Priprema iz Signala i sustava
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Inačica E

3.1-6

a) Ispitajte koji od zadanih signala su periodični te koji im je temeljni period?

1.) $f_1(n) = \cos^2\left(\frac{\pi n}{4}\right)$

$\rightarrow f_1(n) = f_1(n+N)$

$\rightarrow \cos^2\left(\frac{\pi n}{4}\right) = \cos^2\left(\frac{\pi(n+N)}{4}\right)$

$\cos^2\left(\frac{\pi n}{4}\right) = \cos^2\left(\frac{\pi n + \pi N}{4}\right) = \cos^2\left(\frac{\pi n}{4} + \frac{\pi N}{4}\right)$

$\frac{\pi N}{4} = k\pi \;/\cdot 4$

$\cancel{\pi} N = k\cancel{\pi}\cdot 4$

$N = 4\cdot k \;;\; k\in\mathbb{Z} \Rightarrow N_0 = 4$

2.) $f_2(n) = n\sin\left(\frac{\pi n}{4}\right)$

$f_2(n) = f_2(n+N)$

$n\sin\left(\frac{\pi n}{4}\right) = (n+N)\underbrace{\sin\left(\frac{\pi n}{4} + \frac{\pi N}{4}\right)}$

$= (n+N)\cdot\left(\sin\frac{\pi n}{4}\cos\frac{\pi N}{4} + \cos\frac{\pi n}{4}\sin\frac{\pi N}{4}\right)$

$= n\sin\frac{\pi n}{4}\underbrace{\cos\frac{\pi N}{4}}_{1} + \cancel{n\cos\frac{\pi n}{4}\underbrace{\sin\frac{\pi N}{4}}_{\emptyset}} + N\sin\frac{\pi n}{4}\cos\frac{\pi N}{4} + N\cos\frac{\pi n}{4}\sin\frac{\pi N}{4}$

$\rightarrow \emptyset \qquad \rightarrow \emptyset$

$\rightarrow$ nije periodična funkcija

3.) $f_3(n) = \sin\left(\frac{\pi n^2}{4}\right)$

$\rightarrow f_3(n) = f_3(n+N)$

$\sin\left(\frac{\pi n^2}{4}\right) = \sin\left(\frac{\pi n^2}{4} + \frac{\pi n N}{2} + \frac{\pi N^2}{4}\right)$

$\frac{\cancel{\pi} nN}{2} + \frac{\cancel{\pi} N^2}{4} = 2k\cancel{\pi} \;/\cdot 4$

$2nN + N^2 = 8k \rightarrow \boxed{N_0 = 4} \qquad 1(n+2)\in\mathbb{Z}$

3.1-8c - Izračunajte analitički energiju i snagu signala:

1.) $x_1(t) = \sin\left(\frac{\pi t}{3}\right) \rightarrow$ periodičan signal $\rightarrow E = \infty$ //

$\boxed{\omega = \frac{\pi}{3} \rightarrow T = \frac{2\pi}{\omega} = 6}$

$$P_{\tilde{x}_1} = \frac{1}{T}\int_T |x_1(t)|^2 dt$$

$$= \frac{1}{6}\int_0^6 \underbrace{\sin^2\left(\frac{\pi t}{3}\right)}_{\frac{1-\cos(2\cdots)}{2}} dt = \frac{1}{6}\int_0^6 \left[\frac{1-\cos\left(\frac{2\pi t}{3}\right)}{2}\right] dt = \frac{1}{12}\int_0^6 dt - \frac{1}{12}\int_0^6 \cos\left(\frac{2\pi t}{3}\right) dt$$

$$= \frac{1}{12}\left[t\Big|_0^6 - \frac{1}{\frac{2\pi t}{3}}\sin\left(\frac{2\pi t}{3}\right)\Big|_0^6\right] = \frac{6}{12} - 0 = \frac{6}{12} = \underline{\frac{1}{2}} //$$

2.) $x_2(t) = \cos\left(\frac{\pi t}{3}\right) \rightarrow$ periodičan signal $\rightarrow \underline{E = \infty}$ //

$\boxed{\omega = \frac{\pi}{3} \quad T = \frac{2\pi}{\omega} = 6}$

$$P_{\tilde{x}_2} = \frac{1}{T}\int_T |x_2(t)|^2 dt$$

$$= \frac{1}{6}\int_0^6 \underbrace{\left|\cos^2\left(\frac{\pi t}{3}\right)\right|}_{\frac{1+\cos(2\cdots)}{2}} dt = \frac{1}{12}\left[\int_0^6 dt + \underbrace{\int_0^6 \cos\left(\frac{2\pi t}{3}\right) dt}_{\phi}\right]$$

$$= \frac{1}{12}\left[t\Big|_0^6 + \phi\right] = \frac{6}{12} - \frac{0}{12} = \frac{6}{12} = \underline{\frac{1}{2}} //$$

3.2-1 a Izračunajte Fourierov red signala:

4.) $x_1(t) = 220\sin(100\pi t)$

$$X[k] = \frac{1}{T_p}\int_{T_p} x(t)e^{-j\omega_p k t}dt \qquad \omega_p = \frac{2\pi}{T_p} \quad T_p = \frac{2\pi}{\omega_p} = \frac{2\pi}{100\pi} = \frac{1}{50}$$

$$x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{j\omega_p k t}$$

$$+\begin{cases} e^{jx} = \cos x + j\sin x \\ e^{-jx} = \cos(-x) + j\sin(-x) = \cos x - j\sin x \end{cases}$$

$2j\sin x = e^{jx} - e^{-jx}$

$\sin x = \dfrac{e^{jx} - e^{-jx}}{2j}$

$2\cos x = e^{jx} + e^{-jx} \rightarrow \cos x = \dfrac{e^{jx}+e^{-jx}}{2}$ $\Rightarrow x_1(t) = \dfrac{220}{2}\left[e^{100j\pi t} + e^{-100j\pi t}\right]$

$k = 1, -1 \rightarrow$ osnovni harmonici

$X_1 = X_{-1} = 110$

5.) $x_2(t) = 220\sin(100\pi t) + 50\cos(300\pi t + \frac{\pi}{3})$

$\omega = 100\pi$

$300\pi = 3\omega$

$$x_2(t) = \frac{\overset{110}{\cancel{220}}}{\cancel{2}}\left(e^{j\omega t} + e^{-j\omega t}\right) + \frac{\overset{25}{\cancel{50}}}{\cancel{2}j}\left(e^{3j\omega t + \frac{\pi}{3}} - e^{-3j\omega t + \frac{\pi}{3}}\right)$$

$$= 110e^{j\omega t} + 110e^{-j\omega t} + \frac{25}{j}\left(e^{3j\omega t + \frac{\pi}{3}}\right) - \frac{25}{j}\left(e^{-3j\omega t + \frac{\pi}{3}}\right)$$

$$= 110e^{j\omega t} + 110e^{-j\omega t} + \frac{25}{j}e^{3j\omega t}e^{\frac{\pi}{3}} - \frac{25}{j}e^{-3j\omega t}e^{\frac{\pi}{3}}$$

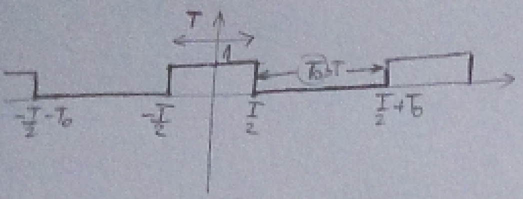
$\rightarrow X_1 = 55$

$X_{-1} = 55$

$X_3 = \dfrac{25}{j}e^{\frac{\pi}{3}}$

$X_{-3} = -\dfrac{25}{j}e^{\frac{\pi}{3}}$

3.2-2 a) Izračunajte rastav u Fourierov red niza pravokutnih impulsa jedinične amplitude trajanja T koji se ponavljaju svakih T_0 za čemu je $T_0 > T$. Korištenjem Parsevalove relacije odredite snagu signala



Parsevalova jednadžba (relacija): $P = \frac{1}{T_p}\int_{T_p} |x(t)|^2 dt = \sum_{k=-\infty}^{\infty} |x[k]|^2$ *

$$x[k] = \frac{1}{T_p}\int_{T_p} x(t) e^{-j\omega_p k t} dt$$

$$x[k] = \frac{1}{T_p}\int_{-\frac{T_p}{2}}^{T_p/2} 1 \cdot e^{-j\omega_0 k t} dt = \frac{1}{T_p} e^{-j\omega_0 k t}\Big|_{-\frac{T}{2}}^{\frac{T}{2}} \cdot -\frac{1}{j\omega_0 k}$$

$$= -\frac{1}{\cancel{j}\omega_0 k}\frac{1}{T_p}\Big[\underbrace{e^{-j\omega_0 k\cdot\frac{T}{2}} - e^{-(-j\omega_0 k\cdot\frac{T}{2})}}_{2\cancel{j}\sin(\omega_0 k\frac{T}{2})}\Big] = \frac{2\cdot T}{T_p\omega_0 k\cdot T}\cdot\sin\left(\omega_0 k\cdot\frac{T}{2}\right) = \frac{2T}{T_p\omega_0 k\cdot T}\cdot\sin\left(\frac{k\omega_0 T}{2}\right)$$

$$= -\frac{T}{T_p}\cdot\frac{2}{k\omega_0 T}\sin\left(\frac{k\omega_0 T}{2}\right) = -\frac{T}{T_p}\cdot\frac{\sin\left(\frac{k\omega_0 T}{2}\right)}{\left(\frac{k\omega_0 T}{2}\right)} = x[k] \rightarrow *$$

$\rightarrow \frac{\sin x}{x}$

$$\Rightarrow P = \frac{1}{T_p}\int_{T_p}|x(t)|^2 dt = \sum_{k=-\infty}^{\infty}\left[\frac{T}{T_p}\frac{\sin\left(\frac{k\omega_0 T}{2}\right)}{\left(\frac{k\omega_0 T}{2}\right)}\right]^2$$