Osnovne trigonometrijske jednakosti

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm\cos x$$
$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp\sin x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$\sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$$
$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$\cos x - \cos y = 2\sin\frac{x+y}{2}\sin\frac{y-x}{2}$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x-y) - \cos(x+y)\big)$$
$$\cos x \cos y = \frac{1}{2}\big(\cos(x-y) + \cos(x+y)\big)$$
$$\sin x \cos y = \frac{1}{2}\big(\sin(x-y) + \sin(x+y)\big)$$

$$\sin(2x) = 2\sin x\cos x$$
$$\cos(2x) = \cos^2 x - \sin^2 x$$
$$2\sin^2 x = 1 - \cos(2x)$$
$$2\cos^2 x = 1 + \cos(2x)$$

Tablice suma i integrala

Konačne sume

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$
$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$
$$\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$$
$$\sum_{i=0}^{n} e^{j(\theta+i\phi)} = \frac{\sin\big((n+1)\phi/2\big)}{\sin(\phi/2)}e^{j(\theta+n\phi/2)}$$
$$\sum_{i=0}^{n}\binom{n}{i} = \sum_{i=1}^{n}\frac{n!}{i!(n-i)!} = 2^n$$

Neodređeni integrali

Racionalne funkcije

$$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)}, \quad 0 < n$$
$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln|ax+b|$$
$$\int \frac{dx}{a^2x^2+b^2} = \frac{1}{ab}\,\mathrm{tg}^{-1}\left(\frac{ax}{b}\right)$$
$$\int \frac{x\,dx}{a^2+x^2} = \frac{1}{2}\ln(a^2+x^2)$$
$$\int \frac{x^2\,dx}{a^2+x^2} = x - a\,\mathrm{tg}^{-1}\left(\frac{x}{a}\right)$$
$$\int \frac{dx}{(a^2+x^2)^2} = \frac{x}{2a^2(a^2+x^2)} + \frac{1}{2a^3}\,\mathrm{tg}^{-1}\left(\frac{x}{a}\right)$$
$$\int \frac{x\,dx}{(a^2+x^2)^2} = \frac{-1}{2(a^2+x^2)}$$
$$\int \frac{x^2\,dx}{(a^2+x^2)^2} = \frac{-x}{2(a^2+x^2)} + \frac{1}{2a}\,\mathrm{tg}^{-1}\left(\frac{x}{a}\right)$$

Trigonometrijske funkcije

$$\int \cos(x)\,dx = \sin(x)$$
$$\int x\cos(x)\,dx = \cos(x) + x\sin(x)$$
$$\int x^2\cos(x)\,dx = 2x\cos(x) + (x^2-2)\sin(x)$$
$$\int \sin(x)\,dx = -\cos(x)$$
$$\int x\sin(x)\,dx = \sin(x) - x\cos(x)$$
$$\int x^2\sin(x)\,dx = 2x\sin(x) + (2-x^2)\cos(x)$$

Eksponencijalne funkcije

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$
$$\int xe^{ax}\,dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax}$$
$$\int x^2e^{ax}\,dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax}$$
$$\int x^3e^{ax}\,dx = \left(\frac{x^3}{a} - \frac{3x^2}{a^2} + \frac{6x}{a^3} - \frac{6}{a^4}\right)e^{ax}$$
$$\int \sin(x)e^{ax}\,dx = \frac{1}{a^2+1}\big(a\sin(x) - \cos(x)\big)e^{ax}$$
$$\int \cos(x)e^{ax}\,dx = \frac{1}{a^2+1}\big(a\cos(x) + \sin(x)\big)e^{ax}$$

Na pismenom ispitu iz *Signala i sustava* dozvoljeno je imati isključivo pribor za pisanje, kalkulator bez bilježaka vezanih uz predmet te ovaj šalabahter. Ovaj šalabahter je dostupan na stranicama predmeta `http://www.fer.hr/predmet/sis2`. © Sveučilište u Zagrebu-FER-ZESOI, 2007. Dozovljeno je umnažanje i distribucija ovog šalabahtera samo ako svaka kopija sadrži gore navedenu informaciju o autorskim pravima te ovu dozvolu o umnažanju.

Određeni integrali

$$\int_{-\infty}^{+\infty} e^{-a^2x^2+bx}\,dx = \frac{\sqrt{\pi}}{a}e^{\frac{b^2}{4a^2}}, \quad a>0$$

$$\int_{0}^{+\infty} x^2e^{-x^2}\,dx = \frac{\sqrt{\pi}}{4}$$

$$\int_{0}^{+\infty} \frac{\sin(x)}{x}\,dx = \frac{\pi}{2}$$

$$\int_{0}^{+\infty} \frac{\sin^2(x)}{x^2}\,dx = \frac{\pi}{2}$$

Laplaceova transformacija

Laplaceova transformacija funkcije $x(t)$ je:

$$\mathcal{L}\big[f(t)\big] = \int_{0^-}^{+\infty} f(t)e^{-st}dt$$

Kažemo da su $x(t)$ i $X(s)$ transformacijski par i pišemo $x(t) \circ\!\!-\!\!\bullet X(s)$.

Tablica $\mathcal{L}$ transformacije

$$1 \circ\!\!-\!\!\bullet \frac{1}{s}$$

$$t \circ\!\!-\!\!\bullet \frac{1}{s^2}$$

$$e^{-at} \circ\!\!-\!\!\bullet \frac{1}{s+a}$$

$$\frac{1}{b-a}(e^{-at}-e^{-bt}) \circ\!\!-\!\!\bullet \frac{1}{(s+a)(s+b)}$$

$$\frac{1}{a-b}(ae^{-at}-be^{-bt}) \circ\!\!-\!\!\bullet \frac{s}{(s+a)(s+b)}$$

$$\frac{1}{a}e^{-bt}\sin(at) \circ\!\!-\!\!\bullet \frac{1}{(s+b)^2+a^2}$$

$$e^{-bt}\Big(\cos(at)-\frac{b}{a}\sin(at)\Big) \circ\!\!-\!\!\bullet \frac{s}{(s+b)^2+a^2}$$

Fourierova transformacija

Fourierova transformacija funkcije $x(t)$ je:

$$\mathcal{F}\big[x(t)\big] = X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,dt$$

Inverzna transformacija je:

$$\mathcal{F}^{-1}\big[X(\omega)\big] = x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)e^{j\omega t}\,d\omega$$

Kažemo da su $x(t)$ i $X(\omega)$ transformacijski par i pišemo $x(t) \circ\!\!-\!\!\bullet X(\omega)$.

Tablica $\mathcal{F}$ transformacije

Neka je:

$$\mu(x) = \begin{cases} 1,\ x>0 \\ 0,\ x<0 \end{cases}$$

$$\mathrm{rect}(x) = \begin{cases} 1, & -\frac{1}{2} < x < \frac{1}{2} \\ 0, & \frac{1}{2} < |x| \end{cases}$$

$$\mathrm{tri}(x) = \begin{cases} 1-|x|, & |x|<1 \\ 0, & |x|>1 \end{cases}$$

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

Uz te oznake važnije transformacije su:

$$1 \circ\!\!-\!\!\bullet 2\pi\,\delta(\omega)$$

$$\delta(t) \circ\!\!-\!\!\bullet 1$$

$$\mu(t) \circ\!\!-\!\!\bullet \pi\,\delta(\omega) + \frac{1}{j\omega}$$

$$\frac{1}{2}\,\delta(t) - \frac{1}{2\pi jt} \circ\!\!-\!\!\bullet \mu(\omega)$$

$$\mathrm{sgn}(t) \circ\!\!-\!\!\bullet \frac{2}{j\omega}$$

$$\mathrm{rect}\Big(\frac{t}{T}\Big) \circ\!\!-\!\!\bullet T\,\mathrm{sinc}\Big(\frac{\omega T}{2\pi}\Big)$$

$$\mathrm{sinc}(at) \circ\!\!-\!\!\bullet \frac{1}{a}\,\mathrm{rect}\Big(\frac{\omega}{2\pi a}\Big)$$

$$\mathrm{tri}\Big(\frac{t}{T}\Big) \circ\!\!-\!\!\bullet T\,\mathrm{sinc}^2\Big(\frac{\omega T}{2\pi}\Big)$$

$$\mathrm{sinc}^2(at) \circ\!\!-\!\!\bullet \frac{1}{a}\,\mathrm{tri}\Big(\frac{\omega}{2\pi a}\Big)$$

$$e^{j\omega_0 t} \circ\!\!-\!\!\bullet 2\pi\,\delta(\omega-\omega_0)$$

$$\delta(t-t_0) \circ\!\!-\!\!\bullet e^{-j\omega t_0}$$

$$\sin(\omega_0 t) \circ\!\!-\!\!\bullet -j\pi\big(\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\big)$$

$$\cos(\omega_0 t) \circ\!\!-\!\!\bullet \pi\big(\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\big)$$

$$\sum_{i=-\infty}^{+\infty}\delta(t-iT_0) \circ\!\!-\!\!\bullet \frac{2\pi}{T_0}\sum_{i=-\infty}^{+\infty}\delta\Big(\frac{\omega}{2\pi}-\frac{i}{T_0}\Big)$$

$$\sin(\omega_0 t)\,\mu(t) \circ\!\!-\!\!\bullet -\frac{j\pi}{2}\big(\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\big)+\frac{j\omega}{\omega_0^2-\omega^2}$$

$$\cos(\omega_0 t)\,\mu(t) \circ\!\!-\!\!\bullet \frac{\pi}{2}\big(\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\big)+\frac{j\omega}{\omega_0^2-\omega^2}$$

$$e^{-at}\,\mu(t) \circ\!\!-\!\!\bullet \frac{1}{a+j\omega}, \quad a>0$$

$$te^{-at}\,\mu(t) \circ\!-\!\bullet \frac{1}{(a+j\omega)^2}, \quad a>0$$

$$t^2e^{-at}\,\mu(t) \circ\!-\!\bullet \frac{2}{(a+j\omega)^3}, \quad a>0$$

$$t^3e^{-at}\,\mu(t) \circ\!-\!\bullet \frac{6}{(a+j\omega)^4}, \quad a>0$$

$$e^{-a|t|} \circ\!-\!\bullet \frac{2a}{a^2+\omega^2}$$

$$e^{-\frac{t^2}{2a^2}} \circ\!-\!\bullet a\sqrt{2\pi}e^{-a^2\omega^2/2}$$

Vremenski diskretna Fourierova transformacija

Vremenski diskretna Fourierova transformacija (DTFT – *Discrete-Time Fourier Transform*) niza $x[n]$ je:

$$\mathcal{F}_{\mathrm{vd}}\big[x[n]\big] = X(\omega) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

Inverzna transformacija je:

$$\mathcal{F}_{\mathrm{vd}}^{-1}\big[X(\omega)\big] = x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(\omega)e^{j\omega n}\,d\omega$$

Niz $x[n]$ i njegov spektar $X(\omega)$ čine transformacijski par $x[n] \circ\!-\!\bullet X(\omega)$.

Tablica $\mathcal{F}_{\mathrm{vd}}$ transformacije

$$\delta[n] \circ\!-\!\bullet 1$$

$$1 \circ\!-\!\bullet \sum_{i=-\infty}^{+\infty} 2\pi\delta(\omega+2\pi i)$$

$$e^{j\omega_0 n} \circ\!-\!\bullet \sum_{i=-\infty}^{+\infty} 2\pi\delta(\omega-\omega_0+2\pi i)$$

$$\mu[n] \circ\!-\!\bullet \frac{1}{1-e^{-j\omega}} + \sum_{i=-\infty}^{+\infty} \pi\delta(\omega+2\pi i)$$

$$a^n\,\mu[n] \circ\!-\!\bullet \frac{1}{1-ae^{-j\omega}}, \quad |a|<1$$

$$na^n\,\mu[n] \circ\!-\!\bullet \frac{ae^{j\omega}}{(e^{-j\omega}-a)^2}, \quad |a|<1$$

$$\sin(\omega_0 n) \circ\!-\!\bullet \sum_{i=-\infty}^{+\infty} j\pi\big(\delta(\omega+\omega_0+2\pi i)-\delta(\omega-\omega_0+2\pi i)\big)$$

$$\cos(\omega_0 n) \circ\!-\!\bullet \sum_{i=-\infty}^{+\infty} \pi\big(\delta(\omega+\omega_0+2\pi i)+\delta(\omega-\omega_0+2\pi i)\big)$$

$$a^n\sin(\omega_0 n)\,\mu[n] \circ\!-\!\bullet \frac{ae^{j\omega}\sin(\omega_0)}{e^{2j\omega}-2ae^{j\omega}\cos(\omega_0)+a^2}, \quad |a|<1$$

$$a^n\cos(\omega_0 n)\,\mu[n] \circ\!-\!\bullet \frac{e^{j\omega}\big(e^{j\omega}-a\cos(\omega_0)\big)}{e^{2j\omega}-2ae^{j\omega}\cos(\omega_0)+a^2}, \quad |a|<1$$

Diskretna Fourierova transformacija

Diskretna Fourierova transformacija konačnog niza $x[n]$ duljine N je:

$$X[k] = \sum_{n=0}^{N-1} x[n]W_N^{nk}, \quad 0\le k\le N-1$$

Pri tome je $W_N^{nk} = e^{-2\pi jnk/N}$. Inverzna transformacija je:

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]W_N^{-nk}, \quad 0\le n\le N-1$$

Niz $x[n]$ i njegov spektar $X[k]$ čine transformacijski par $x[n] \circ\!-\!\bullet X[k]$.

$\mathcal{Z}$-transformacija

$\mathcal{Z}$-transformacija niza $f[n]$ je: $\mathcal{Z}\big[f[n]\big] = \sum_{n=0}^{+\infty} f[n]z^{-n}$

Tablica $\mathcal{Z}$ transformacije

$$\delta[n] \circ\!-\!\bullet 1$$

$$\delta[n-m] \circ\!-\!\bullet z^{-m}$$

$$n \circ\!-\!\bullet \frac{z}{(z-1)^2}$$

$$1^n \circ\!-\!\bullet \frac{1}{1-z^{-1}} = \frac{z}{z-1}$$

$$a^n \circ\!-\!\bullet \frac{1}{1-az^{-1}} = \frac{z}{z-a}$$

$$(n+1)a^n \circ\!-\!\bullet \frac{z^2}{(z-a)^2}$$

$$\frac{(n+1)(n+2)}{2!}a^n \circ\!-\!\bullet \frac{z^3}{(z-a)^3}$$

$$\frac{(n+1)(n+2)\dots(n+m-1)}{(m-1)!}a^n \circ\!-\!\bullet \frac{z^m}{(z-a)^m}$$

$$\frac{n(n-1)(n-2)\dots(n-m+1)}{m!}a^{n-m} \circ\!-\!\bullet \frac{z}{(z-a)^{m+1}}$$

$$a^n-\delta[n] \circ\!-\!\bullet \frac{a}{z-a}$$

$$\sin[an] \circ\!-\!\bullet \frac{z\sin(a)}{z^2-2z\cos(a)+1}$$

$$\cos[an] \circ\!-\!\bullet \frac{z^2-z\cos(a)}{z^2-2z\cos(a)+1}$$

Određivanja početnih uvjeta

Za sustav opisan diferencijalnom jednadžbom

$$y^{(N)}(t) + a_1 y^{(N-1)}(t) + \cdots + a_{N-1} y^{(1)}(t) + a_N y(t) = b_0 u^{(N)}(t) + b_1 u^{(N-1)}(t) + \cdots + b_{N-1} u^{(1)}(t) + b_N u(t)$$

potrebno je odrediti početne uvjete $y(0^+)$, $y'(0^+)$, $y''(0^+)$, $\ldots$, $y^{(N-1)}(0^+)$ iz onih u 0^-. Ako pobuda ne sadrži Diracovu distribuciju rješavamo sustav jednadžbi:

$$\begin{aligned}
\Delta y &= b_0 u(0^+) \\
\Delta y^{(1)} + a_1 \Delta y &= b_0 u^{(1)}(0^+) + b_1 u(0^+) \\
\Delta y^{(2)} + a_1 \Delta y^{(1)} + a_2 \Delta y &= b_0 u^{(2)}(0^+) + b_1 u^{(1)}(0^+) + b_2 u(0^+) \\
\\
\Delta y^{(N-1)} + a_1 \Delta y^{(N-2)} + \cdots + a_{N-1} \Delta y &= b_0 u^{(N-1)}(0^+) + \cdots + b_{N-2} u^{(1)}(0^+) + b_{N-1} u(0^+)
\end{aligned}$$

Pri tome je $\Delta y^{(i)} = y^{(i)}(0^+) - y^{(i)}(0^-)$.

Ako je pobuda $u(t) = \delta(t)$ onda je $y^{(N-1)}(0^+) = y^{(N-1)}(0^-) + 1$, a ostali početni uvjeti se ne razlikuju.