Signali i sustavi – zadaci za aktivnost – tjedan 14

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1. Kontinuirani STABILAN sustav zadan je diferencijalnom jednadžbom:

   $$y''(t) + 5y'(t) + 6y(t) = u(t)$$

   Naći amplitudno-frekvencijsku i fazno-frekvencijsku karakteristiku sustava, te odziv na pobudu $u(t)=5\cos t$. Početni uvjeti su $y(0)=0$ i $y'(0)=1$. Komentirajte izgled odziva za $t >> 0$.

2. Diskretan sustav zadan je jednadžbom diferencija:

   $$y(n) - 2y(n-1) + y(n-2) = u(n)$$

   Naći amplitudno-frekvencijsku i fazno-frekvencijsku karakteristiku sustava, te odziv na pobudu $u(n) = 5$. Početni uvjeti su $y(-2) = 0$ i $y(-1)= 1$. Komentirajte izgled odziva za $n >> 0$.

3. Kontinuirani sustav zadan je diferencijalnom jednadžbom:

   $$y''(t) + 2y'(t) + 5y(t) = u(t)$$

   Pronađite odziv sustava, ako je sustav pobuđen s $u(t) = \sin t$, za $t < 0$ te s $u(t)=2\sin(2t)$, za $t > 0$. Komentirajte odziv sustava za $t >> 0$

4. Kontinuirani sustav zadan je diferencijalnom jednadžbom:

   $$y'(t) + 3y(t) = u(t)$$

   Ako je izlaz iz sustava u trenutku nula jednak nuli, $y(0)=0$, naći odziv sustava na pobudu

   $$u(t)=\left(\sin t+2\sin(2t)+3\sin(3t)+4\sin(4t)\right)\mu(t).$$

   Komentirajte izgled odziva za $t >> 0$.

   UPUTA: Koristite frekvencijsku karakteristiku sustava.

5. Diskretan sustav zadan je jednadžbom diferencija:

   $$y(n) + 0.5y(n-1) = u(n)$$

   Ako je početni uvjet $y(-1)=1$, naći odziv sustava na pobudu

   $$u(n)=\left(\cos(0.5\pi n+0.2\pi)+2\cos(\pi n)+3\cos(1.5\pi n)+4\cos(2\pi n)\right)\mu(n)$$

   Komentirajte izgled odziva za $n >> 0$.

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1) $y''(t) + 5y'(t) + 6y(t) = u(t)$

$u(t) = 5\cos t \Rightarrow \omega = 1$

$y(0) = 0 \quad y'(0) = 1$

$$H(s) = \frac{1}{s^2+5s+6} \qquad H(j\omega) = \frac{1}{(-\omega^2+6)+5j\omega}$$

$$\Rightarrow |H(j\omega)| = \frac{1}{\sqrt{(6-\omega^2)^2+25\omega^2}} = \frac{1}{\sqrt{\omega^4+13\omega^2+36}} = A(\omega$$

za $\omega = 1$ $\quad |H(j)| = \frac{1}{\sqrt{1+13+36}} = \frac{1}{\sqrt{50}} = \frac{1}{5\sqrt{2}}$

$$\varphi(\omega) = -\operatorname{arc\,tg} \frac{5\omega}{6-\omega^2}$$

za $\omega = 1$ $\quad \varphi(1) = -\operatorname{arc\,tg} 1 = -\frac{\pi}{4}$

$$\Rightarrow y_p(t) = 5 \cdot \frac{1}{5\sqrt{2}} \cdot \cos\left(t - \frac{\pi}{4}\right) = \frac{\cos\left(t - \frac{\pi}{4}\right)}{\sqrt{2}}$$

homogena ...

$s^2 + 5s + 6 = 0$

$(s+2)(s+3) = 0$

$s = -2, -3 \quad \Rightarrow y_h(t) = C_1 e^{-2t} + C_2 e^{-3t}$

$$y(t) = c_1 e^{-3t} + c_2 e^{-2t} + \frac{\cos(t - \frac{\pi}{4})}{\sqrt{2}}$$

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$$y(0) = c_1 + c_2 + \frac{1}{2} = 0 \qquad c_1 + c_2 = -\frac{1}{2}$$

$$y'(t) = -3c_1 e^{-3t} - 2c_2 e^{-2t} - \frac{\sin(t - \frac{\pi}{4})}{\sqrt{2}}$$

$$y'(0) = -3c_1 - 2c_2 + \frac{1}{2} = 1$$

$$3c_1 + 2c_2 = -\frac{1}{2}$$

$$-2c_1 - 2c_2 = 1$$

$$c_1 = \frac{1}{2} \qquad c_2 = -1$$

$$y(t) = \frac{1}{2} e^{-3t} - e^{-2t} + \frac{\cos(t - \frac{\pi}{4})}{\sqrt{2}}$$

za $t >> 0$ $\Rightarrow$ ISTITRAJU SE VLASTITE FREKVENCIJE
OSTAJE SAMO y_P
SUSTAV JE STABILAN

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(2) $y(n) - 2y(n-1) + y(n-2) = u(n) \qquad u(n) = 5$

$$H(z) = \frac{1}{1 - 2z^{-1} + z^{-2}}$$

$$z = e^{j\Omega} \Rightarrow H(\Omega) = \frac{1}{1 - 2e^{-j\Omega} + e^{-2j\Omega}}$$

$$\Omega = 0 \Rightarrow H(0) = \frac{1}{1-2+1} = \frac{1}{0} = \infty$$

$\Rightarrow$ SUSTAV JE NESTABILAN

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HOMOGENO ...

$$q^2 - 2q + 1 = 0 \qquad (q-1)^2 = 0$$

$$\Rightarrow y_h(n) = C_1 \cdot (1)^n + C_2 \cdot n \cdot (1)^n$$

PARTIKULARNO ...

$$y_p(n) = K \cdot n^2 \cdot 1^n$$

$$\Rightarrow K \cdot n^2 - 2 \cdot K \cdot (n-1)^2 + K \cdot (n-2)^2 = 5$$

$$-2K + 4K = 5 \Rightarrow K = \frac{5}{2}$$

$$\underline{y(n) = C_1 \cdot 1^n + C_2 \cdot n \cdot 1^n + \frac{5}{2} \cdot n^2 \cdot 1^n}$$

POČ. UVJETI ...

$y(-2) = 0 \qquad y(-1) = 1$

$$\left.\begin{aligned} y(-2) &= C_1 - 2C_2 + 10 = 0 \\ y(-1) &= C_1 - C_2 + \frac{5}{2} = 1 \end{aligned}\right\} \Rightarrow C_2 - 7.5 = 1$$

$$C_2 = 8.5$$

$$C_1 = 7$$

$$\Rightarrow \quad y(n) = 7 \cdot 1^n + 8.5 \cdot n \cdot 1^n + 2.5\, n^2 \cdot 1^n$$

ZA $n \gg 0$ AMPLITUDA TEŽI U BESKONAČNOST

③ $y'' + 2y' + 5y = u$

$$H(s) = \frac{Y(s)}{U(s)} = \frac{1}{s^2+2s+5}$$

→ sustav je stabilan → $s = j\omega$

$$H(j\omega) = \frac{1}{5-\omega^2+2j\omega}$$

$$A(\omega) = |H(j\omega)| = \frac{1}{\sqrt{(5-\omega^2)^2+4\omega^2}}$$

$$\varphi(\omega) = \text{Arctg}\frac{Im}{Re} = -\text{Arctg}\frac{2\omega}{5-\omega^2}$$

→ s obzirom da je $u(t) = \sin t,\ t<0$

$$y_p(t) = A(1)\sin[t+\varphi(1)],\ t<0$$

možemo odmah izračunati partikularno rješenje za $t>0$

$$u(t) = 2\sin 2t,\ t>0$$

$$y_p(t) = 2A(2)\sin[2t+\varphi(2)],\ t>0$$

$$y_p(t) = \begin{cases} \frac{1}{\sqrt{20}} \sin(t - 26.56^\circ), & t<0 \\ \frac{2}{\sqrt{17}} \sin(2t - 75.96^\circ), & t>0 \end{cases}$$

Iz $y_p(t) = \frac{1}{\sqrt{20}} \sin(t - 26.56^\circ)$ računamo

početne uvjet $y(0^-)$ i $y'(0^-)$.

$$y(0^-) = -0.01$$
$$y'(0^-) = 0.2$$

S obzirom da pobuda $u(t), t>0$ ne sadrži diracka $\rightarrow y(0^+) = y(0^-)$, $y'(0^+) = y'(0^-)$

$\rightarrow$ tražimo homogeno rj.

$$s^2 + 2s + 5 = 0 \longrightarrow s_{1/2} = \frac{-2 \pm \sqrt{4 - 20}}{2}$$
$$= -1 \pm 2j$$

$$y_h = e^{-t}(C_1 \cos 2t + C_2 \sin 2t)$$

$$y = e^{-t}[C_1 \cos 2t + C_2 \sin 2t] + \frac{2}{\sqrt{17}} \sin(2t - 75.96^\circ)$$

$$y(0) = C_1 + \frac{2}{\sqrt{17}} \sin(-75.96^\circ) = -0.01$$
$$y'(0) = -C_1 + 2C_2 + \frac{4}{\sqrt{17}} \cos(-75.96^\circ) = 0.2$$
$\Rightarrow$

$c_1 = 0.46 \;,\; c_2 = 0.21$

$$\Rightarrow y(t) = \begin{cases} \frac{1}{\sqrt{20}} \sin(t - 26.56^\circ), & t < 0 \\ e^{-t}\left[0.46 \cos 2t + 0.21 \sin 2t\right] + \frac{2}{\sqrt{17}} \sin(2t - 75.96^\circ), & t \geq 0 \end{cases}$$

Za $t >> 0$ → ostaje nam samo partikularni dio, tj. prisilni odziv. To je posljedica stabilnosti sustava.

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4.

$$y'(t) + 3y(t) = u(t) \qquad y(0) = 0$$

$$u(t) = (\sin t + 2\sin 2t + 3\sin 3t + 4\sin 4t)\,\mu(t)$$

$$H(s) = \frac{1}{s+3} \qquad H(\omega) = \frac{1}{3 + j\omega}$$

$$|H(\omega)| = \frac{1}{\sqrt{3^2 + \omega^2}} \qquad \measuredangle H(\omega) = -\operatorname{arc\,tg} \frac{\omega}{3}$$

$$y_1(t) = \frac{1}{\sqrt{10}} \sin(t - 18.4^\circ) \qquad (\omega = 1)$$

$$y_2(t) = \frac{2}{\sqrt{13}} \sin(2t - 33.7^\circ) \qquad (\omega = 2)$$

$$y_3(t) = \frac{3}{\sqrt{18}} \sin(3t - 45^\circ) \qquad (\omega = 3)$$

$$y_4(t) = \frac{4}{5} \sin(4t - 53.1^\circ) \qquad (\omega = 4)$$

$$y_p(t) = y_1(t) + y_2(t) + y_3(t) + y_4(t)$$

HOMOGENO

$$s + 3 = 0 \qquad s = -3 \qquad y_h(t) = C \cdot e^{-3t}$$

$$y(t) = y_p(t) + y_h(t)$$

$$y(0) = C + \frac{\sin(-18.4^\circ)}{\sqrt{10}} + \frac{2}{\sqrt{13}} \cdot \sin(-33.7^\circ) + \frac{3}{\sqrt{18}} \sin(-45^\circ) + \frac{4}{5} \cdot \sin(-53.1^\circ) = 0$$

$$\Rightarrow \quad C = 1.55$$

$$y(t) = 1.55 \cdot e^{-3t} + \frac{\sin(-18.4^\circ)}{\sqrt{10}} + \frac{2}{\sqrt{13}} \sin(-33.7^\circ) + \frac{3}{\sqrt{18}} \sin(-45^\circ) + \frac{4}{5} \sin(-53.1^\circ)$$

$t >> 0 \Rightarrow$ ISTITRAJU SE VLASTITE FREKVENCIJE,
OSTAJE SAMO ODZIV NA POBUDU

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5. $y(n) + 0.5\, y(n-1) = u(n)$

$$H(z) = \frac{1}{1 + 0.5 z^{-1}} \qquad z = j\Omega \Rightarrow H(\Omega) = \frac{1}{1 + 0.5 e^{-j\Omega}}$$

$$H(0.5\pi) = \frac{1}{1 + 0.5 e^{-0.5\pi}} = \frac{1}{1 + 0.5(-j)} = \left(\underset{=}{1} + \frac{1}{2}\underset{=}{j}\right)\frac{4}{5}$$

$$|H(0.5)| = \sqrt{0.8^2 + 0.4^2} = 0.89$$

$$\measuredangle H(0.5\pi) = -\operatorname{arc\,tg} \frac{0.4}{0.8} = -26.56^\circ$$

$$H(\pi) = \frac{1}{1 + 0.5 e^{-j\pi}} = \frac{1}{1 - 0.5} = 2 = |H(\pi)| = A(\pi)$$

$$\measuredangle H(\pi) = -\operatorname{arc\,tg} 0 = 0^\circ$$

$$H\left(\frac{3\pi}{2}\right) = \frac{1}{1 + 0.5 j} = \left(\underset{=}{1} - \frac{1}{2}\underset{=}{j}\right) \cdot \frac{4}{5}$$

$$\left|H\left(\frac{3\pi}{2}\right)\right| = \sqrt{0.8^2 + 0.4^2} = 0.89$$

$$\measuredangle H\left(\frac{3\pi}{2}\right) = -\operatorname{arc\,tg} \frac{-0.4}{0.8} = 26.56^\circ$$

$$H(2\pi) = \frac{1}{1 + 0.5} = \frac{2}{3} = |H(2\pi)| = A(2\pi)$$

$$\measuredangle H(2\pi) = -\operatorname{arc\,tg} 0 = 0^\circ$$

$$y_p(n) = \Big[0.89 \cos(0.5\pi n + 9.4^\circ) + 4 \cdot \cos n\pi + 2.67 \cos(1.5\pi n + 26.56^\circ) + \frac{8}{3} \cos(2n\pi)\Big]\mu(n)$$

HOMOGENO ...

$$q + 0.5 = 0 \qquad q = -\frac{1}{2} \qquad y_h(n) = C_1 \cdot \left(-\frac{1}{2}\right)^n$$

$$y(n) = c\left(-\frac{1}{2}\right)^n + \Big[0.89\cos\left(\frac{n\pi}{2} + 9.4^\circ\right) + 4\cos(n\pi)$$

$$+ 2.67\cos\left(\frac{3n\pi}{2} + 26.56^\circ\right) + \frac{8}{3}\cos(2n\pi)\Big]$$

$y(-1) = 1 \rightarrow$

$y(n) + 0.5y(n-1) = u(n) \rightarrow$

$$y(0) = -0.5y(-1) + u(0)$$
$$= -0.5 \cdot 1 + 9.8 = 9.3$$

Naime, $u(n) = [\cos(0.5\pi n + 0.2\pi) + 2\cos\pi n + 3\cos 1.5\pi n +$
$+ 4\cos(2\pi n)]\, \mu(n)$

te iz toga računam $u(0)$.

Sada računamo c,

$$y(0) = c + 0.88 + 4 + 2.39 + 2.67 = 9.3$$
$$\Rightarrow c = -0.63$$

$$y(n) = -0.63\left(-\frac{1}{2}\right)^n + \Big[0.89\left(\cos\frac{n\pi}{2} + 62.50^\circ\right) + 4\cos(n\pi) +$$
$$+ 2.67\cos\left(\frac{3n\pi}{2} + 26.56^\circ\right) + \frac{8}{3}\cos(2\pi n)\Big], \quad n \geq 0$$

→ za $n \gg 0$, pošto je sustav stabilan, ostaje samo prisilni odziv.