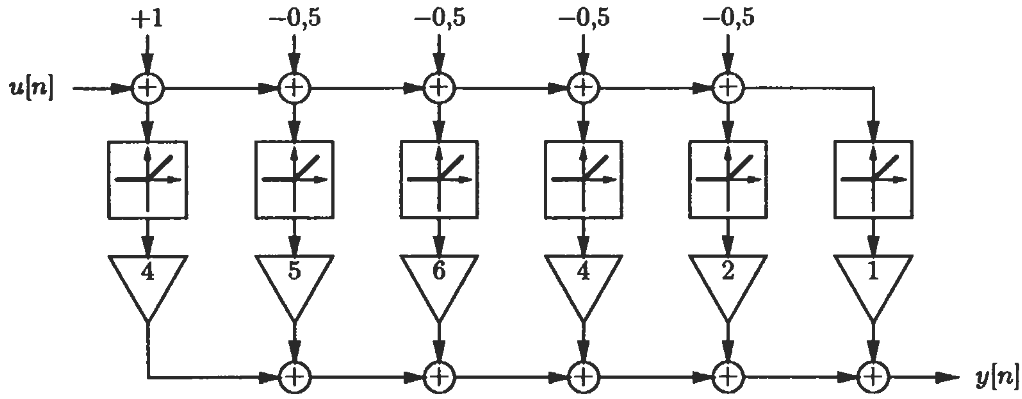
Signali i sustavi

Pismeni ispit – 1. rujna 2004.

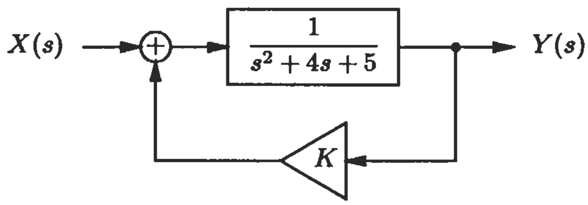
1. Skicirajte ulazno-izlaznu karakteristiku sustava zadanog slikom. Odredite odziv sustava na diskretni signal konačnog trajanja

$$u[n] = \{\ldots, 0, \underline{0}, 1/4, -1/2, 3/4, -1, 3/4, 1/5, 0, 0, 0, \ldots\}.$$



2. Komunikacijski kanal ima skup ulaznih simbola *Ulazi* $= \{0, 1\}$ i skup izlaznih simbola *Izlazi* $= \{0, 1, \perp\}$. Komunikacijski kanal za svaki ulazni simbol na izlazu uglavnom daje taj isti simbol, no ponekad nulu ili jedinicu zamijeni $\perp$ simbolom. Kanal može na izlazu dati najviše tri $\perp$ simbola u nizu. Definirajte nedeterministički automat koji modelira zadani komunikacijski kanal. Funkciju prijelaza možete navesti dijagramom ili tablicom.

3. Za sustav zadan slikom odredite kako polovi ovise o parametru K. Skicirajte amplitudno-frekvencijsku karakteristiku sustava za $K = -8$, $K = 1$ i $K = 10$ te odredite za koje od zadanih vrijednosti parametra K je sustav stabilan.



4. Konvolucijskom sumom riješi jednadžbu diferencija

$$y[n] - 2y[n-1] + y[n-2] = 3 + n + 4^n$$

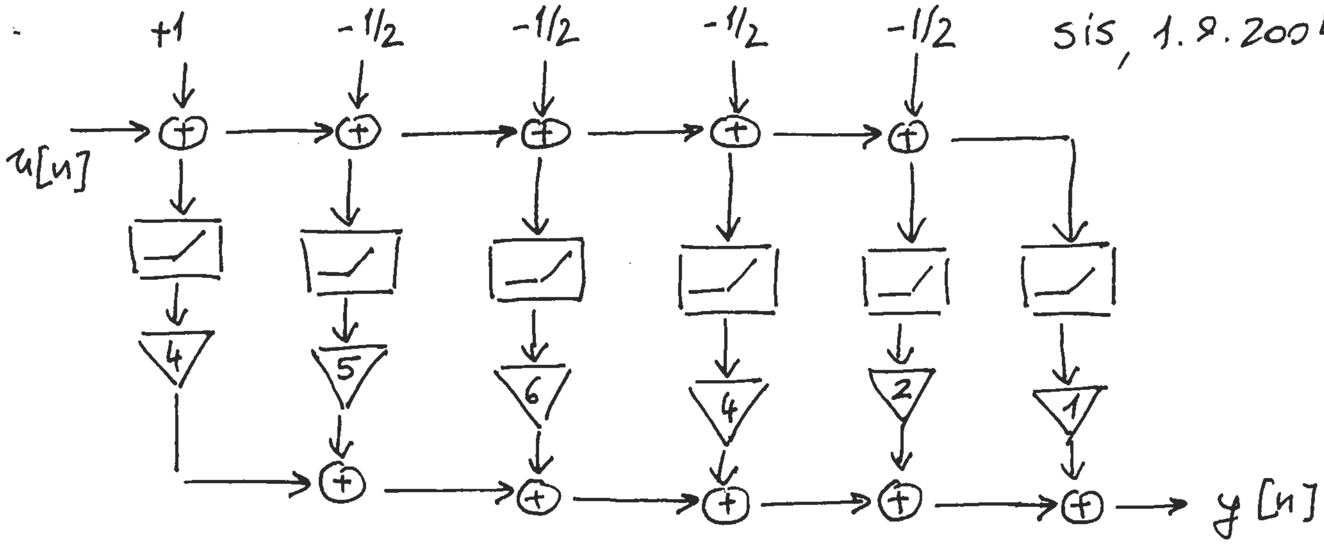
Napomena: Rješavanje drugim metodama neće se uvažiti.

5. Za kontinuirani sustav opisan je diferencijalnom jednadžbom

$$y''(t) + 4y'(t) + 3y(t) = 0.$$

odredi ekvivalentan diskretni sustav koristeći Eulerovu transformaciju uz $T = 1$. Za oba sustava nacrtaj blok-dijagram paralelne realizacije.

① sis, 1.9.2004.



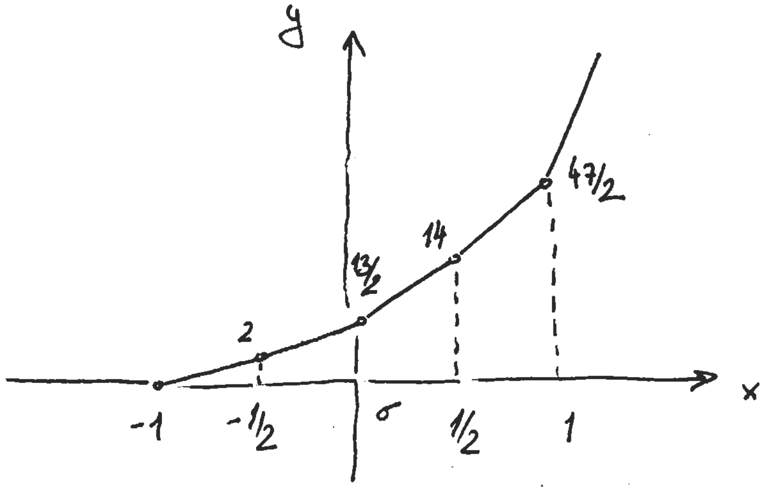
x: -1 $\quad -1/2 \quad 0 \quad 1/2 \quad 1$

$y=0$ ¦ $y=4x+4$ ¦ $y=9x+\frac{13}{2}$ ¦ $y=15x+\frac{13}{2}$ ¦ $y=19x+\frac{9}{2}$ ¦ $y=22x+3/2$

y: $0 \quad 2 \quad 13/2 \quad 14 \quad 47/2$

$u[n] = \{\ldots, \underline{0}, 1/4, -1/2, 3/4, -1, 3/4, 1/5, 0, \ldots\}$

$y[n] = \{\ldots, \underline{13/2}, 41/4, 2, 75/4, 0, 75/4, 19/2, 13/2, \ldots\}$

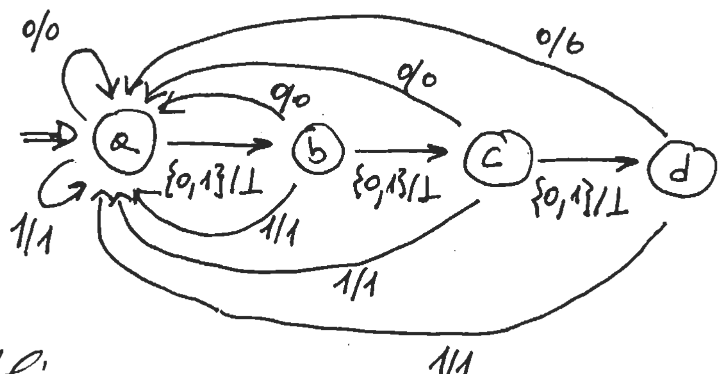


② Ulazi = $\{0, 1\}$

Izlazi = $\{0, 1, \perp\}$

Stanja = $\{a, b, c, d\}$

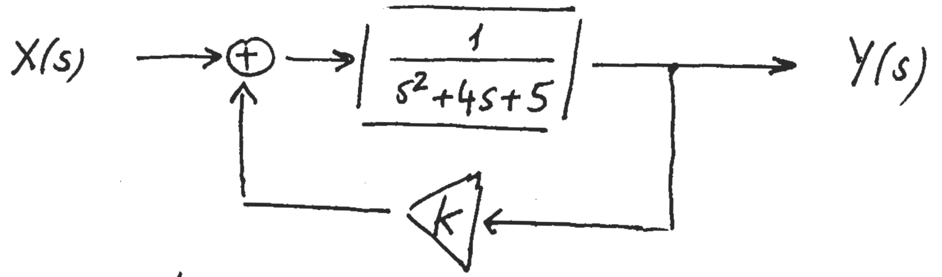
Početno stanje = a



prikazuje se dijagram i tablica

	0	1	odsutan
a	(a,0) , (b,⊥)	(a,1) , (b,⊥)	(a, odsutan)
b	(a,0) , (c,⊥)	(a,1) , (c,⊥)	(b, odsutan)
c	(a,0) , (d,⊥)	(a,1) , (d,⊥)	(c, odsutan)
d	(a,0)	(a,1)	(d, odsutan)

③



$$H(s) = \frac{\frac{1}{s^2+4s+5}}{1 - k\frac{1}{s^2+4s+5}} = \frac{1}{s^2+4s+5-k}, \quad s_{1,2} = -2 \pm \sqrt{k-1}$$

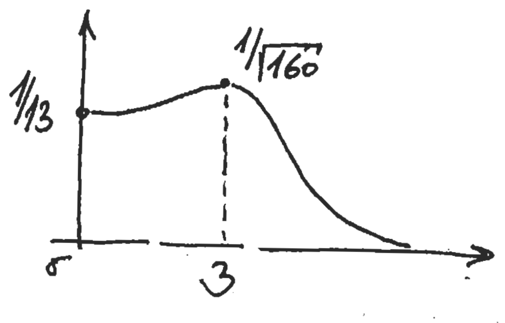
$k = -8$, $s_{1,2} = -2 \pm 3j$, sustav je stabilan

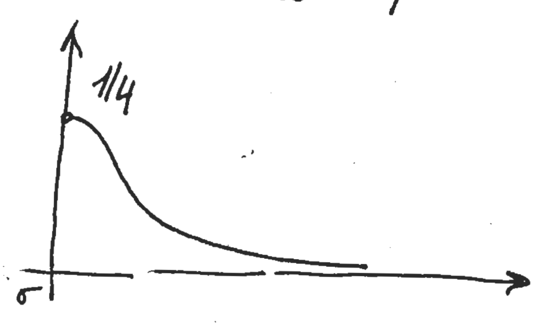
$k = 1$, $s_{1,2} = -2$, sustav je stabilan

$k = 10$, $s_{1,2} = -2 \pm 3$, sustav nije stabilan

$$H(s)\big|_{k=-8} = \frac{1}{s^2+4s+13}, \quad |H(\omega)| = \frac{1}{\sqrt{16\omega^2 + (13-\omega^2)^2}}$$

$$H(s)\big|_{k=1} = \frac{1}{(s+2)^2}, \quad |H(\omega)| = \frac{1}{\omega^2+4}$$





za slučaj $k=10$ sustav je nestabilan te amplitudno-
-frekvencijska karakteristika ne postoji

④:
$$y[n+2]-2y[n+1]+y[n]=3+n+4^n$$

$$\rho^2-2\rho+1=0 \Rightarrow \rho_{1,2}=1$$

$$y_h[n]=A+Bn$$

$$\begin{cases} A-B=0 \\ A+0\cdot B=1 \end{cases}$$

$$A=B=1$$

$$h[n]=1+n$$

$$y[n]=\sum_{i=0}^{n} h[i]u[n-i]=\sum_{i=0}^{n} u[i]h[n-i]=$$

$$=\sum_{i=0}^{n}(3+i+4^i)(1+n-i)=$$

$$=\sum_{i=0}^{n} 3+3n-3i+i+n\cdot i-i^2+4^i+n4^i-i4^i=$$

$$=\sum_{i=0}^{n} 3(n+1)+i(n-2)-i^2+(n+1)4^i-i4^i=$$

$$=n\cdot 3(n+1)+(n-2)\frac{n(n+1)}{2}-\frac{n(n+1)(2n+1)}{6}+(n+1)\frac{1-4^{n+1}}{1-4}$$

$$-\sum_{i=0}^{n} i4^i=$$

$$=\frac{n+1}{6}\left(18n+3n^2-6n-2n^2-n+(-2)\right)+\frac{n+1}{3}4^{n+1}$$

$$-\frac{1}{3}4^{n+1}\left(n-\frac{1}{3}\right)+\frac{4}{3}\cdot\frac{1}{3}=$$

$$=\frac{n+1}{6}\left(n^2+11n-2\right)+4^{n+1}\frac{1}{3}\left(n+1-n+\frac{1}{3}\right)+\frac{4}{9}=$$

$$=\frac{1}{6}\left(n^3+11n^2-2n+n^2+11n-2\right)+\frac{4}{9}+4^{n+1}\frac{1}{3}\cdot\frac{4}{3}=$$

$$=\frac{1}{6}n^3+2n^2+\frac{3}{2}n+\frac{1}{9}+\frac{16}{9}4^{n}$$

$$\sum_{i=1}^{n} i\,4^i = 4^{n+1}\left(n-\frac{1}{3}\right)+\frac{4}{3}$$

$$\Delta^{-1}\beta^k P(k) = \frac{\beta^k}{\beta-1}\left(1-\frac{\beta\Delta}{\beta-1}+\frac{\beta^2\Delta^2}{(\beta-1)^2}-\frac{\beta^3\Delta^3}{(\beta-1)^3}+\dots\right)P(k)$$

$$\Delta^{-1}4^k\cdot k = \frac{4^k}{4-1}\left(1-\frac{4\Delta}{3}+\frac{16\Delta^2}{9}-\dots\right)k =$$

$$= 4^k\,\frac{1}{3}\left(k-\frac{4}{3}\right)$$

$$\sum_{i=1}^{n} i\,4^i = 4^i\,\frac{1}{3}\left(i-\frac{4}{3}\right)\Big|_0^{n+1} = 4^{n+1}\,\frac{1}{3}\left(n-1-\frac{4}{3}\right)-\frac{1}{3}\left(-\frac{4}{3}\right)=$$

$$= 4^{n+1}\left(n-\frac{1}{3}\right)+\frac{4}{3}$$

5

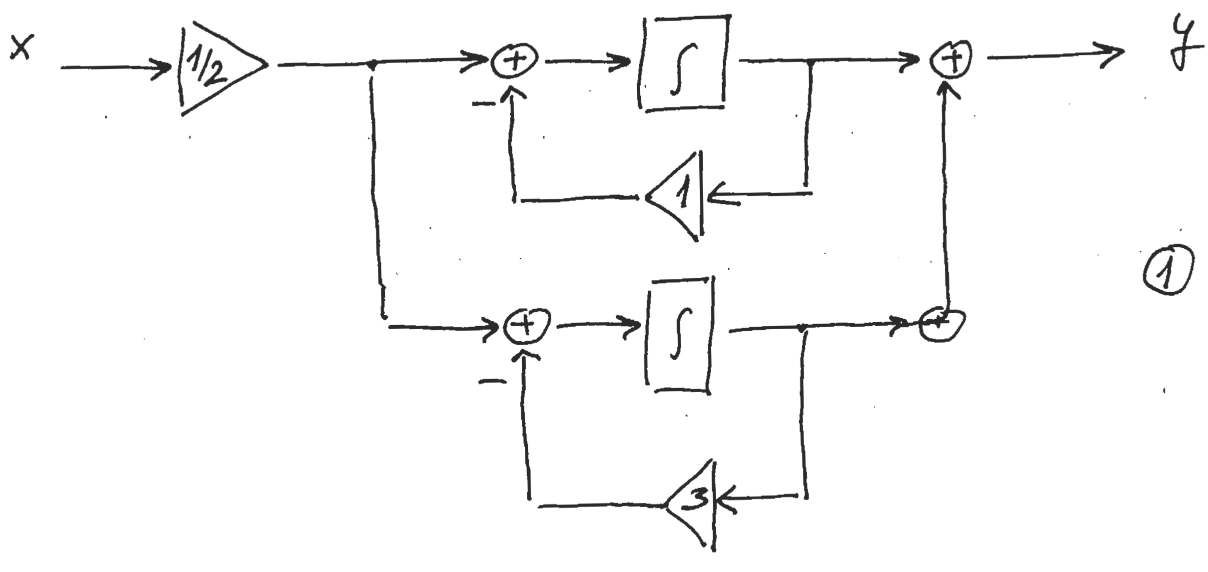
$$y''+4y'+3y=0 \Rightarrow H(s)=\frac{1}{s^2+4s+3}$$

$$s\mapsto z-1,\quad H(z)=\frac{1}{z^2-2z+1+4z-4+3}=\frac{1}{z^2+2z}$$ 1

$$H(s)=\frac{1}{s^2+4s+3}=\frac{1}{(s+1)(s+3)}=\frac{A}{s+1}+\frac{B}{s+3}$$

$$A=\frac{1}{s+3}\Big|_{s=-1}=\frac{1}{2},\quad B=\frac{1}{s+1}\Big|_{s=-3}=-\frac{1}{2}$$ 1

$$H(s)=\frac{1}{2}\,\frac{1}{s+1}-\frac{1}{2}\,\frac{1}{s+3}$$

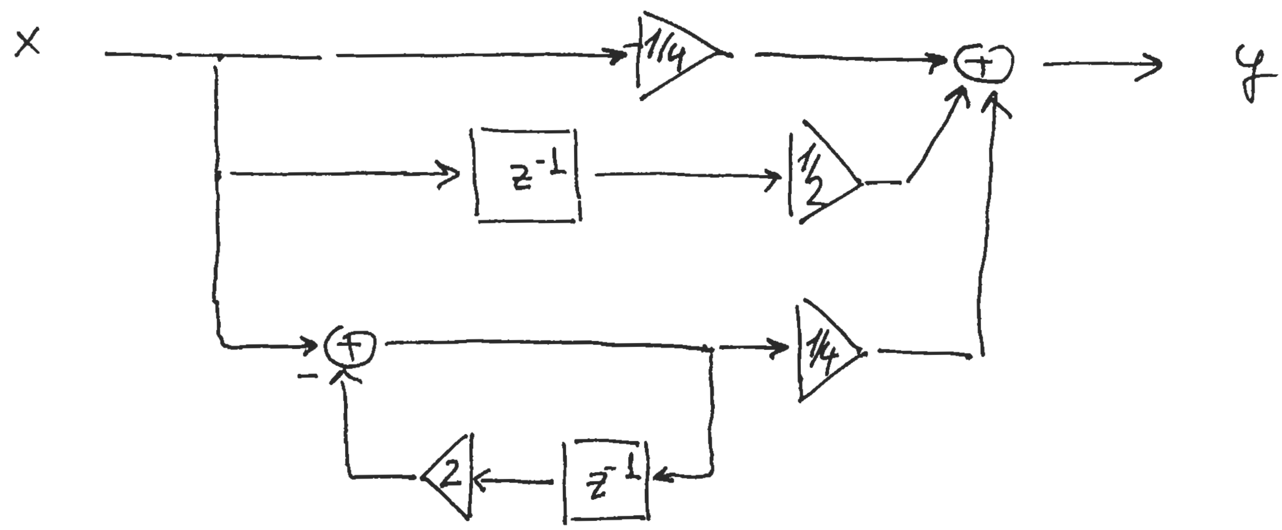


1

$$H(z) = \frac{1}{z(z+2)} = A + \frac{B}{z} + \frac{Cz}{z+2}$$

$$B = \frac{1}{z+2}\Big|_0 = \frac{1}{2}, \quad C = \frac{1}{z^2}\Big|_{-2} = \frac{1}{4}$$

$$\frac{1}{z(z+2)} = A + \frac{z(z+2)+4}{4(z+2)z} \Rightarrow A = -\frac{1}{4} \qquad (1)$$



Signali i sustavi

Pismeni ispit – 1. rujna 2004.

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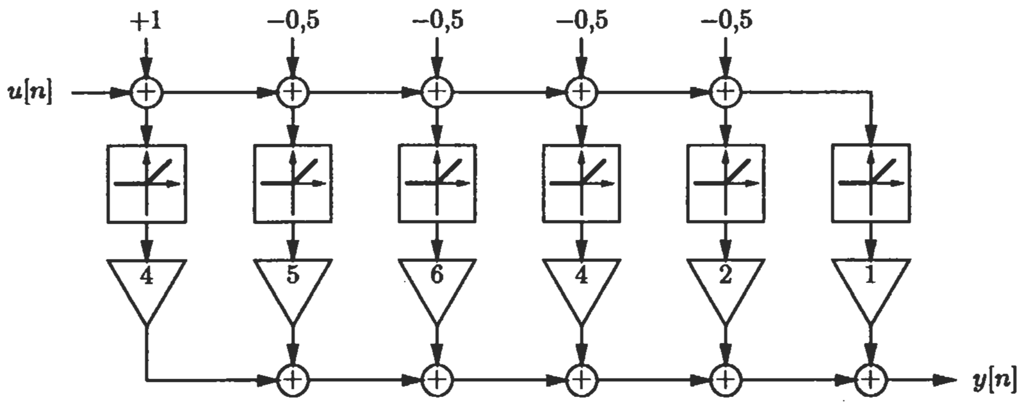
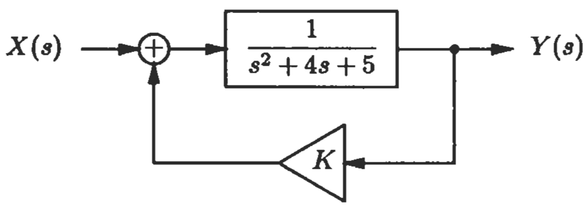


2. Prijenosna funkcija diskretnog sustava je

$$H(z) = \frac{32}{16z^3 - 4z^2 - 4z + 1}.$$

Realizirajte sustav pomoću paralelne realizacije te ispitajte upravljivost i osmotrivost sustava. Nacrtajte simulacijski blok-dijagram te na njemu provjerite da li ste ispravno zaključili o upravljivosti i osmotrivosti zadanog sustava.

3. Za sustav zadan slikom odredite kako polovi ovise o parametru K. Skicirajte amplitudno-frekvencijsku karakteristiku sustava za $K = -8$, $K = 1$ i $K = 10$ te odredite za koje od zadanih vrijednosti parametra K je sustav stabilan.



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$$y[n] - 2y[n-1] + y[n-2] = 3 + n + 4^n$$

Napomena: Rješavanje drugim metodama neće se uvažiti.

5. Za kontinuirani sustav opisan je diferencijalnom jednadžbom

$$y''(t) + 4y'(t) + 3y(t) = 0.$$

odredi ekvivalentan diskretni sustav koristeći Eulerovu transformaciju uz $T = 1$. Za oba sustava nacrtaj blok-dijagram paralelne realizacije.

(2) $$H(z) = \frac{32}{16z^2 - 4z^2 - 4z + 1} = \frac{A}{4z-1} + \frac{B}{2z-1} + \frac{C}{2z+1}$$

$$16z^3 - 4z^2 - 4z + 1 = 4z^2(4z-1) - (4z-1) =$$

$$= (4z-1)(2z-1)(2z+1) \Rightarrow z_1 = \frac{1}{4},\ z_2 = \frac{1}{2},\ z_3 = -\frac{1}{2}$$

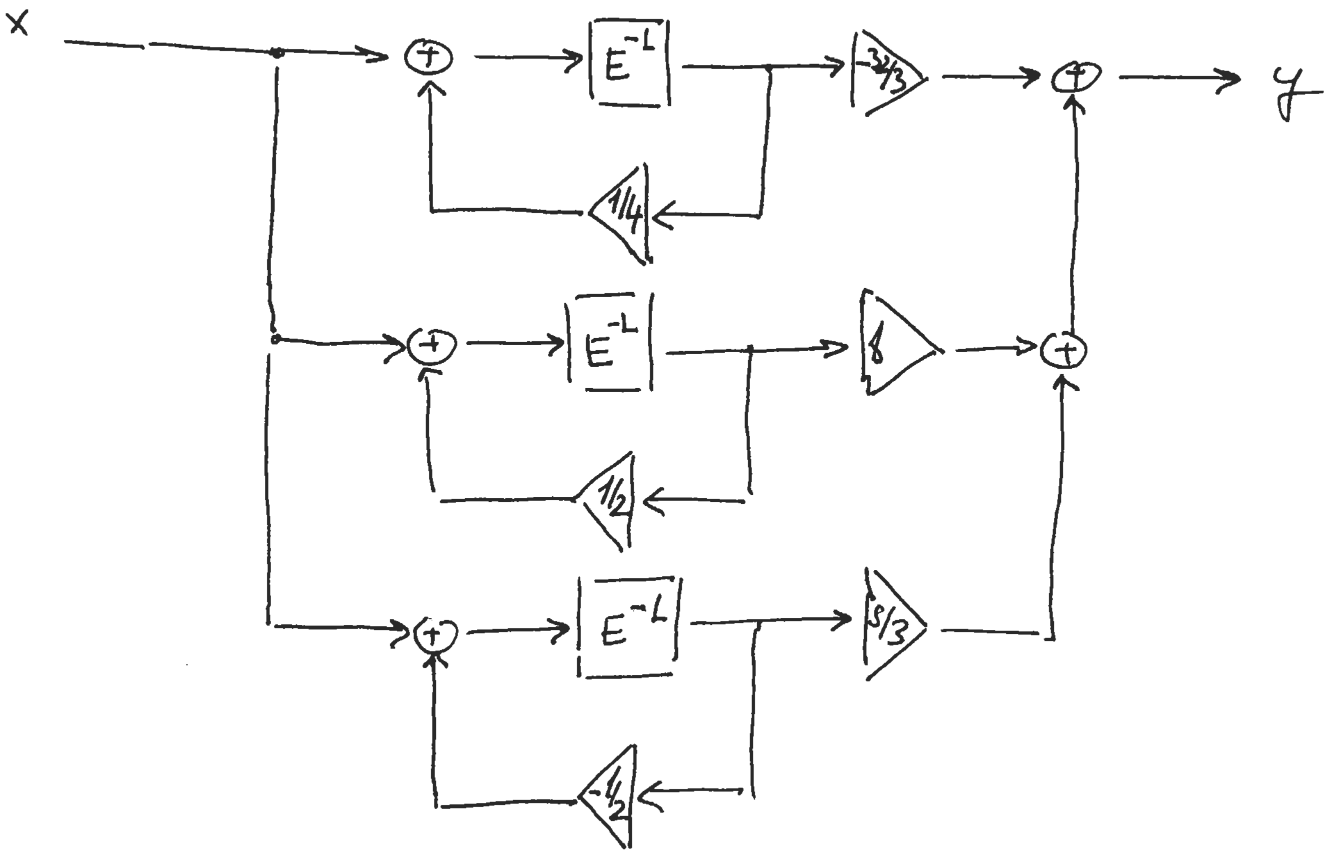
$$A = \frac{32}{4z^2-1}\bigg|_{z=1/4} = \frac{32}{4 \cdot 1/16 - 1} = \frac{32 \cdot 4}{1-4} = -\frac{128}{3}$$

$$B = \frac{32}{(4z-1)(2z+1)}\bigg|_{z=\frac{1}{2}} = \frac{32}{(2-1)(1+1)} = \frac{32}{2} = 16$$

$$C = \frac{32}{(4z-1)(2z-1)}\bigg|_{z=-\frac{1}{2}} = \frac{32}{(-2-1)(-1-1)} = \frac{32}{6} = \frac{16}{3}$$

$$H(z) = -\frac{32}{3}\frac{1}{z-1/4} + 8\frac{1}{z-1/2} + \frac{8}{3}\frac{1}{z+1/2}$$

sustav je upravljiv i osmotriv

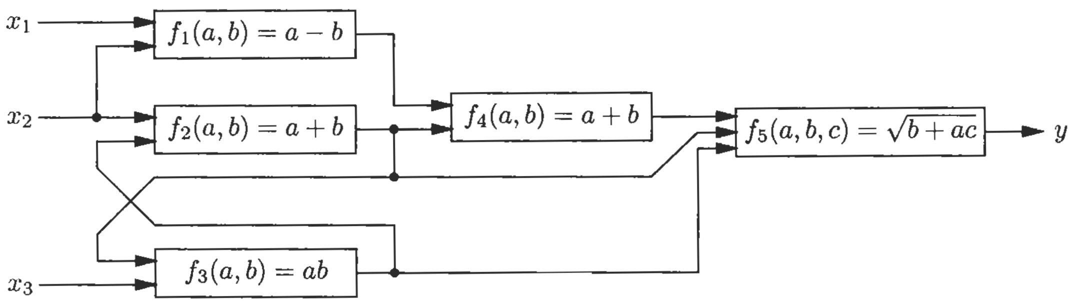


Signali i sustavi

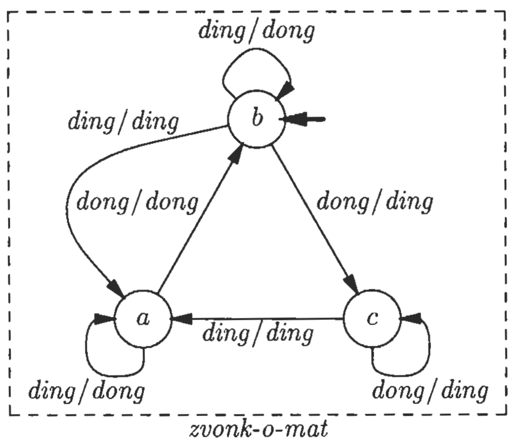
Pismeni ispit – 28. lipnja 2004.

1. Za bezmemorijski sustav zadan slikom odredite spojnu listu. Da li je sustav implicitan? Definirajte novu pomoćnu varijablu q te napišite jednadžbe sustava u obliku

$$q = F(q, x_2, x_3)$$
$$y = G(q, x_1, x_2, x_3)$$



2. Zadan je konačni automat *zvonk-o-mat* čija je funkcija prijelaza dana slikom. Razmotrire spoj zadanog automata u povratnu vezu gdje za ulaz uvodimo nadomjesni znak *djeluj*. Napišite uređenu petorku za tako dobiveni automat (funkciju prijelaza možete navesti dijagramom ili tablično). Ako postoje nedostupna stanja navedite ih!



3. Odziv nepobuđenog sustava drugog reda je

$$y(t) = \frac{1}{5}e^{2t} + \frac{1}{10}e^{-t}.$$

Odredite početna stanja za dani odziv ako sustav nema nula. Odredite matrice **A**, **B**, **C** i **D** te nacrtajte blok-dijagram za paralenu realizaciju ako je $H(0) = -\frac{3}{2}$.

4. Riješi jednadžbu diferencija

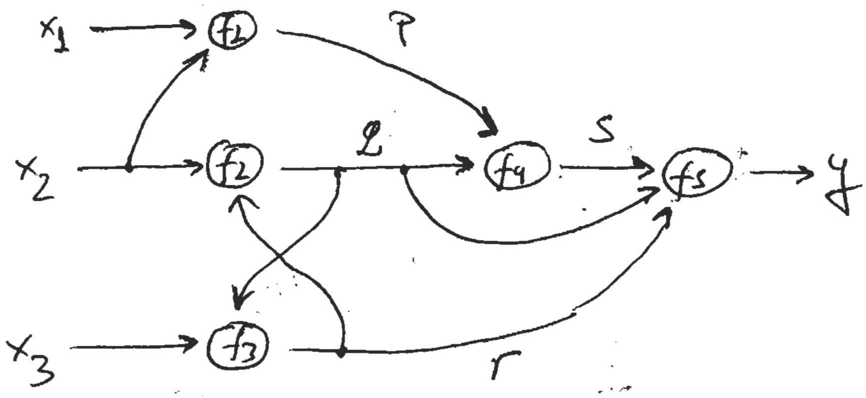
$$8y[n] - 6y[n-1] + y[n-2] = \delta[n] + 2\delta[n-125]$$

uz početne uvjete $y[-1] = 2^{125} + 2^{250}$ i $y[-2] = 1 + 2^{126} + 2^{252}$.

5. Linearni kontinuirani sustav ima dvostruki pol u točci $s = -1$ i nema nula. Maksimalna amplituda impulsnog odziva sustava je $3e^{-1}$. Kolika je maksimalna amplituda impulsnog odziva diskretnog sustava dobivenog bilinearnom transformacijom uz $T = 2$? Da li je dobiveni diskretni sustav stabilan?

SIS, 28. 6. 2004.

(1)



$f_1(a,b) = a - b$

$f_2(a,b) = a + b$

$f_3(a,b) = ab$

$f_4(a,b) = a + b$

$f_5(a,b,c) = \sqrt{b + ac}$

SPOJNA LISTA

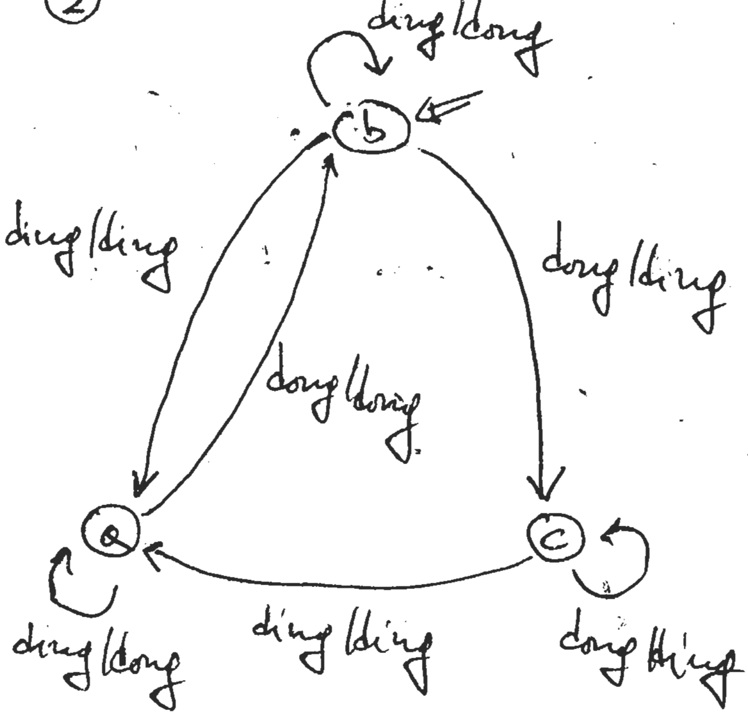
$$
\begin{cases}
p: x_1, x_2 & p = x_1 - x_2 \\
q: x_2, r & q = x_2 + r \\
r: x_3, q & r = q x_3 \\
s: p, q & s = p + q \\
y: s, q, r & y = \sqrt{q + sr}
\end{cases}
$$

Spojnu listu nije moguće sortirati jer sustav nije implicitan

$$
\begin{cases}
y = \sqrt{q + sr} = \sqrt{q + (p+q) q x_3} = \sqrt{q + (x_1 - x_2 + q) q x_3} \\
q = x_2 + r = x_2 + q x_3 \Rightarrow q = \dfrac{x_2}{1 - x_3}
\end{cases}
$$

$$
y = \sqrt{\frac{x_2}{1-x_3} + \left(x_1 - x_2 + \frac{x_2}{1-x_3}\right) \frac{x_2 x_3}{1-x_3}} = f(x_1, x_2, x_3)
$$

(2)



Zvonk-o-mat

ulazi = {ding, dong, ϕ}

izlazi = {ding, dong, ϕ}

stanja = {a, b, c}

početno stanje = {b}

Kako su ulazi jednaki izlazima spoj u povratnu vezu je moguć.

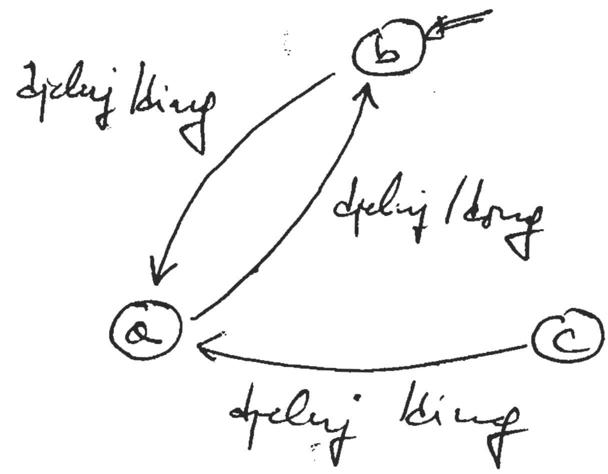
POVRATNA VEZA

ulazi = {djeluj, ϕ}

izlazi = {ding, dong, ϕ}

stanja = {a, b, c}

početno stanje = {b}



	djeluj	ϕ
a	b, dong	a, ϕ
b	a, ding	b, ϕ
c	a, ding	c, ϕ

(3)

$$y(t) = \frac{1}{5} e^{2t} + \frac{1}{10} e^{-t} = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

početna stanja su $y(0)$ i $y'(0)$

$$y(0) = \frac{1}{5} e^{2\cdot 0} + \frac{1}{10} e^{-0} = \frac{2}{10} + \frac{1}{10} = \frac{3}{10}$$

$$y'(0) = \frac{2}{5} e^{2\cdot 0} - \frac{1}{10} e^{-0} = \frac{4}{10} - \frac{1}{10} = \frac{3}{10}$$

polovi su $s_1 = 2$ i $s_2 = -1$

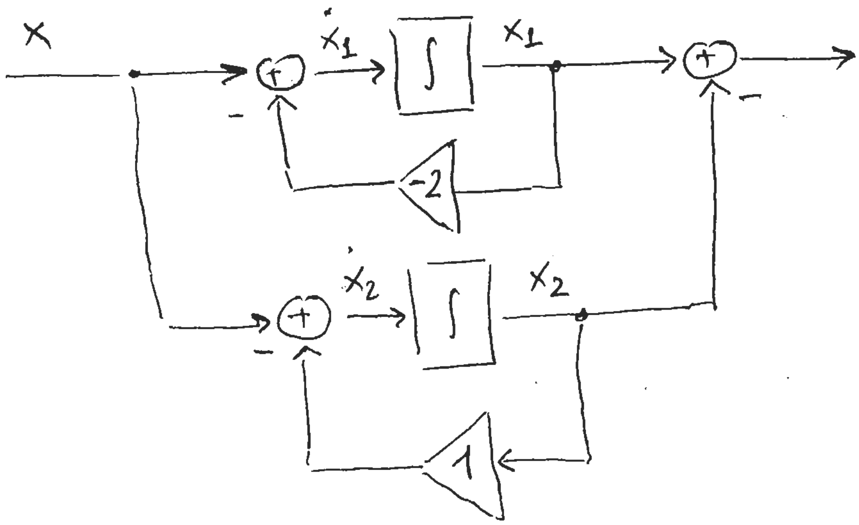
$$H(s) = \frac{c}{(s-s_1)(s-s_2)} = \frac{c}{(s-2)(s+1)} \qquad H(0) = \frac{c}{-2\cdot 1} = -\frac{c}{2} = -\frac{3}{2}$$

Za paralelnu realizaciju tražimo rastav $H(s)$ na parcijalne razlomke

$$H(s) = \frac{3}{(s-2)(s+1)} = \frac{1}{s-2} - \frac{1}{s+1}$$

$$\begin{cases} \dot{x}_1 = 2x_1 + x \\ \dot{x}_2 = -x_2 + x \end{cases} \qquad y = x_1 - x_2$$

③



$$A=\begin{bmatrix}2 & 0\\ 0 & -1\end{bmatrix}$$

$$B=\begin{bmatrix}1\\ 1\end{bmatrix}$$

$$C=\begin{bmatrix}1 & -1\end{bmatrix}$$

$$D=\begin{bmatrix}0\end{bmatrix}$$

④ $8y[n]-6y[n-1]+y[n-2]=\delta[n]+2\delta[n-125]$

$$8\rho^{n}-6\rho^{n-1}+\rho^{n-2}=0$$

$$\rho^{n-2}(8\rho^2-6\rho+1)=0 \Rightarrow \rho_1=\frac{1}{2},\ \rho_2=\frac{1}{4}$$

$$y_h[n]=c_1\left(\frac{1}{2}\right)^n+c_2\left(\frac{1}{4}\right)^n$$

$$\begin{cases} y[-1]=2^{125}+2^{250}\\ y[-2]=1+2^{126}+2^{252}\end{cases}$$

$$\begin{cases} 8y[0]-6(2^{125}+2^{250})+1+2^{126}+2^{252}=1\\ 8y[1]-6y[0]+2^{125}+2^{250}=0\end{cases}$$

$$\Rightarrow y[0]=2^{124}+2^{248},\ y[1]=2^{123}+2^{246}$$

$$\left.\begin{aligned} c_1\left(\tfrac{1}{2}\right)^0+c_2\left(\tfrac{1}{4}\right)^0=2^{124}+2^{248}\\ c_1\left(\tfrac{1}{2}\right)^1+c_2\left(\tfrac{1}{4}\right)^1=2^{123}+2^{246}\end{aligned}\right\} \Rightarrow c_1=2^{124},\ c_2=2^{248}$$

$$y[n]=2^{124}\left(\frac{1}{2}\right)^n+4^{124}\left(\frac{1}{4}\right)^n,\ 0\le n<125$$

$$\begin{cases} y[124]=2^{124}\cdot 2^{-124}+4^{124}4^{-124}=2\\ y[123]=2^{124}2^{-123}+4^{124}4^{-123}=6\end{cases}$$

$$\begin{cases} 8y[125]-6\cdot 2+6=2\\ 8y[126]-6y[125]+2=0\end{cases}$$

$$\Rightarrow y[125]=1,\ y[126]=1/2$$

$$\left.\begin{aligned} c_1 \cdot \left(\tfrac{1}{2}\right)^{125} + c_2 \left(\tfrac{1}{4}\right)^{125} &= 1 \\ c_1 \left(\tfrac{1}{2}\right)^{126} + c_2 \left(\tfrac{1}{4}\right)^{126} &= \tfrac{1}{2} \end{aligned}\right\} \Rightarrow c_1 = 2^{125},\ c_2 = 0$$

$$y[n] = \begin{cases} 2^{124} \left(\tfrac{1}{2}\right)^n + 4^{124} \left(\tfrac{1}{4}\right)^n, & 0 \leq n < 125 \\ 2^{125} \left(\tfrac{1}{2}\right)^n + 0 \left(\tfrac{1}{4}\right)^n, & n \geq 125 \end{cases}$$

5

$$H(s) = \frac{C}{(s+1)^2} \quad \circ\!\!-\!\!\!-\!\!\bigcirc \quad h(t) = C t e^{-t}$$

$$h'(t) = -Cte^{-t} + Ce^{-t} = Ce^{-t}(1-t)$$

$$h'(t) = 0 \Rightarrow t = 1$$

$$3e^{-1} = h(1) = C \cdot 1 e^{-1} \Rightarrow C = 3$$

$$H(s) = \frac{3}{(s+1)^2}$$

BILINEARNA TRANSFORMACIJA UZ $T=2$ $\quad s \mapsto \frac{2}{T}\frac{z-1}{z+1}$

$$H(z) = \frac{3}{\left(\frac{2}{2}\frac{z-1}{z+1} + 1\right)^2} = \frac{3(z+1)^2}{(z-1+z+1)^2} = \frac{3}{4z^2}\left(z^2+2z+1\right) =$$

$$= \frac{3}{4}\left(1 + 2z^{-1} + z^{-2}\right) \quad \circ\!\!-\!\!\!-\!\!\bigcirc \quad h[n] = \frac{3}{4}\left(\delta[n] + 2\delta[n-1] + \delta[n-2]\right)$$

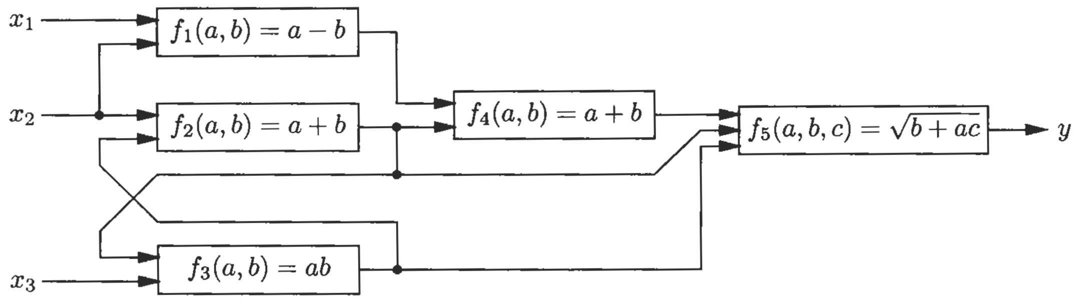
dobiveni diskretni sustav je stabilan

Signali i sustavi

Pismeni ispit – 28. lipnja 2004.

1. Za bezmemorijski sustav zadan slikom odredite spojnu listu. Da li je sustav implicitan? Definirajte novu pomoćnu varijablu q te napišite jednadžbe sustava u obliku

$$q = F(q, x_2, x_3)$$
$$y = G(q, x_1, x_2, x_3)$$



2. Ispitaj upraviljivost i osmotrivost sustava određenog s

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & -2 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \text{i} \quad \mathbf{D} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Nacrtajte paralelnu realizaciju sustava i na njoj provjerite da li ste ispravno zaključili o upravljivosti i osmotrivosti zadanog sustava.

3. Odziv nepobuđenog sustava drugog reda je

$$y(t) = \frac{1}{5}e^{2t} + \frac{1}{10}e^{-t}.$$

Odredite početna stanja za dani odziv ako sustav nema nula. Odredite matrice $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ i $\mathbf{D}$ te nacrtajte blok-dijagram za paralenu realizaciju ako je $H(0) = -\frac{3}{2}$.

4. Riješi jednadžbu diferencija

$$8y[n] - 6y[n-1] + y[n-2] = \delta[n] + 2\delta[n-125]$$

uz početne uvjete $y[-1] = 2^{125} + 2^{250}$ i $y[-2] = 1 + 2^{126} + 2^{252}$.

5. Linearni kontinuirani sustav ima dvostruki pol u točci $s = -1$ i nema nula. Maksimalna amplituda impulsnog odziva sustava je $3e^{-1}$. Kolika je maksimalna amplituda impulsnog odziva diskretnog sustava dobivenog bilinearnom transformacijom uz $T = 2$? Da li je dobiveni diskretni sustav stabilan?

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(2)

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -1 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\det(\lambda I - A) = 0 \Rightarrow \begin{vmatrix} \lambda+1 & 0 & 0 \\ 0 & \lambda+2 & -1 \\ 0 & 0 & \lambda+3 \end{vmatrix} = (\lambda+1)(\lambda+2)(\lambda+3) = 0$$

$$\lambda_1 = -1, \ \lambda_2 = -2, \ \lambda_3 = -3$$

$$(\lambda_1 I - A) v_1 = 0 \Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0, \quad v_1 = \begin{bmatrix} x_1 \\ 0 \\ 0 \end{bmatrix}$$

$$(\lambda_2 I - A) v_2 = 0 \Rightarrow \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0, \quad v_2 = \begin{bmatrix} 0 \\ x_2 \\ 0 \end{bmatrix}$$

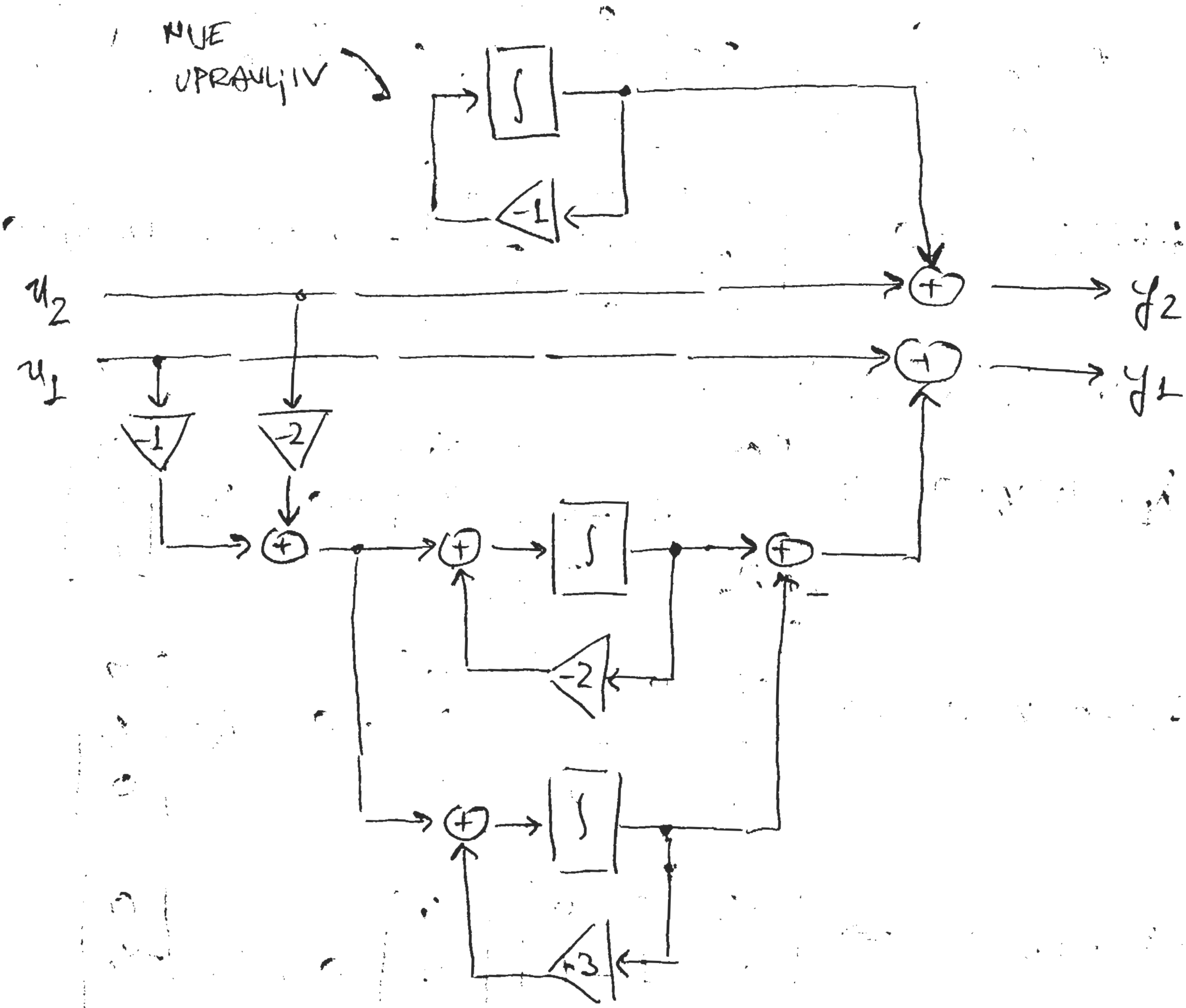
$$(\lambda_3 I - A) v_3 = 0 \Rightarrow \begin{bmatrix} -2 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0, \quad v_3 = \begin{bmatrix} 0 \\ -x_3 \\ x_3 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}, \quad T^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & +1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^* = T^{-1} A T = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad B^* = T^{-1} B = \begin{bmatrix} 0 & 0 \\ -1 & -2 \\ -1 & -2 \end{bmatrix}$$

$$C^* = C \cdot T = \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & 0 \end{bmatrix}, \quad D^* = D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

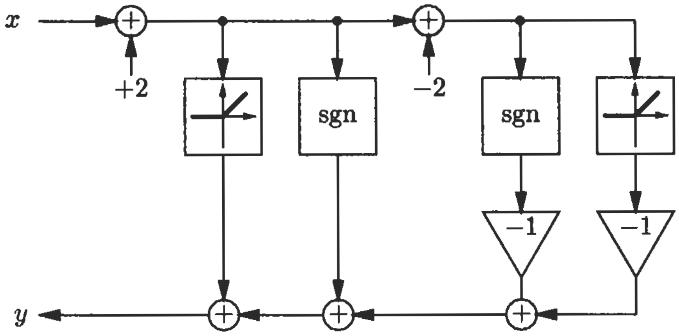
Sustav nije upravljiv, ali je osmotriv



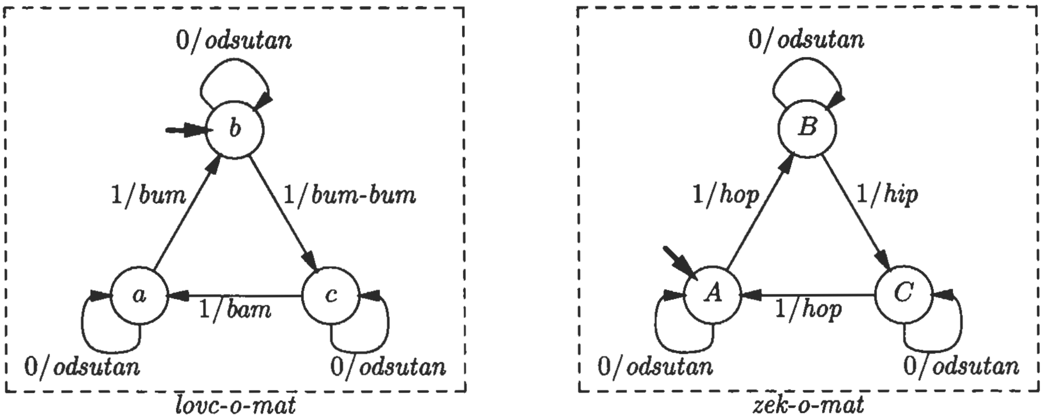
Signali i sustavi

Pismeni ispit – 19. travnja 2004.

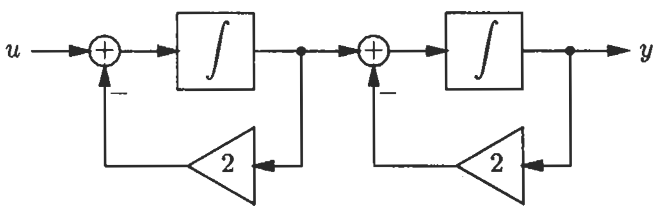
1. Za bezmemorijski sustav zadan slikom odredi ulazno-izlaznu karakteristiku te odziv na pobudu $x(t) = t + 1$.



2. Zadana su dva konačna automata, *zek-o-mat* i *lovc-o-mat*. Kada *lovc-o-mat* puca, *zek-o-mat* bježi od njega u malim skokovima. Za svaki od automata napiši skupove ulaznih i izlaznih simbola te navedi njihova početna stanja. Razmotri spoj zadanih automata u paralelu, ali tako da su ulazi u *zeko-o-mat* i *lovc-o-mat* uvijek jednaki. Napiši uređenu petorku koja definira tako dobiveni automat. Ako postoje nedostupna stanja, navedi ih!



3. Za kontinuirani sustav zadan slikom odredite odziv nepobuđenog i mirnog sustava, te ukupni odziv sustava. Na ulaz sustava dovodimo pobudu $u(t) = 4e^{-t}\,s(t)$. Neka su početni uvjeti $y(0) = 2$ i $y'(0) = 0$. Ispitajte stabilnost sustava.



4. Metodom varijacije parametara riješi jednadžbu diferencija

$$y[n+2] - y[n+1] - 6y[n] = n$$

Napomena: Rješavanje drugim metodama neće se uvažiti.

5. Kontinuirani sustav opisan je diferencijalnom jednadžbom

$$y''(t) - 2y'(t) - 3y(t) = u(t)$$

Pređite na diskretni sustav koristeći *Backward-Eulerovu* transformaciju uz $T = 1$. Odredite impulsni odziv dobivenog diskretnog sustava. Objasniti razloge stabilnosti (ili nestabilnosti) zadanog kontinuiranog i dobivenog diskretnog sustava.

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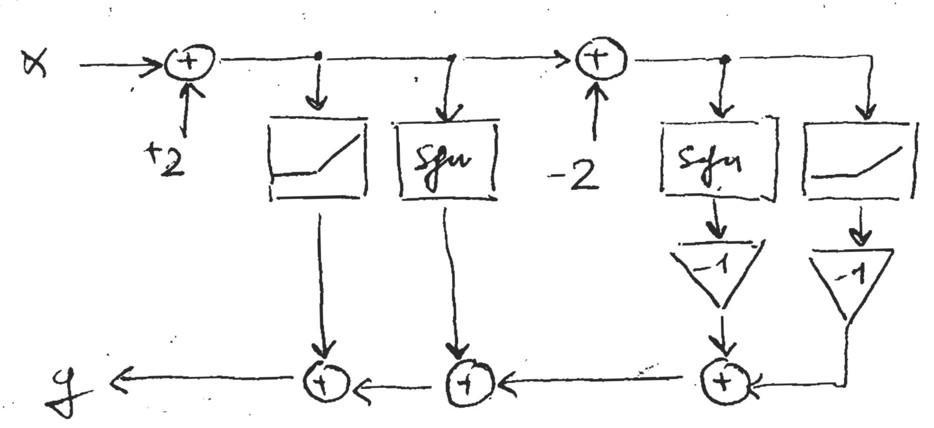
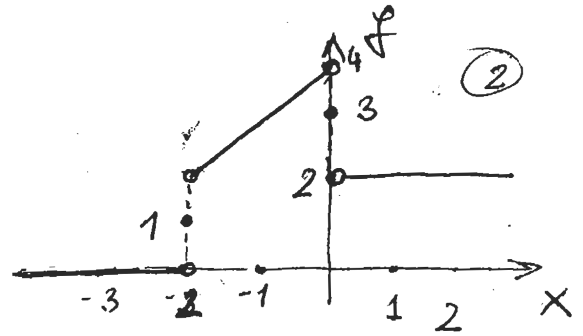
1



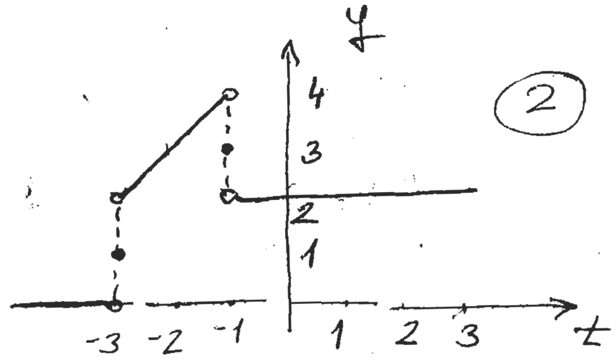
1

točke loma

-2, 0



2



2

2 love-o-mat ulazi = {0, 1, ϕ} → odsutan

izlazi = {bum, bam, bum-bum, ϕ}

početno stanje = {b}

zek-o-mat ulazi = {0, 1, ϕ}

izlazi = {hip, hop, ϕ}

početno stanje = {A}

1

PARALELA:

ulazi = ulazi$_L$ × ulazi$_Z$ = {(0, 0), (0, 1), (0, ϕ),
(1, 0), (1, 1), (1, ϕ),
(ϕ, 0), (ϕ, 1), (ϕ, ϕ)}

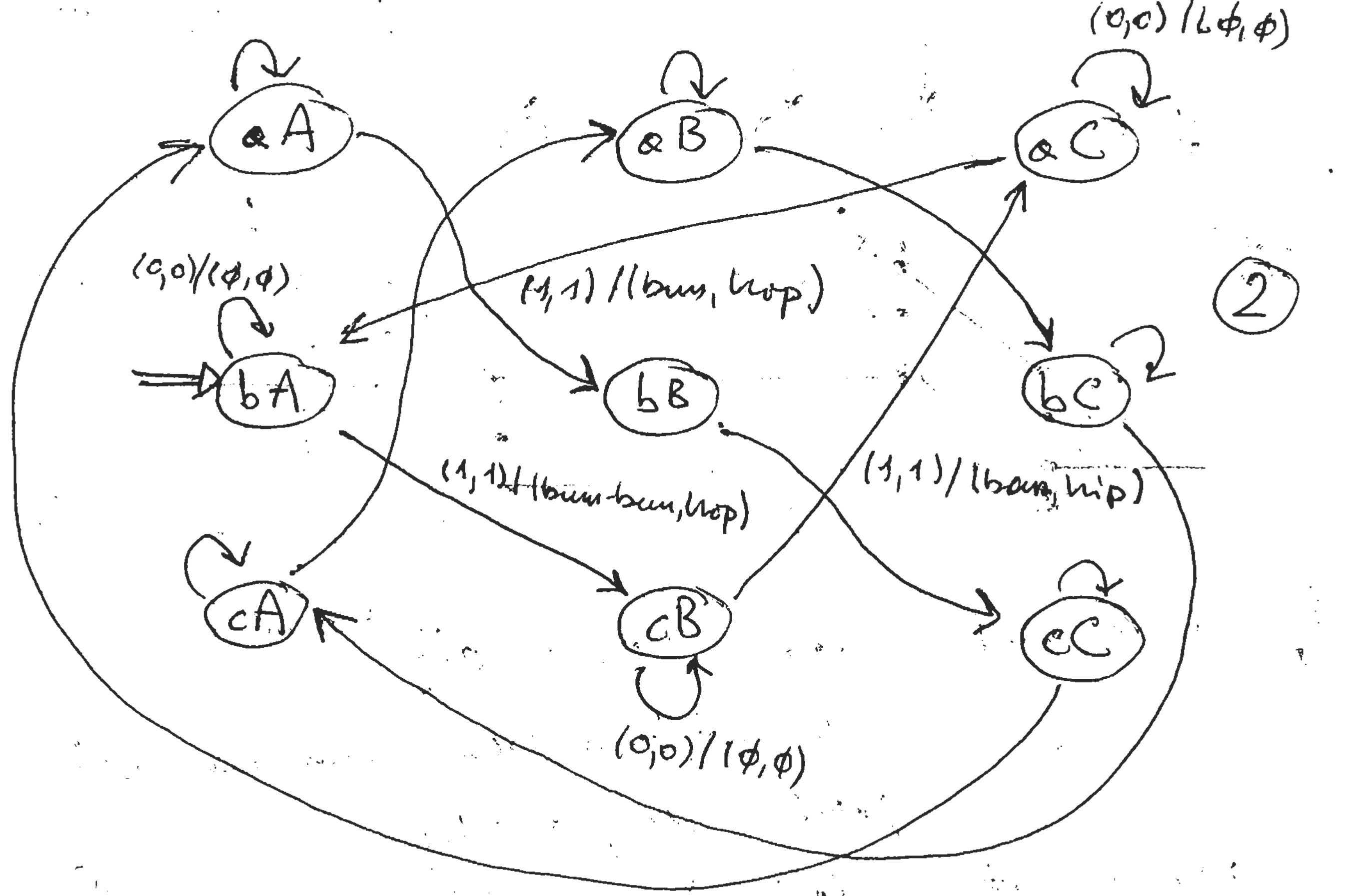
restrikcija na označene parove

1

izlazi = izlazi$_L$ × izlazi$_Z$ = { (bum, hip), (bum, hop), (bum, ϕ),
(bam, hip), (bam, hop), (bam, ϕ),
(bum-bum, hip), (bum-bum, hop), (bum-bum, ϕ),
(ϕ, hip), (ϕ, hop), (ϕ, ϕ) }

stanje = stanje$_L$ x stanje$_Z$ = { (aA), (aB), (aC), (bA), (bB), (bC), (cA), (cB), (cC) }

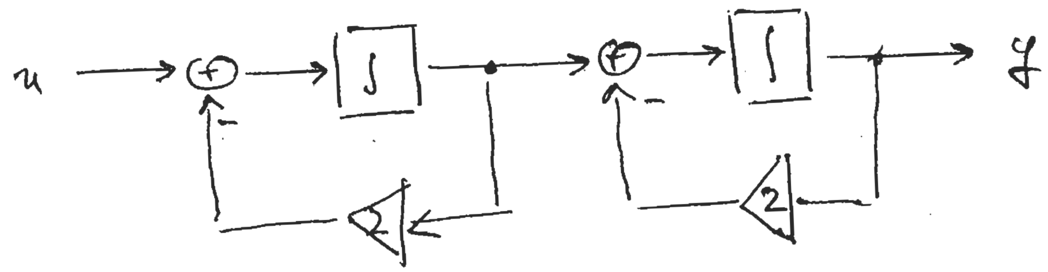
početno stanje = { (bA) }



nedostupna stanja su (aA), (bB), (cC), (aB), (b,c) i (cA)

1

③



POČETNI UVJETI $y(0)=2$, $y'(0)=0$

zadana je kaskadna realizacija

$$H(s)=H_1(s)\,H_2(s)=\frac{1}{s+2}\,\frac{1}{s+2}=\frac{1}{(s+2)^2} \Rightarrow y''+4y'+4y=u$$

$s_{1,2}=-2$ te je zadani sustav stabilan

NEPOBUĐENI: $y''+4y'+4y=0$, $y(0)=2$, $y'(0)=0$

$$y_N=(C_1+C_2t)e^{-2t} \qquad \begin{cases} 2=C_1+0 \\ 0=-2C_1+C_2 \end{cases} \Rightarrow \begin{matrix} C_1=2 \\ C_2=4 \end{matrix}$$

$$y_N=(2+4t)e^{-2t}$$

MIRNI: $y''+4y'+4y=4e^{-t}$, $y(0)=0$, $y'(0)=0$

$$y_P=Ae^{-t} \Rightarrow Ae^{-t}-\underline{4Ae^{-t}}+\underline{4Ae^{-t}}=4e^{-t} \Rightarrow A=4$$

$$y_M=(C_1+C_2t)e^{-2t}+4e^{-t}$$

$$\begin{cases} 0=C_1+4 \\ 0=-2C_1+C_2-4 \end{cases} \Rightarrow \begin{matrix} C_1=-4 \\ C_2=-4 \end{matrix}$$

$$y_M=(-4-4t)e^{-2t}+4e^{-t}$$

$$y=y_N+y_M=-2e^{-2t}+4e^{-t}$$

④ $y[n+2] - y[n+1] - 6y[n] = n$

$$q^2 - q - 6 = 0 \Rightarrow q = \frac{1 \pm \sqrt{1+4\cdot 6}}{2} = \frac{1 \pm 5}{2}$$

$$q_1 = -2, \quad q_2 = +3$$

$$y = C_1 (-2)^n + C_2 3^n$$

$$\begin{cases} \Delta(-2)^{n+1} = (-2)(-2)^{n+1} - (-2)^{n+1} = -3(-2)^{n+1} \\ \Delta 3^{n+1} = 3^{n+1} \cdot 3 - 3^{n+1} = 2 \cdot 3^{n+1} \end{cases}$$

$$\begin{cases} (-2)^{n+1} \Delta C_1[n] + 3^{n+1} \Delta C_2[n] = 0 \qquad / \cdot -2 \\ -3(-2)^{n+1} \Delta C_1[n] + 2 \cdot 3^{n+1} \Delta C_2[n] = n \end{cases}$$

$$\Delta C_1[n] = \frac{n}{-5(-2)^{n+1}}, \quad \Delta C_2[n] = \frac{n}{5 \cdot 3^{n+1}}$$

$$C_1[n] = \frac{1}{\Delta}\left[\frac{n}{10(-2)^n}\right] = \frac{1}{E-1}\left[\left(-\frac{1}{2}\right)^n \frac{n}{10}\right] = \left(-\frac{1}{2}\right)^n \frac{1}{-\frac{1}{2}E-1}\left[\frac{n}{10}\right] =$$

$$= \left(-\frac{1}{2}\right)^n \frac{1}{5} \frac{1}{-E-2}[n] = \frac{1}{5}\left(-\frac{1}{2}\right)^n \frac{1}{-\Delta-3}[n] = -\frac{1}{15}\left(-\frac{1}{2}\right)^n \frac{1}{1+\Delta/3}[n] =$$

$$= -\frac{1}{15}\left(-\frac{1}{2}\right)^n \left(1 - \Delta/3 + \Delta^2/9 - \ldots\right)[n] = -\frac{1}{15}\left(-\frac{1}{2}\right)^n \left(n - \frac{1}{3}\right)$$

$$C_2[n] = \frac{1}{\Delta}\left[\frac{n}{15 \cdot 3^n}\right] = \frac{1}{E-1}\left[\left(\frac{1}{3}\right)^n \frac{n}{15}\right] = \left(\frac{1}{3}\right)^n \frac{1}{\frac{1}{3}E-1}\left[\frac{n}{15}\right] =$$

$$= \left(\frac{1}{3}\right)^n \frac{1}{15} \frac{3}{E-3}[n] = \frac{1}{5}\left(\frac{1}{3}\right)^n \frac{1}{\Delta-2}[n] = -\frac{1}{10}\left(\frac{1}{3}\right)^n \frac{1}{1-\Delta/2}[n] =$$

$$= -\frac{1}{10}\left(\frac{1}{3}\right)^n \left(1 + \Delta/2 + \Delta^2/4 + \ldots\right)[n] = -\frac{1}{10}\left(\frac{1}{3}\right)^n \left(n + \frac{1}{2}\right)$$

$$y[n] = k_1(-2)^n + k_2 \cdot 3^n - \frac{n}{15} + \frac{1}{3 \cdot 15} - \frac{n}{10} - \frac{1}{20} =$$

$$= k_1(-2)^n + k_2 \cdot 3^n - \frac{1}{6}n - \frac{1}{36}$$

(5)

$$y'' - 2y' - 3y = u \Rightarrow H(s) = \frac{1}{s^2-2s-3} = \frac{1}{(s+1)(s-3)}$$

$s_1 = -1$, $s_2 = 3$ kontinuirani sustav nije stabilan jer se pol $s_2 = 3$ nalazi u desnoj poluravnini kompleksne ravnine s

BACKWARD-EULER $\quad s \mapsto \frac{1}{T}(1 - z^{-1})$, $T = 1$

$$H(z) = \frac{1}{(1-z^{-1}+1)(1-z^{-1}-3)} = \frac{z^2}{(2z-1)(-2z-1)} = -\frac{1}{4}\,\frac{z^2}{(z-\frac{1}{2})(z+\frac{1}{2})}$$

$z_1 = \frac{1}{2}$, $z_2 = -\frac{1}{2}$ diskretni sustav je stabilan jer se svi polovi nalaze unutar jedinične kružnice

$$H(z) = -\frac{1}{4}\,\frac{z^2}{(z-\frac{1}{2})(z+1/2)} = \alpha_0 + \alpha_1 \frac{z}{z-1/2} + \alpha_2 \frac{z}{z+1/2}$$

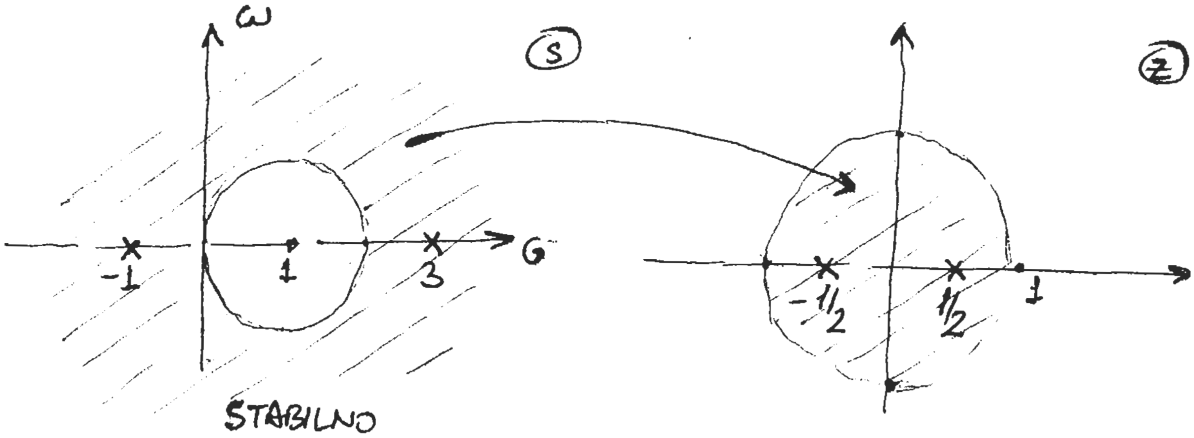
$$\alpha_0 = H(z)\Big|_{z=0} = 0$$

$$\alpha_1 = \frac{z-1/2}{z} H(z)\Big|_{z=1/2} = -\frac{1}{4}\,\frac{1/2}{1/2+1/2} = -\frac{1}{8}$$

$$\alpha_2 = \frac{z+1/2}{z} H(z)\Big|_{z=-1/2} = -\frac{1}{4}\,\frac{-1/2}{-1/2-1/2} = -\frac{1}{8}$$

$$H(z) = -\frac{1}{8}\,\frac{z}{z-1/2} - \frac{1}{8}\,\frac{z}{z+1/2}$$

$$h[n] = -\frac{1}{8}\left(-1/2\right)^n - \frac{1}{8}\left(\frac{1}{2}\right)^n$$



Signali i sustavi

Pismeni ispit – 19. travnja 2004.

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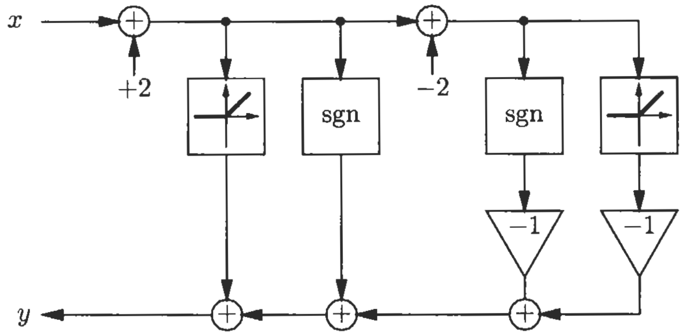

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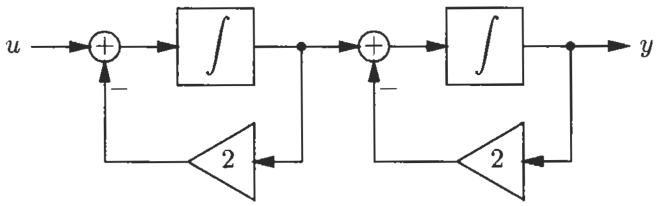


3. Kontinuirani sustav zadan je jednadžbama

$$\dot{y}_1(t) + y_2(t) = u_1(t)$$
$$y_1(t) + \dot{y}_2(t) = u_2(t)$$

Odredite matrice **A**, **B**, **C** i **D** paralelne realizacije. Ispitajte upravljivost i osmotrivost sustava. Nacrtajte paralelnu realizaciju i na njoj provjerite da li ste ispravno zaključili o upravljivosti i osmotrivosti sustava.

4. Konvolucijskom sumacijom riješi jednadžbu diferencija

$$\sqrt{6}y[n] + (\sqrt{2} - \sqrt{3})y[n-1] - y[n-2] = \left(-\frac{1}{\sqrt{5}}\right)\mathrm{s}[n]$$

Napomena: Rješavanje drugim metodama neće se uvažiti.

5. Kontinuirani sustav opisan je diferencijalnom jednadžbom

$$y''(t) - 2y'(t) - 3y(t) = u(t)$$

Pređite na diskretni sustav koristeći *Backward-Eulerovu* transformaciju uz $T = 1$. Odredite impulsni odziv dobivenog diskretnog sustava. Objasniti razloge stabilnosti (ili nestabilnosti) zadanog kontinuiranog i dobivenog diskretnog sustava.

SIS, 19. 4. 2004. STARI PLAN

(3) $\begin{cases} \dot{y}_1 + y_2 = u_1 \\ y_1 + \dot{y}_2 = u_2 \end{cases}$ varijable stanja su $x_1 = y_1 \quad x_2 = y_2$

$\begin{cases} \dot{x}_1 = -x_2 + u_1 \\ \dot{x}_2 = -x_1 + u_2 \end{cases} \qquad \begin{cases} y_1 = x_1 \\ y_2 = x_2 \end{cases}$

$$A = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\det(\lambda I - A) = \begin{vmatrix} \lambda & 1 \\ 1 & \lambda \end{vmatrix} = \lambda^2 - 1 = 0 \Rightarrow \lambda_1 = 1, \lambda_2 = -1$$

$$(\lambda_1 I - A) v_1 = 0 \Rightarrow \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0, \quad v_1 = \begin{bmatrix} x_1 \\ -x_1 \end{bmatrix}$$

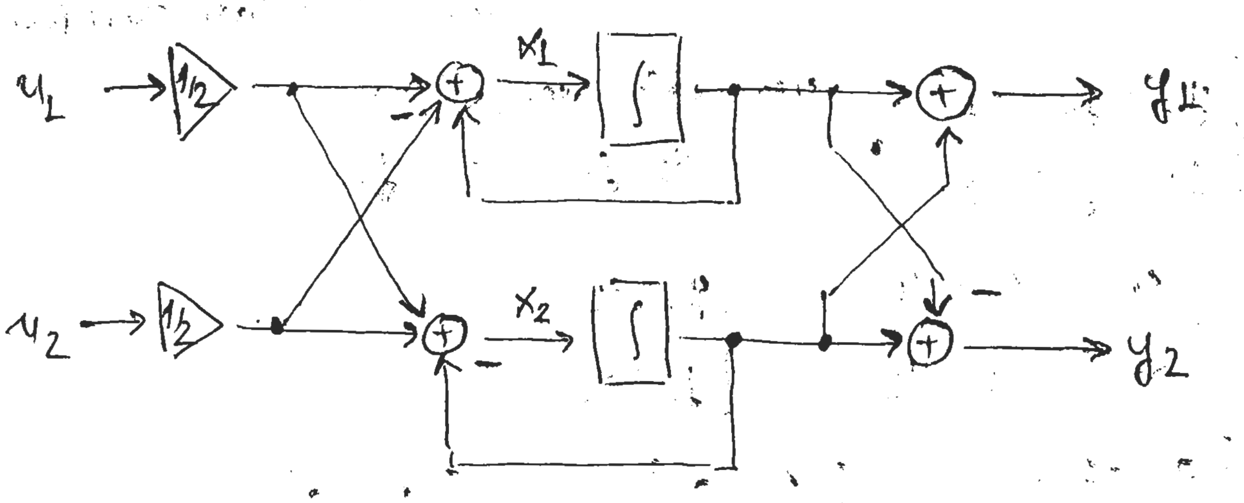
$$(\lambda_2 I - A) v_2 = 0 \Rightarrow \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0, \quad v_2 = \begin{bmatrix} x_2 \\ x_2 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \quad T^{-1} = \frac{\operatorname{adj}(T)}{\det(T)} = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$A^* = T^{-1} A T = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad B^* = T^{-1} B = \frac{1}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

$$C^* = CT = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}, \quad D^* = D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Sustav je upravljiv i osmotriv.



④ $\sqrt{6}\, y[n] + (\sqrt{2}-\sqrt{3})\, y[n-1] - y[n-2] = \left(-\frac{1}{\sqrt{5}}\right)^n \delta[n]$

$$\lambda_{1,2} = \frac{-\sqrt{2}+\sqrt{3} \pm \sqrt{(\sqrt{2}-\sqrt{3})^2+4\sqrt{6}}}{2\sqrt{6}} \Rightarrow \lambda_1 = \frac{+2\sqrt{3}}{2\sqrt{6}} = \frac{1}{\sqrt{2}},\ \lambda_2 = \frac{-2\sqrt{2}}{2\sqrt{6}} = -\frac{1}{\sqrt{3}}$$

$$h[n] = c_1 \left(\frac{1}{\sqrt{2}}\right)^n + c_2 \left(-\frac{1}{\sqrt{3}}\right)^n,\ n \geq 0$$

$$\begin{cases} c_1 \left(\frac{1}{\sqrt{2}}\right)^{-1} + c_2 \left(-\frac{1}{\sqrt{3}}\right)^{-1} = 0 \\ c_1 \left(\frac{1}{\sqrt{2}}\right)^{0} + c_2 \left(-\frac{1}{\sqrt{3}}\right)^{0} = \frac{1}{\sqrt{6}} \end{cases} \Rightarrow \begin{cases} \sqrt{2}\, c_1 - \sqrt{3}\, c_2 = 0 \\ c_1 + c_2 = \frac{1}{\sqrt{2}\sqrt{3}} \end{cases}$$

$$h[n] = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}+\sqrt{3}} \left(\frac{1}{\sqrt{2}}\right)^n + \frac{1}{\sqrt{3}} \frac{1}{\sqrt{2}+\sqrt{3}} \left(-\frac{1}{\sqrt{3}}\right)^n$$

$$y[n] = \sum_{i=1}^{n} \left(\frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}+\sqrt{3}} \left(\frac{1}{\sqrt{2}}\right)^i + \frac{1}{\sqrt{3}} \frac{1}{\sqrt{2}+\sqrt{3}} \left(-\frac{1}{\sqrt{3}}\right)^i \right) \left(-\frac{1}{\sqrt{5}}\right)^{n-i} =$$

$$= \frac{1}{\sqrt{2}+\sqrt{3}} \left(-\frac{1}{\sqrt{5}}\right)^n \sum_{i=1}^{n} \left(\frac{1}{\sqrt{2}} \left(-\frac{\sqrt{5}}{\sqrt{2}}\right)^i + \frac{1}{\sqrt{3}} \left(+\frac{\sqrt{5}}{\sqrt{3}}\right)^i \right) =$$

$$= \frac{1}{\sqrt{2}+\sqrt{3}} \left(-\frac{1}{\sqrt{5}}\right)^n \left(\frac{1}{\sqrt{2}} \frac{1-\left(-\frac{\sqrt{5}}{\sqrt{2}}\right)^{n+1}}{1+\frac{\sqrt{5}}{\sqrt{2}}} + \frac{1}{\sqrt{3}} \frac{1-\left(\frac{\sqrt{5}}{\sqrt{3}}\right)^{n+1}}{1-\frac{\sqrt{5}}{\sqrt{3}}} \right) =$$

$$= \frac{1}{\sqrt{2}+\sqrt{3}} \left(-\frac{1}{\sqrt{5}}\right)^n \left(\frac{1}{\sqrt{2}} \frac{\sqrt{2}+\sqrt{5}\left(-\frac{\sqrt{5}}{\sqrt{2}}\right)^n}{\sqrt{2}+\sqrt{5}} + \frac{1}{\sqrt{3}} \frac{\sqrt{3}-\sqrt{5}\left(\frac{\sqrt{5}}{\sqrt{3}}\right)^n}{\sqrt{3}-\sqrt{5}} \right) =$$

$$= \frac{1}{\sqrt{2}+\sqrt{3}} \frac{\sqrt{5}}{2+\sqrt{10}} \left(\frac{1}{\sqrt{2}}\right)^n - \frac{1}{\sqrt{2}+\sqrt{3}} \frac{\sqrt{5}}{3-\sqrt{15}} \left(-\frac{1}{\sqrt{3}}\right)^n$$

$$+ \frac{1}{(\sqrt{2}+\sqrt{5})(\sqrt{3}-\sqrt{5})} \left(-\frac{1}{\sqrt{5}}\right)^n$$