TJEDAN 9.
SIGNALI I SUSTAVI

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GRUPA 2.E2

1. $y(n) = 2^{u(n)}$

- BEZMEMORIJSKI SUSTAV

$u(n) = \alpha u_1(n) + \beta u_2(n)$

$y(n) = 2^{\alpha u_1(n)} \cdot 2^{\beta u_2(n)}$

- NELINEARAN SUSTAV

$u_1(n) = u(n-M) \qquad y(n-M) = 2^{u(n-M)}$

$y_1(n) = 2^{u(n-M)}$

$y(n-M) = y_1(n)$

- VREMENSKI NEPROMJENJIV

2. $y(t) = u(t^2)$

- MEMORIJSKI SUSTAV

$u(t^2) = \alpha u_1(t^2) + \beta u_2(t^2)$

$y(t) = \alpha u_1(t^2) + \beta u_2(t^2) = \alpha y_1(t) + \beta y_2(t)$

- LINEARAN SUSTAV

$y_1(t) = u(t^2 - T) \qquad y_1(t) \neq y(t-T)$

$y(t-T) = u(t^2 - 2tT + T^2)$

- VREMENSKI PROMJENJIV

3. $y(n) = \sum_{k=-\infty}^{n} \frac{u(k)}{n-k}$

- MEMORIJSKI SUSTAV

$$y_1(n) = \sum_{k=-\infty}^{n} \frac{u_1(k)}{n-k}, \quad y_2(n) = \sum_{k=-\infty}^{n} \frac{u_2(k)}{n-k}$$

$$u(n) = \alpha u_1(n) + \beta u_2(n)$$

$$y(n) = \sum_{k=-\infty}^{n} \frac{\alpha u_1(k) + \beta u_2(k)}{n-k} = \alpha \sum_{k=-\infty}^{n} \frac{u_1(k)}{n-k} + \beta \sum_{k=-\infty}^{n} \frac{u_2(k)}{n-k}$$

- LINEARAN SUSTAV

$$y_1(n) = \sum_{k=-\infty}^{n-M} \frac{u(k)}{n-M-k}, \quad k = m - M \rightarrow m = k + M$$

$$M = m - k$$

$$y_1(n) = \sum_{m=-\infty}^{n-M} \frac{u(m-M)}{n-m}$$

$$y(n-M) = \sum_{m=-\infty}^{n-M}$$

4. $$y(t) = \int_{-\infty}^{+\infty} e^{-\tau} \nu(-\tau)\, u(t-\tau)\, d\tau$$

- MEMORIJSKI SUSTAV

$$y_1(t) = \int_{-\infty}^{+\infty} e^{-\tau} \nu(-\tau)\, u_1(t-\tau)\, d\tau$$

$$y_2(t) = \int_{-\infty}^{+\infty} e^{-\tau} \nu(-\tau)\, u_2(t-\tau)\, d\tau$$

$$u(t-\tau) = \alpha u_1(t-\tau) + \beta u_2(t-\tau)$$

$$y(t) = \int_{-\infty}^{+\infty} e^{-\tau} \nu(-\tau) \left[\alpha u_1(t-\tau) + \beta u_2(t-\tau)\right] d\tau$$

$$y(t) = \alpha \int_{-\infty}^{+\infty} e^{-\tau} \nu(-\tau)\, u_1(t-\tau)\, d\tau + \beta \int_{-\infty}^{\infty} e^{-\tau} \nu(-\tau)\, u_2(t-\tau)\, d\tau$$

$$y(t) = \alpha y_1(t) + \beta y_2(t)$$

- LINEARAN SUSTAV

$$y_1(t) = \int_{-\infty}^{+\infty} e^{-\tau} \mu(-\tau)\, u(t-\tau-T)\, d\tau$$

$$y(t-T) = \int_{-\infty}^{\infty} e^{-\tau} \mu(-\tau)\, u(t-\tau-T)\, d\tau$$

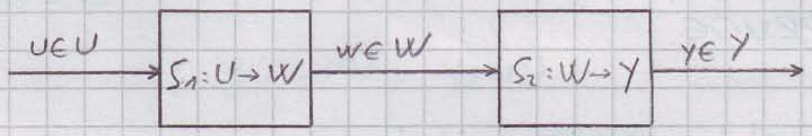
$$y(t-\tau) = y_1(t)$$

- VREMENSKI NEPROMJENJIV

5. S_1, S_2 - spojeni u kaskadu

(a) S_1, S_2 - linearni, vremenski nepromjenjiv

SPOJ ISTO LINEARAN I VREMENSKI NEPROMJENJIV?



$$S_1 := \alpha u_1(t) + \beta u_2(t) \rightarrow \alpha w_1(t) + \beta w_2(t)$$

$$S_2 := \alpha w_1(t) + \beta w_2(t) \rightarrow \alpha y_1(t) + \beta y_2(t)$$

$$S_1 \circ S_2 := \alpha u_1(t) + \beta u_2(t) \rightarrow \alpha y_1(t) + \beta y_2(t)$$

- AKO SU SVI PODSUSTAVI KASKADNOG SPOJA LINEARNI, TADA JE I CIJELI KASKADNI SPOJ LINEARAN

$$S_1 := u(t-t_0) \rightarrow w(t-t_0)$$

$$S_2 := w(t-t_0) \rightarrow y(t-t_0)$$

$$S_1 \circ S_2 := u(t-t_0) \rightarrow y(t-t_0)$$

- AKO SU SVI PODSUSTAVI KASKADNOG SPOJA VREMENSKI NEPROMJENJIVI, TADA JE I CIJELI KASKADNI SPOJ VREMENSKI NEPROMJENJIV

(b) S_1, S_2 - nelinearni

SPOJ ISTO NELINEARAN?

PRIMJER:

$$w(n) = S_1\{u(n)\} = e^{u(n)}$$

$$y(n) = S_2\{w(n)\} = \log(w(n))$$

$$y(n) = S_2(S_1(u(n))) = \log(e^{u(n)}) = u(n)$$

- AKO SU S_1 I S_2 NELINEARNI SUSTAVI NIJE NUŽNO DA ĆE I NJIHOV KASKADNI SPOJ BITI NELINEARAN, NELINEARNOST DRUGOG SUSTAVA MOŽE PONIŠTITI NELINEARNOST PRVOG.

(c) S_1, S_2 - vremenski promjenjivi

SPOJ ISTO VREMENSKI PROMJENJIV?

PRIMJER:

$$w(n) = u(n)\, e^{j\omega_0 n}$$

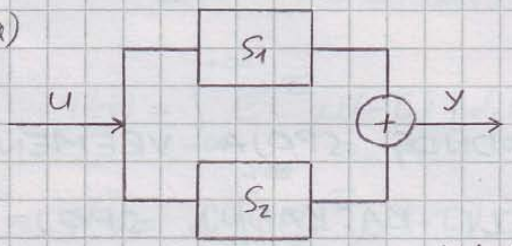
$$y(n) = w(n)\, e^{-j\omega_0 n}$$

$$y(n) = S_2(S_1(u(n))) = u(n) e^{j\omega_0 n} e^{-j\omega_0 n} = u(n)$$

- OBA SUSTAVA VREMENSKI PROMJENJIVA, SPOJ NIJE!

6. S_1, S_2 - spojeni u paralelu

(a)



$$y(n) = S_1(u)(n) + S_2(u)(n)$$

$$u(n) = \alpha u_1(n) + \beta u_2(n)$$

$$y(n) = S_1(\alpha u_1(n) + \beta u_2(n)) + S_2(\alpha u_1(n) + \beta u_2(n))$$

$$y(n) = \alpha S_1(u_1(n)) + \beta S_1(u_2(n)) + \alpha S_2(u_1(n)) + \beta S_2(u_2(n))$$

$$y(n) = \alpha (S_1(u_1(n)) + S_2(u_1(n))) + \beta (S_1(u_1(n)) + S_2(u_2(n)))$$

$$y(n) = \alpha y_1(n) + \beta y_2(n)$$

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6. (b) PRIMJER: NIJE NUŽNO NELINEARAN!

$$S_1(u(n)) = u(n) + 2^n$$

$$S_2(u(n)) = u(n) - 2^n$$

$$y(n) = S_1(u(n)) + S_2(u(n))$$

$$y(n) = 2u(n)$$

(c) PRIMJER:

$$S_1(u(n)) = n \cdot u(n)$$

$$S_2(u(n)) = -(n-1) \cdot u(n)$$

$$y(n) = S_1(u(n)) + S_2(u(n))$$

$$y(n) = u(n)$$

7. SUSTAV A - SISO

$u_1(n) = (-1)^n$, $y_1(n) = A(u_1(n)) = 1$, $\forall n \in$ Cjelobrojni

$u_2(n) = (-1)^{n+1}$, $y_2(n) = A(u_2(n)) = 1$, $\forall n \in$ Cjelobrojni

$$u_1(n) + u_2(n) = (-1)^n - (-1)^n = 0$$

$$y_1(n) + y_2(n) = 2$$

$u_2(n) = u_1(n - n_0)$, $\forall n \in$ Neparni Cjelobrojni

$y_2(n) = y_1(n - n_0)$

- NE MOŽE BITI LINEARAN
- MOŽE BITI VREMENSKI NEPROMJENJIV

SUSTAV B - SISO

$u_3(n) = (-1)^n$, $y_3(n) = B(u_3(n)) = 1$, $\forall n \in$ Cjelobrojni

$u_4(n) = (-1)^{n+1}$, $y_4(n) = B(u_4(n)) = -1$, $\forall n \in$ Cjelobrojni

$u_3(n) + u_4(n) = (-1)^n - (-1)^n = 0$

$y_3(n) + y_4(n) = 1 - 1 = 0$

$u_4(n) = u_3(n - n_0)$, $\forall n_0 \in$ Neparni Cjelobrojni

$y_4(n) \neq y_3(n - n_0)$

- MOŽE BITI LINEARAN
- NE MOŽE BITI VREMENSKI NEPROMJENJIV

8.

$u_1(t) = \mu(t) \rightarrow S(u_1(t)) = y_1(t) = (1 - e^{-2t})\mu(t)$

$u_2(t) = 4\mu(t) - 4\mu(t-1)$

$S_2(u_2(t)) = 4(1 - e^{-2t})\mu(t) - 4(1 - e^{-2(t-1)})\mu(t-1)$

$S_2(u_2(t)) = 4\left[\mu(t) - \mu(t-1) - e^{-2t}(\mu(t) - e^2\mu(t-1))\right]$

DODATNI ZADACI

4.

$y(t) = \frac{u(t)}{1 + u(t-1)}$

$u(t) = \alpha u_1(t) + \beta u_2(t)$

$u(t-1) = \alpha u_1(t-1) + \beta u_2(t-1)$

$y_1(t) = \frac{u_1(t)}{1 + u_1(t-1)}$

$y_2(t) = \frac{u_2(t)}{1 + u_2(t-1)}$

$y(t) = \frac{\alpha u_1(t) + \beta u_2(t)}{1 + \alpha u_1(t-1) + \beta u_2(t-1)}$ → NELINARAN SUSTAV

$y_1(t) = \frac{u(t-T)}{1 + u(t-1-T)}$

$y(t-T) = \frac{u(t-T)}{1 + u(t-T-1)}$

$y_1(t) = y(t-T)$ → VREMENSKI NEPROMJENJIV SUSTAV

- MEMORIJSKI SUSTAV, KAUZALAN
  - zbog $u(t-1)$

3. $y(n) = \left(\frac{1}{2}\right)^n u(3n+2)$

$y_1(n) = \left(\frac{1}{2}\right)^n u_1(3n+2)$ $\quad u(3n+2) = \alpha u_1(3n+2) + \beta u_2(3n+2)$

$y_2(n) = \left(\frac{1}{2}\right)^n u_2(3n+2)$

$y(n) = \left(\frac{1}{2}\right)^n (\alpha u_1(3n+2) + \beta u_2(3n+2))$

$y(n) = \alpha y_1(n) + \beta y_2(n)$

- LINEARAN SUSTAV

$y_1(n) = \left(\frac{1}{2}\right)^n u(3n+2-M)$ $\quad y_1(n) \neq y(n-M)$

$y(n-M) = \left(\frac{1}{2}\right)^{n-M} u(3n+2-3M)$

- VREMENSKI PROMJENJIV
- MEMORIJSKI SUSTAV

2. $y(t) = \int_0^+ u(\tau)\, d\tau$

$u(\tau) = \alpha u_1(\tau) + \beta u_2(\tau)$

$y_1(t) = \int_0^+ u_1(\tau)\, d\tau$ $\quad y(t) = \int_0^+ (\alpha u_1(\tau) + \beta u_2(\tau))\, d\tau$

$y(t) = \alpha y_1(t) + \beta y_2(t)$

$y_2(t) = \int_0^+ u_2(\tau)\, d\tau$

- LINEARAN SUSTAV

$y_1(t) = \int_0^+ u(\tau - T)\, d\tau$ $\quad y_1(t) \neq y(t-T)$

$y(t-T) = \int_T^{+-T} u(\tau)\, d\tau$

- VREMENSKI PROMJENJIV
- MEMORIJSKI SUSTAV

1. $y(t) = \int_{-\infty}^{+} u(\tau)\, d\tau$

$u(\tau) = \alpha u_1(\tau) + \beta u_2(\tau)$

$y(t) = \int_{-\infty}^{+} (\alpha u_1(\tau) + \beta u_2(\tau))\, d\tau$

$y(t) = \alpha y_1(t) + \beta y_2(t)$

$y_1(t) = \int_{-\infty}^{+} u_1(\tau)\, d\tau$

$y_2(t) = \int_{-\infty}^{+} u_2(\tau)\, d\tau$

- LINEARAN SUSTAV

$y_1(t) = \int_{-\infty}^{+} u(\tau - T)\, d\tau$

$y_1(t) \neq y(t-T)$

$y(t-T) = \int_{-\infty}^{+-T} u(\tau)\, d\tau$

- VREMENSKI PROMJENJIV
- MEMORIJSKI SUSTAV