Osnovne trigonometrijske jednakosti

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm\cos x$$
$$\cos\left(x \pm \frac{\pi}{2}\right) = \mp\sin x$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$\sin x - \sin y = 2\sin\frac{x-y}{2}\cos\frac{x+y}{2}$$
$$\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$\cos x - \cos y = 2\sin\frac{x+y}{2}\sin\frac{y-x}{2}$$

$$\sin x \sin y = \frac{1}{2}\big(\cos(x-y) - \cos(x+y)\big)$$
$$\cos x \cos y = \frac{1}{2}\big(\cos(x-y) + \cos(x+y)\big)$$
$$\sin x \cos y = \frac{1}{2}\big(\sin(x-y) + \sin(x+y)\big)$$

$$\sin(2x) = 2\sin x \cos x$$
$$\cos(2x) = \cos^2 x - \sin^2 x$$
$$2\sin^2 x = 1 - \cos(2x)$$
$$2\cos^2 x = 1 + \cos(2x)$$

Tablice suma i integrala

Konačne sume

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$
$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$
$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$
$$\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$$
$$\sum_{i=0}^{n} e^{j(\theta+i\phi)} = \frac{\sin\big((n+1)\phi/2\big)}{\sin(\phi/2)} e^{j(\theta+n\phi/2)}$$
$$\sum_{i=0}^{n} \binom{n}{i} = \sum_{i=1}^{n} \frac{n!}{i!(n-i)!} = 2^n$$

Neodređeni integrali

Racionalne funkcije

$$\int (ax+b)^n \, dx = \frac{(ax+b)^{n+1}}{a(n+1)}, \quad 0 < n$$
$$\int \frac{1}{ax+b} \, dx = \frac{1}{a}\ln|ax+b|$$
$$\int \frac{dx}{a^2x^2+b^2} = \frac{1}{ab}\,\mathrm{tg}^{-1}\left(\frac{ax}{b}\right)$$
$$\int \frac{x\,dx}{a^2+x^2} = \frac{1}{2}\ln(a^2+x^2)$$
$$\int \frac{x^2\,dx}{a^2+x^2} = x - a\,\mathrm{tg}^{-1}\left(\frac{x}{a}\right)$$
$$\int \frac{dx}{(a^2+x^2)^2} = \frac{x}{2a^2(a^2+x^2)} + \frac{1}{2a^3}\,\mathrm{tg}^{-1}\left(\frac{x}{a}\right)$$
$$\int \frac{x\,dx}{(a^2+x^2)^2} = \frac{-1}{2(a^2+x^2)}$$
$$\int \frac{x^2\,dx}{(a^2+x^2)^2} = \frac{-x}{2(a^2+x^2)} + \frac{1}{2a}\,\mathrm{tg}^{-1}\left(\frac{x}{a}\right)$$

Trigonometrijske funkcije

$$\int \cos(x)\,dx = \sin(x)$$
$$\int x\cos(x)\,dx = \cos(x) + x\sin(x)$$
$$\int x^2\cos(x)\,dx = 2x\cos(x) + (x^2-2)\sin(x)$$
$$\int \sin(x)\,dx = -\cos(x)$$
$$\int x\sin(x)\,dx = \sin(x) - x\cos(x)$$
$$\int x^2\sin(x)\,dx = 2x\sin(x) + (2-x^2)\cos(x)$$

Eksponencijalne funkcije

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$$
$$\int xe^{ax}\,dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax}$$
$$\int x^2e^{ax}\,dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax}$$
$$\int x^3e^{ax}\,dx = \left(\frac{x^3}{a} - \frac{3x^2}{a^2} + \frac{6x}{a^3} - \frac{6}{a^4}\right)e^{ax}$$
$$\int \sin(x)e^{ax}\,dx = \frac{1}{a^2+1}\big(a\sin(x) - \cos(x)\big)e^{ax}$$
$$\int \cos(x)e^{ax}\,dx = \frac{1}{a^2+1}\big(a\cos(x) + \sin(x)\big)e^{ax}$$

Određeni integrali

$$\int_{-\infty}^{+\infty} e^{-a^2x^2+bx}\,dx = \frac{\sqrt{\pi}}{a}e^{\frac{b^2}{4a^2}}, \quad a>0$$

$$\int_{0}^{+\infty} x^2e^{-x^2}\,dx = \frac{\sqrt{\pi}}{4}$$

$$\int_{0}^{+\infty} \frac{\sin(x)}{x}\,dx = \frac{\pi}{2}$$

$$\int_{0}^{+\infty} \frac{\sin^2(x)}{x^2}\,dx = \frac{\pi}{2}$$

Laplaceova transformacija

Jednostrana Laplaceova transformacija funkcije $x(t)$ je:

$$\mathcal{L}\big[f(t)\big] = \int_{0^-}^{+\infty} f(t)e^{-st}dt$$

Kažemo da su $x(t)$ i $X(s)$ transformacijski par i pišemo $x(t) \circ\!\!-\!\!\bullet\ X(s)$.

Tablica transformacija

$$1 \circ\!\!-\!\!\bullet\ \frac{1}{s}$$

$$t \circ\!\!-\!\!\bullet\ \frac{1}{s^2}$$

$$e^{-at} \circ\!\!-\!\!\bullet\ \frac{1}{s+a}$$

$$\frac{1}{b-a}(e^{-at}-e^{-bt}) \circ\!\!-\!\!\bullet\ \frac{1}{(s+a)(s+b)}$$

$$\frac{1}{a-b}(ae^{-at}-be^{-bt}) \circ\!\!-\!\!\bullet\ \frac{s}{(s+a)(s+b)}$$

$$\frac{1}{a}e^{-bt}\sin(at) \circ\!\!-\!\!\bullet\ \frac{1}{(s+b)^2+a^2}$$

$$e^{-bt}\big(\cos(at)-\frac{b}{a}\sin(at)\big) \circ\!\!-\!\!\bullet\ \frac{s}{(s+b)^2+a^2}$$

Vremenski kontinuirana Fourierova transformacija

Vremenski kontinuirana Fourierova transformacija (CTFT – *Continuous-Time Fourier Transform*) funkcije $x(t)$ je:

$$\mathrm{CTFT}\big[x(t)\big] = X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,dt$$

Inverzna transformacija je:

$$\mathrm{CTFT}^{-1}\big[X(\omega)\big] = x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)e^{j\omega t}\,d\omega$$

Kažemo da su $x(t)$ i $X(\omega)$ transformacijski par i pišemo $x(t) \circ\!\!-\!\!\bullet\ X(\omega)$.

Dovoljni (ali ne i nužni) uvjeti za postojanje transformacije funkcije $x(t)$ su:

1. Funkcija x(t) zadovoljava Dirichletove uvjete (funkcija je ograničena s konačnim brojem maksimuma i minimuma te konačnim brojem diskontinuiteta u bilo kojem konačnom vremenskom intervalu).
2. $\displaystyle\int_{-\infty}^{+\infty} |x(t)|dt < \infty$

Svojstva Fourierove transformacije

Neka je $x(t) \circ\!\!-\!\!\bullet\ X(\omega)$ i neka su α_i, t_0 i ω_0 konstante. Fourierova transformacija tada zadovoljava sljedeća svojstva:

Linearnost

$$x(t) = \sum_{i=1}^{n}\alpha_i x_i(t) \circ\!\!-\!\!\bullet\ \sum_{i=1}^{n}\alpha_i X_i(\omega) = X(\omega)$$

Dualnost

$$X(t) \circ\!\!-\!\!\bullet\ 2\pi x(-\omega)$$

Pomak u vremenu i frekvenciji

$$x(t-t_0) \circ\!\!-\!\!\bullet\ X(\omega)e^{-j\omega t_0}$$

$$x(t)e^{j\omega_0 t} \circ\!\!-\!\!\bullet\ X(\omega-\omega_0)$$

Skaliranje

$$x(\alpha t) \circ\!\!-\!\!\bullet\ \frac{1}{|\alpha|}X\Big(\frac{\omega}{\alpha}\Big)$$

Deriviranje

$$\frac{d^n x(t)}{dt^n} \circ\!\!-\!\!\bullet\ (j\omega)^n X(\omega)$$

$$(-jt)^n x(t) \circ\!\!-\!\!\bullet\ \frac{d^n X(\omega)}{d\omega^n}$$

Integriranje

$$\int_{-\infty}^{t} x(\tau)\,d\tau \circ\!\!-\!\!\bullet\ \pi X(0)\,\delta(\omega) + \frac{X(\omega)}{j\omega}$$

$$\pi x(0)\,\delta(t) - \frac{x(t)}{jt} \circ\!\!-\!\!\bullet\ \int_{-\infty}^{\omega} X(\xi)\,d\xi$$

Konjugacija

$$x^*(t) \circ\!\!-\!\!\bullet\ X^*(-\omega)$$

$$x^*(-t) \circ\!\!-\!\!\bullet\ X^*(\omega)$$

Konvolucija

$$\int_{-\infty}^{+\infty} x_1(\tau)x_2(t-\tau)\,d\tau \circ\!\!-\!\!\bullet\ X_1(\omega)X_2(\omega)$$

$$x_1(t)x_2(t) \circ\!\!-\!\!\bullet\ \frac{1}{2\pi}\int_{-\infty}^{+\infty} X_1(\xi)X_2(\omega-\xi)\,d\xi$$

Korelacija

$$\int_{-\infty}^{+\infty} x_1^*(\tau)x_2(t+\tau)\,d\tau \circ\!\!-\!\!\bullet\ X_1^*(\omega)X_2(\omega)$$

$$x_1^*(t)x_2(t) \circ\!\!-\!\!\bullet\ \frac{1}{2\pi}\int_{-\infty}^{+\infty} X_1^*(\xi)X_2(\omega+\xi)\,d\xi$$

Parsevalov teorem

$$\int_{-\infty}^{+\infty} x_1^*(t)x_2(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X_1^*(\omega)X_2(\omega)\,d\omega$$

$$\int_{-\infty}^{+\infty} |x(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(\omega)|^2\,d\omega$$

Tablica transformacija

Neka je:

$$\mu(x) = \begin{cases} 1,\ x>0 \\ 0,\ x<0 \end{cases}$$

$$\mathrm{rect}(x) = \begin{cases} 1, & -\frac{1}{2} < \ x\ < \frac{1}{2} \\ 0, & \frac{1}{2} < |x| \end{cases}$$

$$\mathrm{tri}(x) = \begin{cases} 1-|x|,\ |x|<1 \\ 0, \qquad\ \ |x|>1 \end{cases}$$

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

Uz te oznake važnije transformacije su:

$$1 \circ\!\!-\!\!\bullet\ 2\pi\,\delta(\omega)$$

$$\delta(t) \circ\!\!-\!\!\bullet\ 1$$

$$\mu(t) \circ\!\!-\!\!\bullet\ \pi\,\delta(\omega) + \frac{1}{j\omega}$$

$$\frac{1}{2}\,\delta(t) - \frac{1}{2\pi jt} \circ\!\!-\!\!\bullet\ \mu(\omega)$$

$$\mathrm{sgn}(t) \circ\!\!-\!\!\bullet\ \frac{2}{j\omega}$$

$$\mathrm{rect}\Big(\frac{t}{T}\Big) \circ\!\!-\!\!\bullet\ T\,\mathrm{sinc}\Big(\frac{\omega T}{2\pi}\Big)$$

$$\mathrm{sinc}(at) \circ\!\!-\!\!\bullet\ \frac{1}{a}\,\mathrm{rect}\Big(\frac{\omega}{2\pi a}\Big)$$

$$\mathrm{tri}\Big(\frac{t}{T}\Big) \circ\!\!-\!\!\bullet\ T\,\mathrm{sinc}^2\Big(\frac{\omega T}{2\pi}\Big)$$

$$\mathrm{sinc}^2(at) \circ\!\!-\!\!\bullet\ \frac{1}{a}\,\mathrm{tri}\Big(\frac{\omega}{2\pi a}\Big)$$

$$e^{j\omega_0 t} \circ\!\!-\!\!\bullet\ 2\pi\,\delta(\omega-\omega_0)$$

$$\delta(t-t_0) \circ\!\!-\!\!\bullet\ e^{-j\omega t_0}$$

$$\sin(\omega_0 t) \circ\!\!-\!\!\bullet\ -j\pi\big(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\big)$$

$$\cos(\omega_0 t) \circ\!\!-\!\!\bullet\ \pi\big(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\big)$$

$$\sum_{i=-\infty}^{+\infty} \delta(t-iT_0) \circ\!\!-\!\!\bullet\ \frac{2\pi}{T_0}\sum_{i=-\infty}^{+\infty}\delta\Big(\frac{\omega}{2\pi} - \frac{i}{T_0}\Big)$$

$$\sin(\omega_0 t)\,\mu(t) \circ\!\!-\!\!\bullet\ -\frac{j\pi}{2}\big(\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\big) + \frac{j\omega}{\omega_0^2-\omega^2}$$

$$\cos(\omega_0 t)\,\mu(t) \circ\!\!-\!\!\bullet\ \frac{\pi}{2}\big(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\big) + \frac{j\omega}{\omega_0^2-\omega^2}$$

$$e^{-at}\,\mu(t) \circ\!\!-\!\!\bullet\ \frac{1}{a+j\omega}, \quad a>0$$

$$te^{-at}\,\mu(t) \circ\!\!-\!\!\bullet\ \frac{1}{(a+j\omega)^2}, \quad a>0$$

$$t^2e^{-at}\,\mu(t) \circ\!\!-\!\!\bullet\ \frac{2}{(a+j\omega)^3}, \quad a>0$$

$$t^3e^{-at}\,\mu(t) \circ\!\!-\!\!\bullet\ \frac{6}{(a+j\omega)^4}, \quad a>0$$

$$e^{-a|t|} \circ\!\!-\!\!\bullet\ \frac{2a}{a^2+\omega^2}$$

$$e^{-\frac{t^2}{2a^2}} \circ\!\!-\!\!\bullet\ a\sqrt{2\pi}e^{-a^2\omega^2/2}$$

Vremenski kontinuiran Fourierov red

Vremenski kontinuiran Fourierov red (CTFS – *Continuous-Time Fourier Series*) periodike funkcije $x(t)$ s periodom T_p je:

$$\mathrm{CTFS}_{T_p}\big[x(t)\big] = X[k] = \frac{1}{T_p}\int_{T_p} x(t)e^{-j\omega_P kt}\,dt$$

Inverzna transformacija je:

$$\mathrm{CTFS}_{T_p}^{-1}\big[X[k]\big] = x(t) = \sum_{k=-\infty}^{+\infty} X[k]e^{j\omega_P kt}$$

Pri tome je $\omega_P = \frac{2\pi}{T_p}$. Kažemo da su $x(t)$ i $X(k)$ transformacijski par i pišemo $x(t) \circ\!\!-\!\!\bullet\ X[k]$.

Vremenski diskretna Fourierova transformacija

Vremenski diskretna Fourierova transformacija (DTFT – *Discrete-Time Fourier Transform*) niza $x[n]$ je:

$$\mathrm{DTFT}\big[x[n]\big] = X(\omega) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

Inverzna transformacija je:

$$\mathrm{DTFT}^{-1}\big[X(\omega)\big] = x[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X(\omega)e^{j\omega n}\,d\omega$$

Niz $x[n]$ i njegov spektar $X(\omega)$ čine transformacijski par $x[n] \circ\!\!-\!\!\bullet X(\omega)$.

Dovoljan (ali ne i nužni) uvjet za postojanje transformacije niza $x[n]$ je apsolutna sumabilnost:

$$\sum_{n=-\infty}^{+\infty} \big|x[n]\big| < \infty$$

Svojstva vremenski diskretne Fourierove transformacije

Neka je $x[n] \circ\!\!-\!\!\bullet X(\omega)$ i neka su α_i, n_0 i ω_0 konstante. Vremenski diskretna Fourierova transformacija tada zadovoljava sljedeća svojstva:

Linearnost

$$x[n] = \sum_{i=1}^{n} \alpha_i x_i[n] \circ\!\!-\!\!\bullet \sum_{i=1}^{n} \alpha_i X_i(\omega) = X(\omega)$$

Pomak u vremenu i frekvenciji

$$x[n-n_0] \circ\!\!-\!\!\bullet X(\omega)e^{-j\omega n_0}$$

$$x[n]e^{j\omega_0 n} \circ\!\!-\!\!\bullet X(\omega-\omega_0)$$

Deriviranje i diferenciranje

$$\Delta x[n] \circ\!\!-\!\!\bullet (e^{j\omega}-1)X(\omega)$$

$$n^i x[n] \circ\!\!-\!\!\bullet j^i \frac{d^i X(\omega)}{d\omega^i}$$

Sumiranje

$$\sum_{i=-\infty}^{n} x[i] \circ\!\!-\!\!\bullet \frac{1}{1-e^{-j\omega}} X(\omega)$$

Konjugacija

$$x^*[n] \circ\!\!-\!\!\bullet X^*(-\omega)$$

$$x^*[-n] \circ\!\!-\!\!\bullet X^*(\omega)$$

Konvolucija

$$\sum_{i=-\infty}^{+\infty} x_1[i]x_2[n-i] \circ\!\!-\!\!\bullet X_1(\omega)X_2(\omega)$$

$$x_1[n]x_2[n] \circ\!\!-\!\!\bullet \frac{1}{2\pi}\int_{-\pi}^{+\pi} X_1(\xi)X_2(\omega-\xi)\,d\xi$$

Parsevalov teorem

$$\sum_{n=-\infty}^{+\infty} x_1^*[n]x_2[n] = \frac{1}{2\pi}\int_{-\pi}^{+\pi} X_1^*(\omega)X_2(\omega)\,d\omega$$

$$\sum_{n=-\infty}^{+\infty} \big|x[n]\big|^2 = \frac{1}{2\pi}\int_{-\pi}^{+\pi} \big|X(\omega)\big|^2\,d\omega$$

Relacije simetričnosti

Neka je $x[n]$ čisto realan niz i neka je $x[n] \circ\!\!-\!\!\bullet X(\omega)$. Tada je:

$$\frac{1}{2}\big(x[n]+x[-n]\big) \circ\!\!-\!\!\bullet \mathrm{Re}\big[X(\omega)\big]$$

$$\frac{1}{2}\big(x[n]-x[-n]\big) \circ\!\!-\!\!\bullet j\,\mathrm{Im}\big[X(\omega)\big]$$

Također vrijedi:

$$X(\omega) = X^*(-\omega)$$

$$\mathrm{Re}\big[X(\omega)\big] = \mathrm{Re}\big[X(-\omega)\big]$$

$$\mathrm{Im}\big[X(-\omega)\big] = -\,\mathrm{Im}\big[X(\omega)\big]$$

Tablica transformacija

$$\delta[n] \circ\!\!-\!\!\bullet\; 1$$

$$1 \circ\!\!-\!\!\bullet \sum_{i=-\infty}^{+\infty} 2\pi\delta(\omega + 2\pi i)$$

$$e^{j\omega_0 n} \circ\!\!-\!\!\bullet \sum_{i=-\infty}^{+\infty} 2\pi\delta(\omega - \omega_0 + 2\pi i)$$

$$\mu[n] \circ\!\!-\!\!\bullet \frac{1}{1 - e^{-j\omega}} + \sum_{i=-\infty}^{+\infty} \pi\delta(\omega + 2\pi i)$$

$$a^n\,\mu[n] \circ\!\!-\!\!\bullet \frac{1}{1 - ae^{-j\omega}}, \quad |a| < 1$$

$$na^n\,\mu[n] \circ\!\!-\!\!\bullet \frac{ae^{j\omega}}{(e^{-j\omega} - a)^2}, \quad |a| < 1$$

$$\sin(\omega_0 n) \circ\!\!-\!\!\bullet \sum_{i=-\infty}^{+\infty} j\pi\big(\delta(\omega + \omega_0 + 2\pi i) - \delta(\omega - \omega_0 + 2\pi i)\big)$$

$$\cos(\omega_0 n) \circ\!\!-\!\!\bullet \sum_{i=-\infty}^{+\infty} \pi\big(\delta(\omega + \omega_0 + 2\pi i) + \delta(\omega - \omega_0 + 2\pi i)\big)$$

$$a^n \sin(\omega_0 n)\,\mu[n] \circ\!\!-\!\!\bullet \frac{ae^{j\omega}\sin(\omega_0)}{e^{2j\omega} - 2ae^{j\omega}\cos(\omega_0) + a^2}, \quad |a| < 1$$

$$a^n \cos(\omega_0 n)\,\mu[n] \circ\!\!-\!\!\bullet \frac{e^{j\omega}\big(e^{j\omega} - a\cos(\omega_0)\big)}{e^{2j\omega} - 2ae^{j\omega}\cos(\omega_0) + a^2}, \quad |a| < 1$$

Vremenski diskretan Fourierov red

Vremenski diskretan Fourierov red (DTFS – *Discrete-Time Fourier Series*) periodičnog niza $x[n]$ perioda N je:

$$\mathrm{DTFS}_N\big[x[n]\big] = X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-2\pi jkn/N}$$

Inverzna transformacija je:

$$\mathrm{DTFS}_N^{-1}\big[X[k]\big] = x[n] = \sum_{k=0}^{N-1} X[k]e^{2\pi jkn/N}$$

Niz $x[n]$ i njegov spektar $X[k]$ čine transformacijski par $x[n] \circ\!\!-\!\!\bullet X[k]$.

Diskretna Fourierova transformacija

Diskretna Fourierova transformacija (DFT – *Discrete Fourier Transform*) konačnog niza $x[n]$ duljine N je:

$$\mathrm{DFT}_N\big[x[n]\big] = X[k] = \sum_{n=0}^{N-1} x[n]W_N^{nk}, \quad 0 \le k < N$$

Pri tome je $W_N^{nk} = e^{-2\pi jnk/N}$. Inverzna transformacija je:

$$\mathrm{DFT}_N^{-1}\big[X[k]\big] = x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]W_N^{-nk}, \quad 0 \le n < N$$

Niz $x[n]$ i njegov spektar $X[k]$ čine transformacijski par $x[n] \circ\!\!-\!\!\bullet X[k]$.

Svojstva diskretne Fourierove transformacije

Neka je $x[n] \circ\!\!-\!\!\bullet X[k]$ i neka su α_i, n_0 i k_0 konstante. DFT tada zadovoljava sljedeća svojstva:

Linearnost

$$x[n] = \sum_{i=1}^{n} \alpha_i x_i[n] \circ\!\!-\!\!\bullet \sum_{i=1}^{n} \alpha_i X_i[k] = X[k]$$

Dualnost

$$X[n] \circ\!\!-\!\!\bullet Nx\big[\langle -k\rangle_N\big]$$

Cirkularni pomak u vremenu i frekvenciji

$$x\big[\langle n - n_0\rangle_N\big] \circ\!\!-\!\!\bullet X[k]W_N^{kn_0}$$

$$x[n]W_N^{k_0 n} \circ\!\!-\!\!\bullet X\big[\langle k - k_0\rangle_N\big]$$

Cirkularna konvolucija

$$\sum_{i=0}^{N-1} x_1[i]x_2\big[\langle n - i\rangle_N\big] \circ\!\!-\!\!\bullet X_1[k]X_2[k]$$

$$x_1[n]x_2[n] \circ\!\!-\!\!\bullet \frac{1}{N}\sum_{i=0}^{N-1} X_1[i]X_2\big[\langle k - i\rangle_N\big]$$

Parsevalova relacija

$$\sum_{n=0}^{N-1} x_1^*[n]x_2[n] \circ\!\!-\!\!\bullet \frac{1}{N}\sum_{k=0}^{N-1} X_1^*[k]X_2[k]$$

$$\sum_{n=0}^{N-1} \big|x[n]\big|^2 \circ\!\!-\!\!\bullet \frac{1}{N}\sum_{k=0}^{N-1} \big|X[k]\big|^2$$

$\mathcal{Z}$-transformacija

$\mathcal{Z}$-transformacija niza $f[n]$ je: $\mathcal{Z}\big[f[n]\big] = \sum_{n=0}^{+\infty} f[n]z^{-n}$

Svojstva $\mathcal{Z}$ transformacije

Neka je $\mathcal{Z}\big[f[n]\big] = F(z)$ i $\mathcal{Z}\big[g[n]\big] = G(z)$. Tada vrijedi:

Linearnost

$$f[n] = \sum_{i=1}^{n} \alpha_i f_i[n] \circ\!\!-\!\!\bullet \sum_{i=1}^{n} \alpha_i F_i(z) = F(z)$$

Pomak

$$f[n+1] \circ\!\!-\!\!\bullet zF(z) - zf[0]$$

$$f[n+m] \circ\!\!-\!\!\bullet z^m F(z) - \sum_{i=0}^{m-1} f[i] z^{m-i}$$

$$f[n-1] \circ\!\!-\!\!\bullet \frac{1}{z} F(z) + f[-1]$$

$$f[n-m] \circ\!\!-\!\!\bullet z^{-m} F(z) + \sum_{i=0}^{m-1} f[i-m] z^{-i}$$

Skaliranje

$$a^n f[n] \circ\!\!-\!\!\bullet F(\frac{z}{a})$$

Diferenciranje i deriviranje

$$\Delta f[n] \circ\!\!-\!\!\bullet (z-1)F(z)$$

$$nf[n] \circ\!\!-\!\!\bullet -z\frac{dF(z)}{dz}$$

Konvolucija

$$\sum_{i=0}^{+\infty} f[i] g[n-i] \circ\!\!-\!\!\bullet F(z)G(z)$$

Tablica transformacija

$$\delta[n] \circ\!\!-\!\!\bullet 1$$

$$\delta[n-m] \circ\!\!-\!\!\bullet z^{-m}$$

$$n \circ\!\!-\!\!\bullet \frac{z}{(z-1)^2}$$

$$1^n \circ\!\!-\!\!\bullet \frac{1}{1-z^{-1}} = \frac{z}{z-1}$$

$$a^n \circ\!\!-\!\!\bullet \frac{1}{1-az^{-1}} = \frac{z}{z-a}$$

$$(n+1)a^n \circ\!\!-\!\!\bullet \frac{z^2}{(z-a)^2}$$

$$\frac{(n+1)(n+2)}{2!} a^n \circ\!\!-\!\!\bullet \frac{z^3}{(z-a)^3}$$

$$\frac{(n+1)(n+2)\ldots(n+m-1)}{(m-1)!} a^n \circ\!\!-\!\!\bullet \frac{z^m}{(z-a)^m}$$

$$\frac{n(n-1)(n-2)\ldots(n-m+1)}{m!} a^{n-m} \circ\!\!-\!\!\bullet \frac{z}{(z-a)^{m+1}}$$

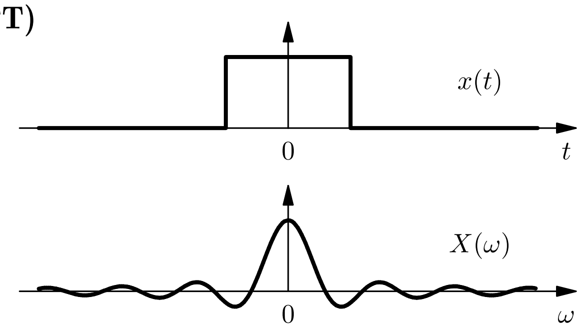
$$a^n - \delta[n] \circ\!\!-\!\!\bullet \frac{a}{z-a}$$

$$\sin[an] \circ\!\!-\!\!\bullet \frac{z\sin(a)}{z^2 - 2z\cos(a) + 1}$$

$$\cos[an] \circ\!\!-\!\!\bullet \frac{z^2 - z\cos(a)}{z^2 - 2z\cos(a) + 1}$$

Pregled Fourierovih transformacija

Vremenski kontinuirana Fourierova transformacija (CTFT)

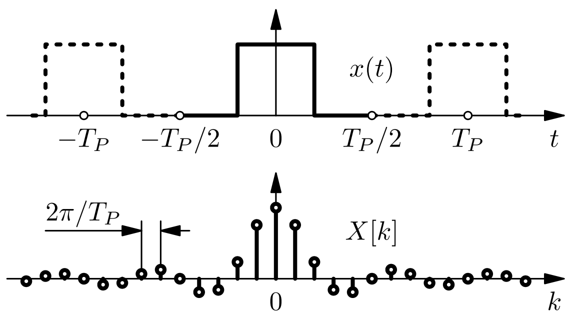


$$X(\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(\omega)e^{j\omega t}\,d\omega$$

$$\int_{-\infty}^{+\infty} x_1(t)x_2^*(t)\,dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X_1(\omega)X_2^*(\omega)$$

Vremenski kontinuiran Fourierov red (CTFS)

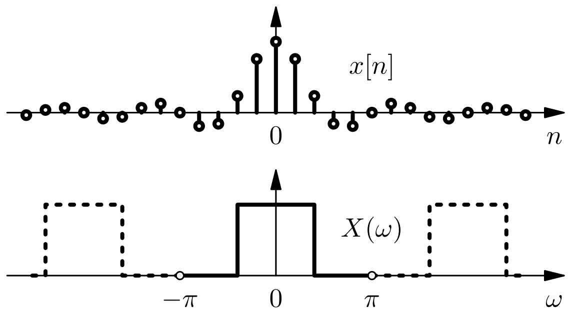


$$X[k] = \frac{1}{T_P}\int_{T_P} x(t)e^{-j\omega_P kt}\,dt$$

$$x(t) = \sum_{k=-\infty}^{+\infty} X[k]e^{j\omega_P kt}$$

$$\frac{1}{T_P}\int_{-\infty}^{+\infty} x(t)x_2^*(t)\,dt = \sum_{k=-\infty}^{+\infty} X_1[k]X_2^*[k]$$

Vremenski diskretna Fourierova transformacija (DTFT)

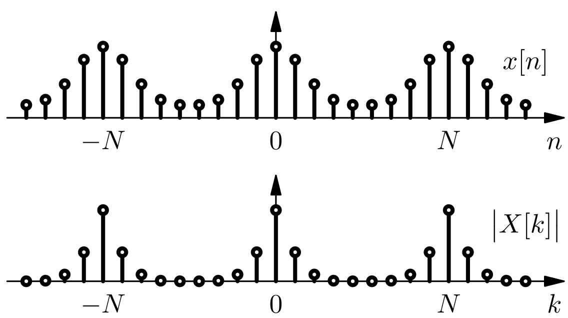


$$X(\omega) = \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}\,d\omega$$

$$\sum_{n=-\infty}^{+\infty} x_1[n]x_2^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(\omega)X_2^*(\omega)\,d\omega$$

Vremenski diskretan Fourierov red (DTFS)

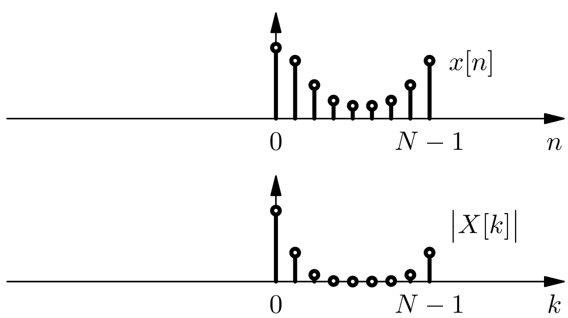


$$X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]e^{-2\pi jkn/N}$$

$$x[n] = \sum_{k=0}^{N-1} X[k]e^{2\pi jkn/N}$$

$$\frac{1}{N}\sum_{n=0}^{N-1} x_1[n]x_2^*[n] = \sum_{k=0}^{N-1} X_1[k]X_2^*[k]$$

Diskretna Fourierova transformacija (DFT)



$$X[k] = \sum_{n=0}^{N-1} x[n]W_N^{kn}, \quad 0 \le k \le N-1$$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]W_N^{-kn}, \quad 0 \le n \le N-1$$

$$\sum_{n=0}^{N-1} x_1[n]x_2^*[n] = \frac{1}{N}\sum_{k=0}^{N-1} X_1[k]X_2^*[k]$$

Određivanja početnih uvjeta

Za sustav opisan diferencijalnom jednadžbom

$$y^{(N)}(t) + a_1 y^{(N-1)}(t) + \cdots + a_{N-1} y^{(1)}(t) + a_N y(t) = b_0 u^{(N)}(t) + b_1 u^{(N-1)}(t) + \cdots + b_{N-1} u^{(1)}(t) + b_N u(t)$$

potrebno je odrediti početne uvjete $y(0^+)$, $y'(0^+)$, $y''(0^+)$, $\ldots$, $y^{(N-1)}(0^+)$ iz onih u 0^-. Ako pobuda ne sadrži Diracovu distribuciju rješavamo sustav jednadžbi:

$$\begin{aligned}
\Delta y &= b_0 u(0^+) \\
\Delta y^{(1)} + a_1 \Delta y &= b_0 u^{(1)}(0^+) + b_1 u(0^+) \\
\Delta y^{(2)} + a_1 \Delta y^{(1)} + a_2 \Delta y &= b_0 u^{(2)}(0^+) + b_1 u^{(1)}(0^+) + b_2 u(0^+) \\
\\
\Delta y^{(N-1)} + a_1 \Delta y^{(N-2)} + \cdots + a_{N-1} \Delta y &= b_0 u^{(N-1)}(0^+) + \cdots + b_{N-2} u^{(1)}(0^+) + b_{N-1} u(0^+)
\end{aligned}$$

Pri tome je $\Delta y^{(i)} = y^{(i)}(0^+) - y^{(i)}(0^-)$.

Ako je pobuda $u(t) = \delta(t)$ onda je $y^{(N-1)}(0^+) = y^{(N-1)}(0^-) + 1$, a ostali početni uvjeti se ne razlikuju.