LJETNI Rok 02.7.2014

1) $x(t) = e^{2t} \cdot \mu(-t) + e^{-3t} \mu(t)$

a) CTFT

b) ENERGIJA

c) Parseval za CTFT

a)

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j\omega t} dt = \int_{-\infty}^{\infty} e^{2t} \cdot e^{-j\omega t} \mu(-t) dt + \int_{-\infty}^{\infty} e^{-3t} \cdot e^{-j\omega t} \cdot \mu(t) dt =$$

$$X(j\omega) = \int_{-\infty}^{0} e^{2t} \cdot e^{-j\omega t} dt + \int_{0}^{\infty} e^{-3t} \cdot e^{-j\omega t} dt = \int_{-\infty}^{0} e^{t(2-j\omega)} dt + \int_{0}^{\infty} e^{-t(3+j\omega t)}$$

$$X(j\omega) = \frac{1}{2-j\omega} \cdot e^{t(2-j\omega)} \Big|_{-\infty}^{0} + \frac{1}{-1(3+j\omega)} \cdot e^{-t(3+j\omega t)} \Big|_{0}^{\infty} =$$

$$= \frac{1}{2-j\omega} + \frac{1}{3+j\omega} = \frac{5}{(2-j\omega)(3+j\omega)} = \frac{5}{\omega^2 - j\omega + 6}$$

b) $$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_{-\infty}^{0} e^{4t} dt + \int_{0}^{\infty} (e^{-3t})^2 dt = \int_{-\infty}^{0} e^{4t} dt + \int_{0}^{\infty} e^{-6t} dt =$$

$$= \frac{1}{4} \cdot e^{4t} \Big|_{-\infty}^{0} + \frac{1}{-6} \cdot e^{-6t} \Big|_{0}^{\infty} =$$

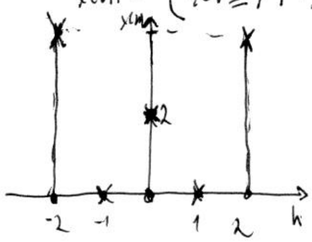
$$= \frac{1}{4}(e^0 - 0) + \frac{1}{-6}(0 - 1) = \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$$

c) $$E_f = \int_{-\infty}^{\infty} |f(t)|^2 dt = \int_{-\infty}^{\infty} f(t) f^*(t) dt = \int_{-\infty}^{\infty} f(t) \cdot \left[\frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} F^*(j\omega) e^{-j\omega t} d\omega\right] dt =$$

$$= \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} F^*(j\omega) \underbrace{\left(\int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt\right)}_{} d\omega = \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} F^*(j\omega) \cdot F(j\omega) d\omega$$

$$= \frac{1}{2\pi} \cdot \int_{-\infty}^{\infty} |F(j\omega)|^2 d\omega$$

a) $x(n) = \{4, 0, \underline{2}, 0, 4\}$



$$X(e^{j\omega}) = \sum_{-\infty}^{+\infty} x(n) \cdot e^{-j\omega n}$$

$$X(e^{j\omega}) = x(-2) \cdot e^{-j\omega(-2)} + x(0)\, e^{0} + x(2) \cdot e^{-j\omega 2} =$$

$$= 4 \cdot e^{2j\omega} + 2 + 4 \cdot e^{-j\omega 2} =$$

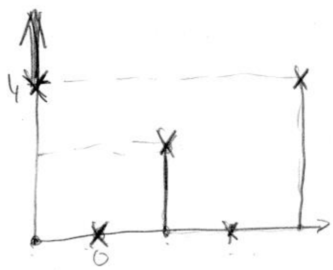
$$= 4 \cdot \frac{(e^{2j\omega} \cdot e^{-2j\omega})}{2} + 2$$

$$= 8 \cdot \cos(2\omega) + 2$$

b) amplitudni spektar $= \sqrt{8 \cdot \cos 2\omega + 2} = \underline{8 \cdot \cos 2\omega + 2}$

fazni spektar $\frac{Im}{Re}$ $\quad \operatorname{arctg} \varphi = \frac{Im}{Re} = 0,$

c) DTFT $= \{\underline{4}, 0, 2, 0, 4\}$

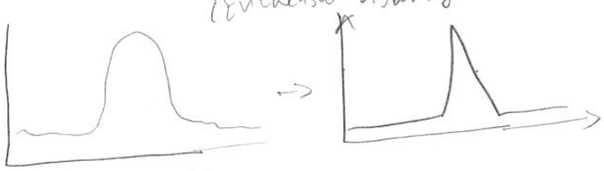


$$X(e^{j\omega}) = \sum_{-\infty}^{+\infty} x(n) \cdot e^{-j\omega n} =$$

$$= 4 \cdot e^{0} + 0 + 2 \cdot e^{-j\omega 2} + 0 + 4 \cdot e^{-j\omega 4} =$$

$$= 4 + 2 \cdot e^{-2j\omega} + 4 \cdot e^{-j\omega 4}$$

d) Skicirati vremenski kontinuirani signal koji je nastao interpolacijom signala $x(n)$ interpolatorom 0. reda.
(vremenski diskretnog)



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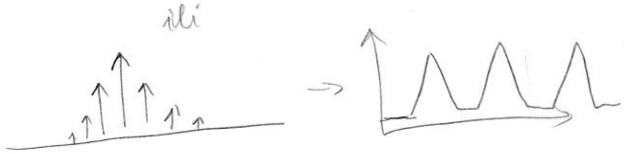

③

$$\xrightarrow{u(n)} \boxed{S_1} \xrightarrow{w(n)} \boxed{S_2} \xrightarrow{y(n)}$$

~~$w(n) + \frac{1}{2} w(n-1)$~~ $\quad w(n) + \frac{1}{2} w(n-1) = 2 \cdot u(n) \quad \ldots S_1$

$y(n) - \frac{1}{5} y(n-1) = 4\, w(n) \quad \ldots S_2$

a) Impulsni odziv sustava S_1 postupkom u vremenskoj domeni

$w(n) + \frac{1}{2} w(n-1) = 2 \cdot u(n)$ $\qquad y_H = C \cdot \left(-\frac{1}{2}\right)^n$

$q^n + \frac{1}{2} q^{n-1} = 0 \,/ \cdot q$ $\qquad w(n) = 2u(n) - \frac{1}{2} w(n-1)$

$w(0) = 2 \cdot \underbrace{u(0)}_{1} - \frac{1}{2} \underbrace{w(0-1)}_{0}$

$q + \frac{1}{2} = 0$

$\boxed{q = -\frac{1}{2}}$ $\qquad w(0) = 2$

$y_H(n) = C \cdot \left(-\frac{1}{2}\right)^n$

$y_H(0) = C \cdot \left(-\frac{1}{2}\right)^0 = C = w(0) = 2$

$w(n) = C \cdot \left(-\frac{1}{2}\right)^n = 2 \cdot \left(-\frac{1}{2}\right)^n$ //

b) impulsni odziv sustava S_2 u z-transf

$y(n) - \frac{1}{5} y(n-1) = 4 \cdot w(n) \,/ \mathcal{Z}$ $\qquad$ impulsni pa je 1

$Y(z) - \frac{1}{5}\left(Y(z) \cdot z^{-1} - \overset{0}{y(-1)}\right) = 4 \cdot W(z)$

$Y(z) - \frac{1}{5} \frac{Y(z)}{z} = 4$

$Y(z)\left(1 - \frac{1}{5z}\right) = 4$

$Y(z) = 4 \cdot \frac{5z}{5z-1} = 4 \frac{1}{\cancel{5}} \frac{\cancel{5}z}{\left(z - \frac{1}{5}\right)} = 4 \cdot \frac{z}{z - \frac{1}{5}}$

$y(n) = 4 \cdot \left(\frac{1}{5}\right)^n$

c) ~~impulsni odziv cijelog sustava, kaskadnog spoja.~~

Kaskada je def kao $S = S_1 \circ S_2 = S_1(S_2(n)) = \cancel{W(y(n))} =$

9) odziv cijelog je umnožak u z-domeni!

$$H_1(z) = 2 \cdot \frac{z}{z+\frac{1}{2}}$$

$$H_2(z) = 4 \cdot \frac{z}{z-\frac{1}{5}}$$

IMPULSNI!

$$8 \cdot \frac{z^2}{\left(z-\frac{1}{5}\right)\left(z+\frac{1}{2}\right)} = \frac{8z^{\cancel{2}}}{\left(z-\frac{1}{5}\right)\left(z+\frac{1}{2}\right)} \cdot \frac{1}{\cancel{z}}$$

RASTAV NA PARC

$$= \frac{A}{\left(z-\frac{1}{5}\right)} + \frac{B}{\left(z+\frac{1}{2}\right)} = \frac{Az+\frac{1}{2}A+Bz-\frac{1}{5}B}{\left(z-\frac{1}{5}\right)\left(z+\frac{1}{2}\right)} =$$

$$= \frac{z(A+B)+\frac{1}{2}A-\frac{1}{5}B}{\left(z-\frac{1}{5}\right)\left(z+\frac{1}{2}\right)}$$

$A+B=8 \qquad B=8-A$

$\frac{1}{2}A-\frac{1}{5}B=0$

$\frac{1}{2}A-\frac{1}{5}(8-A)=0$

$\frac{1}{2}A-\frac{8}{5}+\frac{1}{5}A=0$

$\left(\frac{1}{2}+\frac{1}{5}\right)A=\frac{8}{5} \qquad B=\frac{40}{7}$

$A=\frac{16}{7}$

$$\frac{H_{uk}(z)}{z} = \frac{16/7}{\left(z-\frac{1}{5}\right)} + \frac{40/7}{z+\frac{1}{2}}$$

$$\boxed{y_n = \left(\frac{16}{7}\cdot\left(\frac{1}{5}\right)^n + \frac{40}{7}\cdot\left(-\frac{1}{2}\right)^n\right)\mu(n)}$$

$y(0^-) = 3$

$y'(0^-) = -40$

$u(t) = 3e^{-5t}$

c) prisilni odziv sustava, prirodni odziv, e) mirni odziv sustava

d) odziv nepobuđenog sustava.

c) prisilni odziv

$$\lambda^2 + 11\lambda + 30 = 0$$

$\lambda_1 = -5$ pobuda je $u(t) = 3 \cdot e^{-5t}$.

$\lambda_2 = -6$ dakle partikularno treba biti oblika

$t \cdot k \cdot e^{-5t}$ //

$$y_H = C_1 \cdot e^{-5t} + C_2 \cdot e^{-6t}$$

$$y_P = t \cdot k \cdot e^{-5t}$$

$$y_P' = k \cdot e^{-5t} - 5tk \cdot e^{-5t}$$

$$y_P'' = -5 \cdot k \cdot e^{-5t} + 25 t \cdot k \cdot e^{-5t} - 5 \cdot k \cdot e^{-5t} = -10 k \cdot e^{-5t} + 25 t e^{-5t} \cdot k$$

Uvstimo u početnu.

$$-10 \cdot k \cdot \cancel{e^{-5t}} + 25kt \cdot \cancel{e^{-5t}} + 11\left(k \cdot \cancel{e^{-5t}} - 5 \cdot t \cdot k \cdot \cancel{e^{-5t}}\right) + 30 \cdot t \cdot k \cdot \cancel{e^{-5t}} =$$

$$= 2 \cdot \left(-15 \cdot \cancel{e^{-5t}}\right) + 6 \cdot 3 \cdot \cancel{e^{-5t}} \, / : e^{-5t}$$

$$-10k + 25kt + 11k - 55kt + 30kt = -30 + 18$$

$$k(-10 + 11) + kt(25 - 55 + 30) = -30 + 18$$

$$\boxed{k = -12}$$

$$y(0^+) - y(0^-) = b_0 \cdot (0^+)$$

$$\boxed{y(0^+) = 3}$$

$$y_P = -12t \cdot e^{-5t}$$

$$y'(0^+) - y'(0^-) + a_1 y(0^+) - a_1 \cdot y(0^-) = b_0 u'(0^+) + b_1 \cdot u(0^+)$$

$$y_{TOT} = y_H + y_P = C_1 \cdot e^{-5t} + C_2 \cdot e^{-6t} - 12t e^{-5t}$$

$$y_{TOT}' = -5C_1 e^{-5t} - 6C_2 e^{-6t} - 12 e^{-5t} + 60 e^{-5t} \cdot t$$

$$y'(0^+) + 40 + \cancel{11 \cdot 3} - \cancel{11 \cdot 3} = 0 + 6$$

$$y'(0^+) = 6 - 40 = -34$$

$$y_{TOT}(0) = C_1 + C_2 = 3$$

$$-5C_1 - 6C_2 - 12 = -34$$

$$\boxed{y_{VOT} = -4\cdot e^{-3t} + 7\cdot e^{-6t} - 12t\cdot e^{-5t}}\mu(t)$$ ← ~~odziv~~ prisilnog

b) prirodni odziv je $y_{pr} = (-4\cdot e^{-3t} + 7\cdot e^{-6t})\mu(t)$ } Totalni

prisilni odziv je $y_{pris} = (-12t e^{-5t})\mu(t)$

c) Mirni odziv → početni uvjeti 0

$y(0^-) = y(0^+) = 0$

$y'(0^+) = ? = 6$

$y'(0^+) - y'(0^-) + a_1\cdot y(0^+) - a_1\cdot y(0^-) = b_0\,u'(0^+)$ ...

$y'(0^+) - 0 + 0 - 0 = 0 + 3\cdot 2$

$$\boxed{y'(0^+) = 3\cdot 2 = 6}$$

$y_{mirno} = C_1\cdot e^{-5t} + C_2\cdot e^{-6t} - 12t\cdot e^{-5t}$

$y'_{mirno} = -5C_1 e^{-5t} + (C_2 e^{-6t} - 12e^{-5t} + 60t\cdot e^{-5t}$

$y_{mir}(0) = C_1 + C_2 = 0$ } $C_1 = 18$

$y'_{mir}(0) = -5C_1 - 6C_2 - 12 = 6$ $C_2 = -18$

$y_{mirno} = 18\cdot e^{-5t} - 18\cdot e^{-6t} - 12t\cdot e^{-5t}\mu(t)$

d) Nepobuđeni ??

$y(0^-) = y(0^+) = 3$

$y'(0^+) = y'(0^-) = -40$

$y_{nep} = C_1\cdot e^{-5t} + C_2\cdot e^{-6t}$ //

$y'_{nep} = -5C_1 e^{-5t} + 6C_2 e^{-6t}$

$C_1 + C_2 = 3$

$-5C_1 - 6C_2 = -40$

$C_1 = -22$

$C_2 = 25$

$$\boxed{y_{nep} = (-22\cdot e^{-5t} + 25\cdot e^{-6t})\mu(t)}$$

$$y(n) + a_1 \cdot y(n-1) + a_2 \cdot y(n-2) = b_0 \cdot u(n)$$

$u_1(n) = 1^n \to$ pobuda

$y_1(n) = 1^n \to$ odziv

$u_2(n) = \left(\frac{1}{2}\right)^n$

$y_2(n) = 2 \cdot \left(\frac{1}{2}\right)^n$

$u_3(n) = \left(\frac{1}{4}\right)^n$

$y_3(n) = 4 \cdot \left(\frac{1}{4}\right)^n$

a) parametri $\{a_1, a_2, b_0\}$

1) $1^n + a_1 \cdot 1^{n-1} + a_2 \cdot 1^{n-2} = b_0 \cdot 1^n \;/ : 1^n$

$$1 + a_1 \cdot 1^{-1} + a_2 \cdot 1^{-2} = b_0$$

$$1 + a_1 + a_2 = b_0$$

2) $2 \cdot \cancel{\left(\frac{1}{2}\right)^n} + a_1 \cdot 2 \cdot \left(\frac{1}{2}\right)^{n-1} + a_2 \cdot 2 \cdot \left(\frac{1}{2}\right)^{n-2} = b_0 \cdot \cancel{\left(\frac{1}{2}\right)^n}$

$$2 + a_1 \cdot 2 \cdot 2 + a_2 \cdot 2 \cdot 4 = b_0$$

$$2 + 4a_1 + 8a_2 = b_0$$

3) $4 \cdot \cancel{\left(\frac{1}{4}\right)^n} + a_1 \cdot \left(\frac{1}{4}\right)^{n-1} \cdot 4 + 4a_2 \left(\frac{1}{4}\right)^{n-2} = b_0 \cdot \cancel{\left(\frac{1}{4}\right)^n}$

$$4 + a_1 \cdot 4 \cdot 4 + 4 \cdot 16 a_2 = b_0$$

(1) $a_1 + a_2 - b_0 = -1$
(2) $4a_1 + 8a_2 - b_0 = -2$
(3) $16a_1 + 64a_2 - b_0 = -4$

sustav

$\Downarrow$

riješi se

$$\boxed{a_1 = -\frac{1}{2} \quad a_2 = \frac{1}{14} \quad b_0 = \frac{4}{7}}$$

$$y(n) - \frac{1}{2} y(n-1) + \frac{1}{14} y(n-2) = \frac{4}{7} u(n)$$

b) trebalo bi odrediti odziv na pobudu $u(n) = \left(\frac{1}{8}\right)^n$.
Znači ne bi bilo loše sve riješiti u z domeni. L(1) → poč (0)!

$$Y(z) + a_1 \cdot Y(z) \cdot z^{-1} + a_2 \cdot Y(z)\, z^{-2} = b_0 \cdot U(z)$$

$$Y(z)\left(1 + \frac{a_1}{z} + \frac{a_2}{z^2}\right) = b_0 \cdot U(z)$$

$$\frac{Y(z)}{U(z)} = H(z) = \frac{b_0}{z^2 + a_1 z + a_2} = \frac{z^2 \cdot b_0}{z^2 + a_1 z + a_2}$$

Prethodno ide brže s prijenosom!

$$n) = \left(\frac{1}{8}\right)^n \circ\!\!-\!\!\bullet \frac{z}{z-\frac{1}{8}}$$

$$y(n) = H\left(\frac{1}{8}\right) \cdot \left(\frac{1}{8}\right)^n \qquad H(z) = \frac{z^2 \cdot \frac{4}{7}}{z^2 + \frac{1}{2}z + \frac{1}{14}} \Rightarrow$$

$$H\left(\frac{1}{8}\right) = \frac{\frac{4}{7} \cdot \left(\frac{1}{8}\right)^2}{\left(\frac{1}{8}\right)^2 + \frac{1}{2} \cdot \frac{1}{8} + \frac{1}{14}} = \frac{4}{11} \qquad H\left(\frac{1}{8}\right) = \frac{4}{11}$$

$$y(n) = \frac{4}{11} \cdot \left(\frac{1}{8}\right)^n \, ?$$

rischi ? $u(n) = \left(\frac{1}{8}\right)^n \cdot \mu(n)$

$$y_p = C \cdot \left(\frac{1}{8}\right)^n$$

$$y_p(n-1) = C \cdot \left(\frac{1}{8}\right)^{n-1} = C \cdot 8 \cdot \left(\frac{1}{8}\right)^n$$

$$y_p(n-2) = 64\, C \left(\frac{1}{8}\right)^n$$

$$\left(\frac{1}{8}\right)^n - 4\, C \left(\frac{1}{8}\right)^n + \frac{32}{7} C \left(\frac{1}{8}\right)^n = \frac{4}{7} \cdot \left(\frac{1}{8}\right)^n$$

$$\frac{11}{7} C = \frac{4}{7} \rightarrow C = \frac{4}{11}$$

$$y_{\text{prisicho}} = \frac{4}{11} \cdot \left(\frac{1}{8}\right)^n$$