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1. $$H(z) = \frac{(e^{-2}-e^{-1})z}{(z-e^{-2})(z-e^{-1})}$$

$$H(z) = \frac{Az}{z-e^{-2}} + \frac{Bz}{z-e^{-1}} = \frac{Az(z-e^{-1}) + Bz(z-e^{-2})}{(z-e^{-2})(z-e^{-1})}$$

$$Az^2 - Aze^{-1} + Bz^2 - Bze^{-2} = (e^{-2}-e^{-1})z$$

$$z^2(A+B) + z(-Ae^{-1} - Be^{-2}) = (e^{-2}-e^{-1})z$$

$A+B=0$

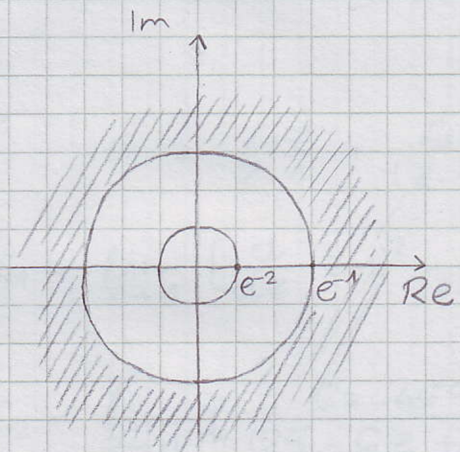
$A=1, \; B=-1$

$\alpha^n \mu(n) \circ\!\!-\!\bullet \; \frac{z}{z-\alpha}, \quad |z|>|\alpha|$

$-\alpha^n \mu(-n-1) \circ\!\!-\!\bullet \; \frac{z}{z-\alpha}, \quad |z|<|\alpha|$

$$H(z) = \frac{z}{z-e^{-2}} - \frac{z}{z-e^{-1}}$$

(a) $|z| > e^{-1}$

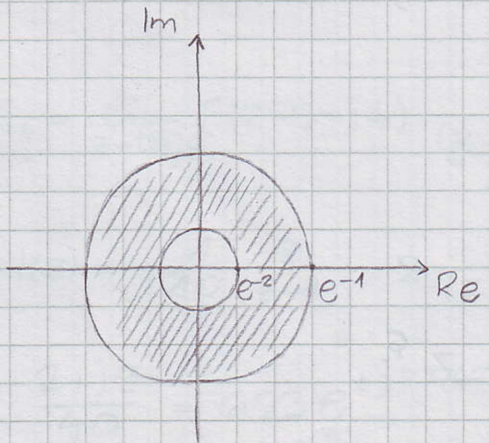


$\alpha_1 = e^{-2}, \; \alpha_2 = e^{-1}$

$h(n) = e^{-2n}\mu(n) - e^{-n}\mu(n)$

$h(n) = (e^{-2n} - e^{-n})\mu(n)$

(b) $e^{-2} < |z| < e^{-1}$

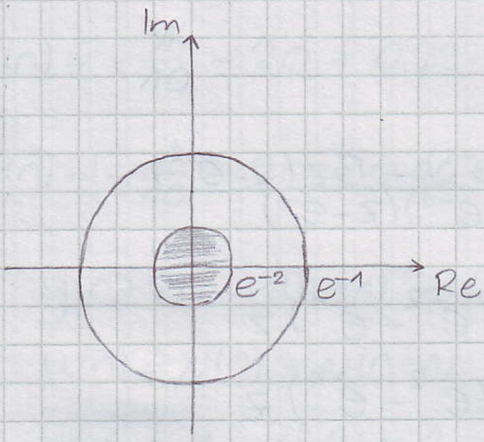


$\alpha_1 = e^{-2}, \; \alpha_2 = e^{-1}$

$h(n) = e^{-2n}\mu(n) + e^{-n}\mu(-n-1)$

(c) $|z| < e^{-2}$

$$\alpha_1 = e^{-2}, \quad \alpha_2 = e^{-1}$$



$$h(n) = -e^{-2n}\mu(-n-1) + e^{-n}\mu(-n-1)$$

$$h(n) = (e^{-n} - e^{-2n})\mu(-n-1)$$

2. $h(n) = \{\ldots, 0, \underline{2}, 1, 0, -1, 0, 0, 0, \ldots\}$

$u(n) = \{\ldots, 0, \underline{0}, 1, 2, 1, 0, 0, \ldots\}$

(a) $y(n) = h(n) * u(n) = \sum_{m=-\infty}^{\infty} h(n-m)\,u(m) = \sum_{m=0}^{n} h(n-m)\,u(m)$

$$y(0) = \sum_{m=0}^{0} h(0-m)\,u(m) = 0$$
$$y(1) = \sum_{m=0}^{1} h(1-m)\,u(m) = 2$$
$$y(2) = \sum_{m=0}^{2} h(2-m)\,u(m) = 5$$
$$y(3) = \sum_{m=0}^{3} h(3-m)\,u(m) = 4$$
$$y(4) = \sum_{m=0}^{4} h(4-m)\,u(m) = 0$$
$$y(5) = \sum_{m=0}^{5} h(5-m)\,u(m) = -2$$
$$y(6) = \sum_{m=0}^{6} h(6-m)\,u(m) = -1$$
$$y(7) = \sum_{m=0}^{7} h(7-m)\,u(m) = 0$$

$$y(n) = \{\ldots, \underline{0}, 2, 5, 4, 0, -2, -1, 0, \ldots\}$$

(b) $H(z) = \sum_{m=-\infty}^{\infty} h(m)\,z^{-m} = 2 + z^{-1} - z^{-3}$

$U(z) = \sum_{m=-\infty}^{\infty} h(m)\,z^{-m} = z^{-1} + 2z^{-2} + z^{-3}$

$$Y(z) = U(z) \cdot H(z)$$

$$Y(z) = 2z^{-1} + 5z^{-2} + 4z^{-3} - 2z^{-5} - z^{-6}$$

$$y(n) = \{\ldots, \underline{0}, 2, 5, 4, 0, -2, -1, 0, \ldots\}$$

3. $$H(z)=\frac{2z(3z-23)}{(25-6z+z^2)(z-1)^2}$$

(a) $$(6z^2-46z):(z^4-8z^3+38z^2-56z+25)=6z^{-2}+2z^{-3}-212z^{-4}\ldots$$

$$\begin{array}{l}
\underline{-6z^2\mp 48z\pm 228\mp 336z^{-1}\pm 150z^{-2}}\\
\quad 2z-228+336z^{-1}-150z^{-2}\\
\quad \underline{-2z\mp 16\pm 76z^{-1}\mp 112z^{-2}\pm 50z^{-3}}\\
\quad\quad -212+260z^{-1}-38z^{-2}-50z^{-3}\\
\quad\quad \ldots\ldots
\end{array}$$

$$h(n)=6\delta(n-2)+2\delta(n-3)-212\delta(n-4)\ldots$$

$$h(n)=\{\ldots,\underline{0},0,6,2,-212,\ldots\}$$

$$h(n=3)=\underline{\underline{2}}$$

(b) $$H(z)=\frac{2z(3z-23)}{(z-1)^2(25-6z+z^2)}$$

$$(z-1)^2=0\ \rightarrow\ z_{1,2}=1$$

$$z^2-6z+25=0\ \rightarrow\ z_{3,4}=\frac{1}{2}(6\pm j8)$$

$$z_{3,4}=3\pm j4$$

$$H(z)=\frac{\mathcal{L}_1 z}{z-1}+\frac{\mathcal{L}_2 z}{(z-1)^2}+\frac{\mathcal{L}_3 z}{z-(3+j4)}+\frac{\mathcal{L}_4 z}{z-(3-j4)}$$

$$\mathcal{L}_1=\lim_{z\to 1}\left[\frac{d}{dz}(z-1)^2\frac{6z-46}{(z-1)^2(25-6z+z^2)}\right]=-\frac{1}{10}$$

$$\mathcal{L}_2=\lim_{z\to 1}\left[\frac{6z-46}{(25-6z+z^2)}\right]=-2$$

$$\mathcal{L}_3=\lim_{z\to(3-j4)}\left[(z-3+j4)\frac{6z-46}{(z-1)^2(z-3-j4)(z-3+j4)}\right]=\frac{2+j9}{40}$$

$$\mathcal{L}_4=\lim_{z\to(3+j4)}\left[(z-3-j4)\frac{6z-46}{(z-1)^2(z-3-j4)(z-3+j4)}\right]=\frac{2-j9}{40}$$

$$H(z)=-\frac{1}{10}\frac{z}{z-1}-2\frac{z}{(z-1)^2}+\frac{2+j9}{40}\frac{z}{z-3+j4}+\frac{2-j9}{40}\frac{z}{z-3-j4}$$

$$\frac{2+j9}{40}=0.23e^{\pm j77.47^\circ}$$

$$3+j4=5e^{\pm j53.13^\circ}$$

$$h(n) = -\frac{1}{10}(1)^n - 2n + 0.23e^{j77.47°}(5)^n e^{-jn53.13°} + 0.23e^{-j77.47°}(5)^n e^{jn53.13°}$$

$$h(n) = -\frac{1}{10} - 2n + 0.23\cdot(5)^n\left(e^{j(77.47° - n53.13°)} + e^{-j(77.47° - n53.13°)}\right)$$

$$h(n) = \left[-\frac{1}{10} - 2n + 0.46\cdot(5)^n \cos(77.47° - n\cdot 53.13°)\right]\mu(n)$$

4. $y(n+2) - y(n+1) = 4u(n+2) - 3u(n+1) + u(n)$

$u(n) = \mu(n) + \mu(n-1)$

$y(-1) = 1, \quad y(-2) = 1$

(a) $$H(z) = \frac{4z^2 - 3z + 1}{z^2 - z}$$

(b) $$\frac{Y_m(z)}{z} = \frac{4z^2 - 3z + 1}{z(z^2 - z)} = \frac{4z^2 - 3z + 1}{z^2(z-1)^2}\cdot\frac{z+1}{z-1} = \frac{4z^3 + z^2 - 2z + 1}{z^2(z-1)^2}$$

$$\frac{Y_m(z)}{z} = \frac{A}{z} + \frac{B}{z^2} + \frac{C}{z-1} + \frac{D}{(z-1)^2}$$

$$z^3(A+C) + z^2(-2A + B - C + D) + z(A - 2B) + B = 4z^3 + z^2 - 2z + 1$$

$A + C = 4$

$-2A + B - C + D = 1$

$A - 2B = -2$

$B = 1$

$A = 0, \quad B = 1, \quad C = 4, \quad D = 4$

$$Y_m(z) = \frac{1}{z} + 4\frac{z}{z-1} + 4\frac{z}{(z-1)^2}$$

$$y_m(n) = \delta(n-1) + 4\mu(n) + 4n\mu(n)$$

(c) $$\frac{Y_0(z)}{z} = \frac{z}{z^2 - z} = \frac{z}{z-1}$$

$$y_0(n) = \mu(n)$$

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5. $y(n) - y(n-2) = u(n)$

(a) $H(z) = \dfrac{1}{1-z^{-2}} = \dfrac{z^2}{z^2-1}$

(b) $u(n) = \mu(n)$

$U(z) = \dfrac{z}{z-1}$

$Y(z) = U(z)\,H(z)$

$Y(z) = \dfrac{z}{z-1} \cdot \dfrac{z^2}{z^2-1}$

$Y(z) = \dfrac{z^3}{(z-1)^2(z+1)}$

$y(0) = \lim\limits_{z\to\infty} Y(z)$

$y(0) = \underline{1}$

$\lim\limits_{N\to\infty} y(N) = \lim\limits_{z\to 1} (1-z^{-1})\,Y(z)$

$= \lim\limits_{z\to 1} \dfrac{z-1}{z} \dfrac{z^3}{(z+1)(z-1)^2}$

$= \lim\limits_{z\to 1} \dfrac{z^2}{z^2-1} = \underline{\infty}$

(c) $\dfrac{Y(z)}{z} = \dfrac{A}{z+1} + \dfrac{B}{z-1} + \dfrac{C}{(z-1)^2}$

$z^2(A+B) + z(-2A+C) + A - B + C = z^2$

$A = \underline{\tfrac{1}{4}}, \; B = \underline{\tfrac{3}{4}}, \; C = \underline{\tfrac{1}{2}}$

$Y(z) = \dfrac{1}{4}\dfrac{z}{z+1} + \dfrac{3}{4}\dfrac{z}{z-1} + \dfrac{1}{2}\dfrac{z}{(z-1)^2}$

$y(n) = \left(\tfrac{1}{4}(-1)^n + \tfrac{3}{4}(1)^n + \tfrac{1}{2}n\right)\underline{\mu(n)}$