9. tjedan – dodatni zadaci

1. zadatak

$$y(t) = \int_{-\infty}^{t} u(\tau)d\tau$$

1) Linearnost?

$$u(t) = \alpha u_1(t) + \beta u_2(t)$$

$$y(t) = \int_{-\infty}^{t} (\alpha u_1(\tau) + \beta u_2(\tau))d\tau = \alpha \int_{-\infty}^{t} u_1(\tau)d\tau + \beta \int_{-\infty}^{t} u_2(\tau)d\tau =$$
$$= \alpha y_1(t) + \beta y_2(t) \rightarrow \boldsymbol{linearan}$$

2) Vremenski stalan?

$$y_1(t) = \int_{-\infty}^{t} u(\tau - T)d\tau = \left| \begin{matrix} \tau - T = a \rightarrow d\tau = da \\ donja\ gr. = -\infty \\ gornja\ gr. = t - T \end{matrix} \right| = \int_{-\infty}^{t-T} u(a)da$$

$$y_2(t - T) = \int_{-\infty}^{t-T} u(\tau)d\tau$$

$$y_1(t) = y_2(t - T) \rightarrow \boldsymbol{vremenski\ stalan}$$

3) Memorijski?

Da! (Integrator)

4) Kauzalnost?

Kauzalan je.

2. zadatak

$$y(t) = \int_{0}^{t} u(\tau)d\tau$$

1) Linearnost?

$$u(t) = \alpha u_1(t) + \beta u_2(t)$$

$$y(t) = \int_{0}^{t} (\alpha u_1(\tau) + \beta u_2(\tau))d\tau = \alpha \int_{0}^{t} u_1(\tau)d\tau + \beta \int_{0}^{t} u_2(\tau)d\tau =$$
$$= \alpha y_1(t) + \beta y_2(t) \rightarrow \boldsymbol{linearan}$$

2) Vremenski stalan?

$$y_1(t) = \int_{0}^{t} u(\tau - T)d\tau = \left| \begin{matrix} \tau - T = a \rightarrow d\tau = da \\ donja\ gr. = -T \\ gornja\ gr. = t - T \end{matrix} \right| = \int_{-T}^{t-T} u(a)da$$

$$y_2(t - T) = \int_{0}^{t-T} u(\tau)d\tau$$

$$y_1(t) \neq y_2(t - T) \rightarrow \boldsymbol{nije\ vremenski\ stalan}$$

3) Memorijski?

Da! (Integrator)

4) Kauzalnost?

Kauzalan je.

3. zadatak

$$y(n) = \left(\frac{1}{2}\right)^n u(3n+2)$$

1) Linearnost?

$$u(n) = \alpha u_1(n) + \beta u_2(n)$$

$$\begin{aligned} y(n) = \left(\frac{1}{2}\right)^n &\left(\alpha u_1(3n+2) + \beta u_2(3n+1)\right) \\ &= \alpha\left[\left(\frac{1}{2}\right)^n u_1(3n+2)\right] + \beta\left[\left(\frac{1}{2}\right)^n u_2(3n+2)\right] \\ &= \alpha y_1(n) + \beta y_2(n) \rightarrow \boldsymbol{linearan} \end{aligned}$$

2) Vremenski stalan?

$$y_1(n) = \left(\frac{1}{2}\right)^n u(3n+2-M)$$

$$y_2(n-M) = \left(\frac{1}{2}\right)^{n-M} u(3(n-M)+2)$$

$$y_1(n) \neq y_2(n-M) \rightarrow \boldsymbol{nije\ vremenski\ stalan}$$

3) Memorijski?

Da! Recimo, za n = 3, izlaz y(3) ovisi o ulazu u(11).

4) Kauzalnost?

Nije kauzalan. Ovisi o budućim stanjima.

4. zadatak

$$y(t) = \frac{u(t)}{1 + u(t-1)}$$

1) Linearnost?

$$u(t) = \alpha u_1(t) + \beta u_2(t)$$

$$y(t) = \frac{\alpha u_1(t) + \beta u_2(t)}{1 + \alpha u_1(t-1) + \beta u_2(t-1)} \neq \alpha y_1(t) + \beta y_2(t)$$

$$\rightarrow \boldsymbol{nije\ linearan}$$

2) Vremenski stalan?

$$y_1(t) = \frac{u(t-T)}{1 + u(t-1-T)}$$

$$y_2(t-T) = \frac{u(t-T)}{1 + u(t-T-1)}$$

$$y_1(t) = y_2(t-T) \rightarrow \boldsymbol{vremenski\ stalan}$$

3) Memorijski?

Da! Npr, za t = 2, y(2) ovisi o u(2) i o u(1).

4) Kauzalnost?

Kauzalan je. Ne ovisi o budućim stanjima.

5. zadatak

$$y(t) = u^2(t)$$

1) Linearnost?

$$u(t) = \alpha u_1(t) + \beta u_2(t)$$

$$y(t) = \big(\alpha u_1(t) + \beta u_2(t)\big)^2 \neq \alpha y_1(t) + \beta y_2(t)$$

$$\rightarrow \boldsymbol{nije\ linearan\ (kvadriranje\ ulaza)}$$

2) Vremenski stalan?

$$y_1(t) = u^2(t - T)$$

$$y_2(t - T) = u^2(t - T)$$

$$y_1(t) = y_2(t - T) \rightarrow \boldsymbol{vremenski\ stalan}$$

3) Memorijski?

Ne! Izlazi u trenutku t ovisi samo o ulazu u trenutku t.

4) Kauzalnost?

Kauzalan je.

5) Inverz?

$$y(t) = u^2(t)$$

$$u(t) = y^2(t)$$

$$y(t) = \pm\sqrt{u(t)}$$

Nema inverz.

6. zadatak

$$y(n) = \sum_{k=-\infty}^{n} u(k)$$

1) Linearnost?

$$u(n) = \alpha u_1(n) + \beta u_2(n)$$

$$y(n) = \sum_{k=-\infty}^{n} \big(\alpha u_1(k) + \beta u_2(k)\big) = \alpha \sum_{k=-\infty}^{n} u_1(k) + \beta \sum_{k=-\infty}^{n} u_2(k)$$

$$= \alpha y_1(n) + \beta y_2(n) \rightarrow \boldsymbol{linearan}$$

2) Vremenski stalan?

$$y_1(n) = \sum_{k=-\infty}^{n} u(k - M) = \left| \begin{matrix} k - M = a \\ donja\ gr. = -\infty \\ gornja\ gr. = n - M \end{matrix} \right| = \sum_{k=-\infty}^{n-M} u(a)$$

$$y_2(n-M) = \sum_{k=-\infty}^{n-M} u(k)$$

$$y_1(n) = y_2(n-M) \rightarrow \boldsymbol{vremenski\ stalan}$$

3) Memorijski?
Da! Akumulator lol, slično ko integrator.

4) Kauzalnost?
Kauzalan.

5) Inverz?

Ima inverz!

$$y(n) = \sum_{k=-\infty}^{n} u(k) = \sum_{k=-\infty}^{n-1} u(k) + u(n) = y(n-1) + u(n)$$

$$y(n) - y(n-1) = u(n)$$

7. zadatak

$$y(n) = nu(n)$$

1) Linearnost?

$$u(n) = \alpha u_1(n) + \beta u_2(n)$$

$$y(n) = n\big(\alpha u_1(n) + \beta u_2(n)\big) = \alpha\big(nu_1(n)\big) + \beta\big(nu_2(n)\big)$$
$$= \alpha y_1(n) + \beta y_2(n) \rightarrow \boldsymbol{linearan}$$

2) Vremenski stalan?

$$y_1(n) = nu(n-M)$$

$$y_2(n-M) = (n-M)u(n-M)$$

$$y_1(n) \neq y_2(n-M) \rightarrow \boldsymbol{nije\ vremenski\ stalan}$$

3) Memorijski?
Ne!

4) Kauzalnost?
Kauzalan.

5) Inverz?
Nema inverz! Ne znam zašto, ali pretpostavljam možda zbog ovoga:

$$y(n) = nu(n)$$

$$u(n) = ny(n)$$

$$y(n) = \frac{1}{n}u(n)$$

pa moguće da kad je n = 0, da je to onda nedefiniran izraz.