MASS. INSTRUKCIJE 25.3.2010

1. MI. 2007/2008 - 31. 3. 2008

1) $f(n) = \cos\left(\frac{47\pi}{7} n + \frac{\pi}{3}\right) = \cos\left(\frac{5\pi}{7} n + \frac{\pi}{3}\right)$

ALIASING

$\omega \in [0, 2\pi]$ tj. $[-\pi, \pi]$

$\omega = \frac{47\pi}{7} - \frac{42\pi}{7} - 2\pi = \frac{33\pi}{7} - 2\pi = \frac{19\pi}{7} - 2\pi = \frac{5\pi}{7}$

$\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$

$\cos\left(\frac{5\pi}{7} n + \frac{\pi}{3}\right) = \boxed{\sin\left(\frac{5\pi}{7} n + \frac{5\pi}{6}\right)}$ D

2) TOTALNA ENERGIJA

$$E_\infty = \lim_{T \to \infty} \int_{-T}^{T} |u(t)|^2 \, dt$$

$$|e^{jx}| = |\cos(x) + j(\sin(x))| = 1$$

$$u(t) = \frac{1}{t} \mu(t-1) = \begin{cases} \frac{1}{t}, & t \geq 1 \\ 0, & t < 1 \end{cases} \qquad \mu(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$E_\infty = \lim_{T \to \infty} \int_{1}^{T} \frac{1}{t^2} \, dt = \lim_{T \to \infty} \left(-\frac{1}{t}\Big|_1^T\right) = \lim_{T \to \infty} \left(\underbrace{-\frac{1}{T}}_{0} + 1\right) = 1$$

3. $v(n) = n\left(\mu(n+6) - \mu(n-6)\right)$

$$\mu(t-a) - \mu(t-b) = \begin{cases} 1, & a \le t \le b \\ 0, & \text{inače} \end{cases}$$ ZA KONTINUIRANE SIGNALE

$$= \begin{cases} 1, & a \le n \le b-1 \\ 0, & \text{inače} \end{cases}$$ ZA DISKRETNE SIGNALE

$$n\left(\mu(n+6) - \mu(n-6)\right) = \begin{cases} n, & -5 \le n \le 5 \\ 0, & \text{inače} \end{cases}$$

$$E_\infty = \lim_{N \to \infty} \sum_{n=-N}^{N} |v(n)|^2 = \sum_{n=-5}^{5} n^2 = 2 \sum_{n=1}^{5} n^2 = \underline{110}$$

4. $x(t) = 5.$

$$P_\infty = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |v(t)|^2 dt = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} 25\, dt = \lim_{T \to \infty} \frac{25 \cdot \cancel{2T}}{\cancel{2T}} = 25$$

⑥ postoje 4 FOURIERA

1. CTFS - za periodične signale
2. CTFT - za aperiodične signale
3. DTFS - za periodične signale
4. DTFT - za aperiodične signale

$$x(t) = e^{-5t} u(t-5)$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_{-\infty}^{\infty} e^{-5t} u(t-5) e^{-j\omega t} =$$

$$= \int_{5}^{\infty} e^{-5t} e^{-j\omega t} = \int_{5}^{\infty} e^{t(-5-j\omega)} dt = \frac{-1}{5+j\omega} e^{t(-5-j\omega)} \Big|_{5}^{\infty} =$$

$$= \frac{-1}{5+j\omega} \Big(\underbrace{e^{-\infty}}_{0} - e^{5(-5-j\omega)} \Big) = \frac{1}{5+j\omega} e^{-25-5j\omega}$$

⑦ $x(t) = \begin{cases} t, & -\pi < t < \pi \\ 0, & \text{inače} \end{cases}$ $\omega = 1$

$$X(j\omega) = \int_{-\pi}^{\pi} t e^{-jt} dt = \left| \begin{matrix} u = t & du = dt \\ v = \int e^{-jt} dt = -\frac{1}{j} e^{-jt} = j e^{-jt} \end{matrix} \right| =$$

$$= j t e^{-jt} - j \int_{-\pi}^{\pi} e^{-jt} dt = j t e^{-jt} - \cancel{j} \frac{(-1)}{\cancel{j}} e^{-jt} \Big|_{-\pi}^{\pi} =$$

$$= \Big(j\pi \underbrace{e^{-j\pi}}_{-1} + \underbrace{e^{-j\pi}}_{-1} \Big) - \Big(-j\pi \underbrace{e^{j\pi}}_{-1} + \underbrace{e^{j\pi}}_{-1} \Big) = -1 - j\pi + 1 - j\pi = \boxed{-2j\pi}$$

⑧ $\delta(t+t_0)$, $t_0 > 0$

$\delta(t) \circ\!\!-\!\!\bullet\ 1$

$\delta(t+t_0) \circ\!\!-\!\!\bullet\ e^{j\omega t_0} \cdot 1 \Rightarrow \phi(\omega) = \omega t_0$

$|A(\omega)| = 1$

⑨

$x(t) \circ\!\!\rightarrow X(j\omega)$ $\qquad$ $x(t-t_0) \circ\!\!\rightarrow e^{-j\omega t_0} X(j\omega)$

$x(at) = \frac{1}{j} X\left(j\frac{\omega}{1}\right)$

$y(t) = x(-t) \circ\!\!\rightarrow y(j\omega) = X(-j\omega)$

$z(t) = y(t-10) \circ\!\!\rightarrow Z(j\omega) = e^{-10j\omega} y(j\omega) = e^{-10j\omega} X(-j\omega)$

⑩

$x(t) \circ\!\!\rightarrow X(j\omega) = |X(j\omega)|\, e^{j\measuredangle X(j\omega)}$

$g(t) = x(t+7) \circ\!\!\rightarrow G(j\omega) = e^{7j\omega} X(j\omega)$

$|G(j\omega)|\, e^{j\measuredangle G(j\omega)} = |X(j\omega)|\, e^{\measuredangle X(j\omega)} \cdot 1\, e^{7j\omega} =$

$= |X(j\omega)|\, e^{(\measuredangle X(j\omega) + 7\omega)}$

$$\boxed{\begin{aligned} |G(j\omega)| &= |X(j\omega)| \\ \measuredangle G(j\omega) &= \measuredangle X(j\omega) + 7 \end{aligned}}$$

(11) $x(t) = 220\cos(50\pi t) + 100\sin\left(200\pi t + \frac{\pi}{6}\right)$

$\omega_1 = 50\pi \quad \omega_2 = 200\pi \Big\} \quad \omega_0 = 50\pi \quad \Rightarrow \omega_2 = 4\omega_0$

$\cos x = \frac{1}{2}\left(e^{jx} + e^{-jx}\right) \quad \sin x = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)$

$x(t) = 220 \cdot \frac{1}{2}\left(e^{j(50\pi t)} + e^{-j(50\pi t)}\right) + 100 \cdot \frac{1}{2j}\left(e^{j(200\pi t)} - e^{-j(200\pi t)}\right.$

$= 110\, e^{j1\omega_0 t} + 110\, e^{j(-1\omega_0 t)} - 50j\, e^{j\frac{\pi}{6}} e^{j(4)\omega_0 t} + 50j\, e^{-j\frac{\pi}{6}} e^{j(-4\omega_0 t)}$

$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\omega_0 t} \qquad X_k = \frac{1}{T_0}\int_{T_0} x(t) e^{-jk\omega_0 t}\, dt$

$X_1 = 110 \quad X_{-1} = 110 \qquad X_4 = -50j\, e^{j\frac{\pi}{6}} = 50\, e^{j\frac{3\pi}{2}} e^{j\frac{\pi}{6}}$

$X_{-4} = 50j\, e^{-j\frac{\pi}{6}} = 50\, e^{j\frac{\pi}{2}} e^{-j\frac{\pi}{6}}$

(12) $x(t) = \cos^2\left(2t + \frac{\pi}{6}\right) = \frac{1}{2}\left(1 + \cos\left(4t + \frac{\pi}{3}\right)\right) = \underset{x_0}{\frac{1}{2}} + \underset{x_1}{\frac{1}{2} e^{j\frac{\pi}{3}}} e^{j4t} +$

$+ \underset{x_{-1}}{\frac{1}{2} e^{-j\frac{\pi}{3}}}\, e^{j(-4)t}$

$\omega_0 = 4$

$\cos^2 x = \frac{1}{2}\left[1 + \cos(2x)\right]$

(15)

DTFT

$$X(e^{j\omega}) = \sum_{n=-4}^{4} n e^{-j\omega} = \overset{-4}{-4e^{4j\omega}} \overset{+3j}{-3e^{3j\omega}} \overset{+2}{-2e^{2j\omega}} \overset{-j}{-e^{j\omega}} + 0 +$$

$$+ \overset{-j}{1e^{-j\omega}} + \overset{-2}{2e^{-2j\omega}} + \overset{+3j}{3e^{-3j\omega}} + \overset{4}{4e^{-4j\omega}} =$$

$$= \cancel{-4} + 3j + \cancel{2} - j - j - \cancel{2} + 3j + \cancel{4} = \boxed{4j}$$

(16)

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega =$$

$$= \frac{1}{2\pi}\int_{-a}^{a} 2 e^{j\omega n} d\omega = \frac{1}{2\pi}\cdot 2 \frac{1}{jn} e^{j\omega n}\Big|_{-a}^{a} =$$

$$= \frac{2}{n\pi}\frac{1}{2j}\left(e^{jna} - e^{-jna}\right) = \frac{2}{n\pi}(\sin(a))$$

(17) Parseivalove jednakosti

$E_\infty = \infty$ $P_\infty = C$ signali energije

$E_\infty = C$ $P_\infty = 0$ signal snage

$E_\infty = \infty$ $P_\infty = \infty$

$$\underline{E} = \frac{1}{2\pi}\int_{-a}^{a} 4\pi^2 d\omega = \cdot 2\pi\omega\Big|_{-a}^{a} = 4a\pi$$

20

Ako je signal:

F.R. — F. Red

KONT. $\xleftrightarrow{F.R.}$ APERIODIČAN

DISKR. $\xleftrightarrow{F.R.}$ PERIODIČAN

171. 2008.

zad. 1

$$x(t) = t(u(t) - u(t-1)) + (-2t+4)(u(t-1) - u(t-2))$$

$$\frac{d}{dt} u(t) = \delta(t)$$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$x'(t) = 1(u(t) - u(t-1)) + t(\delta(t) - \delta(t-1)) - 2(u(t-1) - u(t-2)) +$$

$$+ (-2t+4)(\delta(t-1) - \delta(t-2)) =$$

$$= u(t) - u(t-1) + t\delta(t) - t\delta(t-1) - 2u(t-1) +$$

$$2u(t-2) + (-2t+4)\delta(t-1) - (-2t+4)\delta(t-2) =$$

$$t\delta(t) = 0$$

$$= u(t) - 3u(t-1) + 2u(t-2) + \delta(t-1)$$