1

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$$y(n+1) + 2y(n) = u(n) \Leftrightarrow y(n) + 2y(n-1) = u(n-1) \quad (*)$$

$y(0) = 2$, $n \in \mathbb{N}$

Neka je: $u(n) = \alpha u_1(n) + \beta u_2(n)$

$$y_1(n+1) + 2y_1(n) = u_1(n)$$

$$y_2(n+1) + 2y_2(n) = u_2(n)$$

sustav je linearan ako vrijedi:

$$\forall n \quad y(n) = S(u)(n) = \alpha S(u_1)(n) + \beta S(u_2)(n) \quad (**)$$

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za $n = 1$ (stavljajući u $(*)$):

$$y(1) + 2y(0) = u(0)$$

$$y(1) = u(1-1) - 4 = \alpha u_1(0) + \beta u_2(0) - 4 \neq$$

$$\neq \alpha(u_1(0) - 4) + \beta(u_2(0) - 4) = \alpha y_1(1) + \beta y_2(1) \Rightarrow$$

$\Rightarrow$ ne vrijedi $(**)$ $\Rightarrow$ sustav nije linearan

2

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$$y(n+2) - y(n+1) - y(n) = 0 \quad , y(0)=0 \, , \, y(1)=1$$

$$y(n) = c\,q^n$$

$$c\,q^{n-2}\left(q^2 - q - 1\right) = 0 \quad \Rightarrow \quad q_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

$$y(n) = C_1 \left(\frac{1+\sqrt{5}}{2}\right)^n + C_2 \left(\frac{1-\sqrt{5}}{2}\right)^n$$

$$y(0) = C_1 + C_2 = 0 \quad \Rightarrow \quad C_1 = -C_2$$

$$y(1) = C_1 \frac{1+\sqrt{5}}{2} + C_2 \frac{1-\sqrt{5}}{2} = 1 \quad \Rightarrow$$

$$\Rightarrow \quad C_1 \frac{1+\sqrt{5}}{2} - C_1 \frac{1-\sqrt{5}}{2} = 1 \Rightarrow \sqrt{5}\,C_1 = 1 \Rightarrow C_1 = \frac{\sqrt{5}}{5} = \frac{1}{\sqrt{5}}$$

$$C_2 = -\frac{\sqrt{5}}{5} = \frac{1}{\sqrt{5}}$$

$$\boxed{y(n) = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]}$$

$y(0)=0$, $y(1)=1$, $y(2)=1$, $y(3)=2$, $y(4)=3$, $y(5)=5$

$y(6)=8$, $y(7)=$

→ Ovo je Fibonaccijev niz $F(n) = F(n-1) + F(n-2)$

uz uvjet da je $F(0)=0$ i $F(1)=1$

3

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$y(n+3) - y(n) = 0 \qquad y(0) = y(1) = 0, \; y(2) = 1$

$y(n) = c\, q^n$

$c\, q^{n-3}(q^3 - 1) = 0 \Rightarrow q^3 - 1 = 0$

$(q-1)(q^2+q+1) = 0 \Rightarrow q_1 = 1$

$q_{2,3} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm \sqrt{3}\, j}{2} \Rightarrow q_2 = e^{\frac{2\pi}{3} j}$

$q_3 = e^{-\frac{2\pi}{3} j}$ (*)

zbog (*) rjesenje oblika:

$y(n) = C_1 \cdot 1^n + C_2 1^n \cos\left(\frac{2\pi}{3} n\right) + C_3 \cdot 1^n \sin\left(\frac{2\pi}{3} n\right), \quad C_1, C_2, C_3 \in \mathbb{R}$

$y(0) = C_1 + C_2 = 0 \Rightarrow C_1 = -C_2$ (1)

$y(1) = C_1 + C_2 \cos\left(\frac{2\pi}{3}\right) + C_3 \sin\left(\frac{2\pi}{3}\right) = 0$ (2)

$y(2) = C_1 + C_2 \cos\left(\frac{4\pi}{3}\right) + C_3 \sin\left(\frac{4\pi}{3}\right) = 1$ (3)

$\left.\begin{array}{ll} (2)+(3): & 2C_1 - C_2 = 1 \\ (1): & C_1 = -C_2 \end{array}\right\} \Rightarrow -3C_2 = 1 \Rightarrow C_2 = -\frac{1}{3}$

$C_1 = \frac{1}{3}$

$(3)-(2) \quad -\sqrt{3}\, C_3 = 1 \Rightarrow C_3 = -\frac{1}{\sqrt{3}}$

$\boxed{y(n) = \frac{1}{3} - \frac{1}{3}\cos\left(\frac{2\pi}{3} n\right) - \frac{1}{\sqrt{3}} \sin\left(\frac{2\pi}{3} n\right)}$

3

nastavak:

Članovi niza su cijeli brojevi. Vrijedi da je $y(n+3) = y(n)$, i prva 3 člana su cijeli brojevi, pa se rekurzivnim postupkom zaključuje da su članovi niza cijeli brojevi.

4

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a) sustav opisan jednadžbom diferencija:

$$y(n) - 2y(n-1) - 3y(n-2) = u(n)$$

uz početne uvjete:

$$y(-1) = 3$$

$$y(-2) = 2$$

b)

$$y(n) = 3^n + 5^n + 7 = 3^n + 5^n + 7 \cdot 1^n$$

rješenja karakteristične jednadžbe: $q_1 = 1$ $q_2 = 3$ $q_3 = 5$

$$y(n) = C \cdot q^n$$

$$C \cdot q^{n-3} (q-1) \cdot (q-3) \cdot (q-5) = 0$$

$$C q^{n-3} (q^2 - 4q + 3)(q-5) = 0$$

$$\underbrace{C q^{n-3} (q^3 - 9q^2 + 23q - 15) = 0}$$

$\downarrow$

hom. jed.:

$$y(n) - 9y(n-1) + 23y(n-2) - 15y(n-3) = 0$$

4

nastavak:

→ početni uvjeti:

$$y(0) = 1 + 1 + 7 = 9$$

$$y(1) = 3 + 5 + 7 = 15$$

$$y(2) = 9 + 25 + 7 = 41$$

primjer sustava

$$y(n) - 9y(n-1) + 23y(n-2) - 15y(n-3) = u(n)$$

uz početne uvjete:

$$y(0) = 9$$

$$y(1) = 15$$

$$y(2) = 41$$

5. Naći odziv mirnog sustava opisanog jednadžbom diferencija:

$$3y(n+2)+6y(n+1)+3y(n)=2u(n+1)-5u(n).$$

Sustav je pobuđen nizom impulsa $u(n)=\{\ldots,\underline{0},0,1,2,1,0,0,\ldots\}$, gdje je podvučena vrijednost amplituda impulsa u koraku n=0.

$$3y(n+2)+6y(n+1)+3y(n)=2u(n+1)-5u(n)$$

$$u(n)=\{\ldots,\underline{0},0,1,2,1,0,0\ldots\}$$

$\to n=0$

$$y(n+2)=-2y(n+1)-y(n)+\frac{2}{3}u(n+1)-\frac{5}{3}u(n)$$

$y(n)=0$ za $n<0$ $\Rightarrow$ $y(0)=y(1)=\boxed{0}$

$n=0$ $\quad y(2)=-2y(1)-y(0)+\frac{2}{3}u(1)-\frac{5}{3}u(0)=\boxed{0}$

$n=1$ $\quad y(3)=-2y(2)-y(1)+\frac{2}{3}u(2)-\frac{5}{3}u(1)=\boxed{\frac{2}{3}}$

$n=2$ $\quad y(4)=-2y(3)-y(2)+\frac{2}{3}u(3)-\frac{5}{3}u(2)=$

$\quad =-2\cdot\frac{2}{3}-0+\frac{2}{3}\cdot 2-\frac{5}{3}=\boxed{-\frac{5}{3}}$

$n=3$ $\quad y(5)=-2y(4)-y(3)+\frac{2}{3}u(4)-\frac{5}{3}u(3)=$

$\quad =-2\left(-\frac{5}{3}\right)-\frac{2}{3}+\frac{2}{3}-\frac{5}{3}\cdot 2=\boxed{0}$

$n=4$ $\quad y(6)=-2y(5)-y(4)+\frac{2}{3}u(5)-\frac{5}{3}u(4)=0+\frac{5}{3}+0-\frac{5}{3}=\boxed{0}$

$n=5$ $\quad y(7)=-2y(6)-y(5)+\frac{2}{3}u(6)-\frac{5}{3}u(5)=0$

$y(n)=0, \quad n>5$

$$y_m(n)=\{\ldots\underline{0},0,0,\frac{2}{3},-\frac{5}{3},0,0,0,0\ldots\}$$

6. Na ulaz diskretnog sustava narinut je signal *u(n)*. Korištenjem konvolucijske sumacije naći impulsni odziv ako je poznat odziv mirnog sustava *y(n)*. Zadani su ulazni signal $u(n) = \{..., 0, \underline{1}, 2, 3, 4, 5, 6, 7, ...\}$ i izlazni signal $y(n) = \{..., 0, \underline{0}, -1, 1, 2, 3, 4, 5, 6, ...\}$, gdje je podvučena vrijednost amplituda impulsa u koraku *n=0*.

$$u(n) = (n+1)\mu(n)$$

$$y(n) = (n-1)\mu(n-2) - \delta(n-1)$$

$$y(n) = \sum_{k=-\infty}^{\infty} h(k)\, u(n-k)$$

$$(n-1)\mu(n-2) - \delta(n-1) = \sum_{k=0}^{\infty} h(k)(n+1-k)$$

$n=0$

$$\boxed{h(0) = 0}$$

$n=1$

$$-1 = h(0)\cdot 2 + h(1) \Rightarrow \boxed{h(1) = -1}$$

$n=2$

$$1 = h(0)\cdot 3 + h(1)\cdot 2 + h(2)\cdot 1$$

$$1 = 0 - 2 + h(2)$$

$$\boxed{h(2) = 3}$$

$n=3$

$$2 = h(0)\cdot 4 + h(1)\cdot 3 + h(2)\cdot 2 + h(3)\cdot 1$$

$$2 = 0 - 3 + 6 + h(3)$$

$$\boxed{h(3) = -1}$$

$\underline{n=4}$

$$3 = 5\cdot h(0) + h(1)\cdot 4 + h(2)\cdot 3 + h(3)\cdot 2 + h(4)\cdot 1$$

$$3 = 0 - 4 + 9 - 2 + h(4)$$

$$\boxed{h(4)=0}$$

$\underline{n=5}$

$$4 = 6h(0) + 5h(1) + 4h(2) + 3h(3) + 2h(4) + h(5)$$

$$4 = 0 - 5 + 12 - 3 + 0 + h(5)$$

$$\boxed{h(5)=0}$$

Slutnja ... $h(n)=0, n \geq 4$

Pretpostavimo da za $n > 3$ $h(n) = 0$

$$h(0)=0, \quad h(1)=-1, \quad h(2)=3, \quad h(3)=-1$$

$$h(n \geq 4) = 0$$

Tada za n-ti korak vrijedi:

$$n-1 = \sum_{k=0}^{n} h(k)(n+1-k)$$

$$= h(0)(n+1) + h(1)n + h(2)(n-2) + h(3)(n-2) + \ldots + h(n)$$

$$= 0 - n + 3(n-1) - 1(n-2) + 0 + 0 + 0 + \ldots + 0 + h(n)$$

$$= -n + 3n - 3 - n + 2 + h(n)$$

$$\cancel{n-1} = \cancel{n-1} + h(n)$$

$$0 = h(n) \Rightarrow \boxed{h(n)=0, \quad n>3}$$

$$\boxed{h(n) = \{\underline{0}, -1, 3, -1, 0, 0, \ldots\}}$$

7. Diskretan sustav je opisan jednadžbom diferencija

$$y(n) - 6y(n-1) + 8y(n-2) = 4u(n) .$$

Ako je ulaz u sustav $u(n) = 2\mu(n) - 3n\mu(n)$, nađite prirodni, prisilni te totalni odziv sustava uz početne uvjete $y(-1)=2$, $y(-2)=1$.

3.

$$y(n) - 6y(n-1) + 8y(n-2) = 4u(n)$$

$$u(n) = 2\mu(n) - 3n\mu(n)$$

$$y(-1) = 2$$

$$y(-2) = 1$$

$$y(n) - 6y(n-1) + 8y(n-2) = 8\mu(n) - 12n\mu(n) = (8-12n)\mu(n)$$

$$y_h(n) = c \cdot q^n$$

HOMOGENO

$$\cancel{c} \cdot \cancel{q^n} - 6 \cdot \cancel{c} \cdot \cancel{q^n} \cdot q^{-1} + 8 \cdot \cancel{c} \cdot \cancel{q^n} \cdot q^{-2} = 0$$

$$1 - 6q^{-1} + 8q^{-2} = 0 \quad / \cdot q^2$$

$$q^2 - 6q + 8 = 0$$

$$q_{12} = \frac{6 \pm \sqrt{4}}{2}$$

$$q_1 = 4 \qquad q_2 = 2$$

$$\boxed{y_h(n) = c_1 \cdot 4^n + c_2 \cdot 2^n}$$

PARTIKULARNO

$$y_p = (K_0 + K_1 \cdot n) \cdot 1^n \mu(n)$$

$$(K_0 + K_1 n) - 6 \cdot (K_0 + K_1 (n-1)) + 8 \cdot (K_0 + K_1 \cdot (n-2)) = (8 - 12n)$$

$$K_0 + K_1 n - 6K_0 - 6K_1 n + 6K_1 + 8K_0 + 8K_1 n - 16K_1 = 8 - 12n$$

$$(3K_0 - 10K_1) + 3K_1 n = 8 - 12n$$

$$3K_0 - 10K_1 = 8 \qquad 3K_1 = -12$$

$$3K_0 = 8 - 40 \qquad \boxed{K_1 = -4}$$

$$\boxed{K_0 = -\frac{32}{3}}$$

$$\boxed{y_p = \left(-\frac{32}{3} - 4n\right)}$$

$$y = y_h + y_p$$

$$\boxed{y(n) = c_1 \cdot 4^n + c_2 \cdot 2^n - \frac{32}{3} - 4n}$$

Računanje početnih uvjeta:

Zadana je jednadžba diferencija:

$$y(n) - 6y(n-1) + 8y(n-2) = 4(2\mu(n) - 3n\mu(n))$$

$$y(n) = 6y(n-1) - 8y(n-2) + 8\mu(n) - 12n\mu(n))$$

Kako jedinični step počinje djelovati u $n=0$, potrebno je kao početne uvjete računati $y(0)$ i $y(1)$:

$y(0)=8+12-8=12$

$y(1)=8-12+72-16=52$

Konstante C_1 i C_2 se sada računaju iz tih početnih uvjeta:

$$y(0) = C_1 + C_2 - \frac{32}{3} = 12$$

$$y(1) = 4C_1 + 2C_2 - \frac{32}{3} - 4 = 52$$

Rješavanjem ovih jednadžbi dobije se: $C_1 = \frac{32}{3}$, $C_2 = 12$.

Totalni odziv je prema tome:

$$y(n) = \left(\frac{32}{3}\cdot 4^n + 12\cdot 2^n - \frac{32}{3} - 4n\right)\mu(n)$$

TOTALNI ODZIV

$$y(n) = \left(-\frac{32}{3} - 4n\right)\mu(n) \rightarrow \text{PRISILNI ODZIV}$$

$$y(n) = \left(\frac{32}{3}\cdot 4^n + 12\cdot 2^n\right)\mu(n) \rightarrow \text{PRIRODNI ODZIV}$$