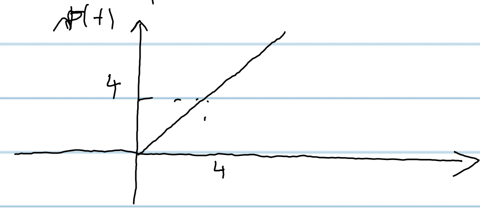
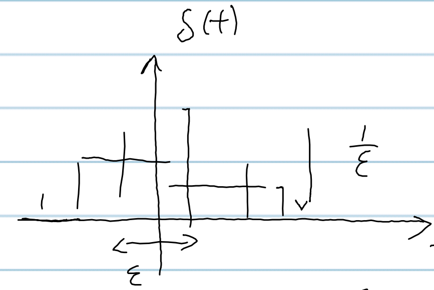
MASOVNE

$u(t)$ - STEP (SKOKOVITA) FUNKCIJA

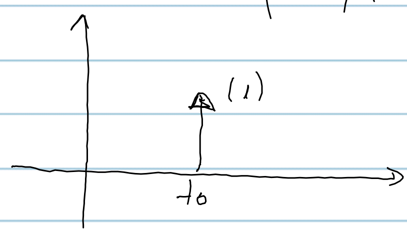




$\varepsilon \to 0 \quad g(t) \to$

$$\delta(t) = \begin{cases} \infty, & t=0 \\ 0, & \text{inače} \end{cases}$$

$$\int_a^b \delta(t-t_0)\,dt = \begin{cases} 1, & a \le t_0 \le b \\ 0, & \text{inače} \end{cases}$$



$$\int_a^b f(t)\,\delta(t-t_0)\,dt = \begin{cases} f(t_0), & a \le t_0 \le b \\ 0, & \text{inače} \end{cases}$$

$$f(t)\,\delta(t-t_0) = f(t_0)\,\delta(t-t_0)$$

$$\frac{d}{dt}u(t) = \delta(t) \longmapsto u(t) = \int_{-\infty}^{t} \delta(t)\,dt$$

$$\frac{d}{dt}r(t) = u(t)$$

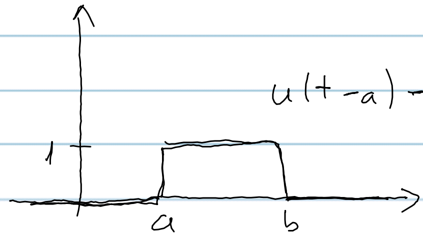
2008

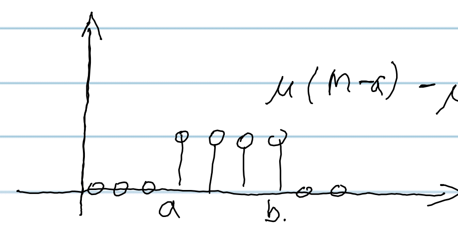
$\boxed{5.}$ $u(t) - u(t-1) + (t-2)^2\left(u(t-2) - u(t-3)\right)$

$$\frac{d}{dt}u(t) = \delta(t) - \delta(t-1) + 2(t-2)\cdot 1\left(u(t-2) - u(t-3)\right) + (t-2)^2 \cdot \left(\delta(t-2) - \delta(t-3)\right)$$

$$= \delta(t) - \delta(t-1) - \delta(t-3) + 2(t-2)\left(u(t-2) - u(t-3)\right)$$

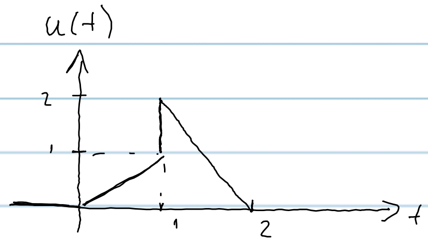
VREMENSKI PROZOR





2009.

$\boxed{5}$



$$\mu(t) = \begin{cases} t, & 0 \le t \le 1 \\ -2(t-2), & 1 \le t \le 2 \\ 0, & \text{inače} \end{cases}$$

$$u(t) = t\,(\mu(t) - \mu(t-1)) + (-2)(t-2)\,(\mu(t-1) - \mu(t-2))$$

$$E = \lim_{T\to\infty} \int_{-T}^{T} |u(t)|^2 dt$$

signali koji imaju konačnu E imaju $P = 0$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |u(t)|^2 dt$$

$$E = \lim_{N\to\infty} \sum_{m=-N}^{N} |u(m)|^2$$

energija periodičnih = 0

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{m=-N}^{N} |u(m)|^2$$

$$P = \frac{1}{T_0} \int_{T_0} |\tilde{x}(t)|^2 dt$$

2009.

$\boxed{2.}$ $x(m) = m\, e^{-j\pi m}\,(\mu(m) - \mu(m-3))$

$$|e^{jx}| = |\cos(x) + j\sin(x)| = \sqrt{\cos^2(x) + \sin^2(x)} = 1$$

$$|x(m)| = |m\,(\mu(m) - \mu(m-3))| = \begin{cases} |m|, & 0 \le m \le 2 \\ 0, & \text{inače} \end{cases}$$

$$E = \sum_{m=0}^{2} m^2 = 5$$

2010.

$\boxed{2.}$ $x(t) = 5e^{j\pi t}$

$|x(t)| = 5$

$$E = \lim_{T\to\infty} \int_{-T}^{T} 25\,dt = 25 \lim_{T\to\infty} 2T = \infty$$

$$P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2\,dt = 25 \lim_{T\to\infty} \frac{2T}{2T} = 25$$

2010.

$\boxed{3}$ $x(n) = 3^{-n}\mu(n) = \left(\frac{1}{3}\right)^n \mu(n)$

$$E = \lim_{N\to\infty} \sum_{0}^{N} \left(\left(\frac{1}{3}\right)^n\right)^2 = \lim_{N\to\infty} \sum_{n=0}^{N} \left(\frac{1}{9}\right)^n = \frac{1}{1-\frac{1}{9}} = \frac{9}{8}$$

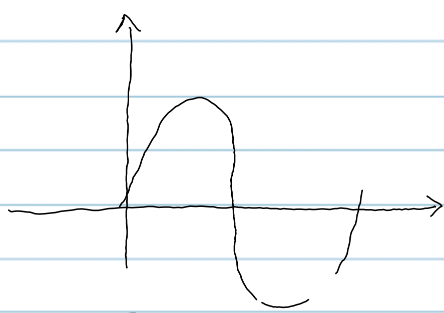
2010.

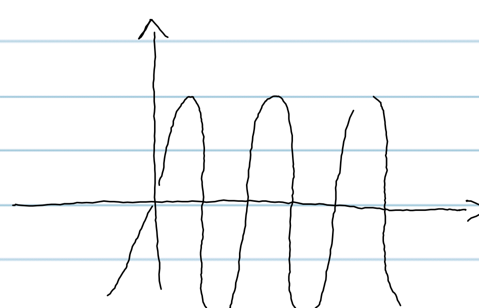
$\boxed{4.}$ $x(n) = (6 + 2^{-n})\mu(n)$

$$(x(n))^2 = (\underbrace{36}_{\frac{36}{2}} + \underbrace{12\cdot 2^{-n}}_{0} + \underbrace{4^{-n}}_{0})\,u(n)$$

$$\frac{36(N+1)}{2N+1} \qquad P = \frac{36}{2}$$

$\sin\left(\frac{\pi}{6}t\right)$ $\qquad$ $\sin\left(\frac{13\pi}{6}t\right)$



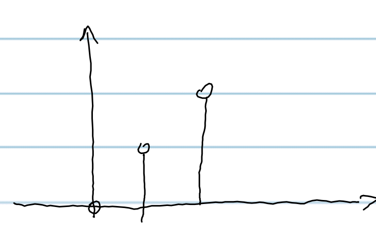


$\sin\left(\frac{\pi}{6}n\right)$

$\{\underline{0}, \frac{1}{2}, \frac{\sqrt{3}}{2}, \dots\}$

$\sin\left(\frac{13\pi}{6}n\right)$

$\{\underline{0}, \frac{1}{2}, \frac{\sqrt{3}}{2}, \dots\}$



$\mathrm{I} \in (-\pi, \pi)$

$$x(t) = A\sin(\omega_c t + \varphi)$$

$$u(n) = x(t)\Big|_{nT_s} = A\sin(\omega_c \underbrace{T_s}_{\frac{2\pi}{\Omega_s}} n + \varphi) \qquad \omega_d = 2\pi \frac{\omega_c}{\Omega_s}$$

$$-\pi < \omega_d < \pi \qquad 2\pi \frac{\omega_c}{\Omega_s} < \pi \rightarrow \Omega_s > 2\omega_c$$

2008.

$\boxed{1}$ $f(n) = \cos\left(\frac{47\pi}{7} n + \frac{\pi}{3}\right)$

$$\frac{47\pi}{7} - 2\pi = \frac{33\pi}{7} - 2\pi = \frac{19\pi}{7} - 2\pi = \frac{5\pi}{7}$$

$$\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$$

$$g(n) = \sin\left(\frac{5\pi}{7} n + \frac{\pi}{3} + \frac{\pi}{2}\right) = \sin\left(\frac{5\pi}{7} n + \frac{5\pi}{6}\right)$$

2009.

$$\cos x = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)$$

$$\sin x = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$$

$\boxed{7.}$ $x(t) = 2\cos(4t) - 2\sin(6t)$

$$\omega_1 = 4 \qquad \omega_2 = 6 \rightarrow \omega_0 = 2$$

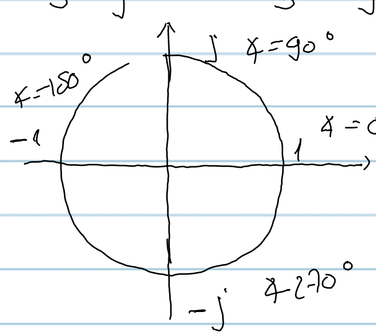
$$x(t) = 2\cdot\frac{1}{2}\left(e^{j2\omega_0 t} + e^{j(-2)\omega_0 t}\right) - 2\,\frac{1}{2j}\left(e^{j3\omega_0 t} - e^{j(-3)\omega_0 t}\right)$$

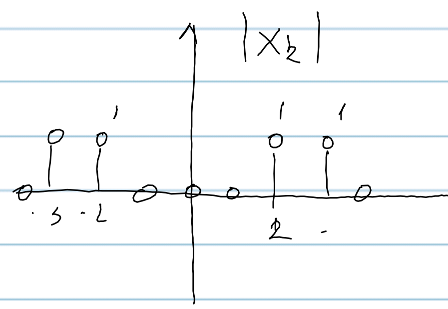
$$= \underbrace{e^{j2\omega_0 t}}_{X_2} + \underbrace{e^{j(-2)\omega_0 t}}_{X_{-2}} + \underbrace{j}_{X_3 =} e^{j3\omega_0 t} - \underbrace{j}_{X_{-3}} e^{j(-3)\omega_0 t}$$

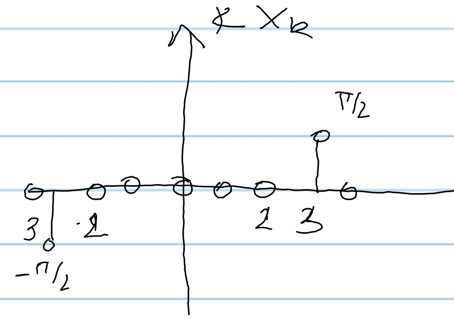
$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t} = \dots + X_{-1} e^{j(-1)\omega_0 t} + X_0 + X_1 e^{j\omega_0 t} + \dots$$

$$X_2 = 1 \qquad X_{-2} = 1$$

$$X_3 = j \qquad X_{-3} = -j \quad \Rightarrow \quad X_3 = 1\, e^{j\frac{\pi}{2}} \qquad X_{-3} = 1\, e^{j\frac{3\pi}{2}} = e^{-j\frac{\pi}{2}}$$







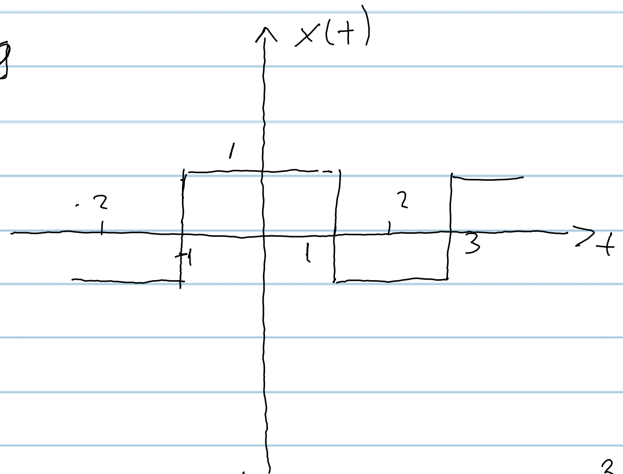
$z = Re(z) + j\,Im(z) = r e^{jt}$

$r = \sqrt{Re^2 + Im^2}$

$\varphi = \operatorname{arctg} \frac{Im}{Re}$

2009

$\boxed{9.}$



Nulti harmonik signala je istosmjerna komponenta signala tj srednja vrijednost

$X_0 = 0$

CTFS:

$$X_1 = \frac{1}{4} \left\{ \int_{-1}^{1} 1\, e^{-j1\frac{\pi}{2}t} dt + \int_{1}^{3} (-1) e^{-j1\cdot\frac{\pi}{2}t} dt \right\}$$

$$= \frac{1}{4} \left\{ \frac{(-1)}{j\frac{\pi}{2}} e^{-j\frac{\pi}{2}t} \Big|_{-1}^{1} + \frac{(-1)(-1)}{j\frac{\pi}{2}} e^{-j\frac{\pi}{2}t} \Big|_{1}^{3} \right\} =$$

$$= \frac{1}{4} \left\{ \frac{(-1)}{j\frac{\pi}{2}} \cdot \left(\underbrace{e^{-j\frac{\pi}{2}}}_{-j} - \underbrace{e^{j\frac{\pi}{2}}}_{j} \right) + \frac{1}{j\frac{\pi}{2}} \left(\underbrace{e^{-j\frac{3\pi}{2}}}_{j} - \underbrace{e^{-j\frac{\pi}{2}}}_{-j} \right) \right.$$

$$= \frac{1}{4} \left(\frac{4}{\pi} + \frac{4}{\pi} \right) = \frac{2}{\pi}$$

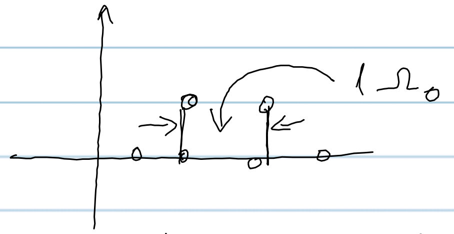
IZ GLAVE:

$x(t) = 2\cos(4t) - 2\sin(6t)$

$X_2 = 1$ $\quad X_{-2} = 1$ $\quad X_3 = 1\, e^{j\frac{\pi}{2}}$ $\quad X_{-3} = 1\, e^{j\frac{3\pi}{2}}$

$$P = \sum_{k=-\infty}^{\infty} |X_k|^2 = 1^2 + 1^2 + 1^2 + 1^2 = 4$$

u službenom: $X_k X_k^* = |X_k|^2$



VREMENSKI — SPEKTAR

PERIODIČAN → DISKRETAN

DISKRETAN → PERIODIČAN $(N=2\pi)$

KONTINUIRAN → APERIODIČAN

APERIODIČAN → KONTINUIRAN

2008.

$\boxed{20.}$ aperiodičan signal

kontinuiran i periodičan spektar

SVOJSTVA FOURIEROVE TRANSFORMACIJE

$x(t) \circ\!\!-\!\!\bullet X(j\omega)$

$x(t-t_0) \circ\!\!-\!\!\bullet X(j\omega)e^{-j\omega t_0}$

$e^{-at}x(t) \circ\!\!-\!\!\bullet X(j(\omega+a))$ $\qquad a=j\omega_0$

$x(at) \circ\!\!-\!\!\bullet \frac{1}{|a|} X\left(j\frac{\omega}{a}\right)$

2008

$\boxed{9.}$ $x(t) \circ\!\!-\!\!\bullet X(j\omega)$

$y(t) = x(-t) \circ\!\!-\!\!\bullet \frac{1}{|-1|} X\left(j\frac{\omega}{-1}\right) = X(-j\omega) = Y(j\omega)$

$z(t) = y(t-10) \circ\!\!-\!\!\bullet e^{-10j\omega}\, Y(j\omega) = e^{-10j\omega} X(-j\omega)$

2008.

$\boxed{10}$

$x(t) = \cos(t)\,\mu(t-2008)$

$g(t) = x(t+7)$

$|X(j\omega)| = a|G(j\omega)|$, $a=?$

$|X(j\Omega)| - |G(j\Omega)| = 0$

$x(t) \circ\!\!-\!\!\bullet X(j\omega)$ → AMP: $|X(j\omega)|$

$g(t) = x(t+7) \circ\!\!-\!\!\bullet e^{j7\omega} X(j\omega) = G(j\omega)$

AMP: $|G(j\omega)| = |X(j\omega)|$

$x(n) \circ\!\!-\!\!\bullet X(e^{j\omega})$

2009.

$\boxed{16}$ $x(n) = \begin{cases} 2009^n, & 0 \le n \le 2009 \\ 0, & \text{inače} \end{cases}$ $\qquad \omega = \pi$

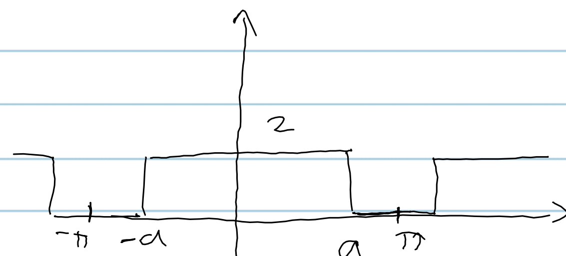
$$X(e^{j\omega}) = \sum_{n=0}^{2008} 2009^n e^{-j\pi n} = \sum_{n=0}^{2008} (2009 \underbrace{e^{-j\pi}}_{-1})^n$$

$$= \sum_{n=0}^{2008} (-2009)^n = \frac{(-2009)^{2009} - 1}{-2009 - 1} = \frac{2009^{2009} + 1}{2010}$$

$$\boxed{\sum_{n=0}^{N} q^n = \frac{q^{N+1} - 1}{q - 1}}$$

2008.

$\boxed{16}$ $X(e^{j\omega}) = \begin{cases} 2, & |\omega| \le a \\ 0, & a < |\omega| < \pi \end{cases}$



$$x(n) = \frac{1}{2\pi} \int_{-a}^{a} 2 e^{j\omega n} d\omega = \frac{1}{\pi} \cdot \frac{1}{jn} e^{j\omega n} \Big|_{-a}^{a} =$$

$$= \frac{1}{\pi n} \cdot \frac{2}{2j} \left(e^{jan} - e^{-jan} \right) = \frac{2}{\pi n} \sin(an)$$

2008.

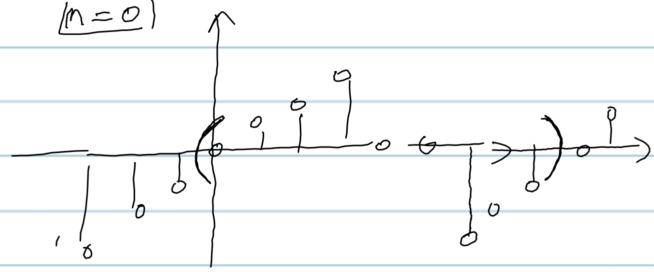
$\boxed{17}$ $X(e^{j\omega}) = \begin{cases} 2\pi, & |\omega| \le a \\ 0, & a < |\omega| < \pi \end{cases}$

$$x(n) = \frac{2}{n} \sin(an) \rightarrow E = \sum_{n=-\infty}^{\infty} \frac{4}{n^2} \sin^2(an)$$

$$E = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(j\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-a}^{a} 4\pi^2 d\omega = 4a\pi$$

$$\tilde{x}(n) = \begin{cases} n, & |n| \leq 3 \\ 0, & n = 4, 5 \end{cases}$$

$$x(k) = \frac{1}{N} \sum_{\boxed{n=0}}^{\boxed{N-1}} x(n) e^{-jk\omega_0 n} \quad \text{(DTFS)}$$



$$x(k) = \frac{1}{9} \sum_{n=0}^{8} x(n) e^{-jk\frac{2\pi n}{9}}$$

$k = 0, \ldots, N-1$