MASS 15.06.

17 6 2008

$\mathcal{L}$ – TRANSFORMACIJA

$$H(s) = \int_{-\infty}^{\infty} h(t)\, e^{-st}\, dt \qquad H(j\Omega) = \int_{-\infty}^{\infty} h(t)\, e^{-j\Omega t}\, dt$$

$$\boxed{X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt}$$

$$\boxed{e^{-at}\mu(t) \circ\!\!-\!\!\circ \frac{1}{s+a}}$$

PR. 1.

$$x(t) = e^{-at}\mu(t)$$

$$X(s) = \int_0^{\infty} e^{-at} e^{-st}\, dt = \int_0^{\infty} e^{-t(s-a)}\, dt = \frac{(-1)}{s+a} e^{-t(s+a)} \Big|_0^{\infty} = \frac{(-1)}{s+a}\left(e^{-\infty(s+a)} - e^{0}\right) =$$

$$X(s) = \frac{1}{s+a}$$

PR. 2.

$$x(t) = -e^{-at}\mu(t)$$

$$X(s) = \int_{-\infty}^{0} -e^{-at} e^{-st}\, dt = -\int_{-\infty}^{0} e^{t(-a-s)}\, dt = \frac{-1}{-s-a} e^{t(-a-s)} \Big|_{-\infty}^{0} = \frac{1}{s+a}\left(e^{0} - e^{-\infty(s-a)}\right) =$$

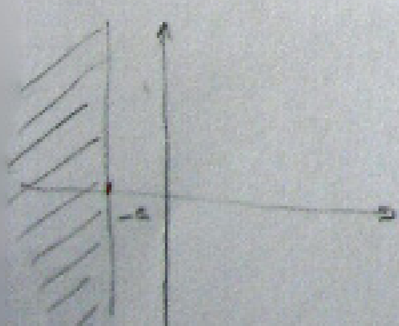
$$X(s) = \frac{1}{s+a}$$

$$\boxed{-e^{-at}\mu(-t) \circ\!\!-\!\!\circ +\frac{1}{s+a}}$$

1) $-e^{-at}\mu(-t)$

$-s-a > 0$

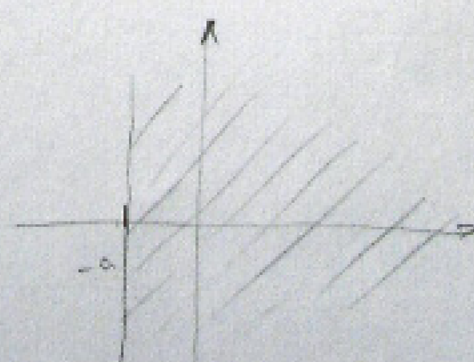
$s < -a$



2) $e^{-at}\mu(t)$

$s+a > 0$

$s > -a$



$$X(s) = \int_0^{\infty} x(t)\, e^{-st}\, dt$$

$$x(t) = w(t) \mp y(t)$$

$$X(s) = W(s) \pm Y(s)$$

$$x(t) \circ\!\!-\!\!\circ X(s)$$

$$x(t) = \delta(t)$$

$$X(s) = \int_0^{\infty} \delta(t)\, e^{-st}\, dt = 1 \quad t_0 = 0$$

$$\boxed{\delta(t) \circ\!\!-\!\!\circ 1}$$

$$\boxed{\int_a^b \delta(t-t_0)\, f(t)\, dt = f(t_0) \quad a \le t_0 \le t}$$

$$x(t-t_0) = \delta(t-t_0)$$

$$X(s) = \int_0^{\infty} \delta(t-t_0)\, e^{-st}\, dt = e^{-st} \cdot 1$$

ako $\quad x(t) \circ\!\!-\!\!\circ X(s)$

$$x(t-t_0) \circ\!\!-\!\!\circ e^{-st_0} \cdot X(s)$$

$$\boxed{\frac{d^i}{dt^i} x(t) \circ\!\!-\!\!\circ s^i X(s) - \sum_{m=1}^{i} s^{i-m} x^{(m-1)}(0^-)}$$

$s^{i-m} \rightarrow$ red potencije

$x^{(m-1)} \rightarrow$ red derivacije

$$x(t) \circ\!\!-\!\!\circ X(s)$$

$$\underset{i=2}{x''(t)} \circ\!\!-\!\!\circ s^2 X(s) - \sum_{m=1}^{2} s^{2-m} x^{(m-1)}(0^-)$$

$$s^2 X(s) - \left(s x(0^-) + x'(0^-)\right)$$

$$e^{-at} \cdot 1 \circ\!\!-\!\!\circ \frac{1}{s+a}$$

$$1 \circ\!\!-\!\!\circ \frac{1}{s}$$

$$t \circ\!\!-\!\!\circ \frac{1}{s^2} \qquad t^3 \circ\!\!-\!\!\circ \frac{3!}{s^4}$$

$$e^{-at} \cdot t \circ\!\!-\!\!\circ \frac{1}{(s+a)^2}$$

$$e^{-at} \cdot t^3 \circ\!\!-\!\!\circ \frac{6}{(s+a)^4}$$

ZADATAK 1.

$$y''(t) + 2y'(t) + y(t) = u''(t) + u'(t) + u(t)$$

$$u(t) = \delta(t) \qquad y(0^-) = y'(0^-) = 0$$

$$y(t) \circ\!\!-\!\!\circ Y(s)$$

$$y'(t) \circ\!\!-\!\!\circ sY(s) - \overset{=0}{y(0^-)} = sY(s)$$

$$y''(t) \circ\!\!-\!\!\circ s^2Y(s) - s\underbrace{y(0^-)}_{=0} - \underbrace{y'(0^-)}_{=0} = s^2Y(s)$$

$$u(t) \circ\!\!-\!\!\circ U(s)$$

$$u'(t) \circ\!\!-\!\!\circ sU(s) - \underbrace{U(0^-)}_{=0} = sU(s)$$

$$u''(t) \circ\!\!-\!\!\circ s^2U(s) - s\underbrace{U(0^-)}_{=0} - \underbrace{U'(0^-)}_{=0} = s^2U(s)$$

$$s^2Y(s) + 2sY(s) + Y(s) = s^2U(s) + sU(s) + U(s)$$

$$Y(s)(s^2+2s+1) = U(s)(s^2+s+1)$$

$$Y(s) = \frac{s^2+s+1}{s^2+2s+1}U(s) = \frac{s^2+s+1}{s^2+2s+1} = 1 + \frac{-s}{s^2+2s+1} = 1 - \frac{s}{s^2+2s-1}$$

$$(s^2+s+1):(s^2+2s+1) = 1$$
$$\underline{-s^2-2s-1}$$
$$-s$$

$$\frac{s}{s^2+2s+1} = \frac{s}{(s+1)^2}$$

$$\frac{s}{(s+1)^2} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2}$$

$$\frac{\dots}{(s^2+1)(s-1)^3} = \frac{As+B}{s^2+1} + \frac{C}{(s-1)} + \frac{D}{(s-1)^2} + \frac{E}{(s-1)^3}$$

$$\frac{s}{(s^2-4s+4)(s^2+1)^2} =$$

$$= \frac{A}{s-2} + \frac{B}{(s-2)^2} + \frac{Cs+D}{s^2+1} +$$

$$+ \frac{Es+F}{(s^2+1)^2}$$

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$$\frac{s}{s^2+2s+1} = \frac{s}{(s+1)^2} = \frac{A}{(s+1)} + \frac{B}{(s+1)^2} = \frac{A(s+1)+B}{(s+1)^2} = \frac{sA+A+B}{(s+1)^2}$$

$$A = 1$$

$$A + B = 0 \rightarrow B = -1$$

$$\frac{1}{s+1} - \frac{1}{(s+1)^2}$$

$$\frac{1}{s+1} \circ\!\!-\!\!\!-\!\!\circ\; e^{-t}\mu(t) \qquad \frac{1}{(s+1)^2} \circ\!\!-\!\!\!-\!\!\circ\; t\,e^{-t}\mu(t)$$

$$\boxed{y(t) = -\mu(t)\left(e^{-t} - t e^{-t}\right) + \delta(t)}$$

PR. 2.6.4, str. 67

$$X(s) = \frac{7s^2-5s-6}{(s+1)(s-2)(s-1)}$$

$$X(s) = \frac{C_{11}}{s+1} + \frac{C_{21}}{s-2} + \frac{C_{31}}{s-1}$$

$$\boxed{\frac{\quad}{(s+1)^2(s-2)(s-1)} = \frac{C_{11}}{s+1} + \frac{C_{12}}{(s+1)^2} + \frac{C_{21}}{s-2} + \frac{C_{31}}{s-1}}$$

$$\boxed{C_{*j} = \frac{1}{(r-j)!} \lim_{s \to p_*} \left[\frac{d^{r-j}}{ds^{r-j}} (s-p_*)^r \cdot X(s) \right]} \Rightarrow \text{ZAPAMTI !!!}$$

r – višestrukost pola

$$C_{11} = \frac{1}{(1-1)!} \lim_{s \to -1} \left[\frac{d^{1-1}}{ds^{1-1}} (s+1)^1 \cdot \frac{7s^2-5s-6}{(s+1)(s-2)(s-1)} \right]$$

$$= \lim_{s \to -1} \left(\frac{7s^2-5s-6}{(s-2)(s-1)} \right)$$

$$C_{11} = 1$$

$$C_{21} = 4$$

$$C_{31} = 2$$

$$X(s) = \frac{1}{s+1} + \frac{4}{s-2} + \frac{2}{s-1}$$

$$\frac{1}{s+1} \circ\!\!-\!\!\circ\ e^{-at}\mu(t)$$

$$4\,\frac{1}{s-2} \circ\!\!-\!\!\circ\ 4e^{2t}\mu(t)$$

$$2\,\frac{1}{s-1} \circ\!\!-\!\!\circ\ 2e^{t}\mu(t)$$

P2.

$$X(s) = \frac{s+1}{(s-1)^2(s+2)}$$

$$= \frac{C_{11}}{(s-1)} + \frac{C_{12}}{(s-1)^2} + \frac{C_{21}}{s+2}$$

$$C_{21} = -\frac{1}{9}$$

$$C_{11} = \frac{1}{(2-1)!}\lim_{s\to 1}\left[\frac{d^{2-1}}{ds^{2-1}}\cancel{(s-1)^2}\cdot\frac{s+1}{\cancel{(s-1)^2}(s+2)}\right]$$

$$= \lim_{s\to 1}\left(\frac{d}{ds}\left(\frac{s+1}{s+2}\right)\right) \qquad \frac{b}{n} = \frac{b'n - bn'}{n^2}$$

$$= \lim_{s\to 1}\left(\frac{s+2-(s+1)\cdot 1}{(s+2)^2}\right) = \frac{1}{9}$$

$$C_{12} = \frac{1}{(2-2)!}\lim_{s\to 1}\left[\frac{d^{2-2}}{ds^{2-2}}\left(\cancel{(s-1)^2}\cdot\frac{s+1}{\cancel{(s-1)^2}(s+2)}\right)\right]$$

$$= \lim_{s\to 1}\frac{s+1}{s+2} = \frac{2}{3}$$

$$X(s) = \frac{1}{9}\cdot\frac{1}{s-1} + \frac{2}{3}\,\frac{1}{(s-1)^2} - \frac{1}{9}\,\frac{1}{s+2}$$

$$\frac{1}{s+a} \circ\!\!-\!\!\circ\ e^{-at}\mu t$$

$$x(t)e^{at} \circ\!\!-\!\!\circ\ X(s+a)$$

$$X(s) = 1 - \frac{s}{(s+1)^2}$$

$$\frac{s}{(s+1)^2} = \frac{C_{11}}{s+1} + \frac{C_{12}}{(s+1)^2}$$

$$C_{11} = \lim_{s\to -1}\left[\frac{d}{ds}\cancel{(s+1)^2}\cdot\frac{s}{\cancel{(s+1)^2}}\right] = 1$$

$$C_{12} = \lim_{s\to -1}\left[\cancel{(s+1)^2}\cdot\frac{s}{\cancel{(s+1)^2}}\right] = -1$$

Pr. 1

$$X(s) = \frac{s}{(s^2+1)(s-1)^2}$$

$$= \frac{s}{(s-j)(s+j)(s-1)^2} = \frac{C_{11}}{s-j} + \frac{C_{21}}{s+j} + \frac{C_{31}}{s-1} + \frac{C_{32}}{(s-1)^2}$$

$$C_{11} = \frac{\cancel{j}}{2\cancel{j}(j-1)^2} = \frac{1}{2(-1-2j+1)} = \frac{1}{-4j}\cdot\frac{j}{j} = \frac{1}{4}j$$

$$C_{21} = \frac{-\cancel{j}}{-2\cancel{j}(-j-1)^2} = \frac{1}{2(-1+2j+1)} = -\frac{1}{4}j$$

$$C_{31} = \lim_{s\to 1}\left[\frac{d}{ds}\left(\cancel{(s-1)^2}\cdot\frac{s}{(s^2+1)\cancel{(s-1)^2}}\right)\right] = \lim_{s\to 1}\frac{s^2+1-2s^2}{(s^2+1)^2} = 0$$

$$C_{32} = \lim_{s\to 1}\left[\frac{s}{s^2+1}\right] = \frac{1}{2}$$

$$= \frac{1}{4}j\,\frac{1}{s-j} - \frac{1}{4}j\,\frac{1}{s+j} + \frac{1}{2}\,\frac{1}{(s-1)^2}$$

$$= \frac{1}{4j} \cdot e^{jt} - \frac{1}{4j} e^{-jt} + \frac{1}{2} e^{t} \cdot t$$

$$= \frac{1}{4j} \cdot \frac{j}{j} \left(e^{jt} - e^{-jt} \right) + \frac{1}{2} e^{t} \cdot t$$

$$= -\frac{1}{2} \sin(t)\, \mu(t) + \frac{1}{2} e^{t}\, t\, \mu(t)$$

Z - TRANSFORMACIJA

$$H(z) = \sum_{m=-\infty}^{\infty} h(m)\, z^{-m}$$

$$X(z) = \sum_{m=-\infty}^{\infty} x(m)\, z^{-m}$$

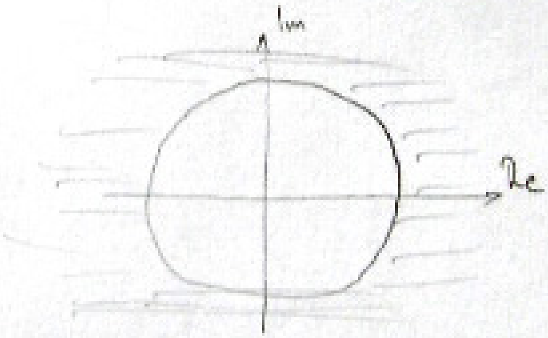
PRIMJER 1.

$$x(n) = \alpha^{n} \mu(n)$$

$$X(z) = \sum_{m=0}^{\infty} \alpha^{m} z^{-m} = \sum_{m=0}^{\infty} \left(\frac{\alpha}{z} \right)^{m} = \frac{1}{1 - \frac{\alpha}{z}} = \frac{z}{z - \alpha}$$

$$\left| \frac{\alpha}{z} \right| < 1$$

$$|z| > |\alpha|$$



PRIMJER 2

$$x(n) = -\alpha^{n} \mu(-n-1) \qquad \mu(-n-1) = \begin{cases} 1, & n \leq -1 \\ 0, & n > -1 \end{cases}$$

$$X(z) = -\sum_{m=-\infty}^{-1} \alpha^{m} z^{-m} \qquad \begin{matrix} -n-1 \geq 0 \\ n \leq -1 \end{matrix}$$

$$m = -t$$

$$X(z) = -\sum_{t=\infty}^{t=1} \alpha^{-t} z^{t} = -\sum_{t=1}^{\infty} \left(\frac{z}{\alpha} \right)^{t} = -\sum_{t=0}^{\infty} \left(\frac{z}{\alpha} \right)^{t} + 1 \qquad = -1 - \sum_{t=0}^{\infty} \left(\frac{z}{\alpha} \right)^{t}$$

$$1-\frac{1}{1-\frac{z}{\alpha}} = 1-\frac{\alpha}{\alpha-z} = \frac{\cancel{\alpha}-z-\cancel{\alpha}}{\alpha-z} = \frac{z}{z-\alpha}$$

$\left|\frac{z}{\alpha}\right|<1$

$|z|<|\alpha|$

$$\alpha^n\mu(n) \circ\!\!-\!\!\bullet \frac{z}{z-\alpha}, \quad |z|>|\alpha|$$

$$-\alpha^n\mu(-n-1) \circ\!\!-\!\!\bullet \frac{z}{z-\alpha}, \quad |z|<|\alpha|$$

ZZV – 16

1)

$$H(z) = \frac{(e^{-2}-e^{-1})\,z}{(z-e^{-2})(z-e^{-1})}$$

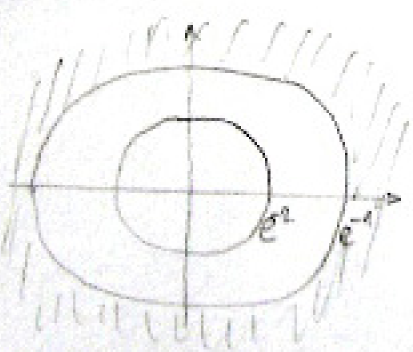
$$H_1(z) = \frac{H(z)}{z} = \frac{e^{-2}-e^{-1}}{(z-e^{-2})(z-e^{-1})}$$

$$H_1(z) = \frac{1}{z-e^{-2}} - \frac{1}{z-e^{-1}}$$

$$H(z) = \frac{z}{z-e^{-2}} - \frac{z}{z-e^{-1}}$$

$\alpha_1 = e^{-2}$

$\alpha_2 = e^{-1}$



$|z|>|\alpha_1|$

$|z|>|\alpha_2|$

$$h(n) = e^{-2n}\mu(n) - e^{-n}\mu(n)$$

$$Z^{-1}(H_1(z)) = \frac{A}{z-e^{-2}} + \frac{B}{z-e^{-1}}$$

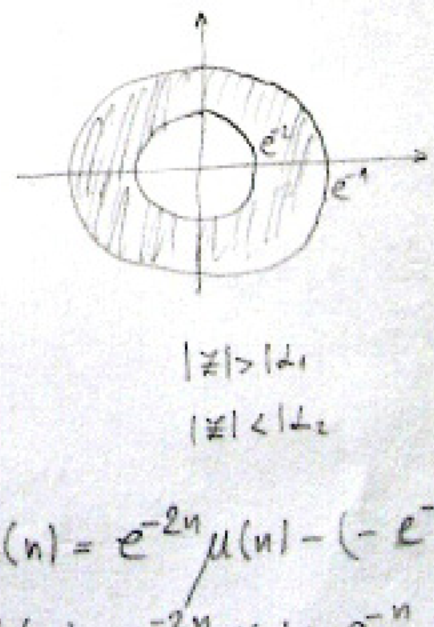
$$\frac{A(z-e^{-1})+B(z-e^{-2})}{(z-e^{-2})(z-e^{-1})} =$$

$$\underbrace{z(A+B)}_{=0} + (-Ae^{-1}-Be^{-2}) = e^{-2}-e^{-1}$$

$A+B=0$

$-A=-1 \quad ; \quad A=1$

$-B=1 \quad ; \quad B=-1$

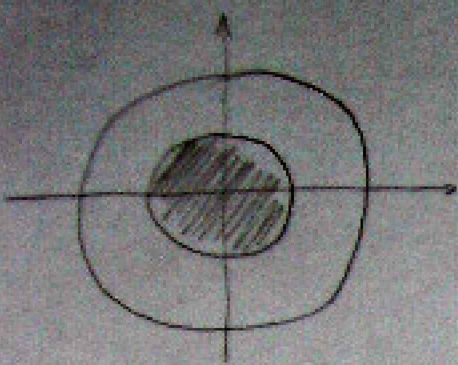


$|z|>|\alpha_1|$

$|z|<|\alpha_2|$

$$h(n) = e^{-2n}\mu(n) - (-e^{-n}\mu(-n-1))$$

$$h(n) = e^{-2n}\mu(n) + e^{-n}\mu(-n-1)$$



$|z| < \alpha_1$ isto je $|z| < \alpha_1$

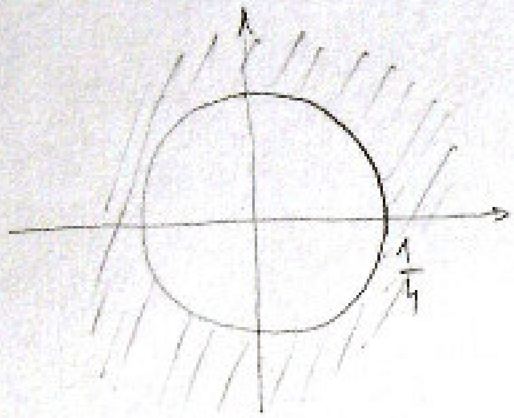
$|z| > \alpha_2$ i za $|z| < \alpha_2$

$$h(n) = -e^{-2n}\mu(n-1) + e^{-n}\mu(-n-1)$$

ZI grupa A

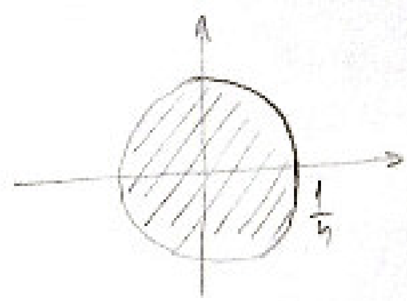
(4) $$H(z) = \frac{1}{4 - z^{-1}} = \frac{1}{4 - \frac{1}{z}} = \frac{z}{4z - 1} = \frac{1}{4}\,\frac{z}{z - \frac{1}{4}}$$

$\alpha = \frac{1}{4}$



$|z| > \alpha$

$$h(n) = \frac{1}{4}\cdot\left(\frac{1}{4}\right)^n\cdot\mu(n)$$



$$h(n) = \frac{1}{4}\cdot\left(-\frac{1}{4}\right)^n\mu(-n-1)$$

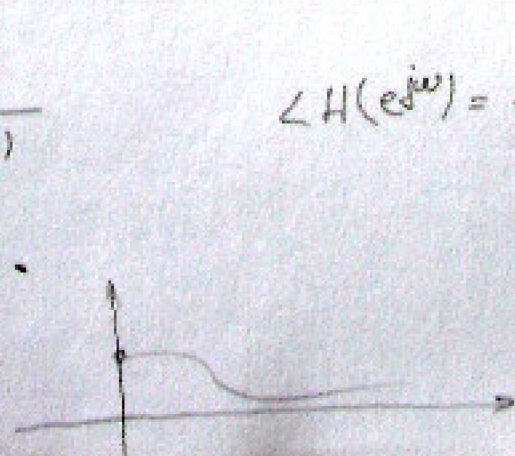
$r = \frac{1}{4} < 1$ sustav je stabilan

$z = e^{j\omega}$

$$H(e^{j\omega}) = \frac{1}{4 - e^{-j\omega}} = \frac{1}{4 - \cos(\omega) + j\sin(\omega)}$$

$$\angle H(e^{j\omega}) = -\operatorname{arctg}\frac{\sin(\omega)}{4 - \cos(\omega)}$$

$$|H(e^{j\omega})| = \frac{1}{\sqrt{17 - 8\cos(\omega)}}$$



SVOJSTVA

$$x(n-j) \circ\!-\!\bullet\; z^{j}\left(X(z)+\sum_{m=-j}^{-1} x(m)\, z^{-m}\right)$$

$$x(n+j) \circ\!-\!\bullet\; z^{j}\left(X(z)+\sum_{m=0}^{j-1} x(m)\, z^{-m}\right)$$

PRIMJER

$$X(z) = \frac{1}{1-0.75z^{-1}} = \frac{z}{z-0.75}$$

$$\boxed{X(z) = \sum_{m=0}^{\infty} x(m)\, z^{-m} = x(0)+x(1)z^{-1}+x(2)z^{-2}+x(3)z^{-3}+\ldots}$$

$$z : (z-0.75) = \underbrace{1}_{x(0)} + \underbrace{0.75}_{x(1)} z^{-1} + \underbrace{0.56}_{x(2)} z^{-1} + \ldots$$

$$-\,z-0.75$$

$$0.75$$

$$-\,0.75 \mp 0.56z^{-1}$$

$$0.56z^{-1}$$

$$0.75^{n}\cdot\mu(n)$$

PR.)

$$X(z) = \frac{z^2}{(z-1)(z-2)(z-3)} \qquad \frac{A}{(z-1)} + \frac{B}{(z-2)} + \frac{C}{(z-3)} =$$

$$= A(z^2-5z+6) + B(z^2-4z+3) + C(z^2-3z+2)$$

$$z^2(A+B+C) + z(-5A-4B-3C)$$

$$6A+3B+2C = z^2$$

$$A+B+C=1$$

$$-5A-4B-3C=0$$

$$A+3B+2C=0 \qquad \boxed{3B+4C=6}$$

$$5A+5B+5C=5 \qquad -2C=-9$$

$$\boxed{B+2C=5} \qquad C=\frac{9}{2}$$

$$-6A-6B-6C=-6 \qquad B=-4$$

$$-3B-4C=-6 \qquad C=\frac{9}{2}$$

$$X_1(z) = \frac{X(z)}{z}$$

$$\mathcal{Z}^{-1}\{X_1(z)\}$$

$$C_{xj} = \frac{1}{(r-j)!} \lim_{z \to p_x} \left[\frac{d^{r-j}}{dz^{r-j}} \left((z-p_x)^r X_1(z) \right) \right]$$

$$y(t) = u(t) * h(t) \circ\!\!-\!\!\circ U(s) \cdot H(s) = Y(s)$$

$$y(t) \circ\!\!-\!\!\circ Y(s)$$

$$y'(t) + y(t) = u(t)$$

$$sY(s) - Y(0^-) + Y(s) = U(s)$$

$$U(s)(s+1) = U(s) + y(0^-)$$

$$Y(s) = \underbrace{\frac{1}{s+1}}_{H(s)} U(s) + \frac{2}{s+1}$$

PRIMJER

$$X(z) = \frac{10z^3 - 37z^2 + 24z + 18}{z(z-3)^2}$$

$$X_1(z) = \frac{10z^3 - 37z^2 + 24z + 18}{z^2(z-3)^2}$$

$$= \frac{C_{11}}{z} + \frac{C_{12}}{z^2} + \frac{C_{21}}{z-3} + \frac{C_{22}}{(z-3)^2}$$

$$C_{11} = \frac{1}{(2-1)!} \lim_{z \to 0} \left\{ \frac{d}{dz} \left[\cancel{z^2} \cdot \frac{10z^3 - 37z^2 + 24z + 18}{\cancel{z^2}(z-3)^2} \right] \right\}$$

$c_{11} = 4$

$c_{12} = 2$

$c_{21} = 5$

$c_{22} = 1$

$$\frac{4}{z} + \frac{2}{z^2} + \frac{5}{z-3} + \frac{1}{(z-3)^2}$$