Signali i sustavi - Zadaci za vježbu

IV. tjedan

1. Pretpostavite da želite uživo, preko Interneta slušati prijenos nekog koncerta. Pri tome Internet ne koristite za nikakav drugi prijenos podataka. Neka je za predstavljanje svakog audio uzorka potrebno 16 bita.

    a. Nalazite se kod kuće i spojeni ste s modemom, 56 kbps (kilobita u sekundi), na Internet. Kojom maksimalnom frekvencijom uzorkovanja može biti diskretiziran audio signal koji slušate?
    b. Koja je frekvencija u pitanju ako se nalazite na 100 Mbps LAN-u?

RJEŠENJE:

a)

v_I=56kbps=56000bps

Za jedan uzorak treba 16 bitova => N= 16 bit

v_I=56000bit/s

Frekvenciju cemo dobiti tako da podjelimo brzinu prijenosa sa kolicinom podataka po jednom uzorku.

$$f_{MAX} = \frac{v_I}{N} = \frac{56000 bit/s}{16 bit} = 3500 Hz$$

$$f_{MAX} = 3.5 kHz$$

b) Analogno prvom slučaju radimo i za drugi slučaj

v_I=100Mbps=100000000bps

$$f_{MAX} = \frac{v_I}{N} = \frac{100000000 bit/s}{16 bit} = 6.25 MHz$$

2. Zadan je diskretan signal $x(n) = \cos\left(\frac{n\pi}{8}\right)$. Nađite dva različita kontinuirana signala koja otipkavanjem daju ovaj diskretan signal. Frekvencija otipkavanja neka je $f_s = 10kHz$.

RJEŠENJE:

Zadan je diskretan signal $x(n) = \cos\left(\frac{n\pi}{8}\right)$.

Njega smo mogli dobiti iz nekog kontinuiranog signala $x(t) = \cos(\omega t)$ otipkavanjem $x(nT_s) = \cos(\omega n T_s)$.

Period otipkavanja je $T_s = \frac{1}{f_s} = 10000^{-1} s$.

Da bi otipkani signal i zadani diskretni signal bili jednaki mora vrijediti:

$$\cos\left(\frac{n\pi}{8}\right) = \cos(\omega n T_s),$$

$$\cos\left(\frac{n\pi}{8}\right) = \cos\left(\frac{2\pi f}{f_s} n\right).$$

$$\frac{n\pi}{8} = 2\pi n \frac{f}{f_s}$$

$$f = \frac{f_s}{16} = \frac{10000}{16} = 625Hz.$$

Početni kontinuirani signal je prema tome bio

$$x(t) = \cos(2\pi \cdot 625t) = \cos(1250t).$$

Primijetite da za neki cijeli broj k vrijedi i (zbog periodičnosti cos):

$$\cos\left(2\pi \frac{f}{f_s} n\right) = \cos\left(2\pi \frac{f + kf_s}{f_s} n\right).$$

Tako možemo izabrati i frekvenciju $f = 625 + 10000 = 10625Hz$ kontinuiranog signala koji će nakon otipkavanja imati jednak diskretan signal.

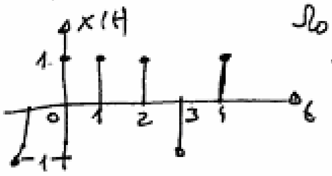
Drugi kontinuirani signal koji otipkavanjem daje početni diskretni je i

$$x(t) = \cos(2\pi \cdot 10625t) = \cos(21250t).$$

Ovo nisu jedini signali koji su rješenje zadatka. Nađite još neki.

3.

$T_0 = 4s$



$\Omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$

$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\frac{\pi}{2}\cdot t}$

$X_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\frac{\pi}{2}t} dt$

$$X_k = \frac{1}{4}\int_{0^-}^{4^-} x(t) e^{-jk\frac{\pi}{2}t} dt = \frac{1}{4}\left[\int_{0^-}^{1^-} e^{-jk\frac{\pi}{2}t}\delta(t) \cancel{dt} + \int_{1^-}^{2^-}\delta(t-1)e^{-jk\frac{\pi}{2}t} dt + \int_{2^-}^{3^-}\delta(t-2)e^{-jk\frac{\pi}{2}t} dt + \int_{3^-}^{4^-} -\delta(t-3)e^{-jk\frac{\pi}{2}t} dt\right]$$

$$= \frac{1}{4}\left[e^{-jk\frac{\pi}{2}\cdot 0} + e^{-jk\frac{\pi}{2}} + e^{-jk\frac{\pi}{2}\cdot 2} - e^{-jk\frac{\pi}{2}\cdot 3}\right]$$

$= \frac{1}{4}\left[1 + \dots\right]$

$\cos k\frac{\pi}{2} - j\sin k\frac{\pi}{2}$ → $(e^{-j\frac{\pi}{2}})^k$ → $(-j)^k$: $1, -j, -1, j, 1, \dots$

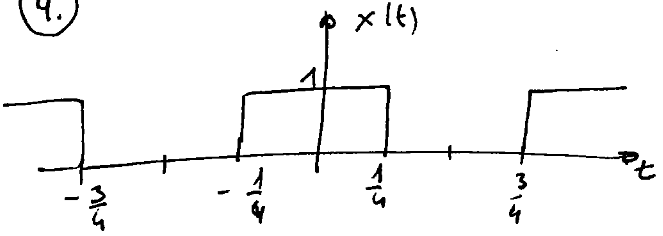
$\cos k\pi - j\sin k\pi$ → $(e^{-\pi j})^k$ → $(-1)^k$

$\cos k\frac{3\pi}{2} - j\sin k\frac{3\pi}{2}$ → $(e^{-j\frac{3\pi}{2}})^k$ → $(j)^k$

$$= \frac{1}{4}\left[1^k + (-j)^k + (-1)^k - j^k\right]$$

Konvergencija:
Fourierov red je konvergentan ako zadovoljava Dirichletove uvjete.

4.



$$x(t)=\sum_{k=-\infty}^{\infty} X_k\, e^{jk\Omega_0 t} \qquad T_0 = 1\ s$$

$$\Omega_0 = \frac{2\pi}{T_0} = 2\pi$$

$$X_k = \int_{-1/4}^{1/4} 1\, e^{-jk\cdot 2\pi t}\,dt$$

$$= \frac{1}{-jk\cdot 2\pi}\, e^{-jk2\pi t}\Big|_{-1/4}^{1/4} = \frac{1}{jk\pi\cdot 2}\left[e^{-jk\cdot\frac{\pi}{2}} - e^{+jk\frac{\pi}{2}}\right]$$

$$\cancel{\cos k\tfrac{\pi}{2}} - j\sin k\frac{\pi}{2} - \cancel{\cos k\tfrac{\pi}{2}} - j\sin k\frac{\pi}{2}$$

$$= \frac{-\cancel{j}\cdot\cancel{2}\sin k\frac{\pi}{2}}{-\cancel{j}k\pi\cdot\cancel{2}} = +\frac{1}{2}\,\frac{\sin\frac{k\pi}{2}}{\frac{k\pi}{2}}$$

$$X_0 = \int_{-1/4}^{1/4} dt = \frac{1}{4}+\frac{1}{4} = \frac{1}{2}$$

SREDNJA SNAGA

$$P_x = \frac{1}{T_0}\int_{T_0} |x(t)|^2 dt = \int_{-1/4}^{1/4} dt = \frac{1}{4}+\frac{1}{4} = \frac{1}{2}$$

$$P_x = \sum_{k=-\infty}^{\infty} |X_k|^2 = X_0^2 + \sum_{k=-\infty}^{0^-} |X_k|^2 + \sum_{k=0^+}^{\infty} |X_k|^2 = X_0^2 + 2\sum_{k=1}^{\infty} |X_k|^2$$

$$|X_0|^2 = \frac{1}{4}$$

$$|X_1|^2 = \left(\frac{\sin\frac{\pi}{2}}{\pi}\right)^2 = \left(\frac{1}{\pi}\right)^2 = 0.10132$$

$$|X_2|^2 = 0$$

$$|X_3|^2 = \left(\frac{-1}{3\pi}\right)^2 = 0.011258$$

$$|X_4|^2 = 0$$

$$|X_5|^2 = \left(\frac{1}{5\pi}\right)^2 = 0.004052$$

$\vdots$

$$P_x = \frac{1}{4} + 2\cdot(0.101 + 0.011 + 0.004\ldots) \to 0.5$$

$$P_x = \frac{1}{4} + 2\cdot\left(\frac{1}{\pi}\right)^2 \underbrace{\sum_{k=0}^{\infty}\left(\frac{1}{2k+1}\right)^2}_{\frac{\pi^2}{8}} = \frac{1}{4} + \cancel{2}\,\frac{1}{\cancel{\pi^2}}\cdot\frac{\cancel{\pi^2}}{\cancel{8}_4} = \frac{2}{4} = \frac{1}{2}$$

5. ZADATAK

(15)

$$x(t) = 10\cos(50\pi t) + 5\sin(100\pi t) + \sin(150\pi t + 2\pi/3) + \cos(200\pi t + \pi/4) \quad \Big\} \text{ FR ?}$$

Moramo odrediti period ovog signala T_0

Frekvencije njegovih komponenti su:

$\omega_1 = 50\pi$ rad/s $\Rightarrow T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{50\pi} = \frac{1}{25}$ [s]

$\omega_2 = 100\pi$ rad/s $\Rightarrow T_2 = \frac{2\pi}{\omega_2} = \frac{1}{50}$ [s]

$\omega_3 = 150\pi$ rad/s $\Rightarrow T_3 = \frac{2\pi}{\omega_3} = \frac{1}{75}$ [s]

$\omega_4 = 200\pi$ rad/s $\Rightarrow T_4 = \frac{2\pi}{\omega_4} = \frac{1}{100}$ [s]

Zajednički period ovih signala T_0, jednak je periodu najsporije komponente T_1 jer.

$$T_1 = T_0 = \frac{1}{25} \text{ [s]}$$

$$T_2 = \frac{T_0}{2}, \quad T_3 = \frac{T_0}{3} \quad T_4 = \frac{T_0}{4}$$

Dakle, osnovni period T_0 pri razvoju ovog signala jednak je $T_0 = \frac{1}{25}$ [s], a osnovna kružna frekvencija je $\frac{2\pi}{T_0} = \Omega_0 = 50\pi$ rad/s

Frekvencije komponenata signala su sljedeće:

$$\omega_1 = 1 \cdot \Omega_0, \quad \omega_2 = 2\Omega_0, \quad \omega_3 = 3\Omega_0, \quad \omega_4 = 4\Omega_0$$

Odredimo sada koeficijente razvoja u FR

$$x(t) = \sum_{k=-\infty}^{\infty} X_k \cdot e^{jk\Omega_0 t}, \text{ gdje:}$$

$$X_k = \frac{1}{T_0}\int_{T_0} x(t)\cdot e^{-jk\Omega_0 t}\,dt$$

Koef. X_k uopće nije potrebno određivati korištenjem formulej izraza, jer je očito da svaka komponenta signala generira jedan par kompl. exponencijala.

Ako cos raspišemo kao sumu exp. imamo:

$$10\cdot\cos(50\pi t) = 10\cdot\cos(\omega_1 t) = 10\cdot\cos(1\cdot\Omega_0 t)$$
$$= \frac{10}{2}\left(e^{j1\cdot\Omega_0 t} + e^{-j1\cdot\Omega_0 t}\right)$$
$$= \left(\frac{10}{2}e^{j\cdot\emptyset}\right)\cdot e^{j1\cdot\Omega_0 t} + \left(\frac{10}{2}e^{j\emptyset}\right)\cdot e^{-j1\cdot\Omega_0 t}$$
$$= X_1\cdot e^{j1\cdot\Omega_0 t} + X_{-1}\cdot e^{-j1\cdot\Omega_0 t}$$

Prepoznajemo da prva komponenta signala se u razvoju u FR vidi na koeficijentima X_1 i X_{-1} koji iznose $X_1 = 5\cdot e^{j\emptyset}$ $X_{-1} = X_1^* = 5\cdot e^{j\emptyset}$

Analogno radimo i za preostale 3 komponente signala:

2. komp.

$$5\cdot\sin(100\pi t) = 5\cdot\sin(\omega_2 t) = 5\cdot\sin(2\cdot\Omega_0 t)$$
$$= 5\cdot\cos\left(2\Omega_0 t - \frac{\pi}{2}\right) = \frac{5}{2}\left(e^{j(2\Omega_0 t - \pi/2)} + e^{-j(2\Omega_0 t - \pi/2)}\right)$$
$$= \left(\frac{5}{2}\cdot e^{-j\frac{\pi}{2}}\right)\cdot e^{j2\Omega_0 t} + \left(\frac{5}{2}e^{j\frac{\pi}{2}}\right)e^{-j2\Omega_0 t}$$
$$= X_2\cdot e^{j2\Omega_0 t} + X_{-2}\cdot e^{-j2\Omega_0 t}$$
$$\Rightarrow X_2 = \frac{5}{2}\cdot e^{-j\frac{\pi}{2}} \qquad X_{-2} = X_2^* = \frac{5}{2}\cdot e^{j\pi/2}$$

3 komp:

$$\sin(150\pi t + 2\pi/3) = \sin\left(\omega_3 t + \frac{2\pi}{3}\right) = \sin\left(3\Omega_0 t + \frac{2\pi}{3}\right)$$
$$= \cos\left(3\Omega_0 t - \frac{\pi}{2} + \frac{2\pi}{3}\right) = \cos\left(3\Omega_0 t + \frac{-3+4}{6}\pi\right) = \cos\left(3\Omega_0 t + \frac{\pi}{6}\right)$$
$$= \frac{1}{2}\left(e^{j(3\Omega_0 t + \pi/6)} + e^{-j(3\Omega_0 t + \pi/6)}\right) = \frac{1}{2}\cdot e^{j\pi/6}\cdot e^{j3\Omega_0 t} + \frac{1}{2}e^{-j\frac{\pi}{6}}e^{-j3\Omega_0 t}$$
$$= X_3 e^{j3\Omega_0 t} + X_{-3}e^{-j3\Omega_0 t} \Rightarrow X_3 = \frac{1}{2}\cdot e^{j\pi/6},\ X_{-3} = X_3^* = \frac{1}{2}e^{-j\pi/6}$$

Konačno i zadnja komponenta:

$$\cos(200\pi t + \pi/4) = \cos(\omega_4 t + \pi/4) = \cos(4\Omega_0 t + \pi/4)$$

$$= \frac{1}{2}\cdot\left[e^{j(4\Omega_0 t + \pi/4)} + e^{-j(4\Omega_0 t+\pi/4)}\right] =$$

$$= \frac{1}{2}\cdot e^{j\pi/4}\cdot e^{j4\Omega_0 t} + \frac{1}{2}\cdot e^{-j\pi/4}\cdot e^{-j4\Omega_0 t}$$

$$= X_4\cdot e^{j4\Omega_0 t} + X_{-4}\cdot e^{-j4\Omega_0 t}$$

$$\Rightarrow X_4 = \frac{1}{2} e^{j\pi/4} \qquad X_{-4} = X_4^* = \frac{1}{2} e^{-j\pi/4}$$

Zaključujemo FR se sastoji od 8 članova za $|k| \in [1,2,3,4]$, dok su svi ostali koef. X_k jednaki $\varnothing$

$$X_k = \begin{cases} 5\cdot e^{j0} \text{ za } k=1, & 5\cdot e^{j\varnothing} \text{ za } k=-1 \\ 5/2\cdot e^{-j\pi/2} \text{ za } k=2 & 5/2\cdot e^{j\pi/2} \text{ za } k=-2 \\ 1/2\cdot e^{j\pi/6} \text{ za } k=3 & 1/2\cdot e^{-j\pi/6} \text{ za } k=-3 \\ 1/2\cdot e^{j\pi/4} \text{ za } k=4 & 1/2\cdot e^{-j\pi/4} \text{ za } k=-4 \\ \varnothing \text{ za sve ostale } k \end{cases}$$

Ovo se može zapisati i pomoću Kroneckerovog deltla impulsa kao:

$$\begin{aligned} X_k = \; & 5\cdot e^{j\varnothing}\cdot\delta[k-1] + 5\cdot e^{j\varnothing}\,\delta[k+1] + \\ & \frac{5}{2}\cdot e^{-j\pi/2}\cdot\delta[k-2] + \frac{5}{2}\, e^{j\pi/2}\,\delta[k+2] + \\ & \frac{1}{2}\cdot e^{j\pi/6}\,\delta[k-3] + \frac{1}{2}\cdot e^{-j\pi/6}\,\delta[k+3] + \\ & \frac{1}{2}\cdot e^{j\pi/4}\,\delta[k-4] + \frac{1}{2}\cdot e^{-j\pi/4}\,\delta[k+4] \end{aligned}$$

Koeficijente X_k smo odredili „prepoznavanjem" koeficijenata uz članove razvoja u FR. Pokažimo da smo do istog rezultata mogli doći i primjenom direktnog izraza za X_k, ...
za ilustraciju uzmimo samo prvu komponentu

00.A.503.A.90

$x_1(t) = 10 \cdot \cos(50\pi t) = 5 \cdot e^{j\Omega_0 t} + 5 \cdot e^{-j\Omega_0 t}$

Ovaj signal $x_1(t)$ bi dao:

$$X_k = \frac{1}{T_0}\int_{T_0} x_1(t)\cdot e^{-jk\Omega_0 t}dt = \frac{1}{T_0}\int_0^{T_0}\left[5\cdot e^{j\Omega_0 t} + 5\cdot e^{-j\Omega_0 t}\right]\cdot e^{-jk\Omega_0 t}dt$$

$$= \frac{1}{T_0}\int_0^{T_0} 5\cdot e^{j(1-k)\Omega_0 t}dt + \frac{1}{T_0}\int_0^{T_0} 5\cdot e^{-j(1+k)\Omega_0 t}dt$$

$$= \frac{5}{T_0}\cdot\frac{1}{j(1-k)\Omega_0}\cdot \underbrace{e^{j(1-k)\Omega_0 t}\Big|_0^{T_0}} + \frac{5}{T_0}\,\frac{1}{-j(1+k)\Omega_0}\cdot e^{-j(1+k)\Omega_0 t}\Big|_0^{T_0}$$

za $k \neq 1$ ovaj integral je jednak nuli jer:

$e^{j(1-k)\Omega_0 T_0} - e^{j0} =$

$e^{j2\pi(1-k)} - 1 =$

$1 - 1 = 0$, jer $1-k \in Z$, a nazivnik $j(1-k)\Omega_0$ je $\neq 0$

Specijalno za $k=1$ integral ima slijedeći oblik:

$$\frac{1}{T_0}\int_0^{T_0} 5\cdot e^{j0\Omega_0 t}dt = \frac{1}{T_0}\int_0^{T_0} 5\,dt = 5$$

Zaključujemo da je prvi član jednak 5 za $k=1$, a jednak nuli za sve ostale k

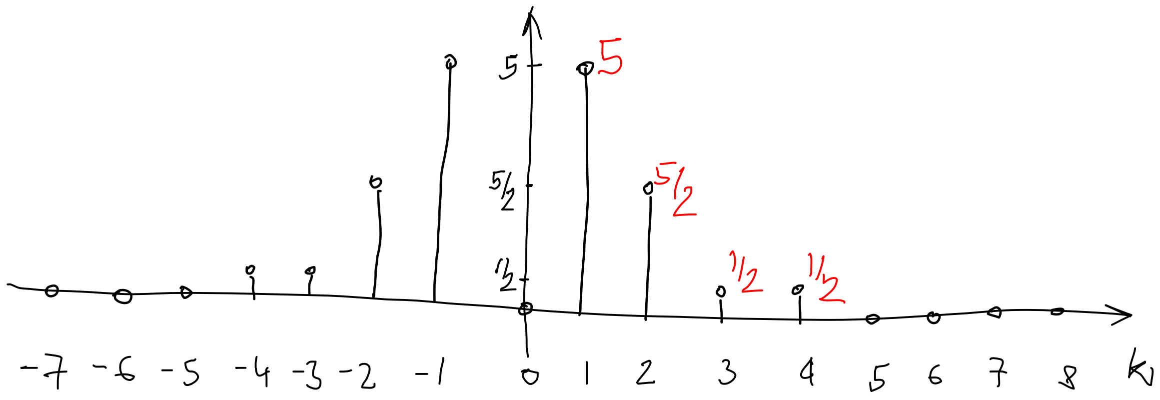
Po analogiji ovaj integral je jednak 5 za $k=-1$, a jednak nuli za sve ostale k

Vidimo da prva komponenta signala $x_1(t)$ ima u razvoju u FR dvije komponente X_1 i X_{-1}, a koeficijenti iznose $X_1 = 5$, $X_{-1} = 5$, tj.

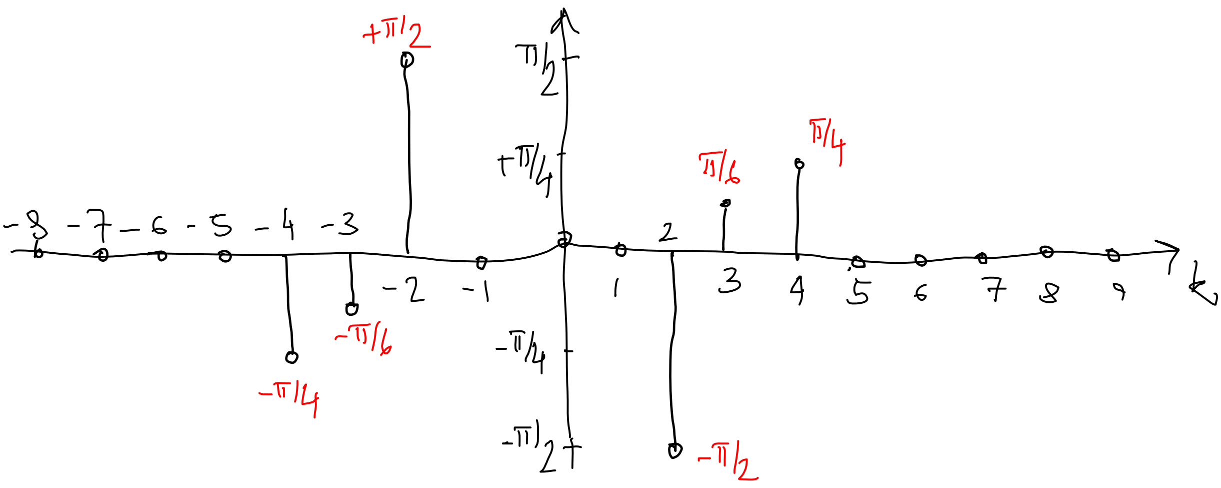
$$X_k = 5\cdot\delta[k-1] + 5\cdot\delta[k+1]$$

Slično možemo napraviti za preostale 3 komp. signala.

AMPLITUDNI SPEKTAR $|X(k)|$



FAZNI SPEKTAR $\sphericalangle X(k)$



Da prilikom otipkavanja ne dođe do pojave preklapanja spektra frekvencija otipkavanja f_s mora biti barem 2 puta viša od najviše frekvencije signala f_{max}

U našem primjeru $f_{max} = \frac{\omega_{max}}{2\pi} = \frac{\omega_4}{2\pi} =$

$= \frac{200\pi}{2\pi} = 100\,Hz$ .... frekvencija 4. komponente

$f_s > 2 \cdot f_{max} = 2 \cdot 100 = 200\,Hz$ ... da nema preklapanja

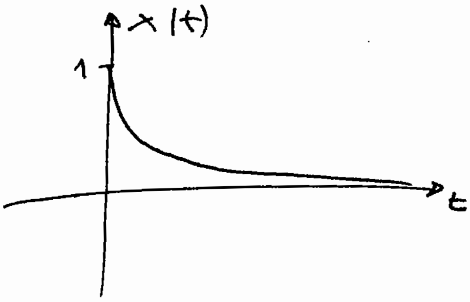
U jednoj grupi $f_s = \frac{1}{T_s} = \frac{1}{0.02} = 50\,Hz$

U drugoj grupi $f_s = \frac{1}{T_s} = \frac{1}{0.01} = 100\,Hz$

Vidimo da je za obje grupe $f_s < 2 \cdot f_{max}$ pa zaključujemo da dolazi do pojave preklapanja spektra, jer je frekvencija otipkavanja nedovoljno visoka.

6.

a) $x(t) = e^{-t}\mu(t)$



$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt$$
$$= \int_{-\infty}^{\infty} e^{-t}\mu(t)\, e^{-j\Omega t}\, dt$$
$$= \int_{0}^{\infty} e^{-(j\Omega+1)t}\, dt$$
$$= \frac{1}{-(1+j\Omega)} \cdot e^{-(j\Omega+1)t}\Big|_0^{\infty}$$
$$= \boxed{\frac{1}{1+j\Omega}} =$$
$$= \frac{1-j\Omega}{1+\Omega^2}$$

AMPLITUDNI SPEKTAR

$$|X(j\Omega)| = \sqrt{\frac{1+\Omega^2}{(1+\Omega^2)^2}} = \frac{1}{\sqrt{1+\Omega^2}}$$

FAZNI SPEKTAR

$$\measuredangle X(j\Omega) = \operatorname{arctg} \frac{\frac{-\Omega}{1+\Omega^2}}{\frac{1}{1+\Omega^2}} = \operatorname{arctg}(-\Omega) = -\operatorname{arctg}\Omega$$

REALNI SPEKTAR

$$\operatorname{Re}(X(j\Omega)) = \frac{1}{1+\Omega^2}$$

IMAGINARNI

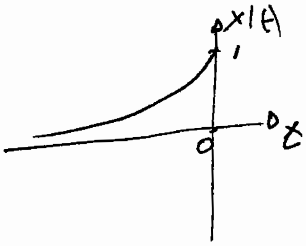
$$\operatorname{Im}(X(j\Omega)) = -\frac{\Omega}{1+\Omega^2}$$

ENERGIJA

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |e^{-t}\mu(t)|^2 dt = \int_0^{\infty} e^{-2t} dt = \frac{1}{-2}\, e^{-2t}\Big|_0^{\infty} = \frac{1}{2}$$

$$E_x = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\Omega)|^2 d\Omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(\frac{1}{1+j\Omega}\right)^2 d\Omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{1+\Omega^2} d\Omega = \frac{1}{2\pi}\cdot\pi = \frac{1}{2}$$

b) $x(t) = e^{t}\mu(-t)$



$$X(j\Omega) = \int_{-\infty}^{\infty} e^{t}\mu(-t)e^{-j\Omega t} dt = \int_{-\infty}^{0} e^{(1-j\Omega)t} dt$$
$$= \frac{1}{1-j\Omega}\cdot e^{(1-j\Omega)t}\Big|_{-\infty}^{0} = \boxed{\frac{1}{1-j\Omega}} = \frac{1+j\Omega}{1+\Omega^2}$$

$$|X(j\Omega)| = \sqrt{\frac{1+\Omega^2}{(1+\Omega^2)^2}} = \frac{1}{\sqrt{1+\Omega^2}}$$

$$\measuredangle X(j\Omega) = \operatorname{arctg} \frac{\Omega}{1+\Omega^2} \Big/ \frac{1}{1+\Omega^2} = \operatorname{arctg}\Omega$$

$$\operatorname{Re}(X(j\Omega)) = \frac{1}{1+\Omega^2}$$

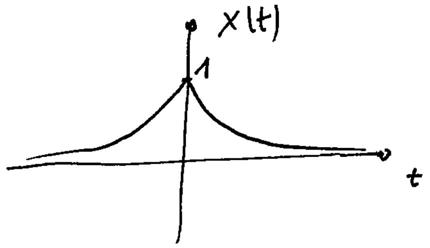
$$\operatorname{Im}(X(j\Omega)) = \frac{\Omega}{1+\Omega^2}$$

ENERGIJA

$$E_x = \int_{-\infty}^{0} |e^{t}|^2 dt = \int_{-\infty}^{0} e^{2t} dt = \frac{1}{2}\, e^{2t}\Big|_{-\infty}^{0} = \frac{1}{2}$$

$$E_x = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\Omega)|^2 d\Omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|\frac{1}{1-j\Omega}\right|^2 d\Omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{1+\Omega^2} d\Omega = \frac{1}{2\pi}\cdot\pi = \frac{1}{2}$$

c) $x(t) = e^{-|t|}$



$$X(j\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} dt$$

$$= \int_{-\infty}^{0} e^{t} e^{-j\Omega t} dt + \int_{0}^{\infty} e^{-t} e^{-j\Omega t} dt$$

$$= \int_{-\infty}^{0} e^{(1-j\Omega)t} dt + \int_{0}^{\infty} e^{-(1+j\Omega)t} dt$$

$$= \frac{1}{1-j\Omega} + \frac{1}{1+j\Omega} = \frac{1+j\Omega+1-j\Omega}{1+\Omega^2}$$

$$= \frac{2}{1+\Omega^2}$$

$$|X(j\Omega)| = \frac{2}{1+\Omega^2}$$

$$\measuredangle |X(j\Omega)| = \operatorname{arctg} \frac{0}{\frac{2}{1+\Omega^2}} = 0$$

$$\operatorname{Re}(X(j\Omega)) = \frac{2}{1+\Omega^2}$$

$$\operatorname{Im}(X(j\Omega)) = 0$$

ENERGIJA

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{0} e^{2t} dt + \int_{0}^{\infty} e^{-2t} dt = \frac{1}{2} e^{2t} \Big|_{-\infty}^{0} + \frac{1}{-2} e^{-2t} \Big|_{0}^{\infty}$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

$$E_x = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\Omega)|^2 d\Omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\frac{2}{1+\Omega^2}\right)^2 d\Omega = \frac{1}{\cancel{2\pi}} \cdot \cancel{4} \cdot \frac{\cancel{\pi}}{\cancel{2}} = 1$$

7. H | $x(t) = e^{2t}\mu(-t)$

a) Fourierova transformacija

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} e^{2t}\mu(-t)\, e^{-j\omega t}\, dt$$

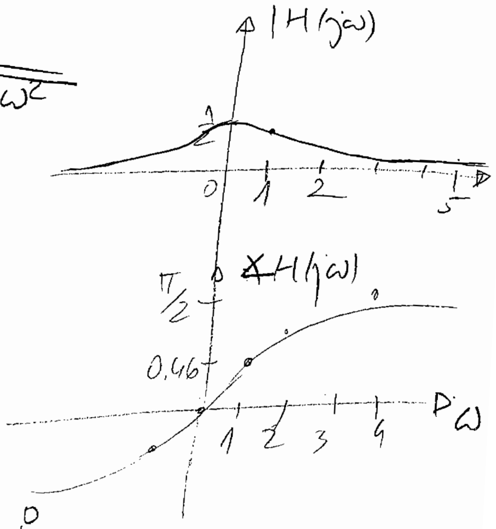
$$= \int_{-\infty}^{0} e^{2t - j\omega t}\, dt$$

$$= \frac{e^{(2-j\omega)t}}{2-j\omega}\Bigg|_{-\infty}^{0} = \frac{1}{2-j\omega} = \frac{1}{2-j\omega}\cdot\frac{2+j\omega}{2+j\omega} = \frac{2+j\omega}{4+\omega^2}$$

b)

$$|H(j\omega)| = \sqrt{\frac{2^2+\omega^2}{(4+\omega^2)^2}} = \sqrt{\frac{\cancel{2^2+\omega^2}}{(4+\omega^2)^{\cancel{2}}}} = \frac{1}{\sqrt{4+\omega^2}}$$

$$\measuredangle H(j\omega) = \text{arctg}\, \frac{\frac{\omega}{\cancel{4+\omega^2}}}{\frac{2}{\cancel{4+\omega^2}}} = \text{arctg}\, \frac{\omega}{2}$$



c)

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

$$= \int_{-\infty}^{\infty} |e^{2t}\mu(t)|^2\, dt = \int_{-\infty}^{0} e^{4t}\, dt = \frac{e^{4t}}{4}\Bigg|_{-\infty}^{0} = \frac{1}{4}$$

d)

$$E_x = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(e^{j\omega})|^2\, d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|\frac{1}{2-j\omega}\right|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{4+\omega^2}\, d\omega =$$

$$= \frac{1}{2\pi}\frac{1}{2}\, \text{arctg}\, \frac{1}{2}\omega \Bigg|_{-\infty}^{\infty} = \frac{1}{2}\frac{1}{2\pi}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = \frac{\pi}{2}\cdot\frac{1}{2\pi} = \frac{1}{4}$$