① $x(n) = n \cdot [u(n) - u(n-32)]$, $P = ?$ $\rightarrow \underbrace{P = 0}$

$n \in \langle 0, 31 \rangle$

$E = \sum_{0}^{31} n^2 = \frac{31 \cdot 32 \cdot 63}{6}$

$\hookrightarrow$ APERIODIČKI

$\Rightarrow P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N \to 0}^{N \to 31} |x(n)|^2$

② $x(t) = 2\cos(2t) + 4\sin(4t)$, $E = ?$ ČIM JE sin, cos $\Rightarrow \underbrace{E = \infty}$

$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \infty$? $\longrightarrow$ PERIODIČKI SIGNALI $\Rightarrow \underbrace{E = \infty}$

③ PARNE / NEPARNE FUNKCIJE: $\underset{\text{PARNO}}{f(x) = f(-x)}$, $\underset{\text{NEPARNO}}{f(x) = -f(-x)}$

a) $f(x) = \sqrt{1-x^2} \Rightarrow \sqrt{1-x^2} = \sqrt{(1-(-x)^2} \Rightarrow$ PARNA

b) $f(x) = |x+1| + |x-1| \Rightarrow |x+1| + |x-1| = |-x+1| + |-x-1| = |-1||x-1| + |-1||x+1| \rightarrow$ PARNA

c) $f(x) = \frac{1-x}{1+x} \Rightarrow \frac{1-x}{1+x} \neq \frac{1-(-x)}{1+(-x)} \Rightarrow$ NIJE PARNA ; $\frac{1-x}{1+x} = -\frac{1-(-x)}{1+(-x)} \rightarrow$ NIJE PARNA

d) $f(x) = \ln \frac{1-x}{1+x}$

e) $f(x) = \log \frac{1-\sin x}{1+\sin x}$

④ $x(n) = \cos\left(\frac{\pi}{8} n^3 + 3\right)$, PERIODIČAN ? $x(n+N) = x(n)$

$x(n+N) = \left(\frac{\pi}{8}(n+N)^3 + 3\right) = \cos\left(\frac{\pi}{8}\underbrace{(n^3 + 3n^2 N + 3nN^2 + N^3)} + 3\right) = \cos\left(\frac{\pi}{8} n^3 + 3 + \underbrace{\frac{\pi}{8}(3n^2 N + 3nN^2 + N^3)}\right)$

$\frac{\cancel{\pi}}{8}(3n^2 N + 3nN^2 + N^3) = 2k\cancel{\pi} \Rightarrow k = \frac{1}{16}(N^3 + 3nN^2 + 3n^2 N) \Rightarrow \underline{N = 2p}$

$k = \frac{1}{16}(8p^3 + 12np^2 + 6n^2 p) = \frac{1}{8}(4p^3 + 6np^2 + 3n^2 p) \Rightarrow \underline{p = 2q}$

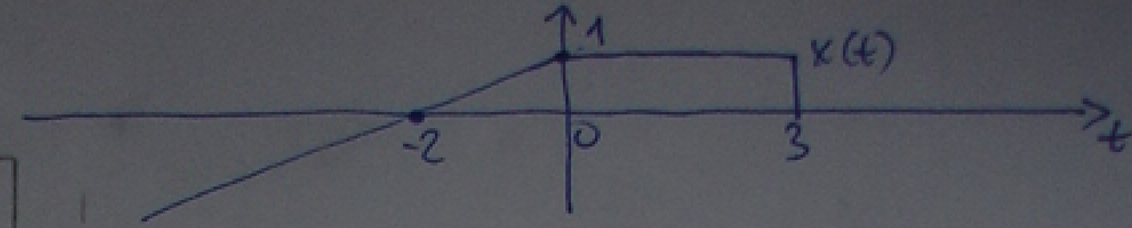
$k = \frac{1}{8}(32q^3 + 24nq^2 + 6n^2 q) = \frac{1}{4}(16q^3 + 12nq^2 + 3n^2 q) \Rightarrow \underline{q = 2r}$

$k = \frac{1}{4}(128r^3 + 48nr^2 + 6n^2 r) = \frac{1}{2}(64r^3 + 24nr^2 + 3n^2 r) \Rightarrow \underline{r = 2x}$

$k = \frac{1}{2}(512x^3 + 96nx^2 + 6n^2 x) = 256x^3 + 48nx^2 + 3n^2 x$

$N = 2p = 4q = 8r = 16x \Rightarrow \underbrace{N_0 = 16}$?

⑤ GENERALIZIRANA DERIVACIJA:



$$x(t)=\left(\tfrac{1}{2}t+1\right)\mu(-t)+1\cdot\left[\mu(t)-\mu(t-3)\right]$$

$$x'(t)=\tfrac{1}{2}\mu(-t)+\left(\tfrac{1}{2}t+1\right)\cdot\underbrace{\delta(-t)}_{-\delta(t)}+\delta(t)-\delta(t-3)$$

$$=\tfrac{1}{2}\mu(-t)-\delta(t-3)-\left(\tfrac{1}{2}t+1\right)\delta(t)+\delta(t)=\tfrac{1}{2}\mu(-t)-\delta(t-3)+\delta(t)\left(-\tfrac{1}{2}t-1+1\right)=\tfrac{1}{2}\mu(-t)-\delta(t-3)-\underbrace{\tfrac{1}{2}t\,\delta(t)}_{=0}$$

$$=\tfrac{1}{2}\mu(-t)-\delta(t-3)$$

⑥ a) $|X_{-1}|^2=|X_1|^2=3$ $\quad P=\sum|X_k|^2=3\cdot 2+7\cdot 2=20$

$|X_2|^2=|X_2|^2=7$

AKO ZADAJU SAMO X_{-1}, MORAŠ U OBZIR UZETI I X_1

b) $|X_{-1}|^2=|X_1|^2=8$ $\quad P=\sum|X_k|^2=8\cdot 2+2\cdot 2=20$

$|X_{-2}|^2=|X_2|^2=2$

c) $|X_{-1}|^2=|X_1|^2=6$ $\quad P=\sum|X_k|^2=6\cdot 2+4\cdot 2=20$

$|X_{-2}|^2=|X_2|^2=4$

d) $|X_{-1}|^2=|X_1|^2=5$ $\quad P=\sum|X_k|^2=5\cdot 2+5\cdot 2=20$

$|X_{-2}|^2=|X_2|^2=5$

e) $|X_{-1}|^2=|X_1|^2=8$ $\quad P=8\cdot 2+1\cdot 2=18$ ✓

$|X_{-2}|^2=|X_2|^2=1$

⑦ $X_1=X_{-1}=1$

$X_4=-j$

$X_{-4}=j$

$$x(t)=\sum X_k e^{j\omega_p k t}$$

$$=\underset{k=1}{1\cdot e^{j\omega_p t}}+\underset{k=-1}{1\,e^{-j\omega_p t}}-\underset{k=4}{j\cdot e^{j\omega_p 4t}}+\underset{k=-4}{j\,e^{-j\omega_p 4t}}=$$

$$=2\cos(\omega_p t)-j\left(e^{j\omega_p 4t}-e^{-j\omega_p 4t}\right)=2\cos(\omega_p t)-j\cdot 2j\cdot\sin(\omega_p 4t)=$$

$$=2\cos(\omega_p t)+2\sin(4\omega_p t)\Rightarrow \text{c) } \omega=2\Rightarrow T=\frac{2\pi}{2}=\cancel{\pi}$$

KAKO IZRAČUNATI ω_p=?

e) $\omega=3\Rightarrow T=\frac{2\pi}{3}$ ✓ → OK

(8) $x(t)=30\cos\left(20t+\frac{\pi}{3}\right)+60\sin\left(80t+\frac{\pi}{4}\right)$, $T_P=\frac{\pi}{10}$ → MORA BITI ZADANI DOBAR T_P KOJI ODGOVARA $\omega_P=20$; $\omega[X_{-4}]=?$, $\angle X_{-4}=?$

$\omega_P=20$ (uz $20t$); $4\omega_P=80$ (uz $80t$)

ω → KRUŽNA FREKV. ; $\angle$ → FAZNI KUT

$$\sin\left(80t+\frac{\pi}{4}\right) \Rightarrow \frac{60}{2j}\left(e^{j\left(80t+\frac{\pi}{4}\right)}-e^{-j\left(80t+\frac{\pi}{4}\right)}\right)$$

$$X_{-4}=\frac{-60}{2j}e^{-j80t}\cdot e^{-j\frac{\pi}{4}}=30je^{-j\frac{\pi}{4}}e^{j(-4)\omega_0 t}$$

$$30e^{-j\frac{\pi}{2}}e^{-j\frac{\pi}{4}}e^{-j(-4)\omega_0 t}$$

$\Rightarrow \omega=80$

$\Rightarrow \angle X_{-4}=\frac{\pi}{4}$

AKO JE $X_{-4}=-30\ldots$

$X_{-4}=30\cdot e^{-j\pi}$

AMPLITUDA $X_{-4}=30$ MORA BITI ČISTI POZITIVNI BROJ

$$\left(X_{-4}-\underbrace{30}_{\text{AMPLITUDA}}e^{j\frac{\pi}{4}}\cdot e^{-j(-4)\omega_0 t}\right.$$

(9) FOURIEROV RED, $T_P=2$ ← KOJI INTERVAL IZABRATI? MOŽE $\langle -1,1\rangle$, $\langle 1,3\rangle$, $\langle -3,-1\rangle$ → POMAKNUTA t FUNKCIJA => $t+\ldots$

$$y=x \Rightarrow X_k=\frac{1}{T_P}\int_{-1}^{1}x(t)e^{-j\omega_P kt} \qquad \omega_P=\frac{2\pi}{T_P}=\pi$$

$$\Rightarrow k=0 \Rightarrow X_0=\frac{1}{2}\int_{-1}^{1}t\cdot e^{0}=\frac{1}{4}\cdot t^2\Big|_{-1}^{1}=\frac{1}{4}\cdot\left(1^2-(-1)^2\right)=0$$
=> ISTOSMJERNA KOMP. - SREDNJA VRIJEDNOST

$$\Rightarrow k\neq 0 \Rightarrow X_k=\frac{1}{2}\cdot\int_{-1}^{1}t\cdot e^{-j\pi kt}dt=\left|\begin{matrix}t=u & dv=e^{-jk\pi t}dt\\ dt=du & v=-\frac{1}{j\pi k}e^{-j\pi tk}\end{matrix}\right|=\frac{-1}{j\pi k}t\cdot e^{-jk\pi t}\Big|_{-1}^{1}+\frac{1}{j\pi k}\cdot\left(-\frac{1}{j\pi k}\right)\cdot e^{-j\pi kt}\Big|_{-1}^{1}=$$

$$=\frac{1}{2}\left[\frac{-1}{j\pi k}\left[e^{-j\pi k}+e^{j\pi k}\right]+\frac{1}{\pi k}\cdot\left[e^{-j\pi k}-e^{j\pi k}\right]\right]=\frac{1}{2}\left[\frac{1}{j\pi k}\cdot 2\cos(\pi k)-\cancel{\frac{1}{\pi k}\cdot 2j\sin(\pi k)}\right]=$$

$$=\frac{1}{\cancel{2}}\cdot\frac{+j\cancel{2}}{k\pi}\cdot\cos(k\pi)=\frac{+j}{k\pi}\cdot\cos(k\pi)$$ ✓

(B)

(10) FOURIEROV RED: $X_k=\sqrt{3}\cdot 2^{-|k|}$, $k\neq 0$; $P=?$ ← KAKO OVO UTJEČE NA REZULTAT?

$X_0=0$, $k\in\mathbb{Z}$

$$\text{ILI } P=\sum_{-\infty}^{\infty}|X_k|^2=\sum_{-\infty}^{\infty}3\cdot 2^{-2|k|}=3\sum_{-\infty}^{\infty}\frac{1}{4}^k=3\cdot\left[\frac{1}{1-\frac{1}{4}}\cdot 2-2\right]=3\cdot\left[\frac{8}{3}-\frac{6}{3}\right]=2$$

→ U SVAKOM INTERVALU BROJIMO 0, ZATO JE MORAMO 2 PUTA ODUZETI

$\hookrightarrow \sum_{0}^{\infty}\left(\frac{1}{4}\right)^k$ => IMAMO 2 INTERVALA: $\langle -\infty,0\rangle$, $\langle 0,+\infty\rangle$

(11) $x(t) \circ\!\!-\!\bullet\, X(\omega)$

✓ $X^*(\omega) \bullet\!\!-\!\circ\, x^*(-t)$ (D)

PREMA PRAVILU KONJUGACIJE: $x^*(t) \circ\!\!-\!\bullet\, X^*(-\omega)$

$x^*(-t) \circ\!\!-\!\bullet\, X^*(\omega)$

(12) $x(t)=e^{-t(j+1)}\mu(t)$, $X(\omega)=?$

$$\Rightarrow e^{-at}\mu(t) \circ\!\!-\!\!\bullet \frac{1}{a+j\omega} \Rightarrow |a=j+1| \Rightarrow e^{-t(j+1)}\mu(t) \circ\!\!-\!\!\bullet \frac{1}{(j+1)+j\omega}=\frac{1}{j(\omega+1)+1} \quad \text{D}$$

(13) KONTINUIRANI, APERIODIČKI SIGNAL => CTFT => FOURIEROV INTEGRAL

$$E=\frac{1}{2\pi}\int_{-\infty}^{\infty}|X(\omega)|^2 d\omega; \; E=\int_{-\infty}^{\infty}|x(t)|^2 dt \Rightarrow \text{D}$$

(14) $x(t)=e^{jt}$; AMPL. SPEKTRA ZA $k=0$ (FOURIEROV RED) => $|X_k|=?$

$$X_k=\frac{1}{T_P}\int_{-T_P/2}^{T_P/2} x(t)e^{-j\omega_P k t}dt \Rightarrow X_0=\frac{1}{T_P}\int_{-T_P/2}^{T_P/2} e^{jt}dt=\frac{1}{T_P}\left[\frac{1}{j}e^{jt}\Big|_{-T_P/2}^{T_P/2}\right]=\frac{-j}{T_P}\cdot\left(e^{jT_P/2}-e^{-jT_P/2}\right)=$$

$$=-\frac{j}{T_P}\cdot 2j\sin\left(\frac{T_P}{2}\right)=\frac{2}{T_P}\cdot\sin\left(\frac{T_P}{2}\right) \Rightarrow T_P=2\pi=\frac{2}{2\pi}\cdot\sin\pi=\emptyset \quad \text{D}$$

$$e^{j(t+T)}=e^{jt}\cdot e^{jT}=x(t)=e^{jt}$$

$$e^{jT}=1 \Rightarrow T=2k\pi=2\pi$$

$$e^{jt}=e^{j\omega_0 t} \Rightarrow X_0, \quad \omega_0=1$$

$$e^{jt+\frac{\pi}{2}} \Rightarrow \text{AMPLITUDA}=e^{\frac{\pi}{2}}$$

$$\Rightarrow \text{FAZA}=e^{j0}$$

FAZA

(15) DTFT => e) $X(e^{j\omega})=\frac{\pi}{3\sqrt{2}} \Rightarrow E=\frac{1}{2\pi}\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}}\frac{\pi^2}{18}=\frac{\pi^2}{36\pi}\cdot\left(\frac{2\pi}{3}-\left(-\frac{2\pi}{3}\right)\right)=\frac{\pi}{36_9}\cdot\frac{4\pi}{3}=\frac{\pi^2}{27}$

=> d) $X(e^{j\omega})=\frac{\pi}{4} \Rightarrow E=\frac{1}{2\pi}\cdot\frac{\pi^2}{16}\cdot\left(\frac{2\pi}{3}-\left(-\frac{2\pi}{3}\right)\right)=\frac{\pi}{32}\cdot\frac{4\pi}{3}=\frac{\pi^2}{24}$ ✓

=> c) $X(e^{j\omega})=\frac{\omega}{3} \Rightarrow E=\frac{1}{2\pi}\cdot 2\int_{-\pi}^{\pi}\dots=\frac{\pi^2}{27}$

2 INTEGRALA

=> b) $\int_0^{2\pi/3}$ PRVI PRAVAC

$\int_{\frac{2\pi}{3}}^{\pi}$ DRUGI PRAVAC

} 2x OBA INTEGRALA

(16) $x(t)=\cos\left(\frac{1}{4}t\right) \Rightarrow x(n)$

~~KONTINUIRANI~~ DISKRETNI, PERIODIČKI SIGNAL => DTFS TRANSFORMACIJA

$x(nT_s)=\cos\left(\frac{1}{4}nT_s\right) \Rightarrow$

$w_0=\frac{T_s}{4} \Rightarrow N=\frac{2\pi}{w_0}=\frac{8\pi}{T_s}$

(17) $k=2$, $\sin\left(\frac{\pi n}{4}\right)$, N = TEMELJNI PERIOD

DISKRETNI, PERIODIČNI => DTFS

$\frac{\pi}{4}N=2k\pi /\cdot 4$

$\underbrace{N=8k=8} \Rightarrow X_k=\frac{1}{8}\sum_{0}^{7}x(n)\cdot e^{\frac{-2\pi jkn}{8}}$

$\Rightarrow k=2 \Rightarrow X_2=\frac{1}{8}\sum_{0}^{7}\sin\left(\frac{\pi n}{4}\right)\cdot e^{-j\frac{\pi n}{4}}=\frac{1}{8}\cdot\sum_{0}^{7}\frac{1}{2j}\left[e^{j\frac{\pi}{4}n}-e^{-j\frac{\pi}{4}n}\right]\cdot e^{-j\frac{\pi}{4}n}=\frac{1}{16j}\cdot\sum_{0}^{7}\left(1-e^{-j\frac{\pi}{2}n}\right)=$

$=\frac{-j}{16}\left[1-e^{0}+(1-e^{-j\frac{\pi}{2}})+(1-e^{-j\pi})+(1-e^{-j\frac{3\pi}{2}})+(1-e^{-j2\pi})+(1-e^{-j\frac{5\pi}{2}})+(1-e^{-j3\pi})+(1-e^{-j\frac{7\pi}{2}})\right]=$

$=\frac{-j}{16}\left[\cancel{(1-1)}+(1+\cancel{j})+(1+1)+(1-\cancel{j})+\cancel{(1-1)}+(1-\cancel{j})+(1+1)+(1+\cancel{j})\right]=$

$=\frac{-j}{16}8=\frac{-j}{2}$ (E)

GRANICE $\langle -1, 1\rangle$

(18) $X(e^{jw})=e^{jw}\left[\mu(w+1)-\mu(w-1)\right]$; VRIJEDNOST $x(n)$ ZA $1,-1$?

$\underbrace{\text{DTFT}} \rightarrow x(n)=\frac{1}{2\pi}\int_{-1}^{1}e^{jw}\cdot e^{jwn}\,dw=\frac{1}{2\pi}\int_{-1}^{1}e^{w(j+jn)}$

$\Rightarrow n=1 \Rightarrow x(1)=\frac{1}{2\pi}\int_{-1}^{1}e^{w\cdot 2j}\,dw=\frac{1}{2\pi}\cdot\frac{1}{2j}\,e^{2jw}\Big|_{-1}^{1}=\frac{1}{2\pi}\cdot\frac{1}{2j}\cdot\left[e^{2j}-e^{-2j}\right]=\frac{1}{2\pi}\cdot\sin(2)$

$\Rightarrow n=-1 \Rightarrow x(-1)=\frac{1}{2\pi}\int_{-1}^{1}e^{w\cdot 0}=\frac{1}{2\pi}\int_{-1}^{1}1=\frac{1}{2\pi}\cdot\left[1-(-1)\right]=\frac{2}{2\pi}=\frac{1}{\pi}$

(19) AMPL. SPEKTAR ZA $\omega=\frac{\pi}{2}$, $x(n)=2^{-n}\mu(n)$

$$X(\omega)=\sum_{-\infty}^{\infty}x(n)\cdot e^{-j\omega n}\Rightarrow X\left(\frac{\pi}{2}\right)=\sum_{0}^{\infty}\underbrace{2^{-n}}_{\left(\frac{1}{2}\right)^{n}}\cdot e^{-j\frac{\pi}{2}n}=1+\frac{1}{2}e^{-j\frac{\pi}{2}}+\frac{1}{4}e^{-j\pi}+\frac{1}{8}e^{-j\frac{3\pi}{2}}+\frac{1}{16}e^{-2j\pi}+\dots=$$

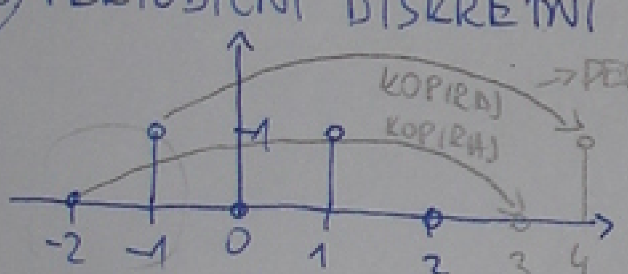
$$=1-\frac{1}{2}j-\frac{1}{4}+\frac{1}{8}j+\frac{1}{16}-\frac{1}{32}j+\dots\Rightarrow$$

$$\left.\begin{aligned}Re[X(\omega)]&=1-\frac{1}{4}+\frac{1}{16}-\frac{1}{64}+\dots\approx 0{,}8\\ Im[X(\omega)]&=-\frac{1}{2}+\frac{1}{8}-\frac{1}{32}+\frac{1}{128}+\dots\approx -0{,}4\end{aligned}\right\}\sqrt{Im^2+Re^2}=0{,}894=\frac{2}{\sqrt{5}}\ \text{(B)}$$

ILI: $X\left(\frac{\pi}{2}\right)=\sum_{0}^{\infty}\frac{1}{2}^{n}\cdot e^{-j\frac{\pi}{2}n}=\sum_{0}^{\infty}\left(\frac{1}{2}e^{-j\frac{\pi}{2}}\right)^{n}=\frac{1}{1-\frac{1}{2}e^{-j\frac{\pi}{2}}}=\frac{1}{1+\frac{1}{2}j}=\frac{2}{2+j}\cdot\frac{2-j}{2-j}=\frac{4-2j}{4+1}$

$$\left.\begin{aligned}Re\left[X\left(\tfrac{\pi}{2}\right)\right]&=\frac{4}{5}\\ Im\left[X\left(\tfrac{\pi}{2}\right)\right]&=j\cdot\frac{-2}{5}\end{aligned}\right\}\sqrt{Re^2+Im^2}=\sqrt{\frac{16}{25}+\frac{4}{25}}=\sqrt{\frac{20}{25}}=\frac{\sqrt{20}}{5}=\frac{\sqrt{4\cdot 5}}{5}=\frac{2\sqrt{5}}{5}=\frac{2}{\sqrt{5}}\ \text{(B)}$$

(20) PERIODIČNI DISKRETNI SIGNAL ?



→PERIODIČNI SIGNAL

$\Rightarrow N=5$

$$\Rightarrow X_k=\frac{1}{N}\sum_{0}^{N-1}x(n)\cdot e^{\frac{-2\pi jkn}{N}}\qquad e^{-j\frac{8\pi}{5}k+2\pi k}=e^{j\frac{2\pi}{5}k}$$

$$\Rightarrow X_k=\frac{1}{5}\sum_{n=0}^{4}x(n)\cdot e^{\frac{-2\pi jkn}{5}}=\frac{1}{5}\left[e^{-j\frac{2\pi}{5}k}+e^{-j\frac{8\pi}{5}k}\right]=\frac{1}{5}\left[e^{-j\frac{2\pi}{5}k}+e^{j\frac{2\pi}{5}k}\right]=\frac{1}{5}\cdot 2\cos\left(\frac{2\pi}{5}k\right)$$

AUTOMATSKI UVRŠTAVAJ ZA $X_0, X_1, X_2, X_3, X_4 \dots$ U RJEŠENJA, JEDNO PO JEDNO