$\boxed{\text{ZAD}}$ $y''(t) + \frac{5}{2} y'(t) + y(t) = u''(t) + u'(t) + u(t)$

MIRAN SUSTAV => poč. uvjeti = 0

$y'(0^-) = 0$

$y(0^-) = 0$

$u(t) = e^{-t} \cos(t)\, \mu(t) \circ\!\!-\!\!\bullet \frac{s+1}{(s+1)^2+1}$

$H(s) = \frac{s^2+s+1}{s^2+\frac{5}{2}s+1}$ $\quad H(j\omega)\Big|_{\omega=1} = \frac{j}{j\frac{5}{2}} = \frac{2}{5}$

$Y_m(s) = H(s) \underbrace{U(s)}_{\text{Laplaceova transf. pobude}}$

1. RAČUNANJE POMOĆU LIMESA

$Y_m(s) = C_0 + \underbrace{C_1 \frac{1}{s-\lambda_1} + C_2 \frac{1}{s-\lambda_2}} + C_3 \frac{1}{s-(\alpha+j\beta)} + C_4 \frac{1}{s-(\alpha-j\beta)}$

2. METODA NEODREĐENIH KOEF

$Y_m(s) = \underline{\qquad\qquad\qquad} + \frac{C_3 s + C_4}{(s+1)^2+1}$

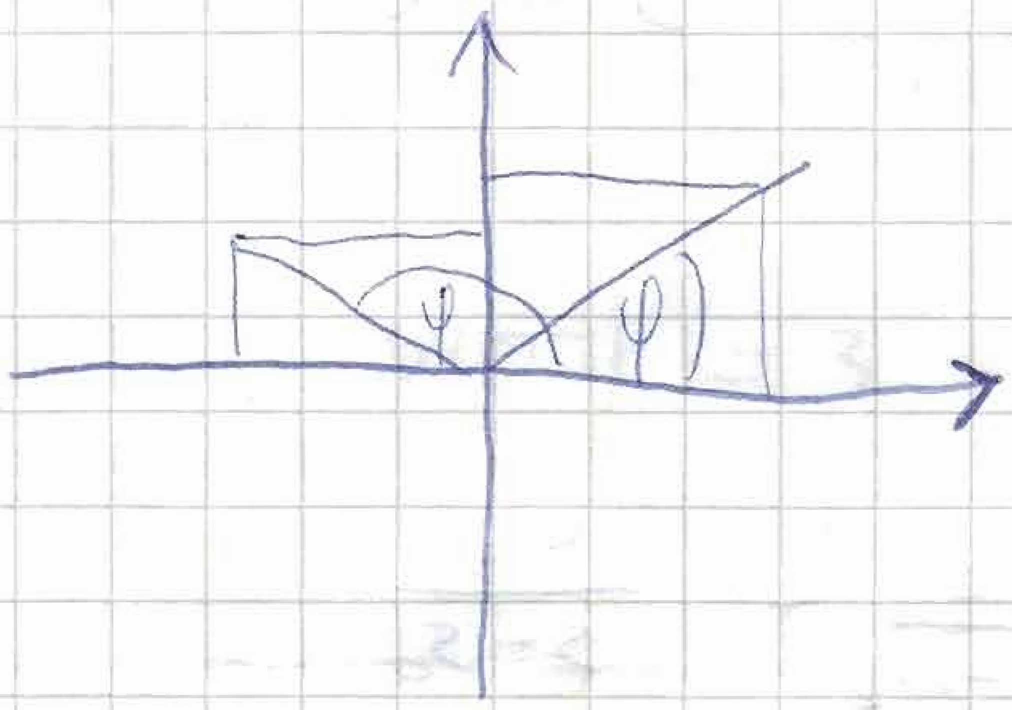
3. POMOĆU FREK. KARAKTERISTIKE

$u = U\cos\omega t$

$Y_{PRIS}(t) = U(t)\, \underbrace{|H(j\omega)|}_{\text{AMPLITUDNA FREK. KAR}} \cos(\omega t + \underbrace{\sphericalangle H(j\omega)}_{\text{FAZNA FREK. KAR}})$

$|H(j\omega)| = \sqrt{Re[H(j\omega)]^2 + Im[H(j\omega)]^2}$

$\sphericalangle H(j\omega) = \text{atan}_2 \frac{Im\, H(j\omega)}{Re\, H(j\omega)}$



$$u(t) = e^{-t}\cos(t)\,\mu(t) = e^{-t}\,\frac{1}{2}\left(e^{jt} + e^{-jt}\right)\mu(t)$$

$$= \frac{1}{2}\left[e^{(-1+j)t} + e^{(-1-j)t}\right]\mu(t)$$

$$H(j\omega)\Big|_{s=(-1+j)} = -\frac{1}{5}(1-3j)$$

$$|H(j\omega)| = \frac{\sqrt{10}}{5}$$

$$\text{atan}_2 \frac{-\frac{3}{5}}{\frac{1}{5}} = \text{atan}_2(-3)$$

$$Y_{PRIS}(t) = 1 \cdot \frac{\sqrt{10}}{5}\, e^{-t}\cos(t+\varphi)$$

$$Y(s) = Y_m(s) + Y_0(s)$$

$$= H(s)\,U(s) + Y_0(s)$$

kad bi imali
poč. uvjete

aka je $\quad u(t) = U e^{st} \qquad s \in R$

$$= U \, |H(s_0)| \, e^{s_0 t}$$

$$Y_m(s) = c_0 + \frac{c_1}{s - \lambda_1} + \frac{c_2}{s - \lambda_2} + \frac{c_3}{s - s_0}$$

$$Y(s) = \underbrace{H(s) \cdot U(s)}_{Y_m(s)} + \underbrace{\frac{s\,y(0^-) + y'(0^-) + y(0^-)}{A(s)}}_{Y_0(s)}$$

$$H(s) = \frac{B(s)}{A(s)} \qquad \begin{matrix} u \\ y \end{matrix}$$

$$A(s) = s^2 + \frac{5}{2}s + 1$$

$$y'(t) = s\,Y(s) - y(0^-)$$

$$y''(t) - s^2 Y(s) - s\,y(0^-) - y'(0^-)$$

AKTIVNOST 16.1

$\boxed{\text{ZAD}}$

$$y''(t) - 5y'(t) + 6y(t) = \underbrace{u(t)}_{u_1(t)} + \underbrace{3u'(t)}_{u_2(t)}$$

$y(0^-) = 3$

$y'(0^-) = 0$

$u(t) = 2\mu(t)$

a) HOMOGENO + PART.

$s^2 - 5s + 6 = 0$

$s_1 = 2 \qquad s_2 = 3$

$y_h(t) = c_1 e^{2t} + c_2 e^{3t}$

$y_p(t) = K \cancel{\mu(t)}$

$6K = 2 \qquad u(t) = 2\mu(t)$

$K = \frac{1}{3}$

$y_1(t) = y_h(t) + y_p(t)$

$= c_1 e^{2t} + c_2 e^{3t} + \frac{1}{3}$

$y(0^-) = c_1 + c_2 + \frac{1}{3} = 3$

nema $\delta(t)$ funkcije pa ne treba preračunati u 0^+ uvjete

$y'(0^-) = 2c_1 + 3c_2 = 0$

$c_1 = 8 \qquad c_2 = -\frac{16}{3}$

$y(t) = y_1(t) + \underbrace{y_2(t)}_{u(t) = 6\delta(t)}$

rastavi na odziv

$y_1(t)$ - odziv na $u_1(t)$

$y_2(t)$ - odziv na $u_2(t)$

$$y_2''(t) - 5y'(t) + 6y_2(t) = 6\delta(t)$$

$$h_2(t) = c_1 e^{2t} + c_2 e^{3t}$$

$$y'(0^-) = 0$$

$$y'(0^+) = 6 + y'(0^-) = 6$$

$$y_2(0^+) = c_1 + c_2$$

$$y_2(0^+) = 2c_1 + 3c_2 = 6$$

$$y_2 = -6e^{2t} + 6e^{3t}$$

b) POMOĆU LA PLACEA

$$y''(t) - 5y'(t) + 6y(t) = u(t) + 3u'(t) \quad / \mathcal{L}$$

$$y(0^-) = 3$$

$$u(t) = 2\mu(t)$$

$$y'(0^-) = 0$$

$$s^2Y(s) - s\,y(0^-) - y'(0^-) - 5(sY(s) - y(0^-)) + 6Y(s) = U(s) + 3s\,U(s) - \underbrace{u(0^-)}_{=0}$$

$$u(t) = 2\mu(t) \multimap \frac{2}{s}$$

$$u'(t) = 2\delta(t) \multimap 2$$

$$Y(s)\underbrace{[s^2 - 5s + 6]}_{H(s)} = \frac{2}{s} + 3s \cdot \frac{2}{s} + 3s - 15$$

$$Y(s) = \frac{3s^2 - 9s + 2}{s(s-2)(s-3)} = c_0 + \frac{c_1}{s} + \frac{c_2}{s-2} + \frac{c_3}{s-3}$$

$c_0 = 0$

$c_1 = \frac{1}{3}$

$c_2 = 2$

$c_3 = \frac{2}{3}$

$\boxed{ZAD}$ Z5.8

$$y''(t) + 4y'(t) + 4y(t) = u(t) \quad / \mathcal{L}$$

$y(0) = 2$

$y'(0) = 0$ $\qquad u(t) = 4e^{-t}\mu(t)$

$$Y(s)(s^2 + 4s + 4) - s\,y(0^-) - y'(0^-) - 4y(0^-) = U(s)$$

$$Y(s) = \underbrace{\frac{1}{s^2+4s+4}U(t)}_{Y_m(s)} + \underbrace{\frac{2s+8}{s^2+4s+4}}_{Y_0(s)}$$

$$Y_m(s) = c_0 + \frac{c_{11}}{s+2} + \frac{c_{12}}{(s+2)^2} + \frac{c_2}{s+1}$$

$$U(s) = 4 \cdot \frac{1}{s+1} \qquad Y_m(s)\ \frac{4}{(s+2)^2(s+1)}$$

$$c_0 = \lim_{s\to\infty} Y_m(s) = 0$$

$$c_{11} = \frac{1}{(2-1)!} \lim_{s\to -2} \left[\frac{4}{(s+2)^2(s+1)}(s+2)^2\right]'$$

$$= \lim_{s\to -2} \frac{-4}{(s+1)^2} = -4$$

$c_{12} = \lim_{s \to -2} \cancel{(s+2)^2} \frac{4}{\cancel{(s+2)^2}(s+1)} = -4$

$c_2 = \lim_{s \to -1} (s+1) \frac{4}{(s+2)^2(s+1)} = 4$

⋮

$\boxed{ZAD}$ A 14.3

$y''(t) + 2y'(t) + 5y(t) = u(t)$

$t < 0 \qquad u(t) = \sin(t)$

$t > 0 \qquad u(t) = 2\sin(2t)$

$y(t) = ? \qquad t \geq 0$

$\lim_{t \to \infty} y(t) = y_{PRIS}(t)$

$y(t) = \underbrace{\underbrace{c_1 e^{s_1 t}}_{\to 0} + \underbrace{c_2 e^{s_2 t}}_{\to 0}} + y_{PRIS}(t)$

$u(t) = \sin t$

$y_{PRIS}(t) = A \sin t + B \cos t$

$y(0^-) = y_{PRIS}(0^-)$

$y'(0^-) = y_{PRIS}'(0^-)$

SUSTAVI S VARIJABLAMA STANJA

$$\dot{x}(t) = A x(t) + B u(t)$$
$$y(t) = C x(t) + D u(t)$$

$$A \cdot B \neq B \cdot A$$
$$(A \cdot B) \cdot C = A \cdot (B \cdot C)$$

$$sA = \begin{bmatrix} s \cdot a_{11} & s \cdot a_{12} \\ s \cdot a_{21} & s \cdot a_{22} \end{bmatrix}$$

$$I = \begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & \ddots \; 1 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} =$$

$$= \begin{bmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} & a_{11} \cdot b_{12} + a_{12} \cdot b_{22} \\ a_{21} \cdot b_{11} + a_{22} \cdot b_{21} & a_{21} \cdot b_{12} + a_{22} \cdot b_{22} \end{bmatrix}$$

$$\phi(s) = (sI - A)^{-1} = \begin{bmatrix} s - a_{11} & -a_{12} \\ -a_{21} & s - a_{22} \end{bmatrix}^{-1} = \frac{\operatorname{adj}(sI - A)}{\det(sI - A)}$$

$$= \frac{1}{\det(sI - A)}$$

$$A = \begin{bmatrix} 0 & -1 \\ 6 & -3 \end{bmatrix} \Rightarrow \phi(s) = \frac{1}{s^2 + 5s + 6}$$

$$sI - A = \begin{bmatrix} s & 1 \\ -6 & s+5 \end{bmatrix}$$

$$\begin{bmatrix} s-a_{12} & a_{12} \\ a_{21} & s-a_{11} \end{bmatrix}^T$$

$$\phi(s) = \frac{1}{s^2+5s+6} \cdot \begin{bmatrix} s+5 & 1 \\ -6 & s \end{bmatrix} = \begin{bmatrix} \frac{s+5}{s^2+5s+6} & \frac{1}{\ } \\ \frac{-6}{\ } & \frac{s}{\ } \end{bmatrix}$$

$\phi(t)$ FUNDAMENTALNA MATRICA

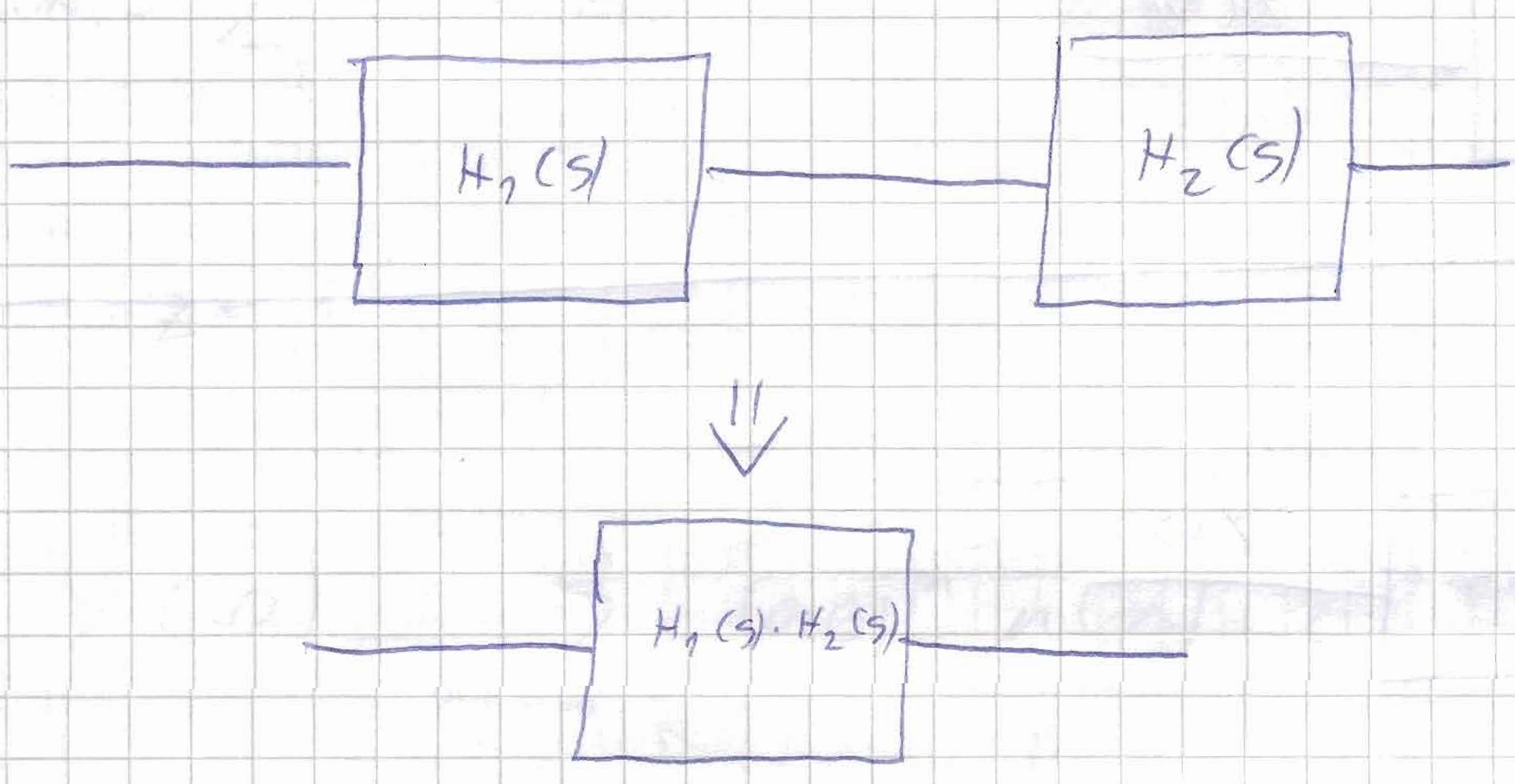
$$X(s) = \phi(s)\, x(0^-) + \phi(s)\, B\, U(s)$$

$$x(0^-) = \begin{bmatrix} x_1(0^-) \\ x_2(0^-) \\ \vdots \end{bmatrix}$$

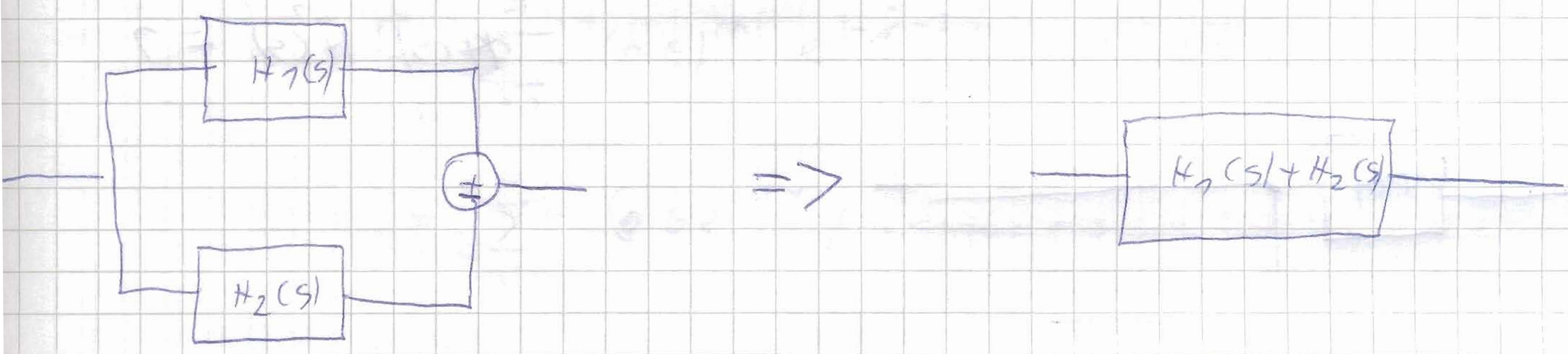
$$Y(s) = C\,\phi(s)\, x(0^-) + C \cdot \phi(s) \cdot B \cdot U(s) + D \cdot U(s)$$

REALIZACIJE SUSTAVA

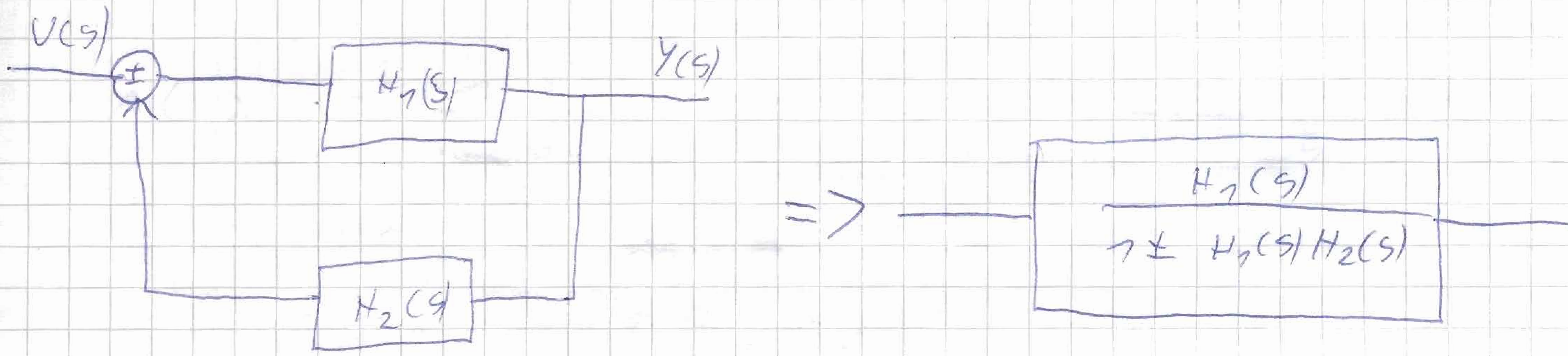
KASKADA

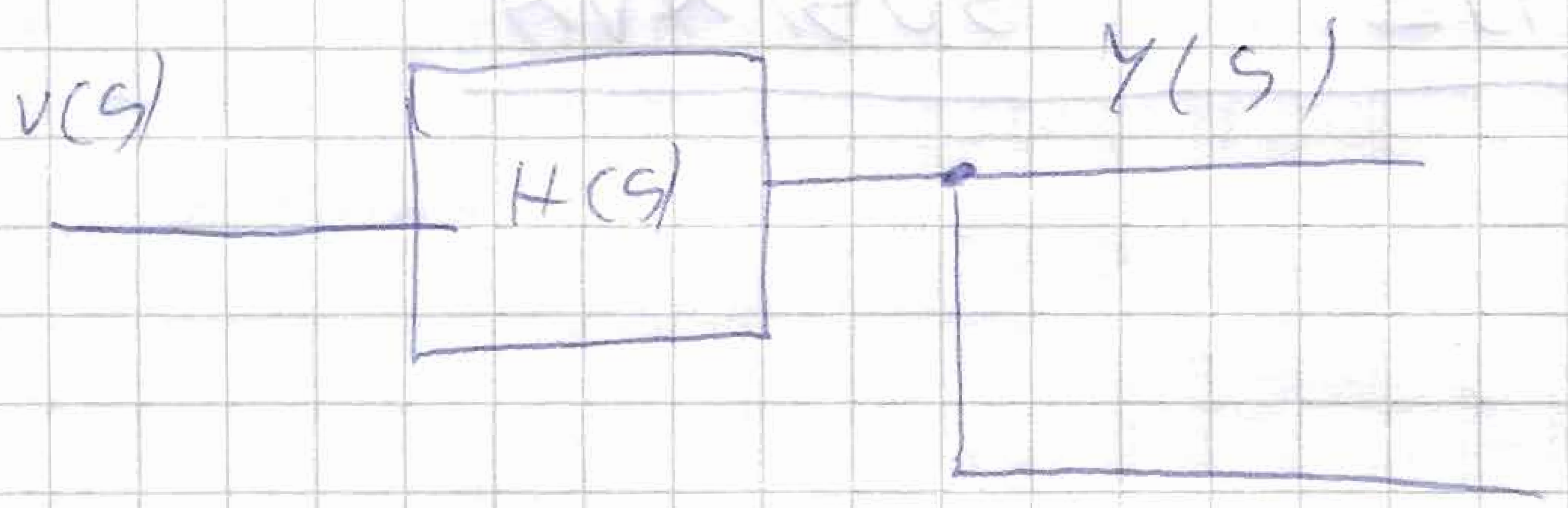


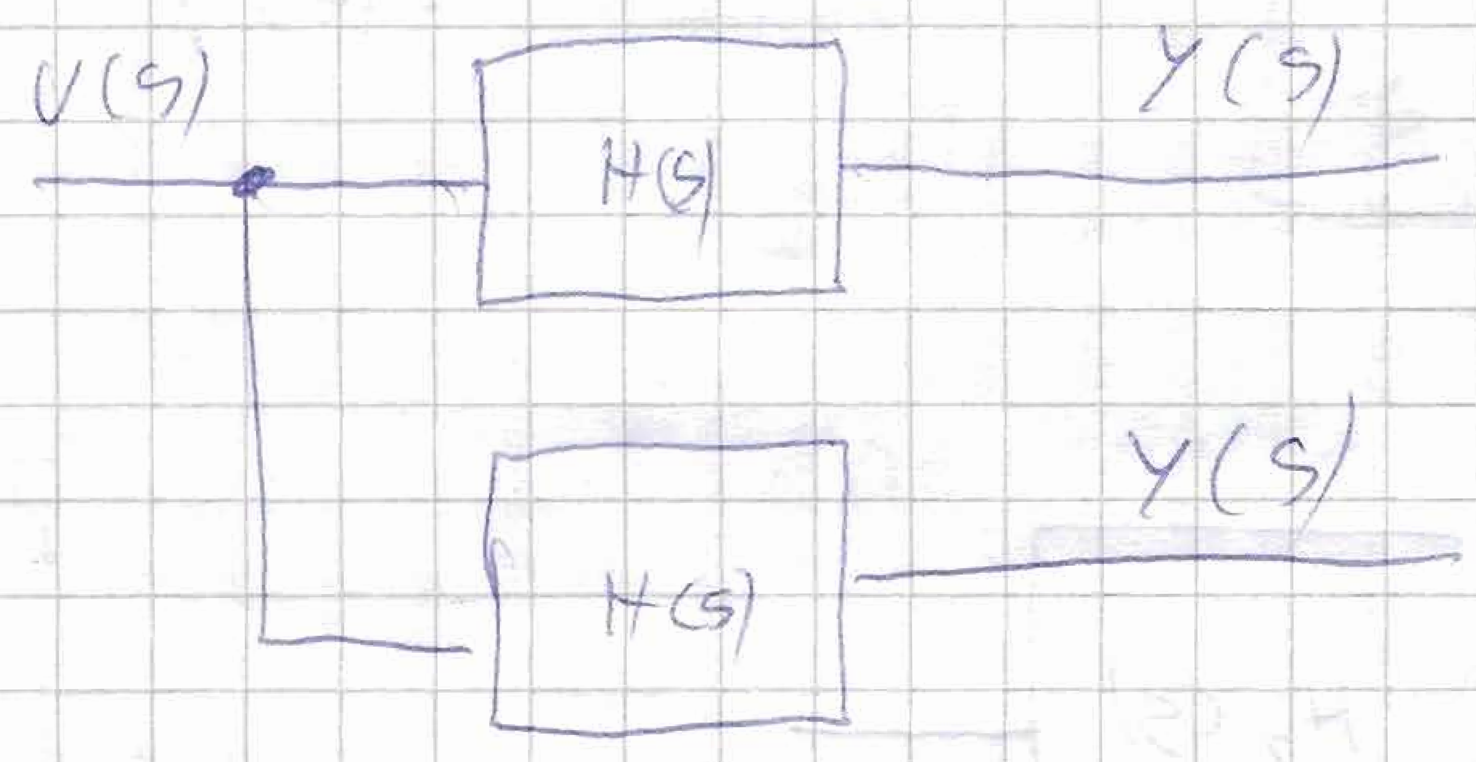
PARALELA

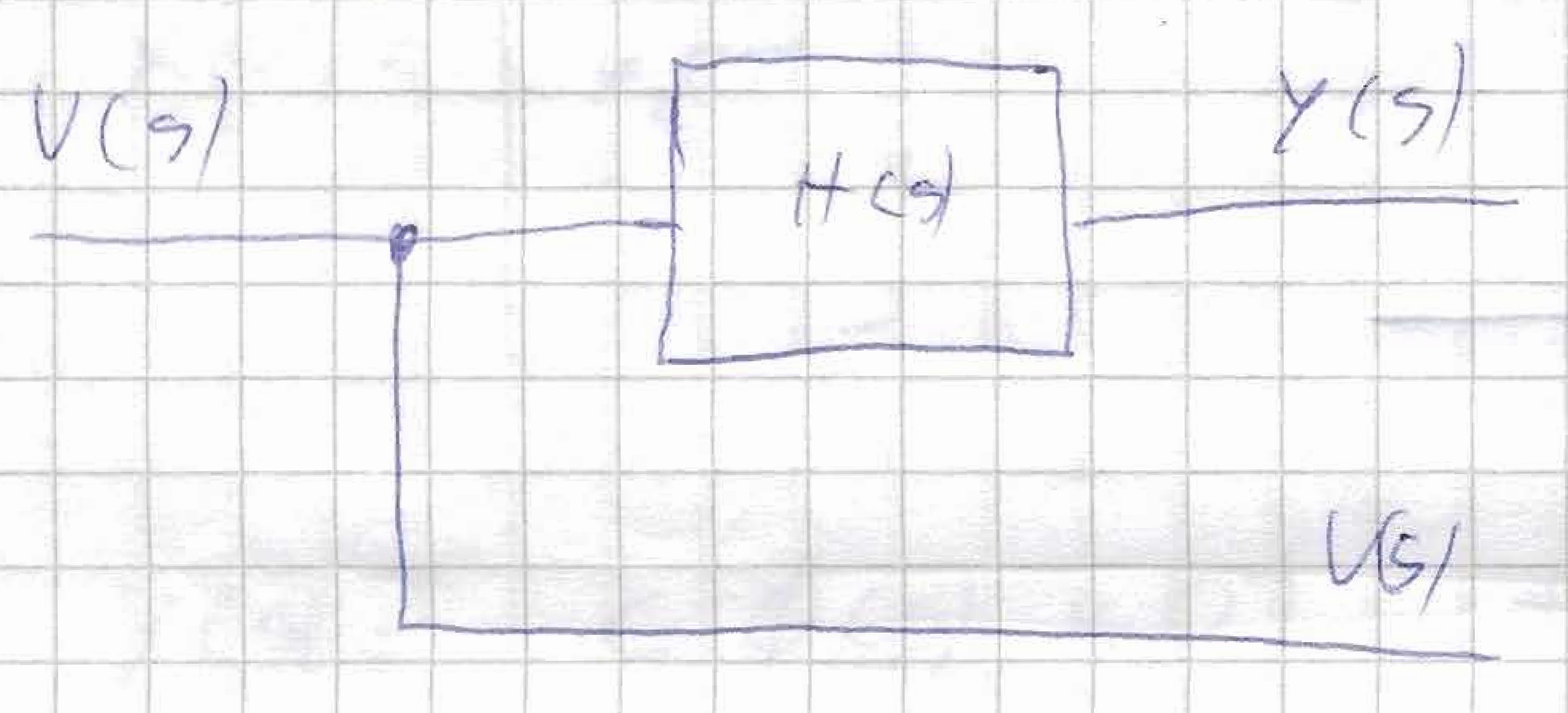


POVRATNA VEZA

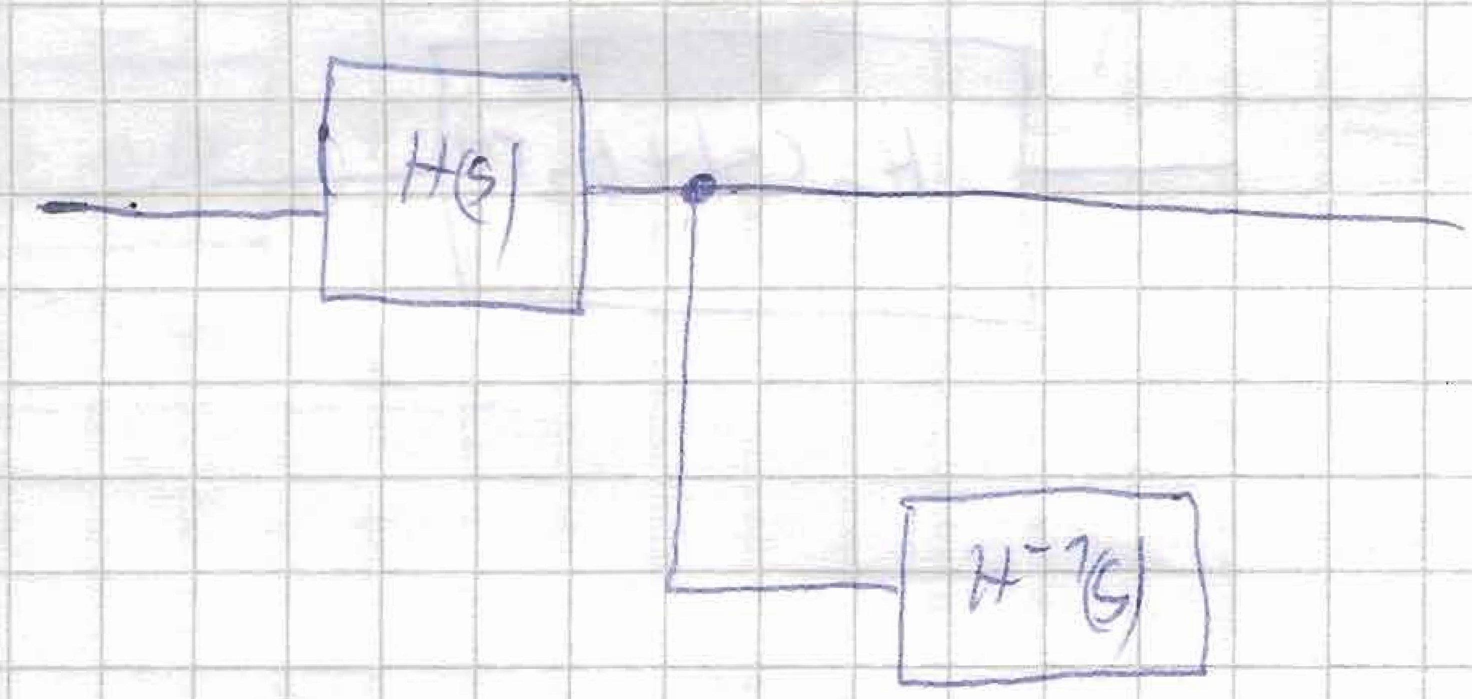








$\cancel{H}(s)\ H^{-1}(s) = 1$



FOURIER

$\boxed{P\ 3.4.1}$ $\quad x(n) = (0.6)^n \mu(n)$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} \qquad \underline{DTFT}$$

$$Z\{x(n)\} = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n} \qquad F\{x(n)\} = \sum_{-\infty}^{\infty} x(n)\, e^{-j\omega n}$$

$$z = e^{j\omega}$$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} (0.6)^n \underbrace{\mu(n)}_{\Downarrow\ n=0} e^{-j\Omega n}$$

$$= \sum_{n=0}^{\infty} (0.6)^n e^{-j\Omega n}$$

$$= \sum_{n=0}^{\infty} \left(0.6 e^{-j\Omega}\right)^n$$

$$\sum_{n=0}^{\infty} \alpha^n = \frac{1}{1-\alpha} \qquad |\alpha| < 1$$

$$X(\Omega) = \frac{1}{1-0.6e^{-j\Omega}} = \frac{1}{1-0.6\left[\cos(\Omega) - j\sin(\Omega)\right]}$$

$$= \frac{1}{\underbrace{1-0.6\cos\Omega}_{Re} + \underbrace{j\,0.6\sin\Omega}_{Im}} \cdot \frac{1-0.6\cos\Omega - j0.6\sin}{-\,\|\,-}$$

$$\left|0.6\, e^{-j\Omega}\right| < 1 \qquad ?$$

$$\left|e^{-j\Omega}\right| = 1$$

$$0.6 < 1 \qquad \checkmark$$

$$|X(\Omega)| = \sqrt{Re\{X(\Omega)\}^2 + Im\{X(\Omega)\}^2}$$

AMPLITUDNI SPEKTAR

$$\measuredangle X(\Omega) = atan_2 \frac{Im\{X(\Omega)\}}{Re\{X(\Omega)\}}$$

FAZNI SPEKTAR

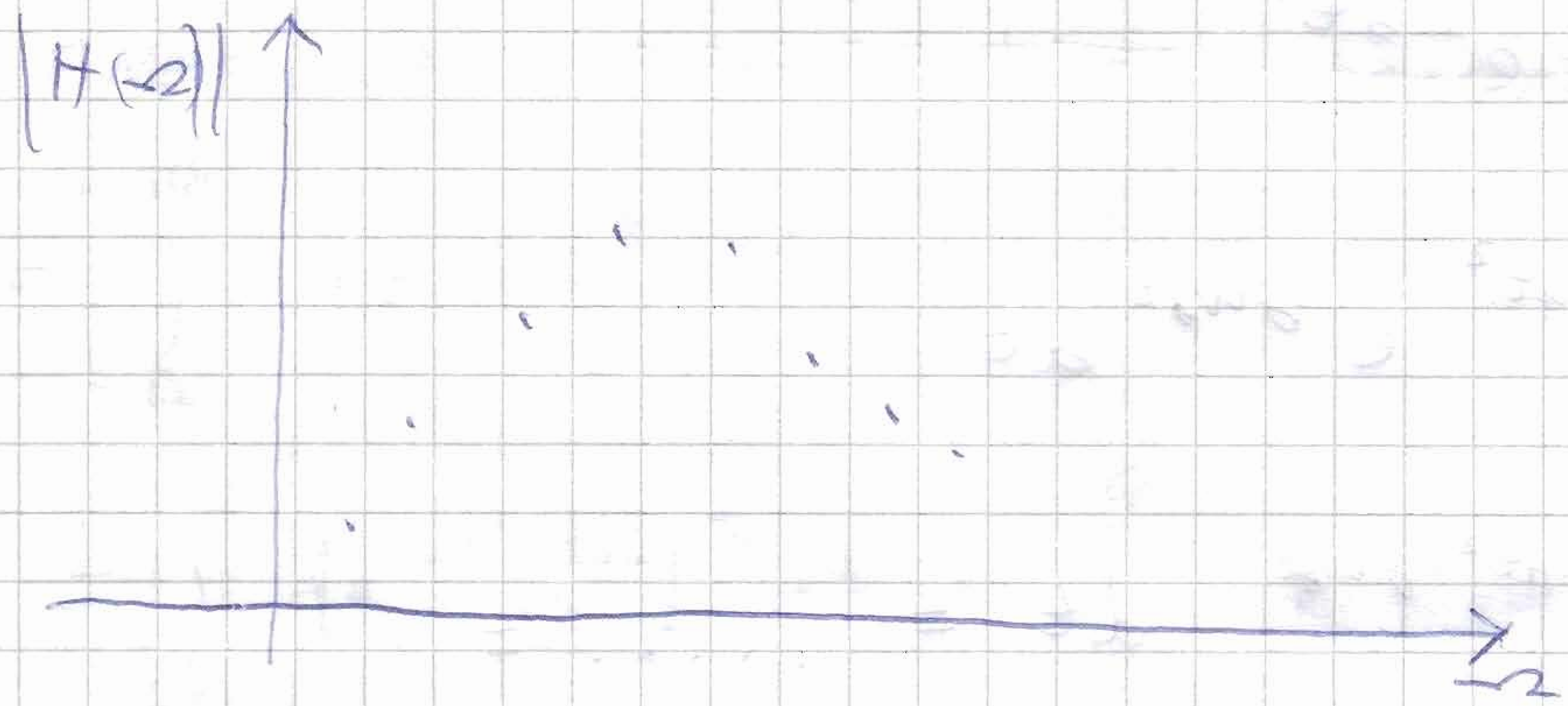
$$|X(\Omega)| = \frac{1}{\sqrt{(1-0.6\cos\Omega)^2 + (0.6\sin\Omega)^2}}$$

$$\measuredangle X(\Omega) = \frac{-0.6\sin\Omega}{1-0.6\cos\Omega}$$

$$\frac{r_1 e^{j\varphi_1} \cdot r_2 e^{j\varphi_2}}{r_3 e^{j\varphi_3} \cdot r_4 e^{j\varphi_4}}$$

$$|A| = \frac{r_1 \cdot r_2}{r_3 \cdot r_4}$$

$$\varphi = r_1 + r_2 - r_3 - r_4$$



PETKOVIĆ

$\boxed{P\,4.3}$

$$x(t) = \begin{cases} \cos(\omega_0 t) & -T \le t \le T \\ 0 & \text{inače} \end{cases}$$

$$\mathcal{L}\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$s = j\omega$$

$$F\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$X(\omega) = \int_{-\infty}^{\infty} \cos(\omega_0 t)\, e^{-j\omega t}\, dt$$

$$= \frac{1}{2}\int_{-T}^{T} \left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right) e^{-j\omega t}\, dt = \dots =$$

$$= \frac{1}{\omega-\omega_0}\sin\left((\omega-\omega_0)T\right) + \frac{1}{\omega+\omega_0}\sin\left((\omega+\omega_0)T\right)$$

PETKOVIĆ

$\boxed{Z\ 4.1}$

$$x(t) = e^{-at^2}$$

$$X(\omega) = \int_{-\infty}^{\infty} e^{-at^2} e^{-j\omega \cdot t} \, dt$$

$$= \int_{-\infty}^{\infty} e^{-at^2 - j\omega t} \, dt = \ldots\ldots =$$

$$= \sqrt{\frac{\pi}{a}} \, e^{-\frac{\omega^2}{4a}}$$

$\boxed{A\ 17.5}$

a) $x(t) = e^{-t} \mu(t)$

$$X(\omega) = \int_{0}^{\infty} e^{-t} e^{-j\omega t} \, dt = \int_{0}^{\infty} e^{(-1-j\omega)t} \, dt$$

$$= \frac{-1}{1+j\omega} e^{(-1-j\omega)t} \Big|_{0}^{\infty}$$

$$= \frac{-1}{1+j\omega} \left(0 - 1\right) = \frac{1}{1+j\omega} \cdot \frac{1-j\omega}{1-j\omega} = \frac{1-j\omega}{1+\omega^2}$$

$$|H(\omega)| = \frac{1}{\sqrt{1^2+\omega^2}}$$

$$\angle H(\omega) = \operatorname{atan}_2 \frac{\frac{-\omega}{1+\omega^2}}{\frac{1}{1+\omega^2}} = \operatorname{atan}_2(-\omega) = -\operatorname{atan}_2(\omega)$$

$$\operatorname{Re}\{X(\omega)\} = \frac{1}{1+\omega^2} \qquad \operatorname{Im}\{X(\omega)\} = \frac{-\omega}{1+\omega^2}$$

ENERGIJA – na dva načina

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{-\infty}^{\infty} |e^{-t} \mu(t)|^2 \, dt = \int_{0}^{\infty} e^{-2t} \, dt = -\frac{1}{2} e^{-2t} \Big|_{0}^{\infty}$$

$$= +\frac{1}{2}$$

$$E = \frac{1}{2\pi} \int_{-\infty}^{\infty} |\cancel{H}(\omega)|^2 \, d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left| \frac{1}{\sqrt{1+\omega^2}} \right|^2 d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{1+\omega^2} \, d\omega = \frac{1}{2\pi} \arctan(\omega) \Big|_{-\infty}^{\infty}$$

$$= \frac{1}{2\pi} \left(\frac{\pi}{2} - \left(-\frac{\pi}{2} \right) \right) = \frac{1}{2}$$