Signali i sustavi

Pismeni ispit – 23. lipnja 2015.

1. **(8 bodova)** Vremenski diskretan kauzalan sustav zadan je jednadžbom diferencija $y(n) - \frac{1}{6}y(n-1) = 4u(n)$ te početnim uvjetom y(-1)=2. Na ulaz sustava dovedena je pobuda $u(n) = (\frac{1}{4})^n \mu(n)$.

   a) **(2 boda)** Odredite prisilni odziv sustava.

   b) **(2 boda)** Odredite prirodni odziv sustava.

   c) **(2 boda)** Odredite odziv mirnog sustava.

   d) **(2 boda)** Odredite odziv nepobuđenog sustava.

2. **(8 bodova)** Vremenski kontinuiran kauzalan LTI sustav zadan je diferencijalnom jednadžbom $y'(t) + 4y(t) = 2u'(t) + u(t)$.

   a) **(6 bodova)** Izračunajte impulsni odziv sustava pomoću Laplaceove transformacije.

   b) **(2 boda)** Odredite prijenosnu funkciju sustava te ispitajte stabilnost sustava.

3. **(8 bodova)** Vremenski diskretan kauzalan sustav zadan je jednadžbom diferencija

$$y(n) - \frac{1}{2}y(n-1) = u(n) + 4u(n-1).$$

   a) **(2 boda)** Odredite prijenosnu funkciju sustava te ispitajte stabilnost sustava.

   b) **(3 boda)** Izračunajte impulsni odziv sustava pomoću Z transformacije.

   c) **(3 boda)** Izračunajte odziv mirnog sustava na pobudu $u(n) = (-4)^n \mu(n)$ pomoću Z transformacije.

4. **(8 bodova)** Vremenski kontinuiran kauzalan LTI sustav zadan je diferencijalnom jednadžbom

$$y''(t) + 7y'(t) + 6y(t) = u(t).$$

   a) **(4 boda)** Izračunajte impulsni odziv sustava postupkom u vremenskoj domeni.

   b) **(4 boda)** Odredite odziv sustava na pobudu $u(t) = e^{-2t}\mu(t)$ metodom konvolucijskog integrala.

5. **(8 bodova)** Zadan je impulsni odziv vremenski diskretnog kauzalnog LTI sustava $h(n) = \left(\frac{3}{2^n} + \frac{2}{3^n}\right)\mu(n)$.

   a) **(3 boda)** Odredite prijenosnu funkciju sustava.

   b) **(2 boda)** Odredite jednadžbu diferencija zadanog sustava.

   c) **(3 boda)** Odredite odziv sustava na svevremensku pobudu $u(n) = 2\cos(\frac{\pi}{2}n)$.

1. $y(n) - \frac{1}{6} y(n-1) = 4u(n)$

$y(-1) = 2$

$u(n) = \left(\frac{1}{4}\right)^n \mu(n)$

a) PARTIKULARNI

$y_p(n) = k\left(\frac{1}{4}\right)^n$

$k\left(\frac{1}{4}\right)^n - \frac{1}{6} k \left(\frac{1}{4}\right)^{n-1} = 4\left(\frac{1}{4}\right)^n$

$k - \frac{1}{6 \cdot 4} \cdot k \cdot 4 = 4$

$3k - 2k = 12$

$k = 12$

$\boxed{y_p(n) = 12\left(\frac{1}{4}\right)^n \mu(n)}$

b) PRIRODNI

$y(n) = y_p(n) + y_h(n)$

$2 - \frac{1}{6} = 0$

$2 = \frac{1}{6}$

$y_h(n) = C\left(\frac{1}{6}\right)^n$

$y(n) = C\left(\frac{1}{6}\right)^n + 12\left(\frac{1}{4}\right)^n$

POČETNI UVJET: $y(n) = 4u(n) + \frac{1}{6} y(n-1)$

$y(0) = 4 \cdot 1 + \frac{1}{6} \cdot 2 = 4 + \frac{1}{3} = \frac{13}{3}$

$y(0) = C + 12 = \frac{13}{3} \longrightarrow C = \frac{13}{3} - 12 = \frac{13 - 36}{3} = \frac{-23}{3}$

$\boxed{y_{PRIRODNI}(n) = -\frac{23}{3}\left(\frac{1}{6}\right)^n \mu(n)}$

c) MIRNI

$y(-1) = 0 \longrightarrow y(0) = 4 \cdot 1 + \frac{1}{6} \cdot 0 = 4$

$y(n) = C\left(\frac{1}{6}\right)^n + 12\left(\frac{1}{4}\right)^n \rightarrow y(0) = C + 12 = 4 \rightarrow C = -8$

$\boxed{y_{MIRNI}(n) = \left[-8\left(\frac{1}{6}\right)^n + 12\left(\frac{1}{4}\right)^n\right]\mu(n)}$

d) NEPOBUDENI

$u(n) = 0$

$y_0(n) = C\left(\frac{1}{6}\right)^n$

$y_0(-1) = C \cdot 6 = 2$

$C = \frac{1}{3}$

$\boxed{y_0(n) = \frac{1}{3}\left(\frac{1}{6}\right)^n}$

2. $y'(t)+4y(t)=2u'(t)+u(t)$

$sY(s)+4Y(s)=2sU(s)+U(s)$

$Y(s)(s+4)=U(s)(2s+1)$

$\frac{Y(s)}{U(s)}=\frac{2s+1}{s+4}$

a) 2a $u(t)=\delta(t) \rightarrow U(s)=1$

$H(s)=\frac{2s+1}{s+4}=2+\frac{-7}{s+4}$

$(2s+1):(s+4)=2$
$-2s\pm8$
-7

$h(t)=2\delta(t)-7e^{-4t}\ \mu(t)$

b) $H(s)=\frac{2s+1}{s+4}$ PRIJENOSNA FUNKCIJA

$s+4=0$

$s=-4$ POL

$\mathrm{Re}\{s\}=-4<0$ SUSTAV JE STABILAN

3. $y(n) - \frac{1}{2} y(n-1) = u(n) + 4u(n-1)$

a) $H(z) = \frac{1 + 4z^{-1}}{1 - \frac{1}{2}z^{-1}} = \frac{z+4}{z - \frac{1}{2}}$

POL $z = \frac{1}{2}$

STABILNOST $|z| < 1$ STABILAN SUSTAV

b) $\frac{H(z)}{z} = \frac{z+4}{z\left(z-\frac{1}{2}\right)} = \frac{A}{z} + \frac{B}{z-\frac{1}{2}}$

$A + B = 1$

$-\frac{1}{2}A = 4 \longrightarrow A = -8$ $\quad \rightarrow B = 1 - A = 9$

$H(z) = \frac{-8z}{z} + \frac{9z}{z - \frac{1}{2}}$

$h(n) = -8\delta(n) + 9\left(\frac{1}{2}\right)^n \mu(n)$

c) $u(n) = (-4)^n \mu(n) \longrightarrow U(z) = \frac{z}{z+4}$

$y(z) = H(z) \cdot U(z) = \frac{z+4}{z-\frac{1}{2}} \cdot \frac{z}{z+4} = \frac{z}{z-\frac{1}{2}}$

$y(n) = \left(\frac{1}{2}\right)^n \mu(n)$ ODZIV MIROVOG SUSTAVA

4. $y''(t) + 7y'(t) + 6y(t) = u(t)$

a) $s^2 + 7s + 6 = 0$

$(s+6)(s+1) = 0$

$s = -6 \qquad s = -1$

$h_a(t) = C_1 e^{-6t} + C_2 e^{-t}$

$h_a'(t) = -6C_1 e^{-6t} - C_2 e^{-t}$

$h_a(0^+) = 0 \quad , \quad h_a'(0^+) = 1$

$h_a(0^+) = C_1 + C_2 = 0$

$h_a'(0^+) = -6C_1 - C_2 = 1$

$C_1 = -C_2$

$6C_2 - C_2 = 1$

$5C_2 = 1$

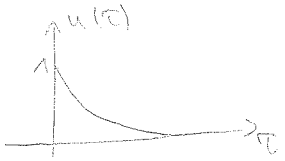
$C_2 = \frac{1}{5} \qquad C_1 = -\frac{1}{5}$

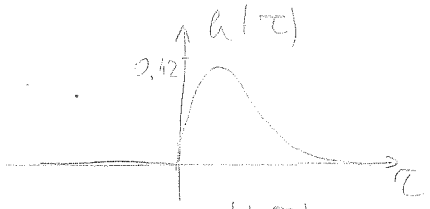
$h_a(t) = -\frac{1}{5} e^{-6t} + \frac{1}{5} e^{-t}$

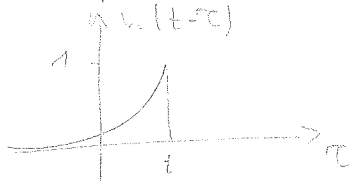
$h(t) = h_a(t)$

$h(t) = \left(-\frac{1}{5} e^{-6t} + \frac{1}{5} e^{-t}\right) \mu(t)$

b) $u(t) = e^{-2t} \mu(t)$







1° $t < 0$

nema preklapanja

$y(t) = 0$

2° $t \geq 0$

$$y(t) = \int_0^t e^{-2(t-\tau)} \left(-\frac{1}{5} e^{-6\tau} + \frac{1}{5} e^{-\tau}\right) d\tau$$

$$= -\frac{1}{5} \int_0^t e^{-2t-4\tau} d\tau + \frac{1}{5} \int_0^t e^{-2t+\tau} d\tau$$

$$= +\frac{1}{5} e^{-2t} \frac{e^{-4\tau}}{+4} \Big|_0^t + \frac{1}{5} e^{-2t} e^{\tau} \Big|_0^t = \frac{1}{20} e^{-2t}\left(e^{-4t} - 1\right) + \frac{1}{5}\left(e^{-t} - e^{-2t}\right)$$

$$= \frac{1}{20} e^{-6t} - \frac{1}{20} e^{-2t} + \frac{1}{5} e^{-t} - \frac{1}{5} e^{-2t}$$

$$y(t) = \left(\frac{1}{20} e^{-6t} - \frac{1}{4} e^{-2t} + \frac{1}{5} e^{-t}\right) \mu(t)$$

5.

$$h[n] = \left(\frac{3}{2^n} + \frac{2}{3^n}\right)\mu[n]$$

$$= \left[3\left(\frac{1}{2}\right)^n + 2\cdot\left(\frac{1}{3}\right)^n\right]\mu[n]$$

a) $$H[z] = 3\,\frac{z}{z-\frac{1}{2}} + 2\,\frac{z}{z-\frac{1}{3}}$$

$$= \frac{5z^2 - 2z}{z^2 - \frac{5}{6}z + \frac{1}{6}} = \frac{5 - 2z^{-1}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}$$

b)

$$H[z] = \frac{Y(z)}{U(z)}$$

$$Y(z)\left(z^2 - \frac{5}{6}z + \frac{1}{6}\right) = U(z)\left(5z^2 - 2z\right)$$

$$Y(z)\left(1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}\right) = U(z)\left(5 - 2z^{-1}\right)$$

$$y[n] - \frac{5}{6}y[n-1] + \frac{1}{6}y[n-2] = 5u[n] - 2u[n-1]$$

c) $$u[n] = 2\cos\frac{\pi}{2}n$$

$$H[e^{i\Omega}] = \frac{5 - 2e^{-i\Omega}}{1 - \frac{5}{6}e^{-i\Omega} + \frac{1}{6}e^{-2i\Omega}}$$

$$H[e^{i\frac{\pi}{2}}] = \frac{5 - 2e^{-i\frac{\pi}{2}}}{1 - \frac{5}{6}e^{-i\frac{\pi}{2}} + \frac{1}{6}e^{-i\pi}} = \frac{5 + 2j}{\frac{7}{6}(1+j)}$$

$$|H(e^{i\frac{\pi}{2}})| = \frac{6}{5}\sqrt{\frac{29}{2}}$$

$$\angle H(e^{i\frac{\pi}{2}}) = \operatorname{arctg}\frac{2}{5} - \operatorname{arctg} 1 = -0.405$$

$$y[n] = 2\cdot\frac{6}{5}\sqrt{\frac{29}{2}}\cos\left(\frac{\pi}{2}n - 0.405\right)$$

$$= 9.14\cos\left(\frac{\pi}{2}n - 0.405\right)$$