

Osnovne trigonometrijske jednakosti

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm \cos x$$

$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp \sin x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \sin \frac{x-y}{2} \cos \frac{x+y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = 2 \sin \frac{x+y}{2} \sin \frac{y-x}{2}$$

$$\sin x \sin y = \frac{1}{2} (\cos(x-y) - \cos(x+y))$$

$$\cos x \cos y = \frac{1}{2} (\cos(x-y) + \cos(x+y))$$

$$\sin x \cos y = \frac{1}{2} (\sin(x-y) + \sin(x+y))$$

$$\sin(2x) = 2 \sin x \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$2 \sin^2 x = 1 - \cos(2x)$$

$$2 \cos^2 x = 1 + \cos(2x)$$

Tablice suma i integrala

Konačne sume

$$\sum_{i=1}^n i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^n i^3 = \frac{n^2(n+1)^2}{6}$$

$$\sum_{i=0}^n x^i = \frac{x^{n+1} - 1}{x - 1}$$

$$\sum_{i=0}^n e^{j(\theta+n\phi)} = \frac{\sin((n+1)\phi/2)}{\sin(\phi/2)} e^{j(\theta+n\phi/2)}$$

$$\sum_{i=0}^n \binom{n}{i} = \sum_{i=1}^n \frac{n!}{i!(n-i)!} = 2^n$$

Neodređeni integrali

Racionalne funkcije

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)}, \quad 0 < n$$

$$\int \frac{1}{ax+b} dx = \frac{1}{a} \ln |ax+b|$$

$$\int \frac{dx}{a^2x^2+b^2} = \frac{1}{ab} \operatorname{tg}^{-1}\left(\frac{ax}{b}\right)$$

$$\int \frac{x dx}{a^2+x^2} = \frac{1}{2} \ln(a^2+x^2)$$

$$\int \frac{x^2 dx}{a^2+x^2} = x - a \operatorname{tg}^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{(a^2+x^2)^2} = \frac{x}{2a^2(a^2+x^2)} + \frac{1}{2a^3} \operatorname{tg}^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{x dx}{(a^2+x^2)^2} = \frac{-1}{2(a^2+x^2)}$$

$$\int \frac{x^2 dx}{(a^2+x^2)^2} = \frac{-x}{2(a^2+x^2)} + \frac{1}{2a} \operatorname{tg}^{-1}\left(\frac{x}{a}\right)$$

Trigonometrijske funkcije

$$\int \cos(x) dx = \sin(x)$$

$$\int x \cos(x) dx = \cos(x) + x \sin(x)$$

$$\int x^2 \cos(x) dx = 2x \cos(x) + (x^2 - 2) \sin(x)$$

$$\int \sin(x) dx = -\cos(x)$$

$$\int x \sin(x) dx = \sin(x) - x \cos(x)$$

$$\int x^2 \sin(x) dx = 2x \sin(x) + (2 - x^2) \cos(x)$$

Eksponencijalne funkcije

$$\int e^{ax} dx = \frac{1}{a} e^{ax}$$

$$\int x e^{ax} dx = \left(\frac{x}{a} - \frac{1}{a^2}\right) e^{ax}$$

$$\int x^2 e^{ax} dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right) e^{ax}$$

$$\int x^3 e^{ax} dx = \left(\frac{x^3}{a} - \frac{3x^2}{a^2} + \frac{6x}{a^3} - \frac{6}{a^4}\right) e^{ax}$$

$$\int \sin(x) e^{ax} dx = \frac{1}{a^2+1} (a \sin(x) - \cos(x)) e^{ax}$$

$$\int \cos(x) e^{ax} dx = \frac{1}{a^2+1} (a \cos(x) + \sin(x)) e^{ax}$$

Određeni integrali

$$\int_{-\infty}^{+\infty} e^{-a^2 x^2 + bx} dx = \frac{\sqrt{\pi}}{a} e^{\frac{b^2}{4a^2}}, \quad a > 0$$

$$\int_0^{+\infty} x^2 e^{-x^2} dx = \frac{\sqrt{\pi}}{4}$$

$$\int_0^{+\infty} \frac{\sin(x)}{x} dx = \frac{\pi}{2}$$

$$\int_0^{+\infty} \frac{\sin^2(x)}{x^2} dx = \frac{\pi}{2}$$

Laplaceova transformacija

Laplaceova transformacija funkcije $x(t)$ je:

$$\mathcal{L}[f(t)] = \int_{0-}^{+\infty} f(t) e^{-st} dt$$

Kažemo da su $x(t)$ i $X(s)$ transformacijski par i pišemo $x(t) \text{---}\bullet X(s)$.

Tablica \mathcal{L} transformacije

$$1 \text{---}\bullet \frac{1}{s}$$

$$t \text{---}\bullet \frac{1}{s^2}$$

$$e^{-at} \text{---}\bullet \frac{1}{s+a}$$

$$\frac{1}{b-a}(e^{-at} - e^{-bt}) \text{---}\bullet \frac{1}{(s+a)(s+b)}$$

$$\frac{1}{a-b}(ae^{-at} - be^{-bt}) \text{---}\bullet \frac{s}{(s+a)(s+b)}$$

$$\frac{1}{a}e^{-bt} \sin(at) \text{---}\bullet \frac{1}{(s+b)^2 + a^2}$$

$$e^{-bt}(\cos(at) - \frac{b}{a} \sin(at)) \text{---}\bullet \frac{s}{(s+b)^2 + a^2}$$

Fourierova transformacija

Fourierova transformacija funkcije $x(t)$ je:

$$\mathcal{F}[x(t)] = X(\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$$

Inverzna transformacija je:

$$\mathcal{F}^{-1}[X(\omega)] = x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) e^{j\omega t} d\omega$$

Kažemo da su $x(t)$ i $X(\omega)$ transformacijski par i pišemo $x(t) \text{---}\bullet X(\omega)$.

Tablica \mathcal{F} transformacije

Neka je:

$$\mu(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}$$

$$\text{rect}(x) = \begin{cases} 1, & -\frac{1}{2} < x < \frac{1}{2} \\ 0, & \frac{1}{2} < |x| \end{cases}$$

$$\text{tri}(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

Uz te oznake važnije transformacije su:

$$1 \text{---}\bullet 2\pi \delta(\omega)$$

$$\delta(t) \text{---}\bullet 1$$

$$\mu(t) \text{---}\bullet \pi \delta(\omega) + \frac{1}{j\omega}$$

$$\frac{1}{2} \delta(t) - \frac{1}{2\pi j t} \text{---}\bullet \mu(\omega)$$

$$\text{sgn}(t) \text{---}\bullet \frac{2}{j\omega}$$

$$\text{rect}\left(\frac{t}{T}\right) \text{---}\bullet T \text{sinc}\left(\frac{\omega T}{2\pi}\right)$$

$$\text{sinc}(at) \text{---}\bullet \frac{1}{a} \text{rect}\left(\frac{\omega}{2\pi a}\right)$$

$$\text{tri}\left(\frac{t}{T}\right) \text{---}\bullet T \text{sinc}^2\left(\frac{\omega T}{2\pi}\right)$$

$$\text{sinc}^2(at) \text{---}\bullet \frac{1}{a} \text{tri}\left(\frac{\omega}{2\pi a}\right)$$

$$e^{j\omega_0 t} \text{---}\bullet 2\pi \delta(\omega - \omega_0)$$

$$\delta(t - t_0) \text{---}\bullet e^{-j\omega t_0}$$

$$\sin(\omega_0 t) \text{---}\bullet -j\pi(\delta(\omega - \omega_0) - \delta(\omega + \omega_0))$$

$$\cos(\omega_0 t) \text{---}\bullet \pi(\delta(\omega - \omega_0) + \delta(\omega + \omega_0))$$

$$\sum_{i=-\infty}^{+\infty} \delta(t - iT_0) \text{---}\bullet \frac{2\pi}{T_0} \sum_{i=-\infty}^{+\infty} \delta\left(\frac{\omega}{2\pi} - \frac{i}{T_0}\right)$$

$$\sin(\omega_0 t) \mu(t) \text{---}\bullet -\frac{j\pi}{2}(\delta(\omega - \omega_0) - \delta(\omega + \omega_0)) + \frac{j\omega}{\omega_0^2 - \omega^2}$$

$$\cos(\omega_0 t) \mu(t) \text{---}\bullet \frac{\pi}{2}(\delta(\omega - \omega_0) + \delta(\omega + \omega_0)) + \frac{j\omega}{\omega_0^2 - \omega^2}$$

$$e^{-at} \mu(t) \text{---}\bullet \frac{1}{a + j\omega}, \quad a > 0$$

$$te^{-at} \mu(t) \bigcirc \bullet \frac{1}{(a+j\omega)^2}, \quad a > 0$$

$$t^2 e^{-at} \mu(t) \bigcirc \bullet \frac{2}{(a+j\omega)^3}, \quad a > 0$$

$$t^3 e^{-at} \mu(t) \bigcirc \bullet \frac{6}{(a+j\omega)^4}, \quad a > 0$$

$$e^{-a|t|} \bigcirc \bullet \frac{2a}{a^2 + \omega^2}$$

$$e^{-\frac{t^2}{2a^2}} \bigcirc \bullet a\sqrt{2\pi}e^{-a^2\omega^2/2}$$

Vremenski diskretna Fourierova transformacija

Vremenski diskretna Fourierova transformacija (DTFT – *Discrete-Time Fourier Transform*) niza $x[n]$ je:

$$\mathcal{F}_{\text{vd}}[x[n]] = X(\omega) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

Inverzna transformacija je:

$$\mathcal{F}_{\text{vd}}^{-1}[X(\omega)] = x[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(\omega)e^{j\omega n} d\omega$$

Niz $x[n]$ i njegov spektar $X(\omega)$ čine transformacijski par $x[n] \bigcirc \bullet X(\omega)$.

Tablica \mathcal{F}_{vd} transformacije

$$\delta[n] \bigcirc \bullet 1$$

$$1 \bigcirc \bullet \sum_{i=-\infty}^{+\infty} 2\pi\delta(\omega + 2\pi i)$$

$$e^{j\omega_0 n} \bigcirc \bullet \sum_{i=-\infty}^{+\infty} 2\pi\delta(\omega - \omega_0 + 2\pi i)$$

$$\mu[n] \bigcirc \bullet \frac{1}{1 - e^{-j\omega}} + \sum_{i=-\infty}^{+\infty} \pi\delta(\omega + 2\pi i)$$

$$a^n \mu[n] \bigcirc \bullet \frac{1}{1 - ae^{-j\omega}}, \quad |a| < 1$$

$$na^n \mu[n] \bigcirc \bullet \frac{ae^{j\omega}}{(e^{-j\omega} - a)^2}, \quad |a| < 1$$

$$\sin(\omega_0 n) \bigcirc \bullet \sum_{i=-\infty}^{+\infty} j\pi(\delta(\omega + \omega_0 + 2\pi i) - \delta(\omega - \omega_0 + 2\pi i))$$

$$\cos(\omega_0 n) \bigcirc \bullet \sum_{i=-\infty}^{+\infty} \pi(\delta(\omega + \omega_0 + 2\pi i) + \delta(\omega - \omega_0 + 2\pi i))$$

$$a^n \sin(\omega_0 n) \mu[n] \bigcirc \bullet \frac{ae^{j\omega} \sin(\omega_0)}{e^{2j\omega} - 2ae^{j\omega} \cos(\omega_0) + a^2}, \quad |a| < 1$$

$$a^n \cos(\omega_0 n) \mu[n] \bigcirc \bullet \frac{e^{j\omega}(e^{j\omega} - a \cos(\omega_0))}{e^{2j\omega} - 2ae^{j\omega} \cos(\omega_0) + a^2}, \quad |a| < 1$$

Diskretna Fourierova transformacija

Diskretna Fourierova transformacija konačnog niza $x[n]$ duljine N je:

$$X[k] = \sum_{n=0}^{N-1} x[n]W_N^{nk}, \quad 0 \leq k \leq N-1$$

Pri tome je $W_N^{nk} = e^{-2\pi jnk/N}$. Inverzna transformacija je:

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]W_N^{-nk}, \quad 0 \leq n \leq N-1$$

Niz $x[n]$ i njegov spektar $X[k]$ čine transformacijski par $x[n] \bigcirc \bullet X[k]$.

\mathcal{Z} -transformacija

\mathcal{Z} -transformacija niza $f[n]$ je: $\mathcal{Z}[f[n]] = \sum_{n=0}^{+\infty} f[n]z^{-n}$

Tablica \mathcal{Z} transformacije

$$\delta[n] \bigcirc \bullet 1$$

$$\delta[n-m] \bigcirc \bullet z^{-m}$$

$$n \bigcirc \bullet \frac{z}{(z-1)^2}$$

$$1^n \bigcirc \bullet \frac{1}{1-z^{-1}} = \frac{z}{z-1}$$

$$a^n \bigcirc \bullet \frac{1}{1-az^{-1}} = \frac{z}{z-a}$$

$$(n+1)a^n \bigcirc \bullet \frac{z^2}{(z-a)^2}$$

$$\frac{(n+1)(n+2)}{2!}a^n \bigcirc \bullet \frac{z^3}{(z-a)^3}$$

$$\frac{(n+1)(n+2)\dots(n+m-1)}{(m-1)!}a^n \bigcirc \bullet \frac{z^m}{(z-a)^m}$$

$$a^n - \delta[n] \bigcirc \bullet \frac{a}{z-a}$$

$$\sin[an] \bigcirc \bullet \frac{z \sin(a)}{z^2 - 2z \cos(a) + 1}$$

$$\cos[an] \bigcirc \bullet \frac{z^2 - z \cos(a)}{z^2 - 2z \cos(a) + 1}$$

Određivanja početnih uvjeta

Za sustav opisan diferencijalnom jednačbom

$$y^{(N)}(t) + a_1 y^{(N-1)}(t) + \dots + a_{N-1} y^{(1)}(t) + a_N y(t) = b_0 u^{(N)}(t) + b_1 u^{(N-1)}(t) + \dots + b_{N-1} u^{(1)}(t) + b_N u(t)$$

potrebno je odrediti početne uvjete $y(0^+)$, $y'(0^+)$, $y''(0^+)$, \dots , $y^{(N-1)}(0^+)$ iz onih u 0^- . Ako pobuda ne sadrži Diracovu distribuciju rješavamo sustav jednačbi:

$$\begin{aligned}\Delta y &= b_0 u(0^+) \\ \Delta y^{(1)} + a_1 \Delta y &= b_0 u^{(1)}(0^+) + b_1 u(0^+) \\ \Delta y^{(2)} + a_1 \Delta y^{(1)} + a_2 \Delta y &= b_0 u^{(2)}(0^+) + b_1 u^{(1)}(0^+) + b_2 u(0^+)\end{aligned}$$

$$\Delta y^{(N-1)} + a_1 \Delta y^{(N-2)} + \dots + a_{N-1} \Delta y = b_0 u^{(N-1)}(0^+) + \dots + b_{N-2} u^{(1)}(0^+) + b_{N-1} u(0^+)$$

Pri tome je $\Delta y^{(i)} = y^{(i)}(0^+) - y^{(i)}(0^-)$.

Ako je pobuda $u(t) = \delta(t)$ onda je $y^{(N-1)}(0^+) = y^{(N-1)}(0^-) + 1$, a ostali početni uvjeti se ne razlikuju.