TJEDAN 10.
SIGNALI I SUSTAVI

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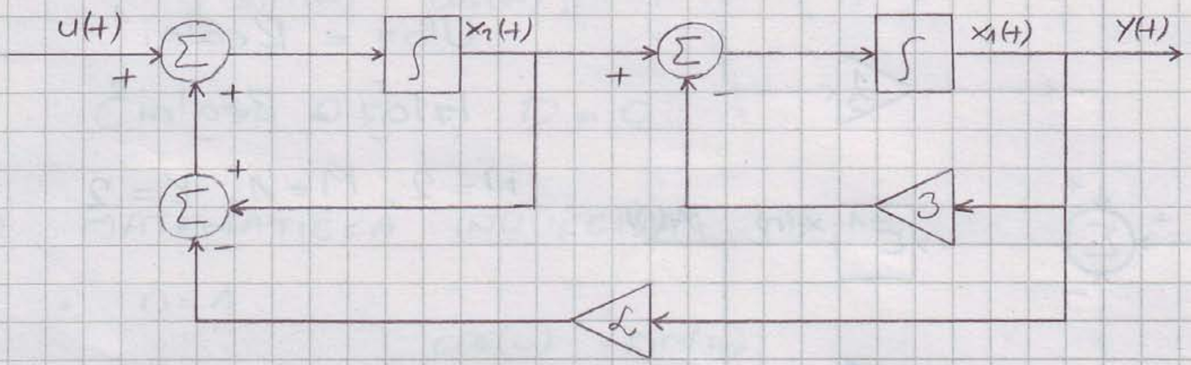
GRUPA 2.E2

1.



$$x_2(t) = \int \left[x_2(t) + u(t) - \mathcal{L} x_1(t) \right] dt \quad / \frac{d}{dt}$$

$$x_1(t) = \int \left[x_2(t) - 3 x_1(t) \right] dt \quad / \frac{d}{dt}$$

$$\dot{x}_2(t) = x_2(t) + u(t) - \mathcal{L} x_1(t)$$

$$\dot{x}_1(t) = x_2(t) - 3 x_1(t)$$

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \begin{bmatrix} -3 & +1 \\ -\mathcal{L} & +1 \end{bmatrix} + u(t) \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$y(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} + D u(t)$$

Rješenje:

$$A = \begin{bmatrix} -3 & +1 \\ -\mathcal{L} & +1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 0$$

Provjera:

$N = 2,\ M = 1,\ K = 1$

Stanja = Realni^2

Ulaz = Realni^1

Izlaz = Realni^1

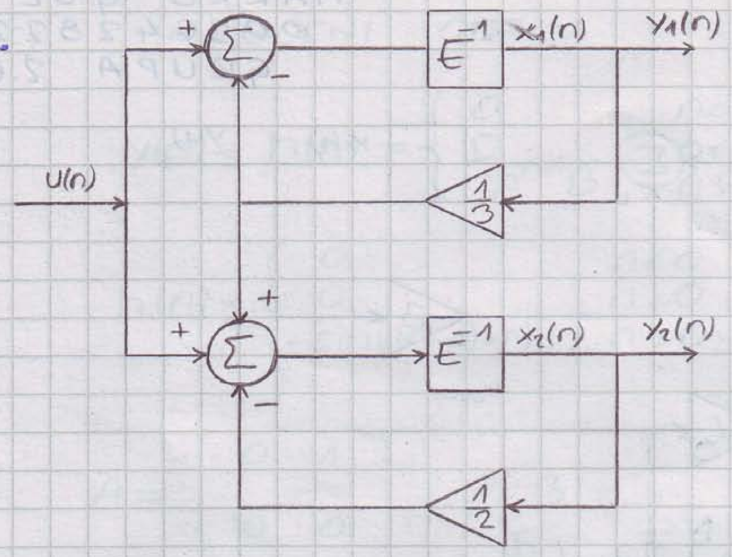
$A \ldots \quad N \times N \to 2 \times 2$ ⊕

$B \ldots \quad N \times M \to 2 \times 1$ ⊕

$C \ldots \quad K \times N \to 1 \times 2$ ⊕

$D \ldots \quad N \times M \to 2 \times 1$ ⊕

3.



Stanja = $Realni^2$

Ulaz = $Realni^1$

Izlaz = $Realni^2$

$N=2,\ M=1,\ K=2$

$$x_1(n+1) = +u(n) - \frac{1}{3}x_1(n)$$

$$x_2(n+1) = +u(n) + \frac{1}{3}x_1(n) - \frac{1}{2}x_2(n)$$

$$y_1(n) = x_1(n)$$

$$y_2(n) = x_2(n)$$

$$x(n+1) = \begin{bmatrix} x_1(n+1) \\ x_2(n+1) \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & 0 \\ \frac{1}{3} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(n)$$

$$y(n) = \begin{bmatrix} y_1(n) \\ y_2(n) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + D \cdot u(n)$$

Rješenje

$$A = \begin{bmatrix} -\frac{1}{3} & 0 \\ \frac{1}{3} & -\frac{1}{2} \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad D = 0$$

Provjera

$N=2,\ M=1,\ K=2$

A ... $N \times N \rightarrow 2 \times 2$ ⊕

B ... $N \times M \rightarrow 2 \times 1$ ⊕

C ... $K \times N \rightarrow 2 \times 2$ ⊕

D ... $N \times M \rightarrow 2 \times 1$ ⊕

3.

$$A = \begin{bmatrix} \cos(w) & -\sin(w) \\ \sin(w) & \cos(w) \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 0$$

(a) MATEMATIČKA INDUKCIJA:

- $n = 1$

$$A = \begin{bmatrix} \cos(w) & -\sin(w) \\ \sin(w) & \cos(w) \end{bmatrix}$$

- $n = k$

$$A = \begin{bmatrix} \cos(kw) & -\sin(kw) \\ \sin(kw) & \cos(kw) \end{bmatrix}$$

- $n = k+1$

$$A^n = A^k \cdot A = \begin{bmatrix} \cos(kw) & -\sin(kw) \\ \sin(kw) & \cos(kw) \end{bmatrix} \begin{bmatrix} \cos(1w) & -\sin(1w) \\ \sin(1w) & \cos(1w) \end{bmatrix}$$

$$= \begin{bmatrix} \cos((k+1)w) & -\sin((k+1)w) \\ \sin((k+1)w) & \cos((k+1)w) \end{bmatrix} = \begin{bmatrix} \cos(nw) & -\sin(nw) \\ \sin(nw) & \cos(nw) \end{bmatrix}$$

(b) • ODZIV STANJA NEPOBUĐENOG SUSTAVA

$$x(n) = A^n x(0), \quad n > 0$$

$$x(n) = \begin{bmatrix} \cos(nw) & -\sin(nw) \\ \sin(nw) & \cos(nw) \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$x(n) = \begin{bmatrix} -\sin(nw) \\ \cos(nw) \end{bmatrix}$$

- ODZIV NEPOBUĐENOG SUSTAVA

$$y(n) = \begin{cases} C\,x(0), & n = 0 \\ C A^n x(0), & n > 0 \end{cases}$$

$$y(n) = \begin{cases} 0, & n = 0 \\ -\sin(nw), & n > 0 \end{cases}$$

(c) • IMPULSNI ODZIV

$$y(n) = h(n) = \begin{cases} 0 & , \ n<0 \\ D & ; \ n=0 \\ CA^{n-1}B & , \ n>0 \end{cases}$$

$$h(n) = \begin{cases} 0 & , \ n<0 \\ 0 & ; \ n=0 \\ -\sin((n-1)w) & ; \ n>0 \end{cases}$$

4.

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad x(2) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$x(n+1) = Ax(n) + Bu(n)$$

$$x(2) = Ax(1) + Bu(1)$$

$$x(1) = Ax(0) + Bu(0)$$

$$x(2) = A(Ax(0) + Bu(0)) + Bu(1)$$

$$x(2) = A \cdot A\, x(0) + A \cdot B\, u(0) + B u(1)$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(0) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(1)$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} u(0) \\ u(1) \end{bmatrix}$$

$$u(0) = 1$$

$$u(1) = 2$$

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5.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$A \cdot A = A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \qquad A^2 \cdot A = A^3 = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$$

$$A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$$

- ODZIV STANJA NEPOBUĐENOG SUSTAVA

(a)
$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$x(n) = A^n \cdot x(0) \rightarrow x(n) = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$x(n) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(b)
$$x(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$x(n) = A^n \cdot x(0) \rightarrow x(n) = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$x(n) = \begin{bmatrix} n \\ 0 \end{bmatrix}$$

(c)
$$x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x(n) = A^n \cdot x(0) \rightarrow x(n) = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$x(n) = \begin{bmatrix} 1+n \\ 1 \end{bmatrix}$$

6.

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \rightarrow A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 0$$

$$x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

- ODZIV NEPOBUĐENOG SUSTAVA

$$y(n) = \begin{cases} Cx(0), & n=0 \\ CA^n x(0), & n>0 \end{cases}$$

$$y(n) = \begin{cases} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, & n=0 \\ \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, & n>0 \end{cases}$$

$$y(n) = \begin{cases} 1, & n=0 \\ 1+n, & n>0 \end{cases}$$

7.

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow A^2 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$x(0) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad x(2) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad D = 0$$

$$\left.\begin{aligned} x(2) &= Ax(1) + Bu(1) \\ x(1) &= Ax(0) + Bu(0) \end{aligned}\right\} x(2) = A \cdot A x(0) + A \cdot B u(0) + B u(1)$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(0) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(1)$$

$$u(0) = -2x_1 - x_2$$

$$u(1) = -x_1 - x_2$$

8. $S\{\mu(n)\} = y(n) = (n+1)\,\mu(n)$

$\delta(n) = \mu(n) - \mu(n-1)$

$$h(n) = y(n) - y(n-1)$$

$$h(n) = (n+1)\mu(n) - n\mu(n-1)$$

$$h(n=5) = 6\cdot\mu(5) - 5\cdot\mu(4)$$

$$h(n=5) = 1$$

9.
$$h(n) = \begin{cases} 1, & n=0,1 \\ 0, & \text{inače} \end{cases}$$

$$y(n) = \sum_{m=-\infty}^{\infty} h(n-m)\,u(m) \quad , \quad h(n-m) = \begin{cases} 1, & m=n,\, n-1 \\ 0, & \text{inače} \end{cases}$$

$$y(n) = \sum_{m=n-1}^{n} h(n-m)\,u(m)$$

$$y(n) = u(n-1) + u(n)$$

10. $u(n) = \mathcal{L}^n \mu(n)$

$h(n) = \beta^n \mu(n)$

$$y(n) = \sum_{m=-\infty}^{\infty} h(m)\,u(n-m)$$

$$y(n) = \sum_{m=-\infty}^{\infty} \beta^m \mu(m)\cdot \mathcal{L}^{n-m}\mu(n-m)$$

$$\mu(m) = \begin{cases} 1, & m \geq 0 \\ 0, & m < 0 \end{cases}$$

$$y(n) = \sum_{m=0}^{\infty} \beta^m \mathcal{L}^{n-m}\mu(n-m)$$

$$\mu(n-m) = \begin{cases} 1, & m \leq n \\ 0, & m > n \end{cases}$$

$$y(n) = \sum_{m=0}^{n} \beta^m \mathcal{L}^{n-m} = \mathcal{L}^n \sum_{m=0}^{n} \left(\frac{\beta}{\mathcal{L}}\right)^m$$

$$y(n) = \mathcal{L}^n \frac{1-\left(\frac{\beta}{\mathcal{L}}\right)^{n+1}}{1-\frac{\beta}{\mathcal{L}}}$$

11. (a) $u(t) * \delta(t) = \int_{-\infty}^{\infty} u(\tau)\,\delta(t-\tau)\,d\tau$

$t \neq \tau \rightarrow \delta(t-\tau) = 0$

ZA $t = \tau$

$= \int_{-\infty}^{\infty} u(\tau)\,\delta(t-\tau)\,d\tau = u(t)\cdot 1 = u(t)$

(b) $u(t) * \delta(t-t_0) = \int_{-\infty}^{\infty} u(\tau)\,\delta(t-t_0-\tau)\,d\tau$

$t-t_0 \neq \tau \rightarrow \delta(t-t_0-\tau) = 0$

ZA $t-t_0 = \tau$

$= \int_{-\infty}^{\infty} u(\tau)\,\delta(t-t_0-\tau)\,d\tau = u(t-t_0)\cdot 1 = u(t-t_0)$

(c) $u(t) * \mu(t) = \int_{-\infty}^{\infty} u(\tau)\,\mu(t-\tau)\,d\tau$

$\mu(t-\tau) = \begin{cases} 1, & t \geq \tau \\ 0, & t < \tau \end{cases}$

$= \int_{-\infty}^{t} u(\tau)\cdot 1\,d\tau$

(d) $u(t) * \mu(t-t_0) = \int_{-\infty}^{\infty} u(\tau)\,\mu(t-t_0-\tau)\,d\tau$

$\mu(t-t_0-\tau) = \begin{cases} 1, & t-t_0 \geq \tau \\ 0, & t-t_0 < \tau \end{cases}$

$= \int_{-\infty}^{t-t_0} u(\tau)\cdot 1\,d\tau$

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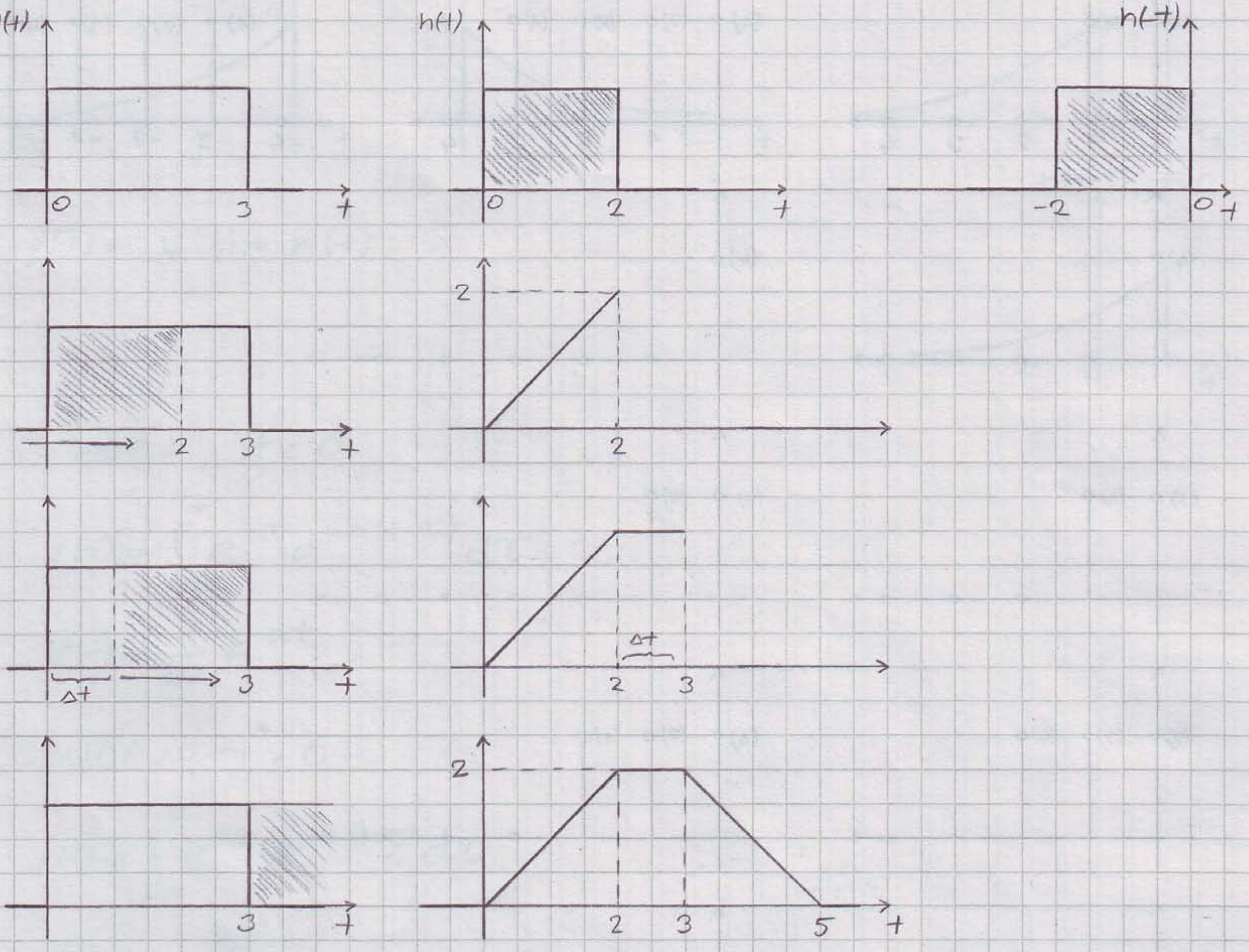
12.

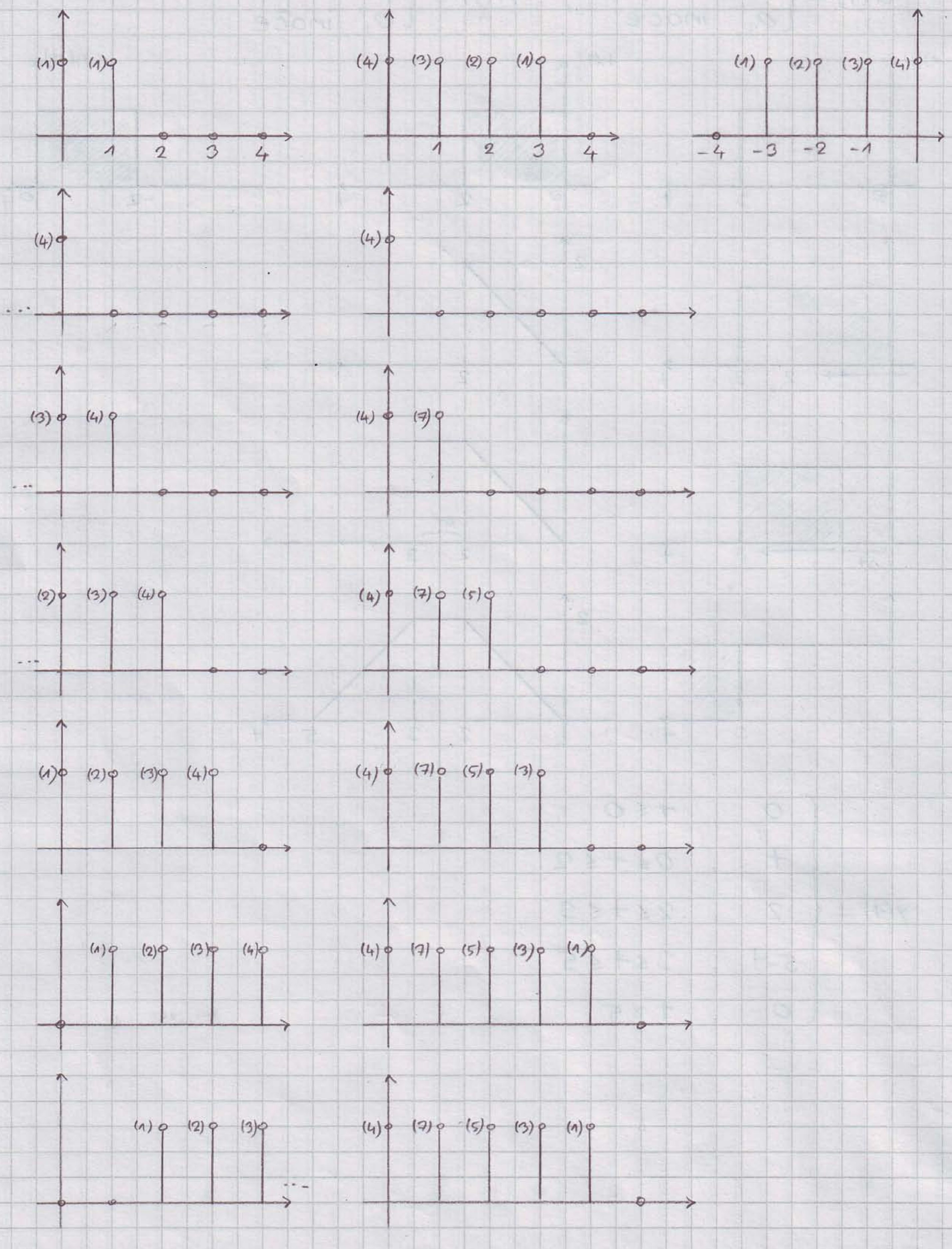
$$u(t) = \begin{cases} 1, & 0 < t \leq 3 \\ 0, & \text{inače} \end{cases}, \quad h(t) = \begin{cases} 1, & 0 < t \leq 2 \\ 0, & \text{inače} \end{cases}$$



$$y(t) = \begin{cases} 0, & t \leq 0 \\ t, & 0 \leq t \leq 2 \\ 2, & 2 \leq t \leq 3 \\ 5-t, & 3 \leq t \leq 5 \\ 0, & t \geq 5 \end{cases}$$

13. $h(n) = 4\delta(n) + 3\delta(n-1) + 2\delta(n-2) + \delta(n-3)$

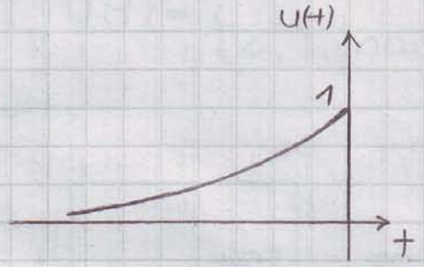
$u(n) = \delta(n) + \delta(n-1)$

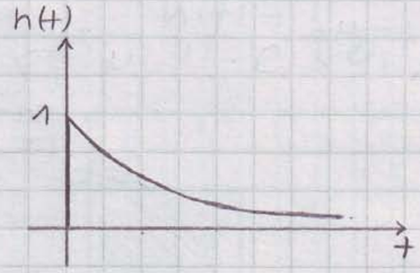


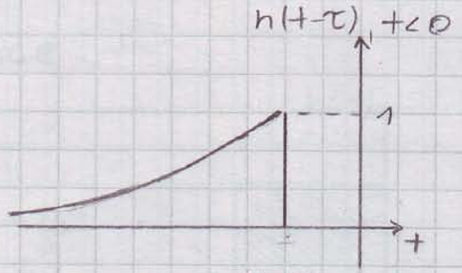
$y(n) = 4\delta(n) + 7\delta(n-1) + 5\delta(n-2) + 3\delta(n-3) + 1\delta(n-4)$

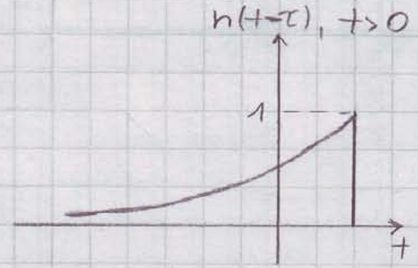
14. $h(t) = e^{-at}\mu(t)$

$u(t) = e^{at}\mu(-t), \quad a > 0$









$y(t) = u(t) * h(t)$

• 1. SLUČAJ, $t < 0$

$$y(t) = \int_{-\infty}^{t} e^{a\tau} e^{-a(t-\tau)}\, d\tau$$

$$y(t) = \frac{1}{2a} e^{at}$$

• 2. SLUČAJ, $t > 0$

$$y(t) = \int_{-\infty}^{0} e^{a\tau} e^{-a(t-\tau)}\, d\tau$$

$$y(t) = \frac{1}{2a} e^{-at}$$

• KONAČNO

$$y(t) = \frac{1}{2a} e^{-a|t|}, \quad a > 0$$

15. $f: \mathbb{Z} \to \mathbb{R}$ $\qquad g(n) = f(n) * f(n)$

$$f(n) = \begin{cases} 1, & n = 0, 1, 2 \\ 0, & \text{inače} \end{cases} \qquad g(3) = ?$$

$$g(3) = \sum_{m=-\infty}^{\infty} f(m) \cdot f(3-m)$$

$$g(3) = f(0) \cdot \overset{=0}{f(3)} + f(1) \cdot \overset{=1}{f(2)} + f(2) \cdot \overset{=1}{f(1)}$$

$$g(3) = 2$$