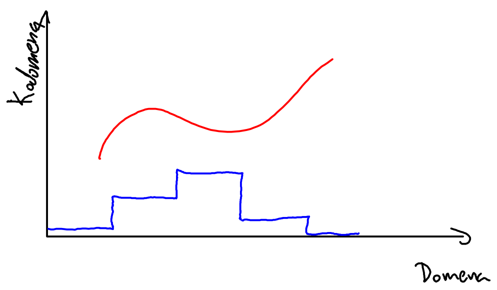
22. ožujak 2009
7:55



$f(x) =$

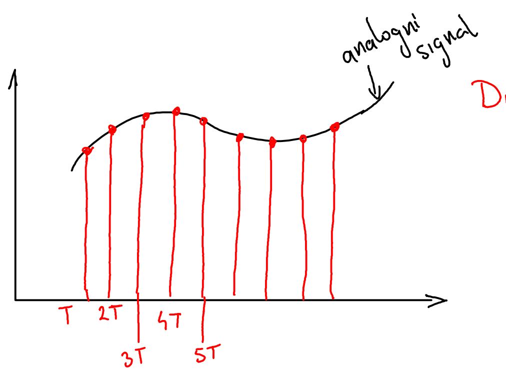
Analogni signal – kontinuiran u domeni i kodomeni

$f: \mathbb{R} \to \mathbb{R}$

Kvantiziran signal –

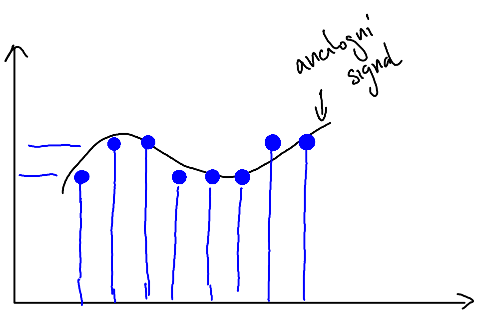
$f: \mathbb{R} \to \mathbb{Z}$

$\underbrace{\text{Analogni i kvantizirani}}$ su **Kontinurani**



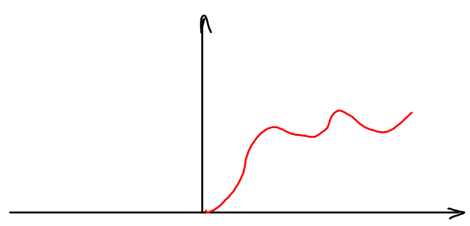
Diskretni signal

$f: \mathbb{Z} \to \mathbb{R}$

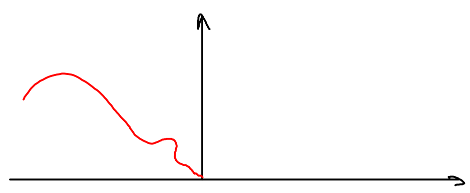


Digitalni signal

$x(t) \to$ kont.
$x(n) \to$ diskretni

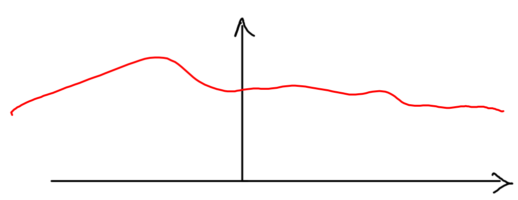


Kauzalni – od važnosti u praksi samo kauzalni



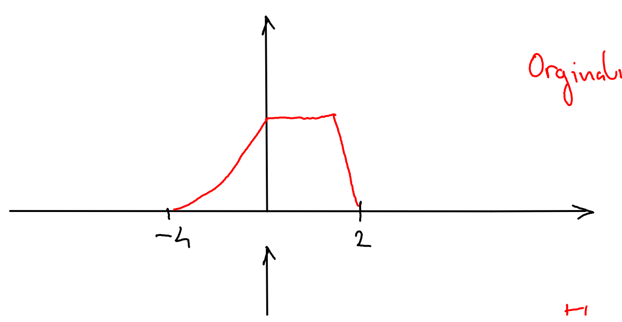
Antikauzalni

Nekauzalnost



Svevremenski

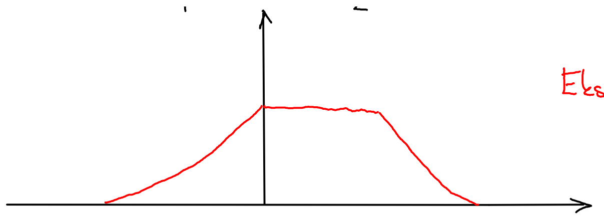
Operacija sa signalima



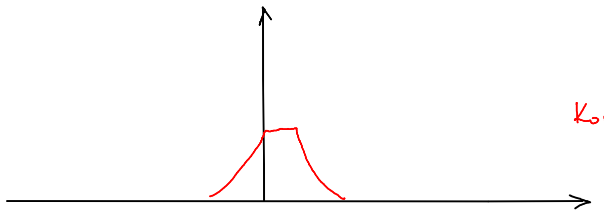
Orginalni signal

$x(at)$

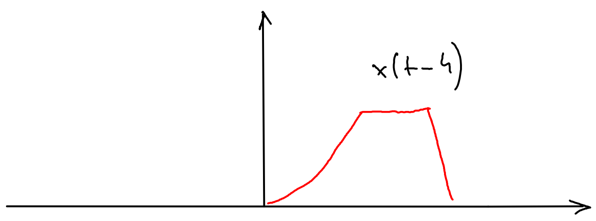
$|a| > 1$ – kompresija
$|a| < 1$ – ekspanzija



Ekspandirani

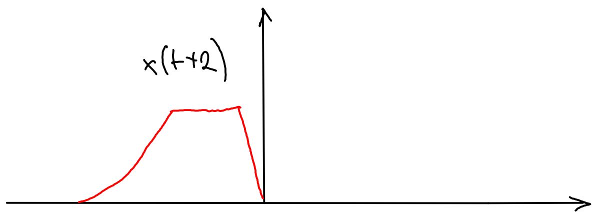


Komprimirani



Pomak

$x(t-t_0) \Rightarrow$ pomak udesno

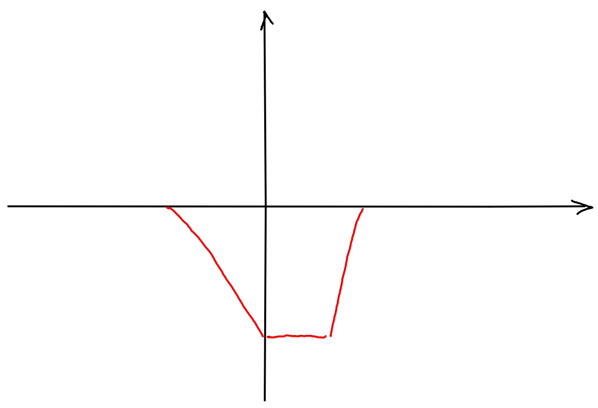


Pomak

$x(t+t_0) \Rightarrow$ pomak ulijevo

Inverzija

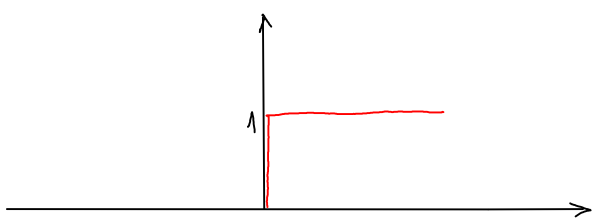
$x(-t)$



Skaliranje

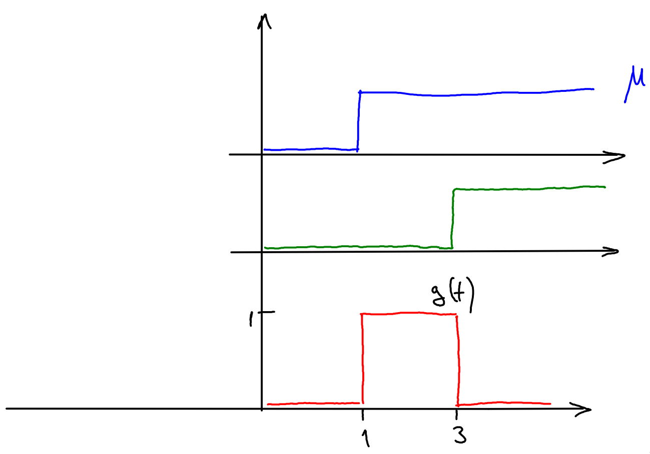
$-2x(t)$

$$\mu(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



$u(t) = \sin t \, \mu(t)$ — $\begin{cases} \sin t & \text{za } t \geq 0 \\ 0 & t < 0 \end{cases}$

$u(t) = \sin t \, \mu(t-5)$ — $\begin{cases} \sin t & \text{za } t \geq 5 \\ 0 & t < 5 \end{cases}$



$\mu(t-1)$

$\mu(t-3)$

$g(t) = \mu(t-1) - \mu(t-3)$

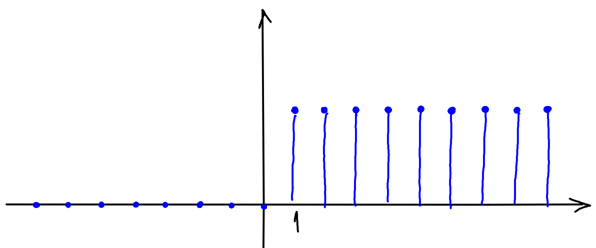
$u(t) = \sin t \, (\mu(t+1) - \mu(t-3))$

$\begin{cases} \sin t & -1 \leq t \leq 3 \\ 0 & \text{za ostalo} \end{cases}$

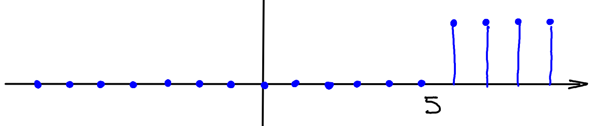
Gate

za diskretne

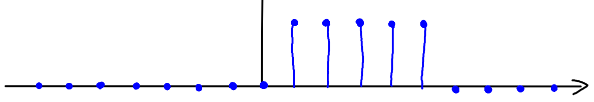
$(\mu(t-1) - \mu(t-3))$



$\mu(n-1)$

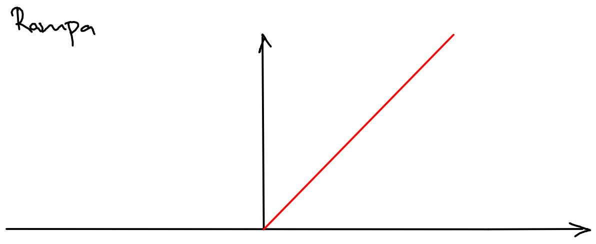


$\mu(n-6)$



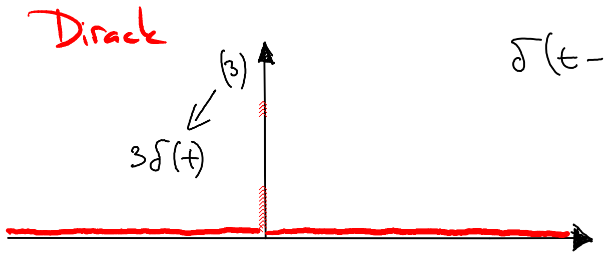
$\mu(n-1) - \mu(n-6)$

Rampa



$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

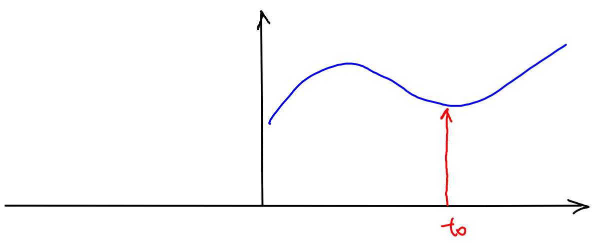
Dirack



$$\delta(t-t_0) = 0, \quad t \neq t_0$$

$$\int_a^b \delta(t-t_0)\,dt = 1$$

$$a \leq t_0 \leq b$$



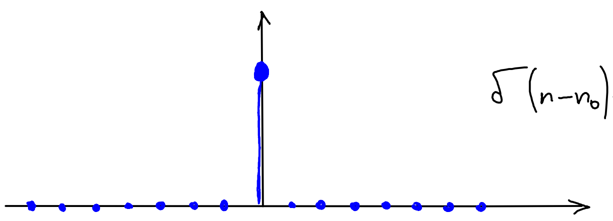
$$a \leq t_0 \leq b$$

$$\int_a^b u(t)\,\delta(t-t_0)\,dt = u(t_0)$$

Dirack "vadi" vrijednost funkcije u t_0

Comb (češalj) $\Rightarrow \sum_{k=-\infty}^{\infty} \delta(t-kT)$

Cronecker



$$\delta(n-n_0) = \begin{cases} 1, & n = n_0 \\ 0, & n \neq n_0 \end{cases}$$

$$\sum_{k=a}^{b} \delta(n-n_0)\,u(n) = u(n_0)$$

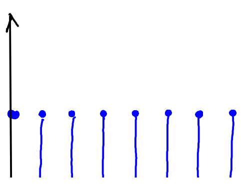
$$a \leq n_0 \leq b$$

$$\frac{d}{dt}\mu(t) = \delta(t) \qquad \frac{d}{dt} r(t) = \mu(t)$$

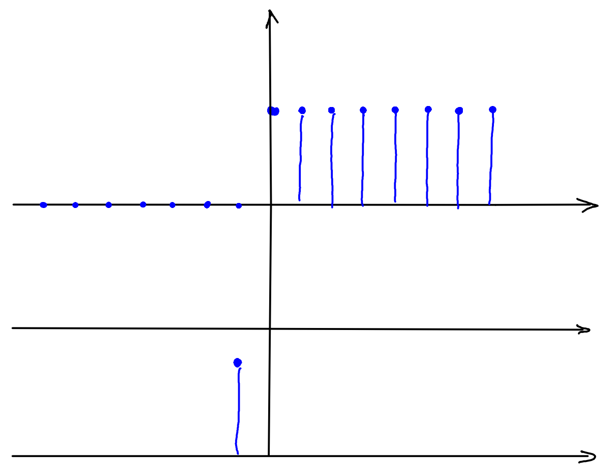
Unazna diferencija

$$\Delta u(n) = u(n+1) - u(n)$$

$$\nabla u(n) = u(n) - u(n+1)$$



step diskretn.



step diskretn.

utkuzna

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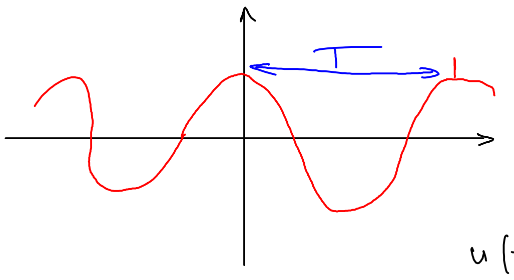
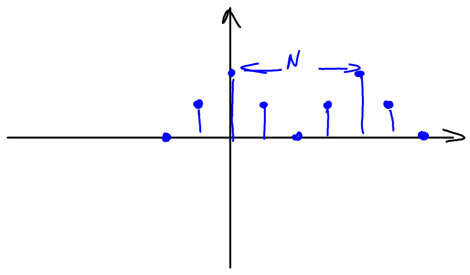
Periodičnost signala



$T \in \mathbb{R}$

$u(t) = u(t + kT)$



$N \in \mathbb{Z}$

$u(n) = u(n + kN)$

Primjer 1:

a) $u(t) = \cos\left(4t + \frac{\pi}{3}\right)$

$u(t) = u(t+T)$

$\cos\left(4t + \frac{\pi}{3}\right) = \cos\left(4(t+T) + \frac{\pi}{3}\right)$

$\cos\left(4t + \frac{\pi}{3}\right) = \cos\left(4t + \frac{\pi}{3} + 4T\right)$

$4T = 2k\pi$

$T = \frac{k\pi}{2}$ $\quad k = 1 \Rightarrow T_0 = \frac{\pi}{2}$

b) $u(t) = \cos(\omega t + \varphi)$

$\cos(\omega t + \varphi) = \cos(\omega t + \varphi + \underbrace{\omega T})$

$\omega T = 2k\pi$

$\omega = \frac{2k\pi}{T}$ $\quad k=1 \Rightarrow \omega = \frac{2\pi}{T}$

$f = \frac{1}{T} \Rightarrow \omega = 2\pi f$

c) $u(t) = \cos^2(\omega t + \varphi) = \frac{1}{2}\left(1 + \cos(2\omega t + 2\varphi)\right)$

$T = \frac{2k\pi}{2\omega} = \frac{k\pi}{\omega}$

d) $u(t) = \cos^2\left(2\pi t + \frac{\pi}{4}\right)$

$T = \frac{1}{2}$

e) $u(t) = \sin(t^2 + 2\pi)$

$u(t) = u(t+T)$

$$\sin(t^2+2\pi)=\sin\left((t+T)^2+2\pi\right)$$
$$=\sin\left(t^2+2tT+T^2+2\pi\right)$$

$$2tT+T^2=2k\pi$$

$$k=\frac{T(2t+T)}{2}$$

nije periodičan $\Rightarrow$ $k\in\mathbb{Z}$, $t,T\in\mathbb{R}$

f) $u(t)=\sin(\pi t)\,\mu(t)$ — nije periodičan

g) $u(t)=e^{j\left(4\pi t+\frac{\pi}{2}\right)}=\cos\left(4\pi t+\frac{\pi}{2}\right)+j\sin\left(4\pi t+\frac{\pi}{2}\right)$

$e^{jx}=\cos x+j\sin x$

$$T=\frac{2k\pi}{\omega}\overset{k=1}{=}\frac{1}{2}$$

- $u(t)=\underbrace{\cos\left(\pi t+\frac{\pi}{4}\right)}_{\omega_1=\pi}+\underbrace{\sin\left(4\pi t+\frac{\pi}{2}\right)}_{\omega_2=4\pi}\to\omega_0=\pi$

  $T_1=2$ $T_2=\frac{1}{2}$

  Tražimo najmanji zajednički period od perioda pa je $T_0=2$

- $u(t)=\underbrace{\cos(2\pi t)}+\cos(2t+\pi)+\cos\left(6t+\frac{\pi}{2}\right)$

  $\omega_1=2\pi$ $\omega_2=2$ $\omega_3=6$

  $T_1=1$ $T_2=\pi$ $T=\frac{\pi}{3}$

  nema perioda jer je nemoguće naći najmanji zajednički

$u(n)=\cos(\omega n+\varphi)$

$N=\frac{2\pi}{\omega}$

- $u(n)=\cos(4n)$ $N=\frac{\pi}{2}\notin\mathbb{Z}$ nije
- $u(n)=\cos(\pi n)$ $\Rightarrow$ $N_0=2\in\mathbb{Z}$ ✓
- $u(n)=\cos(4\pi n)$ $N_0=\frac{2\pi}{4\pi}=\frac{1}{2}$ ✓ ali za $k=2$
- $u(n)=\cos\left(\frac{\pi}{8}n^2\right)$

$$\cos\left(\frac{\pi}{8}n^2\right)=\cos\left(\frac{\pi}{8}n^2+\frac{\pi}{8}\left(2nN+N^2\right)\right)$$

$$\frac{\cancel{\pi}}{8}\left(2nN+N^2\right)=2k\cancel{\pi}$$

$$k=\frac{N(2n+N)}{16}$$ $N=16m$ / Supstitucija

$$=\frac{\cancel{16}m(2n+16m)}{\cancel{16}}=m(2n+16m)$$

$m=1$ $N=16$

$$k=\frac{N(2n+M)}{16}$$ $N=2m$

$$= \frac{m(n+m)}{4}$$

$N = 2p$ $\qquad\qquad \longrightarrow p = 2r$

$$k = \frac{2p(n+2p)}{4} = \frac{p(n+2p)}{2} \qquad k = \frac{2r(n+4r)}{2} = r(n+4r)$$

pa je $N_0 = 8$

Energija signala i snaga signala

$$E = \int_{t_1}^{t_2} |x(t)|^2 dt \qquad E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

$$P = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} |x(t)|^2 dt \qquad P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

Zadatci

- $u(t) = t \quad I[-6, 7]$

$$E = \int_{-6}^{7} t^2 dt = \qquad P = \frac{1}{7 - -6} \int_{-6}^{7} t^2 dt =$$

- $u(t) = t^2 \mu(t) \qquad I[3, 8]$

$$E = \int_{0}^{8} t^4 dt \qquad P = \frac{1}{8-(-3)} \int_{0}^{8} t^4 dt =$$

- $u(t) = t\,(\mu(t+2) - \mu(t-5))$

$$E = \lim_{T\to\infty} \int_{-2}^{5} t^2 dt \qquad P = \lim_{T\to\infty} \frac{1}{2T} \int_{-2}^{5} t^2 dt = 0$$

- $u(t) = e^{-at}\mu(t) \qquad a > 0$

$$E = \lim_{T\to\infty} \int_{0}^{T} e^{-2at} dt = \lim_{T\to\infty} \frac{-1}{2a} e^{-2at} \Big|_0^T =$$

$$= \lim_{T\to\infty} \frac{-e^{-2aT} + e^0}{2a} = \frac{1}{2a}$$

0

$P = 0$

- $u(t) = e^{-at}\mu(t), \quad a < 0$

$$E = \lim_{T\to\infty} \int_{0}^{T} e^{-2at} dt = \lim_{T\to\infty} \frac{-1}{2a} e^{-2at} \Big|_0^T$$

$$= \lim_{T\to\infty} \frac{-e^{-2aT} + e^0}{2a} = \frac{\infty}{2a} = \infty$$

$$P = \lim \frac{1}{2b}\left(\underbrace{\frac{e^{2at}}{2T}}_{\infty} - \underbrace{\frac{e^{0}}{2T}}_{0}\right) = \infty$$

- $u(t) = e^{-|at|} \quad a>0$

$$|t| = \begin{cases} -t, & \text{za } t<0 \\ t, & \text{za } t>0 \end{cases} \qquad u(t) = \begin{cases} e^{-(-at)} = e^{at}, & t<0 \\ e^{-at} = e^{-at} & \geq 0 \end{cases}$$

$$E = \lim_{T\to\infty}\left(\int_{-T}^{0} e^{2at}\,dt + \int_{0}^{T} e^{-2at}\,dt\right) =$$

$$= \lim_{T\to\infty}\left(\frac{1}{2a}\left(\underset{1}{e^{0}} - \underset{0}{e^{-2at}}\right) + \frac{(-1)}{2a}\left(\underset{0}{\cancel{e^{-2at}}} - \underset{1}{\cancel{e^{0}}}\right)\right)$$

$$= \frac{1}{a}$$

- $u(t) = 3e^{j7t}$

$$|u(t)| = |3e^{j7t}| = |3|\,|e^{j7t}| = 3\cdot 1$$

$$|\cos x + j\sin x| = \sqrt{Re^2 + Im^2} = 1$$

$$E = \lim_{T\to\infty}\int_{-T}^{T} 9\,dt = 9\lim_{T\to\infty} 2T = \infty$$

$$P = 2\lim_{T\to\infty} 2T \lim_{T\to\infty}\frac{1}{2T} = 2\lim_{T\to\infty}\frac{2T}{2T} =$$

$$= 2$$

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Diskretni signali

$$E=\sum_{n=n_1}^{n_2}|u(n)|^2 \qquad P=\frac{1}{n_2-n_1+1}\sum_{n=n_1}^{n_2}|u(n)|^2$$

$$E=\lim_{N\to\infty}\sum_{-N}^{N}|u(n)|^2 \qquad P=\lim_{N\to\infty}\frac{1}{n_2-n_1+1}\sum_{-N}^{N}|u(n)|^2$$

- $u(n)=n\,\mu(n-5)$

$I\in[-4,10]$

$$E=\sum_{n=5}^{10}|u(n)|^2=5^2+6^2+7^2+8^2+9^2+10^2$$

$$P=\frac{1}{15}$$

$$\sum_{i=1}^{i=n}i^2=\frac{n(n+1)(2n+1)}{6}$$

$$\sum_{n=0}^{10}-\sum_{n=0}^{4}=\ldots$$

- $u(n)=5\mu(n)$

$$E=\lim_{N\to\infty}\sum_{0}^{N}|5|^2=25\lim_{N\to\infty}\sum_{0}^{N}1=25\lim_{N\to\infty}(n+1)=\infty$$

$$P=25\lim_{N\to\infty}n+1\lim_{N\to\infty}\frac{1}{2n+1}=25\lim_{N\to\infty}\frac{n+1}{2n+1}=\frac{25}{2}$$

- $u(n)=5n$

$$E=\lim_{N\to\infty}\sum_{n=-N}^{N}25n^2=25\lim_{N\to\infty}\sum_{n=-N}^{N}n^2=25\left[\lim_{N\to\infty}\left(\sum_{n=-N}^{-1}n^2+0^2+\sum_{n=1}^{N}n^2\right)\right]$$

$$\sum_{n=1}^{N}n^2=\frac{n(n+1)(2n+1)}{6}$$

$$\sum_{n=-N}^{-1}n^2=\left|\begin{matrix}m=-n & \\ n\to-N & m\to N\\ n\to-1 & m\to1\end{matrix}\right|=\sum_{m=N}^{1}(-m)^2=\sum_{m=1}^{N}m^2$$

- $u(n)=\left(-\frac{1}{3}\right)^n\mu(n)$

$$E=\lim_{N\to\infty}\sum_{i=0}^{\infty}\left(\frac{1}{3}\right)^{2n}=\lim_{N\to\infty}\sum_{n=0}^{N}\left(\frac{1}{9}\right)^n=\frac{1}{1-\frac{1}{9}}=\frac{9}{8}$$

$$\sum_{i=0}^{\infty}q^i=\frac{1}{1-q} \qquad |q|<1$$

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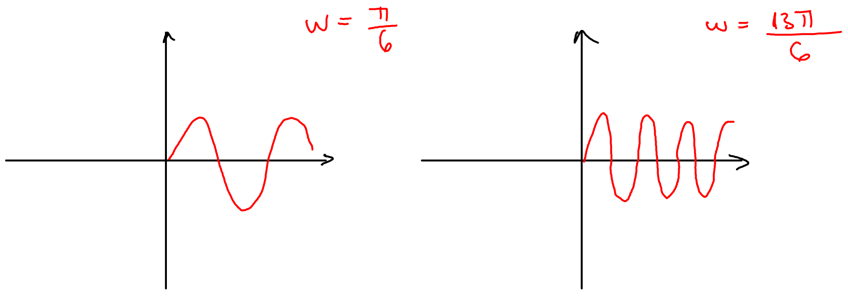
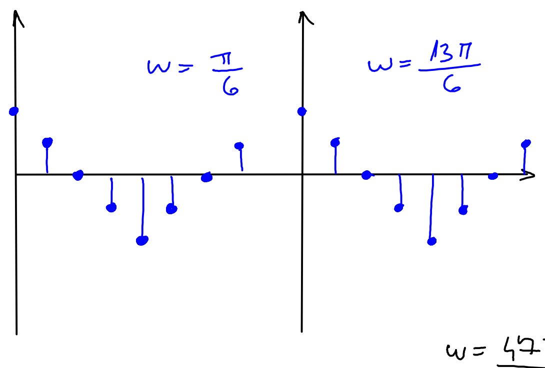
Aliasing





$-\pi \leq \omega < \pi$

$0 \leq \omega \leq 2\pi$

$\omega = \frac{47\pi}{6} \rightarrow \frac{\pi}{6}$

1. zadatak

$$f(n) = \cos\left(\frac{47\pi}{7}n + \frac{\pi}{3}\right) = \cos\left(\frac{5\pi n}{7} + \frac{\pi}{3}\right)$$

$$\frac{47\pi}{7} - \frac{14\pi}{7} - \frac{14\pi}{7} - \frac{14\pi}{7} = \frac{5\pi}{7}$$

$$\cos\left(x - \frac{\pi}{2}\right) = \sin x$$

$$\cos\left(\frac{5\pi n}{7} + \frac{\pi}{3} + \frac{\pi}{2} - \frac{\pi}{2}\right) = \sin\left(\frac{5\pi n}{7} + \frac{\pi}{3} + \frac{\pi}{2}\right)$$

$$= \sin\left(\frac{5\pi n}{7} + \frac{5\pi}{6}\right)$$

Otipkavanje

$$u(t) = \cos(\omega t + \varphi)$$

$$nT_s = \quad \frac{2\pi}{T} \cdot T_s$$

$$u(n) = \cos(\hat{\omega} n T_s + \varphi) = \cos(\Omega n + \varphi)$$

$$-\pi \leq \Omega \leq \pi$$

$$-\cancel{\pi} \leq 2\cancel{\pi}\frac{T_s}{T} \leq \cancel{\pi}$$

$$-1 \leq 2\frac{T_s}{T} \leq 1$$

$$\left|\frac{T_s}{T}\right| \leq \frac{1}{2} \qquad T_s = \frac{1}{f_s} \qquad T = \frac{1}{f}$$

$$\left|\frac{f}{f_s}\right| \leq \frac{1}{2} \implies f_s \geq 2f$$

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Fourierova analiza i sinteza

- Fourierov red

Kontinuirani i periodični

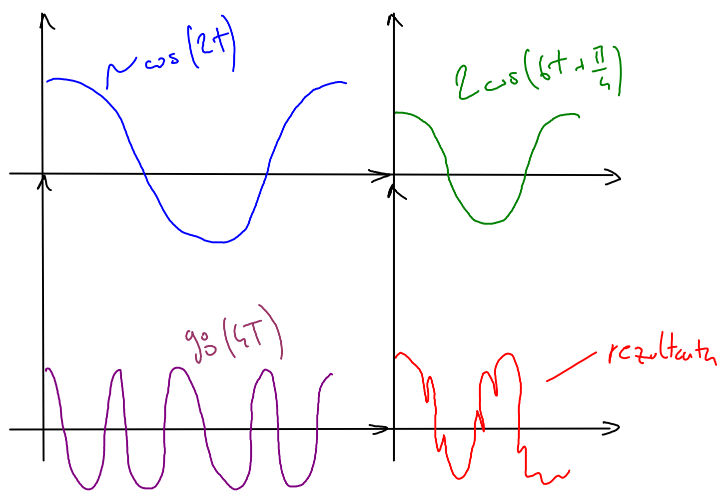
$$x(t) = \cos(2t) + 2\cos\left(6t + \frac{\pi}{4}\right) + 98\cos(4t)$$

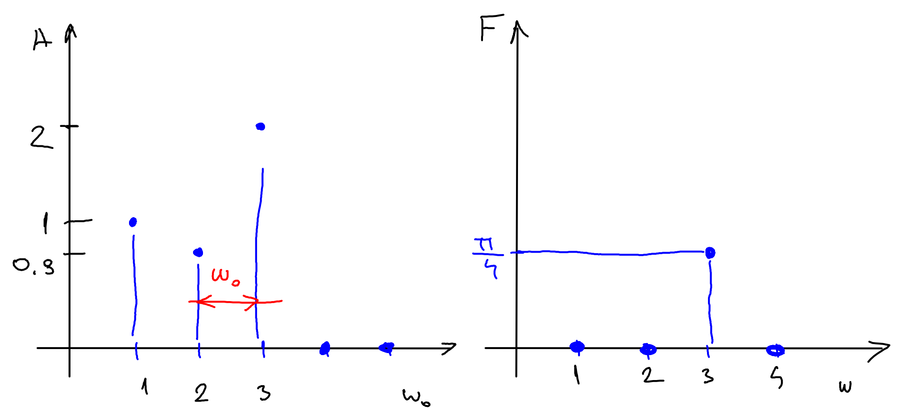
$\omega_1 = 2$ $\omega_2 = 6$ $\omega_3 = 4$

$T_1 = \pi$ $T_2 = \frac{\pi}{3}$ $T_3 = \frac{\pi}{2}$

$\omega_0 = 2t$ $T_0 = \pi$

skica





Periodični signali ⟺ Diskretan spektar
Aperiodični signali ⟺ Kontinuiran spektar

F. red $X_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}\,dt$, $X_k \in \mathbb{C}$

$$X_k = |X_k|e^{j\measuredangle X_k}$$

F. sinteza

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t}\,dt$$

Srednja snaga je: $P = \sum_{k=-\infty}^{\infty} |X_k|^2$ ← Parsevalova jednakost

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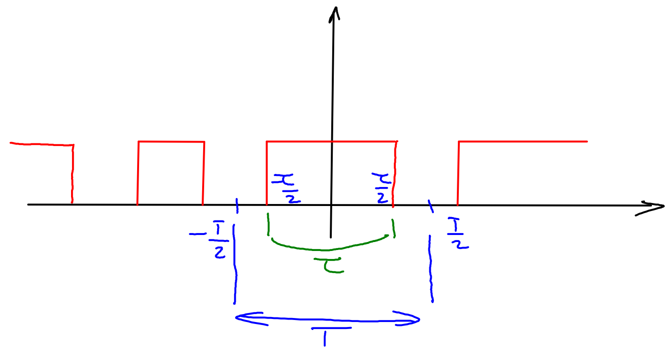
Primjer 1



$$x_k = \frac{1}{T}\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} A e^{-jk\omega_0 t}\,dt = \frac{A}{T}\,\frac{(-1)}{k\omega}\,\frac{1}{j}\, e^{-jk\omega_0 t}\Big|_{-\frac{\tau}{2}}^{\frac{\tau}{2}} =$$

$$= \frac{A}{T}\left(\frac{-1}{j\omega}\right)\left(\frac{1}{j}\right)\left(e^{-j\omega\frac{\tau}{2}} - e^{j\omega\frac{\tau}{2}}\right) = \frac{A}{T}\,\frac{1}{j\omega}\,\frac{2}{2j}\left(e^{j\omega\frac{\tau}{2}} - e^{-j\omega\frac{\tau}{2}}\right) =$$

$$= \frac{2A}{T}\,\frac{1}{k\omega}\,\sin\left(k\omega\frac{\tau}{2}\right) =$$

sinc

$$= \frac{\cancel{2}A}{T}\,\frac{1}{k\omega\frac{\tau}{2}\,\frac{\cancel{2}}{\tau}}\,\sin\left(k\omega\frac{\tau}{2}\right) =$$

$$= A\,\frac{\tau}{T}\,\operatorname{sinc}\left(k\omega\frac{\tau}{2}\right) = A\,\frac{\tau}{T}\,\operatorname{sinc}\left(k\pi\frac{\tau}{T}\right)$$

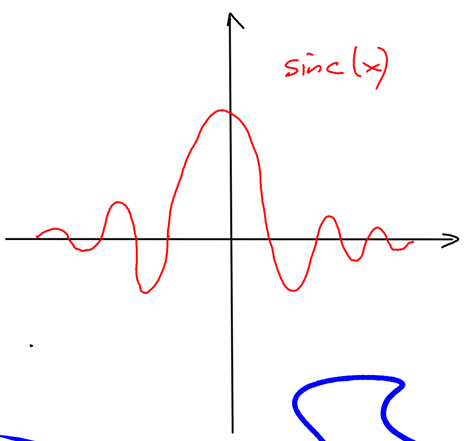
$$\omega = \frac{2\pi}{T}$$

Korisno

$$\cos x = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)$$

$$\sin x = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$$

$$\operatorname{sinc}(x) = \frac{\sin(x)}{x}$$



$$\operatorname{sinc}\left(k\pi\frac{\tau}{T}\right)$$

$$k\pi\frac{\tau}{T} = n\pi$$

$$k = \frac{T}{\tau}n$$

nul točke $\sin\left(k\pi\frac{\tau}{T}\right)$

X_0 – komponenta — srednja vrijednost signala odnosno istosmjerna komponenta

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Primjer: $x(t) = \cos(2t) + 2\cos(6t + \pi/4) + 0.8\cos(4t)$

$\omega_0 = 2$, $T_0 = \pi$

$$x(t) = \frac{1}{2}\left(e^{j2t} + e^{-j2t}\right) + \left(e^{j6t} + e^{-j6t}\right) + 0.8\cdot\frac{1}{2}\left(e^{j4t} + e^{-j4t}\right) =$$

$$= \underbrace{\frac{1}{2}e^{j(1)2t}}_{k=1} + \underbrace{\frac{1}{2}e^{j(-1)2t}}_{k=-1} + \underbrace{e^{\frac{\pi}{4}}e^{j(3)2t}}_{k=3} + \underbrace{e^{\frac{\pi}{4}}e^{j(-3)2t}}_{k=-3}$$

$$+ 0.4\left[\underbrace{e^{j(2)2t}}_{k=2} + \underbrace{e^{j(-2)2t}}_{k=-2}\right]$$

$x_1 = 0.5$ $x_{-1} = 0.5$ $x_2 = 0.4$ $x_{-2} = 0.4$ $x_3 = e^{j\frac{\pi}{4}}$ $x_{-3} = e^{-j\frac{\pi}{4}}$

$$x(t) = \sum^{\infty} x_k e^{jk\omega_0 t} = \ldots\ldots + x_{-2}e^{j(-2)\omega_0 t} + x_{-1}e^{j(-1)\omega_0 t} + \ldots\ldots$$

$$x(t) = \sum_{k=-\infty}^{\infty} x_k e^{jk\omega_0 t} = \dots + x_{-2} e^{j(-2)\omega_0 t} + x_{-1} e^{j(-1)\omega_0 t} + \dots$$

Zadatak

$$x(t) = 220 \cos(50\pi t) + 100 \sin\left(200\pi t + \frac{\pi}{6}\right)$$

$$\omega_0 = 50\pi \quad T = \frac{1}{25}$$

$$= 220 \cdot \frac{1}{2}\left(e^{j50\pi t} + e^{-j50\pi t}\right) + \frac{100}{2j}\left(e^{j200\pi t + \frac{\pi}{6}} - e^{-j200\pi t + \frac{\pi}{6}}\right) =$$

$$= 110\left(e^{j(1)50\pi t} + e^{j(-1)50\pi t}\right) + \frac{50}{j} e^{\frac{\pi}{6}}\left(e^{j(4)50\pi t} - e^{j(-4)50\pi t}\right)$$

$$x_1 = 110 \qquad x_4 = -50j e^{j\frac{\pi}{6}} = 50 e^{j\frac{3\pi}{2}} e^{j\frac{\pi}{6}} = e^{j\frac{5\pi}{3}}$$

$$x_{-1} = 110 \qquad x_{-4} = 50j e^{j\left(-\frac{\pi}{6}\right)} = 50 e^{j\frac{\pi}{2}} e^{j\left(-\frac{\pi}{6}\right)} =$$

$$= 50 e^{j\frac{\pi}{3}}$$

Fourierova transformacija

Kontinuirani Aperiodični:

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{j\omega t}\, dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$E = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2\, d\omega$$

Zad.

- $x(t) = e^{-t}\mu(t)$

$$X(j\omega) = \int_{0}^{\infty} e^{-t} e^{-j\omega t}\, dt = \int_{0}^{\infty} e^{-t(1+j\omega)}\, dt =$$

$$= \frac{-1}{1+j\omega}\, e^{-t(1+j\omega)} \Big|_{0}^{\infty} = \frac{-1}{1+j\omega}\left(e^{-\infty} - e^{0}\right) =$$

$$= \frac{1}{1+j\omega} \cdot \frac{1-j\omega}{1-j\omega} = \frac{1-j\omega}{1+\omega^2} =$$

$$e^{-t}\mu(t) \circ\!\!-\!\!\bullet \frac{1-j\omega}{1+\omega^2}$$

$$X(j\omega) = \frac{1}{1+\omega^2} + j\cdot\frac{(-\omega)}{1+\omega^2}$$

$$Re = \frac{1}{1+\omega^2} \qquad Im = \frac{-\omega}{1+\omega^2}$$

$$|X(j\omega)| = \frac{\sqrt{1+(-\omega)^2}}{1+\omega^2}$$

$$\measuredangle X(j\omega) = \text{arctg}\,\frac{Im}{Re} = \frac{\frac{-\omega}{\cancel{1+\omega^2}}}{\frac{1}{\cancel{1+\omega^2}}} = \text{arctg}(-\omega)$$

$$\text{arctg}\left(\frac{b}{a}\right)$$

$$\text{arctg}\left(\frac{-b}{a}\right) = -\text{arctg}\left(\frac{b}{a}\right)$$

$$\text{arctg}\left(\frac{b}{-a}\right) = \pi - \text{arctg}\left(\frac{b}{a}\right)$$

$$\text{arctg}\left(\frac{-b}{-a}\right) = \pi + \text{arctg}\left(\frac{b}{a}\right)$$

Svojstva F. transformacije

$$x(t) = \delta(t)$$

$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t)\, e^{-j\omega t}\, dt = e^{-j\omega t_0} = e^{0} = 1$$

$$\delta(t) \circ\!\!-\!\!\bullet 1$$

$x(t) = \delta(t-t_0)$

$$X(j\omega) = \int_{-\infty}^{\infty} \delta(t-t_0) e^{-j\omega t} dt = e^{-j\omega t_0} \cdot 1$$

$$\delta(t-t_0) \circ\!\!-\!\!\bullet\ e^{-j\omega t_0} \cdot 1$$

Pa znači :

$$x(t) \circ\!\!-\!\!\bullet\ X(j\omega)$$
$$x(t-t_0) \circ\!\!-\!\!\bullet\ X(j\omega) e^{-j\omega t_0}$$
$$x(at) \circ\!\!-\!\!\bullet\ \frac{1}{|a|} X\left(j\frac{\omega}{a}\right)$$

$\delta(t-t_0)$, $t_0 > 0$

$$\delta(t+t_0) \circ\!\!-\!\!\bullet\ 1 e^{-j\omega(-t_0)} = e^{j\omega t_0}$$

$$\omega t_0 = \varphi(\omega)$$
$$\varphi(\omega) > 0 \text{ za } \omega > 0$$

$x(t) \circ\!\!-\!\!\bullet\ X(j\omega)$ $\qquad |X(j\omega)| - |G(j\omega)| = ?$

$g(t) \circ\!\!-\!\!\bullet\ G(j\omega)$

$$g(t) = x(t-7) \circ\!\!-\!\!\bullet\ G(j\omega) = X(j\omega) e^{-j7\omega}$$

$$|X(j\omega)| - |X(j\omega)| \underbrace{|e^{-j7\omega}|}_{1} = 0$$

$x(t) \circ\!\!-\!\!\bullet\ X(j\omega)$ $\qquad$ Invertirati pa pomaknut u desno za 10

$$x(-t) \circ\!\!-\!\!\bullet\ \frac{1}{|-1|} X\left(j\frac{\omega}{-1}\right) = X(-j\omega)$$

$$\circ\!\!-\!\!\bullet\ X(-j\omega) e^{-j10\Omega}$$

22. ožujak 2009
14:43

Fourierova transformacija

Diskretan Periodičan $\Rightarrow$ Periodičan Kontinuiran

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x(n) e^{-jwn}$$

$$x(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{jw}) e^{jwn} dw$$

$$E = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{jw})|^2 dw$$

Zad.

1) $x(n) = \begin{cases} n, & |n| \leq 4 \\ 0, & \text{inače} \end{cases}$

$X(e^{jw})$, $w = \frac{\pi}{2}$

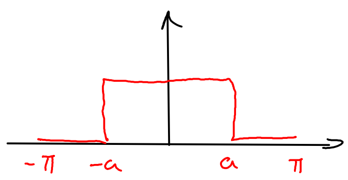
$$X(e^{jw}) = -4e^{j4w} - 3e^{j3w} - 2e^{j2w} - e^{jw} + e^{-jw} + 2e^{-j2w} + 3e^{-j3w} + 4e^{-j4w} =$$

$$= -4 \frac{2j}{2j}\left(e^{j4w} - e^{-j4w}\right) - 3 \frac{2j}{2j}\left(\dots \right) \dots =$$

$$= -8j \sin(4w) - 6j \sin(3w) - 4j \sin(2w) - 2j \sin(w) = 6j - 2j = 4j$$

---

2) $X(e^{jw}) = \begin{cases} 2, & |w| < a \\ 0, & a \leq |w| \leq \pi \end{cases}$



$$x(n) = \frac{1}{2\pi} \int_{-a}^{a} 2e^{jwn} dw = \frac{1}{\pi} \frac{1}{jn} e^{jwn} \Big|_{-a}^{a} =$$

$$= \frac{1}{\pi n} \frac{2}{2j} \left(e^{jan} - e^{-jan}\right) = \frac{2}{\pi n} \sin(an)$$

---

3) $X(e^{jw}) = \begin{cases} 2\pi, & |w| < a \\ 0, & a < |w| < \pi \end{cases}$

$$x(n) = \frac{2 \sin(an)}{n}$$

$$E = \frac{1}{2\pi} \int_{-a}^{a} 4\pi^2 dw = 2\pi\left(a - (-a)\right) = 4a\pi$$

Svojstva FT

$$x(n) \circ\!\!-\!\!\bullet X(e^{j\omega})$$

$$x(n-n_0) \circ\!\!-\!\!\bullet X(e^{j\omega})e^{-j\omega n_0}$$

$$e^{-j\omega_0 n}\, x(n) \circ\!\!-\!\!\bullet X\left(e^{j(\omega+\omega_0)}\right)$$

Primjer:

$$x(n)-x(n-1) \circ\!\!-\!\!\bullet X(e^{j\omega}) - X(e^{j\omega})e^{-j\omega}$$

$$= X(e^{j\omega})\left(1-e^{-j\omega}\right)$$