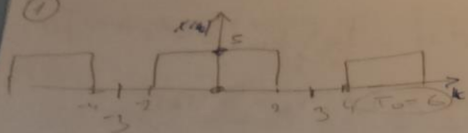
①



a) CTFS + amplitudni spektar

$$X_k = \frac{1}{T_0}\int_{T_0} x(t)\cdot e^{-j\omega_0 k t}\,dt = \frac{1}{6}\int_{-2}^{2} 5\cdot e^{-j\omega_0 k t}\,dt =$$

$$= \frac{5}{6}\int_{-2}^{2} e^{-j\omega_0 k t}\,dt = \frac{5}{6}\cdot\frac{1}{-j\omega_0 k}\, e^{-j\omega_0 k t}\Big|_{t=-2}^{2}$$

$$= \frac{5}{\cancel{6}\,3}\cdot\frac{1}{2(-j)\omega_0 k}\left(e^{-2j\omega_0 k} - e^{2j\omega_0 k}\right)$$

$$= \frac{5}{3}\,\frac{1}{\omega_0 k}\cdot\sin(k\cdot 2\omega_0) = \frac{5}{\cancel{3}}\cdot\frac{1}{k}\cdot\frac{\cancel{3}}{\pi}\cdot\sin\left(\frac{2\pi}{3}k\right)$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{6} = \frac{\pi}{3} \qquad \boxed{X_k = \frac{5}{k\pi}\cdot\sin\left(\frac{2\pi}{3}k\right)}\ (*)$$

$$X_0 = \lim_{k\to 0}\frac{5\cdot\sin\left(\frac{2\pi}{3}k\right)}{k\pi} \stackrel{L'H}{=} \lim_{k\to 0}\frac{5\cdot\frac{2\pi}{3}\cdot\cos\left(\frac{2\pi}{3}k\right)}{\pi} = \frac{10}{3}\approx 3.33$$

Uvrstite u (*)

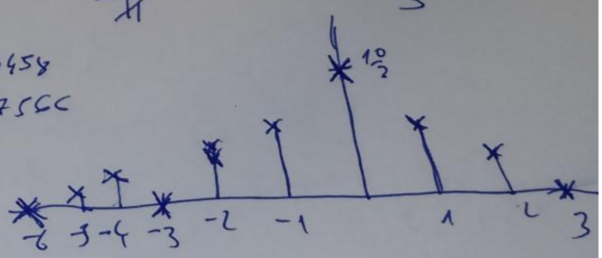
$X_1 = 1.37832$ $\quad |X_4| = |X_{-4}| = 0.34458$

$|X_2| = 0.684$ $\quad |X_5| = |X_{-5}| = 0.27566$

$|X_{-1}| = 1.37832$ $\quad |X_6| = |X_{-6}| = 0$

$|X_{-2}| = 0.689$

$|X_3| = |X_{-3}| = 0$



b) Parseval CTFS

$$P_f = \frac{1}{T_0}\int_{T_0} |f(t)|^2\,dt = \frac{1}{T_0}\int_{T_0} f(t)\cdot f(t)^*\,dt = \frac{1}{T_0}\int_{T_0} f(t)\cdot\left(\sum_{k=-\infty}^{\infty} F_k^*\cdot e^{-jk\omega_0 t}\right)dt$$

$$= \sum_{k=-\infty}^{\infty} F_k^*\cdot\frac{1}{T_0}\int_{T_0} f(t)\cdot e^{-jk\omega_0 t}\,dt = \sum_{k=-\infty}^{\infty} F_k^*\cdot F_k = \sum_{k=-\infty}^{\infty}|F_k|^2 \;//$$

c) Snaga,

$$P = \frac{1}{T_0}\int_{T_0} |f(t)|^2\,dt = \frac{1}{6}\int_{-2}^{2} 25\,dt = \frac{25}{6}\cdot 4 = \frac{50}{3} = 16{,}66667$$

d) generalizirani CTFT → u FORMULE, naša fja je tone rect

nisam siguran, ali

$$f(t) \qquad \mathrm{rect}\left(\frac{t}{T}\right) \circ\!\!-\!\!\bullet\; T\cdot \mathrm{Sin}\left(\frac{\omega T}{2\pi}\right) \quad F(\omega)$$

$$\mathrm{rect}\left(\frac{t}{6}\right) \circ\!\!-\!\!\bullet\; 6\cdot \mathrm{Sin}\left(\omega\frac{\cancel{6}}{\cancel{2}\pi}\right) = 6\cdot \mathrm{sin}\left(\omega\frac{3}{\pi}\right) \;/\cdot 5 = 30\,\mathrm{sin}\left(\omega\frac{3}{\pi}\right)$$

(???)

(2)

$$f(t) = \cancel{4}\,2 \cdot \sin\left(400\pi t + \frac{\pi}{6}\right) - \cancel{4}\cos\left(800\pi t + \frac{\pi}{8}\right) \qquad f_s = 300\,Hz$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

a) $f(n) = ?$

$$f = \frac{1}{T}$$

$$f(nT_s) = 2\sin\left(400\pi \cdot \frac{1}{300}n + \frac{\pi}{6}\right) - \cancel{4}\cos\left(\frac{800\pi}{300}n + \frac{\pi}{8}\right) \qquad T = \frac{1}{f} = N = \frac{1}{300}$$

$$f(n) = 2\sin\left(\frac{4\pi}{3}n + \frac{\pi}{6}\right) - 4\cos\left(\frac{8\pi}{3}n + \frac{\pi}{8}\right)$$ ← 6 bodova ??? 2veda ???

b) DTFS

$$2\sin\left(\frac{4\pi}{3}n + \frac{\pi}{6}\right) = 2\cdot\left(\frac{1}{2j}\left(e^{j\frac{4\pi}{3}n} \cdot e^{j\frac{\pi}{6}} - e^{-\frac{4\pi}{3}jn} \cdot e^{-j\frac{\pi}{6}}\right)\right) =$$

$$= e^{-j\frac{\pi}{2}} \cdot e^{j\frac{4\pi}{3}n} \cdot e^{j\frac{\pi}{6}} - e^{-j\frac{\pi}{2}} \cdot e^{-j\frac{4\pi}{3}n} \cdot e^{-j\frac{\pi}{6}}$$

$$= e^{j\frac{4\pi}{3}n} \cdot e^{-j\frac{\pi}{3}} - e^{-j\frac{4\pi}{3}n} \cdot e^{-j\frac{2\pi}{3}}$$

$$= e^{j\frac{4\pi}{3}n} \cdot e^{-j\frac{\pi}{3}} - e^{-j\frac{4\pi}{3}n} \cdot e^{-j\frac{\pi}{3}}$$

$$4\cos\left(\frac{8\pi}{3}n + \frac{\pi}{8}\right) = \cancel{4}\cdot\frac{1}{\cancel{2}}\left(e^{\frac{8\pi}{3}jn} \cdot e^{j\frac{\pi}{8}} + e^{-\frac{8\pi}{3}jn} \cdot e^{-j\frac{\pi}{8}}\right) =$$

$$= 2e^{\frac{8\pi}{3}jn} \cdot e^{\frac{\pi}{8}j} + 2 \cdot e^{-\frac{8\pi}{3}jn} \cdot e^{-j\frac{\pi}{8}}$$

$$X(n) \overset{\text{TFSN}}{=} X_k = \frac{1}{N} \cdot \sum_{n=0}^{N-1} x(n) \cdot e^{-2\pi jk\frac{n}{N}}$$

$$\frac{8\pi}{3}n = 2\pi \quad \text{djelu} \quad \frac{4\pi}{3} \; ?$$

$$N_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{4\pi/3} = \frac{3}{2} = 1{,}5 \qquad N = \frac{3}{2}$$

F

$$N_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{8\pi/3} = \frac{3}{4} \quad \langle 0{,}75 \; /2 \qquad \Omega_0 = \frac{2\pi}{3/2} = \frac{4\pi}{3} \; !$$

$$X(n) = e^{j\Omega_0 \cdot n \cdot 1} \cdot e^{-j\frac{\pi}{3}} - e^{-j\Omega_0 \cdot n \cdot 1} \cdot e^{-j\frac{\pi}{3}} + 2e^{j\Omega_0 \cdot n \cdot 2} \cdot e^{\frac{\pi}{8}j} + 2e^{-j\Omega_0 \cdot n \cdot 2} \cdot e^{-\frac{\pi}{8}j}$$

$\to k=1$ $\quad \to k=-1$ $\quad \to k=2$ $\quad \to k=-2$

vidi 8, već nekoko

$$X_1 = e^{-j\frac{\pi}{3}}$$

$$X_{-1} = -e^{-j\frac{\pi}{3}}$$

$$X_2 = 2e^{\frac{\pi}{8}j}$$

$$X_{-2} = 2e^{-\frac{\pi}{8}j}$$

③ c) Vremenski oblik ako je idealna interpolacija

1/1 $f(t) = 2\cdot\sin\left(400\pi t + \frac{\pi}{6}\right) - 4\cos\left(800\pi t + \frac{\pi}{8}\right)$

$f_s = 300\,Hz \quad \cancel{f_1 = \frac{2\pi}{\omega}} \quad \omega = 2\pi f \rightarrow f_1 = \frac{\omega_1}{2\pi} = \frac{400\cancel{\pi}}{2\cancel{\pi}} = 200\,Hz$

$f_2 = \frac{\omega_2}{2\pi} = \frac{800\pi}{2\pi} = 400\,Hz$

frekvencija očitavanja nije 2x veća od najveće frekvencije [nije ni od jedne]

a)1 '' $2\sin\left(400\pi t + \frac{\pi}{6}\right) \Rightarrow 2\sin\left(\frac{4\pi}{3}n + \frac{\pi}{6}\right) = 2\sin\left(\frac{4\pi}{3}n + \frac{\pi}{6} + 2k\pi n\right)$

$-\pi < \frac{4\pi}{3} + 2k\pi < \pi$

$-1 < \frac{4}{3} + 2k < 1$

$2k < 1 - \frac{4}{3} =$

$k < -\frac{1}{6}\,(-0,16667)$

$-\frac{7}{6} \quad (-1) \quad -\frac{1}{6}$

$2k > -1 - \frac{4}{3} \Rightarrow k > -\frac{7}{6}\,(-1,6667)$

$\boxed{k = -1}$

$2\sin\left(\frac{4\pi}{3}n + 2\pi n + \frac{\pi}{6}\right) = 2\sin\left(\frac{-2\pi}{3}n + \frac{\pi}{6}\right)$

b) $\frac{\pi}{...}$ ... $\left(\frac{8\pi}{...} ... \frac{\pi}{...} + 2k\pi n\right)$

$$2\sin\left(\frac{4\pi}{3}n - 2\pi n + \frac{\pi}{6}\right) = 2\sin\left(\frac{-2\pi}{3}n + \frac{\pi}{6}\right)$$

$y_4(t)$ II

$y_4'(t)$

$$4\cos\left(800\pi t + \frac{\pi}{8}\right) \Rightarrow 4\cos\left(\frac{8\pi}{3}n + \frac{\pi}{8}\right) = 4\cos\left(\frac{8\pi}{3}n + \frac{\pi}{8} + 2k\pi n\right)$$

$$-\pi < \frac{8\pi}{3} + 2k\pi < \pi$$

$$-1 < \frac{8}{3} + 2k < 1$$

$$2k < 1 - \frac{8}{3}$$

$$k < -\frac{5}{6} \approx -0.8333$$

$$2k > -1 - \frac{8}{3}$$

-1.8 -1 -0.833

$$k > -\frac{11}{6} \approx -1.8333$$

$k = -1$

$$4\cos\left(\frac{8\pi}{3}n - 2\pi n + \frac{\pi}{8}\right) = 4\cos\left(\frac{2\pi}{3}n + \frac{\pi}{8}\right)$$

konačno -> idealni interpolator

$$f(t) = 2\sin\left(-\frac{2\pi}{3}n + \frac{\pi}{6}\right) - 4\cos\left(\frac{2\pi}{3}n + \frac{\pi}{8}\right)$$

$$= 2\sin\left(-\frac{2\pi}{3}\cdot 300t + \frac{\pi}{6}\right) - 4\cos\left(\frac{2\pi}{3}\cdot 300\cdot t + \frac{\pi}{8}\right)$$

$$= 2\sin\left(-200\pi t + \frac{\pi}{6}\right) - 4\cos\left(200t + \frac{\pi}{8}\right)$$

③ $y''(t)+8y'(t)+15y(t)=u'(t)+2u(t)$

1) Početni uvjeti (ako će trebati):

opća → napiši si na čelo da zapamtiš $\boxed{LOL}$!

$y''(t)+a_1y'(t)+a_2y(t)=b_0u''(t)+b_1\cdot u'(t)+b_2\cdot u(t)$

$y(0^+)-y(0^-)=b_0\cdot u(0^+)$

$y'(0^+)-y'(0^-)+a_1\cdot y(0^+)-a_1\cdot y(0^-)=b_0\cdot u'(0^+)+b_1\cdot u(0^+)$

a) odziv mirnog sustava na $u(t)=\mu(t)$ u t → domeni → MIRNI SUSTAV

$y(0^-)=0$!
$y'(0^-)=0$!

1) Polinom

$\lambda^2+8\lambda+15=0$

$\lambda_1=-3$

$\lambda_2=-5$

$y_p=k$!

$y_p'=0$

$y_p''=0$

$0+8\cdot 0+15k=0+2$

$\boxed{k=\frac{2}{15}}$

$y(0^+)=b_0\cdot u(0^+)+y(0^-)$

$y(0^+)=0+0$

$y'(0^+)=1$

formule dolje jednostavnije

$y_H=C_1\cdot e^{-3t}+C_2\cdot e^{-5t}+\frac{2}{15}$

$y_H(0)=C_1+C_2+\frac{2}{15}=0$

$y_H'(0)=-3C_1-5C_2=1$

$y_{mst}=\left(\frac{1}{6}e^{-3t}-\frac{3}{10}e^{-5t}+\frac{2}{15}\right)\mu(t)$

$C_1=-\frac{2}{15}-C_2$

a) Odziv mirnog sustava na $u(t) = \mu(t)$ u $t \to$ domeni $\to$ MIRNI SUSTAV

$y(0^-) = 0$!

$y'(0^-) = 0$!

1) Polinom

$q^2 + 8q + 15 = 0$

$q_1 = -3$

$q_2 = -5$

$y_p = k$!

$y_p' = 0$

$y_p'' = 0$

$0 + 8 \cdot 0 + 15k = 0 + 2$

$\boxed{k = \frac{2}{15}}$

$y(0^+) = b_0 \cdot u(0^+) + y(0^-)$

$y(0^+) = 0 + 0$

$y'(0^+) = 1$

gornje dvije jednadžbe

$y_H = C_1 \cdot e^{-3t} + C_2 \cdot e^{-5t} + \frac{2}{15}$

$y_H(0) = C_1 + C_2 + \frac{2}{15} = 0$

$y_H'(0) = -3C_1 - 5C_2 = 1$

$C_1 = -\frac{2}{15} - C_2$

$-3\left(-\frac{2}{15} - C_2\right) - 5C_2 = 1$

$\frac{2}{5} + 3C_2 - 5C_2 = 1$

$-2C_2 = \frac{3}{5}$

$C_2 = -\frac{3}{10}$

$C_1 = \frac{1}{6}$

$y_{mirni} = \Big(\underbrace{\frac{1}{6} e^{-3t} - \frac{3}{10} e^{-5t}}_{\text{nepotrebno}} + \frac{2}{15}\Big)\mu(t)$

Ovo je mirno!

Ovo je nepotrebno ali da se uvaste ~~8~~ X

(1)

$y(n) - \frac{1}{3}y(n+1) = u(n) + 2u(n-1)$

a) Impulsni odziv u vremenskoj domeni → pozvijeti su nula!

$h(n) = y(n) - u(n) - 2u(n-1)$

b) odziv nepobuđenog ~~sustava~~

$y(0^-) = -1$

$y'(0^-) = 1$

$y''(t) + 8y'(t) + 15y(t) = 0$

$s^2Y - sy(0) - y'(0) + 8(sY - y(0)) + 15Y = 0$

$s^2Y - s(-1) - 1 + 8 \cdot sY - 8(-1) + 15Y = 0$

$s^2Y + s - 1 + 8sY + 8 + 15Y = 0$

$s^2Y + 8sY + 15Y = -7 - s$

$$Y(s^2+8s+15) = \frac{-7-s}{s^2+8s+15} = \frac{-7-s}{(s+3)(s+5)} = \frac{A}{s+3} + \frac{B}{(s+5)}$$

$As + Bs + 5A + 3B = -7 - s$

$5A + 3B = -7 \qquad 5A + 3(-1+A) = -7$

$5A - 3 + 3A = -7$

$$Y(s^2+8s+15) = \frac{-7-s}{s^2+8s+15} = \frac{-7-s}{(s+3)(s+5)} = \left(\frac{A}{s+3} + \frac{B}{(s+5)}\right)$$

$$As + Bs + 5A + 3B = -7 - s$$

$$5A + 3B = -7 \qquad 5A + 3(-1-A) = -7$$

$$A + B = -1 \qquad 5A - 3 - 3A = -7$$

$$-2A = 4$$

$$B = -1 - A \qquad A = -2$$

$$B = -1 + 2 = 1$$

$$Y(s) = \frac{-2}{s+3} + \frac{1}{s+5} \circ\!\!-\!\!\bullet \left(-2e^{-3t} + e^{-5t}\right)\mu(t) = y_0(t)$$

c)

$$Y_{TOTAL} = Y_0 + Y_m = \left(-\frac{11}{6}e^{-3t} + \frac{7}{10}e^{-5t} + \frac{2}{15}\right)\mu(t)$$

ali $-2e^{-3t} + e^{-5t} + \frac{2}{15}$ e nisam sigurna mislim da je

①

$$y(n) - \frac{1}{5} y(n+1) = u(n) + 2u(n-1)$$

a) Impulsni odziv u vremenskoj domeni $\rightarrow$ poč. uvjeti su nula!

$$\alpha^n - \frac{1}{5}\alpha^{n-1} = 0 \qquad y_h(n) = \left(\frac{1}{5}\right)^n \cdot C$$

$$\frac{1}{5} y(n-1) = y(n) - u(n) - 2u(n-1)$$

$$\cancel{\frac{1}{5} y(-1) = 1 - 1 = 0} \ \checkmark$$

$$\boxed{\alpha = \frac{1}{5}}$$

$$y(n) = u(n) + 2u(n-1) + \frac{1}{5} y(n-1)$$

$$y(n) = \delta(n) + 2\delta(n-1) + \frac{1}{5} y(n-1)$$

$$y(0) = \delta(0) + 2\delta(-1) + \frac{1}{5} y(-1) = 1$$

$$1 + 0 + 0 \quad ??$$

$$y(0) = y_h(0) = C \cdot \left(\frac{1}{5}\right)^n = C \cdot \left(\frac{1}{5}\right)^0 = C = 1$$

$h(n)$ je impulsni odziv $\quad h(n) = \left(\frac{1}{5}\right)^n \cdot \mu(n)$

b) prijenosnu funkciju sustava

$$Y - \frac{1}{5} z^{-1} \cdot Y = U + 2 \cdot z^{-1} U$$

$$Y\left(1 - \frac{1}{z}\right) = U\left(1 + \frac{2}{z}\right)$$

$h(n) = \left(\frac{1}{5}\right)^n \cdot \mu(n)$

b) prijenosnu funkciju sustava

$$Y - \frac{1}{5} z^{-1} \cdot Y = U + 2 \cdot z^{-1} U$$

$$Y\left(1 - \frac{1}{5z}\right) = U\left(1 + \frac{2}{z}\right)$$

$$H(z) = \frac{Y}{U} = \frac{1 + \frac{2}{z} / 5z}{1 - \frac{1}{5z} / 5z} = \frac{5z + 10}{5z - 1} = \frac{5 \cdot (z+2)}{5(z - \frac{1}{5})} = \frac{z}{z - \frac{1}{5}} + \frac{2}{z - \frac{1}{5}}$$

c) odziv na $U(t) = \mu(t)$ $\quad \mu(t) \circ\!\!-\!\!\bullet \frac{z}{z-1}$

$$\frac{Y(z)}{z} = \frac{z}{z - \frac{1}{5}} \cdot \frac{1}{z-1} + \frac{2}{z - \frac{1}{5}} \cdot \frac{1}{z-1} = \frac{z}{(z - \frac{1}{5})(z-1)} + \frac{2}{(z - \frac{1}{5})(z-1)}$$

$$= \frac{z + 2}{(z - \frac{1}{5})(z-1)} = \frac{A}{z - \frac{1}{5}} + \frac{B}{z-1} = -\frac{11}{4} \cdot \frac{1}{z - \frac{1}{5}} + \frac{15}{4} \cdot \frac{1}{z-1}$$

$-A - \frac{1}{5} B = 2$ $\quad A = 1 - B$

$A + B = 1$ $\quad -(1-B) - \frac{1}{5} B = 2$

$-1 + B - \frac{1}{5} B = 2$

$\frac{4}{5} B = 3$

$B = \frac{15}{4}$

$A = -\frac{11}{4}$

$$\frac{Y}{z} = -\frac{11}{4} \cdot \frac{1}{z - \frac{1}{5}} + \frac{15}{4} \cdot \frac{1}{z-1}$$

$$Y(z) = -\frac{11}{4} \cdot \frac{z}{z - \frac{1}{5}} + \frac{15}{4} \cdot \frac{z}{z-1}$$

$$y(t) = \left(-\frac{11}{4} \cdot \left(\frac{1}{5}\right)^n + \left(\frac{15}{4}\right)\right) \cdot \mu(n)$$

(5) $y(t) = \int_{t-1}^{t+1} u(\tau)\, d\tau$

a) linearnost i vremenska nepromjenjivost

$u_1(t)$

$u_2(t)$

$u(t) = \alpha u_1(t) + \beta u_2(t)$

$$y(t) = \int_{t-1}^{t+1} \left(\alpha u_1(t) + \beta u_2(t)\right) dt = \alpha \int_{t-1}^{t+1} u_1(t)\, dt + \beta \int_{t-1}^{t+1} u_2(t)\, dt$$

$$y_1(t) = \int_{t-1}^{t+1} \alpha u_1(t)\, dt$$

$+$ linearan je

$$y_2(t) = \int_{t-1}^{t+1} \alpha u_2(t)\, dt$$

$\tau = T - M$

Vremenski nepromj.

$$y_2(t) = \int_{t-1} u_1(\xi)\, d\xi$$

Vremenska nepromj. $\qquad \tau = \tilde{\tau} - M$

$$u_1(t) = u(t-M)$$

$$y_1(t) = \int_{t-1}^{t+1} u(\tau)\, d\tau = \int_{t-1}^{t+1} u(\tilde{\tau} - M)\, d\tilde{\tau}$$

$$y_2(t-M) = \int_{t-M-1}^{t-M+1} u(\tau)\, d\tau$$

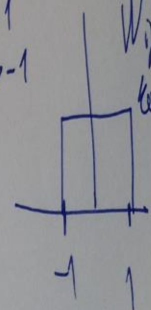
} promjenjiv je (?) //

b) impulsni odziv + kauzalnost

$u(t) = \delta(t)$ → impuls koji 0

$$y(t) = \int_{t-1}^{t+1} \delta(\xi)\, d\xi = 1 \text{ za } \begin{matrix} t-1 \cancel{<} 0 & t<1 \\ t+1>0 & t>-1 \end{matrix}$$

$$= \mu(t+1) - \mu(t-1)$$



Nije kauzalno

c) prijenosna funkcija

$$\mu(t) \Rightarrow \frac{1}{s}$$

$$\mu(t-a) \Rightarrow e^{-as} \frac{1}{s} \;//$$

$$\mu(t+1) \Rightarrow e^{s} \cdot \frac{1}{s}$$

$$\mu(t-1) \Rightarrow e^{-s} \frac{1}{s}$$

$$Y(s) = e^{s} \frac{1}{s} - e^{-s} \frac{1}{s}$$

$$\boxed{H(s) = \frac{1}{s} \cdot \left(e^{s} - e^{-s}\right)}$$

d) frekvenc kar i odzu $u(t) = 2\sin(\pi t + \frac{\pi}{4})$

$$H(j\omega) = \frac{e^{j\omega} - e^{-j\omega}}{j\omega} \cdot \frac{\cancel{j}}{\cancel{j}} = \frac{2j\sin\omega}{j\omega} = 2 \cdot \frac{\sin\omega}{\omega}$$

$u(t) = 2\sin(\pi t + \frac{\pi}{4}) \to \omega = \pi$

$\cancel{H(j\omega)}$ $H(j\cdot\pi) = 2 \cdot \frac{\sin\pi}{\pi} = 0$

$y(t) = 0$