③ $3y(n)+10y(n-1)+3y(n-2)=u(n)$

$$3y(n) \circ\!\!-\!\!\bullet\ 3Y(z)$$

$$10y(n-1) \circ\!\!-\!\!\bullet\ 10\left(z^{-1}Y(z)+y(-1)\right)=$$

$$=10z^{-1}Y(z)+10y(-1)$$

$$3y(n-2)\ \circ\!\!-\!\!\bullet\ 3\left(z^{-2}Y(z)+\sum_{i=0}^{1}y(i-2)z^{-i}\right)=$$

$$=3\left(z^{-2}Y(z)+y(-2)z^{0}+y(-1)z^{-1}\right)=$$

$$=3z^{-2}Y(z)+3z^{-1}y(-1)+3y(-2)$$

$$u(n)\ \circ\!\!-\!\!\bullet\ U(z)$$

$$3Y(z)+10z^{-1}Y(z)+10y(-1)+3z^{-2}Y(z)+3z^{-1}y(-1)+$$
$$+3y(-2)=U(z)$$

$$Y(z)\left(3+10z^{-1}+3z^{-2}\right)=U(z)-10y(-1)-3z^{-1}y(-1)-$$
$$-3y(-2)$$

$$Y(z)=\underbrace{\frac{1}{3+10z^{-1}+3z^{-2}}}_{H(z)}U(z)-\frac{10y(-1)+3z^{-1}y(-1)+3y(-2)}{3+10z^{-1}+3z^{-2}}$$

a) IMPULSNI ODZIV

(6) prijenosna f-ja

$$\boxed{H(z)} = \frac{1}{3+10z^{-1}+3z^{-2}} = \boxed{\frac{z^2}{3z^2+10z+3}} =$$

$$= \frac{z^2}{3\left(z^2+\frac{10}{3}z+1\right)}$$

$$z^2+\frac{10}{3}z+1=0$$

$$z_{1,2} = -\frac{5}{3} \pm \sqrt{\frac{25}{9}-\frac{9}{9}} = -\frac{5}{3} \pm \frac{4}{3}$$

$$z_1 = -\frac{9}{3} = -3$$

$$z_2 = -\frac{1}{3} = -\frac{1}{3}$$

$$H(z) = \frac{1}{\ldots} \quad \frac{z^2}{\ldots}$$

$$H(z) = \frac{1}{3} \cdot \frac{z^2}{(z+3)(z+\frac{1}{3})}$$

$$\frac{H(z)}{z} = \frac{1}{3} \frac{z}{(z+3)(z+\frac{1}{3})} = \frac{A}{z+3} + \frac{B}{z+\frac{1}{3}}$$

$$\frac{1}{3} z = Az + \frac{1}{3}A + Bz + 3B =$$

$$= z(A+B) + \frac{1}{3}A + 3B$$

$$A + B = \frac{1}{3} \quad / \cdot (-3)$$

$$\frac{1}{3}A + 3B = 0$$

$$\left.\begin{aligned} -3A - 3B &= -1 \\ \frac{1}{3}A + 3B &= 0 \end{aligned}\right\} +$$

$$-3A + \frac{1}{3}A = -1$$

$$-\frac{8}{3}A = -1$$

$$\boxed{A = \frac{3}{8}}$$

$$\boxed{B = -\frac{1}{24}}$$

$$\frac{H(z)}{z} = \frac{3}{8}\,\frac{1}{z+3} - \frac{1}{24}\,\frac{1}{z+\frac{1}{3}}$$

$$H(z) = \frac{3}{8}\,\frac{z}{z+3} - \frac{1}{24}\,\frac{z}{z+\frac{1}{3}}$$

$$\boxed{h(n) = \frac{3}{8}(-3)^n - \frac{1}{24}\left(-\frac{1}{3}\right)^n, \; n \geq 0}$$

(c) <u>polovi</u>:

$z_1 = -3$ $\qquad$ $z_2 = -\frac{1}{3}$

$|z_1| = 3$ $\qquad$ $|z_2| = \frac{1}{3}$

(c) polovi:

$$z_1 = -3 \qquad z_2 = -\frac{1}{3}$$

$$\underbrace{|z_1| = 3 \qquad |z_2| = \frac{1}{3}}$$

$$\underbrace{|z| < 1}_{\text{uvjet}} \Rightarrow \boxed{\text{NESTABILAN}}$$

1) $u(n) = \cos(n\pi) = (-1)^n$

MIRNI ODZIV:

$$\boxed{y(-2) = y(-1) = 0}$$

$$u(n) \circ\!\!-\!\!\bullet\, U(z) = \frac{z}{z+1}$$

$$Y(z) = \frac{1}{3}\,\frac{z^2}{(z+3)(z+\frac{1}{3})}\cdot\frac{z}{z+1}$$

$$\frac{Y(z)}{z} = \frac{\frac{1}{3}z^2}{(z+3)(z+\frac{1}{3})(z+1)} = \frac{A}{z+3} + \frac{B}{z+\frac{1}{3}} + \frac{C}{z+1}$$

$$\frac{1}{3}z^2 = A\left(z+\frac{1}{3}\right)(z+1) +$$

$$+ B(z+3)(z+1) +$$

$$+ C(z+3)\left(z+\frac{1}{3}\right)$$

$$+ C(z+3)\left(z+\frac{1}{3}\right)$$

$$\boxed{A = \frac{9}{16}} \quad \boxed{B = \frac{1}{48}} \quad \boxed{C = -\frac{1}{4}}$$

$$\frac{Y(z)}{z} = \frac{9}{16} \cdot \frac{1}{z+3} + \frac{1}{48} \frac{1}{z+\frac{1}{3}} - \frac{1}{4} \cdot \frac{1}{z+1}$$

$$Y(z) = \frac{9}{16} \frac{z}{z+3} + \frac{1}{48} \frac{z}{z+\frac{1}{3}} - \frac{1}{4} \frac{z}{z+1}$$

$$\boxed{y_{MIRNI}(n) = \frac{9}{16}(-3)^n + \frac{1}{48}\left(-\frac{1}{3}\right)^n - \frac{1}{4}(-1)^n, \quad n \geq 0}$$

④ $4y''(t) - y(t) = 4u(t)$

$$y''(t) \circ\!\!-\!\!\bullet\; s^2 Y(s) - s\,y(0^-) - y'(0^-)$$

$$y(t) \circ\!\!-\!\!\bullet\; Y(s)$$

$$u(t) \circ\!\!-\!\!\bullet\; U(s)$$

$$4s^2 Y(s) - 4s\,y(0^-) - 4y'(0^-) - Y(s) = 4U(s)$$

$$Y(s)(4s^2 - 1) = 4U(s) + 4s\,y(0^-) + 4y'(0^-)$$

$$Y(s) = \frac{4}{4s^2 - 1} U(s) + \frac{4s\,y(0^-) + 4y'(0^-)}{4s^2 - 1}$$

(b)

$$4s^2-1 \qquad\qquad 4s^2-1$$

(b) $\boxed{H(s) = \dfrac{4}{4s^2-1}}$

(a) $$H(s) = \frac{4}{4\left(s^2-\frac{1}{4}\right)} = \frac{1}{\left(s-\frac{1}{2}\right)\left(s+\frac{1}{2}\right)} =$$

$$= \frac{A}{s-\frac{1}{2}} + \frac{B}{s+\frac{1}{2}}$$

$$1 = s(A+B) + \frac{1}{2}A - \frac{1}{2}B$$

$$\Rightarrow \boxed{\begin{aligned} A &= 1 \\ B &= -1 \end{aligned}}$$

$$H(s) = \frac{1}{s-\frac{1}{2}} - \frac{1}{s+\frac{1}{2}}$$

$$\boxed{h(t) = \left(e^{\frac{t}{2}} - e^{-\frac{t}{2}}\right)\mu(t)}$$

(a) <u>pólů:</u>

$$4s^2 - 1 = 0$$

$$s^2 - \frac{1}{4} = 0$$

$$\boxed{s_1 = \frac{1}{2}} \quad \boxed{s_2 = -\frac{1}{2}} \rightarrow \boxed{\text{NESTABILAN}}$$

(d) $u(t) = \cos(\frac{1}{2}t)\mu(t)$

$$U(s) = \frac{s}{s^2+\frac{1}{4}}$$

$$\boxed{\begin{array}{l}\text{MIRNI ODZIV} \\ y(0^-) = y'(0^-) = 0\end{array}}$$

$$Y(s) = \frac{4}{4s^2-1} \cdot \frac{s}{s^2+\frac{1}{4}} = \frac{s}{(s-\frac{1}{2})(s+\frac{1}{2})(s^2+\frac{1}{4})}$$

$$= \frac{A}{s-\frac{1}{2}} + \frac{B}{s+\frac{1}{2}} + \frac{Cs+D}{s^2+\frac{1}{4}}$$

$$\rightarrow \boxed{A=1} \quad \boxed{B=1} \quad \boxed{C=-2} \quad \boxed{D=0}$$

$$Y(s) = \frac{1}{s-\frac{1}{2}} + \frac{1}{s+\frac{1}{2}} - 2\frac{s}{s^2+\frac{1}{4}}$$

$$\boxed{y(t) = e^{\frac{t}{2}} + e^{-\frac{t}{2}} - 2\cos(\tfrac{1}{2}t), \quad t \geq 0}$$