Signali i sustavi

Ponovljeni završni ispit – 3. srpnja 2008.

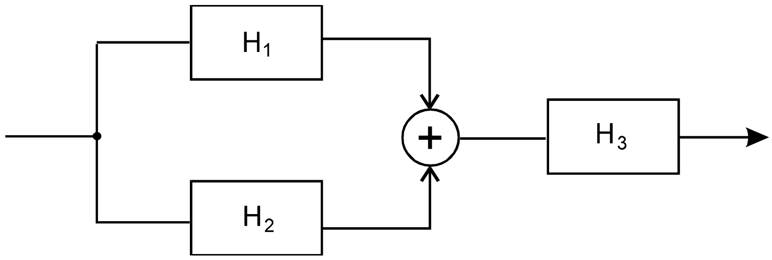
1. Zadan je impulsni odziv kontinuiranog LTI sustava $h(t) = 2te^{-t}\,\mu(t)$. Pronađite:

a) prijenosnu funkciju sustava,

b) amplitudnu i faznu karakteristiku sustava (ne treba crtati),

c) odziv sustava, ako je sustav pobuđen signalom $u(t) = 2\,\mu(t)$ te ako su početni uvjeti $y(0^-) = 2, y'(0^-) = 0$.

2. Zadan je složeni diskretni sustav prema slici. Nađite odziv na jediničnu stepenicu $\mu(n)$, trećeg podsustava ako je poznat impulsni odziv prvog podsustava $h_1(n) = \{\underline{1}, 0, 1, 0, 1, 0, \ldots\}$, odziv na jediničnu stepenicu drugog podsustava $y_2(n) = \frac{1}{4} + \frac{1}{2}n - \frac{1}{4}(-1)^n$, $n \geq 0$, te impulsni odziv cijelog sustava $h(n) = \{\underline{1}, 1\}$



3. Diskretan kauzalan LTI sustav zadan je jednadžbom diferencija:

$$y(n) - \frac{1}{9}y(n-2) = u(n).$$

Odredite:

a) odziv sustava, ako je sustav pobuđen signalom $u(n) = 80 \cdot 3^n\,\mu(n)$ te ako su početni uvjeti $y(-1) = 18, y(-2) = 0$,

b) je li sustav stabilan. Objasnite.

4. Vremenski kontinuiran sustav zadan je matricama **A, B, C, D**:

$$\mathbf{A} = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -3 & -2 \\ 1 & 2 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 4 \end{bmatrix}, \quad \text{i} \quad \mathbf{D} = \begin{bmatrix} 0 & 0 \end{bmatrix}.$$

a) Koliko ovaj sustav ima ulaza, a koliko izlaza,

b) Pronađite prijenosnu matricu sustava,

c) Odredite impulsni odziv.

5. Zadan je signal $x(t) = e^{2t}\,\mu(-t) + e^{-2t}\,\mu(t)$. Odredite:

a) vremenski kontinuiranu Fourierovu transformaciju (CTFT) ovog signala te skicirajte amplitudni spektar,

b) energiju signala.

1. $h(t) = 2te^{-t}\mu(t)$

a) $\boxed{H(s) = \frac{2}{(s+1)^2}}$

b) $H(j\omega) = \frac{2}{(j\omega+1)^2} = \frac{2}{1-\omega^2+2j\omega}$

$|H(j\omega)| = \frac{2}{\sqrt{(1-\omega^2)^2+4\omega^2}} = \frac{2}{\sqrt{1+2\omega^2+\omega^4}} = \frac{2}{\sqrt{(1+\omega^2)^2}}$

$\boxed{|H(j\omega)| = \frac{2}{(1+\omega^2)}}$

$\boxed{\sphericalangle H(j\omega) = -\operatorname{arctg}\frac{2\omega}{1-\omega^2}}$

c) diferencijalna jednadžba:

$\frac{Y(s)}{U(s)} = \frac{2}{(s+1)^2}$

$(s+1)^2 Y(s) = 2U(s)$

$y''(t) + 2y'(t) + y(t) = 2u(t)$

homogeno rješenje

$s^2+2s+1=0$

$s_{1,2}=-1$

$\boxed{y_h(t) = (C_1 + C_2 t)e^{-t}}$

partikularno rješenje

$y''(t)+2y'(t)+y(t) = 4\mu(t)$

$y_p(t) = K$

$K=4$

$\boxed{y_p(t) = 4}$

početni uvjeti

$\boxed{\begin{array}{l} y(0^-) = y(0^+) = 2 \\ y'(0^-) = y'(0^+) = 0 \end{array}}$

totalno rješenje

$y_{tot}(t) = (C_1 + C_2 t)e^{-t} + 4$

$y'_{tot}(t) = -C_1 e^{-t} - C_2 te^{-t} + C_2 e^{-t}$

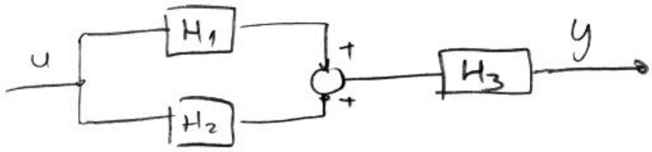
$y_{tot}(0^+) = C_1 + 4 = 2$ $\qquad C_1 = 2-4 = -2$

$y'_{tot}(0^+) = -C_1 + C_2 = 0$ $\qquad C_2 = C_1 = -2$

$\boxed{y_{tot}(t) = [(-2-2t)e^{-t} + 4]\mu(t)}$

A

2.



$$H(z) = (H_1 + H_2)\, H_3$$

$$\boxed{H_3(z) = \frac{H(z)}{H_1(z) + H_2(z)}}$$

$h_1(n) = \{\underline{1}, 0, 1, 0, \dots\}$

$$H_1(z) = z^0 + z^{-2} + z^{-4} + \dots =$$

$$= \sum_{n=0}^{\infty} (z^{-2})^n = \frac{1}{1 - z^{-2}} = \frac{z^2}{z^2 - 1}$$

$y_2(n) = \frac{1}{4} + \frac{1}{2} n - \frac{1}{4}(-1)^n \quad , n \geq 0$

$$Y_2(z) = \frac{1}{4}\frac{z}{z-1} + \frac{1}{2}\frac{z}{(z-1)^2} - \frac{1}{4}\frac{z}{z+1}$$

$$= \frac{z(z^2-1) + 2z(z+1) - z(z-1)^2}{4(z-1)^2(z+1)} = \frac{\cancel{z^3} \cancel{-z} + 2z^2 + \cancel{2z} \cancel{-z^3} + 2z^2 \cancel{-z}}{4(z-1)^2(z+1)}$$

$$= \frac{z^2}{(z-1)^2(z+1)}$$

$u_2(n) = \mu(n)$

$$U_2(z) = \frac{z}{z-1}$$

$$H_2(z) = \frac{Y_2(z)}{U_2(z)} = \frac{z^{\cancel{2}}}{(z-1)^{\cancel{2}}(z+1)} \cdot \frac{\cancel{z-1}}{\cancel{z}} = \frac{z}{(z-1)(z+1)}$$

$h(n) = \{\underline{1}, 1\}$

$$H(z) = z^0 + z^{-1} = 1 + z^{-1} = \frac{1+z}{z}$$

$$H_3(z) = \frac{1+z}{z} \cdot \frac{1}{\frac{z^2}{z^2-1} + \frac{z}{z^2-1}} = \frac{1+z}{z} \cdot \frac{1}{\frac{z(z+1)}{z^2-1}} = \frac{1+z}{z} \cdot \frac{(z-1)\cancel{(z+1)}}{z\cancel{(z+1)}} = \frac{z^2-1}{z^2}$$

$u_3(n) = \mu(n)$

$$U_3(z) = \frac{z}{z-1}$$

$$Y_3(z) = H_3(z) \cdot U_3(z) = \frac{\cancel{(z-1)}(z+1)}{z^{\cancel{2}}} \cdot \frac{\cancel{z}}{\cancel{z-1}} = \frac{z+1}{z} = 1 + z^{-1}$$

$$y_3(n) = \delta(n) + \delta(n-1)$$

$$= \{\underline{1}, 1\}$$

3. A

$$y(n) - \frac{1}{9} y(n-2) = u(n)$$

a) homogeno:

$$q^2 - \frac{1}{9} = 0$$

$$q^2 = \frac{1}{9}$$

$$q_1 = \frac{1}{3} \qquad q_2 = -\frac{1}{3}$$

$$y_h = C_1 \left(\frac{1}{3}\right)^n + C_2 \left(-\frac{1}{3}\right)^n$$

partikularno

$$y(n) - \frac{1}{9} y(n-2) = 80 \cdot 3^n \mu(n)$$

$$y_p(n) = K \cdot 3^n$$

$$K \cdot \cancel{3^n} - \frac{1}{9} \cdot K \cdot \cancel{3^n} \cdot 3^{-2} = 80 \cdot \cancel{3^n}$$

$$K - \frac{1}{81} K = 80$$

$$\frac{80}{81} K = 80$$

$$K = 81$$

$$y_p = 81 \cdot 3^n$$

totalno:

$$y_{tot}(n) = C_1 \left(\frac{1}{3}\right)^n + C_2\left(-\frac{1}{3}\right)^n + 81 \cdot 3^n$$

$$y_{tot}(-1) = 3 C_1 - 3 C_2 + 81 \frac{1}{3} = 3C_1 - 3C_2 + 27 = 18$$

$$y_{tot}(-2) = 9 C_1 + 9 C_2 + 81 \cdot \frac{1}{9} = 9 C_1 + 9 C_2 + 9 = 0$$

$$3C_1 - 3C_2 = -9$$
$$9C_1 + 9C_2 = -9$$

$$C_1 - C_2 = -3$$
$$C_1 + C_2 = -1$$
$$\overline{}$$
$$2C_1 = -4$$
$$C_1 = -2$$

$$C_2 = -1 - C_1 = -1 + 2 = 1$$

$$y_{tot}(n) = \left[-2 \cdot \left(\frac{1}{3}\right)^n + \left(-\frac{1}{3}\right)^n + 81 \cdot 3^n\right] \mu(n)$$

b) $\left.\begin{array}{l} |q_1| = \frac{1}{3} < 1 \\ |q_2| = \frac{1}{3} < 1 \end{array}\right\}$ stabilan sustav

A

4.

$$A = \begin{bmatrix} -4 & 0 \\ 0 & -5 \end{bmatrix} \qquad B = \begin{bmatrix} -3 & -2 \\ 1 & 2 \end{bmatrix} \qquad C = [1 \quad 4] \qquad D = [0 \quad 0]$$

a) $D = [0 \quad 0]$

1 redak → 1 izlaz

2 stupac → 2 ulaza

b)

$$H(s) = C(sI - A)^{-1} B + D$$

$$(sI - A)^{-1} = \begin{bmatrix} s+4 & 0 \\ 0 & s+5 \end{bmatrix}^{-1} = \frac{1}{(s+4)(s+5)} \begin{bmatrix} s+5 & 0 \\ 0 & s+4 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+4} & 0 \\ 0 & \frac{1}{s+5} \end{bmatrix}$$

$$H(s) = [1 \quad 4] \begin{bmatrix} \frac{1}{s+4} & 0 \\ 0 & \frac{1}{s+5} \end{bmatrix} \begin{bmatrix} -3 & -2 \\ 1 & 2 \end{bmatrix} + [0 \quad 0]$$

$$= \begin{bmatrix} \frac{1}{s+4} & \frac{4}{s+5} \end{bmatrix} \begin{bmatrix} -3 & -2 \\ 1 & 2 \end{bmatrix} + [0 \quad 0]$$

$$= \begin{bmatrix} \frac{-3}{s+4} + \frac{4}{s+5} & \frac{-2}{s+4} + \frac{8}{s+5} \end{bmatrix}$$

c)

$$h(t) = \left[(-3e^{-4t} + 4e^{-5t})\mu(t) \quad (-2e^{-4t} + 8e^{-5t})\mu(t) \right]$$

A

5. $x(t) = e^{2t}\mu(-t) + e^{-2t}\mu(t)$

a) CTFT

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \left(e^{2t}\mu(-t) + e^{-2t}\mu(t)\right) e^{-j\Omega t}\, dt$$

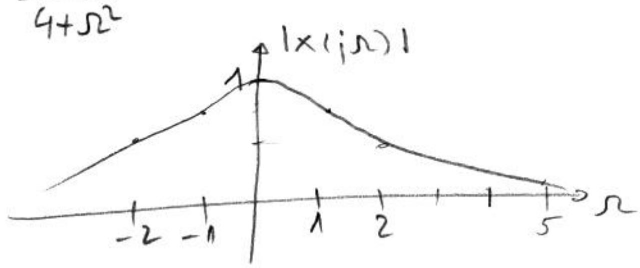
$$= \int_{-\infty}^{\infty} e^{(2-j\Omega)t}\mu(-t)\, dt + \int_{-\infty}^{\infty} e^{-(2+j\Omega)t}\mu(t)\, dt$$

$$= \int_{-\infty}^{0} e^{(2-j\Omega)t}\, dt + \int_{0}^{\infty} e^{-(2+j\Omega)t}\, dt$$

$$= \frac{e^{(2-j\Omega)t}}{2-j\Omega}\Big|_{-\infty}^{0} + \frac{e^{-(2+j\Omega)t}}{-(2+j\Omega)}\Big|_{0}^{\infty} =$$

$$= \frac{1}{2-j\Omega} - \frac{1}{-(2+j\Omega)} = \frac{4}{4+\Omega^2}$$

$$|X(j\Omega)| = \frac{4}{4+\Omega^2}$$



b) $E_x = ?$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} \left(e^{2t}\mu(-t) + e^{-2t}\mu(t)\right)^2 dt$$

$$= \int_{-\infty}^{\infty} e^{4t}\mu^2(-t)\, dt + \underbrace{\int_{-\infty}^{\infty} 2e^{2t}e^{-2t}\mu(-t)\mu(t)}_{\emptyset} + \int_{-\infty}^{\infty} e^{-4t}\mu^2(t)\, dt$$

$$= \int_{-\infty}^{0} e^{4t}\, dt + \int_{0}^{\infty} e^{-4t}\, dt$$

$$= \frac{e^{4t}}{4}\Big|_{-\infty}^{0} + \frac{e^{-4t}}{-4}\Big|_{0}^{\infty} =$$

$$= \frac{1}{4} - \frac{1}{-4} = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Signali i sustavi

Ponovljeni završni ispit – 3. srpnja 2008.

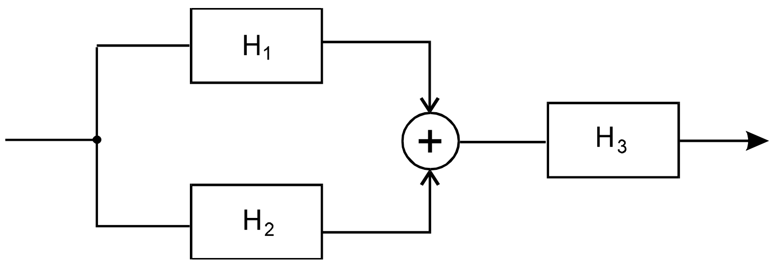
1. Zadan je impulsni odziv kontinuiranog LTI sustava $h(t) = 3te^{-t}\,\mu(t)$. Pronađite:

a) prijenosnu funkciju sustava,

b) amplitudnu i faznu karakteristiku sustava (ne treba crtati),

c) odziv sustava, ako je sustav pobuđen signalom $u(t) = 3\,\mu(t)$ te ako su početni uvjeti $y(0^-) = 3, y'(0^-) = 0$.

2. Zadan je složeni diskretni sustav prema slici. Nađite odziv na jediničnu stepenicu $\mu(n)$, trećeg podsustava ako je poznat impulsni odziv prvog podsustava $h_1(n) = \{\underline{1}, 0, 1, 0, 1, 0, \ldots\}$, odziv na jediničnu stepenicu drugog podsustava $y_2(n) = \frac{1}{4} + \frac{1}{2}n - \frac{1}{4}(-1)^n$, $n \geq 0$, te impulsni odziv cijelog sustava $h(n) = \{\underline{0}, 1\}$



3. Diskretan kauzalan LTI sustav zadan je jednadžbom diferencija:

$$y(n) - \frac{1}{4}y(n-2) = u(n).$$

Odredite:

a) odziv sustava, ako je sustav pobuđen signalom $u(n) = 15 \cdot 2^n\,\mu(n)$ te ako su početni uvjeti $y(-1) = 0, y(-2) = 12$,

b) je li sustav stabilan. Objasnite.

4. Vremenski kontinuiran sustav zadan je matricama **A, B, C, D**:

$$\mathbf{A} = \begin{bmatrix} -3 & 0 \\ 0 & -4 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 & 1 \\ -2 & -3 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 5 & 1 \end{bmatrix}, \quad \text{i} \quad \mathbf{D} = \begin{bmatrix} 0 & 0 \end{bmatrix}.$$

a) Koliko ovaj sustav ima ulaza, a koliko izlaza,

b) Pronađite prijenosnu matricu sustava,

c) Odredite impulsni odziv.

5. Zadan je signal $x(t) = e^{3t}\,\mu(-t) + e^{-3t}\,\mu(t)$. Odredite:

a) vremenski kontinuiranu Fourierovu transformaciju (CTFT) ovog signala te skicirajte amplitudni spektar,

b) energiju signala.

B

1. $h(t) = 3t e^{-t} \mu(t)$

a) $H(s) = \dfrac{3}{(s+1)^2}$

b) $H(j\omega) = \dfrac{3}{(j\omega+1)^2} = \dfrac{3}{1-\omega^2+2j\omega}$

$|H(j\omega)| = \dfrac{3}{\sqrt{(1-\omega^2)^2+4\omega^2}} = \dfrac{3}{\sqrt{1-2\omega^2+\omega^4+4\omega^2}} = \dfrac{3}{\sqrt{(1+\omega^2)^2}}$

$= \dfrac{3}{|1+\omega^2|}$

$\measuredangle H(j\omega) = -\operatorname{arctg} \dfrac{2\omega}{1-\omega^2}$

c) početni uvjeti:

$y(0^+) = y(0^-) = 3$

$y'(0^+) = y'(0^-) = 0$

diferencijalna jednadžba:

$\dfrac{Y(s)}{U(s)} = \dfrac{3}{(s+1)^2}$ $\qquad y''(t) + 2y'(t) + y(t) = 3u(t)$

pobuda: $u(t) = 3\mu(t)$

$y''(t) + 2y'(t) + y(t) = 9\mu(t)$

homogeno

$s^2+2s+1=0$

$s_{1,2} = -1$

$y_h(t) = (C_1 + C_2 t) e^{-t}$

partikularno

$y_p(t) = K$

$K = 9$

totalno

$y_{tot}(t) = (C_1 + C_2 t) e^{-t} + 9$

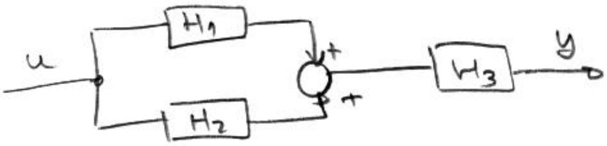
$y_{tot}'(t) = -C_1 e^{-t} - C_2 t e^{-t} + C_2 e^{-t}$

$y_{tot}(0^+) = C_1 + 9 = 3$ $\qquad C_1 = 3 - 9 = -6$

$y_{tot}'(0^+) = -C_1 + C_2 = 0$ $\qquad C_2 = C_1 = -6$

$$\boxed{y_{tot}(t) = \left[(-6-6t)e^{-t} + 9\right]\mu(t)}$$

2. B



$h_1(n) = \{\underline{1}, 0, 1, 0, 1, 0, \ldots\}$

$$H_1(z) = z^0 + z^{-2} + z^{-4} + \ldots$$
$$= \sum_{n=0}^{\infty} (z^{-2})^n = \frac{1}{1-z^{-2}} = \boxed{\frac{z^2}{z^2-1}}$$

$y_2(n) = \frac{1}{4} + \frac{1}{2}n - \frac{1}{4}(-1)^n \qquad n \geq 0$

$u_2(n) = \mu(n)$

$$Y_2(z) = \frac{1}{4} \cdot \frac{z}{z-1} + \frac{1}{2} \frac{z}{(z-1)^2} - \frac{1}{4} \frac{z}{z+1}$$
$$= \frac{z(z^2-1) + 2z(z+1) - z(z-1)^2}{4(z-1)^2(z+1)} = \frac{\cancel{z^3} - \cancel{z} + 2z^2 + \cancel{2z} - \cancel{z^3} + 2z^2 - \cancel{z}}{4(z-1)^2(z+1)}$$
$$= \frac{\cancel{4}z^2}{\cancel{4}(z-1)^2(z+1)} = \boxed{\frac{z^2}{(z-1)^2(z+1)}}$$

$$U_2(z) = \frac{z}{z-1}$$

$$H_2(z) = \frac{Y_2(z)}{U_2(z)} = \frac{z^{\cancel{2}}}{(z-1)^{\cancel{2}}(z+1)} \cdot \frac{\cancel{z-1}}{\cancel{z}} = \boxed{\frac{z}{(z-1)(z+1)}}$$

$$H(z) = [H_1(z) + H_2(z)] \cdot H_3(z) \quad \multimap \quad H_3(z) = \frac{H(z)}{H_1(z) + H_2(z)}$$

$h(n) = \{\underline{0}, 1\}$

$$H(z) = 0 \cdot z^0 + 1 \cdot z^{-1} = \boxed{\frac{1}{z}}$$

$$H_3(z) = \frac{1}{z} \cdot \frac{1}{\frac{z^2}{z^2-1} + \frac{z}{z^2-1}} = \frac{1}{z} \cdot \frac{1}{\frac{z(z+1)}{z^2-1}} = \frac{1}{z} \cdot \frac{(z-1)\cancel{(z+1)}}{z\cancel{(z+1)}} = \boxed{\frac{z-1}{z^2}}$$

$U_3(n) = \mu(n)$

$$U_3(z) = \frac{z}{z-1}$$

$$Y_3(z) = H_3(z) \cdot U_3(z)$$
$$= \frac{\cancel{(z-1)}}{z^{\cancel{2}}} \cdot \frac{\cancel{z}}{\cancel{z-1}} = \boxed{\frac{1}{z}}$$

$$\boxed{\begin{aligned} y_3(n) &= \delta(n-1) \\ &= \{\underline{0}, 1\} \end{aligned}}$$

B

3. $y(n) - \frac{1}{4} y(n-2) = u(n)$

a) $u(n) = 15 \cdot 2^n \mu(n)$

$y(-1) = 0$

$y(-2) = 12$

homogeno

$q^2 - \frac{1}{4} = 0$

$q^2 = \frac{1}{4}$

$q_1 = \frac{1}{2} \quad q_2 = -\frac{1}{2}$

$$\boxed{y_h = C_1 \cdot \left(\tfrac{1}{2}\right)^n + C_2 \left(-\tfrac{1}{2}\right)^n}$$

partikularno

$y_p = K \cdot 2^n$

$K \cdot 2^n - \frac{1}{4} \cdot K \cdot 2^{n-2} = 15 \cdot 2^n$

$K - \frac{1}{4} \cdot K \cdot \frac{1}{4} = 15$

$\frac{15}{16} K = 15$

$\boxed{K = 16} \rightarrow \boxed{y_p = 16 \cdot 2^n}$

totalno

$y_{tot}(n) = C_1 \cdot \left(\frac{1}{2}\right)^n + C_2 \left(-\frac{1}{2}\right)^n + 16 \cdot 2^n$

$y_{tot}(-1) = 2C_1 - 2C_2 + 8 = 0$

$y_{tot}(-2) = 4C_1 + 4C_2 + 4 = 12$

$2C_1 - 2C_2 = -8$

$2C_1 + 2C_2 = 4$

$4C_1 = -4$

$C_1 = -1$

$2C_2 = 4 - 2C_1 = 4 + 2 = 6$

$C_2 = 3$

$$\boxed{y_{tot}(n) = \left[-\left(\tfrac{1}{2}\right)^n + 3\left(-\tfrac{1}{2}\right)^n + 16 \cdot 2^n\right] \mu(n)}$$

b) $\left.\begin{array}{l} |q_1| = \frac{1}{2} < 1 \\ |q_2| = \frac{1}{2} < 1 \end{array}\right\}$ sustav je stabilan.

B

4. $A = \begin{bmatrix} -3 & 0 \\ 0 & -4 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 1 \\ -2 & -3 \end{bmatrix}$ $C = [5 \quad 1]$ $D = [0 \quad 0]$

a) $D = [0 \quad 0]$

1 redak = 1 izlaz

2 stupca = 2 ulaza

b) $H(s) = C \cdot (sI - A)^{-1} \cdot B + D$

$$= [5 \quad 1] \begin{bmatrix} s+3 & 0 \\ 0 & s+4 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 2 & 1 \\ -2 & -3 \end{bmatrix} + [0 \quad 0]$$

$$\begin{bmatrix} s+3 & 0 \\ 0 & s+4 \end{bmatrix}^{-1} = \frac{1}{(s+3)(s+4)} \begin{bmatrix} s+4 & 0 \\ 0 & s+3 \end{bmatrix} = \begin{bmatrix} \frac{1}{s+3} & 0 \\ 0 & \frac{1}{s+4} \end{bmatrix}$$

$$H(s) = [5 \quad 1] \begin{bmatrix} \frac{1}{s+3} & 0 \\ 0 & \frac{1}{s+4} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & -3 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{5}{s+3} & \frac{1}{s+4} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -2 & -3 \end{bmatrix} = \boxed{\begin{bmatrix} \frac{10}{s+3} - \frac{2}{s+4} & \frac{5}{s+3} - \frac{3}{s+4} \end{bmatrix}}$$

c) $h(t) = ?$

$$\boxed{h(t) = \left[(10 e^{-3t} - 2 e^{-4t}) \mu(t) \quad (5 e^{-3t} - 3 e^{-4t}) \mu(t) \right]}$$

5. $x(t) = e^{3t}\mu(-t) + e^{-3t}\mu(t)$

B

a) CTFT

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \left[e^{3t}\mu(-t) + e^{-3t}\mu(t)\right] e^{-j\Omega t}\, dt$$

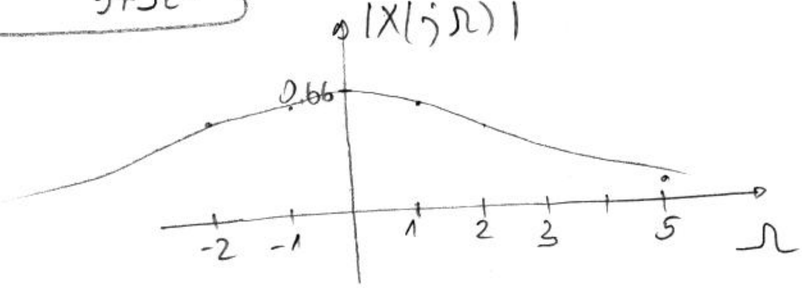
$$= \int_{-\infty}^{\infty} e^{(3-j\Omega)t}\mu(-t)\, dt + \int_{-\infty}^{\infty} e^{-(3+j\Omega)t}\mu(t)\, dt$$

$$= \int_{-\infty}^{0} e^{(3-j\Omega)t}\, dt + \int_{0}^{\infty} e^{-(3+j\Omega)t}\, dt$$

$$= \frac{e^{(3-j\Omega)t}}{3-j\Omega}\Bigg|_{-\infty}^{0} + \frac{e^{-(3+j\Omega)t}}{-(3+j\Omega)}\Bigg|_{0}^{\infty}$$

$$= \frac{1}{3-j\Omega} + \frac{1}{3+j\Omega} = \boxed{\frac{6}{9+\Omega^2}}$$

$$\boxed{|X(j\Omega)| = \frac{6}{9+\Omega^2}}$$



b) $E_x = ?$

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\, dt = \int_{-\infty}^{\infty} \left[e^{3t}\mu(-t) + e^{-3t}\mu(t)\right]^2 dt$$

$$= \int_{-\infty}^{\infty} \Big[e^{6t}\mu^2(-t) + \underbrace{2e^{0t}\mu(-t)\mu(t)}_{0} + e^{-6t}\mu^2(t)\Big]\, dt$$

$$= \int_{-\infty}^{\infty} e^{6t}\mu(-t)\, dt + \int_{-\infty}^{\infty} e^{-6t}\mu(t)\, dt$$

$$= \frac{e^{6t}}{6}\Bigg|_{-\infty}^{0} + \frac{e^{-6t}}{-6}\Bigg|_{0}^{\infty}$$

$$\boxed{E_x = \frac{1}{6} - \frac{1}{-6} = \frac{1}{3}}$$