3\. LAB. VJEŽBA SIGNALI I SUSTAVI

varijanta (B)

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Z.R2

$\boxed{313a}$ $y(-1) = y(-2) = 0$

1) • $y(n) - y(n-1) + 0{,}34\, y(n-2) = u(n)$ impulsni $u(n) = \delta(n)$

$h(n) - h(n-1) + 0{,}34\, h(n-2) = \delta(n)$

hom: $c g^n - c g^{n-1} + 0{,}34\, c g^{n-2} = 0$

$c g^{n-2} \underbrace{(g^2 - g + 0{,}34)}_{=0} = 0$

$g_{1,2} = \dfrac{1 \pm \sqrt{1-1{,}36}}{2} = \dfrac{1}{2} \pm \dfrac{3}{10} j$

$\boxed{g_1 = \dfrac{\sqrt{34}}{10} e^{j \cdot 0{,}54}}$ $\boxed{g_2 = \dfrac{\sqrt{34}}{10} e^{-j\, 0{,}54}}$

$h(n) = \left(\dfrac{\sqrt{34}}{10}\right)^n \left[A \cos(0{,}54 n) + B \sin(0{,}54 n)\right]$

– $y(0) - \cancel{y(-1)} + 0{,}34\, \cancel{y(-2)} = \delta(0) = 1$

$y(0) = 1 = h(0) \Rightarrow \boxed{A = 1}$

$y(1) - y(0) + \cancel{0{,}34\, y(-1)} = \cancel{\delta(1)} = 0$

$y(1) = y(0) = 1$ $\quad h(1) = \dfrac{\sqrt{34}}{10}\left[\cos(0{,}54) + B \sin(0{,}54)\right] = 1$ $\quad \boxed{B = 1{,}67}$

$\Rightarrow \boxed{h(n) = \left(\dfrac{\sqrt{34}}{10}\right)^n \left[\cos(0{,}54 n) + 1{,}67 \sin(0{,}54 n)\right] \mu(n)}$

• $y(n) - y(n-1) + 0{,}34\, y(n-2) = \mu(n)$ odziv na step

– $y_h(n) = \left(\dfrac{\sqrt{34}}{10}\right)^n \left[A \cos(0{,}54 n) + B \sin(0{,}54 n)\right]$

– $y_p(n) = K, \ n \geq 0$ $\quad \cancel{K} - \cancel{K} + 0{,}34\, K = 1$

$\boxed{K = 2{,}941}$ $\quad \boxed{y_p(n) = 2{,}941\, \mu(n)}$

$y(0) - \cancel{y(-1)} + 0{,}34\, \cancel{y(-2)} = \mu(0)$ $\quad \boxed{y(0) = 1}$

$y(1) - y(0) + \cancel{0{,}34\, y(-1)} = \mu(1)$ $\quad \boxed{y(1) = 2}$

$\Rightarrow y(n) = \left(\dfrac{\sqrt{34}}{10}\right)^n \left[A \cos(0{,}54 n) + B \sin(0{,}54 n)\right] + 2{,}941$

$y(0) = A + 2{,}941 = 1$ $\quad \boxed{A = -1{,}941}$

$y(1) = \dfrac{\sqrt{34}}{10}\left[-1{,}941 \cos 0{,}54 + B \sin 0{,}54\right] + 2{,}941 = 2 \Rightarrow \boxed{B = 0{,}274}$

$\Rightarrow \boxed{y(n) = \left[\left(\dfrac{\sqrt{34}}{10}\right)^n \left[-1{,}941 \cos(0{,}54 n) + 0{,}274 \sin(0{,}54 n)\right] + 2{,}941\right] \mu(n)}$

2) • $h(n) - \sqrt{2}\, h(n-1) + h(n-2) = \delta(n)$

hom: $g^2 - \sqrt{2}\, g + 1 = 0$ $\quad g_{1,2} = \dfrac{\sqrt{2} \pm \sqrt{2-4}}{2} = \dfrac{\sqrt{2}}{2} \pm \dfrac{\sqrt{2}}{2} j \Rightarrow \boxed{g_1 = e^{j\frac{\pi}{4}} \quad g_2 = e^{-j\frac{\pi}{4}}}$

$h(n) = A \cos\left(\dfrac{\pi}{4} n\right) + B \sin\left(\dfrac{\pi}{4} n\right)$

– $y(0) - \sqrt{2}\, \cancel{y(-1)} + \cancel{y(-2)} = \delta(0)$ $\quad y(0) = 1 \Rightarrow \boxed{A = 1}$

– $y(1) - \sqrt{2}\, y(0) + y(-1) = \delta(1)$ $\quad y(1) = \sqrt{2} \Rightarrow \cos\dfrac{\pi}{4} + B \sin\dfrac{\pi}{4} = \sqrt{2}$

$\dfrac{\sqrt{2}}{2} + B \dfrac{\sqrt{2}}{2} = \sqrt{2}$ $\quad \boxed{B = -1}$

$\Rightarrow \boxed{h(n) = \left[\cos\left(\dfrac{\pi}{4} n\right) - \sin\left(\dfrac{\pi}{4} n\right)\right] \mu(n)}$

• $y(n) - \sqrt{2}\, y(n-1) + y(n-2) = \mu(n)$

– $y_h(n) = A \cos\left(\dfrac{\pi}{4} n\right) + B \sin\left(\dfrac{\pi}{4} n\right)$

– $y_p(n) = K$ $\quad K - \sqrt{2} K + K = 1$ $\quad K = 1{,}707$ $\quad \boxed{y_p(n) = 1{,}707\, \mu(n)}$

$\rightarrow y(n) = A \cos\left(\dfrac{\pi}{4} n\right) + B \sin\left(\dfrac{\pi}{4} n\right) + 1{,}707$

$y(0) - \sqrt{2}\, \cancel{y(-1)} + \cancel{y(-2)} = \mu(0)$ $\quad \boxed{y(0) = 1}$

$y(1) - \sqrt{2}\, y(0) + \cancel{y(-1)} = \mu(1)$ $\quad \boxed{y(1) = 1 + \sqrt{2}}$

$y(0) = A + 1{,}707 = 1 \Rightarrow \boxed{A = -0{,}707}$

$y(1) = -0{,}707 \dfrac{\sqrt{2}}{2} + B \dfrac{\sqrt{2}}{2} + 1{,}707 = 1 + \sqrt{2} \Rightarrow \boxed{B = 1{,}707}$

$\Rightarrow \boxed{y(n) = \left[-0{,}707 \cos\left(\dfrac{\pi}{4} n\right) + 1{,}707 \sin\left(\dfrac{\pi}{4} n\right) + 1{,}707\right] \mu(n)}$

3) • $h(n) - 2\, h(n-1) + h(n-2) = \delta(n)$

$g^2 - 2g + 1 = 0$ $\quad (g-1)^2 = 0$ $\quad g_1 = g_2 = 1$ $\quad \boxed{h(n) = (C_1 + C_2 n)\, 1^n}$

– $y(0) = \delta(0) = 1$ $\quad h(0) = \boxed{C_1 = 1}$

– $y(1) - 2 y(0) = 0$ $\quad y(1) = 2$ $\quad h(1) = C_1 + C_2 = 2$ $\quad \boxed{C_2 = 1}$

$\Rightarrow \boxed{h(n) = \left[(1 + n)\, 1^n\right] \mu(n)}$

• $y(n) - 2\, y(n-1) + y(n-2) = \mu(n)$

– $y_h(n) = (C_1 + C_2 n)\, 1^n$

– $y_p(n) = K n^2 \mu(n)$

$K n^2 - 2K(n-1)^2 + K(n-2)^2 = 1$

$\cancel{Kn^2} - \cancel{2Kn^2} + \cancel{4Kn} - 2K + \cancel{Kn^2} - \cancel{4Kn} + 4K = 1$

$\boxed{K = \dfrac{1}{2}}$ $\quad \boxed{y_p(n) = \dfrac{1}{2} n^2}$

$\boxed{y(0) = \mu(0) = 1}$

$\boxed{y(1) = \mu(1) + 2\, y(0) = 3}$

$y(n) = (C_1 + C_2 n)\, 1^n + \dfrac{1}{2} n^2$

$y(0) = \boxed{C_1 = 1}$

$y(1) = 1 + C_2 + \dfrac{1}{2} = 3$ $\quad \boxed{C_2 = \dfrac{3}{2}}$

$\Rightarrow \boxed{y(n) = \left[\left(1 + \dfrac{3}{2} n\right) 1^n + \dfrac{1}{2} n^2\right] \mu(n)}$

4) • $h(n) - 4h(n-1) + 13h(n-2) = \delta(n)$

hom $\quad g^2 - 4g + 13 = 0 \qquad g_{1,2} = \frac{4 \pm \sqrt{16-52}}{2} \qquad g_{1,2} = 2 \pm 3j$

$\rightarrow h(n) = (\sqrt{13})^n \left[A\cos(0{,}983n) + B\sin(0{,}983n) \right] \qquad |g| = \sqrt{13} \quad \angle g = 0{,}983$

$y(0) = \delta(0) = 1 \qquad h(0) = \boxed{A = 1}$

$y(1) = 4y(0) = 4$

$h(1) = \sqrt{13}\left[\cos 0{,}983 + B\sin 983\right] \Rightarrow \boxed{B = 0{,}667}$

$$\Rightarrow \boxed{h(n) = \left[\cos(0{,}983n) + 0{,}667\sin(0{,}983n)\right]\mu(n)}$$

• $y(n) - 4y(n-1) + 13y(n-2) = \mu(n)$

– $y_h(n) = (\sqrt{13})^n\left[A\cos(0{,}983n) + B\sin(0{,}983n)\right]$

– $y_p = K \qquad K - 4K + 13K = 1 \qquad \boxed{K = 0{,}1} \qquad \boxed{y_p(n) = 0{,}1\,\mu(n)}$

$\rightarrow y(n) = (\sqrt{13})^n\left[A\cos(0{,}983n) + B\sin(0{,}983n)\right] + 0{,}1$

$y(0) = \mu(0) = 1 \qquad y(0) = A + 0{,}1 = 1 \Rightarrow \boxed{A = 0{,}9}$

$y(1) = \mu(1) + 4y(0) = 5$

$y(1) = \sqrt{13}\left(0{,}9\cos 0{,}983 + B\sin 0{,}983\right) + 0{,}1 = 5 \Rightarrow \boxed{B = 1{,}033}$

$$\Rightarrow \boxed{y(n) = \left[(\sqrt{13})^n\left[0{,}9\cos(0{,}983n) + 1{,}033\sin(0{,}983n)\right] + 0{,}1\right]\mu(n)}$$

314a partikularno za harmonijske pobude

$u(n) = A\cos(\omega_0 n)$ ili $u(n) = A\sin(\omega_0 n)$

$\rightarrow y_p(n) = C_1\cos(\omega_0 n) + C_2\sin(\omega_0 n)$

• kad se frekv. pobude poklopi s vlastitom frekv. sustava → **rezonancija** – nestabilnost

315a

1) $|g_1| = |g_2| = \frac{\sqrt{34}}{10} = 0{,}583 < 1 \rightarrow$ stabilan

2) $|g_1| = |g_2| = 1 \rightarrow$ marginalno stabilan (nije višestruka)

3) $|g_1| = |g_2| = 1$ – dvostruka! → nestabilan

4) $|g_1| = |g_2| = \sqrt{13} > 1 \rightarrow$ nestabilan

321a kauzalnost, linearnost, memorijski ?

1) $y(t) = \int_{-\infty}^{t} u(\tau)\,d\tau \qquad$ INTEGRATOR

LIN $\quad u(t) = \alpha u_1(t) + \beta u_2(t) \qquad y_1(t) = \int_{-\infty}^{t} u_1(\tau)\,d\tau$

$y_2(t) = \int_{-\infty}^{t} u_2(\tau)\,d\tau$

$y(t) = \int_{-\infty}^{t} u(\tau)\,d\tau = \int_{-\infty}^{t}\left[\alpha u_1(\tau) + \beta u_2(\tau)\right]d\tau = \alpha\int_{-\infty}^{t} u_1(\tau)\,d\tau + \beta\int_{-\infty}^{t} u_2(\tau)\,d\tau = \boxed{\alpha y_1(t) + \beta y_2(t)}$

KAUZ $\quad \forall t \in \mathbb{R} \quad y(t) = f\left(u_{[-\infty,t]}\right)(t)$

MEM $\quad y(t) = \int_{-\infty}^{t} u(\tau)\,d\tau = \int_{-\infty}^{t_0} u(\tau)\,d\tau + \int_{t_0}^{t} u(\tau)\,d\tau = y(t_0) + \int_{t_0}^{t} u(\tau)\,d\tau$

sav info o signalu prije t_0

2) $y(t) = \frac{d}{dt}u(t) \qquad$ DERIVATOR

LIN $\quad u(t) = \alpha u_1(t) + \beta u_2(t) \qquad y_1(t) = \frac{d}{dt}u_1(t) \qquad y_2(t) = \frac{d}{dt}u_2(t)$

$y(t) = \frac{d}{dt}u(t) = \frac{d}{dt}\left[\alpha u_1(t) + \beta u_2(t)\right] = \alpha\frac{d}{dt}u_1(t) + \beta\frac{d}{dt}u_2(t) = \boxed{\alpha y_1(t) + \beta y_2(t)}$

KAUZ / MEM $\quad y(t) = \lim_{\Delta t \to 0}\frac{u(t+\Delta t) - u(t)}{\Delta t} \rightarrow$ mem. i nekauzalan

322a impulsni odziv $\qquad u(t) = \delta(t) \qquad y(0^+) = 0 \qquad y'(0^+) = 1$

1) $y''(t) + 2y'(t) + 15y(t) = u(t)$

$h(t) = h_A(t) \qquad h_A''(t) + 2h_A'(t) + 15h_A(t) = \delta(t) \qquad h_A(t) = ce^{st}$

$h_A'(t) = sce^{st}$

$s^2 + 2s + 15 = 0$

$\boxed{s_{1,2} = -1 \pm j\,3{,}741}$

$h_A(t) = e^{-t}\left[A\cos(3{,}741t) + B\sin(3{,}741t)\right]$

$h_A'(t) = -e^{-t}\left[A\cos(3{,}741t) + B\sin(3{,}741t)\right] + e^{-t}\left[-3{,}741A\sin(3{,}741t) + 3{,}741B\cos(3{,}741t)\right]$

$h_A(0^+) = \boxed{A = 0}$

$h_A'(0^+) = -A + 3{,}741B = 1 \Rightarrow \boxed{B = 0{,}267}$

$$\boxed{h(t) = h_A(t) = 0{,}267\,e^{-t}\sin(3{,}741t)\,\mu(t)}$$

3. LAB. VJEŽBA SIGNALI I SUSTAVI Damir Bučar ②

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322a

2) $y''(t) + 17y(t) = u(t)$ $h_A(0^+) = 0$, $h_A'(0^+) = 1$

$h_A''(t) + 17h_A(t) = \delta(t)$

$s^2 + 17 = 0$ $\boxed{s_{1,2} = \pm j\,4{,}123}$ $h_A(t) = A\cos(4{,}123t) + B\sin(4{,}123t)$

$h_A'(t) = -4{,}123A\sin(4{,}123t) + 4{,}123B\cos(4{,}123t)$

$h_A(0^+) = \boxed{A = 0}$

$h_A'(0^+) = 4{,}123B = 1$ $\boxed{B = 0{,}246}$ $\boxed{h(t) = [0{,}246\sin(4{,}123t)]\mu(t)}$

3) $y''(t) = u(t)$

$h_A''(t) = \delta(t)$ $s^2 = 0$ $s_1 = s_2 = 0$

$h_A(t) = C_1e^{0t} + C_2te^{0t} = C_1 + C_2t$

$h_A'(t) = C_2$ $\boxed{C_1 = 0}$ $\boxed{C_2 = 1}$ $h_A(t) = t$

$\boxed{h(t) = t\mu(t)}$

4) $y''(t) - 2y'(t) + 10y(t) = u(t)$

$h_A''(t) - 2h_A'(t) + 10h_A(t) = \delta(t)$

$s^2 - 2s + 10 = 0$ $s_{1,2} = \frac{2 \pm \sqrt{4-40}}{2} = 1 \pm j3$

$h_A(t) = e^t[A\cos 3t + B\sin 3t]$

$h_A'(t) = e^t[A\cos 3t + B\sin 3t] + e^t[-3A\sin 3t + 3B\cos 3t]$

$h_A(0^+) = \boxed{0 = A}$

$h_A'(0^+) = 1 = A + 3B$ $\boxed{B = \frac{1}{3}}$ $\boxed{h(t) = \frac{1}{3}e^t\sin 3t\;\mu(t)}$

322b

1) $s^2 + 2s + 15 = 0$

2) $s^2 + 17 = 0$

3) $s^2 = 0$

4) $s^2 - 2s + 10 = 0$

325a

$\frac{d}{dt}u_{iz}(t) + \frac{1}{RC}u_{iz}(t) = \frac{d}{dt}u_{ul}(t)$ $u_{iz}(t) = y(t)$, $u_{ul}(t) = u(t)$

$y'(t) + \frac{1}{RC}y(t) = u'(t)$ $u(t) = \begin{cases} \sin t, & t < 0^- \\ 1, & t > 0^+ \end{cases}$

∘ za pobudu $u(t) = \sin(t)$ tražimo samo partikularno [hom. se istitralo]

$\boxed{u(0^-) = 0 \quad u(0^+) = 1}$

$y_p(t) = K_1\cos t + K_2\sin t$

$y_p'(t) = -K_1\sin t + K_2\cos t$

$-K_1\sin t + K_2\cos t + \frac{1}{RC}K_1\cos t + \frac{1}{RC}K_2\sin t = -\cos t$ $\left(\frac{1}{RC} = 100\right)$

$\cos t\,[K_2 + 100K_1] + \sin t\,[100K_2 - K_1] = -\cos t$

$K_2 + 100K_1 = -1$

$100K_2 - K_1 = 0 \quad /\cdot 100 \quad |+$

$10001K_2 = -1$ $\boxed{K_2 = -0{,}0001}$ $\boxed{K_1 = -0{,}01}$

$\boxed{y_p(t) = -0{,}01\cos t - 0{,}0001\sin t}$

$t = 0^-$ $\boxed{y(0^-) = -0{,}01}$

$y(0^+) - y(0^-) = b_0u(0^+)$

$\boxed{y(0^+)} = 1\cdot 1 - 0{,}01 = \boxed{0{,}99}$

327a

$y''(t) + 2\xi\Omega_n y'(t) + \Omega_n^2 y(t) = A\Omega_n^2u(t)$ $\boxed{\Omega_n = 0{,}4}$

$H(s) = \frac{A\Omega_n^2}{s^2 + 2\xi\Omega_n s + \Omega_n^2}$ $A = \frac{1}{(0{,}4)^2} = \boxed{6{,}25}$

a) $\xi = -0{,}125$ $H(s) = \frac{6{,}25\cdot 0{,}4^2}{s^2 + 2\cdot(-0{,}125)\cdot 0{,}4s + 0{,}4^2}$

$H(s) = \frac{1}{s^2 - 0{,}1s + 0{,}16}$

$H(j\omega) = \frac{1}{(j\omega)^2 - 0{,}1j\omega + 0{,}16} = \frac{1}{0{,}16 - \omega^2 - 0{,}1j\omega}$

$\boxed{|H(j\omega)| = \frac{1}{\sqrt{(0{,}16-\omega^2)^2 + (0{,}1\omega)^2}}}$ $\boxed{\measuredangle H(j\omega) = \operatorname{arct}\frac{0{,}1\omega}{0{,}16-\omega^2}}$

b) $\xi = 0{,}25$ $H(s) = \frac{1}{s^2 + 0{,}2s + 0{,}16}$ $H(j\omega) = \frac{1}{0{,}16 - \omega^2 + 0{,}2j\omega}$

$\boxed{|H(j\omega)| = \frac{1}{\sqrt{(0{,}16-\omega^2)^2 + (0{,}2\omega)^2}}}$ $\boxed{\measuredangle H(j\omega) = \operatorname{arct}\frac{0{,}2\omega}{0{,}16-\omega^2}}$

c) $\xi = 1$ $H(s) = \frac{1}{s^2 + 0{,}8 + 0{,}16}$ $H(j\omega) = \frac{1}{0{,}16 - \omega^2 + 0{,}8j\omega}$

$\boxed{|H(j\omega)| = \frac{1}{\sqrt{(0{,}16-\omega^2)^2 + (0{,}8\omega)^2}}}$ $\boxed{\measuredangle H(j\omega) = \frac{0{,}8\omega}{0{,}16-\omega^2}}$

$\boxed{327c}$

$u(t) = \cos(2t)$

1) $\sphericalangle H(j\omega) = \operatorname{arctg} \frac{0{,}1\,\omega}{0{,}16-\omega^2} = \operatorname{arctg} \frac{0{,}2}{0{,}16-4} = \underline{-2{,}98^\circ}$

$|H(j\omega)| = \frac{1}{\sqrt{(0{,}16-4)^2 + (0{,}2)^2}} = \underline{0{,}26}$

$y(t) = 0{,}26 \cos(2t - 2{,}98^\circ)$

2) $|H(j\omega)| = \frac{1}{\sqrt{(0{,}16-4)^2 + 0{,}4^2}} = \underline{0{,}259}$

$\sphericalangle H(j\omega) = \operatorname{arctg} \frac{0{,}4}{0{,}16-4} = \underline{-5{,}94^\circ}$

$y(t) = 0{,}259 \cos(2t - 5{,}94^\circ)$

3) $|H(j\omega)| = \frac{1}{\sqrt{(0{,}16-4)^2 + 1{,}6^2}} = \underline{0{,}24}$

$\sphericalangle H(j\omega) = \operatorname{arctg} \frac{0{,}8 \cdot 2}{0{,}16-4} = \underline{-22{,}61^\circ}$

$y(t) = 0{,}24 \cos(2t - 22{,}61^\circ)$