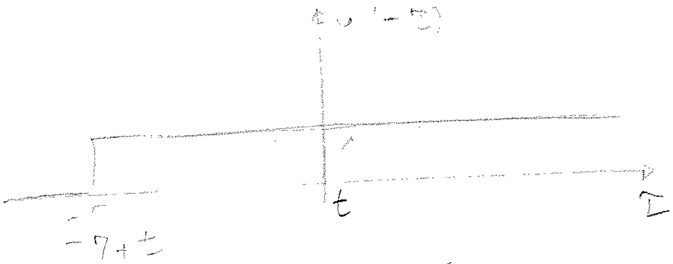
b)



3. $h(t) = 2\delta(t) + 3e^{-2t}\mu(t)$

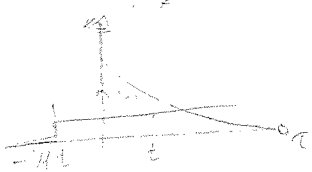
$u(t) = \mu(7-t)$



1° $t \in (-\infty, 7\rangle$

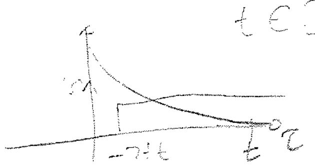
$$y(t) = \int_{-\infty}^{\infty} h(\tau)\,u(t-\tau)\,d\tau = \int_{0}^{\infty} 3e^{-2\tau}\,d\tau + \int_{-7+t}^{\infty} 2\,\delta(\tau)\,d\tau$$



$$= 3\,\frac{e^{-2\tau}}{-2}\Big|_0^{\infty} + 2$$

$$= -\frac{3}{2}(0-1) + 2 = \frac{3}{2} + 2 = \frac{7}{2}$$

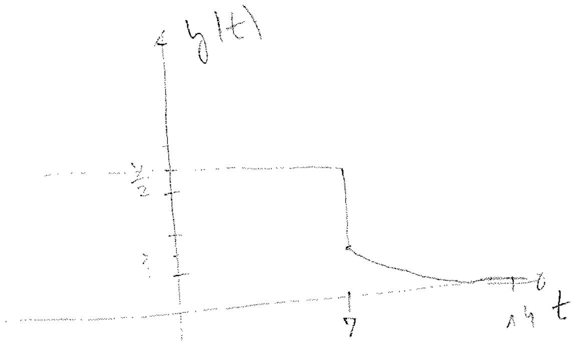
2° $t \in [7, \infty\rangle$



$$y(t) = \int_{-\infty}^{\infty} h(\tau)\,u(t-\tau)\,d\tau = \int_{-7+t}^{\infty} 3e^{-2\tau}\,d\tau$$

$$= 3\,\frac{e^{-2\tau}}{-2}\Big|_{-7+t}^{\infty} = -\frac{3}{2}\left(0 - e^{-2(-7+t)}\right)$$

$$= \frac{3}{2}e^{14-2t} = \frac{3}{2}e^{14}e^{-2t}$$



4. $2y(n) - y(n-1) = 3u(n) - u(n-1)$

$u(n) = \frac{5}{9}\left(-\frac{1}{3}\right)^n \mu(n)$

$y(-1) = 4$

$\boxed{A}$

a) $2Y(z) - Y(z)\cdot z^{-1} = 3U(z) - z^{-1}U(z)$

$Y(z)(2 - z^{-1}) = U(z)(3 - z^{-1})$

$H(z) = \frac{Y(z)}{U(z)} = \frac{3 - z^{-1}}{2 - z^{-1}} = \frac{3z - 1}{2z - 1}$

POLOVI $2z - 1 = 0$, $z = \frac{1}{2}$ NULE $3z - 1 = 0$, $z = \frac{1}{3}$

b) Kriterij unutrašnje stabilnosti sustava:

asimptotski stabilan sustav ako je $|s_i| < 1, \forall i$

stabilan - marginalno stabilan ako je pol na jediničnoj kružnici

nestabilan - ako postoji $|s_i| > 1$ ili više polova točno na istom mjestu na jedn. kruž.

za zadani sustav - pol je $\frac{1}{2}$

$\left|\frac{1}{2}\right| < 1$ sustav je asimptotski stabilan

c) $U(z) = ?$

$u(n) = \frac{5}{9}\left(-\frac{1}{3}\right)^n \mu(n)$

$U(z) = \frac{5}{9}\,\frac{z}{z + \frac{1}{3}}$

d) $U(z) = \frac{5}{3} \frac{z}{3z+1}$

$H(z) = \frac{3z-1}{2z-1}$

$y(z) = H(z) \cdot U(z) = \frac{5}{3} \frac{z}{3z+1} \cdot \frac{3z-1}{2z-1} = \frac{5}{18} \frac{z}{z+\frac{1}{3}} \frac{3z-1}{z-\frac{1}{2}}$

$\frac{y(z)}{z} = \frac{5}{18} \frac{3z-1}{(z+\frac{1}{3})(z-\frac{1}{2})} = \frac{A}{z+\frac{1}{3}} + \frac{B}{z-\frac{1}{2}}$

$A+B = \frac{5}{18_6} \cdot \cancel{3} \;/\cdot 1$

$-\frac{1}{2}A + \frac{1}{3}B = -\frac{5}{18} \;/\cdot 18$

$-6A - 6B = -5$

$-9A + 6B = -5$

$-15A = -10$

$A = \frac{2}{3}$

$B = \frac{5}{6} - A$

$= \frac{5}{6} - \frac{2}{3} = \frac{5-4}{6}$

$B = \frac{1}{6}$

$y(z) = \frac{2}{3} \frac{z}{z+\frac{1}{3}} + \frac{1}{6} \frac{z}{z-\frac{1}{2}}$

$y(n) = \frac{2}{3} \cdot (-\frac{1}{3})^n + \frac{1}{6} (\frac{1}{2})^n \mu(n)$

e) $2y(n) - y(n-1) = 3u(n) - u(n-1)$

$2y(n) - y(n-1) = 0$ nepobuđeni

$2q - 1 = 0$

$q = \frac{1}{2}$

$y_n(n) = C(\frac{1}{2})^n$

$y_n(-1) = C \cdot 2 = 4$

$C = 2$

$y_n(n) = 2(\frac{1}{2})^n$

f) $y_t = y_m + y_n$

$= \frac{2}{3}(-\frac{1}{3})^n + \frac{1}{6}(\frac{1}{2})^n + 2(\frac{1}{2})^n$

$\boxed{y_t(n) = \frac{2}{3}(-\frac{1}{3})^n + \frac{13}{6}(\frac{1}{2})^n}$ za $n \geq 0$

5. $y''(t)+6y'(t)+25y(t)=u'(t)$

$u(t)=12\sin(5t)\mu(t)$

$y(0^-)=y'(0^-)=0$

$\boxed{A}$

a) $s^2Y(s)+6sY(s)+25Y(s)=sU(s)$

$$H(s)=\frac{Y(s)}{U(s)}=\frac{s}{s^2+6s+25}$$

POLOVI

$s^2+6s+25=0$

$$s_{1,2}=\frac{-6\pm\sqrt{36-100}}{2}\Rightarrow\frac{-6\pm\sqrt{-64}}{2}=-3\pm4j$$

NULE

$s=0$

$s=\infty$

b) Kriterij unutarnje stabilnosti:

asimptotski stabilan za $Re\{s_i\}<0$, ki

marginalno stabilan za $Re\{s_i\}\leq0$, ki kratnosti 1

nestabilan za bar jedan $Re\{s_i\}>0$ ili višestruki pol na Im osi

$\left.\begin{matrix}Re\{s_1\}<0\\ Re\{s_2\}<0\end{matrix}\right\}$ STABILAN SUSTAV

c) $$H(j\omega)=\frac{j\omega}{-\omega^2+6j\omega+25}=\frac{j\omega}{25-\omega^2+6j\omega}$$

d) $\omega=5$

$$H(j\omega)=\frac{j5}{25-25+6\cdot j\cdot5}=\frac{\cancel{j}\cancel{5}}{\cancel{30}_6\cancel{j}}=\frac{1}{6}$$

$\measuredangle H(j\omega)=0$

$|H(j\omega)|=\frac{1}{6}$

$y(t)=12\cdot\frac{1}{6}\sin5t\,\mu(t)=2\sin5t\,\mu(t)$

e) homogeno

$s^2+6s+25=0$

$s_{1,2}=-3\pm4j$

$y_h(t)=C_1e^{(3-4j)t}+C_2e^{(3+4j)t}$

$y_t(t)=C_1e^{(3-4j)t}+C_2e^{(3+4j)t}+2\sin5t$

$y_t'(t)=(-3-4j)C_1e^{(3-4j)t}+C_2(-3+4j)e^{(3+4j)t}+10\cos5t$

$y_t(0^+)=C_1+C_2=0$

$y_t'(0^+)=(-3-4j)C_1+C_2(-3+4j)+10=0$

početni uvjeti

$$y'' + 6y' + 25y = 0 \cdot u'' + 1 \cdot u' + 0 \cdot u$$

$y(0^-) = 0$
$y'(0^-) = 0$

$$y(0^+) - y(0^-) = 0$$
$$y(0^+) = 0$$
$$y'(0^+) - y'(0^-) = 1 \cdot u(0^+) = 0$$
$$y'(0^+) = y(0^-) = 0$$

$C_1 = -C_2$

$$(-3-4j)(-C_2) + (-3+4j)C_2 + 10 = 0$$
$$4jC_2 + 4jC_2 + 10 = 0$$
$$8jC_2 = -10$$
$$C_2 = -\frac{\cancel{10}^{5}}{\cancel{8}_{4}j} \cdot \frac{j}{j} = \frac{5}{4}j$$
$$C_1 = -\frac{5}{4}j$$

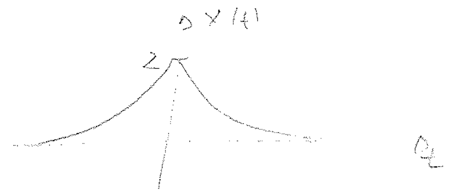
$$y_t(t) = -\frac{5}{4}j e^{(-3-4j)t} + \frac{5}{4}j e^{(-3+4j)t} + 2\sin 5t$$

$$= \frac{5}{4}j e^{-3t}(-\cancel{\cos 4t} + j\sin 4t + \cancel{\cos 4t} + j\sin 4t) + 2\sin 5t$$

$$\boxed{y_t(t) = \left(-\frac{5}{2}e^{-3t}\sin 4t + 2\sin 5t\right)\mu(t)}$$

1. $x(t) = 2e^{-3|t|}$

$$= \begin{cases} 2e^{+3t} & -\infty < t < 0 \\ 2e^{-3t} & 0 \le t < \infty \end{cases}$$



a) signal je kont. aperiodičan → treba koristiti CTFT

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} 2e^{3t} e^{-j\omega t} dt + \int_{0}^{\infty} 2e^{-3t} e^{-j\omega t} dt$$

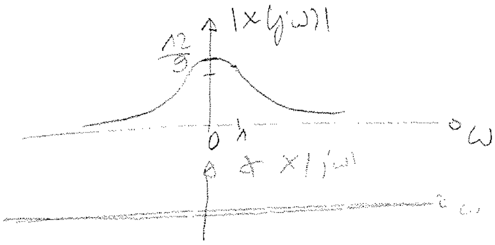
$$= 2\int_{-\infty}^{0} e^{(3-j\omega)t} dt + 2\int_{0}^{\infty} e^{-(3+j\omega)t} dt$$

$$= 2\frac{e^{(3-j\omega)t}}{3-j\omega}\Big|_{-\infty}^{0} + 2\frac{e^{-(3+j\omega)t}}{-(3+j\omega)}\Big|_{0}^{\infty}$$

$$= \frac{2}{3-j\omega} + 2\frac{1}{3+j\omega} = \frac{6+2j\omega+6-2j\omega}{9+\omega^2} = \frac{12}{9+\omega^2}$$

b) $|X(j\omega)| = \frac{12}{9+\omega^2}$

$\measuredangle X(j\omega) = 0$



c) $$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty}\left(\frac{12}{9+\omega^2}\right)^2 d\omega = \frac{144}{2\pi}\int_{-\infty}^{\infty}\frac{1}{(9+\omega^2)^2} d\omega$$

$$= \frac{144}{2\pi}\left(\frac{\omega}{2\cdot 9(9+\omega^2)} + \frac{1}{2\cdot 27}\operatorname{arctg}\frac{\omega}{3}\right)\Big|_{-\infty}^{\infty}$$

$$= \frac{144}{2\pi}\left(\frac{1}{2\cdot 27}\cdot\frac{\pi}{2} - \frac{1}{2\cdot 27}\cdot\frac{-\pi}{2}\right) = \frac{\overset{36}{\cancel{144}}}{2\pi}\cdot\frac{\pi}{2\cdot \cancel{27}_3} = \frac{4}{3}$$

d) $x(t) \to X(j\omega) = \frac{12}{9+\omega^2}$

$x(2t+3) \to ?$

$$X_1(j\omega) = \int_{-\infty}^{\infty} x(2t+3)\, e^{-j\omega t} dt = \left| \begin{matrix} a = 2t+3 \\ da = 2dt \\ t = \frac{1}{2}(a-3) \end{matrix} \right| = \frac{1}{2}\int_{-\infty}^{\infty} x(a)\, e^{-j\frac{\omega}{2}(a-3)} da$$

$$= \frac{1}{2}e^{j\frac{\omega}{2}3}\int_{-\infty}^{\infty} x(a)\, e^{-j\frac{\omega}{2}a} da = \frac{1}{2}e^{j\frac{\omega}{2}\cdot 3}X\left(j\frac{\omega}{2}\right)$$

$$= \frac{1}{2}e^{j\frac{3\omega}{2}}\cdot\frac{12}{9+\frac{\omega^2}{4}} = e^{j\frac{3\omega}{2}}\frac{24}{36+\omega^2}$$

2. $x(t) = 1 + \sin 28\pi t + \cos 38\pi t$

a) TEOREM OČITAVANJA:

Vremenski kont. signal $x(t)$, $\forall t \in \mathbb{R}$ s frekv ne većim od f_{max}, može biti egzaktno rekonstruiran iz svojih očitaka $x(n) = x(nT)$, $\forall n \in \mathbb{Z}$, ako je očitavanje provedeno frekv. $f_s = \frac{1}{T}$ koja je veća od $2 \cdot f_{max}$.

b) $\sin 28\pi t$

$28\pi T = 2k\pi$

$T_1 = \frac{1}{14}k$

$f_1 = 14$

$\cos 38\pi t$

$38\pi T = 2k\pi$

$T_2 = \frac{k}{19}$

$f_2 = 19$

$\rightarrow f_2 > f_1$

da ne bi došlo do preklapanja spektra $f_s > 2f_2 \rightarrow f_s > 38$ Hz

preklapanje spektra je za $f_s \leq 38$ Hz

c) $T_s = \frac{1}{6}$

$x(nT_s) = 1 + \sin(28\pi nT_s) + \cos(38\pi nT_s)$

$x(n) = 1 + \sin\left(\frac{28}{6}\pi n\right) + \cos\left(\frac{38}{6}\pi n\right)$

d) $\frac{28}{6}\pi N = 2k\pi$

$N_1 = \frac{12}{28}k = \frac{3}{7}k$

$N_1 = 3$

$\frac{38}{6}\pi N = 2k\pi$

$N_2 = \frac{12}{38}k = \frac{6}{19}k$

$N_2 = 6$

$\rightarrow N = 6$

periodičan je s periodom $N=6$

e) DTFS

$x(n) = 1 + \sin\left(\frac{28}{6}\pi n\right) + \cos\left(\frac{38}{6}\pi n\right)$

$= 1 + \frac{1}{2j} e^{+\frac{28}{6}\pi n j} - \frac{1}{2j} e^{-\frac{28}{6}\pi n j} + \frac{1}{2} e^{j\frac{38}{6}\pi n} + \frac{1}{2} e^{-j\frac{38}{6}\pi n}$

$= 1 + \frac{1}{2j} e^{j\frac{2\pi}{6} n \cdot 14} - \frac{1}{2j} e^{-j\frac{2\pi}{6} n \cdot 14} + \frac{1}{2} e^{j\frac{2\pi}{6} \cdot n \cdot 19} + \frac{1}{2} e^{-j\frac{2\pi}{6} \cdot n \cdot 19}$

$= 1 + \frac{1}{2j} e^{j\frac{2\pi}{6} \cdot 2} - \frac{1}{2j} e^{j\frac{2\pi}{6} n \cdot 4} + \frac{1}{2} e^{j\frac{2\pi}{6} \cdot n \cdot 1} + \frac{1}{2} e^{-j\frac{2\pi}{6} \cdot n \cdot 5}$

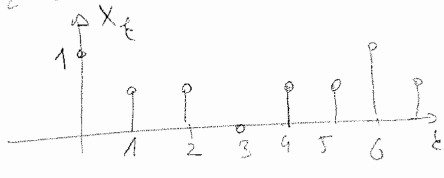
$X_0 = 1$

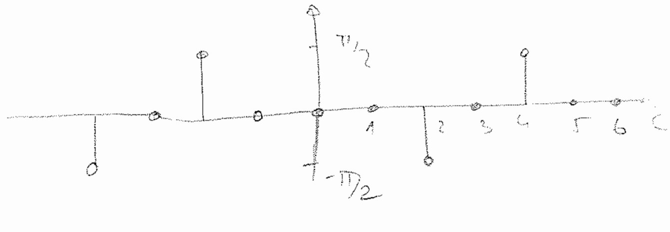
$X_1 = \frac{1}{2}$

$X_2 = \frac{1}{2j} = -\frac{j}{2} = \frac{1}{2} e^{-j\frac{\pi}{2}}$

$X_4 = -\frac{1}{2j} = \frac{1}{2}j = \frac{1}{2} e^{j\frac{\pi}{2}}$

$X_5 = \frac{1}{2}$



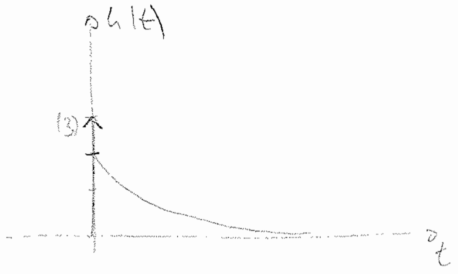


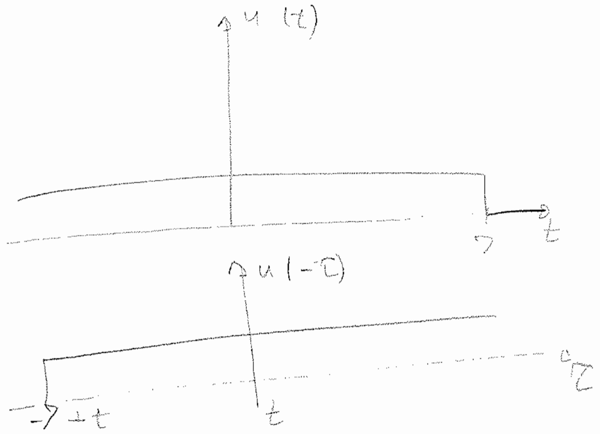
$\boxed{B}$

3.

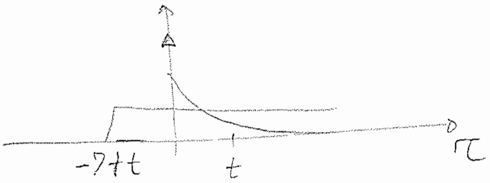
$$h(t) = 3\delta(t) + 2e^{-3t}\mu(t)$$

$$u(t) = \mu(7-t)$$





1°

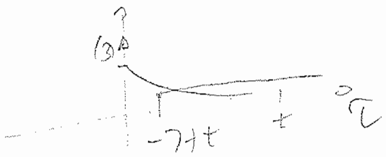


$t \in \langle -\infty, 7 \rangle$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)u(t-\tau)\,d\tau = \int_{0}^{\infty} 2e^{-3\tau}\,d\tau + \int_{-7+t}^{\infty} 3\delta(\tau)\cdot 1\,d\tau$$

$$= 3 + 2\,\frac{e^{-3\tau}}{-3}\Big|_{0}^{\infty} = 3 - \frac{2}{3}(0-1) = 3 + \frac{2}{3} = \frac{11}{3}$$

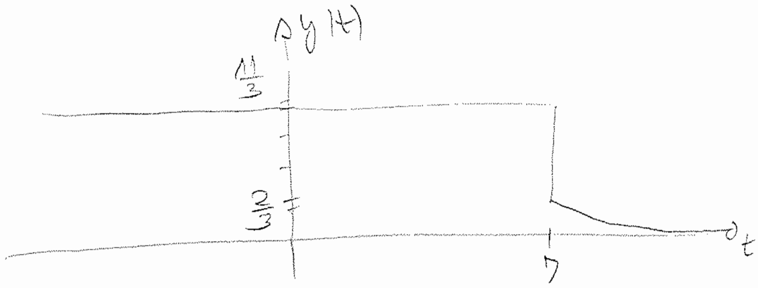
2°



$t \in [7, +\infty\rangle$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)u(t-\tau)\,d\tau = \int_{-7+t}^{\infty} 2e^{-3\tau}\,d\tau = 2\,\frac{e^{-3\tau}}{-3}\Big|_{-7+t}^{\infty}$$

$$= -\frac{2}{3}\left(0 - e^{21-3t}\right) = \frac{2}{3}e^{21}\cdot e^{-3t}$$



4. $3y(n) - y(n-1) = 2u(n) - u(n-1)$ [B]

$u(n) = \frac{5}{4}\left(-\frac{1}{2}\right)^n$

$y(-1) = 9$

a) $3Y(z) - z^{-1}Y(z) = 2U(z) - z^{-1}U(z)$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{2 - z^{-1}}{3 - z^{-1}} = \frac{2}{3}\,\frac{z - \frac{1}{2}}{z - \frac{1}{3}}$$

NULA $z = \frac{1}{2}$ POL $z = \frac{1}{3}$

b) Kriterij unutrašnje stabilnosti

asimptotski stabilno ako je $|s_i| < 1$, $\forall i$

marginalno stabilno ako je $|s_i| \leq 1$, $\forall i$, kratnosti 1

nestabilno ako postoji $|s_i| > 1$ ili višestruki pol na jedn. kružnici

$|z| = \left|\frac{1}{3}\right| < 1$ STABILNO

c) $u(n) = \frac{5}{4}\left(-\frac{1}{2}\right)^n \mu(n)$

$$U(z) = \frac{5}{4} \cdot \frac{z}{z + \frac{1}{2}}$$

d) $Y(z) = H(z) \cdot U(z)$

$$= \frac{2}{3}\,\frac{z - \frac{1}{2}}{z - \frac{1}{3}} \cdot \frac{5}{4}\,\frac{z}{z + \frac{1}{2}} = \frac{5}{6}\,\frac{z\left(z - \frac{1}{2}\right)}{\left(z - \frac{1}{3}\right)\left(z + \frac{1}{2}\right)}$$

$$\frac{Y(z)}{z} = \frac{\frac{5}{6}\left(z - \frac{1}{2}\right)}{\left(z - \frac{1}{3}\right)\left(z + \frac{1}{2}\right)} = \frac{A}{z - \frac{1}{3}} + \frac{B}{z + \frac{1}{2}}$$

$A + B = \frac{5}{6} \quad /\cdot 6$ $\qquad -6A - 6B = -5$ $\qquad A = \frac{5}{6} - B = \frac{5}{6} - 1$

$\frac{1}{2}A - \frac{1}{3}B = -\frac{5}{12} \quad /\cdot 12 \longrightarrow$ $\qquad 6A - 4B = -5$

$-10B = -10$ $\qquad A = -\frac{1}{6}$

$B = 1$

$$Y(z) = \frac{-\frac{1}{6}z}{z - \frac{1}{3}} + \frac{z}{z + \frac{1}{2}}$$

$$y(n) = \left(-\frac{1}{6}\left(\frac{1}{3}\right)^n + \left(-\frac{1}{2}\right)^n\right)\mu(n)$$

e)

$$3y(n) - y(n-1) = 0$$

$$3q - 1 = 0$$

$$q = \frac{1}{3}$$

$$y_h(n) = C\left(\frac{1}{3}\right)^n$$

$$y_h(-1) = C \cdot 3 = 9$$

$$C = 3$$

$$\boxed{y_h(n) = 3\left(\frac{1}{3}\right)^n}$$

f)

$$y_t(n) = 3\left(\frac{1}{3}\right)^n - \frac{1}{6}\left(\frac{1}{3}\right)^n + \left(-\frac{1}{2}\right)^n$$

$$\boxed{y_t(n) = \left(\frac{17}{6}\left(\frac{1}{3}\right)^n + \left(-\frac{1}{2}\right)^n\right)\mu(n)}$$

B

5. $y''(t) + 8y'(t) + 25y(t) = u'(t)$

$u(t) = 16 \sin 5t \, \mu(t)$

$y(0^-) = y'(0^-) = 0$

a) $s^2 Y(s) + 8sY(s) + 25Y(s) = sU(s)$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{s}{s^2+8s+25}$$

NULE $s=0$, $s=\infty$

POLOVI $s^2+8s+25=0$

$$s_{1,2} = \frac{-8 \pm \sqrt{64-100}}{2} = -4 \pm 3j$$

b) Kriterij unutrašnje stabilnosti:
asimptotski stabilan za $Re\{s_i\}<0$, $\forall i$
marginalno stabilan za $Re\{s_i\} \leq 0$, $\forall i$, kratnosti 1
nestabilan za bar jedan $Re\{s_i\}>0$ ili višestruki pol na Im osi

$\left. \begin{array}{l} Re\{s_1\}<0 \\ Re\{s_2\}<0 \end{array} \right\}$ STABILAN SUSTAV

c) $$H(j\omega) = \frac{j\omega}{-\omega^2+8j\omega+25} = \frac{j\omega}{25-\omega^2+8j\omega}$$

d) $\omega = 5$

$$H(j5) = \frac{j5}{25-25+8j5} = \frac{1}{8}$$

$|H(j5)| = \frac{1}{8}$

$\measuredangle H(j5) = 0$

$y(t) = 16 \cdot \frac{1}{8} \sin 5t \, \mu(t) = 2 \sin 5t \, \mu(t)$

e) homogeno

$y_h(t) = C_1 e^{(-4+3j)t} + C_2 e^{(-4-3j)t}$

$y_t(t) = C_1 e^{(-4+3j)t} + C_2 e^{(-4-3j)t} + 2 \sin 5t$

$y_t'(t) = C_1 (-4+3j) e^{(-4+3j)t} + C_2(-4-3j) e^{(-4-3j)t} + 10 \cos 5t$

$y_t(0^+) = C_1 + C_2$

$y_t'(0^+) = C_1(-4+3j) + C_2(-4-3j) + 10$

Početni uvjeti

$$y'' + 8y' + 25y = 0 \cdot u'' + 1 \cdot u' + 0 \cdot u$$

$$y(0^+) - y(0^-) = 0$$

$$y(0^+) = 0$$

$$y'(0^+) - y'(0^-) = 1 \cdot u(0^+) = 0$$

$$y'(0^+) = 0$$

$$c_1 + c_2 = 0 \quad \to \quad c_1 = -c_2$$

$$c_1(-4+3j) + c_2(-4-3j) + 10 = 0$$

$$-c_2(-4+3j) + c_2(-4-3j) + 10 = 0$$

$$-6c_2 j = -10$$

$$c_2 = \frac{10 j}{6 j j} = -\frac{5}{3} j$$

$$c_1 = \frac{5}{3} j$$

$$y_t(t) = \frac{5}{3} j e^{(-4+3j)t} - \frac{5}{3} j e^{(-4-3j)t} + 2 \sin 5t$$

$$= \frac{5}{3} j e^{-4t}\left(e^{3jt} - e^{-3jt}\right) + 2 \sin 5t$$

$$= \frac{5}{3} j e^{-4t} \cdot 2j \sin 3t + 2 \sin 5t$$

$$\boxed{y_t(t) = \left(-\frac{10}{3} e^{-4t} \sin 3t + 2 \sin 5t\right) \mu(t)}$$