1. $y(n) - \frac{1}{4} y(n-1) = u(n)$ A

a) $H(z) = \frac{1}{1-\frac{1}{4} z^{-1}} = \frac{z}{z-\frac{1}{4}}$

POL $z - \frac{1}{4} = 0$

$z = \frac{1}{4}$ $|z| < 1$ SUSTAV JE STABILAN

b) $H(e^{j\Omega}) = \frac{1}{1-\frac{1}{4} e^{-j\Omega}} = \frac{1}{1-\frac{1}{4}\cos\Omega + \frac{1}{4}\sin\Omega \cdot j}$

$|H(e^{j\Omega})| = \frac{1}{\sqrt{(1-\frac{1}{4}\cos\Omega)^2 + (\frac{1}{4}\sin\Omega)^2}}$

$= \frac{1}{\sqrt{1-\frac{1}{2}\cos\Omega + \frac{1}{16}\cos^2\Omega + \frac{1}{16}\sin^2\Omega}} = \frac{1}{\sqrt{\frac{17}{16} - \frac{1}{2}\cos\Omega}}$

c) $u(n) = 2\cos\left(\frac{\pi}{2} n + \frac{\pi}{4}\right)$

$y_p(n) = 2 \cdot |H(e^{j\Omega})| \cdot \cos\left(\Omega n + \frac{\pi}{4} + \angle H(e^{j\Omega})\right)$

$H(e^{j\frac{\pi}{2}}) = \frac{1}{1-\frac{1}{4}\cos\frac{\pi}{2} + \frac{1}{4}\sin\frac{\pi}{2}\cdot j} = \frac{1}{1+\frac{1}{4} j}$

$|H(e^{j\frac{\pi}{2}})| = \frac{1}{\sqrt{1+\frac{1}{16}}} = \frac{4}{\sqrt{17}} = 0.97$

$\angle H(e^{j\frac{\pi}{2}}) = -\operatorname{arctg}\frac{\frac{1}{4}}{1} = -\operatorname{arctg}\frac{1}{4} = -0.25$

$y_p(n) = \frac{8}{\sqrt{17}} \cos\left(\frac{\pi}{2} n + \frac{\pi}{4} - 0.25\right)$

$\boxed{y_p(n) = 1.94 \cos\left(\frac{\pi}{2} n + 0.54\right)}$

A

$$y'(t) + 10\, y(t) = u(t)$$

a) IMPULSNI ODZIV

$h_a(0^+) = 1$

$h(t) = ?$

$h(t) = e^{-10t}\,\mu(t)$

$s + 10 = 0$

$s = -10$

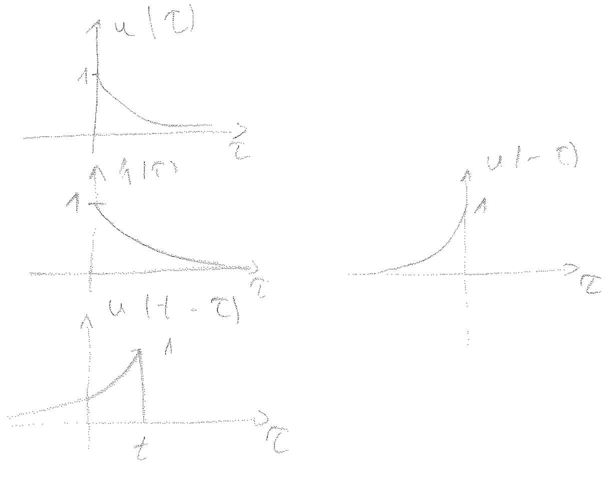
$y_h(t) = C e^{-10t}$

$h_a(t) = C e^{-10t}$

$h_a(0^+) = C = 1$

$h_a(t) = e^{-10t}$

b) $u(t) = e^{-2t}\mu(t)$



$$y(t) = h(t) * u(t)$$

$$= \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau$$

$$= \int_{-0}^{t} e^{-10\tau}\, e^{-2(t-\tau)} d\tau$$

$$= e^{-2t}\int_{0}^{t} e^{-8\tau}\, d\tau$$

$$= e^{-2t} \left. \frac{e^{-8\tau}}{-8} \right|_{0}^{t} = e^{-2t} \cdot \frac{-1}{8}\left(e^{-8t} - 1\right)$$

$$y(t) = -\frac{1}{8} e^{-10t} + \frac{1}{8} e^{-2t} \quad \text{za} \quad t \geq 0$$

A

c) nepobuđeni

$y(0^-)=2$

$y_n(t) = Ce^{-10t}$

$y_n(0^-) = C = 2$

$y_0(t) = 2e^{-10t}$

totalni

$y_t(t) = y_m(t) + y_0(t)$

$= -\frac{1}{8}e^{-10t} + \frac{1}{8}e^{-2t} + 2e^{-10t}$

$\boxed{y_t(t) = \frac{15}{8}e^{-10t} + \frac{1}{8}e^{-2t}}$ za $t>0$

3. $y(n) - \frac{1}{6} y(n-1) - \frac{1}{6} y(n-2) = u(n) - 3u(n-1) + 2u(n-2)$

a) $Y(z) - \frac{1}{6} z^{-1} Y(z) - \frac{1}{6} Y(z) z^{-2} = U(z) - 3z^{-1} U(z) + 2z^{-2} U(z)$

$$H(z) = \frac{1-3z^{-1}+2z^{-2}}{1-\frac{1}{6}z^{-1}-\frac{1}{6}z^{-2}} = \frac{z^2-3z+2}{z^2-\frac{1}{6}z-\frac{1}{6}}$$

$$= \frac{z^2-3z+2}{(z-\frac{1}{2})(z+\frac{1}{3})}$$

$$\frac{H(z)}{z} = \frac{1}{z}\,\frac{z^2-3z+2}{(z-\frac{1}{2})(z+\frac{1}{3})} = \frac{A}{z} + \frac{B}{z-\frac{1}{2}} + \frac{C}{z+\frac{1}{3}}$$

$$Az^2 - \frac{1}{6}Az - \frac{1}{6}A + Bz^2 + \frac{1}{3}Bz + Cz^2 - \frac{1}{2}Cz = z^2 - 3z + 2$$

$A + B + C = 1$

$-\frac{1}{6}A + \frac{1}{3}B - \frac{1}{2}C = -3$

$-\frac{1}{6}A = 2$

$A = -12$

$B + C = 13$

$\frac{1}{3}B - \frac{1}{2}C = -3 + \frac{1}{6}\cdot(-12) = -5$

$\frac{2}{3}B - C = -10$

$\frac{5}{3}B = +3$

$B = \frac{9}{5}$

$C = 13 - B = \frac{56}{5}$

$$H(z) = -12 + \frac{9}{5}\,\frac{z}{z-\frac{1}{2}} + \frac{56}{5}\,\frac{z}{z+\frac{1}{3}}$$

$$h(n) = -12\,\delta(n) + \left[\frac{9}{5}\left(\frac{1}{2}\right)^n + \frac{56}{5}\left(\frac{1}{3}\right)^n\right]\mu(n)$$

b) $u(n) = \{\underline{2}, 0, 2, 0, \dots\}$

$$U(z) = 2z^0 + 2z^{-2} + 2z^{-4} + \dots = 2\cdot\sum_{k=0}^{\infty}(z^{-2})^k$$

$$= 2\cdot\frac{1}{1-z^{-2}} = 2\,\frac{z^2}{z^2-1}$$

$$Y(z) = H(z)\cdot U(z) = \frac{(z-1)(z-2)}{(z-\frac{1}{2})(z+\frac{1}{3})}\cdot\frac{2z^2}{(z-1)(z+1)} = \frac{2z^2(z-2)}{(z-\frac{1}{2})(z+\frac{1}{3})(z+1)}$$

$$\frac{Y(z)}{z} = \frac{2z^2-4z}{(z-\frac{1}{2})(z+\frac{1}{3})(z+1)} = \frac{A}{z-\frac{1}{2}} + \frac{B}{z+\frac{1}{3}} + \frac{C}{z+1}$$

$$Az^2 + \frac{1}{3}Az + Az + \frac{1}{3}A + Bz^2 - \frac{1}{2}Bz + Bz - \frac{1}{2}B + Cz^2 - \frac{1}{6}Cz - \frac{1}{6}C = 2z^2 - 4z$$

$A + B + C = 2$

$\frac{1}{3}A + A - \frac{1}{2}B + B - \frac{1}{6}C = -4$

$\frac{1}{3}A - \frac{1}{2}B - \frac{1}{6}C = 0$

$\frac{4}{3}A + \frac{1}{2}B - \frac{1}{6}C = -4$

$A = -\frac{6}{5}, \quad B = -\frac{14}{5}, \quad C = 6$

$$Y(z) = -\frac{6}{5}\,\frac{z}{z-\frac{1}{2}} - \frac{14}{5}\,\frac{z}{z+\frac{1}{3}} + \frac{6z}{z+1}$$

$$y(n) = -\frac{6}{5}\left(\frac{1}{2}\right)^n - \frac{14}{5}\left(-\frac{1}{3}\right)^n + 6(-1)^n \quad \text{za } n \geq 0$$

1

4. $y''(t) + 11y'(t) + 10y(t) = u'(t) - 2u(t)$

$y(0^-) = 1$

$y'(0^-) = 2$

$u(t) = e^t \mu(t)$

$u(t) = e^t$
$u'(t) = e^t$

homogeno

$s^2 + 11s + 10 = 0$

$(s+10)(s+1) = 0$

$s_1 = -10 \quad s_2 = -1$

$y_h(t) = C_1 e^{-10t} + C_2 e^{-t}$

partikularno

$y_p(t) = Ce^t$

$y_p'(t) = Ce^t$

$y_p''(t) = Ce^t$

$Ce^t + 11Ce^t + 10Ce^t = e^t - 2e^t$

$22Ce^t = -e^t$

$22C = -1$

$C = -\frac{1}{22}$

$y_p = -\frac{1}{22} e^t \mu(t)$

totalni odziv

$y_t(t) = C_1 e^{-10t} + C_2 e^{-t} - \frac{1}{22} e^t \qquad t \geq 0$

početni uvjeti

$y(0^+) - y(0^-) = 0 \cdot u(0^+) \longrightarrow y(0^+) = 1$

$y'(0^+) - y'(0^-) + 11(y(0^+) - y(0^-)) = 0 + 1(u(0^+) - u(0^-))$

$y'(0^+) - 2 = 1 \cdot 1$

$y'(0^+) = 3$

4.

totalni odziv

$$y_t(0^+) = C_1 + C_2 - \frac{1}{22} = 1$$

$$y_t'(t) = -10C_1 e^{-10t} - C_2 e^{-t} - \frac{1}{22} e^{t}$$

$$y_t'(0^+) = -10C_1 - C_2 - \frac{1}{22} = 3$$

$$-9C_1 - \frac{\cancel{2}\,1}{\cancel{22}\,11} = 4$$

$$-9C_1 = 4 + \frac{1}{11} = \frac{44+1}{11} = \frac{45}{11}$$

$$C_1 = \frac{45}{-9\cdot 11} = \frac{-5}{11}$$

$$C_2 = 1 + \frac{1}{22} - C_1 = \frac{23}{22} + \frac{5}{11} = \frac{23+10}{22} = \frac{33}{22} = \frac{3}{2}$$

$$\boxed{y_t(t) = \left[-\frac{5}{11} e^{-10t} + \frac{3}{2} e^{-t} - \frac{1}{22} e^{t}\right]\mu(t)}$$

A

5 $\hfill$ 1

a) $h(n) = \left(\frac{1}{3^n} + \frac{1}{9^n}\right)\mu(n)$

$$H(z) = \frac{z}{z-\frac{1}{3}} + \frac{z}{z-\frac{1}{9}} = \frac{z^2 - \frac{1}{9}z + z^2 - \frac{1}{3}z}{(z-\frac{1}{3})(z-\frac{1}{9})} = \frac{2z^2 - \frac{4}{9}z}{z^2 - \frac{4}{9}z + \frac{1}{27}}$$

b) $u(n) = 3^n$

$y(n) = H(z)\cdot U z^n \longrightarrow z = 3$

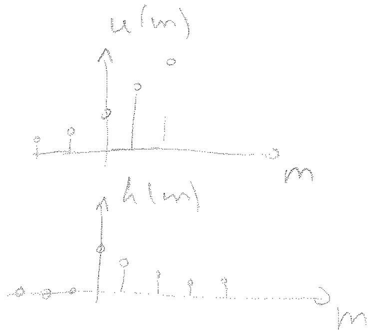
$y(n) = H(3)\cdot U \cdot 3^n$

$\quad = H(3)\cdot 3^n$

$$H(3) = \frac{2\cdot 3^2 - \frac{4}{9}\cdot 3}{3^2 - \frac{4}{9}\cdot 3 + \frac{1}{27}} = \frac{\frac{162-12}{9}}{\frac{243-36+1}{27}} = \frac{\frac{150}{\cancel{9}}}{\frac{208}{\cancel{27}\,3}} = \frac{150\cdot 3}{208} = \frac{225}{104}$$

$y(n) = \frac{225}{104}\cdot 3^n$

c) $u(n) = 3^n$



$y(n) = u(n) * h(n)$

$$= \sum_{m=-\infty}^{\infty} u(m)\, h(n-m)$$

$$= \sum_{m=-\infty}^{\infty} h(m)\, u(n-m)$$

$$= \sum_{m=-\infty}^{\infty}\left[\left(\frac{1}{3}\right)^m + \left(\frac{1}{9}\right)^m\right]\mu(m)\cdot 3^{n-m}$$

$$= 3^n \sum_{m=0}^{\infty} 3^{-2m} + 3^{(-2-1)m} = 3^n\left[\frac{1}{1-3^{-2}} + \frac{1}{1-3^{-3}}\right]$$

$$= 3^n\left[\frac{1}{1-\frac{1}{9}} + \frac{1}{1-\frac{1}{27}}\right] = 3^n\left[\frac{1}{\frac{8}{9}} + \frac{1}{\frac{26}{27}}\right]$$

$$= 3^n\left[\frac{9}{8} + \frac{27}{26}\right] = 3^n\cdot\frac{117+108}{104} = \frac{225}{104}\cdot 3^n$$