1.

A

$h(t) = \delta(t) + 2e^{-4t}\mu(t)$

a) $s = -4$

zbog $\delta(t)$ - postoji derivacija ulaza

$y'(t) + a_1 y(t) = b_0 u'(t) + b_1 u(t)$

$b_0 = 1$

$a_1 = +4$

$h_a'(t) + h_a(t) \cdot 4 = 0$

$h_a = Ce^{-4t}$

$h_a(0^+) = 1$

$h_a(0^+) = C \cdot 1 = 1$

$C = 1$

$h_a(t) = e^{-4t} \rightarrow h_a'(t) = -4e^{-4t}$

$h(t) = b_0 \cdot \delta(t) + b_0 h_a'(t) + b_1 h_a(t)$

$= b_0 \cdot \delta(t) - 4b_0 e^{-4t} + b_1 e^{-4t}$

$b_0 = 1$ $\quad -4b_0 + b_1 = 2$

$-4 + b_1 = 2$

$b_1 = 6$

$y'(t) + 4y(t) = u'(t) + 6u(t)$

$\boxed{H(s) = \frac{s+6}{s+4}}$

ili

$h(t) = \delta(t) + 2e^{-4t}\mu(t)$

u Laplaceovoj domeni

$\downarrow$

$H(s) = 1 + 2 \cdot \frac{1}{s+4}$

$H(s) = \frac{s+6}{s+4}$

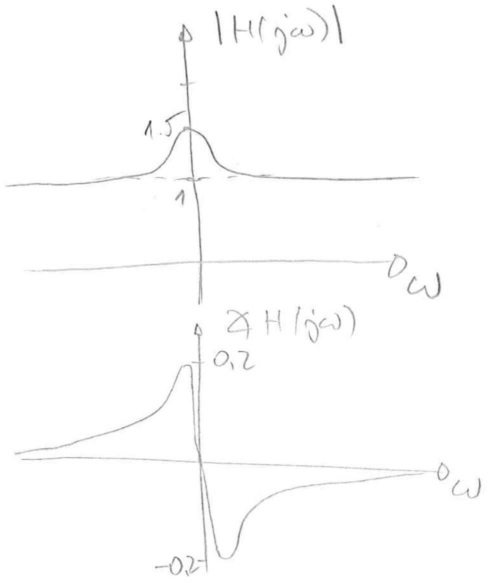
POL $s = -4 < 0$

STABILAN SUSTAV

b) $H(j\omega) = \frac{j\omega + 6}{j\omega + 4}$

$|H(j\omega)| = \sqrt{\frac{36+\omega^2}{16+\omega^2}}$

$\measuredangle H(j\omega) = \text{arctg}\,\frac{\omega}{6} - \text{arctg}\,\frac{\omega}{4}$



c) $\boxed{y'(t) + 4y(t) = u'(t) + 6u(t)}$

A

2.

$$y(n) + \frac{1}{5} y(n-1) = u(n)$$

a) $H(z) = \dfrac{1}{1+\frac{1}{5}z^{-1}} = \dfrac{z}{z+\frac{1}{5}}$

$z_1 = -\frac{1}{5}$ POL $|z_1| < 1$ STABILNO

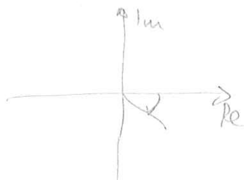
b) $H(e^{j\Omega}) = \dfrac{1}{1+\frac{1}{5}e^{-j\Omega}} = \dfrac{1}{1+\frac{1}{5}\cos\Omega - j\frac{1}{5}\sin\Omega}$

$u(n) = 2\cos\frac{\pi}{2}n$

$\Omega = \frac{\pi}{2}$

$H(e^{j\frac{\pi}{2}}) = \dfrac{1}{1+\frac{1}{5}\cos\frac{\pi}{2} - j\frac{1}{5}\sin\frac{\pi}{2}} = \dfrac{1}{1-j\frac{1}{5}}$

$|H(e^{j\frac{\pi}{2}})| = \dfrac{1}{\sqrt{1+\frac{1}{25}}} = \sqrt{\dfrac{1}{\frac{26}{25}}} = \dfrac{5}{\sqrt{26}}$



$\measuredangle H(e^{j\pi/2}) = -\mathrm{arctg}\,\dfrac{-\frac{1}{5}}{\frac{1}{1}} = -\mathrm{arctg}\,\dfrac{-1}{5} = 0.197$

$y(n) = 2 \cdot \frac{5}{\sqrt{26}} \cos\left(\frac{\pi}{2}n + 0.197\right)$

$= 1.96 \cos\left(\frac{\pi}{2}n + 0.197\right)$

c) LTI → h

pobuda u → odziv $y = h * u$

za ulaz $\delta(n)$ → izlaz je $h(n)$

za ulaz $u(0)\delta(n)$, gdje je $u(0)$ neki broj → izlaz je $u(0)h(n)$

zbog svojstva homogenosti

neka je ulaz zbroj impulsa različitih amplituda

$u(n) = \ldots + u(-1)\delta(n+1) + u(0)\delta(n) + u(1)\delta(n-1) + \ldots$

$= \sum_{m=-\infty}^{\infty} u(m)\delta(n-m)$

za ulaz $u(1)\delta(n-1)$ → na izlazu je $u(1)h(n-1)$ → zbog vremenske stalnosti

za ulaz $u(n)$ je tada (zbog linearnosti)

$y(n) = \ldots + u(-1)h(n+1) + u(0)h(n) + u(1)h(n-1) + \ldots$

$= \sum_{m=-\infty}^{\infty} u(m)h(n-m) = u * h = h * u$

3. A

$$y''(t)+4y'(t)+5y(t)=2u'(t)+u(t)$$

a) $u(t)=\begin{cases}-t, & t<0\\ -t+1, & t>0\end{cases}$

homogeno

$s^2+4s+5=0$

$s_{1,2}=\frac{-4\pm\sqrt{16-4\cdot5}}{2}=\frac{-4\pm\sqrt{-4}}{2}=-2\pm j$

$\boxed{y_h(t)=C_1e^{(-2-j)t}+C_2e^{(-2+j)t}}$

1° $t<0$

partikularno

$y_p(t)=k_0+k_1t$ 1° $u_1(t)=-t$

$u_1'(t)=-1$

$y_p'(t)=k_1$

$4k_1+5(k_0+k_1t)=-2-t$

$4k_1+5k_0=-2$ $5k_0=-2-4k_1$

$5k_1=-1$ $5k_0=-2+\frac{4}{5}=\frac{-6}{5}$

$k_1=-\frac{1}{5}$ $k_0=-\frac{6}{25}$

$\boxed{y_{p1}(t)=-\frac{6}{25}-\frac{1}{5}t}$

$\boxed{\begin{aligned}&t<0:\\ &y(t)=-\frac{6}{25}-\frac{1}{5}t\end{aligned}}$

u blizini nule se homogeno istitralo pa je za

$y_{p1}'(t)=-\frac{1}{5}$

$u(0^-)=0$

$u(0^+)=1$

početni uvjeti za poslije nule:

$y_{p1}(0^-)=-\frac{6}{25}$ $y(0^+)=-\frac{6}{25}$

$y_{p1}'(0^-)=-\frac{1}{5}$

$y'(0^+)-y_{p1}'(0^-)=2\cdot(u(0^+)-u(0^-))=2$

$y'(0^+)=2+(-\frac{1}{5})=\frac{9}{5}$

2° $t>0$

$u_2(t)=-t+1$

$y_p=k_0+k_1t$ $u_2'(t)=-1$

$y_p'=k_1$

$4k_1+5(k_0+k_1t)=-2-t+1=-1-t$

$4k_1+5k_0=-1$ $5k_0=-1-4k_1$

$5k_1=-1$ $k_0=(-1+\frac{4}{5})\cdot\frac{1}{5}=\frac{-1}{25}$

$k_1=-\frac{1}{5}$

$\boxed{y_{p2}=-\frac{1}{25}-\frac{1}{5}t}$

3.

A

$$y(t) = c_1 e^{(-2-j)t} + c_2 e^{(-2+j)t} - \frac{1}{25} - \frac{1}{5}t$$

$$y(0^+) = c_1 + c_2 - \frac{1}{25} = -\frac{6}{25}$$

$$y'(t) = (-2-j)c_1 e^{(-2-j)t} + c_2(-2+j)e^{(-2+j)t} - \frac{1}{5}$$

$$y'(0^+) = (-2-j)c_1 + c_2(-2+j) - \frac{1}{5} = \frac{9}{5}.$$

$$c_1 + c_2 = -\frac{1}{5}$$

$$(-2-j)c_1 + (-2+j)c_2 = 2$$

$$-(-2-j)c_1 - (-2-j)c_2 = +\frac{1}{5}(-2-j)$$

$$(-2+j+2+j)c_2 = 2 - \frac{2}{5} - \frac{1}{5}j$$

$$2jc_2 = \frac{8-j}{5}$$

$$c_2 = \frac{8-j}{10j} = \frac{8j+1}{-10} = -\frac{1}{10} - \frac{8}{10}j$$

$$c_1 = -\frac{1}{5} - c_2 = -\frac{1}{5} + \frac{1}{10} + \frac{8}{10}j = \frac{-1}{10} + \frac{8}{10}j$$

$$c_1 = -\frac{1}{10} + \frac{8}{10}j$$

za $t > 0$

$$\boxed{y(t) = \left(-\frac{1}{10} + \frac{8}{10}j\right)e^{(-2-j)t} - \left(\frac{1}{10} + \frac{8}{10}j\right)e^{(-2+j)t} - \frac{1}{25} - \frac{1}{5}t}$$

ukupno rješenje

$$\boxed{y(t) = \begin{cases} -\frac{6}{25} - \frac{1}{5}t, & t < 0 \\ \left(-\frac{1}{10} + \frac{8}{10}j\right)e^{(-2-j)t} - \left(\frac{1}{10} + \frac{8}{10}j\right)e^{(-2+j)t} - \frac{1}{25} - \frac{1}{5}t, & t > 0 \end{cases}}$$

3b) A

$$y'(t) + a_1 y(t) = b_0 u'(t) + b_1 u(t) \quad / \int_{0^-}^{t}$$

$$\int_{0^-}^{t} y'(\tau)\,d\tau + a_1 \int_{0^-}^{t} y(\tau)\,d\tau = b_0 \int_{0^-}^{t} u'(\tau)\,d\tau + b_1 \int_{0^-}^{t} u(\tau)\,d\tau$$

$$y(t) - y(0^-) + a_1 \int_{0^-}^{t} y(\tau)\,d\tau = b_0\, u(t) - b_0\, u(0^-) + b_1 \int_{0^-}^{t} u(\tau)\,d\tau$$

ze $t = 0^+$:

$$y(0^+) - y(0^-) + a_1 \underbrace{\int_{0^-}^{0^+} y(\tau)\,d\tau}_{\emptyset} = b_0 \left(u(0^+) - u(0^-)\right) + b_1 \underbrace{\int_{0^-}^{0^+} u(\tau)\,d\tau}_{\emptyset}$$

$$y(0^+) - y(0^-) = b_0 \left(u(0^+) - u(0^-)\right)$$

$$\boxed{y(0^+) = b_0\, u(0^+) - b_0\, u(0^-) + y(0^-)}$$

1

4. $H(z) = \dfrac{9z^{-1} - z^{-2}}{9 - z^{-2}}$

a) $H(z) = \dfrac{9z - 1}{9z^2 - 1}$

$H_1(z) = \dfrac{H(z)}{z} = \dfrac{9z-1}{z(3z-1)(3z+1)} = \dfrac{z - \frac{1}{9}}{z(z-\frac{1}{3})(z+\frac{1}{3})}$

$= \dfrac{A}{z} + \dfrac{B}{z-\frac{1}{3}} + \dfrac{C}{z+\frac{1}{3}}$

$A(z^2 - \frac{1}{9}) + Bz(z+\frac{1}{3}) + Cz(z-\frac{1}{3}) = z - \frac{1}{9}$

$A + B + C = 0$ $\qquad B + C = -1$ $\qquad C = -1 - B$

$\frac{1}{3}B - \frac{1}{3}C = 1$ $\longrightarrow$ $B - C = 3$ $\qquad = -1 - 1$

$-\frac{1}{9}A = -\frac{1}{9}$ $\qquad 2B = 2$ $\qquad C = -2$

$A = 1$ $\qquad B = 1$

$H_1(z) = \dfrac{1}{z} + \dfrac{1}{z-\frac{1}{3}} + \dfrac{-2}{z+\frac{1}{3}}$

$H(z) = 1 + \dfrac{z}{z-\frac{1}{3}} + \dfrac{-2z}{z+\frac{1}{3}}$

$\boxed{h(n) = \delta(n) + (\tfrac{1}{3})^n \mu(n) - 2(-\tfrac{1}{3})^n \mu(n)}$

b) $Y(z) = \dfrac{9z^{-1} - z^{-2}}{9 - z^{-2}} U(z)$

$\boxed{9y(n) - y(n-2) = 9u(n-1) - u(n-2)}$

c) $u(n) = \{\underline{1}, 0, \frac{1}{9^2}, 0, \frac{1}{9^4}, 0, \dots 0, \frac{1}{9^{2k}}, \dots\}$

$U(z) = 1 + \frac{1}{9^2} z^{-2} + \frac{1}{9^4} z^{-4} + \dots + \frac{1}{9^{2k}} z^{-2k} + \dots$

$= \sum_{k=0}^{\infty} \left(\dfrac{1}{9^2 z^2}\right)^k = \dfrac{1}{1 - \frac{1}{9^2 z^2}} = \dfrac{1}{\frac{81z^2 - 1}{81z^2}} = \dfrac{81z^2}{81z^2 - 1} = \dfrac{z^2}{z^2 - \frac{1}{81}}$

$y(-1) = 1$

$y(-2) = 0$

4.

A

$$y(n) - \frac{1}{9} y(n-2) = u(n-1) - \frac{1}{9} u(n-2)$$

$$Y(z) - \frac{1}{9}\left[z^{-2}Y(z) + z^{-1}y(-1) + y(-2)\right] = z^{-1}U(z) + \cancel{u(-1)} - \frac{1}{9}\left(z^{-2}U(z) + z^{-1}\cancel{u(-1)} + \cancel{u(-2)}\right)$$

$$Y(z)\left(1 - \frac{1}{9}z^{-2}\right) = U(z)\left(z^{-1} - \frac{1}{9}z^{-2}\right) + \frac{1}{9}z^{-1}\cdot 1 + \frac{1}{9}\cdot 0$$

$$Y(z) = \frac{z^{-1} - \frac{1}{9}z^{-2}}{1 - \frac{1}{9}z^{-2}} \cdot U(z) + \frac{\frac{1}{9}z^{-1}}{1 - \frac{1}{9}z^{-2}}$$

$$= \frac{z - \frac{1}{9}}{z^2 - \frac{1}{9}} \cdot U(z) + \frac{\frac{1}{9}z}{z^2 - \frac{1}{9}}$$

MIRNI

$$Y_m(z) = \frac{z - \frac{1}{9}}{z^2 - \frac{1}{9}} \cdot \frac{z^2}{z^2 - \frac{1}{81}}$$

$$= \frac{\cancel{(z - \frac{1}{9})}\, z^2}{(z - \frac{1}{3})(z + \frac{1}{3})\cancel{(z - \frac{1}{9})}(z + \frac{1}{9})}$$

$$\frac{Y_m(z)}{z} = \frac{z}{(z - \frac{1}{3})(z + \frac{1}{3})(z + \frac{1}{9})} = \frac{A}{z - \frac{1}{3}} + \frac{B}{z + \frac{1}{3}} + \frac{C}{z + \frac{1}{9}}$$

$$A = \lim_{z \to \frac{1}{3}} \cancel{(z - \frac{1}{3})} \frac{z}{\cancel{(z - \frac{1}{3})}(z + \frac{1}{3})(z + \frac{1}{9})} = \frac{\frac{1}{\cancel{3}}}{\frac{2}{\cancel{3}} \cdot \frac{4}{9}} = \frac{9}{8}$$

$$B = \lim_{z \to -\frac{1}{3}} \cancel{(z + \frac{1}{3})} \frac{z}{(z - \frac{1}{3})\cancel{(z + \frac{1}{3})}(z + \frac{1}{9})} = \frac{+\frac{1}{\cancel{3}}}{+\frac{2}{\cancel{3}} \cdot \frac{-3+1}{9}} = \frac{-9}{4}$$

$$C = \lim_{z \to -\frac{1}{9}} \frac{z}{z^2 - \frac{1}{9}} = \frac{-\frac{1}{9}}{\frac{1}{81} - \frac{1}{9}} = \frac{-\frac{1}{\cancel{9}}}{\frac{1-9}{\cancel{81}\,9}} = \frac{-9}{-8} = \frac{9}{8}$$

$$Y_m(z) = \frac{9}{8}\frac{z}{z - \frac{1}{3}} - \frac{9}{4}\frac{z}{z + \frac{1}{3}} + \frac{9}{8}\frac{z}{z + \frac{1}{9}}$$

$$\boxed{y_m(n) = \left[\frac{9}{8}\left(\frac{1}{3}\right)^n - \frac{9}{4}\left(-\frac{1}{3}\right)^n + \frac{9}{8}\left(-\frac{1}{9}\right)^n\right]\mu(n)}$$

NEPOBUĐENI

$$Y_0(z) = \frac{1}{9}\frac{z}{(z - \frac{1}{3})(z + \frac{1}{3})}$$

$$\frac{Y_0(z)}{z} = \frac{\frac{1}{9}}{(z - \frac{1}{3})(z + \frac{1}{3})} = \frac{A}{z - \frac{1}{3}} + \frac{B}{z + \frac{1}{3}}$$

$A + B = 0$

$\frac{1}{3}A - \frac{1}{3}B = \frac{1}{9}$

$A - B = \frac{1}{3}$

$2A = \frac{1}{3}$

$A = \frac{1}{6}$

$B = -\frac{1}{6}$

$$Y_0(z) = \frac{\frac{1}{6}z}{z - \frac{1}{3}} + \frac{-\frac{1}{6}z}{z + \frac{1}{3}}$$

$$\boxed{y_0(n) = \frac{1}{6}\left(\frac{1}{3}\right)^n - \frac{1}{6}\left(-\frac{1}{3}\right)^n}$$

A

5. a)

$$y'(t) + a_1 y(t) = b_0 u(t)$$

$$H(s) = \frac{b_0}{s + a_1}$$

$$u_1(t) = e^{-2t} \longrightarrow y_1(t) = 2e^{-2t}$$

$$u_2(t) = e^{-3t} \longrightarrow y_2(t) = 3e^{-3t}$$

prijenosna funkcija je definirana:

$$H(s) = \left.\frac{y(t)}{u(t)}\right|_{u(t)=e^{st}} \longleftrightarrow y = H(s) \cdot U$$

$$s \in \mathbb{C}$$

$$H(s) = \frac{y}{U}$$

za prvu pobudu:

$$U_1 = 1 \qquad y_1 = 2 \qquad s = -2$$

$$H(-2) = \frac{2}{1} = \frac{b_0}{-2+a_1}$$

za drugu pobudu:

$$U_2 = 1 \qquad y_2 = 3 \qquad s = -3$$

$$H(-3) = \frac{3}{1} = \frac{b_0}{-3+a_1}$$

nepoznanice:

$$\left.\begin{aligned} b_0 &= (-2+a_1)\cdot 2 \\ b_0 &= (-3+a_1)\cdot 3 \end{aligned}\right\} \quad \begin{aligned} -4+2a_1 &= -9+3a_1 \\ -4+9 &= 3a_1 - 2a_1 \\ a_1 &= 5 \end{aligned} \qquad \begin{aligned} b_0 &= 2(-2+5) \\ &= 2\cdot 3 = 6 \end{aligned}$$

prijenosna funkcija:

$$\boxed{H(s) = \frac{6}{s+5}}$$

impulsni odziv

$$\boxed{h(t) = 6e^{-5t}\mu(t)}$$

A

b) $u(t) = e^{-4t}$

$s = -4$

$y = H(-4) \cdot U$

$H(-4) = \frac{6}{-4+5} = 6$

$y = 6 \cdot 1 = 6$

$\boxed{y(t) = 6\, e^{-4t}}$

c) $u(t) = e^{-4t} \mu(t)$

$U(s) = \frac{1}{s+4}$

$Y(s) = H(s) \cdot U(s)$

$= \frac{6}{s+5} \cdot \frac{1}{s+4}$

$= \frac{A}{s+5} + \frac{B}{s+4}$

$A + B = 0$

$4A + 5B = 6$

$-4B + 5B = 6$

$B = 6 \qquad A = -6$

$Y(s) = \frac{-6}{s+5} + \frac{6}{s+4}$

$\boxed{y(t) = \left(-6\, e^{-5t} + 6 e^{-4t}\right) \mu(t)}$

1. B

$$h(t) = \delta(t) + 4e^{-2t}\mu(t)$$

a) Prijenosna funkcija

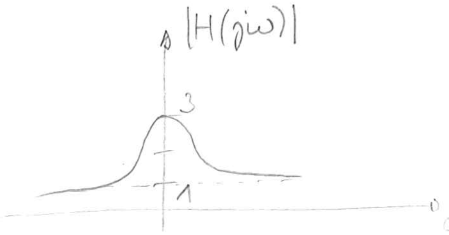
$$H(s) = 1 + 4\,\frac{1}{s+2}$$

$$= \frac{s+2+4}{s+2}$$

$$H(s) = \frac{s+6}{s+2}$$

stabilnost

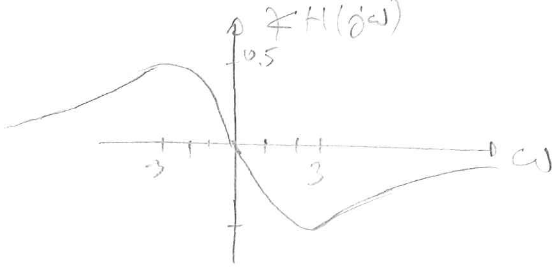
pol $s=-2 < 0$ stabilan sustav



b) $$H(j\omega) = \frac{j\omega+6}{j\omega+2}$$

$$|H(j\omega)| = \sqrt{\frac{36+\omega^2}{4+\omega^2}}$$

$$\measuredangle H(j\omega) = \operatorname{arctg}\frac{\omega}{6} - \operatorname{arctg}\frac{\omega}{2}$$



c) $$H(s) = \frac{Y(s)}{U(s)} = \frac{s+6}{s+2}$$

$$Y(s)\cdot(s+2) = (s+6)\,U(s)$$

$$y'(t) + 2y(t) = u'(t) + 6u(t)$$ diferencijalna jednadžba

2. $y(n) + \frac{1}{3} y(n-1) = u(n)$ B

a) $H(z) = \dfrac{1}{1+\frac{1}{3}z^{-1}} = \dfrac{z}{z+\frac{1}{3}}$

POLOVI: $z = -\frac{1}{3}$

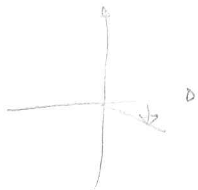
$|z| < 1$ SUSTAV JE STABILAN

b) $H(e^{j\Omega}) = \dfrac{1}{1+\frac{1}{3}e^{-j\Omega}} = \dfrac{1}{1+\frac{1}{3}\cos\Omega - j\frac{1}{3}\sin\Omega}$

$u(n) = 2\sin\frac{\pi}{2}n$

$\Omega = \frac{\pi}{2}$

$H(e^{j\frac{\pi}{2}}) = \dfrac{1}{1+\frac{1}{3}\cos\frac{\pi}{2} - j\frac{1}{3}\sin\frac{\pi}{2}} = \dfrac{1}{1-j\frac{1}{3}}$



$|H(e^{j\frac{\pi}{2}})| = \dfrac{1}{\sqrt{1+\frac{1}{9}}} = \dfrac{1}{\sqrt{\frac{10}{9}}} = \dfrac{3}{\sqrt{10}}$

$\measuredangle H(e^{j\frac{\pi}{2}}) = -\mathrm{arctg}\,\dfrac{-\frac{1}{3}}{1} = 0.3218$

$y(n) = 2 \cdot \frac{3}{\sqrt{10}} \sin\left(\frac{\pi}{2}n + 0.3218\right)$

$= \frac{6}{\sqrt{10}} \sin\left(\frac{\pi}{2}n + 0.3218\right)$

c) kao u A grupi

3. $$y''(t) + 2y'(t) + 2y(t) = 2u'(t) + u(t)$$

B

a) $$u(t) = \begin{cases} -t, & t<0 \\ -t+1, & t>0 \end{cases}$$

1° $t<0$

partikularno

$u(t) = -t$ $\qquad u'(t) = -1$

$y_p(t) = k_0 + k_1 t$

$y_p'(t) = k_1$

$2k_1 + 2k_0 + 2k_1 t = -2 - t$

$2k_1 = -1$ $\qquad 2k_0 + 2k_1 = -2$

$k_1 = -\frac{1}{2}$ $\qquad k_0 - \frac{1}{2} = -2$ $\qquad k_0 = -2 + \frac{1}{2} = \frac{-3}{2}$

$y_p(t) = -\frac{3}{2} - \frac{1}{2}t$

homogeno rješenje se istitralo, pa je odziv za $t<0$

$$\boxed{y(t) = -\frac{3}{2} - \frac{1}{2}t}$$

Oko nule računamo početne uvjete:

$y(0^-) = -\frac{3}{2} - \frac{1}{2}\cdot 0 = -\frac{3}{2}$

$y'(t) = -\frac{1}{2}$

$y'(0^-) = -\frac{1}{2}$

početni uvjeti poslije nule:

$y(0^+) - y(0^-) = 0 \rightarrow y(0^+) = -\frac{3}{2}$

$y'(0^+) - y'(0^-) + 0 = 2\cdot(u(0^+) - u(0^-))$

$y'(0^+) = 2\cdot(1-0) + y'(0^-) = 2 - \frac{1}{2} = \frac{3}{2}$

2° $t>0$

partikularno: $\qquad u(t) = -t+1$

$\qquad u'(t) = -1$

$y_p(t) = k_0 + k_1 t$

$y_p'(t) = k_1$

$2k_1 + 2k_0 + 2k_1 t = -2 - t + 1$ $\qquad y_p(t) = -\frac{1}{2}t$

$2k_1 + 2k_0 = -1$ $\qquad 2k_0 = -1 - 2k_1$

$2k_1 = -1$ $\qquad = -1 + 1$

$k_1 = -\frac{1}{2}$ $\qquad k_0 = 0$

homogeno: B

$$s^2+2s+2=0$$

$$s_{1,2}=\frac{-2\pm\sqrt{4-4\cdot 2}}{2}=\frac{-2\pm 2j}{2}=-1\pm j$$

$$y_h(t)=c_1 e^{(-1-j)t}+c_2 e^{(-1+j)t}$$

totalno rješenje za $t>0$

$$y(t)=c_1 e^{(-1-j)t}+c_2 e^{(-1+j)t}-\frac{1}{2}t$$

$$y'(t)=(-1-j)c_1 e^{(-1-j)t}+c_2(-1+j)c_2 e^{(-1+j)t}-\frac{1}{2}$$

$$y(0^+)=c_1+c_2=-\frac{3}{2}$$

$$y'(0^+)=(-1-j)c_1+c_2(-1+j)-\frac{1}{2}=\frac{3}{2}$$

$$-(-1-j)c_1-(-1-j)c_2=+\frac{3}{2}(-1-j)$$

$$(-1-j)c_1+(-1+j)c_2-\frac{1}{2}=\frac{3}{2}$$

$$(1+j-1+j)c_2=\cancel{-\frac{3}{2}}-\frac{3}{2}j+\cancel{\frac{3}{2}}+\frac{1}{2}$$

$$2j\,c_2=\frac{1}{2}-\frac{3}{2}j$$

$$c_2=\frac{1}{4j}-\frac{3j}{2\cdot 2j}=-\frac{1}{4}j-\frac{3}{4}$$

$$c_1=-\frac{3}{2}-c_2=-\frac{3}{2}+\frac{1}{4}j+\frac{3}{4}=-\frac{3}{4}+\frac{1}{4}j$$

$$\boxed{y(t)=\left(-\frac{3}{4}+\frac{1}{4}j\right)e^{(-1-j)t}+\left(-\frac{1}{4}j-\frac{3}{4}\right)e^{(-1+j)t}-\frac{1}{2}t \quad \text{za } t>0}$$

ukupno rješenje

$$\boxed{y(t)=\begin{cases}-\frac{3}{2}-\frac{1}{2}t, & t<0\\ \left(-\frac{3}{4}+\frac{1}{4}j\right)e^{(-1-j)t}+\left(-\frac{1}{4}j-\frac{3}{4}\right)e^{(-1+j)t}-\frac{1}{2}t, & t>0\end{cases}}$$

b) kao u A grupi

4. B

$$H(z)=\frac{4z^{-1}-z^{-2}}{4-z^{-2}}$$

a) impulsni odziv

$$H(z)=\frac{4z^{-1}-z^{-2}}{4-z^{-2}}=\frac{4z-1}{4z^2-1}=\frac{z-\frac{1}{4}}{z^2-\frac{1}{4}}=\frac{z-\frac{1}{4}}{(z-\frac{1}{2})\ (z+\frac{1}{2})}$$

$$\frac{H(z)}{z}=\frac{z-\frac{1}{4}}{z(z-\frac{1}{2})(z+\frac{1}{2})}=\frac{A}{z}+\frac{B}{z-\frac{1}{2}}+\frac{C}{z+\frac{1}{2}}$$

$$A=\lim_{z\to 0}\frac{z-\frac{1}{4}}{(z-\frac{1}{2})(z+\frac{1}{2})}=\frac{-\frac{1}{4}}{-\frac{1}{2}\cdot\frac{1}{2}}=1$$

$$B=\lim_{z\to\frac{1}{2}}\frac{z-\frac{1}{4}}{z(z+\frac{1}{2})}=\frac{\frac{1}{2}-\frac{1}{4}}{\frac{1}{2}(\frac{1}{2}+\frac{1}{2})}=\frac{\frac{1}{4}}{\frac{1}{2}\cdot 1}=\frac{1}{2}$$

$$C=\lim_{z\to-\frac{1}{2}}\frac{z-\frac{1}{4}}{z(z-\frac{1}{2})}=\frac{-\frac{1}{2}-\frac{1}{4}}{-\frac{1}{2}(-\frac{1}{2}-\frac{1}{2})}=\frac{\frac{-2-1}{4}}{-\frac{1}{2}\cdot\frac{-2}{2}}=\frac{-\frac{3}{4}}{\frac{1}{2}}=-\frac{6}{4}=-\frac{3}{2}$$

$$H(z)=1+\frac{1}{2}\ \frac{z}{z-\frac{1}{2}}-\frac{3}{2}\ \frac{z}{z+\frac{1}{2}}$$

$$\boxed{h(n)=\delta(n)+\frac{1}{2}\left(\frac{1}{2}\right)^n\mu(n)-\frac{3}{2}\left(-\frac{1}{2}\right)^n\mu(n)}$$

b) diferencijske jednadžbe

$$H(z)=\frac{Y(z)}{U(z)}=\frac{4z^{-1}-z^{-2}}{4-z^{-2}}$$

$$Y(z)\ (4-z^{-2})=(4z^{-1}-z^{-2})U(z)$$

$$4y(n)-y(n-2)=4u(n-1)-u(n-2)$$

$$\boxed{y(n)-\frac{1}{4}y(n-2)=u(n-1)-\frac{1}{4}u(n-2)}$$

c) $u(n)=\{\underline{1},\ 0,\ \frac{1}{4^2},\ 0,\ \frac{1}{4^4},\ 0,\dots\}$

$$U(z)=1+\frac{1}{4^2}z^{-2}+\frac{1}{4^4}z^{-4}+\frac{1}{4^6}z^{-6}+\dots$$

$$=\sum_{k=0}^{\infty}\left(\frac{1}{4^2z^2}\right)^k=\frac{1}{1-\frac{1}{4^2z^2}}=\frac{1}{\frac{16z^2-1}{4^2z^2}}=\frac{16z^2}{16z^2-1}=\frac{z^2}{z^2-\frac{1}{16}}$$

početni uvjeti $y(-1)=1$
$y(-2)=0$

B

4.

$$y(n) - \frac{1}{4} y(n-2) = u(n-1) - \frac{1}{4} u(n-2)$$

$$Y(z) - \frac{1}{4}\left[z^{-2}Y(z) + z^{-1}y(-1) + y(-2)\right] = z^{-1}U(z) + \cancel{u(-1)}$$
$$- \frac{1}{4}\left[U(z)\, z^{-2} + \cancel{z^{-1}u(-1)} + \cancel{u(-2)}\right]$$

$$Y(z)\left(1 - \frac{1}{4}z^{-2}\right) = U(z)\left(z^{-1} - \frac{1}{4}z^{-2}\right) + \frac{1}{4} z^{-1} y(-1) + \frac{1}{4}\cancel{y(-2)}$$

$$Y(z) = \frac{z^{-1} - \frac{1}{4}z^{-2}}{1 - \frac{1}{4}z^{-2}} U(z) + \frac{\frac{1}{4}z^{-1}}{1 - \frac{1}{4}z^{-2}}$$

MIRNI

$$Y_m(z) = \frac{z^{-1} - \frac{1}{4}z^{-2}}{1 - \frac{1}{4}z^{-2}} \cdot \frac{z^2}{z^2 - \frac{1}{16}} = \frac{\cancel{z - \frac{1}{4}}}{z^2 - \frac{1}{4}} \frac{z^2}{\cancel{(z - \frac{1}{4})}(z + \frac{1}{4})}$$

$$\frac{Y_m(z)}{z} = \frac{z}{(z - \frac{1}{2})(z + \frac{1}{2})(z + \frac{1}{4})} = \frac{A}{z - \frac{1}{2}} + \frac{B}{z + \frac{1}{2}} + \frac{C}{z + \frac{1}{4}}$$

$$A = \lim_{z \to \frac{1}{2}} \frac{z}{(z + \frac{1}{2})(z + \frac{1}{4})} = \frac{\frac{1}{2}}{(\frac{1}{2} + \frac{1}{2})(\frac{1}{2} + \frac{1}{4})} = \frac{\frac{1}{2}}{1 \cdot \frac{3}{4}} = \frac{4}{6} = \frac{2}{3}$$

$$B = \lim_{z \to -\frac{1}{2}} \frac{z}{(z - \frac{1}{2})(z + \frac{1}{4})} = \frac{-\frac{1}{2}}{(-\frac{1}{2} - \frac{1}{2})(-\frac{1}{2} + \frac{1}{4})} = \frac{-\frac{1}{2}}{-1 \cdot \frac{-2+1}{4}} = \frac{-\frac{1}{2}}{\frac{1}{4}} = -2$$

$$C = \lim_{z \to -\frac{1}{4}} \frac{z}{(z - \frac{1}{2})(z + \frac{1}{2})} = \frac{-\frac{1}{4}}{(-\frac{1}{4} - \frac{1}{2})(-\frac{1}{4} + \frac{1}{2})} = \frac{-\frac{1}{4}}{\frac{-1-2}{4} \cdot \frac{-1+2}{4}} = \frac{-\cancel{\frac{1}{4}}}{\frac{-3}{4} \cdot \cancel{\frac{1}{4}}} = \frac{4}{3}$$

$$Y_m(z) = \frac{2}{3} \frac{z}{z - \frac{1}{2}} - 2 \frac{z}{z + \frac{1}{2}} + \frac{4}{3} \frac{z}{z + \frac{1}{4}}$$

$$\boxed{y_m(n) = \left[\frac{2}{3}\left(\frac{1}{2}\right)^n - 2\left(-\frac{1}{2}\right)^n + \frac{4}{3}\left(-\frac{1}{4}\right)^n\right]\mu(n)}$$

NEPOBUĐENI

$$Y_0(z) = \frac{\frac{1}{4}z^{-1}}{1 - \frac{1}{4}z^{-2}} = \frac{\frac{1}{4}z}{z^2 - \frac{1}{4}} =$$

$$\frac{Y_0(z)}{z} = \frac{\frac{1}{4}}{(z - \frac{1}{2})(z + \frac{1}{2})} = \frac{A}{z - \frac{1}{2}} + \frac{B}{z + \frac{1}{2}}$$

$A + B = 0 \to B = -A$

$\frac{1}{2}A - \frac{1}{2}B = \frac{1}{4}$

$\frac{1}{2}A + \frac{1}{2}A = \frac{1}{4}$

$A = \frac{1}{4}$ $\quad B = -\frac{1}{4}$

$$Y_0(z) = \frac{1}{4} \frac{z}{z - \frac{1}{2}} - \frac{1}{4} \frac{z}{z + \frac{1}{2}}$$

$$\boxed{y_0(n) = \frac{1}{4}\left(\frac{1}{2}\right)^n - \frac{1}{4}\left(-\frac{1}{2}\right)^n}$$

5. $y'(t) + a_1 y(t) = b_0 u(t)$ B

$$H(s) = \frac{b_0}{s + a_1}$$

$$u_1(t) = e^{-3t} \longrightarrow y_1(t) = 3e^{-3t}$$

$$u_2(t) = e^{-4t} \longrightarrow y_2(t) = 4e^{-4t}$$

a) prijenosna funkcija:

$$H(s) = \left.\frac{y(t)}{u(t)}\right|_{u(t) = Ue^{st}} \longrightarrow \underset{\substack{\downarrow \\ s \in \mathbb{C}}}{y = H(s) \cdot U} \longrightarrow H(s) = \frac{Y}{U}$$

za 1. pobudu

$U_1 = 1$ $\quad y_1 = 3$ $\quad s = -3$

$$H(-3) = \frac{3}{1} = \frac{b_0}{-3 + a_1}$$

za 2. pobudu

$U_2 = 1$ $\quad y_2 = 4$ $\quad s = -4$

$$H(-4) = \frac{4}{1} = \frac{b_0}{-4 + a_1}$$

nepoznanice

$$\frac{b_0}{a_1 - 3} = 3 \qquad \frac{b_0}{a_1 - 4} = 4$$

$$b_0 = 3a_1 - 9 \qquad b_0 = 4a_1 - 16$$

$$3a_1 - 9 = 4a_1 - 16$$

$$a_1 = -9 + 16 = 7 \qquad b_0 = 4 \cdot 7 - 16 = 28 - 16 = 12$$

prijenosna funkcija

$$H(s) = \frac{12}{s + 7}$$

impulsni odziv

$$h(t) = 12e^{-7t} \mu(t)$$

5. b) B

$u(t) = e^{-5t}$

$s = -5$

$y = H(-5) \cdot u$

$H(-5) = \frac{12}{-5+7} = 6$

$y = 6 \cdot 1 = 6$

$\boxed{y(t) = 6\, e^{-5t}}$

c) $u(t) = e^{-5t} \mu(t)$

$U(s) = \frac{1}{s+5}$

$y(s) = \frac{12}{s+7} \cdot \frac{1}{s+5}$

$= \frac{A}{s+7} + \frac{B}{s+5}$

$A + B = 0 \quad \rightarrow \quad A = -B$

$5A + 7B = 12 \qquad A = -6$

$-5B + 7B = 12$

$2B = 12$

$B = 6$

$y(s) = \frac{-6}{s+7} + \frac{6}{s+5}$

$\boxed{y(t) = \left(-6\, e^{-7t} + 6\, e^{-5t}\right)\mu(t)}$