

## Problem 2

$$y'(t) + 4y(t) = u(t) + 2u'(t) \quad u(t) = p(t)$$

$$y(0^-) = 2$$

$$a) y(0^+) = b u(0^+) + y(0^-) \Rightarrow 2 \cdot 1 + 2 = 4$$

b)

$$s + 4 = 0 \quad s = -4 \quad y_h(t) = C e^{-4t}$$

partikuläres

$$y_p = k \quad u(t) = p(t) \quad 4k = 1 \quad k = \frac{1}{4}$$

Impulsantwort

$$u(t) = 2\delta(t)$$

$$h_A'(t) + 4h_A(t) = 2\delta(t) \quad h_A(t) = y_h(t) = C e^{-4t} \quad h_A(0^+) = 1$$

$$h_A(0) = 1 = C \quad y_A(t) = e^{-4t}$$

$$y(t) = C e^{-4t} + \frac{1}{4} + e^{-4t}$$

$$y(0^+) = 4 = C + \frac{1}{4} + 1 \quad C = 4 + 1 - \frac{1}{4} = \frac{16-1}{4} = \frac{15}{4}$$

$$y(t) = \left( \frac{15}{4} e^{-4t} + \frac{1}{4} + e^{-4t} \right) = \left( \frac{15}{4} e^{-4t} + \frac{1}{4} \right) p(t)$$

$$c) sY(s) - y(0^-) + 4Y(s) = U(s) + 2sU(s) - 2u(0^-)$$

$$Y(s)(s+4) - 2 = U(s)(1+2s) - 2 \cdot 0$$

$$Y(s) = \frac{U(s)(1+2s)}{s+4} + \frac{2}{s+4} \quad U(s) = \frac{1}{s}$$

$$Y(s) = \frac{1+2s}{s(s+4)} + \frac{2}{s+4} = \frac{1+2s+2s}{s(s+4)} = \frac{1+4s}{s(s+4)}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s+4}$$

$$A(s+4) + Bs = 1+4s \quad 4A = 1 \quad A = \frac{1}{4}$$

$$A + B = 4 \quad \frac{1}{4} + B = 4 \quad B = 4 - \frac{1}{4} = \frac{15}{4}$$

$$Y(s) = \frac{1}{4} \frac{1}{s} + \frac{15}{4} \frac{1}{s+4}$$

$$y(t) = \left( \frac{1}{4} + \frac{15}{4} e^{-4t} \right) p(t)$$

d) System stabil, reelle Pole  $< 0$