MARKO GULIN
0036428227

1. $y''(t) + 5y'(t) + 6y(t) = u(t)$

$u(t) = 5\cos(t)$

$y(0) = 0, \quad y'(0) = 1$

- PRIJENOSNA FUNKCIJA

$$H(s) = \frac{1}{s^2 + 5s + 6}$$

- FREKVENCIJSKA KARAKTERISTIKA

$s = j\omega$

$$H(j\omega) = \frac{1}{(j\omega)^2 + 5\cdot(j\omega) + 6} = \frac{1}{6 - \omega^2 + j5\omega}$$

$$H(j\omega) = \frac{6 - \omega^2 - j5\omega}{\omega^4 + 13\omega^2 + 36}$$

$$|H(j\omega)| = \frac{1}{\sqrt{\omega^4 + 13\omega^2 + 36}}$$

$$\angle H(j\omega) = -\operatorname{arctg} \frac{5\omega}{6 - \omega^2}$$

- OPĆA HOMOGENA JEDNADŽBA

$y_h(t) = Ce^{st}$

$Ce^{st}(s^2 + 5s + 6) = 0, \quad Ce^{st} \neq 0$

$s^2 + 5s + 6 = 0$

$(s+3)(s+2) = 0 \rightarrow s_1 = -3 \quad s_2 = -2$

$$y_h(t) = C_1 e^{-3t} + C_2 e^{-2t}$$

- PARTIKULARNA JEDNADŽBA

$$u(t) = 5\cos(t) \rightarrow y_p(t) = A\cos(t+\Theta)$$

$$w = 1$$

$$A = 5 \cdot |H(jw)| \Big|_{w=1} \qquad \Theta = \angle H(jw) \Big|_{w=1}$$

$$A = 5 \cdot \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{2} \qquad \Theta = -45^\circ$$

$$y_p(t) = \frac{\sqrt{2}}{2}\cos(t-45^\circ)$$

- TOTALNI ODZIV

$$y(t) = y_n(t) + y_p(t)$$

$$y(0) = 0, \quad y'(0) = 1$$

$$y(t) = C_1 e^{-3t} + C_2 e^{-2t} + \frac{\sqrt{2}}{2}\cos(t-45^\circ)$$

$$y'(t) = -3C_1 e^{-3t} - 2C_2 e^{-2t} - \frac{\sqrt{2}}{2}\sin(t-45^\circ)$$

$$y(0) = C_1 + C_2 + \frac{1}{2} = 0$$

$$y'(0) = -3C_1 - 2C_2 + \frac{1}{2} = 1$$

$$\left.\begin{aligned} C_1 + C_2 &= -\frac{1}{2} \\ -3C_1 - 2C_2 &= \frac{1}{2} \end{aligned}\right\} C_1 = \frac{1}{2}, \; C_2 = -1$$

$$y(t) = \frac{1}{2}e^{-3t} - e^{-2t} + \frac{\sqrt{2}}{2}\cos(t-45^\circ)$$

- STABILNOST SUSTAVA

IZ VLASTITIH FREKVENCIJA SUSTAVA ZAKLJ. DA JE SUSTAV STABILAN.
U TRENUTKU $t \gg 0$ HOMOGENA JEDNADŽBA POSTAJE JEDNAKA NULI, TE "PREŽIVI" SAMO PARTIKULARNI DIO TOTALNOG ODZIVA.

2. $y(n) - 2y(n-1) + y(n-2) = u(n)$

$u(n) = 5$

$y(-2) = 0, \; y(-1) = 1$

- PRIJENOSNA FUNKCIJA

$$H(z) = \frac{1}{1 - 2z^{-1} + z^{-2}}$$

- FREKVENCIJSKA KARAKTERISTIKA

$$z = e^{j\Omega}$$

$$H(e^{j\Omega}) = \frac{1}{1 - 2e^{-j\Omega} + e^{-2j\Omega}}$$

$$H(e^{j\Omega}) = \frac{1}{1 - 2\cos(\Omega) + j2\sin(\Omega) + \cos(2\Omega) - j\sin(2\Omega)}$$

$$|H(e^{j\Omega})| = \frac{1}{\sqrt{6 - 8\cos(\Omega) + 2\cos(2\Omega)}}$$

$$\angle H(e^{j\omega}) = \mathrm{arctg}\, \frac{\sin(2\Omega) - 2\sin(\Omega)}{1 - 2\cos(\Omega) + \cos(2\Omega)}$$

- OPĆA HOMOGENA JEDNADŽBA

$$y_h(n) = Cq^n$$

$$Cq^n(1 - 2q^{-1} + q^{-2}) = 0$$

$$Cq^{n-2}(q^2 - 2q + 1) = 0 \; ; \; Cq^{n-2} \neq 0$$

$$q^2 - 2q + 1 = 0$$

$$(q-1)^2 = 0 \rightarrow q_{1,2} = 1$$

$$y_h(n) = (C_1 + C_2 n)(1)^n$$

- PARTIKULARNA JEDNADŽBA

$u(n) = 5 \rightarrow y_p(n) = A \cdot n^2 \cdot (1)^n$

$w = 0$

$A = 5 \cdot |H(e^{jw})| \Big|_{w=0} \cdot n^2 \cdot (1)^n$

$A = 5 \cdot \frac{1}{0} \cdot n^2 \cdot (1)^n$

ZA $w=0$ FREKVENCIJSKA KARAKTERISTIKA NIJE DEFINIRANA

$y_p(n) = A n^2 (1)^n$

$A n^2 - 2A(n-1)^2 + A(n-2)^2 = 5$

$A(\cancel{n^2} - \cancel{2n^2} + \cancel{4n} - 2 + \cancel{n^2} - \cancel{4n} + 4) = 5$

$A = \frac{5}{2}$

$y_p(n) = \frac{5}{2} n^2$

- TOTALNI ODZIV

$y(n) = y_h(n) + y_p(n)$

$y(-2) = 0, \ y(-1) = 1$

$y(n) = (C_1 + C_2 n)(1)^n + \frac{5}{2} n^2$

$$\left.\begin{array}{l} y(-2) = C_1 - 2C_2 + 10 = 0 \\ y(-1) = C_1 - C_2 + \frac{5}{2} = 1 \end{array}\right\} \ C_1 = 7, \ C_2 = -8.5$$

$y(n) = (7 - 8.5n)(1)^n + \frac{5}{2} n^2$

- STABILNOST SUSTAVA

SUSTAV JE NESTABILAN.

U TRENUTKU $t \gg 0$, ODZIV TEŽI U BESKONAČNO.

MARKO GULIN
0036428227

3. $y''(t) + 2y'(t) + 5y(t) = u(t)$

$u(t) = \sin(t), \; t<0$

$u(t) = 2\sin(2t), \; t>0$

- PRIJENOSNA FUNKCIJA

$$H(s) = \frac{1}{s^2+2s+5}$$

- FREKVENCIJSKA KARAKTERISTIKA

$s = jw$

$$H(jw) = \frac{1}{(jw)^2 + 2\cdot jw + 5} = \frac{1}{5 - w^2 + j2w}$$

$$H(jw) = \frac{5 - w^2 - j2w}{w^4 - 6w^2 + 25}$$

$$|H(jw)| = \frac{1}{\sqrt{w^4 - 6w^2 + 25}}$$

$$\angle H(jw) = -\operatorname{arctg}\frac{2w}{5-w^2}$$

- OPĆA HOMOGENA JEDNADŽBA

$y_h(t) = Ce^{st}$

$Ce^{st}(s^2+2s+5) = 0, \quad Ce^{st} \neq 0$

$s^2 + 2s + 5 = 0 \qquad s_{1,2} = \frac{-2 \pm j4}{2}$

$s_1 = -1 + j2, \; s_2 = -1 - j2$

$$y_h(t) = e^{-t}(C_1\cos(2t) + C_2\sin(2t))$$

- PARTIKULARNA JEDNADŽBA

1.) $u_1(t) = \sin(t) \rightarrow y_{P1}(t) = A\sin(t + \Theta)$

$w = 1$

$A = 1 \cdot |H(jw)|\Big|_{w=1} \qquad \Theta = \angle H(jw)\Big|_{w=1}$

$A = 1 \cdot \frac{\sqrt{5}}{10} \qquad \Theta = -26.56^\circ$

$y_{P1}(t) = \frac{\sqrt{5}}{10}\sin(t - 26.56^\circ)$

2.) $u_2(t) = 2\sin(2t) \rightarrow y_{P2}(t) = A\sin(2t + \Theta)$

$w = 2$

$A = 2 \cdot |H(jw)|\Big|_{w=2} \qquad \Theta = \angle H(jw)\Big|_{w=2}$

$A = \frac{2\sqrt{17}}{17} \qquad \Theta = -75.96^\circ$

$y_{P2}(t) = \frac{2\sqrt{17}}{17}\sin(2t - 75.96^\circ)$

- POČETNI UVJETI

$y_{P1}(0^-) = y(0^-) = -0.01$

$y'_{P1}(0^-) = y'(0^-) = 0.2$

$b_0 = 0, \; b_1 = 0, \; b_2 = 1$

$y(0^+) = b_0 u(0^+) + y(0^-)$

$y(0^+) = y(0^-) = -0.01$

$y'(0^+) = b_0 u'(0^+) + b_1 u(0^+) - a_1(y(0^+) - y(0^-)) + y'(0^-)$

$y'(0^+) = y'(0^-) = 0.2$

$y(0^+) = -0.01$

$y'(0^+) = 0.2$

- TOTALNI ODZIV

$$y(t)=\begin{cases} y_{P1}(t), & t<0 \\ y_h(t)+y_{P2}(t), & t>0 \end{cases}$$

$$y(0^+)=-0.01,\ y'(0^+)=0.2$$

ZA $t>0$

$$y(t)=e^{-t}(C_1\cos(2t)+C_2\sin(2t))+\frac{2\sqrt{17}}{17}\sin(2t-75.96^\circ)$$

$$y'(t)=e^{-t}(\cos(2t)(-C_1+2C_2)+\sin(2t)(-2C_1-2C_2))+\frac{2\sqrt{17}}{17}\sin(2t-75.96^\circ)$$

$$y(0)=C_1=0.46$$

$$y'(0)=-C_1+2C_2=-0.035$$

$$C_1=0.46,\ C_2=0.21$$

$$y(t)=\begin{cases} \frac{\sqrt{5}}{10}\sin(t-26.56^\circ), & t<0 \\ e^{-t}(0.46\cos(2t)+0.21\sin(2t))+\frac{2\sqrt{17}}{17}\sin(2t-75.96^\circ), & t>0 \end{cases}$$

- STABILNOST SUSTAVA

IZ VLASTITIH FREKVENCIJA SUSTAVA ZAKLJUČUJEMO DA JE SUSTAV STABILAN.

U TRENUTKU $t\gg0$ HOMOGENA JEDNADŽBA POSTAJE JEDNAKA NULI, TE "PREŽIVI" SAMO PARTIKULARNI DIO TOTALNOG ODZIVA.

4. $y'(t) + 3y(t) = u(t)$

$y(0) = 0$

$u(t) = (\sin(t) + 2\sin(2t) + 3\sin(3t) + 4\sin(4t))\,\mu(t)$

- PRIJENOSNA FUNKCIJA

$$H(s) = \frac{1}{s+3}$$

- FREKVENCIJSKA KARAKTERISTIKA

$$s = jw$$

$$H(jw) = \frac{1}{jw+3} = \frac{3-jw}{3+w^2}$$

$$|H(jw)| = \frac{1}{\sqrt{9+w^2}}$$

$$\angle H(jw) = -\operatorname{arctg}\frac{w}{3}$$

- OPĆA HOMOGENA JEDNADŽBA

$$y_h(t) = Ce^{st}$$

$$Ce^{st}(s+3) = 0 \,,\quad Ce^{st} \neq 0$$

$$s+3 = 0 \rightarrow s = -3$$

$$y_h(t) = C_1 e^{-3t}$$

- PARTIKULARNA JEDNADŽBA

$$u_1(t) = \sin(t) \rightarrow y_{P1}(t) = \frac{\sqrt{10}}{10}\sin(t - 18.43^\circ)$$

$$u_2(t) = 2\sin(2t) \rightarrow y_{P2}(t) = \frac{2\sqrt{13}}{13}\sin(2t - 33.69^\circ)$$

$$u_3(t) = 3\sin(3t) \rightarrow y_{P3}(t) = \frac{\sqrt{2}}{2}\sin(3t - 45.00^\circ)$$

$$u_4(t) = 4\sin(4t) \rightarrow y_{P4}(t) = \frac{4}{5}\sin(4t - 53.13^\circ)$$

$$y_P(t) = y_{P1}(t) + y_{P2}(t) + y_{P3}(t) + y_{P4}(t)$$

MARKO GULIN
0036428227

- TOTALNI ODZIV

$$y(t) = y_h(t) + y_p(t)$$

$$y(t) = C_1 e^{-3t} + \frac{\sqrt{10}}{10}\sin(t-18.43^\circ) + \frac{2\sqrt{13}}{13}\sin(2t-33.69^\circ) + \frac{\sqrt{2}}{2}\sin(3t-45.00^\circ) + \frac{4}{5}\sin(4t-53.13^\circ)$$

$$y(0) = 0$$

$$y(0) = C_1 = 1.55$$

$$C_1 = 1.55$$

$$y(t) = 1.55 e^{-3t} + \frac{\sqrt{10}}{10}\sin(t-18.43^\circ) + \frac{2\sqrt{13}}{13}\sin(2t-33.69^\circ) + \frac{\sqrt{2}}{2}\sin(3t-45.00^\circ) + \frac{4}{5}\sin(4t-53.13^\circ)$$

- STABILNOST SUSTAVA

IZ VLASTITIH FREKVENCIJA SUSTAVA ZAKLJUČUJEMO DA JE SUSTAV STABILAN.
U TRENUTKU $t \gg 0$ HOMOGENA JEDNADŽBA POSTAJE JEDNAKA NULI, TE „PREŽIVI" SAMO PARTIKULARNI DIO TOTALNOG ODZIVA.

5. $y(n) + 0.5y(n-1) = u(n)$

$$y(-1) = 1$$

$$u(n) = \left(\cos(0.5\pi n + 0.2\pi) + 2\cos(\pi n) + 3\cos(1.5\pi n) + 4\cos(2\pi n)\right)\mu(n)$$

- PRIJENOSNA FUNKCIJA

$$H(z) = \frac{1}{1+0.5z^{-1}} = \frac{z}{z+0.5}$$

- FREKVENCIJSKA KARAKTERISTIKA

$$z = e^{j\Omega}$$

$$H(e^{j\Omega}) = \frac{1}{1+0.5e^{-j\Omega}}$$

$$H(e^{j\Omega}) = \frac{1}{1+0.5\cos(\Omega) - j0.5\sin(\Omega)}$$

$$|H(e^{j\Omega})| = \frac{2}{\sqrt{5+4\cos(\Omega)}}$$

$$\angle H(e^{j\Omega}) = \operatorname{arctg}\frac{\sin(\Omega)}{2+\cos(\Omega)}$$

- OPĆA HOMOGENA JEDNADŽBA

$$y_h(n) = Cq^n$$

$$Cq^{n-1}(q+0.5) = 0, \quad Cq^{n-1} \neq 0$$

$$q + 0.5 = 0 \rightarrow q = -0.5$$

$$y_h(n) = C_1 \cdot \left(-\frac{1}{2}\right)^n$$

- PARTIKULARNA JEDNADŽBA

$$u_1(n) = \cos(0.5\pi n + 0.2\pi) \rightarrow y_{P_1}(n) = A\cos(0.5\pi n + \Theta)$$

$$w = 0.5\pi$$

$$A = |H(e^{j\Omega})|\Big|_{\Omega=0.5\pi} \qquad \Theta = 0.2\pi + \angle H(e^{j\Omega})\Big|_{\Omega=0.5\pi}$$

$$A = \frac{2\sqrt{5}}{5} \qquad \Theta = 62.57^\circ$$

$$u_2(n) = 2\cos(\pi n) \rightarrow y_{P_2}(n) = A\cos(\pi n + \Theta)$$

$$w = \pi$$

$$A = 2 \cdot |H(e^{j\Omega})|\Big|_{\Omega=\pi} \qquad \Theta = \angle H(e^{j\Omega})\Big|_{\Omega=\pi}$$

$$A = 4 \qquad \Theta = 0^\circ$$

$$u_3(n) = 3\cos(1.5\pi n) \rightarrow y_{P_3}(n) = A\cos(1.5\pi n + \Theta)$$

$$w = 1.5\pi$$

$$A = 3 \cdot |H(e^{j\Omega})|\Big|_{\Omega=1.5\pi} \qquad \Theta = \angle H(e^{j\Omega})\Big|_{\Omega=1.5\pi}$$

$$A = \frac{6\sqrt{5}}{5} \qquad \Theta = -26.57^\circ$$

$$u_4(n) = 4\cos(2\pi n) \rightarrow y_{P_4}(n) = A\cos(2\pi n + \theta)$$

$$w = 2\pi$$

$$A = 4 \cdot |H(e^{j\Omega})|\Big|_{\Omega=2\pi} \qquad \theta = \angle H(e^{j\Omega})\Big|_{\Omega=2\pi}$$

$$A = \frac{8}{3} \qquad \theta = 0^\circ$$

$$y_P(n) = y_{P_1}(n) + y_{P_2}(n) + y_{P_3}(n) + y_{P_4}(n)$$

- TOTALNI ODZIV

$$y(n) = y_h(n) + y_p(n)$$

$$y(n) = u(n) - 0.5y(n-1)$$

$$y(0) = u(0) - 0.5y(-1)$$

$$y(0) = 9.3$$

$$y(n) = C_1 \cdot \left(-\frac{1}{2}\right)^n + \frac{2\sqrt{5}}{5}\cos(0.5\pi n + 62.57^\circ) + 4\cos(\pi n) + \frac{6\sqrt{5}}{5}\cos(1.5\pi n) + \frac{8}{3}\cos(2\pi n)$$

$$y(0) = C_1 = -0.46$$

$$C_1 = -0.46$$

$$y(n) = (-0.46)\left(-\frac{1}{2}\right)^n + \frac{2\sqrt{5}}{5}\cos(0.5\pi n + 62.57^\circ) + 4\cos(\pi n) + \frac{6\sqrt{5}}{5}\cos(1.5\pi n) + \frac{8}{3}\cos(2\pi n)$$

- STABILNOST SUSTAVA

IZ VLASTITIH FREKVENCIJA SUSTAVA ZAKLJUČUJEMO DA JE SUSTAV STABILAN

U TRENUTKU $t \gg 0$ HOMOGENA JEDNADŽBA POSTAJE JEDNAKA NULI, TE "PREŽIVI" SAMO PARTIKULARNI DIO TOTALNOG ODZIVA.