(2.) $y''(t) + 5y'(t) + 6y(t) = u(t)$

a/ $u(t) = \delta(t)$

$\delta(t) \circ\!\!-\!\!\bullet\ 1$

$$s^2 Y(s) - s\overset{0}{\overset{\shortparallel}{y(0^-)}} - \overset{0}{\overset{\shortparallel}{y'(0^-)}} + 5\cdot(sY(s) - \overset{0}{\overset{\shortparallel}{Y(0^-)}}) + 6Y(s) = 1$$

$$s^2 Y(s) + 5s Y(s) + 6Y(s) = 1$$

$$Y(s)(s^2 + 5s + 6) = 1$$

$$Y(s) = \frac{1}{s^2+5s+6} = \frac{1}{(s+2)(s+3)} = \frac{A}{s+2} + \frac{B}{s+3}$$

$$A(s+3) + B(s+2) =$$

$$= As + 3A + Bs + 2B = s(A+B) + (+3A + 2B)$$

$$A + B = 0 \quad /(-2)$$

$$+3A + 2B = 1$$

$$\boxed{A = 1} \quad \boxed{B = -1}$$

$$\boxed{Y(s) = \frac{1}{s+2} - \frac{1}{s+3}}$$

$$y(t) = (e^{-2t} - e^{-3t})\mu(t)$$

b) $u(t) = (12t+16)\mu(t)$

$y(0^-) = 3 \quad y'(0^-) = -8$

$y''(t) + 5y'(t) + 6y(t) = u(t)$

$y(t) = Ce^{st}$

$Ce^{st}(s^2+5s+6) = 0$

$s^2+5s+6=0 \Rightarrow s_{1,2} = \dfrac{-5 \pm \sqrt{25-24}}{2}$

$s_1 = -2 \quad s_2 = -3$

$y_h(t) = C_1e^{-2t} + C_2e^{-3t}$

$y_p(t) = K_0 + K_1t$

$y_p'(t) = K_1 \qquad y_p''(t) = 0$

$5K_1 + 6K_0 + 6K_1t = 12t + 16$

$6K_1 = 12 \Rightarrow \boxed{K_1 = 2}$

$5K_1 + 6K_0 = 16$

$10 + 6K_0 = 16$

$6K_0 = 6 \Rightarrow \boxed{K_0 = 1}$

$y_p(t) = 1 + 2t, \; t \geq 0$

$y(t) = C_1e^{-2t} + C_2e^{-3t} + 1 + 2t, \; t \geq 0 \qquad y'(t) = -2C_1e^{-2t} - 3C_2e^{-3t} + 2$

$y(0^-) = 3 \quad y'(0^-) = -8 \quad a_1 = 5 \quad a_2 = 6 \quad b_0 = 0 \quad b_1 = 0 \quad b_2 = 1$

$y(0^-) = y(0^+) = 3$

$y'(0^+) - y'(0^-) + a_1 \cdot (y(0^-) - y(0^+)) = b_0u'(0^+) + b_1u(0^+)$

$y'(0^+) = y'(0^-) = -8$

$$\left.\begin{aligned} y(0^+) &= C_1 + C_2 + 1 = 3 \\ y'(0^+) &= -2C_1 - 3C_2 + 2 = -8 \end{aligned}\right\} \Rightarrow \begin{aligned} C_1 + C_2 &= 2 \quad /\cdot(-2) \\ -2C_1 - 3C_2 &= -10 \end{aligned}$$

$\boxed{C_2 = 6} \; \boxed{C_1 = -4}$

$y(t) = -4e^{-2t} + 6e^{-3t} + 1 + 2t, \; t \geq 0$

③ $h(t) = 2te^{-t}\mu(t)$

a/ prijenosna

- impulsni u vrem. → prijenosno u Laplac.

$2te^{-t} \rightarrow 2 \cdot \frac{1}{(s+1)^2}$

$$H(s) = \frac{2}{(s+1)^2}$$

b/ - odziv sustava ⇒ $u(t) = 2\mu(t)$

$y(0^-) = 2$

$y'(0^-) = 0$

$(s+1)^2 = s^2 + 2s + 1$

$2u(t) \equiv 4\mu(t)$

$$\underline{y''(t) + 2y'(t) + y(t) = 2u(t)}$$

$$s^2Y(s) - sy(0^-) - y'(0^-) + 2(sY(s) - y(0^-)) + Y(s) = \frac{4}{s}$$

$$s^2Y(s) - 2s + 2sY(s) - 4 + Y(s) = \frac{4}{s}$$

$$Y(s)(s^2 + 2s + 1) = \frac{4}{s} + 2s + 4 \quad / \cdot s$$

$$Y(s) \cdot s \cdot (s^2 + 2s + 1) = 4 + 2s^2 + 4s$$

$$Y(s) = \frac{2s^2 + 4s + 4}{s(s^2 + 2s + 1)} = \frac{2s^2 + 4s + 4}{s(s+1)(s+1)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$$

$$A(s+1)^2 + B \cdot s(s+1) + Cs = As^2 + 2As + A + Bs^2 + Bs + Cs =$$
$$= s^2(A+B) + s(2A + B + C) + A$$

$A = 4$

$2A + B + C = 4$

$A + B = 2$

$B = 2 - 4 = -2$

$6 + C = 4$

$C = 4 - 6 = -2$

$$Y(s) = \frac{4}{s} + \frac{-2}{s+1} + \frac{-2}{(s+1)^2}$$

$$y(t) = 4\mu(t) - \left[2e^{-t} + 2te^{-t}\right]\mu(t) =$$

$$= \underline{\left[e^{-t}(-2 - 2t) + 4\right]\mu(t)}$$

b/ LAPLACE

$$y''(t) + 5y'(t) + 6y(t) = u(t) \quad , \; u(t) = (12t + 16)\mu(t)$$

$$s^2Y(s) - sy(0^-) - y'(0^-) + 5\cdot(sY(s) - y(0^-)) + 6Y(s) = \frac{12}{s^2} + \frac{16}{s}$$

$$s^2Y(s) - 3s + 8 + 5sY(s) - 15 + 6Y(s) = \frac{12}{s^2} + \frac{16}{s}$$

$$Y(s)(s^2 + 5s + 6) = \frac{12}{s^2} + \frac{16}{s} + 3s + 7 \;/\cdot s^2$$

$$Y(s)\, s^2(s^2+5s+6) = 12 + 16s + 3s^3 + 7s^2$$

$$Y(s) = \frac{3s^3 + 7s^2 + 16s + 12}{s^2(s^2+5s+16)} = \frac{3s^3 + \cancel{7}s^2 + 16s + 12}{s^2(s+2)(s+3)} =$$

$$= \frac{A + Bs}{s^2} + \frac{C}{s+2} + \frac{D}{s+3}$$

$$(A+Bs)(s+2)(s+3) + Cs^2(s+3) + Ds^2(s+2) =$$

$$= (A+Bs)(s^2+5s+6) + Cs^3 + 3Cs^2 + Ds^3 + 2Ds^2 =$$

$$= As^2 + 5As + 6A + Bs^3 + 5Bs^2 + 6Bs + Cs^3 + 3Cs^2 + Ds^3 + 2Ds^2 =$$

$$= s^3(B + C + D) + s^2(A + 5B + 3C + 2D) + s(5A + 6B) + 6A$$

$$6A = 12 \Rightarrow \boxed{A = 2}$$

$$B + C + D = 3$$
$$5B + 3C + 2D = 5$$
$$6B = 6 \Rightarrow \boxed{B = 1}$$

$$C + D = 2 \;/\cdot(2)$$
$$3C + 2D = 0$$
$$2C + 2D = 4$$
$$3C + 2D = 0$$
$$-C = 4$$
$$\boxed{C = -4} \qquad D = 6$$

$$Y(s) = \frac{2+s}{s^2} + \frac{-4}{s+2} + \frac{6}{s+3} = \frac{2}{s^2} + \frac{1}{s} - 4\frac{1}{s+2} + 6\frac{1}{s+3}$$

$$y(t) = (2t + 1 - 4e^{-2t} + 6e^{-3t})\mu(t)$$

4.

$$y'(t) + 4y(t) = u(t) + 2u'(t)$$

$$u(t) = \mu(t)$$

$$y(0^-) = 2$$

a/ $y(0^-) = 2$

$y(0^+) = ?$

$a_1 = 4 \quad b_0 = 2 \quad b_1 = 1$

$$y(0^+) - y(0^-) = b_0 u(0^+)$$

$$y(0^+) = b_0 u(0^+) + y(0^-) =$$

$$= 2 \cdot 1 + 2 = 4$$

b/ OPĆA HOMOGENA

$$\left.\begin{array}{l} y(t) = Ce^{st} \\ Ce^{st}(s+4) = 0 \\ s_1 = -4 \end{array}\right\} y_h(t) = C_1 \cdot e^{-4t}, t \geq 0$$

PARTIKUL.

$$y_p = K$$

$$K' + 4K = 1K + 2K'$$

$$4K = 1 \Rightarrow \boxed{K = \frac{1}{4}} \quad y_p(t) = \frac{1}{4}$$

$$y(t) = \left(C_1 e^{-4t} + \frac{1}{4}\right)\mu(t)$$

$$y(0^+) = C_1 + \frac{1}{4} = 4$$

$$C_1 = \frac{16}{4} - \frac{1}{4} = \frac{15}{4}$$

$$y(t) = \left(\frac{15}{4}e^{-4t} + \frac{1}{4}\right)\mu(t)$$

c/ $y'(t) + 4y(t) = \mu(t) + 2\mu'(t)$

$$sY(s) - y(0^-) + 4Y(s) = \frac{1}{s} + 2$$

$$sY(s) - 2 + 4Y(s) = \frac{1}{s} + 2$$

$$Y(s)\,(s+4) = \frac{1}{s} + 4 \;/\cdot s$$

$$Y(s)\cdot s\cdot(s+4) = 1 + 4s$$

$$Y(s) = \frac{4s+1}{s(s+4)} = \frac{A}{s} + \frac{B}{s+4}$$

$$A\cdot(s+4) + Bs = As + 4A + Bs = s(A+B) + 4A \Rightarrow$$

$$4A = 1 \Rightarrow A = \frac{1}{4}$$

$$A + B = 4$$

$$B = 4 - \frac{1}{4} = \frac{15}{4}$$

$$Y(s) = \frac{1}{4}\cdot\frac{1}{s} + \frac{15}{4}\cdot\frac{1}{s+4}$$

$$\boxed{y(t) = \left(\frac{1}{4}t + \frac{15}{4}e^{-4t}\right)\mu(t)}$$

d/ $H(s) = \frac{2s+1}{s+4}$

$$s+4=0 \Rightarrow s_1 = -4 < 0$$

– sustav je stabilan!