TJEDAN 8. MARKO GULIN
SIGNALI I SUSTAVI 0036428227
GRUPA 2.E2

1. $x(t) = \sin(8000\pi t) + 2\cos(24000\pi t + \frac{\pi}{3}) + \sin(16000\pi t)$

$F_s = 10$ kHz

DA NEBI DOŠLO DO ALIASINGA: $\Omega_s \geq 2\Omega_{SIGNALA}$

$F_s \geq 2F_{SIGNALA}$

$x_1(t) = \sin(8000\pi t)$, $F_1 = 4000$ Hz NO ALIASING!

$x_2(t) = 2\cos(24000\pi t + \frac{\pi}{3})$, $F_2 = 12000$ Hz ALIASING!

$x_3(t) = \sin(16000\pi t)$, $F_3 = 8000$ Hz ALIASING!

2. KOMPONENTA

$x_2(n) = 2\cos(2\pi \cdot \frac{12000}{10000} n + \frac{\pi}{3})$

$x_2(n) = 2\cos(2.4\pi n + \frac{\pi}{3})$

$w_2^{(n)} = w_0^{(2)} + 2k\pi$

$w_0^{(2)} = w_2^{(n)} - 2k\pi \qquad w_0 \in [-\pi, \pi]$

$w_0^{(2)} = 0.4\pi \rightarrow k = 1$

3. KOMPONENTA

$x_3(n) = \sin(2\pi \cdot \frac{8000}{10000} n)$

$x_3(n) = \sin(1.6\pi n)$

$w_3^{(n)} = w_0^{(3)} + 2k\pi$

$w_0^{(3)} = w_3^{(n)} - 2k\pi \qquad w_0 \in [-\pi, \pi]$

$w_0^{(3)} = -0.4\pi \rightarrow k = 1$

KONAČNO:

$x(n) = \sin(0.8\pi n) + 2\cos(0.4\pi n) + \sin(-0.4\pi n)$

$x(n) = \sin(0.8\pi n) + 2\cos(0.4\pi n) + \sin(0.4\pi n + \pi)$

REKONSTRUKCIJA:

$F_1 = 4$ kHz, $F_2 = \frac{0.4\pi}{2\pi} 10$ kHz, $F_3 = \frac{0.4\pi}{2\pi} 10$ kHz

$F_2 = 2$ kHz $\qquad F_3 = 2$ kHz

$x(t) = \sin(8000\pi t) + 2\cos(4000\pi t + \frac{\pi}{3}) + \sin(4000\pi t + \pi)$

2. (a) $x(n) = \delta(n)$

$$X(k) = \sum_{n=0}^{N-1} \delta(n) e^{-j\frac{2\pi}{N}kn} = e^{0} = 1 \quad , \quad k = 0, 1, \dots, N-1$$

(b) $x(n) = \delta(n-n_0)$, $\quad 0 < n_0 < N$

$$X(k) = \sum_{n=0}^{N-1} \delta(n-n_0) e^{-j\frac{2\pi}{N}kn} = e^{-j\frac{2\pi}{N}kn_0}, \quad k = 0, 1, \dots, N-1$$

3. (a) $x(n) = \mu(n) - \mu(n-N)$

$$X(k) = \sum_{n=0}^{N-1} (\mu(n) - \mu(n-N)) e^{-j\frac{2\pi}{N}kn}$$

$$X(k) = \sum_{n=0}^{N-1} e^{-j\frac{2\pi}{N}kn}, \quad i := e^{-j\frac{2\pi}{N}k}$$

$$X(k) = \sum_{n=0}^{N-1} i^n = \frac{1-i^N}{1-i}$$

$$X(k) = \frac{1-e^{-j2\pi k}}{1-e^{-j\frac{2\pi}{N}k}} = 0 \quad , \quad k \neq 0$$

za $k = 0 \rightarrow X(k) = \sum_{n=0}^{N-1} e^{0} = \sum_{n=0}^{N-1} 1 = N-1+1 = N$

(b) $x(n) = \mu(n) - \mu(n-n_0)$, $\quad 0 < n_0 < N$

$$X(k) = \sum_{n=0}^{N-1} (\mu(n) - \mu(n-n_0)) e^{-j\frac{2\pi}{N}kn}$$

$$X(k) = \sum_{n=0}^{n_0-1} e^{-j\frac{2\pi}{N}kn}, \quad i := e^{-j\frac{2\pi}{N}k}$$

$$X(k) = \sum_{n=0}^{n_0-1} i^n = \frac{1-i^{n_0}}{1-i}$$

$$X(k) = \frac{1-e^{-j\frac{2\pi}{N}kn_0}}{1-e^{-j\frac{2\pi}{N}k}}$$

4. $N = 4$

(a) $x(n) = \cos(\frac{\pi}{2}n)$, $\quad n = 0, 1, 2, 3$

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N}kn}, \quad X(k) = \sum_{n=0}^{3} \cos(\tfrac{\pi}{2}n) e^{-j\frac{\pi}{2}kn}$$

$$X(k) = \cos(0) \cdot e^{0} + \underbrace{\cos(\tfrac{\pi}{2})}_{=0} e^{-jk\frac{\pi}{2}} + \cos(\pi) e^{-j\pi k} + \underbrace{\cos(\tfrac{3\pi}{2})}_{=0} e^{-j\frac{3\pi}{2}k}$$

$$X(k) = 1 - 1 \cdot (\cos(\pi k) - j\sin(\pi k)) = 1 - \cos(k\pi)$$

$$X(0) = 0, \quad X(1) = 2, \quad X(2) = 0, \quad X(3) = 2$$

(b) $x(n) = \left(\frac{1}{2}\right)^n, \quad n = 0, 1, 2, 3$

$$X(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N}kn}, \quad X(k) = \sum_{n=0}^{3} \left(\frac{1}{2}\right)^n \cdot e^{-j\frac{\pi}{2}kn}$$

$$X(k) = 1 \cdot e^0 + \frac{1}{2} e^{-j\frac{\pi}{2}k} + \frac{1}{4} e^{-j\pi k} + \frac{1}{8} e^{-j\frac{3\pi}{2}k}$$

$$X(k) = 1 + \frac{1}{2} e^{-j\frac{\pi}{2}k} + \frac{1}{4} e^{-j\pi k} + \frac{1}{8} e^{-j\frac{3\pi}{2}k}$$

$$X(0) = \frac{15}{8}, \quad X(1) = \frac{3}{4} - \frac{3}{8}j, \quad X(2) = \frac{1}{8}, \quad X(3) = \frac{3}{4} + \frac{3}{8}j$$

5. $X(k) = \frac{3}{4} + \frac{3}{8} e^{-j\frac{\pi}{2}k} - \frac{3}{4} e^{-j\pi k} - \frac{3}{8} e^{-j\frac{3\pi}{2}k}, \quad k = 0, 1, 2, 3$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn}$$

$$X(k) = \sum_{n=0}^{3} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$X(k) = x(0) e^0 + x(1) e^{-j\frac{\pi}{2}k} + x(2) e^{-j\pi k} + x(3) e^{-j\frac{3\pi}{2}k}$$

$$X(k) = \frac{3}{4} e^0 + \frac{3}{8} e^{-j\frac{\pi}{2}k} - \frac{3}{4} e^{-j\pi k} - \frac{3}{8} e^{-j\frac{3\pi}{2}k}$$

$$x(n) = \left\{ \frac{3}{4}, \frac{3}{8}, -\frac{3}{4}, -\frac{3}{8} \right\}$$

6.
$$x(n) = \begin{cases} e^{j\Omega_0 n}, & 0 \le n \le N-1 \\ 0, & \text{inače} \end{cases}$$

(a) $X(e^{jw}) = \sum_{n=0}^{N-1} x(n) e^{-jwn}$

$$X(e^{jw}) = \sum_{n=0}^{N-1} e^{jn(\Omega_0 - w)} = \frac{1 - e^{j(\Omega_0 - w)N}}{1 - e^{j(\Omega_0 - w)}}$$

(b) $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$

$$X(k) = \sum_{n=0}^{N-1} e^{j\Omega_0 n} e^{-jwkn}$$

$$X(k) = \sum_{n=0}^{N-1} e^{jn(\Omega_0 - wk)} = \frac{1 - e^{j(\Omega_0 - wk)N}}{1 - e^{j(\Omega_0 - wk)}}$$

7. $F_s = 20\ kHz \qquad \Delta f = \frac{F_s}{N} = 20\ Hz$

$N = 1000$