3.1-1a)

$x_1(n) = \{ \ldots 0, 0, 1, \underline{1}, 1, 0, 0 \ldots \}$

$N=3$

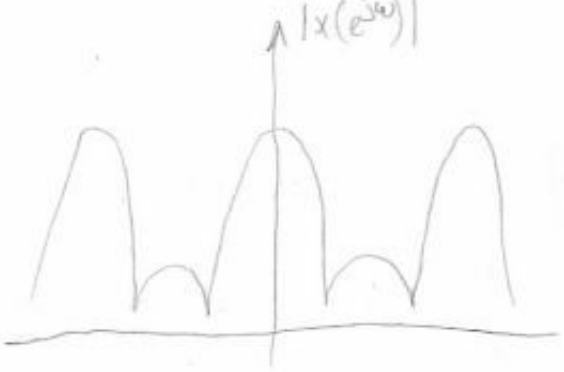
$$x(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) \cdot e^{-j\omega n}$$

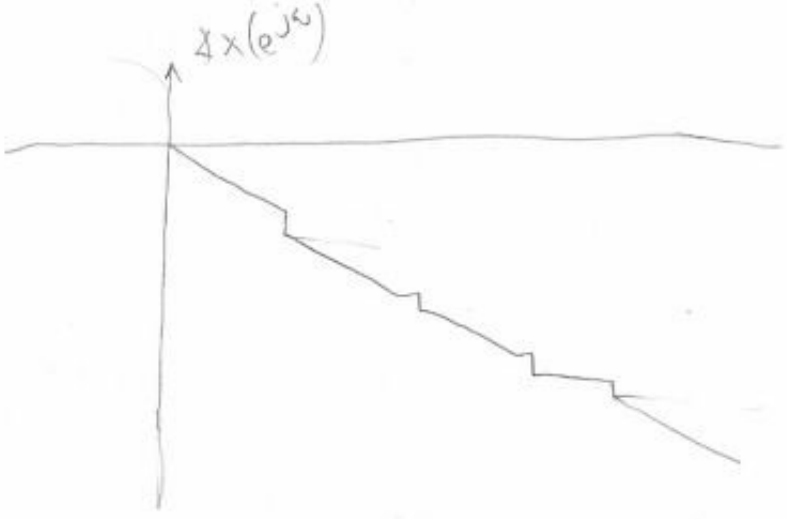
$$x(e^{j\omega}) = \sum_{n=-1}^{1} e^{-j\omega n} = e^{j\omega} + 1 + e^{-j\omega} = 1 + 2 \cdot \frac{1}{2}\left(e^{j\omega} + e^{-j\omega}\right)$$

$$\boxed{x(e^{j\omega}) = 1 + 2\cos(\omega)} \quad \Rightarrow \; Im = 0$$

$$Re = 1 + 2\cos(\omega)$$

$$\measuredangle x(e^{j\omega}) = \frac{Im}{Re} = 0$$





3.2-1 a) Spektar vremenski diskretnog signala dobivenog otipkavanjem s periodom T je kontinuirana funkcija periodična s periodom 2π. Pri tome vremenski kontinuiranoj sinusoidi kružne frekvencije Ω odgovara diskretna sinusoida frekvencije ω i vrijedi $\omega = \Omega T$. Općenito, veza između spektra kontinuiranog signala $x_c(j\Omega)$ i spektra diskretnog signala $x(e^{j\omega})$ dana je izrazom:

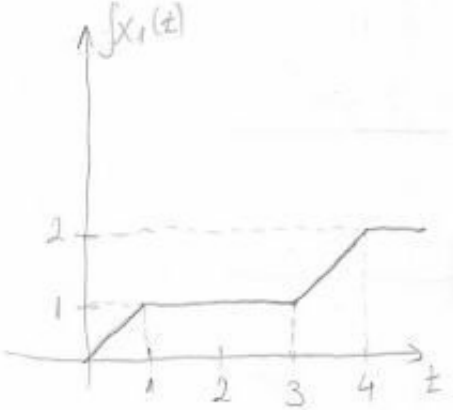
$$X(e^{j\omega}) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X_c\left(j\frac{\omega}{T} - j\frac{2\pi n}{T}\right)$$

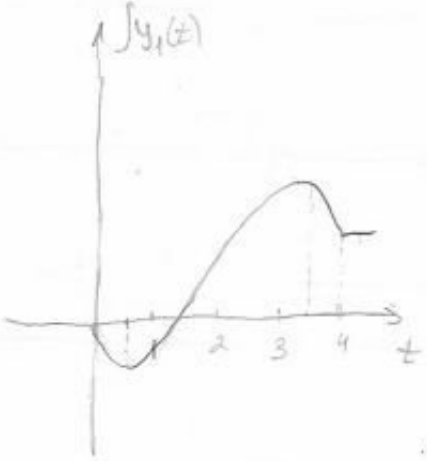
$f = 8\,kHz$

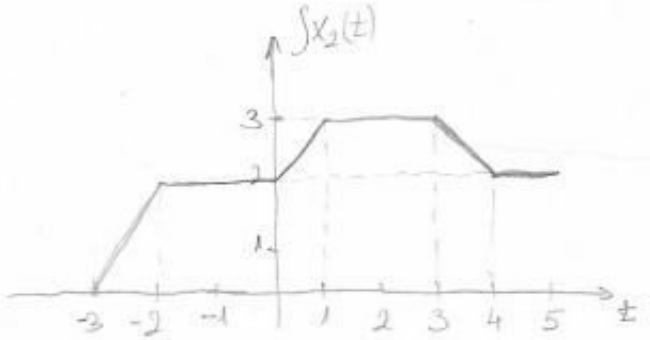
$f_s > 2 \cdot f \Rightarrow \boxed{f_s > 16\,kHz}$

$\omega = \frac{2\pi}{f} \Rightarrow \boxed{\omega_s > 2{,}5 \cdot 10^{-4}\pi}$

3.3-1a)







3.3.-3a) 1. linearnost

$$y(t) = v(t)$$

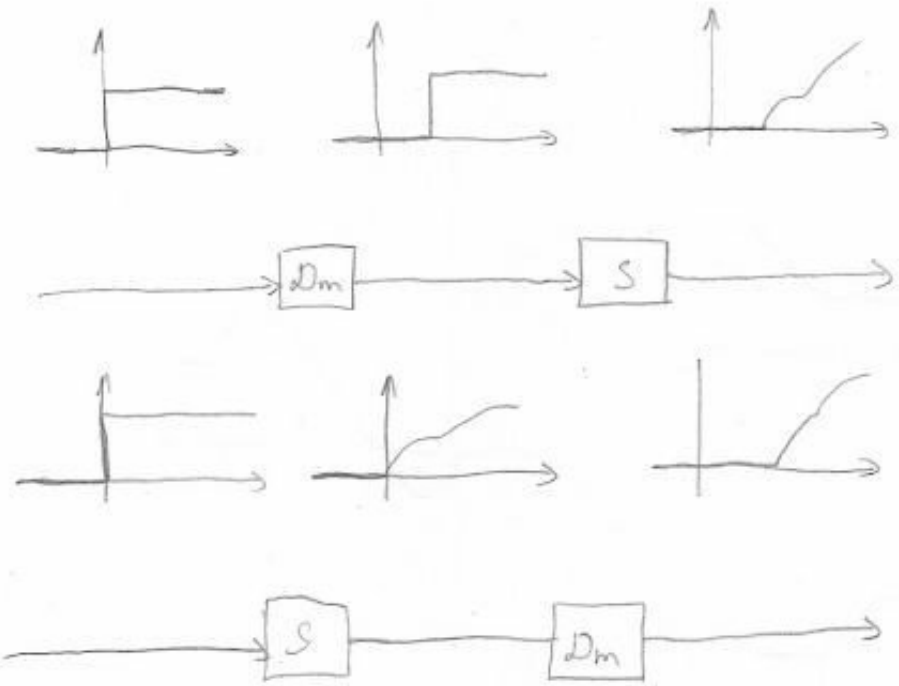
$$v(t) = \alpha u_1(t) + \beta u_2(t)$$

$$y_1(t) = v_1(t), \quad y_2(t) = v_2(t)$$

$$y(t) = \alpha v_1(t) + \beta v_2(t)$$

$$y(t) = \alpha y_1(t) + \beta y_2(t)$$

2. vremenska nepromjenjivost sustava



3. memorijski sustav

ako za računanje trenutnog odziva treba pozivati neku prošlu ili buduću vrijednost signala, $u(n \pm m)$, pri čemu je $M \in \mathbb{Z}$, $M \neq 0$, tada je sustav memorijski

3.3-3b) 1) $y(t) = \frac{1}{v(t)}$ → bezmemorijski

$y_1(t) = \frac{1}{v_1(t)}$, $y_2(t) = \frac{1}{v_2(t)}$

$v(t) = \alpha v_1(t) + \beta v_2(t)$

$y(t) \neq \frac{1}{\alpha v_1(t) + \beta v_2(t)}$ → nelinearan

$y_1(t) = \frac{1}{v(t-T)}$

$y(t-T) = \frac{1}{v(t-T)}$

$y_1(t) = y(t-T)$ → vremenski nepromjenjiv

2) $y(t) = t^2 v(t) + 2$ → bezmemorijski

$y_1(t) = t^2 v_1(t) + 2$, $y_2(t) = t^2 v_2(t) + 2$

$v(t) = \alpha v_1(t) + \beta v_2(t)$

$y(t) = t^2 \cdot [\alpha v_1(t) + \beta v_2(t)] + 2 = \alpha t^2 v_1(t) + \beta^2 v_2(t) + 2$

⇒ nelinearan

$y_1(t) = t^2 v(t-T) + 2$

$y(t-T) = (t-T)^2 \cdot v(t-T) + 2$

$y_1(t) \neq y(t-T)$ → vremenski promjenjiv

3) $y(t) = v(t-1)$ → memorijski

$y_1(t) = v_1(t-1), \quad y_2(t) = v_2(t-1)$

$v(t) = \alpha v_1(t-1) + \beta v_2(t-1)$

$y(t) = \alpha v_1(t-1) + \beta v_2(t-1) = \alpha y_1(t) + \beta y_2(t)$ → linearan

$y_1(t) = v(t-T-1)$ $\qquad y_1(t) = y(t-T)$ → vremenski nepromjenjiv

$y(t-T) = v(t-T-1)$

3.3-4 a) $x_1(t) = \mu(t) - \mu(t-1) + \mu(t-3) - \mu(t-4)$

$y_1(t) = (2t-1)(\mu(t) - \mu(t-2)) + (7-2t)\cdot(\mu(t-2) - \mu(t-4))$

$x_2(t) = 2\cdot[\mu(t+3) - \mu(t+2)] + \mu(t) - \mu(t-1) - [\mu(t-3) - \mu(t-4)]$

$x_2(t) = 2x_1(t+3) - x_1(t)$

(zbog linearnosti)

$y_2(t) = 2y_1(t+3) - y_1(t)$

$y_2(t) = 2\left[(2t-1+3)(\mu(t+3) - \mu(t+3-2)) + (7-2t+3)\cdot(\mu(t+3-2) - \mu(t+3-4))\right] -$

$-\left[(2t-1)(\mu(t) - \mu(t-2)) + (7-2t)(\mu(t-2) - \mu(t-4))\right]$

$y_2(t) = (4t+4)\cdot g(-3,-1) + (20-4t)\cdot g(-1,1) - (2t-1)g(0,2) - (7-2t)\cdot g(2,4)$

3.3-5 b) $x(n+1) = 1{,}07\,[x(n) + 1000]$

$x(n+1) = 1{,}07\,x(n) + 1070$

$x(0) = b$

$x^1(n) = a \cdot 1{,}07^n$

$b = 1{,}07b + 1070 \Rightarrow b = -15\,285{,}7$

$x(n) = a \cdot 1{,}07^n - 15\,285{,}7 \quad x(0) = 0$

$a = 14\,285{,}7$

$x(n) = 14\,285{,}7 \cdot 1{,}07^n - 15\,285{,}7$

$x(15) = 24\,129$ eurá