1.

A

$$x(t) = \sin \frac{5\pi}{3} t \left(\mu(t) - \mu(t-6)\right)$$

a) $f_s = 1\,Hz$

$T_s = 1\,s$

$$x(nT) = \sin \frac{5\pi}{3} nT \left(\mu(nT) - \mu(nT-6)\right)$$

$$x(n) = \sin \frac{5\pi}{3} n \left(\mu(n) - \mu(n-6)\right)$$

$$= \{\underline{0}, -\tfrac{\sqrt{3}}{2}, -\tfrac{\sqrt{3}}{2}, 0, \tfrac{\sqrt{3}}{2}, \tfrac{\sqrt{3}}{2}\}$$

b) DTFT

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n} =$$

$$= -\frac{\sqrt{3}}{2} e^{-j\Omega} - \frac{\sqrt{3}}{2} e^{-j\Omega 2} + \frac{\sqrt{3}}{2} e^{-j\Omega 4} + \frac{\sqrt{3}}{2} e^{-j\Omega 5}$$

$$= -\frac{\sqrt{3}}{2} e^{-j3\Omega}\left(e^{+j2\Omega} + e^{j\Omega} - e^{-j\Omega} - e^{-j2\Omega}\right)$$

$$= -\frac{\sqrt{3}}{2} e^{-j3\Omega}\left(2j\sin 2\Omega + 2j\sin\Omega\right)$$

$$= -\sqrt{3}\, j\, e^{-j3\Omega}\left(\sin\Omega + \sin 2\Omega\right)$$

c) DFT $N=6$

$$X_k = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{6}kn}$$

$$= -\frac{\sqrt{3}}{2} e^{-j\frac{2\pi}{6}k\cdot 1} - \frac{\sqrt{3}}{2} e^{-j\frac{2\pi}{6}k\cdot 2} + \frac{\sqrt{3}}{2} e^{-j\frac{2\pi}{6}k4} + \frac{\sqrt{3}}{2} e^{-j\frac{2\pi}{6}k5}$$

$$= -\frac{\sqrt{3}}{2} e^{-j\frac{2\pi}{6}k3}\left(e^{j\frac{2\pi}{6}k2} + e^{j\frac{2\pi}{6}k} - e^{-j\frac{2\pi}{6}k} - e^{-j\frac{2\pi}{6}k2}\right)$$

$$= -\frac{\sqrt{3}}{2} e^{-j\frac{6\pi}{6}k}\left(2j\sin\frac{4\pi}{6}k + 2j\sin\frac{2\pi}{6}k\right)$$

$$= -\frac{\sqrt{3}}{2}\left(\cos\pi k - j\sin\pi k\right)\left(2j\sin\frac{2\pi}{3}k + 2j\sin\frac{\pi}{3}k\right)$$

$$= -\sqrt{3}\, j \cos\pi k\left(\sin\frac{\pi}{3}k + \sin\frac{2\pi}{3}k\right)$$

$X_0 = -\sqrt{3}j \cdot (0+0) = 0$

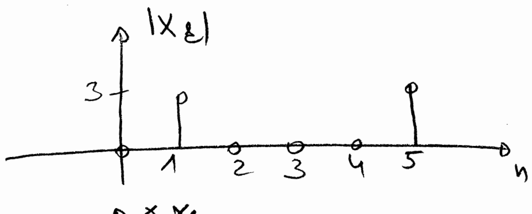
$X_1 = -\sqrt{3}\, j\, (-1)\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) = 3j$

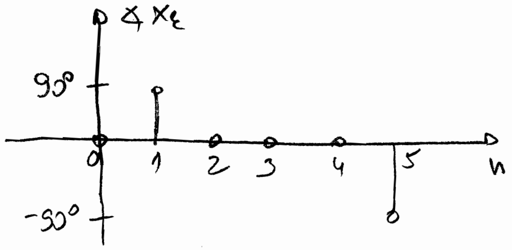
$X_2 = -\sqrt{3}\, j \cdot 1 \cdot \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) = 0$

$X_3 = -\sqrt{3}\, j\, (-1)(0) = 0$

$X_4 = -\sqrt{3}\, j \cdot 1 \cdot \left(-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) = 0$

$X_5 = -\sqrt{3}\, j \cdot (-1) \cdot \left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) = -3j$





2. $y(n) - \frac{1}{4} y(n-2) = u(n)$ (A)

a) PRIJENOSNA FUNKCIJA

$$Y(z) - \frac{1}{4} z^{-2} Y(z) = U(z)$$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{1}{1-\frac{1}{4}z^{-2}} = \frac{z^2}{z^2-\frac{1}{4}}$$

b) $z^2 - \frac{1}{4} = 0$

$z^2 = \frac{1}{4}$

$z_{1,2} = \pm \frac{1}{2}$

$|z_{1,2}| < 1$ SUSTAV JE STABILAN

FREKVENCIJSKE KARAKTERISTIKE

$$H(e^{j\Omega}) = \frac{1}{1-\frac{1}{4}e^{-2j\Omega}} = \frac{4}{4-e^{-2j\Omega}} = \frac{4}{4-\cos 2\Omega + j\sin 2\Omega} = 4 \cdot \frac{4-\cos 2\Omega - j\sin 2\Omega}{17-8\cos 2\Omega}$$

AMPLITUDNO FREKV.

$$|H(e^{j\Omega})| = \frac{4}{\sqrt{(4-\cos 2\Omega)^2 + \sin^2 2\Omega}} = \frac{4}{\sqrt{-8\cos 2\Omega + 17}}$$

FAZNO FREKV.

$$\sphericalangle H(e^{j\Omega}) = -\operatorname{arctg} \frac{\sin 2\Omega}{4-\cos 2\Omega}$$

c) STACIONARNO STANJE = prisilni odziv = partikularno rješenje

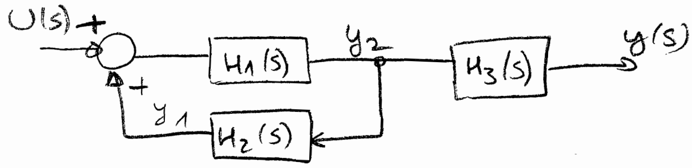
$u(n) = \sin\left(\frac{\pi}{2}n + \frac{\pi}{2}\right)$ $\Omega = \frac{\pi}{2}$ $H(e^{j\Omega}) = \frac{4}{4-\cos\pi + j\sin\pi} = \frac{4}{4+1} = \frac{4}{5}$

$$|H(e^{j\frac{\pi}{2}})| = \frac{4}{\sqrt{17-8\cos 2\cdot\frac{\pi}{2}}} = \frac{4}{\sqrt{17+8}} = \frac{4}{5}$$

$$\sphericalangle H(e^{j\frac{\pi}{2}}) = -\operatorname{arctg} \frac{\sin 2\cdot\frac{\pi}{2}}{4-\cos 2\cdot\frac{\pi}{2}} = 0$$

$$y(n) = \frac{4}{5}\sin\left(\frac{\pi}{2}n + \frac{\pi}{2}\right)$$

3. (A)



$h_1(t) = \cos 2t\, \mu(t)$ $\qquad$ $h_2(t) = 13\,\mu(t)$ $\qquad$ $h_3(t) = -48 e^{-5t} \mu(t)$

$H_1(s) = \frac{s}{s^2+4}$ $\qquad$ $H_2(s) = \frac{13}{s}$ $\qquad$ $H_3(s) = -48 \cdot \frac{1}{s+5}$

$(U + y_1) H_1 = y_2$

$y_1 = H_2 \cdot y_2$ $\qquad$ $y = y_2 \cdot H_3$

$(U + H_2 y_2) H_1 = y_2$ $\qquad$ $y = \frac{H_1 H_3}{1 + H_1 H_2} \cdot U$

$UH_1 = -H_1 H_2 y_2 + y_2$

$\frac{y_2}{U} = \frac{H_1}{1 - H_1 H_2}$

$$\boxed{H(s) = \frac{H_1(s) H_3(s)}{1 - H_1(s) H_2(s)}}$$

PRIJENOSNA FUNKCIJA

$$\boxed{H(s) = \frac{\frac{s}{s^2+4} \cdot \frac{-48}{s+5}}{1 - \frac{s}{s^2+4} \cdot \frac{13}{\cancel{s}}} = \frac{\frac{-48s}{\cancel{(s^2+4)}(s+5)}}{\frac{s^2-9}{\cancel{s^2+4}}} = \frac{-48s}{(s^2-9)(s+5)}}$$

b) POLOVI

$s^2 - 9 = 0$ $\qquad$ $s_3 = -5$ $\qquad$ Re $s = 3$ je > 0 → SUSTAV je NESTABILAN.

$s^2 = 9$

$s_{1,2} = \pm 3$

c) $u(t) = \mu(t)$

$U(s) = \frac{1}{s}$

$$y(s) = H(s)\,U(s) = \frac{-48\cancel{s}}{(s-3)(s+3)(s+5)} \cdot \frac{1}{\cancel{s}} = \frac{-48}{(s-3)(s+3)(s+5)}$$

$$= \frac{A}{s-3} + \frac{B}{s+3} + \frac{C}{s+5}$$

$As^2 + 8sA + 15A + Bs^2 + 2Bs - 15B + Cs^2 - 9C = -48$

$A + B + C = 0$ $\qquad$ $C = -A - B$ $\qquad$ $C = -A + 4A = 3A$

$8A + 2B = 0$ $\qquad$ $2B = -8A$ $\qquad$ $B = -4A$

$15A - 15B - 9C = -48$ $\qquad$ $15A - 15 \cdot (-4A) - 9 \cdot 3A = -48$

$(15 + 60 - 27) A = -48$

$48A = -48$

$A = -1$ $\qquad$ $B = 4$ $\qquad$ $C = -3$

$y(s) = \frac{-1}{s-3} + \frac{4}{s+3} - \frac{3}{s+5}$

$$\boxed{y(t) = \left(-e^{3t} + 4e^{-3t} - 3e^{-5t}\right)\mu(t)}$$

4.

$\boxed{A}$

$$y_1(n+1) - 2y_2(n) = u(n)$$
$$-2y_1(n) + y_2(n+1) = u(n)$$

a) $x_1(n) = y_1(n)$ $\quad$ $x_2(n) = y_2(n)$ $\rightarrow$ $x_1(n+1) = y_1(n+1) = u(n) + 2y_2(n) = 2x_2(n) + u(n)$

$x_2(n+1) = y_2(n+1) = u(n) + 2y_1(n) = 2x_1(n) + u(n)$

$$\begin{bmatrix} x_1(n+1) \\ x_2(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(n)$$

$$\begin{pmatrix} y_1(n) \\ y_2(n) \end{pmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u(n)$$

$$A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

b) $\phi(z) = z(zI - A)^{-1}$

$$= z\begin{bmatrix} z & -2 \\ -2 & z \end{bmatrix}^{-1} = \frac{z}{z^2-4}\begin{bmatrix} z & 2 \\ 2 & z \end{bmatrix} = \begin{bmatrix} \frac{z^2}{(z-2)(z+2)} & \frac{2z}{(z-2)(z+2)} \\ \frac{2z}{(z-2)(z+2)} & \frac{z^2}{(z-2)(z+2)} \end{bmatrix}$$

$\frac{Az}{z-2} + \frac{Bz}{z+2} = \frac{z^2}{(z-2)(z+2)}$

$A + B = 1$

$2A - 2B = 0 \rightarrow A = B = \frac{1}{2}$

$\frac{Az}{z-2} + \frac{Bz}{z+2} = \frac{2z}{(z-2)(z+2)}$

$A + B = 0 \quad A = -B$

$2A - 2B = 2 \quad -4B = 2$

$B = -\frac{1}{2} \quad A = \frac{1}{2}$

$$\boxed{\phi(z) = \begin{pmatrix} \frac{\frac{1}{2}}{z-2} + \frac{\frac{1}{2}}{z+2} & \frac{\frac{1}{2}}{z-2} - \frac{\frac{1}{2}}{z+2} \\ \frac{\frac{1}{2}}{z-2} - \frac{1/2}{z+2} & \frac{1/2}{z-2} + \frac{1/2}{z+2} \end{pmatrix}} \rightarrow \boxed{\phi(n) = \begin{pmatrix} \frac{1}{2}(2)^n + \frac{1}{2}(-2)^n & \frac{1}{2}2^n - \frac{1}{2}(-2)^n \\ \frac{1}{2}(2)^n - \frac{1}{2}(-2)^n & \frac{1}{2}2^n + \frac{1}{2}(-2)^n \end{pmatrix} \mu(n)}$$

c) $H(z) = C(zI - A)^{-1}B + D$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{z^2-4}\begin{bmatrix} z & 2 \\ 2 & z \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{z+2}{z^2-4} \\ \frac{z+2}{z^2-4} \end{bmatrix} = \boxed{\begin{bmatrix} \frac{1}{z-2} \\ \frac{1}{z-2} \end{bmatrix}}$$

d) $$\frac{H(z)}{z} = \begin{bmatrix} \frac{A}{z} + \frac{B}{z-2} \\ \frac{A}{z} + \frac{B}{z-2} \end{bmatrix}$$

$A + B = 0$

$-2A = 1$

$A = -\frac{1}{2}$

$B = \frac{1}{2}$

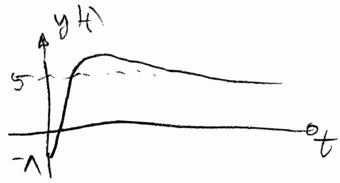
$$\frac{H(z)}{z} = \begin{bmatrix} -\frac{1}{2z} + \frac{1/2}{z-2} \\ -\frac{1}{2z} + \frac{1/2}{z-2} \end{bmatrix}$$

$$H(z) = \begin{bmatrix} -\frac{1}{2} + \frac{1/2\, z}{z-2} \\ -\frac{1}{2} + \frac{1/2\, z}{z-2} \end{bmatrix}$$

$$\boxed{h(n) = \begin{bmatrix} -\frac{1}{2}\delta(n) + \frac{1}{2}2^n\mu(n) \\ -\frac{1}{2}\delta(n) + \frac{1}{2}2^n\mu(n) \end{bmatrix}}$$

5.

$$H(s)=\frac{K}{s^2+as+b}$$



$u(t)=\mu(t)$

$u(t)=\sin 12t \rightarrow y(t)=\frac{1}{\sqrt{2}}\sin\left(12t-\frac{3\pi}{4}\right)$

(A)

a) STABILAN JE
jer na ograničenu pobudu $\mu(t)$ daje ograničen odziv sa stacionarnim stanjem 5.

b)
$$H(s)=\frac{K}{s^2+as+b}=\frac{Y(s)}{U(s)}$$

$$y''(t)+ay'(t)+by(t)=K\,u(t)$$

$y_p=\frac{1}{\sqrt{2}}\sin\left(12t-\frac{3\pi}{4}\right) \quad \rightarrow u(t)=\sin 12t$

$y_p'=\frac{12}{\sqrt{2}}\cos\left(12t-\frac{3\pi}{4}\right)$

$y_p''=-\frac{144}{\sqrt{2}}\sin\left(12t-\frac{3\pi}{4}\right)$

$$-\frac{144}{\sqrt{2}}\sin\left(12t-\frac{3\pi}{4}\right)+\frac{12a}{\sqrt{2}}\cos\left(12t-\frac{3\pi}{4}\right)+\frac{b}{\sqrt{2}}\sin\left(12t-\frac{3\pi}{4}\right)=K\sin 12t$$

$$=K\sin\left(12t-\frac{3\pi}{4}+\frac{3\pi}{4}\right)$$

$$=K\sin\left(12t-\frac{3\pi}{4}\right)\cos\frac{3\pi}{4}+K\sin\frac{3\pi}{4}\cos\left(12t-\frac{3\pi}{4}\right)$$

$-\frac{144}{\cancel{\sqrt{2}}}+\frac{b}{\cancel{\sqrt{2}}}=K\cos\frac{3\pi}{4}=K\cdot\cancel{\frac{-\sqrt{2}}{2}}$ $\qquad -144+b=-K$

$\frac{12a}{\sqrt{2}}=K\sin\frac{3\pi}{4}=+K\frac{\sqrt{2}}{2}$ $\qquad 12a=+K$

$K=+12a$

$u(t)=\mu(t)$

$y_p(t)=5\mu(t)$

$y_p'(t)=0$

$y_p''(t)=0$

$5b=K\cdot 1$

$K=5b$

$\rightarrow -144+b=-5b$

$-144=-6b$

$b=24$

$K=5\cdot 24$

$K=120$

$a=\frac{K}{12}=\frac{120}{12}$

$a=10$

$$\boxed{H(s)=\frac{120}{s^2+10s+24}}$$

c) $\boxed{y''(t)+10y'(t)+24y(t)=120\,u(t)}$

d) $\boxed{y(0^+)=-1}$

$y(1)=5.128$

POLOVI

$s^2+10s+24=0$

$(s+6)(s+4)=0$

$s_1=-6$

$s_2=-4$

$y_h=C_1e^{-6t}+C_2e^{-4t}$

$y_t=C_1e^{-6t}+C_2e^{-4t}+5$

$\boxed{y_t(0)=C_1+C_2+5=-1}$

$y_t(1)=e^{-6}C_1+e^{-4}C_2+5=5.128$

$C_1+C_2=-6 \quad /\cdot(-e^{-6})$

$e^{-6}C_1+e^{-4}C_2=0.128$

$-e^{-6}C_1-e^{-6}C_2=+6e^{-6}$

$(e^{-4}-e^{-6})C_2=0.128+6e^{-6}$

$C_2=9.02\approx 9$

$C_1=-6-C_2$

$=-6-9$

$C_1=-15$

$y_t(t)=\left(-15e^{-6t}+9e^{-4t}+5\right)\mu$

$y_t'(t)=-15\cdot(-6)e^{-6t}+9\cdot(-4)e^{-4t}$

$=\left(90e^{-6t}-36e^{-4t}\right)\mu(t)$

$\boxed{y_t'(0^+)=90-36=54}$

1. $x(t) = \sin \frac{4\pi}{3} t \, (\mu(t) - \mu(t-6))$

3

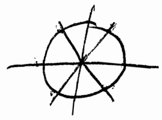
a) $f_s = 1\,Hz$

$T_s = 1\,s$

$x(nT) = \sin\left(\frac{4\pi}{3} nT\right)(\mu(nT) - \mu(nT-6))$

$x(n) = \sin \frac{4\pi}{3} n \, (\mu(n) - \mu(n-6))$

$= \{\underline{0}, -\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}, 0, -\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2}\}$



b) DTFT

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\Omega n} =$$

$$= -\frac{\sqrt{3}}{2} e^{-j\Omega} + \frac{\sqrt{3}}{2} e^{-j2\Omega} - \frac{\sqrt{3}}{2} e^{-j4\Omega} + \frac{\sqrt{3}}{2} e^{-j5\Omega}$$

$$= -\frac{\sqrt{3}}{2} e^{-j3\Omega}\left(e^{j2\Omega} - e^{j\Omega} + e^{-j\Omega} - e^{-2j\Omega}\right)$$

$$= -\frac{\sqrt{3}}{2} e^{-j3\Omega}\left(2j \sin 2\Omega - 2j \sin \Omega\right)$$

$$= -\sqrt{3} j \, e^{-j3\Omega}\left(\sin 2\Omega - \sin \Omega\right)$$

c) DFT $N=6$

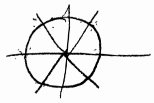
$$X_k = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$= -\frac{\sqrt{3}}{2} e^{-j\frac{2\pi}{6}k} + \frac{\sqrt{3}}{2} e^{-j2\frac{2\pi}{6}k} - \frac{\sqrt{3}}{2} e^{-j4\frac{2\pi}{6}k} + \frac{\sqrt{3}}{2} e^{-j5\frac{2\pi}{6}k}$$

$$= -\frac{\sqrt{3}}{2} e^{-j\frac{2\pi}{6}3k}\left(e^{j\frac{2\pi}{6}\cdot 2k} - e^{j\frac{2\pi}{6}k} + e^{-j\frac{2\pi}{6}k} - e^{-j\frac{2\pi}{6}k\cdot 2}\right)$$

$$= -\frac{\sqrt{3}}{2} e^{-j\frac{6\pi}{6}k}\left(2j \sin\left(\frac{2\pi}{6}\cdot 2k\right) - 2j \sin \frac{2\pi}{6} k\right)$$

$$= -\sqrt{3} j \, (\cos \pi k - j \sin \pi k)\left(\sin \frac{2\pi}{3} k - \sin \frac{\pi}{3} k\right)$$



$$= -\sqrt{3} j \cos \pi k \left(\sin \frac{2\pi}{3} k - \sin \frac{\pi}{3} k\right)$$

$X_0 = -\sqrt{3} \cdot j \cdot 0 = 0$

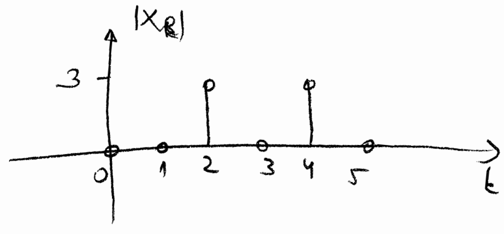
$X_1 = -\sqrt{3} \cdot j \, (-1) \cdot \left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) = 0$

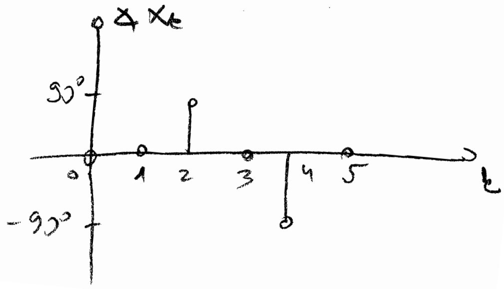
$X_2 = -\sqrt{3} j \cdot \left(-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right) = 3j$

$X_3 = 0$

$X_4 = -\sqrt{3} j \left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) = -3j$

$X_5 = -\sqrt{3} j (-1)\left(-\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right) = 0$





2. $y(n) - \frac{1}{9} y(n-2) = u(n)$ $\boxed{B}$

a) $Y(z) - \frac{1}{9} z^{-2} Y(z) = U(z)$

$Y(z)\left(1 - \frac{1}{9} z^{-2}\right) = U(z)$

$H(z) = \frac{Y(z)}{U(z)} = \frac{1}{1-\frac{1}{9}z^{-2}} = \frac{z^2}{z^2 - \frac{1}{9}}$

b) $H(e^{j\Omega}) = \frac{1}{1-\frac{1}{9}e^{-2j\Omega}} = \frac{1}{1-\frac{1}{9}\cos 2\Omega + \frac{1}{9} j \sin 2\Omega} = \frac{9}{9-\cos 2\Omega + j\sin 2\Omega}$

$|H(e^{j\Omega})| = \frac{1}{\sqrt{(1-\frac{1}{9}\cos 2\Omega)^2 + (\frac{1}{9}\sin 2\Omega)^2}} = \frac{1}{\sqrt{1 - \frac{2}{9}\cos 2\Omega + \frac{1}{81}}}$

$= \frac{1}{\sqrt{\frac{82}{81} - \frac{2}{9}\cos 2\Omega}} = \frac{9}{\sqrt{82 - 18\cos 2\Omega}}$

$\measuredangle H(e^{j\Omega}) = -\text{arctg}\, \frac{\sin 2\Omega}{9 - \cos 2\Omega}$

POLOVI

$z^2 - \frac{1}{9} = 0$

$(z - \frac{1}{3})(z + \frac{1}{3}) = 0$

$z = \frac{1}{3}$ $\quad |z| < 1$ $\quad$ sustav je stabilan

$z = -\frac{1}{3}$ $\quad$ IMA frekvencijsku karakteristiku

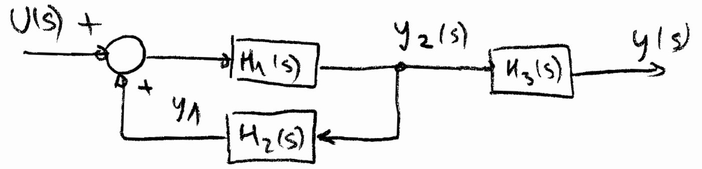
c) $u(n) = \sin\left(\frac{\pi}{3} n + \frac{\pi}{3}\right)$

$\Omega = \frac{\pi}{3}$

$|H(e^{j\frac{\pi}{3}})| = \frac{9}{\sqrt{82 - 18\cos\frac{2\pi}{3}}} = \frac{9}{\sqrt{82 - 18\cdot(-\frac{1}{2})}} = \frac{9}{\sqrt{82+9}} = \frac{9}{\sqrt{91}} = 0.943$

$\measuredangle H(e^{j\frac{\pi}{3}}) = -\text{arctg}\, \frac{\sin\frac{2\pi}{3}}{9 - \cos\frac{2\pi}{3}} = -\text{arctg}\, \frac{\frac{\sqrt{3}}{2}}{9 - \frac{\sqrt{3}}{2}} = -0.091$

$y(n) = 0.943 \sin\left(\frac{\pi}{3} n + \frac{\pi}{3} - 0.091\right) = 0.943 \sin\left(\frac{\pi}{3} n + 0.956\right)$

3. 

B

$h_1(t) = \cos 3t\, \mu(t)$ $\qquad h_2(t) = 25\, \mu(t)$ $\qquad h_3(t) = -72\, e^{-5t} \mu(t)$

$H_1(s) = \frac{s}{s^2+9}$ $\qquad H_2(s) = \frac{25}{s}$ $\qquad H_3(s) = -72 \frac{1}{s+5}$

$(U + y_1) H_1 = y_2$

$y_1 = H_2 y_2$

$(U + H_2 y_2) H_1 = y_2$

$\frac{y_2}{U} = \frac{H_1}{1 - H_1 H_2}$

$y = y_2 H_3$

$y = \frac{H_1 H_3}{1 - H_1 H_2} U$

$\boxed{H(s) = \frac{H_1(s) H_2(s)}{1 - H_1(s) H_2(s)}}$

$$H(s) = \frac{\frac{s}{\cancel{s^2+9}} \cdot \frac{-72}{s+5}}{1 - \frac{\cancel{s}}{s^2+9} \cdot \frac{25}{\cancel{s}}} = \frac{-72 s}{(s+5)(s^2-16)} = \boxed{\frac{-72 s}{(s+5)(s-4)(s+4)} = H(s)}$$

b) POLOVI

$s_1 = -5$

$s_2 = 4 \rightarrow > 0$ NESTABILNO

$s_3 = -4$

c) $u(t) = \mu(t)$

$U(s) = \frac{1}{s}$

$$y(s) = H(s) U(s) = \frac{\cancel{s}}{(s+5)(s-4)(s+4)} \cdot \frac{1}{\cancel{s}} = \frac{-72}{(s+5)(s-4)(s+4)}$$

$$= \frac{A}{s+5} + \frac{B}{s-4} + \frac{C}{s+4}$$

$A s^2 - 16A + B s^2 + 9 s B + 20 B + C s^2 + C s - 20 C = -72$

$A + B + C = 0$ $\qquad A + B - 9B = 0$ $\qquad A - 8B = 0$ $\qquad A = 8B$

$9B + C = 0$ $\qquad C = -9B$

$-16A + 20B - 20C = -72$ $\qquad -16 \cdot 8B + 20B - 20 \cdot (-9) B = -72$

$(-128 + 20 + 180) B = -72$

$72 B = -72$

$B = -1$

$A = -8$

$C = 9$

$y(s) = \frac{-8}{s+5} - \frac{1}{s-4} + \frac{9}{s+4}$

$\boxed{y(t) = (-8 e^{-5t} - e^{4t} + 9 e^{-4t}) \mu(t)}$

4. $y_1(n+1) - 4\, y_2(n) = u(n)$

$-4y_1(n) + y_2(n+1) = u(n)$

B

a) $x_1(n) = y_1(n)$ $\quad x_1(n+1) = y_1(n+1) = u(n) + 4y_2(n) = 4x_2(n) + u(n)$

$x_2(n) = y_2(n)$ $\quad x_2(n+1) = y_2(n+1) = u(n) + 4y_1(n) = 4x_1(n) + u(n)$

$$\begin{bmatrix} x_1(n+1) \\ x_2(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(n)$$

$$\begin{bmatrix} y_1(n) \\ y_2(n) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} u(n)$$

$A = \begin{bmatrix} 0 & 4 \\ 4 & 0 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $\quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

b) $\phi(z) = z(zI - A)^{-1}$

$$= z \begin{bmatrix} z & -4 \\ -4 & z \end{bmatrix}^{-1} = \frac{z}{z^2-16} \begin{bmatrix} z & 4 \\ 4 & z \end{bmatrix} = \begin{bmatrix} \frac{z^2}{(z-4)(z+4)} & \frac{4z}{(z-4)(z+4)} \\ \frac{4z}{(z-4)(z+4)} & \frac{z^2}{(z-4)(z+4)} \end{bmatrix}$$

$$\frac{z^2}{(z-4)(z+4)} = \frac{Az}{z-4} + \frac{Bz}{z+4}$$

$A+B=1$

$4A-4B=0$

$A=B=\frac{1}{2}$

$$\frac{4z}{(z-4)(z+4)} = \frac{Az}{z-4} + \frac{Bz}{z+4}$$

$A+B=0$

$4A-4B=4$

$A=-B$

$A-B=1$

$-2B=1$

$B=-\frac{1}{2}$ $\quad A=\frac{1}{2}$

$$\boxed{\phi(z) = \begin{bmatrix} \frac{1}{2}\frac{z}{z-4} + \frac{1}{2}\frac{z}{z+4} & \frac{1}{2}\frac{z}{z-4} - \frac{1}{2}\frac{z}{z+4} \\ \frac{1}{2}\frac{z}{z-4} - \frac{1}{2}\frac{z}{z+4} & \frac{1}{2}\frac{z}{z-4} + \frac{1}{2}\frac{z}{z+4} \end{bmatrix}}$$

$$\boxed{\phi(n) = \begin{bmatrix} \frac{1}{2}(4)^n + \frac{1}{2}(-4)^n & \frac{1}{2}4^n - \frac{1}{2}(-4)^n \\ \frac{1}{2}4^n - \frac{1}{2}(-4)^n & \frac{1}{2}4^n + \frac{1}{2}(-4)^n \end{bmatrix} \mu(n)}$$

c) $H(z) = C(zI - A)^{-1}B + D$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z & 4 \\ 4 & z \end{bmatrix} \frac{1}{z^2-16} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} z+4 \\ z+4 \end{bmatrix} \frac{1}{(z-4)(z+4)}$$

$$\boxed{H(z) = \begin{bmatrix} \frac{1}{z-4} \\ \frac{1}{z-4} \end{bmatrix}}$$

d) $$\frac{H(z)}{z} = \begin{bmatrix} \frac{1}{z(z-4)} \\ \frac{1}{z(z-4)} \end{bmatrix} = \begin{bmatrix} \frac{A}{z} + \frac{B}{z-4} \\ \frac{A}{z} + \frac{B}{z-4} \end{bmatrix} = \begin{bmatrix} -\frac{1}{4z} + \frac{1}{4}\frac{1}{z-4} \\ -\frac{1}{4z} + \frac{1}{4}\frac{1}{z-4} \end{bmatrix}$$

$A+B=0$

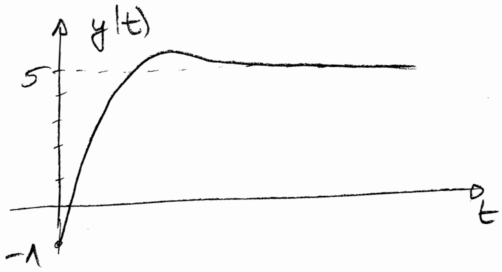
$-4A=1$

$A=-\frac{1}{4}$

$B=\frac{1}{4}$

$$H(z) = \begin{bmatrix} -\frac{1}{4} + \frac{1}{4}\frac{z}{z-4} \\ -\frac{1}{4} + \frac{1}{4}\frac{z}{z-4} \end{bmatrix} \rightarrow \boxed{h(n) = \begin{bmatrix} -\frac{1}{4}\delta(n) + \frac{1}{4}4^n\mu(n) \\ -\frac{1}{4}\delta(n) + \frac{1}{4}4^n\mu(n) \end{bmatrix}}$$

5.



B

a) sustav je STABILAN zato što na ograničenu pobudu ($\mu(t)$) daje ograničen odziv, sa stacionarnim stanjem.

b) $H(s) = \dfrac{K}{s^2+as+b} = \dfrac{Y}{U}$ $\qquad K \cdot u = y'' + ay' + by$

- $u(t) = \mu(t)$ $\qquad y_p(t) = 5\mu(t)$ $\qquad b \cdot K_1 = K \cdot 1$
  $U(s) = \frac{1}{s}$ $\qquad y_p = K_1$ $\qquad b \cdot 5 = K$
  $y_p' = 0$ $\qquad K = 5b$
  $y_p'' = 0$

- $u(t) = \sin 6t$

$$y_p(t) = \frac{1}{\sqrt{2}} \sin\left(6t - \frac{3\pi}{4}\right)$$

$$y_p' = \frac{1}{\sqrt{2}} 6 \cos\left(6t - \frac{3\pi}{4}\right)$$

$$y_p'' = -\frac{36}{\sqrt{2}} \sin\left(6t - \frac{3\pi}{4}\right)$$

$$-\frac{36}{\sqrt{2}} \sin\left(6t-\frac{3\pi}{4}\right) + a\frac{6}{\sqrt{2}}\cos\left(6t-\frac{3\pi}{4}\right) + b\frac{1}{\sqrt{2}}\sin\left(6t-\frac{3\pi}{4}\right) = K \cdot \sin 6t$$

$$= K \sin\left(6t - \frac{3\pi}{4} + \frac{3\pi}{4}\right)$$

$$= K \sin\left(6t-\frac{3\pi}{4}\right)\cos\frac{3\pi}{4} + K\cos\left(6t-\frac{3\pi}{4}\right)\sin\frac{3\pi}{4}$$

$$= -\frac{K\sqrt{2}}{2}\sin\left(6t-\frac{3\pi}{4}\right) + \frac{\sqrt{2}}{2}K\cos\left(6t-\frac{3\pi}{4}\right)$$

$-36 + b = -K$ $\qquad -36 + b = -5b$
$6a = K$ $\qquad -36 = -6b$
$K = 5b$ $\qquad b = 6 \qquad K = 30 \qquad 6a = 30$
$a = 5$

$$H(s) = \frac{30}{s^2+5s+6}$$

c) $y''(t) + 5y'(t) + 6y(t) = 30u(t)$

d) Iz danog odziva moguće je odrediti početne uvjete:

$y(t=1) = 5.471$

$\boxed{y(t=0) = -1}$

$s^2+5s+6=0$
$(s+2)(s+3)=0$

$y_h(t) = C_1 e^{-2t} + C_2 e^{-3t}$

$y_t(t) = C_1 e^{-2t} + C_2 e^{-3t} + 5$

$y_t(0) = C_1 + C_2 + 5 = -1$

$y_t(1) = e^{-2}C_1 + e^{-3}C_2 + 5 = 5.471$

$C_1 + C_2 = -6 \quad / \cdot (-e^{-2})$
$e^{-2}C_1 + e^{-3}C_2 = 0.471$
$-e^{-2}C_1 - e^{-2}C_2 = 6e^{-2}$
$(e^{-3} - e^{-2})C_2 = 0.471 + 6e^{-2}$
$C_2 = -14.957 \approx -15$

$C_1 = -6 - C_2$
$= -6 + 15$
$C_1 = 9$

$y_t(t) = (+9e^{-2t} - 15e^{-3t} + 5)\mu(t)$

$y_t'(t) = 9 \cdot (-2)e^{-2t} - 15 \cdot (-3)e^{-3t}$
$= -18e^{-2t} + 45e^{-3t}$

$y_t'(0) = -18 + 45$

$\boxed{y'(t=0) = 27}$