① $x_1(t)=2$, $x_2(t)=\sin\left(\frac{2\pi}{T}\cdot t\right)$, $x_3(t)=\sin\left(\frac{2\pi}{T}t^2\right)$, $T>0$

Periodičan vremenski kontinuiran signal, perioda T_0, definiran je kao:

$x:\mathbb{R}\to\mathbb{R}$

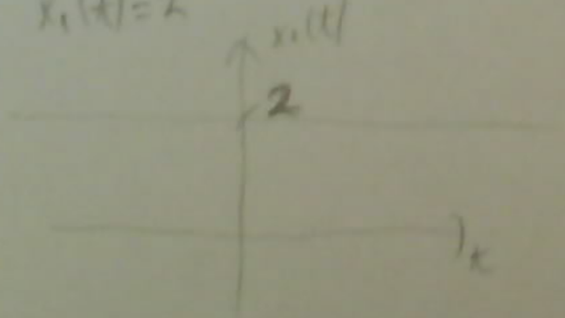
$\forall t\in\mathbb{R},\ T_0\in\mathbb{R}_+ \quad x(t)=x(t+T_0)$.

Signal x ponavlja se svakih T_0, ali i svakih $2T_0, 3T_0, \ldots$, pa vrijedi

$x(t+r\cdot T_0)=x(t)$, za $r\in\mathbb{Z}$

a) Ispitati periodičnost, odrediti temeljni period.

$x_1(t)=2$



Signal je periodičan, jer vrijedi da je $x(t)=x(t+T_0)$, ali nema točno određen p

jer jednakost vrijedi za svaki $T_0\in\mathbb{R}$.

$x_2(t)=\sin\left(\frac{2\pi}{T}\cdot t\right)$

$x_2(t)\overset{!}{=}x_2(t+T_0)$

$\sin(2\pi\ \ldots + 2k\pi) = \sin\left(\frac{2\pi}{T}(t+T_0)\right)$

$$x_2(t) = \sin\left(\frac{2\pi}{T} \cdot t\right)$$

$$x_2(t) \stackrel{?}{=} x_2(t + T_0)$$

$$\sin\left(\frac{2\pi}{T} \cdot t + 2k\pi\right) = \sin\left(\frac{2\pi}{T}\left(t + T_0\right)\right)$$

$$\cancel{\frac{2\pi}{T} \cdot t} + 2k\pi = \cancel{\frac{2\pi}{T} \cdot t} + \frac{2\pi}{T} \cdot T_0$$

$$\frac{T_0}{T} = k$$

$T_0 = k \cdot T$ $\Rightarrow$ traži se najmanji pozitivni T_0, pa k mora imati vrijednost 1.

$$\boxed{T_0 = T}$$

$$x_3(t) = \sin\left(\frac{2\pi}{T} \cdot t^2\right)$$

$$\sin\left(\frac{2\pi}{T} \cdot t^2 + 2k\pi\right) \stackrel{?}{=} \sin\left(\frac{2\pi}{T} \cdot (t + T_0)^2\right)$$

$$\cancel{\frac{2\pi}{T} \cdot t^2} + 2k\pi = \cancel{\frac{2\pi}{T} \cdot t^2} + \frac{2\pi}{T} \cdot 2 \cdot t \cdot T_0 + \frac{2\pi}{T} \cdot T_0^2$$

$$\frac{2 \cdot t \cdot T_0}{T} + \frac{T_0^2}{T} = k$$

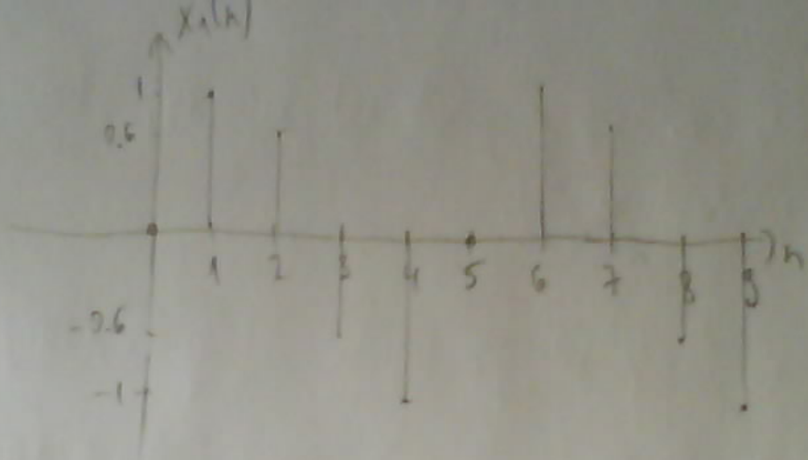
$$T_0 \cdot \left(\frac{T_0}{T} + \frac{2 \cdot t}{T}\right) = k$$

Ne postoji konstantna vrijednost T_0 koja bi dala za k prirodan broj, pa $\sin\left(\frac{2\pi}{T} \cdot t^2\right)$ nije periodičan.

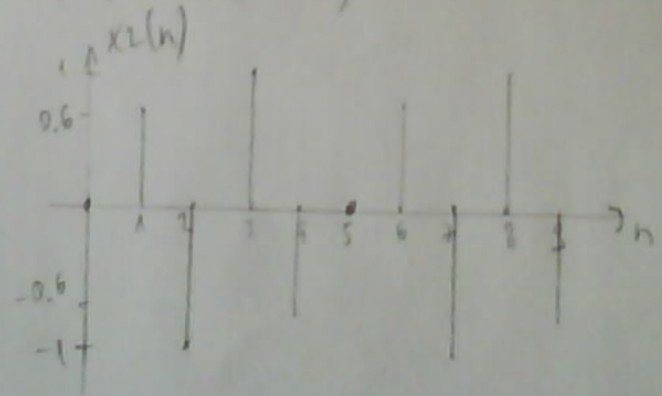
3. a) $x_k(n) = \sin(w_k \cdot n)$, $w_k = \frac{2\pi k}{5}$

$k \in \{1, 2, 4, 6\}$, $n \in [0, 8]$

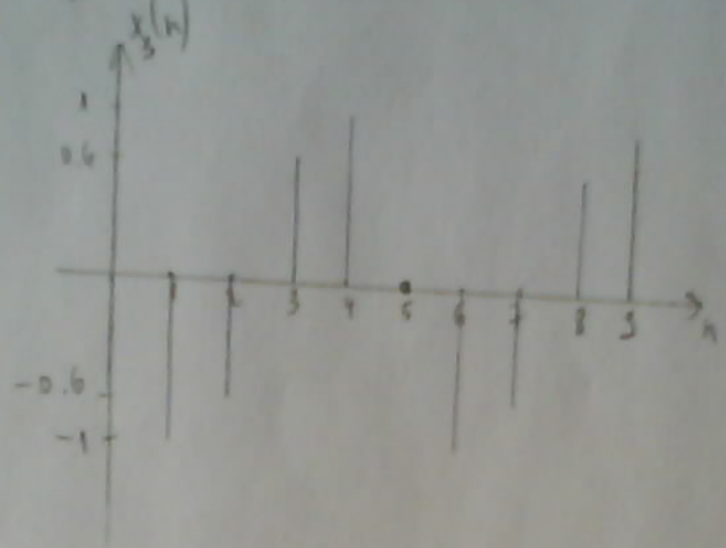
$$x_1 = \sin\left(\frac{2\pi \cdot 1}{5} \cdot n\right)$$



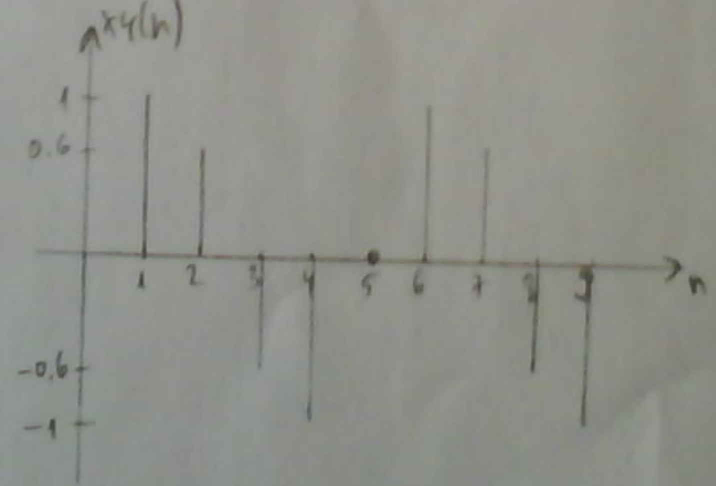
$$x_2 = \sin\left(\frac{2\pi}{5} \cdot 2 \cdot n\right) = \sin\left(\frac{4}{5}\pi \cdot n\right)$$

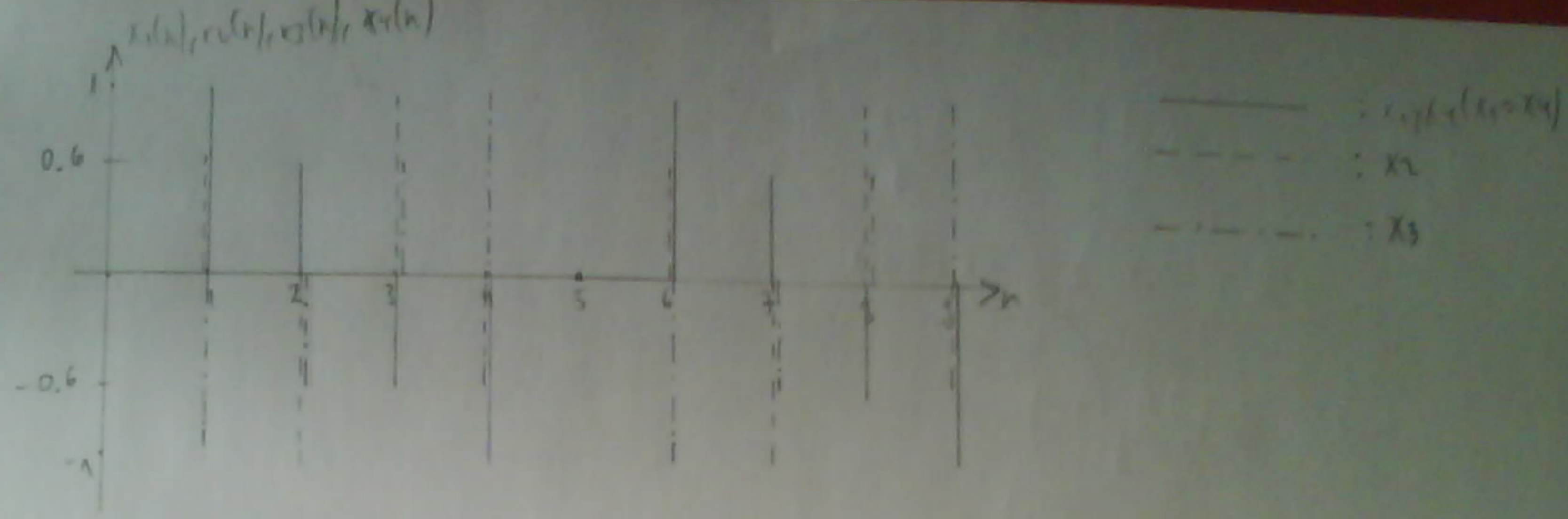


$$x_3 = \sin\left(\frac{8\pi}{5} \cdot n\right)$$



$$x_4 = \sin\left(\frac{12}{5}\pi \cdot n\right)$$





b) Nacrtana su 3 različita signala: x_1 (koji je jednak x_4), x_2 i x_3, s time da je x_3 zapravo x_1 pomnožen s -1 (odnosno pomaknut za π).

c) Jednaki su signali x_1 i x_4. To je zato jer vrijedi $\sin\left(\frac{2}{5}\pi n + 2n\pi\right) = \sin\left(\frac{12}{5}n\pi\right)$ tj. pomak za $2n\pi$ je zapravo opet isti signal.

5.

$x(t) = 110 \sin(120\pi t) + 50 \cdot \cos(360\pi t + \frac{\pi}{3})$, $T = \frac{1}{60}$

$T_0 = 2T = \frac{1}{30}$

$\omega_0 = 60\pi$

$x(t)$ - signal

$T_0 = \frac{2\pi}{\omega_0}$ - period signala

X_k - spektar

t - vrijeme

k - red harmonika

CTFS:

$$X_k = \frac{1}{T_0} \int_{T_0} x(t) \cdot e^{-j\omega_0 k t} \, dt$$

$$x(t) = \sum_{k=-\infty}^{+\infty} X_k \cdot e^{j\omega_0 \cdot k t}$$

$\frac{1}{j} = e^{-j\frac{\pi}{2}} = -j$

$$x(t) = \frac{110}{2j} \cdot \left(e^{j120\pi t} - e^{-j120\pi t}\right) + \frac{50}{2} \cdot \left(e^{j\cdot(360\pi t + \frac{\pi}{3})} + e^{-j(360\pi t + \frac{\pi}{3})}\right) =$$

$$= 55 \cdot e^{-j\frac{\pi}{2}} \cdot e^{j120\pi t} + 55 \cdot e^{j\frac{\pi}{2}} \cdot e^{-j120\pi t} + 25 \cdot e^{j\frac{\pi}{3}} \cdot e^{j360\pi t} + 25 \cdot e^{-j\frac{\pi}{3}} \cdot e^{-j360\pi t}$$

$k = 2$, $X_2 = 55 \cdot e^{-j\frac{\pi}{2}}$ $\qquad$ $k = 6$, $X_6 = 25 \cdot e^{j\frac{\pi}{3}}$

$k = -2$, $X_{-2} = 55 \cdot e^{j\frac{\pi}{2}}$ $\qquad$ $k = -6$, $X_{-6} = 25 \cdot e^{-j\frac{\pi}{3}}$

- naredbe:

```
syms To k t
x=110*sin((4*pi*t)/To)+ 50*cos(12*pi*t/To + pi/3)
Fk= int(x*exp(-j*2*pi*k*t/To),-To/2,To/2)/To
Fk= subs(Fk,To,1/30)
```

- dobiveno: Fk= -(50*sin(pi*k)*(-3*3^(1/2)+(k*i)/2)*i)/(pi*(k^2-36)) + (30*(-(22*sin(pi*k)^2)/3 + (11*sin(2*pi*k)*i)/3)*(2*sin((pi*k)/2)^2 + sin(pi*k)*i -1))/(pi*(k^2-4))

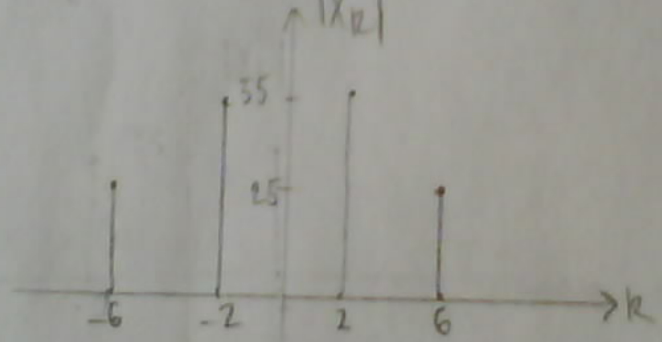
```
A= zeros(1,51);
for n=-25:25
```

```
Fk = subs(Fk, T0, 1/30)
```

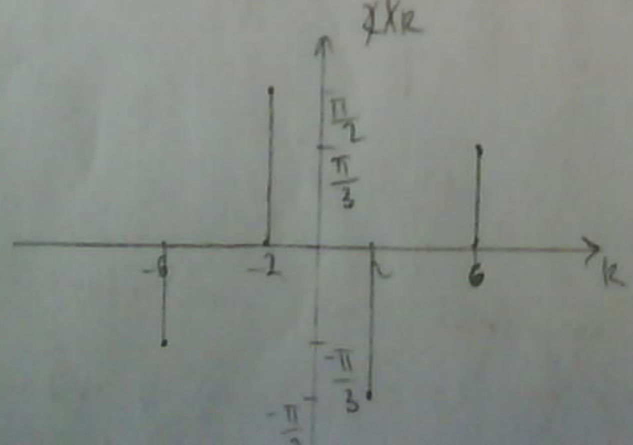
- dobiveno: Fk = -(50*sin(pi*k)*(-3*3^(1/2) + (k*i)/2)*i)/(pi*(k^2-36)) + (30*(-(22*sin(pi*k)^2)/3 + (11*sin(2*pi*k)*i)/3)*(2*sin((pi*k)/2)^2 + sin(pi*k)*i - 1))/(pi*(k^2-4))

```
A=zeros(1,51);
for n=-25:25
A(n+26)=subs(limit(Fk, k, n));
end
stem([-25:25], abs(A));
figure;
stem([-25:25], angle(A));
```

- amplitudni spektar:



- fazni spektar:



1.

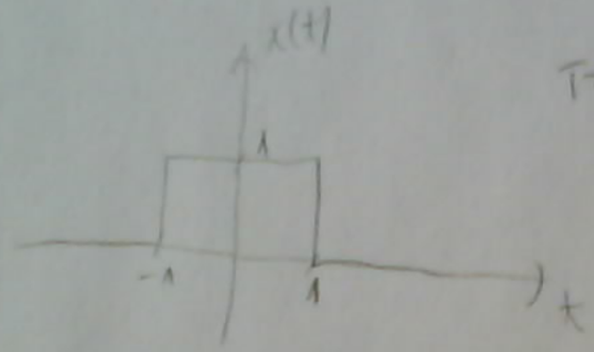
CTFT:

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) \cdot e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) \cdot e^{j\omega t} d\omega$$

$X(j\omega)$ - spektar
$x(t)$ - signal
t - vrijeme
ω - kružna frekvencija

a)



$T = 2$

$$X(j\omega) = \int_{-\frac{T}{2}}^{\frac{T}{2}} 1 \cdot e^{-j\omega \cdot t} dt = -\frac{e^{-j\omega t}}{j\omega}\Bigg|_{-\frac{T}{2}}^{\frac{T}{2}} = \frac{-1}{j\omega} \cdot \left(e^{-j\omega\frac{T}{2}} - e^{j\omega\frac{T}{2}}\right) =$$

$$= \frac{\left(e^{j\omega\frac{T}{2}} - e^{-j\omega\frac{T}{2}}\right) \cdot 2}{2j\omega} = 2 \cdot \frac{\sin(\omega)}{\omega}$$

b)

$$x = -\left(-\sin\omega + \cos(\omega) \cdot j\right) \cdot \frac{1}{\omega} + \left(\sin(\omega) + \cos(\omega) \cdot j\right) \cdot \frac{1}{\omega}$$

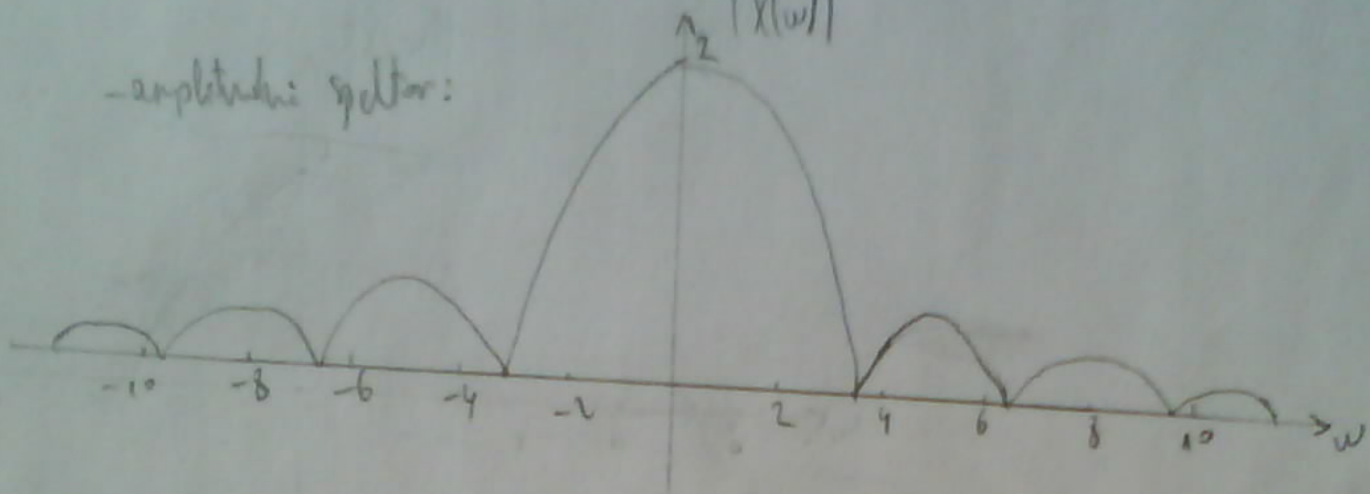
c)

- amplitudni spektar:

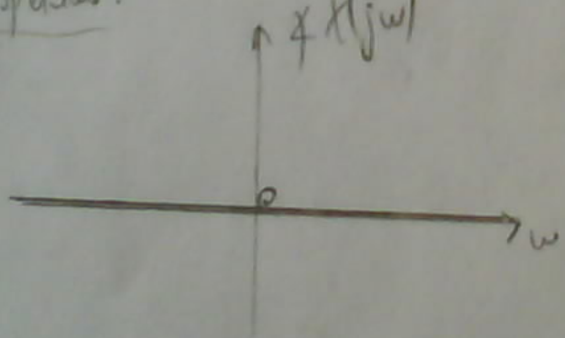
$$X = -\left(-\sin w + \cos(w)\cdot j\right)\cdot\frac{1}{w} + \left(\sin(w) + \cos(w)\cdot j\right)\cdot\frac{1}{w}$$

c)

- amplitudni spektar:



- fazni spektar:



Razlika između ovog signala i onog iz 6. c) je u amplitudi glavne latice, koja je veća u 7. zadatku. Oba spektra s porastom w, odnosno teže u 0.

3.

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

$x(t)$ - signal
$X(j\omega)$ - spektar
t - vrijeme
ω - kružna frekvencija

a)

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt =$$

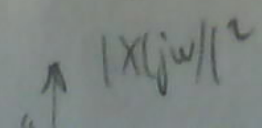
$$= \int_{-1}^{1} 1 \cdot e^{-j\omega t} dt = \frac{-e^{-j\omega t}}{j\omega} \Big|_{-1}^{1} =$$

$$= \frac{-1}{j\omega} \cdot (e^{-j\omega} - e^{j\omega}) = \frac{(e^{j\omega} - e^{-j\omega})}{2j \cdot \omega} \cdot 2$$

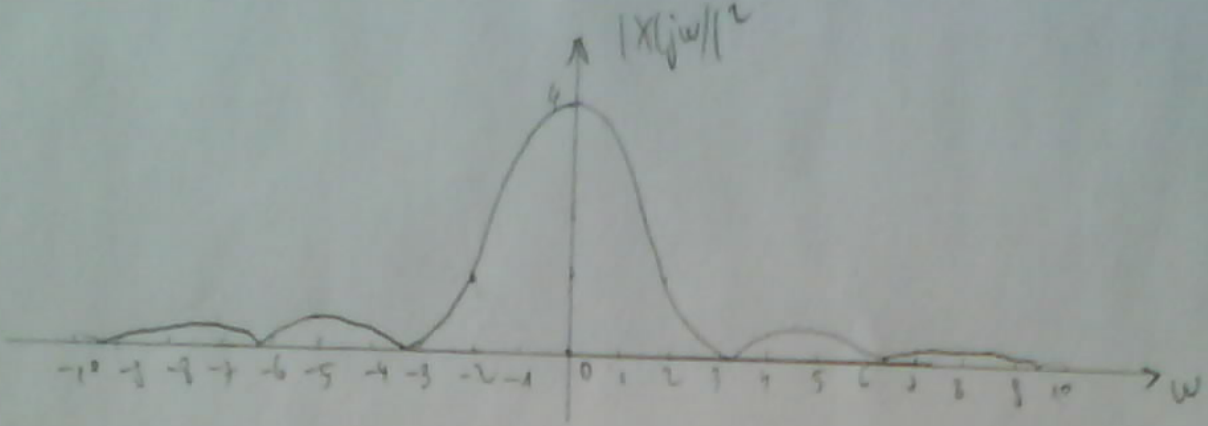
$$= 2 \cdot \frac{\sin(\omega)}{\omega}$$

$$|X(j\omega)|^2 = 4 \cdot \frac{\sin^2 \omega}{\omega^2}$$

```
spg = (4*abs(sin(w))^2) / abs(w)^2
```



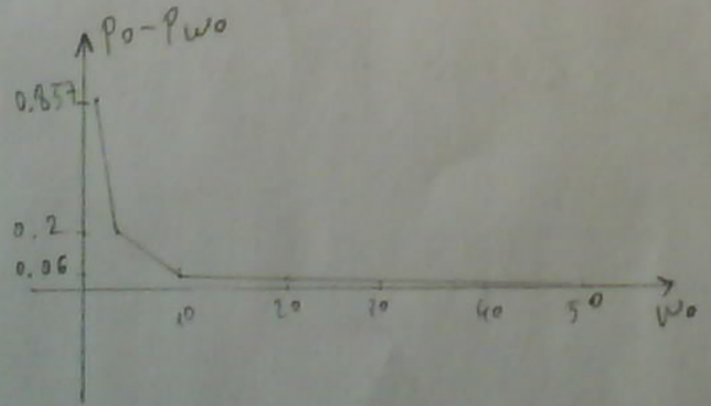
spg = (4 * abs(sin(w))^2) / abs(w)^2



b) $\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{2} = \pi$

$$P_{\omega_0} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |X(j\omega)|^2 d\omega = 1.80565$$

$$P_0 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = 2$$



Ako wo teži u beskonačno, tada proračun snage postaje sve precizniji, pa razlika Po-Pwo teži u 0.

(11.)

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2$$

$x(n)$ - signal

$X(e^{j\omega})$ - spektar

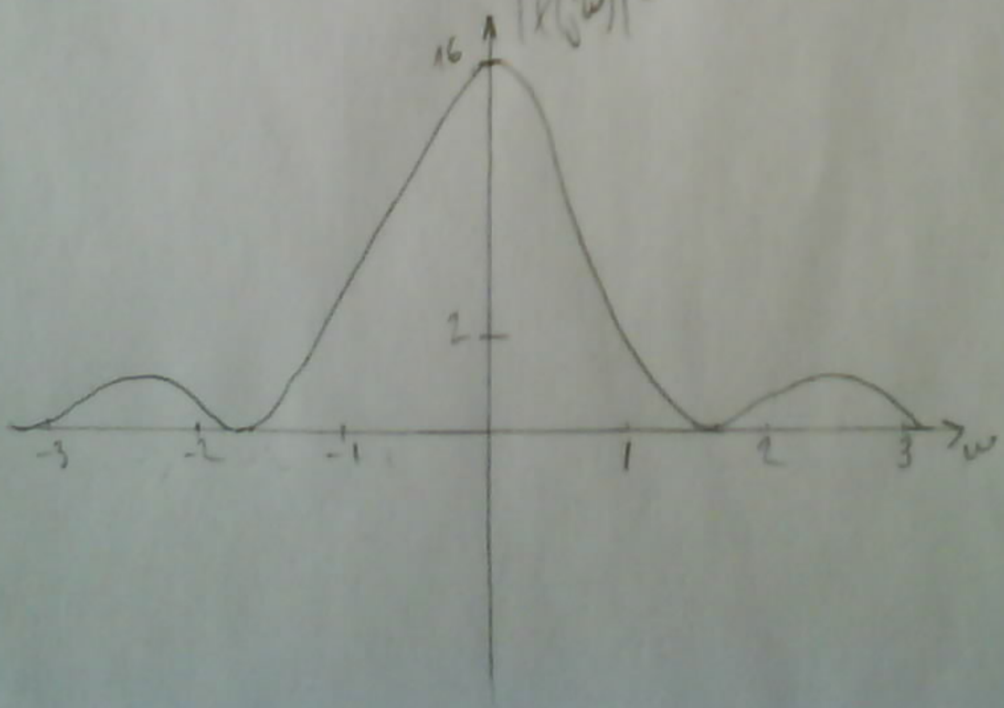
t - vrijeme

ω - kružna frekvencija

$$x(n) = \{ \ldots, 0, 0, 0, \underline{1}, 1, 1, 1, 0, 0, 0 \ldots \}$$

a)

```
w=[-pi:0.01:pi]
x=[1 1 1 1]
X=freqz(x,1,w)
plot(w, abs(power(X,2)))
```



12.

DTFS:

$$X_k = \frac{1}{N} \sum_{n=0}^{N-1} x(n) \cdot e^{-2\pi j \, k \cdot \frac{n}{N}}$$

$$x(n) = \sum_{k=0}^{N-1} X_k \cdot e^{2\pi j \frac{k \cdot n}{N}}$$

$x(n)$ - signal

X_k - spektar

t - vrijeme

k - red harmonika

a) $x_1(t) = \sin(2\pi \cdot 700 t)$, $f_s = 8000\,Hz$, $N = 8000$ uzoraka

$$x_1(n) = x_1(n \cdot T_s) = \sin\left(2\pi \cdot 700 \cdot \frac{1}{8000} \cdot n\right)$$

$$x_1(n) = \sin\left(\frac{7\pi}{40} \cdot n\right)$$

b) naredbe:

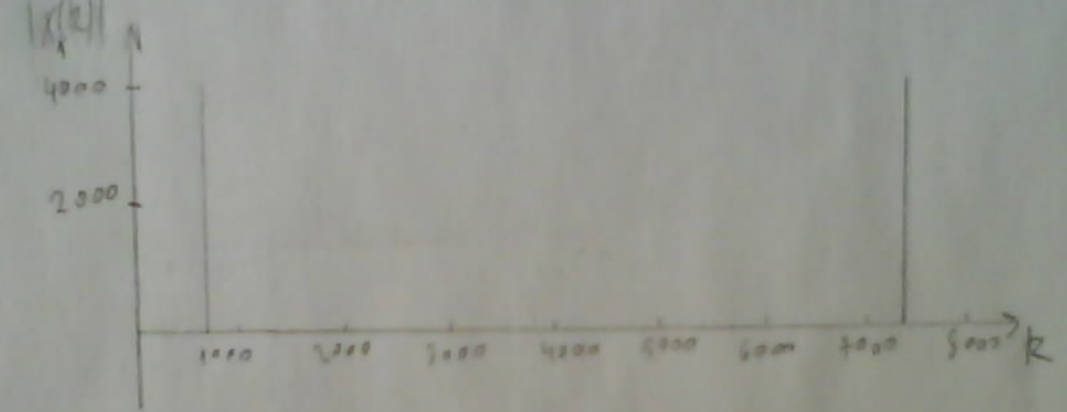
```
t = [0:0.000125:0.999875]
x = sin (2*pi*700*t)
X = fft(x)
```

A = abs(X)

N = length(X)

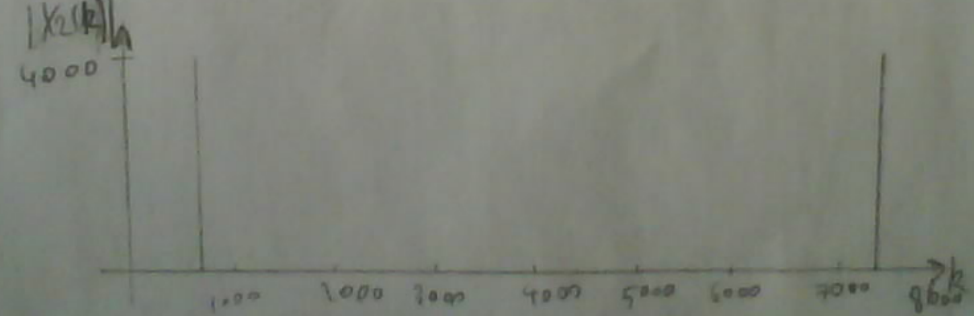
w = ([1:N]-1)/N * 8000

plot(w, A)

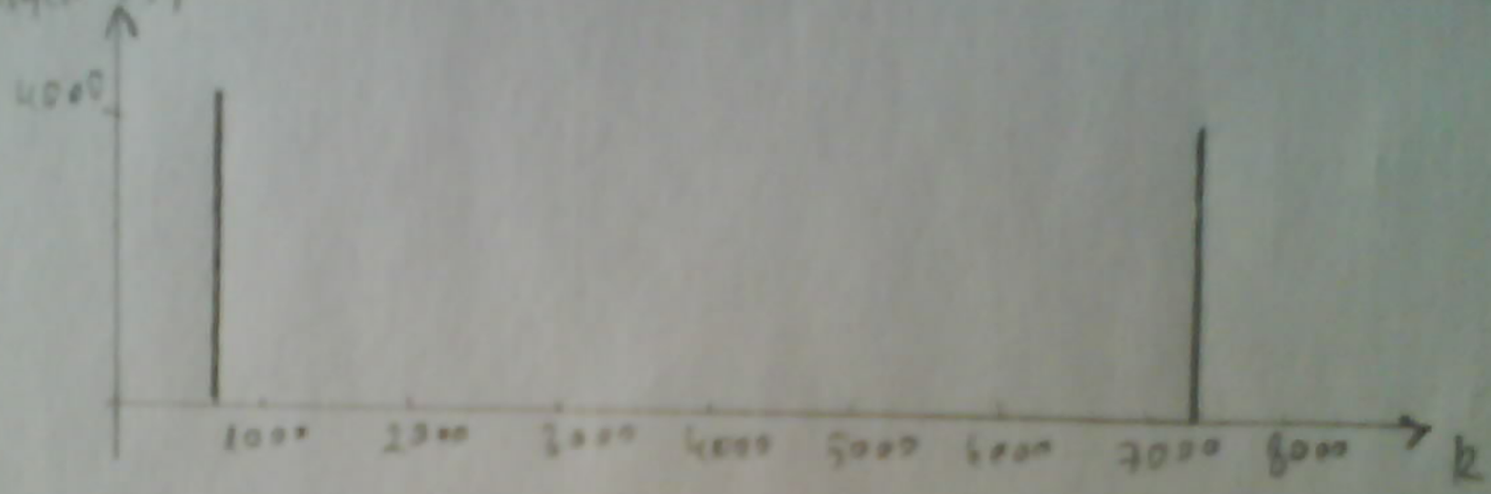


c) $f_s = 8kHz$, $x_2(t) = \sin(2\pi \cdot 705 \cdot t)$ $N = 8000$

$x_2(n \cdot T_s) = \sin\left(\frac{2\pi \cdot 705}{8000} \cdot n\right)$



d) $|X_1(k)+X_2(k)|$



f) Spektri iz a), b) i d) podzadatka su jednaki.