(1.) $x(t) = e^{-2t}[\mu(t-1) - \mu(t-3)]$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{1}^{3} e^{-4t} dt = -\frac{1}{4} e^{-4t} \Big|_{1}^{3} = \frac{1}{4}(e^{-4} - e^{-12})$$

(2.) $x(t) = 5e^{j\pi t}$

$$P = \lim_{L \to \infty} \frac{1}{L} \int_{-L/2}^{L/2} |x(t)|^2 dt = 25$$

(3.) $x(n) = 3^{-n}\mu(n)$

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=0}^{\infty} 3^{-2n} = \sum_{n=0}^{\infty} (3^{-2})^n = \frac{1}{1-3^{-2}} = \frac{9}{8}$$

$$\sum_{n=0}^{\infty} q^n = \frac{1}{1-q}$$

(4.) $x(n) = (6 + 2^{-n})\mu(n)$

$$P = \lim_{M \to \infty} \frac{1}{2M+1} \sum_{n=-M}^{M} |x(n)|^2 = \lim_{M \to \infty} \frac{1}{2M+1} \sum_{n=0}^{M} (36 + 12 \cdot 2^{-n} + 2^{-2n})$$

$$= \frac{36}{2}$$

5. a) $x(t) = \sin(3t) + \sin(3\pi t)$

$3T = 2\pi$ $\quad$ $3\pi T = 2\pi$

$T = \frac{2\pi}{3}$ $\quad$ $T = \frac{2}{3}$

b) $\frac{3}{4}\pi(2\pi N + N^2) = 2\pi$

nije periodična

c) $2N = 2\pi$

$N = \pi$

nije periodična (N mora biti cijeli broj)

d) $\frac{3}{4}\pi(2\pi T + T^2)$

nije periodična

e) $\cos n + j\sin n$

$N = 2\pi$

nije periodična

6. $x'(t) = -\frac{\pi}{2}\sin\left(\frac{\pi}{2}t\right)(\mu(t-1) - \mu(1-t))$

7. $4\cos\left(\pi t + \frac{\pi}{4}\right) = 2e^{j2\omega_0 t} \cdot e^{j\frac{\pi}{4}} + 2e^{-j2\omega_0 t} e^{-j\frac{\pi}{4}}$

$\omega_0 = \frac{\pi}{2}$

$X_2 = 2e^{j\frac{\pi}{4}} = 2\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right) \quad |X_2| = 2, \ \sphericalangle X_2 = \operatorname{arctg} 1 = \frac{\pi}{4}$

$X_{-2} = 2e^{-j\frac{\pi}{4}} = 2\left(\cos\frac{\pi}{4} - j\sin\frac{\pi}{4}\right) \quad |X_{-2}| = 2, \ \sphericalangle X_{-2} = \operatorname{arctg}(-1) = -\frac{\pi}{4}$

(8.) PETFS

$$P = \sum_{k=-\infty}^{\infty} |X(k)|^2 = 8$$

(9.) $x(t) = 10\sin(7\pi t) + 4\cos(\pi t) = 10\sin(7\omega_0 t) + 4\cos(\omega_0 t)$

$\omega_0 = \pi$

$$= \frac{5}{j}\left(e^{7j\omega_0 t} - e^{-7j\omega_0 t}\right) + 2\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$$

4

(10) $T_0 = 6$

$\omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$

$$X(k) = \frac{1}{T}\int_{T_p} x(t)e^{-jw_p kt}\,dt$$

$$x(k) = \frac{1}{6}\int_{-1}^{1} 2\cdot e^{-j\frac{\pi}{3}kt}\,dt$$

$$x(0) = \frac{1}{3}\cdot 2 = \frac{2}{3}$$

$$x(3) = \frac{1}{6}\int_{-1}^{1} 2\cdot e^{-j\pi t}\,dt = -\frac{1}{3j}\,e^{-j\pi t}\Big|_{-1}^{1} =$$

$$= -\frac{1}{3j}\left(e^{-j\pi} - e^{j\pi}\right) = 0$$

11. $CTFT(x(t)) = e^{-3j\omega}\mu(t)$

$CTFT(x(t-4)) = ?$

$x(t-4) \circ\!\!-\!\!\bullet\; X(\omega)e^{-j4\omega}$

$CTFT(x(t-4)) = e^{-7j\omega}\mu(\omega)$

12. CTFT

$$x(t) = \begin{cases} e^{-t}, & t \in \langle 0, 2\pi \rangle \\ 0, & \text{inaczej} \end{cases}$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt = \int_{0}^{2\pi} e^{-t(1+\omega)}dt = \frac{-1}{1+\omega}e^{-t(1+\omega)}\Big|_{0}^{2\pi}$$

$$= \frac{1}{1+\omega}\left(1 - e^{-2\pi(1+\omega)}\right)$$

13. $x(t) = e^{-t(j+1)}\mu(t) \circ\!\!-\!\!\bullet \frac{1}{j+1+j\omega} = \frac{1}{1+j(\omega+1)}$.

$e^{-at}\mu(t) \circ\!\!-\!\!\bullet \frac{1}{a+j\omega}$

(14.) $X(e^{j\Omega}) = \begin{cases} 3, & |\Omega| < a \\ 0, & a < |\Omega| < \pi \end{cases}$

DTFT

$$E = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega = \frac{9}{2\pi}\int_{-a}^{a} d\omega = \frac{9a}{\pi}$$

(15.) $x(n) = \begin{cases} \sin\left(\frac{\pi}{4}n\right), & -5 \le n \le 5 \\ 0, & \text{inače} \end{cases}$

$$X\left(\Omega = \frac{\pi}{2}\right) = ?$$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$X\left(\Omega = \frac{\pi}{2}\right) = \sum_{n=-5}^{5} \sin\frac{\pi}{4}n \cdot \left(\cos\frac{\pi}{2}n - j\sin\frac{\pi}{2}n\right)$$

$n=-5$ $\quad \sin(-225)\,(0 - j\sin(-450)) = j\frac{\sqrt{2}}{2}\cdot j$

$n=-4$ $\quad \sin 0 \cdot \ldots = 0$

$n=-3$ $\quad \sin(-135)\cdot(0 - j\sin(-270)) = -\frac{\sqrt{2}}{2}(-j) = \frac{\sqrt{2}}{2}j$

$n=-2$ $\quad \sin(-90)\cdot(\cos(-180) - 0) = A$

$n=-1$ $\quad \sin(-45)\cdot(0 - j\sin(-90)) = -\frac{\sqrt{2}}{2}\cdot j$

$n=0$ $\quad \sin 0 \cdot \ldots = 0$

$n=1$ $\quad \sin(45)\,(0 - j\sin(90)) = \frac{\sqrt{2}}{2}(-j) = -\frac{\sqrt{2}}{2}j$

$n=2$ $\quad \sin(90)\cdot(\cos(180) - 0) = -A$

$n=3$ $\quad \sin(135)\cdot(0 - j\sin(270)) = \frac{\sqrt{2}}{2}j$

$n=4$ $\quad \sin 0 \cdot \ldots = 0$

$n=5$ $\quad \sin(225)\,(0 - j\sin(450)) = -\frac{\sqrt{2}}{2}(-j) = \frac{\sqrt{2}}{2}j$

$$\Sigma = 2\frac{\sqrt{2}}{2}j = \sqrt{2}j$$

(16.) $x(e^{j\Omega}) = \cos(2\Omega) + \cos(5\Omega)$

$$= \frac{e^{j2\Omega} + e^{-j2\Omega} + e^{j5\Omega} + e^{-j5\Omega}}{2}$$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\Omega)e^{j\Omega n}\,d\Omega$$

$$x(n) = \frac{1}{4\pi}\int_{-\pi}^{\pi}\left(e^{j\Omega(n+2)} + e^{j\Omega(n-2)} + e^{j\Omega(n+5)} + e^{j\Omega(n-5)}\right)d\Omega$$

$$= \frac{1}{4\pi}\left(\frac{1}{j(n+2)}e^{j\Omega(n+2)} + \frac{1}{j(n-2)}e^{j\Omega(n-2)} + \frac{1}{j(n+5)}e^{j\Omega(n+5)} + \frac{1}{j(n-5)}e^{j\Omega(n-5)}\right)\Bigg|_{-\pi}^{\pi}$$

$$= \frac{1}{4\pi}\left[\frac{2\sin(\pi(n+2))}{j(n+2)} + \frac{2\sin(\pi(n-2))}{j(n-2)} + \frac{2\sin(\pi(n+5))}{j(n+5)} + \frac{2\sin(\pi(n-5))}{j(n-5)}\right]$$

$$= 0$$

(17.) $E = \frac{1}{2\pi}\int_{-\pi}^{\pi} dw = 1$

(18) $x_k = \cos\left(\frac{\pi}{2}k\right)$

$N = 4$

$$x(n) = \sum_{k=0}^{N-1} X(k)e^{2\pi j kn/N}$$

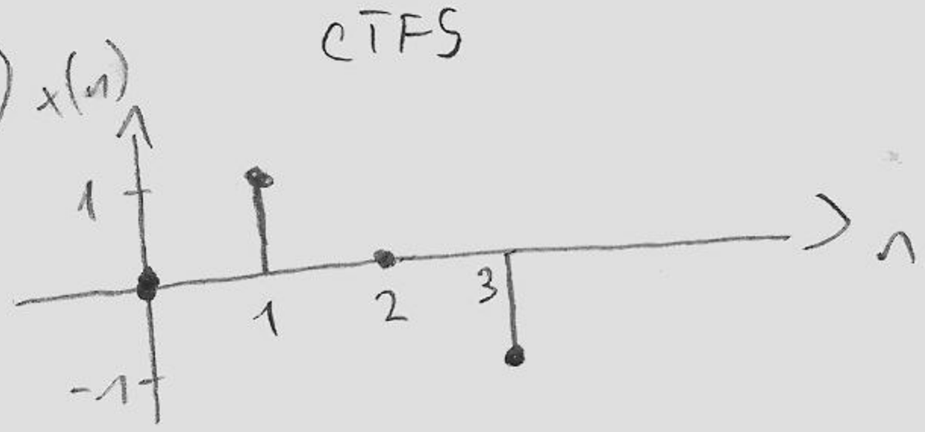
$$x(n) = \sum_{k=0}^{3} \cos\left(\frac{\pi}{2}k\right)e^{j2\pi kn/4}$$

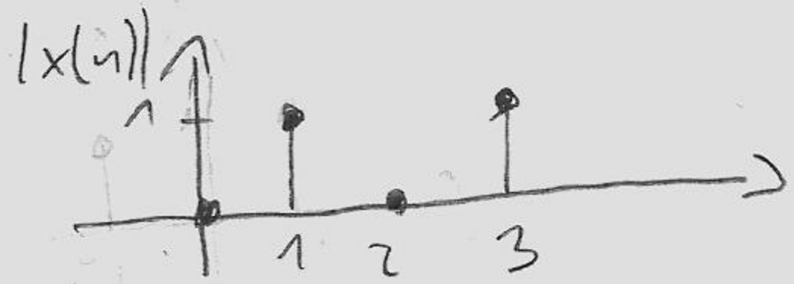
$$= 1 + e^{j\pi n} = 1 - \cos\pi n - \underbrace{j\sin\pi n}_{0} = 1 - (-1)^n$$

$\downarrow$

za $k=0$

(19.) CTFS





$x(0) = 0$

$x(1) = 1$

$x(2) = 0$

$x(3) = -1$

$$x(k) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-2\pi j k n / N}$$

$$= \frac{1}{2} \sum_{n=0}^{1} x(n) e^{-2\pi j k n / N}$$

$$= \frac{1}{2} \left(e^{-j\frac{\pi}{2}k} \right)$$

$$= \frac{1}{2} \left(\underbrace{\cos\left(\frac{\pi}{2}k\right)}_{0} - j \sin\left(\frac{\pi}{2}k\right) \right)$$

$$= -\frac{1}{2} j \sin\left(\frac{\pi}{2}k\right)$$

(20) $x(n) = \frac{1}{1+n+n^2}$ → aperiodican, diskretan → DTFT