Signali i sustavi

Auditorne vježbe 12.

Matrični prikaz sustava

TRANSFORMACIJA VARIJABLI STANJA

$$\begin{cases} x' = Ax + B\cdot u \\ y = Cx + D\cdot u \end{cases}$$

$$x = P\cdot z$$
$$x' = P\cdot z'$$

- P je regularna matrica ($\exists P^{-1}$ Û det P¹0)

$$\begin{cases} P^{-1}\cdot / \quad Pz' = APz + B\cdot u \\ \qquad\qquad y = CPz + D\cdot u \end{cases}$$

nastavak

$$z' = \underbrace{P^{-1}AP}_{\mathrm{A}^*}\cdot z + \underbrace{P^{-1}B}_{\mathrm{B}^*}\cdot u$$

$$y = \underbrace{CP}_{\mathrm{C}^*}\cdot z + \underbrace{D}_{\mathrm{D}^*}\cdot u$$

$$\begin{cases} z' = A^*z + B^*\cdot u \\ y = C^*z + D^*\cdot u \end{cases}$$

nastavak

$$A^* = P^{-1}AP \qquad C^* = CP$$
$$B^* = P^{-1}B \qquad D^* = D$$

- Vrijedi:
- $\det(sI - A) = \det(sI - A^*)$
- Karakteristične vrijednosti matrica A i A^* su nepromjenjene.
- Sustav je isti, ali je opisan preko drugih varijabli stanja.
- Polovi (frekvencije sustava) su isti za A i A^*.

nastavak

- Svaka regularna matrica *P* daje novi izbor stanja sustava.
- Mi ćemo odabrati takvu regularnu matricu *P* koja će varijable stanja transformirati u <u>kanonske varijable stanja</u>.
- Ako matrica *P* transformira matricu *A* u dijagonalnu (kanonske varijable stanja) onda se matrica *P* zove <u>modalna</u> i označava s *M.*
- Kako naći matricu *M*?

nastavak

- Transformacija vektora $\vec{x}$ u vektor $\vec{y}$:

$$\boxed{\vec{y} = A\vec{x}}$$

- *A* je matrica, matrični zapis linearnog operatora koji vektoru pridružuje vektor.
- Da li postoji takav vektor $\vec{x}$ da transformacija $A\vec{x}$ daje vektor $\vec{y}$ istog smjera kao $\vec{x}$?

$\boxed{A\vec{x} = \vec{y}}$ $\boxed{A\vec{x} = s\vec{x}}$

Ako vektor $\bar{y}$ nije promijenio smjer, tada je *s* skalar.

nastavak

- Drukčije pisano: $\boxed{(sI - A)\vec{x} = 0}$ homogena algebarska jednadžba
- trivijalno rješenje: $\boxed{\vec{x} = 0}$
- netrivijalno rješenje dobijemo za $\det(sI - A) = 0$ što rezultira polinomom kojeg zovemo karakteristični polinom matrice *A*.
- Ako je rang matrice *A* jednak *n* polinom je *n*–tog reda.
- Nule karakterističnog polinoma s_i, $i = 1, n$ zovu se svojstvene vrijednosti , a vektori

$$\boxed{\vec{x}_i, \quad i = 1, n \quad A\,\vec{x}_i = s_i\,\vec{x}_i}$$

- karakteristični vektori matrice *A.*

nastavak

- Formirajmo matricu *P* pomoću karakterističnih vektora matrice *A.*

$$\boxed{P = [\vec{x}_1 \quad \vec{x}_2 \quad \dots \quad \vec{x}_n] \quad \vec{x}_i, i = 1, n}$$

karakteristični vektori

- tada je: $A \cdot P = P \cdot A^*$

$$A \cdot P = A[\vec{x}_1 \quad \vec{x}_2 \quad \dots \quad \vec{x}_n] = [A\vec{x}_1 \quad A\vec{x}_2 \quad \dots \quad A\vec{x}_n]$$

$$= [s_1\vec{x}_1 \quad s_2\vec{x}_2 \quad \dots \quad s_n\vec{x}_n] = \underbrace{[\vec{x}_1 \quad \vec{x}_2 \quad \dots \quad \vec{x}_n]}_{P} \underbrace{\begin{bmatrix} s_1 & & & 0 \\ & s_2 & & \\ & & \ddots & \\ 0 & & & s_n \end{bmatrix}}_{A^*}$$

nastavak

- $A \cdot P = P \cdot A^*$
- A^* — dijagonalna $\Rightarrow$ $P = M$ — modalna matrica sačinjena od svojstvenih vektora matrice *A.*
- $A \cdot M = M \cdot A^*$
- Pomnožimo slijeva sa M^{-1}:
- $M^{-1} \cdot A \cdot M = A^*$.

Zadatak 1.

Zadana je matrica $\mathbf{A}$. Treba naći modalnu matricu $\mathbf{M}$.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 2 & 3 & 2 \end{bmatrix}$$

Odrediti vlastite (svojstvene) vrijednosti.

Zadatak 1. – Svojstvene vrijednosti

$$\det(s\mathbf{I}-\mathbf{A}) = \begin{vmatrix} s-1 & 0 & 0 \\ -1 & s-1 & 0 \\ -2 & -3 & s-2 \end{vmatrix} = 0$$

$$\begin{vmatrix} s-1 & 0 & 0 \\ -1 & s-1 & 0 \\ -2 & -3 & s-2 \end{vmatrix} = (s-1)^2(s-2) \qquad \begin{aligned} s_1 &= 2 \\ s_2 = s_3 &= 1 \end{aligned}$$

Formiranje matrice $\mathbf{A}^*$

$$\mathbf{A}^* = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Jordanov blok
(Jordanova klijetka)

Zadatak 1.

- Odrediti vlastite vektore, matricu $\mathbf{M}$. Vrijedi

$$\mathbf{AM} = \mathbf{MA}^*$$

$$\mathbf{A}[\mathbf{x}_1 \quad \mathbf{x}_2 \quad \mathbf{x}_3] = [\mathbf{x}_1 \quad \mathbf{x}_2 \quad \mathbf{x}_3]\begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 1 \\ 0 & 0 & s_3 \end{bmatrix}$$

$$[\mathbf{Ax}_1 \quad \mathbf{Ax}_2 \quad \mathbf{Ax}_3] = [s_1\mathbf{x}_1 \quad s_2\mathbf{x}_2 \quad s_3\mathbf{x}_3 + \mathbf{x}_2]$$

$$\mathbf{Ax}_1 = s_1\mathbf{x}_1 \qquad (1)$$
$$\mathbf{Ax}_2 = s_2\mathbf{x}_2 \qquad (2)$$
$$\mathbf{Ax}_3 = s_3\mathbf{x}_3 + \mathbf{x}_2 \qquad (3)$$

Zadatak 1.

- (1)

$$\begin{bmatrix}1&0&0\\1&1&0\\2&3&2\end{bmatrix}\cdot\begin{bmatrix}x_{11}\\x_{21}\\x_{31}\end{bmatrix}=2\cdot\begin{bmatrix}x_{11}\\x_{21}\\x_{31}\end{bmatrix}$$

$$\left\{\begin{matrix}x_{11}=2x_{11}\\x_{11}+x_{21}=2x_{21}\\2x_{11}+3x_{21}+2x_{31}=2x_{31}\end{matrix}\right\}\Rightarrow\left\{\begin{matrix}x_{11}=0\\x_{21}=0\\x_{31}\text{ proizvoljno } x_{31}=1\quad(\neq 0)\end{matrix}\right\}$$

$$\vec{x}_1=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

Zadatak 1.

- (2)

$$\begin{bmatrix}1&0&0\\1&1&0\\2&3&2\end{bmatrix}\cdot\begin{bmatrix}x_{12}\\x_{22}\\x_{32}\end{bmatrix}=1\cdot\begin{bmatrix}x_{12}\\x_{22}\\x_{32}\end{bmatrix}$$

$$\left\{\begin{matrix}x_{12}=x_{12}\\x_{12}+x_{22}=x_{22}\\2x_{12}+3x_{22}+2x_{32}=x_{32}\end{matrix}\right\}\Rightarrow\left\{\begin{matrix}x_{12}=0,\quad x_{22}=1\,(\text{proizvoljno})\\x_{32}=-3\end{matrix}\right\}$$

$$\vec{x}_2=\begin{bmatrix}0\\1\\-3\end{bmatrix}$$

Zadatak 1.

- (3)

$$\begin{bmatrix}1&0&0\\1&1&0\\2&3&2\end{bmatrix}\cdot\begin{bmatrix}x_{13}\\x_{23}\\x_{33}\end{bmatrix}=1\cdot\begin{bmatrix}x_{13}\\x_{23}\\x_{33}\end{bmatrix}+\begin{bmatrix}x_{12}\\x_{22}\\x_{32}\end{bmatrix}$$

$$\left.\begin{matrix}x_{13}=x_{13}+x_{12}\\x_{13}+x_{23}=x_{23}+x_{22}\\2x_{13}+3x_{23}+2x_{33}=x_{33}+x_{32}\end{matrix}\right\}\Rightarrow\left.\begin{matrix}x_{13}=x_{13}\\x_{13}+x_{23}=x_{23}+1\\2x_{13}+3x_{23}+2x_{33}=x_{33}-3\end{matrix}\right\}\Rightarrow$$

$$\left.\begin{matrix}\Rightarrow x_{13}=1, x_{23}=1(\mathit{proizvoljno})\\x_{33}=-8\end{matrix}\right\}\Rightarrow x_3=\begin{bmatrix}1\\1\\-8\end{bmatrix}$$

Zadatak 1.

$$M = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & -3 & -8 \end{bmatrix}$$

Posebni slučaj

- Specijalni slučaj: korijeni (svojstvene vrijednosti) matrice *A* su različiti.
- Recept: stupci matrice *M* mogu se uzeti jednaki ili proporcionalni bilo kojem stupcu adjungirane pridružene matrice $\mathrm{adj}(s_i I - A)$ koji nije nul–stupac.

Adjungirana matrica?

- $\mathrm{adj}(A) = ?$
- $\mathrm{adj}(A) = [x_{ij}]^{\mathrm{T}}$,
- $x_{ij} = (-1)^{i+j} \cdot D_{ij}$.
- D_{ij} = determinanta podmatrice A dobivena izbacivanjem i–tog retka i j–tog stupca.
- $^{\mathrm{T}}$ transponiranje — i–ti redak postaje i–ti stupac.

Zadatak 2.

Zadana je matrica:

$$A=\begin{bmatrix}2 & -2 & 3\\ 1 & 1 & 1\\ 1 & 3 & -1\end{bmatrix}$$

Naći modalnu matricu *M.*

$$sI-A=\begin{bmatrix}s-2 & 2 & -3\\ -1 & s-1 & -1\\ -1 & -3 & s+1\end{bmatrix}$$

$$\det(sI-A)=0$$

Zadatak 2. - nastavak

- $s^3-2s^2-5s+6=(s-1)(s+2)(s-3)=0$
- $s_1=1$
- $s_2=-2$
- $s_3=3$

$$\operatorname{adj}(sI-A)=\begin{bmatrix}s^2-4 & -2s+7 & 3s-5\\ s+2 & s^2-s-5 & s+1\\ s+2 & 3s-8 & s^2-3s+4\end{bmatrix}$$

Pojašnjenje:

$$sI-A=\begin{bmatrix}s-2 & 2 & -3\\ -1 & \boxed{\begin{matrix}s-1 & -1\\ -3 & s+1\end{matrix}}\end{bmatrix}$$

Zadatak 2. - nastavak

- $s^3-2s^2-5s+6=(s-1)(s+2)(s-3)=0$
- $s_1=1$
- $s_2=-2$
- $s_3=3$

$$\operatorname{adj}(sI-A)=\begin{bmatrix}s^2-4 & -2s+7 & 3s-5\\ s+2 & s^2-s-5 & s+1\\ s+2 & 3s-8 & s^2-3s+4\end{bmatrix}$$

Pojašnjenje:

$$sI-A=\begin{bmatrix}s-2 & & -3\\ \cancel{-1} & \cancel{s-1} & \cancel{-1}\\ -1 & \cancel{-3} & s+1\end{bmatrix}$$

Zadatak 2. - nastavak

- $s^3 - 2s^2 - 5s + 6 = (s - 1)(s + 2)(s-3) = 0$
- $s_1 = 1$
- $s_2 = -2$
- $s_3 = 3$

$$\mathrm{adj}(sI - A) = \begin{bmatrix} s^2-4 & -2s+7 & 3s-5 \\ s+2 & s^2-s-5 & s+1 \\ s+2 & 3s-8 & s^2-3s+4 \end{bmatrix}$$

Pojašnjenje:

$$\text{minus} \quad sI - A = \begin{bmatrix} s-2 & 2 & -3 \\ -1 & s-1 & -1 \\ -1 & -3 & s+1 \end{bmatrix}$$

itd.

Zadatak 2.- nastavak

- $s = s_1 = 1$

$$\mathrm{adj}(s_1 I - A) = \begin{bmatrix} -3 & 5 & -2 \\ 3 & -5 & 2 \\ 3 & -5 & 2 \end{bmatrix}$$

- $s = s_2 = -2$

$$\mathrm{adj}(s_2 I - A) = \begin{bmatrix} 0 & 11 & -11 \\ 0 & 1 & -1 \\ 0 & -14 & 14 \end{bmatrix}$$

Zadatak 2. - nastavak

- $s = s_3 = 3$

$$\mathrm{adj}(s_3 I - A) = \begin{bmatrix} 5 & 1 & 4 \\ 5 & 1 & 4 \\ 5 & 1 & 4 \end{bmatrix}$$

$$M = \begin{bmatrix} -1 & 11 & 1 \\ 1 & 1 & 1 \\ 1 & -14 & 1 \end{bmatrix}$$

$$A^* = M^{-1} \cdot A \cdot M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

Posebni slučaj

- Specijalni slučaj: direktna realizacija
- Recept

$$A = \begin{bmatrix} 0 & 1 & & 0 \\ & 0 & 1 & \\ & & \ddots & 1 \\ -a_0 & -a_1 & -a_2 & -a_n \end{bmatrix}$$

Nastavak

- Neka su s_i, $i = 1,...,n$ jednostruki polovi.
- Tada M konstruiramo na slijedeći način:

$$M = \begin{bmatrix} 1 & 1 & \dots & 1 \\ s_1 & s_2 & \dots & s_n \\ s_1^2 & s_2^2 & \dots & s_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ s_1^{n-1} & s_2^{n-1} & \dots & s_n^{n-1} \end{bmatrix}$$

Nastavak

- Slučaj višestrukih polova npr.:
- $s_1 = s_2 = s_3 = s_4$,
- $s_5 = s_6$,
- s_7

$$\begin{bmatrix} 1 & 0 & \\ s_1 & 1 & \\ s_1^2 & \frac{2s_1}{1!} & \\ s_1^3 & \frac{3s_1^2}{1!} & \\ s_1^4 & \frac{4s_1^3}{1!} & \\ s_1^5 & \frac{5s_1^4}{1!} & \\ s_1^6 & \frac{6s_1^5}{1!} & \end{bmatrix}$$

derivacija prethodnog stupca, podijeljena s 1 faktorijela

Nastavak

- Slučaj višestrukih polova npr.:
- $s_1 = s_2 = s_3 = s_4$,
- $s_5 = s_6$,
- s_7

$$\begin{bmatrix} 1 & 0 & 0 \\ s_1 & 1 & 0 \\ s_1^2 & \frac{2s_1}{1!} & 1 \\ s_1^3 & \frac{3s_1^2}{1!} & \frac{6s_1}{2!} \\ s_1^4 & \frac{4s_1^3}{1!} & \frac{12s_1^2}{2!} \\ s_1^5 & \frac{5s_1^4}{1!} & \frac{20s_1^3}{2!} \\ s_1^6 & \frac{6s_1^5}{1!} & \frac{30s_1^4}{2!} \end{bmatrix}$$

derivacija prethodnog stupca, podijeljena s 2 faktorijela

Nastavak

- Slučaj višestrukih polova npr.:
- $s_1 = s_2 = s_3 = s_4$,
- $s_5 = s_6$,
- s_7

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ s_1 & 1 & 0 & 0 & s_5 & 1 & s_7 \\ s_1^2 & \frac{2s_1}{1!} & 1 & 0 & s_5^2 & \frac{2s_5}{1!} & s_7^2 \\ s_1^3 & \frac{3s_1^2}{1!} & \frac{6s_1}{2!} & 1 & s_5^3 & \frac{3s_5^2}{1!} & s_7^3 \\ s_1^4 & \frac{4s_1^3}{1!} & \frac{12s_1^2}{2!} & \frac{24s_1}{3!} & s_5^4 & \frac{4s_5^3}{1!} & s_7^4 \\ s_1^5 & \frac{5s_1^4}{1!} & \frac{20s_1^3}{2!} & \frac{60s_1^2}{3!} & s_5^5 & \frac{5s_5^4}{1!} & s_7^5 \\ s_1^6 & \frac{6s_1^5}{1!} & \frac{30s_1^4}{2!} & \frac{120s_1^3}{3!} & s_5^6 & \frac{6s_5^5}{1!} & s_7^6 \end{bmatrix}$$

Zadatak 3.

- Zadana je matrica A, naći M i A*

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$$

  - Jasno, radi se o direktnoj realizaciji.

$$sI - A = \begin{bmatrix} s & -1 & 0 \\ 0 & s & -1 \\ -1 & 3 & s-3 \end{bmatrix}$$

Nastavak

- det (sI - A) = $s^3 - 3s^2 + 3s - 1$,
- = $(s - 1)^3$.
- $s_1 = s_2 = s_3 = 1$.
- Slijedi matrica M:

$$M = \begin{bmatrix} 1 & 0 & 0 \\ s_1 & 1 & 0 \\ s_1^2 & \frac{2s_1}{1!} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix}$$

Nastavak

- Matrica A* je naravno:

$$A^* = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Provjeriti da je $A^* = M^{-1}$ A M !

Odziv linearnih sustava

$\mathbf{x}' = \mathbf{Ax} + \mathbf{Bu}$, $t_0 = 0$.
$s\mathbf{X}(s) - \mathbf{x}(0) = \mathbf{A}\,\mathbf{X}(s) + \mathbf{B}\,\mathbf{U}(s)$,
$(s\mathbf{I} - \mathbf{A})\,\mathbf{X}(s) = \mathbf{x}(0) + \mathbf{B}\,\mathbf{U}(s)$.
Pomnožimo slijeva sa $(s\mathbf{I} - \mathbf{A})^{-1}$:
$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{x}(0) + (s\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{B}\,\mathbf{U}(s)$,
$\mathbf{\Phi}(s) = (s\mathbf{I} - \mathbf{A})^{-1}$,

- matrica karakterističnih frekvencija.

$\mathbf{X}(s) = \mathbf{\Phi}(s)\,\mathbf{x}(0) + \mathbf{\Phi}(s)\,\mathbf{B}\,\mathbf{U}(s)$. (1)
$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$,
$\mathbf{Y}(s) = \mathbf{C}\,\mathbf{X}(s) + \mathbf{D}\,\mathbf{U}(s)$,
$\mathbf{Y}(s) = \mathbf{C}\,\mathbf{\Phi}(s)\,\mathbf{x}(0) + [\mathbf{C}\,\mathbf{\Phi}(s)\,\mathbf{B} + \mathbf{D}]\,\mathbf{U}(s)$. (2)

Nastavak ...

- $\mathbf{H}(s) = \mathbf{C}\Phi(s)\mathbf{B} + \mathbf{D}$,
  - transfer matrica.
- Pretvorimo (1) u donje područje

$$\mathbf{x}(t) = \mathbf{\Phi}(t)\cdot\mathbf{x}(0) + \int_0^t \mathbf{\Phi}(t-\tau)\mathbf{B}\mathbf{u}(\tau)d\tau.$$

- $\mathbf{\Phi}(t)$ – fundamentalna (prijelazna) matrica.
- Pretvorimo (2) u donje područje

$$\mathbf{y}(t) = \mathbf{C}\mathbf{\Phi}(t)\cdot\mathbf{x}(0) + \int_0^t \mathbf{C}\mathbf{\Phi}(t-\tau)\mathbf{B}\mathbf{u}(\tau)d\tau + \mathbf{D}\mathbf{u}(t).$$

Zadatak 4.

- Zadane su matrice A, B, C, D kontinuiranog sustava, te pobuda *u*. Odredi odziv sustava i napiši matricu impulsnog odziva.

$$\mathbf{A} = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix},\quad \mathbf{B} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix},\quad \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix},\quad \mathbf{D} = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

$$\mathbf{u}(t) = \begin{bmatrix} 2\delta(t) \\ S(t) \end{bmatrix},\quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

Rješenje

$$\mathbf{\Phi}(s) = (s\mathbf{I} - \mathbf{A})^{-1},\qquad \mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})}\cdot adj(\mathbf{A}),$$

$$s\mathbf{I} - \mathbf{A} = \begin{bmatrix} s & 2 \\ -1 & s+3 \end{bmatrix},\qquad adj(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s+3 & -2 \\ 1 & s \end{bmatrix},$$

$$\det(s\mathbf{I} - \mathbf{A}) = (s+1)(s+2),$$

$$\mathbf{\Phi}(s) = (s\mathbf{I} - \mathbf{A})^{-1} = \frac{1}{(s+1)(s+2)}\cdot\begin{bmatrix} s+3 & -2 \\ 1 & s \end{bmatrix}.$$

Nastavak ...

$$\mathbf{\Phi}(s)=\begin{bmatrix} -\frac{1}{s+2}+\frac{2}{s+1} & \frac{2}{s+2}-\frac{2}{s+1} \\ -\frac{1}{s+2}+\frac{1}{s+1} & \frac{2}{s+2}-\frac{1}{s+1} \end{bmatrix}$$ matrica karakterističnih frekvencija

- Transformacijom $\Phi(s)$ u $\Phi(t)$ dobivamo :

$$\mathbf{\Phi}(t)=\begin{bmatrix} -e^{-2t}+2e^{-t} & 2e^{-2t}-2e^{-t} \\ -e^{-2t}+e^{-t} & 2e^{-2t}-e^{-t} \end{bmatrix}$$ prijelazna ili fundamentalna matrica

Nastavak ...

- Skraćeno zapisano :

$$\mathbf{\Phi}(t)=\begin{bmatrix} \varphi_{11}(t) & \varphi_{12}(t) \\ \varphi_{21}(t) & \varphi_{22}(t) \end{bmatrix},$$

$$\mathbf{C\Phi}(t)=\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{bmatrix}=\begin{bmatrix} \varphi_{11} & \varphi_{12} \end{bmatrix}.$$

Nastavak ...

- Skraćeno zapisano :

$$\mathbf{\Phi}(t)=\begin{bmatrix} \varphi_{11}(t) & \varphi_{12}(t) \\ \varphi_{21}(t) & \varphi_{22}(t) \end{bmatrix},$$

$$\mathbf{C\Phi}(t)=\begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} \varphi_{11} & \varphi_{12} \\ \varphi_{21} & \varphi_{22} \end{bmatrix}=\begin{bmatrix} \varphi_{11} & \varphi_{12} \end{bmatrix},$$

$$\mathbf{C\Phi}(t)\mathbf{B}=\begin{bmatrix} \varphi_{11} & \varphi_{12} \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}=\begin{bmatrix} 2\varphi_{11} & \varphi_{12} \end{bmatrix}.$$

Nastavak ...

$$\mathbf{y}(t)=\left[\varphi_{11}(t)\quad \varphi_{12}(t)\right]\begin{bmatrix}x_1(0)\\x_2(0)\end{bmatrix}+\int_0^t\left[2\varphi_{11}(t-\tau)\quad \varphi_{12}(t-\tau)\right]\begin{bmatrix}2\delta(\tau)\\S(\tau)\end{bmatrix}d\tau+$$
$$+\left[0\quad 1\right]\begin{bmatrix}2\delta(t)\\S(t)\end{bmatrix}.$$

$$\mathbf{y}(t)=\varphi_{11}(t)x_1(0)+\varphi_{12}(t)x_2(0)+$$
$$+\int_0^t\left[4\varphi_{11}(t-\tau)\delta(\tau)+\varphi_{12}(t-\tau)S(\tau)\right]d\tau+$$
$$+0\cdot 2\delta(t)+1\cdot S(t).$$

Nastavak ...

$$y(t)=\left(-e^{-2t}+2e^{-t}\right)x_1(0)+\left(2e^{-2t}-2e^{-t}\right)x_2(0)+$$
$$+4\int_0^t -e^{-2(t-\tau)}\delta(\tau)d\tau+4\int_0^t 2e^{-(t-\tau)}\delta(\tau)d\tau+$$
$$+\int_0^t 2e^{-2(t-\tau)}S(\tau)d\tau+\int_0^t -2e^{-(t-\tau)}S(\tau)d\tau+0\cdot 2\delta(t)+1\cdot S(t).$$

$$\int_0^t f(t)\delta(t)dt=f(0).$$

Nastavak ...

$$\mathbf{y}(t)=\underbrace{\left[2x_1(0)+2x_2(0)\right]e^{-t}+\left[2x_2(0)-x_1(0)\right]e^{-2t}}_{\text{slobodni odziv}}+$$
$$\underbrace{+10e^{-t}S(t)-5e^{-2t}S(t)+0\cdot 2\delta(t)+1\cdot S(t)}_{\text{prisilni odziv}}.$$

Nastavak ...

- Transfer matrica

$$
\begin{aligned}
\mathbf{H}(s) &= \mathbf{C\Phi B} + \mathbf{D} \\
&= [2\varphi_{11}(s) \quad \varphi_{12}(s)] + [0 \quad 1] \\
&= [2\varphi_{11}(s) \quad \varphi_{12}(s) + 1] \\
&= \left[-\frac{2}{s+2} + \frac{4}{s+1} \quad \frac{2}{s+2} - \frac{2}{s+1} + 1 \right].
\end{aligned}
$$

Nastavak ...

- Transformacijom H(s) u *h*(t) dobivamo

$$\mathbf{h}(t) = \left[-2e^{-2t} + 4e^{-t} \quad 2e^{-2t} - 2e^{-t} + \delta(t) \right].$$

- Broj redaka od $\mathbf{H}(s) \equiv$ broj izlaza.
- Broj stupaca od $\mathbf{H}(s) \equiv$ broj ulaza.

$$H_{ij}(s) = \frac{i\text{ - ti izlaz}}{j\text{ - ti ulaz}}, \qquad \mathbf{H}(s) = \left[H_{ij}(s) \right]$$