

$$\textcircled{1.} \quad x(t) = \sin(\omega_0 t) \cdot \left[\mu(t) - \mu\left(t - \frac{2\pi}{\omega_0}\right) \right]$$

$$(a) \quad X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_0^{\frac{2\pi}{\omega_0}} \sin(\omega_0 t) e^{-j\omega t} dt = \dots =$$

$$= \frac{\omega_0 \left(1 - e^{-j \frac{2\pi\omega}{\omega_0}} \right)}{\omega_0^2 - \omega^2}$$

$$(b) \quad \omega_0 = 2\pi \Rightarrow X(j\omega) = \frac{2\pi (1 - e^{-j\omega})}{4\pi^2 - \omega^2} \quad \xrightarrow{\cos\omega - j\sin\omega}$$

$$|X(j\omega)| = 2\pi \frac{\sqrt{(1 - \cos\omega)^2 + \sin^2\omega}}{|4\pi^2 - \omega^2|} = \frac{2\pi \sqrt{2 - 2\cos\omega}}{|4\pi^2 - \omega^2|}$$

$$|X(j\pi)| = \frac{2\pi \sqrt{4}}{3\pi^2} = \frac{4}{3\pi}$$

1. e) $\boxed{E} = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_0^{\frac{2\pi}{\omega_0}} \sin^2(\omega_0 t) dt =$

$$= \int_0^{\frac{2\pi}{\omega_0}} \frac{1 - \cos(2\omega_0 t)}{2} dt = \frac{1}{2} \cdot \frac{2\pi}{\omega_0} - \frac{1}{2} \cdot \frac{\sin(2\omega_0 t)}{2\omega_0} \Big|_0^{\frac{2\pi}{\omega_0}} =$$

$$= \frac{\pi}{\omega_0} - \frac{1}{4\omega_0} (\sin(4\pi) - \sin(0)) = \boxed{\frac{\pi}{\omega_0}}$$

(d) Signal je realan \Rightarrow vrijedi $X^*(j\omega) = X(-j\omega)$.

$$\underline{X^*(j\omega)} = \left(\frac{\omega_0 (1 - e^{-j \frac{2\pi\omega}{\omega_0}})}{\omega_0^2 - \omega^2} \right)^* = \frac{\omega_0}{\omega_0^2 - \omega^2} (1 - e^{j \frac{2\pi\omega}{\omega_0}})$$

$$\underline{X(-j\omega)} = \frac{\omega_0 (1 - e^{j \frac{2\pi}{\omega_0} (-\omega)})}{\omega_0^2 - (-\omega)^2} = \frac{\omega_0}{\omega_0^2 - \omega^2} (1 - e^{j \frac{2\pi\omega}{\omega_0}})$$

$$(2.) x(t) = e^{-|2t+1|}$$

$$\begin{aligned} a) \boxed{X(j\omega)} &= \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt = \int_{-\infty}^{-\frac{1}{2}} e^{2t+1} e^{-j\omega t} dt + \\ &+ \int_{-\frac{1}{2}}^{\infty} e^{-2t-1} e^{-j\omega t} dt = \frac{je^{j\frac{\omega}{2}}}{\omega + j2} - \frac{je^{j\frac{\omega}{2}}}{\omega - j2} = \\ &= je^{j\frac{\omega}{2}} \frac{\omega - j2 - \omega - j2}{\omega^2 + 4} = \boxed{\frac{4e^{j\frac{\omega}{2}}}{\omega^2 + 4}} \end{aligned}$$

ili preko službenog šablonika:

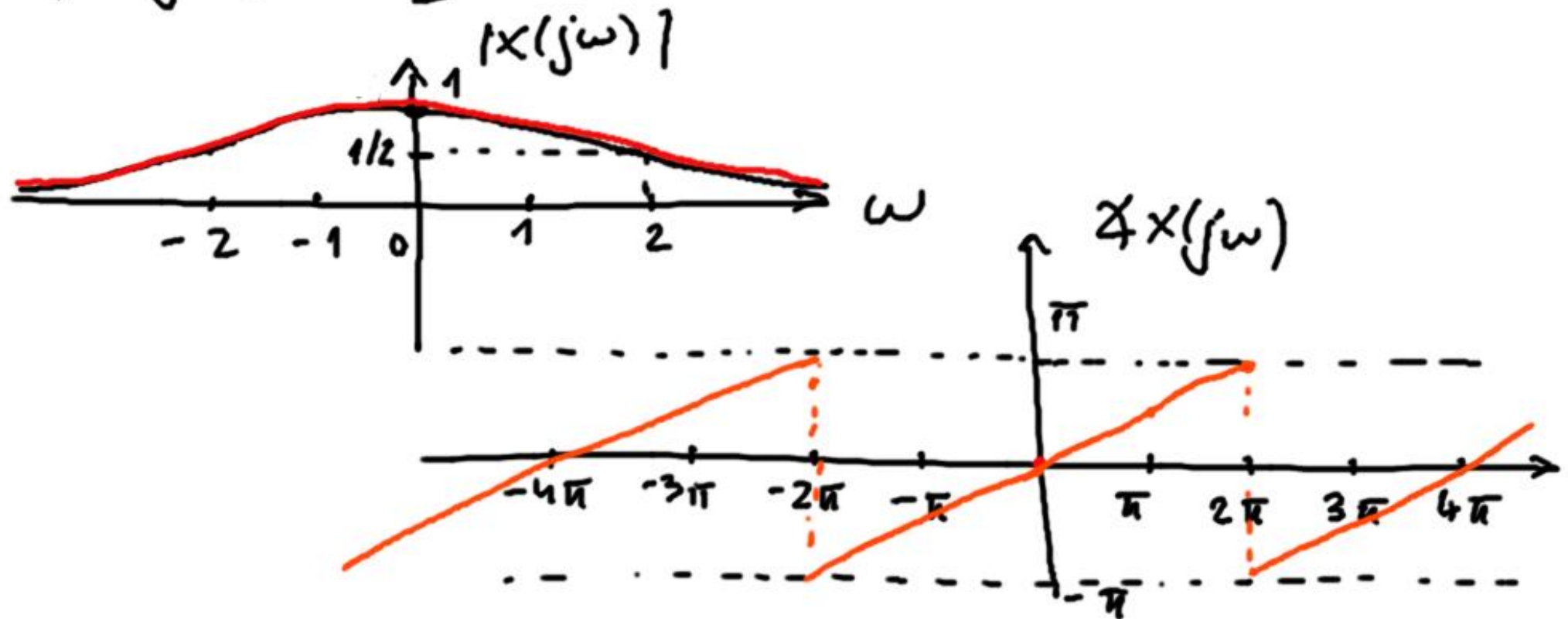
$$e^{-a|t|} \rightarrow \frac{2a}{a^2 + \omega^2} \Rightarrow e^{-2|t|} \rightarrow \frac{2 \cdot 2}{2^2 + \omega^2} = \frac{4}{4 + \omega^2}$$

$$x(t - t_0) \rightarrow X(j\omega) e^{-j\omega t_0} = \frac{4}{4 + \omega^2} \cdot e^{-j\omega \cdot (-\frac{1}{2})} = \boxed{\frac{4e^{j\frac{\omega}{2}}}{\omega^2 + 4}}$$

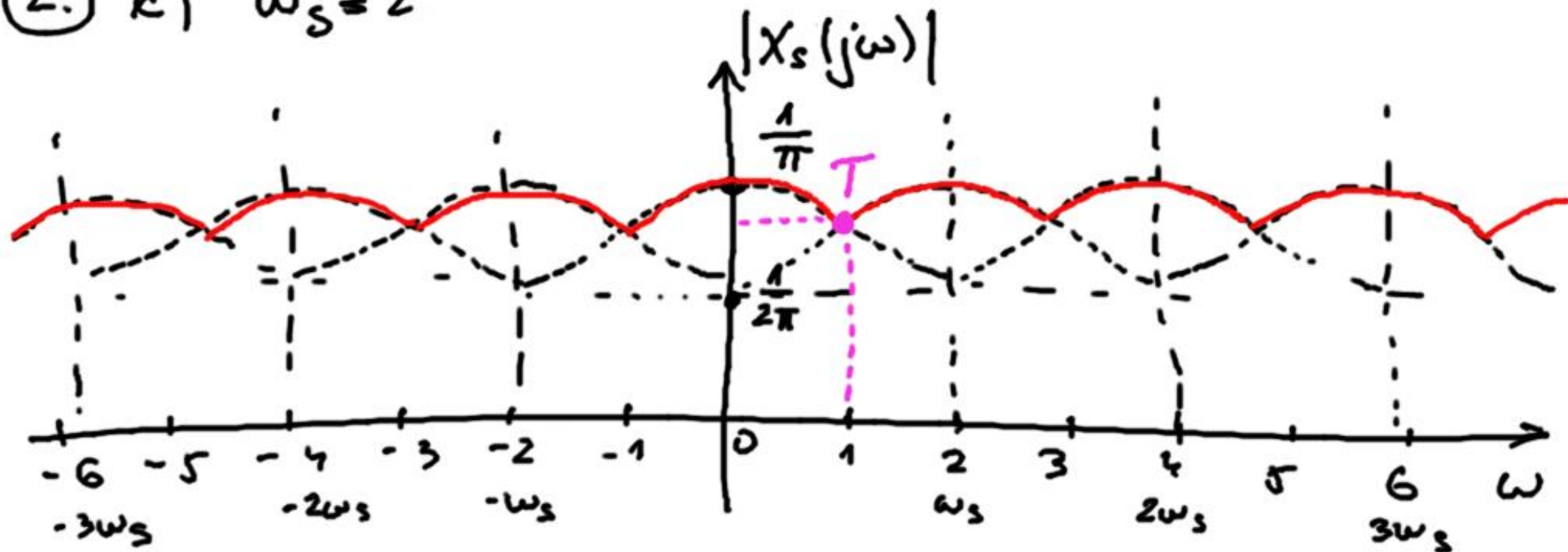
$$(1.) \quad b) \quad X(j\omega) = \frac{4e^{j\frac{3}{2}\omega}}{\omega^2+4} = |X(j\omega)| \cdot e^{j\angle X(j\omega)}$$

$$|X(j\omega)| = \left| \frac{4e^{j\frac{3}{2}\omega}}{\omega^2+4} \right| = \frac{|4e^{j\frac{3}{2}\omega}|}{|\omega^2+4|} = \frac{4}{\omega^2+4}$$

$$\angle X(j\omega) = \frac{\omega}{2} \text{ (unutar } (-\pi, \pi) \text{ uvijek neka bude)}$$



②. c) $\omega_s = 2$



$$|X_s(j\omega)| = \frac{|X(j\omega)|}{T_s} = \frac{|X(j\omega)|}{\frac{2\pi}{\omega_s}} = \frac{|X(j\omega)|}{\pi}$$

$\omega = 1 \Rightarrow$ (ljubičasta točka na grafu)

$$\boxed{|X_s(j1)|} = \frac{|X(j1)|}{\pi} = \frac{\frac{4}{1^2+4}}{\pi} = \boxed{\frac{4}{5\pi}}$$

② d) $E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{-\frac{1}{2}} e^{4t+2} dt + \int_{-\frac{1}{2}}^{\infty} e^{-4t-2} dt =$
 $= \dots = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

e) $y(t) = x(t - \frac{1}{2}) = e^{-|2(t - \frac{1}{2}) + 1|} = e^{-|2t|} = e^{-2|t|}$

$Y(j\omega) = (\text{službeni šala baluter} \rightarrow e^{at} \rightarrow \frac{2a}{a^2 + \omega^2}) = \frac{4}{\omega^2 + 4}$

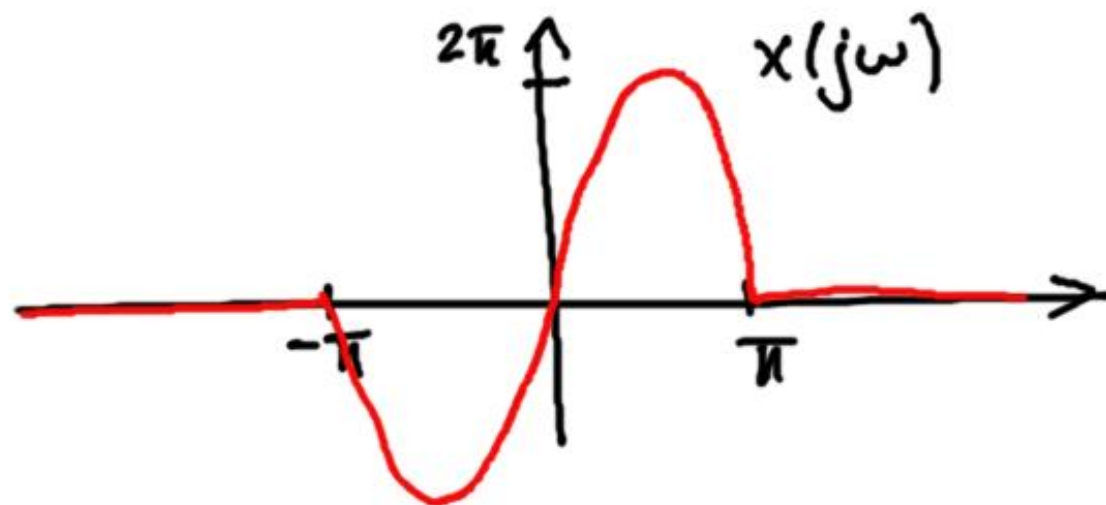
ili

$y(t) = x(t - \frac{1}{2}) \rightarrow X(j\omega) \cdot e^{-j\omega \frac{1}{2}} = \frac{4e^{j\frac{\omega}{2}}}{\omega^2 + 4} \cdot e^{-j\frac{\omega}{2}} = \frac{4}{\omega^2 + 4}$

②. f) Signal $y(t)$ je realan i paran $\rightarrow Y^*(j\omega) = Y(j\omega)$

$$Y^*(j\omega) = \left(\frac{4}{\omega^2 + 4} \right)^* = \frac{4}{\omega^2 + 4} = Y(j\omega)$$

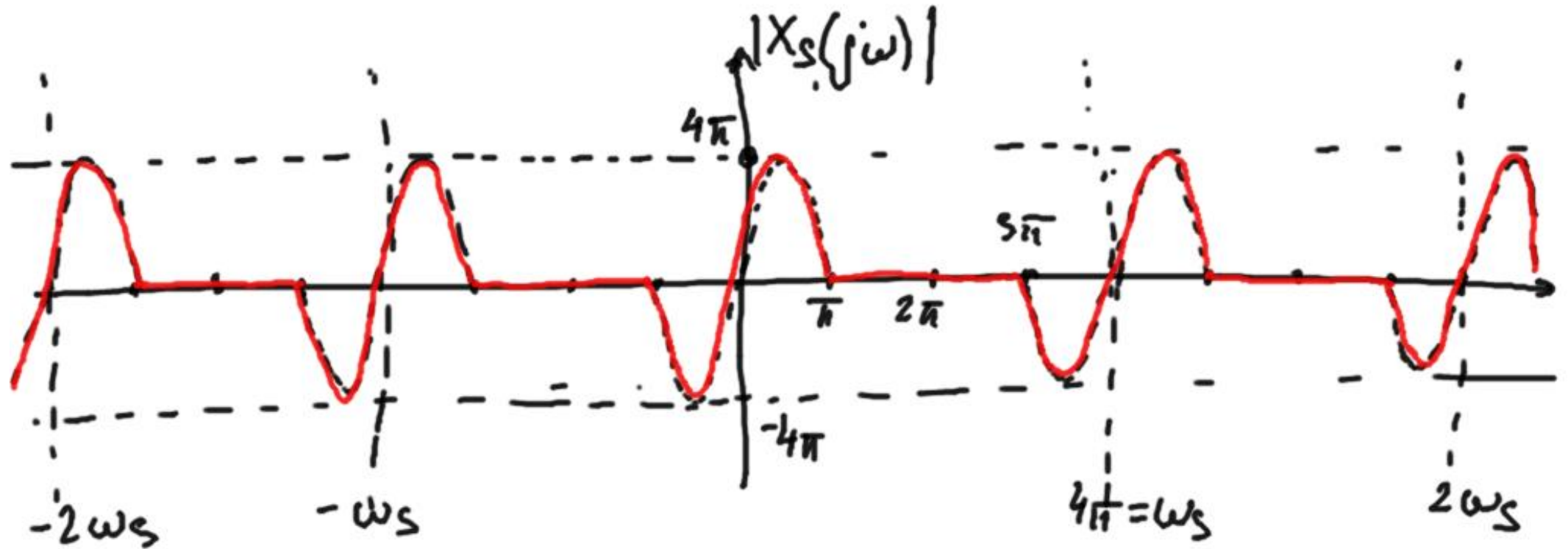
(3.)



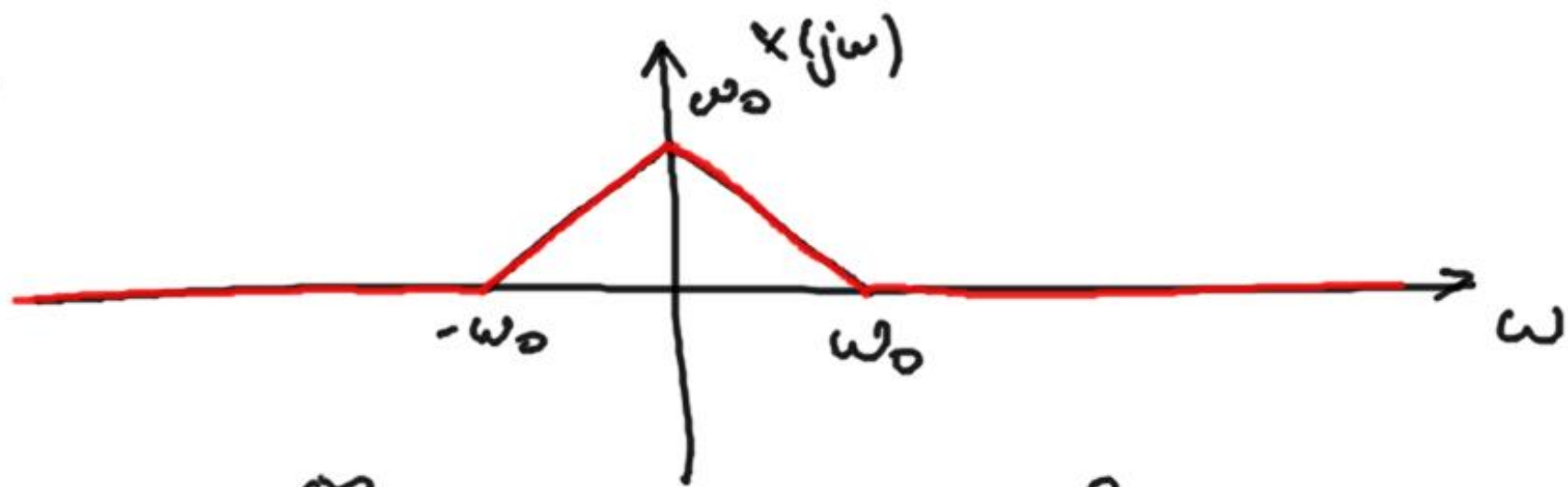
$$(a) \boxed{x(t)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} x(j\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-\pi}^{\pi} 2\pi \sin(\omega) e^{j\omega t} d\omega =$$
$$= \dots = \boxed{-\frac{2j \sin(\pi t)}{t^2 - 1}}$$

$$(b) \boxed{E} = \frac{1}{2\pi} \int_{-\infty}^{\infty} |x(j\omega)|^2 d\omega = \frac{1}{2\pi} \int_{-\pi}^{\pi} 4\pi^2 \sin^2(\omega) d\omega = \dots = \boxed{2\pi^2}$$

$$(3.) c) |X_s(j\omega)| = \frac{|X(j\omega)|}{T_s} = \frac{|X(j\omega)|}{\frac{1}{2}} \quad \omega_s = \frac{2\pi}{T_s} = 4\pi$$



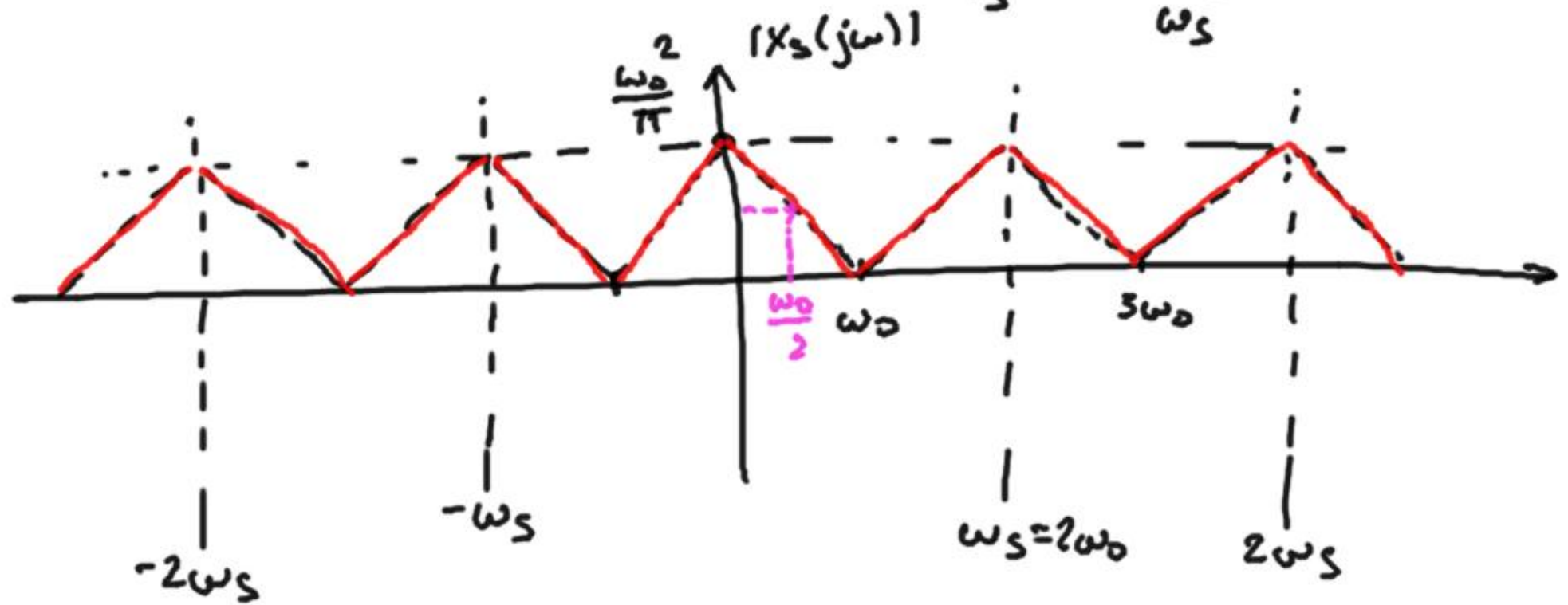
(4.)



$$(a) \boxed{x(t)} = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \left(\int_{-\omega_0}^0 (\omega + \omega_0) e^{j\omega t} d\omega + \int_0^{\omega_0} (-\omega + \omega_0) e^{j\omega t} d\omega \right) = \dots = \boxed{\frac{1 - \cos(\omega_0 t)}{\pi t^2}}$$

$$(b) \boxed{E} = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega = \frac{1}{2\pi} \left(\int_{-\omega_0}^{\omega_0} |\omega + \omega_0|^2 d\omega + \int_0^{\omega_0} |-\omega + \omega_0|^2 d\omega \right) = \dots = \boxed{\frac{\omega_0^3}{3\pi}}$$

4. a) $\omega_s = 2\omega_0$ $|X_s(j\omega)| = \frac{|X(j\omega)|}{T_s} = \frac{|X(j\omega)|}{\frac{2\pi}{\omega_s}} = \frac{\omega_0}{\pi} |X(j\omega)|$

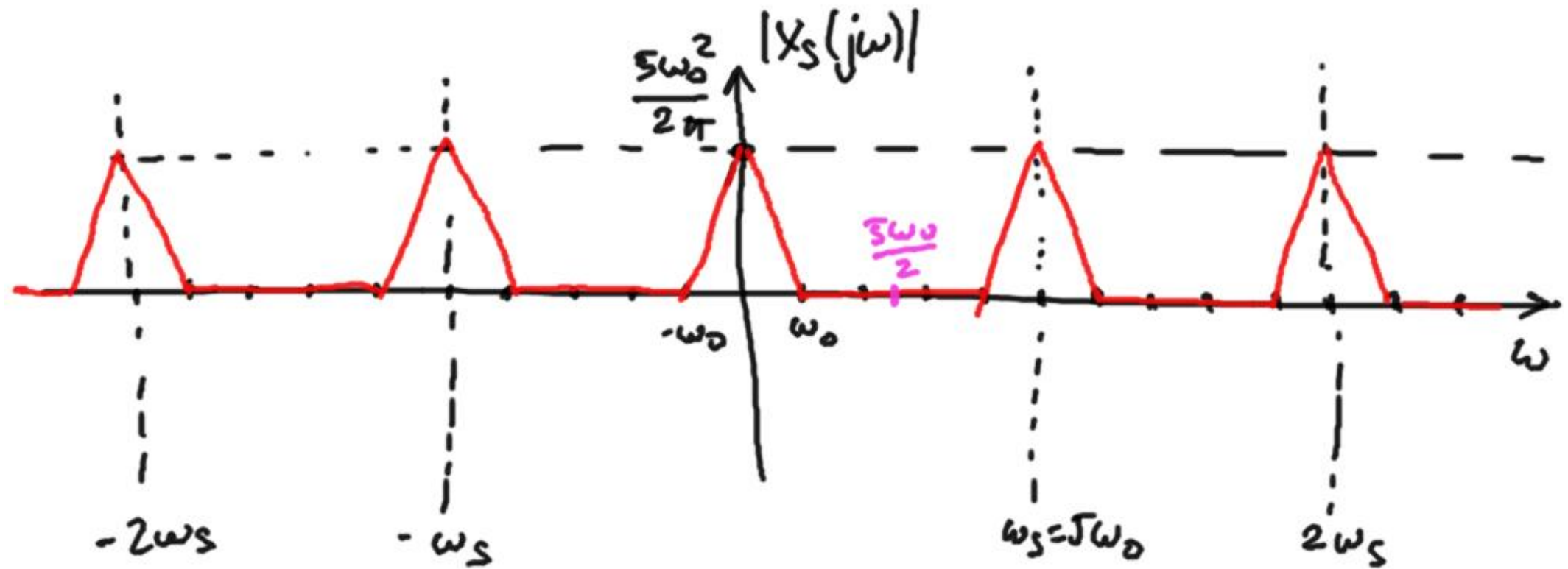


$|X_s(j\frac{\omega_0}{2})| = (\text{ljubi čista točka na grafu}) =$

$$= \frac{\omega_0}{\pi} \cdot |X(j\frac{\omega_0}{2})| = \frac{\omega_0}{\pi} \cdot (-\omega + \omega_0) \Big|_{\omega = \frac{\omega_0}{2}} =$$

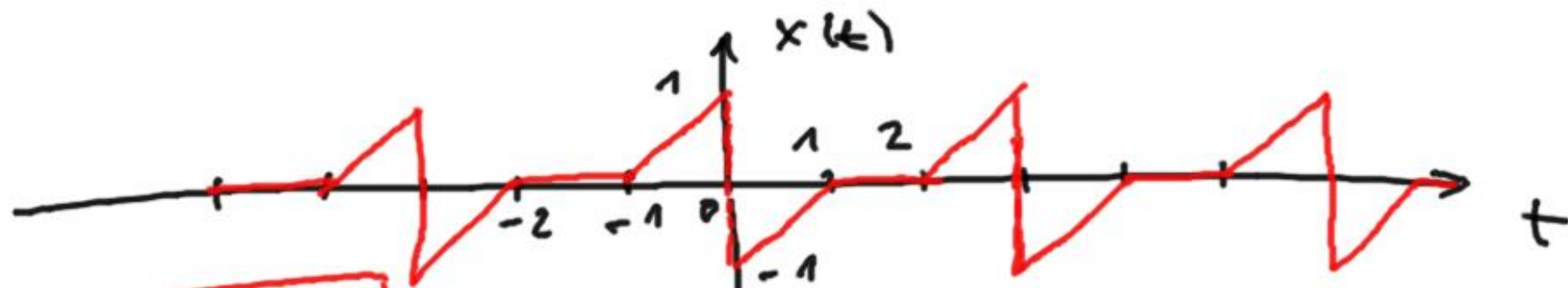
$$= \frac{\omega_0}{\pi} \cdot (-\frac{\omega_0}{2} + \omega_0) = \boxed{\frac{\omega_0^2}{2\pi}}$$

(4. d) $\omega_s = 5\omega_0 \rightarrow |X_s(j\omega)| = \frac{|X(j\omega)|}{T_s} = \frac{|X(j\omega)|}{\frac{2\pi}{\omega_s}} = \frac{5\omega_0}{2\pi} |X(j\omega)|$



$|X_s(j\frac{5\omega_0}{2})| = (\text{люб. точка}) = 0$

(5.) a) Kontinuiran i periodičan \rightarrow CTFS



$x_0 = \dots = 0 \Rightarrow$ srednja vrijednost je 0 //

$$X_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-j\omega_0 k t} dt = \frac{1}{4} \left(\int_{-1}^0 (t+1) e^{-j\omega_0 k t} dt + \int_0^1 (t-1) e^{-j\omega_0 k t} dt \right) = \frac{1}{4} \frac{2j(k\omega_0 - \sin(k\omega_0))}{k^2 \omega_0^2} =$$

$$= \left[\omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2} \right] = 2j \frac{(k \frac{\pi}{2} - \sin(k \frac{\pi}{2}))}{k^2 \pi^2}, k \neq 0$$

5. b) $\boxed{P} = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt = \frac{1}{4} \left(\int_{-1}^0 (t+1)^2 dt + \int_0^1 (t-1)^2 dt \right) =$

$= \dots = \boxed{\frac{1}{6}}$

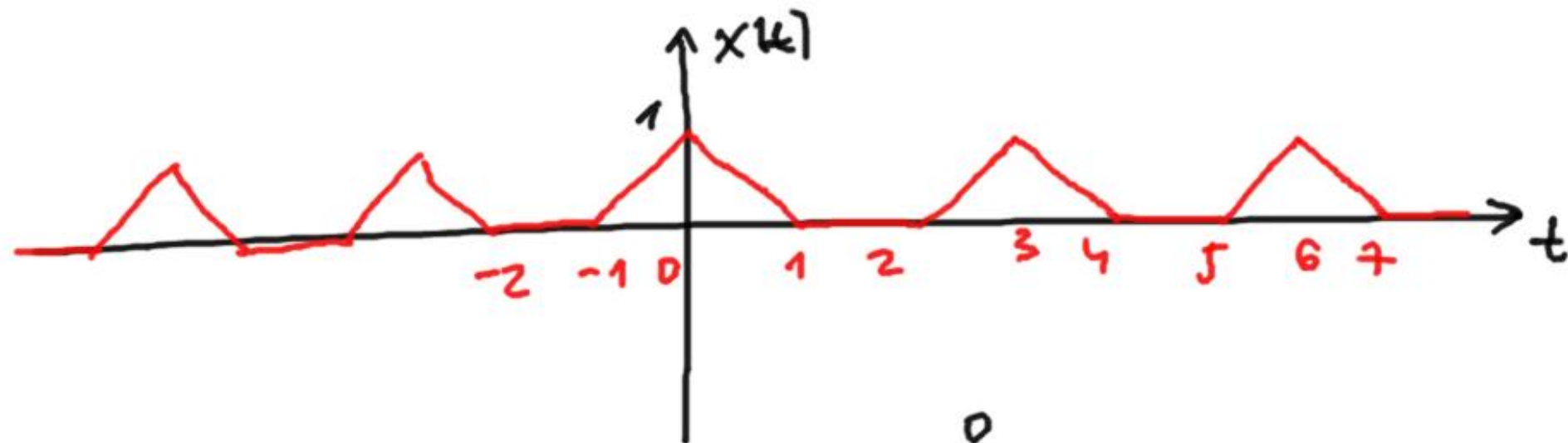
zboj absolutne
vymednosti
se promijeni

c) signal je realan $\rightarrow X_a^* = X_{-a}$

$$X_a^* = \left(2j \frac{\left(\frac{2\pi}{2} - \sin\left(\frac{2\pi}{2}\right) \right)}{2^2 \pi^2} \right)^* = -2j \frac{\frac{2\pi}{2} - \sin\left(\frac{2\pi}{2}\right)}{2^2 \pi^2}$$

$$X_{-a} = 2j \frac{\left(-\frac{2\pi}{2} - \sin\left(-\frac{2\pi}{2}\right) \right)}{(-2)^2 \pi^2} = -2j \frac{\frac{2\pi}{2} - \sin\left(\frac{2\pi}{2}\right)}{2^2 \pi^2}$$

6. a) Kontinuiran i periodičan \rightarrow CTFS



$$\begin{aligned}
 \boxed{X_2} &= \frac{1}{T_0} \int_{T_0} x(t) e^{-j\omega_0 t} dt = \frac{1}{4} \left(\int_{-1}^0 (t+1) e^{-j\omega_0 t} dt + \right. \\
 &\quad \left. + \int_0^1 (-t+1) e^{-j\omega_0 t} dt \right) = \frac{2(1 - \cos(2\omega_0))}{4 \cdot 2^2 \omega_0^2} = \\
 &= \boxed{2 \frac{1 - \cos(\frac{2\pi}{2})}{2^2 \pi^2}}, \quad 2 \neq 0
 \end{aligned}$$

$$\begin{aligned}
 \boxed{X_0} &= \frac{1}{4} \left(\int_{-1}^0 (t+1) dt + \int_0^1 (-t+1) dt \right) = \\
 &= \boxed{\frac{1}{4}}
 \end{aligned}$$

$$\textcircled{6} \text{ b) } \boxed{\overline{P}} = \frac{1}{T_0} \int_{T_0} |x(t)|^2 dt = \frac{1}{4} \left(\int_{-1}^0 |t+1|^2 dt + \int_0^1 |-t+1|^2 dt \right) =$$

$$= \dots = \boxed{\frac{1}{6}}$$

c) signal je realan i paran $\Rightarrow X_e^* = X_e$

$$\boxed{X_e^*} = \left(2 \frac{1 - \cos\left(\frac{2\pi}{2}\right)}{2^2 \pi^2} \right)^* = 2 \frac{1 - \cos\left(\frac{2\pi}{2}\right)}{2^2 \pi^2} = \boxed{X_e}$$

7. a) $x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j\omega_0 k t} =$

$$= X_{-4} e^{-j4\omega_0 t} + X_{-2} e^{-j2\omega_0 t} + X_0 + X_2 e^{j2\omega_0 t} +$$

$$+ X_4 e^{j4\omega_0 t} = \frac{j}{4} e^{-j4\omega_0 t} + \frac{1}{2} e^{-j2\omega_0 t} + 1 +$$

$$+ \frac{1}{2} e^{j2\omega_0 t} - \frac{j}{4} e^{j4\omega_0 t} =$$

$$= 1 + \frac{e^{j2\omega_0 t} + e^{-j2\omega_0 t}}{2} + \frac{1}{2} \frac{e^{j4\omega_0 t} - e^{-j4\omega_0 t}}{2j} =$$

$$= 1 + \cos(2\omega_0 t) + \frac{1}{2} \sin(4\omega_0 t)$$

7. b) $\boxed{P} = \sum_{a=-\infty}^{\infty} |x_a|^2 = \left(\frac{1}{4}\right)^2 + \left(\frac{1}{2}\right)^2 + 1^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2 =$
 $= \frac{1}{8} + \frac{1}{2} + 1 = \frac{1+4+8}{8} = \boxed{\frac{13}{8}}$

c) Koristimo sluzbene šalab elter i svojstva:

$$\sin(\omega_0 t) \longrightarrow -j\pi [\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$$

$$\cos(\omega_0 t) \longrightarrow \pi [\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$$

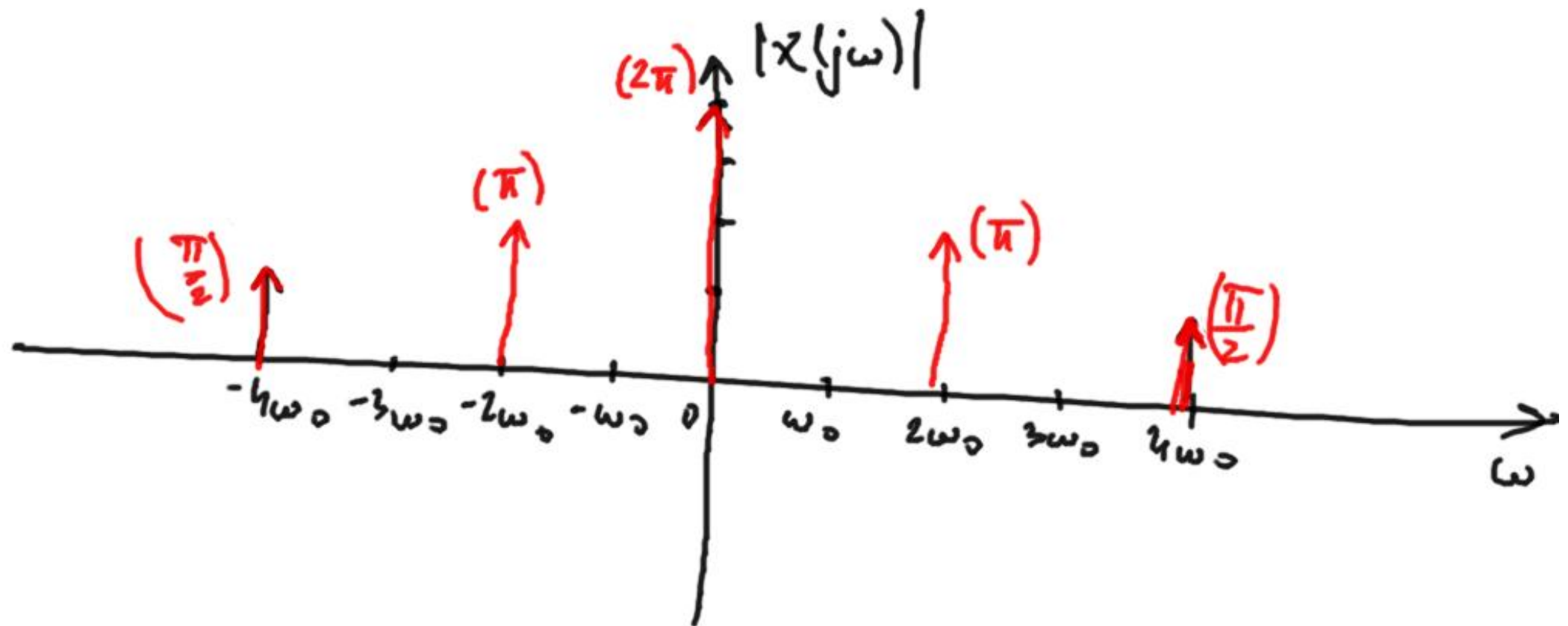
$$1 \longrightarrow 2\pi\delta(\omega)$$

$$X(j\omega) = 2\pi\delta(\omega) + \pi[\delta(\omega - 2\omega_0) + \delta(\omega + 2\omega_0)] -$$

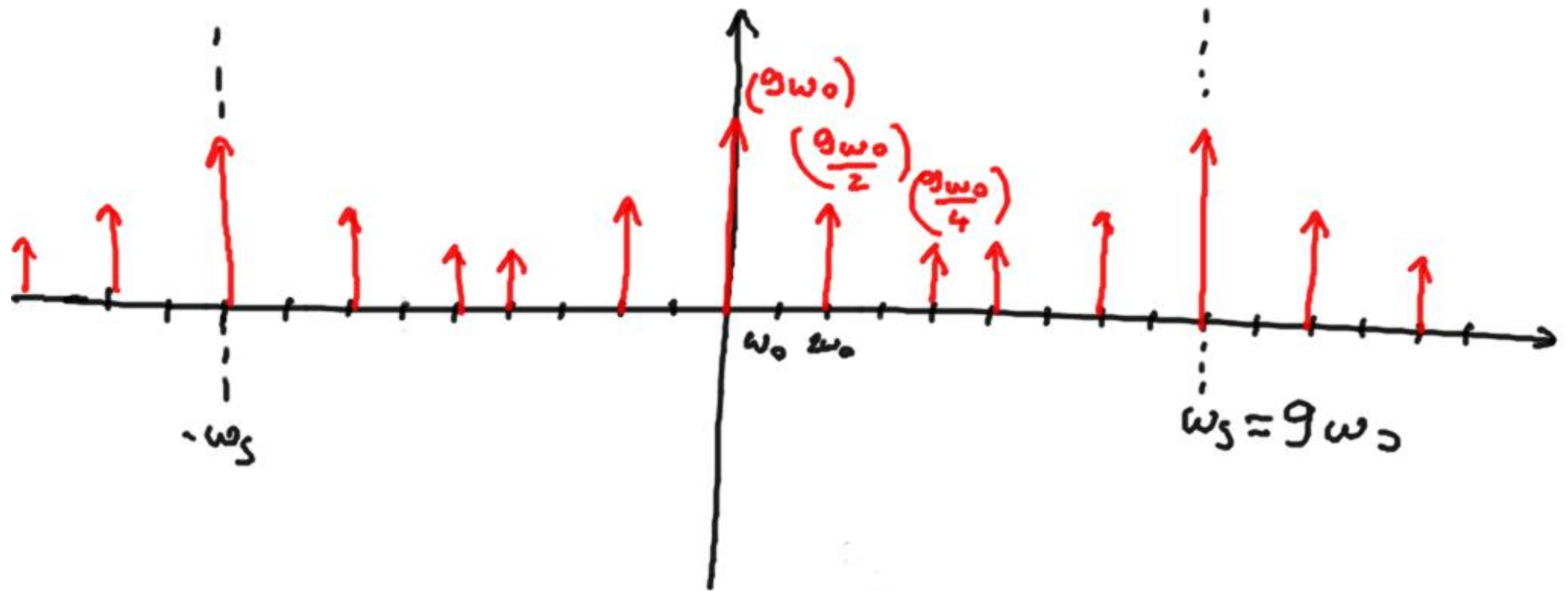
$$- \frac{j\pi}{2} [\delta(\omega - 4\omega_0) - \delta(\omega + 4\omega_0)]$$

7. e) nastavak

$$|X(j\omega)| = \sqrt{\left[2\pi\delta(\omega) + \pi\delta(\omega-2\omega_0) + \pi\delta(\omega+2\omega_0) \right]^2 + \dots + \left[-\frac{\pi}{2}\delta(\omega-4\omega_0) + \frac{\pi}{2}\delta(\omega+4\omega_0) \right]^2}$$



4. d) $\omega_s = 9\omega_0$ $|X_s(j\omega)| = \frac{|X(j\omega)|}{T_s} = \frac{|X(j\omega)|}{\frac{2\pi}{\omega_s}} = \frac{9\omega_0}{2\pi} |X(j\omega)|$



$$\textcircled{8.} \text{ a) } x(n) = x(nT_s) = 1 + \cos\left(2\omega_0 \cdot n \cdot \frac{\pi}{8\omega_0}\right) + \frac{1}{2} \sin\left(4\omega_0 \cdot n \cdot \frac{\pi}{8\omega_0}\right) =$$

$$= 1 + \cos\left(\frac{n\pi}{4}\right) + \frac{1}{2} \sin\left(\frac{n\pi}{2}\right)$$

$$N_1 = \frac{2\pi}{\frac{\pi}{4}} = 8k = (k=1) = 8$$

$$N_2 = \frac{2\pi}{\frac{\pi}{2}} = 4k = (k=1) = 4$$

$$N_0 = \text{NZV}(4, 8) = 8$$

\Rightarrow signal je diskretan i periodičan! \rightarrow DTFS

(8.) a) nastavak

$$N_0 = 8 \Rightarrow \omega_0 = \frac{2\pi}{N_0} = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$\omega_1 = \omega_0$$

$$\omega_2 = 2\omega_0$$

$$x(n) = 1 + \cos(\omega_0 n) + \frac{1}{2} \sin(2\omega_0 n) = 1 + \frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2} + \frac{1}{2} \frac{e^{j2\omega_0 n} - e^{-j2\omega_0 n}}{2j}$$

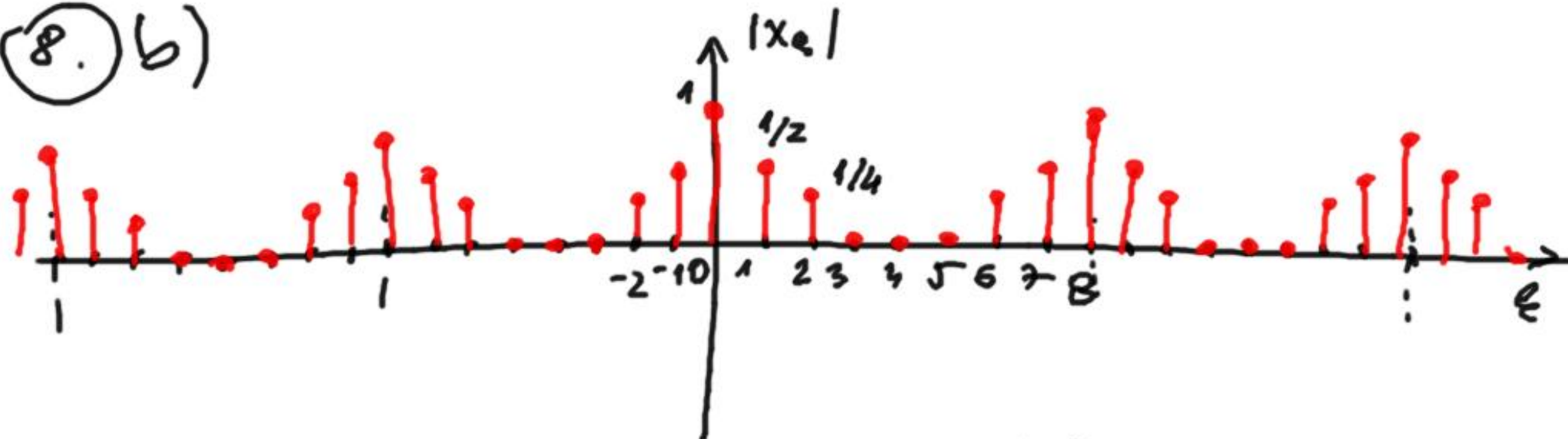
$$\Rightarrow X_0 = 1 = 1e^{j0}$$

$$X_1 = X_{-1} = \frac{1}{2} = \frac{1}{2}e^{j0}$$

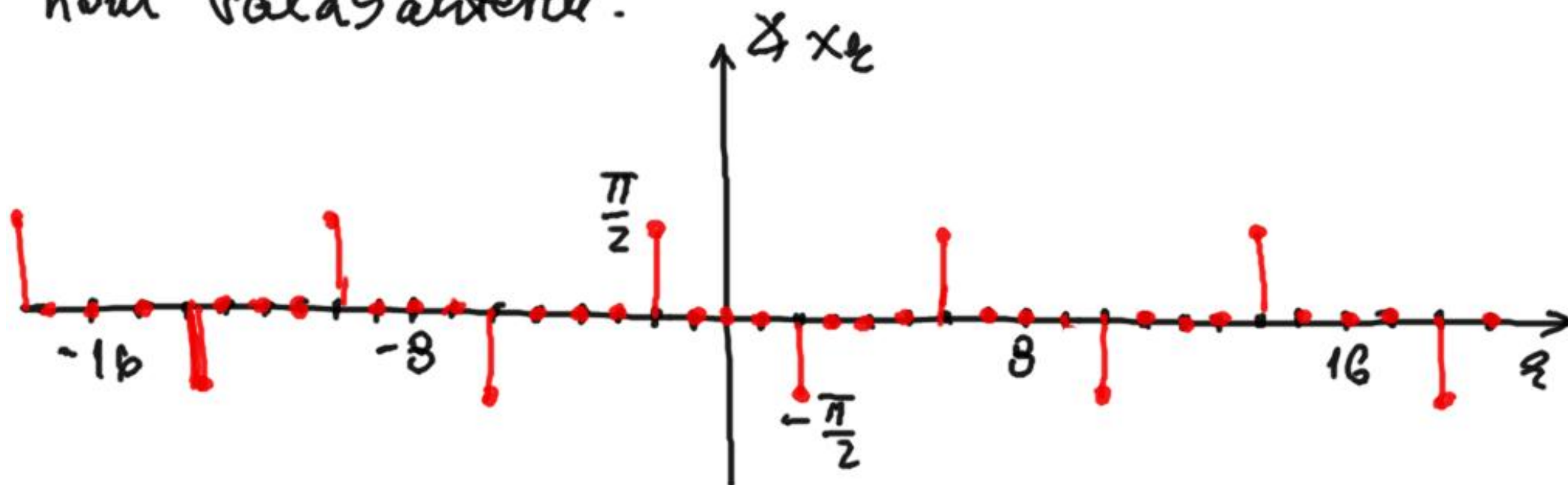
$$X_2 = \frac{1}{4j} = \frac{1}{4}(-j) = \frac{1}{4}e^{-j\frac{\pi}{2}}$$

$$X_{-2} = -\frac{1}{4j} = \frac{1}{4}j = \frac{1}{4}e^{j\frac{\pi}{2}}$$

8. b)



Ne zaboraviti da je spektar periodičan s istim periodom kao i signal !!! To se vidi i na sledećem talasnom obliku.



8. a) $\boxed{P} = \sum_{k=0}^{N-1} |X_k|^2 = \sum_{k=0}^7 |X_k|^2 =$
 $= 1^2 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{2}\right)^2 =$
 $= 1 + \frac{1}{2} + \frac{1}{8} = \boxed{\frac{13}{8}}$

d) signal je realan $\rightarrow X_k^* = X_{-k}$

$$k = -2 \Rightarrow X_{-2}^* = X_{-(-2)} = X_2 \Rightarrow X_{-2}^* = \left(\frac{j}{4}\right)^* = -\frac{j}{4} = X_2 //$$

$$k = -1 \Rightarrow X_{-1}^* = X_{-(-1)} = X_1 \Rightarrow X_{-1}^* = \left(\frac{1}{2}\right)^* = \frac{1}{2} = X_1 //$$

$$k = 0 \Rightarrow X_0^* = X_0 \Rightarrow X_0^* = 1^* = 1 = X_0 //$$

$$k = 1 \Rightarrow X_1^* = X_{-1} \Rightarrow X_1^* = \left(\frac{1}{2}\right)^* = \frac{1}{2} = X_{-1} //$$

$$k = 2 \Rightarrow X_2^* = X_{-2} \Rightarrow X_2^* = \left(-\frac{j}{4}\right)^* = \frac{j}{4} = X_{-2} //$$

$$(9.) a) \omega_0 = 5\pi \Rightarrow x(t) = 1 + \cos(10\pi t) + \frac{1}{2} \sin(20\pi t)$$

$$\begin{aligned} x(n) &= x(nT_s) = 1 + \cos(10\pi \cdot nT_s) + \frac{1}{2} \sin(20\pi nT_s) = \\ &= 1 + \cos\left(10n\pi \cdot \frac{2\pi}{\omega_s}\right) + \frac{1}{2} \sin\left(20n\pi \cdot \frac{2\pi}{\omega_s}\right) = \\ &= 1 + \cos\left(10n\pi \cdot \frac{2\pi}{60\pi}\right) + \frac{1}{2} \sin\left(20n\pi \cdot \frac{2\pi}{60\pi}\right) = \\ &= 1 + \cos\left(\frac{n\pi}{3}\right) + \frac{1}{2} \sin\left(\frac{2n\pi}{3}\right) \end{aligned}$$

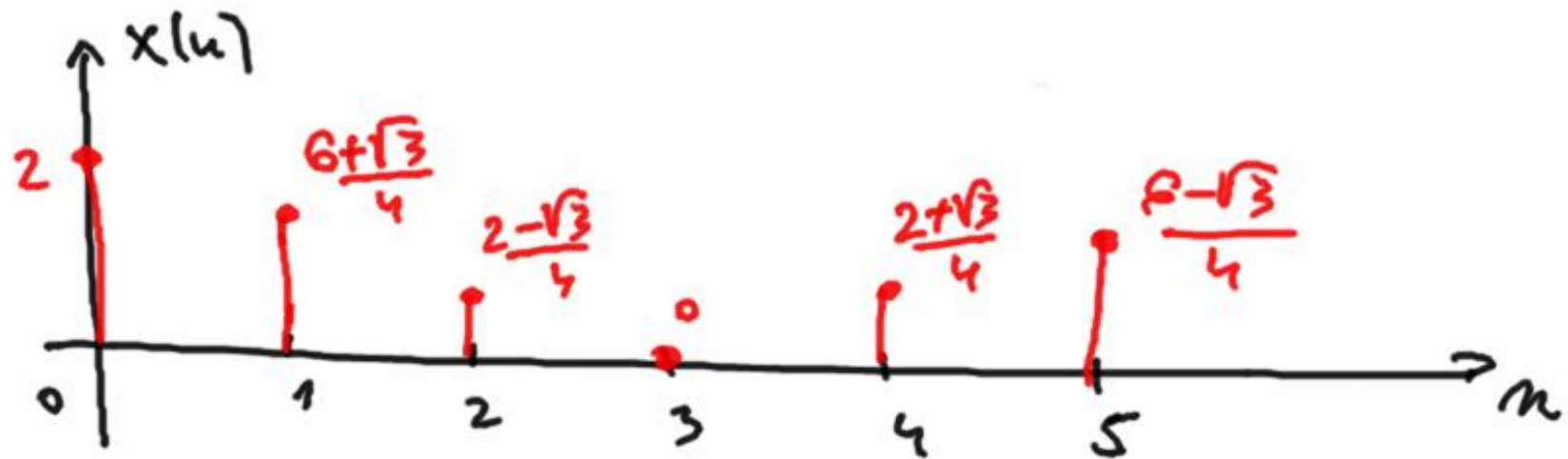
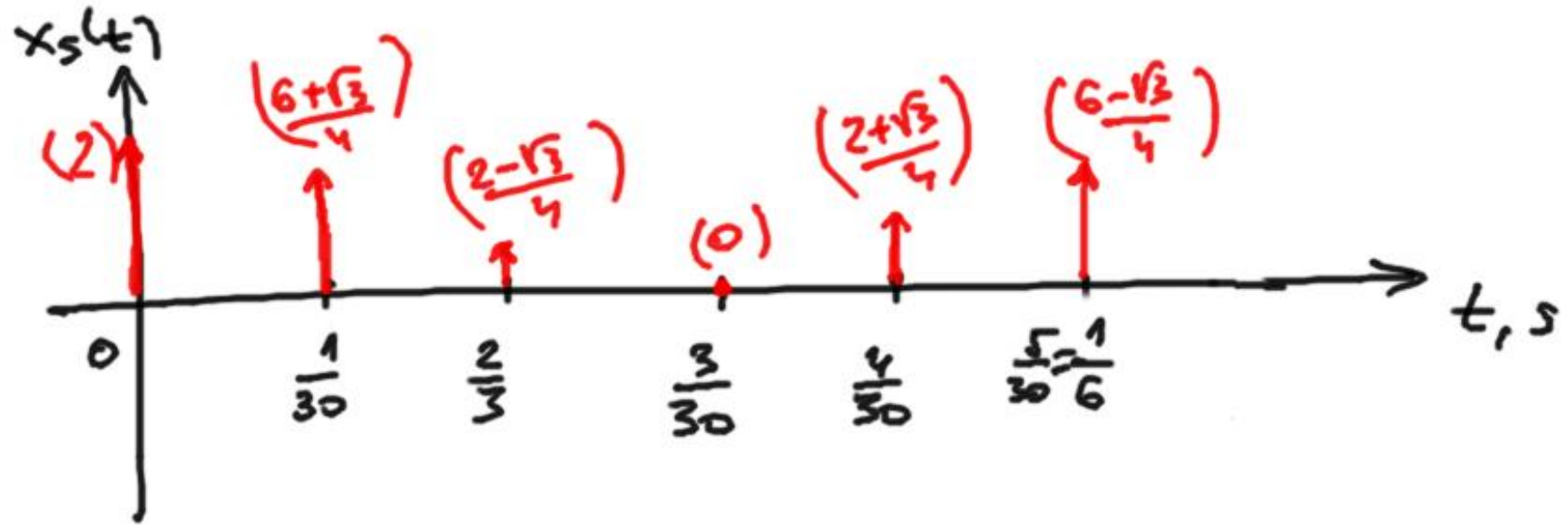
$\frac{1}{6}$ sek. vrimlyes \rightarrow period otipkanafa $T_s = \frac{2\pi}{\omega_s} = \frac{1}{30}$ sekundi

$$\Rightarrow \frac{\frac{1}{6}}{\frac{1}{30}} = \frac{30}{6} = 5 \text{ rezmarka} \Rightarrow 6 \text{ očitaka}$$

$$x(0) = \dots = 2 \quad x(2) = \frac{2-\sqrt{3}}{4} \quad x(4) = \frac{2+\sqrt{3}}{4}$$

$$x(1) = \dots = \frac{6+\sqrt{3}}{4} \quad x(3) = 0 \quad x(5) = \frac{6-\sqrt{3}}{4}$$

9. a) nastavok



9. a) nastavak 2

$$\overbrace{X(k)}^{\text{DFT}} = \sum_{n=0}^{N-1} x(n) e^{-\frac{2\pi j k n}{N}} =$$

$$= x(0) + x(1) e^{-\frac{2\pi j k}{6}} + x(2) e^{-\frac{2\pi j k \cdot 2}{6}} + x(3) e^{-\frac{2\pi j k \cdot 3}{6}} +$$

$$+ x(4) e^{-\frac{2\pi j k \cdot 4}{6}} + x(5) e^{-\frac{2\pi j k \cdot 5}{6}} =$$

$$= 2 + \frac{6+\sqrt{3}}{4} e^{-j\frac{k\pi}{3}} + \frac{2-\sqrt{3}}{4} e^{-j\frac{2k\pi}{3}} + \frac{2+\sqrt{3}}{4} e^{-j\frac{4k\pi}{3}} +$$

$$+ \frac{6-\sqrt{3}}{4} e^{-j\frac{5k\pi}{3}}$$

Ne da mi se računati :D

$$X(1) = \dots =$$

$$X(3) = \dots$$

$$X(5) = \dots$$

$$X(2) = \dots =$$

$$X(4) = \dots$$

(9.) b) Uzorci su frekvencijski razmaknuti za $\frac{\omega_s}{N}$:

$$\frac{\omega_s}{N} = \frac{60\pi}{6} = 10\pi$$

Datelo: $X(0) \Rightarrow 0 \text{ rad/s}$

$$X(4) = 40\pi \text{ rad/s}$$

$$X(5) = 50\pi \text{ rad/s}$$

$$X(1) \Rightarrow 10\pi \text{ rad/s}$$

$$X(2) \Rightarrow 20\pi \text{ rad/s}$$

$$X(3) \Rightarrow 30\pi \text{ rad/s}$$

→ ovo je za $\omega = 10\pi \text{ rad/s}$

→ toj frekv. odgovara

$$X(1) = \frac{6+\sqrt{3}}{4}$$

(9.) c) Očitano $\omega_s = 60\pi$.

Broj razmaka = 5

$$\boxed{\omega_0} = \frac{\omega_s}{\text{br. raz.}} = \frac{60\pi}{5} = \boxed{12\pi \text{ rad/s}}$$

d) $\omega_0 = 2\pi \text{ rad/s}$
 $\omega_s = 60\pi \text{ rad/s}$

$$\text{br. razmaka} = \frac{\omega_s}{\omega_0} = \frac{60\pi}{2\pi} = 30 \rightarrow N = 31$$

$$\boxed{T_p} = \frac{N}{f_s} = \frac{31}{\frac{\omega_s}{2\pi}} = \frac{31 \cdot 2\pi}{60\pi} = \boxed{1,0333 \text{ s}}$$