datum / date 2.5.2007.

$$Y(n+N) + a_1 Y(n+N-1) + a_2 (n+N-2) \cdots a_N Y(n) = b_M u(n+M) + b_{M-1} u(n+M-1) +$$

$Y(n+N) = E^N Y(n)$ $\quad u(n+N) = E^N u(n)$ operatorski zapis

$Y(n-2) = E^{-2} Y(n)$

$y(n) + 3y(n-1) - 4y(n-2) = u(n)$

i dovedea ulaz $u(n) = n\mu(n)$

Traže se: $y(0)$, $y(1)$, $y(2)$, $y(3)$

SVE osim $y(n)$ ide na desnu stranu [iterativna (step-by-step) metoda]

$y(n) = n\mu(n) - 3y(n-1) + 4y(n-2)$

pa onda uvrštavamo

$y(0) = 0\mu(0) - 3y(0-1) + 4y(0-2) = 0 - 3 + 4 = 1$

$y(1) = 1\mu(1) - 3y(2-1) + 4y(1-2) = 1 - 3 + 4 = 2$

i na ishi način uvrštavanjem dobijamo

$y(2) =$

$y(3) =$

$y = y_H + y_P$

$y(n) - 4y(n-2) = n\mu(n)$

početni uvjeti: $y(-1) = y(-2) = 0$

$y(n) = n\mu(n)$

$q - 4 = 0$

$q_1 = 2$

$q_2 = -2$

① y_h

$y(n) - 4y(n-2) = 0 \qquad y(n) = q^n$

$q^n - 4q^{n-2} = 0 \qquad q^{n-2}(q^2 - 4) = 0$

$$y_h = C \cdot 2^n + D \cdot (-2)^n$$

② y_p

$y_p = An\mu(n) + B\mu(n) \Rightarrow$ iz tablice

$y(n-2) = A(n-2)\mu(n-2) + B\mu(n-2)$

$An\mu(n) + B\mu(n) - 4A(n-2)\mu(n-2) - 4B\mu(n-2) = n\mu(n)$

$An + B - 4A(n-2) - 4B = n$

$An + B - 4An + 8A - 4B = n$

$A - 4A = 1 \Rightarrow A = -\frac{1}{3}$

$B + 8A - 4B = 0 \Rightarrow B = -\frac{8}{9}$

$$y_p = -\frac{1}{3}n - \frac{8}{9}$$

odziv sustava $= y_h + y_p = C \cdot 2^n + D(-2)^n - \frac{1}{3}n - \frac{8}{9}$

$y = y(n) = n\mu(n) + 4y(n-2)$

$y(0) = 0 \cdot 1 + 4 \cdot 1 \cdot 0 = 0$

$y(1) = 1 \cdot 1 + 4 \cdot 0 = 1$

$y = C \cdot 2^n + D(-2)^n - \frac{1}{3}n - \frac{8}{9}$

$0 = C + D - \frac{8}{9} \Rightarrow C + D = \frac{8}{9}$

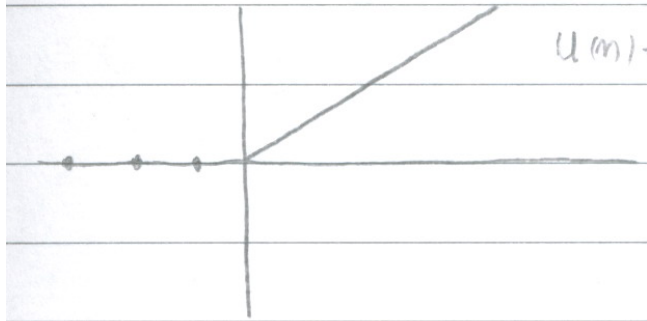
$1 = 2C - 2D - \frac{1}{3} - \frac{8}{9} \Rightarrow 2C - 2D = \frac{20}{9} \qquad C - D = \frac{10}{9}$

$4C = \frac{16}{9} + \frac{20}{9}$

$4C = 4 \Rightarrow C = 1 \Rightarrow D =$

$$y = 1 \cdot 2^n - \frac{1}{9}(-2)^n - \frac{1}{3}n - \frac{8}{9}$$

Recimo jednadžba



$u(n) = \delta(n-1) + 2\delta(n-2) + \delta(n-3)$

uvrstiti poč. uvjete

$y(n) + a\,y(n-1) + b\,y(n-2) = u(n)$

$0 \qquad -4$ prošli zad.

$y(n) = 4y(n-2) + u(n)$

$y(0) = 4 \cdot 0 + 0 = 0$

$y(1) = 4 \cdot 0 + \delta(n=1) = 1$

$y(2) = 4 \cdot 0 + 2 = 2$

$y(3) = 4 \cdot 1 + 1 = 5$

sustav $y(n+2) + ay(n-1) + by(n) = u(n)$

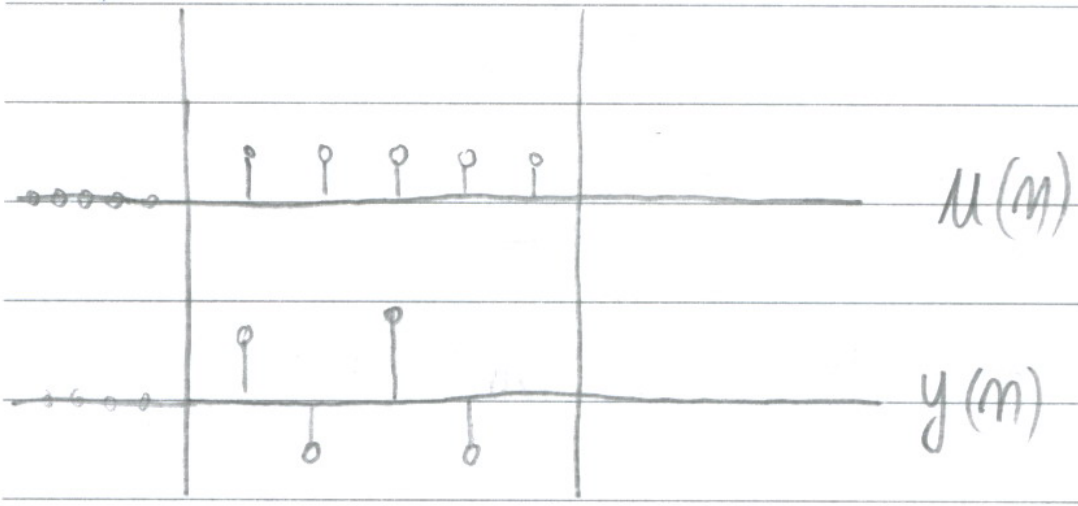
MIRAN SUSTAV

dok nema pobude $y(n+2) + ay(n-1) + by(n) = 0$

poč uvjeti

$y(-2) = y(-1) = 0$

$u(n) = \delta(n) \neq 0$



↳ prekid pobude

NEPOBUĐEN SUSTAV

$u(n) = 0$

$y(n_0-1)$, $y(n_0-2)$ } bilo što

datum / date

Zad:

$y(n) - 7y(n-1) + 12y(n-2) = 3^n$

$y(-1) = 7 \quad y(-2) = 2$

$y = y_h + y_n$

① y_n

$y(n) - 7y(n-1) + 12(n-2) = 0 \qquad y(n) = g^n$

$g^n - 7g^{n-1} + 12g^{n-2} = 0$

$g^{n-2}(g^2 - 7g + 12) = 0$

$g_{1,2} = \frac{7 \pm \sqrt{49-48}}{2} \rightarrow g_1 = 4, \; g_2 = 3$

$y = A4^n + B3^n$

$2 = A4^{-2} + B3^{-2}$

$7 = A4^{-1} + B3^{-1}$

$2 = \frac{1}{16}A + \frac{1}{3}B \;/\cdot(-4)$

$7 = \frac{1}{4}A + \frac{1}{3}B \;\Big/ +$

$-8+7 = -\frac{4}{9}A - \frac{4}{9}B + \frac{1}{3}B$

$-1 = -\frac{1}{9}B \Rightarrow B = 9$

$A = 16$

$y_n = A4^n + B3^n = 16 \cdot 4^n + 9 \cdot 3^n$ → odziv nepobuđenog sustava

② y_m

$y_p = C \cdot n \cdot 3^n$

$C \cdot n \cdot 3^n - 7C(n-1) \cdot 3^{n-1} + 12C(n-2) \cdot 3^{n-2} = 3^n$

$C \cdot n \, 3^n - 7Cn \cdot 3^{n-1} + 7C3^{n-1} + n\,12C \cdot 3^{n-2} - 24C3^{n-2} = 3^n \;/3^{n-2}$

$9Cn - 21Cn + 21C + 12nC - 24C = 9$

$-3C = 9 \Rightarrow C = -3$

$y_p = -3n \cdot 3^n$

$y_m = (-3)n \cdot 3^n + A4^n + B3^n =$ prijelazna se uvjeta $y(-1) = y(-2) = 0$

$A_2 = 16 \quad B_2 = -15$

$y_m = -3n \cdot 3^n + 16 \cdot 4^n + (-15) \cdot 3^n$ → odziv mirnog sustava

$y = y_m + y_n = 32 \cdot 4^n - 6 \cdot 3^n - 3n \cdot 3^n$

prisilni odziv (izdvoji se dio koji pripada partikularnom rješenju)

$-3n \cdot 3^n$

prirodni odziv

$32 \cdot 4^n - 6 \cdot 3^n$

(4) drugi način za dobiti odziv mirnoga sustava (metodom konvolucijske sume)

$$y(n) = \sum_{k=0}^{n} h(n-k)\, u(k) \qquad n \geq 0$$

h - impulsni odziv

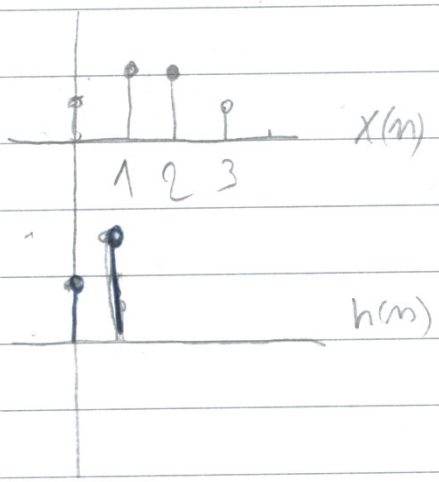
Zad $h(n) = \delta(n) + 2\delta(n-1)$

$x(n) = \delta(n) + 2\delta(n-1) + 2\delta(n-2) + \delta(n-3)$

$L_1 + L_2 = 1 + 5$

L_1 → duljina ulaza

L_2 → duljina odziva



$y(0) = \sum_{k=0}^{0} x(k)\, h(0-k) = x(0)\, h(0) = 1$

$y(1) = \sum_{k=0}^{1} x(k)\, h(1-k) = x(0)\, h(1) + x(1)\, h(0) = 2 + 2 = 4$

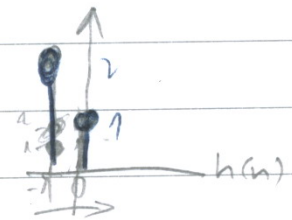
$y(2) = \sum x(k)\, h(2-k) = x(0)\, h(2) + x(1)\, h(1) + x(2)\, h(0) = 0 + 4 + 2 = 6$

$y(3) = 5$

$y(4) = 2$

Rješen na graf. način

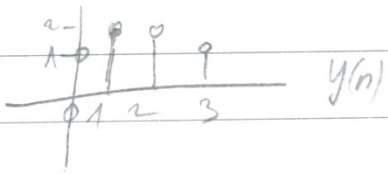
1. okrenemo graf h



za $y(0)$ i sjeku se za 1

$y(1)$ 4

$y(2)$ 6



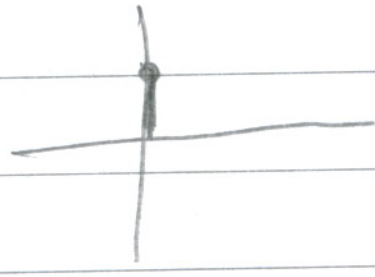
(6) $y(n) + 6y(n+1) = u(n)$ $\quad y(-1) = y(-2) = 0$

$u(n) = \delta(n)$

$g^n + 6g^{n-1} = 0 \mid : g^{n-1}$

$g = -6$

$y(n) = C(-6)^n$



I $y(0) = \delta(0) - 6y(0-1) = 1$

II $y(0) = C(-6)^0 = 1 = C$

$y(n) = (-6)^n$

Što se dogodilo?

U $n=0$ doveli smo ulaznu pobudu (dirac) koja je djelovala samo u tome trenutku.

Iracionali smo I.

U II smo odredili C i iz toga $y(n)$