A

1. $y(n) - \frac{1}{6} y(n-1) = 2u(n)$

$y(-1) = 2$

$u(n) = \left(\frac{1}{2}\right)^n \mu(n)$

a) $y_p = K\left(\frac{1}{2}\right)^n$

$K\left(\frac{1}{2}\right)^n - \frac{1}{6} \cdot K \left(\frac{1}{2}\right)^{n-1} = 2 \cdot \left(\frac{1}{2}\right)^n$

$K - \frac{1}{\cancel{6}_3} \cdot K \cdot \cancel{2} = 2$

$\boxed{y_{PRISILNI}(n) = 3\left(\frac{1}{2}\right)^n \mu(n)}$

$3K - K = 6$

$2K = 6$

$K = 3$

b) $q - \frac{1}{6} = 0$ $\quad y(n) = C\left(\frac{1}{6}\right)^n + 3\left(\frac{1}{2}\right)^n$

$q = \frac{1}{6}$

$y_h(n) = C\left(\frac{1}{6}\right)^n$

POČETNI UVJET: $y(n) = \frac{1}{6} y(n-1) + 2u(n)$

$y(0) = \frac{1}{6} \cdot 2 + 2 \cdot 1 = \frac{1}{3} + 2 = \frac{7}{3}$

$y(0) = C \cdot 1 + 3 \cdot 1 = \frac{7}{3}$

$C = \frac{7}{3} - 3 = \frac{7-9}{3} = -\frac{2}{3}$

$y(n) = -\frac{2}{3}\left(\frac{1}{6}\right)^n + 3\left(\frac{1}{2}\right)^n$

$\boxed{y_{PRIRODNI}(n) = -\frac{2}{3}\left(\frac{1}{6}\right)^n \mu(n)}$

A

c) MIRNI

$y(-1)=0$

$y(0)=\frac{1}{6}\cdot 0+2\cdot 1=2$

$y_m(n)= C\left(\frac{1}{6}\right)^n + 3\left(\frac{1}{2}\right)^n$

$y_m(0)= C+3 = 2$

$C=-1$

$$\boxed{y_m(n)=\left[3\left(\frac{1}{2}\right)^n-\left(\frac{1}{6}\right)^n\right]\mu(n)}$$

d) NEPOBUĐENI

$y(-1)=2$

$u(n)=0$

$y_n(n)= C\left(\frac{1}{6}\right)^n$

$y_0(-1)= C\cdot 6 = 2$

$C=\frac{2}{6}=\frac{1}{3}$

$$\boxed{y_0(n)=\frac{1}{3}\left(\frac{1}{6}\right)^n}$$

A

2.

$$y'(t) + 6y(t) = 2u'(t) + u(t)$$

$$sY(s) + 6Y(s) = 2sU(s) + U(s)$$

$$Y(s)(s+6) = U(s)(2s+1)$$

$$Y(s) = \frac{2s+1}{s+6} U(s)$$

a) za $u(t) = \delta(t) \longrightarrow U(s) = 1$

$$H(s) = \frac{2s+1}{s+6}$$

$$(2s+1) : (s+6) = 2$$
$$\frac{-2s \pm 12}{-11}$$

$$= 2 + \frac{-11}{s+6}$$

$$h(t) = 2\delta(t) - 11e^{-6t} \mu(t)$$

b) PRIJENOSNA FUNKCIJA $H(s) = \frac{2s+1}{s+6}$

POL $s+6=0$
$s=-6$

STABILNOST $\mathrm{Re}\{s\} = -6 < 0$

SUSTAV JE STABILAN

3. $y(n) - \frac{1}{4} y(n-1) = u(n) + 2u(n-1)$

a) $$H(z) = \frac{1 + 2z^{-1}}{1 - \frac{1}{4} z^{-1}} = \frac{z+2}{z - \frac{1}{4}}$$

POL $z = \frac{1}{4}$

STABILNOST $|z| = \frac{1}{4} < 1$ STABILNO

b) $H(z)$ u vremenskoj domeni je impulsni odziv $h(n)$

$$H(z) = \frac{z+2}{z - \frac{1}{4}}$$

$$\frac{H(z)}{z} = \frac{z+2}{(z - \frac{1}{4})z} = \frac{A}{z - \frac{1}{4}} + \frac{B}{z}$$

$$A + B = 1 \qquad \longrightarrow A = 1 - B$$

$$-\frac{1}{4} B = 2 \longrightarrow B = -8 \qquad A = 9$$

$$\frac{H(z)}{z} = \frac{9}{z - \frac{1}{4}} - \frac{8}{z}$$

$$H(z) = \frac{9z}{z - \frac{1}{4}} - 8 \quad \circ\!\!-\!\!\bullet \quad \boxed{h(n) = 9 \cdot \left(\frac{1}{4}\right)^n \mu(n) - 8\,\delta(n)}$$

c) $u(n) = (-2)^n \mu(n) \longrightarrow U(z) = \frac{z}{z+2}$

$$Y(z) = H(z) \cdot U(z)$$

$$= \frac{\cancel{z+2}}{z - \frac{1}{4}} \cdot \frac{z}{\cancel{z+2}} = \frac{z}{z - \frac{1}{4}}$$

$\boxed{y(n) = \left(\frac{1}{4}\right)^n \mu(n)}$ ODZIV MIRNOG SUSTAVA

4. $y''(t) + 6y'(t) + 5y(t) = u(t)$

a) $s^2 + 6s + 5 = 0$

$(s+5)(s+1) = 0$

$s = -5 \qquad s = -1$

$h_a(t) = y_h(t) = C_1 e^{-5t} + C_2 e^{-t}$

$h_a'(t) = -5C_1 e^{-5t} - C_2 e^{-t}$

$h_a(0^+) = 0$

$h_a'(0^+) = 1$

$h_a(0^+) = C_1 + C_2 = 0 \rightarrow C_1 = -C_2$

$h_a'(0^+) = -5C_1 - C_2 = 1 \rightarrow 5C_2 - C_2 = 1 \rightarrow 4C_2 = 1$

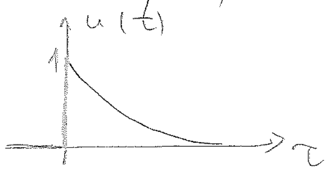
$C_2 = \frac{1}{4} \qquad C_1 = -\frac{1}{4}$

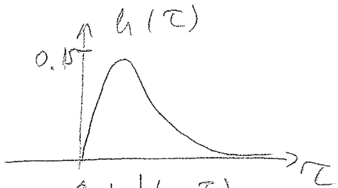
$h_a(t) = -\frac{1}{4} e^{-5t} + \frac{1}{4} e^{-t}$

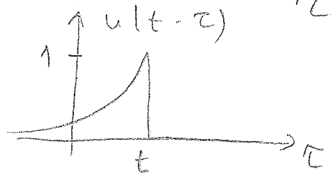
$h(t) = h_a(t)$

$\boxed{h(t) = \left(-\frac{1}{4} e^{-5t} + \frac{1}{4} e^{-t}\right)\mu(t)}$

b) $u(t) = e^{-2t}\mu(t)$







1° $t < 0$ nema preklapanja

$y(t) = 0$

2° $t \geq 0$ $\qquad y(t) = \int_{-\infty}^{\infty} h(\tau)u(t-\tau)\,d\tau$

$y(t) = \int_0^t e^{-2(t-\tau)}\left(-\frac{1}{4}e^{-5\tau} + \frac{1}{4}e^{-\tau}\right)d\tau$

$= -\frac{1}{4}\int_0^t e^{-2t-3\tau}\,d\tau + \frac{1}{4}\int_0^t e^{-2t+\tau}\,d\tau$

$= -\frac{1}{4}e^{-2t}\left.\frac{e^{-3\tau}}{-3}\right|_0^t + \frac{1}{4}e^{-2t}\left.e^{\tau}\right|_0^t$

$= \frac{1}{12}e^{-2t}\left(e^{-3t} - 1\right) + \frac{1}{4}e^{-2t}\left(e^{t} - 1\right)$

$= \frac{1}{12}e^{-5t} - \frac{1}{12}e^{-2t} + \frac{1}{4}e^{-t} - \frac{1}{4}e^{-2t}$

$\boxed{y(t) = \frac{1}{12}e^{-5t} - \frac{1}{3}e^{-2t} + \frac{1}{4}e^{-t} \qquad t \geq 0}$

1

5.

$$h(n) = \left(\frac{3}{2^n} + \frac{2}{3^n}\right)\mu(n)$$

$$= \left(3\cdot\left(\frac{1}{2}\right)^n + 2\cdot\left(\frac{1}{3}\right)^n\right)\mu(n)$$

a) $$H(z) = 3\cdot\frac{z}{z-\frac{1}{2}} + 2\cdot\frac{z}{z-\frac{1}{3}}$$

$$= \frac{3z(z-\frac{1}{3}) + 2z(z-\frac{1}{2})}{(z-\frac{1}{2})(z-\frac{1}{3})} = \frac{3z^2 - z + 2z^2 - z}{(z-\frac{1}{2})(z-\frac{1}{3})}$$

$$= \frac{5z^2-2z}{z^2-\frac{5}{6}z+\frac{1}{6}}$$

b) $$H(z) = \frac{Y(z)}{U(z)} = \frac{5z^2-2z}{z^2-\frac{5}{6}z+\frac{1}{6}} = \frac{5-2z^{-1}}{1-\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$$

$$Y(z)\left(z^2-\frac{5}{6}z+\frac{1}{6}\right) = U(z)(5z^2-2z)$$

$$y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2) = 5u(n) - 2u(n-1)$$

c) $$u(n) = 4\cos\frac{\pi}{2}n$$

$$H(e^{i\Omega}) = \frac{5-2e^{-i\Omega}}{1-\frac{5}{6}e^{-i\Omega}+\frac{1}{6}e^{-2i\Omega}}$$

$$H(e^{i\frac{\pi}{2}}) = \frac{5-2e^{-i\frac{\pi}{2}}}{1-\frac{5}{6}e^{-i\frac{\pi}{2}}+\frac{1}{6}e^{-2i\frac{\pi}{2}}} = \frac{5-2(\cos\frac{\pi}{2}-i\sin\frac{\pi}{2})}{1-\frac{5}{6}(\cos\frac{\pi}{2}-i\sin\frac{\pi}{2})+\frac{1}{6}(\cos\pi - i\sin\pi)}$$

$$= \frac{5+2i}{1+\frac{5}{6}j-\frac{1}{6}} = \frac{5+2i}{\frac{5}{6}+\frac{5}{6}i}$$

$$|H(e^{i\frac{\pi}{2}})| = \frac{\sqrt{25+4}}{\frac{5}{6}\sqrt{1+1}} = \frac{6}{5}\sqrt{\frac{29}{2}}$$

$$\sphericalangle H(e^{i\frac{\pi}{2}}) = \operatorname{arctg}\frac{2}{5} - \operatorname{arctg}1 = -0.405$$

$$y(n) = 4\cdot\frac{6}{5}\sqrt{\frac{29}{2}}\cdot\cos\left(\frac{\pi}{2}n - 0.405\right)$$

$$= 18.278\cos\left(\frac{\pi}{2}n - 0.405\right)$$