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1. $y'(t) + 5y(t) + 2 = u(t) \rightarrow y'(t) = u(t) - 5y(t) - 2$

$u(t) = \alpha u_1(t) + \beta u_2(t)$

$y_1'(0) = u_1(0) - 2$
$y_2'(0) = u_2(0) - 2$

$y'(0) = u(0) - 2$
$y'(0) = \alpha u_1(0) + \beta u_2(0) - 2$
$y'(0) = \alpha y_1'(0) + \beta y_2'(0) + 2 \rightarrow$ SUSTAV NIJE LINEARAN.

U SLUČAJU DA JE SUSTAV LINEARAN, I JOŠ K TOMU I VREMENSKI NEPROMJENJIV (LTI), MOGLI BISMO PRONAĆI ODZIV NA BILO KOJU POBUDU u_2 KOJA SE MOŽE PRIKAZATI KAO LINEARNA KOMBINACIJA IZ PRVOG SIGNALA u_1 (MORAMO ZNATI ODZIV NA POBUDU u_1)

2. $y'''(t) - y''(t) + y'(t) + 39y(t) = u''(t) + 2u(t)$

- TRAŽENJE VLASTITIH FREKVENCIJA

$$y_h(t) = Ce^{st}$$

$$Ce^{st}(s^3 - s^2 + s + 39) = 0, \quad Ce^{st} \neq 0$$

$$s^3 - s^2 + s + 39 = 0$$

JEDNO RJEŠENJE MORAMO POGODITI, A DRUGA DVA ĆEMO DOBITI RJEŠAVANJEM KVADRATNE JEDNADŽBE KOJU DOBIJEMO SLJEDEĆIM POSTUPKOM

$$(s^3 - s^2 + s + 39) : (s - s_0) = a_0 s^2 + a_1 s + a_2$$

POGOĐENO RJEŠENJE: $s_1 = -3$

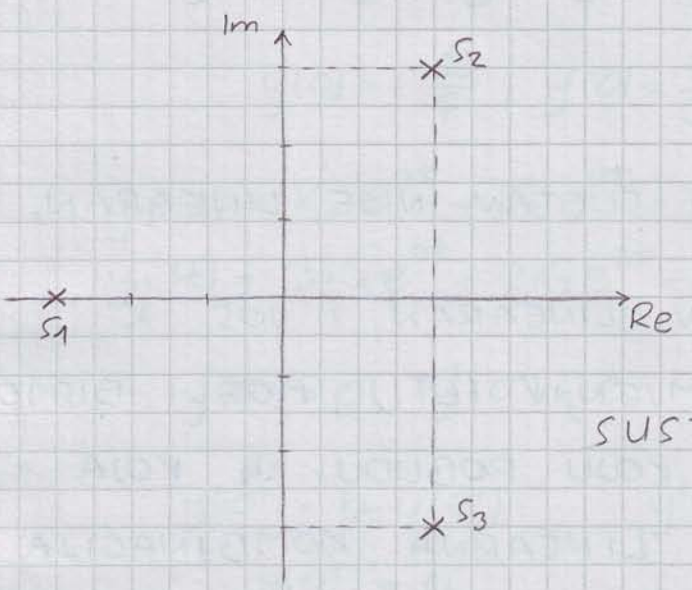
$$(s^3 - s^2 + s + 39) : (s+3) = s^2 - 4s + 13$$

$$\begin{array}{l}
\underline{-s^3 \pm 3s^2} \\
\quad -4s^2 + s \\
\quad \underline{\mp 4s^2 \mp 12s} \\
\qquad 13s + 39 \\
\qquad \underline{-13s \pm 39} \\
\qquad\quad 0
\end{array}$$

$$s^2 - 4s + 13 = 0 \qquad s_{2,3} = \frac{4 \pm \sqrt{16-52}}{2}$$

$$s_{2,3} = \frac{4 \pm j6}{2}$$

$$s_2 = 2 + j3, \quad s_3 = 2 - j3$$



SUSTAV JE NESTABILAN.

3. $y'(t) + y(t) = u'(t) + 2u$

$u(t) = 3\mu(t)$

- OPĆA HOMOGENA JEDNADŽBA

$$y_h(t) = Ce^{st}$$

$$Ce^{st}(s+1) = 0, \quad Ce^{st} \neq 0$$

$$s + 1 = 0 \rightarrow s = -1$$

$$y_h(t) = C_1 e^{-t}$$

- PARTIKULARNA JEDNADŽBA

$$u(t) = 3\mu(t) \rightarrow y_p(t) = K$$

$$K = 6, \ t > 0 \rightarrow y_p(t) = 6, \ t > 0$$

$y(t) = y_h(t) + y_p(t)$

$y(t) = C_1 e^{-t} + 6, \; t > 0$

(a) $y(0^-) = 9$

$y(0^+) = b_0 u(0^+) + y(0^-)$

$y(0^+) = 12$

$y(0) = C_1 + 6 = 12$

$C_1 = 6$

$y(t) = (6e^{-t} + 6)\,\mu(t)$

(b) $y(0^+) = 9$

$y(0) = C_1 + 6 = 9$

$C_1 = 3$

$y(t) = (3e^{-t} + 6)\,\mu(t)$

4. $y_h(t) = \frac{1}{2} e^{3t} + \frac{5}{3} e^{2t}$

$u(t) = \frac{5}{2} e^{2t}, \; t \geq 0$

$s_1 = 3, \; C_1 = \frac{1}{2}$

$s_2 = 2, \; C_2 = \frac{5}{3}$

$y''(t) + a_1 y'(t) + a_0 y(t) = b_2 u(t)$

$(s-3)(s-2) = 0$

$s^2 - 5s + 6 = 0 \rightarrow a_1 = -5$

$a_0 = 6$

$y''(t) - 5y'(t) + 6y(t) = u(t)$

$u(t) = \frac{5}{2} e^{2t} \rightarrow y_p(t) = A t e^{2t}$

$y_p'(t) = A e^{2t}(1 + 2t)$

$y_p''(t) = A e^{2t}(4 + 4t)$

$$Ae^{2t}(4+4t-5-10t+6t) = \frac{5}{2}e^{2t}$$

$$A = -\frac{5}{2} \rightarrow y_p(t) = -\frac{5}{2}te^{2t}$$

$$y(t) = \frac{1}{2}e^{3t} + \frac{5}{3}e^{2t} - \frac{5}{2}te^{2t}$$
$$y'(t) = \frac{3}{2}e^{3t} + \frac{10}{3}e^{2t} - \frac{5}{2}e^{2t}(1+2t)$$

$$y(0) = \frac{1}{2} + \frac{5}{3} = -\frac{13}{6}$$
$$y'(0) = \frac{3}{2} + \frac{10}{3} - \frac{5}{2} = \frac{7}{3}$$

$$y(0) = -\frac{13}{6}, \quad y'(0) = \frac{7}{3}$$

$$y_m(t) = K_1e^{3t} + K_2e^{2t} - \frac{5}{2}te^{2t}$$
$$y_m'(t) = 3K_1e^{3t} + 2K_2e^{2t} - \frac{5}{2}e^{2t}(1+2t)$$

$$y(0^-) = 0, \quad y'(0^-) = 0$$

$$y(0^+) = b_0u(0^+) \qquad y'(0^+) = b_0u'(0^+) + b_1u(0^+)$$
$$y(0^+) = 0 \qquad y'(0^+) = 0$$

$$\left.\begin{aligned} y_m(0) &= K_1 + K_2 = 0 \\ y_m'(0) &= 3K_1 + 2K_2 - \frac{5}{2} = 0 \end{aligned}\right\} K_1 = \frac{5}{2}, \quad K_2 = -\frac{5}{2}$$

$$y_m(t) = \frac{5}{2}e^{3t} - \frac{5}{2}e^{2t} - \frac{5}{2}te^{2t}, \quad t \geq 0$$

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5. $y'(t) + 2y(t) = u(t)$

$u(t) = A\cos(w_0 t)\cdot \mu(t)$

$y(0^-) = 0 \;,\; b_0 = 0 \quad b_1 = 1$

$y(0^+) = b_0 u(0^+) + y(0^-)$

$y(0^+) = 0$

- OPĆA HOMOGENA JEDNADŽBA

$y_h(t) = Ce^{st}$

$Ce^{st}(s+2) = 0 \;,\; Ce^{st} \neq 0$

$s + 2 = 0 \rightarrow s = -2$

$y_h(t) = C_1 e^{-2t}$

- PARTIKULARNA JEDNADŽBA

$u(t) = A\cos(w_0 t) \rightarrow y_p(t) = K_1\cos(w_0 t) + K_2 \sin(w_0 t)$

$y_p'(t) = -w_0 K_1 \sin(w_0 t) + w_0 K_2 \cos(w_0 t)$

$-w_0 K_1 \sin(w_0 t) + w_0 K_2 \cos(w_0 t) + 2K_1\cos(w_0 t) + 2K_2 \sin(w_0 t) = A\cos(w_0 t)$

$\cos(w_0 t)(w_0 K_2 + 2K_1) + \sin(w_0 t)(-w_0 K_1 + 2K_2) = A\cos(w_0 t)$

$$\left.\begin{aligned} w_0 K_2 + 2K_1 &= A \\ -w_0 K_1 + 2K_2 &= 0 \end{aligned}\right\} \quad K_1 = \frac{2}{4+w_0^2}A \;,\; K_2 = \frac{w_0}{4+w_0^2}A$$

$$y_p(t) = \frac{2}{4+w_0^2}A\cos(w_0 t) + \frac{w_0}{4+w_0^2}A\sin(w_0 t)$$

- TOTALNI ODZIV

$$y(t) = y_h(t) + y_p(t)$$

$$y(t) = C_1 e^{-2t} + \frac{2}{4+w_0^2} A\cos(w_0 t) + \frac{w_0}{4+w_0^2} A\sin(w_0 t)$$

$$y(0) = C_1 + \frac{2}{4+w_0^2} A = 0$$

$$C_1 = -\frac{2}{4+w_0^2} A$$

$$y(t) = \frac{-2}{4+w_0^2} A e^{-2t} + \frac{2}{4+w_0^2} A\cos(w_0 t) + \frac{w_0}{4+w_0^2} A\sin(w_0 t)$$

- ODZIV NA POBUDU $u_1(t) = B\cos(w_0(t-1))\,\mu(t-1)$

KORISTIMO ČINJENICU DA JE SUSTAV LTI.

$$y_1(t) = y(t-1)$$

$$y_1(t) = \frac{-2}{4+w_0^2} B e^{-2(t-1)} + \frac{2}{4+w_0^2} B\cos(w_0(t-1)) + \frac{w_0}{4+w_0^2} A\sin(w_0(t-1))$$

6. $y'(t) - y(t) = -u(t)$

$u(t) = \mu(-t) \rightarrow u(t) = 1,\ t \leq 0$

- OPĆA HOMOGENA JEDNADŽBA

$$y_h(t) = Ce^{st}$$

$$Ce^{st}(s-1) = 0,\quad Ce^{st} \neq 0$$

$$s-1 = 0 \rightarrow s = 1$$

$$y_h(t) = C_1 e^{t}$$

- PARTIKULARNA JEDNADŽBA

$$u(t) = \mu(-t) \rightarrow y_p(t) = K$$

$$K = 1,\ t<0 \rightarrow y_p(t) = 1,\ t<0$$

$y(t) = y_h(t) + y_p(t)$ $\qquad$ $y(t) = y_h(t)$

$y(t) = C_1 e^{t} + 1, \quad t < 0$ $\qquad$ $y(t) = C_1 e^{t}, \quad t > 0$

(a) $y(0^+) = 0 \rightarrow y(0^-) = y(0^+) = 0$

$y(0^-) = C_1 + 1 = 0$

$C_1 = -1$

$y(t) = -e^{t} + 1, \quad t < 0$

$y(0^+) = C_1 = 0$

$C_1 = 0$

$y(t) = 0, \quad t > 0$

KONAČNO:

$y(t) = (1 - e^{t})\,\mu(-t)$

(b) $y(0^+) = 1 \rightarrow y(0^-) = y(0^+) = 0$

$y(0^-) = C_1 + 1 = 1$

$C_1 = 0$

$y(t) = 1, \quad t < 0$

$y(0^+) = C_1 = 1$

$C_1 = 1$

$y(t) = e^{t}, \quad t > 0$

KONAČNO:

$y(t) = e^{t} + (1 - e^{t})\,\mu(t)$

7. $y'(t) + 2y(t) = u'(t) + u(t)$

$h(t) = -e^{-2t}\mu(t) + \delta(t)$

$h'(t) = e^{-2t}\left(-\delta(t) + 2\mu(t)\right) + \delta'(t)$

$-e^{-2t}\delta(t) + 2e^{-2t}\mu(t) + \delta'(t) - 2e^{-2t}\mu(t) + 2\delta(t) = \delta'(t) + \delta(t)$

$\delta(t)\left(2 - e^{-2t}\right) + \delta'(t) = \delta(t) + \delta'(t)$

u $t = 0 \rightarrow \delta(t) + \delta'(t) = \delta(t) + \delta'(t)$

8. $y''(t) + 2y'(t) + y(t) = u(t)$

$y(0^-) = y'(0^-) = 0$

$y(0^+) = y(0^-) = 0$

$y'(0^+) = y'(0^-) = 0$

$y''(t) = u(t) - 2y'(t) - y(t)$

$y''(0) = u(0) - 2y'(0) - y(0)$

$y''(0) = u(0)$

$u(t) = \mathcal{L}\, u_1(t) + \mathcal{B} u_2(t)$

$y_1''(0) = \mathcal{L} u_1(0)$, $y_2''(0) = \mathcal{B} u_2(0)$

$y''(0) = \mathcal{L} u_1(0) + \mathcal{B} u_2(0)$

$y''(0) = \mathcal{L} y_1''(0) + \mathcal{B} y_2''(0) \rightarrow$ SUSTAV JE LINEARAN

- OPĆA HOMOGENA JEDNADŽBA

$y_h(t) = C e^{st}$

$C e^{st}(s^2 + 2s + 1) = 0$, $C e^{st} \neq 0$

$s^2 + 2s + 1 = 0 \rightarrow s_{1,2} = -1$

$y_h(t) = (C_1 + C_2 t) e^{-t}$

(a) $u(t) = t\mu(t) \rightarrow u(t) = t$, $t \geq 0$

$y_p(t) = K_0 + K_1 t$, $y_p'(t) = K_1$, $y_p''(t) = 0$

$2K_1 + K_0 + K_1 t = t$

$$\left.\begin{aligned} 2K_1 + K_0 &= 0 \\ K_1 &= 1 \end{aligned}\right\} \quad K_1 = 1 , \; K_0 = -2$$

$y_p(t) = -2 + t$, $t \geq 0$

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$$y(t) = y_n(t) + y_p(t)$$
$$y(t) = (C_1 + C_2 t)e^{-t} - 2 + t$$
$$y'(t) = (-C_1 + C_2 - C_2 t)e^{-t} + 1$$

$$y(0) = C_1 - 2 = 0$$
$$C_1 = 2$$
$$y'(0) = -C_1 + C_2 + 1 = 0$$
$$C_2 = 1$$

$$y(t) = (2+t)e^{-t} - 2 + t, \quad t \geq 0$$

(b) $u_b(t) = \mu(t)$

$$u_a(t) = t\mu(t) \rightarrow u_a'(t) = \mu(t) + t\delta(t)$$
$$t\delta(t) = 0, \ \forall t$$
$$u_a'(t) = \mu(t) = u_b(t)$$

ZBOG LINEARNOSTI SUSTAVA MOŽEMO PISATI

$$y_b(t) = y_a'(t)$$
$$y(t) = (-1-t)e^{-t} + 1, \quad t \geq 0$$

(c) $u_c(t) = \delta(t)$

$$u_b(t) = \mu(t) \rightarrow u_b'(t) = \delta(t) = u_c(t)$$

$$y_c(t) = y_b'(t)$$
$$y(t) = te^{-t}, \quad t \geq 0$$

(d) $u_d(t) = t\mu(t) + \mu(t) + \delta(t)$

$$u_d(t) = u_a(t) + u_b(t) + u_c(t) \rightarrow y_d(t) = y_a(t) + y_b(t) + y_c(t)$$
$$y(t) = (1+t)e^{-t} - 1 + t, \quad t \geq 0$$

9. $y'(t) + a y(t) = u(t)$

$y(0^-) = 0 \rightarrow y(0^+) = 0$

- OPĆA HOMOGENA JEDNADŽBA

$y_h(t) = Ce^{st}$

$Ce^{st}(s+a) = 0, \quad Ce^{st} \neq 0$

$s + a = 0 \rightarrow s = -a$

$y_h(t) = C_1 e^{-at}$

(a) $u(t) = \delta(t)$

$y'(t) + a_1 y(t) = b_0 u'(t) + b_1 u(t)$

$a_1 = a, \quad b_1 = 1$

$b_0 = 0$

$h_A'(t) + a h_A(t) = \delta(t)$

$h_A(0^+) = 1$

$h_A'(t) + a h_A(t) = 0$

$Ce^{st}(s+a) = 0, \quad Ce^{st} \neq 0$

$s + a = 0 \rightarrow s = -a$

$h_A(t) = C_1 e^{-at}$

$h_A(0^+) = C_1 = 1$

$C_1 = 1$

$h_A(t) = e^{-at}$

$h(t) = \sum_{m=0}^{M} (b_{N-m} D^m) h_A(t) \qquad M = 0, \; N = 1$

$h(t) = e^{-at}, \; t \geq 0$

(b) $u(t) = \mu(t) \rightarrow y_p(t) = A$

$$y_p'(t) = 0$$

$$aA = 1$$

$$A = \frac{1}{a},\ t \geq 0 \rightarrow y_p(t) = \frac{1}{a},\ t \geq 0$$

$$y(t) = y_h(t) + y_p(t)$$

$$y(t) = C_1 e^{-at} + \frac{1}{a}$$

$$y(0^+) = C_1 + \frac{1}{a} = 0$$

$$C_1 = -\frac{1}{a}$$

$$y(t) = \frac{1}{a}(1 - e^{-at}),\ t \geq 0$$

(c) $u(t) = \mu(t)$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau$$

$$y(t) = \int_{-\infty}^{\infty} e^{-a\tau} \mu(\tau) \cdot \mu(t-\tau)\, d\tau$$

$$y(t) = \left\{ \int_0^t e^{-a\tau} d\tau \right\} \mu(t)$$

$$y(t) = \frac{1}{a}(1 - e^{-at})\, \mu(t)$$

(d) $u_a(t) = \delta(t)$

$u_b(t) = \mu(t) \rightarrow u_a(t) = u_b'(t)$

SUSTAV JE LINEARAN TE STOGA MOŽEMO PISATI

$$y_a(t) = y_b'(t)$$

$$y(t) = e^{-at}\mu(t)$$

10. $y'(t) + 2y(t) = u'(t) + u(t)$

$N = M = 1$

$a_1 = 2, \; b_0 = b_1 = 1$

$u(t) = \delta(t)$

$h_A'(t) + 2h_A(t) = \delta'(t) + \delta(t)$

$h_A'(t) + 2h_A(t) = 0$

$Ce^{st}(s+2) = 0 \; , \; Ce^{st} \neq 0$

$s + 2 = 0 \rightarrow s = -2$

$h_A(t) = C_1 e^{-2t} \; , \; h_A(0^+) = 1$

$h_A(0^+) = C_1 = 1$

$C_1 = 1$

$h_A(t) = e^{-2t} \; , \; t \geq 0$

$$h(t) = \sum_{m=0}^{M} (b_{N-m} D^m) h_A(t) + \delta(t)$$

$$h(t) = b_1 D^0 h_A(t) + b_0 D^1 h_A(t) + \delta(t)$$

$$h(t) = h_A'(t) + h_A(t) + \delta(t)$$

$$h(t) = -e^{-2t} + \delta(t) \; , \; t \geq 0$$

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11. $y''(t) + 6y'(t) + 13y(t) = u'(t) + 4u(t)$

$N = 2,\ M = 1$

$a_1 = 6,\ a_2 = 13$

$b_0 = 0,\ b_1 = 1,\ b_2 = 4$

$h_A''(t) + 6h_A'(t) + 13h_A(t) = \delta'(t) + 4\delta(t)$

$h_A(0^+) = 0,\ h_A'(0^+) = 1$

$h_A''(t) + 6h_A'(t) + 13h_A(t) = 0$

$Ce^{st}(s^2 + 6s + 13) = 0,\ Ce^{st} \neq 0$

$s^2 + 6s + 13 = 0 \rightarrow s_{1,2} = -3 \pm j2$

$s_1 = -3 + j2,\ s_2 = -3 - j2$

$h_A(t) = Ae^{-3t}e^{+j2t} + Be^{-3t}e^{-j2t}$

$h_A(t) = e^{-3t}(Ae^{+j2t} + Be^{-j2t})$

$h_A(t) = e^{-3t}(C_1\cos(2t) + C_2\sin(2t))$

$h_A'(t) = e^{-3t}(C_1(-3\cos(2t) - 2\sin(2t)) + C_2(-3\sin(2t) + 2\cos(2t)))$

$$\left.\begin{aligned} h_A(0) &= C_1 = 0 \\ h_A'(0) &= 2C_2 = 1 \end{aligned}\right\} C_1 = 0,\ C_2 = \frac{1}{2}$$

$h_A(t) = \frac{1}{2}e^{-3t}\sin(2t)$

$h_A'(t) = e^{-3t}\left(\cos(2t) - \frac{3}{2}\sin(2t)\right)$

$h(t) = b_2 h_A(t) + b_1 h_A'(t)$

$h(t) = e^{-3t}\left(\cos(2t) + \frac{1}{2}\sin(2t)\right),\ t \geq 0$

12. $u_1(t) = \mu(t)$

$y_1(t) = \cos(w_0 t)\,\mu(t)$

$u_2(t) = \delta(t)$

$$u_2(t) = u_1'(t) \xrightarrow{LTI} y_2(t) = y_1'(t)$$

$$y_2(t) = -w_0 \sin(w_0 t)\,\mu(t) + \delta(t)$$

$$h(t) = -w_0 \sin(w_0 t)\,\mu(t) + \delta(t)$$

13. $y(t) = \int_{-\infty}^{t} u(\tau)\,\delta(\tau)$

(a) $u(t) = \delta(t)$

$$h(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$$

$$h(t) = \mu(t)$$

(b) $y'(t) = u(t)$

$s = 0 \rightarrow$ RUBNO STABILAN

15. $y(t) = \int_{0}^{1} u(t-h)\,dh, \ \forall t \in \mathbb{R}$

SUPSTITUCIJA:

$t - h = \tau \rightarrow h = t - \tau$

$dh = -d\tau$

GRANICE:

$h = 1 \rightarrow t - \tau = 1$ $\quad$ $h = 0 \rightarrow t - \tau = 0$

$\tau = t - 1$ $\quad$ $\tau = t$

$$y(t) = \int_{t}^{t-1} -u(\tau)\,d\tau$$

$$y(t) = \int_{t-1}^{t} u(\tau)\,d\tau$$

(a) $u(t) = \delta(t)$

$$h(t) = \int_{t-1}^{t} \delta(\tau)\,d\tau = \int_{0}^{t} \delta(\tau)\,d\tau + \int_{t-1}^{0} \delta(\tau)\,d\tau$$

$$h(t) = \int_{0}^{t} \delta(\tau)\,d\tau - \int_{0}^{t-1} \delta(\tau)\,d\tau$$

$$h(t) = \mu(t) - \mu(t-1)$$

(b) $u(t) = \sin(\frac{\pi}{2}t), \ \forall t \in \mathbb{R}$

$$y(t) = \int_{t-1}^{t} \sin(\tfrac{\pi}{2}t)\,dt$$

$$y(t) = \frac{2}{\pi} \cos(\tfrac{\pi}{2}t)\Big|_{t}^{t-1}$$

$$y(t) = \frac{2}{\pi}\left(\cos(\tfrac{\pi}{2}t - \tfrac{\pi}{2}) - \cos(\tfrac{\pi}{2}t)\right)$$

$$y(t) = \frac{2}{\pi}\left(\sin(\tfrac{\pi}{2}t) - \cos(\tfrac{\pi}{2}t)\right)$$