Zadatak 1. (1)

Zadan je sustav:

$$y[n] - 0.5y[n-1] = u[n]$$

Traži se prisilni odziv na harmonijsku pobudu oblika:

$$u[n] = A_1 \cdot \cos(\omega_1 n + \Theta_1) + A_2 \cdot \cos(\omega_2 n + \Theta_2)$$

Prisilni odziv mirnog kauzalnog LTI sustava na harmonijsku pobudu najlakše se nalazi primjenom frekvencijske karakteristike sustava $H(e^{j\omega})$. Prisilni odziv mora biti sljedećeg oblika:

$$y[n] = Y_1 \cdot \cos(\omega_1 n + \Psi_1) + Y_2 \cdot \cos(\omega_2 n + \Psi_2)$$

gdje

$$\left.\begin{aligned} Y_i &= A_i \cdot |H(e^{j\omega_i})| \\ \Psi_i &= \Theta_i + \measuredangle H(e^{j\omega_i}) \end{aligned}\right\} \quad i = 1, 2$$

Dakle dovoljno je odrediti $H(e^{j\omega})$ i evaluirati je za $\omega = \omega_1$, $\omega = \omega_2$.

Zbog toga krećemo od pitanja B)

$$H(e^{j\omega}) = H(z)\Big|_{z=e^{j\omega}}$$

$$y[n] - 0.5y[n-1] = u[n] \;\xrightarrow{\mathcal{Z}}\; Y(z)(1-0.5z^{-1}) = U(z)$$

$$H(z) = \frac{Y(z)}{U(z)} = \frac{1}{1-0.5z^{-1}}$$

$$H(e^{j\omega}) = \frac{1}{1-0.5 \cdot e^{-j\omega}}$$

$$= \frac{1}{1-0.5\cos\omega + j0.5\sin\omega}$$

Nađimo amplitudno-frekv. karakt.

$$|H(e^{j\omega})| = \frac{1}{\sqrt{(1-\frac{1}{2}\cos\omega)^2 + (\frac{1}{2}\sin\omega)^2}}$$

$$= \frac{1}{\sqrt{1-\cos\omega + \underbrace{\frac{1}{4}\cos^2\omega + \frac{1}{4}\sin^2\omega}_{1/4}}} =$$

$$= \frac{1}{\sqrt{\frac{5}{4} - \cos\omega}}$$

$$\measuredangle H(e^{j\omega}) = -\operatorname{atan}_2\left(\frac{1}{2}\sin\omega,\ 1-\frac{1}{2}\cos\omega\right)$$

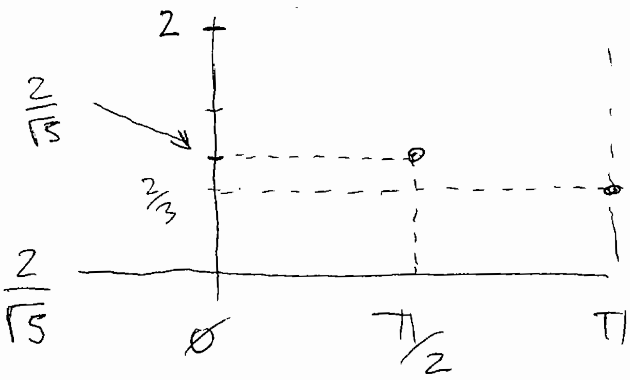
$$= \operatorname{atan}_2\left(-\frac{1}{2}\sin\omega,\ 1-\frac{1}{2}\cos\omega\right)$$

$$= \operatorname{atan}_2\left(-\sin\omega,\ 2-\cos\omega\right)$$

Skicirajmo A-F i F-F karakteristiku u nekoliko točaka. Obzirom da nam za A) dio zadatka treba $H(e^{j\omega_1})$ & $H(e^{j\omega_2})$ za $\omega_1 = \frac{\pi}{2}$ & $\omega_2 = \pi$, neka dvije frekvencije budu upravo te, a dodajmo i treću za npr. $\omega_0 = \emptyset$.

$$|H(e^{j\emptyset})| = \frac{1}{\sqrt{\frac{5}{4} - \frac{4}{4}}} = 2$$

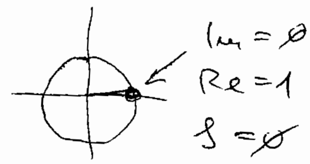
$$|H(e^{j\omega_1})| = |H(e^{j\frac{\pi}{2}})| = \frac{1}{\sqrt{\frac{5}{4} - \emptyset}} = \frac{2}{\sqrt{5}}$$



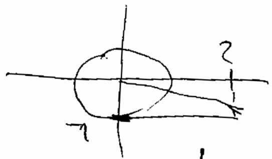
$$|H(e^{j\omega_2})| = |H(e^{j\pi})| = \frac{1}{\sqrt{\frac{5}{4} + \frac{4}{4}}} = \frac{2}{3}$$

Slično određujemo F-F karakteristiku:

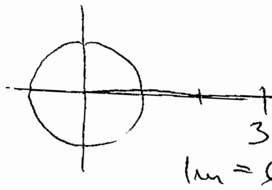
$$\measuredangle H(e^{j0}) = \operatorname{atan2}(-\sin 0,\ 2-\cos 0) = \operatorname{atan2}(0,\ 1) = 0$$

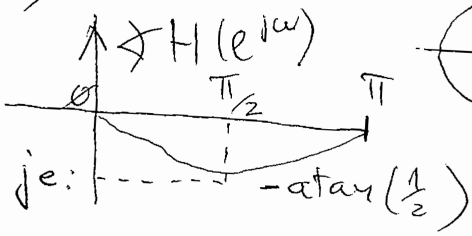


$$\measuredangle H(e^{j\pi/2}) = \operatorname{atan2}\left(-\sin\tfrac{\pi}{2},\ 2-\cos\tfrac{\pi}{2}\right) = \operatorname{atan2}(-1,\ 2) = -\operatorname{atan}(1/2)$$



$$\measuredangle H(e^{j\pi}) = \operatorname{atan2}(-\sin\pi,\ 2-\cos\pi) = \operatorname{atan2}(0,\ 3) = 0$$





Dakle prisilni odziv je:

$$y_{pris}[n] = A_1 \cdot |H(e^{j\omega_1})| \cos(\omega_1 n + \Theta_1 + \measuredangle H(e^{j\omega_1})) + A_2 |H(e^{j\omega_2})| \cdot \cos(\omega_2 n + \Theta_2 + \measuredangle H(e^{j\omega_2}))$$

$$= A_1 \cdot \frac{2}{\sqrt{5}} \cdot \cos\left(\omega_1 n + \Theta_1 + \left(-\operatorname{atan}(1/2)\right)\right) + $$

$$+ A_2 \cdot \frac{2}{3} \cdot \cos(\omega_2 n + \Theta_2 + 0)$$

U obje grupe je $\omega_1 = \pi/2$, $\omega_2 = \pi$, a konstante A_1, A_2, Θ_1 i Θ_2 iznose ovisno o grupi:

GRUPA A

$A_1 = 2$ $\quad \Theta_1 = -\frac{\pi}{2}$

$A_2 = 3$ $\quad \Theta_2 = \frac{\pi}{10} - \frac{\pi}{2}$

GRUPA B

$A_1 = 3$ $\quad \Theta_1 = \frac{\pi}{10} - \frac{\pi}{2}$

$A_2 = 2$ $\quad \Theta_2 = -\frac{\pi}{2}$

jer:

$$\sin\alpha = \cos\left(\alpha - \frac{\pi}{2}\right)$$

Dakle konačno rješenje pišemo:

za grupu A.

$$y_{pris}[n] = \frac{4}{\sqrt{5}} \cdot \cos\left(\frac{\pi}{2}n - \frac{\pi}{2} - \operatorname{atan}\left(\frac{1}{2}\right)\right) + 2\cos\left(\pi n + \frac{\pi}{10} - \frac{\pi}{2} + \phi\right)$$

$$= \frac{4}{\sqrt{5}} \cdot \cos\left(\frac{\pi}{2}n - \left(\frac{\pi}{2} + \operatorname{atan}\frac{1}{2}\right)\right) + 2\cos\left(\pi n - \frac{2\pi}{5}\right)$$

ili pomoću sinusa

$$= \frac{4}{\sqrt{5}} \cdot \sin\left(\frac{\pi}{2}n - \operatorname{atan}\left(\frac{1}{2}\right)\right) + 2\sin\left(\pi n + \frac{\pi}{10}\right)$$

Za grupu B.

$$y_{pris}[n] = \frac{6}{\sqrt{5}}\cos\left(\frac{\pi}{2}n + \frac{\pi}{10} - \frac{\pi}{2} - \operatorname{atan}\frac{1}{2}\right) + \frac{4}{3}\cos\left(\pi n - \frac{\pi}{2} + \phi\right)$$

$$= \frac{6}{\sqrt{5}} \cdot \cos\left(\frac{\pi}{2}n - \frac{2\pi}{5} - \operatorname{atan}\frac{1}{2}\right) + \frac{4}{3}\cos\left(\pi n - \frac{\pi}{2}\right)$$

$$= \frac{6}{\sqrt{5}} \sin\left(\frac{\pi}{2}n + \frac{\pi}{10} - \operatorname{atan}\frac{1}{2}\right) + \frac{4}{3}\sin\left(\pi n\right)$$

ZADATAK 2.

GRUPA A	GRUPA B
$z_1 = \frac{1}{2}$	$z_1 = -\frac{1}{2}$
$z_2 = -\frac{1}{2}$	$z_2 = \frac{1}{2}$
$z_3 = \frac{3}{4}$	$z_3 = -\frac{3}{4}$

Sustav

$$H(z) = \frac{z^3}{(z-z_1)(z-z_2)(z-z_3)}$$

realizirati kaskadnom realizacijom, te odrediti matrice A, B, C, D

Sustav razbijamo na kaskadu tri sustava prvog reda (zbog realnih polova)

$$H(z) = \underbrace{\frac{z}{z-z_1}}_{H_1(z)} \cdot \underbrace{\frac{z}{z-z_2}}_{H_2(z)} \cdot \underbrace{\frac{z}{z-z_3}}_{H_3(z)} = H_1(z) \cdot H_2(z) \cdot H_3(z)$$

ili sa negativnim potencijama varijable z:

$$H(z) = \frac{1}{1-z_1 z^{-1}} \cdot \frac{1}{1-z_2 z^{-1}} \cdot \frac{1}{1-z_3 z^{-1}} = H_1(z) \cdot H_2(z) \cdot H_3(z)$$

Sve tri sekcije imaju identičan oblik:

$$H_i(z) = \frac{1}{1-z_i z^{-1}} \qquad i = 1, 2, 3$$

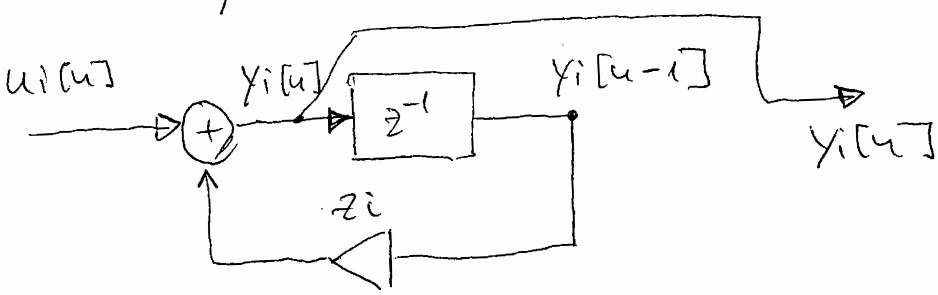
Odredimo realizaciju općenite sekcije $H_i(z)$

$$H_i(z) = \frac{Y_i(z)}{U_i(z)} = \frac{1}{1-z_i z^{-1}} \quad / \cdot U_i(z)(1-z_i z^{-1})$$

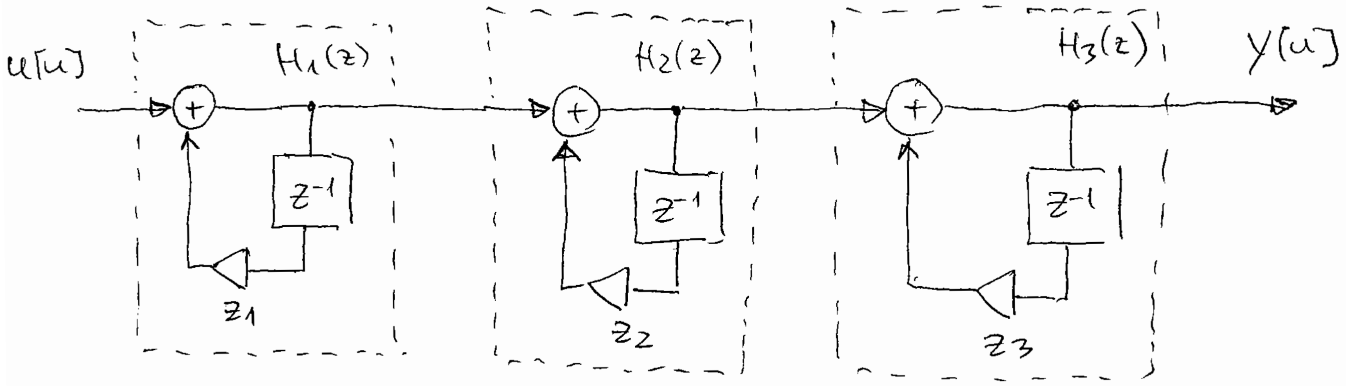
$$Y_i(z)(1-z_i z^{-1}) = U_i(z) \quad z^{-1}$$

$$y_i[n] - z_i y_i[n-1] = u_i[n]$$

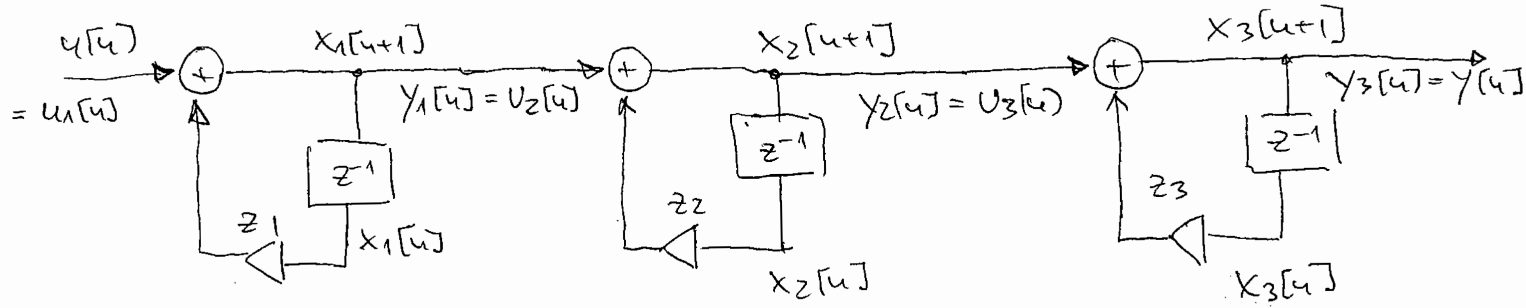
$$y_i[n] = u_i[n] + z_i \cdot y_i[n-1]$$



Cijeli sustav $H(z)$ je kaskada H_1, H_2 & H_3



Prema konvenciji sa predavanja (19. cjelina) kao varijable stanja odabiremo izlaze elemenata za kašnjenje, gdje indeks varijable stanja x_i odgovara indeksu sekcije 1. reda H_i



Pišemo jedadžbe:

$$x_i[n+1] = u_i[n] + z_i \cdot x_i[n] \qquad i = 1, 2, 3$$

$$x_1[n+1] = u[n] + z_1 \cdot x_1[n]$$

$$x_2[n+1] = u_2[n] + z_2 x_2[n] \qquad u_2[n] = y_1[n] = x_1[n+1]$$

$$= u[n] + z_1 x_1[n] + x_2[n] \cdot z_2$$

$$x_3[n+1] = u_3[n] + z_3 x_3[n] \qquad u_3[n] = y_2[n] = x_2[n+1]$$

$$= u[n] + z_1 x_1[n] + z_2 x_2[n] + z_3 x_3[n]$$

ili matrično:

$$\begin{bmatrix} x_1[n+1] \\ x_2[n+1] \\ x_3[n+1] \end{bmatrix} = \overbrace{\begin{bmatrix} z_1 & 0 & 0 \\ z_1 & z_2 & 0 \\ z_1 & z_2 & z_3 \end{bmatrix}}^{A} \begin{bmatrix} x_1[n] \\ x_2[n] \\ x_3[n] \end{bmatrix} + \overbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}^{B} \cdot u[n]$$

Napišimo i izlazne jednadžbe:

$$y[n] = y_3[n] = x_3[n+1]$$
$$= u[n] + z_1 x_1[n] + z_2 x_2[n] + z_3 x_3[n]$$

ili matrično:

$$\left[y[n]\right] = \underbrace{\begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix}}_{C} \begin{bmatrix} x_1[n] \\ x_2[n] \\ x_3[n] \end{bmatrix} + \underbrace{[1]}_{D} \cdot u[n]$$

Dakle matrice koje opisuju naslađenu realizaciju sustava su:

$$A = \begin{bmatrix} z_1 & 0 & 0 \\ z_1 & z_2 & 0 \\ z_1 & z_2 & z_3 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} z_1 & z_2 & z_3 \end{bmatrix} \quad D = [1]$$

Uvrstimo vrijednosti polova

Grupa A

$$A = \begin{bmatrix} 1/2 & 0 & 0 \\ 1/2 & -1/2 & 0 \\ 1/2 & -1/2 & 3/4 \end{bmatrix} \qquad C = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{3}{4} \end{bmatrix}$$

Grupa B

$$A = \begin{bmatrix} -1/2 & 0 & 0 \\ -1/2 & 1/2 & 0 \\ -1/2 & 1/2 & -3/4 \end{bmatrix} \qquad C = \begin{bmatrix} -1/2 & \frac{1}{2} & -\frac{3}{4} \end{bmatrix}$$

3. A)

$$A=\begin{bmatrix}0 & -4\\ -4 & 0\end{bmatrix}\qquad B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\qquad C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\qquad D=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}$$

a) $H(s)=C(sI-A)^{-1}B+D$

$$(sI-A)^{-1}=\begin{bmatrix}s & 4\\ 4 & s\end{bmatrix}^{-1}=\frac{\begin{bmatrix}s & -4\\ -4 & s\end{bmatrix}}{s^2-16}=\begin{bmatrix}\frac{s}{s^2-16} & \frac{-4}{s^2-16}\\ \frac{-4}{s^2-16} & \frac{s}{s^2-16}\end{bmatrix}$$

$$H(s)=\begin{bmatrix}\frac{s}{s^2-16} & \frac{-4}{s^2-16}\\ \frac{-4}{s^2-16} & \frac{s}{s^2-16}\end{bmatrix}$$

b) $h(t)=?$

$$\frac{s}{s^2-16}=\frac{A}{s-4}+\frac{B}{s+4}\qquad\qquad \frac{-4}{s^2-16}=\frac{A}{s-4}+\frac{B}{s+4}$$

$$A+B=1\qquad\qquad A+B=0\quad A=-B$$

$$-4B+4A=0\qquad\qquad -4B+4A=-4$$

$$A=B=\frac{1}{2}\qquad\qquad -8B=-4$$

$$B=\frac{1}{2}\qquad A=-\frac{1}{2}$$

$$H(s)=\begin{bmatrix}\frac{\frac{1}{2}}{s-4}+\frac{\frac{1}{2}}{s+4} & \frac{-\frac{1}{2}}{s-4}+\frac{\frac{1}{2}}{s+4}\\ \frac{-\frac{1}{2}}{s-4}+\frac{\frac{1}{2}}{s+4} & \frac{\frac{1}{2}}{s-4}+\frac{\frac{1}{2}}{s+4}\end{bmatrix}$$

$$h(t)=\begin{bmatrix}\left(\frac{1}{2}e^{4t}+\frac{1}{2}e^{-4t}\right)\mu(t) & \left(-\frac{1}{2}e^{4t}+\frac{1}{2}e^{-4t}\right)\mu(t)\\ \left(-\frac{1}{2}e^{4t}+\frac{1}{2}e^{-4t}\right)\mu(t) & \left(\frac{1}{2}e^{4t}+\frac{1}{2}e^{-4t}\right)\mu(t)\end{bmatrix}$$

c) $u(t)=\begin{bmatrix}4\mu(t)\\ \delta(t)\end{bmatrix}$

$$U(s)=\begin{bmatrix}\frac{4}{s}\\ 1\end{bmatrix}$$

$$y(s)=H(s)\cdot U(s)=\begin{bmatrix}\frac{4}{s^2-16}-\frac{4}{s^2-16}\\ \frac{-16}{s(s^2-16)}+\frac{s}{s^2-16}\end{bmatrix}=\begin{bmatrix}0\\ \frac{\cancel{-16+s^2}}{s(\cancel{s^2-16})}\end{bmatrix}=\begin{bmatrix}0\\ \frac{1}{s}\end{bmatrix}$$

$$y(t)=\begin{bmatrix}0\\ \mu(t)\end{bmatrix}$$

B 3.

$$A=\begin{bmatrix}0 & -5\\ -5 & 0\end{bmatrix} \quad B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad D=\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$$

a)

$$\dot{x}=Ax+Bu \rightarrow sX=Ax+Bu$$
$$y=Cx+Du \qquad (sI-A)X=BU$$
$$X=(sI-A)^{-1}BU$$
$$y=(C(sI-A)^{-1}B+D)u$$

$$H(s)=C(sI-A)^{-1}B+D$$

$$sI-A=\begin{bmatrix}s & 5\\ 5 & s\end{bmatrix}$$

$$(sI-A)^{-1}=\left(\begin{array}{cc|cc}s & 5 & 1 & 0\\ 5 & s & 0 & 1\end{array}\right)\sim\left[\begin{array}{cc|cc}1 & \frac{5}{s} & \frac{1}{s} & 0\\ 5 & s & 0 & 1\end{array}\right]\sim\left[\begin{array}{cc|cc}1 & \frac{5}{s} & \frac{1}{s} & 0\\ 0 & s-\frac{25}{s} & -\frac{5}{s} & 1\end{array}\right]$$

$$\sim\left[\begin{array}{cc|cc}1 & \frac{5}{s} & \frac{1}{s} & 0\\ 0 & 1 & -\frac{5}{\cancel{s}}\cdot\frac{\cancel{s}}{-s^2+25} & \frac{s}{s^2-25}\end{array}\right]\sim\left[\begin{array}{cc|cc}1 & 0 & \frac{1}{s}+\frac{-25}{(25-s^2)s} & \frac{-5s}{s(s^2-25)}\\ 0 & 1 & \frac{5}{25-s^2} & \frac{s}{s^2-25}\end{array}\right]$$

$$\sim\left[\begin{array}{cc|cc}1 & 0 & \frac{s^2-25+25}{\cancel{s}(s^2-25)} & \frac{-5}{(s^2-25)}\\ 0 & 1 & \frac{5}{25-s^2} & \frac{s}{s^2-25}\end{array}\right]$$

$$C(sI-A)^{-1}B=\begin{bmatrix}\frac{s}{s^2-25} & \frac{-5}{s^2-25}\\ \frac{5}{25-s^2} & \frac{s}{s^2-25}\end{bmatrix}=H(s)$$

b)

ili

$$(sI-A)^{-1}=\frac{\operatorname{adj}(sI-A)}{\det(sI-A)}=\frac{1}{s^2-25}\cdot\begin{bmatrix}s & -5\\ -5 & s\end{bmatrix}=\begin{bmatrix}\frac{s}{s^2-25} & \frac{-5}{s^2-25}\\ \frac{-5}{s^2-25} & \frac{s}{s^2-25}\end{bmatrix}$$

$$\frac{s}{s^2-25}=\frac{A}{s-5}+\frac{B}{s+5} \qquad \frac{-5}{s^2-25}=\frac{A}{s-5}+\frac{B}{s+5}$$

$$A+B=1 \qquad A+B=0 \qquad A=-B$$
$$5A-5B=0 \qquad -5B+5A=-5$$
$$A=B=\frac{1}{2} \qquad -5B-5B=-5$$
$$B=\frac{1}{2}$$
$$A=-\frac{1}{2}$$

$$H(s)=\begin{bmatrix}\frac{\frac{1}{2}}{s-5}+\frac{\frac{1}{2}}{s+5} & \frac{-\frac{1}{2}}{s-5}+\frac{\frac{1}{2}}{s+5}\\ \frac{-\frac{1}{2}}{s-5}+\frac{\frac{1}{2}}{s+5} & \frac{\frac{1}{2}}{s-5}+\frac{1/2}{s+5}\end{bmatrix}$$

$$h(t)=\begin{bmatrix}(\frac{1}{2}e^{5t}+\frac{1}{2}e^{-5t})\mu(t) & (-\frac{1}{2}e^{5t}+\frac{1}{2}e^{-5t})\mu(t)\\ (-\frac{1}{2}e^{5t}+\frac{1}{2}e^{-5t})\mu(t) & (\frac{1}{2}e^{5t}+\frac{1}{2}e^{-5t})\mu(t)\end{bmatrix}$$

c)

$$u(t) = \begin{bmatrix} 5\mu(t) \\ \delta(t) \end{bmatrix}$$

$$U(s) = \begin{bmatrix} \frac{5}{s} \\ 1 \end{bmatrix}$$

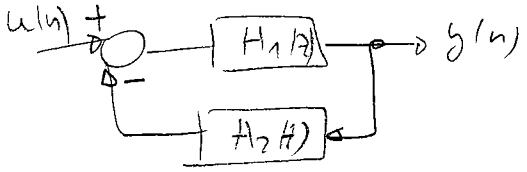
$$Y(s) = H(s)\,U(s)$$

$$= \begin{bmatrix} \frac{s}{s^2-25} & \frac{-5}{s^2-25} \\ \frac{-5}{s^2-25} & \frac{s}{s^2-25} \end{bmatrix} \begin{bmatrix} \frac{5}{s} \\ 1 \end{bmatrix}$$

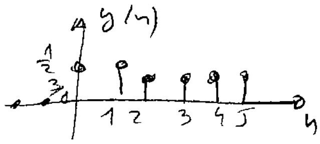
$$= \begin{bmatrix} \frac{5}{s^2-25} - \frac{5}{s^2-25} \\ \frac{-25}{s(s^2-25)} + \frac{s}{s^2-25} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{\cancel{-25+s^2}}{s(\cancel{s^2-25})} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{s} \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 0 \\ \mu(t) \end{bmatrix}$$

4. A



$$u(n)=\mu(n) \rightarrow U(z)=\frac{z}{z-1}$$



$$y(n)=\left\{\frac{1}{2},\frac{1}{2},\frac{3}{8},\frac{3}{8},\dots\right\}$$

$$y(z)=\frac{1}{2}\cdot z^{0}+\frac{1}{2}z^{-1}+\frac{3}{8}\left(z^{-2}+z^{-3}+z^{-4}+\dots\right)$$

$$=\frac{1}{2}\left(1+\frac{1}{z}\right)+\frac{3}{8}z^{-2}\left(1+z^{-1}+z^{-2}+\dots\right)$$

$$\sum_{n=0}^{\infty} z^{-n}=\frac{1}{1-z^{-1}}=\frac{z}{z-1}$$

$$=\frac{1}{2}\cdot\frac{1+z}{z}+\frac{3}{8}\frac{\cancel{z}}{(z-1)z^{\cancel{2}}}$$

$$=\frac{4(z-1)(z+1)+3}{8z(z-1)}=\boxed{\frac{4z^2-1}{8z(z-1)}=\frac{(z-\frac{1}{2})(z+\frac{1}{2})}{2z(z-1)}=\frac{z^2-\frac{1}{4}}{2z(z-1)}}$$

$$H(z)=\frac{y(z)}{U(z)}=\frac{(z-\frac{1}{2})(z+\frac{1}{2})}{2z(\cancel{z-1})}\cdot\frac{\cancel{z-1}}{z}=\boxed{\frac{z^2-\frac{1}{4}}{2z^2}=\frac{4z^2-1}{8z^2}}$$

$$H_2(n)=\frac{1}{2}\left(-\frac{1}{2}\right)^n+\frac{1}{2}\left(\frac{1}{2}\right)^n$$

$$=\boxed{\frac{1}{2}\frac{z}{z+\frac{1}{2}}+\frac{1}{2}\frac{z}{z-\frac{1}{2}}}=\frac{z^2-\cancel{\frac{1}{2}z}+z^2+\cancel{\frac{1}{2}z}}{2(z+\frac{1}{2})(z-\frac{1}{2})}=\boxed{\frac{z^2}{(z-\frac{1}{2})(z+\frac{1}{2})}}$$

$$\frac{\cancel{2}z^2}{\cancel{2}(z^2-\frac{1}{4})}=\frac{4z^2}{4z^2-1}$$

$$H(z)=\frac{y(z)}{U(z)}$$

$$\left(U(z)-H_2(z)\,y(z)\right)H_1(z)=y(z)$$

$$H_1(z)U(z)-H_1(z)H_2(z)y(z)=y(z)$$

$$U(z)H_1(z)=y(z)\left(1+H_1(z)H_2(z)\right)$$

$$\frac{y(z)}{U(z)}=H=\frac{H_1(z)}{1+H_1(z)H_2(z)} \Rightarrow H(z)+H_1(z)H_2(z)H(z)=H_1(z)$$

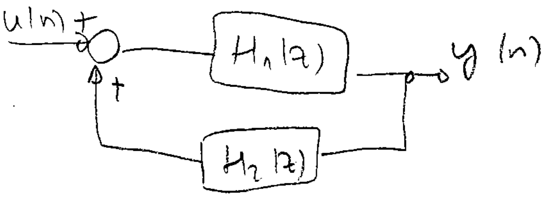
$$H_1(z)\left[-H_2H+1\right]=+H$$

$$H_1(z)=\frac{H(z)}{1-H(z)H_2(z)}$$

$$=\frac{\frac{(z-\frac{1}{2})(z+\frac{1}{2})}{2z^2}}{1-\frac{\cancel{(z-\frac{1}{2})}\cancel{(z+\frac{1}{2})}}{2\cancel{z^2}}\cdot\frac{\cancel{z^2}}{\cancel{(z-\frac{1}{2})}\cancel{(z+\frac{1}{2})}}}=\frac{\frac{z^2-\frac{1}{4}}{\cancel{2}z^2}}{\frac{1}{\cancel{2}}}=\frac{z^2-\frac{1}{4}}{z^2}$$

$$\boxed{H_1(z)=\frac{z^2-\frac{1}{4}}{z^2}=\frac{4z^2-1}{4z^2}}$$

4.



$u(n) = \mu(n)$

$U(z) = \frac{z}{z-1}$

$y(n) = \{\frac{1}{2}, \frac{1}{2}, \frac{3}{8}, \frac{3}{8}, \frac{3}{8}, \dots\}$

$Y(z) = \frac{(z-\frac{1}{2})(z+\frac{1}{2})}{2z(z-1)}$

$H(z) = \frac{(z-\frac{1}{2})(z+\frac{1}{2})}{2z^2}$

$H_2(z) = \frac{z^2}{(z-\frac{1}{2})(z+\frac{1}{2})}$

$$[U(z) + H_2(z)Y(z)]\,H_1(z) = Y(z)$$

$$U \cdot H_1 + H_1 H_2 Y = Y$$

$$U H_1 = Y(1 - H_1 H_2)$$

$$\frac{Y(z)}{U(z)} = \frac{H_1(z)}{1 - H_1(z)H_2(z)} = H(z)$$

$$H_1 = H - H_1 H_2 H$$

$$H_1(1 + H_2 H) = H$$

$$H_1 = \frac{H}{1 + H_2 H} = \frac{\frac{(z-\frac{1}{2})(z+\frac{1}{2})}{2z^2}}{1 + \frac{\cancel{z^2}}{\cancel{(z-\frac{1}{2})(z+\frac{1}{2})}} \cdot \frac{\cancel{(z-\frac{1}{2})(z+\frac{1}{2})}}{2\cancel{z^2}}}$$

$$H_1(z) = \frac{\frac{z^2-\frac{1}{4}}{\cancel{2}z^2}}{\frac{3}{\cancel{2}}} = \frac{(z^2-\frac{1}{4})}{3z^2}$$

5. $x(t) = e^{2t}\mu(-t)$

a) Fourierova transformacija

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\,dt$$

$$= \int_{-\infty}^{\infty} e^{2t}\mu(-t)\, e^{-j\omega t}\,dt$$

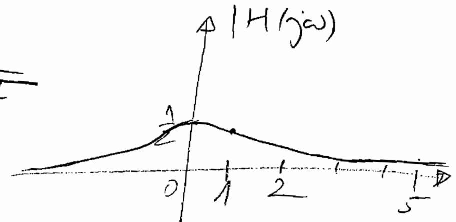
$$= \int_{-\infty}^{0} e^{2t-j\omega t}\,dt$$

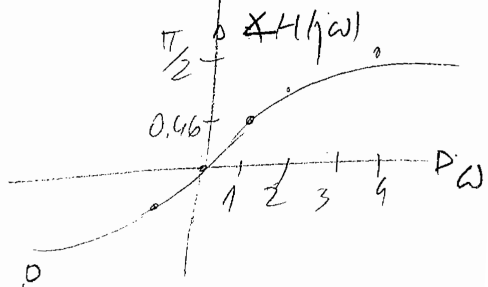
$$= \frac{e^{(2-j\omega)t}}{2-j\omega}\Bigg|_{-\infty}^{0} = \frac{1}{2-j\omega} = \frac{1}{2-j\omega}\,\frac{2+j\omega}{2+j\omega} = \frac{2+j\omega}{4+\omega^2}$$

b)

$$|H(j\omega)| = \sqrt{\frac{2^2+\omega^2}{(4+\omega^2)^2}} = \sqrt{\frac{\cancel{2^2+\omega^2}}{(4+\omega^2)^{\cancel{2}}}} = \frac{1}{\sqrt{4+\omega^2}}$$

$$\measuredangle H(j\omega) = \operatorname{arctg}\frac{\frac{\omega}{\cancel{4+\omega^2}}}{\frac{2}{\cancel{4+\omega^2}}} = \operatorname{arctg}\frac{\omega}{2}$$





c)

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

$$= \int_{-\infty}^{\infty} |e^{2t}\mu(t)|^2\,dt = \int_{-\infty}^{0} e^{4t}\,dt = \frac{e^{4t}}{4}\Bigg|_{-\infty}^{0} = \frac{1}{4}$$

d)

$$E_x = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(e^{j\omega})|^2\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|\frac{1}{2-j\omega}\right|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{4+\omega^2}\,d\omega =$$

$$= \frac{1}{2\pi}\frac{1}{2}\operatorname{arctg}\frac{1}{2}\omega\Bigg|_{-\infty}^{\infty} = \frac{1}{2}\frac{1}{2\pi}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = \frac{\pi}{2}\cdot\frac{1}{2\pi} = \frac{1}{4}$$

5. B

$$x(t) = e^{4t}\mu(-t)$$

a) Fourierova transformacija

$$X(j\omega) = \int_{-\infty}^{\infty} e^{4t}\mu(-t)\, e^{-j\omega t}\,dt$$

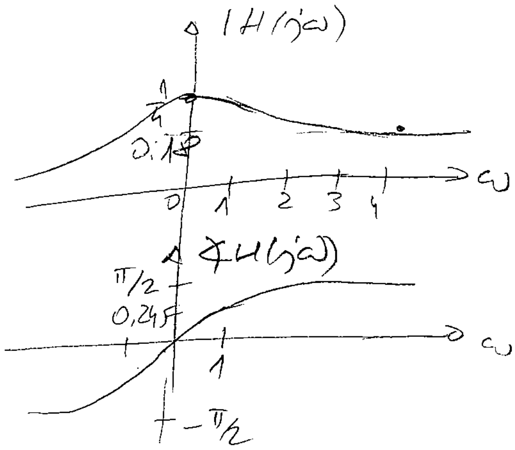
$$= \int_{-\infty}^{0} e^{4t} e^{-j\omega t}\,dt$$

$$= \int_{-\infty} e^{(4-j\omega)t}\,dt$$

$$= \frac{e^{(4-j\omega)t}}{4-j\omega}\Big|_{-\infty}^{0} = \frac{1}{4-j\omega} = \frac{4+j\omega}{16+\omega^2}$$

b) $$|H(j\omega)| = \sqrt{\frac{1}{16+\omega^2}}$$

$$\sphericalangle H(j\omega) = \mathrm{arctg}\,\frac{\cancel{\frac{\omega}{16+\omega^2}}}{\frac{4}{16+\omega^2}} = \mathrm{arctg}\,\frac{\omega}{4}$$



c) $$E_x = \int_{-\infty}^{\infty} |x(t)|^2\,dt$$

$$= \int_{-\infty}^{\infty} |e^{4t}\mu(-t)|^2\,dt$$

$$= \int_{-\infty}^{0} e^{8t}\,dt = \frac{e^{8t}}{8}\Big|_{-\infty}^{0} = \frac{1}{8}$$

d) $$E_x = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \left|\frac{1}{4-j\omega}\right|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{1}{16+\omega^2}\,d\omega$$

$$= \frac{1}{2\pi}\,\frac{1}{4}\,\mathrm{arctg}\,\frac{\omega}{4}\Big|_{-\infty}^{\infty} = \frac{1}{2\pi}\cdot\frac{1}{4}\left(\frac{\pi}{2}+\frac{\pi}{2}\right) = \frac{1}{8}$$