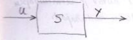
MASS – SIS – 2. ciklus (1)

SUSTAVI

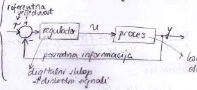


$f: x \to y \Rightarrow y = f(x)$
↓
ponašanje sustava

$s: u \to y \rightarrow y = S(u)$

-podjela 1: 1. kontinuirani → ulazni i izlazni signal kontinuiran
2. diskretni → ulazni i izlazni signal diskretan
3. hibridni → pojavljuju se i kontinuirani i diskretni signali

hibridni sustav



idealni slučaj: $r = y$
→ željeno = ostvareno

-podjela 2: (broj uli) 1. SISO → 1 ulaz, 1 izlaz
2. MIMO → više ulaza, više izlaza

→podjela 3: (zakon superpozicije) 1. linearni
2. nelinearni

→podjela 4: 1. vremenski promjenjivi
2. vremenski nepromjenjivi

→podjela 5: (prema predviđanju) 1. kauzalni → ovise o trenutnom/prošlom stanju
2. nekauzalni → gledaju samo budućnost

→podjela 6: 1. memorijski
2. bezmemorijski

→ primjeri

- $y(t) = u(t-4)$

  $t = 2 \quad y(2) = u(-2)$

  ⇒ kauzalan, memorijski

- $y(t) = u(t+4)$

  $t = 2 \quad y(2) = u(6)$

  ⇒ nekauzalan, memorijski

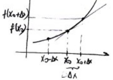
- $y(t) = u(t^2)$

  ⇒ nekauzalan, memorijski

⇒ nekauzalan sustav ne može biti bezmemorijski

⇒ kauzalan sustav je bezmemorijski ako gleda u sadašnjosti

→ DERIVATOR



$$k = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_0+\Delta x) - f(x_0)}{\cancel{x_0} + \Delta x - \cancel{x_0}} = \lim_{\Delta x \to 0} \frac{f(x_0+\Delta x) - f(x_0)}{\Delta x}$$

⇒ nekauzalan

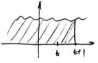
$$k = \lim_{\Delta x \to 0} \frac{f(x_0) - f(x_0 - \Delta x)}{\Delta x}$$

⇒ kauzalan

→ INTEGRATOR

$$y(t) = \int_{-\infty}^{t} u(\tau)\,d\tau$$ → memorijski ($y(t)$ ovisno pobudi u t i prošlosti)

$$y(t) = \int_{-\infty}^{t+1} u(\tau)\,d\tau$$ ⇒ nekauzalan



→ DIFERENCIJA

$$\Delta y(n) = y(n+1) - y(n)$$

$$\nabla y(n) = y(n) - y(n-1)$$

$$y(n) = \sum_{k=-\infty}^{n} u(k)$$

→ Zadaci

① $y(t) = 3u(t+4)$, linearan ?

→ linearnost: načelo aditivnosti, načelo homogenosti → $S(\alpha u) = \alpha S(u)$, $S(\alpha u_1 + \beta u_2) = S(\alpha u_1) + S(\beta u_2)$

$\Rightarrow S(\alpha u_1 + \beta u_2) = \alpha S(u_1) + \beta S(u_2)$

• $u(t) = \alpha u_1(t) + \beta u_2(t)$

$u(t+4) = \alpha u_1(t+4) + \beta u_2(t+4) \} \longrightarrow y(t) = 3u(t+4)$

• $y_1(t) = 3u_1(t+4) \qquad y_2(t) = 3u_2(t+4)$

$\{ S(\alpha u_1 + \beta u_2) = y \;,\; S(u_1) = y_1 \;,\; S(u_2) = y_2 \}$

$$\Rightarrow y(t) = 3(\alpha u_1(t+4) + \beta u_2(t+4))$$
$$= 3\alpha u_1(t+4) + 3\beta u_2(t+4)$$
$$= \alpha \underbrace{3u_1(t+4)}_{y_1(t)} + \beta \underbrace{3u_2(t+4)}_{y_2(t)}$$
$$= \alpha y_1(t) + \beta y_2(t)$$

$y = \alpha y_1 + \beta y_2$ // → sustav je linearan!

② $y(t) = t u(t^2)$, linearan?

• $u(t) = \alpha u_1(t) + \beta u_2(t)$

$u(t^2) = \alpha u_1(t^2) + \beta u_2(t^2)$

• $y_1(t) = t u_1(t^2)$, $y_2(t) = t u_2(t^2)$

$$y(t) = t(\alpha u_1(t^2) + \beta u_2(t^2))$$
$$= \alpha \underbrace{t u_1(t^2)}_{y_1(t)} + \beta \underbrace{t u_2(t^2)}_{y_2(t)}$$

$\rightarrow y = \alpha y_1(t) + \beta y_2(t) \Rightarrow$ sustav je linearan!

③ $y(n) = 2^{u(n)}$, linearan?

• $u(n) = \alpha u_1(n) + \beta u_2(n)$

• $y_1(n) = 2^{u_1(n)}$, $y_2(n) = 2^{u_2(n)}$

$$y(n) = 2^{\alpha u_1(n) + \beta u_2(n)}$$
$$= \underbrace{2^{\alpha u_1(n)}}_{\neq y_1(n)} \cdot \underbrace{2^{\beta u_2(n)}}_{\neq y_2(n)} \Rightarrow \text{sustav je nelinearan!}$$

④ $y(t) = u(t) + 4$, linearan?

- $u(t) = \alpha u_1(t) + \beta u_2(t)$
- $y_1(t) = u_1(t) + 4$, $y_2(t) = u_2(t) + 4$

$y(t) = \alpha u_1(t) + \beta u_2(t) + 4 = \alpha(u_1(t)+4) + \beta(u_2(t)+4)$ proujera

$= \alpha(u_1(t)+4) + \beta(u_1(t)+4) - 4\alpha - 4\beta + 4$

$= \alpha u_1(t) + 4\alpha + \beta u_2(t) + \beta 4 + 4$

$= \alpha y_1(t) + \beta y_2(t) + \underbrace{4(1-\alpha-\beta)}_{REP}$ → nelinearan!

⑤ $y(n) = \frac{n}{u(n)}$, linearan?

- $u(n) = \alpha u_1(n) + \beta u_2(n)$
- $y_1(n) = \frac{n}{u_1(n)}$, $y_2(n) = \frac{n}{u_2(n)}$

$y(n) = \frac{n}{\alpha u_1(n) + \beta u_2(n)} \neq$

→ treba dobit. $\alpha \frac{n}{u_1} + \beta \frac{n}{u_2} = \frac{n(\alpha u_2 + \beta u_1)}{u_1 u_2}$

→ nelinearan!

⑥ $y(t) = \int_{-\infty}^{t} u(\tau)d\tau$, linearan?

- $u(t) = \alpha u_1(t) + \beta u_2(t) = \alpha u_1(\tau) + \beta u_2(\tau)$
- $y_1(t) = \int_{-\infty}^{t} u_1(\tau)d\tau$, $y_2(t) = \int_{-\infty}^{t} u_2(\tau)d\tau$

$y(t) = \int_{-\infty}^{t} (\alpha u_1(\tau) + \beta u_2(\tau))d\tau =$

$= \alpha \underbrace{\int_{-\infty}^{t} u_1(\tau)d\tau}_{y_1(t)} + \beta \underbrace{\int_{-\infty}^{t} u_2(\tau)d\tau}_{y_2(t)}$

$= \alpha y_1(t) + \beta y_2(t)$ → linearan!

⑦ $y(n) = \frac{u(n)}{1 + u(n-1)}$ → problem u nazivniku → nelinearno

⑧ $y(n) = \left(\frac{1}{2}\right)^n u(3n+2)$ →

$u(3n+2) = \alpha u_1(3n+2) + \beta u_2(3n+2)$

$y_1(n) = \left(\frac{1}{2}\right)^n u_1(3n+2)$, $y_2(n) = \left(\frac{1}{2}\right)^n u_2(3n+2)$

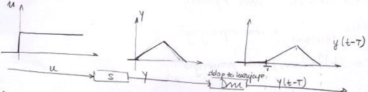
$\rightarrow y(n) = \alpha\left(\frac{1}{2}\right)^n u_1(3n+2) + \beta\left(\frac{1}{2}\right)^n u_2(3n+2) = \alpha y_1 + \beta y_2 \rightarrow$ linearan

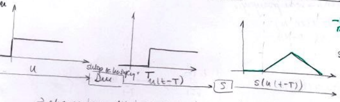
⑤ $y(t) = u(t) + u(t-1) \rightarrow$ linearno

VREMENSKA PROMJENJIVOST

$S(u(t)) = y(t)$

$S(u(t-T)) = y(t-T)$

① 

② 

-vremenski nepromjenjiv

$S(u(t-T))$

$\Rightarrow$ ako na kraju dobijemo nešto drugačije od zelenog grafa $\rightarrow$ sustav nije stabilan, tj. vremenski je promjenjiv

$S \rightarrow$ svugdje u pravcu $-T \mid -N \rightarrow$ bez obzira na varijablu

→ primjeri

- $y(t) = 3u(t+4)$

$S(u(t-T)) = 3u(t+4-T)$
$y(t-T) = 3u(t-T+4)$ } $= \rightarrow$ sustav vremenski nepromjenjiv

- $y(t) = tu(t^2)$

$S(u(t-T)) = tu(t^2-T)$
$y(t-T) = (t-T)u((t-T)^2)$ } $\neq \Rightarrow$ vremenski promjenjiv

- $y(n) = 2^{u(n)}$

$S(u(n-N)) = 2^{u(n-N)}$
$y(n-N) = 2^{u(n-N)}$ } $= \Rightarrow$ vremenski nepromjenjiv

- $y(t) = u(t)+4$

$$S(u(t-T)) = u(t-T)+4$$
$$y(t-T) = u(t-T)+4$$
$\Rightarrow$ vr. nepromjenjiv

- $y(n) = \dfrac{n}{u(n)}$

$$S(u(n-N)) = \frac{n}{u(n-N)}$$
$$y(n-N) = \frac{n-N}{u(n-N)}$$
$\neq \rightarrow$ vr. promjenjiv

- $y(t) = u(t) + u(t-1) \rightarrow$ vrem. nepromjenjiv
- $y(n) = \dfrac{u(n)}{1+u(n-1)} \rightarrow$ vrem. nepromjenjiv
- $y(n) = \left(\frac{1}{2}\right)^n u(3n+2) \Rightarrow$ vrem. promjenjiv
- $y(n) = u(2n) \rightarrow$ vrem. promjenjiv

$$S(u(n-N)) = u(2n-N)$$
$$y(n-N) = u(2(n-N))$$

$\rightarrow$ zadaci

① $y(t) = \int_{\tau=-\infty}^{\tau=t} u(\tau)d\tau$ $\left\{ \sum_{k=1}^{t=100} u(k) \longleftrightarrow \int_{\tau=1}^{100} u(\tau)d\tau \right\}$

$$S(u(t-T)) = \int_{-\infty}^{t} u(\tau-T)d\tau$$
$$y(t-T) = \int_{-\infty}^{t-T} u(\tau)d\tau$$

supstitucija!
$a = \tau - T$
$da = d\tau$

$\tau = \cancel{t}$

$S(u(t-T)) = \int u(a)da = \left| \begin{matrix} \tau = t \\ a+T = t \\ a = t-T \\ \tau = -\infty \\ a+T = -\infty \\ a = -\infty \end{matrix} \right| \begin{matrix} \tau \to t, a \to T-t \\ \tau \to -\infty, a \to -\infty \end{matrix} \left| = \int_{-\infty}^{t-T} u(a)da \right.$

$\tau = -\cancel{\infty}$

$$S(u(t-T)) = \int_{-\infty}^{t-T} u(a)da = \left| \begin{matrix} a = b \\ da = db \end{matrix} \right| = \int_{-\infty}^{t-T} u(b)db$$

$$y(t-T) = \int_{-\infty}^{t-T} u(\tau)d\tau = \left| \begin{matrix} \tau = b \\ d\tau = db \end{matrix} \right| = \int_{-\infty}^{t-T} u(b)dd$$

vrem. nepromjenjiv

② $y(t) = \int_{1}^{t} u(\tau)d\tau$

$S(u(t-T)) = \int_{1}^{t} u(\tau-T)d\tau$

$y(t-T) = \int_{1}^{t-T} u(\tau)d\tau$

$a = \tau - T$, $\tau = 1 \to a = 1-T$

$d\tau = da$, $\tau = t \to a = t-T$

$S(u(t-T)) = \int_{1-T}^{t-T} u(a)da$

$y(t-T) = \int_{1}^{t-T} u(\tau)d\tau$

} vrem. promjenjiv

③ $y(n) = \sum_{k=-\infty}^{n} \frac{u(k)}{n-k}$

$S(u(n-N)) = \sum_{k=-\infty}^{n} \frac{u(k-N)}{n-k}$

$y(n-N) = \sum_{k=-\infty}^{n-N} \frac{u(k)}{n-N-k}$

$a = k-N \quad k = a+N$

$k = -\infty \quad a = -\infty$

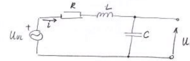
$k = n \quad a = n-N$

$S(u(n-N)) = \sum_{a=-\infty}^{n-N} \frac{u(a)}{n-(a+N)} = \sum_{a=-\infty}^{n-N} \frac{u(a)}{n-N-a}$

$y(n-N) = \sum_{k=-\infty}^{n-N} \frac{u(k)}{n-N-k} = /k=a/ = \sum_{a=-\infty}^{n-N} \frac{u(a)}{n-N-a}$

$=$ → vr. nepromjenjiv

Kontinuirani sustavi → ne u ispitu (2. MI)



n-ti red → n-1 - red

$$U_{UL} = U_R + U_L + U_C$$

$$U_R = i \cdot R$$

$$U_L = L \cdot \frac{di}{dt}$$

$$U_C = \frac{1}{C}\int_0^t i(\tau)d\tau = U_{IZ}$$

$$U_{UL} = i \cdot R + L \cdot \frac{di}{dt} + \underbrace{\frac{1}{C}\int_0^t i(\tau)d\tau}_{U_{IZ}}$$

$$U_{UL} = i \cdot R + L\frac{di}{dt} + U_{IZ} = C \cdot U_{IZ}' \cdot R + L \cdot \frac{d}{dt}\left(C \cdot U_{IZ}'\right) + U_{IZ} =$$

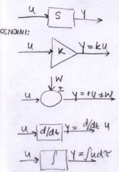
$$U_{IZ} = \frac{1}{C}\int_0^t i(\tau)d\tau \qquad = CR\,U_{IZ}' + CL \cdot U_{IZ}'' + U_{IZ} =$$

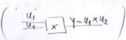
$$C\,U_{IZ} = \int_0^t i(\tau)d\tau \;/\;' \qquad \Rightarrow U_{UL} = CR \cdot U_{IZ}' + C \cdot L\,U_{IZ}'' + U_{IZ}$$

$$C\,U_{IZ}' = i \qquad u = U_{UL},\; y = U_{IZ}$$

$$u = CLy'' + CRy' + y \longrightarrow \text{dif. jedn.}$$

Blokovski dijagram





-primjeri:

- $y'' - 3y' + 5y = u$

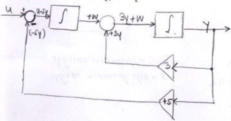
$y'' = u + 3y' - 5y \quad / \iint$

$y = \iint u \, d\tau \, d\tau + \iint 3y' \, d\tau \, d\tau - \iint 5y \, d\tau \, d\tau$

$y = \iint u \, d\tau \, d\tau + \int 3y \, d\tau - \iint 5y \, d\tau \, d\tau$

$y = \int \left[3y + \int u \, d\tau - \int 5y \, d\tau \right] d\tau$

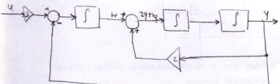
$y = \int \left[3y + \underbrace{\int (u - 5y) \, d\tau}_{w} \right] d\tau$ → modeliranje

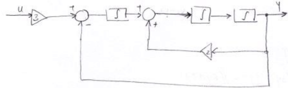


- $y''' - 2y' + y = 3u$

$y''' = 3u + 2y' - y \quad / \iiint$

$y = \iint \left[2y + \overbrace{\int (3u - y) \, d\tau}^{w} \right] d\tau \, d\tau$



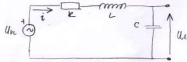


$$y = \int \left[\int \left(2y + \int (3u - y)\, d\tau \right) d\tau \right] d\tau \quad /'''$$

$$y''' = 2y' + 3u - y$$

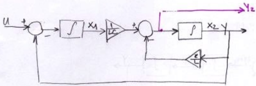
$$y''' - 2y' + y = 3u$$

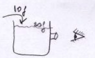
Varijable stanja



L, C = spremnici energije
R = nije spremnik energije

$$LCy'' + RCy' + y = u$$





→ izlaz ovisi o trenutnom stanju sustava i pobudi
x – stanje sustava
u – ulazna pobuda
y – izlaz

– dif. jedn. n-tog reda ⇒ n dif. jedn. 1. reda

$$\dot{x} = x' = Ax + Bu$$

$$y = Cx + Du$$

$x_1 = \int (u-y)\,d\tau \;/'$

$\dot{x}_1 = u - y \Rightarrow$ slika: $y = x_2 \Rightarrow \dot{x}_1 = u - x_2$

$x_2 = \int \left(\frac{1}{LC}x_1 - \frac{R}{L}y\right)d\tau \;/'$

$\dot{x}_2 = \frac{1}{LC}x_1 - \frac{R}{L}y \Rightarrow$ slika $y = x_2 \Rightarrow \dot{x}_2 = \frac{1}{LC}x_1 - \frac{R}{L}x_2$

$\Rightarrow \dot{x}, x, u \rightarrow$ vektori

$$\left.\underbrace{\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}}_{[n\times 1]} = \underbrace{\begin{array}{cc} x_1 & x_2 \\ \hline \end{array}\hspace{-2.5em}\begin{bmatrix}0 & -1\\ \frac{1}{LC} & -\frac{R}{L}\end{bmatrix}}_{A[n\times n]} \underbrace{\begin{bmatrix}x_1\\ x_2\end{bmatrix}}_{[n\times 1]} + \underbrace{\begin{bmatrix}1\\ 0\end{bmatrix}}_{B[n\times m]}\cdot\underbrace{[u]}_{[1\times 1]}\right\} *_1 \qquad (m=1)$$

$\dot{x}_1 = -x_2 + u$
$\dot{x}_2 = \frac{1}{LC}x_1 - \frac{R}{L}x_2$

x = varijabla stanja $\Rightarrow n$ = broj var. stanja
u = pobuda $\Rightarrow m$ = broj pobuda
y = izlaz $\Rightarrow k$ = broj izlaza

$\Rightarrow$ matrica A: $n\times n \rightarrow$ stupac: zastupljenost var. stanja
$\Rightarrow$ matrica B: $n \times m$

$y_2 \ldots \quad y_2 = \frac{1}{LC}x_1 - \frac{R}{L}x_2$

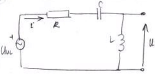
(scope $\rightarrow$ simulink $\rightarrow$ izlaz)

$$\left.\underbrace{\begin{bmatrix}y_1\\ y_2\end{bmatrix}}_{[k\times 1]} = \underbrace{\begin{bmatrix}0 & 1\\ \frac{1}{LC} & -\frac{R}{L}\end{bmatrix}}_{C[k\times n]} \underbrace{\begin{bmatrix}x_1\\ x_2\end{bmatrix}}_{[n\times 1]} + \underbrace{\begin{bmatrix}0\\ 0\end{bmatrix}}_{D[k\times m]}\underbrace{[u]}_{[m\times 1]}\right\} *_2$$

$y_1 = x_2$
$y_2 = \frac{1}{LC}x_1 - \frac{R}{L}x_2$

$*_1 + *_2 \rightarrow$ zapis sustava n-dif. jedn preko matrica sustava A, B, C, D

A – matrica dinamike sustava $= [n\times n]$
B = ulazna matrica sustava $= [n \times m]$
C = izlazna matrica sustava $= [k\times n]$
D = ulazno-izlazna matrica sustava $[k \times m]$



$$u_{ul} = u_R + u_C + u_L$$

$$u_L = u_{iz}$$

$$u_{ul} = iR + \frac{1}{C}\int_0^t i(\tau)d\tau + L\frac{di}{dt} -$$

$$= iR + \frac{1}{C}\int_0^t i(\tau)d\tau + u_{iz}$$

$$u_{iz} = L\frac{di}{dt} \Rightarrow i = \frac{1}{L}\int u_{iz}\,d\tau$$

$$\Rightarrow u_{ul} = \frac{R}{L}\int u_{iz}\,d\tau + \frac{1}{LC}\iint u_{iz}\,d\tau\, d\tau + u_{iz} \quad /''$$

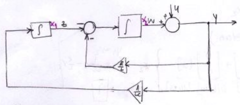
$$u_{ul}'' = \frac{R}{L}u_{iz}' + \frac{1}{LC}u_{iz} + u_{iz}''$$

$$y'' + \frac{R}{L}y' + \frac{1}{LC}y = u'' \Rightarrow \text{modeliranje}$$

$$y = \iint u'' - \frac{R}{L}\iint y' - \frac{1}{LC}\iint y$$

$$y = u - \frac{R}{L}\int y - \frac{1}{LC}\iint y$$

$$y = u + \int \underbrace{\left(-\frac{R}{L}y - \overbrace{\int \frac{1}{LC}y\,d\tau}^{z}\right)}_{w} d\tau$$



$$x_1 = \int \frac{1}{LC}y\,d\tau \Rightarrow \dot{x}_1 = \frac{1}{LC}y = \frac{1}{LC}(u + x_2) = \frac{1}{LC}u + \frac{1}{LC}x_2$$

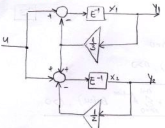
$$x_2 = \int\left(-x_1 - \frac{R}{L}y\right) \Rightarrow \dot{x}_2 = -x_1 - \frac{R}{L}u - \frac{R}{L}x_2$$

$$y = u + x_2$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & \frac{1}{LC}\\ -1 & \frac{-R}{L}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\frac{1}{LC}\\ \frac{-R}{L}\end{bmatrix}[u]$$

$$[y] = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\end{bmatrix}\cdot[u]$$

Diskretni sustavi → ISPIT



$$x(k+1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k) + Du(k)$$

$$x_1(k) = \underbrace{E^{-1}}_{\text{sklop za kašnjenje}}\left(u(k) - \frac{1}{3}x_1(k)\right) = u(k-1) - \frac{1}{3}x_1(k-1)$$

(1) $x_1(k+1) = u(k) - \frac{1}{3}x_1(k)$

$$x_2(k) = E^{-1}\left(\frac{1}{3}x_1(k) - \frac{1}{2}x_2(k) + u(k)\right) = \frac{1}{3}x_1(k-1) - \frac{1}{2}x_2(k-1) + u(k-1)$$

(2) $x_2(k+1) = u(k) + \frac{1}{3}x_1(k) - \frac{1}{2}x_2(k)$

(3) $y_1(k) = x_1(k)$

(4) $y_2(k) = x_2(k)$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \underbrace{\begin{bmatrix} -\frac{1}{3} & 0 \\ \frac{1}{3} & -\frac{1}{2} \end{bmatrix}}_{A} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{B} [u(k)]$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 0 \end{bmatrix}}_{D} [u(k)]$$

→ zadaci

→ Odziv sustava

$$y(n)=\begin{cases} Cx(0)+Du(0) & , n=0 \\ C A^n x(0)+\sum_{m=0}^{n-1} C A^{n-1-m}\cdot Bu(m) & , n>0 \end{cases}$$

Odziv stanja sustava ← $x(n)=A^n x(0)+\sum_{m=0}^{n-1} A^{n-1-m} Bu(m) \ , n>0$

①

$x_1(k+1)=x_1(k)+x_2(k)$ $\{k-n\}$

$x_2(k+1)=x_2(k)$

$x_1(0)=1 , x_2(0)=3$

$y(k)=x_1(k)-u(k)$

$u(k)=4\mu(k)$

stanje u 50 koraku?

$x_1(1)=x_1(0)+x_2(0)=1+3=4$

$x_2(1)=x_2(0)=3$

$x_1(2)=x_1(1)+x_2(1)=4+3=7$

$x_2(2)=x_1(1)=3$

$x_1(3)=x_1(2)+x_2(2)=\ldots$

⋮

→ jednostavnije:

$A=\begin{bmatrix}1&1\\0&1\end{bmatrix}, \ B=\begin{bmatrix}0\\0\end{bmatrix}, \ C=\begin{bmatrix}1&0\end{bmatrix}, D=[-1]$

– stanje nepobuđenog sustava ⇒ $u=0$

$x(0)=\begin{bmatrix}1\\3\end{bmatrix}$

$x(n)=A^n x(0)+\cancel{\sum_{m=0}^{n-1} A^{n-1-m} Bu(m)}$

$=A^n x(0)=\begin{bmatrix}1&n\\0&1\end{bmatrix}\cdot x(0)=$

$=\begin{bmatrix}1&n\\0&1\end{bmatrix}\begin{bmatrix}1\\3\end{bmatrix}=\begin{bmatrix}1+3n\\3\end{bmatrix}$

$x(50)=\begin{bmatrix}151\\3\end{bmatrix} \Rightarrow x_1(50)=151$

$x_2(50)=3$

$A\cdot A=\begin{bmatrix}1&1\\0&1\end{bmatrix}\begin{bmatrix}1&1\\0&1\end{bmatrix}=\begin{bmatrix}1&2\\0&1\end{bmatrix}$

$AAA=\begin{bmatrix}1&2\\0&1\end{bmatrix}\begin{bmatrix}1&1\\0&1\end{bmatrix}=\begin{bmatrix}1&3\\0&1\end{bmatrix}$

$A^4=\begin{bmatrix}1&3\\0&1\end{bmatrix}\begin{bmatrix}1&1\\0&1\end{bmatrix}=\begin{bmatrix}1&4\\0&1\end{bmatrix}$

$A^n=\begin{bmatrix}1&n\\0&1\end{bmatrix}$

③

② (6. → zadaci to vježbu → 10 b.)

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D = [0]$

$x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Odziv nepobuđenog sustava? , $n > 0$

$y(n) = CA^n x(0)$

$A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

$y(n) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & n \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = [1+n]$, $n > 0$

③ (7. → zad. za vj. → 10 b.)

$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $D = [1]$

$x(0) = \begin{bmatrix} a \\ b \end{bmatrix}$, $x(2) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$u(0), u(1) = ?$

$x_1(k+1) = x_1(k) + x_2(k)$
$x_2(k+1) = x_1(k) + u(k)$ } iz matrica!

$x_1(2) = x_1(1) + x_2(1) = 0$
$x_2(2) = x_1(1) + u(1) = 0$

$x_1(1) = x_1(0) + x_2(0) = a+b$
$x_2(1) = x_1(0) + u(0) = a + u(0)$

$\Rightarrow x_1(1) + x_2(1) = 0$
$a+b+a+u(0) = 0$
$\underline{u(0) = -2a-b}$

$\Rightarrow x_1(1) + u(1) = 0$
$a+b+u(1) = 0$
$\underline{u(1) = -a-b}$

④ (zad. z. vj. → 10 b.)

$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $x(2) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$u(0), u(1) = ?$

$x_1(k+1) = x_2(k)$
$x_2(k+1) = u(k)$

$x_1(1) = x_2(0) = 0$
$x_2(1) = \underline{u(0) = 1}$

$x_1(2) = x_2(1) = 1$
$x_2(2) = \underline{u(1) = 2}$