Signali i sustavi

Završni ispit (grupa A) – 1. srpnja 2009.

1. Zadan je kontinuirani LTI sustav opisan jednadžbom $y''(t) + 3y'(t) + 2y(t) = 3u(t)$.

a) Odredite prijenosnu funkciju sustava.

b) Izračunajte inverznu Laplaceovu transformaciju prijenosne funkcije! Što ona predstavlja?

c) Odredite polove i nule sustava te ispitajte stabilnost.

d) Odredite frekvencijsku karakteristiku te pomoću nje izračunajte odziv sustava u stacionarnom stanju na pobudu $u(t) = \sin(3t)$.

2. Kontinuirani kauzalni LTI sustav opisan jednadžbom $y'(t) + 6y(t) = u'(t) + 3u(t)$ pobuđen je signalom $u(t) = 2\,\mu(t)$. Poznat je početni uvjet $y(0^-) = 3$.

a) Odredite početni uvjet $y(0^+)$ u trenutku $t = 0^+$.

b) Izračunajte odziv sustava na zadanu pobudu za $t > 0$.

3. Zadan je diskretni kauzalni LTI sustav opisan jednadžbom $y(n) + y(n-2) = 2u(n)$.

a) Postupkom u vremenskoj domeni odredite impulsni odziv sustava.

b) Odredite prijenosnu funkciju sustava.

c) Odredite polove i nule sustava te ispitajte stabilnost.

d) Što je rezonancija? Odredite odziv mirnog sustava na pobudu $u(n) = \cos(\frac{n\pi}{2})\,\mu(n)$ (bilo kojim postupkom)!

4. Vremenski DISKRETNI sustav s više ulaza i izlaza (MIMO) opisan je matricama

$$\mathbf{A} = \begin{bmatrix} 0 & -4 \\ -4 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{i} \quad \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

a) Odredite prijenosnu matricu sustava.

b) Odredite matricu impulsnog odziva sustava.

5. Zadan je signal $x(t) = 4\cos(3t) + 6\sin(6t)$.

a) Izračunajte i skicirajte amplitudni i fazni spektar zadanog signala. Koju Fourierovu transformaciju ste koristili i zašto?

b) Koja je najmanja frekvencija otipkavanja potrebna da ne dođe do preklapanja spektra (eng. *aliasing*)?

c) Odredite otipkani signal za frekvenciju otipkavanja $f_s = \frac{12}{\pi}$.

d) Izračunajte i skicirajte amplitudni i fazni spektar dobivenog otipkanog signala. Koju Fourierovu transformaciju ste koristili i zašto?

e) Izračunajte snage oba signala.

1. $y''(t) + 3y'(t) + 2y(t) = 3u(t)$ A

a) $s^2Y(s) + 3sY(s) + 2Y(s) = 3U(s)$

$Y(s)(s^2+3s+2) = U(s)$

$H(s) = \frac{Y(s)}{U(s)} = \frac{3}{s^2+3s+2}$

b) $H(s) = \frac{3}{s^2+3s+2} = \frac{3}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$

$As + 2A + Bs + B = 3$

$A + B = 0$

$2A + B = 3$

$A = 3$

$B = -3$

$= \frac{3}{s+1} - \frac{3}{s+2}$

$h(t) = (3e^{-t} - 3e^{-2t})\mu(t)$

To je IMPULSNI ODZIV sustava.

c) POLOVI $s=-1$ <0

$s=-2$ <0 STABILAN SUSTAV

NULE nema

d) $H(j\Omega) = \frac{3}{(j\Omega)^2 + 3j\Omega + 2} = \frac{3}{2-\Omega^2+3\Omega j}$

$|H(j\Omega)| = \frac{3}{\sqrt{(2-\Omega^2)^2+9\Omega^2}} = \frac{3}{\sqrt{4+\Omega^4+5\Omega^2}} = \frac{3}{\sqrt{(\Omega^2+1)(\Omega^2+4)}}$

$\measuredangle H(j\Omega) = -\operatorname{arctg}\frac{3\Omega}{2-\Omega^2}$

$u(t) = \sin 3t$

$y(t) = |H(j3)| \cdot \sin(3t + \measuredangle H(j3))$

$|H(j3)| = \frac{3}{\sqrt{(9+1)(9+4)}} = \frac{3}{\sqrt{130}}$

$\measuredangle H(j3) = -\operatorname{arctg}\frac{9}{2-9} = -\operatorname{arctg}\frac{9}{-7} = \operatorname{arctg}\frac{9}{7} = -2.23 = -127.88°$

$y(t) = |H(j3)| \cdot \sin(3t - 2.23)$

2. $y'(t) + 6y(t) = u'(t) + 3u(t)$ A

$u(t) = 2\mu(t)$

$y(0^-) = 3$

a) $y(0^+) - y(0^-) = b_0 u(0^+) = 1 \cdot 2 = 2$

$y(0^+) = 2 + y(0^-) = 5$

b) $y'(t) + 6y(t) = 2\delta(t) + 6\mu(t)$

homogeno

$y'(t) + 6y(t) = 0$

$s + 6 = 0$

$s = -6$

$y(t) = Ce^{-6t}$

partikularno

$y'(t) + 6y(t) = 6\mu(t)$

$y_p(t) = K$

$6K = 6$

$K = 1$

$y_p(t) = 1$

impulsni odziv

$h_1'(t) + 6h_1(t) = 0$

$h_1(t) = C_1 e^{-6t}$

$h_1(0^+) = 1$

$h_1(0^+) = C_1 = 1$

$h_1(t) = e^{-6t}$

$h'(t) + 6h(t) = 2\delta(t)$

$h(t) = 2e^{-6t}$

TOTALNI ODZIV

$y_T(t) = (Ce^{-6t} + 2e^{-6} + 1)\mu(t)$

POČETNI UVJETI

$y_T(0^+) = C + 2 + 1 = 5$

$C = 2$

$y_T(t) = (4e^{-6t} + 1)\mu(t)$

A

3. $y(n)+y(n-2)=2u(n)$

a) $y(n)+y(n-2)=0$

$z^2+1=0$

$z_{1,2}=\pm i$

$h(n)=C_1 i^n + C_2(-i)^n$

$=C_1 e^{j\pi/2 n}+C_2 e^{-j\pi/2 n}$

početni uvjeti $y(n)=2\delta(n)-y(n-2)$

$h(0)=2-h(-2)=2$

$h(1)=0-h(-1)=0$

$h(0)=C_1+C_2=2$

$h(1)=C_1 i - C_2 i = 0$

$C_1=C_2=1$

$h(n)=i^n+(-i)^n$

$=e^{j\pi/2 n}+e^{-j\frac{\pi}{2}n}$

$=2\cos\frac{\pi}{2}n$

b) $Y(z)+z^{-2}Y(z)=2U(z)$

$Y(z)(1+z^{-2})=2U(z)$

$H(z)=\frac{Y(z)}{U(z)}=\frac{2}{1+z^{-2}}=\frac{2z^2}{z^2+1}$

c) NULE $z^2=0$ $z_{1,2}=0$

POLOVI $z^2+1=0$

$z^i=\pm i$

MARGINALNO / GRANIČNO STABILAN SUSTAV

d) REZONANCIJA → kada se frekv. pobude poklopi sa vlastitim frekv. sustava.

$u(n)=\left(\cos\frac{n\pi}{2}\right)\mu(n)$

homogeno: $y(n)=C_1 i^n + C_2(-i)^n$

partikularno: $y_p(n)=An\cos\frac{n\pi}{2}+Bn\sin\frac{n\pi}{2}$

$An\cos\frac{n\pi}{2}+Bn\sin\frac{n\pi}{2}+A(n-2)\underbrace{\cos\frac{\pi}{2}(n-2)}_{-\cos\frac{\pi}{2}n}+B(n-2)\underbrace{\sin\frac{\pi}{2}(n-2)}_{-\sin\frac{\pi}{2}n}=2\cos\frac{n\pi}{2}$

$2A\cos\frac{\pi}{2}n+2B\sin\frac{\pi}{2}n=2\cos\frac{\pi n}{2}$

$A=1$ $B=0$

$y_p(n)=n\cos\frac{n\pi}{2}$

TOTALNO: $y(n)=C_1 i^n+C_2(-i)^n+n\cos\frac{n\pi}{2}$

POČETNI UVJETI MIRNI

$y(-1)=0$

$y(-2)=0$

$y(0)=2\cdot\cos\frac{0\pi}{2}-y(-2)=2$

$y(1)=2\cos\frac{\pi}{2}-y(-1)=0$

$y(0)=C_1+C_2=2$

$y(1)=C_1 i - C_2 i+\cos\frac{\pi}{2}=0$

$C_1=C_2=1$

$y_{mirni}(n)=\left(i^n+(-i)^n+n\cos\frac{n\pi}{2}\right)\mu(n)$

$=\left(2\cos\frac{n\pi}{2}+n\cos\frac{n\pi}{2}\right)\mu(n)$

4. $A=\begin{bmatrix}0 & -4\\ -4 & 0\end{bmatrix}$ $B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ $D=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$ **A**

a)
$$\dot{x}(n+1)=Ax(n)+Bu(n)$$
$$x(z)=(zI-A)^{-1}Bu(z)$$
$$y(n)=Cx(n)+Du(n)$$
$$y(z)=(C(zI-A)^{-1}B+D)u(z)$$
$$H(z)=\frac{y(z)}{u(z)}=C(zI-A)^{-1}B+D$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}z & 4\\ 4 & z\end{bmatrix}^{-1}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$
$$=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\frac{1}{z^2-16}\begin{bmatrix}z & -4\\ -4 & z\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$
$$=\frac{1}{z^2-16}\begin{bmatrix}z & -4\\ -4 & z\end{bmatrix}=\begin{bmatrix}\frac{z}{(z-4)(z+4)} & \frac{-4}{(z-4)(z+4)}\\ \frac{-4}{(z-4)(z+4)} & \frac{z}{(z-4)(z+4)}\end{bmatrix}$$

b)
$$\frac{H_{11}}{z}=\frac{1}{(z-4)(z+4)}=\frac{A}{z-4}+\frac{B}{z+4}$$
$$A+B=0$$
$$4A-4B=1$$
$$8A=1$$
$$A=\frac{1}{8}\qquad B=-\frac{1}{8}$$
$$H_{11}(z)=\frac{\frac{1}{8}z}{z-4}-\frac{1}{8}\frac{z}{z+4}$$
$$h_{11}(n)=\left(\frac{1}{8}4^n-\frac{1}{8}(-4)^n\right)\mu(n)$$
$$\frac{H_{21}}{z}=-\frac{4}{z(z-4)(z+4)}=\frac{A}{z-4}+\frac{B}{z+4}+\frac{C}{z}=-\frac{1}{8}\frac{1}{z-4}-\frac{1}{8}\frac{1}{z+4}+\frac{1}{4}\frac{1}{z}$$
$$A+B+C=0$$
$$4A-4B=0$$
$$-16C=-4$$
$$C=\frac{1}{4}$$
$$A=-\frac{1}{8}\qquad B=-\frac{1}{8}$$
$$H_{21}(z)=-\frac{1}{8}\frac{z}{z-4}-\frac{1}{8}\frac{z}{z+4}+\frac{1}{4}$$
$$h_{21}(n)=\left(-\frac{1}{8}4^n-\frac{1}{8}(-4)^n\right)\mu(n)+\frac{1}{4}\delta(n)$$
$$h(n)=\begin{bmatrix}\left(\frac{1}{8}4^n-\frac{1}{8}(-4)^n\right)\mu(n) & \left(-\frac{1}{8}4^n-\frac{1}{8}(-4)^n\right)\mu(n)+\frac{1}{4}\delta(n)\\ \left(-\frac{1}{8}4^n-\frac{1}{8}(-4)^n\right)\mu(n)+\frac{1}{4}\delta(n) & \left(\frac{1}{8}4^n-\frac{1}{8}(-4)^n\right)\mu(n)\end{bmatrix}$$

5. $x(t) = 4\cos 3t + 6\sin 6t$

A

a) $\cos 3(t+T) = \cos 3t$ $\quad$ $\sin(6(t+T)) = \sin 6t$

$3T = 2k\pi$ $\quad$ $6T = 2k\pi$

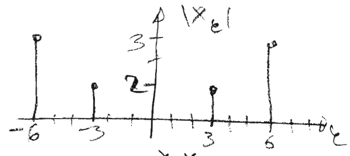
$T = \frac{2}{3}\pi$ $\quad$ $T = \frac{\pi}{3}$

$T = \frac{2}{3}\pi$

periodičan signal - Vremenski kontinuirani Fourierov red

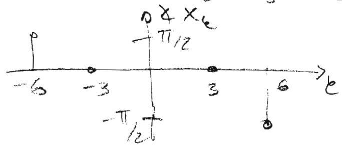
$$x(t) = 4\,\frac{e^{j3t}+e^{-j3t}}{2} + 6\,\frac{e^{j6t}-e^{-j6t}}{2j}$$

$$= 2e^{j3t} + 2e^{-j3t} - 3je^{j6t} + 3je^{-j6t}$$



$X_3 = 2$ $\quad$ $X_6 = -3j = 3e^{-j\pi/2}$

$X_{-3} = 2$ $\quad$ $X_{-6} = 3j = 3e^{j\pi/2}$



b) Najveća frekvencija u sustavu je $\omega = 6$

Minimalna frekv. otipkavanja je $\omega = 12 \Rightarrow f = \frac{\omega}{2\pi} = \frac{12}{2\pi} = \frac{6}{\pi}$

c) $f_s = \frac{12}{\pi} \to T_s = \frac{\pi}{12}$

$x(nT_s) = 4\cos(3nT_s) + 6\sin(6nT_s)$

$x(n) = 4\cos(3n\frac{\pi}{12}) + 6\sin(6n\frac{\pi}{12})$

$= 4\cos(\frac{\pi}{4}n) + 6\sin(\frac{\pi}{2}n)$

d) $\cos(\frac{\pi}{4}(n+N)) = \cos\frac{\pi}{4}n$ $\quad$ $\sin\frac{\pi}{2}(n+N) = \sin\frac{\pi}{2}n$

$\frac{\cancel{\pi}}{4}N = 2k\cancel{\pi}$ $\quad$ $\frac{\pi}{2}N = 2k\pi$

$N = 8$ $\quad$ $N = 4$

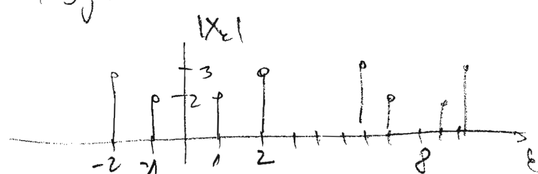
$N = 8$

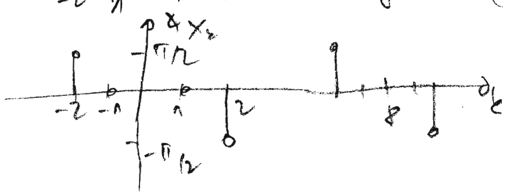
$$x(n) = 4\,\frac{e^{j\frac{\pi}{4}n}+e^{-j\frac{\pi}{4}n}}{2} + 6\,\frac{e^{j\frac{\pi}{2}n}-e^{-j\frac{\pi}{2}n}}{2j}$$

$$= 2e^{j\frac{2\pi}{8}n} + 2e^{-j\frac{2\pi}{8}n} - 3je^{j\frac{2\pi}{8}\cdot n\cdot 2} + 3je^{-j\frac{2\pi}{8}n\cdot 2}$$

$X_1 = 2$ $\quad$ $X_2 = -3j = 3e^{-j\pi/2}$

$X_{-1} = 2$ $\quad$ $X_{-2} = 3j = 3e^{j\pi/2}$





e) $P_{kontinuirani} = \sum_{i=-\infty}^{\infty} X_i^2 = 3^2 + 2^2 + 2^2 + 3^2 = 26$

$P_{diskretni} = \sum_{k=0}^{N-1} X_k^2 = 2^2 + 3^2 + 2^2 + 3^2 = 4+9+4+9 = 26$

Signali i sustavi

Završni ispit (grupa B) – 1. srpnja 2009.

1. Zadan je kontinuirani LTI sustav opisan jednadžbom $y''(t) + 4y'(t) + 3y(t) = 2u(t)$.

a) Odredite prijenosnu funkciju sustava.

b) Izračunajte inverznu Laplaceovu transformaciju prijenosne funkcije! Što ona predstavlja?

c) Odredite polove i nule sustava te ispitajte stabilnost.

d) Odredite frekvencijsku karakteristiku te pomoću nje izračunajte odziv sustava u stacionarnom stanju na pobudu $u(t) = \sin(3t)$.

2. Kontinuirani kauzalni LTI sustav opisan jednadžbom $y'(t) + 8y(t) = u'(t) + 4u(t)$ pobuđen je signalom $u(t) = 2\,\mu(t)$. Poznat je početni uvjet $y(0^-) = 3$.

a) Odredite početni uvjet $y(0^+)$ u trenutku $t = 0^+$.

b) Izračunajte odziv sustava na zadanu pobudu za $t > 0$.

3. Zadan je diskretni kauzalni LTI sustav opisan jednadžbom $y(n) + y(n-2) = 4u(n)$.

a) Postupkom u vremenskoj domeni odredite impulsni odziv sustava.

b) Odredite prijenosnu funkciju sustava.

c) Odredite polove i nule sustava te ispitajte stabilnost.

d) Što je rezonancija? Odredite odziv mirnog sustava na pobudu $u(n) = \sin(\frac{n\pi}{2})\,\mu(n)$ (bilo kojim postupkom)!

4. Vremenski DISKRETNI sustav s više ulaza i izlaza (MIMO) opisan je matricama

$$\mathbf{A} = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \text{i} \quad \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

a) Odredite prijenosnu matricu sustava.

b) Odredite matricu impulsnog odziva sustava.

5. Zadan je signal $x(t) = 2\cos(2t) + 4\sin(4t)$.

a) Izračunajte i skicirajte amplitudni i fazni spektar zadanog signala. Koju Fourierovu transformaciju ste koristili i zašto?

b) Koja je najmanja frekvencija otipkavanja potrebna da ne dođe do preklapanja spektra (eng. *aliasing*)?

c) Odredite otipkani signal za frekvenciju otipkavanja $f_s = \frac{8}{\pi}$.

d) Izračunajte i skicirajte amplitudni i fazni spektar dobivenog otipkanog signala. Koju Fourierovu transformaciju ste koristili i zašto?

e) Izračunajte snage oba signala.

B

1. $y''(t) + 4y'(t) + 3y(t) = 2u(t)$

a) $s^2 Y(s) + 4sY(s) + 3Y(s) = 2U(s)$

$Y(s)(s^2+4s+3) = 2U(s)$

$H(s) = \frac{Y(s)}{U(s)} = \frac{2}{s^2+4s+3}$

b) $H(s) = \frac{2}{(s+3)(s+1)} = \frac{A}{s+3} + \frac{B}{s+1}$

$As + A + Bs + 3B = 2$

$A + B = 0 \quad A = -B$

$A + 3B = 2$

$2B = 2$

$B = 1 \quad A = -1$

$H(s) = \frac{-1}{s+3} + \frac{1}{s+1}$

$h(t) = (-e^{-3t} + e^{-t})\mu(t)$

To je impulsni odziv.

c) POLOVI $s_1 = -3 \quad < 0$ SUSTAV JE STABILAN

$s_2 = -1 \quad < 0$

NULE nema

d) $H(j\Omega) = \frac{2}{(j\Omega)^2 + 4j\Omega + 3} = \frac{2}{3 - \Omega^2 + 4\Omega j}$

$|H(j\Omega)| = \frac{2}{\sqrt{(3-\Omega^2)^2 + 16\Omega^2}} = \frac{2}{\sqrt{9 - 6\Omega^2 + \Omega^4 + 16\Omega^2}} = \frac{2}{\sqrt{9 + 10\Omega^2 + \Omega^4}}$

$\angle H(j\Omega) = -\text{arctg}\frac{4\Omega}{3-\Omega^2}$ $\angle H(j3) = -\text{arctg}\frac{12}{-6} = -116.57° = -2.03$

$u(t) = \sin 3t$

$|H(j3)| = \frac{2}{\sqrt{9 + 90 + 81}} = \frac{2}{\sqrt{180}}$

$y(t) = \frac{2}{\sqrt{180}} \sin(3t - 2.03)$

B

2. $y'(t) + 8y(t) = u'(t) + 4u(t)$

$u(t) = 2\mu(t)$

$y(0^-) = 3$

a) $y(0^+) - y(0^-) = 1 \cdot u(0^+)$

$y(0^+) = 3 + 2 = 5$

b) HOMOGENA

$y'(t) + 8y(t) = 0$

$s = -8$

$y_h(t) = Ce^{-8t}$

PARTIKULARNO

$y_p = K$

$8K = 4 \cdot 2$

$K = 1$

IMPULSNI ODZIV

$h_A' + 8h_A = 0$

$h_A(t) = Ce^{-8t}$

$h_A(0^+) = 1$

$h_A(0^+) = C = 1$

$h_A(t) = e^{-8t}$

$h(t) = 2e^{-8t}\mu(t)$

TOTALNI ODZIV

$y_T(t) = Ce^{-8t} + 1 + 2e^{-8t}$

$y_T(0^+) = C + 1 + 2 = 5$

$C = 2$

$y_T(t) = (4e^{-8t} + 1)\mu(t)$

3. $y(n) + y(n-2) = 4u(n)$

B

a) $y(n) + y(n-2) = 0$

$z^2 + 1 = 0$

$z_{1,2} = \pm i$

$h(n) = C_1 i^n + C_2 (-i)^n$

početni uvjeti $y(n) = 4\delta(n) - y(n-2)$

$h(0) = 4$

$h(1) = 4 \cdot 0 - h(-1) = 0$

$h(0) = C_1 + C_2 = 4$

$h(1) = C_1 i - C_2 i = 0$

$C_1 = C_2 = 2$

$h(n) = 2i^n + 2(-i)^n$

$= 2e^{j\frac{\pi}{2}n} + 2e^{-j\frac{\pi}{2}n} = 2(\cos\frac{\pi}{2}n + \cancel{j\sin\frac{\pi}{2}n} + \cos\frac{\pi}{2}n - \cancel{j\sin\frac{\pi}{2}n})$

$= 4\cos\frac{\pi}{2}n$

b) $z^0 Y(z) + z^{-2} Y(z) = 4U(z)$

$Y(z)(1 + z^{-2}) = 4U(z)$

$H(z) = \frac{Y(z)}{U(z)} = \frac{4}{1+z^{-2}} = \frac{4z^2}{z^2+1}$

c) NULE $z^2 = 0$ $\quad z_{1,2} = 0$

POLOVI $z^2 + 1 = 0$

$z_{1,2} = \pm i$

GRANIČNO (MARGINALNO) STABILAN SUSTAV

d) REZONANCIJA → kada se frekvencija pobude poklopi sa vlastitim frekvencijama sustava.

Ako je sustav bio marginalno stabilan - odziv počne stalno rasti

$u(n) = \sin\left(\frac{n\pi}{2}\right)\mu(n)$

homogeno: $y(n) = C_1 i^n + C_2 (-i)^n$

partikularno: $y_p(n) = An\cos\frac{n\pi}{2} + Bn\sin\frac{n\pi}{2}$

$An\cos\frac{\pi}{2}n + Bn\sin\frac{\pi}{2}n + A(n-2)\underbrace{\cos\frac{\pi}{2}(n-2)}_{-\cos\frac{\pi}{2}n} + B(n-2)\underbrace{\sin\frac{\pi}{2}(n-2)}_{-\sin\frac{\pi}{2}n} = 4\sin\frac{n\pi}{2}$

$A(\cancel{n\cos\frac{\pi}{2}n} - \cancel{n\cos\frac{\pi}{2}n} + 2\cos\frac{\pi}{2}n) + B(\cancel{n\sin\frac{\pi}{2}n} - \cancel{n\sin\frac{\pi}{2}n} + 2\sin\frac{\pi}{2}n) = 4\sin\frac{n\pi}{2}$

$A\cos\frac{\pi}{2}n + B\sin\frac{\pi}{2}n = 2\sin\frac{n\pi}{2}$

$A = 0 \qquad B = 2$

$y_p(n) = 2n\sin\frac{n\pi}{2}$

$y_{TOT}(n) = C_1 i^n + C_2(-i)^n + 2n\sin\frac{n\pi}{2}$

$y_{TOT}(0) = C_1 + C_2 = 0 \qquad C_1 = -C_2$

$y_{TOT}(1) = C_1 i - C_2 i + 2 = 4$

$C_1 i - C_2 i = 2$

$-C_2 i - C_2 i = 2$

$-2i C_2 = 2 \quad /:(-2i)$

$C_2 = \frac{1}{-i} \cdot \frac{i}{i} = i \qquad C_1 = -i$

POČETNI UVJETI

$y(-1) = 0$

$y(-2) = 0$

$y(0) = 4\sin\left(\frac{0\pi}{2}\right) - y(-2) = 0$

$y(1) = 4\sin\left(\frac{\pi}{2}\right) - y(-1) = 4$

$y_{TOT}(n) = (-i \cdot i^n + i(-i)^n + 2n\sin\frac{n\pi}{2})\mu(n)$

$= (-i^{n+1} + i(-i)^n + 2n\sin\frac{n\pi}{2})\mu(n)$

$= (2\cos(\frac{\pi}{2}n - \frac{\pi}{2}) + 2n\sin\frac{n\pi}{2})\mu(n)$

$= (2\sin\frac{\pi}{2}n + 2n\sin\frac{\pi}{2}n)\mu(n)$

4. B

$$A = \begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}$$
$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

a)

$$H(z) = \frac{Y(z)}{U(z)} = C(zI - A)^{-1}B + D$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z & 2 \\ 2 & z \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \frac{1}{z^2-4} \begin{bmatrix} z & -2 \\ -2 & z \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} \frac{z}{z^2-4} & \frac{-2}{z^2-4} \\ \frac{-2}{z^2-4} & \frac{z}{z^2-4} \end{bmatrix}$$

b)

$$H_{11}(z) = \frac{z}{z^2-4} = H_{22}(z)$$

$$\frac{H_{11}(z)}{z} = \frac{1}{(z-2)(z+2)} = \frac{A}{z-2} + \frac{B}{z+2} = \frac{1}{4}\frac{1}{z-2} - \frac{1}{4}\frac{1}{z+2}$$

$A + B = 0$ $\qquad B = -\frac{1}{4}$

$2A - 2B = 1$

$4A = 1$

$A = \frac{1}{4}$

$$H_{11}(z) = \frac{1}{4}\frac{z}{z-2} - \frac{1}{4}\frac{z}{z+2} \quad \to \quad h_{11}(n) = \left(\frac{1}{4}2^n - \frac{1}{4}(-2)^n\right)\mu(n)$$

$$H_{21}(z) = H_{12}(z) = -\frac{2}{z^2-4}$$

$$\frac{H_{21}(z)}{z} = \frac{-2}{z(z-2)(z+2)} = \frac{A}{z} + \frac{B}{z-2} + \frac{C}{z+2}$$

$$Az^2 - 4A + Bz^2 + 2Bz + Cz^2 - 2Cz = -2$$

$A + B + C = 0$ $\qquad B + C = \frac{1}{2}$ $\qquad C = \frac{1}{2} - \frac{1}{4}$

$2B - 2C = 0$ $\qquad 2B + 2C = 1$ $\qquad C = \frac{1}{4}$

$-4A = -2$ $\qquad 4B = 1$

$A = \frac{1}{2}$ $\qquad B = \frac{1}{4}$

$$H_{21}(z) = \frac{1}{2} + \frac{1}{4}\frac{z}{z-2} + \frac{1}{4}\frac{z}{z+2} \quad \to \quad h_{21}(n) = \frac{1}{2}\delta(n) + \left(\frac{1}{4}2^n + \frac{1}{4}(-2)^n\right)\mu(n)$$

$$h(n) = \begin{bmatrix} \left(\frac{1}{4}2^n - \frac{1}{4}(-2)^n\right)\mu(n) & -\left(\frac{1}{4}2^n + \frac{1}{4}(-2)^n\right)\mu(n) + \frac{1}{2}\delta(n) \\ -\left(\frac{1}{4}2^n + \frac{1}{4}(-2)^n\right)\mu(n) + \frac{1}{2}\delta(n) & \left(\frac{1}{4}2^n - \frac{1}{4}(-2)^n\right)\mu(n) \end{bmatrix}$$

B.

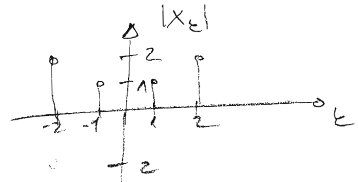
5. $x(t) = 2\cos 2t + 4\sin 4t$

a) $\cos(2(t+T)) = \cos 2t$ $\qquad$ $\cos(4(t+T)) = \cos 4t$

$2T = 2k\pi$ $\qquad$ $4T = 2k\pi$

$T = \pi$ $\qquad$ $T = \frac{\pi}{2}$

$T = \pi$

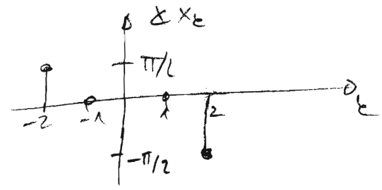


periodičan signal

Vremenski kontinuirani Fourierov red

$$x(t) = 2\,\frac{e^{j2t} + e^{-j2t}}{2} + 4\,\frac{e^{j4t} - e^{-j4t}}{2j}$$

$$= e^{j2t} + e^{-j2t} - 2je^{j4t} + 2je^{-j4t}$$



$X_1 = 1$ $\qquad$ $X_2 = -2j = 2e^{-j\frac{\pi}{2}}$

$X_{-1} = 1$ $\qquad$ $X_{-2} = +2j = 2\,e^{j\frac{\pi}{2}}$

b) Najveća frekvencija je $\omega = 4 = 2\cdot 2$

Min frekv. otipkavanja je $\omega = 8$ rad/s $\Rightarrow f = \frac{\omega}{2\pi} = \frac{8}{2\pi} = \frac{4}{\pi}$

c) $f_s = \frac{8}{\pi} \rightarrow T_s = \frac{\pi}{8}$

$$x(nT_s) = 2\cos(2nT_s) + 4\sin(4nT_s)$$

$$= 2\cos\left(2n\tfrac{\pi}{8}\right) + 4\sin\left(4n\tfrac{\pi}{8}\right) = 2\cos\left(\tfrac{\pi}{4}n\right) + 4\sin\left(\tfrac{\pi}{2}n\right)$$

$\cos\left(\frac{\pi}{4}(n+N)\right) = \cos\frac{\pi}{4}n$ $\qquad$ $\sin\left(\frac{\pi}{2}(n+N)\right) = \sin\frac{\pi}{2}n$

$\frac{\pi}{4}N = 2k\pi$ $\qquad$ $\frac{\pi}{2}N = 2k\pi$

$N = 8$ $\qquad$ $N = 4$

period $N = 8$

signal je periodičan - koristit će se DTFS

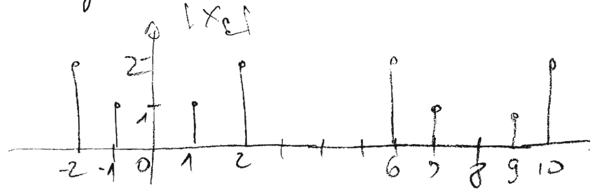
d) $x(n) = 2\cos\frac{\pi}{4}n + 4\sin\frac{\pi}{2}n$

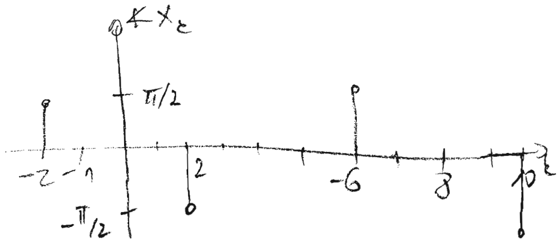
$$= 2\,\frac{e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}}{2} + 4\,\frac{e^{j\frac{\pi}{2}n} - e^{-j\frac{\pi}{2}n}}{2j}$$

$$= e^{j\frac{2\pi}{8}\cdot 1n} + e^{-j\frac{2\pi}{8}\cdot 1\cdot n} - 2je^{j\frac{2\pi}{8}\cdot 2n} + 2je^{-j\frac{2\pi}{8}\cdot 2n}$$

$X_1 = 1$ $\qquad$ $X_2 = -2j = 2e^{-j\pi/2}$

$X_{-1} = 1$ $\qquad$ $X_{-2} = +2j = 2\,e^{j\pi/2}$





e) $P_{kont} = \sum X_i^2 = 2^2 + 1^2 + 1^2 + 2^2 = 10$

$P_{disk} = \sum_{k=0}^{N-1} X_k^2 = 1^2 + 2^2 + 2^2 + 1^2 = 10$