$\underline{1}$ $6y(n) - 5y(n-1) + y(n-2) = u(n)$

homogena:

$$6Cq^{n} - 5Cq^{n-1} + Cq^{n-2} = 0$$

$$\left(1 - \frac{5}{q} + \frac{1}{q^2}\right) = 0$$

$$\frac{6q^2 - 5q + 1}{q^2} = 0$$

$$q_{1,2} = \frac{5 \pm \sqrt{25 - 4 \cdot 6}}{2}$$

$$= \frac{5 \pm 1}{12}$$

$q_1 = \frac{1}{2}$ $q_2 = \frac{1}{3}$ – stabilan impulsni odziv:

- koristimo homogene: $-\delta(n) = u(n)$ i $P.U. = 0$

$h(n) = C_1\left(\frac{1}{2}\right)^n + C_2\left(\frac{1}{3}\right)^n$

$6h(n) - 5h(n-1) + h(n-2) = \delta(n)$

$h(n) = \delta(n)\cdot\frac{1}{6} + h(n-1)\cdot\frac{5}{6} - \frac{1}{6}h(n-2)$

$h(0) = C_1 + C_2 = \frac{1}{6} \Rightarrow C_2 = \frac{1}{6} - C_1$

$h(0) = 1\cdot\frac{1}{6} + \frac{5}{6}\cdot 0 - \frac{1}{6}\cdot 0 = \frac{1}{6}$

$h(1) = \frac{1}{2}C_1 + \frac{1}{3}C_2 = \frac{5}{36}$

$h(1) = 0\cdot\frac{1}{6} + \frac{5}{6}\cdot\frac{1}{6} - \frac{1}{6}\cdot 0 = \frac{5}{36}$

$\frac{1}{2}C_1 + \frac{1}{18} - \frac{1}{3}C_1 = \frac{5}{36}$

$\frac{1}{6}C_1 = \frac{1}{12}$

$C_1 = \frac{1}{2}$

$C_2 = -\frac{1}{3}$

$h(n) = \left(\frac{1}{2}\left(\frac{1}{2}\right)^n - \frac{1}{3}\cdot\left(\frac{1}{3}\right)^n\right)u(n)$

[illegible]:

$$y_m(n) = C_1 \cdot \left(\frac{1}{2}\right)^n + C_2\left(\frac{1}{3}\right)^n + y_p(n)$$

$$y_m(n) = C_1\left(\frac{1}{2}\right)^n + C_2\left(\frac{1}{3}\right)^n + \frac{1}{2}$$

$$y_m(0) = C_1 + C_2 + \frac{1}{2} = \frac{1}{6} \Rightarrow C_2 = -\frac{1}{3} - C_1$$

$$y_m(1) = \frac{1}{2}C_1 + \frac{1}{3}C_2 + \frac{1}{2} = \frac{11}{36}$$

$$\frac{1}{2}C_1 - \frac{1}{9} - \frac{1}{3}C_1 + \frac{1}{2} = \frac{11}{36}$$

$$\frac{1}{6}C_1 = -\frac{1}{12}$$

$$C_1 = -\frac{1}{2}$$

$$C_2 = -\frac{5}{6}$$

$$y_m(n) = \left(-\frac{1}{2}\left(\frac{1}{2}\right)^n - \frac{5}{6}\left(\frac{1}{3}\right)^n + \frac{1}{2}\right)\mu(n)$$

$$u(n) = \mu(n) \Rightarrow K = y_p(n)$$

$$6K - 5K + K = 1$$

$$K = \frac{1}{2}$$

$$y_m(n) = \frac{1}{6}\mu(n) + \frac{5}{6}y_m(n-1) - \frac{1}{6}y_m(n-2)$$

$$y_m(0) = \frac{1}{6}\underset{1}{\mu(0)} + \frac{5}{6}\underset{0}{y_m(-1)} - \frac{1}{6}\underset{0}{y_m(-2)} = \frac{1}{6}$$

$$y_m(1) = \frac{1}{6}\mu(1) + \frac{5}{6}\overset{\frac{1}{6}}{y(0)} - \frac{1}{6}\overset{0}{y(-1)} = \frac{11}{36}$$

4. $y(n) - y(n-2) = u(n)$

impulsni odziv: $u(n) = \delta(n)$ - p.u = 0

$z^n - z^{n-2} = 0$ - homogena

$$\frac{z^2 - 1}{z^2} = 0$$

$z_{1,2} = \pm 1$

$y_h(n) = C_1 + C_2(-1)^n$

- nestabilan

- ulazni odziv za jed. step.:

$u(n) = \mu(n)$

$y_{ul}(n) = C_1 + C_2(-1)^n + y_p(n)$

$y_{ul}(n) = C_1 + C_2(-1)^n + \frac{1}{2}n$

$y_{ul}(n) = \mu(n) + y_{ul}(n-2)$

, 0 \ zbog p.u = 0

$y(n) = \delta(n) + y(n-2)$

$y(0) = 1 + 0 = 1$

$y(1) = 0 + 0 = 0$

$C_1 + C_2 = 1$

$C_1 - C_2 = 0 \Rightarrow C_1 = C_2$

$C_1 = \frac{1}{2}, \quad C_2 = \frac{1}{2}$

$h(n) = \left(\frac{1}{2} + \frac{1}{2}(-1)^n\right)\mu(n)$

partikularno:

$y_p(n) = Kn$

- pojašnjenje:

pobuda	$y_p(n)$
$A r^n, r \neq z_i$	$K r^n$
$A r^n, r = z_i$	$K n^M r^n$

$u(n) = \mu(n)$

$y_m(n) = C_1 + C_2(-1)^n + y_p(n)$

$y_m(n) = C_1 + C_2(-1)^n + \frac{1}{2}n$

$y_m(n) = \mu(n) + y_m(n-2)$

$y_m(0) = \mu(0) + y_m(-2)$ zbog p.u. = 0

$y_m(1) = \mu(1) + y_m(-1)$

$C_1 + C_2 = 1 \qquad C_2 = 1 - C_1$

$C_1 - C_2 + \frac{1}{2} = 1$

$C_1 - 1 + C_1 + \frac{1}{2} = 1$

$2C_1 = \frac{3}{2}$

$C_1 = \frac{3}{4} \qquad C_2 = \frac{1}{4}$

$y_m(n) = \left(\frac{3}{4} + \frac{1}{4}(-1)^n + \frac{1}{2}\right)\mu(n)$

partikularno:

$y_p(n) = Kn$

- objašnjenje:

pobuda	$y_p(n)$
$A r^n,\ r \neq z_i$	$K r^n$
$A r^n,\ r = z_i$	$K n^M r^n$

M – broj koji govori koliko puta vrijedi jednakost

r – frekvencija pobude (ovdje = 1)

z_i – korijeni karakteristične funkc.

npr. $z_{1,2} = 2$ i $u(n) = 2\mu(n)$

onda bi bilo $y_p(n) = Kn^2$

kontaš?

$Kn - K(n-2) = 1$

$Kn - Kn + 2K = 1$

$K = \frac{1}{2}$

5 $y(n) - 5y(n-1) + 6y(n-2) = u(n)$

impulsni odziv: $u(n) = \delta(n)$ - p.u. = 0

- homogena:

$C_2^n - 5C_2^{n-1} + 6C_2^{n-2} = 0$

$$\frac{1 - 5_2 + 6}{2^2} = 0$$

$$2_{1,2} = \frac{5 \pm \sqrt{25-24}}{2}$$

$2_1 = 3$, $2_2 = 2$

- nestabilan

$y_h(n) = C_1 \cdot 3^n + C_2 \cdot 2^n$

$y(n) = 5y(n-1) - 6y(n-2) + \delta(n)$

1) $y(0) = 5 \cdot 0 - 6 \cdot 0 + 1 = 1$

2) $y(1) = 5 \cdot 1 - 6 \cdot 0 + 0 = 5$

1) $C_1 + C_2 = 1 \Rightarrow C_2 = 1 - C_1$

2) $3C_1 + 2C_2 = 5$

$3C_1 + 2 - 2C_1 = 5$

$C_1 = 3$

$C_2 = -2$

$h(n) = (3 \cdot 3^n - 2 \cdot 2^n)\mu(n)$

mirni odziv:

$u(n) = \mu(n)$ - p.u. = 0

$y_{m}(n) = C_1 3^n + C_2 2^n + \frac{1}{2}$

$y_p(n) = K$

$K - 5K + 6K = 1$

- nestabilan

$y_h(n) = C_1 3^n + C_2 2^n$

mirni odziv:

$u(n) = \mu(n)$ - p.u. = 0

$y_m(n) = C_1 3^n + C_2 2^n + \frac{1}{2}$

$C_1 + C_2 + \frac{1}{2} = 1 \Rightarrow C_2 = \frac{1}{2} - C_1$

$3C_1 + 2C_2 + \frac{1}{2} = 6$

$3C_1 + 1 - 2C_1 + \frac{1}{2} = 6$

$C_1 = \frac{9}{2}$

$C_2 = -4$

$y_m(n) = \left(\frac{9}{2} 3^n - 4 \cdot 2^n + \frac{1}{2}\right)\mu(n)$

$C_2 = -2$

$h(n) = (3 \cdot 3^n - 2 \cdot 2^n)\mu(n)$

$y_p(n) = K$

$K - 5K + 6K = 1$

$K = \frac{1}{2}$

$y_m(n) = 5y(n-1) - 6y(n-2) + \mu(n)$

$y_m(0) = 5 \cdot 0 - 6 \cdot 0 + 1 = 1$

$y_m(1) = 5 \cdot 1 - 6 \cdot 0 + 1 = 6$

$y_h(n) = C_1$ ...

$= C_1 + C_2$ ...

- za k-struki ...

$z = z_1 = \dots = z_k$

$y_h(n) = (C_1 + C_2$ ...

mirni od...

$u(n) = \mu(n)$ -

$y_m(n) = C_1 +$ ...

$C_1 + 0 + \frac{1}{2} = 1$ ...

$C_1 + C_2 + \frac{1}{2} =$ ...

$C_2 = 3 - 1$ ...

$y_m(n) = \left(\frac{1}{2}\right.$ ...

7. $y(n) - 2y(n-1) + y(n-2) = u(n)$

impulsni odziv

$z^n - 2z^{n-1} + z^{n-2} = 0$

$$\frac{z^2 - 2z + 1}{z^2} = 0$$

$$z_{1,2} = \frac{2 \pm \sqrt{4-4}}{2} = 1$$

$z_{1,2} = 1$

$y_h(n) = C_1 (1)^n + C_2 \cdot n (1)^n$

$= C_1 + C_2 \cdot n$

za k-struki realni korijen

$z_1 = z_2 = \dots = z_k$

$y_h(n) = (C_1 + C_2 n + \dots + C_k n^{k-1}) z_1^n$

$h(n) = 2h(n-1) - h(n-2) + \delta(n)$

$h(0) = 2 \cdot 0 - 0 + 1 = 1$

$h(1) = 2 \cdot 1 - 0 + 0 = 2$

$C_1 + 0 = 1 \Rightarrow C_1 = 1$

$C_1 + C_2 = 2 \Rightarrow C_2 = 1$

$h(n) = (1 + n)\mu(n)$

prisilni odziv:

$u(n) = \mu(n)$ - p.u. = 0

$y_h(n) = C_1 + C_2 \cdot n + \frac{1}{2}$

$y_p(n) = K n^2$ - objašnjeno u 4 zad.

$Kn^2 - 2K(n-1)^2 + K(n-2)^2 = 1$

$Kn^2 - 2Kn^2 + 4Kn - 2K + Kn^2 - 4Kn + 4K = 1$

- za k-struki realni korijen

$\lambda = \lambda_i \ldots = \lambda_k$

$y_h(n) = (C_1 + C_2 n + \ldots + C_k n^{k-1}) \lambda_1^n$

nulti odziv:

$u(n) = \mu(n) \quad - p.u. = 0$

$y_{m}(n) = C_1 + C_2 \cdot n + \frac{1}{2}$

$C_1 + 0 + \frac{1}{2} = 1 \Rightarrow \quad C_1 = \frac{1}{2}$

$C_1 + C_2 + \frac{1}{2} = 3$

$C_2 = 3 - 1 = 2$

$y_{m}(n) = \left(\frac{1}{2} + \frac{1}{2} n + \frac{1}{2}\right) \mu(n)$

$y_p(n) = K \cdot n^2 \quad$ - objašnjeno u 4. zad.

$Kn^2 - 2K(n-1)^2 + K(n-2)^2 = 1$

$\cancel{Kn^2} - \cancel{2Kn^2} + \cancel{4Kn} - 2K + \cancel{Kn^2} - \cancel{4Kn} + 4K = 1$

$\underline{K = \frac{1}{2}}$

$y_m(n) = 2y(n-1) - y(n-2) + \mu(n)$

$y_m(0) = 2 \cdot 0 - 0 + 1 = 1$

$y_m(1) = 2 \cdot 1 - 0 + 1 = 3$

8. zadatak:

$y(n) - 4y(n-1) + 4y(n-2) = u(n)$

impulsni odziv:

$C q^n - 4C q^{n-1} + 4C q^{n-2} = 0$ - homogena

$$\frac{q^2 - 4q + 4}{q^2} = 0$$

$q_{1,2} = 2$ - dvostruki

$y_h(n) = C_1 2^n + C_2 \cdot n \cdot 2^n$

- objašnjeno u 7. zad.

$h(n) = 4h(n-1) - 4h(n-2) + \delta(n)$

$h(0) = 4 \cdot 0 - 4 \cdot 0 + 1 = 1$

$h(1) = 4 \cdot 1 - 4 \cdot 0 + 0 = 4$

$C_1 + 0 = 1 \Rightarrow C_1 = 1$

$2C_1 + 2C_2 = 4$

$2C_2 = 2$

$C_2 = 1$

$h(n) = (2^n + 2^n n)\mu(n)$

$= 2^n(1+n)\mu(n)$

mirni odziv:

$u(n) = \mu(n)$ - p.u. = 0

$y_p(n) = K$

$K - 4K + 4K = 1$

$K = 1$

$y_m(n) = C_1 2^n + C_2 n 2^n + 1$

$y_m(n) = 4y_m(n-1) - 4y_m(n-2) + \mu(n)$

$C_1 + 1 = 1 \Rightarrow C_1 = 0$

$= 2^n(1+n)\mu(n)$

Βηματική απόκριση:

$u(n)=\mu(n)$ $-$ ρ.υ. $=0$

$y_p(n)=K$

$K-4K+4K=1$

$K=1$

$y_m(n)=4y_m(n-1)-4y_m(n-2)+\mu(n)$

$y_m(0)=4\cdot 0-4\cdot 0+1=1$

$y_m(1)=4\cdot 1-4\cdot 0+1=5$

$y_m(n)=C_1 2^n+C_2\cdot n 2^n+1$

$C_1+1=1 \Rightarrow \underline{C_1=0}$

$2C_2+1=5$

$\underline{C_2=2}$

$y_m(n)=(2n\cdot 2^n+1)\mu(n)$

3. $4y(n) - 4y(n-1) + y(n-2) = u(n)$

impulsni odziv:

$u(n) = \delta(n)$ - p.u. = 0

homogena:

$$4c_1 q^n - 4c_1 q^{n-1} + c_1 q^{n-2} = 0$$

$$\frac{4q^2 - 4q + 1}{q^2} = 0$$

$$q_{1,2} = \frac{4}{8} = \frac{1}{2}$$

- stabilan

$$y_h(n) = c_1\left(\frac{1}{2}\right)^n + c_2 n\left(\frac{1}{2}\right)^n$$

$$h(n) = h(n-1) - \frac{1}{4}h(n-2) + \frac{1}{4}\delta(n)$$

$$h(0) = 0 - \frac{1}{4}0 + \frac{1}{4} = \frac{1}{4}$$

$$h(1) = \frac{1}{4} - \frac{1}{4}0 + \frac{1}{4}0 = \frac{1}{4}$$

$$c_1 = \frac{1}{4}$$

$$c_1\frac{1}{2} + \frac{1}{2}c_2 = \frac{1}{4}$$

$$c_2 = \frac{1}{4}$$

$$h(n) = \left(\frac{1}{4}\left(\frac{1}{2}\right)^n + \frac{1}{4}n\left(\frac{1}{2}\right)^n\right)\mu(n)$$

jedinični odziv:

$u(n) = \mu(n)$ - p.u. = 0

$$y_{m}(n) = c_1\left(\frac{1}{2}\right)^n + c_2 \cdot n\left(\frac{1}{2}\right)^n + y_p(n)$$

$$= c_1\left(\frac{1}{2}\right)^n + c_2 \cdot n\left(\frac{1}{2}\right)^n + 1$$

$y_p(n) = K$

$4K - 4K + K = 1$

$K = 1$

$$y_m(n) = y_m(n-1) - \frac{1}{4}y_m(n-2) + \frac{1}{4}\mu(n)$$

-stabilan

$y_h(n) = C_1 \left(\frac{1}{2}\right)^n + C_2 \cdot n \left(\frac{1}{2}\right)^n$

Prinudni odziv:

$u(n) = \mu(n) \quad -p.u. = 0$

$y_{pn}(n) = C_1 \left(\frac{1}{2}\right)^n + C_2 \cdot n \left(\frac{1}{2}\right)^n + y_p(n)$

$= C_1 \left(\frac{1}{2}\right)^n + C_2 \cdot n \left(\frac{1}{2}\right)^n + 1$

$C_1 + 1 = \frac{1}{4} \Rightarrow C_1 = -\frac{3}{4}$

$\frac{1}{2} C_1 + \frac{1}{2} C_2 + 1 = \frac{1}{2}$

$C_2 = -\frac{1}{4}$

$y_{pn}(n) = \left(\frac{1}{2} \left(\frac{1}{2}\right)^n - \frac{1}{4} n \left(\frac{1}{2}\right)^n + 1\right) \mu(n)$

$h(n) = \left(\frac{1}{4} \left(\frac{1}{2}\right)^n + \frac{1}{4} n \left(\frac{1}{2}\right)^n\right) \mu(n)$

$y_p(n) = K$

$4K - 4K + K = 1$

$K = 1$

$y_{pn}(n) = y_{pn}(n-1) - \frac{1}{4} y_{pn}(n-2) + \frac{1}{4} \mu(n)$

$y_{pn}(0) = 0 - \frac{1}{4} \cdot 0 + \frac{1}{4} = 1$

$y_{pn}(1) = \frac{1}{4} - \frac{1}{4} \cdot 0 + \frac{1}{4} = \frac{1}{2}$

$$y_{tot2} = y_n + y_n = 3\left(\frac{1}{3}\right)^n - 2\left(\frac{1}{3}\right)^n + 3\cdot 2^{-n} = \left(\left(\frac{1}{3}\right)^n + 3\cdot 2^{-n}\right)\mu(n) = y_{tot1}(n)$$

DODATNO $y_p(n)$ ako $u(n)$ nije jedinična stepenica (osim ako je stvarno tako zadano)

1. $y_p(n) = K\cdot n\cdot 3^{-n}$

$$6kn3^{-n} - 5k(n-1)3^{-n+1} + k(n-2)3^{-n+9} = 3^{-n}$$

5. $y_p(n) = k2^n$

$$k2^n - 5\cdot k\cdot 2^{n-1} + 6k\cdot 2^{n-2} = 2^n$$

8. $y_p(n) = kn^2\cdot 2^n$

$$kn^2 2^n - 4\cdot k(n-1)^2\cdot 2^{n-1} + 4k(n-2)^2 2^{n-2} = 2^n$$

9. $y_p(n) = kn^2\cdot 2^{-n}$

$$4kn^2\cdot 2^{-n} - 4k(n-1)^2 2^{-n+1} + k(n-2)^2 2^{-n+2} = 2^{-n}$$