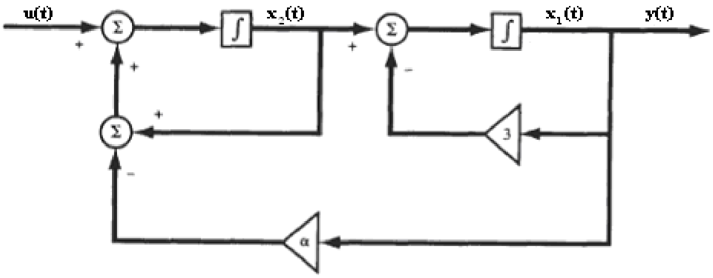
Signali i sustavi - Zadaci za vježbu

X. tjedan

VARIJABLE STANJA

1. Vremenski kontinuirani LTI sustav dan je Slikom 1. Nađite model s varijablama stanja $x_1(t)$ i $x_2(t)$ kako su odabrane na slici (matrice *A, B, C* i *D*).



Slika 1.

Rješenje:

$$x_2(t) = \int \left[x_2(t) + u(t) - \alpha x_1(t) \right] dt \quad / \frac{d}{dt}$$

$$x_1(t) = \int \left[x_2(t) - 3 x_1(t) \right] dt \quad / \frac{d}{dt}$$

$$\dot{x}_2(t) = x_2(t) + u(t) - \alpha x_1(t)$$

$$\dot{x}_1(t) = x_2(t) - 3 x_1(t)$$

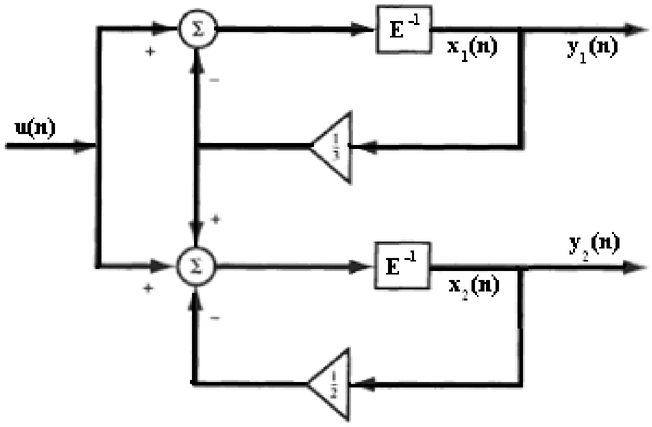
$$y(t) = x_1(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -\alpha & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} + D u(t)$$

$$\boxed{A = \begin{bmatrix} -3 & 1 \\ -\alpha & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 0}$$

2. Zadan je vremenski diskretan LTI sustav prema slici 2. Nađite model s varijablama stanja ovog sustava (matrice *A, B,C* i *D*). Ulaz u sustav je *u(n)*, stanja su $x_1(n)$ i $x_2(n)$, dok su izlazi $y_1(n)$ i $y_2(n)$.



Slika 2.

<u>Rješenje:</u>

Na ulazu elementa za kašnjenje: $x_1(n+1)$, odnosno $x_2(n+1)$.

Izrazimo te varijable pomoću vrijednosti koje ulaze u sumator:

$$x_1(n+1) = +u(n) - \frac{1}{3}x_1(n)$$

$$x_2(n+1) = +u(n) + \frac{1}{3}x_1(n) - \frac{1}{2}x_2(n)$$

Izlazi iz sustava:

$$y_1(n)=x_1(n)$$

$$y_2(n)=x_2(n)$$

Ukoliko ove jednadžbe zapišemo u matričnom obliku, dobivamo:

$$\begin{bmatrix} x_1(n+1) \\ x_2(n+1) \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & 0 \\ \frac{1}{3} & -\frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(n)$$

$$\begin{bmatrix} y_1(n) \\ y_2(n) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix} + 0 \cdot u(n)$$

Prema tome, matrice glase:

$$A = \begin{bmatrix} -\frac{1}{3} & 0 \\ \frac{1}{3} & -\frac{1}{2} \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 1 \end{bmatrix},\ C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ D = 0\,.$$

3. Audio oscilator je sustav koji proizvodi sinusoidalni signal dane frekvencije ω. Ovaj sustav je moguće prikazati pomoću modela s varijablama stanja:

$$A=\begin{bmatrix}\cos(\omega) & -\sin(\omega)\\ \sin(\omega) & \cos(\omega)\end{bmatrix},\ B=\begin{bmatrix}0\\1\end{bmatrix},\ C=\begin{bmatrix}1 & 0\end{bmatrix},\ D=0.$$

a. Matematičkom indukcijom dokažite: $A^n=\begin{bmatrix}\cos n\omega & -\sin n\omega\\ \sin n\omega & \cos n\omega\end{bmatrix}$.

b. Nađite odziv stanja nepobuđenog sustava, te odziv nepobuđenog sustava, ako je početno stanje $x(0)=\begin{bmatrix}0\\1\end{bmatrix}$.

c. Nađite impulsni odziv mirnog sustava.

Rješenje:

3. $A=\begin{bmatrix}\cos(w) & -\sin(w)\\ \sin(w) & \cos(w)\end{bmatrix}$ $B=\begin{bmatrix}0\\1\end{bmatrix}$ $C=[1\ 0]$ $D=0$

a) 1. $n=1$

$A=\begin{bmatrix}\cos(w) & -\sin(w)\\ \sin(w) & \cos(w)\end{bmatrix}$

2. $n=k$ $A=\begin{bmatrix}\cos(kw) & -\sin(kw)\\ \sin(kw) & \cos(kw)\end{bmatrix}$

3. $n=k+1$

$$A^n=A^k\cdot A=\begin{bmatrix}\cos(kw) & -\sin(kw)\\ \sin(kw) & \cos(kw)\end{bmatrix}\begin{bmatrix}\cos(w) & -\sin(w)\\ \sin(w) & \cos(w)\end{bmatrix}$$

$$=\begin{bmatrix}\cos(kw)\cos(w)-\sin(kw)\sin(w) & -\cos(kw)\sin(w)-\sin(kw)\cos(w)\\ \sin(kw)\cos(w)+\cos(kw)\sin(w) & -\sin(kw)\sin(w)+\cos(kw)\cos(w)\end{bmatrix}$$

$$=\frac{1}{2}\begin{bmatrix}\cos[(k+1)w]+\cancel{\cos[(k-1)w]}-\cancel{\cos[(k-1)w]}+\cos[(k+1)w] & -\sin[(k+1)w]+\cancel{\sin[(k-1)w]}-\sin[(k+1)w]-\cancel{\sin[(k-1)w]}\\ \sin[(k+1)w]-\cancel{\sin[(1-k)w]}+\sin[(k+1)w]-\cancel{\sin[(k-1)w]} & -\cancel{\cos[(k-1)w]}+\cos[(k+1)w]+\cos[(k+1)w]+\cancel{\cos[(k-1)w]}\end{bmatrix}$$

$$=\begin{bmatrix}\cos[(k+1)w] & -\sin[(k+1)w]\\ \sin[(k+1)w] & \cos[(k+1)w]\end{bmatrix}=\begin{bmatrix}\cos(nw) & -\sin(nw)\\ \sin(nw) & \cos(nw)\end{bmatrix}$$

b) odziv stanja nepobuđenog sustava:

$x(n)=A^n x(0)$, $n>0$

$x(n)=\begin{bmatrix}\cos(nw) & -\sin(nw)\\ \sin(nw) & \cos(nw)\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}$

$x(n)=\begin{bmatrix}-\sin(nw)\\ \cos(nw)\end{bmatrix}$

odziv nepobuđenog sustava:

$$y(n)=\begin{cases}Cx(0) & n=0\\ CA^n x(0) & n>0\end{cases}$$

$$y(n)=\begin{cases}[1\ 0]\begin{bmatrix}0\\1\end{bmatrix} & n=0\\ [1\ 0]\begin{bmatrix}\cos(nw) & -\sin(nw)\\ \sin(nw) & \cos(nw)\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} & n>0\end{cases}$$

$$y(n)=\begin{cases}0 & n=0\\ -\sin(nw) & n>0\end{cases}$$

c) impulsni odziv.

$$y(n)=h(n)=\begin{cases}0, & n<0\\ D, & n=0\\ CA^{n-1}B, & n>0\end{cases}=\begin{cases}0 & n<0\\ 0 & n=0\\ [1\ 0]\begin{bmatrix}\cos[(n-1)w] & -\sin[(n-1)w]\\ \sin[(n-1)w] & \cos[(n-1)w]\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} & n>0\end{cases}$$

$$y(n)=\begin{cases}0 & n<0\\ 0 & n=0\\ -\sin[(n-1)w] & n>0\end{cases}$$

4. Dana je matrica *A* vremenski diskretnog SISO LTI sustava $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, te vektor $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Pretpostavite da je $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Nađite ulaznu sekvencu *u(0), u(1)* takve da je stanje u drugom koraku $x(2) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

Rješenje:

4. $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad x(2) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$x(n+1) = Ax(n) + Bu(n)$$

$$x(2) = Ax(1) + Bu(1)$$

$$x(1) = Ax(0) + Bu(0)$$

$$x(2) = A(Ax(0) + Bu(0)) + Bu(1) =$$

$$= A^2 \cdot x(0) + ABu(0) + Bu(1)$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}u(0) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(1)$$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}u(0) + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u(1)$$

$$u(0) = 1 \qquad u(1) = 2$$

5. Dana je matrica *A* vremenski diskretnog SISO sustava $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$. Nađite A^n, te odziv stanja nepobuđenog sustava, ukoliko su početna stanja:

   a. $x(0)=\begin{bmatrix}1\\ 0\end{bmatrix}$

   b. $x(0)=\begin{bmatrix}0\\ 1\end{bmatrix}$

   c. $x(0)=\begin{bmatrix}1\\ 1\end{bmatrix}$

Rješenje:

Lako je uočiti pravilo:

$$A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix} \rightarrow A^2=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix} \rightarrow A^3=\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix} \rightarrow A^4=\begin{bmatrix}1 & 4\\ 0 & 1\end{bmatrix} \rightarrow \ldots \rightarrow A^n=\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}.$$

Dokaz da je ovo stvarno istina je jednostavan preko matematičke indukcije.

Baza: $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$

Korak: $A^k=\begin{bmatrix}1 & k\\ 0 & 1\end{bmatrix}$

Pretpostavka: $A^{n+1}=\begin{bmatrix}1 & n+1\\ 0 & 1\end{bmatrix}$

Izvod: $A^{n+1}=A\cdot A^n=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}\cdot\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}=\begin{bmatrix}1 & n+1\\ 0 & 1\end{bmatrix}$, čime je ovo dokazano.

Odziv stanja nepobuđenog sustava nalazi se iz $x(n)=A^n x(0),\quad n>0$, uz *u(n)=0.*

a. $x(n)=\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}\cdot\begin{bmatrix}1\\ 0\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}$

b. $x(n)=\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}\cdot\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}n\\ 1\end{bmatrix}$

c. $x(n)=\begin{bmatrix}1 & n\\ 0 & 1\end{bmatrix}\cdot\begin{bmatrix}1\\ 1\end{bmatrix}=\begin{bmatrix}n+1\\ 1\end{bmatrix}$

6. Zadan je LTI sustav opisan matricama $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \;\; 0]$ i $D = [0]$. Koliko iznosi odziv nepobuđenog sustava za $n \geq 0$ uz početne uvjete $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$?

Rješenje:

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \rightarrow A^2 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \rightarrow A^n = \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$$

$$y(n) = \begin{cases} Cx(0), & n = 0 \\ CA^n x(0), & n > 0 \end{cases}$$

$$y(n) = [1 \quad 0] \begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = [1 \quad n] \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1 + n$$

7. Zadan je LTI sustav opisan matricama $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = [1 \;\; 1]$ i $D = [1]$. Ukoliko su početni uvjeti $x(0) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ pronađite prve dvije vrijednosti *u(0)* i *u(1)* ulaznog signala tako da se sustav u koraku dva nađe u stanju $x(2) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$.

Rješenje:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow A^2 = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}; \; I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$x(2) = A^2 x(0) + ABu(0) + IBu(1)$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(0) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(1)$$

$$= \begin{bmatrix} 2x_1 + x_2 + u(0) \\ x_1 + x_2 + u(1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$u(0) = -2x_1 - x_2 \, ;$$
$$u(1) = -x_1 - x_2$$

KONVOLUCIJA

8. Odziv diskretnog LTI sustava na jediničnu stepenicu je $y(n)=(n+1)\mu(n)$. Odredite impulsni odziv ovog sustava. Kolika je vrijednost imulsnog odziva u $n=5$?

Rješenje:

$$y(n)=(n+1)\mu(n) \leftarrow \mu(n)$$
$$h(n) \leftarrow \delta(n)=\mu(n)-\mu(n-1)$$
$$h(n)=y(n)-y(n-1)$$
$$=(n+1)\mu(n)-n\mu(n-1)$$
$$h(5)=6\cdot\mu(5)-5\mu(4)=1$$

9. Zadan je vremenski diskretan LTI sustav impulsnim odzivom:

$$h(n)=\begin{cases}1, & n=0,1\\ 0, & \text{inace}\end{cases}$$

Nađite ulazno – izlaznu relaciju (jednadžbu diferencija) za ovaj sustav.

Rješenje:

$$h(n)=\begin{cases}1, & n=0,1\\ 0, & \text{INAČE}\end{cases}$$

$$y(n)=\sum_{m=-\infty}^{+\infty} h(n-m)\cdot u(m), \qquad h(n-m)=\begin{cases}1, & m=n,\ n-1\\ 0, & \text{INAČE}\end{cases}$$

$$y(n)=\sum_{m=n-1}^{n} h(n-m)\cdot u(m)$$

$$y(n)=h(1)\cdot u(n-1)+h(0)\cdot u(n)$$

$$y(n)=u(n-1)+u(n)$$

10. Nađite odziv diskretnog sustava na pobudu $u(n)=\alpha^n \mu(n)$, ako je poznat impulsni odziv sustava $h(n)=\beta^n \mu(n)$.

Rješenje:

6.

$$u(n)=\alpha^n \mu(n) \qquad h(n)=\beta^n \mu(n)$$

$$y(n)=\sum_{k=-\infty}^{\infty} h(k)u(n-k)$$

$$=\sum_{k=-\infty}^{\infty} \beta^k \mu(k)\cdot \alpha^{n-k}\mu(n-k)$$

$$\left(\mu(k)=\begin{cases}0, & k<0\\ 1, & k\geq 0\end{cases}\right)$$

$$=\sum_{k=0}^{\infty} \beta^k \alpha^{n-k}\mu(n-k) \quad \left[\mu(n-k)=\begin{cases}0 & k>n\\ 1 & k\leq n\end{cases}\right]$$

$$=\sum_{k=0}^{n} \beta^k \alpha^{n-k} = \sum_{k=0}^{n}\left(\frac{\beta}{\alpha}\right)^k \alpha^n = \alpha^n \sum_{k=0}^{n}\left(\frac{\beta}{\alpha}\right)^k$$

IMAMO GEOMETRIJSKI NIZ S $q=\frac{\beta}{\alpha}$

$$\Rightarrow \quad y(n)=\alpha^n \frac{1-\left(\frac{\beta}{\alpha}\right)^{n+1}}{1-\frac{\beta}{\alpha}}$$

Za $\alpha=\beta$ → $y(n)=\alpha^n(n+1)$

11. Dokažite svojstva konvolucije vremenski kontinuiranog sustava:
    a. $u(t)*\delta(t)=u(t)$
    b. $u(t)*\delta(t-t_0)=u(t-t_0)$
    c. $u(t)*\mu(t)=\int\limits_{-\infty}^{t}u(\tau)d\tau$
    d. $u(t)*\mu(t-t_0)=\int\limits_{-\infty}^{t-t_0}u(\tau)d\tau$

Rješenje:

a) $u(t)*\delta(t)=\int\limits_{-\infty}^{\infty}u(\tau)\delta(t-\tau)d\tau \quad , \ t\neq\tau \ , \ \delta(t-\tau)=0$

$$\int\limits_{-\infty}^{\infty}u(\tau)\delta(t-\tau)d\tau = u(t)\cdot\int\limits_{-\infty}^{\infty}\delta(t-\tau)d\tau = u(t)\cdot 1 = \underline{\underline{u(t)}}$$

b) $u(t)*\delta(t-t_0)=\int\limits_{-\infty}^{\infty}u(\tau)\delta(t-t_0-\tau)d\tau$

za $\tau\neq t-t_0 \ , \ \delta(t-t_0-\tau)=0$

$$\int\limits_{-\infty}^{\infty}u(\tau)\delta(t-t_0-\tau)d\tau = u(t-t_0)\int\limits_{-\infty}^{\infty}\delta(t-t_0-\tau)d\tau =$$

$$= u(t-t_0)\cdot 1 = \underline{\underline{u(t-t_0)}}$$

c) $u(t)*\mu(t)=\int\limits_{-\infty}^{\infty}u(\tau)\mu(t-\tau)d\tau = \left| \mu(t-\tau)=\begin{cases}0, & \tau>t\\ 1, & \tau\leq t\end{cases}\right|$

$$=\int\limits_{-\infty}^{t}u(\tau)\cdot 1\, d\tau + 0 = \underline{\underline{\int\limits_{-\infty}^{t}u(\tau)d\tau}}$$

d) $u(t)\cdot\mu(t-t_0)=\int\limits_{-\infty}^{\infty}u(\tau)\mu(t-t_0-\tau)d\tau = \left|\mu(t-t_0-\tau)=\begin{cases}0, & \tau>t-t_0\\ 1, & \tau\leq t-t_0\end{cases}\right|$

$$=\int\limits_{-\infty}^{t-t_0}u(\tau)\cdot 1\, d\tau + 0 = \underline{\underline{\int\limits_{-\infty}^{t-t_0}u(\tau)d\tau}}$$

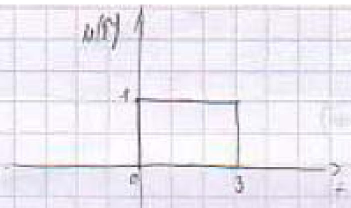
12. Nađite odziv kontinuiranog sustava na pobudu $u(t)=\begin{cases}1, & 0<t\le 3\\ 0, & \text{inace}\end{cases}$, ako je impulsni odziv

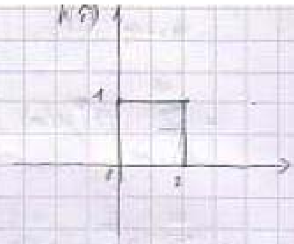
$$h(t)=\begin{cases}1, & 0<\mathrm{t}\le 2\\ 0, & \text{inace}\end{cases}.$$

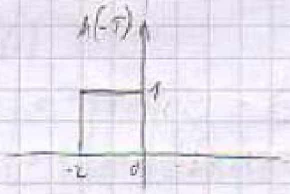
Rješenje:

8) $u(t)=\begin{cases}1 & , 0<t\le 3\\ 0 & , \text{inače}\end{cases}$

$h(t)=\begin{cases}1 & , 0<t\le 2\\ 0 & , \text{inače}\end{cases}$



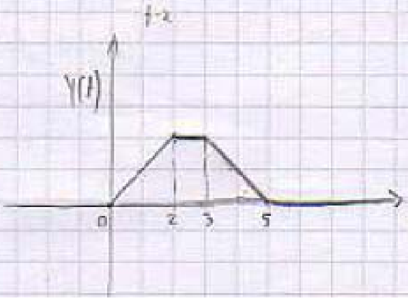




$u(\tau)$ i $h(t-\tau)$ se preklapaju u 3 intervala:

- $t\in[0,2]$ djelomično $y(t)=\int_0^t u(\tau)h(t-\tau)\,d\tau=\int_0^t d\tau=t$
- $t\in[2,3]$ potpuno $y(t)=\int_{t-2}^t u(\tau)h(t-\tau)\,d\tau=\int_{t-2}^t d\tau=2$
- $t\in[3,5]$ djelomično $y(t)=\int_{t-2}^3 u(\tau)h(t-\tau)\,d\tau=\int_{t-2}^3 d\tau=5-t$

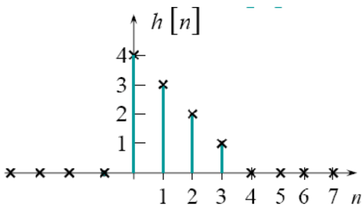
$$y(t)=\begin{cases}0 & , t\le 0\\ t & , 0\le t\le 2\\ 2 & , 2\le t\le 3\\ 5-t & , 3\le t\le 5\\ 0 & , t\ge 5\end{cases}$$



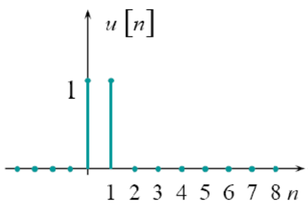
13. Korištenjem konvolucijske sumacije odredite odziv diskretnog sustava zadanog impulsnim odzivom $h(n) = 4\delta(n) + 3\delta(n-1) + 2\delta(n-2) + \delta(n-3)$. Sustav je pobuđen s $u(n) = \delta(n) + \delta(n-1)$.

Rješenje:

Zadani impulsni odziv i pobudu sustava možemo prikazati grafički:



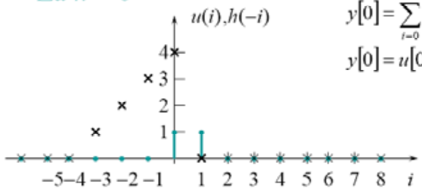
impulsni odziv



pobuda

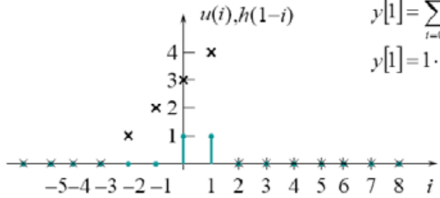
Traženu konvoluciju ćemo isto tako interpretirati grafički iz: $y(n) = \sum_{i=0}^{n} u(i)h(n-i)$.

Za $n = 0$



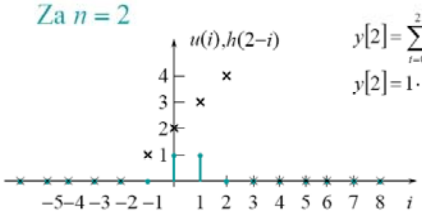
$$y[0] = \sum_{i=0}^{0} u(i)h(-i),$$
$$y[0] = u[0]h[0] = 1 \cdot 4 = 4.$$

Za $n = 1$



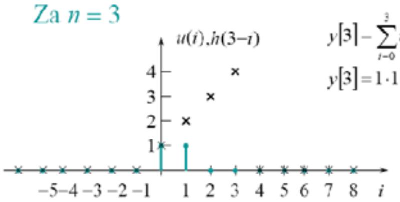
$$y[1] = \sum_{i=0}^{1} u(i)h(1-i),$$
$$y[1] = 1 \cdot 3 + 1 \cdot 4 = 7.$$

Za $n = 2$



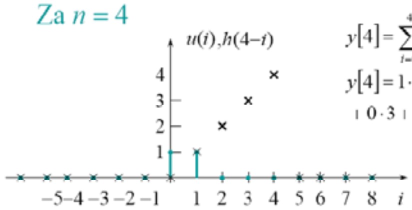
$$y[2] = \sum_{i=0}^{2} u(i)h(2-i),$$
$$y[2] = 1 \cdot 2 + 1 \cdot 3 + 0 \cdot 4 = 5.$$

Za $n = 3$



$$y[3] = \sum_{i=0}^{3} u(i)h(3-i),$$
$$y[3] = 1 \cdot 1 + 1 \cdot 2 + 0 \cdot 3 + 0 \cdot 4 = 3.$$

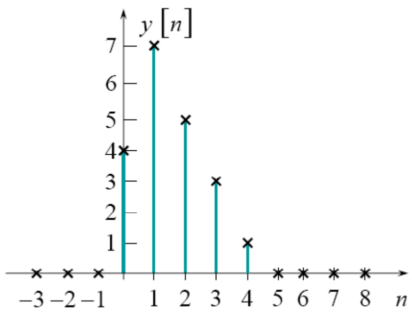
Za $n = 4$



$$y[4] = \sum_{i=0}^{4} u(i)h(4-i),$$
$$y[4] = 1 \cdot 0 + 1 \cdot 1 + 0 \cdot 2 + 0 \cdot 3 + 0 \cdot 4 = 1.$$

Dalje je *y(5)=y(6)=...=0*.

Pa je odziv:



14. Izračunajte izlaz *y(t)* za dani vremenski kontinuirani LTI sustav čiji su impulsni odziv *h(t)* i ulaz *u(t)* dani s

$$h(t) = e^{-a\,t}\mu(t)$$

$$u(t) = e^{a\,t}\mu(-t),\ a > 0\,.$$

<u>Rješenje:</u>

Kako bi se pronašao odziv sustava, mora se upotrijebiti konvolucija:

$$y(t) = u(t) * h(t) = \int_{-\infty}^{\infty} u(\tau)h(t-\tau)d\tau\,.$$

Zadani ulaz *u(τ)* dan je na slici, kao i *h(t- τ)* za dva slučaja *t<0* i *t>0*. Sa slika je vidljivo da se za *t<0*, *u(τ)* i *h(t- τ)* preklapaju u području $\tau = -\infty$ do $\tau = t$:

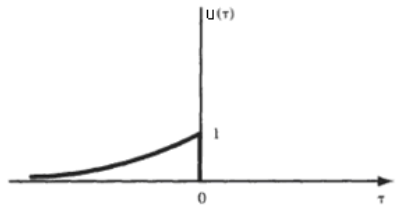
$$y(t) = \int_{-\infty}^{t} e^{a\tau}e^{-a(t-\tau)}d\tau = e^{-at}\int_{-\infty}^{t} e^{2a\tau}d\tau = \frac{1}{2a}e^{at}\,.$$

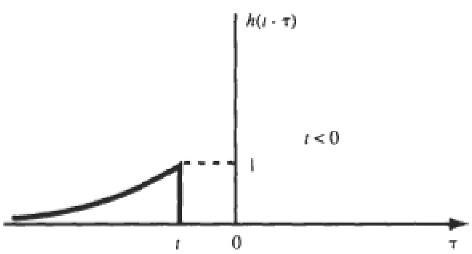
Za *t>0* slike se preklapaju u području $\tau = -\infty$ do $\tau = 0$:

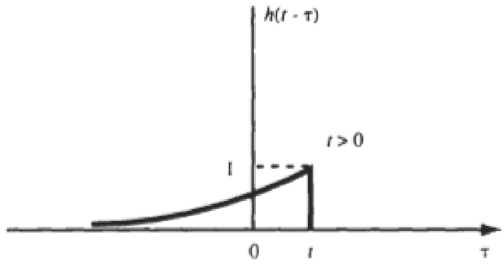
$$y(t) = \int_{-\infty}^{0} e^{a\tau}e^{-a(t-\tau)}d\tau = e^{-.at}\int_{-\infty}^{0} e^{2a\tau}d\tau = \frac{1}{2a}e^{-at}\,.$$

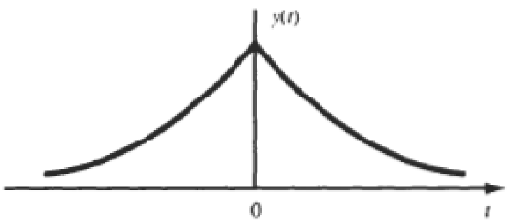
Kombinirajući ova dva odziva, ukupni odziv može biti zapisan:

$$y(t) = \frac{1}{2a}e^{-a|t|},\ \ a > 0\,.$$









15. Zadan je diskretni signal $f: Z \rightarrow R$ kao $f(n) = \begin{cases} 1, & n = 0,1,2 \\ 0, & \text{inače} \end{cases}$. Promatramo signal *q(n)* koji je definiran kao konvolucija *q(n)=f(n)*f(n)*. Koliko iznosi *q(3)*?

Rješenje:

$$f(n) = \begin{cases} 1, & n = 0,1,2 \\ 0, & \text{inače} \end{cases}$$

$$q(n) = f(n) * f(n) = \sum_{k=-\infty}^{\infty} f(k) f(n-k)$$

$$q(3) = \sum_{k=-\infty}^{\infty} f(k) f(3-k) = \ldots + f(0) f(3) + f(1) f(2) + f(2) f(1) + f(3) f(0) + f(4) f(-1) + \ldots$$

$$= 1 \cdot 0 + 1 \cdot 1 + 1 \cdot 1 + 0 \cdot 1 + 0 \cdot 0 = 2$$