Maso 2 ciklus

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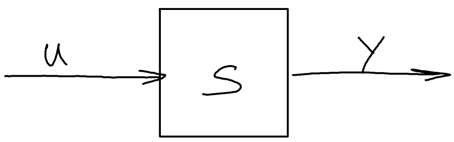
7:52

Sustavi



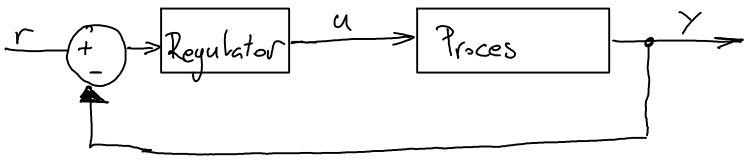
$f: x \to y \Rightarrow y = f(x)$

$S: u \to y \Rightarrow y = S(u)$

Podjela sustava:

1) Kontinuirane – u, y su kontinuirani
2) Diskretni – u, y su diskretni
3) Hibridni – u, y mogu biti diskretni i kontinuirani

Slika: priča o hibridnim sustavima – nebitno za SIS



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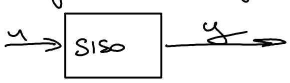
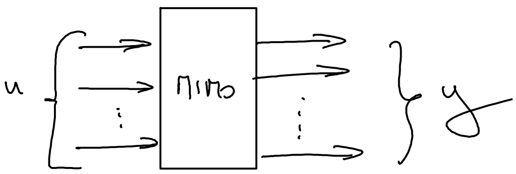
1) SISO – jedan ulaz, jedan izlaz



2) MIMO – više ulaza, više izlaza



1) Linearni
2) Nelinearni

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1) Vremenski promjenjivi
2) Vremenski nepromjenjivi +

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1) Kauzalni +

1) Kauzalni +

2) Nekauzalni

1) Memorijski

2) Bezmemorijski

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- $y(t) = u(t-4)$

  $t=2 \quad y(2) = u(-2)$

  sustav je kauzalan jer ovisi o pobudi iz "prošlosti"

  Memorijski sustav

- $y(t) = u(t+4)$

  $t=2$

  $y(2) = u(6)$

  Nekauzalan

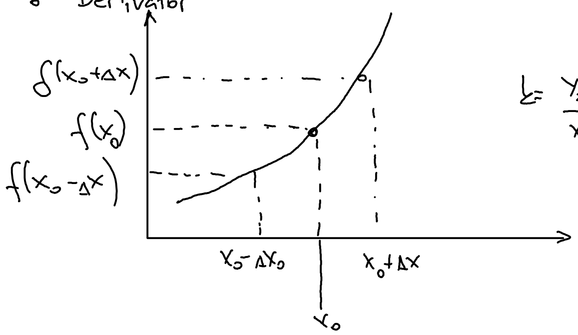
  Memorijski

- $y(t) = u(t^2)$

  Nekauzalan

  Memorijski

- Derivator



$$k = \frac{y_2 - y_1}{x_2 - x_1}$$

$$k = \lim_{x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{x_0 + \Delta x - x_0}$$

– Nekauzalan

$$k = \lim \frac{f(x_0) - f(x_0 - \Delta x)}{\Delta x}$$

– kauzalan

Na labosima uzimamo da je nekauzalan

- Integrator

$$y(t) = \int_{-\infty}^{t} u(\tau)\, d\tau \qquad \text{– kauzalan, memorijski}$$

$$y(t) = \int^{t+1} u(\tau)\, d\tau \qquad \text{– nekauzalan, memorijski}$$

$$y(t) = \int_{-\infty} u(\tau)\, d\tau$$ – nekauzalan, memorijski

Kod diskretnih

$$\Delta y(n) = y(n+1) - y(n)$$

$$\nabla y(n) = y(n) - y(n+1)$$

$$y(n) = \sum_{k=-\infty}^{n} u(k)$$

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Primjer 1:

$$S(\alpha u) = \alpha S(u)$$

$$S(\alpha u_1 + \beta u_2) = S(\alpha u_1) + S(\beta u_2)$$

Sustav je linearan ako zadovoljava svojstvo homogenosti i aditivnosti.

$$y(t) = 3u(t+4)$$

- $u(t) = \alpha u_1(t) + \beta u_2(t)$

$$u(t+4) = \alpha u_1(t+4) + \beta u_2(t+4)$$

- $y_1(t) = 3u_1(t+4)$; $y_2(t) = 3u_2(t+4)$

$S(u_1) = y_1$
$S(u_2) = y_2$

$$y(t) = 3\left(\alpha u_1(t+4) + \beta u_2(t+4)\right)$$

$$y(t) = \alpha \underbrace{3u_1(t+4)}_{y_1} + \beta \underbrace{u_2(t+4)}_{y_2} = \alpha y_1 + \beta y_2$$

$\Downarrow$

sustav je linearan

Primjer 2:

$$y(t) = t\,u(t^2)$$

Linearnost

- $u(t) = \alpha u_1(t) + \beta u_2(t)$

$$u(t) = \alpha u_1(t^2) + \beta u_2(t^2)$$

- $y_1(t) = t u_1(t^2)$; $y_2(t) = t u_2(t^2)$

$$y(t) = t\left(\alpha u_1(t^2) + \beta u_2(t^2)\right) = \alpha t u_1(t^2) + \beta t u_2(t^2) =$$

$$= \alpha y_1(t) + \beta y_2(t)$$

$\Downarrow$

linearan

Primjer 3:

$u(n)$

$$y(n) = 2^{u(n)}$$

Linearan

$$u(n) = \alpha u_1(n) + \beta u_2(n)$$

$$y_1(n) = 2^{u_1(n)} \; ; \; y_2(n) = 2^{u_2(n)}$$

$$y(n) = 2^{\alpha u_1(n) + \beta u_2(n)} = 2^{\alpha u_1(n)} \cdot 2^{\beta u_2(n)}$$

sustav je nelinearan

Primjer 4:

$$y(t) = u(t) + 4$$

Linearnost

- $u(t) = \alpha u_1(t) + \beta u_2(t)$

$$y_1(t) = u_1(t) + 4 \; ; \; y_2(t) = u_2(t) + 4$$

$$y(t) = \alpha u_1(t) + \beta u_2(t) + 4$$

$$y(t) = \alpha(u_1(t) + 4) + \beta(u_2(t) + 4) - 4\alpha - 4\beta + 4$$

$$y = \alpha y_1 + \beta y_2 + 4(1 - \alpha - \beta)$$

Nelinearan

Primjer 5:

$$y(n) = \frac{n}{u(n)}$$

- linearnost
  - $u(n) = \alpha u_1(n) + \beta u_2(n)$
  - $y_1(n) = \dfrac{n}{u_1(n)} \; ; \; y_2(n) = \dfrac{n}{u_2(n)}$

$$y(n) = \frac{n}{\alpha u_1(n) + \beta u_2(n)} \overset{?}{\neq} \alpha \frac{n}{u_1(n)} + \beta \frac{n}{u_2(n)} = \frac{n(\alpha u_2 + \beta u_1)}{u_1 u_2}$$

ne može biti
jednako stoga sustav
je nelinearan

Primjer 6:

$$y(t) = \int_{-\infty}^{t} f(\tau)\, d\tau$$

- $u(t) = \alpha u_1(t) + \beta u_2(t)$

  $u(\tau) = \alpha u_1(\tau) + \beta u_2(\tau)$

$$y(t) = \int_{-\infty}^{t} \alpha u_1(\tau) + \beta u_2(\tau)\, d\tau$$

$$y_1(t) = \int_{-\infty}^{t} u_1(\tau)\, d\tau$$

$$y_2(t) = \int_{-\infty}^{t} u_2(\tau)\, d\tau$$

$$y(t) = \int_{-\infty}^{t} \alpha u_1(\tau)\, d\tau + \int_{-\infty}^{t} \beta u_2(\tau)\, d\tau$$

$$= \alpha y_1 + \beta y_2 \Rightarrow \text{sustav je linearan}$$

Primjeri

- $y(n) = \dfrac{u(n)}{1 + u(n-1)}$ $\Rightarrow$ nelinearan – problem nazivnik

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- $y(n) = \left(\frac{1}{2}\right)^n u(3n+2)$

$$u(3n+2) = \alpha u_1(3n+2) + \beta u_2(3n+2)$$

$$y_1(n) = \left(\frac{1}{2}\right)^n u_1(3n+2)$$

$$y_2(n) = \left(\frac{1}{2}\right)^n u_2(3n+2)$$

$$\Rightarrow y(n) = \alpha \underbrace{\left(\frac{1}{2}\right)^n u_1(3n+2)}_{y_1} + \beta \underbrace{\left(\frac{1}{2}\right)^n u_2(3n+2)}_{y_2} = \alpha y_1 + \alpha y_2$$

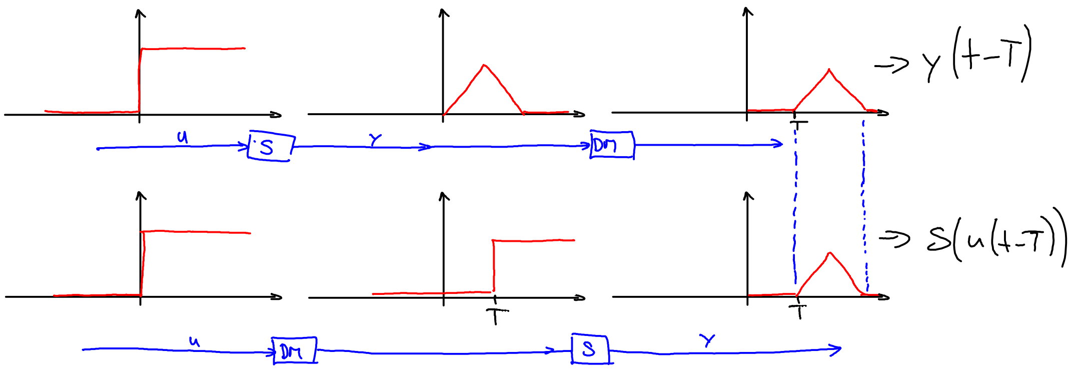
linearno

- $y(t) = u(t) - u(t-1)$ – linearan

Vremenska promjenjivost

$$S(u(t)) = y(t)$$

$$S(u(t-\tau)) = y(t-\tau)$$



Primjer 1:

$$y(t) = 3u(t+4)$$

$$\left.\begin{array}{l} S(u(t-T)) = 3u(t+4) \\ y(t-T) = 3u(t-T+4) \end{array}\right\} \Rightarrow \neq$$

vremenski nepromjenjiv

Primjer 2:

$$y(t) = t u(t^2)$$

$$\left.\begin{array}{l} S(u(t-T)) = t u(t^2-T) \\ y(t-T) = (t-T)\, u((t-T)^2) \end{array}\right\}$$

vremenski promjenjiv

Primjer 3:

$$y(n) = 2^{u(n)}$$

$$\left.\begin{array}{l} S(u(n-N)) = 2^{u(n-N)} \\ y(n-N) = 2^{u(n-N)} \end{array}\right\}$$

vremenski nepromjenjiv

Primjer 4:

$$y(t) = u(t) + 4$$

$$S(u(t-T)) = u(t-T)+4$$
$$y(t-T) = u(t-T)+4$$
vrem. nepromjenjiv

Primjer 5:

$$y(n) = \frac{n}{u(n)}$$

$$S(u(n-N)) = \frac{n}{u(n-N)}$$
$$y(n-N) = \frac{n-N}{u(n-N)}$$
$\neq$ vremenski promjenjiv

Primjeri

- $y(t) = u(t) - u(t-1)$ Vrem. nepromjenjiv
- $y(n) = \frac{u(n)}{1+u(n-1)}$ Nepromjenjiv
- $y(n) = \left(\frac{1}{2}\right)^n u(3n+2)$ promjenjiv
- $y(n) = u(2n)$

$$S(u(n-N)) = u(2n-N)$$
$$y(n-N) = u(2(n-N))$$
$\neq$ vrem. promjenjiv

Zadatak 1:

$$y(t) = \int_{-\infty}^{t} u(\tau)\,d\tau$$

$$S\big(u(t-T)\big) = \int_{\tau=-\infty}^{\tau=t} u(\tau-T)\,d\tau$$

$$y(t-T) = \int_{-\infty}^{t-T} u(\tau)\,d\tau$$

Supstitucija

$a = \tau - T$ $\qquad \tau = t$

$da = d\tau$ $\qquad a + T = t \Rightarrow a = t - T$

$\qquad \tau \to t, \ a \to t - T$

$$S\big(u(t-T)\big) = \int_{-\infty}^{\tau=\cancel{t}} u(a)\,da \Leftarrow \text{neispravo}$$

ispravno

$$S\big(u(t-T)\big) = \int_{-\infty}^{t-T} u(a)\,da = \Longrightarrow \quad a = b$$

$$=$$

$$y(t-T) = \int_{-\infty}^{t-T} u(\tau)\,d\tau = \Longrightarrow \quad \tau = b$$

$$S\big(u(t-T)\big) = \int_{-\infty}^{t-T} u(b)\,db$$

$$y(t-T) = \int_{-\infty}^{t-T} u(b)\,db$$

vremenski nepromjenjiv

Zadatak 2:

$$y(t) = \int_{1}^{t} u(\tau)\,d\tau$$

$$S\left(u(t-T)\right) = \int_{1}^{t} u(\tau-T)\,d\tau = \left| \begin{array}{l} \tau - T = a \\ d\tau = da \\ \tau = t \\ a+T = t \\ a = t-T \end{array} \right| = \int_{1-T}^{t-T} u(a)\,da$$

$$y(t-T) = \int_{1}^{t-T} u(\tau)\,d\tau$$

$\neq$

vremenski promjenjiv

Zadatak 3:

$$y(n) = \sum_{k=-\infty}^{n} \frac{u(k)}{n-k}$$

samo pobudu mjenjamo

$$S\left(u(k-N)\right) = \sum_{k=-\infty}^{n} \frac{u(k-N)}{n-k} = \left| \begin{array}{l} k-N=a \\ k=a+N \\ k=N \\ a+N=n \\ a=n-N \end{array} \right| = \sum_{a=-\infty}^{n-N} \frac{u(a)}{n-a-N} =$$

$$= \sum_{a=-\infty}^{n-N} \frac{u(a)}{n-a-N}$$

$$y(n-N) = \sum_{k=-\infty}^{n-N} \frac{u(k)}{n-k-N} = \left| k=a \right| = \sum_{a=-\infty}^{n-N} \frac{u(a)}{n-a-N}$$

$=$

vrem. nepromjenjivo

mjenjamo s n-ove

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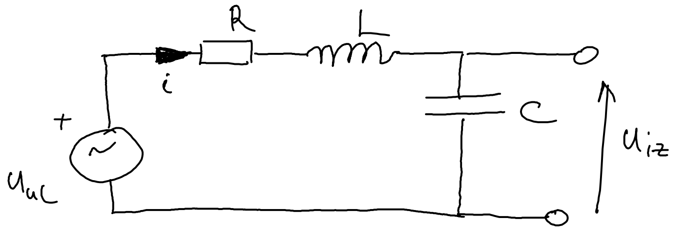
Kontinuirani sustavi



$$u_{ul} = u_R + u_L + u_C$$

$u_R = iR$

$u_L = \frac{di}{dt} L$

$u_{iz} = u_C = \frac{1}{C}\int_0^t i(\tau)d\tau = u_{iz}$

$$u_{ul} = iR + L\frac{di}{dt} + \underbrace{\frac{1}{C}\int_0^t i(\tau)d\tau}_{u_{iz}}$$

$$u_{ul} = iR + \frac{di}{dt}L + u_{iz}$$

$u_{iz} = \frac{1}{C}\int_0^t i(\tau)d\tau$

$Cu_{iz} = \int_0^t i(\tau)d\tau \Rightarrow Cu_{iz}' = i$

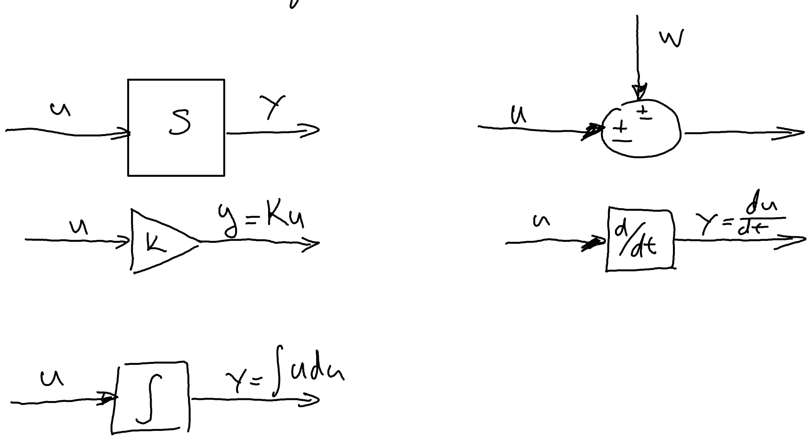
$$u_{ul} = Cu_{iz}'R + L\frac{d(Cu_{iz}')}{dt} + u_{iz}$$

$$u_{ul} = Cu_{iz}'R + LCu_{iz}'' + u_{iz}$$

$u = u_{ul} \quad y = u_{iz}$

$$u = LCy'' + RCy' + y$$

Blokovski diagram



$$y'' - 3y' + 5y = u$$

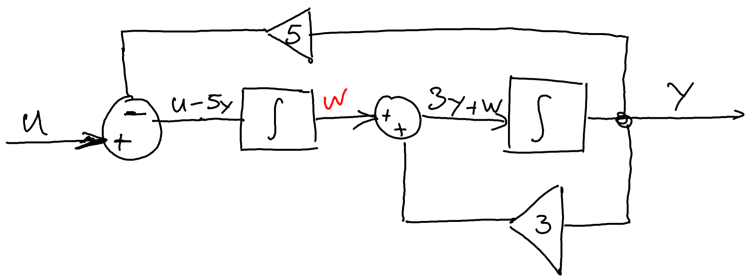
$$y'' = u + 3y' - 5y \quad / \iint$$

$$y = \iint u \, d\tau \, d\tau + 3 \iint y' \, d\tau \, d\tau + 5 \iint y \, d\tau \, d\tau$$

$$y = \iint u \, d\tau \, d\tau + 3 \int y \, d\tau - 5 \iint y \, d\tau \, d\tau$$

$$= \int \left(3y + \int u \, d\tau - 5 \int y \, d\tau \right) d\tau =$$

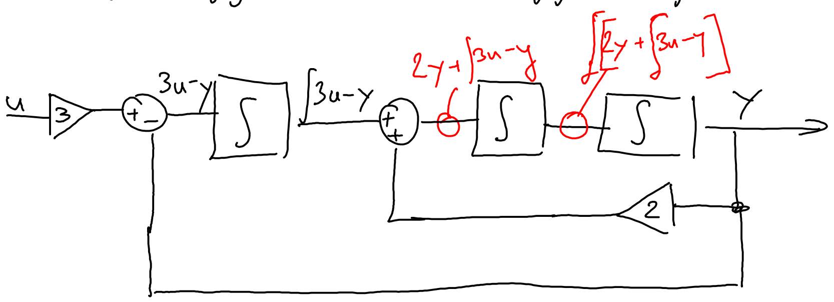
$$y = \int \left[3y + \underbrace{\int (u - 5y) \, d\tau}_{w} \right] d\tau$$



$$y''' - 2y' + y = 3u$$

$$y''' = 3u + 2y' - y \quad / \iiint$$

$$y = \iiint 3u + 2y' - y = \iint 2y + \int 3u - y$$



Dobivanje diff. jed. iz diagrama

$$y = \int \left[\int \left[2y + \int (3u - y) d\tau \right] d\tau \right] d\tau \quad \Big/ '''$$

$$y''' = 2y' + 3u - y$$

$$y''' - 2y' + y = 3u \quad \checkmark$$

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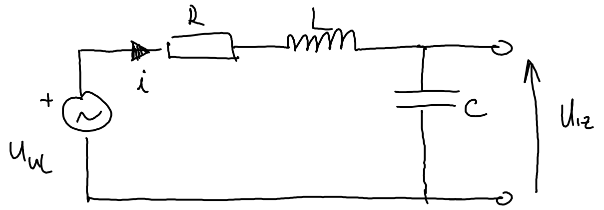
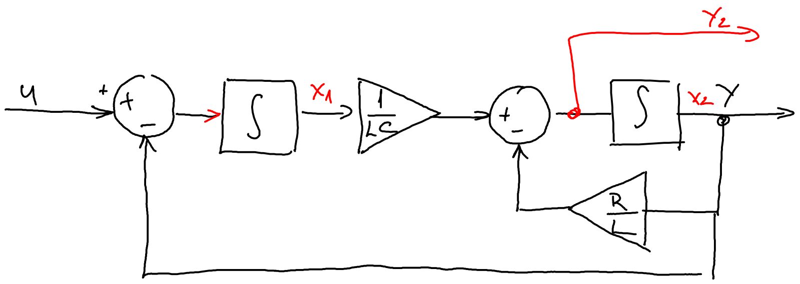
Varijable stanja



$$LCy'' + RCy' + y = u$$



x – stanje sustava

u – ulazna pobuda

y – izlaz

Vektori

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$x_1 = \int (u - y)\, d\tau \Big/ '$$

$$\dot{x}_1 = u - y$$

$$\boxed{\dot{x}_1 = u - x_2} \qquad \boxed{y = x_2}$$

$$x_2 = \int \left(\frac{1}{LC} x_1 - \frac{R}{L} y \right) d\tau \Big/ '$$

$$\boxed{\dot{x}_2 = \frac{1}{LC} x_1 - \frac{R}{L} x_2}$$

$$\dot{x}_1 = u - x_2 \qquad \dot{x}_2 = \frac{1}{LC} x_1 - \frac{R}{L} x_2$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{array}{cc} x_1 & x_2 \\ \left[\begin{matrix} 0 & -1 \\ \frac{1}{LC} & -\frac{R}{L} \end{matrix}\right. & \left.\right] \end{array} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \begin{bmatrix} u \end{bmatrix}$$

$[n \times 1]$ $\quad A[n \times n]$ $\quad [n \times 1]$ $\quad B[n \times m]\,[m \times 1]$ — Veličine matrica

x – stanje sustava – n

u – ulazna pobuda – m

y - broj izlaza – k

$$\underbrace{\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}}_{k \times 1} = \underbrace{\begin{bmatrix} 0 & 1 \\ \frac{1}{LC} & -\frac{R}{L} \end{bmatrix}}_{k \times n} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{n \times 1} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{k \times m} \underbrace{\begin{bmatrix} u \end{bmatrix}}_{m \times 1}$$

$$y_1 = x_2$$

$$y_2 = \frac{1}{LC} x_1 - \frac{R}{L} x_2$$

A – matrica dinamike sustava $n \times n$

B – ulazna matrica sustava $n \times m$

C – izlazna matrica sustava $k \times n$

D – ulazno-izlazna dimenzija sustava $k \times m$

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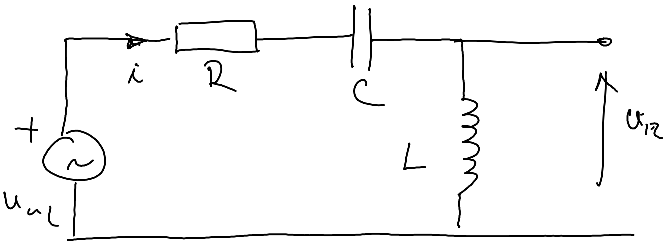


$$u_{ul} = u_R + u_C + u_L$$

$$u_{ul} = iR + \frac{1}{C}\int_0^t i(\tau)\,d\tau + \frac{di}{dt}L$$

$$u_{iz} = \frac{di}{dt}L$$

$$\frac{u_{iz}}{L} = \frac{di}{dt}$$

$$i = \int \frac{u_{iz}}{L}\,d\tau$$

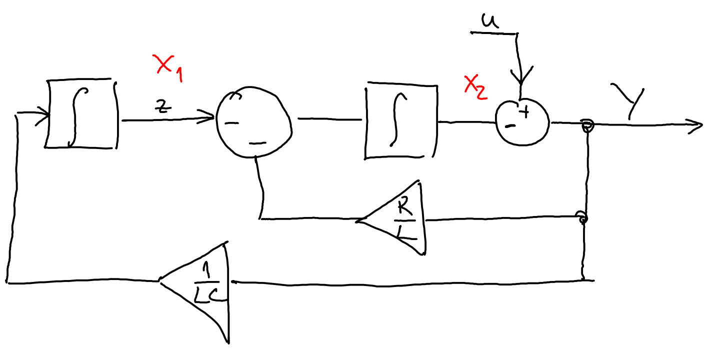
$$u_{ul} = \frac{R}{L}\int u_{iz}\,d\tau + \frac{1}{LC}\iint u_{iz}\,d\tau\,d\tau + u_{iz} \Big|''$$

$$u_{ul}'' = \frac{R}{L}u_{iz}' + \frac{1}{LC}u_{iz} + u_{iz}''$$

$$u'' = y'' + \frac{R}{L}y' + \frac{1}{LC}y$$

$$y = \iint u'' - \frac{R}{L}\iint y' - \frac{1}{LC}\iint y$$

$$y = u - \int\Big[-\frac{R}{L}y - \underbrace{\frac{1}{LC}\int y}_{z}\Big]$$



$$x_1 = \int \frac{1}{LC}y\,d\tau \qquad x_2 = \int\Big(-x_1 - \frac{R}{L}y\Big)d\tau$$

$$\dot{x}_1 = \frac{1}{LC}y = \frac{1}{LC}u - \frac{1}{LC}x_2 \qquad \dot{x}_2 = -x_1 - \frac{R}{L}y = -x_1 - \frac{R}{L}u - \frac{R}{L}x_2$$

$$\dot{x}_1 = \frac{1}{LC} y = \frac{1}{LC} u - \frac{1}{LC} x_2 \qquad \dot{x}_2 = -x_1 - \frac{R}{L} y = -x_1 - \frac{R}{L} u - \frac{R}{L} x_2$$

$$\boxed{\dot{x}_1 = \frac{1}{LC} u - \frac{1}{LC} x_2} \qquad \boxed{\dot{x}_2 = -x_1 - \frac{R}{L} u - \frac{R}{L} x_2}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{LC} \\ -1 & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \end{bmatrix} \begin{bmatrix} u \end{bmatrix}$$

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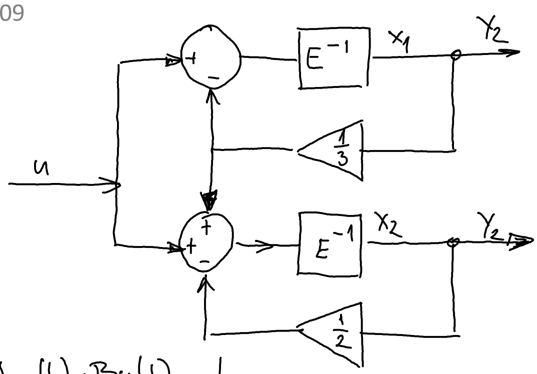

$$\boxed{\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \end{aligned}}$$

sklop za kašnjenje

$$x_1(k) = E^{-1}\left[u(k) - \frac{1}{3}x_1(k)\right]$$

$$x_1(k) = u(k-1) - \frac{1}{3}x_1(k-1)$$

$$\boxed{x_1(k+1) = u(k) - \frac{1}{3}x_1(k)}$$

$$\boxed{y_1(k) = x_1(k)}$$

$$x_2(k) = E^{-1}\left[u(k) - \frac{1}{2}x_2(k) + \frac{1}{3}x_1(k)\right]$$

$$x_2(k) = u(k-1) - \frac{1}{2}x_2(k-1) + \frac{1}{3}x_1(k-1)$$

$$\boxed{x_2(k+1) = u(k) - \frac{1}{2}x_2(k) + \frac{1}{3}x_1(k)}$$

$$\boxed{y_2(k) = x_2(k)}$$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & 0 \\ \frac{1}{3} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \left[u(k)\right]$$

$$\begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \left[u[k]\right]$$

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Diskretni sustavi

Odziv sustava

$$y(n) = \begin{cases} Cx(0) + Du(0) & , n = 0 \\ CA^n x(0) + \sum_{m=0}^{n-1} CA^{n-1-m} Bu(m) & , n > 0 \end{cases}$$

Odziv stanja sustava

$$x(n) = A^n x(0) + \sum_{m=0}^{n-1} A^{n-1-m} Bu(m) \quad , n > 0$$

Primjer

$$x_1(k+1) = x_1(k) + x_2(k)$$
$$x_2(k+1) = x_1(k)$$
$$x(0) = 1$$
$$x_2(0) = 3$$

$$y(k) = y(k) - u(k)$$
$$u(k) = 4\mu(k)$$

$$x_1(1) = x_1(0) + x_2(0) = 1+3 = 4$$
$$x_2(1) = x_2(0) = 3$$

$$x_1(2) = x_1(1) + x_2(1) = 4+3 = 7$$
$$x_2(2) = x_2(1) = 3$$

$\vdots$

$$A = \begin{array}{c} \begin{array}{cc} x_1 & x_2 \end{array} \\ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \end{array}$$

$$B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 \end{bmatrix}$$

odziv stanja nepobuđenog sustava ?

$$x(n) = A^n x(0) + \cancel{\sum_{m=0}^{n-1} A^{n-1-m} Bu(m)} \quad , n > 0$$

pošto nema pobude to je jednako 0

$$x(n) = A^n x(0)$$

$$A \cdot A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^2 \cdot A = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 \end{bmatrix}$$

$$A = \begin{array}{cc} x_1 & x_2 \\ \end{array}$$
$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 \end{bmatrix}$$

odziv stanja nepobuđenog sustava ?

$$x(n) = A^n x(0) + \cancel{\sum_{m=0}^{n-1} A^{n-1-m} Bu(m)}, \quad n \geq 0$$

pošto nema pobude to je jednako 0

$$x(n) = A^n x(0)$$

$$A \cdot A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

$$A^2 \cdot A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$\boxed{A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}} \Rightarrow \quad x(n) = \underset{2\times \boxed{2 \quad 2} \times 1}{\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix}} = \begin{bmatrix} 3n+1 \\ 3 \end{bmatrix}$$

$$x_1(50) = 151$$
$$x_2(50) = 3$$

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$$A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\quad B=\begin{bmatrix}0\\1\end{bmatrix}\quad C=\begin{bmatrix}1 & 0\end{bmatrix}$$

$$D=[0]\qquad x(0)=\begin{bmatrix}1\\1\end{bmatrix}$$

Odziv nepobuđenog sustava?

$$y(n)=CA^n x(0)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=$$

$1\times\boxed{2\quad 2}\times 2$

$1\times\boxed{2\quad 2}\times 1$

$$A\cdot A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$$

$\vdots$

$$A^n=\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix}1 & n\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}=n+1$$

$1\times\boxed{2\quad 2}\times 1$

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$$A=\begin{array}{c}\begin{array}{cc}x_1 & x_2\end{array}\\ \begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix}\end{array}\quad B=\begin{array}{c}u\\ \begin{bmatrix}0\\1\end{bmatrix}\end{array}\quad C=\begin{bmatrix}1 & 1\end{bmatrix}\quad D=\begin{bmatrix}1\end{bmatrix}$$

$$x(0)=\begin{bmatrix}a\\b\end{bmatrix}\qquad x(2)=\begin{bmatrix}0\\0\end{bmatrix}$$

$u(0)=?$
$u(1)=?$

$$x_1(k+1)=x_1(k)+x_2(k)\Rightarrow\quad x_1(2)=x_1(1)+x_2(1)=0$$

$$x_2(k+1)=x_1(k)+u(k)\Rightarrow\quad x_2(2)=x_1(1)+u(1)=0$$

$$x_1(1)=x_1(0)+x_2(0)=a+b$$

$$x_2(1)=x_1(0)+u(0)=a+u(0)$$

$x_1(1)+x_2(1)=0$

$a+b+a+u(0)=0$

$u(0)=-2a-b$

$x_1(1)+u(1)=0$

$a+b+u(1)=0$

$u(1)=-a-b$

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$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$u(0) = u(1) = ?$

$x(2) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$x_1(t+1) = x_2(t)$

$x_2(t+1) = u(t)$

$x_1(2) = x_2(1) = 1$

$x_2(2) = u(1) = 2$

$x_1(1) = x_2(0) = 0$

$x_2(1) = u(0) = 1$