Signali i sustavi

Međuispit (grupa A) – 24. travnja 2013.

1. (9 bodova) Zadani su signali $x_1(t) = 2^{-t}\,\mu(t)$ i $x_2(n) = \sin\left(\frac{\pi}{3}n\right)$.

a) **(2 boda)** Definirajte totalnu energiju i totalnu snagu vremenski kontinuiranog signala.

b) **(2 boda)** Izračunajte totalnu energiju i totalnu snagu signala $x_1(t)$.

c) **(2 boda)** Definirajte totalnu energiju i totalnu snagu vremenski diskretnog signala.

d) **(3 boda)** Izračunajte totalnu energiju i totalnu snagu signala $x_2(n)$.

2. (9 bodova) Zadan je vremenski kontinuirani signal $x(t) = e^{3t}\big(\mu(t) - \mu(t-6)\big)$.

a) **(4 boda)** Postoji li vremenski kontinuirana Fourierova transformacija (CTFT) signala $x(t)$? Ako postoji, pokažite zašto postoji, a ako ne postoji, pokažite zašto ne postoji!

b) **(5 bodova)** Ako transformacija postoji izračunajte je (nije potrebno računati amplitudu i fazu), a ako ne postoji, pokažite da Fourierov integral divergira!

3. (9 bodova) Zadan je vremenski diskretan signal $x(n) = 3^{-|n|}$, gdje je $n \in \mathbb{Z}$.

a) **(4 boda)** Izračunajte vremenski diskretnu Fourierovu transformaciju (DTFT) signala $x(n)$.

b) **(2 boda)** Odredite amplitudni i fazni spektar.

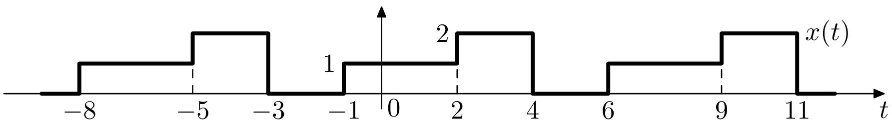
c) **(3 boda)** Odredite na kojim frekvencijama Ω amplitudni spektar $\left|X(e^{j\Omega})\right|$ poprima minimalne, a na kojima maksimalne vrijednosti.

4. (9 bodova) Vremenski kontinuiran signal $x(t)$ perioda $T = 7$ zadan je slikom.

a) **(4 boda)** Odredite rastav signala $x(t)$ u vremenski kontinuirani Fourierov red (CTFS).

b) **(2 boda)** Navedite svojstvo simetričnosti spektra X_k realnog signala $x(t)$. Pokažite da dobiveni spektar X_k zadovoljava taj uvjet!

c) **(3 boda)** Skicirajte amplitudni i fazni spektar X_k za $-3 \le k \le 3$.



5. (9 bodova) Promatramo vremenski diskretan signal konačnog trajanja oblika $x[n] = \{\underline{-2}, 0, 2, -2, 0, 2, -2, 0, 2, -2, 0, 2, \ldots\}$ gdje se uzorak $\{-2, 0, 2\}$ ponavlja m-puta. Neka je trajanje signala $N = 3m$, $m \in \mathbb{N}$.

a) **(2 boda)** Izračunajte diskretnu Fourierovu transformaciju DFT_N signala $x[n]$ u N točaka.

b) **(1 bod)** Za koje k je transformacija signala $X[k]$ različita od nule?

c) **(2 boda)** Korištenjem spektra $X[k]$ raspišite signal $x[n]$ kao zbroj kosinusoida.

d) **(2 boda)** Ako je promatrani signal $x[n]$ dobiven očitavanjem vremenski kontinuiranog signala $x(t)$ s frekvencijom očitavanja $f_S = 10\,\mathrm{kHz}$ koje spektralne komponente se nalaze u signalu $x(t)$?

e) **(2 boda)** Odredite periodičan vremenski kontinuirani signal $x(t)$ dobiven idealnom rekonstrukcijom iz signala $x[n]$.

GRUPA A

1

a) TOTALNA ENERGIJA: $E_\infty = \int_{-\infty}^{\infty} |f(t)|^2 dt$

TOTALNA SNAGA: $P_\infty = \lim_{L \to \infty} \frac{1}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} |f(t)|^2 dt$

b) $E_\infty = \int_{-\infty}^{\infty} (2^{-t} \mu(t))^2 dt = \int_{0}^{\infty} \left(\frac{1}{4}\right)^t dt = \left.\frac{\left(\frac{1}{4}\right)^t}{\ln \frac{1}{4}}\right|_0^\infty =$

$= \frac{0 - 1}{-\ln 4} = \frac{1}{\ln 4}$

ENERGIJA JE KONAČNA $\Rightarrow$ $P_\infty = 0$

c) TOTALNA ENERGIJA: $E_\infty = \sum_{n=-\infty}^{\infty} |y[n]|^2$

TOTALNA SNAGA: $P_\infty = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |y[n]|^2$

d) $P_\infty = P_T = \frac{1}{T} \cdot \sum_{T} \sin^2\left(\frac{\pi n}{3}\right) =$

$= \frac{1}{6} \cdot \left(0 + \frac{3}{4} + \frac{3}{4} + 0 + \frac{3}{4} + \frac{3}{4}\right) = \frac{1}{2}$

$\Rightarrow$ SNAGA JE $\neq 0$ $\Rightarrow$ $E_\infty = \infty$

GRUPA A

(2) $x(t) = e^{3t}\left(\mu(t) - \mu(t-6)\right)$

a) Možemo pokazati da je signal kvadratno-integrabilan (iz L^2) ili da zadovoljava Dirichletove uvjete:

$$L^2: \quad \int_{-\infty}^{+\infty} x^2(t)\,dt = \int_0^6 e^{6t}\,dt = \frac{1}{6}e^{6t}\Big|_0^6 = \frac{1}{6}\left(e^{36}-1\right) < \infty$$

DIRICHLET:

a) $$\int_{-\infty}^{+\infty} |x(t)|\,dt = \int_0^6 e^{3t}\,dt = \frac{1}{3}e^{3t}\Big|_0^6 = \frac{1}{3}\left(e^{18}-1\right) < \infty$$

b) Funkcija ima konačan broj ekstrema (jedan ekstrem u $t = 6^-$), dva diskontinuiteta i vrijedi $|x(t)| \leq e^{18}$.

b) $$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,dt = \int_0^6 e^{3t}e^{-j\omega t}\,dt =$$

$$= \frac{1}{3-j\omega}e^{(3-j\omega)t}\Big|_0^6 = \frac{1}{3-j\omega}\left(e^{6(3-j\omega)} - 1\right)$$

(3) $x(n) = 3^{-|n|} \; ; n \in \mathbb{Z}.$

a) $$X(e^{i\Omega}) = \sum_{n=-\infty}^{\infty} 3^{-|n|} e^{-i\Omega n} =$$

$$= \sum_{n=-\infty}^{-1} 3^{n} e^{-i\Omega n} + \sum_{n=0}^{\infty} 3^{-n} e^{-i\Omega n} \quad \left| \begin{array}{c} m=-n \\ \text{ZA PRVU} \\ \text{SUMU} \end{array} \right|$$

$$= \sum_{m=0}^{\infty} \left(\frac{1}{3e^{-i\Omega}}\right)^{m} - 1 + \sum_{n=0}^{\infty} \left(\frac{1}{3e^{i\Omega}}\right)^{n}$$

$$= \frac{1}{1-\frac{1}{3e^{-i\Omega}}} - 1 + \frac{1}{1-\frac{1}{3e^{i\Omega}}} =$$

$$= \frac{3}{3-e^{i\Omega}} + \frac{3}{3-e^{-i\Omega}} - 1 =$$

$$= \frac{\cancel{9} - \cancel{3e^{-i\Omega}} + \cancel{9} - 3\cancel{e^{i\Omega}} - \cancel{9} + \cancel{3e^{i\Omega}} + \cancel{3e^{-i\Omega}} - \cancel{1}}{9 - 3e^{i\Omega} - 3e^{-i\Omega} + 1}$$

$$= \frac{8}{10 - 6\cos\Omega} = \frac{4}{5 - 3\cos\Omega}$$

b) $$|X(e^{i\Omega})| = \frac{4}{5-3\cos\Omega} \; ; \; \measuredangle X(e^{i\Omega}) = \emptyset$$

c) $$\Omega_{max} = 2m\pi \; ; \; m \in \mathbb{Z}$$

$$\Omega_{min} = \pi + 2m\pi \; ; \; m \in \mathbb{Z}.$$

4. A

a) $T = 7 \qquad \omega_0 = \frac{2\pi}{7}$

$$\cancel{X}_k = \frac{1}{7}\left[\int_{-1}^{2} e^{-j\frac{2\pi}{7}kt}dt + 2\int_{2}^{4} e^{-j\frac{2\pi}{7}kt}dt\right]$$

$$= \frac{e^{-j\frac{4\pi}{7}k} - e^{-j\frac{2\pi}{7}k}}{-j2\pi k} + 2\,\frac{e^{-j\frac{8\pi}{7}k} - e^{-j\frac{4\pi}{7}k}}{-j2\pi k}$$

$$= \frac{e^{-j\frac{3\pi}{7}k}e^{-j\frac{\pi}{7}k} - e^{+j\frac{3\pi}{7}k}e^{-j\frac{\pi}{7}k}}{-j2\pi k} + 2\,\frac{e^{-j\frac{6\pi}{7}k}e^{-j\frac{2\pi}{7}k} - e^{-j\frac{6\pi}{7}k}\cdot e^{j\frac{2\pi}{7}k}}{-j2\pi k}$$

$$= \frac{\sin\frac{3\pi}{7}k}{\pi k}e^{-j\frac{\pi}{7}k} + 2\,\frac{\sin(\frac{2\pi}{7}k)}{\pi k}e^{-j\frac{6\pi}{7}k}$$

$$= \frac{3}{7}\,\frac{\sin\frac{3\pi}{7}k}{\frac{3\pi}{7}k}e^{-j\frac{\pi}{7}k} + \frac{4}{7}\,\frac{\sin\frac{2\pi}{7}k}{\frac{2\pi}{7}k}e^{-j\frac{6\pi}{7}k}$$

b) $X_k^* = X_{-k}$

$$X_k^* = \frac{3}{7}\,\frac{\sin\frac{3\pi}{7}k}{\frac{3\pi}{7}k}e^{j\frac{\pi}{7}k} + \frac{4}{7}\,\frac{\sin\frac{2\pi}{7}k}{\frac{2\pi}{7}k}e^{j\frac{6\pi}{7}k}$$

$$X_{-k} = \frac{3}{7}\,\frac{\sin\frac{3\pi}{7}k}{\frac{3\pi}{7}k}e^{j\frac{\pi}{7}k} + \frac{4}{7}\,\frac{\sin\frac{2\pi}{7}k}{\frac{2\pi}{7}k}e^{j\frac{6\pi}{7}k}$$

c) $X_0 = \frac{1}{7}\int_{-1}^{2} dt + \frac{2}{7}\int_{2}^{4} dt = \frac{3+4}{7} = 1$

$$A = \frac{3}{7}\,\frac{\sin\frac{3\pi}{7}k}{\frac{3\pi}{7}k} \quad ; \quad B = \frac{4}{7}\,\frac{\sin\frac{2\pi}{7}k}{\frac{2\pi}{7}k}$$

$$X_k = A\cdot e^{-j\frac{\pi}{7}k} + B\,e^{-j\frac{6\pi}{7}k} = A\cos\frac{\pi}{7}k - jA\sin\frac{\pi}{7}k + B\cos\frac{6\pi}{7}k - jB\sin\frac{6\pi}{7}k$$

$$X_k = \left(A\cos\frac{\pi}{7}k + B\cos\frac{6\pi}{7}k\right) + j\left(-A\sin\frac{\pi}{7}k - B\sin\frac{6\pi}{7}k\right)$$

NASTAVAK →

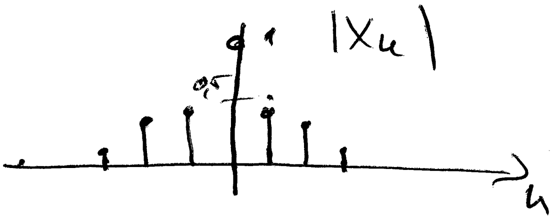
4.A.

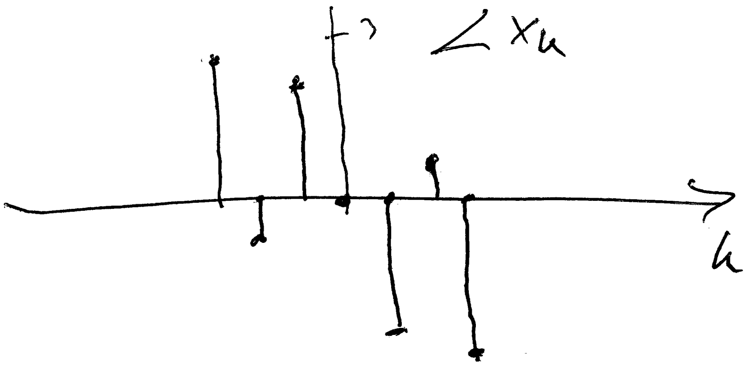
KALKULATOROM

$X_{-3} = -0{,}0389 + j0{,}0089 = 0{,}0399\, e^{j2{,}9162} = X_3^*$

$X_{-2} = 0{,}2365 - j0{,}1886 = 0{,}3025\, e^{-j0{,}6732} = X_2^*$

$X_{-1} = -0{,}1688 + 0{,}3506 = 0{,}3891\, e^{j2{,}0195} = X_1^*$





⑤ $x(n) = \underbrace{\{-2, 0, 2, \dots\}}_{N = 3m \; ; \; m \in \mathbb{N}}$ GRUPA A

a) $X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$

$= -2 + 2 \cdot e^{-i\frac{2\pi}{N}k \cdot 2} - 2 \cdot e^{-i\frac{2\pi}{N}k\cdot 3} + 2 e^{-i\frac{2\pi}{N}k\cdot 5} + \dots +$
$+ (-2) \cdot e^{-i\frac{2\pi}{N}\cdot k\cdot(N-3)} + 2 e^{-i\frac{2\pi}{N}k(N-1)}$; $k \in \{0 - N-1\}$.

$= -2\left(1 + e^{-i\frac{2\pi}{N}k\cdot 3} + \dots + e^{-i\frac{2\pi}{N}k(N-3)}\right) + 2\left(e^{-i\frac{2\pi}{N}k\cdot 2} + \dots + e^{-i\frac{2\pi}{N}k(N-1)}\right)$

$= -2\left(1 + \dots + e^{-i\frac{2\pi}{N}k(N-3)}\right) + 2 e^{-i\frac{2\pi}{N}k\cdot 2}\left(1 + e^{-i\frac{2\pi}{N}k\cdot 3} + \dots + e^{-i\frac{2\pi}{N}k(N-3)}\right)$

$= -2\left(1 - e^{-j\frac{4\pi}{N}k}\right)\left(1 + \underbrace{e^{-i\frac{2\pi}{N}k\cdot 3}}_{q} + \dots + e^{-i\frac{2\pi}{N}k(N-3)}\right)$

$1 + q + q^2 + \dots + q^{\frac{N-3}{3}} = \frac{1 - q^{\frac{N}{3}}}{1 - q}$ $\quad q \neq 1$

$= -2\left(1 - e^{-i\frac{4\pi}{N}k}\right) \frac{1 - e^{-i\frac{2\pi}{N}\cdot k\cdot 3\cdot\frac{N}{3}}}{1 - e^{-i\frac{2\pi}{N}k3}}$; $e^{-i\frac{2\pi}{N}k3} \neq 1$

$= -2\left(1 - e^{-i\frac{4\pi}{N}k}\right) \frac{1 - e^{-i2\pi k}}{1 - e^{-i\frac{2\pi}{N}k3}} = \emptyset$.

Za $e^{-i\frac{2\pi}{N}k\cdot 3} = 1 \Rightarrow \cos\frac{2\pi}{N}k3 - j\sin\frac{2\pi}{N}k3 = 1 \Leftrightarrow$

$\frac{2\pi}{N}k3 = 2m\pi$; $m \in \mathbb{Z} \Rightarrow k = \frac{N}{3}\cdot m$; $m \in \mathbb{Z}$.

No kako je $k \in \{0, \dots, N-1\} \Rightarrow k = \{\frac{N}{3}, \frac{2N}{3}\}$

$X(\frac{N}{3}) = -2\left(1 - e^{-i\frac{4\pi}{N}\cdot\frac{N}{3}}\right) \cdot \left(\underbrace{1 + 1 + \dots + 1}_{N/3}\right) = -\frac{2N}{3}\left(1 - e^{-i\frac{4\pi}{3}}\right)$

$$X\left(\frac{2N}{3}\right) = -2\left(1 - e^{-j\frac{4\pi}{N}\cdot\frac{2N}{3}}\right)\underbrace{(1+\dots+1)}_{N/3} = -\frac{2N}{3}\left(1 - e^{-j\frac{8\pi}{3}}\right)$$

Prema tome:

$$X\left(\frac{N}{3}\right) = \frac{2N}{3}\left(e^{j\frac{2\pi}{3}} - 1\right) \; ; \; X\left(\frac{2N}{3}\right) = \frac{2N}{3}\left(e^{-j\frac{2\pi}{3}} - 1\right)$$

$$X(k) = 0 \; , \text{ za } k \in \{0, \dots, N-1\} \setminus \left\{\frac{N}{3}, \frac{2N}{3}\right\}.$$

b) Iz a) $\Rightarrow$ za $k \in \left\{\frac{N}{3}, \frac{2N}{3}\right\}$.

c) $$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}kn} \quad ; n \in \{0 - N-1\}.$$

$$x(n) = \frac{1}{N}\cdot\frac{2N}{3}\left(e^{j\frac{2\pi}{3}} - 1\right) e^{j\frac{2\pi}{N}\cdot\frac{N}{3}\cdot n} + \frac{1}{N}\cdot\frac{2N}{3}\left(e^{-j\frac{2\pi}{3}} - 1\right) e^{+j\frac{2\pi}{N}\cdot\frac{2N}{3}\cdot n}$$

$$x(n) = -\frac{4\sqrt{3}}{3}\cos\left(\frac{2\pi}{3}n - \frac{\pi}{6}\right) \; ; n \in \{0 - N-1\}.$$

d) Harmonijskoj komponenti k odgovara diskretna frekvencij

$$\Omega_k = \frac{2\pi}{N}\cdot k \Rightarrow \Omega_{\frac{N}{3}} = \frac{2\pi}{3} \; ; \; \Omega_{\frac{2N}{3}} = \frac{4\pi}{3} = -\frac{2\pi}{3}$$

Iz $\omega_k \cdot T_s = \Omega_k \Rightarrow f_{\frac{N}{3}} = \frac{10}{3}$ kHz ; $f_{\frac{2N}{3}} = -\frac{10}{3}$ kHz.

e) Iz c) uz $n = \frac{t}{T_s} \Rightarrow x(t) = -\frac{4\sqrt{3}}{3}\cos\left(\frac{2\pi}{3}\cdot 10^3 t - \frac{\pi}{6}\right)$

Signali i sustavi

Međuispit (grupa B) – 24. travnja 2013.

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a) **(4 boda)** Postoji li vremenski kontinuirana Fourierova transformacija (CTFT) signala $x(t)$? Ako postoji, pokažite zašto postoji, a ako ne postoji, pokažite zašto ne postoji!

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c) **(3 boda)** Odredite na kojim frekvencijama Ω amplitudni spektar $\left|X(e^{j\Omega})\right|$ poprima minimalne, a na kojima maksimalne vrijednosti.

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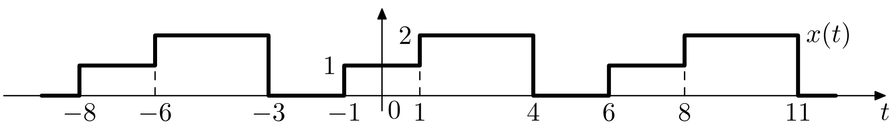

5. (9 bodova) Promatramo vremenski diskretan signal konačnog trajanja oblika $x[n] = \{\underline{-3}, 0, 3, -3, 0, 3, -3, 0, 3, -3, 0, 3, \ldots\}$ gdje se uzorak $\{-3, 0, 3\}$ ponavlja m-puta. Neka je trajanje signala $N = 3m$, $m \in \mathbb{N}$.

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d) **(2 boda)** Ako je promatrani signal $x[n]$ dobiven očitavanjem vremenski kontinuiranog signala $x(t)$ s frekvencijom očitavanja $f_S = 10\,\text{kHz}$ koje spektralne komponente se nalaze u signalu $x(t)$?

e) **(2 boda)** Odredite periodičan vremenski kontinuirani signal $x(t)$ dobiven idealnom rekonstrukcijom iz signala $x[n]$.

GRUPA B

1

a) TOTALNA ENERGIJA: $E_\infty = \int_{-\infty}^{\infty} |f(t)|^2 \, dt$

TOTALNA SNAGA: $P_\infty = \lim_{L \to \infty} \frac{1}{L} \int_{-\frac{L}{2}}^{\frac{L}{2}} |f(t)|^2 \, dt$

b) $E_\infty = \int_{-\infty}^{\infty} (3^{-t} \mu(t))^2 \, dt = \int_{0}^{\infty} \left(\frac{1}{9}\right)^t dt = \frac{\left(\frac{1}{9}\right)^t}{\ln \frac{1}{9}} \Bigg|_0^\infty =$

$= \frac{0 - 1}{-\ln 9} = \frac{1}{\ln 9}$

ENERGIJA JE KONAČNA $\Rightarrow P_\infty = 0$

c) TOTALNA ENERGIJA: $E_\infty = \sum_{n=-\infty}^{\infty} |y[n]|^2$

TOTALNA SNAGA: $P_\infty = \lim_{M \to \infty} \frac{1}{2M+1} \sum_{n=-M}^{M} |y[n]|^2$

d) $P_\infty = P_T = \frac{1}{T} \cdot \sum_{T} \cos^2\left(\frac{\pi n}{3}\right) =$

$= \frac{1}{6} \cdot \left(1 + \frac{1}{4} + \frac{1}{4} + 1 + \frac{1}{4} + \frac{1}{4}\right) = \frac{1}{2}$

$\Rightarrow$ SNAGA JE $\neq 0 \Rightarrow E_\infty = \infty$

GRUPA B

(2) $$x(t) = e^{2t}\left(\mu(t) - \mu(t-8)\right)$$

a) Moramo pokazati da je signal kvadratno-integrabilan ($\in L^2$) ili da zadovoljava Dirichletove uvjete:

$$L^2: \quad \int_{-\infty}^{+\infty} x^2(t)\,dt = \int_0^8 e^{4t}\,dt = \frac{1}{4}e^{4t}\Big|_0^8 = \frac{1}{4}\left(e^{32}-1\right) < \infty$$

DIRICHLET:

a) $$\int_{-\infty}^{+\infty} |x(t)|\,dt = \int_0^8 e^{2t}\,dt = \frac{1}{2}e^{2t}\Big|_0^8 = \frac{1}{2}\left(e^{16}-1\right) < \infty$$

b) Funkcija ima konačan broj ekstrema (jedan ekstrem u $t = 8^-$), ima dva diskontinuiteta i vrijedi $|x(t)| \leq e^{16}$.

b) $$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,dt = \int_0^8 e^{2t}e^{-j\omega t}\,dt =$$

$$= \frac{1}{2-j\omega}e^{(2-j\omega)t}\Big|_0^8 = \frac{1}{2-j\omega}\left(e^{8(2-j\omega)}-1\right)$$

③ $x(n) = 2^{-|n|} \quad n \in \mathbb{Z}$ GRUPA B

a) $$X(e^{i\Omega}) = \sum_{n=-\infty}^{\infty} 2^{-|n|} e^{-i\Omega n}$$

$$= \sum_{n=-\infty}^{-1} 2^{n} e^{-i\Omega n} + \sum_{n=0}^{\infty} 2^{-n} e^{-i\Omega n}$$ | $m=-n$ ZA PRVU SUMU |

$$= \sum_{n=0}^{\infty} \left(\frac{1}{2e^{i\Omega}}\right)^n + \sum_{m=0}^{\infty} \left(\frac{e^{i\Omega}}{2}\right)^m - 1$$

$$= \frac{1}{1-\frac{1}{2e^{i\Omega}}} + \frac{1}{1-\frac{e^{i\Omega}}{2}} - 1 =$$

$$= \frac{2e^{i\Omega}}{2e^{i\Omega}-1} + \frac{2}{2-e^{i\Omega}} - 1 =$$

$$= \frac{2}{2-e^{-i\Omega}} + \frac{2}{2-e^{i\Omega}} - 1 =$$

$$= \frac{\cancel{4}\cancel{-2e^{i\Omega}} + 4 \cancel{-2e^{-i\Omega}} \cancel{-4} - 1 + \cancel{2e^{-i\Omega}} \cancel{+2e^{i\Omega}}}{4+1-2e^{-i\Omega}-2e^{i\Omega}}$$

$$= \frac{3}{5-4\cos\Omega}$$

b) $|X(e^{i\Omega})| = \frac{3}{5-4\cos\Omega}$; $\measuredangle X(e^{i\Omega}) = \emptyset$.

c) $\Omega_{max} = 2m\pi$; $m \in \mathbb{Z}$

$\Omega_{min} = \pi + 2m\pi$; $m \in \mathbb{Z}$

4. B

a) $T = 7 \qquad \omega_0 = \frac{2\pi}{7}$ ... 1 bod

$$X_k = \frac{1}{7}\int_{-1}^{1} e^{-j\omega_0 kt}\,dt + \frac{2}{7}\int_{1}^{4} e^{-j\omega_0 kt}\,dt \quad \ldots\ldots \text{ 1 bod}$$

$$= \frac{1}{7}\,\frac{e^{-j\omega_0 k} - e^{+j\omega_0 k}}{-j\omega_0 k} + \frac{2}{7}\,\frac{e^{-j4\omega_0 k} - e^{-j\omega_0 k}}{-j\omega_0 k} \quad \ldots\ldots \text{ 1 bod}$$

$$= \left\{ \begin{array}{l} \text{ili} \quad \frac{2}{7}\,\frac{\sin \omega_0 k}{\omega_0 k} + \frac{4}{7} e^{-j\frac{5}{2}\omega_0 k}\,\frac{\sin \frac{3}{2}\omega_0 k}{\omega_0 k} \\ \text{ili} \quad \frac{2}{7}\,\mathrm{sinc}\,\frac{2}{7}k + \frac{6}{7} e^{-j\frac{5\pi}{7}k}\,\mathrm{sinc}\,\frac{3}{7}k \end{array} \right\} \text{ 1 bod}$$

b) $X_k^* = X_{-k}$ ... 1 bod

npr. $$\frac{2}{7}\,\mathrm{sinc}\,\frac{2}{7}k + \frac{6}{7} e^{+j\frac{5\pi}{7}k}\,\mathrm{sinc}\,\frac{3}{7}k = \frac{2}{7}\,\overbrace{\mathrm{sinc}\left(-\tfrac{2}{7}k\right)}^{\text{parna}} + \frac{6}{7} e^{\overbrace{-j\frac{5\pi}{7}(-k)}^{+}}\,\underbrace{\mathrm{sinc}\left(-\tfrac{3}{7}k\right)}_{\text{parna}}$$

✓ .... 1 bod

c) $$X_0 = \frac{1}{7}\int_{-1}^{1} dt + \frac{2}{7}\int_{1}^{4} dt = \frac{2+6}{7} = \frac{8}{7} \quad \ldots \text{ 1 bod}$$

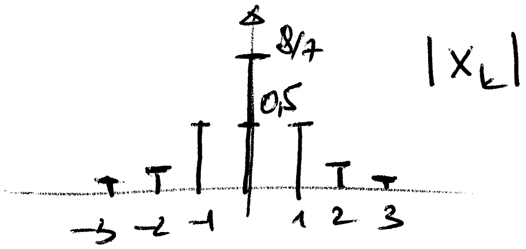
KALKULATOR:

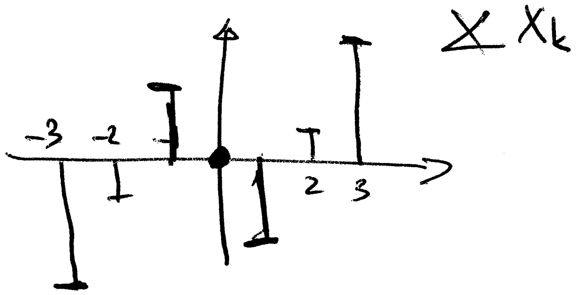
$X_{-1} = -0{,}14 + j0{,}49 = 0{,}5\,e^{+j1{,}85} = X_1^*$

$X_{-2} = 0{,}12 - j0{,}13 = 0{,}18\,e^{-j0{,}82} = X_2^*$

$X_{-3} = -0{,}1 - j0{,}07 = 0{,}13\,e^{j4{,}0} = X_3^*$

.... 2 boda





GRUPA B

(5) $x[n] = \{-3, 0, 3, -3, 0, 3, -3, 0, 3, \dots\}$, $N = 3m$, $m \in \mathbb{N}$

a) $DFT_N[x[n]] = \sum_{n=0}^{N-1} x[n] W_N^{nk} =$

$$= \sum_{n=0}^{N/3-1} \underbrace{x[3n]}_{=-3} W_N^{3nk} + \sum_{n=0}^{N/3-1} \underbrace{x[3n+1]}_{=\phi} W_N^{(3n+1)k} + \sum_{n=0}^{N/3-1} \underbrace{x[3n+2]}_{=3} W_N^{(3n+2)k} =$$

$$= -3 \sum_{n=0}^{N/3-1} W_N^{3nk} + \phi + 3 \sum_{n=0}^{N/3-1} W_N^{2k} \cdot W_N^{3nk} =$$

$$= -3 \frac{1 - W_N^{3Nk}}{1 - W_N^{3k}} + 3 W_N^{2k} \cdot \frac{1 - W_N^{3Nk}}{1 - W_N^{3k}}$$

$1 - W_N^{3Nk} = 1 - e^{j2\pi \frac{3Nk}{N}} = 1 - 1 = 0$ te se gornja dva člana gube za svaki k za koji je $W_N^{3k} \neq 1$.

$W_N^{3k} = e^{-j2\pi \frac{3k}{N}} = 1 \Rightarrow k = 0, \frac{N}{3}, \frac{2}{3}N$

Sada je $X[k] = 0$ za $k \neq 0, N/3, \frac{2}{3}N$

$$X[0] = -3 \cdot N/3 + 3 W_N^{2 \cdot 0} \cdot N/3 = 0$$

$$X[N/3] = -3 \cdot N/3 + 3 W_N^{2N/3} \cdot N/3 = N(-1 + e^{-j4\pi/3})$$

$$X[2N/3] = -3 \cdot N/3 + 3 W_N^{2\frac{2N}{3}} \cdot N/3 = N(-1 + e^{-j8\pi/3})$$

b) $X[k]$ je ϕ za sve k osim $k = N/3 + \ell N$ i $k = \frac{2N}{3} + \ell N$, $\ell \in \mathbb{Z}$.

c) $$x[n] = \frac{1}{N} \cdot \left(N(-1 + e^{j2\pi/3}) e^{j2\pi \frac{n N/3}{N}} + N(-1 + e^{-j2\pi/3}) e^{j2\pi \frac{n 2N/3}{N}} \right) =$$

$$= (-1 + e^{j2\pi/3}) e^{j2\pi/3 n} + (-1 + e^{-j2\pi/3}) e^{-j2\pi/3 n} =$$

$$= -2\cos\left(\frac{2\pi}{3}n\right) + 2\cos\left(\frac{2\pi}{3}n + \frac{2\pi}{3}\right)$$

d) Iz rastava pod c) vidimo da diskretne harmonijske komponente imaju frekvenciju $\pm \frac{2\pi}{3}$.
Uz $f_s = 10$ kHz dobivamo:

$$f = f_s \cdot \frac{\pm 2\pi/3}{2\pi} = \pm f_s/3 = \pm \frac{10}{3} \text{ kHz}$$

e) $$x[n] = 2\cos\left(\frac{2\pi}{3}n + \frac{2\pi}{3}\right) - 2\cos\left(\frac{2\pi}{3}n\right) =$$
$$= 4\sin\left(\frac{4\pi}{3}n + \frac{2\pi}{3}\right) \cdot \sin\left(-\frac{2\pi}{3}\right) =$$
$$= 2\sqrt{3}\sin\left(\frac{2\pi}{3}n - \frac{2\pi}{3}\right) = 2\sqrt{3}\cos\left(\frac{2\pi}{3}n - \frac{7\pi}{6}\right)$$

Sada je:

$$x(t) = 2\sqrt{3}\cos\left(\frac{2\pi}{3}t \cdot f_s - \frac{7\pi}{6}\right) =$$
$$= 2\sqrt{3}\cos\left(\frac{2\pi}{3} \cdot 10^3 \cdot t - \frac{7\pi}{6}\right)$$