Zadatak

$$H(s) = \frac{Y(s)}{U(s)} = \frac{s^2+3s+2}{(s+3)(s^2+2s+2)} = \frac{s^2+3s+2}{s^3+2s^2+2s+3s^2+6s+6} = \frac{s^2+3s+2}{s^3+5s^2+8s+6}$$

direktna

$$Y(s) = s^2+3s+2 \cdot \frac{1}{s^3+5s^2+8s+6} U(s)$$

$$V(s) = \frac{1}{s^3+5s^2+8s+6} U(s) \qquad \dddot{v}(t) + 5\ddot{v}(t) + 8\dot{v}(t) + 6v(t) = u(t)$$

$$x_1(t) = v(t)$$
$$x_2(t) = \dot{v}(t)$$
$$x_3(t) = \ddot{v}(t)$$

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = x_3(t)$$
$$\dot{x}_3(t) = -5x_3(t) - 8x_2(t) - 6x_1(t) + u(t)$$

$$Y(s) = (s^2+3s+2)V(s) = \ddot{v}(t) + 3\dot{v}(t) + 2v(t)$$
$$= x_3(t) + 3x_2(t) + 2x_1(t)$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -6 & -8 & -5\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 1\end{bmatrix}u(t) \qquad y(t) = \begin{bmatrix}2 & 3 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + [0]u(t)$$

paralelna

$$H(s) = \frac{s^2+3s+2}{(s+3)(s^2+2s+2)}$$

$$= \frac{A}{s+3} + \frac{Bs+C}{s^2+2s+2} = A(s^2+2s+2) + (Bs+C)(s+3)$$

$$s^2+3s+2 = As^2 + 2As + 2A + Bs^2 + 3Bs + Cs + 3C$$

$$1 = A + B \qquad A = 1 - B \Rightarrow 1 - \frac{3}{5} = \frac{2}{5}$$
$$3 = 2A + 3B + C \qquad 3 = 2 - 2B + 3B + C \Rightarrow C = 1 - B \Rightarrow 1 - \frac{3}{5} = \frac{2}{5}$$
$$2 = 2A + 3C \qquad 2 = 2 - 2B + 3 - 3B \Rightarrow -3 = -5B \qquad B = \frac{3}{5}$$

$$H(s) = \frac{2}{5}\frac{1}{s+3} + \frac{\frac{3}{5}s + \frac{2}{5}}{s^2+2s+2}$$

$$Y(s) = \frac{2}{5}\frac{1}{s+3}U(s) = \frac{2}{5}V(s) \qquad V(s) = \frac{1}{s+3}U(s) \qquad \dot{v}(t) + 3v(t) = u(t) \qquad v(t) = x_1$$

$$\dot{x}_1(t) = -3x_1(t) + u(t) \qquad y_1(t) = x_1(t) = \frac{2}{5}x_1$$

$$Y(s) = \left(\frac{3}{5}s + \frac{2}{5}\right)\frac{1}{s^2+2s+2}U(s) \qquad V_2(s) = \frac{1}{s^2+2s+2}U(s) \qquad \ddot{v}_2(t) + 2\dot{v}_2(t) + 2v_2(t) = u(t)$$

$$x_2 = v_2(t)$$
$$x_3 = \dot{x}_2(t) = \dot{v}_2(t)$$
$$\dot{x}_3 = -2x_3(t) - 2x_2(t) + u(t)$$

$$Y(s) = \left(\frac{3}{5}s + \frac{2}{5}\right)V_2(s) = \frac{3}{5}\dot{v}_2(t) + \frac{2}{5}v_2(t)$$
$$= \frac{3}{5}x_3(t) + \frac{2}{5}x_2(t)$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}-3 & 0 & 0\\ 0 & 0 & 1\\ 0 & -2 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}1\\ 0\\ 1\end{bmatrix} u(t) \qquad y(t) = \begin{bmatrix}\frac{2}{7} & \frac{2}{5} & \frac{3}{5}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}\ \end{bmatrix} u(t)$$

Kaskada

$$H(s) = \frac{s^2+3s+2}{(s+3)(s^2+2s+2)}$$

$$H(s) = \frac{1}{(s+3)} \cdot \frac{s^2+3s+2}{s^2+2s+2} = \frac{V_1(s)}{U(s)} \cdot \frac{Y(s)}{V_1(s)}$$

$$V_1(s) = \frac{1}{s+3} U(s) \qquad \dot{v}_1(t) + 3v_1(t) = u(t) \qquad \dot{x}_1(t) = -3x_1(t) + u(t) \qquad v_1(t) = x_1(t)$$

$$\frac{Y(s)}{V_1(s)} = \frac{s^2+3s+2}{s^2+2s+2} \Rightarrow \qquad Y(s) = s^2+3s+2 \ \frac{1}{s^2+2s+2} V_1(s) = (s^2+3s+2)(Z(s))$$

$$Z(s) = \frac{1}{s^2+2s+2} V_1(s) \Rightarrow \qquad \ddot{z}(t) + 2\dot{z}(t) + 2z(t) = v_1(t)$$

$$x_2 = z(t)$$
$$x_3 = \dot{x}_2 = \dot{z}(t)$$
$$\dot{x}_3 = \ddot{x}_2 = \ddot{z} = -2x_3 - 2x_2 + x_1$$

$$y(s) = (s^2+3s+2)\,Z(s) = \ddot{z}(t) + 3\dot{z}(t) + 2z(t)$$
$$= -2x_3 - \cancel{2x_2} + x_1 + 3x_3 + \cancel{2x_2}$$
$$= x_1 + x_3$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}-3 & 0 & 0\\ 0 & 0 & 1\\ 1 & -2 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}1\\ 0\\ 0\end{bmatrix} u(t) \qquad y(t) = \begin{bmatrix}1 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\end{bmatrix} u(t)$$