SIS - 14.06 2008. MASS B1

Diferencijske jednadžbe

opća – $y(n) + a_1 y(n-1) + a_2 y(n-2) = u(n)$ ⇒ 2. reda

$E^0 y(n)$

$E^{(-1)} y(n) = y(n-1)$

Pr.

$y(n-1) + y(n-4) = u(n)$ ⇒ DIF. JED. 3. reda

$y(n) + y(n-3) = u(n+1)$

PRIMJER:

$y(n) - 4\,y(n-2) = u(n)$ $\quad u(n) = n\mu(n) = \begin{cases} n, & n \geq 0 \\ 0, & \text{inače.} \end{cases}$

P.U. $y(-1) = 1$

$y(-2) = 2$

$y(0) - 4\,y(-2) = u(0)$

$y(0) = 8$

$y(1) - 4\,y(-1) = u(1)$

$y(1) = 5$

$y(2) - 4\,y(0) = u(2)$

$y(2) = 34$

- radi odziv sustava u 100 - koraku !?

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totalni odziv = opće homogeno + partikularno rješenje

* OPĆE HOMOGENO → ovisi samo o sustavu

$y_h(n) = C q^n$ → vrijedi za sve sustave (supstitucija)

$y_h(n-1) = C q^{n-1}$

$y(n) - 4y(n-2) = u(n)$

$y_h(n) - 4y_h(n-2) = 0$

$Cq^n - 4Cq^{n-2} = 0$

neka exp. fja ← $Cq^{n-2}(q^2 - 4) = 0$

$q^2 - 4 = 0$ → karak. jedn. sustava

→ karak. polinom sustava

$q^2 = 4 \Rightarrow \left.\begin{array}{l} q_1 = 2 \\ q_2 = -2 \end{array}\right\}$ vlastite frekv. sustava

$\boxed{q^3 - 1 = 0 \Rightarrow q^3 = 1 \to q = 1}$ → neki primjer.

$y_h(n) = C_1 q_1^n + C_2 q_2^n \to \boxed{y_h(n) = C_1 2^n + C_2 (-2)^n}$

OPĆA HOMOGENA JED.

$u(n) = A_0 + A_1 n$ → to je neki polinom

H: $u(n) = A_0 + A_1 n + A_2 n^2$

$y_p(n) = K_0 + K_1 n$

* PART. RJEŠENJE

$y_p(n) = K_0 + K_1 n \to y_p(n-2) = K_0 + K_1(n-2)$

$y(n) - 4y(n-2) = u(n)$

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$$y_p(n) - 4y_p(n-2) = u(n)$$

$$K_0 + K_1 n - 4\left(K_0 + K_1(n-2)\right) = n$$

$$\underline{K_0} + K_1 n - 4\underline{K_0} - 4K_1 n + \underline{8K_1} = n$$

$$-3K_0 + 8K_1 - 3K_1 n = A_0 + A_1 n \quad , A_0 = 0 \; ; \; A_1 = 1$$

$$\left.\begin{aligned} -3K_0 + 8K_1 &= 0 \\ -3K_1 &= 1 \end{aligned}\right\} \quad K_0 = -\frac{8}{9} \quad , K_1 = -\frac{1}{3}$$

$$\boxed{y_p(n) = -\frac{8}{9} - \frac{1}{3}n \quad , n \geq 0} \Rightarrow \text{vrijedi samo } n \geq 0$$

* TOTALNI ODZIV

$$y(n) = y_h(n) + y_p(n)$$

$$y(n) = C_1 2^n + C_2(-2)^n - \frac{8}{9} - \frac{1}{3}n$$

$$y(0) = 8, \quad y(1) = 5$$

$$\left.\begin{aligned} y(0) &= C_1 + C_2 - \frac{8}{9} = 8 \\ y(1) &= 2C_1 - 2C_2 - \frac{8}{9} - \frac{1}{3} = 5 \end{aligned}\right\} \quad C_1 = 6 \quad ; \quad C_2 = \frac{26}{9}$$

$$\boxed{y(n) = 6 \cdot 2^n + \frac{26}{9}(-2)^n - \frac{8}{9} - \frac{1}{3}n \quad , n \geq 0}$$

PRIMJER 2:

$$y(n) - \frac{3}{16}\,y(n-2) + \frac{1}{32}\,y(n-3) = 0$$

$\Rightarrow$ nema pobude, nema parti. rj.

$\Rightarrow$ 3. reda

$y(-1) = 12$; $y(-2) = 8$; $y(-3) = 16$

* OPĆE HOMOGENO RJEŠENJE

$$y_h(n) = C q^n$$

$$C q^{n-3}\left(q^3 - \frac{3}{14}\,q + \frac{1}{32}\right) = 0 \;\Rightarrow\; q^3 - \frac{3}{16}\,q + \frac{1}{32} = 0$$

$q_1 = \frac{1}{4}$; $q_2 = \frac{1}{4}$; $q_3 = -\frac{1}{2}$

$$y_h(n) = (C_1 + C_2 n)\,q_1^n + C_3\,q_3^n$$

$$\boxed{(C_1 + C_2 n + C_3 n^2)\,q^n \Rightarrow \text{za trostruki korjen}}$$

$$y_h(n) = (C_1 + C_2 n)\left(\frac{1}{4}\right)^n + C_3\left(-\frac{1}{2}\right)^n \Rightarrow \text{OPĆE HOMOGENO RJ.}$$

$$y(n) = y_h(n)$$

uvrstimo i dobijemo $C_1 = 3$; $C_2 = 1$; $C_3 = -2$

$$\boxed{y(n) = (3 + n)\left(\frac{1}{4}\right)^n - 2\left(-\frac{1}{2}\right)^n}$$

PRIMJER 3: "caka zadatak by Marko"

$$y(n) - 2y(n-1) + y(n-2) = u(n)$$

$$u(n) = n\mu(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$$y(-1) = 1, \quad y(-2) = 2$$

* OPĆE HOMOG RJ.

$$y_h(n) = Cq^n$$

$$Cq^{n-2}(q^2 - 2q + 1) = 0 \rightarrow q^2 - 2q + 1 = 0 \rightarrow q_1 = 1$$

$$q_2 = 1$$

$$y_h(n) = (C_1 + C_2 n)(1)^n$$

$$\boxed{y_h(n) = C_1 + C_2 n}$$

* PARTIKULARNO RJ.

$$u(n) = A_0 + A_1 n$$

$$y_p(n) = K_0 + K_1 n$$

$$K_0 + K_1 n - 2(K_0 + K_1(n-1)) + K_0 + K_1(n-2) = n$$

$$\cancel{K_0} + \cancel{K_1 n} - \cancel{2K_0} - \cancel{2K_1 n} + \cancel{2K_1} + \cancel{K_0} + \cancel{K_1 n} - \cancel{2K_1} = n$$

$$n = 0$$

ovo je KRIVO !!!

- ali ovo je točno

$$u(n) = n \rightarrow u(n) = \underbrace{n}_{\text{polinom}}(1)^n$$

$u(n) = n^M r^n \rightarrow y_p(n) = K^n r^n n^{m}$ → koliko se puta r pojavlju kod vlastite fvek.

$y_p(n) = (K_0 + K_1 n)n^2$ $\quad u(n) = K^n n^2$

$y_p(n) = (K_0 + K_1 n)\, n^2$

$K_0 = \frac{1}{2} \quad K_1 = \frac{1}{6}$

- objašnjenje zašto je to tako !

POBUDA	PARTIK. RJ.
A	K
n^m	$K_0 + K_1 n + \ldots + K_M n^M$
$A \cdot r^n$	$K \cdot r^n \cdot n^{m} \longrightarrow$ koliko je puta r frekvencija sustava

pr. primjera

$g_{1,2} = 1 \;,\; g_3 = 2$

$u(n) = 5 \quad \mu(n) = 5\,(1)^n\,\mu(n)$

$$5 r^n \rightarrow K r^n \cdot n^m = K \cdot n^2$$

ili

$g_{1,2} = 1 \;,\; g_3 = 2$

$u(n) = 5 \cdot (2)^n$

$$y_p(n) = K (2)^n n^m = K \cdot (2)^n \cdot n$$

ili

$g_{1,2} = 1 \;,\; g_3 = 2$

$u(n) = 5 \cdot (5)^n$

$$y_p(n) = K \cdot (5)^n \cdot n^m$$

da je pisalo

$u(n) = 5n$

$q_{1,2} = 1 \; ; \; q_3 = 2$

$u(n) = 5n \cdot (1)^n$

$y_p(n) = (k_0 + k_1 n) \cdot (1)^n \cdot n^m \Rightarrow y_p(n) = (k_0 + k_1)\, n^2$

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povratak na zadatak

$k_0 = \frac{1}{2} \; ; \; k_1 = \frac{1}{6}$

$$\boxed{y_p(n) = \left(\frac{1}{2} + \frac{1}{6} n\right) n^2 \; , \; n \geq 0}$$

$y(n) = C_1 + C_2 n + \left(\frac{1}{2} + \frac{1}{6} n\right) n^2 \; , \; n \geq 0$

$y(0) = 0 \; ; \; y(1) = 0$

$C_1 = 0 \; ; \; C_2 = -\frac{2}{3}$

* VAŽNI PRIMJER

$q_{1,2} = \frac{-b \pm \sqrt{\quad}}{2a}$

$q_1 = 3 + j4 \qquad q_2 = 3 - j4$

$y_h(n) = C_1 (3 + j4)^n + C_2 (3 - j4)^n$

prebaci u polarni oblik

$q_1 = |q_1| e^{j\angle q_1} \qquad |q_1| = \sqrt{3^2 + 4^2} = 5$

$\angle q_1 = \mathrm{arctg}\, \frac{4}{3} = 53{,}13^\circ$

$q_1 = -3 - j4$

$|q_1| = 5$

$\angle q_1 = 53{,}13^\circ + 180^\circ$

$\angle q_1 = 233{,}13^\circ$

$q_1 = 5 \cdot e^{j53,13}$ $\qquad q_2 = 5 \cdot e^{-j53,13}$

$$y_h(n) = C_1 5^n e^{j53,13^\circ n} + C_2 5^n e^{-j53,13^\circ n}$$
$$= 5^n \left(C_1 e^{j53,13^\circ n} + C_2 e^{-j53,13^\circ n}\right)$$
$$= 5^n \left(A \cos(53,13^\circ n) + B \sin(53,13^\circ n)\right)$$

Mirni odziv

- $y_m(n)$

$y_m(n) = y_h(n) + y_p(n)$

$y(n) - 4y(n-2) = u(n) \quad \rightarrow * \; y_h(n) = C_1 2^n + C_2 (-2)^n \qquad * \; y(-1) = 1$

$* \; y_p(n) = -\frac{8}{9} - \frac{1}{3} n \;, \; n \geq 0 \qquad * \; y(-2) = 2$

$y_m(n) = C_1 (2)^n + C_2 (-2)^n - \frac{8}{9} - \frac{1}{3} n \;, \; n \geq 0$

$y(-1) = y(-2) = 0$

$y(0) = 0 \;;\; y(1) = 1 \;\rightarrow$ ovo su sada P.U.

$$\left.\begin{array}{l} y_m(0) = C_1 + C_2 - \frac{8}{9} = 0 \\ y_m(1) = 2C_1 - 2C_2 - \frac{8}{9} - \frac{1}{9} = 1 \end{array}\right\} C_1 = 1 \;,\; C_2 = -\frac{1}{9}$$

Nepobudeni odziv

- $y_0(n)$

$y_0(n) = y_h(0)$

$y_0(n) = C_1 (2)^n + C_2 (-2)^n$

$y(-1) = 1 \;;\; y(2) = 2$

$$\left.\begin{array}{l} y_0(-1) = \frac{1}{2} C_1 - \frac{1}{2} C_2 = 1 \\ y_0(-2) = \frac{1}{4} C_1 + \frac{1}{4} C_2 = 2 \end{array}\right\} \begin{array}{l} C_1 = 5 \\ C_2 = 3 \end{array}$$

$y_0(n) = 5(2)^n + 3(-2)^n$

$$\boxed{y_m(n) = 2^n + \frac{1}{9}(-2)^n - \frac{8}{9} - \frac{1}{3}n} \quad , n \geq 0$$

$$\boxed{y_o(n) = 5(2)^n + 3(-2)^n}$$

$$y(n) = 6(2)^n + \frac{26}{9}(-2)^n - \frac{8}{9} - \frac{1}{3}n$$

partikularni dio rješenja

to je
prirodni odziv

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Impulsni odziv

$u(n) = \delta(n)$

$y(n) \Rightarrow h(n)$

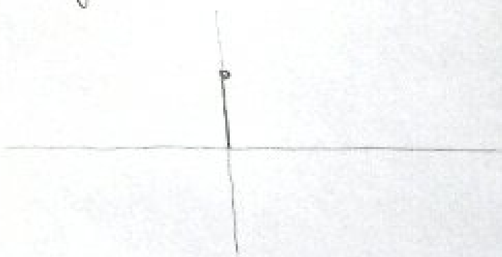
$y(n) - 4y(n-2) = u(n)$

$y(-1) = 1 \quad ; \quad y(-2) = 2$

$y_h(n) = C_1 2^n + C_2 (2)^n$

$h(n) - 4h(n-2) = \delta(n)$

$h(n) = y_h(n)$



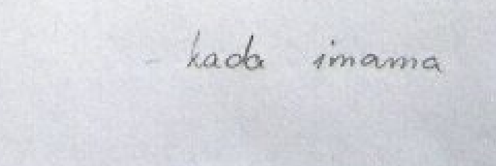
$y(0) - 4y(-2) = \delta(0) \Rightarrow y(0) = 9$

$y(1) - 4y(-1) = \delta(1) \Rightarrow y(1) = 4$

$C_1 = \frac{11}{2} \quad ; \quad C_2 = \frac{7}{2}$

$$\boxed{h(n) = \frac{11}{2} 2^n + \frac{7}{2}(-2)^n \quad , n \geq 0}$$

- kada imama 2 δ



zadano je : (za sliku) 1 - iterativno

$y(-1), y(-2) \xrightarrow{1} y(0), y(1)$

① $h(n) = y_h(n) \quad , \quad 0 \le n < 100$

② $h(98), h(99) \xrightarrow{1} h(100), h(101)$

$h(n) = y_h(n) \quad , \quad n \ge 100$

- primjer sa 2 Diraca - 147 str pr. 13.-6.

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Diferencijalne jednadžbe

- sve radimo za vremensku domenu

$$y''(t) + a_1 y'(t) + a_2 y(t) = b_0 u''(t) + b_1 u'(t) + b_2 u(t)$$

$\Rightarrow$ 2. REDA

$$y(t) = y_h(t) + y_p(t)$$

* $y_h(t) = Ce^{st} \quad , \quad y_h'(t) = sCe^{st} \quad , \quad y_h''(t) = s^2 C e^{st}$

* PRIMJER 1.

$$y''(t) - 4y(t) = u(t)$$

$$u(t) = t\,\mu(t)$$

$$y(0^-) = 1 \quad , \quad y'(0^-) = 2$$

* opće homogeno rješenje

$$s^2 C e^{st} - 4Ce^{st} = 0 \rightarrow Ce^{st}(s^2 - 4) = 0$$

$$s^2 - 4 = 0 \rightarrow s_1 = 2$$

$$s_2 = -2$$

$$y_h(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

$$\boxed{y_h(t) = C_1 e^{2t} + C_2 e^{-2t}}$$

POBUDA	PART. RJEŠENJE
A	K
t^M	$K_0 + K_1 t + \ldots + K_m t^n$
$t^M e^{s_0 t}$	$(K_0 + K_1 t + \ldots + K_m t^M)\, e^{s_0 t}\, t^{(m)}$ → koliko je puta s_0 vlastita frek. sustava

$A\cos(\omega_0 t) \rightarrow K_0 \cos(\omega_0 t) + K_1 \sin(\omega_0 t)$

$A\cos(\omega_0 t) \rightarrow K \cos(\omega_0 t + \varphi)$

$A\sin(\omega_0 t) \rightarrow K_0 \cos(\omega_0 t) + K_1 \sin(\omega_0 t)$

$A\sin(\omega_0 t) \rightarrow K \sin(\omega_0 t + \varphi)$

dokaz

$$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$$

$$= \underbrace{K\cos(\phi)}_{K_0} \cos(\omega_0 t) - \underbrace{K\sin(\phi)}_{K_1} \sin(\omega_0 t)$$

$$\boxed{C_1 e^{j\omega_0 t} + C_2 e^{-j\omega_0 t}}$$

$$A\cos(\omega_0 t) + B\sin(\omega_0 t) = C_1(\cos(\omega_0 t) + j\sin(\omega_0 t)) + C_2(\cos(\omega_0 t) - j\sin(\omega_0 t)) = \cos(\omega_0 t)\underbrace{(C_1 + C_2)}_{A} + \sin(\omega_0 t)\underbrace{(C_1 - C_2)}_{B}$$

$$t^m e^{s_0 t} \rightarrow$$

$$\rightarrow (K_0 + K_1 t + \ldots + K_m t^{m})\, e^{s_0 t}\, t^{M}$$

$y(t) = t\mu(t) = t, \quad t \geq 0$

$y''(t) - 4y(t) = u(t)$

$y_p(t) = (K_0 + K_1 t)\, e^{0t}\, t^0 = K_0 + K_1 t$

$y_p''(t) - 4y_p(t) = u(t)$

$y_p'(t) = K_1$

$y_p''(t) = 0$

$-4K_0 - 4K_1 t = t$

$K_0 = 0$

$K_1 = -\frac{1}{4}$

$$\boxed{y_p(t) = -\frac{1}{4} t, \quad t \geq 0}$$

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$$y(t) = C_1 e^{2t} + C_2 e^{-2t} - \frac{1}{4} t \quad , t \geq 0$$

P.U. $\quad y(0^-) = 1 \quad , \quad y'(0^-) = 2$

$$y'(t) = 2C_1 e^{2t} - 2C_2 e^{-2t} - \frac{1}{4}$$

$$y(0^+) - y(0^-) = b_0 u(0^+)$$

$$y'(0^+) - y'(0^-) + a_1 (y(0^+) - y(0^-)) = b_0 u'(0^+) + b_1 u(0^+)$$

$$y''(t) + a_1 y'(t) + a_2 y(t) = b_0 u''(t) + b_1 u'(t) + b_2 u(t)$$

$$y'''(t) + a_1 y''(t) + a_2 y'(t) + a_3 y(t) = b_0 u'''(t) + b_1 u''(t) + \ldots$$

$b_0 = 0 \quad , \quad b_1 = 0 \quad , \quad b_2 = 1$

$a_1 = 0 \quad , \quad a_2 = -4$

$y(0^+) = 1 \qquad u(t) = t\,\mu(t) \longrightarrow u(0^+) = 0$

$y'(0^+) = 2 \qquad u'(t) = \mu(t) + t\,\delta(t) \rightarrow u'(0^+) = 0$

$$\left.\begin{aligned} y(0^+) &= C_1 + C_2 = 1 \\ y'(0^+) &= 2C_1 - 2C_2 - \frac{1}{4} = 2 \end{aligned}\right\} \quad \begin{aligned} C_1 &= \frac{17}{16} \\ C_2 &= -\frac{1}{16} \end{aligned}$$

$$\boxed{y(t) = \frac{17}{16} e^{2t} - \frac{1}{16} e^{-2t} - \frac{1}{4} t \quad , t \geq 0}$$

Mirni odziv

$$y_m(t) = y_h(t) + y_p(t)$$

$$y(0^-) = \ldots = y^{(n-1)}(0^-) = 0 \rightarrow y(0^+) =$$

$$y(0^+) = \ldots$$

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PRIMJER

$$y''(t) - 2y'(t) + y(t) = 0 \qquad y(0^-) = 0 \; ; \; y'(0^-) = 1 \rightarrow y(0^+) = 0 \, , \; y'(0^+) = 1$$

$$u(t) = t\mu(t)$$

1. opće homogeno rj

$$y_h(t) = Ce^{st}$$

$$Ce^{st}(s^2 - 2s + 1) = 0 \qquad s_1 = 1 \, , \; s_2 = 1 \Big\} \boxed{y_h(t) = (C_1 + C_2 t)e^t}$$

$$(C_1 + C_2 t)e^{s_1 t}$$

2. partikularno rj.

$$u(t) = t \quad , t \geq 0$$

$$y_p(t) = K_0 + K_1 t$$

$$y_p'(t) = K_1 \quad ; \; y_p''(t) = 0$$

$$0 - 2K_1 + K_0 + K_1 t = t$$

$$\left.\begin{aligned} -2K_1 + K_0 &= 0 \\ K_1 &= 1 \end{aligned}\right\} \begin{aligned} K_0 &= 2 \\ K_1 &= 1 \end{aligned} \qquad \boxed{y_p(t) = 2 + t \quad , t \geq 0}$$

$$y_m(t) = (C_1 + C_2 t)e^t + 2 + t \quad , t \geq 0$$

$$y(0^-) = y'(0^-) = 0 \rightarrow y(0^+) = y'(0^+) = 0$$

$$y_m'(t) = e^t(C_1 + C_2 t) + e^t(C_2) + 1$$

$$y_m'(t) = e^t(C_1 + C_2 + C_2 t) + 1$$

$y_m(0^+) = C_1 + 2 = 0$

$y_m'(0^+) = C_1 + C_2 + 1 = 0$

$C_1 = -2 \; ; \quad C_2 = 1$

$$\boxed{y_m(t) = e^t(-2+t) + 2 + t \quad , t \geq 0}$$

$t \geq 0$ kod totalnog i mirnog sustava

Nepobuđeni odziv

$- \; y_0(t)$

$y_0(t) = y_h(t)$

$y_0(t) = (C_1 + C_2 t) e^t \qquad , \; y(0^-) = 0$

$y_0(0^-) = C_1 = 0$

$y_0'(t) = e^t (C_1 + C_2 + C_2 t) \quad , \; y'(0^-) = 1$

$y_0'(0^-) = C_1 + C_2 = 1 \quad \rightarrow \quad C_1 = 0$

$C_2 = 1$

$$\boxed{y_0(t) = t e^t}$$

$$y(t) = \underbrace{e^t(-2+2t)}_{\text{prirodni odziv}} \underbrace{+2+t}_{\text{prisilni odziv}} \quad , t \geq 0$$

PRIMJER 5.4, str. 47.

$$y''(t) + 0.2\,y'(t) + 0.1\,y(t) = u(t)$$

$$u(t) = 3\cos(1.8t)$$

$$y(0) = -10 \;;\; y'(0) = -5$$

① $s^2 + 0.2s + 0.1 = 0 \;\rightarrow\; s_1 = -0.1 + j0.3$

$$s_2 = -0.1 - j0.3$$

$$y_h(t) = C_1 e^{s_1 t} + C_2 e^{s_2 t}$$

$$= C_1 e^{(-0.1 + j0.2)t} + C_2 e^{(-0.1 - j0.3)t}$$

$$= e^{-0.1t}\left(C_1 e^{j0.3t} + C_2 e^{j0.3t}\right)$$

$$\boxed{y_h(t) = e^{-0.1t}\left(A\cos(0.3t) + 3\sin(0.3t)\right)}$$

② $u(t) = 3\cos(1.8t) \;\rightarrow\; y_p(t) = K_0\cos(1.8t) + K_1\sin(1.8t)$

$$y_p'(t) = -1.8K_0\sin(1.8t) + 1.8K_1\cos(1.8t)$$

$$y_p''(t) = -3{,}24K_0\cos(1.8t) - 3{,}24K_1\sin(1.8t)$$

$$\cos(1.8t)\left(-3{,}24K_0 + 0.36K_1 + 0{,}1K_0\right) + \sin(1.8t)\left(-3{,}24K_1 - 0{,}36K_0 + 0{,}1K_1\right) = 3\cos(1.8t)$$

Impulsni odziv

$u(t) = \delta(t)$

$y(t) \to h(t)$

$y''(t) + 2y'(t) + 5y(t) = u'(t) + 3u(t)$

1. $s^2 + 2s + 5 = 0 \to s_{1,2} = \frac{-2 \pm \sqrt{-16}}{2} \to s_1 = -1 + j2$

$s_2 = -1 - j2$

$y_h(t) = e^{-t}\left(A\cos(2t) + B\sin(2t)\right) - h_A(t)$

$h_A(0^+) = 0$

$h_A'(t) = e^{-t}\left(A(\cos(2t)) - 2\sin(2t)\right) + B\left(\sin(2t) + 2\cos(2t)\right)$

$h_A'(0^+) = 1$

$h_A(0^+) = A = 0 \Rightarrow A = 0$

$h_A'(0^+) = 2B = 1 \to B = \frac{1}{2}$

$$\boxed{h_A(t) = \frac{1}{2}e^{-t}\sin(2t)}$$

$$\boxed{h(t) = \sum_{m=0}^{M}\left(b_{N-m}D^m\right)h_A(t) + b_0\delta(t)}$$

N – red derivacije izlaza

M – red derivacije ulaza

$D^1 h_A(t) = h_A'(t)$

$D^2 h_A(t) = h_A''(t)$

$D^0 h_A(t) = h_A(t)$

$N = 2$, $M = 1$

$b_0 = 0$, $b_1 = 1$, $b_2 = 3$

$y'(t) + y(t) = u(t)$

$h_A(0^+) = 1$

$y'''(t) + y''(t) + y'(t) + y(t) = u(t)$

$h_A(0^+) = 0$

$h_A'(0^+) = 0$

$h_A''(0^+) = 1$

$h(t) = \sum_{m=0}^{1}\left(b_{2-m}D^M\right)h_A(t)$

$h(t) = b_2 D^0 h_A(t) + b_1 D^1 h_A(t)$

$h(t) = b_2 h_A(t) + b_1 h_A'(t)$

$$h_1(t) = \frac{1}{2} e^{-t} \sin(2t)$$

$$h_1'(t) = e^{-t}\left(-\frac{1}{2}\sin(2t) + \cos(2t)\right)$$

$$\boxed{h(t) = e^{-t}\left(\sin(2t) + \cos(2t)\right), \quad t \geq 0}$$

Prijenosne funkcije

$h(t)$

$u(t) \overset{\text{odziv}}{\Longrightarrow} \quad y(t) = h(t) * u(t) = \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau = \int_{-\infty}^{\infty} h(\tau) e^{st} e^{-s\tau}\, d\tau$

$u(t) = e^{st}, \quad \cos(\omega_0 t) = e^{j\omega_0 t} + e^{-j\omega_0 t}$

$$y(t) = e^{st} \underbrace{\int_{-\infty}^{\infty} h(\tau) e^{-s\tau}\, d\tau}_{H(s)}$$

$$\boxed{X(s) = \int_{-\infty}^{\infty} x(\tau)\, e^{-s\tau}\, d\tau}$$

LAP. TRANS.

IZVOD s

$u(t) = U e^{st}$

$y(t) = Y e^{st}$

$$y^{(n)}(t) + a_1 y^{(n-1)}(t) + \ldots + a_n y(t) = b_0 u^{(n)}(t) + \ldots + b_n u(t)$$

$$s^n Y e^{st} + a_1 s^{n-1} Y e^{st} + \ldots + a_n Y e^{st} = b_0 s^n U e^{st} + \ldots + b_n U e^{st}$$

$$Y\left(s^n + a_1 s^{n-1} + \ldots + a_n\right) = U\left(b_0 s^n + \ldots + b_n\right)$$

$$Y = \underbrace{\boxed{\frac{b_0 s^n + \ldots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \ldots + a_n}}}_{H(s)} \cdot U$$

- imamo sustav

$$y''(t)+5y'(t)+6y(t)=u(t) \rightarrow H(s)=\frac{1}{s^2+5s+6}$$

$$s=j\Omega$$

$$H(j\Omega)=\frac{1}{6-\Omega^2+5j\Omega}$$

$$z=a+jb$$

$$|z|=\sqrt{a^2+b^2}$$

$$\angle z=\operatorname{arctg}\frac{b}{a}$$

$$z_1=\frac{1}{a+jb}$$

$$|z_1|=\frac{|1|}{\sqrt{a^2+b^2}}$$

$$z=\frac{c}{a+jb}\cdot\frac{a-jb}{a-jb}=\frac{c(a-jb)}{a^2+b^2}$$

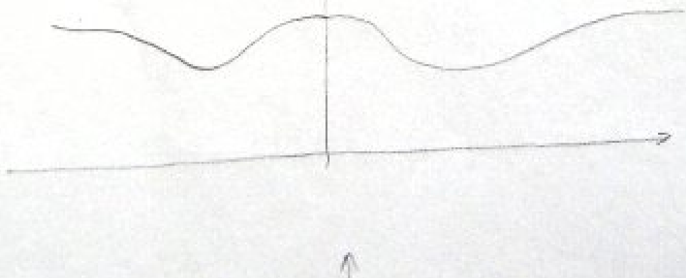
$$Re=\frac{ca}{a^2+b^2} \qquad Im=\frac{-cb}{a^2+b^2}$$

$$\angle z=\operatorname{arctg}-\frac{b}{a}$$

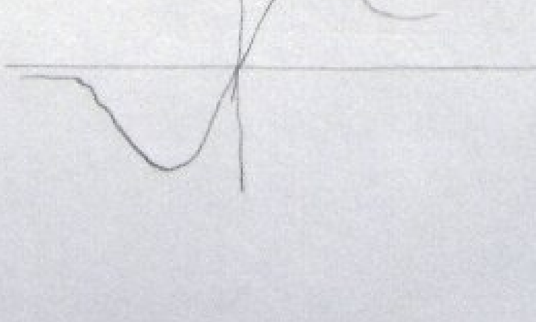
$$\angle z=-\operatorname{arctg}\frac{b}{a}$$

$$\left.\begin{aligned}\angle H(j\Omega)&=-\operatorname{arctg}\frac{5\Omega}{6-\Omega^2}\\&=\operatorname{arctg}\frac{5\Omega}{\Omega^2-6}\end{aligned}\right\}\text{ fazno frekv. karak.}$$

$$|H(j\Omega)|=\frac{1}{\sqrt{(6-\Omega^2)^2+(5\Omega)^2}}=\frac{1}{\sqrt{\Omega^4+13\Omega^2+36}}\ \Big\}\text{ amplitudno frekv. karak.}$$



amp.



faz.

Stabilnost sustava

* KONTINUIRANI SUSTAVI

  * JEDNOSTRUKI KORJENI

    - sustav je stabilan $Re\{S_i\} < 0 \;, \forall S_i$

  * GRANIČNO - STABILAN

    - $Re\{S_i\} = 0 \;, \exists S_i$ - ako postoji jedan kojem je $Re = 0$

  * NESTABILAN

    - $Re\{S_i\} > 0 \;, \exists S_i$

* VIŠESTRUKI KORJEN

  * STABILAN

    $Re\{S_i\} < 0$

  * NESTABILAN

    $Re\{S_i\} \geq 0 \;, \exists S_i$

npr.

$S_{1,2} = 0 \quad S_3 = -2$ → NESTABILAN

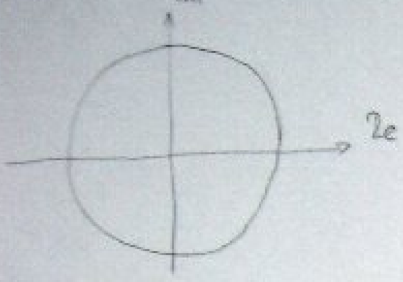
$S_1 = 0 \quad S_{1,2} = -1$ → GRANIČNO STABILAN

$S_1 = -1 + j2 \quad S_2 = -1 - j2$ → STABILAN

$S_1 = -j \quad S_2 = -j$ → GRANIČNO STABILAN

* STABILNOST SUSTAVA

DISKRETNI SUSTAVI



$y_h(n) = 5\, q^n$

$\lim_{n \to \infty} q^n = 0 \;;\; |q| < 1$

$\left(\frac{1}{2}\right)^n = \frac{1}{2^n}$

$2^n = \infty$

* jednostruki

* stabilnost

$|q_i| < 1 \;,\; \forall q_i$

* granično-stabilan

$|q_i| = 1 \;,\; \exists q_i$

* nestabilan

$|q_i| \geq 1 \;,\; \exists q_i$

npr: $\left.\begin{array}{l} q_1 = 2 - j3 \\ q_2 = 2 + j3 \end{array}\right\}$ $\sqrt{4+9} = \sqrt{13} > 1$ pa je nestabilan

$q_1 = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} j \Rightarrow$ granično-stabilan

* višestruki

* stabilan

$|q_i| < 1 \;,\; \forall q_i$

* nestabilan

$|q_i| \geq 1 \;,\; \exists q_i$

$q_1 = j$

$q_2 = j$

$q_3 = -j$

ZADATAK 1. 15 bj.

$$y''(t) + 5y'(t) + 6y(t) = u(t)$$

$s_1 = -3$; $s_2 = -2$; sustav je stabilan

$$H(s) = \frac{1}{s^2 + 5s + 6}$$

$$A(\omega) = \text{amplituda} = \frac{1/1}{\sqrt{36 + 13\omega^2 + \omega^4}}$$

$$H(j\Omega) = \frac{1}{6 - \Omega^2 + 5\Omega}$$

$$P(\omega) = -\mathrm{arctg}\,\frac{5\Omega}{6 - \Omega^2}$$

$$= \mathrm{arctg}\,\frac{5\Omega}{\Omega^2 - 6}$$

$y(0) = 0$; $y'(0) = 1$

$$\boxed{y_h(t) = C_1 e^{-3t} + C_2 e^{-2t}}$$

$$u(t) = 5\cos\underset{\omega=1}{(t)} \longrightarrow y_p(t) = K\cos(t + \phi)$$

$$K = 5 \cdot A(\omega)\Big|_{\omega=1} \qquad \phi = 0 + P(\omega)\Big|_{\omega=1}$$

$$K = \frac{\sqrt{2}}{2} \qquad \phi = -45^\circ$$

$$\boxed{y_p(t) = \frac{\sqrt{2}}{2}\cos(t - 45^\circ)\ ,\ t \geq 0}$$

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$$y''(t) + 2y'(t) + 5y(t) = u(t)$$

$$u(t) = \begin{cases} \sin(t), & t<0 \\ 2\sin(2t), & t>0 \end{cases}$$

$s_1 = -1 + j2 \;,\; s_2 = -1 - j2 \;\Rightarrow$ sustav je stabilan

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$$\boxed{y_h(t) = e^{-t}\left(A\cos(2t) + B\sin(2t)\right)}$$

$$H(s) = \frac{1}{s^2 + 2s + 5} \qquad H(j\Omega) = \frac{1}{5 - \Omega^2 + 2j\Omega}$$

$$A(\omega) = \frac{1}{\sqrt{\Omega^4 - 6\Omega^2 + 25}}$$

$$P(\omega) = -\operatorname{arctg}\frac{2\Omega}{5-\Omega^2}$$

1) $u_1(t) = \sin(t)$

$$\phi = 0^\circ + P(\omega)\Big|_{\omega=1}$$

$$y_p(t) = K\sin(t+\phi) \qquad \phi = -26{,}56^\circ$$

$$K = 1 \cdot A(\omega)\Big|_{\omega=1}$$

$$K = \frac{\sqrt{5}}{10}$$

$$y_{p1}(t) = \frac{\sqrt{5}}{10}\sin\left(t - 26{,}56^\circ\right),\; t\leq 0$$

2) $u_2(t) = 2\sin(2t)$

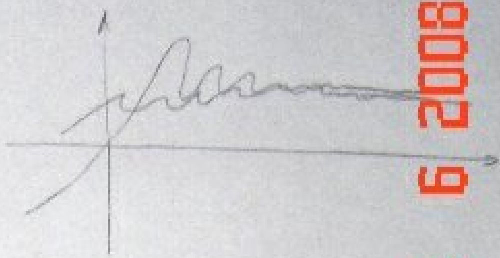
$$y_{p2}(t) = K\sin(2t+\phi) \qquad \phi = 0^\circ + P(\omega)\Big|_{\omega=2}$$

$$K = 2\cdot A(\omega)\Big|_{\omega=2} \qquad \phi = -75{,}96^\circ$$

$$K = \frac{2\sqrt{17}}{17}$$

$$y_{p2}(t) = \frac{2\sqrt{17}}{17}\sin\left(2t - 75{,}96^\circ\right),\; t\geq 0$$

$$y(t) = \begin{cases} \cancel{y_h(t)} + y_{p_1}(t) & , t<0 \\ \\ y_h(t) + y_{p_2}(t) & , t>0 \end{cases}$$



opdemito

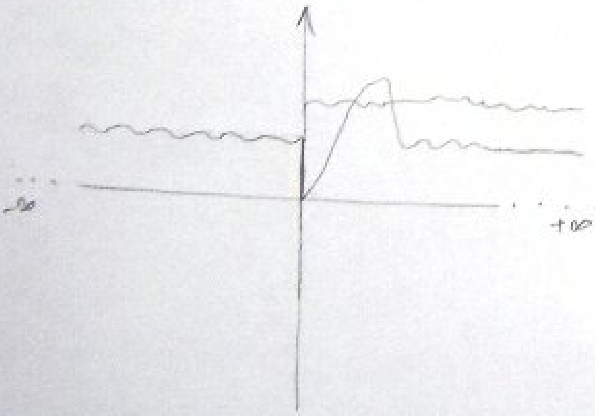
$$u(t) = U \cos(\omega_0 t + \phi_0) \longrightarrow y_p(t) = K \cos(\omega_0 t + \phi)$$

$$K = U \cdot A(\omega)\Big|_{\omega=\omega_0}$$

$$\phi = \phi_0 + P(\omega)\Big|_{\omega=\omega_0}$$

$$u(t) = U e^{st} \longrightarrow y_p(t) = K \cdot e^{st}$$

$$K = U \cdot A(\omega)\Big|_{\omega=0}$$



4

$$y'(t) + 3y(t) = u(t)$$

$$u(t) = \left(\sin(t) + 2\sin(2t) + 3\sin(3t) + 4\sin(4t)\right)\mu(t)$$

ako je diskretni ovog tipa

$$y(n) + y(n-1) + y(n-2) = u(n)$$

$$y(n) = z^n \qquad\qquad z = e^{j\omega}$$

$$y(n-2) = z^{n-2} = z^n \cdot z^{-2}$$

$$u(n) = z^n$$

$$y\, z^n\left(1 + z^{-1} + z^{-2}\right) = z^n y$$

$$y = \underbrace{\frac{1}{1 + z^{-1} + z^{-2}}}_{H(z)}\, u$$