TJEDAN 11.
SIGNALI I SUSTAVI

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GRUPA 2.E2

1. $y(n+1) + 2y(n) = u(n)$

$y(0) = 2, \; n \in \mathbb{N}_0$

$y(n) + 2y(n-1) = u(n-1)$

Pišemo:

$u(n) = \alpha u_1(n) + \beta u_2(n) \qquad y_1(n+1) + 2y_1(n) = u_1(n)$

$\qquad\qquad\qquad\qquad\qquad\qquad y_2(n+1) + 2y_2(n) = u_2(n)$

Sustav je linearan ako vrijedi:

$y(n) = S u(n) = \alpha S u_1(n) + \beta S u_2(n)$

Računamo:

$n=1 \qquad y(1) + 2y(0) = u(0)$

$\qquad\qquad y(1) = u(0) - 4 = \alpha u_1(0) + \beta u_2(0) - 4$

$\alpha y_1(1) + \beta y_2(1) \neq \alpha u_1(0) + \beta u_2(0) - 4$

- SUSTAV NIJE LINEARAN

2. $y(n+2) - y(n+1) - y(n) = 0$

$y(0) = 0, \; y(1) = 1$

Pretpostavimo:

$y(n) = cq^n$

$cq^{n+2} - cq^{n+1} - cq^n = 0$

$cq^n(q^2 - q - 1) = 0 \rightarrow q^2 - q - 1 = 0, \quad q_{12} = \frac{1 \pm \sqrt{5}}{2}$

$y(n) = C_1 \cdot \left(\frac{1+\sqrt{5}}{2}\right)^n + C_2\left(\frac{1-\sqrt{5}}{2}\right)^n$

$y(0) = 0 \rightarrow C_1 + C_2 = 0, \quad C_1 = -C_2$

$y(1) = 1 \rightarrow 1.618\, C_1 + 0{,}618\, C_2 = 1$

$C_1 = \frac{\sqrt{5}}{5}, \quad C_2 = \frac{-\sqrt{5}}{5}$

$$y(n) = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$$

$y(0)=0,\ y(1)=1,\ y(2)=1,\ y(3)=2,\ y(4)=3,\ y(5)=5$

$\Rightarrow$ NE prepoznajem da je ovo Fibonaccijev niz!

3. $y(n+3) - y(n) = 0$

$y(0) = y(1) = 0,\ y(2) = 1$

$y(n) = cq^n$

$cq^{n+3} - cq^n = 0$

$cq^n(q^3-1) = 0 \rightarrow (q-1)(q^2+q+1) = 0$

$q_1 = 1$

$q^2+q+1 = 0 \rightarrow q_{2,3} = \frac{-1 \pm j\sqrt{3}}{2}$

$q_2 = -\frac{1}{2} + j\frac{\sqrt{3}}{2} \rightarrow q_2 = e^{j\frac{2\pi}{3}}$

$q_3 = -\frac{1}{2} - j\frac{\sqrt{3}}{2} \rightarrow q_3 = e^{-j\frac{2\pi}{3}}$

$y(n) = C_1 \cdot 1^n + C_2 \cdot 1^n \cdot \cos(\frac{2\pi}{3}n) + C_3 \cdot 1^n \cdot \sin(\frac{2\pi}{3}n),\ C_1, C_2, C_3 \in \mathbb{R}$

$y(0) = C_1 + C_2 = 0$

$y(1) = C_1 + C_2 \cdot \frac{-1}{2} + C_3 \cdot \frac{\sqrt{3}}{2} = 0$

$y(2) = C_1 + C_2 \cdot \frac{-1}{2} + C_3 \cdot -\frac{\sqrt{3}}{2} = 0$

$\hookrightarrow C_1 = \frac{1}{3},\ C_2 = -\frac{1}{3},\ C_3 = -\frac{\sqrt{3}}{3}$

$y(n) = \frac{1}{3} - \frac{1}{3}\cos(\frac{2\pi}{3}n) - \frac{1}{\sqrt{3}}\sin(\frac{2\pi}{3}n)$

$y(n+3) = y(n)$, PRVA TRI ČLANA CIJELI BROJEVI,
MOŽEMO ZAKLJUČITI DA SU I
OSTALI ČLANOVI CIJELI BROJEVI

4. (a) $y(0)=0$, $y(1)=1$, $y(2)=2$, $y(3)=1$

$$5y(n)-4y(1)+4y(2)-4y(3)=u(n)$$

(b) $y(n)=3^n+5^n+7\cdot 1^n$

$$q_1=1,\quad q_2=3,\quad q_3=5$$

$$y(n)=cq^n$$

$$cq^{n-3}(q-1)(q-3)(q-5)=0$$
$$cq^{n-3}(q^3-9q^2+23q-15)=0$$

$$y(n)-9y(n-1)+23y(n-2)-15y(n-3)=0$$

POČETNI UVJETI:

$$y(0)=9,\quad y(1)=15,\quad y(2)=41$$

PRIMJER SUSTAVA UZ NAVEDENE POČ. UVJETE:

$$y(n)-9y(n-1)+23y(n-2)-15y(n-3)=u(n)$$

5. $3y(n+2)+6y(n+1)+3y(n)=2u(n+1)-5u(n)$

$$u(n)=\{\dots,\underline{0},0,1,2,1,0,0,\dots\}$$

$$3y(n+2)=-6y(n+1)-3y(n)+2u(n+1)-5u(n)\quad /:3$$
$$y(n+2)=-2y(n+1)-y(n)+\frac{2}{3}u(n+1)-\frac{5}{3}u(n)$$

$y(n)=0,\ n<0$

$n=0$ $\quad y(2)=-2y(1)-y(0)+\frac{2}{3}u(1)-\frac{5}{3}u(0)$

$y(2)=0$

$n=1$ $\quad y(3)=-2y(2)-y(1)+\frac{2}{3}u(2)-\frac{5}{3}u(1)$

$y(3)=\frac{2}{3}$

$n=2$ $\quad y(4)=-2y(3)-y(2)+\frac{2}{3}u(3)-\frac{5}{3}u(2)$

$y(4)=-\frac{5}{3}$

$n=3 \quad y(5) = -2y(4) - y(3) + \frac{2}{3}u(4) - \frac{5}{3}u(3)$

$y(5) = 0$

$n=4 \quad y(6) = -2y(5) - y(4) + \frac{2}{3}u(5) - \frac{5}{3}u(4)$

$y(6) = 0$

$n=5 \quad y(7) = -2y(6) - y(5) + \frac{2}{3}u(6) - \frac{5}{3}u(5)$

$y(7) = 0$

$y(n) = \{\ldots, 0, 0, 0, \frac{2}{3}, -\frac{5}{3}, 0, 0, 0, 0, \ldots\}$

6. $u(n) = \{\ldots, 0, \underline{1}, 2, 3, 4, 5, 6, 7, \ldots\}$

$y(n) = \{\ldots, 0, \underline{0}, -1, 1, 2, 3, 4, 5, 6, \ldots\}$

$u(n) = (n+1)\mu(n)$

$y(n) = (n-1)\mu(n-2) - \delta(n-1)$

$$y(n) = \sum_{m=-\infty}^{\infty} h(m)\, u(n-m) \rightarrow \quad y(n) = \sum_{m=0}^{\infty} h(m)(n+1-m)$$

$$(n-1)\mu(n-2) - \delta(n-1) = \sum_{m=0}^{\infty} h(m) \cdot u(n-m)$$

$n=0 \quad (-1)\cdot\mu(-2) - \delta(-1) = \sum_{m=0}^{\infty} h(m)\cdot u(0-m)$

$h(0) = 0$

$n=1 \quad (1-1)\mu(1-2) - \delta(1-1) = \sum_{m=0}^{\infty} h(m)\cdot u(1-m)$

$h(0)\cdot u(1) + h(1)\cdot u(0) = -1$

$h(1) = -1$

$n=2 \quad (2-1)\mu(2-2) - \delta(2-1) = \sum_{m=0}^{\infty} h(m)\cdot u(2-m)$

$h(0)u(2) + h(1)u(1) + h(2)u(0) = 1$

$(-1)\cdot 2 + h(2) = 1$

$h(2) = 3$

$n=3 \quad (3-1)\mu(3-2) - \delta(3-1) = \sum_{m=0}^{\infty} h(m)\cdot u(3-m)$

$h(0)u(3) + h(1)u(2) + h(2)u(1) + h(3)u(0) = 2$

$(-1)\cdot 3 + 3\cdot 2 + h(3) = 2$

$h(3) = -1$

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$n=4 \quad (4-1)\mu(4-2) - \delta(4-1) = \sum_{m=0}^{\infty} h(m)\, u(4-m)$

$h(0)u(4) + h(1)u(3) + h(2)u(2) + h(3)u(1) + h(4)u(0) = 3$

$(-1)\cdot 4 + 3\cdot 3 + (-1)\cdot 2 + h(4) = 3$

$h(4) = 0$

$n=5 \quad (5-1)\mu(5-2) - \delta(5-1) = \sum_{m=0}^{\infty} h(m)\, u(5-m)$

$h(0)u(5) + h(1)u(4) + h(2)u(3) + h(3)u(2) + h(4)u(1) + h(5)u(0) = 4$

$(-1)\cdot 5 + 3\cdot 4 + (-1)\cdot 3 + h(5) = 4$

$h(5) = 0$

$h(n) = \{\ldots, \underline{0}, -1, 3, -1, 0, 0, \ldots\}$

7. $y(n) - 6y(n-1) + 8y(n-2) = 4u(n)$

$u(n) = 2\mu(n) - 3n\mu(n)$

$y(-1) = 2, \quad y(-2) = 1$

$y(n) - 6y(n-1) + 8y(n-2) = (8-12n)\mu(n)$

$y_h(n) = Cq^n$

HOMOGENO:

$C\cdot q^n - 6Cq^{n-1} + 8Cq^{n-2} = 0 \quad /\cdot q^2$

$Cq^{n+2} - 6Cq^{n+1} + 8Cq^n = 0$

$Cq^n(q^2 - 6q + 8) = 0$

$q_{1,2} = \frac{+6 \pm 2}{2}, \quad q_1 = 4$

$q_2 = 2$

$y_h(n) = C_1\cdot 4^n + C_2\cdot 2^n$

PARTIKULARNO:

$$y_P = (K_0 + K_1 n) \cdot 1^n \mu(n)$$

$$(K_0 + K_1 n) - 6(K_0 + K_1(n-1)) + 8(K_0 + K_1(n-2)) = (8-12n)$$

$$K_0 + K_1 n - 6K_0 - 6K_1 n + 6K_1 + 8K_0 + 8K_1 n - 16K_1 = 8 - 12n$$

$$(3K_0 - 10K_1) + 3K_1 n = 8 - 12n$$

$$K_1 = -4, \quad K_0 = -\frac{32}{3}$$

$$y_P = \left(-\frac{32}{3} + 4n\right)$$

$$y = y_h + y_P$$

$$y = C_1 \cdot 4^n + C_2 \cdot 2^n - \frac{32}{3} + 4n$$

$$y(n) - 6y(n-1) + 8y(n-2) = 4(2\mu(n) - 3n\mu(n))$$

$$y(n) = 6y(n-1) - 8y(n-2) + 8\mu(n) - 12n\mu(n)$$

$$y(0) = 6y(-1) - 8y(-2) + 8\mu(0) - 12 \cdot 0 \cdot \mu(0)$$

$$y(0) = 12$$

$$y(1) = 6y(0) - 8y(-1) + 8\mu(1) - 12\mu(1)$$

$$y(1) = 52$$

$$y(0) = C_1 + C_2 - \frac{32}{3} = 12$$

$$y(1) = 4C_1 + 2C_2 - \frac{32}{3} - 4 = 52$$

$$C_1 = \frac{32}{3}, \quad C_2 = 12$$

TOTALNI ODZIV:

$$y(n) = \left(\frac{32}{3} \cdot 4^n - 12 \cdot 2^n - \frac{32}{3} - 4n\right)\mu(n)$$

PRISILNI ODZIV:

$$y(n) = \left(-\frac{32}{3} - 4n\right)\mu(n)$$

PRIRODNI ODZIV:

$$y(n) = \left(\frac{32}{3} 4^n + 12 \cdot 2^n\right)\mu(n)$$