2. MEĐUISPIT 2009./2010.

GRUPA A

APOLJON: 8., 15., 20.

CATRONIA: 1.-7., 9.-14., 16.-19.

1.

$x(n) = \{\underline{3}, 0, -3{,}0\}$

$X(k) \ = \sum_{n=0}^{N-1} x(n) \cdot e^{-\frac{2\pi jkn}{N}} =$

$= \sum_{n=0}^{3} x(n) \cdot e^{-\frac{2\pi jkn}{4}} =$

$= \ x(0) \cdot e^0 + x(1) \cdot e^{-\frac{2\pi jk}{4}} + x(2) \cdot e^{-2\pi jk \cdot \frac{2}{4}} + x(3) \cdot e^{-2\pi jk \cdot \frac{3}{4}} =$

$= 3 + 0 - 3e^{-\pi jk} + 0$

$= 3 - 3e^{-\pi jk}$

$X(0) = 3 - 3 \cdot e^0 = \mathbf{0}$

$X(1) = 3 - 3 \cdot e^{-\pi jk} = 3 - 3(cos(\pi) - j\, sin(\pi)) = 3 - 3(-1 - 0) = \mathbf{6}$

$X(2) = 3 - 3 \cdot e^{-2\pi jk} = 3 - 3(cos(2\pi) - j\, sin(2\pi)) = 3 - 3(1 - 0) = \mathbf{0}$

$X(3) = 3 - 3 \cdot e^{-3\pi jk} = 3 - 3(cos(3\pi) - j\, sin(3\pi)) = 3 - 3(-1 - 0) = \mathbf{6}$

$\boldsymbol{X(k) = \{\underline{0}, 6, 0, 6\}}$

2.

$X(k) = \{\underline{2}, 8, 2, 8, 2, 8\}$

$x(n) \ = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \cdot e^{\frac{2\pi jkn}{N}} =$

$= \frac{1}{6} \sum_{k=0}^{5} X(k) \cdot e^{\frac{2\pi jkn}{6}}$

$x(0) \ = \frac{1}{6} \sum_{k=0}^{5} X(k) \cdot e^0 = \frac{1}{6}(2 + 8 + 2 + 8 + 2 + 8) = \frac{30}{6} = 5$

$x(3) \ = \frac{1}{6} \sum_{k=0}^{5} X(k) \cdot e^{\frac{2\pi jk \cdot 3}{6}} = \frac{1}{6} \sum_{k=0}^{5} X(k) \cdot e^{\pi jk} = \frac{1}{6} \sum_{k=0}^{5} X(k) \cdot (-1)^k = \frac{1}{6}(2 - 8 + 2 - 8 + 2 - 8)$

$= \frac{-18}{6} = \mathbf{-3}$

$\boldsymbol{x(n) = \{\underline{5}, 0, 0, -3, 0, 0\}}$

3.

$DTFT \quad Y(e^{j\omega}) = \ \sum_{n=-\infty}^{\infty} y(n) \cdot e^{-j\omega n}$

y(n) je jednak x(n) samo za n veći od nula i manje od N-1.

$DTFT \quad Y(e^{j\omega}) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\omega n}$

$DTF \quad X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-\frac{2\pi jkn}{N}}$

$$X(k) = Y(e^{j\Omega})$$

$$\sum_{n=0}^{N-1} x(n) \cdot e^{-j\omega n} = \sum_{n=0}^{N-1} x(n) \cdot e^{-\frac{2\pi jkn}{N}}$$

$$e^{-j\omega n} = e^{-\frac{2\pi jkn}{N}}$$

$$-j\omega n = -\frac{2\pi jkn}{N}$$

$$\boldsymbol{\omega = 2\pi \frac{k}{N}}$$

4.

$-j < j\omega < j \quad => \quad -1 < \omega < 1$

$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$

$= \frac{1}{2\pi}\int_{-1}^{1} 1 \cdot e^{j\omega t}\, d\omega$

$= \frac{1}{2\pi}\frac{1}{jt}\, e^{j\omega t}\Big|_{-1}^{1}$

$= \frac{1}{\pi t}\frac{1}{2j}(e^{jt} - e^{-jt})$

$= \frac{1}{\pi t} \sin t$

$T_s = \pi$

$x(t) := x(nT_s) = \frac{1}{n\pi^2} \sin n\pi$

$e^{-j\omega} = -j$

$\omega = -\frac{\pi}{2}$

$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n}$

za $n \neq 0$, zbog $\sin(n\pi) \to 0$

za n=0: $\quad Y(e^{j\omega}) = x(0) = lim_{n\to 0} \frac{\sin n\pi}{n\pi^2} = \left| \lim_{n\to 0} \frac{0}{0} \right| = lim_{n\to 0} \frac{\pi \cdot \cos n\pi}{\pi^2} = \boldsymbol{\frac{1}{\pi}}$

5.

za vremenski stalan sustav za svaki T vrijedi:

ako ulaz pomaknemo za T i pustimo ga kroz sustav, dobit ćemo isto kao da pustimo kroz sustav ulaz, a onda izlaz pomaknemo za T.

$$\boldsymbol{\forall T: S(u(t-T)) = y(t-T)}$$

6.

LINEARNOST:

$u(n) = \alpha u_1(n) + \beta u_2(n)$

$y_1(n) = \sum_{k=0}^{n} u_1(k)$, $y_2(n) = \sum_{k=0}^{n} u_2(k)$

$$\begin{aligned} y(n) &= \sum_{k=0}^{n} u(k) \\ &= \sum_{k=0}^{n}(\alpha u_1(k) + \beta u_2(k)) \\ &= \alpha \sum_{k=0}^{n} u_1(k) + \beta \sum_{k=0}^{n} u_2(k) \\ &= \alpha y_1(n) + \beta y_2(n) \end{aligned}$$

=> **LINEARAN**

VREMENSKA PROMJENJIVOST:

$u(n-N) \quad \rightarrow \quad y(n) = \sum_{k=0}^{n} u(k-N)$

$y(n-N) = \sum_{k=0}^{n-N} u(k)$

$$y(n) = \sum_{k=0}^{n} u(k-N) = \left| \begin{matrix} p = k - N \\ k = 0 => p = -N \\ k = n => p = n - N \end{matrix} \right| = \sum_{p=-N}^{n-N} u(p) \neq y(n-N)$$

=> **VREMENSKI PROMJENJIV**

MEMORIJA:

y(n) ovisi i o ulazima različitim od u(n) (ovisi o svim ulazima od nultog do trenutnog) => **MEMORIJSKI**

7.

LINEARNOST:

$u(n) = \alpha u_1(n) + \beta u_2(n)$

$y_1(n) = \sum_{k=-\infty}^{n} u_1(k)$, $\qquad y_2(n) = \sum_{k=-\infty}^{n} u_2(k)$

$$\begin{aligned} y(n) &= \sum_{k=-\infty}^{n} u(k) \\ &= \sum_{k=-\infty}^{n}(\alpha u_1(k) + \beta u_2(k)) \\ &= \alpha \sum_{k=-\infty}^{n} u_1(k) + \beta \sum_{k=-\infty}^{n} u_2(k) \\ &= \alpha y_1(n) + \beta y_2(n) \end{aligned}$$

=> **LINEARAN**

VREMENSKA PROMJENJIVOST:

$u(n-N) \quad \rightarrow \quad y(n) = \sum_{k=0}^{n} u(k-N)$

$y(n-N) = \sum_{k=-\infty}^{n-N} u(k)$

$$y(n) = \sum_{k=-\infty}^{n} u(k-N) = \left| \begin{matrix} p = k - N \\ k \rightarrow -\infty => p \rightarrow -\infty \\ k = n => p = n - N \end{matrix} \right| = \sum_{p=-\infty}^{n-N} u(p) = y(n-N)$$

=>**VREMENSKI NEPROMJENJIV**

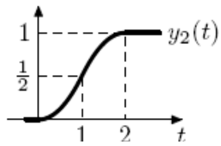
MEMORIJA:

y(n) ovisi i o ulazima različitim od u(n) (ovisi o svim ulazima prije trenutnog) => **MEMORIJSKI**

8.

$$u_2(t) = \int u_1(\tau)\, d\tau \xRightarrow{LTI\ sustav} y_2(t) = \int y_1(\tau)\, d\tau:$$

a)



9.

$$x(t) * y(t) = \int_{-\infty}^{\infty} e^{-3\tau}\mu(\tau)e^{-2(t-\tau)}\mu(t-\tau)\, d\tau$$

$$= \left| \begin{matrix} \mu(t-\tau) = \begin{cases} 1, & \tau < t \\ 0, & \tau > t \end{cases} \\ \mu(\tau) = \begin{cases} 1, & \tau > 0 \\ 0, & \tau < 0 \end{cases} \end{matrix} \right|$$

$$= \int_0^t e^{-3\tau}e^{-2(t-\tau)}\, d\tau$$

$$= \int_0^t e^{-3\tau-2t+2\tau}\, d\tau$$

$$= e^{2t}\int_0^t e^{-\tau}\, d\tau$$

$$= -e^{-2t}e^{-\tau}|_0^t$$

$$= -e^{-2t}(e^{-t} - 1)$$

$$\boldsymbol{= e^{-2t} - e^{-3t}}$$

10.

$$x(t) * \mu(t) = \int_{-\infty}^{\infty} x(\tau)\mu(t-\tau)\, d\tau$$

$$= \left| \mu(t-\tau) = \begin{cases} 1, & \tau < t \\ 0, & \tau > t \end{cases} \right|$$

$$= \int_{-\infty}^{t} x(\tau)\, d\tau$$

11.

$h(n) = n\mu(n)\,, \qquad u(n) = \mu(n)$

$y(n) = h(n) * u(n) = \sum_{m=-\infty}^{\infty} m\,\mu(m)\mu(n-m) = \sum_{m=0}^{n} m \; = \frac{n(n+1)}{2}$

$\frac{n(n+1)}{2} = 2019045$ => $\boldsymbol{n = 2009}$

12.

$$\left(sin(n) * \delta(n+1)\right)\delta(n-2) = |x(n) * \delta(n-n_0) = x(n-n_0)|$$
$$= sin(n+1)\,\delta(n-2)$$
$$= |x(n)\delta(n-n_0) = x(n_0)|$$
$$= sin(2+1)\,\delta(n-2)$$
$$\boldsymbol{= sin\,(3)\delta(n-2)}$$

13. – 16.

$y(n) - \frac{1}{4}y(n-1) = u(n)\,,$ $\quad u(n) = \left(\frac{1}{2}\right)^n \mu(n)$

1. HOMOGENA JEDNADŽBA

$q - \frac{1}{4} = 0$ => $q = \frac{1}{4}$ => $y_h(n) = C\left(\frac{1}{4}\right)^n$

2. $u(n) = \left(\frac{1}{2}\right)^n \mu(n)$ => $y_p(n) = K\left(\frac{1}{2}\right)^n$

u početnu: $K\left(\frac{1}{2}\right)^n - \frac{1}{4}K\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^n$

$K - \frac{1}{2}K = 1$ => $\boldsymbol{K = 2}$

$y_p(n) = 2\left(\frac{1}{2}\right)^n = \left(\frac{1}{2}\right)^{n-1}$

3. TOTALNI ODZIV $\quad y_{totalni}(n) = y_h(n) + y_p(n) = C\left(\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n-1}$

iz početnih uvjeta: $y(-1) = 4 = C\left(\frac{1}{4}\right)^{-1} + \left(\frac{1}{2}\right)^{-1-1} = 4K + 4 = 4$ => $K = 0$

$$\boldsymbol{y_t(n) = \left(\frac{1}{2}\right)^{n-1} \mu(n)}$$ => TOTALNI ODZIV

$$\boldsymbol{y_h(n) = 0}$$ => PRIRODNI ODZIV

4. MIRAN ODZIV uz ulaz $\quad u(n) = \left(\frac{1}{4}\right)^n \mu(n)$

Sad je partikularno rješenje: $\quad y_p(n) = Kn\left(\frac{1}{4}\right)^n$ *jer je* $q = \left(\frac{1}{4}\right)$

$Kn\left(\frac{1}{4}\right)^n - \frac{1}{4}K(n-1)\left(\frac{1}{4}\right)^{n-1} = \left(\frac{1}{4}\right)^n$

$Kn - K(n-1) = 1$

$Kn - Kn + K = 1$

$\boldsymbol{K = 1}$

$\boldsymbol{y_p(n) = n\left(\frac{1}{4}\right)^n}$

$y(n) = y_h(n) + y_p(n) = C\left(\frac{1}{4}\right)^n + n\left(\frac{1}{4}\right)^n$

Za miran sustav za svaki x<0 vrijedi y(x) = 0 : $y(-1) = 0 = C\left(\frac{1}{4}\right)^{-1} + n\left(\frac{1}{4}\right)^{-1} = 4C - 4 = 0\,,$ **C=1**

$$\boldsymbol{y_m(n) = \left(\frac{1}{4}\right)^n + n\left(\frac{1}{4}\right)^n = (n+1)\left(\frac{1}{4}\right)^n \mu(n)}$$

5. *IMPULSNI ODZIV*

$$y(n) = \frac{1}{4}y(n-1) + u(n)$$

$$h(n) = \frac{1}{4}h(n-1) + \delta(n)$$

$$h(0) = \frac{1}{4}h(-1) + \delta(0) = \frac{1}{4} * 0 + 1 = 1$$

$$h(1) = \frac{1}{4}h(0) + \delta(1) = \frac{1}{4} * 1 + 0 = \frac{1}{4}$$

$$h(2) = \frac{1}{4}h(1) + \delta(2) = \frac{1}{4} * \frac{1}{4} + 0 = \frac{1}{16}$$

Intuitivno zaključujemo $\boldsymbol{h(n) = \left(\frac{1}{4}\right)^n \mu(n)}$ => IMPULSNI ODZIV

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17. - 19.

$y''(t) - 2y'(t) + 2y(t) = u(t)$

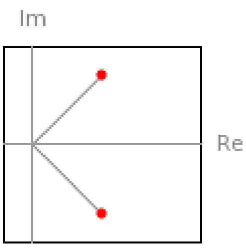
$u(t) = 15e^{-t}\,\mu(n)$

$y(0^-) = y(0^+) = 9, \qquad y'(0^-) = y'(0^+) = 3$

$s^2 - 2s + 2 = 0$

$s_1 = 1 - j\,, \qquad s_2 = 1 + j$

=> **NESTABILAN**



Rješenje karakteristične jednadžbe je oblika $s = \sigma + j\omega$ *pa je* y_h *oblika* $y_h(n) = e^{\sigma t}(A\sin\omega t + B\cos\omega t)$

$$y_h(n) = e^{t}(A\sin t + B\cos t)$$

Ulaz je $u(t) = 15e^{-t}\,\mu(n)$ *pa je* $y_p(n) = Ce^{-t}$, $\quad y'_p(n) = -Ce^{-t}$, $\quad y''_p(n) = Ce^{-t}$

$$y_p''(t) - 2y_p'(t) + 2y_p(t) = u(t)$$

$$Ce^{-t} + 2Ce^{-t} + 2Ce^{-t} = 15e^{-t} \qquad => \qquad \boldsymbol{C = 3}$$

$$\boldsymbol{y_p(n) = 3e^{-t}\mu(n)}$$ => PRISILNI ODZIV

Za nepobuđeni sustav rješavamo homogenu jednadžbu uz početne uvjete.

$y_h(n) = e^t A \sin t + e^t B \cos t$

$y'_h(n) = e^t A \sin t + e^t A \cos t + e^t B \cos t - e^t B \sin t$

$y(0) = A \sin 0 + B \cos 0 = 9$

$y'(0) = A \sin 0 + A \cos 0 + B \cos 0 - B \sin 0 = 3$

$\boldsymbol{B = 9}$

$A + B = 3$ $\quad$ $\boldsymbol{A = -6}$

$$\boldsymbol{y_n(n) = (9e^t \cos t - 6e^t \sin t)\mu(n)}$$ => ODZIV NEPOBUĐENOG SUSTAVA

20.

Obzirom da je N=M=1 (stupanj derivacije i s lijeve i desne strane je isti), onda se koristi formula:

$$b_0\delta(t) + \sum_{m=0}^{M}(b_{N-m}D^m)h_A(t), \quad t \geq 0, \quad M = N$$

Obzirom da je Bo = 1 (koeficijent uz u'(t)), znači da u rješenju mora biti Dirac, a to je jedino rješenje pod d).

Inače, da nije bio dirac ponuđen u samo jednom rješenju, najprije se našao hA(t), koji je zapravo yh(n) (hom. jedn.), gdje su početni uvjeti hA(0-) = 0 i hA(0+) = 1 (svi početni uvjeti su jednaki 0, osim tamo gdje je (N-1) derivacija u 0+).

I onda bi se taj hA(t) uvrštavao u jednadžbu iznad ☺