B

1

$$y(n) - \frac{1}{6} y(n-1) = u(n)$$

a)

$$H(z) = \frac{1}{1 - \frac{1}{6} z^{-1}} = \frac{z}{z - \frac{1}{6}}$$

POL $z = \frac{1}{6}$ $|z| < 1$ STABILNO

b)

$$H(e^{j\Omega}) = \frac{1}{1 - \frac{1}{6} e^{-j\Omega}} = \frac{1}{1 - \frac{1}{6}\cos\Omega + \frac{1}{6} j \sin\Omega}$$

c)

$$u(n) = 2\cos\left(\frac{\pi}{2} n + \frac{\pi}{4}\right)$$

$$y_p(n) = |H(e^{j\Omega})| \cdot 2 \cdot \cos\left(\frac{\pi}{2} n + \measuredangle H(e^{j\Omega}) + \frac{\pi}{4}\right)$$

$$\Omega = \frac{\pi}{2}$$

$$H(e^{j\frac{\pi}{2}}) = \frac{1}{1 - \frac{1}{6}\cos\frac{\pi}{2} + \frac{1}{6} j \sin\frac{\pi}{2}} = \frac{1}{1 + \frac{1}{6} j}$$

$$|H(e^{j\pi/2})| = \frac{1}{\sqrt{1 + \frac{1}{36}}} = \frac{6}{\sqrt{37}} = 0.986$$

$$\measuredangle H(e^{j\pi/2}) = -\mathrm{arctg}\frac{\frac{1}{6}}{1} = -0.165$$

$$y_p(n) = 0.986 \cdot 2 \cdot \cos\left(\frac{\pi}{2} n + \frac{\pi}{4} - 0.165\right)$$

$$\boxed{y_p(n) = 1.97 \cos\left(\frac{\pi}{2} n + 0.62\right)}$$

2. $y'(t) + 12y(t) = u(t)$

a) IMPULSNI ODZIV

homogeno

$s + 12 = 0$

$s = -12$

$y_h(t) = Ce^{-12t}$

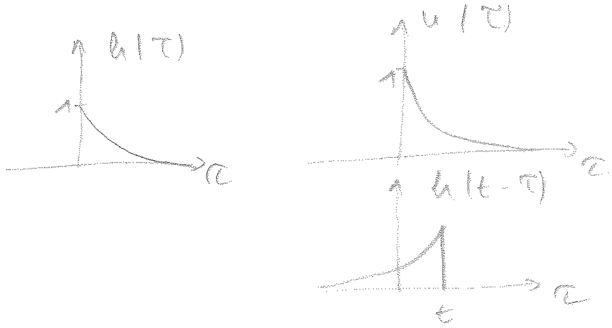
$h_a(0^+) = 1$

$h_a(t) = Ce^{-12t}$

$h_a(0^+) = C = 1$

$\boxed{h(t) = e^{-12t}\mu(t)}$

b) $u(t) = e^{-2t}\mu(t)$



$$y_m(t) = u(t) * h(t)$$

$$= \int_{-\infty}^{\infty} u(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{0}^{t} e^{-2\tau} \cdot e^{-12(t-\tau)}\, d\tau$$

$$= e^{-12t} \int_{0}^{t} e^{10\tau}\, d\tau = e^{-12t} \cdot \frac{e^{10\tau}}{10}\Big|_0^t = \frac{1}{10}\left(e^{-12t} \cdot e^{10t} - e^{-12t} e^{0}\right)$$

$$\boxed{y_m(t) = \frac{1}{10}e^{-2t} - \frac{1}{10}e^{-12t} \quad \text{za} \quad t>0}$$

c) $y(0^-) = 2$

$y_h(t) = Ce^{-12t}$

$y_0(t) = Ce^{-12t}$

$y_0(0^-) = 2 = C$

$y_0(t) = 2e^{-12t}$

$$y_t = y_m + y_0$$

$$= \frac{1}{10}e^{-2t} - \frac{1}{10}e^{-12t} + 2e^{-12t}$$

$$\boxed{y_t = \frac{1}{10}e^{-2t} + \frac{19}{10}e^{-12t} \quad \text{za} \quad t>0}$$

3.

$$y(n) + \frac{1}{6} y(n-1) - \frac{1}{6} y(n-2) = u(n) - 3u(n-1) + 2u(n-2)$$

a) $$y(z) = \underbrace{\frac{1 - 3z^{-1} + 2z^{-2}}{1 + \frac{1}{6} z^{-1} - \frac{1}{6} z^{-2}}}_{H(z)} \cdot U(z)$$ $\quad -\frac{1}{2} + \frac{1}{3} \quad \frac{-3+2}{6}$

$$H(z) = \frac{z^2 - 3z + 2}{(z + \frac{1}{2})(z - \frac{1}{3})}$$

$$\frac{H(z)}{z} = \frac{1}{z} \cdot \frac{z^2 - 3z + 2}{(z + \frac{1}{2})(z - \frac{1}{3})} = \frac{A}{z} + \frac{B}{z + \frac{1}{2}} + \frac{C}{z - \frac{1}{3}}$$

$$Az^2 + \frac{1}{6} Az - \frac{1}{6} A + Bz^2 - Bz\frac{1}{3} + Cz^2 + \frac{1}{2} Cz = z^2 - 3z + 2$$

$A + B + C = 1$ $\qquad A = -12$

$\frac{1}{6} A - \frac{1}{3} B + \frac{1}{2} C = -3$ $\qquad B + C = 1 - A = 13$

$-\frac{1}{6} A = 2$ $\qquad -\frac{1}{3} B + \frac{1}{2} C = -3 - \frac{1}{6} \cdot (-12) = -3 + 2 = -1 \quad | \cdot 3$

$-B + \frac{3}{2} C = -3$ $\qquad B = 13 - B = 9$

$\frac{5}{2} C = 10$

$C = 4$

$$H(z) = \frac{-12z}{z} + \frac{9z}{z + \frac{1}{2}} + \frac{4z}{z - \frac{1}{3}}$$

$$\boxed{h(n) = -12\,\delta(n) + \left[9\left(-\frac{1}{2}\right)^n + 4\left(\frac{1}{3}\right)^n\right]\mu(n)}$$

b) $u(n) = \{4, 0, 4, 0, \ldots\}$

$$U(z) = 4 + 4z^{-2} + 4z^{-4} + \ldots = 4 \sum_{k=0}^{\infty} (z^{-2})^k = 4 \cdot \frac{1}{1 - z^{-2}} = \frac{4z^2}{z^2 - 1}$$

$$y(z) = H(z) \cdot U(z) = \frac{\cancel{(z-1)}(z-2)}{(z + \frac{1}{2})(z - \frac{1}{3})} \cdot \frac{4z^2}{\cancel{(z-1)}(z+1)} = \frac{4z^2(z-2)}{(z + \frac{1}{2})(z - \frac{1}{3})(z+1)}$$

$$\frac{y(z)}{z} = \frac{4z(z-2)}{(z + \frac{1}{2})(z - \frac{1}{3})(z+1)} = \frac{A}{z + \frac{1}{2}} + \frac{B}{z - \frac{1}{3}} + \frac{C}{z+1}$$

$$Az^2 - \frac{1}{3} Az + Az - \frac{1}{3} A + Bz^2 + \frac{1}{2} Bz + Bz + \frac{1}{2} B + Cz^2 + \frac{1}{6} Cz - \frac{1}{6} C = 4z^2 - 8$$

$A + B + C = 4$ $\qquad A = -12$

$\frac{2}{3} A + \frac{3}{2} B + \frac{1}{6} C = -8$ $\qquad B = -2$

$-\frac{1}{3} A + \frac{1}{2} B - \frac{1}{6} C = 0$ $\qquad C = 18$

$$y(z) = \frac{-12z}{z + \frac{1}{2}} + \frac{-2z}{z - \frac{1}{3}} + \frac{18z}{z+1}$$

$$\boxed{y(n) = \left[-12\left(-\frac{1}{2}\right)^n - 2\left(\frac{1}{3}\right)^n + 18(-1)^n\right]\mu(n)}$$

4. $y''(t) + 9y'(t) + 8y(t) = u'(t) - 2u(t)$ 9

$y(0^-) = 1$
$y'(0^-) = 2$ $u(t) = e^t \mu(t)$

početni uvjeti

$y(0^+) - y(0^-) = 0 \cdot u(0^+) \rightarrow y(0^+) = 1$

$y'(0^+) - y'(0^-) + 9(y(0^+) - y(0^-)) = 0 + 1(u(0^+) - u(0^-))$

$y'(0^+) - 2 = 1$
$y'(0^+) = 3$

homogeno rješenje

$s^2 + 9s + 8 = 0$
$(s+8)(s+1) = 0$

$s_1 = -8 \quad s_2 = -1$

$y_h(t) = C_1 e^{-8t} + C_2 e^{-t}$

partikularno

$y_p(t) = K e^t$ $u(t) = e^t$
$y_p'(t) = K e^t$ $u'(t) = e^t$
$y_p''(t) = K e^t$

$Ke^t + 9Ke^t + 8Ke^t = e^t - 2e^t$

$18K = -1$

$K = -\frac{1}{18}$

$y_p(t) = -\frac{1}{18} e^t \mu(t)$

totalno rješenje

$y_t(t) = C_1 e^{-8t} + C_2 e^{-t} - \frac{1}{18} e^t$
$y_t'(t) = -8C_1 e^{-8t} - C_2 e^{-t} - \frac{1}{18} e^t$

$y_t(0^+) = C_1 + C_2 - \frac{1}{18} = 1$
$y_t'(0^+) = -8C_1 - C_2 - \frac{1}{18} = 3$

$-7C_1 - \frac{2}{18} = 4$

$-7C_1 = \frac{2}{18} + 4 = \frac{1}{9} + 4 = \frac{37}{9}$

$C_1 = -\frac{37}{63}$

$C_2 = 1 + \frac{1}{18} - C_1$
$= \frac{19}{18} + \frac{37}{63}$
$= \frac{133 + 74}{126} = \frac{207}{126} = \frac{69}{42}$
$= \frac{23}{14}$

$y_t(t) = \left[-\frac{37}{63} e^{-8t} + \frac{23}{14} e^{-t} - \frac{1}{18} e^t\right] \mu(t)$

5. a) $h(n) = \left(\frac{1}{2^n} + \frac{1}{4^n}\right)\mu(n)$ B

$$H(z) = \frac{z}{z-\frac{1}{2}} + \frac{z}{z-\frac{1}{4}} = \frac{z^2 - \frac{1}{4}z + z^2 - \frac{1}{2}z}{\left(z-\frac{1}{2}\right)\left(z-\frac{1}{4}\right)} = \frac{2z^2 - \frac{3}{4}z}{z^2 - \frac{3}{4}z + \frac{1}{8}}$$

b) $u(n) = 2^n$

$y(n) = H(z)\, U z^n$

$y(n) = H(2) \cdot 2^n$

$$H(2) = \frac{2\cdot 2^2 - \frac{3}{4}\cdot 2}{2^2 - \frac{3}{4}\cdot 2 + \frac{1}{8}} = \frac{\frac{32-6}{4}}{\frac{32-12+1}{8}} = \frac{\frac{26}{4}}{\frac{21}{8}} = \frac{52}{21}$$

$$\boxed{y(n) = \frac{52}{21}\cdot 2^n}$$

c) $u(n) = 2^n$

$$y(n) = u(n) * h(n)$$

$$= \sum_{m=-\infty}^{\infty} u(m)h(n-m)$$

$$= \sum_{m=-\infty}^{\infty} u(n-m)\, h(m)$$

$$= \sum_{m=-\infty}^{\infty} \left[\left(\frac{1}{2}\right)^m + \left(\frac{1}{4}\right)^m\right]\mu(m)\cdot 2^{n-m}$$

$$= \sum_{m=0}^{\infty} 2^{-m}\cdot 2^{n-m} + 2^{-2m}\cdot 2^{n-m}$$

$$= 2^n\left[\sum_{m=0}^{\infty}(2^{-2})^m + \sum_{m=0}^{\infty}(2^{-3})^m\right] =$$

$$= 2^n\cdot\left[\frac{1}{1-2^{-2}} + \frac{1}{1-2^{-3}}\right] = 2^n\cdot\left[\frac{1}{1-\frac{1}{4}} + \frac{1}{1-\frac{1}{8}}\right]$$

$$= 2^n\cdot\left[\frac{1}{\frac{3}{4}} + \frac{1}{\frac{7}{8}}\right] = 2^n\cdot\left[\frac{4}{3} + \frac{8}{7}\right] = 2^n\cdot\frac{28+24}{21}$$

$$\boxed{y(n) = \frac{52}{21}\cdot 2^n}$$