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Signali i Sustavi
~Priprema za 3. labos~

~~3.1-3a~~, ~~3.1-4a~~, ~~3.1-5a~~, ~~3.2-1a~~, ~~3.2-2a~~, ~~3.2-2b~~, ~~3.2-5a~~, 3.2-7a, 3.2-7c

$\boxed{3.1-3a}$ ① $y(n)-0{,}8y(n-1)+0{,}2y(n-2)=u(n)$ $\quad y(-1)=0,\ y(-2)=0$

$\underline{u(n)=\delta(n)}$

$y(n)=u(n)+0{,}8y(n-1)-0{,}2y(n-2) \Rightarrow h(n)=\delta(n)+0{,}8\,h(n-1)-0{,}2\,h(n-2)$

$h(0)=\delta(0)+0{,}8h(-1)-0{,}2h(-2) \rightarrow \underline{h(0)=1}$

$q^2-0{,}8q+0{,}2=0 \rightarrow q_1=0{,}4+j0{,}2$ $\quad \varphi=\operatorname{arctg}\frac{0{,}2}{0{,}4}=26{,}56^\circ$

$\rightarrow q_2=0{,}4-j0{,}2$ $\quad |q|=0{,}447$

$h(n)=0{,}447^n\left(A\cos(26{,}56^\circ n)+B\sin(26{,}56^\circ n)\right)$

$h(0)=\boxed{1=A}$

$h(-1)=0{,}447^{-1}\left(\cos(-26{,}56^\circ)-B\sin(26{,}56^\circ)\right)=2-B=0 \rightarrow \boxed{B=2}$

$\underline{h(n)=0{,}447^n\left(\cos(26{,}56^\circ n)+2\sin(26{,}56^\circ n)\right)}$ //

$\underline{u(n)=\mu(n)}$ $\Rightarrow$ $y_h=0{,}447^n\left(A\cos(26{,}56^\circ n)+B\sin(26{,}56^\circ n)\right)$

$y_p=K \Rightarrow K-0{,}8K+0{,}2K=1$

$\underline{K=2{,}5}$ //

$y(0)=1 \Rightarrow A=-1{,}5$ $\quad y(-1)=0=0{,}447^{-1}A\cos(-26{,}56^\circ)+0{,}447^{-1}B\sin(-26{,}56^\circ)+2{,}5$

$B=-0{,}5$

$\underline{y(n)=0{,}447^n\left(-1{,}5\cos(26{,}56^\circ n)-0{,}5\sin(26{,}56^\circ n)\right)}$ //

2) $y(n) - \sqrt{3}\,y(n-1) + y(n-2) = u(n)$ $\quad \underline{u(n) = \delta(n)}$ $\quad h(-1) = h(-2) = 0$

$h(n) = \delta(n) + \sqrt{3}\,h(n-1) - h(n-2)$

$q^2 - \sqrt{3}\,q + 1 = 0$

$\underline{q_{1,2} = \frac{\sqrt{3}}{2} \pm \frac{1}{2} j}$ $\quad \varphi = \frac{\pi}{6}$

$\underline{|q| = 1}$

$h(n) = A\cos\left(\frac{\pi}{6} n\right) + B\sin\left(\frac{\pi}{6} n\right)$

$h(-1) = A\frac{\sqrt{3}}{2} - B\frac{1}{2} = 0$ $\quad \underline{h(0) = A = 1} \Rightarrow \underline{B = \sqrt{3}}$

$$\underline{h(n) = \cos\left(\frac{\pi}{6} n\right) + \sqrt{3}\sin\left(\frac{\pi}{6} n\right)}$$

$\underline{u(n) = \mu(n)}$ $\quad y_h = A\cos\left(\frac{\pi}{6} n\right) + B\sin\left(\frac{\pi}{6} n\right)$ $\quad y_p = K$

$K - \sqrt{3}K + K = 1 \Rightarrow \underline{K = \frac{1}{2-\sqrt{3}} = y_p}$

$y(-1) = 0 = A\frac{\sqrt{3}}{2} - \frac{1}{2}B + \frac{1}{2-\sqrt{3}} \Rightarrow y(0) = 1 = A + \frac{1}{2-\sqrt{3}} \Rightarrow \underline{A = -1-\sqrt{3}}$

$\underline{B = 1+\sqrt{3}}$

$$\underline{y(n) = \left[(-1-\sqrt{3})\cos\left(\frac{\pi}{6} n\right) + (1+\sqrt{3})\sin\left(\frac{\pi}{6} n\right)\right]\mu(n)}$$

3. $y(n) - 2y(n-1) + y(n-2) = u(n)$ $\quad u(n) = \delta(n)$

$h(0) = 1$ $\quad q^2 - 2q + 1 = 0$ $\quad \underline{q_{1,2} = \frac{2 \pm \sqrt{4-4}}{2} = 1}$

$h(n) = c q^n$ $\quad h(n) = C_1 q^n + C_2 n q^n = (1)^n (C_1 + C_2 n)$

$y(0) = 2y(-1) + y(-2) = 0$

$y(1) = 2y(0) + y(-1) = 0$ $\quad h(0) = 1 \quad h(1) = 2 \quad C_1 = C_2 = 1$

$$h(n) = (1)^n (1+n)\mu(n)$$

$u(n) = \mu(n)$

$y_h(n) = 1^n (C_1 + C_2 n)$

$y_p(n) = (k \cdot n^2)$

$kn^2 - 2k(n-1)^2 + k(n-2)^2 = 0$

$-2k + 4k = 1 \rightarrow k = \frac{1}{2} \quad y_p(n) = \frac{1}{2} n^2$

$y(n) = (1)^n (C_1 + C_2 n) + \frac{1}{2} n^2 \qquad y(-1) = y(-2) = 0$

$y(0) = C_1 \quad y(0) = 1 \rightarrow \boxed{C_1 = 1}$

$y(1) = C_1 + C_2 + \frac{1}{2} = 3 \rightarrow \boxed{C_2 = \frac{3}{2}}$

$y(n) = \left[(1)^n \left(1 + \frac{3}{2} n\right) + \frac{1}{2} n^2\right] \mu(n)$

4. $y(n) - 2y(n-1) + 5y(n-2) = u(n) \qquad u(n) = \delta(n)$

$q^2 - 2q + 5 = 0$

$q_{1,2} = 1 \pm j2 \qquad \varphi = 63{,}43^\circ$

$h(n) = (\sqrt{5})^n (A \cos(63{,}43^\circ n) + B \sin(63{,}43^\circ n))$

$h(0) = 1 \qquad h(-1) = 0 \rightarrow A = 1$

$h(-1) = \frac{1}{\sqrt{5}} (A \cos(63{,}43^\circ) - B \sin(63{,}43^\circ)) = 0 \quad \boxed{B = 0{,}5}$

$h(n) = (\sqrt{5})^n (\cos(63{,}43^\circ n) + 0{,}5 \sin(63{,}43^\circ n))$ imp. odziv

$u(n) = \mu(n) \qquad y(n) = (\sqrt{5})^n (A \cos(63{,}43^\circ n) + B \sin(63{,}43^\circ n))$

$y_p(n) = k \qquad k - 2k + 5k = 1 \qquad k = \frac{1}{4} \quad y_p(n) = \frac{1}{4}$

$y(0) = 1 \quad , y(-1) = 0$

$A + \frac{1}{4} = 1 \rightarrow \boxed{A = \frac{3}{4}}$

$y(-1) = \frac{1}{\sqrt{5}} \left(\frac{3}{4} \cos(63{,}43^\circ) - B \sin(63{,}43^\circ)\right) + \frac{1}{4} = 0 \quad \rightarrow B = 1$

$y(n) = (\sqrt{5})^n \left(\frac{3}{4} \cos(63{,}43^\circ n) + \sin(63{,}43^\circ n)\right) + \frac{1}{4}$

3.1-4a Partikularno rješenje za tipične harmonijske pobude

$u(n) = A\cos(\omega_0 n) \rightarrow y_p(n) = C_1\cos(\omega_0 n) + C_2\sin(\omega_0 n)$

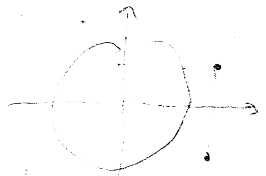
$u(n) = A\sin(\omega_0 n) \rightarrow$ —— // ——

→ kada se frekvencija pobude poklopi sa karakterističnim frekvencijama → sustav nestabilan ⇒ REZONANCIJA

3.1-5a Stabilnost sustava:

(1) $q_{1,2} = 0{,}4 \pm j0{,}2$ → unutar jed. kružnice → stabilan

(2) $q_{1,2} = \frac{\sqrt{3}}{2} \pm j\frac{1}{2}$



→ NESTABILAN

(3) $q_{1,2} = 1$ → NESTABILAN

(4) $q_{1,2} = 1 \pm j2$ → NESTABILAN

3.2-1a Kauzalnost, linearnost i memorija:

(1) $y(t) = \int_{-\infty}^{t} u(\tau)d\tau$ (integrator)

LINEARAN → zbog linearnosti f-je integracije

MEMORIJSKI → ovisi o prošlosti zbog granica integracije

KAUZALAN → ovisi isključivo o prošlosti i sadašnjosti

(2) $y(t) = \frac{d}{dt}u(t)$ (derivator)

LINEARAN → $\frac{d}{dt}[\alpha u_1(t) + \beta u_2(t)] = \alpha\frac{d}{dt}u_1(t) + \beta\frac{d}{dt}u_2(t)$

MEMORIJSKI → zbog prirode f-je moramo znati buduću vrijednost

NEKAUZALAN

3.2-2a) $u(n) = \delta(n)$

1. $y''(t) + 2y'(t) + 25y(t) = u(t)$ $\quad a_0 = 1,\ a_1 = 2,\ a_2 = 25,\ b_2 = 1$

$s^2 + 2s + 25 = 0 \qquad s_{1,2} = -1 \pm j2\sqrt{6} \qquad h_A(0^+) = 0,\ h_A'(0^+) = 1$

$$h_A(t) = e^{-t}\left(A\cos(2\sqrt{6}t) + B\sin(2\sqrt{6}t)\right)$$

$$h_A'(t) = -Ae^{-t}\cos(2\sqrt{6}t) + Ae^{-t}(-\sin(2\sqrt{6}t))\cdot 2\sqrt{6} - Be^{-t}\sin(2\sqrt{6}t) + Be^{-t}\cos(2\sqrt{6}t)\cdot 2\sqrt{6}$$

$h'(0^+) = -A + 2\sqrt{6}B = 1 \qquad h(0^+) = 0 = A$

$$\boxed{B = \frac{\sqrt{6}}{12}}$$

$$h(t) = b_2 D^0 h_A(t) = 1\, e^{-t}\frac{\sqrt{6}}{12}\sin(2\sqrt{6}t)$$

2. $y''(t) + 23y(t) = u(t)$ $\quad a_0 = 1,\ a_2 = 23,\ b_2 = 1$

$s^2 + 23 = 0 \qquad s_1 = j\sqrt{23} \quad s_2 = -j\sqrt{23}$

$$h_A(t) = A\cos(\sqrt{23}t) + B\sin(\sqrt{23}t)$$

$$h_A'(t) = -A\sin(\sqrt{23}t)\cdot\sqrt{23} + B\sin(\sqrt{23}t)\cdot\sqrt{23}$$

$h_A(0^+) = A = 0 \qquad h_A'(0^+) = B\sqrt{23} = 1 \rightarrow B = \frac{\sqrt{23}}{23}$

$$h_A(t) = \frac{\sqrt{23}}{23}\sin(\sqrt{23}t)$$

$$h(t) = b_2 D^0 h_A(t) = \frac{\sqrt{23}}{23}\sin(\sqrt{23}t) \qquad t > 0$$

3. $y''(t) = u(t) \qquad s_{1,2} = 0 \qquad h_A'(0^+) = 1 \quad h_A(0^+) = 0$

$h_A(t) = C_1 + C_2 t \qquad h_A'(t) = C_2 \qquad h_A(0^+) = C_1 = 0,\ C_2 = 1$

$$h_A(t) = t,\ t > 0$$

$$h(t) = t$$

(4) $\overset{a_0}{y''}(t) - \overset{a_1}{2}y'(t) + \overset{a_2}{17}y(t) = \overset{b_2}{u}(t)$

$h_A'(0^+) = 1$

$h_A(0^+) = 0$

$s^2 - 2s + 17 = 0$

$s_{1/2} = 1 \pm j4$

$h_A(t) = e^t (A\cos 4t) + jB\sin(4t)$

$h_A'(t) = e^t (A\cos(4t) + jB\sin(4t) + 4A\sin(4t) + j4B\cos(4t))$

$= e^t (\cos(4t)(A + j4B) + \sin(4t)(jB - 4A))$

$\underline{A = 0} \; (h_A(0^+) = 0)$

$h_A'(0^+) = A + j4B = 1 \quad B = \frac{1}{4j}$

$h_A(t) = e^t (j \cdot \frac{1}{4j} \sin(4t) = \underline{\frac{1}{4} e^t \sin(4t)}$

$\underline{h(t) = \frac{1}{4} e^t \sin(4t) \quad , t > 0}$

$\boxed{B.2\text{-}26}$ Karakteristični polinomi

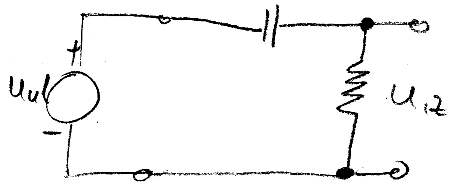
1) $s^2 + 2s + 25 = 0$

2) $s^2 + 17 = 0$

3) $s^2 = 0$

4) $s^2 - 2s + 17 = 0$

3.2-5a $\frac{du_{iz}(t)}{dt} + \frac{1}{RC} u_{iz}(t) = \frac{du_{ul}(t)}{dt}$



$$u(t) = \begin{cases} \sin t, & -\infty < t < 0^- \\ 1, & 0^+ < t < +\infty \end{cases}$$

$$y'(t) + \frac{1}{RC} y(t) = u'(t)$$

za $u(t) = \sin(t)$ → nema $y_h(t)$, samo $y_p(t)$ (zbog $-\infty < t < 0^-$)

$$y_p(t) = K_1 \cos(t) + K_2 \sin(t)$$
$$y'_p(t) = -K_1 \sin(t) + K_2 \cos(t)$$

$$-K_1 \sin(t) + K_2 \cos(t) + \frac{1}{RC} \cdot K_1 \cos(t) + \frac{1}{RC} K_2 \sin t = -\cos(t)$$

$R = 1k\Omega$ $C = 10\mu F$

$\frac{1}{RC} = 100$ //

$$\cos(t)\left[K_2 + 100K_1\right] + \sin(t)\left[100K_2 - K_1\right] = -\cos(t)$$

$K_2 + 100K_1 = -1$ → $100K_2 = K_1$

$K_2 + 100 \cdot 100K_2 = -1$ → $K_2 = 0{,}0001$

$\boxed{K_1 = -0{,}01}$ //

$y_p(t) = -0{,}01\cos(t) - 0{,}0001\sin(t)$ $t<0$

$y(0^-) = -0{,}01$ → $y(0^+) - y(0^-) = b_0(u(0^+))$

$\underline{y(0^+) = 0{,}99}$ //

3.2-7a

frekv. karakteristike

$$y''(t) + 2\xi \Omega_n y'(t) + \Omega_n^2 y(t) = A \Omega_n^2 u(t)$$

$$\underline{\xi = \overset{1}{-0{,}125}\ ;\ \overset{2}{0{,}25}\ ;\ \overset{3}{1}}$$

$$H(s) = \frac{A \Omega_n^2}{s^2 + 2\xi \Omega_n s + \Omega_n^2}$$

$\Omega_n = 0{,}4$

$A = \frac{1}{(0{,}4)^2} = \underline{6{,}25}$ iz zadatka

① $\xi = -0{,}125$

$$H(s) = \frac{6{,}25 \cdot 0{,}4^2}{s^2 + 2 \cdot (-0{,}125) \cdot 0{,}4 s + 0{,}4^2} = \frac{1}{s^2 - 0{,}1 s + 0{,}16} \qquad s = j\omega$$

$$H(j\omega) = \frac{1}{-\omega^2 - 0{,}1 j\omega + 0{,}16} = \frac{1}{0{,}16 - \omega^2 + 0{,}1 j\omega}$$

$$|H(j\omega)| = \frac{1}{\sqrt{(0{,}16 - \omega^2)^2 + (0{,}1\omega)^2}}$$

② $\xi = 0{,}25$

$$H(s) = \frac{1}{s^2 - 0{,}25 s + 0{,}4 s + 0{,}16} = \frac{1}{s^2 + 0{,}2 s + 0{,}16}$$

$$H(j\omega) = \frac{1}{0{,}16 - \omega^2 - 0{,}2 j\omega}$$

$$|H(j\omega)| = \frac{1}{\sqrt{(0{,}16 - \omega^2)^2 + (0{,}2\omega)^2}}$$

③ $\xi = 1$

$$H(j\omega) = \frac{1}{0{,}16\omega^2 + 0{,}8 j\omega} \qquad |H(j\omega)| = \frac{1}{\sqrt{(0{,}16 - \omega^2)^2 + (0{,}8\omega)^2}}$$

$\boxed{3.2-7c}$ signal je: $3\sin(2t)$ $\omega=2$

a) $A\sin(2t+\varphi)$ $\xi=-0{,}125$

$$A = |H(j\omega)| = 3\cdot\frac{1}{\sqrt{(96-4)^2+(9\cdot 2)^2}} = 0{,}776$$

$$\varphi = \operatorname{arctg}\frac{-0{,}1\cdot 2}{96-4} = -2{,}93^\circ$$

$$\underline{y_1(t) = 0{,}776\sin(2t-2{,}93^\circ)}$$

b) $\xi=0{,}25$

$$A = 3\cdot\frac{1}{\sqrt{(96-4)^2+(9\cdot 2\cdot 2)^2}} = 0{,}777 \qquad \varphi = 5{,}95^\circ$$

$$\underline{y_2(t) = 0{,}777\sin(2t+5{,}95^\circ)}$$

c) $\xi=1$

$$A = 3\cdot\frac{1}{\sqrt{(96-4)^2+(98\cdot 2)^2}} = 0{,}721 \qquad \varphi = 22{,}6^\circ$$

$$\underline{y_3(t) = 0{,}721\cdot\sin(2t+22{,}6^\circ)}$$