Zadaci za vježbu – tjedan 17

1. Zadan je kontinuiran sustav $y''(t)–5y'(t)+6y(t) = u(t)+3u'(t)$, y(0-)=3, $y'(0-)=0$. Pronađite odziv sustava na pobudu $u(t)=2\mu(t)$.
   a. U vremenskoj domeni (homogeno + partikularno)
   b. Pomoću Laplaceove transformacije

2. Zadan je diskretan linearan sustav: $y(n) + 2y(n-2) = u(n)$

   Pronađite:

   a. Opis sustava u prostoru stanja (varijable stanja i matrice *A, B, C, D* paralelne realizacije). Jesu li matrice *A*, *B*, *C*, *D* jednoznačno određene? Objasnite!

   b. Impulsni odziv sustava.

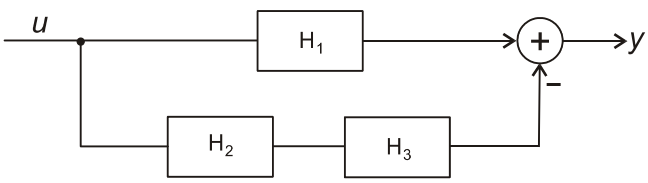
   c. Polove sustava iz matrice *A*. Je li definirana frekvencijska karakteristika sustava?

3. Kontinuiran sustav zadan je prijenosnom funkcijom:

$$H(s) = \frac{s^2+3s+4}{(s+2)(s^2+2s+1)}$$

   Napišite jednadžbe stanja i izlaznu jednadžbu zadanog sustava koristeći kaskadnu realizaciju. Opišite matricu *A* koju ste dobili kaskadnom realizacijom. Je li dobivena realizacija jedinstvena?

4. Složeni mirni diskretni sustav zadan je slikom:



   Koliki je impulsni odziv drugog podsustava $h_2(n)$ ako je impulsni odziv prvog podsustava $h_1(n) = \{\underline{0}, 0, 1, 3, 3, 3, 3, \ldots\}$, impulsni odziv trećeg podsustava $h_3(n) = \{\underline{0}, 0, 1, 2, 0, 1, 2, 0, \ldots\}$, te prijenosna funkcija sustava $H(z) = 0$?

5. Kontinuirani sustav zadan je jednadžbama:

$$y'_1+3y_2=u_1$$
$$3y_1+y'_2=u_2$$

   Naći matrice *A*, *B*, *C*, *D*, prijenosnu funkciju, impulsni odziv i odziv na pobudu $u(t)=\begin{bmatrix} 3S(t) \\ \delta(t) \end{bmatrix}$.

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TJEDAN 15

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1. $y''(t) - 5y'(t) + 6y(t) = u(t) + 3u'(t)$

$y(0^-) = 3$

$y'(0^-) = 0$

$u(t) = 2\mu(t)$

a) $y(t) = y_h(t) + y_p(t) = ?$

b) $y(t) = \mathcal{L}^{-1}\{Y(s)\} = ?$

a) $y_p(t) = k$

$6k = 2$

$k = \frac{1}{3}$

$y_p(t) = \frac{1}{3}$

$y_h(t) = Ce^{st}$ pretpostavljeni oblik

$s^2Ce^{st} - 5s^2Ce^{st} + 6Ce^{st} = 0$

$Ce^{st}(s^2 - 5s^2 + 6) = 0$

$s_{1,2} = \frac{5 \pm \sqrt{25-24}}{2} = \frac{5}{2} \pm \frac{1}{2}$

$s_1 = 3 \quad , s_2 = 2$

$y_h(t) = C_1e^{3t} + C_2e^{2t}$

$y(t) = \frac{1}{3} + C_1e^{3t} + C_2e^{2t}$

početni uvjeti: $y(0^+) - y(0^-) = b_0u(0^+)$

$y(0^+) = y(0^-) = 3$

$y'(0^+) - y'(0^-) + a_1\underbrace{(y(0^+) - y(0^-))}_{=0} = b_0u'(0^+) + b_1u(0^+)$

$y'(0^+) = 6$

$y'(t) = 3C_1e^{3t} + 2C_2e^{2t}$

$y'(0^+) = 3C_1 + 2C_2 = 6$

$y(t) = C_1e^{3t} + C_2e^{2t} + \frac{1}{3}$

$y(0^+) = C_1 + C_2 + \frac{1}{3} = 3$

$3C_1 + 2C_2 = 6$

$C_1 + C_2 = \frac{8}{3} \Rightarrow C_1 = \frac{8}{3} - C_2 = \frac{2}{3}$

$8 - 3C_2 + 2C_2 = 6$

$C_2 = 2$

$\boxed{y(t) = \frac{1}{3} + \frac{2}{3}e^{3t} + 2e^{2t}}$ za $t \geq 0$

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1. b) $x'' = s^2X(s) - sx(0^-) - x'(0^-)$
$x' = sX(s) - x(0^-)$

prva i druga derivacija u vremenu signala $x(t)$ iz tablica Laplaceove transformacije

$s^2Y(s) - 3s - 0 - 5(sY(s) - 3) + 6Y(s) = \frac{2}{s} + 6$

$s^2Y(s) - 5sY(s) + 6Y(s) = \frac{2}{s} + 6 + 3s - 15$

$Y(s)(s^2 - 5s + 6) = 3s^2 - 9s + 2$

$$Y(s) = \frac{3s^2 - 9s + 2}{s(s^2 - 5s + 6)} = \frac{A}{s} + \frac{B}{s-2} + \frac{C}{s-3}$$

$A(s-2)(s-3) + Bs(s-3) + Cs(s-2) = As^2 - 5As + 6A + Bs^2 - 3Bs + Cs^2 - 2Cs$

$s^2(A+B+C) + s(-5A - 3B - 2C) + 6A = 3s^2 - 9s + 2$

$A + B + C = 3$

$-5A - 3B - 2C = -9$

$6A = 2 \Rightarrow A = \frac{1}{3}$

$B + C = \frac{8}{3} \quad \Rightarrow B = \frac{8}{3} - C = \frac{6}{3} = 2$

$3B + 2C = 9 - \frac{5}{3}$

$8 - 3C + 2C = \frac{22}{3}$

$-C = -\frac{2}{3}$

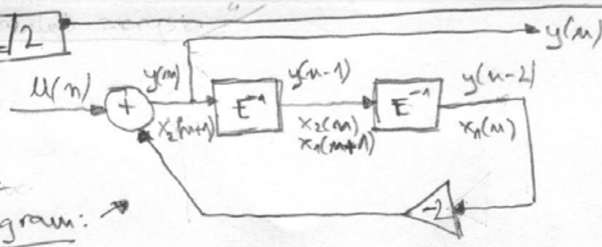
$C = \frac{2}{3}$

$A = \frac{1}{3} \quad B = 2 \quad C = \frac{2}{3}$

$$\boxed{\begin{aligned} Y(s) &= \frac{1}{3s} + \frac{2}{s-2} + \frac{2}{3} \cdot \frac{1}{s-3} \\ y(t) &= \left[\frac{1}{3} + 2e^{2t} + \frac{2}{3}e^{3t}\right]\mu(t) \end{aligned}}$$

$\boxed{1/2}$

2.



blok dijagram:

SEKCIJA DRUGOG REDA (PARALELNI)!

uz uvjet: $\left(a_{11}=0,\ b_{11}=0,\ b_{01}=1 \right)$

$$y(n) + 2y(n-2) = u(n)$$

$$y(n) = -2y(n-2) + u(n) \quad \Big\} \mathcal{Z}$$

$$Y(z) = -2z^{-2}Y(z) + U(z)$$

• realizacija sekcijom drugog reda ($b_{12}=0$, $a_{12}=0$)

varyable stanja?

$$x(n) = \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix}$$

SISO sustav: $x(n+1) = A\,x(n) + B\,u(n)$
$\qquad\qquad y(n) = C\,x(n) + D\,u(n)$

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a)

$$\left.\begin{aligned} y(n) &= x_2(n+1) \\ y(n-1) &= x_2(n) = x_1(n+1) \\ y(n-2) &= x_1(n) \end{aligned}\right\}$$

$$y(n) = -2x_1(n) + 0 \cdot x_2(n) + u(n)$$

$$\boxed{D = 1}$$

$$\boxed{C = [-2\ \ 0] = \begin{bmatrix} -2 \\ 0 \end{bmatrix}^T}$$

$$y(n) = [-2\ \ 0]\begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + 1 \cdot u(n)$$

$$y(n) = C\,x(n) + D \cdot u(n)$$

$$x_2(n+1) = y(n)$$
$$x_1(n+1) = x_2(n)$$

$\to$
$$\left[\begin{aligned} x_2(n+1) &= -2x_1(n) + 0 \cdot x_2(n) + u(n) \\ x_1(n+1) &= 0\,x_1(n) + x_2(n) + 0 \cdot u(n) \end{aligned}\right.$$

$$\begin{bmatrix} x_1(n+1) \\ x_2(n+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(n)$$

$$\boxed{A = \begin{bmatrix} 0 & 1 \\ -2 & 0 \end{bmatrix}} \quad \boxed{B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}}$$

$$x(n+1) = A\,x(n) + B\,u(n)$$

• nije jednoznačno – zamjenimo mjesta $x_1(n)$, i $x_2(n)$ → nove matrice A, B, C, D

b)

"impulsni odziv"

$$H(z) = C[zI - A]^{-1}B + D =$$

$$zI - A = \begin{bmatrix} z & -1 \\ 2 & z \end{bmatrix}; \quad [zI - A]^{-1} = \frac{1}{z^2+2}\begin{bmatrix} z & -2 \\ 1 & z \end{bmatrix}^T = \begin{bmatrix} \frac{z}{z^2+2} & \frac{1}{z^2+2} \\ \frac{-2}{z^2+2} & \frac{z}{z^2+2} \end{bmatrix}$$

$$H(z) = C[zI - A]^{-1}B + D = \begin{bmatrix} \frac{-2z}{z^2+2} & \frac{-2}{z^2+2} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1 = 1 - \frac{2}{z^2+2} = \frac{z^2}{z^2+2} = \frac{1}{1+2z^{-2}}$$

$$\to \boxed{h(n) = \sqrt{2}^{\,n}\cos\left(\frac{\pi}{2}n\right)\mu(n)}$$

(NESTABILAN)

c) frekvencijska karakt. [isto kao i u dijelu $\boxed{1/2}$] (iz prijenosne funkc. H(s))

$$zI - A = \begin{bmatrix} z & -1 \\ 2 & z \end{bmatrix}; \quad \underbrace{\det(zI - A)}_{\text{vadimo korijene}} = z^2 + 2 \Rightarrow \boxed{z_{1,2} = \pm j\sqrt{2}} \text{ polovi}$$

Pošto je sustav nestabilan frekvencijska karakteristika nije definirana

(3) kont. sustav

kaskada

$$H(s) = \frac{s^2+3s+4}{(s+2)(s^2+2s+1)} = \boxed{\text{kaskadna realizacija}} = \left(\frac{1}{s+2}\right)\cdot\left(\frac{s^2+3s+4}{s^2+2s+1}\right) = \overbrace{H_1(s)\cdot H_2(s)}$$

"prijenosna f."

sekcija prvog + sekcija drugog reda !

SKLOP ZA INTEGRIRANJE

$\boxed{\int} = \boxed{\frac{1}{s}}$

kaskada

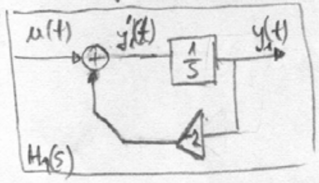
$$\frac{1}{s+2} = \frac{Y_1(s)}{U(s)}$$

$\rightarrow sY_1(s) + 2Y_1(s) = U(s)$

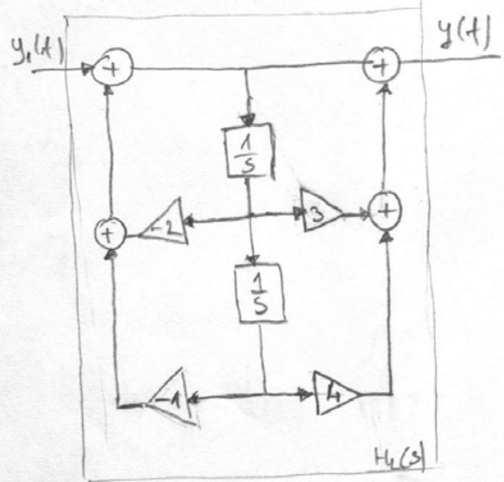
$\rightarrow y_1'(t) + 2y_1(t) = u(t)$

$y_1'(t) = -2y(t) + u(t)$

"direktna real." (uvjet: $b_0 = 0$)

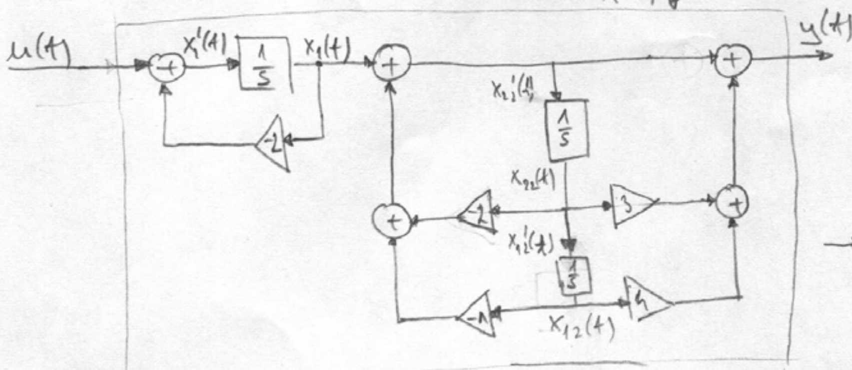


$$\frac{s^2+3s+4}{s^2+2s+1}$$ (s predavanja) "direktna real."



$H(s) = H_1(s)\cdot H_2(s)$

H(s)



- $x_1'(t) = -2x_1(t) + u(t)$
- $x_{12}'(t) = x_{22}(t)$
- $x_{22}'(t) = x_1(t) + x_{12}(t) - 2x_{22}(t)$

$y(t) = x_1(t) - 2x_{22}(t) - x_{12}(t) + 3x_{22}(t) + 4x_{12}(t)$

- $y(t) = x_1(t) + x_{22}(t) + 3x_{12}(t)$

$$\begin{bmatrix} x_1'(t) \\ x_{12}'(t) \\ x_{22}'(t) \end{bmatrix} = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_{12}(t) \\ x_{22}(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$\downarrow$ A $\qquad$ $\downarrow$ B

$$y(t) = \begin{bmatrix} 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_{12}(t) \\ x_{22}(t) \end{bmatrix} + 0\cdot u(t)$$

$\downarrow$ C $\qquad$ $\downarrow$ D

* – dobivena realizacija nije jedinstvena jer smo sustav mogli realizirati preko drugih prijenosnih fja. (sekcija prvog reda)

npr. $\left(\frac{1}{s-2}\right)\cdot\left(\frac{1}{s+1}\right)\left(\frac{s^2+3s+4}{s+1}\right) = H_1\cdot H_2\cdot H_3$

pa bi matrice A,B,C,D bile drugačije

– također odabir var. stanja može biti drugačiji

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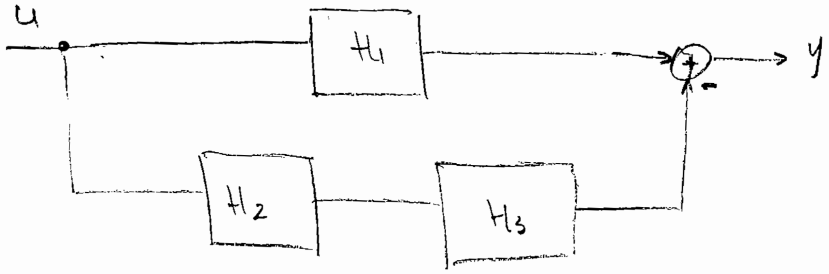
4. $h_1(n)=\{\underline{0},0,1,3,3,\dots\}$

$h_3(n)=\{\underline{0},0,1,2,0,1,2,0\dots\}$

$H(z)=0$

$h_2(n)=?$



$$H_1(z)=\frac{1}{z^2}+3\left(\frac{1}{z^3}+\frac{1}{z^4}+\dots\right)=\frac{1}{z^2}+3\sum_{i=3}^{\infty}\frac{1}{z^i}=\sum_{i=2}^{\infty}\frac{1}{z^i}+2\sum_{i=3}^{\infty}\frac{1}{z^i}=$$

$$=\frac{1}{z^2}\frac{1}{1-\frac{1}{z}}+\frac{2}{z^3}\frac{1}{1-\frac{1}{z}}=\frac{z+2}{z^2(z-1)}$$

$$H_3(z)=\frac{1}{z^2}+\frac{2}{z^3}+\frac{1}{z^5}+\frac{2}{z^6}+\dots=\frac{1}{z^2}\sum_{i=0}^{\infty}\frac{1}{z^{3i}}+2\sum_{i=1}^{\infty}\frac{1}{z^{3i}}=$$

$$=\frac{1}{z^2}\frac{1}{1-\frac{1}{z^3}}+\frac{2}{z^3}\cdot\frac{1}{1-\frac{1}{z^3}}=\frac{z+2}{z^3-1}=\frac{z+2}{(z-1)(z^2+z+1)}$$

$$H(z)=H_1(z)-H_2(z)H_3(z)=0$$

$$H_2(z)=\frac{H_1(z)}{H_3(z)}=\frac{\cancel{(z-1)}(z^2+z+1)}{z^2\cancel{(z-1)}}=\frac{z^2+z+1}{z^2}=1+\frac{1}{z}+\frac{1}{z^2}$$

$$h_2(n)=\delta(n)+\delta(n-1)+\delta(n-2)=\{\underline{1},1,1,0\dots\}$$

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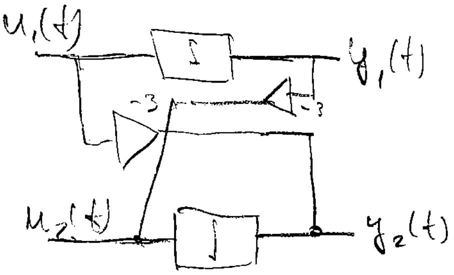
5.

$$y_1' + 3y_2 = u_1$$
$$3y_1 + y_2' = u_2$$
$$u(t) = \begin{bmatrix} 3S(t) \\ \delta(t) \end{bmatrix}$$

$A, B, C, D = ?$
$H(s), h(t) = ?$
$y(t) = ?$

$$X' = AX(t) + Bu(t)$$
$$y = CX(t) + Du(t)$$

$$\left.\begin{aligned} x_1' &= -3x_2 + u_1 \\ x_2' &= -3x_1 + u_2 \end{aligned}\right\} \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{B} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$



$$\left.\begin{aligned} y_1 &= x_1 \\ y_2 &= x_2 \end{aligned}\right\} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{C} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \underset{D}{\underset{\downarrow}{0}} \cdot u$$

$$H(s) = \left[C(sI - A)^{-1} + D\right] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \left(\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & -3 \\ -3 & 0 \end{bmatrix} \right)^{-1} =$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} s & 3 \\ 3 & s \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{s}{s^2-9} & \frac{-3}{s^2-9} \\ \frac{-3}{s^2-9} & \frac{s}{s^2-9} \end{bmatrix}$$

$$H(s) = \begin{bmatrix} \frac{s}{s^2-9} & -\frac{3}{s^2-9} \\ -\frac{3}{s^2-9} & \frac{s}{s^2-9} \end{bmatrix} \xrightarrow{\mathcal{L}^{-1}} h(t) = \begin{bmatrix} \operatorname{ch} 3t & -\operatorname{sh} 3t \\ -\operatorname{sh} 3t & \operatorname{ch} 3t \end{bmatrix}$$

$$U(s) = \begin{bmatrix} \frac{3}{s} \\ 1 \end{bmatrix}$$

$$y(s) = H(s)\,U(s) = \begin{bmatrix} \frac{s}{s^2-9} & -\frac{3}{s^2-9} \\ -\frac{3}{s^2-9} & \frac{s}{s^2-9} \end{bmatrix} \begin{bmatrix} \frac{3}{s} \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{s} \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 0 \\ \mu(t) \end{bmatrix}$$