(C)

1MI 2009

(2.) $x(n) = n e^{-j\pi n}(\mu(n) - \mu(n-3))$

$E = ?$

$$E = \sum_{-\infty}^{\infty} |x(n)|^2 = \sum_{0}^{2} n^2 = 1^2 + 2^2 = 5$$

(4.) a) $E = 3 \cdot \int_0^1 t^2 dt = 1$

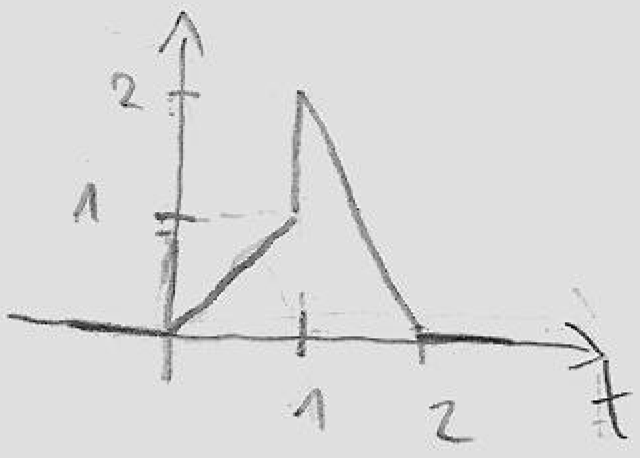
b) $E = \int_{-3}^{0} \frac{1}{9} t^2 dt = \frac{t^3}{27} \Big|_{-3}^{0} = 1$

c) $E = \int_2^3 dt = 1$

d) $E = \int_0^2 \frac{1}{4} t^2 dt + \int_0^1 t^2 dt = \frac{2}{3} + \frac{1}{3} = 1$

e) $E = \frac{1}{2} \int_0^3 dt = \frac{3}{2}$

(5.)



$x(t) = t(\mu(t) - \mu(t-1)) + (-2t+4)(\mu(t-1) - \mu(t-2))$

$x'(t) = \mu(t) - \mu(t-1) - \delta(t-1) - 2(\mu(t-1) - \mu(t-2))$
$+ 2\delta(t-1)$

$x'(t) = \mu(t) - 3\mu(t-1) + 2\mu(t-2) + \delta(t-1)$

6. $X_{-1} = 2j+4$

$X_1 = -2j+4$

$P = ?$

$$x(t) = (2j+4)e^{-j\omega t} + (-2j+4)e^{j\omega_0 t}$$

$$P = \frac{1}{T_p}\int_{-\infty}^{\infty} |x(t)|^2\,dt = \sum_{k=-1}^{1} |X(k)|^2 = 40$$

CTFS

↓

pogledat službeni šalabahter

7. $x(t) = 2\cos(4t) - 2\sin(6t)$

$\omega_0 = 2$

$\frac{2\pi}{T_0} = 2 \quad T_0 = \pi$

$= 2\cos(2\omega_0 t) - 2\sin(3\omega_0 t)$

$= \frac{2}{2}\left(e^{j2\omega_0 t} + e^{-j2\omega_0 t}\right) - \frac{2}{2j}\left(e^{j3\omega_0 t} - e^{-j3\omega_0 t}\right)$

$= \underset{\substack{\downarrow \\ X_1}}{e^{j2\omega_0 t}} + e^{-j2\omega_0 t} + \underset{\substack{\downarrow \\ X_3}}{j e^{j3\omega_0 t}} - j e^{-j3\omega_0 t}$

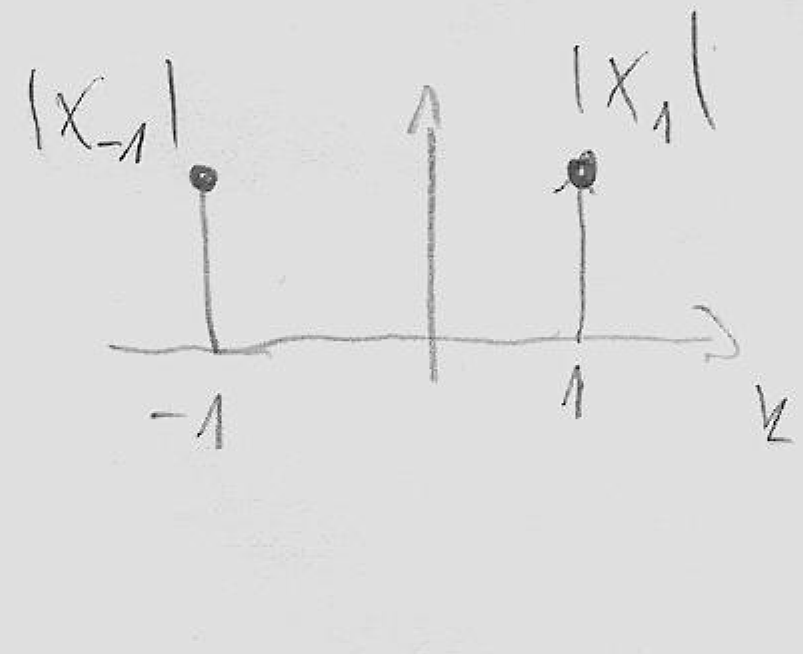
8. $x(t) = 2\cos(\omega_0 t + \frac{\pi}{4})$

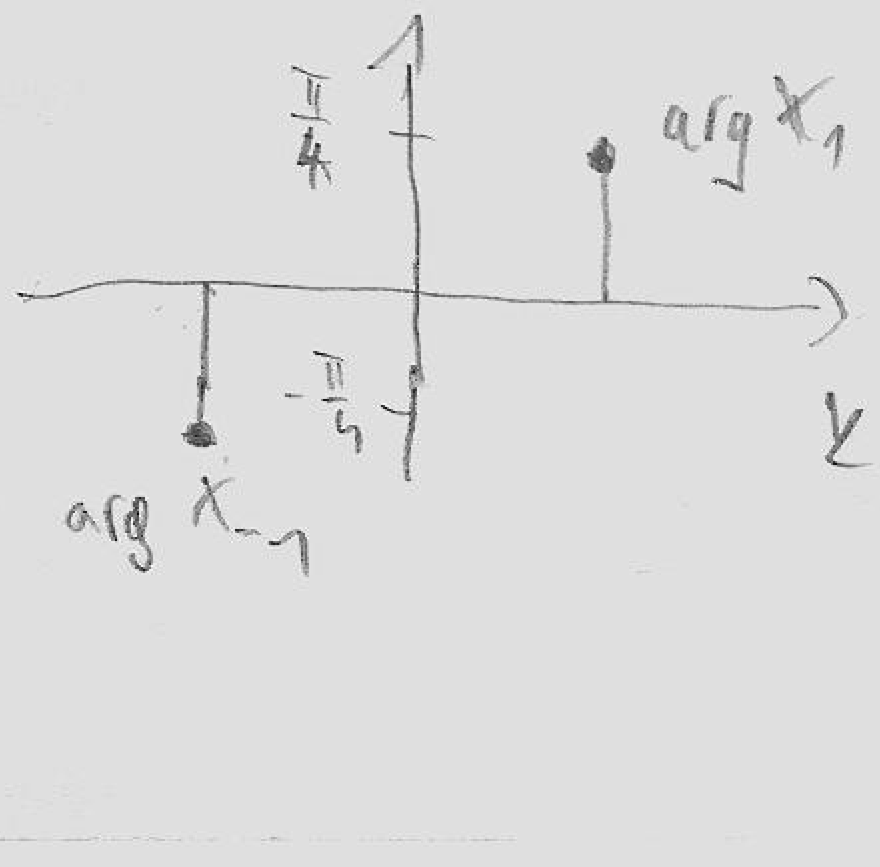
$= \frac{2}{2}\left(e^{j\omega_0 t}\cdot e^{j\frac{\pi}{4}} + e^{-j\omega_0 t}\cdot e^{-j\frac{\pi}{4}}\right)$

$= e^{j\frac{\pi}{4}}\cdot e^{j\omega_0 t} + e^{-j\pi/4}\cdot e^{-j\omega_0 t}$

$X_1 = e^{j\frac{\pi}{4}} = \cos\frac{\pi}{4} + j\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$

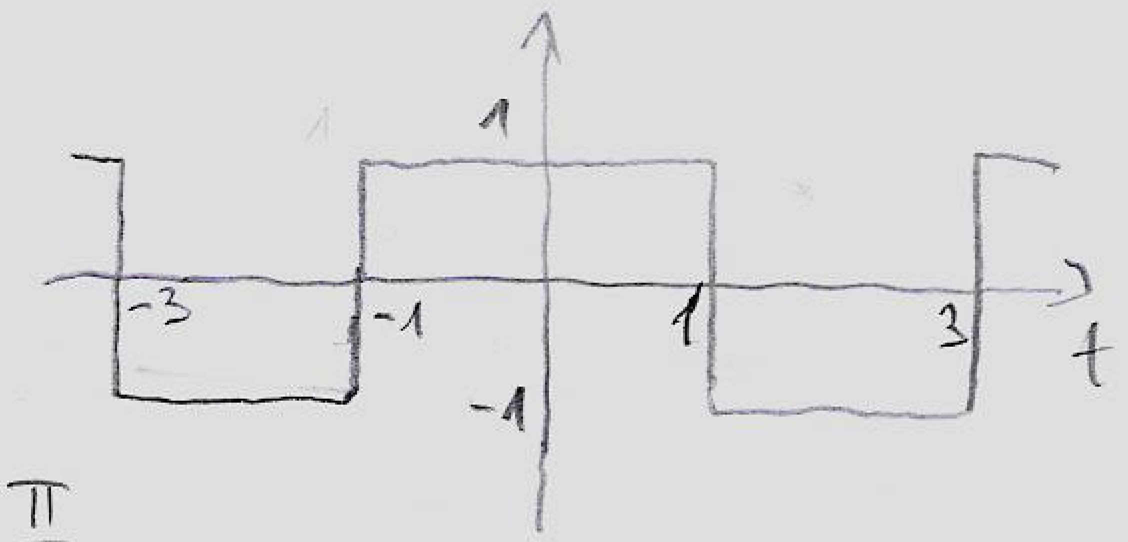
$X_{-1} = e^{-j\frac{\pi}{4}} = \cos\frac{\pi}{4} - j\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$





(9.) $T_p = 4$

$(X_0, X_1) = ?$



$\omega_p = \frac{2\pi}{T} = \frac{\pi}{2}$

$$X[k] = \frac{1}{T_p} \int_{T_p} x(t) e^{-j\omega_p k t} dt$$

$$X(0) = \frac{1}{4} \left[\int_{-1}^{1} dt - \int_{1}^{3} dt \right] = 0$$

$$X(1) = \frac{1}{4} \left[\int_{-1}^{1} e^{-j\frac{\pi}{2} t} dt - \int_{1}^{3} e^{-j\frac{\pi}{2} t} dt \right]$$

$$= \frac{1}{4} \left[-\frac{2}{j\pi} e^{-j\frac{\pi}{2} t} \Big|_{-1}^{1} + \frac{2}{j\pi} e^{-j\frac{\pi}{2} t} \Big|_{1}^{3} \right]$$

$$= \frac{1}{4} \left[\underbrace{-\frac{2}{j\pi} \left(e^{-j\frac{\pi}{2}} - e^{j\frac{\pi}{2}} \right)}_{\frac{2}{j\pi}\left(e^{j\frac{\pi}{2}} - e^{-j\frac{\pi}{2}} \right)} + \frac{2}{j\pi} \left(e^{-j\frac{3\pi}{2}} - e^{-j\frac{\pi}{2}} \right) \right]$$

$$= \frac{1}{4} \left[\frac{4}{\pi} \sin\frac{\pi}{2} + \frac{2}{\cancel{j}\pi} \left(\underbrace{\cos\frac{3\pi}{2}}_{0} - \cancel{j}\sin\frac{3\pi}{2} - \underbrace{\cos\frac{\pi}{2}}_{0} + \cancel{j}\sin\frac{\pi}{2} \right) \right]$$

$$= \frac{1}{4} \left[\frac{4}{\pi} + \frac{4}{\pi} \right] =$$

$$X(1) = \frac{2}{\pi}$$

(10.) $f(t)= \begin{cases} e^{-3t}, & t \geq 0 \\ 0, & \text{inače} \end{cases}$

$$F(\omega)=\int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$F(\omega)=\int_{0}^{\infty} e^{-t(3+j\omega)} dt = -\frac{1}{3+j\omega} e^{-t(3+j\omega)} \Big|_{0}^{\infty} = \frac{1}{3+j\omega}$$

(11.) $x(t)=3\mu(t)+1$

$$X(\omega)=\int_{-\infty}^{\infty} (3\mu(t)+1) e^{-j\omega t} dt = 3\int_{0}^{\infty} e^{-j\omega t} dt + 1\int_{-\infty}^{\infty} e^{-j\omega t} dt$$

$$= -\frac{3}{j\omega} e^{-j\omega t} \Big|_{0}^{\infty} - \frac{1}{j\omega} e^{-j\omega t} \Big|_{-\infty}^{\infty} = \frac{3}{j\omega}$$

(13.) $x(t)$ i $X(j\Omega)$ F. trans. par

$$x(t) \circ\!\!-\!\!\bullet X(j\Omega)$$

1.) komprimiramo za 2

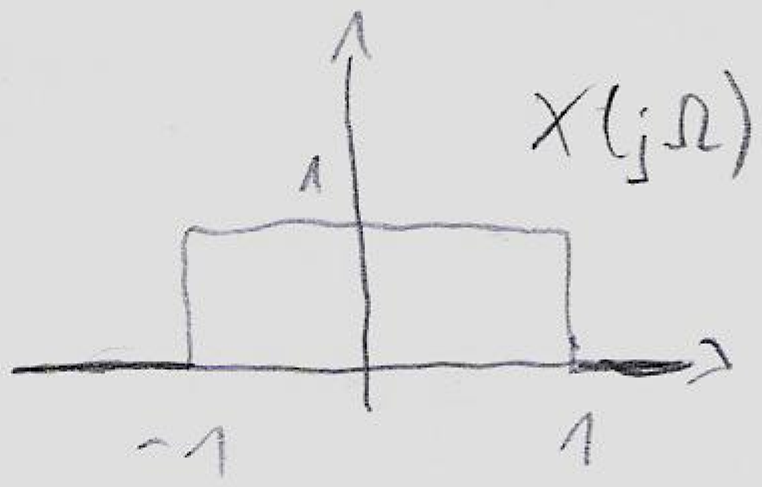
$$x(2t) \circ\!\!-\!\!\bullet \frac{1}{2} X\left(\frac{j\omega}{2}\right)$$

2.) pomak u desno za 10

$$x(t-10) \circ\!\!-\!\!\bullet X(j\omega) e^{-10j\omega}$$

$$x(2t-10) = \frac{1}{2} X\left(\frac{j\omega}{2}\right) \cdot e^{-10j\omega}$$

(14.) $x(t=\frac{\pi}{2})=?$



$$x(t)=\frac{1}{2\pi}\int_{-1}^{1}e^{jwt}dw=\frac{1}{t\pi 2j}\cdot e^{jwt}\Big|_{-1}^{1}=\frac{1}{jw2\pi}\left(e^{jt}-e^{-jt}\right)=\frac{1}{t\pi}\sin(t)$$

$$x\left(\frac{\pi}{2}\right)=\frac{2}{\pi^2}$$

(16.) DTFT

$$x(n)=\begin{cases}2009^n, & 0\leq n<2009\\ 0, & \text{inače}\end{cases}\qquad \text{za} \quad w=\pi$$

$X(w)=?$

$$X(w)=\sum_{n=-\infty}^{\infty}x(n)e^{-jwn}$$

$$X(\pi)=\sum_{n=0}^{2008}2009^n e^{-j\pi n}=\sum_{n=0}^{2008}2009^n\cdot(\cos\pi n-j\underbrace{\sin\pi n}_{0})$$

$$=\sum_{n=0}^{2008}2009^n\cdot\cos\pi n=\sum_{n=0}^{2008}2009^n\cdot(-1)^n=\sum_{n=0}^{2008}(-2009)^n$$

$$=\frac{-2009^{2009}-1}{-2010}=\frac{1+2009^{2009}}{2010}$$

(17.) $x(n) = \{3, 1, \underline{0}, 1, 3\}$ za $\omega = \frac{\pi}{2}$

$x_{-2}\ x_{-1}\ x_0\ x_1\ x_2$

$X(\omega) = ?$

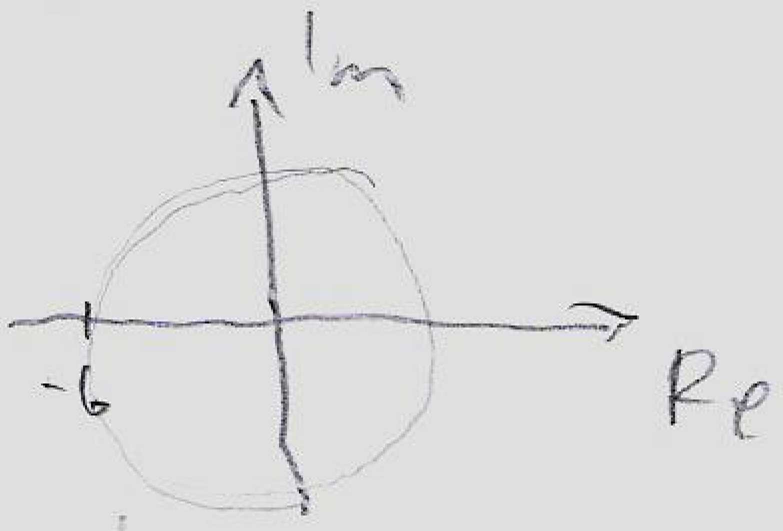
$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x(n) e^{-j\frac{\pi}{2} n}$$

$$= 3 \cdot e^{j\pi} + e^{j\frac{\pi}{2}} + e^{-j\frac{\pi}{2}} + 3e^{-j\pi}$$

$$= \underbrace{2\cos\frac{\pi}{2}}_{0} + 6\cos\pi = -6$$

$Re[X(\omega)] = -6$

$Im[X(\omega)] = 0$



$tg\varphi = \frac{0}{-6}$

$\varphi = arctg\, 0$

$\varphi = 0$ ili π

$\varphi = \pi$

18. DTFT

$$X(e^{j\omega}) = \cos\omega + \cos 3\omega = \frac{e^{j\omega} + e^{-j\omega} + e^{j3\omega} + e^{-j3\omega}}{2}$$

$x(1) + x(-1) = ?$

$$x(1) = \frac{1}{4\pi} \int_{-\pi}^{\pi} (e^{j\omega} + e^{-j\omega} + e^{j3\omega} + e^{-j3\omega}) e^{j\omega} d\omega$$

$$= \frac{1}{4\pi} \int_{-\pi}^{\pi} (e^{j2\omega} + 1 + e^{j4\omega} + e^{-j2\omega}) d\omega$$

$$= \frac{1}{4\pi} \Big[\underbrace{\frac{1}{2j}(e^{j2\pi} - e^{-j2\pi})}_{0} + 2\pi + \underbrace{\frac{1}{4j}(e^{j4\pi} - e^{j4\pi})}_{0} - \underbrace{\frac{1}{2j}(e^{-j2\pi} - e^{j2\pi})}_{0} \Big]$$

$$= \frac{1}{2}$$

$$x(-1) = \frac{1}{2}$$

$$x(1) + x(-1) = 1$$

19. $x(t) = \cos\left(\frac{\pi}{2} t\right) =$ DTFS

$T_s = \frac{1}{2}$ $\quad \frac{\pi}{2} T = 2k\pi$

$N = T_{potip}$ $\quad T = 4$

$x_0, x_1 = ?$

$$x(n) = \cos\left(\frac{2\pi}{T} \cdot T_s n\right) = \cos\left(\frac{\pi}{4} n\right) = \frac{e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}}{2}$$

$$\frac{\pi}{4} N = 2\pi$$

$$N = 8$$

$$X(k) = \frac{1}{N} \sum_{n=0}^{7} x(n) e^{-2\pi j kn/N}$$

$$X(0) = \frac{1}{8} \sum_{n=0}^{7} \cos\left(\frac{\pi}{4} n\right) = \cancel{1} + \cancel{\frac{\sqrt{2}}{2}} + 0 - \cancel{\frac{\sqrt{2}}{2}} - \cancel{1} - \cancel{\frac{\sqrt{2}}{2}} + 0 + \cancel{\frac{\sqrt{2}}{2}} = 0$$

$$X(1) = \frac{1}{8} \sum_{n=0}^{7} \left[\cos\left(\frac{\pi}{4} n\right) \cdot e^{-j\frac{\pi}{4}n}\right] = \frac{1}{8} \sum_{n=0}^{7} \left[\frac{1 + e^{-j\frac{\pi}{2}n}}{2}\right]$$

$$= \frac{1}{2}$$

20. DTFS

$N=5$

$X_k = \{2, 3, 0, 3, 2\}$ (uzorci u jednom periodu)

$\quad\;\; 0 \;\; 1 \;\; 2 \;\; 3 \;\; 4$

$P_x = ?$, (snaga signala)

$$x(n) = \sum_{k=0}^{N-1} X(k) e^{2\pi j k n / N}$$

$$x(n) = \sum_{k=0}^{4} X(k) e^{2\pi j k n / 5}$$

$$= 2 + 3 \cdot e^{j \frac{2\pi}{5} n} + 3 \cdot e^{j \frac{6\pi}{5} n} + 2 \cdot e^{j \frac{8\pi}{5} n}$$

$$P_x = \frac{1}{N} \sum_{k=0}^{N-1} |x(n)|^2 = \sum_{n=0}^{N-1} |X(k)|^2 = 26$$