PRIPREMA ZA 3. LABOS IZ

SIGNALA I SUSTAVA

z 313 a) 1.) $y(n) - y(n-1) + 0{,}34\, y(n-2) = u(n)$ $\qquad u(n) = \delta(n)$

$$H(z) = \frac{1}{1 - z^{-1} + 0{,}34 z^{-2}} = \frac{z^2}{z^2 - z + 0{,}34}$$

$a^2 = 0{,}34$

$a = \sqrt{0{,}34}$

$\Omega_0 = 0{,}54$

$$= \frac{z(z - 0{,}5)}{z^2 - z + 0{,}34} + \frac{0{,}5 \cdot z}{z^2 - z + 0{,}34}$$

$$= \frac{z(z - \cos 0{,}54 \cdot \sqrt{0{,}34})}{z^2 - 2\sqrt{0{,}34}\cos 0{,}54 + 0{,}34} + 1{,}67 \cdot \frac{\sqrt{0{,}34} \cdot \sin 0{,}54}{z^2 - 2\sqrt{0{,}34}\cos 0{,}54 + 0{,}34}$$

$$h(n) = a^n \left[\cos \Omega_0 n + 1{,}67 \sin \Omega_0 n\right] \mu(n)$$

$$= \sqrt{0{,}34}^{\,n} \left[\cos 0{,}54 n + 1{,}67 \sin 0{,}54 n\right] \mu(n)$$

$u(n) = \mu(n)$

$$Y(z) = H(z) \cdot \frac{z}{z-1} = \frac{z^3}{(z^2 - z + 0{,}34)(z-1)}$$

$$\frac{Y(z)}{z} = \frac{Az + B}{z^2 - z + 0{,}34} + \frac{C}{z-1}$$

$Az^2 - Az + Bz - B + Cz^2 - Cz + 0{,}34C = z^2$

$A + C = 1 \qquad -B + 0{,}34C = 0$

$-A - C + B = 0 \qquad C - 1 - C + 0{,}34C = 0$

$\underline{A = -1{,}94} \quad \underline{B = 1} \quad \underline{C = 2{,}94}$

$$Y(z) = -\frac{1{,}94 z^2 + z}{z^2 - z + 0{,}34} + 2{,}94 \frac{z}{z-1}$$

$$= -1{,}94 \frac{z(z - 0{,}5)}{z^2 - z + 0{,}34} + \frac{0{,}03 z}{z^2 - z + 0{,}34} + 2{,}94 \frac{z}{z-1}$$

$$y(n) = \left(\sqrt{0{,}34}^{\,n} \cdot \left[-1{,}94 \cos 0{,}54 n - 3{,}23 \sin 0{,}54\right] + 2{,}94\right) \mu(n)$$

z313 a) 2.) $y(n) - \sqrt{2}\, y(n-1) + y(n-2) = u(n)$

$u(n) = \delta(n)$

$$H(z) = \frac{z^2}{z^2 - \sqrt{2} z + 1} \qquad \left| \begin{array}{l} a = 1 \\ \Omega_0 = \frac{\pi}{4} \end{array} \right|$$

$$= \frac{z\left(z - \cos\frac{\pi}{4}\right)}{z^2 - \sqrt{2} z + 1} + \frac{z \sin\frac{\pi}{4}}{z^2 - \sqrt{2} z + 1}$$

$$h(n) = \left(\cos\frac{\pi}{4} n + \sin\frac{\pi}{4} n\right) \mu(n)$$

$u(n) = \mu(n)$

$$Y(z) = \left(\frac{z^2}{z^2 - \sqrt{2} z + 1}\right)\left(\frac{z}{z-1}\right)$$

$$\frac{Y(z)}{z} = \frac{Az + B}{z^2 - \sqrt{2} z + 1} + \frac{C}{z-1}$$

$$Az^2 - Az + Bz - B + Cz^2 - \sqrt{2} Cz + C = z^2$$

$$A + C = 1 \quad -A + B - \sqrt{2} C = 0 \quad -B + C = 0$$

$$C - 1 + C - \sqrt{2} C = 0$$

$$A = 1 - C = -0{,}707 \qquad C = 1{,}707 \qquad B = 1{,}707$$

$$Y(z) = \frac{-0{,}707 z^2 + 1{,}707 z}{z^2 - \sqrt{2} z + 1} + 1{,}707 \frac{z}{z-1}$$

$$= -0{,}707 \frac{z(z - 0{,}707)}{z^2 - \sqrt{2} z + 1} + 1{,}707 \cdot \frac{0{,}707 z}{z^2 - \sqrt{2} z + 1} + 1{,}707 \frac{z}{z-1}$$

$$y(n) = \left(-0{,}707 \cos\frac{\pi}{4} n + 1{,}707 \sin\frac{\pi}{4} n + 1{,}707 \frac{z}{z-1}\right) \mu(n)$$

z 313 a) 3.) $y(n) - 2y(n-1) + y(n-2) = u(n)$

$u(n) = \delta(n)$

$$H(z) = \frac{z^2}{z^2 - 2z + 1} = 1 + \frac{2z - 1}{z^2 - 2z + 1}$$

$$\frac{H(z)}{z} = \frac{A}{z-1} + \frac{B}{(z-1)^2} = \frac{z}{(z-1)^2}$$

$Az - A + B = z$

$A = 1 \quad B = 1$

$$H(z) = \frac{z}{z-1} + \frac{z}{(z-1)^2}$$

$$h(n) = (n+1)\,\mu(n)$$

$u(n) = \mu(n)$

$$Y(z) = H(z) \cdot \frac{z}{z-1} = \frac{z^3}{(z-1)^3}$$

$$\frac{Y(z)}{z} = \frac{A}{z-1} + \frac{B}{(z-1)^2} + \frac{C(z+1)}{(z-1)^3}$$

$Az^2 - 2Az + A + Bz - B + Cz + C = z^2$

$\boxed{A = 1}$

$-2 + B + C = 0 \qquad 1 + C - B = 0$

$2C = 1$

$\boxed{C = \frac{1}{2}} \qquad \boxed{B = \frac{3}{2}}$

$$Y(z) = \frac{z}{z-1} + \frac{3}{2}\frac{z}{(z-1)^2} + \frac{1}{2}\frac{z(z+1)}{(z-1)^3}$$

$$y(n) = \left(1 + \frac{3}{2}n + \frac{1}{2}n^2\right)\mu(n)$$

z 313 a) 4)

$$y(n) - 4y(n-1) + 13\,y(n-2) = u(n) \qquad u(n) = \delta(n)$$

$$H(z) = \frac{z^2}{z^2-4z+13} = \left\{ \begin{array}{l} a^2 = 13 \quad a = \sqrt{13} \\ \Omega_0 = 0{,}98 \end{array} \right\}$$

$$= \frac{z\,(z - \overset{\cos\Omega_0}{0{,}555})}{z^2-4z+13} + 0{,}668 \cdot \frac{\overset{\sin\Omega_0}{0{,}83}\,z}{z^2-4z+13}$$

$$h(n) = (\sqrt{13})^n \left(\cos 0{,}98\,n + 0{,}668 \sin 0{,}98\,n\right) \mu(n)$$

$$u(n) = \mu(n)$$

$$Y(z) = \left(\frac{z}{z-1}\right) \cdot H(z) = \frac{z^3}{z^2-4z+13}$$

$$\frac{Y(z)}{z} = \frac{Az+B}{z^2-4z+13} + \frac{C}{z-1}$$

$$\left\{ \begin{array}{l} Az^2 - Az + Bz - B + Cz^2 - 4Cz + 13C = z^2 \\ A + C = 1 \\ -A + B - 4C = 0 \\ -B + 13C = 0 \end{array} \right. \qquad C - 1 + 9C = 0$$

$$C = \frac{1}{10} \quad A = \frac{9}{10} \quad B = \frac{13}{10}$$

$$Y(z) = \frac{\frac{9}{10}z^2 + \frac{13}{10}z}{z^2-4z+13} + \frac{1}{10}\,\frac{z}{z-1}$$

$$= \frac{9}{10}\,\frac{z(z-0{,}555)}{z^2-4z+13} + \frac{1{,}8}{0{,}83}\,\frac{0{,}83\,z}{z^2-4z+13} + \frac{1}{10}\,\frac{z}{z-1}$$

$$y(n) = (\sqrt{13})^n \left(0{,}9\cos 0{,}98\,n + 2{,}17 \sin 0{,}98\,n + 0{,}1\right) \mu(n)$$

Z 314) a)

pobuda :

$u(n) = A\cos(\Omega_0 n)$

$u(n) = A\sin(\Omega_0 n)$

part. rješenje :

$y_p = C_1 \cos \Omega_0 n + C_2 \sin \Omega_0 n$

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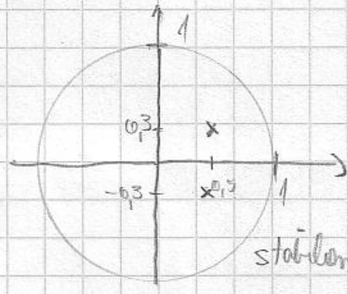
- kada se frekvencija pobude (Ω_0) poklapi sa vlastitom freq. sustava, part. rješenje ima oblik $y_{p_1}(n) = n \cdot y_p$, tj. sustav se linearno raspiruje → nestabilan je
- pojava se zove REZONANCIJA

Z 315 a)

1. $y(n) - y(n-1) + 0{,}34\, y(n-2) = u(n)$

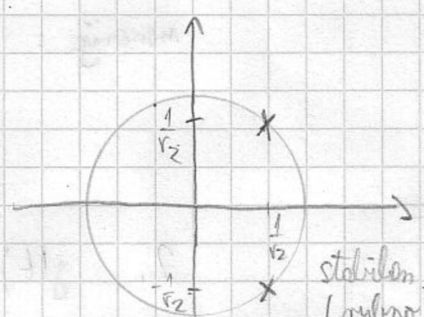
$$H(z) = \frac{z^2}{\left(z-\left(\frac{1}{2}-0{,}3i\right)\right)\left(z-\left(\frac{1}{2}+0{,}3i\right)\right)}$$



stabilan

2. $y(n) - \sqrt{2}\, y(n-1) + y(n-2) = u(n)$

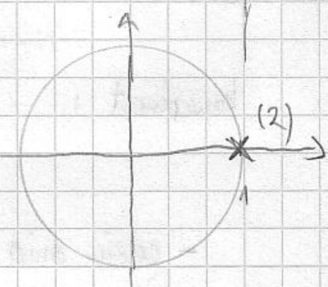
$$H(z) = \frac{z^2}{\left(z-\left(\frac{1}{\sqrt2}-\frac{1}{\sqrt2}i\right)\right)\left(z-\left(\frac{1}{\sqrt2}+\frac{1}{\sqrt2}i\right)\right)}$$



stabilan (rubno)

3. $y(n) - 2y(n-1) + y(n-2) = u(n)$

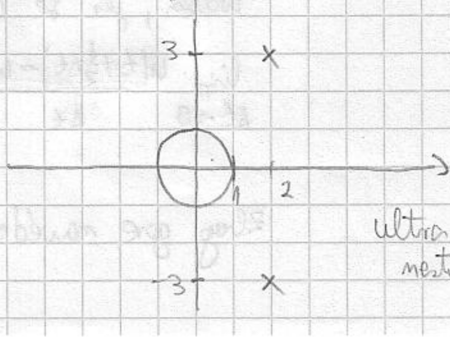
$$H(z) = \frac{z^2}{(z-1)^2}$$



nestabilan

4. $y(n) - 4\cdot y(n-1) + 13\, y(n-2) = u(n)$

$$H(z) = \frac{z^2}{(z-(2-3i))(z+(2+3i))}$$



ultra nestabilan

Z 321 a) 1. $y(t) = \int_{-\infty}^{t} u(\tau)\, d\tau$

kauzalnost : $y(t) = S\left(u_{(-\infty, t]}\right)(t)$ ulaza

- ovisi samo o prošlosti signala, pa je kauzalan !

linearnost : $u(t) = \alpha u_1(t) + \beta u_2(t)$

$y_1(t) = S(u_1(t))$, $y_2(t) = S(u_2(t))$

$$y(t) = \int_{-\infty}^{t} \alpha u_1(t) + \beta u_2(t)\, dt$$

$$= \alpha \underbrace{\int_{-\infty}^{t} u_1(t)\, dt}_{y_1} + \beta \underbrace{\int_{-\infty}^{t} u_2(t)\, dt}_{y_2} = \alpha y_1(t) + \beta y_2(t)$$

LINEARAN JE

memorija : odziv ovisi o cijeloj prošlosti pobude, pa je sustav memorijski

2. $y(t) = \frac{d}{dt} u(t)$

kauzalnost : $y(t) = \frac{d}{dt} u(t) = \lim_{\Delta t \to 0} \frac{u(t+\Delta t) - u(t)}{\Delta t}$

- odziv ovisi o bliskoj budućnosti (točnije i o bliskoj prošlosti ulaza, jer se derivacija može i ovako definirati : $\lim_{\Delta t \to 0} \frac{u(t + \frac{1}{2}\Delta t) - u(t - \frac{1}{2}\Delta t)}{\Delta t}$) pa je sustav NEKAUZALAN.

zbog gore navedenoga, sustav je i MEMORIJSKI linearnost :

$$u(t) = \alpha u_1(t) + \beta u_2(t)$$

$$y_1(t) = \frac{d}{dt} u_1(t) \quad , \quad y_2(t) = \frac{d}{dt} u_2(t)$$

$$y(t) = \frac{d}{dt}\left(\alpha u_1(t) + \beta u_2(t)\right) = \alpha \underbrace{\frac{d}{dt} u_1(t)}_{y_1(t)} + \beta \underbrace{\frac{d}{dt} u_2(t)}_{y_2(t)}$$

$$= \alpha y_1(t) + \beta y_2(t)$$

LINEARAN JE

Z 322 a) 1. $y''(t) + 2y'(t) + 15 y(t) = u(t)$ $\qquad u(t) = \delta(t)$

$$H(s) = \frac{1}{s^2 + 2s + 15} = \frac{1}{(s+1)^2 + 14}$$

$$= \frac{1}{\sqrt{14}} \frac{\sqrt{14}}{(s+1)^2 + (\sqrt{14})^2}$$

$$h(t) = \mathcal{L}^{-1}(H(s)) = \left(\frac{1}{\sqrt{14}} \cdot e^{-t} \sin \sqrt{14} t\right) \mu(t)$$

2. $y''(t) + 17 y(t) = u(t)$

$$H(s) = \frac{1}{s^2 + 17} = \frac{1}{\sqrt{17}} \cdot \frac{\sqrt{17}}{s^2 + (\sqrt{17})^2}$$

$$h(t) = \mathcal{L}^{-1}(H(s)) = \frac{1}{\sqrt{17}} \sin(\sqrt{17} t) \, \mu(t)$$

3.) $y''(t) = u(t)$

$$H(s) = \frac{1}{s^2}$$

$$h(t) = \mathcal{L}^{-1}(H(s)) = t\,\mu(t)$$

4.) $y''(t) - 2y'(t) + 10y(t) = u(t)$

$$H(s) = \frac{1}{s^2 - 2s + 10} = \frac{1}{(s-1)^2 + 9}$$

$$= \frac{1}{3} \cdot \frac{3}{(s-1)^2 + 3^2}$$

$$h(t) = \mathcal{L}^{-1}(H(s)) = \frac{1}{3} e^{t} \sin 3t \; \mu(t)$$

z 322 b) k.j. ovih sustava su nazivnici prijenosnih fja $H(s)$

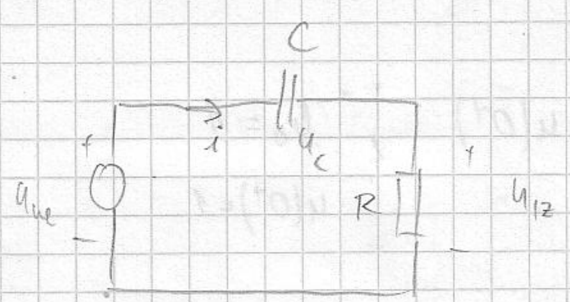
1. $s^2 + 2s + 15 = 0$

2. $s^2 + 17 = 0$

3. $s^2 = 0$

4. $s^2 - 2s + 10 = 0$

z 325 a)



$u_{ul} = u_C + u_{iz}$ $u_C = \frac{1}{C}\int i\,dt$

$i = \frac{u_{iz}}{R}$

$u_{iz} + \frac{1}{RC}\int u_{iz}\,dt = u_{ul} \Big/ \frac{d}{dt}$

$u_{iz}' + \frac{1}{RC}u_{iz} = u_{ul}'$ → da, jdba štima

$R = 1k\Omega \quad , \quad C = 10\,\mu F$

$y'(t) + 100\,y(t) = u'(t)$ $u(t) = \begin{cases} \sin(t), & -\infty < t < 0^- \\ 1, & 0^+ < t < +\infty \end{cases}$

ajmo prvo naći poč. uvjete!

Sustav je u stac. stanju od $-\infty$ do 0 zbog djelovanja harmonijske pobude. Početni uvjeti su se istitrali pa je potrebno naći samo part. rješ.

$y_p = A\sin t + B\cos t$

$A\cos t - B\sin t + 100A\sin t + 100B\cos t = \cos t$

$A + 100B = 1$ $-B + 100A = 0$

$10001\,A = 1$

$\boxed{A = \frac{1}{10\,001}}$ $\boxed{B = \frac{100}{10\,001}}$

$y_p = 0{,}0001\sin t + 0{,}01\cos t$

$t = 0^-$ $y(0^-) = 0{,}01$

$$y(0^+) - y(0^-) = b_0 \cdot u(0^+) \quad , \quad b_0 = 1$$
$$u(0^+) = 1$$

$$y(0^+) = 0{,}01 + 1$$

$$y(0^+) = 1{,}01$$

Z 327 a) $y'' + 2\xi \Omega_m y' + \Omega_m^2 y = A\Omega_m^2 u$

$\Omega_m = 0{,}4 \quad A = \frac{1}{\Omega_m^2}$

$$y''(t) + 0{,}8\,\xi\, y'(t) + 0{,}16\, y(t) = u(t)$$

$$H(s) = \frac{1}{s^2 + 0{,}8\xi s + 0{,}16}$$

$$H(j\omega) = \frac{1}{(0{,}16 - \omega^2) + 0{,}8\xi \cdot \omega \cdot j}$$

1.) $\xi = -0{,}125$

$$H(j\omega) = \frac{1}{0{,}16 - \omega^2 - 0{,}1\,\omega\, j}$$

$$|H(j\omega)| = \frac{1}{\sqrt{(0{,}16-\omega^2)^2 + (0{,}1\omega)^2}} \qquad \arg(H(j\omega)) = -\operatorname{arctg} \frac{-0{,}1\,\omega}{(0{,}16 - \omega^2)}$$

2.) $\xi = 0{,}25$

$$H(jw) = \frac{1}{0{,}16 - w^2 + 0{,}2jw}$$

$$|H(jw)| = \frac{1}{\sqrt{(0{,}16-w^2)^2 + (0{,}2w)^2}} \qquad \arg(H(jw)) = -\operatorname{arctg}\frac{0{,}2w}{0{,}16-w^2}$$

3) $\xi = 1$

$$H(jw) = \frac{1}{0{,}16 - w^2 + 0{,}8jw}$$

$$|H(jw)| = \frac{1}{\sqrt{(0{,}16-w^2)^2 + (0{,}8w)^2}} \qquad \arg(H(jw)) = -\operatorname{arctg}\frac{0{,}8w}{0{,}16-w^2}$$

z 327 c) $u(t) = \cos t \rightarrow w = 1$

1) $$y(t) = \frac{1}{\sqrt{(0{,}16-1)^2 + 0{,}1^2}} \cos\left(t + \operatorname{arctg}\frac{0{,}1}{0{,}16-1}\right)$$

$$= 1{,}114 \cos(t - 0{,}1185)$$

2) $$y(t) = \frac{1}{\sqrt{(0{,}16-1)^2 + 0{,}2^2}} \cos\left(t - \operatorname{arctg}\frac{0{,}2}{0{,}16-1}\right)$$

$$= 1{,}158 \cos(t + 0{,}2337)$$

3) $$y(t) = \frac{1}{\sqrt{(0{,}16-1)^2 + 0{,}8^2}} \cos\left(t - \operatorname{arctg}\frac{0{,}8}{0{,}16-1}\right)$$

$$= 0{,}862 \cos(t - 0{,}761)$$