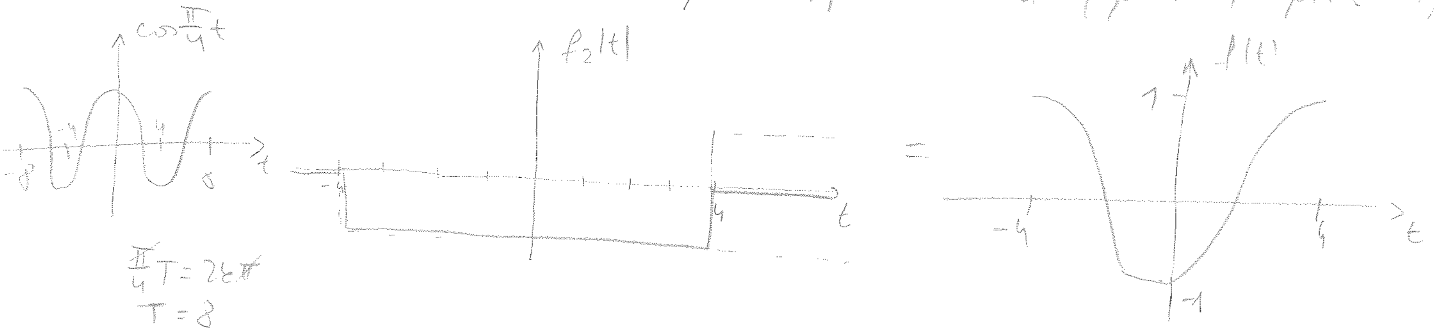
1. $f(t) = \cos\frac{\pi}{4}t\,\left(\mu(t-4) - \mu(t+4)\right) = -\cos\frac{\pi}{4}t\,\left(\mu(t+4) - \mu(t-4)\right)$



$\frac{\pi}{4}T = 2\pi$
$T = 8$

a) $$E = \int_{-\infty}^{\infty} |f(t)|^2\,dt = \int_{-4}^{4} \left(\cos\frac{\pi}{4}t\right)^2 dt = \int_{-4}^{4} \frac{1+\cos\frac{\pi}{2}t}{2}\,dt$$

$$= \left(\frac{1}{2}t + \frac{1}{2}\frac{\sin\frac{\pi t}{2}}{\frac{\pi}{2}}\right)\Bigg|_{-4}^{4} = \left(\frac{1}{2}t + \frac{1}{\pi}\sin\frac{\pi}{2}t\right)\Bigg|_{-4}^{4}$$

$$= \frac{1}{2}(4+4) + \frac{1}{\pi}\left(\sin\frac{\pi\cdot 4}{2} - \sin\frac{\pi(-4)}{2}\right) = 4 + \frac{1}{\pi}\left(\sin 2\pi - \sin(-2\pi)\right) = 4$$

b) CTFT

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\,dt$$

$$= -\int_{-4}^{4} \cos\frac{\pi}{4}t\; e^{-j\omega t}\,dt = -\int_{-4}^{4} \frac{e^{+j\frac{\pi}{4}t} + e^{-j\frac{\pi}{4}t}}{2}\cdot e^{-j\omega t}\,dt$$

$$= -\frac{1}{2}\int_{-4}^{4}\left(e^{j(-\omega+\frac{\pi}{4})t} + e^{j(-\omega-\frac{\pi}{4})t}\right)dt$$

$$= -\frac{1}{2}\left(\frac{e^{j(-\omega+\frac{\pi}{4})t}}{j(-\omega+\frac{\pi}{4})} + \frac{e^{-j(\omega+\frac{\pi}{4})t}}{-j(\omega+\frac{\pi}{4})}\right)\Bigg|_{-4}^{4}$$

$$= -\frac{1}{2}\left(\frac{e^{j(-\omega+\frac{\pi}{4})\cdot 4} - e^{-j(-\omega+\frac{\pi}{4})\cdot 4}}{j(-\omega+\frac{\pi}{4})} + \frac{e^{-j(\omega+\frac{\pi}{4})4} - e^{+j(\omega+\frac{\pi}{4})4}}{-j(\omega+\frac{\pi}{4})}\right)$$

$$= -\frac{1}{\cancel{2}}\left(\frac{\cancel{2}\cancel{j}\sin(-\omega+\frac{\pi}{4})\cdot 4}{\cancel{j}(-\omega+\frac{\pi}{4})} + \frac{-\cancel{2}\cancel{j}\sin(\omega+\frac{\pi}{4})\cdot 4}{-\cancel{j}(\omega+\frac{\pi}{4})}\right)$$

$$= \frac{\sin(-4\omega+\pi)}{\omega-\frac{\pi}{4}} - \frac{\sin(4\omega+\pi)}{\omega+\frac{\pi}{4}}$$

1. c) LINEARNOST za CTFT

$$a f(t) + b\, g(t) \xleftrightarrow{CTFT} a\, F(j\omega) + b\, G(j\omega)$$

$$CTFT\left\{a f(t) + b\, g(t)\right\} = \int_{-\infty}^{\infty} \big(a f(t) + b\, g(t)\big) e^{-j\omega t}\, dt$$

$$= a \int_{-\infty}^{\infty} f(t) e^{-j\omega t}\, dt + b \int_{-\infty}^{\infty} g(t) e^{-j\omega t}\, dt$$

$$= a\, F(j\omega) + b\, G(j\omega)$$

2. $f(n) = \cos\frac{\pi}{4}n + \sin\frac{\pi}{4}n$

$$\frac{\pi}{4}N = 2k\pi \rightarrow N = 8$$

a) DTFS

$$f(n) = \frac{1}{2}e^{j\frac{\pi}{4}n} + \frac{1}{2}e^{-j\frac{\pi}{4}n} + \frac{1}{2j}e^{j\frac{\pi}{4}n} - \frac{1}{2j}e^{-j\frac{\pi}{4}n}$$

$$= \left(\frac{1}{2} + \frac{1}{2j}\right)e^{j\frac{\pi}{4}n} + \left(\frac{1}{2} - \frac{1}{2j}\right)e^{-j\frac{\pi}{4}n}$$

$$= \frac{1-j}{2}e^{j\frac{2\pi}{8}\cdot 1\cdot n} + \frac{1+j}{2}e^{j\frac{2\pi}{8}\cdot(-1)\cdot n}$$

$$= \frac{1-j}{2}e^{j\frac{2\pi}{8}\cdot 1\cdot n} + \frac{1+j}{2}e^{j\frac{2\pi}{8}\cdot 7\cdot n}$$

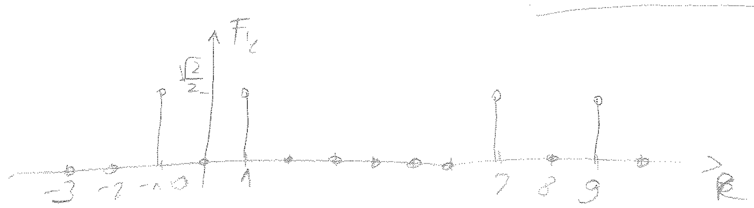
$$\boxed{F_1 = \frac{1-j}{2}} \qquad \boxed{F_7 = \frac{1+j}{2}}$$

$$|F_1| = \sqrt{\frac{1}{4}+\frac{1}{4}} = \frac{\sqrt{2}}{2} \qquad |F_7| = \frac{\sqrt{2}}{2}$$

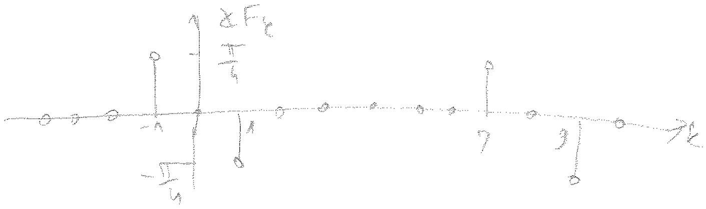
$$\measuredangle F_1 = \operatorname{arctg}\frac{-\frac{1}{2}}{\frac{1}{2}} = -\frac{\pi}{4} \qquad \measuredangle F_7 = \frac{\pi}{4}$$

$$\boxed{F_1 = \frac{\sqrt{2}}{2}e^{j\frac{\pi}{4}}} \qquad \boxed{F_7 = \frac{\sqrt{2}}{2}e^{+j\frac{\pi}{4}}}$$

b)



PERIODIČKI SE PONAVLJA



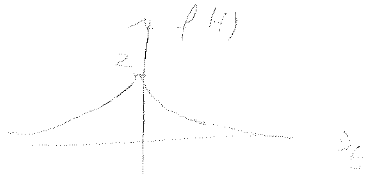
c) DTFT – iz tablica

$$\cos\frac{\pi}{4}n \xrightarrow{DTFT} \sum_{i=-\infty}^{\infty} \pi\left[\delta\left(\Omega + \frac{\pi}{4} + 2\pi i\right) + \delta\left(\Omega - \frac{\pi}{4} + 2\pi i\right)\right]$$

$$\sin\frac{\pi}{4}n \xrightarrow{DTFT} \sum_{i=-\infty}^{\infty} j\pi\left[\delta\left(\Omega + \frac{\pi}{4} + 2\pi i\right) - \delta\left(\Omega - \frac{\pi}{4} + 2\pi i\right)\right]$$

$$F(e^{j\Omega}) = \sum_{i=-\infty}^{\infty} (\pi + j\pi)\,\delta\left(\Omega + \frac{\pi}{4} + 2\pi i\right) + (\pi - j\pi)\,\delta\left(\Omega - \frac{\pi}{4} + 2\pi i\right)$$

3. $f(t) = 2e^{-4|t|}$



a) CTFT

$$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t}\, dt$$

$$F(j\omega) = \int_{-\infty}^{0} 2e^{4t} e^{-j\omega t}\, dt + \int_{0}^{\infty} 2e^{-4t} e^{-j\omega t}\, dt$$

$$= \frac{2e^{(4-j\omega)t}}{4-j\omega}\Big|_{-\infty}^{0} + \frac{2e^{-(4+j\omega)t}}{-(4+j\omega)}\Big|_{0}^{\infty}$$

$$= \frac{2}{4-j\omega} + \frac{2}{4+j\omega}$$

$$= \frac{8+2j\omega+8-2j\omega}{16+\omega^2} = \frac{16}{16+\omega^2}$$

b) DTFT

$T = \frac{1}{4}$

$$f(n) = 2e^{-4|nT|} = 2e^{-4|\frac{1}{4}n|} = 2e^{-|n|}$$

$$F(e^{j\Omega}) = \sum_{n=-\infty}^{-1} 2e^{n} e^{-j\Omega n} + \sum_{n=0}^{\infty} 2e^{-n} e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{-1} 2e^{(1-j\Omega)n} + \sum_{n=0}^{\infty} 2e^{-(1+j\Omega)n}$$

$$= \sum_{n=1}^{\infty} 2e^{-(1-j\Omega)n} + \sum_{n=0}^{\infty} 2e^{-(1+j\Omega)n}$$

$$= \sum_{n=0}^{\infty} 2e^{(-1+j\Omega)n} - 2 + \sum_{n=0}^{\infty} 2e^{-(1+j\Omega)n}$$

$$= \frac{2}{1-e^{-1+j\Omega}} - 2 + \frac{2}{1-e^{-(1+j\Omega)}}$$

c) Kada $f(t) = 2e^{-4|t|}$ očitamo periodom očitavanja $T = \frac{1}{4}$ dolazi do pojave aliasinga.

$T = \frac{1}{4} \rightarrow \omega_s = \frac{2\pi}{T} = 8\pi \rightarrow$ frekvencija očitavanja je 8π

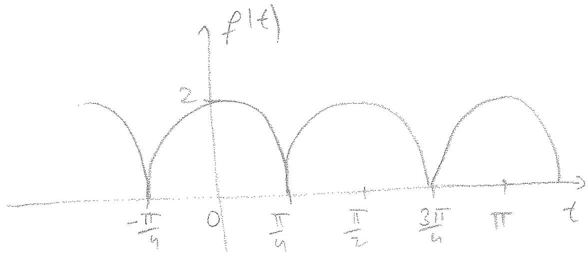
da ne bi imali aliasing maksimalna frekv. signala mora biti $< 4\pi$

$F(j\omega) = \frac{16}{16+\omega^2} \rightarrow$ ne postoji max frekvencija signala

$=$ imamo aliasing

4. $f(t) = 2\,|\cos(2t)|$

a) CTFS



$$F_k = \frac{1}{T}\int_T f(t)\, e^{-j\frac{2\pi}{T}kt}\,dt$$

$2T = 2k\pi$

$T = \pi$ za cijeli $\cos 2t$

zbog $|\cdot| \longrightarrow T = \frac{\pi}{2}$

$$F_k = \frac{1}{\pi/2}\int_{-\pi/4}^{\pi/4} 2\cdot\cos 2t\; e^{-j\frac{2\pi}{\pi/2}kt}\,dt$$

$$= \frac{4}{\pi}\int_{-\pi/4}^{\pi/4} \frac{e^{j2t}+e^{-j2t}}{2}\, e^{-j\cdot 4kt}\,dt$$

$$= \frac{2}{\pi}\left[\int_{-\pi/4}^{\pi/4} e^{j(2t-4kt)}\,dt + \int_{-\pi/4}^{\pi/4} e^{-j(2+4k)t}\,dt\right]$$

$$= \frac{2}{\pi}\left[\left.\frac{e^{j(2-4k)t}}{j(2-4k)}\right|_{-\pi/4}^{\pi/4} + \left.\frac{e^{-j(2+4k)t}}{-j(2+4k)}\right|_{-\pi/4}^{\pi/4}\right]$$

$$= \frac{2}{\pi}\left[\frac{e^{j(2-4k)\frac{\pi}{4}} - e^{-j(2-4k)\frac{\pi}{4}}}{j(2-4k)} + \frac{e^{-j(2+4k)\frac{\pi}{4}} - e^{j(2+4k)\frac{\pi}{4}}}{-j(2+4k)}\right]$$

$$= \frac{2}{\pi}\left[\frac{2\cancel{j}\sin(2-4k)\frac{\pi}{4}}{\cancel{j}(2-4k)} + \frac{-2\cancel{j}\sin(2+4k)\frac{\pi}{4}}{-\cancel{j}(2+4k)}\right]$$

$$= \frac{4}{\pi}\left[\frac{\sin(1-2k)\frac{\pi}{2}}{2(1-2k)} + \frac{\sin(1+2k)\frac{\pi}{2}}{2(1+2k)}\right]$$

$$= \frac{2}{\pi}\left[\frac{\sin(1-2k)\frac{\pi}{2}}{1-2k} + \frac{\sin(1+2k)\frac{\pi}{2}}{1+2k}\right]$$

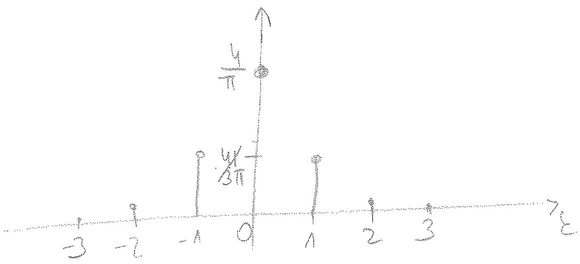
$$= \frac{2}{\pi}\cdot\frac{\sin(1-2k)\frac{\pi}{2} + \sin(1+2k)\frac{\pi}{2} + 2k\left[\sin(1-2k)\frac{\pi}{2} - \sin(1+2k)\frac{\pi}{2}\right]}{1-4k^2}$$

$$= \frac{2}{\pi(1-4k^2)}\cdot\left(2\sin\frac{\pi}{2}\cos(-k\pi) + 2k\cdot 2\,\cancel{\cos\frac{\pi}{2}\sin(-k\pi)}\right)$$

$$= \frac{4\cos k\pi}{\pi(1-4k^2)} = \frac{4}{\pi(1-4k^2)}\cdot(-1)^k$$

b)

$$|F_k| = \left| \frac{4(-1)^k}{\pi(1-4k^2)} \right|$$



$$|F_0| = \left|\frac{4\cdot 1}{\pi\cdot 1}\right| = \frac{4}{\pi}$$

$$|F_{-1}| = \left|\frac{4(-1)^{-1}}{\pi(1-4(-1)^2)}\right| = \left|\frac{-4}{-3\pi}\right| = \frac{4}{3\pi}$$

$$|F_1| = \left|\frac{4\cdot(-1)}{\pi(1-4\cdot 1)}\right| = \frac{4}{3\pi}$$

$$|F_2| = \left|\frac{4\cdot 1}{\pi(1-4\cdot 4)}\right| = \left|\frac{4}{-15\pi}\right| = \frac{4}{15\pi}$$

c)

$$P = \frac{1}{T_0}\int_{T_0} |f(t)|^2\,dt = \sum_{k=-\infty}^{\infty} |F_k|^2$$

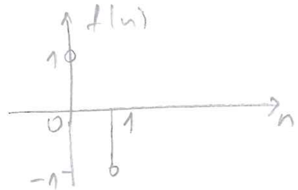
$$P = \frac{1}{T_0}\int_{T_0} f(t)\, f^*(t)\,dt = \frac{1}{T_0}\int_{T_0} f(t)\left(\sum_{k=-\infty}^{\infty} F_k^* e^{-jk\omega_0 t}\right)dt$$

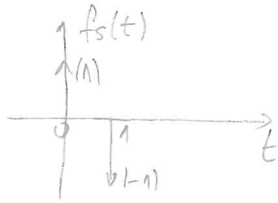
$$= \sum_{k=-\infty}^{\infty} F_k^* \underbrace{\frac{1}{T_0}\int_{T_0} f(t)\, e^{-jk\omega_0 t}\,dt}_{F_k} = \sum_{k=-\infty}^{\infty} |F_k|^2$$

5. $f(n) = \{\ldots\ 0, 0, 0, \underline{1}, -1, 0, 0, \ldots\}$

a) idealna interpolacija

$T = \frac{1}{2}$





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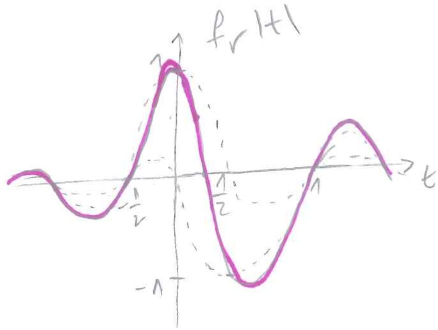


$$\left[\delta(t) - \delta(t-1)\right] * \frac{\sin \frac{\pi t}{T}}{\frac{\pi t}{T}} =$$

$$= \frac{\sin \frac{\pi t}{T}}{\frac{\pi t}{T}} - \frac{\sin \frac{\pi (t-1)}{T}}{\frac{\pi (t-1)}{T}}$$

uz $T = \frac{1}{2}$

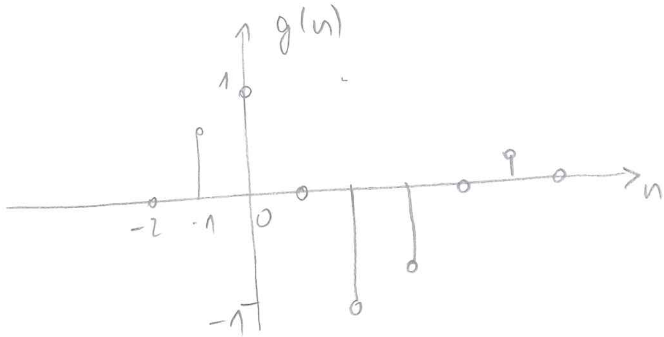
$$f_r(t) = \frac{\sin 2\pi t}{2\pi t} - \frac{\sin 2\pi\left(t - \frac{1}{2}\right)}{2\pi\left(t - \frac{1}{2}\right)}$$

b) $g(n)$ uz $T = \frac{1}{4}$

$t = nT$

$$g(nT) = \frac{\sin 2\pi \cdot \frac{n}{4}}{2\pi \cdot \frac{n}{4}} - \frac{\sin 2\pi\left(\frac{n}{4} - 1\right)}{2\pi\left(\frac{n}{4} - 1\right)} = \frac{\sin \frac{\pi}{2} n}{\frac{\pi}{2} n} - \frac{\sin \frac{\pi}{2}(n-2)}{\frac{\pi}{2}(n-2)}$$

n	g(n)
-2	0
-1	0.849
0	1
1	0
2	-1
3	-0.849
4	0
5	0.34
6	0



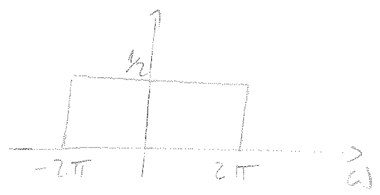
5. a) $f(t)$

$T = 1$

(SISIS-97/9 9)

$$f(t) = \frac{\sin 2\pi t}{2\pi t} - \frac{\sin 2\pi\left(t-\frac{1}{2}\right)}{2\pi\left(t-\frac{1}{2}\right)}$$

$$\frac{\sin 2\pi t}{2\pi t} = \operatorname{sinc} 2t \xleftrightarrow{CTFT} \frac{1}{2}\operatorname{rect}\left(\frac{\omega}{4\pi}\right)$$ (ŠAVABAHITE)



$$\operatorname{rect}\frac{\omega}{4\pi} = \begin{cases} 1, & -\frac{1}{2} < \frac{\omega}{4\pi} < \frac{1}{2} \\ 0, & \text{inače} \end{cases}$$

$$= \begin{cases} 1, & -2\pi < \omega < 2\pi \\ 0, & \text{inače} \end{cases}$$

$\omega_g = 2\pi$

kako ne bi bilo ALIASINGA frekvencija očitanja mora biti bar $2 \cdot 2\pi = 4\pi$

$T = 1 \rightarrow f = 1\,Hz \rightarrow \omega = 2\pi f = 2\pi$

zadana frekvencija očitavanja je $2\pi < 4\pi$,

uvjeti Shannonovog teorema nisu zadovoljeni.