1. Računanje inverza matrice.

Općenito za kvadratne matrice 2x2 vrijedi:

$$\text{Za } A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \text{ inverz glasi: } \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} = \frac{1}{\det A}\begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}, \text{ uz } \det A = a_{11}a_{22} - a_{12}a_{21}.$$

Općenito za kvadratne matrice 3x3 vrijedi:

$$\text{Za A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \text{ inverz glasi: } \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}^{-1} = \frac{1}{\det A}\begin{pmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \\ -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} \\ \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}^{T},$$

$$\text{uz } \det A = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} =$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})$$

Sada se jasnije vidi što se misli kada se kaže:

Adjugirana matrica je transponirana matrica kofaktora.

Adjugirana matrica jest:

$$adj(A) = \begin{pmatrix} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix} \\ -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix} \\ \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} & -\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix} & \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \end{pmatrix}^{T}$$

Na ispitu će vam najvjerojatnije doći samo matrica 2x2. Primjer za matricu 3x3 je napravljen radi kompletnosti.

2. Izravno računanje $\phi(s)$ ili $\phi(z)$ iz matrice A:

$$\phi(s) = (sI - A)^{-1}$$
$$H(s) = C\phi(s)B + D$$

Za kontinuirane sustave vrijedi:

$\dot{x}(t) = Ax(t) + Bu(t),$

$y(t) = Cx(t) + Du(t).$

Za diskretne sustave vrijedi:

$x(n+1) = Ax(n) + Bu(n),$

$y(n) = Cx(n) + Du(n).$

Općenito za kvadratnu matricu A oblika $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ vrijedi:

$$\phi(s) = \frac{1}{\det(sI - A)} \begin{pmatrix} s - a_{22} & a_{12} \\ a_{21} & s - a_{11} \end{pmatrix},$$

$$\det(sI - A) = (s - a_{22})(s - a_{11}) - a_{12}a_{21} = s^2 - (a_{11} + a_{22})s + a_{11}a_{22} - a_{12}a_{21}.$$

Konačno dobijemo:

$$\phi(s) = \begin{pmatrix} \dfrac{s - a_{22}}{(s - a_{22})(s - a_{11}) - a_{12}a_{21}} & \dfrac{a_{12}}{(s - a_{22})(s - a_{11}) - a_{12}a_{21}} \\ \dfrac{a_{21}}{(s - a_{22})(s - a_{11}) - a_{12}a_{21}} & \dfrac{s - a_{11}}{(s - a_{22})(s - a_{11}) - a_{12}a_{21}} \end{pmatrix}$$

Za diskretne sustave, jedina promjena jest z umjesto s-a.

3. Množenje matrica:

Općenito vrijedi:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix}$$

Posebni slučaj (za brzo rješavanje):

Ako je neka od matrica jedinična (*I*), onda dobijemo slijedeće:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix},$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}.$$

Vidimo da dobijemo onu matricu koja nije jedinična.