SIS - 14.06.2008. MASS B1

Diferencijske jednadžbe

opća - $y(n) + a_1 y(n-1) + a_2 y(n-2) = u(n)$ $\Rightarrow$ 2. reda

$E^0 y(n)$

$E^{(-1)} y(n) = y(n-1)$

Pr.

$y(n-1) + y(n-4) = u(n)$ $\Rightarrow$ DIF. JED. 3. reda

$y(n) + y(n-3) = u(n+1)$

PRIMJER:

$y(n) - 4y(n-2) = u(n)$ $\quad u(n) = n\mu(n) = \begin{cases} n, & n \geq 0 \\ 0, & \text{inače.} \end{cases}$

P.U. $y(-1) = 1$

$y(-2) = 2$

$y(0) - 4y(-2) = u(0)$

$y(0) = 8$

$y(1) - 4y(-1) = u(1)$

$y(1) = 5$

$y(2) - 4y(0) = u(2)$

$y(2) = 34$

- radi odziv sustava u 100-koraku !?

totalni odziv = opće homogeno + partikularno rješenje

* OPĆE HOMOGENO → ovisi samo o sustavu

$y_h(n) = C q^n$ → vrijedi za sve sustave (supstitucija)

$y_h(n-1) = C q^{n-1}$

$y(n) - 4y(n-2) = u(n)$

$y_h(n) - 4y_h(n-1) = 0$

$Cq^n - 4Cq^{n-2} = 0$

neka exp. fja ← $Cq^{n-2}(q^2 - 4) = 0$

$q^2 - 4 = 0$ → karak. jedn. sustava

q^2 → karak. polinom sustava

$q^2 = 4 \rightarrow \left.\begin{matrix} q_1 = 2 \\ q_2 = -2 \end{matrix}\right\}$ vlastite frekv. sustava

$\boxed{q^3 - 1 = 0 \rightarrow q^3 = 1 \rightarrow q = 1}$ → neki primjer.

$y_h(n) = C_1 q_1^n + C_2 q_2^n \rightarrow \boxed{y_h(n) = C_1 2^n + C_2 (-2)^n}$

OPĆA HOMOGENA JED.

$u(n) = \underset{0}{A_0} + \underset{1}{A_1} n$ → to je neki polinom

H: $u(n) = A_0 + A_1 n + A_2 n^2$

$y_p(n) = K_0 + K_1 n$

* PART. RJEŠENJE

$y_p(n) = K_0 + K_1 n \rightarrow y_p(n-2) = K_0 + K_1(n-2)$

$y(n) - 4y(n-2) = u(n)$

$$y_p(n) - 4y_p(n-2) = u(n)$$

$$K_0 + K_1 n - 4\left(K_0 + K_1(n-2)\right) = n$$

$$\underline{K_0} + K_1 n - 4\underline{K_0} - 4K_1 n + \underline{8K_1} = n$$

$$-3K_0 + 8K_1 - 3K_1 n = A_0 + A_1 n \quad , \; A_0 = 0 \; ; \; A_1 = 1$$

$$\left.\begin{aligned} -3K_0 + 8K_1 &= 0 \\ -3K_1 &= 1 \end{aligned}\right\} \; K_0 = -\frac{8}{9} \; , \; K_1 = -\frac{1}{3}$$

$$\boxed{y_p(n) = -\frac{8}{9} - \frac{1}{3}n \quad , \; n \geq 0} \Rightarrow \text{vrijedi samo } n \geq 0$$

* TOTALNI ODZIV

$$y(n) = y_h(n) + y_p(n)$$

$$y(n) = C_1 2^n + C_2(-2)^n - \frac{8}{9} - \frac{1}{3}n$$

$$y(0) = 8 \; , \; y(1) = 5$$

$$\left.\begin{aligned} y(0) &= C_1 + C_2 - \frac{8}{9} = 8 \\ y(1) &= 2C_1 - 2C_2 - \frac{8}{9} - \frac{1}{3} = 5 \end{aligned}\right\} \; C_1 = 6 \; ; \; C_2 = \frac{26}{9}$$

$$\boxed{y(n) = 6 \cdot 2^n + \frac{26}{9}(-2)^n - \frac{8}{9} - \frac{1}{3}n \quad , \; n \geq 0}$$

PRIMJER 2:

$$y(n) - \frac{3}{16}y(n-2) + \frac{1}{32}y(n-3) = 0$$

$\Rightarrow$ nema pobude, nema parti. rj. ☺

$\Rightarrow$ 3. reda

$$y(-1) = 12 \; ; \; y(-2) = 8 \; ; \; y(-3) = 16$$

* OPĆE HOMOGENO RJEŠENJE

$$y_h(n) = Cq^n$$

$$Cq^{n-3}\left(q^3 - \frac{3}{14}q + \frac{1}{32}\right) = 0 \Rightarrow q^3 - \frac{3}{16}q + \frac{1}{32} = 0$$

$$q_1 = \frac{1}{4} \; ; \; q_2 = \frac{1}{4} \; ; \; q_3 = -\frac{1}{2}$$

* $y_h(n) = (C_1 + C_2 n)q_1^n + C_3 q_3^n$

$\boxed{(C_1 + C_2 n + C_3 n^2)\, q^n \Rightarrow \text{za trostruki korjen}}$

$$y_h(n) = (C_1 + C_2 n)\left(\frac{1}{4}\right)^n + C_3\left(-\frac{1}{2}\right)^n \Rightarrow \text{OPĆE HOMOGENO RJ.}$$

$$y(n) = y_h(n)$$

uvrstimo i dobijemo $C_1 = 3$; $C_2 = 1$; $C_3 = -2$

$$\boxed{y(n) = (3+n)\left(\frac{1}{4}\right)^n - 2\left(-\frac{1}{2}\right)^n}$$

PRIMJER 3 : "caka zadatak by Marko"

$$y(n) - 2y(n-1) + y(n-2) = u(n)$$

$$u(n) = n\mu(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$y(-1) = 1 \;, \; y(-2) = 2$$

* OPĆE HOMOG. RJ

$$y_h(n) = C q^n$$

$$C q^{n-2}\left(q^2 - 2q + 1\right) = 0 \to q^2 - 2q + 1 = 0 \to q_1 = 1$$

$$q_2 = 1$$

$$y_h(n) = (C_1 + C_2 n)(1)^n$$

$$\boxed{y_h(n) = C_1 + C_2 n}$$

* PARTIKULARNO RJ.

$$u(n) = A_0 + A_1 n$$

$$y_p(n) = K_0 + K_1 n$$

$$K_0 + K_1 n - 2\left(K_0 + K_1(n-1)\right) + K_0 + K_1(n-2) = n$$

$$\cancel{K_0} + \cancel{K_1 n} - \cancel{2K_0} - \cancel{2K_1 n} + \cancel{2K_1} + \cancel{K_0} + \cancel{K_1 n} - \cancel{2K_1} = n$$

$$n = 0$$

ovo je KRIVO !!!

- ali ovo je točno

$$u(n) = n \to u(n) = n\,(1)^n$$

polinom

$$u(n) = n^M r^n \to y_p(n) = K^n r^n n^{(m)}$$ -> koliko se puta r pojavlju kod

$$y_p(n) = (K_0 + K_1 n)\, n^2$$

$$K_0 = \frac{1}{2} \qquad K_1 = \frac{1}{6}$$

- objašnjenje zašto je to tako !

POBUDA	PARTIK. RJ.
A	K
n^m	$K_0 + K_1 n + \ldots + K_M n^M$
$A \cdot r^n$	$K \cdot r^n \cdot n^{m}$ → koliko je puta r frekvencija sustava

<u>npr. primjera</u>

$q_{1,2} = 1 \;,\; q_3 = 2$

* $u(n) = 5\,\mu(n) = 5\,(1)^n \mu(n)$

$$5 r^n \rightarrow K r^n \cdot n^m = K \cdot n^2$$

ili

$q_{1,2} = 1 \;,\; q_3 = 2$

$u(n) = 5 \cdot (2)^n$

$$y_p(n) = K\,(2)^n\, n^m = K \cdot (2)^n \cdot n$$

ili

$q_{1,2} = 1 \;,\; q_3 = 2$

$u(n) = 5 \cdot (5)^n$

$$y_p(n) = K \cdot (5)^n \cdot n^m$$

da je pisalo

$$\mu(n) = 5n$$

$$q_{1,2} = 1 \; ; \; q_3 = 2$$

$$\mu(n) = 5n \cdot (1)^n$$

$$y_p(n) = (k_0 + k_1 n) \cdot (1)^n \cdot n^m \Rightarrow y_p(n) = (k_0 + k_1)\, n^2$$

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povratak na zadatak

$$k_0 = \frac{1}{2} \; ; \; k_1 = \frac{1}{6}$$

$$\boxed{y_p(n) = \left(\frac{1}{2} + \frac{1}{6}n\right) n^2 \; , \; n \geq 0}$$

$$y(n) = C_1 + C_2 n + \left(\frac{1}{2} + \frac{1}{6}n\right) n^2 \; , \; n \geq 0$$

$$y(0) = 0 \; ; \; y(1) = 0$$

$$C_1 = 0 \; ; \; C_2 = -\frac{2}{3}$$

* VAŽNI PRIMJER

$$q_{1,2} = \frac{-b \pm \sqrt{\quad}\; j}{2a}$$

$$q_1 = 3 + j4 \qquad q_2 = 3 - j4$$

$$y_h(n) = C_1 (3+j4)^n + C_2 (3-j4)^n$$

prebaci u polarni oblik

$$q_1 = |q_1|\, e^{j\angle q_1} \qquad |q_1| = \sqrt{3^2 + 4^2} = 5$$

$$\angle q_1 = \operatorname{arctg} \frac{4}{3} = 53{,}13^\circ$$

$$q_1 = -3 - j4$$

$$|q_1| = 5$$

$$\angle q_1 = 53{,}13^\circ + 180^\circ$$

$q_1 = 5 \cdot e^{j53,13}$ $\qquad$ $q_2 = 5 \cdot e^{-j53,13}$

$$y_h(n) = C_1 5^n e^{j53,13^\circ n} + C_2 5^n e^{-j53,13^\circ n}$$

$$= 5^n \left(C_1 e^{j53,13^\circ n} + C_2 e^{-j53,13^\circ n} \right)$$

$$= 5^n \left(A \cos(53,13^\circ n) + B \sin(53,13^\circ n) \right)$$

Mirni odziv

- $y_m(n)$

$y_m(n) = y_h(n) + y_p(n)$

$y(n) - 4\,y(n-2) = u(n)$ $\quad \rightarrow$ * $y_h(n) = C_1 2^n + C_2 (-2)^n$ $\qquad$ * $y(-1) = 1$

* $y_p(n) = -\frac{8}{9} - \frac{1}{3} n \,, \; n \geq 0$ $\qquad$ * $y(-2) = 2$

$y_m(n) = C_1 (2)^n + C_2 (-2)^n - \frac{8}{9} - \frac{1}{3} n \quad , n \geq 0$

$y(-1) = y(-2) = 0$

$y(0) = 0 \; ; \; y(1) = 1 \quad \Rightarrow$ ovo su sada P.U.

$$\left.\begin{aligned} y_m(0) &= C_1 + C_2 - \frac{8}{9} = 0 \\ y_m(1) &= 2C_1 - 2C_2 - \frac{8}{9} - \frac{1}{9} = 1 \end{aligned}\right\} \; C_1 = 1 \,, \; C_2 = -\frac{1}{9}$$

Nepobuđeni odziv

- $y_o(n)$

$y_o(n) = y_h(0)$

$y_o(n) = C_1 (2)^n + C_2 (-2)^n$

$y(-1) = 1 \; ; \; y(-2) = 2$

$$\left.\begin{aligned} y_o(-1) &= \frac{1}{2} C_1 - \frac{1}{2} C_2 = 1 \\ y_o(-2) &= \frac{1}{4} C_1 + \frac{1}{4} C_2 = 2 \end{aligned}\right\} \; C_1 = 5 \,, \; C_2 = 3$$

$y_o(n) = 5(2)^n + 3(-2)^n$

$$\boxed{y_{\infty}(n) = 2^n + \frac{1}{9}(-2)^n - \frac{8}{9} - \frac{1}{3}n} \quad , n \geq 0$$

$$\boxed{y_0(n) = 5(2)^n + 3(-2)^n}$$

$$y(n) = \underbrace{6(2)^n + \frac{26}{9}(-2)^n}_{\text{to je prirodni odziv}} \underbrace{- \frac{8}{9} - \frac{1}{3}n}_{\text{partikularni dio rješenja}}$$

Impulsni odziv

$u(n) = \delta(n)$

$y(n) \rightarrow h(n)$

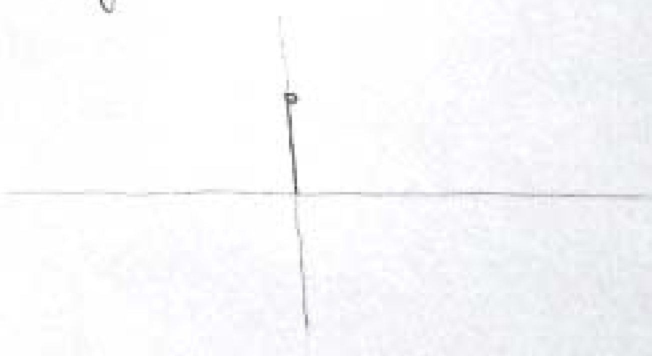
$y(n) - 4y(n-2) = u(n)$

$y(-1) = 1 \quad ; \quad y(-2) = 2$

$y_h(n) = C_1 2^n + C_2 (2)^n$

$h(n) - 4h(n-2) = \delta(n)$

$h(n) = y_h(n)$



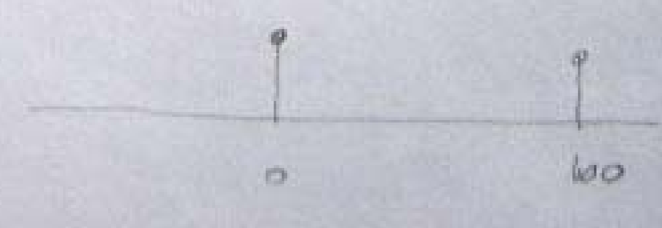
$y(0) - 4y(-2) = \delta(0) \Rightarrow y(0) = 9$

$y(1) - 4y(-1) = \delta(1) \Rightarrow y(1) = 4$

$C_1 = \frac{11}{2} \quad ; \quad C_2 = \frac{7}{2}$

$$\boxed{h(n) = \frac{11}{2} 2^n + \frac{7}{2}(-2)^n \quad , n \geq 0}$$

– kada imamo 2 δ



zadano je : (za slike) 1 - iterativno

$y(-1), y(-2)$ ① $y(0), y(1)$

① $h(n) = y_h(n)$, $0 \leq n < 100$

② $h(98), h(99) \xrightarrow{1} h(100), h(101)$

$h(n) = y_h(n)$, $n \geq 100$

- primjer sa 2 Diraca - 147 str pr. 13-6.

Diferencijalne jednadžbe

- sve radimo za vremensku domenu

$$y''(t) + a_1 y'(t) + a_2 y(t) = b_0 u''(t) + b_1 u'(t) + b_2 u(t)$$

$\Rightarrow$ 2. REDA

$$y(t) = y_h(t) + y_p(t)$$

* $y_h(t) = Ce^{st}$, $y_h'(t) = sCe^{st}$, $y_h''(t) = s^2 C e^{st}$

* PRIMJER 1.

$$y''(t) - 4y(t) = u(t)$$

$$u(t) = t\mu(t)$$

$$y(0^-) = 1 \quad , \quad y'(0^-) = 2$$

* opće homogeno rješenje

$$s^2Ce^{st} - 4Ce^{st} = 0 \rightarrow Ce^{st}(s^2 - 4) = 0$$

$$s^2 - 4 = 0 \rightarrow s_1 = 2$$

$$s_2 = -2$$

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$y_h(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$

$$\boxed{y_h(t) = C_1 e^{2t} + C_2 e^{-2t}}$$

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POBUDA	PART. RJEŠENJE
A	K
t^M	$K_0 + K_1 t + \dots + K_m t^n$
$t^M e^{s_0 t}$	$(K_0 + K_1 t + \dots + K_m t^M)\, e^{s_0 t}\, t^{(m)}$ ⇒ koliko je puta s_0 vlastita frek. sustava
$A\cos(\omega_0 t)$, $A\sin(\omega_0 t)$	$K_0 \cos(\omega_0 t) + K_1 \sin(\omega_0 t)$
$A\cos(\omega_0 t)$	$K \cos(\omega_0 t + \varphi)$
$A\sin(\omega_0 t)$	$K \sin(\omega_0 t + \varphi)$

dokaz

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$= \underbrace{K\cos(\phi)}_{K_0}\cos(\omega_0 t) - \underbrace{K\sin(\phi)}_{K_1}\sin(\omega_0 t)$$

$y(t) = t\,\mu(t) = t \,,\ t \geq 0$

$y''(t) - 4y(t) = \mu(t)$

$y_p(t) = (K_0 + K_1 t)\, e^{0t} \cdot t^0 = K_0 + K_1 t$

$y_p''(t) - 4y_p(t) = \mu(t)$

$y_p'(t) = K_1$

$y_p''(t) = 0$

$-4K_0 - 4K_1 t = t$

$K_0 = 0$

$K_1 = -\frac{1}{4}$

$$\boxed{y_p(t) = -\frac{1}{4}t \,,\ t \geq 0}$$

$t^m e^{s_0 t} \to$

$\to (K_0 + K_1 t + \dots + K_m t^m)\, e^{s_0 t}\, t^{(m)}$

$C_1 e^{j\omega_0 t} + C_2 e^{-j\omega_0 t}$

$A\cos(\omega_0 t) + B\sin(\omega_0 t) =$

$C_1(\cos(\omega_0 t) + j\sin(\omega_0 t)) +$

$+ C_2(\cos(\omega_0 t) - j\sin(\omega_0 t)) =$

$\cos(\omega_0 t)\underbrace{(C_1 + C_2)}_{A} + \sin(\omega_0 t)\underbrace{(C_1 - C_2)}_{B}$

$$y(t) = C_1 e^{2t} + C_2 e^{-2t} - \frac{1}{4}t \quad , t \geq 0$$

P.U $\quad y(0^-) = 1 \quad , y'(0^-) = 2$

$$y'(t) = 2C_1 e^{2t} - 2C_2 e^{-2t} - \frac{1}{4}$$

$$y(0^+) - y(0^-) = b_0 u(0^+)$$

$$y'(0^+) - y'(0^-) + a_1(y(0^+) - y(0^-)) = $$
$$= b_0 u'(0^+) + b_1 u(0^+)$$

$$\boxed{\begin{aligned} &y''(t) + a_1 y'(t) + a_2 y(t) = b_0 u''(t) + b_1 u'(t) + b_2 u(t) \\ &y'''(t) + a_1 y''(t) + a_2 y'(t) + a_3 y(t) = b_0 u'''(t) + b_1 u''(t) + \ldots \end{aligned}}$$

$b_0 = 0 \quad , b_1 = 0 \quad , b_2 = 1$

$a_1 = 0 \quad , a_2 = -4$

$y(0^+) = 1 \qquad u(t) = t\,\mu(t) \longrightarrow u(0^+) = 0$

$y'(0^+) = 2 \qquad u'(t) = \mu(t) + t\,\delta(t) \rightarrow u'(0^+) = 0$

$$\left.\begin{aligned} y(0^+) &= C_1 + C_2 = 1 \\ y'(0^+) &= 2C_1 - 2C_2 - \frac{1}{4} = 2 \end{aligned}\right\} \quad \begin{aligned} C_1 &= \frac{17}{16} \\ C_2 &= -\frac{1}{16} \end{aligned}$$

$$\boxed{y(t) = \frac{17}{16} e^{2t} - \frac{1}{16} e^{-2t} - \frac{1}{4}t \quad , t \geq 0}$$

Mirni odziv

$$y_m(t) = y_h(t) + y_p(t)$$

$$y(0^-) = \ldots = y^{(n-1)}(0^-) = 0 \rightarrow y(0^+) =$$

$$y(0^+) = \ldots$$

PRIMJER

$$y''(t) - 2y'(t) + y(t) = 0 \qquad y(0^-) = 0\ ;\ y'(0^-) = 1 \rightarrow y(0^+) = 0\ ,\ y'(0^+) = 1$$

$$u(t) = t\mu(t)$$

1. opće homogeno rj.

$$y_h(t) = Ce^{st}$$

$$Ce^{st}(s^2 - 2s + 1) = 0 \qquad s_1 = 1\ ,\ s_2 = 1 \ \Big\}\ \boxed{y_h(t) = (C_1 + C_2 t)e^t}$$

$$(C_1 + C_2 t)e^{s_1 t}$$

2. partikularno rj.

$$u(t) = t \quad , t \geq 0$$

$$y_p(t) = K_0 + K_1 t$$

$$y_p'(t) = K_1 \quad ; \ y_p''(t) = 0$$

$$0 - 2K_1 + K_0 + K_1 t = t$$

$$\left.\begin{array}{l} -2K_1 + K_0 = 0 \\ K_1 = 1 \end{array}\right\} \begin{array}{l} K_0 = 2 \\ K_1 = 1 \end{array} \qquad \boxed{y_p(t) = 2 + t\ ,\ t \geq 0}$$

$$y_m(t) = (C_1 + C_2 t)e^t + 2 + t \quad , t \geq 0$$

$$y(0^-) = y'(0^-) = 0 \rightarrow y(0^+) = y'(0^+) = 0$$

$$y_m'(t) = e^t(C_1 + C_2 t) + e^t(C_2) + 1$$

$$y_m'(t) = e^t(C_1 + C_2 + C_2 t) + 1$$

$y_m(0^+) = C_1 + 2 = 0$

$y_m'(0^+) = C_1 + C_2 + 1 = 0$

$C_1 = -2 \ ; \ C_2 = 1$

$$\boxed{y_m(t) = e^t(-2+t) + 2 + t \ , \ t \geq 0}$$

$\boxed{t \geq 0 \text{ kod totalnog i mirnog sustava}}$

Nepobuđeni odziv

- $y_0(t)$

$y_0(t) = y_h(t)$

$y_0(t) = (C_1 + C_2 t)e^t \quad , \ y(0^-) = 0$

$y_0(0^-) = C_1 = 0$

$y_0'(t) = e^t(C_1 + C_2 + C_2 t) \quad , \ y'(0^-) = 1$

$y_0'(0^-) = C_1 + C_2 = 1 \rightarrow C_1 = 0$

$C_2 = 1$

$$\boxed{y_0(t) = t e^t}$$

$$y(t) = \underbrace{e^t(-2+2t)}_{\text{prirodni odziv}} \underbrace{+2+t}_{\text{prisilni odziv}} \ , \ t \geq 0$$

PRIMJER 5.4 , str. 47.

$$y''(t) + 0.2\,y'(t) + 0.1\,y(t) = u(t)$$

$$u(t) = 3\cos(1.8t)$$

$$y(0) = -10 \;;\; y'(0) = -5$$

①

$$s^2 + 0.2s + 0.1 = 0 \;\rightarrow\; s_1 = -0.1 + j0.3$$

$$s_2 = -0.1 - j0.3$$

$$y_h(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

$$= C_1 e^{(-0.1 + j0.2)t} + C_2 e^{(-0.1 - j0.3)t}$$

$$= e^{-0.1t}\left(C_1 e^{j0.3t} + C_2 e^{j0.3t}\right)$$

$$\boxed{y_h(t) = e^{-0.1t}\left(A\cos(0.3t) + B\sin(0.3t)\right)}$$

②

$$u(t) = 3\cos(1.8t) \;\rightarrow\; y_p(t) = K_0\cos(1.8t) + K_1\sin(1.8t)$$

$$y_p'(t) = -1.8K_0\sin(1.8t) + 1.8K_1\cos(1.8t)$$

$$y_p''(t) = -3{,}24K_0\cos(1.8t) - 3{,}24K_1\sin(1.8t)$$

$$\cos(1.8t)\left(-3{,}24K_0 + 0{,}36K_1 + 0{,}1K_0\right) + \sin(1.8t)\left(-3{,}24K_1 - 0{,}36K_0 + 0{,}1K_1\right) = 3\cos(1.8t)$$

Impulsni odziv

$u(t) = \delta(t)$

$y(t) \rightarrow h(t)$

$y''(t) + 2y'(t) + 5y(t) = u'(t) + 3u(t)$

1. $s^2 + 2s + 5 = 0 \rightarrow s_{1,2} = \frac{-2 \pm j4}{2} \rightarrow s_1 = -1 + j2$

$s_2 = -1 - j2$

$y_h(t) = e^{-t}\left(A\cos(2t) + B\sin(2t)\right) = h_A(t)$

$h_A(0^+) = 0$

$h_A'(0^+) = 1$

$h_A'(t) = e^{-t}\left(A(\cos(2t) - 2\sin(2t)) + B(\sin(2t) + 2\cos(2t))\right)$

$h_A'(0^+) = A = 0 \rightarrow A = 0$

$h_A'(0^+) = 2B = 1 \rightarrow B = \frac{1}{2}$

$$\boxed{h_A(t) = \frac{1}{2} e^{-t} \sin(2t)}$$

$$\boxed{h(t) = \sum_{m=0}^{M} \left(b_{N-m} D^m\right) h_A(t) + b_0 \delta(t)}$$

N – red derivacije izlaza

M – red derivacije ulaza

$N = 2,\ M = 1$

$b_0 = 0,\ b_1 = 1,\ b_2 = 3$

$D^1 h_A(t) = h_A'(t)$

$D^2 h_A(t) = h_A''(t)$

$D^0 h_A(t) = h_A(t)$

$y'(t) + y(t) = u(t)$

$h_A(0^+) = 1$

$y'''(t) + y''(t) + y'(t) + y(t) = u(t)$

$h_A(0^+) = 0$

$h_A'(0^+) = 0$

$h_A''(0^+) = 1$

$h(t) = \sum_{m=0}^{1} \left(b_{2-m} D^M\right) h_A(t)$

$h(t) = b_2 D^0 h_A(t) + b_1 D^1 h_A(t)$

$h(t) = b_2 h_A(t) + b_1 h_A'(t)$

$$h_1(t) = \frac{1}{2}e^{-t}\sin(2t)$$

$$h_1'(t) = e^{-t}\left(-\frac{1}{2}\sin(2t) + \cos(2t)\right)$$

$$\boxed{h(t) = e^{-t}\left(\sin(2t) + \cos(2t)\right) \quad , t \geq 0}$$

Prijenosne funkcije

$h(t)$

$$u(t) \xrightarrow{\text{ODZIV}} y(t) = h(t) * u(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau) e^{st} e^{-s\tau}\, d\tau$$

$$u(t) = e^{st}, \quad \cos(\omega_0 t) = e^{j\omega_0 t} + e^{-j\omega_0 t}$$

$$y(t) = e^{st} \underbrace{\int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau}_{H(s)}$$

$$\boxed{X(s) = \int_{-\infty}^{\infty} x(\tau)\, e^{-s\tau}\, d\tau}$$

LAP. TRANS.

IZVOD :

$u(t) = U e^{st}$

$y(t) = y e^{st}$

$$y^{(n)}(t) + a_1 y^{(n-1)}(t) + \ldots + a_n y(t) = b_0 u^{(n)}(t) + \ldots + b_n u(t)$$

$$s^n y e^{st} + a_1 s^{n-1} y e^{st} + \ldots + a_n y e^{st} = b_0 s^n U e^{st} + \ldots + b_n U e^{st}$$

$$y\left(s^n + a_1 s^{n-1} + \ldots + a_n\right) = U\left(b_0 s^n + \ldots + b_n\right)$$

$$y = \underbrace{\boxed{\frac{b_0 s^n + \ldots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \ldots + a_n}}}_{H(s)} \cdot U$$

- imamo sustav

$$y''(t) + 5y'(t) + 6y(t) = u(t) \rightarrow H(s) = \frac{1}{s^2 + 5s + 6}$$

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$s = j\Omega$

$$H(j\Omega) = \frac{1}{6 - \Omega^2 + 5j\Omega}$$

$z = a + jb$

$|z| = \sqrt{a^2 + b^2}$

$\angle z = \mathrm{arctg}\, \frac{b}{a}$

$$z_1 = \frac{1}{a + jb}$$

$$|z_1| = \frac{|1|}{\sqrt{a^2 + b^2}}$$

$$z = \frac{c}{a + jb} \cdot \frac{a - jb}{a - jb} = \frac{c(a - jb)}{a^2 + b^2}$$

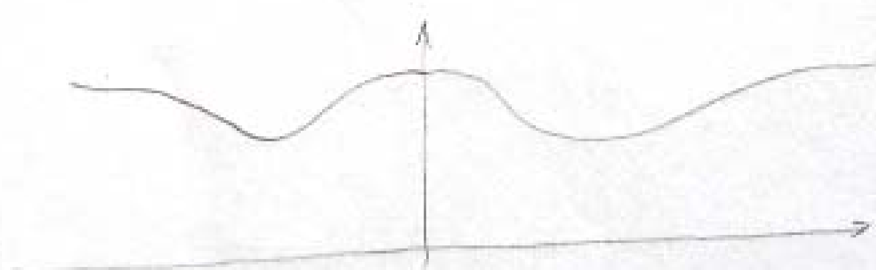
$$Re = \frac{ca}{a^2 + b^2} \qquad Im = \frac{-cb}{a^2 + b^2}$$

$\angle z = \mathrm{arctg} - \frac{b}{a}$

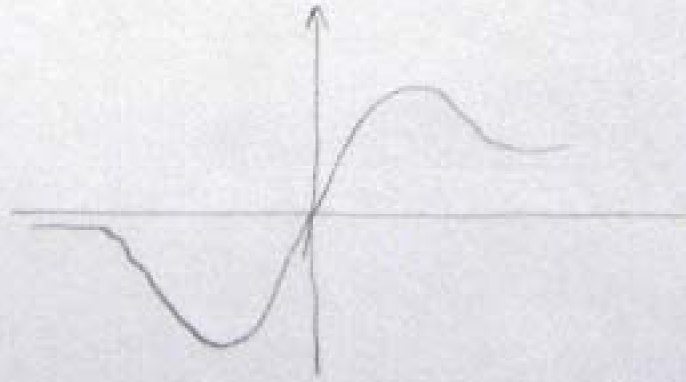
$\angle z = -\mathrm{arctg}\, \frac{b}{a}$

$$\left.\begin{aligned} \angle H(j\Omega) &= -\mathrm{arctg}\, \frac{5\Omega}{6 - \Omega^2} \\ &= \mathrm{arctg}\, \frac{5\Omega}{\Omega^2 - 6} \end{aligned}\right\} \text{fazno frekv. karak.}$$

$$\left.|H(j\Omega)| = \frac{1}{\sqrt{(6 - \Omega^2)^2 + (5\Omega)^2}} = \frac{1}{\sqrt{\Omega^4 + 13\Omega^2 + 36}}\right\} \text{amplitudno frekv. karak.}$$



amp.



faz.

Stabilnost sustava

* KONTINUIRANI SUSTAVI

  * JEDNOSTRUKI KORJENI

    - sustav je stabilan $Re\{S_i\}<0$, $\forall S_i$

  * GRANIČNO - STABILAN

    - $Re\{S_i\}=0$, $\exists S_i$ - ako postoji jedan kojem je $Re=0$

  * NESTABILAN

    - $Re\{S_i\}>0$, $\exists S_i$

* VIŠESTRUKI KORJEN

  * STABILAN

    $Re\{S_i\}<0$

  * NESTABILAN

    $Re\{S_i\}\geq 0$, $\exists S_i$

npr.

$S_{1,2}=0$ $\quad S_3=-2$ $\Rightarrow$ NESTABILAN

$S_1=0$ $\quad S_{1,2}=-1$ $\Rightarrow$ GRANIČNO STABILAN

$S_1=-1+j2$ $\quad S_2=-1-j2$ $\Rightarrow$ STABILAN

$S_1=-j$ $\quad S_2=-j$ $\Rightarrow$ GRANIČNO STABILAN

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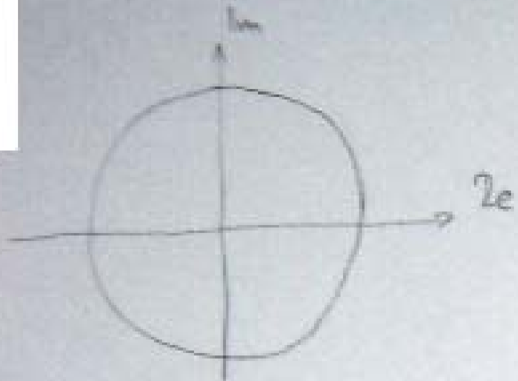
* STABILNOST SUSTAVA

DISKRETNI SUSTAVI



$y_h(n) = 5\, g^n$

$\lim_{n\to\infty} g^n = 0 \;; \quad |g| < 1$

$\left(\frac{1}{2}\right)^n = \frac{1}{2^n}$

$2^n = \infty$

* jednostruki

* stabilnost

$|g_i| < 1 \;, \forall g_i$

* granično-stabilan

$|g_i| = 1 \;, \exists g_i$

* nestabilan

$|g_i| \geq 1 \;, \exists g_i$

npr: $\left.\begin{array}{l} g_1 = 2 - j3 \\ g_2 = 2 + j3 \end{array}\right\}$ $\sqrt{4+9} = \sqrt{13} > 1$ pa je nestabilan

$g_1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} j \Rightarrow$ granično-stabilan

* višestruki

* stabilan

$|g_i| < 1 \;, \forall g_i$

* nestabilan

$|g_i| \geq 1 \;, \exists g_i$

$g_1 = j$

$g_2 = j$

$g_3 = -j$

ZADATAK 1. 15 g.

$y''(t) + 5y'(t) + 6y(t) = u(t)$

$s_1 = -3$; $s_2 = -2$; sustav je stabilan

$H(s) = \frac{1}{s^2 + 5s + 6}$

$A(\omega) =$ amplituda $= \frac{1/1}{\sqrt{36 + 13\omega^2 + \omega^4}}$

$H(j\Omega) = \frac{1}{6 - \Omega^2 + 5\Omega}$

$P(\omega) = -\text{arctg}\,\frac{5\Omega}{6 - \Omega^2}$

$= \text{arctg}\,\frac{5\Omega}{\Omega^2 - 6}$

$y(0) = 0$; $y'(0) = 1$

$$\boxed{y_h(t) = C_1 e^{-3t} + C_2 e^{-2t}}$$

$\underset{\omega=1}{u(t) = 5\cos(t)} \longrightarrow y_p(t) = K\cos(t + \phi)$

$K = 5 \cdot A(\omega)\Big|_{\omega=1}$ $\qquad \phi = 0 + P(\omega)\Big|_{\omega=1}$

$K = \frac{\sqrt{2}}{2}$ $\qquad \phi = -45°$

$$\boxed{y_p(t) = \frac{\sqrt{2}}{2}\cos(t - 45°) \ , t \geq 0}$$

9 lip 2008

$$y''(t) + 2y'(t) + 5\,y(t) = u(t)$$

$$u(t) = \begin{cases} \sin(t), & t<0 \\ 2\sin(2t), & t>0 \end{cases}$$

$s_1 = -1 + j2$, $s_2 = -1 - j2$ $\Rightarrow$ sustav je stabilan

$$\boxed{y_h(t) = e^{-t}\left(A\cos(2t) + B\sin(2t)\right)}$$

$$H(s) = \frac{1}{s^2 + 2s + 5} \qquad H(j\Omega) = \frac{1}{5 - \Omega^2 + 2j\Omega}$$

$$A(\omega) = \frac{1}{\sqrt{\Omega^4 - 6\Omega^2 + 25}}$$

$$P(\omega) = -\operatorname{arctg} \frac{2\Omega}{5 - \Omega^2}$$

1) $u_1(t) = \sin(t)$

$$y_p(t) = K \sin(t + \phi) \qquad \phi = 0^\circ + P(\omega)\big|_{\omega=1}$$

$$\phi = -26{,}56^\circ$$

$$K = 1 \cdot A(\omega)\big|_{\omega=1}$$

$$K = \frac{\sqrt{5}}{10}$$

$$y_{p1}(t) = \frac{\sqrt{5}}{10}\sin(t - 26{,}56^\circ), \quad t \leq 0$$

2) $u_2(t) = 2\sin(2t)$

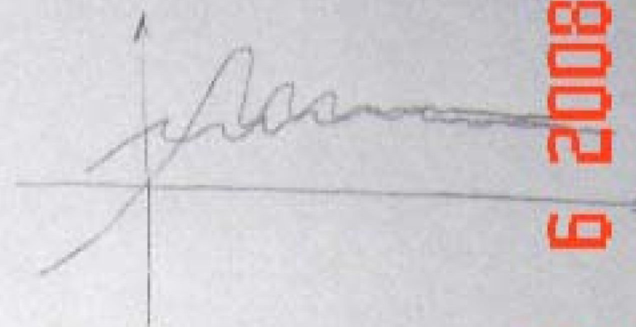
$$y_{p2}(t) = K \sin(2t + \phi) \qquad \phi = 0^\circ + P(\omega)\big|_{\omega=2}$$

$$K = 2 \cdot A(\omega)\big|_{\omega=2} \qquad \phi = -75{,}96^\circ$$

$$K = \frac{2\sqrt{17}}{17}$$

$$y_{p2}(t) = \frac{2\sqrt{17}}{17}\sin(2t - 75{,}96^\circ), \quad t \geq 0$$

$$y(t) = \begin{cases} y_h(t) + y_p(t), & t<0 \\ \\ y_h(t) + y_{p_2}(t), & t>0 \end{cases}$$



apdemto

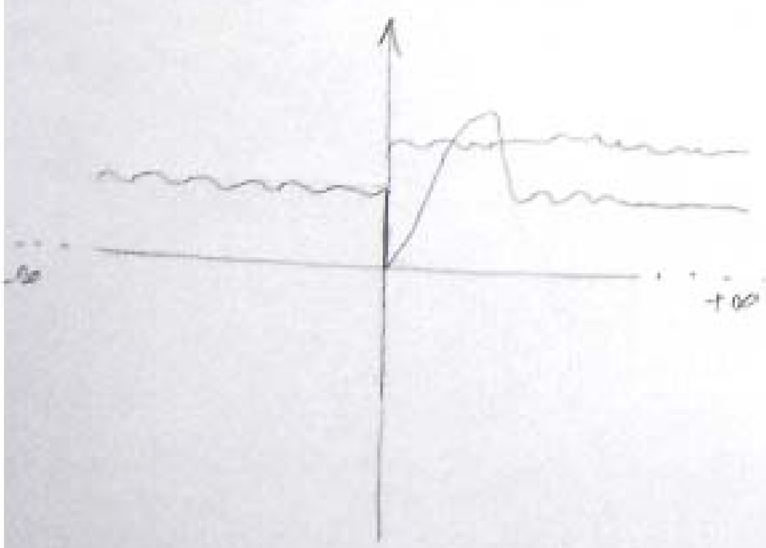
$$u(t) = U\cos(\omega_0 t + \phi_0) \longrightarrow y_p(t) = K\cos(\omega_0 t + \phi)$$

$$K = U \cdot A(\omega)\Big|_{\omega=\omega_0}$$

$$\phi = \phi_0 + P(\omega)\Big|_{\omega=\omega_0}$$

$$u(t) = Ue^{st} \longrightarrow y_p(t) = K \cdot e^{st}$$

$$K = U \cdot A(\omega)\Big|_{\omega=0}$$



(1) $y'(t) + 3y(t) = u(t)$

$u(t) = \left(\sin(t) + 2\sin(2t) + 3\sin(3t) + 4\sin(4t)\right)\mu(t)$

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ako je diskretni ovog tipa

$y(n) + y(n-1) + y(n-2) = u(n)$

$y(n) = z^n$ $\qquad\qquad z = e^{j\omega}$

$y(n-2) = z^{n-2} = z^n \cdot z^{-2}$

$u(n) = z^n$

$y z^n (1 + z^{-1} + z^{-2}) = z^n y$

$$y = \underbrace{\frac{1}{1+z^{-1}+z^{-2}}}_{H(z)} u$$

Kopirajt by Cartman