2. MI GRUPA A

Zadatak 1. DFT_6 $x(n)=\{0,1,0,0,0,-1\}$ $N=6$ $k=3$

$$X(k)=\sum_{n=0}^{N-1} x(n)e^{-j\frac{2\pi}{N}kn} = X(1)+X(5) = e^{-j\frac{2\pi}{6}k\cdot 1} - e^{-j\frac{2\pi}{6}k\cdot 5}$$

$$= e^{-j\frac{k\pi}{3}} - e^{-j\frac{5\pi k}{3}}$$

$$X(3) = e^{-j\pi} - e^{-j5\pi} = (-1) - (-1) = \underline{\underline{0}}$$

A

Zadatak 2. $IDFT_4$ $X(k)=\{j,-j,-1,j\}$ $n=1$ $N=4$

$$x(n)=\frac{1}{N}\sum_{k=0}^{N-1}X(k)e^{j\frac{2\pi}{N}kn} = \frac{1}{4}\left[\sum_{k=0}^{3}X(k)e^{j\frac{2\pi}{4}k\cdot n}\right]$$

$$=\frac{1}{4}\left[1\cdot e^{j\frac{2\pi}{4}0\cdot n} - j\,e^{j\frac{2\pi}{4}n} - 1\cdot e^{j\frac{2\pi}{4}\cdot 2n} + j\cdot e^{j\frac{2\pi}{4}3\cdot n}\right]$$

$$=\frac{1}{4}\left[e^{0} - j\,e^{j\frac{\pi}{2}n} - e^{j\pi n} + j\cdot e^{\frac{3\pi}{2}n}\right]$$

$$x(1)=\frac{1}{4}\left[1 - j\,e^{j\frac{\pi}{2}} - e^{j\pi} + j\cdot e^{\frac{3\pi}{2}}\right] = \frac{1}{4}\left[1 - j(j) - (-1) + j(-j)\right] = \frac{1}{4}(1+1+1+1) = \frac{4}{4} = \underline{1}$$

B

Zadatak 3.
??
$X(j\Omega)$ CTFT $\Omega_s = 2\Omega_{max}$ $|X(e^{j\omega})|$ DTFT $\omega=0$

Zadatak 4.

$$x(t) = \sin(\underset{f_1}{20\pi t}) + \sin(\underset{f_2}{70\pi t}) + \sin(\underset{f_3}{150\pi t})$$ $f_s = 100\,Hz$, $\frac{f_s}{2} = 50\,Hz$

$f_1 = 10\,Hz$ $f_2 = 35\,Hz$ $f_3 = 75\,Hz$

za f_1 i f_2 nema preklapanja spektra f_1 i $f_2 < \frac{f_s}{2}$

$$x_r(t) = \sin(20\pi t) + \sin(70\pi t)$$

D

Zadatak 5.

Sustav nije aditivan (E)

Zadatak 6.

???

$S[x(n)] = \sin(\lambda n) x^2(n)$ $\lambda = ?$

Zadatak 7.

a) $y(t) = \sin(u(t))$

$u(t) = \alpha u_1(t) + \beta u_2(t) \Rightarrow y(t) = \sin(\alpha u_1(t)) + \sin(\beta u_2(t))$
$y(t) = \alpha y_1(t) + \beta y_2(t) = \alpha \sin u_1(t) + \beta \sin u_2(t)$
$\neq$

b) $y = t\,u(t)$

$u(t) = \alpha u_1(t) + \beta u_2(t) \Rightarrow y = t(\alpha u_1(t)) + t\beta u_2(t)$
$y(t) = \alpha y_1(t) + \beta y_2(t) = \alpha t u_1(t) + \beta t u_2(t)$ } = linearan

c) $y = \cos(u(t-1))$

$u(t) = \alpha u_1(t) + \beta u_2(t) \Rightarrow y = \cos(\alpha u_1(t-1)) + \cos(\beta u_2(t-1))$
$y(t) = \alpha y_1(t) + \beta y_2(t) \Rightarrow y = \alpha \cos(u(t-1)) + \beta \cos(u(t-1))$ } $\neq$

d) $y = u(t) + \cos(t)$

$u(t) = \alpha u_1(t) + \beta u_2(t) \Rightarrow y = \alpha u_1(t) + \cos(t) + \beta u_2(t)$
$y(t) = \alpha y_1 + \beta y_2 = \alpha(u_1(t) + \cos(t)) + \beta(u_2(t) + \cos(t))$ } $\neq$

e) $y(t) = \sin(u(t) - 1)$

$u(t) = \alpha u_1(t) + \beta u_2(t) \Rightarrow y = \sin(\alpha u_1(t) - 1) + \sin(\beta u_2(t) - 1)$
$y(t) = \alpha y_1(t) + \beta y_2(t) \Rightarrow y = \alpha(\sin(u(t) - 1)) + \beta(\sin(u_2(t) - 1))$ } $\neq$

(B)

Zadatak 8.

$$y(t) = \int_t^\infty u(\tau)\,d\tau$$

$u(t) = \alpha u_1(t) + \beta u_2(t) \Rightarrow y(t) = \alpha \int_t^\infty u_1(\tau)\,d\tau + \beta \int_t^\infty u_2(\tau)\,d\tau$

$y(t) = \alpha y_1(t) + \beta y_2(t) \Rightarrow y(t) = \alpha \int_t^\infty u_1(\tau)\,d\tau + \beta \int_t^\infty u_2(\tau)\,d\tau$

} = linearan

nepromjenjivosti

$S(u(t-T)) = \int_{\tau=t}^{\tau=\infty} u(\tau - T)\,d\tau$ $y(t-T) = \int_{t-T}^{\infty} u(\tau)\,d\tau$

$a = \tau - T$ $\tau = t$
$da = d\tau$ $a + T = t \Rightarrow a = t - T$
$\tau \to t,\ a \to t - T$

$S(u(t-T)) = \int_{t-T}^{\infty} u(a)\,da$ } $a = b$
$y(t) = \int_{t-T}^{\infty} u(\tau)\,d\tau$ } $\tau = b$
} $S = y \Rightarrow$ vrem. nepron.

$\rightarrow$ integral $\rightarrow$ memorijski
+ vrem. nepromj. (D)

Zadatak 12.

$$x(n) = \{1, 2\} \qquad y(n) = \{6, -1, -4, -1\}$$

$$h(n) = x(n) * y(n) = \sum_{m=-\infty}^{\infty} x(m)\, y(n-m) =$$

$$h(0) = \sum_{m=0}^{0} x(m)\, y(n-m) = x(0)\cdot y(0) = 1\cdot 6 = 6$$

$$h(1) = \sum_{m=0}^{1} x(m)\, y(1-m) = x(0)\cdot y(1) + x(1)\, y(0) = 1\cdot(-1) + 2\cdot 6 = -1+12 = 11$$

$$h(2) = \sum_{m=0}^{2} x(m)\, y(2-m) = x(0)\, y(2) + x(1)\, y(1) + x(2)\, y(0) = 1\cdot(-4) + 2(-1) + 0\cdot 6 = -4-2 = -6$$

A

Zadatak 13.

$$h(n) = 3^n \mu(n) \qquad u(n) = 2^n \mu(n)$$

$$y(n) = h(n) * u(n) = \sum_{m=-\infty}^{\infty} h(m)\, u(n-m) =$$

$$= \sum_{m=-\infty}^{\infty} 3^m \mu(m)\, 2^{n-m} \mu(n-m) = \sum_{m=0}^{n} 2^{n-m}\, 3^m = 2^n \sum_{m=0}^{n} \left(\frac{3}{2}\right)^m = 2^n \frac{\left(\frac{3}{2}\right)^{n+1} - 1}{\frac{3}{2} - 1}$$

$$= 2^n \frac{\left(\frac{3}{2}\right)^{n+1} - 1}{\frac{1}{2}} = 2^{n+1}\left[3^{n+1}\, 2^{-n-1} - 1\right] = \left(3^{n+1} - 2^{n+1}\right)\mu(n)$$

B

$\boxed{\text{Zadatak 15.}}$

$$(3n+3) * \delta(2n-4)$$

$$(3n+3) * \delta(2(n-2)) = 3(n-2)+3 = 3n-6+3 = 3n-3$$ (E)

$\boxed{\text{Zadatak 16.}}$

$$y(n) - 7y(n-1) + 10y(n-2) = \mu(n) \qquad \mu(n) = (4n+7) \qquad y(-1) = 0 \quad y(-2) = \frac{8}{5}$$

homogeni

$$q^2 - 7q + 10 = 0 \qquad q_{1,2} = \frac{7 \pm \sqrt{49-40}}{2} = \frac{7 \pm 3}{2} = \begin{matrix} 5 \\ 2 \end{matrix}$$

$$y_h(n) = C_1(2)^n + C_2(5)^n$$

$\boxed{\text{partikularno}}$:

$$y(t) = 4n+7 \qquad y_p(n) = K_0 + K_1 n \qquad y_p(n-1) = K_0 + K_1(n-1) = K_0 + K_1 n - K_1$$
$$y_p(n-2) = K_0 + K_1(n-2) = K_0 + K_1 n - 2K_1$$

$$\cancel{K_0} + K_1 n - 7\cancel{K_0} - 7K_1 n + 7\cancel{K_1} + 10\cancel{K_0} + 10K_1 n - 20\cancel{K_1} = 4n+7$$
$$K_0 - 7K_0 + 7K_1 + 10K_0 - 20K_1 = 7 \Rightarrow \quad 4K_0 = 7 + 13K_1 \qquad (K_0 = 5)$$
$$K_1 - 7K_1 + 10K_1 = 4$$
$$4K_1 = 4 \Rightarrow K_1 = 1$$

$$y_p(n) = (5+n)\,\mu(n) \rightarrow \boxed{\text{prisilni odziv}}$$ (E)

$\boxed{\text{Zadatak 17.}}$

$$y_h(n) = C_1(2)^n + C_2(5)^n \qquad y_p(n) = (5+n)\mu(n)$$

$$y_T(n) = C_1(2)^n + C_2(5)^n + n + 5$$

$$\underline{y(n) = 4n+7 + 7y(n-1) - 10y(n-2)}$$

$$y(0) = 7 + 7y(-1) - 10y(-2) = 7 + 0 - 10\,\frac{8}{5} = 7 - 16 = -9$$
$$y(1) = 4+7 + 7y(0) - 10y(-1) = 11 + 7\cdot(-9) - 10\cdot 0 = 11 - 63 = -52$$

$$y_T(0) = -9 = C_1 + C_2 + 5 \qquad -14 - C_2 = C_1 \Rightarrow C_1 = -14 + 10 = -4$$
$$y(1) = -52 = 2C_1 + 5C_2 + 6 \qquad -58 = -28 - 2C_2 + 5C_2 \qquad 3C_2 = -30$$
$$C_2 = -10$$

$$y_T(n) = (\underbrace{-4(2)^n - 10(5)^n}_{\text{prirodni odziv}} + 5 + n)\,\mu(n) \rightarrow \text{Totalni odziv}$$ (B)

$\boxed{\text{Zadatak 18.}}$

$$y_h(n) = C_1(2)^n + C_2(5)^n$$

$$y(n) = 7y(n-1) - 10y(n-2)$$

$$y(0) = 7y(-1) - 10y(-2) = 0 - 16 = -16$$
$$y(1) = 7y(0) - 10y(-1) = -16\cdot 7 = -112$$

$$y(0) = C_1 + C_2 = -16 \qquad C_1 = -16 - C_2 = -16 + \frac{80}{3} = \frac{-48+80}{3} = \frac{32}{3}$$
$$y(1) = 2C_1 + 5C_2 = -112 \qquad -32 - 2C_2 + 5C_2 = -112 \qquad 3C_2 = -112 + 32$$
$$C_2 = -\frac{80}{3}$$

$$y_n(n) = \frac{32}{3}(2)^n - \frac{80}{3}(5)^n$$ (D)

$\hookrightarrow$ nepobuđeni