SiS AUDITORNE

datum / date 4. 5. 2007.

$$y''(t) + a_1 y'(t) + a_2 y(t) = b_0 u''(t) + b_1 u'(t) + b_2 u(t) \quad \Big/ \int_{0^-}^{t}$$

$$y'(t) - \overset{\text{konst}}{y'(0^-)} + a_1 y(t) - a_1 y(0^-) + a_2 \int_{0^-}^{t} y(\tau)\,d\tau$$
$$= b_0 u'(t) + b_1 u(t) + b_2 \int_{0^-}^{t} u(\tau)\,d\tau \quad \Big/ \int_{0^-}^{t}$$

$$\int_{0^-}^{t} y'(\tau)\,d\tau - \overset{\text{konst}}{y'(0^-)} \int_{0^-}^{t} d\tau + a_1 \int_{0^-}^{t} y(\tau)\,d\tau - a_1 y(0^-) \int_{0^-}^{t} d\tau$$
$$+ a_2 \int_{0^-}^{t} \int_{0^-}^{\tau} y(\lambda)\,d\lambda\,d\tau = b_0 \int_{0^-}^{t} u'(\tau)\,d\tau + b_1 \int_{0^-}^{t} u(\tau)\,d\tau +$$
$$+ b_2 \int_{0^-}^{t} \int_{0^-}^{\tau} u(\lambda)\,d\lambda\,d\tau$$

uvrstimo $t = 0^+$ $\quad$ (reminder: $\int_{0^-}^{t} y'(\tau)\,d\tau = y(0^+) - y(0^-)$)

$$y(0^+) - y(0^-) - \cancel{y'(0^-) \int_{0^-}^{0^+} d\tau} + \cancel{a_1 \int_{0^-}^{0^+} y(\tau)\,d\tau} + \cancel{a_2 \int_{0^-}^{0^+} \int_{0^-} y(\lambda)\,d\lambda\,dt}$$
$$= b_0 u(0^+)$$

$$\boxed{y(0^+) - y(0^-) = b_0 u(0^+)}$$

$$y'(t) - y'(0^-) + a_1 y(t) - a_1 y(0^-) = b_0 u'(t) + b_1 u(t) + b_2 \int_{0^-}^{t} u(\tau)\,d\tau$$

$$\boxed{y'(0^+) - y'(0^-) + a_1 y(0^+) - a_1 y(0^-) = b_0 u'(0^+) + b_1 u(0^+)}$$

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Zad.

$y''(t) + 2y'(t) + y(t) = u'(t) - u(t)$ $\qquad u(t) = \cos(t)\,\mu(t)$

određuje prirodni odziv $\qquad u'(t) = -\sin(t)$

POČETNI UVJETI: $y'(0^-) = 1 \quad y(0^-) = -1$

$y_t(t) = y_h(t) + y_p(t)$ totalni odziv = homogeno (prirodno) + partikularno (prisilno)

$y_h(t) = Ce^{st} \quad y_h'(t) = Cse^{st}$

$y_h''(t) = Cs^2e^{st}$ - naći potrebne derivacije hom. jedn.

$y_h''(t) + 2y_h'(t) + y_h(t) = 0$

$Cs^2e^{st} + 2Cse^{st} + Ce^{st} = 0$ - odrediti 's' (karakteristične frekvencije)

$Ce^{st}[s^2 + 2s + 1] = 0$

$s^2 + 2s + 1 = 0 \Rightarrow \begin{matrix} s_1 = -1 \\ s_2 = -1 \end{matrix}$ sada smo odredili karakt. frekvencije

$y_h(t) = C_1e^{st} + C_2 \cdot te^{st} = C_1e^{-t} + C_2te^{-t}$ - homogeno rješenje

$C_1, C_2, \dots, C_m$ možemo odrediti tek nakon što smo odredili totalni odziv

$y_p(t) = K_1\cos(\omega t) + K_2\sin(\omega t)$ (ovisi o vrsti pobude)

$= K_1\cos(t) + K_2\sin(t)$

$y_p'(t) = -K_1\sin(t) + K_2\cos(t)$ - naći potrebne derivacije part. jedn.

$y_p''(t) = -K_1\cos(t) - K_2\sin(t)$

- uvrstiti u početnu jednadžbu

$-K_1\cos(t) - K_2\sin(t) - 2K_1\sin(t) + 2K_2\cos(t) + K_1\cos(t) + K_2\sin(t) = -\sin(t) - \cos(t)$

$\cos(t)(\cancel{-K_1} + 2K_2 \cancel{+ K_1}) + \sin(t)(\cancel{-K_2} - 2K_1 \cancel{+ K_2}) = -\sin(t) - \cos(t)$

$2K_2 = -1 \rightarrow K_2 = -\frac{1}{2}$ - našli smo koeficijente za part. rješenje

$-2K_1 = -1 \qquad K_1 = \frac{1}{2}$

$y_p = \frac{1}{2}\cos(t) - \frac{1}{2}\sin(t)$ - partikularno rješenje

$y_t(t) = C_1e^{-t} + C_2te^{-t} + \frac{1}{2}\cos(t) - \frac{1}{2}\sin(t)$

- preostaje nam pronaći C_1 i C_2

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$y(0^+) - y(0^-) = b_0 u(0^+) = 0$ (jer je koef. uz 2. der. $u(t) = 0$)

$\rightarrow y(0^+) = y(0^-) = -1$

$y'(0^+) - y'(0^-) + \cancel{a_1 y(0^+)} - \cancel{a_1 y(0^+)} = \cancel{b_0 u'(0^+)} + b_1 u(0^+)$

$y'(0^+) - y'(0^-) = 1$

$y'(0^+) = 1 + y'(0^-) = 1 + 1 = 2$

$y_t(t) = C_1 e^{-t} + C_2 t e^{-t} + \frac{1}{2}\cos(t) - \frac{1}{2}\sin(t)$ — deriviramo i

$y_t'(t) = -C_1 e^{-t} + C_2 e^{-t} - C_2 t e^{-t} - \frac{1}{2}\sin(t) - \frac{1}{2}\cos(t)$ uvrstimo poč. uvjete

$y(t)(0^+) = -1 = C_1 + \frac{1}{2} \rightarrow C_1 = -\frac{3}{2}$

$y_t'(0^+) = 2 = -C_1 + C_2 - \frac{1}{2} = \frac{3}{2} + C_2 - \frac{1}{2} = 2$

$C_2 + 1 = 2$ — našli smo koeficijente za hom. rješenje

$C_2 = 1$

$$\boxed{y_t(t) = -\frac{3}{2} e^{-t} + t e^{-t} + \frac{1}{2}\cos(t) - \frac{1}{2}\sin(t)}$$

— totalni odziv

vrijedi za $t \geq 0$

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Isti zad., drugi način

$y_t(t) = y_0(t) + y_m(t) = y_h(t) + y_p(t)$ POČ. UVJETI: $y'(0^-) = 1$

↳ mirni sustav $y(0^-) = -1$

↳ nepobuđeni sustav

$y_0(t) = C_1 e^{-t} + C_2 t e^{-t}$

$y(0^+) = y(0^-) + \cancel{b_0 u(0^+)}$ - nema pobude pa je zato skriženo

$y'(0^+) - y'(0^-) + \cancel{a_1 y(0^+) - a_1 y(0^-)} = \cancel{b_0 u'(0^+) + b_1 u(0^+)}$

$y'(0^+) = y'(0^-)$

$y_0'(t) = -C_1 e^{-t} + C_2 e^{-t} - C_2 t e^{-t}$

$y(0^+) = -1 = C_1$

$y'(0^+) = 1 = -C_1 + C_2 \rightarrow C_2 = 0$

$y_0(t) = -e^{-t}$

$y_m(t) = C_1 e^{-t} + C_2 t e^{-t} + \frac{1}{2}\cos t - \frac{1}{2}\sin t$

$y(0^-) = y'(0^-) = 0$

$y(0^+) - y(0^-) + \cancel{b_0 u(0^+)} = 0$

$y'(0^+) - y'(0^-) + a_1 y(0^+) - a_1 y(0^-) = \cancel{b_0 u'(0^+)} + b_1 u(0^+)$

$y'(0^+) = b_1 u(0^+) = 1 \cdot 1 = 1$

$y_m'(t) = -C_1 e^{-t} + C_2 e^{-t} - C_2 t e^{-t} - \frac{1}{2}\sin t - \frac{1}{2}\cos t$

$y_m(0^+) = 0 = C_1 + \frac{1}{2} \rightarrow C_1 = -\frac{1}{2}$

$y_m'(0^+) = 1 = -C_1 + C_2 - \frac{1}{2}$

$1 = \frac{1}{2} + C_2 - \frac{1}{2} \rightarrow C_2 = 1$

$y_m(t) = -\frac{1}{2} e^{-t} + t e^{-t} + \frac{1}{2}\cos t - \frac{1}{2}\sin t$

$y_t(t) = -e^{-t} - \frac{1}{2} e^{-t} + t e^{-t} + \frac{1}{2}\cos t - \frac{1}{2}\sin t$

$= -\frac{3}{2} e^{-t} + t e^{-t} + \frac{1}{2}\cos t - \frac{1}{2}\sin t$

$u(t)=\delta(t)$

$A(D)\,y(t)=B(D)\cdot u(t)$

$\hookrightarrow (D^N+a_1D^{N-1}+\dots+a_{N-1}D+a_N)\,y(t)=(b_{N-M}D^M+b_{N-M-1}D^{N-1})$ - derivator

$h(t)$ - impulsni odziv

$h(t)=A_0\,\delta(t)+\sum_{j=1}^{N}C_je^{s_jt}$

za $N=M$ dirac postoji, za $N\neq M$ ne $(A_0=0)$

$(D^N+a_1D^{N-1}+\dots+a_N)\,h_A(t)=\delta(t)$

$h(t)=(b_{N-M}D^M+b_{N-M+1}D^{M-1}+b_{n-1}D+b_N)\,h_A(t)$

$h_A^{(N-1)}(t)=1$

Treći način rješavanja onog zadatka

$y''(t)+2y'(t)+y(t)=u'(t)-u(t)$

$h_A(t)=Ce^{st}$

$s^2+2s+1=0 \Rightarrow s_{1,2}=-1 \Rightarrow h_A(t)=C_1e^{-t}+C_2te^{-t}$

$h_A(0^+)=0$

$h_A'(0^+)=1$

$h_A'(t)=-C_1e^{-t}+C_2e^{-t}-C_2te^{-t}$

$h_A(0)=0=C_1+C_2 \rightarrow C_1=0$

$h_A'(0)=1=\quad C_2 \rightarrow C_2=1$

$\boxed{h_A(t)=te^{-t}}$

$h(t)=h_A'(t)-h_A(t)=e^{-t}-te^{-t}-te^{-t}=e^{-t}-2te^{-t}$ - impulsni odziv sustava

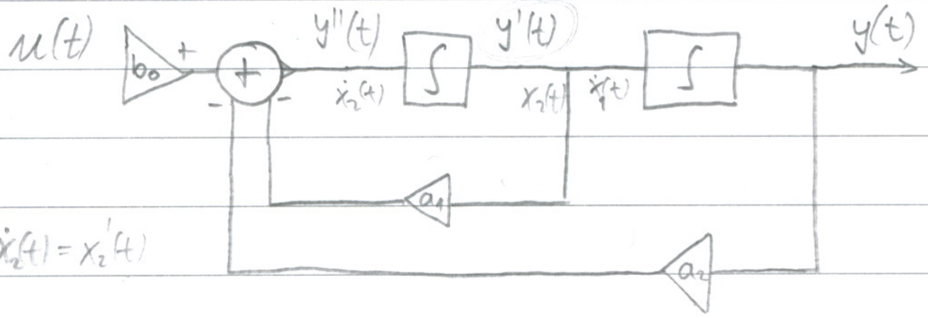
$y_m(t)=h(t)*u(t) \qquad h(t)*u(t)=\int_{-\infty}^{\infty}h(\tau)u(t-\tau)\,d\tau$

$$y''(t) + a_1 y'(t) + a_2 y(t) = b_0 u(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix} u(t)$$

$$y(t) = [c_1 \quad c_2] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + d\,u(t)$$

$$y''(t) = b_0 u(t) - a_1 y'(t) - a_2 y(t)$$



$\dot{x}_2(t) = x_2'(t)$

$$\dot{x}_2(t) = y''(t)$$
$$x_2(t) = y'(t) = \dot{x}_1(t)$$
$$x_1(t) = y(t)$$

$$\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ b_0 \end{bmatrix} \cdot u(t)$$

$$\dot{x}_2(t) = b_0 u(t) - a_1 \overset{x_2(t)}{y'(t)} - a_2 \overset{x_1(t)}{y(t)} = b_0 u(t) - a_1 x_2(t) - a_2 x_1(t)$$

$$\dot{x}_1(t) = x_2(t)$$

$$y(t) = [\,1 \quad 0\,] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

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Zad

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} u(t)$$

$$\cancel{y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + 0\,u(t)}$$

$x_1(t) = K_1 e^{st} \Rightarrow \dot{x}_1(t) = sK_1 e^{st}$

$x_2(t) = K_2 \cdot e^{st} \Rightarrow \dot{x}_2(t) = sK_2 e^{st}$

$sK_1 e^{st} = K_2 e^{st}$

$sK_2 e^{st} = 3K_1 e^{st} - 2K_2 e^{st} + 4u(t) \;/ e^{st}$

$sK_1 = K_2$

$sK_2 = 3K_1 - 2K_2$

$s^2 K_1 = 3K_1 - 2sK_1 \quad | : K_1, \quad K_1 \neq 0$

$s^2 + 2s - 3 = 0 \Rightarrow s_1 = 1 \qquad s_2 = -3$

$x_1(t) = K_{11} e^{t} + K_{12} e^{-3t}$

$x_2(t) = K_{21} e^{t} + K_{22} e^{-3t}$

$$\left.\begin{aligned} x_1(0^-) &= 1 \\ x_2(0^-) &= 0 \\ \dot{x}_1(0^-) &= 0 \\ \dot{x}_2(0^-) &= 0 \end{aligned}\right\} \text{zadano}$$

$x_1(0^-) = K_{11} + K_{12} = 1$

$\dot{x}_1(t) = K_{11} e^{t} - 3K_{12} e^{-3t}$

$\dot{x}_1(0^-) = K_{11} - 3K_{12} = 0$

$x_2(0^-) = K_{21} + K_{22} = 0$

$x_2(t) = K_{21} e^{t} - 3K_{22} e^{-3t}$

$\dot{x}_2(0^-) = K_{21} - 3K_{22} = 0$

$K_{11} = 3K_{12} = \frac{3}{4}$

$3K_{12} + K_{12} = 1$

$K_{12} = \frac{1}{4}$

$x_1(t) = \frac{3}{4} e^{t} + \frac{1}{4} e^{-3t}$

$x_2(t) = 0$

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$$y''(t) + a_1 y'(t) + a_2 y(t) = b_0 u(t)$$

$s^2 + a_1 s + a_2 = 0$

$s_{1,2} = \frac{-a_1 \pm \sqrt{a_1^2 - 4}}{2} = \frac{-\cancel{2}\delta\omega_n \pm \sqrt{4\delta^2\omega^2 - \cancel{4}1\omega_n}}{\cancel{2}}$

$= -\delta\omega_n \pm \omega_n\sqrt{\delta^2 - 1} = s_{1,2}$

$a_1 = 2\delta\omega_n$

$a_2 = \omega_n^2$

$b_0 = A\omega_n^2$

$\delta = 1 \Rightarrow s_{1,2} = -\omega_n \pm \cancel{\omega_n\sqrt{0}}$

KRIT. PRIGUŠEN

$\delta = 0 \Rightarrow s_{1,2} = \cancel{0} \pm j\omega_n$

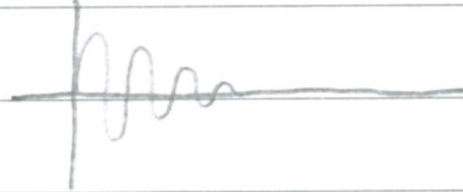
NEPRIGUŠEN SUSTAV

$\delta > 1 \Rightarrow s_{1,2} = -\delta\omega_n \pm \omega_n\sqrt{\delta^2 - 1}$ NESTABILAN

NADKRITIČNO PRIGUŠEN SUSTAV

$\delta < 1 \Rightarrow s_{1,2} = -\delta\omega_n \pm j\omega_n\sqrt{1 - \delta^2}$

PODKRITIČNO PRIGUŠEN SUSTAV



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$$y''(t) + 0.2\,y'(t) + 0.16\,y(t) = 0$$

$$a_2 = \omega_n^2 = 0{,}16$$

$$\omega_n = 0.4$$

$$a_1 = 2\delta\omega_n = 0.2$$

$$\delta = \frac{0{,}2}{0{,}8} = \frac{1}{4} = 0{,}25$$

$$s^2 + 0.2s + 0.16 = 0$$

$$y(t) = K_1 e^{(-0.1 + j0.4\sqrt{1-0.25^2})t} + K_2 e^{(-0.1 - j0.4\sqrt{1-0.25^2})t}$$

$$y(t) = K_1 e^{(-0.1 + j0.39)t} + K_2 e^{(-0.1 - j\cdot 0.39)t}$$

$$\cos\omega t = \frac{e^{j\omega t} + e^{-j\omega t}}{2} \qquad \sin\omega t = \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$y(t) = e^{-0.1t}\left(K_1 e^{j0.39t} + K_2 e^{-j0.39t}\right)$$