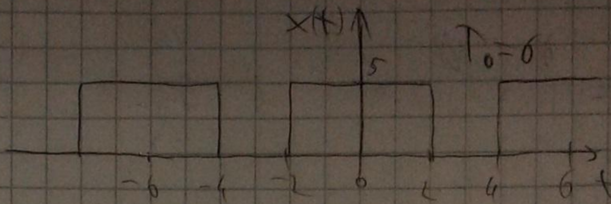
Jesenski 2015.

1.

$T_p = 6$



a) CTFS? amp. spektar.

$$X[k] = \frac{1}{T_p}\int_{T_p} x(t)\, e^{-j\omega_p k t}\, dt = \frac{5}{6}\int_{-2}^{2} e^{-j\omega_p k t}\, dt =$$

$$= \frac{5}{6}\cdot\frac{1}{-j\omega_p k}\, e^{-j\omega_p k t}\Big|_{-2}^{2} = \frac{5}{6}\,\frac{1}{-j\omega_p k}\left(e^{-2j\omega_p k} - e^{2j\omega_p k}\right) =$$

$$= \frac{5}{6}\,\frac{1}{j\omega_p k}\left(e^{2j\omega_p k} - e^{-2j\omega_p k}\right) = \frac{5}{3}\,\frac{1}{j\omega_p k}\sin(2\omega_p k)$$

$$\omega_p = 2\pi / T_p = \frac{\pi}{3}$$

$$\Rightarrow X[k] = \frac{5}{j\pi k}\sin\left(\frac{2\pi}{3}k\right)$$

b) $P = ?$

$$P = \frac{1}{6}\int_{-2}^{2} 25\, dt = \frac{25}{6}\, t\Big|_{-2}^{2} = \frac{25}{3} + \frac{25}{3} = \frac{50}{3}$$

c) $$\frac{1}{T_p}\int_{T_p} x(t)\, x^*(t)\, dt = \sum_{k=-\infty}^{\infty} X_1[k]\, X_2^*[k]$$

$$\frac{1}{T_p}\int_{T_p} x(t)\, x^*(t)\, dt = \frac{1}{T_p}\int_{T_p} x(t)\cdot\left(\sum_{k=-\infty}^{\infty} X^*[k]\, e^{-j\omega_p k t}\right) dt =$$

$$= \sum_{k=-\infty}^{\infty} X^*[k]\cdot\left(\frac{1}{T_p}\int_{T_p} x(t)\, e^{-j\omega_p k t}\, dt\right) = \sum_{k=-\infty}^{\infty} X^*[k]\cdot X[k]$$

d) CTFT 3

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = 5\int_{-2}^{2} e^{-j\omega t}\, dt = 5\,\frac{1}{-j\omega}\, e^{-j\omega t}\Big|_{-2}^{2} =$$

$$= \frac{5}{-j\omega}\left(e^{-2j\omega} - e^{2j\omega}\right) = \frac{5}{j\omega}\left(e^{2j\omega} - e^{-2j\omega}\right) = \frac{10}{j\omega} \sin(2\omega)$$

2.

$$f(t) = 2\sin\left(400\pi t + \frac{\pi}{6}\right) - 4\cos\left(800\pi t + \frac{\pi}{6}\right), \quad f_s = 300\,Hz$$

a) $f(n) = ?$

$$f(n) = 2\sin\left(400\pi \frac{n}{f_s} + \frac{\pi}{6}\right) - 4\cos\left(800\pi \frac{n}{f_s} + \frac{\pi}{6}\right) =$$

$$= 2\sin\left(\frac{4}{3}\pi n + \frac{\pi}{6}\right) - 4\cos\left(\frac{8}{3}\pi n + \frac{\pi}{8}\right)$$

b) DTFS $(f(n))$?

$$\frac{3}{4}\cancel{\pi} = \frac{2\cancel{\pi}k}{N} \Rightarrow N = \frac{2k}{\frac{3}{4}} = \frac{8k}{3} \in \mathbb{Z} \Rightarrow k = 3 \Rightarrow \underline{N = 8}$$

$$X[k] = \frac{1}{8}\sum_{n=0}^{7} f(n)\, e^{-2\pi j k \frac{n}{N}} =$$

$$= \frac{1}{4}\sum_{n=0}^{7} \sin\left(\frac{4}{3}\pi n + \frac{\pi}{6}\right) e^{-2\pi j k \frac{n}{N}} - \frac{1}{2}\sum_{n=0}^{7} \cos\left(\frac{4}{3}\pi n + \frac{\pi}{6}\right) e^{-2\pi j k \frac{n}{N}} = X[k]$$

$$\cancel{\sum_{n=0}^{7} \sin\left(\frac{4}{3}\pi n + \frac{\pi}{6}\right) e^{-2\pi j k \frac{n}{N}} = \sin\frac{\pi}{6} + \sin\frac{3\pi}{2} e^{-2\pi j k \frac{1}{N}} +}$$

$$\sum_{n=0}^{7} \sin\left(\frac{4}{3}\pi n + \frac{\pi}{6}\right) e^{-2\pi j k \frac{n}{N}} = \sum_{n=0}^{7} \frac{1}{2}\left(e^{\frac{4}{3}\pi n + \frac{\pi}{6} - 2\pi j k \frac{n}{N}} - e^{-\frac{4}{3}\pi n - \frac{\pi}{6} - 2\pi j k \frac{n}{N}}\right)$$

... nemam pojma

1) $\delta(t) = 2\sin\left(\frac{4}{3}\pi t + \frac{\pi}{6}\right) - 4\cos\left(\frac{8}{3}\pi t + \frac{\pi}{8}\right) \rightarrow$ vjerojatno krivo

2. $y''(t) + 8y'(t) + 15y(t) = u'(t) + 2u(t)$

a) $u(t) = \mu(t)$, $y(0^-) = y'(0^-) = 0$

$u'(t) = \delta(t)$

$s^2 + 8s + 15 = 0$

$s_{1,2} = \frac{-8 \pm \sqrt{64 - 60}}{2} = \frac{-8 \pm 2}{2} = -3, -5$

$\Rightarrow y_h = C_1 e^{-3t} + C_2 e^{-5t}$

$y_p = K$

$y'_p = y''_p = 0$ $\Rightarrow 0 + 0 + 15K = 1 \Rightarrow K = \frac{1}{15}$

$\Rightarrow y_p = \frac{1}{15}\mu(t)$

$\Rightarrow y_u = C_1 e^{-3t} + C_2 e^{-5t} + \frac{1}{15}\mu(t)$

$y(0^+) - y(0^-) = 0 \Rightarrow \underline{y(0^+)} = y(0^-) = \underline{0}$

$y'(0^+) - y'(0^-) + 8 \cdot 0 = 0 + 1 \cdot 1$

$\Rightarrow \underline{y'(0^+)} = 0 + 0 + 0 + 1 = \underline{1}$

$y(0^+) = C_1 + C_2 + \frac{1}{15} = 0 \quad / \cdot 3$

$y'(0^+) = -3C_1 - 5C_2 = 1$

$3C_1 + 3C_2 = -\frac{1}{5}$

$-3C_1 - 5C_2 = 1$ $\quad / +$

$-2C_2 = \frac{4}{5}$

$\Rightarrow \underline{C_2 = -\frac{2}{5}} \Rightarrow \underline{C_1} = -\frac{1}{15} + \frac{2}{5} = -\frac{1}{15} + \frac{6}{15} = \frac{5}{15} = \underline{\frac{1}{3}}$

$\Rightarrow y_u = \frac{1}{3}e^{-3t} - \frac{2}{5}e^{-5t} + \frac{1}{15}\mu(t)$

b) $y_{nep} = ?$, $y(0^-)=-1$, $y'(0)=1$

$$y''(t)+8y'(t)+15y(t)=0$$

$$s^2+8s+15=0$$

$$s_{1,2}=-3,-5$$

$$\Rightarrow y_{nep}=C_1e^{-3t}+C_2e^{-5t}$$

$$y(0)=C_1+C_2=-1 \quad /\cdot 3$$

$$y'(0^-)=-3C_1-5C_2=1$$

$$3C_1+3C_2=-3$$

$$-3C_1-5C_2=1$$

$$-2C_2=-2$$

$$\Rightarrow C_2=1 \Rightarrow C_1=-2$$

$$\Rightarrow y_{nep}=-2e^{-3t}+e^{-5t}$$

nepotrebno → piše da treba u laplasu, a ne ovako

$$\cancel{\Rightarrow Y(s)=\frac{\frac{5}{2}}{s+3}-\frac{\frac{3}{2}}{s+5}}$$

$$\cancel{\Rightarrow y_{nep}(t)=\frac{5}{2}e^{-3t}-\frac{3}{2}e^{-5t}}$$

$$y(t) \circ\!\!-\!\!\bullet\ Y(s)$$

$$y'(t) \circ\!\!-\!\!\bullet\ sY(s)-y(0)$$

$$y''(t) \circ\!\!-\!\!\bullet\ s^2Y(s)-sy(0)-y'(0)$$

$$\Rightarrow$$

$$s^2Y(s)+s-1+8sY(s)+8+15Y(s)=0$$

$$Y(s)(s^2+8s+15)=-7-s$$

$$Y(s)=\frac{-7-s}{s^2+8s+15}=\frac{A}{s+3}+\frac{B}{s+5}$$

$$=\frac{As+5A+Bs+3B}{(s+3)(s+5)}$$

$$\Rightarrow A+B=-1 \Rightarrow A=-1-B$$

$$5A+3B=-7$$

$$-5-5B+3B=-7$$

$$-2B=-2$$

$$B=1 \Rightarrow A=-1-1=-2$$

$$\Rightarrow Y(s)=\frac{-2}{s+3}+\frac{1}{s+5}$$

$$\Rightarrow y_{nep}(t)=-2e^{-3t}+e^{-5t}$$

c) Totalni odziv $u(t)=\mu(t)$, $y(0)=-1$, $y'(0^-)=1$

$$y_{tot}=y_m+y_{nep}=\frac{1}{3}e^{-3t}-\frac{2}{5}e^{-5t}+\frac{1}{15}\mu(t)-2e^{-3t}+e^{-5t}=$$

$$=-\frac{5}{3}e^{-3t}+\frac{3}{5}e^{-5t}+\frac{1}{15}\mu(t)$$

4. $y(n) - \frac{1}{5}y(n-1) = u(n) + 2u(n-1)$

a) $h(n) = ?$, $u(n) = \delta(n)$

$q^n - \frac{1}{5}q^{n-1} = 0 \quad /: q^{n-1}$

$q - \frac{1}{5} = 0 \Rightarrow q = \frac{1}{5}$

$\Rightarrow h(n) = \left(\frac{1}{5}\right)^n$? ?

b) $H(z) = ?$ stabilnost?

$y(n) \circ\!\!-\!\!\bullet\ Y(z)$

$y(n-1) \circ\!\!-\!\!\bullet\ z\,Y(z) = \ldots$

$Y(z) - \frac{1}{5}z^{-1}Y(z) = U(z)$

$Y(z)\left(1 - \frac{1}{5}z^{-1}\right) = U(z)$

$Y(z) = \frac{U(z)}{1-\frac{1}{5}z^{-1}} \Rightarrow H(z) = \frac{1}{1-\frac{1}{5}z^{-1}} = \frac{z}{z-\frac{1}{5}} \Rightarrow$ sustav je stabilan

? ?

c) $u(n) = \mu(n)$

$U(z) = \frac{z}{z-1} + 2\frac{1}{z}\frac{z}{z-1} + 0 = \frac{z}{z-1} + \frac{2}{z-1} = \frac{2+z}{z-1}$

~~$Y(z) = H(z)\cdot U(z) = \frac{z}{z-\frac{1}{5}}\left(\frac{z}{z-1} + \frac{2}{z-1}\right) = \frac{z^2}{(z-\frac{1}{5})(z-1)} + \frac{2z}{(z-\frac{1}{5})(z-1)}$~~

$$\frac{z}{(z-\frac{1}{5})(z-1)} = \frac{A}{z-\frac{1}{5}} + \frac{B}{z-1} = \frac{Az - A + Bz - \frac{1}{5}B}{(z-\frac{1}{5})(z-1)}$$

$\Rightarrow \quad A + B = 1$

$\quad -A - \frac{1}{5}B = 0 \Rightarrow A = -\frac{1}{5}B$

$\Rightarrow -\frac{1}{5}B + B = 1 \Rightarrow B = \frac{5}{4} \Rightarrow A = -\frac{1}{4}$

$$\Rightarrow \frac{z}{(z-\frac{1}{5})(z-1)} = \frac{-\frac{1}{4}}{z-\frac{1}{5}} + \frac{\frac{5}{4}}{z-1}$$

$$\Rightarrow \frac{z^2}{(z-\frac{1}{5})(z-1)} = -\frac{1}{4}\frac{z}{z-\frac{1}{5}} + \frac{5}{4}\frac{z}{z-1}$$

$$Y(z) = H(z) \cdot U(z) = \frac{z}{z-\frac{1}{5}} \cdot \frac{2+z}{z-1} = \frac{2z+z^2}{(z-\frac{1}{5})(z-1)}$$

$$\frac{Y(z)}{z} = \frac{2+z}{(z-\frac{1}{5})(z-1)} = \frac{A}{z-\frac{1}{5}} + \frac{B}{z-1} = \frac{Az - A + Bz - \frac{1}{5}B}{(z-\frac{1}{5})(z-1)}$$

$\Rightarrow \quad A + B = 1$

$\quad -A - \frac{1}{5}B = 2 \Rightarrow A = -2 - \frac{1}{5}B$

$\quad \frac{4}{5}B = 3$

$\quad \Rightarrow B = \frac{15}{4} \Rightarrow A = -2 - \frac{3}{4} = -\frac{11}{4}$

$$\Rightarrow \frac{Y(z)}{z} = \frac{-\frac{11}{4}}{z-\frac{1}{5}} + \frac{\frac{15}{4}}{z-1}$$

$$\Rightarrow Y(z) = -\frac{11}{4}\frac{z}{z-\frac{1}{5}} + \frac{15}{4}\frac{z}{z-1} \Rightarrow y(k) = -\frac{11}{4}\left(\frac{1}{5}\right)^k + \frac{15}{4}(1)^k$$

5. $y(t) = \int_{t-1}^{t+1} u(\tau)\,d\tau$

a) linearnost i vrem. promjenjivost ?

$y_1(t) = \int_{t-1}^{t+1} u_1(\tau)\,d\tau \quad , \quad y_2(t) = \int_{t-1}^{t+1} u_2(\tau)\,d\tau$

$y(t) = \int_{t-1}^{t+1} (a u_1(\tau) + b u_2(\tau))\,d\tau = a \int_{t-1}^{t+1} u_1(\tau)\,d\tau + b \int_{t-1}^{t+1} u_2(\tau)\,d\tau =$

$= a y_1(t) + b y_2(t) \Rightarrow$ sustav je linearan

dalje ne znam

b) $u(t) = \delta(t)$

$y(t) = \int_{t-1}^{t+1} \delta(\tau)\,d\tau = \mu(\tau)\Big|_{t-1}^{t+1} = \mu(t+1) - \mu(t-1) = g_{[-1,1>}(t)$

$\rightarrow$ sustav je nekauzalan jer $y(t) \neq 0 \ \forall t < 0 \wedge \forall t > 0$

sustav je nekauzalan jer y(t) različito od 0 za svaki t<0 i y(t) različito od 0 za svaki t>0

c) $u(t) = \mu(t+1) - \mu(t-1) \circ\!\!-\!\!\bullet \ H(s) = \int_{-1}^{1} 1 e^{-st}\,dt =$

$= \frac{1}{-s} e^{-st}\Big|_{-1}^{1} = -\frac{1}{s}(e^{-s} - e^{s}) = \frac{1}{s}(e^{s} - e^{-s}) \rightarrow$??

nisam baš siguran

d) $\sphericalangle(H(s)) = \frac{Im(H(s))}{Re(H(s))} = 0$

$\boxed{\text{ovo sve vjerojatno krivo}}$

$u(t) = 2\sin(\pi t + \frac{\pi}{4})$

$y(t) = 2\int_{t-1}^{t+1} \sin(\pi\tau + \frac{\pi}{4})\,d\tau = -\frac{2}{\pi}\cos(\pi\tau + \frac{\pi}{4})\Big|_{t-1}^{t+1} =$

$= -\frac{2}{\pi}\left(\cos(t\pi + \frac{5\pi}{4}) - \cos(t\pi - \frac{3\pi}{4})\right)$