Signali i sustavi
Pismeni ispit – 17. lipnja 2015.

1. (8 bodova) Vremenski diskretan kauzalan sustav zadan je jednadžbom diferencija $y(n) - \frac{1}{4}y(n-1) = u(n)$.

a) **(2 boda)** Izračunajte prijenosnu funkciju sustava te ispitajte stabilnost sustava.

b) **(2 boda)** Odredite frekvencijsku karakteristiku sustava.

c) **(4 boda)** Izračunajte prisilni odziv sustava na svevremensku pobudu $u(n) = 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$ upotrebom frekvencijske karakteristike sustava.

2. (8 bodova) Vremenski kontinuiran kauzalan sustav zadan je diferencijalnom jednadžbom $y'(t) + 10y(t) = u(t)$.

a) **(3 boda)** Izračunajte impulsni odziv sustava postupkom u vremenskoj domeni.

b) **(3 boda)** Odredite odziv sustava na kauzalnu pobudu $u(t) = e^{-2t}\,\mu(t)$ metodom konvolucijskog integrala.

c) **(2 boda)** Odredite totalni odziv sustava na pobudu $u(t) = e^{-2t}\,\mu(t)$, ako je početni uvjet sustava $y(0^-) = 2$.

3. (8 bodova) Vremenski diskretan kauzalan sustav zadan je jednadžbom diferencija

$$y(n) - \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = u(n) - 3u(n-1) + 2u(n-2).$$

a) **(4 boda)** Odredite impulsni odziv sustava pomoću Z transformacije.

b) **(4 boda)** Izračunajte odziv mirnog sustava na pobudu $u(n) = \{\underline{2}, 0, 2, 0, 2, 0, \ldots, 2, 0, \ldots\}$.

4. (8 bodova) Vremenski kontinuiran kauzalan sustav zadan je diferencijalnom jednadžbom

$$y''(t) + 11y'(t) + 10y(t) = u'(t) - 2u(t)$$

te početnim uvjetima $y(0^-) = 1$ i $y'(0^-) = 2$. Na ulaz sustava dovedena je pobuda $u(t) = e^{t}\,\mu(t)$. Odredite totalni odziv sustava.

5. (8 bodova) Zadan je impulsni odziv vremenski diskretnog kauzalnog LTI sustava $h(n) = \left(\frac{1}{3^n} + \frac{1}{9^n}\right)\mu(n)$.

a) **(2 boda)** Odredite prijenosnu funkciju sustava.

b) **(3 boda)** Odredite odziv sustava na svevremensku pobudu $u(n) = 3^n$ koristeći prijenosnu funkciju sustava.

c) **(3 boda)** Odredite odziv sustava na svevremensku pobudu $u(n) = 3^n$ metodom konvolucijskog zbroja.

1. $y(n) - \frac{1}{4} y(n-1) = u(n)$ A

a) $H(z) = \frac{1}{1 - \frac{1}{4} z^{-1}} = \frac{z}{z - \frac{1}{4}}$

POL $z - \frac{1}{4} = 0$

$z = \frac{1}{4}$

$|z| < 1$ SUSTAV JE STABILAN

b) $H(e^{j\Omega}) = \frac{1}{1 - \frac{1}{4} e^{-j\Omega}} = \frac{1}{1 - \frac{1}{4}\cos\Omega + \frac{1}{4}\sin\Omega \cdot j}$

$|H(e^{j\Omega})| = \frac{1}{\sqrt{(1 - \frac{1}{4}\cos\Omega)^2 + (\frac{1}{4}\sin\Omega)^2}}$

$= \frac{1}{\sqrt{1 - \frac{1}{2}\cos\Omega + \frac{1}{16}\cos^2\Omega + \frac{1}{16}\sin^2\Omega}} = \frac{1}{\sqrt{\frac{17}{16} - \frac{1}{2}\cos\Omega}}$

c) $u(n) = 2\cos\left(\frac{\pi}{2} n + \frac{\pi}{4}\right)$

$y_p(n) = 2 \cdot |H(e^{j\Omega})| \cdot \cos\left(\Omega n + \frac{\pi}{4} + \measuredangle H(e^{j\Omega})\right)$

$H(e^{j\frac{\pi}{2}}) = \frac{1}{1 - \frac{1}{4}\cos\frac{\pi}{2} + \frac{1}{4}\sin\frac{\pi}{2} \cdot j} = \frac{1}{1 + \frac{1}{4} j}$

$|H(e^{j\frac{\pi}{2}})| = \frac{1}{\sqrt{1 + \frac{1}{16}}} = \frac{4}{\sqrt{17}} = 0.97$

$\measuredangle H(e^{j\frac{\pi}{2}}) = -\operatorname{arctg} \frac{\frac{1}{4}}{1} = -\operatorname{arctg} \frac{1}{4} = -0.25$

$y_p(n) = \frac{8}{\sqrt{17}} \cos\left(\frac{\pi}{2} n + \frac{\pi}{4} - 0.25\right)$

$\boxed{y_p(n) = 1.94 \cos\left(\frac{\pi}{2} n + 0.54\right)}$

2 A

$y'(t) + 10\, y(t) = u(t)$

a) IMPULSNI ODZIV

$h_a(0^+) = 1$

$h(t) = ?$

$s + 10 = 0$

$s = -10$

$y_h(t) = C e^{-10t}$

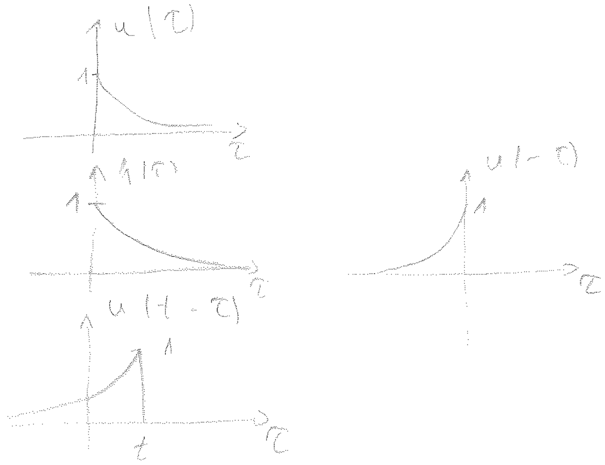
$h_a(t) = C e^{-10t}$

$h_a(0^+) = C = 1$

$h_a(t) = e^{-10t}$

$h(t) = e^{-10t} \mu(t)$

b) $u(t) = e^{-2t} \mu(t)$



$$y(t) = h(t) * u(t)$$

$$= \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau$$

$$= \int_{0}^{t} e^{-10\tau} e^{-2(t-\tau)} d\tau$$

$$= e^{-2t} \int_{0}^{t} e^{-8\tau} d\tau$$

$$= e^{-2t} \left. \frac{e^{-8\tau}}{-8} \right|_0^t = e^{-2t} \cdot \frac{-1}{8} \left(e^{-8t} - 1 \right)$$

$$y(t) = -\frac{1}{8} e^{-10t} + \frac{1}{8} e^{-2t} \quad \text{za} \quad t \geq 0$$

A

c) neposrudeni

$y(0^-)=2$

$y_n(t) = Ce^{-10t}$

$y_n(0^-) = C = 2$

$y_0(t) = 2e^{-10t}$

totalni

$y_t(t) = y_m(t) + y_0(t)$

$= -\frac{1}{8}e^{-10t} + \frac{1}{8}e^{-2t} + 2e^{-10t}$

$\boxed{y_t(t) = \frac{15}{8}e^{-10t} + \frac{1}{8}e^{-2t}}$ za $t>0$

2. $y(n) - \frac{1}{6} y(n-1) - \frac{1}{6} y(n-2) = u(n) - 3u(n-1) + 2u(n-2)$

a) $Y(z) - \frac{1}{6} z^{-1} Y(z) - \frac{1}{6} Y(z) z^{-2} = U(z) - 3z^{-1} U(z) + 2z^{-2} U(z)$

$$H(z) = \frac{1 - 3z^{-1} + 2z^{-2}}{1 - \frac{1}{6} z^{-1} - \frac{1}{6} z^{-2}} = \frac{z^2 - 3z + 2}{z^2 - \frac{1}{6} z - \frac{1}{6}}$$

$$= \frac{z^2 - 3z + 2}{(z - \frac{1}{2})(z + \frac{1}{3})}$$

$$\frac{H(z)}{z} = \frac{1}{z} \cdot \frac{z^2 - 3z + 2}{(z - \frac{1}{2})(z + \frac{1}{3})} = \frac{A}{z} + \frac{B}{z - \frac{1}{2}} + \frac{C}{z + \frac{1}{3}}$$

$$Az^2 - \frac{1}{6} Az - \frac{1}{6} A + Bz^2 + \frac{1}{3} Bz + Cz^2 - \frac{1}{2} Cz = z^2 - 3z + 2$$

$A + B + C = 1$

$-\frac{1}{6} A + \frac{1}{3} B - \frac{1}{2} C = -3$

$-\frac{1}{6} A = 2$

$A = -12$

$B + C = 13$

$\frac{1}{3} B - \frac{1}{2} C = -3 + \frac{1}{6} \cdot (-12) = -5$

$\frac{2}{3} B - C = -10$

$\frac{5}{3} B = +3$

$B = \frac{9}{5}$

$C = 13 - B = \frac{56}{5}$

$$H(z) = -12 + \frac{9}{5} \frac{z}{z - \frac{1}{2}} + \frac{56}{5} \frac{z}{z + \frac{1}{3}}$$

$$h(n) = -12\, \delta(n) + \left[\frac{9}{5} \left(\frac{1}{2}\right)^n + \frac{56}{5} \left(\frac{1}{3}\right)^n\right] \mu(n)$$

b) $u(n) = \{\underline{2}, 0, 2, 0, \dots\}$

$$U(z) = 2z^0 + 2z^{-2} + 2z^{-4} + \dots = 2 \cdot \sum_{k=0}^{\infty} (z^{-2})^k$$

$$= 2 \cdot \frac{1}{1 - z^{-2}} = 2 \frac{z^2}{z^2 - 1}$$

$$Y(z) = H(z) \cdot U(z) = \frac{\cancel{(z-1)}(z-2)}{(z - \frac{1}{2})(z + \frac{1}{3})} \cdot \frac{2z^2}{(z-1)(z+1)} = \frac{2z^2(z-2)}{(z - \frac{1}{2})(z + \frac{1}{3})(z+1)}$$

$$\frac{Y(z)}{z} = \frac{2z^2 - 4z}{(z - \frac{1}{2})(z + \frac{1}{3})(z+1)} = \frac{A}{z - \frac{1}{2}} + \frac{B}{z + \frac{1}{3}} + \frac{C}{z+1}$$

$$Az^2 + \frac{1}{3} Az + Az + \frac{1}{3} A + Bz^2 - \frac{1}{2} Bz + Bz - \frac{1}{2} B + Cz^2 - \frac{1}{6} Cz - \frac{1}{6} C = 2z^2 - 4z$$

$A + B + C = 2$

$\frac{1}{3} A + A - \frac{1}{2} B + B - \frac{1}{6} C = -4$

$\frac{1}{3} A - \frac{1}{2} B - \frac{1}{6} C = 0$

$\frac{4}{3} A + \frac{1}{2} B - \frac{1}{6} C = -4$

$A = -\frac{6}{5}$, $B = -\frac{14}{5}$, $C = 6$

$$Y(z) = \frac{-\frac{6}{5} z}{z - \frac{1}{2}} - \frac{\frac{14}{5} z}{z + \frac{1}{3}} + \frac{6z}{z+1}$$

$$y(n) = -\frac{6}{5} \left(\frac{1}{2}\right)^n - \frac{14}{5} \left(-\frac{1}{3}\right)^n + 6(-1)^n \quad \text{za } n \geq 0$$

1

4. $y''(t) + 11y'(t) + 10y(t) = u'(t) - 2u(t)$

$y(0^-) = 1$

$y'(0^-) = 2$

$u(t) = e^t \mu(t)$

$u(t) = e^t$

$u'(t) = e^t$

homogeno

$s^2 + 11s + 10 = 0$

$(s+10)(s+1) = 0$

$s_1 = -10 \qquad s_2 = -1$

$y_h(t) = C_1 e^{-10t} + C_2 e^{-t}$

partikularno

$y_p(t) = Ce^t$

$y_p'(t) = Ce^t$

$y_p''(t) = Ce^t$

$Ce^t + 11Ce^t + 10Ce^t = e^t - 2e^t$

$22Ce^t = -e^t$

$22C = -1$

$C = -\frac{1}{22}$

$y_p = -\frac{1}{22} e^t \mu(t)$

totalni odziv

$y_t(t) = C_1 e^{-10t} + C_2 e^{-t} - \frac{1}{22} e^t \qquad t \geq 0$

početni uvjeti

$y(0^+) - y(0^-) = 0 \cdot u(0^+) \longrightarrow y(0^+) = 1$

$y'(0^+) - y'(0^-) + 11(y(0^+) - y(0^-)) = 0 + 1(u(0^+) - u(0^-))$

$y'(0^+) - 2 = 1 \cdot 1$

$y'(0^+) = 3$

A

4.

totalni odziv

$$y_t(0^+) = C_1 + C_2 - \frac{1}{22} = 1$$

$$y_t'(t) = -10 C_1 e^{-10t} - C_2 e^{-t} - \frac{1}{22} e^{t}$$

$$y_t'(0^+) = -10 C_1 - C_2 - \frac{1}{22} = 3$$

$$-9C_1 - \frac{\cancel{2}^{1}}{\cancel{22}_{11}} = 4$$

$$-9C_1 = 4 + \frac{1}{11} = \frac{44+1}{11} = \frac{45}{11}$$

$$C_1 = \frac{45}{-9 \cdot 11} = \frac{-5}{11}$$

$$C_2 = 1 + \frac{1}{22} - C_1 = \frac{23}{22} + \frac{5}{11} = \frac{23+10}{22} = \frac{33}{22} = \frac{3}{2}$$

$$\boxed{y_t(t) = \left[-\frac{5}{11} e^{-10t} + \frac{3}{2} e^{-t} - \frac{1}{22} e^{t}\right]\mu(t)}$$

5

a) $h(n) = \left(\frac{1}{3^n} + \frac{1}{9^n}\right)\mu(n)$

$$H(z) = \frac{z}{z-\frac{1}{3}} + \frac{z}{z-\frac{1}{9}} = \frac{z^2 - \frac{1}{9}z + z^2 - \frac{1}{3}z}{(z-\frac{1}{3})(z-\frac{1}{9})} = \frac{2z^2 - \frac{4}{9}z}{z^2 - \frac{4}{9}z + \frac{1}{27}}$$

b) $u(n) = 3^n$

$y(n) = H(z) \cdot U z^n \longrightarrow z = 3$

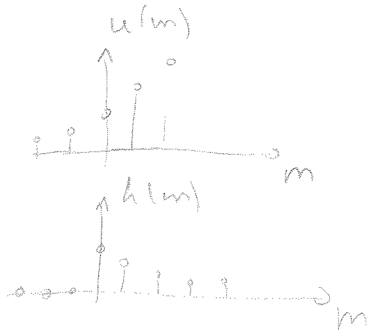
$y(n) = H(3) \cdot U \cdot 3^n$

$= H(3) \cdot 3^n$

$$H(3) = \frac{2 \cdot 3^2 - \frac{4}{9} \cdot 3}{3^2 - \frac{4}{9} \cdot 3 + \frac{1}{27}} = \frac{\frac{162-12}{9}}{\frac{243-36+1}{27}} = \frac{\frac{150}{\cancel{9}}}{\frac{208}{\cancel{27}\,3}} = \frac{150 \cdot 3}{208} = \frac{225}{104}$$

$y(n) = \frac{225}{104} \cdot 3^n$

c) $u(n) = 3^n$



$$y(n) = u(n) * h(n)$$
$$= \sum_{m=-\infty}^{\infty} u(m)\, h(n-m)$$
$$= \sum_{m=-\infty}^{\infty} h(m)\, u(n-m)$$
$$= \sum_{m=-\infty}^{\infty} \left[\left(\frac{1}{3}\right)^m + \left(\frac{1}{9}\right)^m\right]\mu(m) \cdot 3^{n-m}$$
$$= 3^n \sum_{m=0}^{\infty} 3^{-2m} + 3^{(-2-1)m} = 3^n \left[\frac{1}{1-3^{-2}} + \frac{1}{1-3^{-3}}\right]$$
$$= 3^n \cdot \left[\frac{1}{1-\frac{1}{9}} + \frac{1}{1-\frac{1}{27}}\right] = 3^n \left[\frac{1}{\frac{8}{9}} + \frac{1}{\frac{26}{27}}\right]$$
$$= 3^n \left[\frac{9}{8} + \frac{27}{26}\right] = 3^n \cdot \frac{117+108}{104} = \frac{225}{104} \cdot 3^n$$

Signali i sustavi

Pismeni ispit – 17. lipnja 2015.

1. (8 bodova) Vremenski diskretan kauzalan sustav zadan je jednadžbom diferencija $y(n) - \frac{1}{6}y(n-1) = u(n)$.

a) **(2 boda)** Izračunajte prijenosnu funkciju sustava te ispitajte stabilnost sustava.

b) **(2 boda)** Odredite frekvencijsku karakteristiku sustava.

c) **(4 boda)** Izračunajte prisilni odziv sustava na svevremensku pobudu $u(n) = 2\cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right)$ upotrebom frekvencijske karakteristike sustava.

2. (8 bodova) Vremenski kontinuiran kauzalan sustav zadan je diferencijalnom jednadžbom $y'(t) + 12y(t) = u(t)$.

a) **(3 boda)** Izračunajte impulsni odziv sustava postupkom u vremenskoj domeni.

b) **(3 boda)** Odredite odziv sustava na kauzalnu pobudu $u(t) = e^{-2t}\,\mu(t)$ metodom konvolucijskog integrala.

c) **(2 boda)** Odredite totalni odziv sustava na pobudu $u(t) = e^{-2t}\,\mu(t)$, ako je početni uvjet sustava $y(0^-) = 2$.

3. (8 bodova) Vremenski diskretan kauzalan sustav zadan je jednadžbom diferencija

$$y(n) + \frac{1}{6}y(n-1) - \frac{1}{6}y(n-2) = u(n) - 3u(n-1) + 2u(n-2).$$

a) **(4 boda)** Odredite impulsni odziv sustava pomoću Z transformacije.

b) **(4 boda)** Izračunajte odziv mirnog sustava na pobudu $u(n) = \{\underline{4}, 0, 4, 0, 4, 0, \ldots, 4, 0, \ldots\}$.

4. (8 bodova) Vremenski kontinuiran kauzalan sustav zadan je diferencijalnom jednadžbom

$$y''(t) + 9y'(t) + 8y(t) = u'(t) - 2u(t)$$

te početnim uvjetima $y(0^-) = 1$ i $y'(0^-) = 2$. Na ulaz sustava dovedena je pobuda $u(t) = e^{t}\,\mu(t)$. Odredite totalni odziv sustava.

5. (8 bodova) Zadan je impulsni odziv vremenski diskretnog kauzalnog LTI sustava $h(n) = \left(\frac{1}{2^n} + \frac{1}{4^n}\right)\mu(n)$.

a) **(2 boda)** Odredite prijenosnu funkciju sustava.

b) **(3 boda)** Odredite odziv sustava na svevremensku pobudu $u(n) = 2^n$ koristeći prijenosnu funkciju sustava.

c) **(3 boda)** Odredite odziv sustava na svevremensku pobudu $u(n) = 2^n$ metodom konvolucijskog zbroja.

B

1

$$y(n) - \frac{1}{6} y(n-1) = u(n)$$

a)

$$H(z) = \frac{1}{1 - \frac{1}{6} z^{-1}} = \frac{z}{z - \frac{1}{6}}$$

pol $\quad z = \frac{1}{6} \qquad |z| < 1 \qquad$ STABILNO

b)

$$H(e^{j\Omega}) = \frac{1}{1 - \frac{1}{6} e^{-j\Omega}} = \frac{1}{1 - \frac{1}{6}\cos\Omega + \frac{1}{6} j \sin\Omega}$$

c)

$$u(n) = 2\cos\left(\frac{\pi}{2} n + \frac{\pi}{4}\right)$$

$$y_p(n) = |H(e^{j\Omega})| \cdot 2 \cdot \cos\left(\frac{\pi}{2} n + \measuredangle H(e^{j\Omega}) + \frac{\pi}{4}\right)$$

$$\Omega = \frac{\pi}{2}$$

$$H(e^{j\frac{\pi}{2}}) = \frac{1}{1 - \frac{1}{6}\cos\frac{\pi}{2} + \frac{1}{6} j \sin\frac{\pi}{2}} = \frac{1}{1 + \frac{1}{6} j}$$

$$|H(e^{j\pi/2})| = \frac{1}{\sqrt{1 + \frac{1}{36}}} = \frac{6}{\sqrt{37}} = 0.986$$

$$\measuredangle H(e^{j\pi/2}) = -\mathrm{arctg} \frac{\frac{1}{6}}{1} = -0.165$$

$$y_p(n) = 0.986 \cdot 2 \cdot \cos\left(\frac{\pi}{2} n + \frac{\pi}{4} - 0.165\right)$$

$$\boxed{y_p(n) = 1.97 \cos\left(\frac{\pi}{2} n + 0.62\right)}$$

(B)

2. $y'(t) + 12y(t) = u(t)$

a) IMPULSNI ODZIV

homogeno

$s + 12 = 0$

$s = -12$

$y_h(t) = C e^{-12t}$

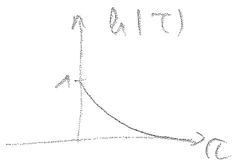
$h_a(0^+) = 1$

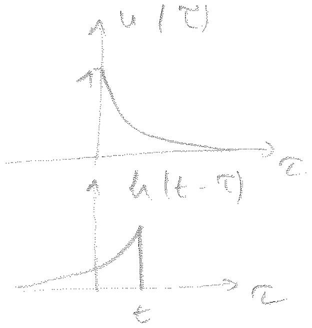
$h_a(t) = C e^{-12t}$

$h_a(0^+) = C = 1$

$$\boxed{h(t) = e^{-12t} \mu(t)}$$

b) $u(t) = e^{-2t} \mu(t)$





$$y_m(t) = u(t) * h(t)$$
$$= \int_{-\infty}^{\infty} u(\tau) h(t-\tau)\, d\tau$$
$$= \int_0^t e^{-2\tau} \cdot e^{-12(t-\tau)} d\tau$$
$$= e^{-12t} \int_0^t e^{10\tau} d\tau = e^{-12t} \cdot \frac{e^{10\tau}}{10} \Big|_0^t = \frac{1}{10}\left(e^{-12t} \cdot e^{10t} - e^{-12t} e^0\right)$$

$$\boxed{y_m(t) = \frac{1}{10} e^{-2t} - \frac{1}{10} e^{-12t} \quad \text{za} \quad t > 0}$$

c) $y(0^-) = 2$

$y_h(t) = C e^{-12t}$

$y_0(t) = C e^{-12t}$

$y_0(0^-) = 2 = C$

$y_0(t) = 2 e^{-12t}$

$$y_t = y_m + y_0$$
$$= \frac{1}{10} e^{-2t} - \frac{1}{10} e^{-12t} + 2 e^{-12t}$$

$$\boxed{y_t = \frac{1}{10} e^{-2t} + \frac{19}{10} e^{-12t} \quad \text{za} \quad t > 0}$$

3.

$$y(n) + \frac{1}{6} y(n-1) - \frac{1}{6} y(n-2) = u(n) - 3u(n-1) + 2u(n-2)$$

a) $$Y(z) = \underbrace{\frac{1-3z^{-1}+2z^{-2}}{1+\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}}}_{H(z)} \cdot U(z)$$

$-\frac{1}{2} + \frac{1}{3}$

$\frac{-3+2}{6}$

$$H(z) = \frac{z^2-3z+2}{(z+\frac{1}{2})(z-\frac{1}{3})}$$

$$\frac{H(z)}{z} = \frac{1}{z} \cdot \frac{z^2-3z+2}{(z+\frac{1}{2})(z-\frac{1}{3})} = \frac{A}{z} + \frac{B}{z+\frac{1}{2}} + \frac{C}{z-\frac{1}{3}}$$

$$Az^2 + \frac{1}{6}Az - \frac{1}{6}A + Bz^2 - Bz\frac{1}{3} + Cz^2 + \frac{1}{2}Cz = z^2 - 3z + 2$$

$A+B+C=1$ $\qquad A=-12$

$\frac{1}{6}A - \frac{1}{3}B + \frac{1}{2}C = -3$ $\qquad B+C = 1-A = 13$

$-\frac{1}{6}A = 2$ $\qquad -\frac{1}{3}B + \frac{1}{2}C = -3 - \frac{1}{6}\cdot(-12) = -3+2 = -1 \quad |\cdot 3$

$-B + \frac{3}{2}C = -3$ $\qquad B = 13 - B = 9$

$\frac{5}{2}C = 10$

$C = 4$

$$H(z) = \frac{-12z}{z} + \frac{9z}{z+\frac{1}{2}} + \frac{4z}{z-\frac{1}{3}}$$

$$\boxed{h(n) = -12\,\delta(n) + \left[9\left(-\frac{1}{2}\right)^n + 4\left(\frac{1}{3}\right)^n\right]\mu(n)}$$

b) $u(n) = \{4, 0, 4, 0, \dots\}$

$$U(z) = 4 + 4z^{-2} + 4z^{-4} + \dots = 4\sum_{k=0}^{\infty}(z^{-2})^k = 4\cdot\frac{1}{1-z^{-2}} = \frac{4z^2}{z^2-1}$$

$$Y(z) = H(z)\cdot U(z) = \frac{\cancel{(z-1)}(z-2)}{(z+\frac{1}{2})(z-\frac{1}{3})} \cdot \frac{4z^2}{\cancel{(z-1)}(z+1)} = \frac{4z^2(z-2)}{(z+\frac{1}{2})(z-\frac{1}{3})(z+1)}$$

$$\frac{Y(z)}{z} = \frac{4z(z-2)}{(z+\frac{1}{2})(z-\frac{1}{3})(z+1)} = \frac{A}{z+\frac{1}{2}} + \frac{B}{z-\frac{1}{3}} + \frac{C}{z+1}$$

$$Az^2 - \frac{1}{3}Az + Az - \frac{1}{3}A + Bz^2 + \frac{1}{2}Bz + Bz + \frac{1}{2}B + Cz^2 + \frac{1}{6}Cz - \frac{1}{6}C = 4z^2 - 8$$

$A+B+C = 4$ $\qquad A=-12$

$\frac{2}{3}A + \frac{3}{2}B + \frac{1}{6}C = -8$ $\qquad B=-2$

$-\frac{1}{3}A + \frac{1}{2}B - \frac{1}{6}C = 0$ $\qquad C=18$

$$Y(z) = \frac{-12z}{z+\frac{1}{2}} + \frac{-2z}{z-\frac{1}{3}} + \frac{18z}{z+1}$$

$$\boxed{y(n) = \left[-12\left(-\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n + 18(-1)^n\right]\mu(n)}$$

4. $y''(t) + 9y'(t) + 8y(t) = u'(t) - 2u(t)$

$y(0^-) = 1$

$y'(0^-) = 2$

$u(t) = e^t \mu(t)$

početni uvjeti

$y(0^+) - y(0^-) = 0 \cdot u(0^+) \quad \rightarrow y(0^+) = 1$

$y'(0^+) - y'(0^-) + 9(y(0^+) - y(0^-)) = 0 + 1(u(0^+) - u(0^-))$

$y'(0^+) - 2 = 1$

$y'(0^+) = 3$

homogeno rješenje

$s^2 + 9s + 8 = 0$

$(s+8)(s+1) = 0$

$s_1 = -8 \quad s_2 = -1$

$y_h(t) = C_1 e^{-8t} + C_2 e^{-t}$

partikularno

$y_p(t) = Ke^t$

$y_p'(t) = Ke^t$

$y_p''(t) = Ke^t$

$u(t) = e^t$

$u'(t) = e^t$

$Ke^t + 9Ke^t + 8Ke^t = e^t - 2e^t$

$18K = -1$

$K = -\frac{1}{18}$

$y_p(t) = -\frac{1}{18} e^t \mu(t)$

totalno rješenje

$y_t(t) = C_1 e^{-8t} + C_2 e^{-t} - \frac{1}{18} e^t$

$y_t'(t) = -8C_1 e^{-8t} - C_2 e^{-t} - \frac{1}{18} e^t$

$y_t(0^+) = C_1 + C_2 - \frac{1}{18} = 1$

$y_t'(0^+) = -8C_1 - C_2 - \frac{1}{18} = 3$

$-7C_1 - \frac{2}{18} = 4$

$-7C_1 = \frac{2}{18} + 4 = \frac{1}{9} + 4 = \frac{37}{9}$

$C_1 = -\frac{37}{63}$

$C_2 = 1 + \frac{1}{18} - C_1$

$= \frac{19}{18} + \frac{37}{63}$

$= \frac{133 + 74}{126} = \frac{207}{126} = \frac{69}{42}$

$= \frac{23}{14}$

$y_t(t) = \left[-\frac{37}{63} e^{-8t} + \frac{23}{14} e^{-t} - \frac{1}{18} e^t\right] \mu(t)$

5. a) $h(n) = \left(\frac{1}{2^n} + \frac{1}{4^n}\right)\mu(n)$

B

$$H(z) = \frac{z}{z-\frac{1}{2}} + \frac{z}{z-\frac{1}{4}} = \frac{z^2 - \frac{1}{4}z + z^2 - \frac{1}{2}z}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)} = \frac{2z^2 - \frac{3}{4}z}{z^2 - \frac{3}{4}z + \frac{1}{8}}$$

b) $u(n) = 2^n$

$y(n) = H(z)\, U z^n$

$y(n) = H(2) \cdot 2^n$

$$H(2) = \frac{2\cdot 2^2 - \frac{3}{4}\cdot 2}{2^2 - \frac{3}{4}\cdot 2 + \frac{1}{8}} = \frac{\frac{32-6}{4}}{\frac{32-12+1}{8}} = \frac{\frac{26}{4}}{\frac{21}{8}} = \frac{52}{21}$$

$$\boxed{y(n) = \frac{52}{21}\cdot 2^n}$$

c) $u(n) = 2^n$

$$y(n) = u(n) * h(n)$$

$$= \sum_{m=-\infty}^{\infty} u(m)h(n-m)$$

$$= \sum_{m=-\infty}^{\infty} u(n-m)\, h(m)$$

$$= \sum_{m=-\infty}^{\infty} \left[\left(\frac{1}{2}\right)^m + \left(\frac{1}{4}\right)^m\right]\mu(m)\cdot 2^{n-m}$$

$$= \sum_{m=0}^{\infty} 2^{-m}\cdot 2^{n-m} + 2^{-2m}\cdot 2^{n-m}$$

$$= 2^n\left[\sum_{m=0}^{\infty}(2^{-2})^m + \sum_{m=0}^{\infty}(2^{-3})^m\right] =$$

$$= 2^n \cdot \left[\frac{1}{1-2^{-2}} + \frac{1}{1-2^{-3}}\right] = 2^n \cdot \left[\frac{1}{1-\frac{1}{4}} + \frac{1}{1-\frac{1}{8}}\right]$$

$$= 2^n \cdot \left[\frac{1}{\frac{3}{4}} + \frac{1}{\frac{7}{8}}\right] = 2^n \cdot \left[\frac{4}{3} + \frac{8}{7}\right] = 2^n \cdot \frac{28+24}{21}$$

$$\boxed{y(n) = \frac{52}{21}\cdot 2^n}$$