1.

$$\frac{\delta y}{\delta t} + 5y + 2 = u(t) \quad , \quad y(0) = 0$$

→ sustav je očigledno nelinearan.

Ako na ulaz dovedemo linearnu kombinaciju dva signala, na izlazu to očigledno nećemo dobiti → razlog je $\underline{2}$.

→ Ako je sustav linearan (specijalno i vremenski nepromjenjiv → LTI), tada što god napravili ulaznom signalu (linearne transformacije) → to vrijedi napravi i izlazu. tj.

Ako je S LTI te

$$Y_1 = S\{U_1\}, \text{ tada ako je}$$

$U_2 = T\{U_1\}$, T – također LTI, onda vrijedi da je

$$\underline{Y_2 = T\{Y_1\}} \quad \rightarrow \text{tj. ako znamo}$$

odziv na U_1, i ako znamo da U_2 možemo dobiti iz U_1 → linearnim transformacijama tada Y_2 možemo dobiti iz Y_1.

2. Zadan je kontinuirani sustav

$$y'''(t) - y''(t) + y'(t) + 39y = u''(t) + 2u(t) .$$

Ispitajte da li je ovaj sustav stabilan.

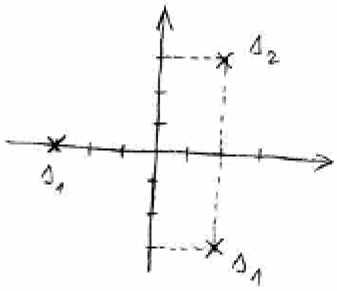
$$y'''(t) - y''(t) + y'(t) + 39\,y(t) = u''(t) + 2u(t)$$

$$y_h(t) = Ce^{st}$$
$$y'_h(t) = Cs\,e^{st}$$
$$y''_h(t) = Cs^2e^{st}$$
$$y'''_h(t) = Cs^3e^{st}$$

$$Ce^{st}\left[s^3 - s^2 + s + 39\right] = 0$$
$$s^3 - s^2 + s + 39 = 0$$
$$\boxed{s_1 = -3}$$

$$(s^3 - s^2 + s + 39) : (s+3) = s^2 - 4s + 13$$
$$-s^3 - 3s^2$$
$$-4s^2 + s$$
$$4s^2 + 12s$$
$$13s + 39$$
$$-(13s + 39)$$
$$=$$

$$(s+3)(s^2 - 4s + 13) = 0$$
$$s_{2,3} = \frac{4 \pm 6j}{2}$$
$$s_2 = 2 + 3j$$
$$s_3 = 2 - 3j$$



$$y_h(t) = C_1e^{-3t} + C_2\,e^{(2+3j)t} + C_3\,e^{(2-3j)t}$$

$$\lim_{t\to\infty} y_h(t) = 0 + \lim_{t\to\infty}\left(C_2\,e^{(2+3j)t} + C_3\,e^{(2-3j)t}\right) = \infty$$

SUSTAV JE NESTABILAN

ILI

Postoje komplekt. frekv. za koje je $Re\{s\} > 0$

SUSTAV JE NESTABILAN

3.

$$y' + y = u' + 2u.$$

$$u(t) = 3\mu(t), \quad \underline{y(0)}$$

a) $y(0^-) = 9$, b) $y(0^+) = 9$.

$$y_h = ce^{-t}$$

$t \in (0, \infty) \rightarrow y_p = k \rightarrow$

$$k = 6 \rightarrow$$

$$y_p = (ce^{-t} + 6)\mu(t), \quad \underline{t \in \mathbb{R}}.$$

$$\rightarrow \quad \cancel{-ce^{-t}\mu(t)} + (c+6)\delta(t) +$$

$$+ \cancel{(ce^{-t}+6)}\mu(t) = 3\delta(t) \cancel{+6\mu(t)}$$

$$\rightarrow \underline{c = -3}$$

$$\rightarrow y = ce^{-t} + (6 - 3e^{-t})\mu(t)$$

a) $y(0^-) = c + 0 = 9 \rightarrow (\underline{\mu(0^-) = 0})$

$$c = 9$$

$$y = 9e^{-t} + (6 - 3e^{-t})\mu(t) \quad \underline{t \in \mathbb{R}}$$

$$\rightarrow y = (6 + 6e^{-t})\mu(t) \quad \underline{t \in (0, \infty)}.$$

7b)

$$y = ce^{-t} + (6 - 3e^{-t})u(t)$$

$$y(0^+) = 9 \qquad (u(0^-) = 9)$$

$$y(0^+) = c + 3 = 9 \longrightarrow \underline{c = 6}.$$

$$y = 6e^{-t} + (6 - 3e^{-t})u(t), \quad \underline{t \in \mathbb{R}}.$$

$$y = (6 + 3e^{-t})u(t), \quad \underline{t \in (0, \infty)}.$$

4. Kontinuirani sustav opisan je diferencijalnom jednadžbom čije je homogeno rješenje

$$y_h(t) = \frac{1}{2}e^{3t} + \frac{5}{3}e^{2t}.$$

Sustav nema nula (u diferencijalnoj jednadžbi ne postoje derivacije ulaza). Odredite tu diferencijalnu jednadžbu. Odredite ukupan i odziv mirnog sustava ako se sustav pobudi s $u(t) = \frac{5}{2}e^{2t}, \quad t \ge 0$. Odredite početna stanja. Ispitajte stabilnost sustava.

4.

$$y_h(t) = \frac{1}{2}e^{3t} + \frac{5}{3}e^{2t}$$

$$u(t) = \frac{5}{2}e^{2t}, \quad t \ge 0$$

$$\boxed{\begin{aligned} s_1 &= 3 \\ s_2 &= 2 \end{aligned}} \qquad c_1 = \frac{1}{2} \qquad c_2 = \frac{5}{3}$$

$$\left.\begin{aligned} s_1 &> 0 \\ s_2 &> 0 \end{aligned}\right\} \text{SUSTAV NIJE STABILAN}$$

$$(s-3)\cdot(s-2) = 0$$

$$s^2 - 5s + 6 = 0$$

$$\boxed{y''(t) - 5y'(t) + 6y(t) = u(t)}$$

– PARTIKULARNO

$$y''(t) - 5y'(t) + 6y(t) = \frac{5}{2}e^{2t}$$

pretpostavimo rješenje:

$$y_p = A \cdot t \cdot e^{2t}$$

$$y'(t) = Ae^{2t} + 2Ate^{2t}$$

$$y''(t) = 2Ae^{2t} + 2Ae^{2t} + 4Ate^{2t}$$

$$2A\cancel{e^{2t}} + 2A\cancel{e^{2t}} + 4At\cancel{e^{2t}} - 5A\cancel{e^{2t}} - 10At\cancel{e^{2t}} + 6At\cancel{e^{2t}} = \frac{5}{2}\cancel{e^{2t}}$$

$$-A = \frac{5}{2}$$

$$\boxed{A = -\frac{5}{2}}$$

$$\boxed{y_p = -\frac{5}{2}te^{2t}}$$

$$\boxed{y = y_H + y_p = \frac{1}{2}e^{3t} + \frac{5}{3}e^{2t} - \frac{5}{2}\cdot t\cdot e^{2t}}$$

NASTAVAK (7)

$$y(t) = K_1 \cdot e^{3t} + K_2 \cdot e^{2t} - \frac{5}{2} \cdot t \cdot e^{2t}$$

$$y'(t) = 3K_1 e^{3t} + 2K_2 e^{2t} - \frac{5}{2} e^{2t} - 5t e^{2t}$$

$y(0) = 0$ $\qquad$ $y'(0) = 0$

$K_1 + K_2 = 0$ $\qquad$ $3K_1 + 2K_2 - \frac{5}{2} = 0$

$K_1 = -K_2$ $\qquad$ $-3K_2 + 2K_2 = \frac{5}{2}$

$$\boxed{K_2 = \frac{-5}{2}}$$

$$\boxed{K_1 = \frac{5}{2}}$$

$$\boxed{y(t) = \frac{5}{2} e^{3t} - \frac{5}{2} e^{2t} - \frac{5}{2} t e^{2t}}$$

$\Rightarrow$ ODZIV MIRNOG SUSTAVA

POČETNA STANJA:

$$\boxed{y(0) = \frac{1}{2} + \frac{5}{3} = \frac{13}{6}}$$

$$y'(t) = \frac{3}{2} e^{3t} + \frac{10}{3} e^{2t} - \frac{5}{2} e^{2t} - 5t e^{2t}$$

$$y'(0) = \frac{3}{2} + \frac{10}{3} - \frac{5}{2} = \frac{-6+20}{6} = \frac{\cancel{14}^{7}}{\cancel{6}_{3}} = \frac{7}{3}$$

$$\boxed{y'(0) = \frac{7}{3}}$$

$y(0^-)=y(0^+)=y(0)$

$y'(0^-)=y'(0^+)=y'(0)$

5. Riješite diferencijalnu jednadžbu

$$y'(t) + 2y(t) = u(t),$$

ako je ulaz

$$u(t) = A \cdot \cos(\omega_0 t) \cdot \mu(t),$$

pri čemu je A realna konstanta i uzimajući da su početni uvjeti jednaki nula. Bez dodatnog računanja odredite rješenje ove jednadžbe ako je ulaz

$$u(t) = B \cdot \cos\omega_0 (t-1) \cdot \mu(t-1).$$

$$y'(t) + 2y(t) = u(t)$$

$$y(t) = y_h(t) + y_p(t)$$

$y_h = ?$ $\qquad u(t) = A\cos(\omega_0 t)\mu(t)$

$$h'(t) + 2h(t) = 0 \qquad h(t) = Ce^{st}$$

$$s\,Ce^{st} + 2Ce^{st} = 0 \qquad Ce^{st}(s+2) = 0 \qquad \boxed{s = -2}$$

$$h(t) = Ce^{st}$$

$y_p = ?$

$$y_p(t) = K_1\cos(\omega_0 t) + K_2\sin(\omega_0 t)$$

$$y_p'(t) = -\omega_0 K_1\sin(\omega_0 t) + \omega_0 K_2\cos(\omega_0 t)$$

$$-\omega_0 K_1\sin(\omega_0 t) + \omega_0 K_2\cos(\omega_0 t) + 2K_1\cos(\omega_0 t) +$$
$$+ 2K_2\sin(\omega_0 t) = A\cos(\omega_0 t)$$

$$\sin(\omega_0 t)\left[-\omega_0 K_1 + 2K_2\right] + \cos(\omega_0 t)\left[\omega_0 K_2 + 2K_1\right] =$$

$$= A\cos(\omega_0 t)$$

$$-\omega_0 K_1 + 2K_2 = 0 \quad \Rightarrow \quad 2K_2 = \omega_0 K_1 \Rightarrow K_2 = \frac{\omega_0}{2} K_1$$

$$\omega_0 K_2 + 2K_1 = A \qquad \frac{\omega_0^2}{2} K_1 + 2K_1 = A \quad K_1\left(\frac{\omega_0^2}{2} + 2\right) = A$$

$$K_1 = \frac{A}{\frac{\omega_0^2}{2} + 2} \qquad K_2 = \frac{\frac{\omega_0}{2} A}{\frac{\omega_0^2}{2} + 2}$$

$$y_p(t) = K_1 \cos(\omega_0 t) + K_2 \sin(\omega_0 t) =$$

$$= \frac{A}{\frac{\omega_0^2}{2} + 2} \cos(\omega_0 t) + \frac{\frac{\omega_0}{2} A}{\frac{\omega_0^2}{2} + 2} \sin(\omega_0 t) =$$

$$= A\left[\frac{2}{\omega_0^2 + 4} \cos(\omega_0 t) + \frac{\omega_0}{\omega_0^2 + 4} \sin(\omega_0 t)\right]$$

$$y(t) = y_h(t) + y_p(t) = Ce^{-2t} + A\left[\frac{2}{\omega_0^2 + 4} \cos(\omega_0 t) + \frac{\omega_0}{\omega_0^2 + 4} \sin(\omega_0 t)\right]$$

$$y(0) = C + A\left(\frac{2}{\omega_0^2 + 4}\right) = 0 \Rightarrow C = \frac{-2A}{\omega_0^2 + 4}$$

$$y(t) = \frac{-2A}{\omega_0^2 + 4} e^{-2t} + A\left[\frac{2}{\omega_0^2 + 4} \cos(\omega_0 t) + \frac{\omega_0}{\omega_0^2 + 4} \sin(\omega_0 t)\right]$$

ZA $u(t) = B\cos\omega(t-1)\, \mu(t-1)$

$$\boxed{y(t) = \frac{-2B}{\omega_0^2 + 4} e^{-2(t-1)} + B \cdot \left[\frac{2}{\omega_0^2 + 4} \cos\omega_0(t-1) + \frac{\omega_0}{\omega_0^2 + 4} \sin\omega_0(t-1)\right]}$$

$\boxed{6.}$

$$\frac{dy}{dt} - y(t) = -u(-t)$$

a) $y(0) = 0$

$$y_h = ce^t$$

$t \in (-\infty, 0) \to y_p = 1 \to$ $t \in (0, \infty)$

tražim $t \in (-\infty, 0]$

$$y_p = ce^t + 1 \to$$

$$y_p = (ce^t + 1)u(-t) \to \text{za } \underline{t \in \mathbb{R}}.$$

$$y_p' - y_p = -u(-t) \to$$

$$\cancel{ce^t u(-t)} - (c+1)\delta(t) - \cancel{(ce^t+1)u(-t)} = -u(-t)$$

$$\underline{c = -1}$$

$$\to y_p = (1 - e^t)u(-t) \quad \underline{t \in \mathbb{R}}.$$

$$\to \underline{y_h = ce^t}$$

$$y = ce^t + (1 - e^t)u(-t)$$

$\downarrow$ $y(0) = 0$

$c = 0$

a) $\to y = (1 - e^t)u(-t)$

b) $y(0) = 1 \to y = e^t + (1 - e^t)u(-t)$

7. Kontinuirani sustav zadan je diferencijalnom jednadžbom

$$y'(t) + 2y(t) = u(t) + u'(t)\,.$$

Provjerite, bez rješavanja zadane diferencijalne jednadžbe, je li impulsni odziv ovog sustava $h(t) = -e^{-2t}\mu(t) + \delta(t)$.

$\boxed{4}$ $y'(t)+2y(t)=u'(t)+u(t)$

$\underline{h(t)=-e^{-2t}\mu(t)+\delta(t)}$ $\quad \Rightarrow h(t)=$

$$h'(t)+2h(t)=\delta'(t)+\delta(t)$$

$$h'(t)=+e^{-2t}\cdot 2\mu(t)-e^{-2t}\delta(t)+\delta'(t)$$

$$\cancel{2e^{-2t}\mu(t)}-e^{-2t}\delta(t)+\delta'(t)\cancel{-2e^{-2t}\mu(t)}+2\delta(t)=\delta'(t)+\delta(t)$$

vrijednost funkcije $-e^{-2t}$ u točki $t=0$

$$-e^{-2t}\Big|_{t=0}=-e^{0}=-1$$

$$-\delta(t)+\delta'(t)+2\delta(t)=\delta'(t)+\delta(t)$$

$$\underline{\delta'(t)+\delta(t)=\delta'(t)+\delta(t)}$$

8. $y'' + 2y' + y = u, \quad y(0)=0, \quad y'(0)=0$

a) $u(t) = t\mu(t)$

k. jedba $\rightarrow$ $\lambda^2 + 2\lambda + 1 = 0$

$(\lambda+1)^2 = 0 \rightarrow \underline{\lambda_{1,2} = -1}$

$y_h = (c_1 + c_2 t)e^{-t}$

$y_p = A + Bt \rightarrow$

$2B + A + Bt = t \rightarrow \underline{B=1}$

$\underline{A=-2}$

$y = (c_1 + c_2 t)e^{-t} + t - 2$

$+ \quad y(0)=0, \quad y'(0)=0 \rightarrow$

$y = (te^{-t} + 2e^{-t} + t - 2)\mu(t)$

b) s obzirom da je $\mu(t) = \frac{d}{dt}[t\mu(t)]$

$\rightarrow \quad y_b = y_a' = (-e^{-t} - te^{-t} + 1)\mu(t)$

c) $\rightarrow$ Anal. $y_c = y_b' = te^{-t}\mu(t)$

d) $\rightarrow$ lin. komb. $\rightarrow (e^{-t} + te^{-t} + t - 1)\mu(t)$

9. Zadan je vremenski kontinuirani sustav

$$\frac{dy(t)}{dt} + ay(t) = u(t),$$

gdje je $u(t)$ ulaz, $y(t)$ izlaz, a a konstanta. Početni uvjet $y(0^-)=0$.

a) Naći impulsni odziv zadanog sustava.
b) Naći odziv na jedinični skok, bez korištenja poznatog impulsnog odziva
c) Naći odziv na jedinični skok, uz poznati impulsni odziv iz a) dijela zadatka.
d) Naći odziv na impuls, uz poznati odziv na jedinični skok iz b) dijela zadatka.

Rješenje:

a) Za početak je potrebno odrediti koeficijent $b_0=0$, te redove $N=1$ i $M=0$. Rješava se prvi podsustav:

$$h_A'(t) + ah_A(t) = \delta(t),$$

odnosno homogena jednadžba

$$h_A'(t) + ah_A(t) = 0,$$

uz početni uvjet $h_A(0^+) = 1$ (izvod početnih uvjeta – vidi predavanje).

Rješenje ove jednadžbe je $h_A(t) = Ce^{-at}$.

Konstanta C se dobiva iz početnog uvjeta i iznosi: $h_A(0^+) = Ce^{-at} = C = 1$.

Pa je $h_A(t) = e^{-at}$.

Kako je $b_0=0$ i $M=0$ automatski proizlazi da je ukupni impulsni odziv zadanog sustava

$$h(t) = e^{-at}\mu(t).$$

b) Potrebno je riješiti jednadžbu $y_\mu'(t) + ay_\mu(t) = \mu(t)$.

Na temelju 13. predavanja, prikaznica 19.-29. određuje se početni uvjet $y(0^+)=0$.
Prvo se rješava homogena jednadžba:

$$y_{\mu h}'(t) + ay_{\mu h}(t) = 0 \rightarrow s+a=0 \rightarrow s=-a \rightarrow y_{\mu h} = Ce^{-at}.$$

Partikularno rješenje se pretpostavi iz dane pobude:

$$y_{\mu p}(t) = K\mu(t) \rightarrow aK=1 \rightarrow K = \frac{1}{a}.$$

Totalni odziv: $y_\mu = y_{\mu h} + y_{\mu p} = \left(Ce^{-at} + \frac{1}{a}\right)\mu(t)$.

Konstanta C dolazi iz početnih uvjeta: $y_\mu(0^+) = C + \frac{1}{a} = 0 \rightarrow C = -\frac{1}{a}$.

Odziv na jediničnu stepenicu iznosi: $y_\mu(t) = \frac{1}{a}\left(1 - e^{-at}\right)\mu(t)$.

c) Drugi način određivanja odziva na jediničnu stepenicu je pomoću konvolucijskog integrala.

$$y_\mu(t)=\int_{-\infty}^{\infty}h(\tau)u(t-\tau)d\tau=\int_{-\infty}^{\infty}\left[e^{-a\tau}\mu(\tau)\right]u(t-\tau)d\tau=$$
$$=\left[\int_0^t e^{-a\tau}d\tau\right]\mu(t)=\frac{1}{a}\left(1-e^{-at}\right)\mu(t)$$

Vidljivo je da je isto rješenje kao u *b)* dijelu zadatka.

d) Ako je poznat odziv na step, vrlo lako je naći odziv na impuls, jer je poznata relacija da je impuls derivacija stepa, pa je prema tome, i impulsni odziv derivacija odziva na step.

$$h(t)=y_\mu'(t)=\frac{d}{dt}\left[\frac{1}{a}\left(1-e^{-at}\right)\mu(t)\right]=$$
$$=e^{-at}\mu(t)+\frac{1}{a}(1-e^{-at})\mu'(t)=e^{-at}\mu(t)+\frac{1}{a}(1-e^{-at})\delta(t)$$

Svojstvo delta impulsa je da „izvlači“ vrijednost funkcije u nuli, pa slijedi:

$$h(t)=e^{-at}\mu(t)+\frac{1}{a}(1-e^{-at})\delta(t)=e^{-at}\mu(t)+\frac{1}{a}(1-1)\delta(t)=e^{-at}\mu(t)$$

što je identično a) dijelu zadatka.

10. Naći impulsni odziv vremenski kontinuiranog sustava: $\dfrac{dy(t)}{dt}+2y(t)=u(t)+\dfrac{du(t)}{dt}$.

Rješenje: $h(t)=-e^{-2t}\mu(t)+\delta(t)$.

11. Kauzalni LTI kontinuirani sustav zadan je diferencijalnom jednadžbom:

$$y''(t) + 6y'(t) + 13y(t) = u'(t) + 4u(t)\,.$$

Nađite impulsni odziv ovog sustava.

$\boxed{5}$ $y''(t) + 6y'(t) + 13y(t) = u'(t) + 4u(t)$ $\Rightarrow h''(t) + 6h'(t) + 13h(t) = \delta'(t) + 4\delta(t)$

$h_a''(t) + 6h'(t) + 13h(t) = \delta(t)$

$h_a(t) = ce^{st}$

$cs^2e^{st} + 6sce^{st} + 13ce^{st} = 0$

$ce^{st}(s^2 + 6s + 13) = 0$

$s^2 + 6s + 13 = 0$

$s_{1,2} = \dfrac{-6 \pm j4}{2} = -3 \pm j2$

$s_1 = -3 + j2$

$s_2 = -3 - j2$

$$\boxed{\begin{aligned} b_0 &= 0 \\ b_1 &= 1 \\ b_2 &= 4 \end{aligned}}$$

$h_a(t) = c_1e^{(-3+j2)t} + c_2e^{(-3-j2)t}$

$h_a'(t) = c_1e^{(-3+j2)t} \cdot (-3+j2) + c_2e^{(-3-j2)t} \cdot (-3-j2)$

$h_a(0^+) = 0 \quad h_a'(0^+) = 1$

$h_a(0^+) = c_1 + c_2 = 0 \longrightarrow c_1 = -c_2$

$h_a'(0^+) = c_1(-3+j2) + c_2(-3-j2) = 1$

$-c_2(-3+j2) + c_2(-3-j2) = 1$

$\cancel{3c_2} - j2c_2 - \cancel{3c_2} - j2c_2 = 1$

$-j4c_2 = 1$

$c_2 = \dfrac{-0{,}25}{j} \cdot \dfrac{j}{j}$

$c_2 = 0{,}25j \qquad c_1 = -0{,}25j$

$h_a(t) = -j0{,}25e^{(-3+j2)t} + j0{,}25e^{(-3-j2)t}$

$\rightarrow h_a'(t) = -j0{,}25e^{(-3+j2)t} \cdot (-3+j2) + j0{,}25e^{(-3-j2)t} \cdot (-3-j2)$

$h(t) = b_0h_a''(t) + b_1h_a'(t) + b_2h_a(t)$

$h(t) = -j0{,}25e^{(-3+j2)t}(-3+j2) + j0{,}25e^{(-3-j2)t}(-3-j2) - 4j \cdot 0{,}25e^{(-3+j2)t} + 4j \cdot 0{,}25e^{(-3-j2)t}$

$h(t) = j0{,}75e^{(-3+j2)t} + 0{,}5e^{(-3+j2)t} - j0{,}75e^{(-3-j2)t} + 0{,}5e^{(-3-j2)t} - je^{(-3+j2)t} + je^{(-3-j2)t}$

$$\boxed{h(t) = -j0{,}25e^{(-3+j2)t} + j0{,}25e^{(-3-j2)t} + 0{,}5e^{(-3+j2)t} + 0{,}5e^{(-3-j2)t}}$$

12. Koristimo svojstvo iz prethodnog zadatka

$$Y(t) = \cos\omega_0(t)\mu(t) = T\{\mu(t)\}$$

$$T\{\delta(t)\} = ?$$

S obzirom da je sustav linearan, te s obzirom da je $\delta(t) = \frac{d}{dt}(\mu(t)) \rightarrow$

$$T\{\delta(t)\} = \frac{d}{dt}(\cos\omega_0 t)\mu(t)) =$$

$$= -\omega_0 \sin\omega_0 t\, \mu(t) + \delta(t)$$

13.

$$y(t) = \int_{-\infty}^{t} u(\tau)d\tau \rightarrow$$

a) $\int_{-\infty}^{t} \delta(\tau)d\tau = \mu(t)$

b) stabilnost sustava $\rightarrow$ dva uvjeta.

i) $\rightarrow$ impulsni odziv u beskonačnosti teži u $1 \neq 0 \rightarrow$ marginalna stabilnost.

ii) $y(t) = \int_{-\infty}^{t} u(\tau)d\tau \;/\frac{d}{dt} \rightarrow y' = u$

$\rightarrow$ vlastita frekv. $s = 0 \rightarrow$ marg. stab.

14.

$$y(t) = \int_0^1 u(t-h)\,dh, \quad \forall t \in \mathbb{R}$$

a) impulsni odziv $\longrightarrow$

$$h(t) = \int_0^1 \delta(t-h)\,dh, \quad \begin{aligned} t-h &= k \\ dh &= -dk \end{aligned}$$

$$h(t) = -\int_t^{t-1} \delta(k)\,dk = \int_{t-1}^{t} \delta(k)\,dk =$$

$$= \int_0^t \delta(k)\,dk - \int_0^{t-1} \delta(k)\,dk =$$

$$= \mu(t) - \mu(t-1)$$

b) $u(t) = \sin\frac{\pi}{2}t$

$$y = \int_0^1 \sin\frac{\pi}{2}(t-h)\,dh = \quad \left| \begin{aligned} \frac{\pi}{2}(t-h) &= k \\ dh &= -\frac{2}{\pi}dk \end{aligned} \right.$$

$$= -\frac{2}{\pi}\int_{\frac{\pi}{2}t}^{\frac{\pi}{2}(t-1)} \sin k\,dk = +\frac{2}{\pi}\cos k \Big|_{\frac{\pi}{2}t}^{\frac{\pi}{2}(t-1)}$$

$$= \frac{2}{\pi}\left[\cos\frac{\pi}{2}t - \cos\left(\frac{\pi}{2}t - \frac{\pi}{2}\right)\right]$$

$$= \frac{2}{\pi}\left[\cos\frac{\pi}{2}t - \sin\frac{\pi}{2}t\right]$$