$\boxed{1M1\ 2010}$

① $x(t) = e^{-2t}(\mu(t-1) - \mu(t-3))$ D

$$E = \int_1^3 e^{-4t}\, dt = \frac{1}{4} e^{-4t}\Big|_1^3 = \frac{1}{4}\left(e^{-12} - e^{-4}\right)$$

② $x(t) = 5e^{j\pi t}$

$$P_\infty = \lim_{L\to\infty} \frac{1}{L} \int_{-L/2}^{L/2} 25\, dt = 25$$

③ $x(n) = (3^{-n}\mu(n))$

$$E = \sum_{n=0}^{\infty} 9^{-n} = \frac{1}{1-\frac{1}{9}} = \frac{9}{8}$$

④ $x(n) = (6+2^{-n})\mu(n)$

$$P = \lim_{M\to\infty} \frac{1}{2M+1} \sum_{n=0}^{N} (36 + 12\cdot 2^{-n} + 2^{-2n}) = \frac{36}{2}$$

⑤ $x(t) = \sin\left(\frac{3}{4}\pi n^2\right) = \sin\left(\frac{3}{4}\pi (n+N)^2\right)$

$= \sin\left(\frac{3\pi}{4}\left(n^2 + 2nN + N^2\right)\right)$

$\frac{3\cancel{\pi}}{4}\left(2nN + N^2\right) = 2k\cancel{\pi}$

$k = \frac{3}{8} N (2n + N)$

za $N = 8$

$k = 3(2n+8)$ periodičan

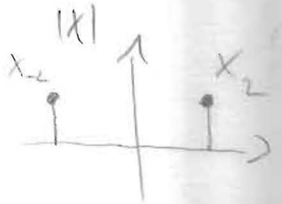
⑥ $x(t) = \cos\left(\frac{\pi}{2} t\right)\left(\mu(t-1) - \mu(1-t)\right)$

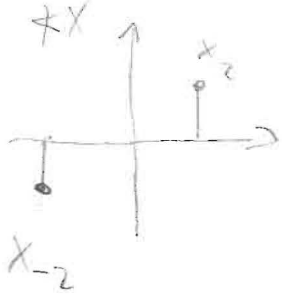
$x'(t) = -\frac{\pi}{2} \sin\left(\frac{\pi}{2} t\right)\left(\mu(t-1) - \mu(1-t)\right) -$

⑦ $\omega_0 = \frac{\pi}{2}$

$|X_2| = 2 \quad \measuredangle X_2 = \frac{\pi}{4}$

$|X_{-2}| = 2 \quad \measuredangle X_{-2} = -\frac{\pi}{4}$





$4\cos\left(\pi t + \frac{\pi}{4}\right) = 2e^{j2\omega_0 t} \cdot e^{j\frac{\pi}{4}} + 2e^{-j2\omega_0 t} \cdot e^{-j\frac{\pi}{4}}$

$X_2 = 2e^{j\frac{\pi}{4}} = 2\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right) = \sqrt{2} + j\sqrt{2} \Rightarrow |X_2| = 2, \measuredangle X_2 = \operatorname{arctg} 1 = \frac{\pi}{4}$

$X_{-2} = 2e^{-j\frac{\pi}{4}} = 2\left(\cos\frac{\pi}{4} - j\sin\frac{\pi}{4}\right) = \sqrt{2} - j\sqrt{2} \Rightarrow |X_{-2}| = 2, \measuredangle X_{-2} = -\frac{\pi}{4}$

⑧ CTFS

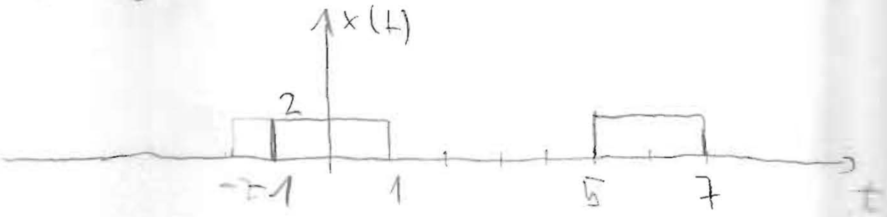
$E = \sum_{k=-\infty}^{\infty} |X_k|^2 = 8$

(9) $x(t) = 10\sin(7\pi t) + 4\cos(\pi t)$

$\omega_0 = \pi$

4 člana $\neq 0$

(10) $T_0 = 6$



CTFS

$\omega_0 = \frac{\pi}{3}$

$$X_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-j\omega_0 k t}\,dt$$

$$= \frac{1}{6}\int_{-1}^{1} 2\cdot e^{-\frac{\pi}{3}kt}\,dt$$

$$X(0) = \frac{2}{3}$$

$$X(3) = \frac{1}{3}\int_{-1}^{1} e^{-\pi t}\,dt = -\frac{1}{3\pi}e^{-\pi t}\Big|_{-1}^{1} = -\frac{1}{3\pi}\left(e^{-\pi} - e^{\pi}\right)$$

$$= \frac{1}{3\pi}\left(e^{\pi} - e^{\pi}\right) = 0$$

(11) CTFT

$\mathrm{CTFT}\{x(t)\} = e^{-3j\omega}\mu(\omega)$

$x(t) = e \quad \omega,$

$\mathrm{CTFT}(x(t-4)) = ?,$

$x(t-4) \circ\!\!-\!\!\bullet\ X(j\omega)e^{-j4\omega}$

$\mathrm{CTFT}(x(t-4)) = e^{-7j\omega}\mu(\omega)$

(12) CTFT

$$x(t) = \begin{cases} e^{-t}, & t \in \langle 0, 2\pi \rangle \\ 0, & \text{inače} \end{cases}$$

$$X(j\omega) = \int_0^{2\pi} e^{-t(1+j\omega)}\, dt = -\frac{1}{1+j\omega} e^{-t(1+j\omega)} \Big|_0^{2\pi} \ldots$$

$$= -\frac{1}{1+j\omega}\left[e^{-2\pi(1+j\omega)} - 1\right] = \frac{1}{1+j\omega}\left[1 - e^{-2\pi(1+j\omega)}\right]$$

(13) $x(t) = e^{-t(j+1)} \mu(t) \circ\!\!-\!\!\bullet \frac{1}{1+j+j\omega} = \frac{1}{1+j(\omega+1)}$

(14)

$$X(e^{j\Omega}) = \begin{cases} 3, & |\Omega| \leq a \\ 0, & a < |\Omega| < \pi \end{cases}$$

$$E(=)\frac{1}{2\pi} \int_{-a}^{a} 9\, d\Omega = \frac{9a}{\pi}$$

(15)

$$x(n) = \begin{cases} \sin\left(\frac{\pi}{4} n\right), & -5 < n < 5 \\ 0, & \text{inače} \end{cases}$$

$$x(n) = \sin\left(\frac{\pi}{4} n\right) = \frac{e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}}{2j}$$

$$\sum X_k = 0$$

(16) $X(e^{j\Omega}) = \cos(2\Omega) + \cos(5\Omega)$

$$= \frac{e^{j2\Omega}+e^{-j2\Omega}}{2} + \frac{e^{j5\Omega}+e^{-j5\Omega}}{2}$$

$N=1$

$x(2) = \frac{1}{2}$

$x(0), x(1), x(3), x(4) = 0$

DTFT

signal $x(n) \rightarrow$ diskretan, aperiodičan

spektar $X(e^{j\Omega}) \rightarrow$ kontinuiran, periodičan

(17) DTFT

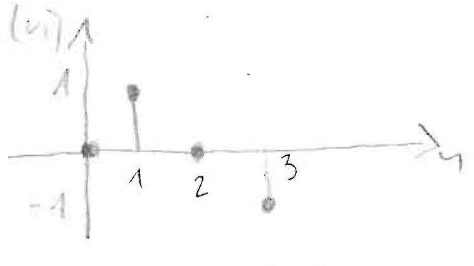
$$E = \sum_{k=-\infty}^{\infty} |x(n)|^2 = 1$$

(18) $X_k = \cos\left(\frac{\pi}{2}k\right)$ uz $N=4$

$$x(n) = \sum_{k=0}^{3} \cos\left(\frac{\pi}{2}k\right) e^{j2\pi kn/4}$$

$$= 1 - e^{j\pi n} = 1 - \cos\pi n - j\sin\pi n = 1-(-1)^n$$

(19)



$$X(k) = \frac{1}{N}\sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

$$= \frac{1}{4}\sum_{n=0}^{3} x(n) e^{-j\frac{\pi}{2}kn}$$

$$= \frac{1}{4}\left(e^{-j\frac{\pi}{2}k} - e^{-j\frac{3\pi}{2}k}\right) = \frac{1}{4}\left(-j\sin\frac{\pi}{2}k + j\sin\frac{3\pi}{2}k\right)$$

$$= \frac{1}{4}\left(-j\sin\frac{\pi}{2}k - j\sin\frac{\pi}{2}k\right) = -\frac{j}{2}\sin\frac{\pi}{2}k$$

(20) DTFT

(16) frekvencijska domena → aperiodičan, diskretan → CTFS

$$x(e^{j\Omega}) = \cos(2\Omega) + \cos(5\Omega) = \frac{e^{j2\Omega} + e^{-j2\Omega} + e^{j5\Omega} + e^{-j5\Omega}}{2}$$

I. način $x(n) = \frac{1}{4\pi}\int_{-\pi}^{\pi} x(e^{j\Omega}) \cdot e^{j\Omega n}\, d\Omega$

$$= \frac{1}{4\pi}\int_{-\pi}^{\pi} \left(e^{j\Omega(n+2)} + e^{j\Omega(n-2)} + e^{j\Omega(n+5)} + e^{j(n-5)}\right) d\Omega$$

$$\int_{-\pi}^{\pi} e^{jx}\, dx = \frac{1}{j} e^{jx}\Big|_{-\pi}^{\pi} = \frac{1}{j}\left(e^{j\pi} - e^{-j\pi}\right) = 0$$

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jedino za $n = 2, -2$ i $n = 5, -5$ imamo pod integralom $e^0 = 1$

$$x(2) = \frac{1}{4\pi}\int_{-\pi}^{\pi} 1 \cdot d\Omega = x(5) = \frac{1}{4\pi}\int_{-\pi}^{\pi} 1 \cdot d\Omega = \frac{1}{2}$$

II. način

DTFT → spektar kontinuiran i periodičan

$$x(e^{j\Omega}) = \frac{e^{j2\Omega} + e^{-j2\Omega} + e^{j5\Omega} + e^{-j5\Omega}}{2}$$

$N = 1$

$$x(e^{j\Omega}) = \frac{e^{j2N\Omega} + e^{-j2N\Omega} + e^{j5N\Omega} + e^{-j5\Omega}}{2}$$

$$x(-2) = x(2) = x(-5) = x(-5) = \frac{1}{2}$$