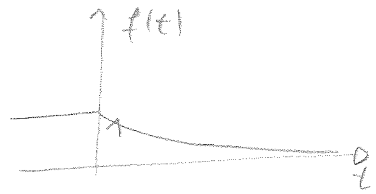
A

1. $f(t) = \begin{cases} 1, & \text{za } t<0 \\ e^{-2t}, & \text{za } t \geq 0 \end{cases}$



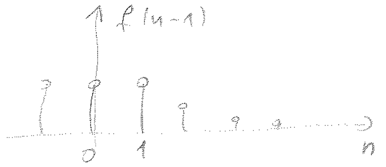
a) $f(n) = ?$

T_s period očitavanja

$f(n) = \begin{cases} 1, & \text{za } n<0 \\ e^{-2T_s \cdot n}, & \text{za } n \geq 0 \end{cases}$

b) $f_{ad}(n) = ?$

$f_{ad}(n) = \frac{1}{T_s} \{ f(n) - f(n-1) \}$



$f_{ad}(n) = \frac{1}{T_s} \{ 1 - 1 \} = 0 \quad \text{za} \quad n \leq 0$

$f_{ad}(n) = \frac{1}{T_s} \{ e^{-2T_s n} - e^{-2T_s(n-1)} \} = \frac{1}{T_s} e^{-2T_s n} (1 - e^{2T_s}) \quad \text{za} \quad n > 0$

c) $f_d(n) = ?$

$f'(t) = \begin{cases} 0, & t<0 \\ -2e^{-2t}, & t \geq 0 \end{cases}$

$f_d(n) = \begin{cases} 0, & n<0 \\ -2e^{-2T_s n}, & n \geq 0 \end{cases}$

d)

$$E_g = \sum_{n=-\infty}^{\infty} |f_d(n) - f_{ad}(n)|^2$$

$$= \sum_{n=-\infty}^{-1} |0-0|^2 + \sum_{n=1}^{\infty} \left| \frac{1}{T_s} \left(e^{-2T_s n} - e^{-2T_s n} \cdot e^{2T_s} \right) - \left(-2 e^{-2T_s n} \right) \right|^2 + \left(-2e^{-2T_s \cdot 0} - 0 \right)^2$$

$$= \sum_{n=1}^{\infty} \left| \left(\frac{1}{T_s} - \frac{1}{T_s} e^{2T_s} + 2 \right) e^{-2T_s n} \right|^2 + 4$$

$$= \left(\frac{1}{T_s} - \frac{1}{T_s} e^{2T_s} + 2 \right)^2 \cdot \sum_{n=1}^{\infty} \left(e^{-4T_s} \right)^n + 4$$

$$= \left(\frac{1}{T_s} - \frac{1}{T_s} e^{2T_s} + 2 \right)^2 \cdot \sum_{n=0}^{\infty} \left(e^{-4T_s} \right)^n - \left(\frac{1}{T_s} - \frac{1}{T_s} e^{2T_s} + 2 \right)^2 \cdot 1 + 4$$

$$= \left(\frac{1 - e^{2T_s} + 2T_s}{T_s} \right)^2 \cdot \frac{1}{1 - e^{-4T_s}} - \frac{(1 - e^{2T_s} + 2T_s)^2}{T_s^2} + 4$$

$$= 4 + \frac{(1 - e^{2T_s} + 2T_s)^2 \, (1 - 1 + e^{-4T_s})}{T_s^2 (1 - e^{-4T_s})}$$

$$= 4 + \frac{(1 - e^{2T_s} + 2T_s)^2 \cdot e^{-4T_s}}{T_s^2 (1 - e^{-4T_s})}$$

2. $f(n) = \cos\frac{\pi}{2}n \cdot (\mu(n) - \mu(n-4)) = \{\underline{1}, 0, -1, 0\}$

A

a) DTFT

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$$= e^{-j\Omega \cdot 0} - e^{-j\Omega \cdot 2}$$

$$= e^{-j\Omega}\left(e^{j\Omega} - e^{-j\Omega}\right)$$

$$= e^{-j\Omega}\left(2j\sin\Omega\right)$$

$$= 2j\sin\Omega\, e^{-j\Omega}$$

$$= 2\sin\Omega\, e^{j\frac{\pi}{2}} e^{-j\Omega}$$

$$= 2\sin\Omega\, e^{j(-\Omega + \frac{\pi}{2})}$$

b) Spektar je periodičan sa 2π

$$X(e^{j(\Omega + 2k\pi)}) = 2\sin(\Omega + 2k\pi) \cdot e^{j(-(\Omega + 2k\pi) + \frac{\pi}{2})}$$

$$= 2\sin(\Omega + 2k\pi) \cdot e^{j(-\Omega + \frac{\pi}{2})} \cdot \underbrace{e^{-j \cdot 2k\pi}}_{\underbrace{\cos 2k\pi}_{1} - j\underbrace{\sin 2k\pi}_{\emptyset}}$$

$$= 2\sin\Omega \cdot e^{j(-\Omega + \frac{\pi}{2})}$$

c) DTFS

$$g(n) = \cos\frac{\pi}{2}n$$

$$= \frac{1}{2} e^{j\frac{\pi}{2}n} + \frac{1}{2} e^{-j\frac{\pi}{2}n}$$

Period $g(n)$:

$$\frac{\pi}{2}N = 2k\pi$$

$$N = 4k$$

$$N_0 = 4$$

$$g(n) = \frac{1}{2} e^{j\frac{2\pi}{4} \cdot n \cdot 1} + \frac{1}{2} e^{j\frac{2\pi}{4} \cdot n \cdot (-1)}$$

$$= \frac{1}{2} e^{j\frac{2\pi}{4} n \cdot 1} + \frac{1}{2} e^{j\frac{2\pi}{4} \cdot n \cdot 3}$$

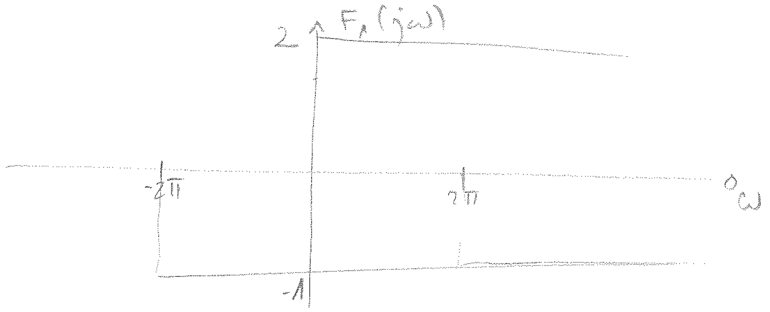
$$\boxed{G_1 = \frac{1}{2} \quad G_3 = \frac{1}{2}}$$

A

3

$$F(j\omega) = j\left[-\mu(\omega+2\pi) + 2\mu(\omega) - \mu(\omega-2\pi)\right] = j \cdot F_1(j\omega)$$

a) $$f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$



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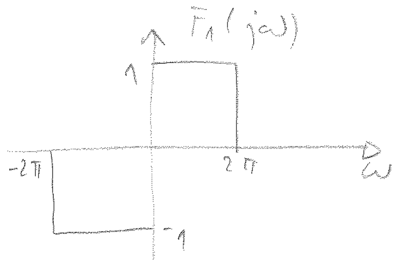


$$f(t) = \frac{1}{2\pi}\int_{-2\pi}^{0} (-j)\, e^{j\omega t}\, d\omega + \frac{1}{2\pi}\int_{0}^{2\pi} j\, e^{j\omega t}\, d\omega$$

$$= \frac{1(-j)}{2\pi} \cdot \left.\frac{e^{j\omega t}}{jt}\right|_{-2\pi}^{0} + \frac{1}{2\pi}\cdot \cancel{j}\, \left.\frac{e^{j\omega t}}{\cancel{j}t}\right|_{0}^{2\pi}$$

$$= \frac{-1}{2\pi t}\left(1 - e^{-2\pi j t}\right) + \frac{1}{2\pi t}\left(e^{j\cdot 2\pi \cdot t} - e^{j\cdot 0}\right)$$

$$= -\frac{1}{2\pi t}\left(1+1\right) + \frac{1}{2\pi t}\left(e^{-j2\pi t} + e^{j2\pi t}\right)$$

$$= -\frac{\cancel{2}}{\cancel{2}\pi t} + \frac{1}{2\pi t}\cdot 2\cos 2\pi t$$

$$= 2\cdot\frac{\cos 2\pi t - 1}{2\pi t}$$

$$f(0) = \frac{1}{2\pi}\left[\int_{-2\pi}^{0}(-j)\,d\omega + \int_{0}^{2\pi} j\,d\omega\right]$$

$$= \frac{1}{2\pi}\left[-j\omega\Big|_{-2\pi}^{0} + j\omega\Big|_{0}^{2\pi}\right]$$

$$= \frac{1}{2\pi}\left[-j\cdot 0 - j\cdot 2\pi + j\cdot 2\pi - j\cdot 0\right]$$

$$f(0) = 0$$

b) $$g(t) = f(t-4)$$

$$G(j\omega) = F(j\omega)\, e^{-j\omega 4}$$

$$= j e^{-j\omega\cdot 4}\left[-\mu(\omega+2\pi) + 2\mu(\omega) - \mu(\omega-2\pi)\right]$$

3\. c) Parsevalova relacija za CTFT

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$$

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} f(t) \cdot f^*(t) dt =$$

$$= \int_{-\infty}^{\infty} f(t) \cdot \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} F^*(j\omega) e^{-j\omega t} d\omega \right] dt$$

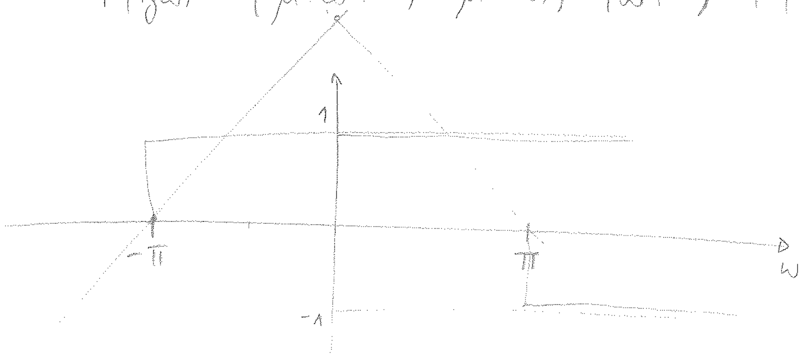
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} F^*(j\omega) \left[\int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \right] d\omega$$

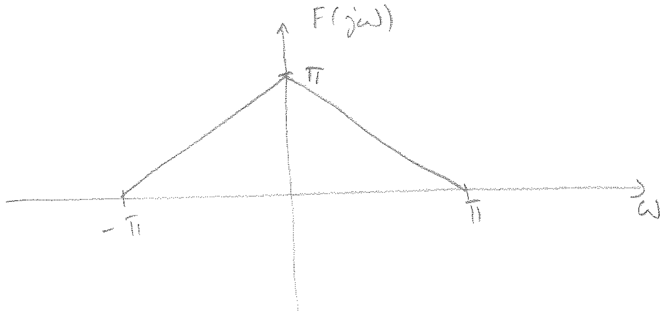
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} F^*(j\omega) \cdot F(j\omega) d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$$

A

4. $F(j\omega) = (\mu(\omega+\pi) - \mu(\omega))\cdot(\omega+\pi) + (-\omega+\pi)(\mu(\omega)-\mu(\omega-\pi))$



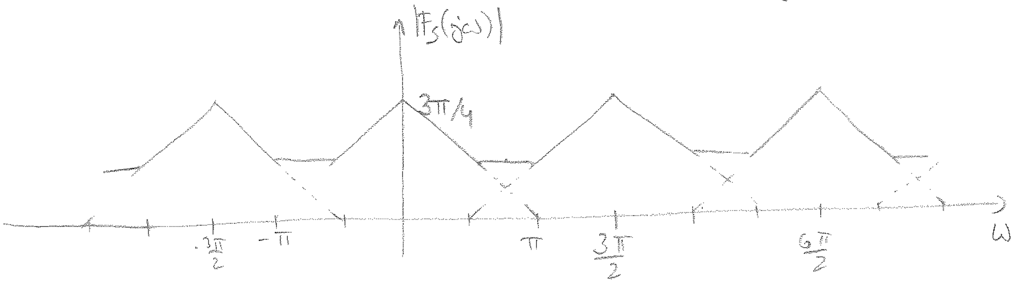


a) Maksimalna frekvencija u signalu je π.
Kako postoji maksimalna frekvencija koja je konačan broj – ovaj signal se može jednoznačno očitati u vremenskoj domeni.

Minimalna frekvencija očitavanja je $2\cdot\omega_{max} = 2\pi$

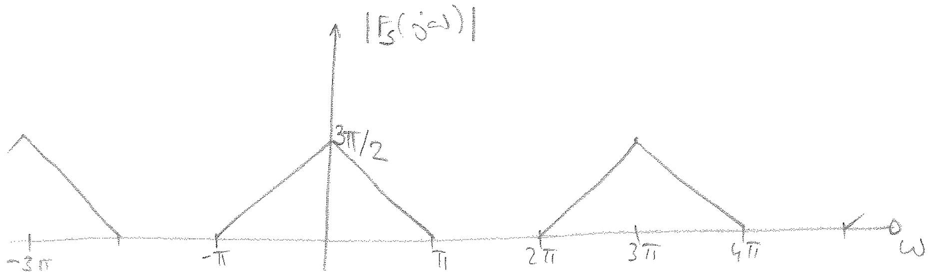
b) $\omega_s = \frac{3\pi}{2} \rightarrow T_s = \frac{2\pi}{\omega_s} = \frac{2\pi}{\frac{3\pi}{2}} = \frac{4\pi}{3\pi} = \frac{4}{3}$

$\omega_s < 2\pi$ dolazi do aliasinga



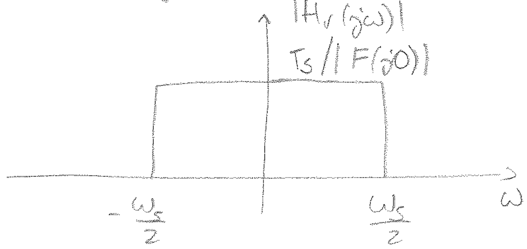
c) $\omega_s = 3\pi \rightarrow T_s = \frac{2\pi}{\omega_s} = \frac{2\pi}{3\pi} = \frac{2}{3}$

$\omega_s > 2\pi$ neće doći do aliasinga



4 d) Rekonstrukcija kontinuiranog signala iz očitanog

- u frekvencijskoj domeni množenjem idealnim filtrom



$$X(j\omega) = X_s(j\omega) \cdot H_r(j\omega)$$

- u vremenskoj domeni je to konvolucija očitanog kontinuiranog signala s $ICTFT\{H_r(j\omega)\}$

$$h_r(t) = ICTFT\{H_r(j\omega)\} = \frac{1}{2\pi}\int_{-\omega_s/2}^{\omega_s/2} \frac{T_s}{|F(j0)|} e^{j\omega t} d\omega$$

$$= \frac{T_s}{2\pi|F(j0)|} \cdot \left.\frac{e^{j\omega t}}{jt}\right|_{-\omega_s/2}^{\omega_s/2}$$

$$= \frac{T_s}{2\pi|F(j0)|jt}\left(e^{j\frac{\omega_s}{2}t} - e^{-j\frac{\omega_s}{2}t}\right)$$

$$= \frac{T_s \cdot 2j \sin\frac{\omega_s}{2}t}{2j\pi t \cdot |F(j0)|}$$

$$= \frac{\cancel{2\pi} \cdot 2 \sin\frac{\omega_s}{2}t}{\cancel{2\pi}\, t \cdot |F(j0)| \cdot \omega_s}$$

$$= \frac{1}{|F(j0)|} \cdot \frac{\sin\frac{\omega_s}{2}t}{\frac{\omega_s}{2}t}$$

$$x(t) = x_s(t) * h_r(t)$$

5. $f(t)$, $g(t)$

$g(t) = f(at)$, $a > 0$

A

a) $E_f = E(f(t)) < \infty$

$E_g = ?$

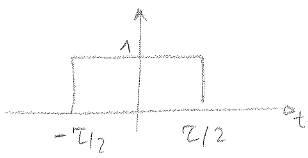
$$E_f = \int_{-\infty}^{\infty} |f(t)|^2 \, dt$$

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 \, dt = \int_{-\infty}^{\infty} |f(at)|^2 \, dt = \left| \begin{array}{ll} at = \tau & \\ a\,dt = d\tau & \\ t = -\infty & \tau = -\infty \\ t = \infty & \tau = \infty \end{array} \right|$$

$$= \int_{-\infty}^{\infty} |f(\tau)|^2 \frac{1}{a} \, d\tau = \frac{1}{a} \int_{-\infty}^{\infty} |f(\tau)|^2 \, d\tau$$

$$= \frac{1}{a} E_f$$

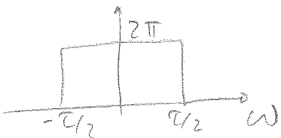
b) $h(t) = \dfrac{\sin 10t}{10t}$



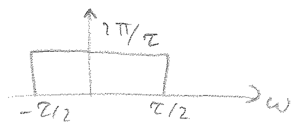
$\xleftrightarrow{CTFT}$ $\tau \cdot \dfrac{\sin \frac{\tau}{2}\omega}{\frac{\tau}{2}\omega}$

SVOJSTVO DUALNOSTI

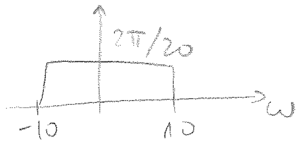
$\tau \dfrac{\sin \frac{\tau}{2} t}{\frac{\tau}{2} t}$ $\xleftrightarrow{CTFT}$



$\dfrac{\sin \frac{\tau}{2} t}{\frac{\tau}{2} t}$ $\xleftrightarrow{CTFT}$



$\dfrac{\sin 10t}{10t}$ $\xleftrightarrow{CTFT}$



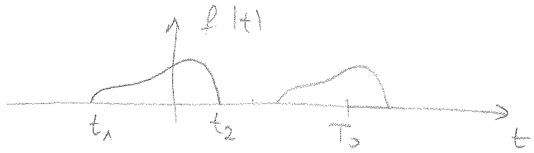
$$E_h = \frac{1}{2\pi} \int_{-\infty}^{\infty} |H(j\omega)|^2 \, d\omega$$

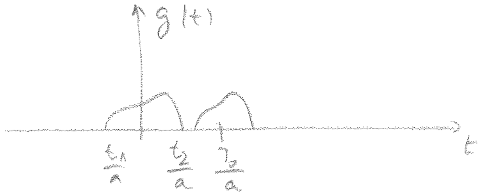
$$= \frac{1}{2\pi} \int_{-10}^{10} \left| \frac{2\pi}{\cancel{20}_{10}} \right|^2 d\omega$$

$$= \frac{1}{\cancel{2\pi}} \frac{\pi^{\cancel{2}}}{10^2} \, \omega \Big|_{-10}^{10} = \frac{\pi}{2 \cdot 100} (10 + 10) = \frac{2\cancel{0}\pi}{20\cancel{0}} = \frac{\pi}{10}$$

c) $f(t)$ periodičan za T_0

$g(t) = f(at)$





$$P_f = \frac{1}{T_0}\int_{T_0} |f(t)|^2\,dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |f(t)|^2\,dt$$

$$P_g = \frac{1}{\frac{T_0}{a}} \int_{T_0/a} |f(at)|^2\,dt$$

$$= \frac{a}{T_0}\int_{-T_0/2a}^{T_0/2a} |f(at)|^2\,dt = \left|\begin{array}{ll} at = \tau & t = -\frac{T_0}{2a} \rightarrow \tau = -\frac{T_0}{2a}\cdot a = -\frac{T_0}{2} \\ a\,dt = d\tau & \\ dt = \frac{1}{a}\,d\tau & t = \frac{T_0}{2a} \rightarrow \tau = \frac{T_0}{2a}\cdot a = \frac{T_0}{2} \end{array}\right|$$

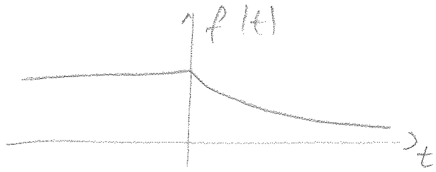
$$= \frac{a}{T_0}\int_{-T_0/2}^{T_0/2} |f(\tau)|^2 \frac{1}{a}\,d\tau = \frac{a}{T_0}\cdot\frac{1}{a}\int_{-T_0/2}^{T_0/2} |f(\tau)|^2\,d\tau$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |f(\tau)|^2\,d\tau$$

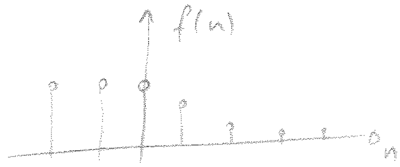
$$= P_f$$

B

1. $f(t) = \begin{cases} 1, & t < 0 \\ e^{-4t}, & t \geq 0 \end{cases}$



a) $f(n) = \begin{cases} 1 & n < 0 \\ e^{-4T_s \cdot n}, & n \geq 0 \end{cases}$



b) $f_{ad}(n) = \frac{1}{T_s} \left(f(n) - f(n-1) \right)$

$f_{ad}(n) = \frac{1}{T_s} (1 - 1) = 0 \quad , \quad n \leq 0$

$f_{ad}(n) = \frac{1}{T_s} \left(e^{-4T_s \cdot n} - e^{-4T_s(n-1)} \right) =$

$= \frac{1}{T_s} e^{-4T_s n} \left(1 - e^{4T_s} \right) \quad , \quad n > 0$

c) $f'(t) = \begin{cases} 0, & t < 0 \\ -4e^{-4t}, & t \geq 0 \end{cases}$

$f_d(n) = \begin{cases} 0, & n < 0 \\ -4e^{-4T_s \cdot n}, & n \geq 0 \end{cases}$

d) $$E_g = \sum_{n=-\infty}^{\infty} \left| f_d(n) - f_{ad}(n) \right|^2$$

$$= \sum_{n=-\infty}^{-1} |0 - 0|^2 + \left| -4e^{-4T_s \cdot 0} - 0 \right|^2 + \sum_{n=1}^{\infty} \left| -4e^{-4T_s \cdot n} - \frac{1}{T_s} e^{-4T_s n} (1 - e^{4T_s}) \right|^2$$

$$= 16 + \sum_{n=1}^{\infty} \left| e^{-4T_s \cdot n} \left(-4 - \frac{1}{T_s} (1 - e^{4T_s}) \right) \right|^2$$

$$= 16 + \sum_{n=0}^{\infty} \left| e^{-4T_s \cdot n} \right|^2 \cdot \left| -4 - \frac{1}{T_s} (1 - e^{4T_s}) \right|^2 - \left(-4 - \frac{1}{T_s} (1 - e^{4T_s}) \right)^2$$

$$= 16 - \left(4 + \frac{1 - e^{4T_s}}{T_s} \right)^2 + \left(4 + \frac{1 - e^{4T_s}}{T_s} \right)^2 \cdot \sum_{n=0}^{\infty} \left(e^{-8T_s} \right)^n$$

$$= 16 - \frac{(4T_s + 1 - e^{4T_s})^2}{T_s^2} + \frac{(4T_s + 1 - e^{4T_s})^2}{T_s^2} \cdot \frac{1}{1 - e^{-8T_s}}$$

$$= \frac{16T_s^2 (1 - e^{-8T_s}) - (1 - e^{-8T_s})(4T_s + 1 - e^{4T_s})^2 + (4T_s + 1 - e^{4T_s})^2}{T_s^2 (1 - e^{-8T_s})}$$

$$= \frac{16T_s^2 (1 - e^{-8T_s}) + (4T_s + 1 - e^{4T_s})^2 e^{-8T_s}}{T_s^2 (1 - e^{-8T_s})}$$

$\boxed{B}$

2.

$$f(n) = \cos\left(\tfrac{3\pi}{2} n\right)\left(\mu(n) - \mu(n-4)\right) = \{\underline{1}, 0, -1, 0\}$$

a) DTFT

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$$= 1 \cdot e^{-j\Omega 0} - 1\, e^{-j\Omega 2}$$

$$= e^{-j\Omega}\left(e^{j\Omega} - e^{-j\Omega}\right)$$

$$= 2j \sin\Omega\, e^{-j\Omega}$$

$$= 2 \sin\Omega\, e^{j(-\Omega + \frac{\pi}{2})}$$

b) Spektar je periodičan sa 2π.

Dokaz:

$$X(e^{j(\Omega + 2k\pi)}) = 2 \sin(\Omega + 2k\pi)\, e^{j(-(\Omega + 2k\pi) + \frac{\pi}{2})}$$

$$= 2 \sin(\Omega + 2k\pi) \cdot e^{j(-\Omega + \frac{\pi}{2})} \cdot \underbrace{e^{-j2k\pi}}_{\underbrace{\cos 2k\pi}_{1} - j \underbrace{\sin 2k\pi}_{0}}$$

$$= 2 \sin\Omega \cdot e^{j(-\Omega + \frac{\pi}{2})}$$

c) DTFS

$$g(n) = \cos \tfrac{3\pi}{2} n$$

Period $\quad \frac{3\pi}{2} N = 2k\pi$

$$N = \frac{4k}{3}$$

$$N_0 = 4$$

$$g(n) = \frac{1}{2} e^{j\frac{3\pi}{2} n} + \frac{1}{2} e^{-j\frac{3\pi}{2} n}$$

$$= \frac{1}{2} e^{j\frac{2\pi}{4} \cdot n \cdot 3} + \frac{1}{2} e^{j\frac{2\pi}{4} \cdot n \cdot (-3)}$$

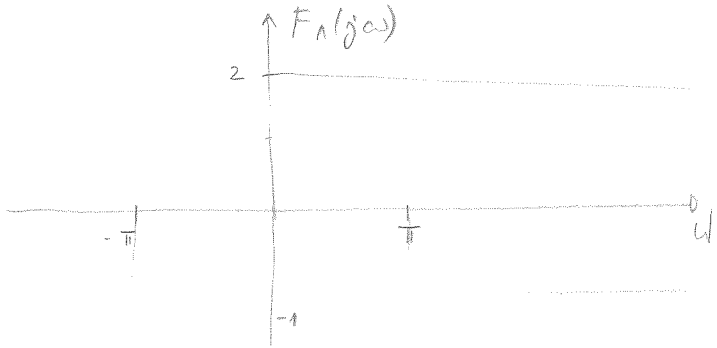
$$= \frac{1}{2} e^{j\frac{2\pi}{4} \cdot n \cdot 3} + \frac{1}{2} e^{j\frac{2\pi}{4} \cdot n \cdot 1}$$

$\boxed{G_1 = \frac{1}{2}}$ $\qquad$ $\boxed{G_3 = \frac{1}{2}}$

$\boxed{B}$

3. $F(j\omega) = j\left[-\mu(\omega+\pi) + 2\mu(\omega) - \mu(\omega-\pi)\right] = j \cdot F_1(j\omega)$

a) $f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{i\omega t} d\omega$



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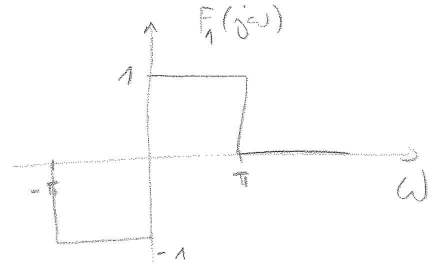


$$f(t) = \frac{1}{2\pi} \int_{-\pi}^{0} (-j) e^{i\omega t} d\omega + \frac{1}{2\pi} \int_{0}^{\pi} j\, e^{i\omega t} d\omega$$

$$= \frac{1 \cdot (-j)}{2\pi} \cdot \frac{e^{i\omega t}}{jt}\Big|_{-\pi}^{0} + \frac{j}{2\pi} \frac{e^{i\omega t}}{jt}\Big|_{0}^{\pi}$$

$$= \frac{-j}{2\pi j t}\left(e^{0} - e^{-jt\pi}\right) + \frac{j}{2\pi j t}\left(e^{j\pi t} - e^{0}\right)$$

$$= \frac{1}{2\pi t}\left(e^{-i\pi t} + e^{i\pi t}\right) - \frac{1}{2\pi t} - \frac{1}{2\pi t}$$

$$= \frac{2\cos \pi t}{2\pi t} - \frac{2}{2\pi t}$$

$$= \frac{\cos \pi t}{\pi t} - \frac{1}{\pi t}, \quad t \neq 0$$

$$f(0) = \frac{1}{2\pi} \int_{-\pi}^{0} (-j) d\omega + \frac{1}{2\pi} \int_{0}^{\pi} j\, d\omega$$

$$= \frac{-j}{2\pi} \omega\Big|_{-\pi}^{0} + \frac{j}{2\pi} \omega\Big|_{0}^{\pi}$$

$$= \frac{-j}{2\pi}(0+\pi) + \frac{j}{2\pi}(\pi - 0)$$

$f(0) = 0$

b) $G(j\omega) = ?$ in tablici: $x(t-t_0) \xleftrightarrow{CTFT} X(j\omega) e^{-j\omega t_0}$

$g(t) = f(t-5)$

$$G(j\omega) = F(j\omega) e^{-j\omega \cdot 5}$$

$$= j \cdot e^{-j\omega 5} \cdot \left[-\mu(\omega+\pi) + 2\mu(\omega) - \mu(\omega - \pi)\right]$$

3. c) Parsevalova relacija za CTFT

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$$

$$E = \int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} f(t) \cdot f^*(t) dt =$$

$$= \int_{-\infty}^{\infty} f(t) \cdot \left[\frac{1}{2\pi} \int_{-\infty}^{\infty} F^*(j\omega) e^{-j\omega t} d\omega\right] dt$$

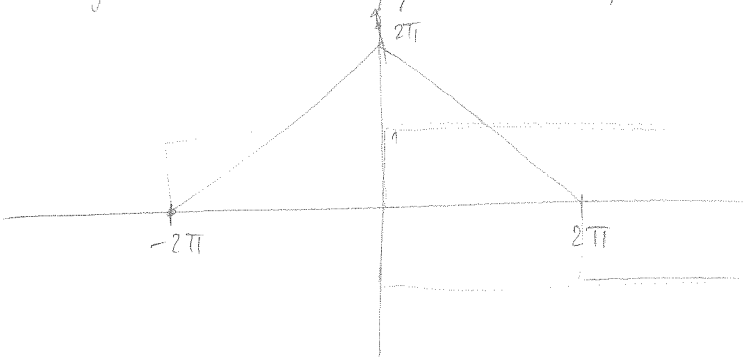
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} F^*(j\omega) \left[\int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt\right] d\omega$$

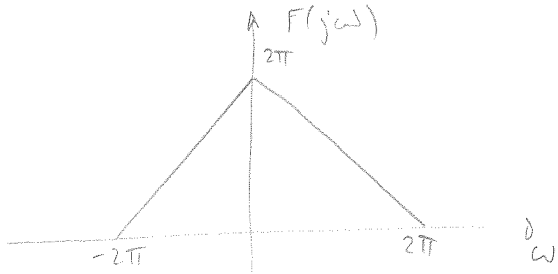
$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} F^*(j\omega) \cdot F(j\omega) \, d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$$

B

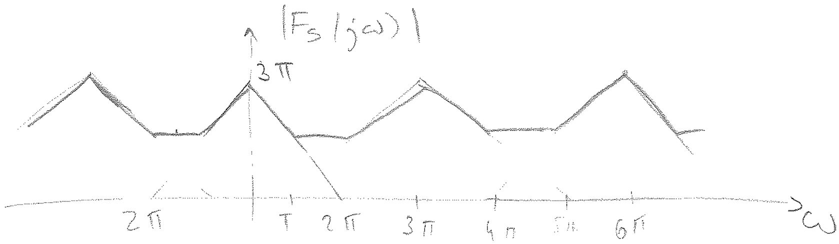
4. $F(j\omega) = (\omega + 2\pi)\left(\mu(\omega+2\pi) - \mu(\omega)\right) + (-\omega + 2\pi)\left(\mu(\omega) - \mu(\omega - 2\pi)\right)$





a) Maksimalna frekvencija u signalu je 2π.
Kako maksimalna frekvencija postoji (konačan broj) - ovaj signal se može jednoznačno očitati u vremenskoj domeni.
Minimalna frekvencija očitavanja je $2 \cdot \omega_{max} = 4\pi$

b) $\omega_s = 3\pi$



$T_s = \frac{2\pi}{\omega_s} = \frac{2\pi}{3\pi} = \frac{2}{3}$

$\omega_s < 2\pi$ dolazi do aliasinga

amplituda u $\omega = 0$:

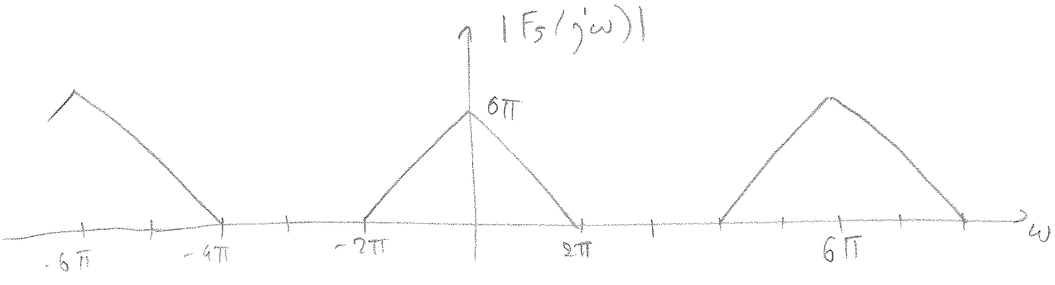
$2\pi \frac{1}{T_s} = 2\pi \cdot \frac{3}{2} = 3\pi$

c) $\omega_s = 6\pi$

$\omega_s > 2\pi$ neće doći do aliasinga

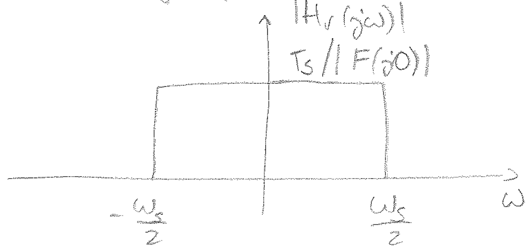
$T_s = \frac{2\pi}{6\pi} = \frac{1}{3}$

amplituda $\frac{2\pi}{T_s} = 2\pi \cdot 3 = 6\pi$



4 d) Rekonstrukcija kontinuiranog signala iz očitanog

- u frekvencijskoj domeni množenjem idealnim filtrom



$$X(j\omega) = X_s(j\omega) \cdot H_r(j\omega)$$

- u vremenskoj domeni je to konvolucija očitanog kontinuiranog signala s $ICTFT\{H_r(j\omega)\}$

$$h_r(t) = ICTFT\{H_r(j\omega)\} = \frac{1}{2\pi}\int_{-\omega_s/2}^{\omega_s/2} \frac{T_s}{|F(j0)|} e^{j\omega t} d\omega$$

$$= \frac{T_s}{2\pi |F(j0)|} \cdot \left.\frac{e^{j\omega t}}{jt}\right|_{-\omega_s/2}^{\omega_s/2}$$

$$= \frac{T_s}{2\pi |F(j0)| jt} \left(e^{j\frac{\omega_s}{2}t} - e^{-j\frac{\omega_s}{2}t}\right)$$

$$= \frac{T_s \cdot 2j \sin\frac{\omega_s}{2}t}{2j\pi t \cdot |F(j0)|}$$

$$= \frac{\cancel{2\pi} \cdot 2 \sin\frac{\omega_s}{2}t}{\cancel{2\pi}\, t \cdot |F(j0)| \cdot \omega_s}$$

$$= \frac{1}{|F(j0)|} \cdot \frac{\sin\frac{\omega_s}{2}t}{\frac{\omega_s}{2}t}$$

$$x(t) = x_s(t) * h_r(t)$$

5. $g(t) = f(at), \quad a > 0$

B

a) $E_f = E(f(t)) < \infty$

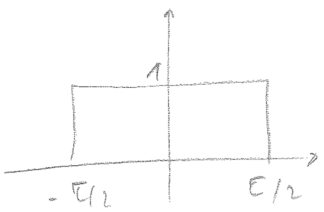
$$E_f = \int_{-\infty}^{\infty} |f(t)|^2 \, dt$$

$$E_g = \int_{-\infty}^{\infty} |g(t)|^2 dt = \int_{-\infty}^{\infty} |f(at)|^2 \, dt = \left| \begin{array}{ll} at = \tau & \\ a\,dt = d\tau & \\ t = -\infty & \tau = -\infty \\ t = \infty & \tau = \infty \end{array} \right|$$

$$= \int_{-\infty}^{\infty} \frac{1}{a} |f(\tau)|^2 d\tau = \frac{1}{a} \int_{-\infty}^{\infty} |f(\tau)|^2 d\tau$$

$$= \frac{1}{a} E_f$$

b) $h(t) = \dfrac{\sin 20t}{20t}$



$\xleftrightarrow{CTFT}$ $\tau \dfrac{\sin \frac{\tau}{2}\omega}{\frac{\tau}{2}\omega}$

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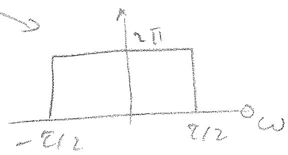
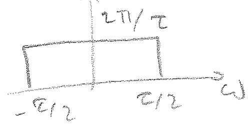
$\tau \dfrac{\sin \frac{\tau}{2} t}{\frac{\tau}{2} t}$ $\xleftrightarrow{CTFT}$



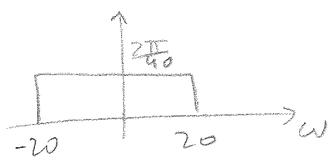
$\dfrac{\sin \frac{\tau}{2} t}{\frac{\tau}{2} t}$ $\xleftrightarrow{CTFT}$



$\frac{\tau}{2} = 20$

$\tau = 40$

$\dfrac{\sin 20t}{20t}$ $\xleftrightarrow{CTFT}$



$$E_h = \frac{1}{2\pi} \int_{-\infty}^{\infty} |H(j\omega)|^2 d\omega$$

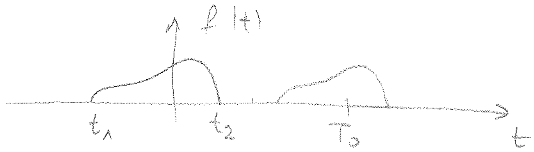
$$= \frac{1}{2\pi} \int_{-20}^{20} \left| \frac{\cancel{2}\pi}{\cancel{40}_{20}} \right|^2 d\omega$$

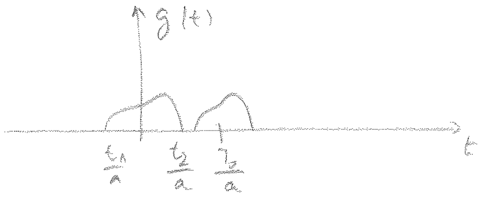
$$= \frac{1}{2\pi} \cdot \frac{\pi^2}{20 \cdot 20} \, \omega \Big|_{-20}^{20} = \frac{\pi}{2 \cdot 20 \cdot 20} (20 + 20) = \frac{2 \cdot \cancel{20} \cdot \pi}{2 \cdot \cancel{20} \cdot 20} = \frac{\pi}{20}$$

ili $E_h = \int_{-\infty}^{\infty} \left(\frac{\sin 20t}{20t}\right)^2 dt = \left| \begin{array}{l} 20t = \tau \\ 20 dt = d\tau \end{array} \right| = \int_{-\infty}^{\infty} \left(\frac{\sin \tau}{\tau}\right)^2 \frac{1}{20} d\tau = \frac{1}{20} \cdot 2 \cdot \underbrace{\int_0^{\infty} \left(\frac{\sin \tau}{\tau}\right)^2 d\tau}_{\text{TABLICA } \frac{\pi}{2}} = \frac{1}{20} \cdot 2 \cdot \frac{\pi}{2} = \frac{\pi}{20}$

c) $f(t)$ periodičan sa T_0

$g(t) = f(at)$





$$P_f = \frac{1}{T_0} \int_{T_0} |f(t)|^2 dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |f(t)|^2 dt$$

$$P_g = \frac{1}{\frac{T_0}{a}} \int_{T_0/a} |f(at)|^2 dt$$

$$= \frac{a}{T_0} \int_{-T_0/2a}^{T_0/2a} |f(at)|^2 dt = \left| \begin{array}{ll} at = \tau & t = -\frac{T_0}{2a} \rightarrow \tau = -\frac{T_0}{2a} \cdot a = -\frac{T_0}{2} \\ a\,dt = d\tau & \\ dt = \frac{1}{a} d\tau & t = \frac{T_0}{2a} \rightarrow \tau = \frac{T_0}{2a} \cdot a = \frac{T_0}{2} \end{array} \right|$$

$$= \frac{a}{T_0} \int_{-T_0/2}^{T_0/2} |f(\tau)|^2 \frac{1}{a} d\tau = \frac{a}{T_0} \cdot \frac{1}{a} \int_{-T_0/2}^{T_0/2} |f(\tau)|^2 d\tau$$

$$= \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |f(\tau)|^2 d\tau$$

$$= P_f$$