NAPOMENA: Od niže navedenih zadataka, u prvi međuispit ulaze zadaci 1, 5, 6, 7. Dakle, prilikom spremanja prvog međuispita možete slobodno zanemariti zadatke 2, 3, 4 – za njihovo rješavanje potrebna su znanja iz osme cjeline predavanja, koja ne ulazi u prvi međuispit!

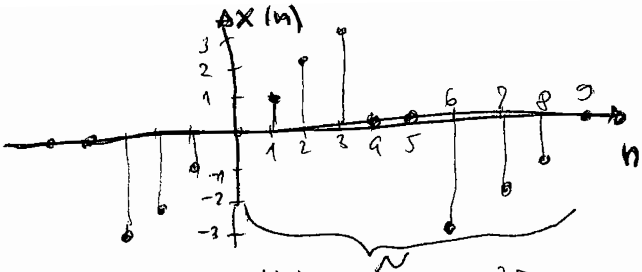
1. Funkciju sgn možemo zapisati preko step funkcije
$$\mathrm{sgn}(t) = 2\mu(t) - 1$$
Koristeći svojstvo linearnosti Fourierove transformacije dobivamo:
$$F\left(\mathrm{sgn}(t)\right) = 2F\left(\mu(t)\right) - F\left(1\right)$$

Uz oznaku $F\left(\mathrm{sgn}(t)\right) = X(j\Omega)$, te prema tabličnim izrazima za Fourierove transformate step funkcije i konstante slijedi,
$$X\left(j\Omega\right) = 2\left(\pi\delta(\Omega) + \frac{1}{j\Omega}\right) - 2\pi\delta(\Omega),$$
$$X\left(j\Omega\right) = \frac{1}{j\Omega}$$

2. Osnovni period signala je $N_0 = 24$, prema tome $\Omega_0 = \frac{2\pi}{24} = \frac{\pi}{12}$
Primjenom Eulerove formule
$$x(n) = \frac{1}{2}\left(e^{j\frac{\pi}{3}n} + e^{-j\frac{\pi}{3}n}\right) + \frac{1}{2j}\left(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right)$$
$$= \frac{1}{2}e^{-j4\Omega_0 n} + j\frac{1}{2}e^{-j3\Omega_0 n} - j\frac{1}{2}e^{j3\Omega_0 n} + \frac{1}{2}e^{j4\Omega_0 n}$$

Prema tome, $X_{-4} = \frac{1}{2}$, $X_{-3} = j\frac{1}{2}$, $X_3 = -j\frac{1}{2}$, $X_4 = \frac{1}{2}$

3.



$N = 9$

$$X_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)\, e^{-jk\frac{2\pi}{N}\cdot n}$$

$$X_k = \frac{1}{9}\sum_{n=0}^{8} x(n)\, e^{-jk\frac{2\pi}{9}\cdot n}$$

$$= \frac{1}{9}\left[0 + e^{-jk\frac{2\pi}{9}} + 2e^{-jk\frac{4\pi}{9}} + 3e^{-jk\frac{6\pi}{9}} - 3e^{-jk\frac{12\pi}{9}} - 2e^{-jk\frac{14\pi}{9}} - e^{-jk\frac{16\pi}{9}}\right]$$

$$= \frac{1}{9}\Big[\cancel{\cos\frac{2\pi}{9}k} - j\sin\frac{2\pi}{9}k + \cancel{2\cos\frac{4\pi}{9}k} - 2j\sin\frac{4\pi}{9}k + \cancel{3\cos\frac{6\pi}{9}k} - 3j\sin\frac{6\pi}{9}k$$

$$-3\left(\cancel{\cos\frac{12\pi}{9}k} - j\sin\frac{12\pi}{9}k\right) - 2\left(\cancel{\cos\frac{14\pi}{9}k} - j\sin\frac{14\pi}{9}k\right) - 1\left(\cancel{\cos\frac{16\pi}{9}k} - j\sin\frac{16\pi}{9}\right.$$

$\Rightarrow$

$$\cos k\frac{2\pi}{9} = \cos\frac{16\pi}{9}k$$

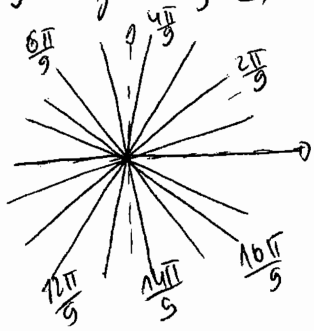
$$\cos\frac{4\pi}{9}k = \cos\frac{14\pi}{9}k$$

$$\cos\frac{6\pi}{9}k = \cos\frac{12\pi}{9}k$$

$$\sin\frac{2\pi}{9}k = -\sin\frac{16\pi}{9}k$$

$$\sin\frac{4\pi}{9}k = -\sin\frac{14\pi}{9}k$$

$$\sin\frac{6\pi}{9}k = -\sin\frac{12\pi}{9}k$$



$$X_k = \frac{1}{9}\left[-2j\sin\frac{2\pi}{9}k - 4j\sin\frac{4\pi}{9}k + 6j\sin\frac{12\pi}{9}k\right]$$

$$= -\frac{2j}{9}\left[\sin\frac{2\pi}{9}k + 2\sin\frac{4\pi}{9}k + 3\sin\frac{6\pi}{9}k\right]$$

DISKRETAN PERIODIČAN SPEKTAR

$$P_x = \frac{1}{N}\sum_{n=0}^{N-1}|x(n)|^2 = \sum_{k=0}^{N-1}|X_k|^2$$

$$= \frac{1}{9}\sum_{n=0}^{8}|x(n)|^2 = \frac{1}{9}\left[0 + 1^2 + 2^2 + 3^2 + 0^2 + 0^2 + (-3)^2 + (-2)^2 + (-1)^2\right]$$

$$= \frac{1}{9}\left[1 + 4 + 9 + 9 + 4 + 1\right] = \underline{\frac{28}{9}}$$

4.

$x(t) = 2\cos(200\pi t) + 3\cos(500\pi t)$

$F_s = 1000\,Hz$

$T_s = \frac{1}{F_s} = 10^{-3}\,s$

otipkani signal

$x(n) = 2\cos(200\pi \cdot nT_s) + 3\cos(500\pi \cdot nT_s)$

$= 2\cos\left(\frac{\pi}{5}n\right) + 3\cos\left(\frac{\pi}{2}n\right)$

periodičan s periodom $N_1 = 10$

$N_2 = 4$

suma je periodična sa $N = 20$

$x(n) = 2\cos\left(\frac{2\pi}{20}\cdot 2n\right) + 3\cos\left(\frac{2\pi}{20}\cdot 5n\right)$

$= \frac{2}{2}e^{j\frac{2\pi}{20}2n} + \frac{2}{2}e^{-j\frac{2\pi}{20}\cdot 2n} + \frac{3}{2}e^{j\frac{2\pi}{20}5n} + \frac{3}{2}e^{-j\frac{2\pi}{20}5n}$

$e^{-j\frac{2\pi}{20}\cdot 2n} = e^{j\frac{2\pi}{20}\cdot 18n}$

$e^{-j\frac{2\pi}{20}5n} = e^{j\frac{2\pi}{20}15n}$

$x(n) = e^{j\frac{2\pi}{20}2n} + e^{j\frac{2\pi}{20}18n} + \frac{3}{2}e^{j\frac{2\pi}{20}5n} + \frac{3}{2}e^{j\frac{2\pi}{20}15n}$

FOURIEROV RED

$X_k = \frac{1}{N}\sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn}$

$= \frac{1}{20}\sum_{k=0}^{19} x(n)e^{-j\frac{2\pi}{20}kn}$

$x(n) = \sum_{k=0}^{N-1} X_k e^{j\frac{2\pi}{N}kn}$

$= \sum_{k=0}^{19} X_k e^{j\frac{2\pi}{20}kn}$

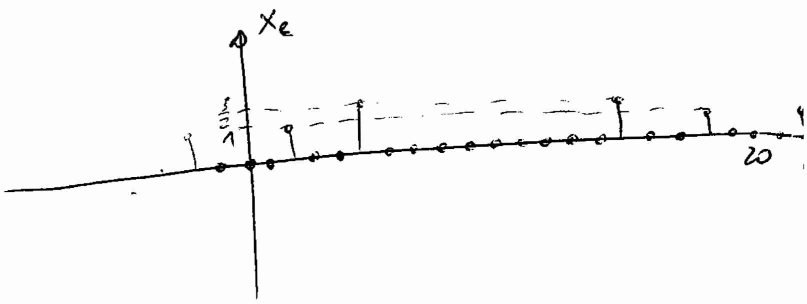
za $k = 2$ $\quad X_k = 1$

$k = 18$ $\quad X_k = 1$

$k = 5$ $\quad X_k = \frac{3}{2}$

$k = 15$ $\quad X_k = \frac{3}{2}$

za ostale $k \rightarrow X_k = 0$

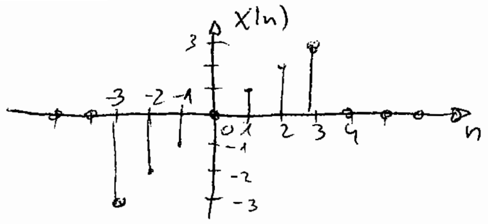


5. Signal je periodičan s osnovnim periodom $T_p = 4 \Rightarrow \Omega_p = \frac{2\pi}{4} = \frac{\pi}{2}$

$$X_m = \frac{1}{4}\int_{0^-}^{4^-} \delta(t) e^{-jm\frac{\pi}{2}t}\,dt = \frac{1}{4},\ \forall m \in \mathbb{Z}$$

6

$$x(n)=\begin{cases} n, & |n|\le 3 \\ 0, & \text{inače}\end{cases}$$



$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$= -3e^{+j\omega\cdot 3} - 2e^{2j\omega} - e^{j\omega} + e^{-j\omega} + 2e^{-j\omega\cdot 2} + 3e^{-3j\omega}$$

$$= 3(-\cancel{\cos 3\omega} - j\sin 3\omega + \cancel{\cos 3\omega} - j\sin 3\omega)$$

$$+2(-\cancel{\cos 2\omega} - j\sin 2\omega + \cancel{\cos 2\omega} - j\sin 2\omega)$$

$$+\cancel{\cos\omega} - j\sin\omega - \cancel{\cos\omega} - j\sin\omega$$

$$= -2j(3\sin 3\omega + 2\sin 2\omega + \sin\omega)$$

KONTINUIRANI PERIODIČAN SPEKTAR

ENERGIJA

$$E_x = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega$$

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \dots + |-3|^2 + |-2|^2 + |-1|^2 + 0^2 + 1^2 + 2^2 + 3^2 + 0 + \dots$$

$$= 9+4+1+1+4+9 = \boxed{28}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |-2j(3\sin 3\omega + 2\sin 2\omega + \sin\omega)|^2 d\omega =$$

$$= \frac{1}{\cancel{2}\pi}\int_{-\pi}^{\pi} \cancel{4}^{\,2}(3\sin 3\omega + 2\sin 2\omega + \sin\omega)^2 d\omega$$

$$9\sin^2 3\omega + 4\sin^2 2\omega + \sin^2\omega + 6\sin 3\omega\cdot\sin 2\omega + 3\sin 3\omega\sin\omega$$

$$+ 6\sin 3\omega\sin 2\omega + 3\sin 3\omega\sin\omega$$

$$+ 2\sin 2\omega\sin\omega + 2\sin 2\omega\sin\omega$$

$$= \dots = \boxed{28}$$

7. S obzirom da je zadan Fourierov transformat, za izračun signala u vremenskoj domeni, koristimo inverznu Fourierovu transformaciju

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} x(e^{j\omega})e^{j\omega n}d\omega =$$

$$= \frac{1}{2\pi}\int_{-w}^{w} e^{j\omega n}d\omega = \frac{\sin wn}{\pi n}$$

Prema tome vrijedi:

$$\frac{\sin wn}{\pi n} \leftrightarrow \begin{cases} 1, & |\omega| \le w \\ 0, & w < |\omega| \le \pi \end{cases}$$