JIC 2012

A - grupa

1. $x(t) = e^{-2t}\mu(t)$

a) $E = \int_{-\infty}^{+\infty} |x(t)|^2 dt = \int_0^{+\infty} e^{-4t} dt = \frac{e^{-4t}}{-4}\Big|_0^{+\infty} = \frac{1}{4}$

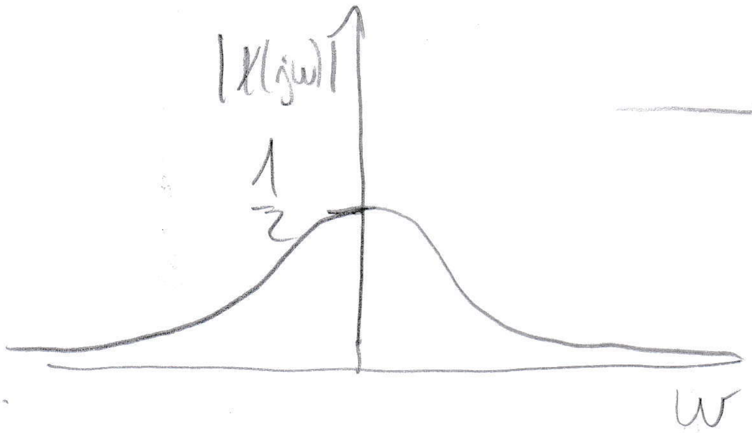
b) $X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt = \int e^{-2t} e^{-j\omega t} dt =$

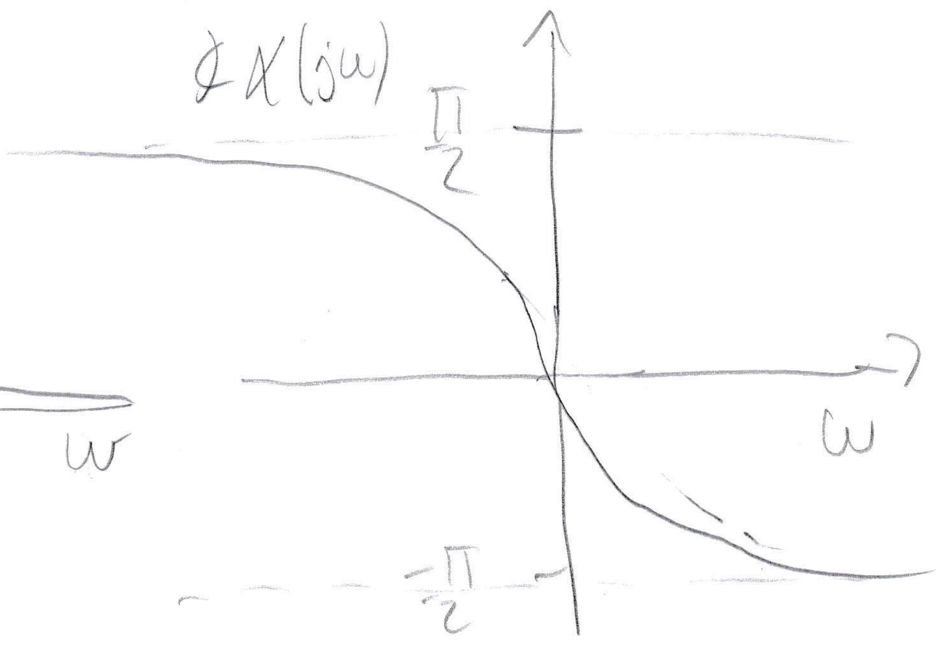
$= \int_0^{+\infty} e^{-t(2+j\omega)} dt = \frac{e^{-t(2+j\omega)}}{-(2+j\omega)}\Big|_0^{+\infty} = \frac{1}{2+j\omega}$

c) $|X(j\omega)| = \frac{|\text{Brojnik}|}{|\text{Nazivnik}|} = \frac{1}{\sqrt{4+\omega^2}}$

$\measuredangle X(j\omega) = \text{arctg}\frac{\text{Im}\{\text{Brojnik}\}}{\text{Re}\{\text{Brojnik}\}} - \text{arctg}\frac{\text{Im}\{\text{Nazivnik}\}}{\text{Re}\{\text{Nazivnik}\}}$

$= \text{arctg}\frac{0}{1} - \text{arctg}\frac{\omega}{2} = -\text{arctg}\frac{\omega}{2}$





2. $x(t) = e^{-2t}\mu(t)$

a) Aperiodičan $\rightarrow T = \infty \rightarrow \omega = \frac{2\pi}{T} = 0 = \omega_{MAX}$

možemo očitati s bilo kojom frekvencijom

jer je $\omega_S \geq 2\omega_{MAX} \geq 0$

b) $y(n) = x(nT_S) = e^{-2n\cdot 2}\mu(2n) = e^{-4n}\mu(2n)$

$T_S = 2$

c) $E = \sum_{-\infty}^{+\infty} |y(n)|^2 = \sum_{0}^{+\infty} e^{-8n} = \frac{1}{1+e^{-8n}}$

d) $X(e^{j\omega}) = \sum_{-\infty}^{+\infty} x(n)e^{-j\omega n} = \sum_{0}^{+\infty} e^{-4n}e^{-j\omega n} =$

$= \sum_{0}^{+\infty} e^{(-4-j\omega)n} = \frac{1}{1-e^{-4-j\omega}} = \frac{1}{1-e^{-4}e^{-j\omega}}$

$= \frac{1}{1-e^{-4}(\cos(\omega) - j\sin(\omega))}$

e) $\left|X(e^{j\omega})\right| = \frac{1}{\sqrt{(1-e^{-4}\cos(\omega))^2 + \sin^2(\omega)}}$

$\omega = 0 \quad |X(e^{j0})| = \frac{1}{1-e^{-4}}$

$\omega = \pi \quad |X(e^{j\pi})| = \frac{1}{1+e^{-4}}$

$\omega = -\pi \quad |X(e^{-j\pi})| = \frac{1}{1-e^{-4}}$

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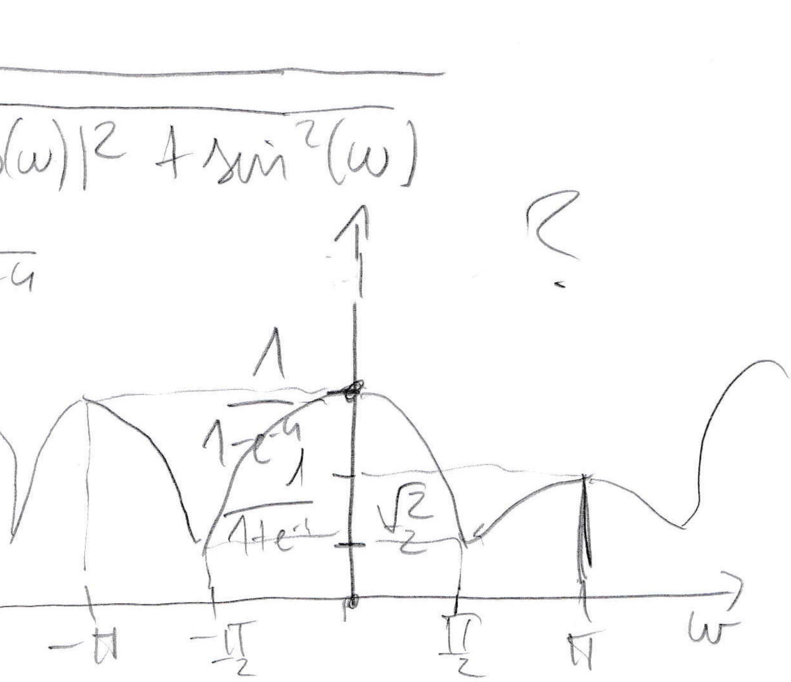


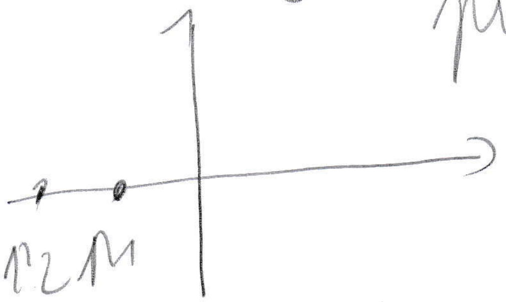
3. $y''(t)+3y'(t)+2y(t)=u'(t)+u(t)$

$y(0^-)=1$

$y'(0^-)=0$

a) $s^2+3s+2=0$ $s_{1,2}=\frac{-3\pm\sqrt{9-8}}{2}=\frac{-3\pm1}{2}=-1,-2$

Sustav je stabilan, svi polovi u lijevoj poluravnini tj. vrijedi



$Re\{p_i\}<0, \ \forall i$

b) $Y_{NEP} \Rightarrow u(t)=0$

$s^2Y(s)-sy(0)-y'(0)+3[sY(s)-y(0)]+2Y(s)=0$

$s^2Y(s)-s+3sY(s)-3+2Y(s)=0$

$Y_{NEP}(s)=\frac{s+3}{s^2+3s+2}=\frac{s+3}{(s+1)(s+2)}$

$=\frac{A}{s+1}+\frac{B}{s+2}=\frac{A(s+2)+B(s+1)}{(s+1)(s+2)}$

$\left.\begin{array}{l} A+B=1 \ |\cdot(-1) \\ 2A+B=3 \end{array}\right| +$ $A=2$, $B=-1$

$$Y(s)=\frac{2}{s+1}-\frac{1}{s+2} \rightarrow y_{NEP}(t)=(2e^{-t}-e^{-2t})\mu(t)$$

c) $u(t)=4\mu(t)$

I. NAČIN

$b_0=0 \quad a_1=1 \quad b_2=1$

$y(0^+)-y(0^-)=b_0u(0^+)=0 \Rightarrow y(0^+)=1$

$y'(0^+)-y(0^-)+\underbrace{a_1\Delta y}_{0}=\underbrace{b_0}_{0}u'(0^+)+\underbrace{b_1u(0^+)}_{4}$

$y'(0^+)=4$

$y_H=C_1e^{-t}+C_2e^{-2t}$

$y_P=K \Rightarrow \quad 0+0+2K=0+4 \Rightarrow K=2$

$y_{TOT}(t)=C_1e^{-t}+C_2e^{-2t}+2$

$y_{TOT}'(t)=-C_1e^{-t}-2C_2e^{-2t}$

$y_{TOT}(0^+)=1=C_1+C_2+2$

$y'_{TOT}(0^+)=4=-C_1-2C_2$

$5=-C_2+2$

$C_2=-3$

$C_1=2$

$y_{TOT}(t)=(2e^{-t}-3e^{-2t}+2)\mu(t)$

$y_{TOT}(t)=y_M(t)+y_{NEP}(t)$

$y_{NEP}(t)=y_{TOT}(t)-y_M(t)=$

$(2e^{-t}-3e^{-2t}+2-2e^{-t}+e^{-2t})=$

$=(-2e^{-2t}+2)\mu(t)$

II. NAČIN

$$y(0^-) = 0 \quad \Rightarrow \quad y(0^+) = 0$$
$$y'(0^-) = 0 \qquad\quad y'(0^+) = 4$$

$$y_M(t) = C_1 e^{-t} + C_2 e^{-2t} + 2$$
$$y_M'(t) = -C_1 e^{-t} - 2C_2 e^{-2t}$$

$$\left.\begin{aligned} y_M(0) &= 0 = C_1 + C_2 + 2 \\ y_M'(0) &= 4 = -C_1 - 2C_2 \end{aligned}\right| +$$

$$4 = -C_2 + 2$$
$$C_2 = -2$$
$$C_1 = 0$$

$$y_M(t) = (-2e^{-2t} + 2)\mu(t)$$

d) $y_{TOT} = y_M + y_{NEP}$
$$= (2e^{-t} - 3e^{-2t} + 2)\mu(t)$$

4. $y(n) - \frac{1}{2} y(n-1) = u(n) + u(n-1)$

a) $z - \frac{1}{2} = 0 \quad z = \frac{1}{2}$

Stabilan je, za sve polove je $|p| < 1$.

b) <u>U vremenskoj</u>

$h(n) = C z^n = C \cdot \left(\frac{1}{2}\right)^n$

$\Rightarrow$ sustav homogen za $n \geq 1$

$\Rightarrow$ postavljam $h(n<0) = 0$

$h(n) = \delta(n) + \delta(n-1) + \frac{1}{2} y(n-1)$

$h(0) = \underset{1}{\delta(0)} + \underset{0}{\delta(-1)} + \underset{0}{\frac{1}{2} y(-1)} = 1$

$h(1) = \underset{0}{\delta(1)} + \underset{1}{\delta(0)} + \frac{1}{2} \underset{1}{y(0)} = \frac{3}{2}$

$h(1) = \frac{3}{2} = C \cdot \left(\frac{1}{2}\right) \Rightarrow C = 3$

$h(n) = 3 \cdot \left(\frac{1}{2}\right)^n \cdot u(n-1) + \delta(n)$

Da bi zadovoljio $\leftarrow$ uvjet $h(0) = 1$

U Z domeni

$$Y(z) - \frac{1}{2} z^{-1} Y(z) = X(z) + z^{-1} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + z^{-1}}{1 - \frac{1}{2} z^{-1}} = \frac{\frac{z+1}{z}}{\frac{2z-1}{2z}} = \frac{2(z+1)}{2(z-\frac{1}{2})}$$

$$= \frac{z+1}{z-\frac{1}{2}} = \frac{z}{z-\frac{1}{2}} + 1 \cdot \frac{2 \cdot \frac{1}{2}}{z - \frac{1}{2}} =$$

$$= \left(\frac{1}{2}\right)^n + 2 \cdot \left[\left(\frac{1}{2}\right)^n - \delta(n)\right] =$$

$$= 3 \cdot \left(\frac{1}{2}\right)^n - 2\delta(n)$$

Dobije se različito, ali nađe je bitno samo da je $h(0) = 1$.

c) $u(n) = \cos\left(\frac{\pi}{2} n\right) \mu(n)$

$h(n) = 3\left(\frac{1}{2}\right)^n \mu(n-1) + \delta(n)$

$$y(n) = \sum_{m=-\infty}^{+\infty} u(m) h(n-m) =$$

$$= \sum_{m=-\infty}^{+\infty} \cos\left(\frac{\pi}{2} m\right) \mu(m) \cdot \left[3\left(\frac{1}{2}\right)^{n-m} \mu(n-m-1) + \delta(n-m)\right]$$

$$= \sum_{m=-\infty}^{+\infty} \cos\left(\frac{\pi}{2} m\right) \mu(m) \delta(n-m) + \sum_{m=-\infty}^{+\infty} \cos\left(\frac{\pi}{2} m\right) \mu(m) 3 \cdot \left(\frac{1}{2}\right)^{n-m} \mu(n-m-1)$$

$$= \sum_{0}^{+\infty} \cos\left(\frac{\pi}{2}m\right) \delta(n-m) + \sum_{0}^{n-1} \cos\left(\frac{\pi}{2}m\right) \cdot 3 \cdot \left(\frac{1}{2}\right)^{n-m}$$

$$= \cos\left(\frac{\pi}{2}n\right) \overbrace{\sum_{m=0}^{+\infty} \delta(n-m)}^{1} + 3 \cdot \left(\frac{1}{2}\right)^{n} \cdot \sum_{0}^{n-1} \cos\left(\frac{m\pi}{2}\right) \cdot \left(\frac{1}{2}\right)^{-m}$$

$$= \cos\left(\frac{\pi}{2}n\right) + 3 \cdot \left(\frac{1}{2}\right)^{n} \cdot \underbrace{\sum_{0}^{n-1} \left(\frac{e^{-j\frac{\pi}{2}m} + e^{j\frac{\pi}{2}m}}{2}\right) \cdot \left(\frac{1}{2}\right)^{-m}}_{S_1}$$

$$S_1 = \frac{1}{2}\left(\sum_{0}^{n-1} \left(e^{-j\frac{\pi}{2}} \cdot \frac{1}{2}\right)^{-m} + \sum_{0}^{n-1} \left(e^{j\frac{\pi}{2}} \cdot 2\right)^{m}\right) =$$

$$= \frac{1}{2}\left[\sum_{0}^{n-1} \left(\frac{2}{e^{j\frac{\pi}{2}}}\right)^{m} + \sum_{0}^{n-1} \left(2e^{j\frac{\pi}{2}}\right)^{m}\right] =$$

$$\left| e^{j\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) + j \sin\left(\frac{\pi}{2}\right) = j \right]$$

$$= \frac{1}{2}\left(\frac{\left(\frac{2}{j}\right)^{n} - 1}{\left(\frac{2}{j}\right) - 1} + \frac{(2j)^{n} - 1}{2j - 1}\right) =$$

$$= \ldots =$$

$$y(n) = \left[\frac{3}{5}\left(\frac{1}{2}\right)^{n} + \frac{2}{5}\cos\left(\frac{\pi}{2}n\right) + \frac{6}{5}\sin\left(\frac{\pi}{2}n\right)\right]\mu(n)$$

JA OVO NEMREM RAZRIJEŠIT

d) $u(n) = \cos(\frac{\pi}{2} n)$

ODZIV MIRNOG SUSTAVA

$$Y(z) - \frac{1}{2} z^{-1} Y(z) = U(z) + U(z) z^{-1}$$

$$Y(z)\left(\frac{2z-1}{2\cancel{z}}\right) = U(z)\left(\frac{z+1}{\cancel{z}}\right)$$

$$Y(z) = U(z)\left(2\frac{z+1}{2z-1}\right)$$

$$u(n) = \cos(\frac{\pi}{2} n) \circ\!\!-\!\!\bullet\ U(z) = \frac{z^2 - z\cos(\frac{\pi}{2})}{z^2 - 2z\cos(\frac{\pi}{2}) + 1}$$

(Tablice)

$$U(z) = \frac{z^2}{z^2+1}$$

$$Y(z) = \frac{\cancel{2}(z+1)}{\cancel{2}\cdot(z-\frac{1}{2})} \cdot \frac{z^2}{z^2+1} = \frac{z^2(z+1)}{(z-\frac{1}{2})(z^2+1)}$$

$$\frac{Y(z)}{z} = \frac{z^2+z}{(z-\frac{1}{2})(z^2+1)} = \frac{A}{z-\frac{1}{2}} + \frac{Bz+C}{z^2+1}$$

$$A(z^2+1) + (Bz+C)(z-\frac{1}{2}) = z^2+z$$

$$A(z^2+1) + B(z^2 - \frac{1}{2}z) + C(z-\frac{1}{2}) = z^2+z$$

$$\left.\begin{aligned} A+B&=1 \\ -\frac{1}{2}B+C&=1 \\ A-\frac{1}{2}C&=0 \end{aligned}\right\} \quad \begin{aligned} &A = \frac{1}{2}C \Rightarrow C = 2A \\ &A+B=1 \\ &-\frac{1}{2}B+2A=1 \,/\cdot 2 \end{aligned} \Big|+ \quad 5A=3$$

$$A = \frac{3}{5} \quad B = \frac{2}{5}$$

$$C = \frac{6}{5}$$

$$\frac{Y(z)}{z} = \frac{\frac{3}{5}}{z-\frac{1}{2}} + \frac{\frac{2}{5}z+\frac{6}{5}}{z^2+1}$$

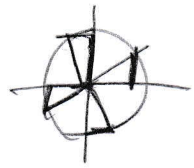
$$Y(z) = \frac{3}{5}\frac{z}{z-\frac{1}{2}} + \frac{\frac{2}{5}z^2+\frac{6}{5}z}{z^2+1} =$$

$$= \frac{3}{5}\frac{z}{z-\frac{1}{2}} + \frac{2}{5}\frac{z^2}{z^2+1} + \frac{6}{5}\frac{z}{z^2+1}$$

$$\Rightarrow y(n) = \left[\frac{3}{5}\left(\frac{1}{2}\right)^n + \frac{2}{5}\cos\left(\frac{\pi}{2}n\right) + \frac{6}{5}\sin\left(\frac{\pi}{2}n\right)\right] u(n)$$

5. $u(t) = \mu(t) \Rightarrow y_{p1}(t) = -2\mu(t)$

$u(t) = 2\sin(t) \Rightarrow y_{p2}(t) = 4\sin(t + \frac{\pi}{2})$



LTI $\rightarrow$ drugi red bez nula

$$y''(t) + a_1 y'(t) + a_2 y(t) = b_2 u(t)$$

$y_{p1} = -2\mu(t)$

$$0 + a_1 0 + a_2 \cdot (-2) = b_2 \cdot 1$$

$y_{p2} = 4\sin(t + \frac{\pi}{2}) = 4\cos(t)$

$$-4\cos(t) - a_1 4\sin(t) + 4a_2\cos(t) = b_2 \cdot 4 \cdot \cos(t)$$

$$\left.\begin{aligned} -2a_2 &= b_2 \\ -4a_1 &= 0 \\ 4a_2 - 4 &= 4b_2 \end{aligned}\right\} \quad \begin{aligned} &a_1 = 0 \\ &4a_2 - 4 = -8a_2 \\ &12a_2 = 4 \quad a_2 = \frac{1}{3} \end{aligned} \quad b_2 = -\frac{2}{3}$$

$$\Rightarrow y''(t) + \frac{1}{3} y(t) = \boxed{-\frac{2}{3} u(t)}$$ b) dio zadatka

$$s^2 Y(s) + \frac{1}{3} Y(s) = -\frac{2}{3} U(s)$$

$$Y(s)\left(s^2 + \frac{1}{3}\right) = -\frac{2}{3} U(s)$$

$$H(s) = \frac{Y(s)}{U(s)} = \boxed{\frac{-2/3}{s^2 + \frac{1}{3}}} \Rightarrow$$ a) dio zadatka

c) $H(s) = \dfrac{-\frac{2}{3}}{s^2+\frac{1}{3}} = -\dfrac{2}{3} \cdot \dfrac{\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{1}}{(s^2)+\left(\frac{1}{\sqrt{3}}\right)^2} =$

$= \dfrac{-2\sqrt{3}}{3} \dfrac{\omega}{s^2+\omega^2} \multimap h(t) = \left(\dfrac{-2\sqrt{3}}{3} \sin\left(\dfrac{1}{\sqrt{3}} t\right)\right) u(t)$

Ovaj zadatak se može rješavati i tako da se ovo sve umotane u amplitudni i fazni prijenosne funkcije, ali mislim da je ovako manje posla.