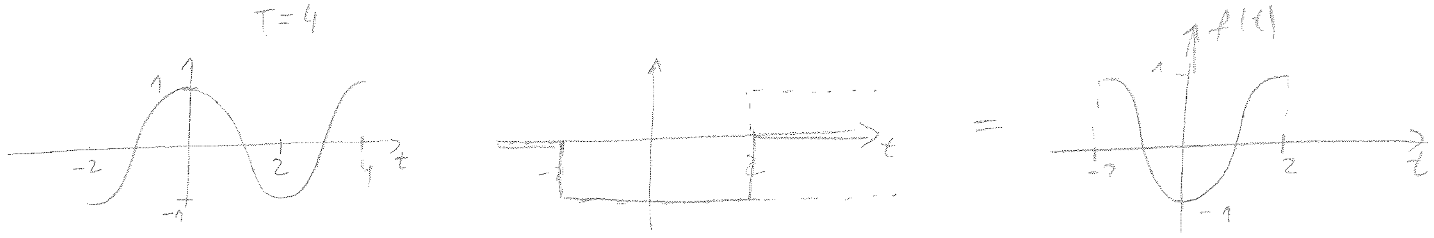
1. $f(t) = \cos\frac{\pi}{2}t \quad (\mu(t-2) - \mu(t+2))$

$\cancel{\frac{\pi}{2}}T = 2\cancel{\pi}$

$T = 4$



a) $E = \int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-2}^{2} (-\cos\frac{\pi}{2}t)^2 dt = \int_{-2}^{2} \frac{1+\cos\pi t}{2} dt$

SISAb-g5/9
[illegible]2/3

$= \left(\frac{1}{2}t + \frac{1}{2}\frac{\sin\pi t}{\pi}\right)\Big|_{-2}^{2} = \frac{1}{2}(2-(-2)) + \frac{1}{2\pi}(\underbrace{\sin 2\pi}_{1} - \underbrace{\sin(-2\pi)}_{1})$

$= \frac{1}{2}\cdot 4 = 2$

SISAb-g5/2-5

b) CTFT

$F(j\omega) = \int_{-\infty}^{\infty} f(t)\, e^{-j\omega t} dt$

$= \int_{-2}^{2} (-\cos\frac{\pi}{2}t)\, e^{-j\omega t} dt = -\int_{-2}^{2} \frac{1}{2}(e^{j\frac{\pi}{2}t} + e^{-j\frac{\pi}{2}t})\, e^{-j\omega t} dt$

$= -\frac{1}{2}\int_{-2}^{2} \left[e^{j(\frac{\pi}{2}-\omega)t} + e^{-j(\frac{\pi}{2}+\omega)t}\right] dt$

$= -\frac{1}{2}\frac{e^{j(\frac{\pi}{2}-\omega)t}}{j(\frac{\pi}{2}-\omega)}\Big|_{-2}^{2} - \frac{1}{2}\frac{e^{-j(\frac{\pi}{2}+\omega)t}}{-j(\frac{\pi}{2}+\omega)}\Big|_{-2}^{2}$

$= -\frac{1}{2j(\frac{\pi}{2}-\omega)}\cdot\left(e^{j(\frac{\pi}{2}-\omega)\cdot 2} - e^{-j(\frac{\pi}{2}-\omega)\cdot 2}\right) + \frac{1}{2j(\frac{\pi}{2}+\omega)}$
$\left(e^{-j(\frac{\pi}{2}+\omega)\cdot 2} - e^{j(\frac{\pi}{2}+\omega)2}\right)$

$= \frac{-1}{\cancel{2j}(\frac{\pi}{2}-\omega)}\cdot\cancel{2j}\sin(\frac{\pi}{2}-\omega)\cdot 2 + \frac{1}{\cancel{2j}(\frac{\pi}{2}+\omega)}(-\cancel{2j})\sin(\frac{\pi}{2}+\omega)\cdot 2$

$= \frac{\sin 2(\frac{\pi}{2}-\omega)}{\omega - \frac{\pi}{2}} - \frac{\sin(\frac{\pi}{2}+\omega)\cdot 2}{\frac{\pi}{2}+\omega}$

SIS16 - g6/15

1. c) LINEARNOST za CTFT

$$a f(t) + b g(t) \xleftrightarrow{CTFT} a F(j\omega) + b G(j\omega)$$

$$CTFT \{ a f(t) + b g(t) \} = \int_{-\infty}^{\infty} (a f(t) + b g(t)) e^{-j\omega t} dt$$

$$= a \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt + b \int_{-\infty}^{\infty} g(t) e^{-j\omega t} dt$$

$$= a F(j\omega) + b G(j\omega)$$

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2011-35 / 3

2. $f(n) = \cos\frac{\pi}{4}n - \sin\frac{\pi}{4}n$

$\hookrightarrow$ PERIOD $\frac{\pi}{4}N = 2k\pi \longrightarrow N = 8$

a) DTFS

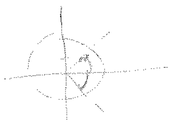
$$f(n) = \frac{1}{2}\left(e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}\right) - \frac{1}{2j}\left(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right)$$

$$= \left(\frac{1}{2} - \frac{1}{2j}\right)e^{j\frac{\pi}{4}n} + \left(\frac{1}{2} + \frac{1}{2j}\right)e^{-j\frac{\pi}{4}n}$$

$$= \frac{1+j}{2}e^{j\frac{\pi}{4}\cdot n} + \frac{1-j}{2}e^{-j\frac{\pi}{4}n}$$

$$= \frac{1+j}{2}e^{j\frac{2\pi}{8}\cdot n\cdot 1} + \frac{1-j}{2}e^{j\frac{2\pi}{8}\cdot n\cdot 7}$$

$\boxed{F_1 = \frac{1+j}{2}}$ $\boxed{F_7 = \frac{1-j}{2}}$

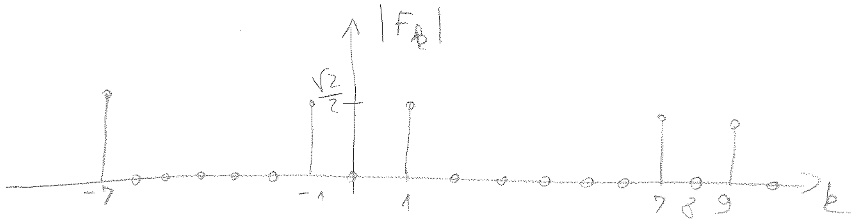


$|F_1| = \sqrt{\frac{1}{4}+\frac{1}{4}} = \frac{\sqrt{2}}{2}$ $|F_7| = \sqrt{\frac{1}{4}+\frac{1}{4}} = \frac{\sqrt{2}}{2}$

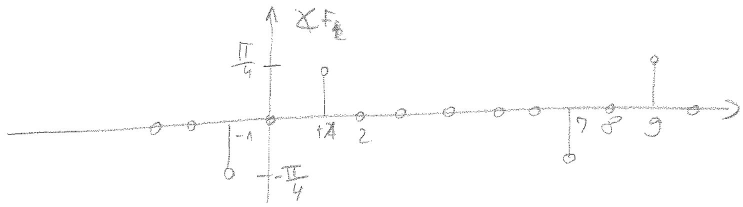
$\measuredangle F_1 = \operatorname{arctg}\frac{\frac{1}{2}}{\frac{1}{2}} = \frac{\pi}{4}$ $\measuredangle F_7 = -\frac{\pi}{4}$

$\boxed{F_1 = \frac{\sqrt{2}}{2}e^{j\frac{\pi}{4}}}$ $\boxed{F_7 = \frac{\sqrt{2}}{2}e^{-j\frac{\pi}{4}}}$

b)



PERIODIČKI SE PONAVLJA



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ŠALABAHTER

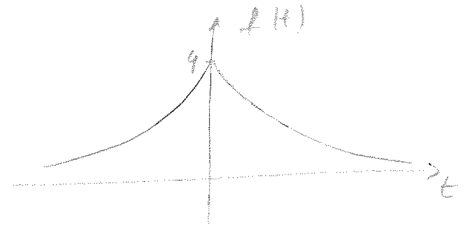
c) DTFT – iz tablica

$$\cos\frac{\pi}{4}n \xrightarrow{DTFT} \sum_{n=-\infty}^{\infty}\pi\left[\delta\left(\Omega+\frac{\pi}{4}+2\pi n\right) + \delta\left(\Omega-\frac{\pi}{4}+2\pi n\right)\right]$$

$$\sin\frac{\pi}{4}n \xleftrightarrow{DTFT} \sum_{n=-\infty}^{\infty}j\pi\left[\delta\left(\Omega+\frac{\pi}{4}+2\pi n\right) - \delta\left(\Omega-\frac{\pi}{4}+2\pi n\right)\right]$$

$$F(e^{j\Omega}) = \sum_{n=-\infty}^{\infty}(\pi - j\pi)\delta\left(\Omega+\frac{\pi}{4}+2\pi n\right) + (\pi+j\pi)\delta\left(\Omega-\frac{\pi}{4}+2\pi n\right)$$

3. $f(t) = 4e^{-2|t|}$



a) CTFT

max. 2,4 / 2,6

$$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{0} 4e^{2t} e^{-j\omega t} dt + \int_{0}^{+\infty} 4e^{-2t} e^{-j\omega t} dt$$

$$= 4 \left. \frac{e^{(2-j\omega)t}}{2-j\omega} \right|_{-\infty}^{0} + \left. \frac{4e^{(-2-j\omega)t}}{-2-j\omega} \right|_{0}^{\infty}$$

$$= \frac{4}{2-j\omega} - \frac{4}{-2-j\omega} = \frac{4}{2-j\omega} + \frac{4}{2+j\omega}$$

$$= \frac{8+4j\omega+8-4j\omega}{4+\omega^2} = \frac{16}{4+\omega^2}$$

b) DTFT

$T = \frac{1}{2}$

$$f(n) = 4e^{-2|nT|} = 4e^{-2\left|\frac{n}{2}\right|} = 4e^{-|n|} = \begin{cases} 4e^{n}, & n<0 \\ 4e^{-n}, & n\geq 0 \end{cases}$$

max. 4,5 / 4

$$F(e^{j\Omega}) = \sum_{n=-\infty}^{-1} 4e^{n} e^{-j\Omega n} + \sum_{n=0}^{\infty} 4e^{-n} e^{-j\Omega n}$$

$$= \sum_{n=1}^{\infty} 4e^{-(1-j\Omega)n} + \sum_{n=0}^{\infty} 4e^{-(1+j\Omega)n}$$

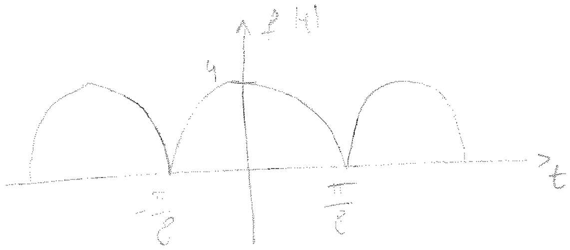
$$= \sum_{n=0}^{\infty} 4e^{-(1-j\Omega)n} - 4 + \sum_{n=0}^{\infty} 4e^{-(1+j\Omega)n}$$

$$= \frac{4}{1-e^{-(1-j\Omega)}} - 4 + \frac{4}{1-e^{-(1+j\Omega)}}$$

c) Da.

Spektar signala $f(t) = 4e^{-2|t|}$ je $\frac{16}{4+\omega^2}$, a on postoji za svaki ω i različit je od 0. Dakle, ne postoji ω_{max}. Da ne bi došlo do aliasinga frekvencija otipkavanja mora biti $2 \cdot \omega_{max}$, a to nije moguće. Zadana frekvencija je manja od toga.

4. $f(t) = 4\,|\cos 4t|$



CLS 96–99 / 28–29
44–46

a) CTFS

$\cos 4t$

$\hookrightarrow \quad 4T = 2k\pi$

$T = \frac{\pi}{2}$

ali: $|\cos 4t| \rightarrow T = \frac{\pi}{4} \rightarrow$ took ie $f(t) = 4\cos 4t$

$$F_k = \frac{1}{T}\int_T f(t)\, e^{-j\frac{2\pi}{T}kt}\,dt$$

$$= \frac{1}{\pi/4}\int_{-\pi/8}^{\pi/8} 4\cos 4t\; e^{-j\frac{2\pi}{\pi/4}kt}\,dt$$

$$= \frac{16}{\pi}\int_{-\pi/8}^{\pi/8} \frac{1}{2}\left(e^{j4t} + e^{-j4t}\right) e^{-j8kt}\,dt$$

$$= \frac{8}{\pi}\int_{-\pi/8}^{\pi/8}\left[e^{j(4-8k)t} + e^{-j(4+8k)t}\right]dt$$

$$= \frac{8}{\pi}\left[\frac{e^{j(4-8k)t}}{j(4-8k)} + \frac{e^{-j(4+8k)t}}{-j(4+8k)}\right]_{-\pi/8}^{\pi/8}$$

$$= \frac{8}{\pi}\left[\frac{e^{j(4-8k)\frac{\pi}{8}} - e^{-j(4-8k)\frac{\pi}{8}}}{j(4-8k)} + \frac{e^{-j(4+8k)\frac{\pi}{8}} - e^{j(4+8k)\frac{\pi}{8}}}{-j(4+8k)}\right]$$

$$= \frac{8}{\pi}\left[\frac{2\cancel{j}\sin(4-8k)\frac{\pi}{8}}{\cancel{j}(4-8k)} + \frac{-2j\sin(4+8k)\frac{\pi}{8}}{-\cancel{j}(4+8k)}\right]$$

$$= \frac{16}{\pi}\left[\frac{\sin(4-8k)\frac{\pi}{8}}{4-8k} + \frac{\sin(4+8k)\frac{\pi}{8}}{4+8k}\right]$$

$$= \frac{16}{\pi}\;\frac{4\left[\sin(4+8k)\frac{\pi}{8} + \sin(4-8k)\frac{\pi}{8}\right] + 8k\left[\sin(4-8k)\frac{\pi}{8} - \sin(4+8k)\frac{\pi}{8}\right]}{16 - 64k^2}$$

$$= \frac{16}{\pi}\;\frac{4\cdot 2\sin\frac{8\pi}{8\cdot 2}\cos\frac{16k\pi}{8\cdot 2} + 8k\cdot 2\cos\frac{8\pi}{8\cdot 2}\sin\frac{-16k\pi}{8\cdot 2}}{16-64k^2}$$

$$= \frac{\cancel{16}}{\pi}\;\frac{2\cos k\pi}{\cancel{16}(1-4k^2)} = \frac{8}{\pi}\;\frac{\cos k\pi}{1-4k^2}$$

4b)

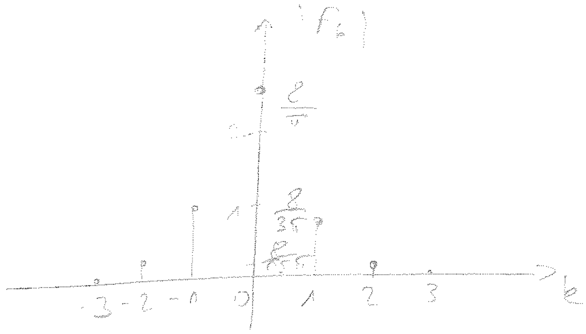
$$|F_k| = \left| \frac{8}{\pi} \frac{\cos k\pi}{1-4k^2} \right|$$

$$|F_0| = \frac{8}{\pi}$$

$$|F_1| = \left| \frac{8}{\pi} \frac{\cos \pi}{1-4} \right| = \left| \frac{-8}{-3\pi} \right| = \frac{8}{3\pi}$$

$$|F_2| = \left| \frac{8 \cos 2\pi}{\pi (1-16)} \right| = \frac{8}{15\pi}$$

$$|F_3| = \left| \frac{8 \cos 3\pi}{\pi (1-4 \cdot 9)} \right| = \frac{8}{35\pi}$$



c) $$P = \frac{1}{T_0} \int_{T_0} |f(t)|^2 dt = \sum_{k=-\infty}^{\infty} |F_k|^2$$ SIS/6-94/26-27

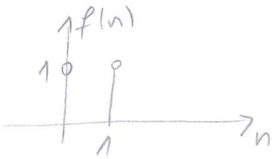
$$P = \frac{1}{T_0} \int_{T_0} f(t) \cdot f^*(t)\, dt = \frac{1}{T_0} \int_{T_0} f(t) \left(\sum_{k=-\infty}^{\infty} F_k^* e^{-ik\omega_0 t} \right) dt$$

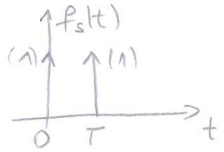
$$= \sum_{k=-\infty}^{\infty} F_k^* \underbrace{\frac{1}{T_0} \int_{T_0} f(t) e^{-ik\omega_0 t} dt}_{F_k} = \sum_{k=-\infty}^{\infty} |F_k|^2$$

5. $f(n) = \{1, 1\}$

$\boxed{\text{S1S16\_97/16-22}}$

a) $T = \frac{1}{2}$





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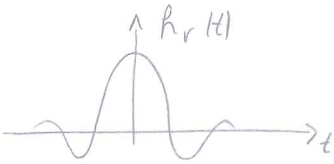

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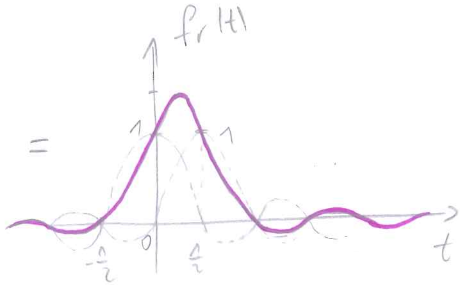


$$\left(\delta(t) + \delta(t-T)\right) * \left(\frac{\sin\frac{\pi t}{T}}{\frac{\pi t}{T}}\right)$$

$$= \frac{\sin\frac{\pi t}{T}}{\frac{\pi t}{T}} + \frac{\sin\frac{\pi(t-T)}{T}}{\frac{\pi(t-T)}{T}}$$

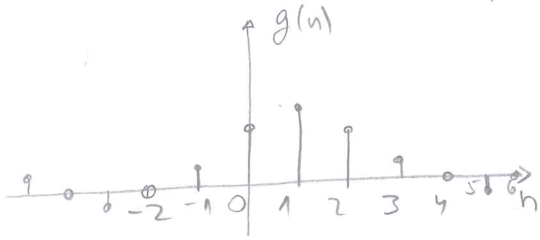
$$= \frac{\sin\frac{\pi t}{T}}{\pi t / T} + \frac{\sin\left(\frac{\pi t}{T} - \pi\right)}{\frac{\pi t}{T} - \pi} = \frac{\sin\frac{\pi t}{T}}{\pi t / T} + \frac{-\sin\frac{\pi t}{T}}{\frac{\pi t}{T} - \pi}$$

$$= \sin\frac{\pi t}{T}\left(\frac{T}{\pi t} - \frac{T}{\pi(t-T)}\right) = T \sin\frac{\pi t}{T}\left(\frac{\cancel{\pi t} - \pi T - \cancel{\pi t}}{\pi t\,(\pi t - \pi T)}\right)$$

$$= \frac{-\cancel{\pi} T^2 \sin\frac{\pi t}{T}}{\cancel{\pi} t\,(\pi t - \pi T)} = -\frac{T^2 \sin\frac{\pi t}{T}}{t\ \pi(t-T)}$$

$$f_r(t) = -\frac{1}{4}\ \frac{\sin 2\pi t}{t\pi\left(t - \frac{1}{2}\right)} = \frac{\sin 2\pi t}{2\pi t} + \frac{\sin 2\pi\left(t - \frac{1}{2}\right)}{2\pi\left(t - \frac{1}{2}\right)}$$

b) $$g(nT) = -\frac{1}{\cancel{4}}\ \frac{\sin 2\pi\cdot\frac{1}{4}n}{\cancel{\frac{1}{4}}n\,\pi\left(\frac{1}{4}n - \frac{1}{2}\right)} = -\frac{\sin\frac{\pi n}{2}}{\frac{1}{4}n\pi(n-2)} = \frac{\sin\frac{\pi}{2}n}{\frac{\pi}{2}n} + \frac{\sin\frac{\pi}{2}(n-2)}{\frac{\pi}{2}(n-2)}$$



n	$g(n)$
-2	0
-1	$\frac{1}{-\frac{\pi}{4}\cdot(-3)} = \frac{4}{3\pi}$
0	$\frac{-\pi/2}{-\pi/2} = 1$
1	$\frac{-1}{\frac{\pi}{4}\cdot(-1)} = \frac{4}{\pi}$
2	$\frac{\pi/2}{\pi/2} = 1$
3	$\frac{1}{\frac{3\pi}{4}} = \frac{4}{3\pi}$

5. a) $f(t)$

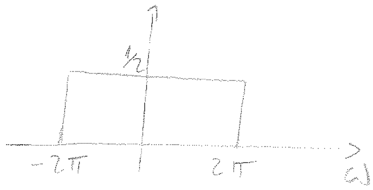
$T = 1$

(SISIG-97/9-3)

$$f(t) = \frac{\sin 2\pi t}{2\pi t} + \frac{\sin 2\pi\left(t - \frac{1}{2}\right)}{2\pi\left(t - \frac{1}{2}\right)}$$

$$\frac{\sin 2\pi t}{2\pi t} = \operatorname{sinc} 2t \xleftrightarrow{CTFT} \frac{1}{2}\operatorname{rect}\left(\frac{\omega}{4\pi}\right)$$ (ŠALABAHTER)

$$\operatorname{rect}\frac{\omega}{4\pi} = \begin{cases} 1, & -\frac{1}{2} < \frac{\omega}{4\pi} < \frac{1}{2} \\ 0, & \text{inače} \end{cases} = \begin{cases} 1, & -2\pi < \omega < 2\pi \\ 0, & \text{inače} \end{cases}$$



$\omega_g = 2\pi$

$\Rightarrow$ kako ne bi bilo ALIASINGA frekvencija očitavanja mora biti bar $2 \cdot 2\pi = 4\pi$

$T = 1 \rightarrow f = 1\,Hz \rightarrow \omega = 2\pi f = 2\pi$

zadana frekvencija očitavanja je $2\pi < 4\pi$,

uvjeti Shannonovog teorema nisu zadovoljeni.